A SPIRITUAL-MATHEMATICAL LENS FOR THE SELF-DIRECTED LEARNING OF ETHNOMATHEMATICS

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Abstract - The recent emphasis to include indigenous knowledge in the national curriculum of South Africa necessitates a discourse between education theory and practice. As part of this discourse, we propose a focus on mathematics education through the possible metacognitive affordances (among others metacognitive awareness) of ethnomathematics, thereby conceptualising a spiritual-mathematical lens for the self-directed learning of mathematics. Such a lens could offer a spiritual view of how cultures influence mathematics and also enable researchers to study the mathematical ideas flowing from cultures' indigenous knowledge systems. Our application of this lens is the result of analysis of literature in the philosophies of education and of mathematics, also ethnomathematics. Such a lens, it is reasoned, can be used to reveal the link between metacognitive awareness and meaningful teaching and learning in mathematics. Indigenous knowledge systems discussed in this paper include Eastern (e.g. mind-heart-spirit dualism) and Western (e.g. life-world) indigenous knowledge views as examples for the learning of ethnomathematics. In practice, this lens can be used to develop cultural artefact-worksheet interventions that might assist mathematics teachers in the understanding and application of indigenous knowledge systems in the self-directed learning of ethnomathematics.

Keywords: spirituality, ethnomathematics, metacognition, indigenous knowledge, awareness, self-directed learning

Introduction

Academics in the humanities and social sciences seem to harbour a secret suspicion, namely that acknowledging spirituality as a dimension of their intelligibility is, somewhat, crossing into forbidden territory. It is therefore reasonable to address mathematical phenomena and their connection with the spiritual, from a hermeneutic perspective (Kafle, 2011), with caution. Put differently, there is some parallel between the conscious manifestations of mathematical thinking (or awareness of mathematical thought) and the inner awareness of the meaning of mathematical symbols and operations and their functions in life. We have, on the one hand, the mathematical symbols, patterns and operators which are governed by scientific rationalisation and logical thought while, on the other hand, we also search for meaning and purpose of what we do as living, thinking and metacognitively self-and culturally-aware individuals. Little research has so far been done on how one can take on such a holistic approach to the teaching mathematics (Dalbotten, 2014).

At least implicitly, the connection between the mathematical and spiritual exists in a domain, which Capra (1996) argues, where interdependence, rationality and spirituality meet. In mathematics education, both implicit and explicit mathematical knowledge and skills can enable rationalisation and theoretical explanation of mathematics (Maxwell & Chahine, 2013) thereby promoting deep meaningful learning as a form of spiritual focus. Together with the search for meaningful mathematics

there is also a movement afoot in mathematics education to acknowledge the role of indigenous knowledge systems in the school curriculum. The RSA Department of Education, for instance, insists that "*acknowledging the rich history and heritage of this country as important contributors to nurturing the values contained in the Constitution*" be endorsed in the curriculum (DoE, 2012:5).

At the same time, there is a seeming conflict at a transcendent level between western and eastern science's views on indigenous knowledge in terms of their meaningfulness for mathematics. The problem with infusing indigenous knowledge into the curriculum and respecting cultures' underpinning philosophies is twofold. Educators who have been raised, taught and qualified through a westernised scientific view do not, necessarily, have the relevant background, knowledge and understanding of exactly what indigenous knowledge to include in the curriculum, or how to do so. The other part of this problem is that we believe that mathematics, a cultural tool, (Clarkson & Atweh, 2003) is embedded in every culture and that every culture has some form of spiritual awareness (d'Aquili & Newberg, 1998). The mathematics education research literature attaches value to the creed of a culturally relevant curriculum (Maxwell & Chahine, 2013, yet exactly whose culture, or whose indigenous knowledge should be included, remains indistinct. Ignorance of the relationship between the spiritual or transcendent complexities of cultural mathematics, and the perception of mathematics as a cultural product (DoE, 2012) or ethnomathematics can often be ascribed to the teacher's or scholar's scientific orientation. A scientific orientation that overlooks the true holistic meaning of education, which lies at the heart of, what Glasberg (2003:278) calls "a spiritual approach to the world", can impact on one's self-directedness.

In view of the above, the research issue addressed here is: What is the conceptual relationship between spirituality and mathematics in the context of indigenous knowledge when learning ethnomathematics, and how can this relationship promote self-directed learning? In order to find an answer to this problem, this article offers a theoretical *spiritual-mathematics lens* that serves to explain the conceptual relationship between spirituality and mathematics, particularly when learning ethnomathematics and attempting to be self-directed in the learning process. Such a lens can assist mathematics education researchers to infuse indigenous knowledge in the curriculum and thereby serve learners' self-directed learning of ethnomathematics. Document analysis was conducted: the concepts spirituality, metacognition and mathematics were contextualised within the theoretical framework of situated embodied cognition and intuition. This was followed by the findings flowing from the resultant three tiers theory, namely: (i) ethnomathematics as an abstract representation of a person/culture's life world, (ii) metacognitive awareness as an expression of embodied cognition and (iii) spirituality as a sphere of mathematical intuition. In the discussion we construct, in view of this conceptual-theoretical framework, a spiritual-mathematical lens. We conclude the paper with thoughts about future directions to embrace a culturally approachable way of learning mathematics, and in the process attend to the issue of self-directedness in this learning process.

Conceptual-theoretical framework

The concept of spirituality is embedded in anthropology and culture (Kellermeier, 2012). Spirituality offers a sense of meaning to the other aspects of a person's life world, including his or her education. It is within this life world that includes a person's culture that an individual experiences traditional indigenous knowledge (Kasavin, 2009). People reflect on and become aware of the experience and represent the experience abstractly through an analysing and mathematising process (Cobb et al., 1997). For example, awareness of one's one worldview can be connected with the meaning attributed

to one's own existence as embedded both in self-awareness and co-awareness . Such a connection would involve the beliefs, intuitions, existential choices and practices that represent a person's being and purpose in life (Delaney, 2005), which transcend the mere logical realm (*I have to do X),* to a more, transformatively speaking, meta-physical external world (*because of Y and Z*). This is especially true of a society that wants educators to engage students actively in learning, and expect that what the latter are learning is applicable and, hence, meaningful to their lives. In order to explain the meaning of mathematics and to promote scholarly-caring and serviceable citizenship, we have to consider what this spiritual "meaning" implies. Experiences may include perceiving mathematics to be everywhere, or using mathematics for everything in everyday life, or knowing how mathematical concepts are embedded in various crafts and artefacts in diverse cultures (Maxwell & Chahine, 2013:68). It is this awareness that Zeev (2002) describes as intuitive mathematics.

Ethnomathematics as an expression of indigenous knowledge

Mathematics refers to the concepts and practices embedded in the processes of abstracting reality, representing it through mathematical symbols and numbers. When this expression takes the form of a multicultural interpretation on mathematics, then the science of mathematics can be understood from a multicultural societal view (Katsap & Silverman, 2008). This culturally aware mathematics, is denoted by the term *ethnomathematics*, and is expressed and made tangible in socio-cultural surroundings. Put differently,it involves the language, social norms and symbols of a particular culture in processes such as counting, weighing and measuring, sorting and categorizing (Katsap & Silverman, 2008 & D'Ambrosio, 2001). Ethnomathematics therefore refers to the way in which different cultures express mathematical thinking to think and talk about their reality (Barton, 1996 & D'Ambrosio, 2001).

The cultural norms and traditional knowledge being expressed through this cultural or ethno-mathematics seem to lie within the indigenous knowledge systems that the culture holds. For instance, Western cultures largely accept a disjunctive mechanical view of the universe and consider mathematics as an exact science that represents the systems and associations of this universe as abstract representations of a person's life world (Kasavin, 2009). This view is largely founded on the mechanics of a Newtonian worldview. Contradictory to this view is the Eastern *I Ch'ing* philosophy of Daoism, a knowledge system that provides tools with which to grasp the origin and processes of reality (Sheng, 2009). In contrast to the Newtonian, it provides a subtle, almost uncertain substrate understanding of a unified universe where all knowledge is in union. It is reminiscent of Euclid and Pythagoras's view on mathematics as a universal knowledge system.

Some examples of African indigenous games include *string figures* such as *Tchadji* (known as Mancala type games) and *Morabaraba* (a three-in-a- row type game), shown in Figure 1. The game is played in three stages. First, each player starts with 12 'cows' (or tokens) either side of a clear board. Each then takes turns to place one 'cow' in a hole or on a circle. The aim is to create rows of three tokens, being vertical, diagonal or horizontal. 'Cows' may only be placed on unoccupied 'junctions'.

Figure 1. The first author took these original photos while mathematics teachers who attended a short course on ethnomathematics were playing Morabaraba at the Mpabatho cultural heritage museum in Rustenburg, South Africa.

When a row of three tokens is achieved, then the player may remove one of the opponent's pieces. This can be done vertically or horizontally, making a row of three tokens. A player cannot remove a piece if there are other 'cows' on the 'board' left. Only one of the opponent's 'cows' may be removed at a time, even if two (or more) lines of three-in-a-row are achieved with a single move. Stage two requires each player to make new lines, or to reposition their 'cows'. 'Cows' may be moved back and forth to the same two holes repeatedly. Stage three results in a player that has lost all but three 'cows', he or she may move a 'cow' to any vacant hole.

Morabaraba is an example of how cultural artefacts indigenous knowledge system can be immersed such that it relates to mathematics in the learning of ethnomathematics. Such mathematical concepts such as probability, counting, adding, subtracting, symmetry – all can be learned through the use of this (as one example) a an indigenous knowledge system's cultural artefact. There is also the argument that all mathematics is European and all the rest (i.e. the counting, measuring and deducing ways of other cultures) is purely anthropology (Crump, 1990). It is this view that separates Western and Eastern cultures' mathematics education anthropologically, and which seems to suggest that spirituality and mathematics are in a perceptual war. To overcome this potential conflict, Maxwell and Chahine's (2013) ethno mathematical ideas propose that a philosophical postmodernist transformative mathematics pedagogy is needed which infuses the indigenous knowledge of different cultures within the mathematics curriculum. To do so, will require a conceptual analysis of the term indigenous knowledge which D'Ambrosio (2001) defines as a knowledge system embedded in the cultural background of people who have particular traditional knowledge about their cultures. Indigenous knowledge is therefore created based on our cultural perceptions of reality or our local ontology (i.e. within a particular context) and is often expressed through cultural artefacts such as music, art and board games. There are various historical records indicating that the art of teaching and learning and the science of mathematics have long been acknowledged as vehicles for indigenous knowledge growth through cultural activities (e.g. craftsmanship, music and organizations of parenthood and family duties and responsibilities). Abstract representations of reality that manifest in cultural artefacts can embody the ethnomathematics of a culture and serve as expressions of the indigenous knowledge of a particular culture. Even though not all members of a particular culture have had schooling in concepts of mathematics they are able to express mathematics in them (Maxwell & Chahine, 2013 & Clarke, 1998).

Embodied mathematical metacognitive awareness

Since it is likely that all cultures express their indigenous knowledge through their cultural traditions, religions or cultural artefacts, as explained above, one can argue that they also abstractly represent this indigenous knowledge through their embodied awareness (Maxwell & Chahine, 2013). Embodied awareness of the traditional and cultural mathematical knowledge is then representative of their indigenous knowledge and can be reflected in mathematical concepts. Expressing one's indigenous knowledge through mathematics therefore requires an awareness of knowledge which Flavell (1976) calls metacognitive knowledge, or Efklides (2011) calls metacognitive awareness – knowledge about knowledge. This can happen even without understanding the mathematical concepts themselves. Put differently, the indigenous knowledge and mathematics located in one's metacognitive cultural awareness, functions on two psychosomatic levels. Fi (1985) identifies this as the *object-level* and *meta-level* relationship, as illustrated in Figure 2.

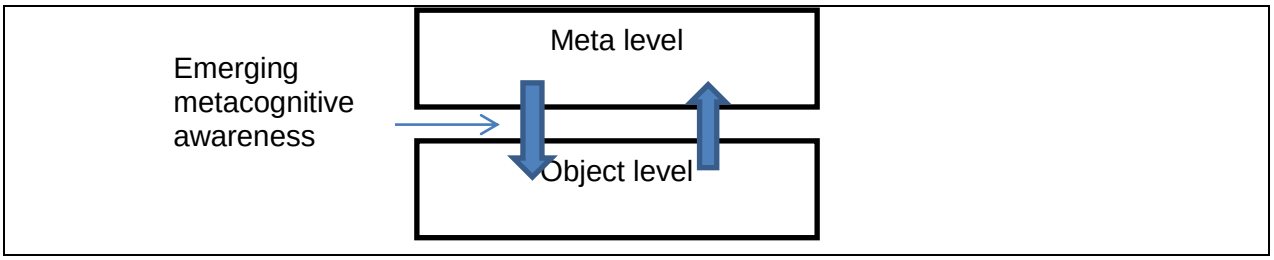

Figure 2 Metacognitive awareness on the Object and Meta level

On the object level objects such as cultural artefacts are constructed from indigenous knowledge. This knowledge of how to construct, paint, or play in the case of games, with the artefact serves as the object level on which an individual can reflect on the meanings, experiences and other cultural traditions or norms that the object relates to. Take for example the Eastern view where Tibetan cultural' practice involves a sophisticated method of divination called *Mo* (Kellermeier, 2012). Traditionally, shamans would cast a six faced dice with sacred phrases of *Manjushri*, one on each side (e.g. AH, RA, PA, TSA, NA & DHI), to make decisions about life world questions, such as family, property and relationships. In themselves, these phrases do not have any specific meanings; however they represent knowledge that educated (or enlightened) beings possess (Kellermeier, 2012). This example shows how an Eastern indigenous knowledge system (Tebatan *Manjushri* divination) manifests in a cultural artefact (dice of *Mo*) and in mathematical concepts (in this case probability). There are six sides to this dice. Casting it twice will result in 36 possibilities of readings that can be made in the *Mo* divination system. Infusion of this system, for instance, as a cultural-artefact worksheet in mathematics education would yield a descriptive, anthropological and mathematising activity for students to analyse. Veenman, Van Hout-Wolters and Afflerbach (2006) describe this object level as the level on which cognitive activity (e.g. remembering, association etc.) takes place as directed or regulated through meta level knowledge.

Analysis would then occur on the meta-level. This is where perception or awareness is formed about the nature of the cognitive knowledge at the object level. Thus, there are two levels of metacognitive awareness: the awareness of the cognitive knowledge on the object level and the awareness of the regulation of this knowledge on the meta level. Gassner (2009) points out that metacognitive awareness not only directs or steers cognitive knowledge but also holds a form of *intuitive knowledge*, as can be explained in terms the philosophy of John Locke (as edited by Fraser, 1894 & Kelly & Tallon, 1972). In the *Mo* divination system, asking 11 questions will result in (11 x 36) 396 types of possible

answers to questions. One then has to still decide what possible readings can be regarded as enlightened or intuitive knowledge. In the Western sense, this intuitive knowledge is similar to Dewey's (1944) *reflective self-awareness* concept. Awareness relates to the cognition of mathematics on the objective level and the metacognitive awareness of the mathematical procedures on the meta level through intuition as a substance of spirituality. Maker and Anuruthwong (2003) conceptualised intuitive learning as *the miracle of learning* and claims that cultural indigenous knowledge (knowledge from the environment), mathematical objectives of the curriculum (i.e. the learning outcomes) and metacognitive awareness (the learning processes involved) all contribute to one's intuitive state.

The intuitive spiritual substance of mathematics

Intuition is often described as those thinking processes that oppose logic and reason (Maker & Anuruthwong, 2003). However, intuitive knowing refers to knowing something (almost) immediately, that is, knowing something without following typical steps or procedures of thinking and argument first. Maker (2001) claims that intuition can be regarded as a first language or metacognitive language (Jagals, 2015), a language of the mind. Symbols, mathematical operators and words therefore embody our second language or intuition. The problem that some successful mathematicians have is that they often struggle to express their intuitive understanding (their first language) in a second language (Maker & Anuruthwong, 2003) and hence merely say that *it makes intuitive sense*. Tisdell (2008:28) phrased this as *"spirituality is always greater than which can be described in language".* This intuitive sense, Shea (2014) avers, is a meta-representation of the inner workings of the mind and implies that there is a secular, non-dogmatic spirituality of the mind associated with mathematical learning.

Spirituality can be part of one's religion, yet it is not synonymous with it (Parsian & Dunning, 2009). Whereas religion involves the cultural traditions and rituals through which we can live out our faith (source), spirituality incorporates beliefs, intuitions, practices and rituals at a deeper meaning or higher purpose (Delaney, 2005). Similarly to the Western concept of meaningful or realistic mathematics education (Cobb et al., 1997), spirituality involves the search for a meaningful existence. This substance of meaningfulness involves some of the typical experiences that relate to the learning of mathematics. For example, not knowing where to start when solving a problem, doubting one's progress, predicting the answers and an awareness of the power of moving onwards and obtaining a feeling of knowing are all characteristic traits of both spirituality (Kasavin, 2009) and mathematics (Efklides, 2011). Mathematics is also a living subject which involves the search for and understanding of patterns, conducting experimentations and observations in which the trained practitioner (i.e. the teacher and learner) engages with and comprehends the nature of the numbers and symbols (Schoenfeld, 2013:4). The mathematician also contemplates about their purpose in everyday life (Kellermeier, 2012). The teacher not only experiences mathematics mentally, but also demonstrates it, seemingly naturally (or intuitively), through conscious experiences in the body as a form of embodied thinking, or embodied cognition (Clarke, 1998). The teacher, therefore, may or may not attribute a deeper meaning to mathematics education, as a domain of their everydayness in the classroom's life world (Kasavin, 2009), yet have some cultural or traditional intuitive sense of being.

One can also argue, as Delaney (2005) has, that spirituality overlaps with religion and that religion can be seen as an organised system of beliefs. Besides this link between spirituality and religion, there exists also spiritual links in a multitude of holistic relationships between all humans, signifying the relationships within and across cultures (Crump, 1990). A mechanistic worldview that separates the mathematician's body, mind and spirit offers only a view on the human experience of thinking as, metaphorically speaking, a machine that could be broken down and examined by other machines. This

technological-materialist view embraces a quantitative and positivist self-directed approach to mathematics. Also, the holistic metacognitive and mathematical state of a civilization depends on this spiritual-mathematical intersection.

The conceptual overlap between spirituality, metacognitive awareness and mathematics is contextualised, in this paper, by the theory of embodied cognition developed by Clark (1998). As Wilson (2002) describes, embodied situated cognition refers to the mental processes or cognitive activities that take place in a particular situation, or life world (Kasavan, 2009), and characteristically encompass the beliefs, experiences, attitudes and intentions that shape the individual's perspective on experience. The awareness of and perspective formed about these experiences, in a particular cultural context, can then be expressed through the abstract language of mathematics. The concepts spirituality, metacognition and ethnomathematics are ordered by *the sphere of intuition* (by Sergienko, 2014) which transcends at the top of the abstractness of mathematics towards the search for something meaningful, or spiritual. This abstraction of reality relates to Schopenhauer's expression of the "*welt und vorstellung"* (Schopenhauer, 1818 & Kamata, 1988), or *"world of representation*" (translated by Payne, 1969:170). In this article, this expression serves as a philosophical underpinning of the spiritual-mathematical lens, and is tied conceptually with situated embodied cognition (Clarke, 1998 & Wilson, 2002) as Figure 3 shows.

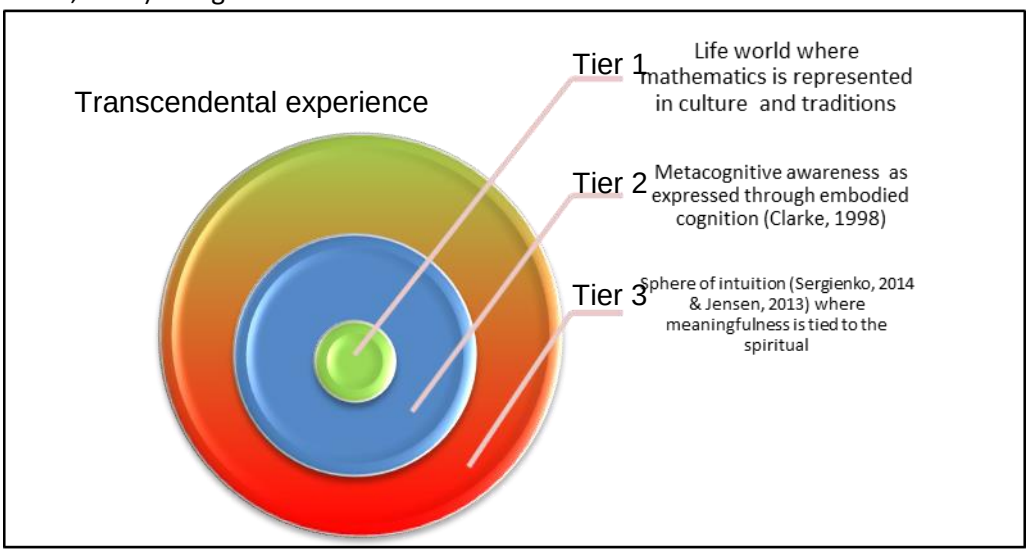

Figure 3 Conceptual-theoretical framework of the spiritual-mathematical lens

The conceptual-theoretical framework as presented in Figure 3 illustrates the three tiers on which the concepts spirituality, metacognition and mathematics lie. At the core, tier 1 represents the life world of the individual or culture, and includes indigenous knowledge as expressed mathematically through cultural traditions (Maxwell & Chahine, 2013). These could include representations of geometrical patterns, symmetry and repeated addition as classic examples of ethnomathematics topics. Cultural artefacts, therefore, seem to embody the indigenous knowledge that the culture holds and can be expressed through art, music, or games in an abstract mathematical way. Typically these artefacts contain symbols, numbers or patterns that directly or indirectly imply some expression or representation of mathematical thinking that hold an intuitive "spatioqualitative imagery" or spiritual metacognitive awareness of the underlying mathematics. Meezenbroek et al. (2012) explain this intuitive state as a spiritual experience that result from intrapersonal connectedness within oneself,

interpersonal connectedness with others and the natural environment and transpersonal connectedness with the unseen, God, or a greater power than the self.

Shapes like circles and triangles are common in culture's expression of indigenous knowledge and the meaning that symbols, numbers or patterns hold for a particular culture is represented intuitively through situated embodied cognition (Clarke, 1998). One can theorise that the embodiment of indigenous knowledge can create awareness of the connection between the mathematics and its spirituality, which lies in the culture's traditions (Meezenbroek et al., 2012). This awareness can serve as a way to recreate ideas, reflect in a shared environment and preserve shared beliefs. We believe that these implicit and explicit mathematics experiences can be rationalised through Western theoretical explanations (e.g. situated embodied cognition) of a person's life world as well as through Eastern philosophy in terms of an intuitive spirit-mind dualism (or *Kokora*) in mathematics education. In doing so, we promote a spiritual-mathematical lens for the learning of ethnomathematics by employing the following empirical design.

Self-directed learning of ethnomathematics

Self-directed learning has been and practiced for over 50 years and has been defined as a personal character of someone who is self-directed and autonomous in their learning, or as a process or a way of regulating learning experiences (Knowles, 1975). As a personal attribute, self-directed learning refers to an individual's predisposition towards learning, and can, much like spirituality, uplift one's wellbeing in the learning of ethnomathematics. Self-directed learning as a process is an attitude to learning that is directed by the learner. Knowles (1975) also described a process which could form the basis for the learning of ethnomathematics (or any learning for that matter) to follow in planning self-directed learning experiences. Creating an atmosphere of respect and support can denote a spiritual awareness to the learning process. Diagnosing learning needs can also relate to the need for meaningful learning whereas the formulating of learning goals refers to an intuitive state of one's knowledge. This often involves identifying resources for learning and choosing and implementing appropriate learning strategies while evaluating the level to which the ethnomathematics learning objective has been met. Similarly, these characteristics of a self-directed learning also reflect the necessary skills to conduct an ethnomathematics investigation as a descriptive, archaeological, mathematizing and analytic activity. We therefore argue that a spiritual-mathematical lens on the learning experience can result in a transcendental awareness of one's indigenous knowledge. Therefore, metacognitive awareness of the sources that instigate a meaningful existence can, at the same time, intuitively link with our transcendental and spiritual nature and promote self-directed learning of ethnomathematics.

Discussion

In this paper we explored the relationship between spirituality, metacognition and mathematics to conceptualise a spiritual-mathematical lens for the self-directed learning of ethnomathematics. Artcles on spirituality, metacognition and ethnomathematics were collected by searching for these descriptors in the title, abstract and keywords from publications since 2006. Relevant new references were also identified in these articles and were followed. In addition we searched for "indigenous knowledge", "mathematics education" or "education" in combination with "spirit", "transcendent" or "connectedness". After sampling relevant articles we followed the data analysis procedures by Jagals and Van der Walt (2016) to analyse the articles and conceptualise the spiritual-mathematical relationship depicted in them. In so doing we revealed the interrelated and complex connection between spirituality, metacognition and mathematics as a network. Figure 4 shows the network

diagram produced using the Harel-Koren Fast Multiscale representation (Jagals & Van der Walt, 2016) of the three concepts.

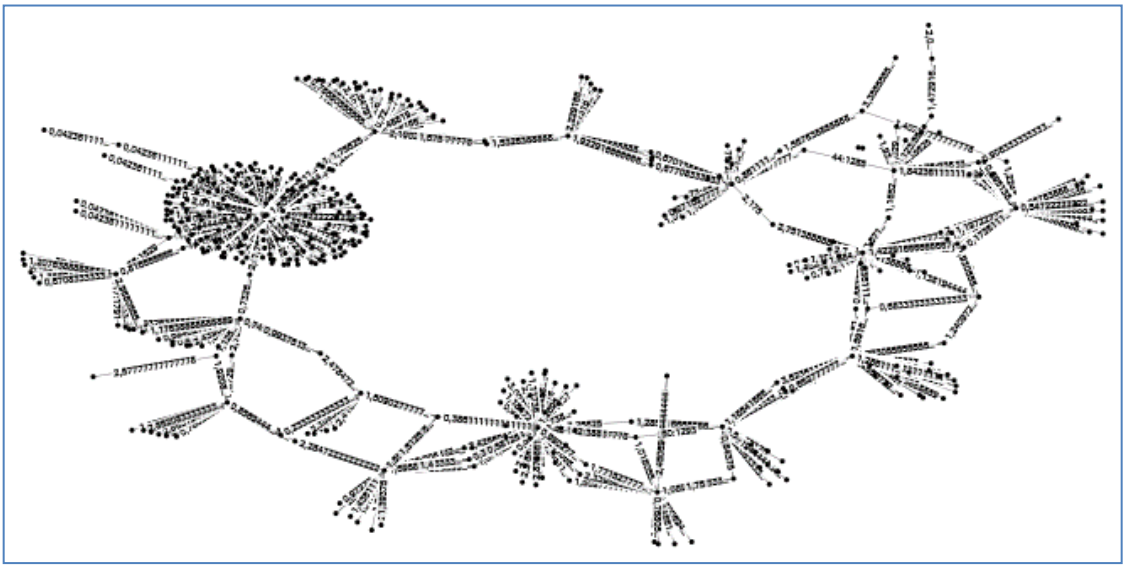

Figure 4 *Representation of the coded segments of the concepts spirituality, metacognition and mathematics.*

We noticed that there were clusters of smaller (or inner) networks in Figure 4's network view which indicated high density in the strengths of the ties between the codes. In particular, Figure 5 displays the largest cluster as a sampled illustration of the density of this network view.

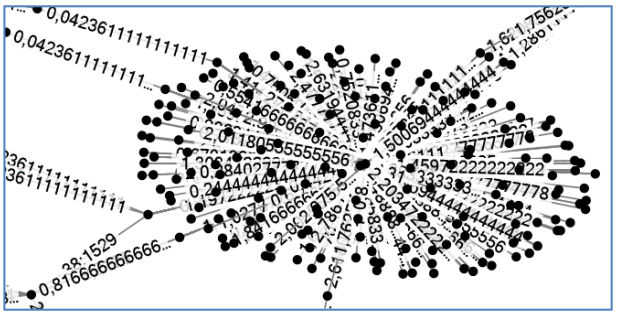

Figure 5 *Sampled cluster with high density indicating a strong relationship between the coded segments of the articles*

To declutter the sample set further, we arranged the network displayed in Figure 5 circularly and depicted each concept (spirituality, metacognition and mathematics) according to the three theoretical tiers depicted in Figure 3 of the conceptual-theoretical framework: (i) *ethnomathematics as an abstract representation of a person/culture's life world,* (ii) *metacognitive awareness as an expression of embodied cognition* and (iii) *spirituality as a sphere of mathematical intuition*. We did this in order to position the concepts on separate consecutive theoretical layers as illustrated in Figure 6, to show that this sample set contains coded data of spirituality, metacognition and mathematics and to reveal that this set is not clustered around a single concept.

Tier 1 - *ethnomathematics as an abstract representation of a person/culture's life world*

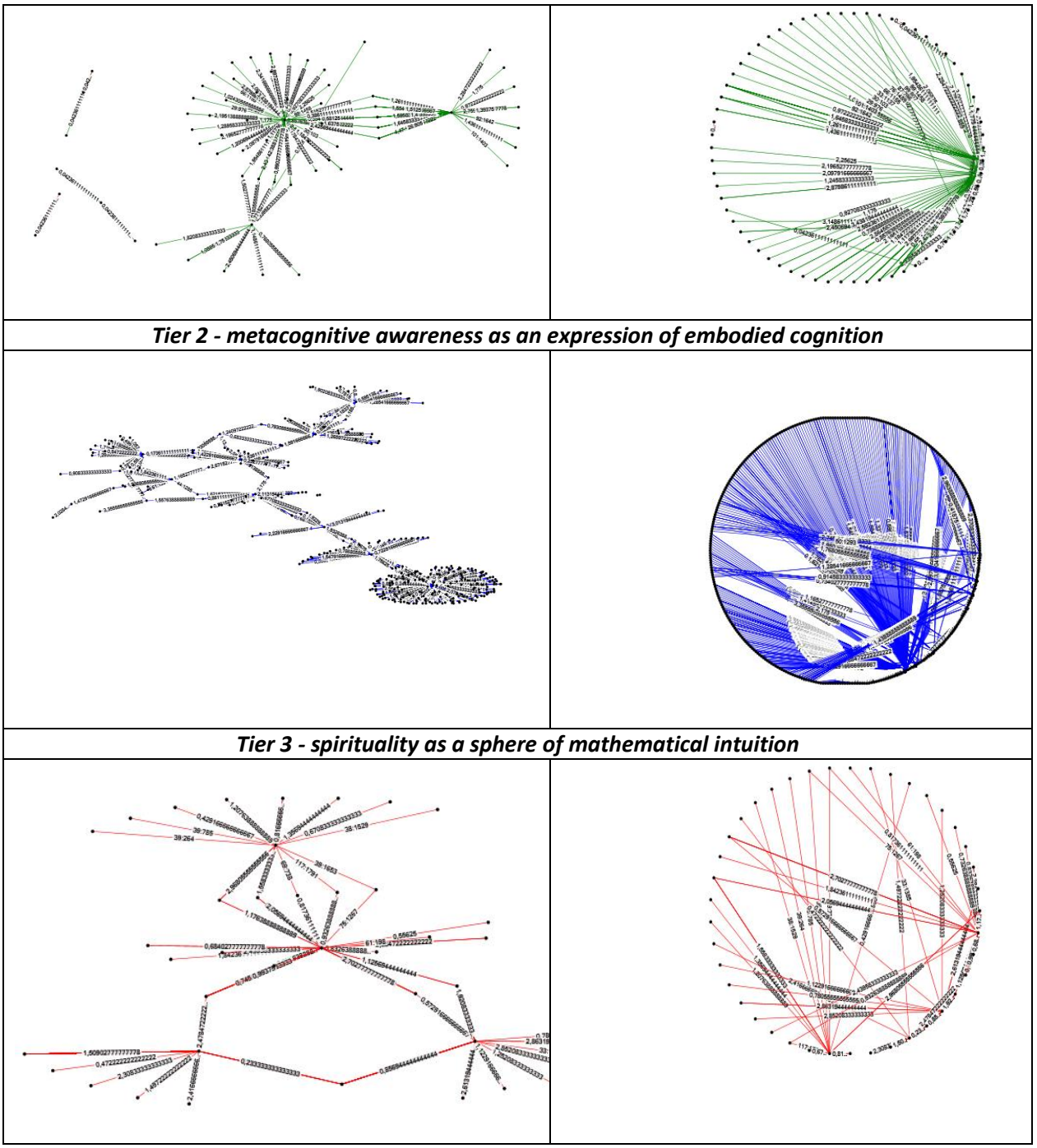

Figure 6 *Outline of the three tiers that constitute the network views of the spiritual-mathematical lens*

In light of the document analysis, the networks produced between the concepts are represented in Figure 5 as a series of relationships. To conceptualise the spiritual-mathematical lens, we aligned the concepts and these network relations to the three tiers of the conceptual-theoretical framework of this paper. Each tier is therefore discussed below in terms of these network views. We also conceptualised each tier as metaphorical filters that, together, can produce a collective view on the spiritual-mathematical lens as illustrated in Figure 6. In order to develop the "lens" and align it theoretically with the illustration in Figure 2, we compiled a collective view of the three tiers. To do so we aligned them using NodeXL's circle-function and revealed six regions (or zones) that identify where the three tiers overlap (see Figure 7). Culture, a dimension of ethnomathematics and indigenous knowledge, therefore overlaps with mathematics as a scientific discipline. Metacognitive awareness of the

spirituality relates to consciousness of one's connectedness to a greater purpose, and that is part of the search for meaning as a self-directed experience when learning ethnomathematics.

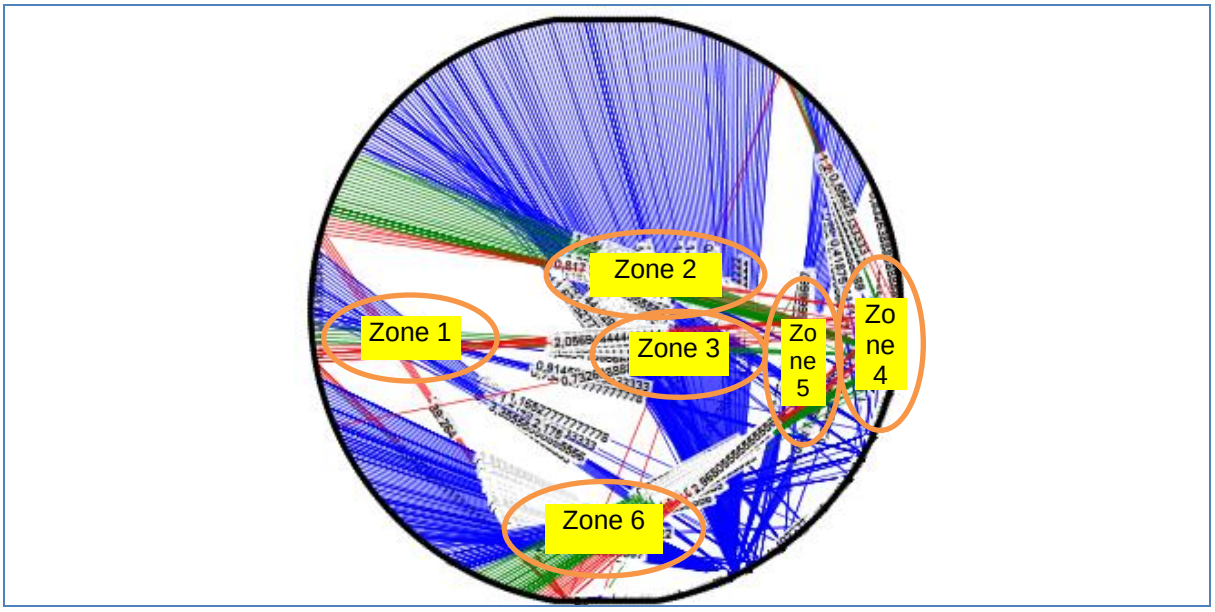

Figure 7 *A spiritual-mathematical lens illustrating the zones where spirituality (red), metacognition (blue) and mathematics (green) overlap*

The main findings of this document analysis point towards spirituality as a sense of oneness, a complete representation of the connectedness with oneself, others and a higher deity or deeper awareness of a meaningful and purposeful life. One's transcendental experiences can lie on the object level or meta level even beyond one's indigenous knowledge, yet having knowledge of this spiritual domain results in knowledge of the person and this is a form of metacognitive awareness. One would then have metacognitive awareness of the spiritual consciousness related to specific cultural artefacts and infuse this knowledge into one's learning of mathematics, thereby making the learning more self-directed. This spiritual-mathematical lens can therefore be viewed from two sides. From the one side the lens offers a view into one's own object level reflection (or metacognitive reflection) into one's soul to filter through the cultural and traditional indigenous knowledge to become aware of the mathematics within the indigenous knowledge system. This "becoming aware" results in the metacognitive awareness of the embodied mathematical knowledge that resides in the object level artefacts (Maxwell & Chahine, 2013). Through such awareness on the meta level, one can intuitively connect awareness of intrapersonal, interpersonally and transpersonal experiences to acquire transcendent spiritual consciousness and spiritual intelligence. In practice this lens can be used to develop cultural artefact-worksheet interventions that seek to assist mathematics teachers in the understanding and applying of indigenous knowledge systems in the teaching and self-directed learning of ethnomathematics. Our discussion now shifts towards the three theoretical tiers that construct this spiritual-mathematical lens.

Tier 1 - Ethnomathematics as an abstract representation of a person/culture's life world

In the first tier we conceptualised ethnomathematics as a construct embedded in a person's life world. Practically, this would imply that a mathematics teacher who instils a particular indigenous knowledge system in his/her curriculum will do so by taking examples or artefacts from cultural traditions or

practices and mathematize those artefacts in order to contextualise identified mathematical concepts that the artefact is perceived to hold. Tier 1 therefore represents the abstraction of reality which serves as the basis from which the other two tiers evolve. Although this tier is not intended to answer the research question alone, it does provide a context and framework on which to reflect about the self-directed learning of ethnomathematics, and to acknowledge the traditional knowledge system or culture where this ethnomathematics originates. We therefore pose that in the first tier one can become aware of the indigenous knowledge drawn upon in a specific culture to determine possible transcendental experiences associated with the self-directed learning of ethnomathematics and result in a type of *ethnolearning*. To support this, Knowles (1975) explains that creating interest and effort will embody self-directedness through interaction with the environment. Even though the discussion of the networks that portray the tiers in Figure 6 does not reflect this relationship directly, it can help explain the nature of reality and indigenous knowledge systems as a source for mathematizing and ethnomathematics activities, which promote future learning. Tier 1 enables the teacher or scholar to interpret reality as a representation of mathematical ideas, and act as an ethnomathematics representation of a person or culture's life world.

Tier 2 - Metacognitive awareness as an expression of embodied cognition

The current paper shows through documents analysis that during metacognitive awareness there emerges either awareness of the knowledge domain (e.g. procedural, declarative or conditional) or knowledge level (e.g. person, task or strategy) about the ethnomathematics and the learning process. For example Wilson and Clarke (2004) explain that those who do not develop new knowledge are likely to be unfamiliar with the language that other members use to express their reasoning. This suggests that merely interacting does not promote or foster metacognitive knowledge; it necessitates a shared understanding of what this knowledge entails and seems to depend on the metacognitive nature of intuition. Through this reasoning one can argue that the environment becomes part of our cognitive system (Wilson & Clarke, 2004), and metacognitive system. Furthermore, the knowledge that flows from the indigenous knowledge systems, that are also fused culturally in ethnomathematics (refer to Tier 1), are reflected upon to create metacognitive awareness of the relationship between the world and the mind. This is often done intuitively as part of one's self-directed learning. Alter and Oppenheimer (2009), for example, explain that through reflection one develops a self-theory, a theory of knowing oneself and one's life world. This knowledge of the self, or metacognitive awareness, articulates also the knowledge of the context in which the life world exists and, as Kasavin (2009) explains, this drives us beyond the boundaries of the physical to the psyche. It is, therefore, through this metacognitive awareness that one can express the knowledge one has about one's indigenous knowledge systems and the ethnomathematics that these systems portray, even without having the meta level vocabulary to express one's mathematical thinking. Maxwell and Chahine (2013) explain that this metacognitive awareness also refers to cultural awareness, awareness of self, and awareness of mathematical properties, awareness of the intuitive, transcendental and spiritual experiences. This becoming aware can lead to a moment of enlightenment if, for example, the learner can view cultural indigenous games and make sense or meaning of mathematics on a different level, or through a cultural lens. It then unfolds another world of possibilities by creating a sense of deeper meaning, a search for purpose and connectedness, at least culturally. Such metacognitive experiences can manifest consequently in the transcendental, spiritual and intuitive experiences, even beyond one's own life world.

Tier 3 - Spirituality as a sphere of mathematical intuition

The concept of spirituality, Meezenbroek et al. (2012) explains, suggests that individuals must think intuitively and reflect on the greater meaning of what they do. However, when individuals come together and share their ideas, play cultural indigenous games or do mathematics, they express this intuitive thinking by building on these complex inner structures. Metacognitive experiences therefore appear to improve the capacity to engage and succeed in the self-directed learning of ethnomathematics. We do acknowledge that there are other equally important contributing factors to the self-directed learning of mathematics, yet the affordance of a spiritual, transencedal intuitive awareness could promote deeper learning, towards the sustainment of future learning. The metacognitive awareness of indigenous knowledge (in the case of Morabaraba, for example) can produce awareness of knowledge, strategies and skills and this reacts the internal cognitive categories structure (the object level) and creates awareness of the deeper, inner, self-directedness of a person's learning. Self-directed ethnolearning therefore is the result of the metacognitive awareness of the indigenous knowledge system's embedded mathematical concepts. In the case of board games (for example see Figure 1), one experiences a sense of mathematical intuition as you monitor, reflect on and compare your "cows" with that of your opponent. You also create a sense of curiosity when being immersed in the ethno world – such as experiencing the traditions of other cultures, and foster admiration and care for other cultures' immersion of mathematics, knowing then that no-one is alone, all is connected, be it through a transcendental enlightened experiences or through the mathematics embedded in our cultural artefacts. Truly, a new paradigm of spiritual-mathematical self-directed learning is upon us.

Future directions

A strong commitment to understand the underlying intuitive states of mathematical and spiritual knowledge, from both Western and Eastern indigenous views, was the rationale for this paper. We argued that a conceptualised spiritual-mathematical lens could benefit the greater mathematics education research community by modelling a way through which the purely abstractness of mathematics can be humbled to acknowledge the anthropology of different cultures (Crump, 1990). Given that so much of our scientifically oriented culture is founded on western (ised) logical (scientifically) based principles, it would be embarrassing for humanities and social scientists to generally accept only one cultural or scientific way of reasoning. To overcome this, we advise that a spiritual-mathematical lens could benefit researchers who embrace a culturally approachable way of teaching. In particular, the scarcity of published articles that met the criteria for this paper hinders the conceptual-theoretical framework from being implemented in different contexts, although it opens the door to greater opportunities for ethnomathematics and self-directed learning researchers. Without as much as acknowledging the cultural knowledge and experience of our students, to make learning more accessible and appropriate for them, one can stand a chance of losing a spirit of curiosity and a desire for knowing as well as deprive meaningful learning experiences as a cornerstone of holistic self-directed learning. What we need now is to celebrate the diversity of mathematics and realise alternatives to its teaching and learning. It is this compassion for educating the soul that should serve as the foundation of our social responsibility as educationalists, and it is this intuitive state of spiritual and transcendental awareness that we should look for in our education. Only then can we truly connect as a community and capitalise our traditional and cultural knowledge through our education.

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