GRADE 9 LEARNERS' UNDERSTANDING OF THE CONCEPT OF THE EQUAL SIGN: A CASE STUDY OF A SECONDARY SCHOOL IN SOSHANGUVE

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ABSTRACT - This paper reports on 49 Grade 9 learners' understanding and interpretation of the concept of the equal sign and how they transit from arithmetic to an algebraic equation. A case study using a sequential mixed method research design was conducted in a secondary school in Soshanguve, a township in Gauteng, South Africa. Out of the 49 learners who wrote a test on the concept of the equal sign, eight were chosen for an interview. The study reveals that Grade 9 learners in this school interpret the equal sign as a "do something" and unidirectional (one-sided) sign, not as the concept that represents an equivalent (concept of keeping both sides of the equal sign equal) of two quantities. The researchers attributed misinterpretation of the equal sign to how learners had been taught the concept of number sentences at lower grades, where greater emphasis was placed on rules rather than the meaning of a concept. It is thus recommended that learners should be encouraged to solve equations through inspection and trial-and-improvement before solving them procedurally in order to develop relational rather than instrumental understanding.

Keywords: Understanding and interpretation, equal sign, arithmetic, algebra, equation

1. INTRODUCTION

Studies have been conducted worldwide about learners' understanding of the equal sign in elementary and middle schools (Hattikudur & Alibali, 2010; Jones, I., Inglis, M., Gilmore, C., & Dowens, M. (2012). 2012; Barrody & Ginsburg, 1988; Essien & Setati, 1996). Linchevski and Herscovics (1996) argue that teachers and learners should be made aware of the demarcation between arithmetic and algebra. In other words, teachers should be able to understand the gap between arithmetic and algebra to enhance learners' readiness to learn secondary school mathematics. The gap between arithmetic and algebra may explain learners' lack of readiness and the poor results achieved in algebra (Linchevski & Herscovics, 1996). The difficulties secondary school learners experience in learning algebra may involve the misinterpretation of the equal sign in given expressions or equations. Unfortunately, most mathematics teachers and textbook authors are not aware that learners misinterpret the equal sign. Common Core State Standard Organization (2010a) argued that learners who passed Grade 7 are expected to master the content and skills to be well-prepared for Grade 8 algebra.

1.2 The notion of the equal sign

Learners should learn the meaning of the equal sign to be an equivalence relation between two objects such as numbers or expressions (Jones, Inglis, Gilmore & Dowens, 2012). The notion of the equal sign in this instance is correlated with arithmetic competence and assists learners to master algebra and other mathematical concepts (Mathews, Rittle-John, Taylor & McEldon, 2012). Jones et al (2012), Li, Ding, Capraro and Capraro, (2008) and Mathews, Rittle-John, Taylor and McEldon (2012) postulate that learners should have a flexible understanding of the equal sign for arithmetic competence and algebra. Essien and Setati (2006) concur that the equal sign is used to indicate the relationship between quantities or values in mathematics. In other words, learners should view the equal sign as a relational symbol to compare numbers or expressions in order to solve mathematical problems effectively. On the other hand, learners view the equal sign as an operation: they interpret it as the total of adding two quantities or getting the answer to a particular mathematical problem (Rittle-Johnson & Alibali, 1999).

Essen and Setati (2006) explored Grades 8 and 9 learners' understanding of the equal sign in the South African context and found that the learners viewed the equal sign as a do-something or a unidirectional sign. The researchers found that Grades 8 and 9 learners viewed the equal sign as a tool to compute the answer rather than a relational symbol to compare the quantities. Essien (2009) further argued in his study that the equal sign is introduced operationally, which mainly focuses on addition and subtraction of numbers. The National Council of Teachers of Mathematics (2000) suggests that learners should learn about the equal sign in the early grades as it is regarded as an algebraic concept. This means that learners should be able to interpret the equal sign with understanding instead of having narrow knowledge that creates difficulties in understanding.

Although an operational understanding of the equal sign can be sufficient for solving standard equations, it contributes to the learners' solution of more complex problems (Hattikudur & Alibali, 2010). Perry, Church and Goldin-Meadow (1988) found that even late elementary learners still view the equal sign as operational instead of the relation of quantities. Hattikudur and Alibali (2010) suggest that learners who hold an operational understanding encounter difficulties in the transition to algebra. Furthermore, learners who hold operational understandings perform poorly compared to those who have a relational understanding (Knuth, Stephens, McNeil & Alibali, 2006). Grobman and Alibali (2007) argue that most learners who hold an operational understanding learn less from lessons involving linear equations. Studies conducted in Western countries also revealed that learners understand the equal sign not as relational, but rather as the operational meaning of expressions or numbers, to "work out the answer" (Baroody & Ginsburg, 1983; Knuth, Stephens, McNeil, & Alibali, 2006).

1.3 Sources of learners' operational understanding

The equal sign has been defined and introduced differently in textbooks around the world (Jones et al, 2012). Studies conducted in the United States of America (USA) and China on teachers' guidebooks or textbooks were compared, and it was found that the equal sign in the USA was rarely defined and is often used interchangeably with computations to get the answer (e.g., Li, Ding, Capraro & Capraro, 2008). The researchers revealed that the textbooks showed arithmetic equations as canonical, which is expression = answer. The textbooks contained little information for teachers to explain the equal sign to the learners when teaching. Similarly, textbook explanations of the equal sign were studied in South Africa, where Essien (2009) found that textbooks introduced the equal sign operationally using addition and subtraction of numbers or expressions. Moreover, the teaching approaches used during teaching and learning differ when teachers teach the equal sign as operation equals the answer (McNeil, 2008; McNeil & Alibali, 2005). Hattikudur and Alibali (2010) found that the instructional approaches used in the USA cannot help learners develop a relational understanding of the equal sign. These types of approaches hinder learners' understanding of the equal sign as they promote an operational understanding of the equal sign (Franke & Levi, 2003; McNeil, 2007). Learners who present the equal sign in an operational way are likely to offer relational interpretation to define the symbol (=) in a nonstandard context (e.g. 3+4=5+2) (McNeil & Alibali, 2005a). Hattikudur and Alibali (2010) postulate that learners who are taught the equal sign as "operation equals answer", may foster the incorrect view of the equal sign as an operational symbol rather a relational one. Li et al (2008) argue that learners are rarely taught to interpret the meaning of the equal sign.

The purpose of this study is to replicate other studies on learners' interpretation of the equal sign in a Grade 9 mathematics class in a South African context in a selected school in Soshanguve. Furthermore, the study also intends to understand how learners transit from arithmetic to algebra when using the equal sign in arithmetic ($9 + 6 = \blacksquare + 5$) and in algebra ($9 + 6 = x + 5$). We anticipated that the interpretation of the algebraic equation would reveal the learners' lack of understanding of the equal sign. No prior study has been conducted in South Africa that focused on how learners move from arithmetic to an algebraic equation, except those that focused on learners' understanding and interpretation of the equal sign (Essien & Setati, 2006). Hence, this study is based on previous international research on the interpretation of the equal sign and the transition of arithmetic to algebra (Hattikudur & Alibali, 2010; Jones et al, 2012) in a South African context. The study intends to respond to the following research questions:

How do Grade 9 learners understand and interpret the equal sign?
What is the transition from arithmetic to algebra in relation to the equal sign?

In this paper, we argue that the teaching and learning of arithmetic and algebraic equations should not be dichotomised. Although the two mathematical concepts appear to be incompatible, they are in fact complimentary. To develop this argument, we drew from Sfard's (1991) theoretical framework on procedural and structural conceptions, which were regarded as relevant to understand the learner's conception of the equal sign and the transition from arithmetic to an algebraic equation.

2. THEORETICAL FRAMEWORK

Sfard's (1991) theoretical framework of procedural and structural conceptions is useful in explaining how learners understand and interpret the notion of the equal sign and how learners transit from an arithmetic identity to an algebraic equation. Sfard indicates that any analysis of different mathematical definitions and representations brings us to the conclusion that abstract notions, such as the equal sign, a number or function, can be conceived in two fundamentally different ways: structurally as an object, and operationally as processes (p. 1). She argues that the two approaches, although ostensibly incompatible, are in fact complementary. In some instances, especially in textbooks, a concept could be defined as if the mathematical notion referred to is an object. This is a structural definition. Seeing a mathematical concept as an object means being capable of referring to it as if it were a real thing – a static structure, existing somewhere in space and time, recognising an idea at a glance and manipulating it as whole, without going into detail. But the same concept can also be defined in terms of its computational process, algorithms and actions rather than about objects. This reflects an operational conception of a notion. The operational is dynamic, sequential, and detailed.

In this paper, we argue that the notion of the equal sign can be interpreted structurally, as a static relationship between two magnitudes, and also operationally, as a concise description of some computation (Kieran, 1981). It can be interpreted procedurally if it is viewed by learners as a symbol that requires them to provide an answer, and it can also be viewed by learners as a symbol that represents an equivalent of two quantities, that is, the right hand side (RHS) = the left hand side (LHS). In other words, it can be regarded as a symbol of identity or as a "command" for executing the operation appearing on its right hand side. Hence in this study, we were interested in investigating how learners interpret the notion of the equal sign. Sfard (1991) provides a summary of the operational and structural conceptions of any notion in mathematics in table 1.

Table 1: Summary of operational and structural conceptions

	Operational conception	**Structural conception**
General characteristics	A mathematical entity is conceived as a product of a certain process or is identified with the process itself	A mathematical entity is conceived as a static structure – as if it were a real object
Internal representation	Is supported by verbal representation	Is supported by visual imagery
Its place in concept development	Develops in first stages of concept formation	Evolves from the operation conception
Its role in cognitive process	Is necessary, but not sufficient, for effective problem-solving and learning	Facilitates all cognitive processes (learning, problem-solving)

3. METHODOLOGY

This study is drawn from a bigger project, the Mathematics Teaching and Learning Intervention Programme (MTLIP), housed in the Department of Mathematics Education at the University of South Africa. The programme focuses on two provinces: Limpopo and Gauteng. Twenty (20) secondary schools in each province were chosen based on their performance in the Annual National Assessment. The programme is divided into two phases: the administration of a pre-diagnostic test in the schools to identify mathematical concepts in which learners did not perform well; and after the results have been analysed, the training of teachers in the schools on the problematic mathematical concepts identified. This paper reports on the first data collected during the administration of the pre-diagnostic test in one of the schools participating in the MTLIP in Soshanguve, Gauteng.

This study followed an explanatory sequential mixed methods design to gather both quantitative and qualitative data (Creswell, 2014). The researchers collected quantitative data in the first phase of this study and analysed the data: in the second phase we collected qualitative data as a follow-up to get an in-depth understanding of the quantitative results. Firstly, a pre-diagnostic test comprising thirteen (13) question items was administered to 49 Grade 9 learners in the selected secondary school in Soshanguve. The test was aimed at assessing learners' understanding and interpretation of the equal sign and the relationship between arithmetic and algebraic equations. Thereafter, a purposive sample of eight Grade 9 learners participated in semi-structured interviews to explore the in-depth understanding of the equal sign and how they can move from arithmetic to algebra. Grade 9 test items provided to learners are indicated in Figure 1. All the questions have been adopted from (Van de Walle (2015).

Grade 9 test

Write the missing number in each square box below

Question 1

$= 8$ ☐

$4 + 5 =$ ☐ $- 1$

☐ $+ 5 =$ ☐ $+ 8$

$3 \times 7 =$ ☐ $\times 3$

Question 2

2. Solve for n in the following equation.

2.1 $7 + n = 6 + 9$

2.2 $126 - 37 = n - 40$

2.3 $4800 \div 25 = n \times 48$

2.4 $4n + 4 = 4n + 1$

2.5 $n + 15 = 4n$

Question 3

Is Mathematical statement below correct? If yes, say why. If no, say why you think it is not.

3.1 $7 = 5 + 2$

Reasons:

3.2 $4 + 5 = 9 + 1$

Reasons:

3. 3 $674 - 389 = 664 - 379$

Reasons:

3.4 $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$

Reasons:
3.5 $64 \div 14 = 32 \div 28$
Reasons:

Figure 1: Test that was given to the learners

4. FINDINGS AND DISCUSSION

4.1 Analysis of Table 2

Table 2: Analysis of the results

Questions	Correct	%	Incorrect	%	Analysis of incorrect responses
Question 1 : Arithmetic identity / equation					
1.1	47	96%	2	4%	Answer 10 (1) Answer 1 (1)
1.2	28	57%	21	43%	Answer 9 (10) Answer 8 (9) Answer 5 (1) Answer 1 (1)
1.3	38	78%	11	22%	Answer 13 (6) Answer 0 (2) Answer 18 (2) Answer -5 (1)
1.4	35	71%	14	29%	Answer 21 (11) Answer 63 (2) Answer 28 (1)
Question 2 : Algebraic equation					
2.1	12	24%	37	76%	N= 22 or 22n (14) N= 15 or 15n (5) N= 16 or 16n (4) N= other (14)
2.2	8	16%	41	84%	N= 49 (20) N= 89 (5) N= other (24)
2.3	8	16%	41	84%	N= 192 x 48 (20) N= n x 48 (3) N= other (26)
2.4	0	0%	49	100%	N= 13 or 13n (16) N= 3 or 3n (7) N= 9 or 9n (4) N = other (22)
2.5	5	10%	44	90%	N= 19 (15) N= 4 or 4n (12) N= 15 (3) N= others (14)
Question 3 : True and False arithmetic identity / equation					
3.1	48	98%	1	2%	No, you cannot start with an answer, then follow with a question
3.2	39	80%	10	20%	Yes, because 4 + 5 is equal to 9
3.3	23	47%	26	53%	No, the answer must be 254
3.4	2	4%	47	96%	Yes, because, I add 1

					plus 1 is 2 and 2 plus 3 is 5

A brief analysis of how learners performed per question is provided below. However, more analysis on how the themes emerged from the data is captured in the Annexure A below

4.1.1 Question 1

Table 2 indicates that learners performed better in questions 1 and 3, which were about an arithmetic equation, than in question 2, which was about an algebraic equation. Furthermore, some Grade 9 learners still have difficulty in finding an unknown represented by a box in an arithmetic equation. For example, 34% out of 43% of learners who got the answer incorrect indicated that the answer to question 1.2 was 9 or 8. These learners do not interpret the equal sign correctly. Similarly, 12% out of 22% of learners who got the answer incorrect indicated that the answer to question 1.2 is 13. Again, 22% out of 29% of learners who got the answer incorrect indicated that the answer to question 1.2 is 21. They interpreted the equal sign as a "provide, give an answer" verb instead of a symbol that represents an equivalent of two expressions, that is, the RHS = LHS. This idea was confirmed by learners' interviews below.

4.1.2 Question 2

As indicated in the preceding paragraph and table 2, most learners did not perform well in the algebraic equation compared to the arithmetic equation. In question 2.1, 76% of learners gave the incorrect answer. Learners provided a range of answers to the question, but the most prevailing answer was either 22 or 22n (14 learners out of 49). Five learners indicated that the answer for question 2.1 was 15, while four learners indicated that the answer was either 16 or 16n, and another 14 learners gave incorrect answers that could not be coded. Eighty-four percent (84%) of learners got question 2.2 incorrect. Out of the 41 learners who got question 2.2 incorrect, 20 learners indicated that their answer was 49, while five learners indicated that their answer was 89. The responses of 24 of the learners to question 2.2 could not be coded. In question 2.3, 84% of learners gave the incorrect answer. Twenty (20) learners indicated that the answer of 192 x 48 = 9 216 was correct, three learners indicated the answer was n x 48, and the answers of 26 learners were not coded because there were inconsistences compared to others. Surprisingly, no learner noticed that there was no solution in question 2.4. Instead, learners just added numbers that were in the question, irrespective of whether they were like terms or not. For example, most learners (16), through just adding numbers, indicated that the answer was either 13 or 13n. Similarly, 90% of learners got question 2.5 incorrect, because they just added numbers in the question without considering whether they are like terms or not.

4.1.3 Question 3

Table 2 indicates that learners performed well in all questions except in questions 3.3 and 3.4. Most learners (53%) indicated that the answer is "No" because the answer must be 254. These learners just subtracted 389 from 674 to get 254, ignoring the left hand side numbers of the equation. One learner indicated in question 3.1 that the answer was no, because you cannot start with an answer, and follow with a question. Although question 3.4 was a fraction, it was intended to check if learners could interpret the concept of the equal sign. Surprisingly, only 2 out of 49 learners got this question correct. Most learners (96) indicated that the answer is "Yes, because, I add 1 plus 1 is 2 and 2 plus 3 is 5."

4.2 Interview findings

Table 2 shows how learners performed in the given tasks; however, it does not describe the thought processes of learners. As a result, we interviewed learners to access their thinking based on the provided responses. Four themes emerged from the analysis of the interviews: articulation of the equal sign as "equals to" instead of "equal to"; the cognitive gap between the arithmetic equation and the algebraic equation; the interpretation of the equal sign as "give me an answer, do something" as opposed to equivalence; and just adding, subtracting, multiplying and dividing unlike terms together. The four themes are discussed below.

4.2.1 Articulation of the equal sign as "equals to" instead of "equal to"

Learners do not know how to articulate the equal sign. They articulate it as "equals to" instead of "equal to". If one cannot articulate a concept, it is unlikely that one will grasp its meaning. This is apparent in the excerpt below.

Researcher: Would you please explain to us how you got you answer for 1.2 as 9?
Learner 1 (L1): The answer is 4 + 5 is equals to 9.
Researcher: How do you understand this sign: = ? (The researcher referring to the equal sign)
L1: Equals to.
Researcher: What did you say? Equal to or equals to?
L: I said equals to.

Although the interviewer could not trace the origin of this incorrect articulation of the equal sign, it appears that this erroneous articulation has been sustained in class without correction because it was not evident in only learner 1, but also in other learners. The incorrect articulation of the equal sign was also indicated by learners 7 and 8 as indicated below.

L7: I did the same because 3 x 7 is equals to 21.
L4: I said 4n + 4 is equals 8n and added to 4n and 1 and gave 13n.
L8: The question said 4 + 5 and a box and -1, so 4 + 5 is equals to 9 and 9 minus one is equals to 8.
When asked about the meaning of the equal sign, L1 said, "No I don't know what this sign means."
L4: Yes, sir, it says I must give an answer of 126 – 37, which is 89 and 89 – 40 is 49.
L1: No I don't know what this sign means.

These learners do not know the meaning of the equal sign. Some have interpreted the equal sign as a "do-something" sign rather than a symbol indicating equivalence (Essien & Setati, 2006; Hattikudur & Alibali, 2010; Stephens, Knuth, Blanton, Isler, Gardiner & Marum, 2013). The operational conception of the equal sign can be attributed to incorrect articulation.

4.2.2 The cognitive gap between an arithmetic and algebraic equation

Table 2 indicates that learners performed better in questions 1 and 3, which were arithmetic equations, than in question, 2 which was an algebraic equation. When learners were asked about the relationship between the two questions, most did not articulate any relationship.

Researcher: What is the relationship between questions 1 and 2? Do you see any similarities or commonalities?
L3: No, sir, these questions are not the same. Question 1 has a box and question 2 has an n.
Researcher: Question 1 has a box and question 2 has an n? What do these symbols represent?
L6: A box means I must give an answer and n means I must solve for n.
Researcher: So, there is no relationship between these two symbols?
L3: No, sir, one is a box and the other one is an alphabet.
L6: I can't see any similarities between questions 1 and 2.

It is clear from the above extract that L3 and L6 did not see any similarities between questions 1 and 2. In fact, L3 saw differences rather than similarities: question 1 with a box and question 2 with alphabet n. When asked about the representation of these symbols, L6 indicated that a "box means I must give an answer and n means I must solve for n". These learners do not see the box and the n as representing a number. This is further evidenced by their responses in question 2. They both responded in questions 2.1 and 2.3 as n = 22n and n = 13n respectively. This is further supported by the learners' performance as indicated in table 2. Most learners performed better in questions 1 and 3, which were about the arithmetic equation, than in question 2, which was about the algebraic equation.

The difference in performance between questions 1 and 2 suggests a cognitive gap between arithmetic and algebra. Learners could recognise algebra as a generalised arithmetic (Herscovics & Linchevski, 1996; Wu, 2001; Wheeler, 1996; Ussikin, 1998; Braithwaite, Goldstone, Van Der Maas & Landy, 2016)

4.2.3 The interpretation of the equal sign as "give me an answer, do something" as opposed to equivalence

In questions 1.2, 1.3 and 1.4, most learners indicated that answers to these questions are 9, 13, and 21 respectively. The interview showed that some learners interpreted the equal sign as "give an answer or do something", but not as a symbol that represents an equivalent of two expressions.

Researcher: Okay, what does the equal sign mean? What does it tell you?
L5: It tells me to give an answer for 4 plus 5, which is 9.
Researcher: Okay, but there is -1 on the right hand side of this equation; where did you take it to?
L5: No, I did not use it because it is not part of an answer.

The remark above by L5 indicates that the equal sign means "give an answer", for example "4 + 5 is 9". When L5 was asked about the negative 1 on the right hand side of an equation, L5 said, "I did not use it because it is not part of an answer." This implies that this learner only considers numbers that are behind the equal sign and the rest are ignored. A similar pattern was also noticed in L7's responses below.

Researcher: In 1.3 you gave your answer as 13. Can you explain how you got it?
L7: It is because I added 5 and 8 and it gave me 13.
Researcher: Okay, what about this other 5? (The researcher referring to 5 on the left hand side of the equation.)
L7: I did not use it because I want an answer of 5 plus 8.
Researcher: What about 1.4?
L7: I did the same because 3 x 7 equals to 21.

When asked about how he did question 1.4, L 4 said, "I multiplied 7 and 3 and it gave me 2." This is also an indication of learners interpreting the equal sign as a "do something" sign irrespective of an operation they are working with. This was confirmed by L2 who said: "I did not use it because the question says 3 x 7, so 3 x 7 is 21." Learners in the above excerpt understand the equal sign as a "give the answer" signal. They have an operational conception of the equal sign rather than a structural conception. Herscovics and Kieran (1980) argue that students view the equal sign as a "do something signal" rather than as a tool that means "same as", that is, RHS = LHS. Essien and Setati (2006) confirmed that Grades 8 and 9 learners see the equal sign as a tool for writing the answer rather than as a relational symbol to compare quantities They regard the equal sign as a "command" for executing the operation appearing the right side (Sfard, 1991). The interpretation of the equal sign as an invitation to an answer has been explored by many researchers (Essien & Setati, 2006; Essien, 2009; Hattikudur & Alibali, 2010; Stephens, Knuth, Blanton, Isler, Gardiner & Marum, 2013; Bush & Karp, 2013; Jones, Inglis, Gilmore & Dowens, 2012).

In question 1.3, learners 5 and 8 just added and subtracted numbers without any understanding of the equal sign. They used numbers and the operation sign at their disposal. This is confirmed in the following remarks below by L5 and L8.

Researcher: In question 2.2 you gave your answer as 8; can you explain why?
L5: I said four + five is 9 and 9 – 1 is 8.
L 8: The question said 4 +5 and a box and -1, so 4 +5 is equals to 9 and 9 minus one is equals to 8.

4.2.4 Just adding, subtracting, multiplying and dividing unlike terms together

Table 2 indicates that learners did not perform well in question 2, which was about an algebraic equation as opposed to question 1, which was about an arithmetic equation. Learners could not make a connection between the arithmetic and the algebraic equation. They could not see that an unknown in an arithmetic equation, which is represented by a box, and an unknown in an algebraic equation, which is represented by a letter, mean the same thing. Thus, learners could not make sense of a variable. When asked to solve for n in question 2, some learners just added numbers that are in the equation. When asked about how she did question 2.1, L3 said, "I have added 6 + 9 and it gave me 15, and added 7 to get 22n." L4 did the same by subtracting numbers that were in the equation because there was a minus operation sign in the equation. When asked about how she worked out question 2.2, L4 said, "126 – 37 is equal to 89 and 89 – 40 gave me 49." The excerpt below is evidence that the usage of variables and numbers was not limited to addition and subtraction signs, but also extended to multiplication and division operation signs.

Researcher: You provided your answer in 2.3 as 91216; can you tell us how you went about this problem?
L5: I said how many times does 52 get into 4800 and it gave me 192, and I times 192 by 48 and it gave me 91216.
L6: I divided 4800 by 25 to get 192 and multiplied 192 by 48 to get 91216.
Researcher: What about question 2.4?
L4: I said 4n + 4 is equals 8n and added to 4n and 1 and gave 13n.
L6: I added 4n + 4n to get 8n, and 4 + 1 gave me 5, so 8n plus 5 gave me 13.
Researcher: What about 2.5?
L1: I have added 15 + 4 and it gave me 19n.
Researcher: Why 19n?
L1: Because an answer must have an n.

The above excerpt is an indication that learners do not realise that a variable or a letter represents any number that can make an equation true. Learners who have provided a solution were without a letter or a variable, for example n = 22, they did not even bother to test the equation by substituting the claimed value to the equation to see if it made sense. Although the researcher did not ask learners what the equation means, it was clear that learners did not know. Sfard (1991) argues that the equal sign is usually interpreted as requiring some action rather than signifying equivalence between two expressions, leading to the blunder that $x + 8 = 8x$, as some of the learners proposed in the above excerpt. Sfard argues that individuals must move from an operational or computational orientation, that is, for example, seeing $x + 8$ as the process whereby 8 is added to the number x, to a structural orientation, as seeing $x + 8$ as an object, a "whole", a "thing", an "answer".

5. CONCLUSION

This paper dealt with Grade 9 learners' understanding and interpretation of the concept of the equal sign and how they transited from arithmetic to an algebraic equation. We argue that the teaching and learning of arithmetic and algebraic equations should not be dichotomised. Although the two mathematical concepts appear to be incompatible, they are in fact complementary. Four themes emerged from this study. Firstly, the study reveals that Grade 9 learners in this school interpret the concept of the equal sign as a "do something" and as a unidirectional (one-sided) sign, but not as a concept that represents an equivalent (concept of keeping both sides of the equal sign equal) of two quantities. The researchers attributed a wrong interpretation of the equal sign to how learners have been taught the concept of number sentences at lower grades, putting more emphasis on rules than the meaning of a concept. Most learners did not see the equal sign as a symbol of identity, but as a "command" for executing the operation appearing on its right side. Their interpretation of the equal sign was an operational – computational process, with algorithms and actions than interpreted structurally, as a static relation between two magnitudes. Secondly, there is a cognitive gap between the arithmetic and the algebraic equation. It is therefore recommended that in order to bridge the gap, the concept of a variable should be introduced in the early grades. Thirdly, most learners articulated the equal sign as equals to. This encouraged them to develop a procedural understanding rather than a conceptual understanding of the concept. Fourthly, the misunderstanding of the equal sign resulted in learners adding, subtracting, multiplying and dividing like and unlike terms together. We therefore recommend that for learners to develop a relational rather than an instrumental understanding, they should be encouraged to solve equations through inspections and trial and improvement, before solving them procedurally.

REFERENCES

Baroody, A. J., & Ginsburg, H. P. (1983). The effects of instruction on children's understanding of the ''equals'' sign. *Elementary School Journal*, 84, 199–212.

Carpenter, T., Franke, M., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Portsmouth: Heinemann.

Creswell. J. W. (2014). *Research Design. Qualitative, Quantitative, & Mixed Methods Approaches*. Singapore: Sage.

Didis, M. G., & Erbas, A. K. (2015). Performance and Difficulties of Students in Formulating and Solving Quadratic Equations with One Unknown. *Educational Sciences: Theory & Practice*, 15(4), 1137-1150.

Essien, A., & Setati, M (2006). Revisiting the equal sign: Some Grade 8 and 9 learners' interpretations. *African Journal of Research in SMT Education,* 10(1), 47-58.

Essien, A. (2009). An Analysis of the Introduction of the equal sign in Three Grade 1 Textbooks. *Pythagoras,* 69, 28-35.

Grobman, K. H., & Alibali, M. W. (2007). Understanding the equal sign and learning algebra. Paper presented at the biennial meeting of the Society for Research in Child Development, Boston.

Hattikudur, A., & Alibali, M. W. (2010). Learning about the equal sign: Does comparing with inequality symbols help? *Journal of Experimental Child Psychology,* 107, 15-30.

Herscovics, N., & Kieran, C. (1980). Constructing meaning for the concept of equation. *Mathematics Teacher*, 73 (8), 572-581.

Jones, I., Inglis, M., Gilmore, C., & Dowens, M. (2012). Substitution and sameness: Two components of a relational conception of the equals sign. *Journal of Experimental Child Psychology,* 113, 166-176.

Knuth, E. J., Stephens, A. C., McNeil, N. M., & Alibali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education,* 36, 297–312.

Li, X., Ding, M., Capraro, M. M., & Capraro, R. M. (2008). Sources of differences in children's understandings of mathematical equality: Comparative analysis of teacher guides and student texts in China and the United States. *Cognition and Instruction,* 26, 195-217.

Linchevski, L., & Herscovics, N. (1996). Crossing the cognitive gap between arithmetic and algebra: Operating on the unknown in the context of equations. *Educational Studies in Mathematics,* 30(1), 39-65.

Matthews, P. G., Rittle-Johnson, B., Taylor, Researcher. S& McEldoon, K. (2012). Measure for measure: What combining diverse measures reveals about children's understanding of the equal sign as an indicator of mathematical equality. *Journal for Research in Mathematics Education*, 43, 316-350.

McNeil, N. M. (2007). U-shaped development in math: 7-year-olds outperform 9-year-olds on equivalence problems. *Developmental Psychology*, 43, 687-695.

McNeil, N. M. (2008). Limitations to teaching children 2 + 2 = 4: Typical arithmetic problems can hinder learning of mathematical equivalence. *Child Development,* 79, 1524-1537.

McNeil, N. M., & Alibali, M. W. (2005). Why won't you change your mind? Knowledge of operational patterns hinders learning and performance on equations. *Child Development,* 76, 883-899.

McNeil, N. M., & Alibali, M. W. (2005a). Knowledge change as a function of mathematics experience: All contexts are not created equal. *Journal of Cognition and Development*, 6, 285-306.

NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Perry, M., Church, R. B., & Goldin-Meadow, S. (1988). Transitional knowledge in the acquisition of concepts. *Cognitive Development,* 3, 359-400.

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology,* 91, 175-189.

Rittle-Johnson, B., Matthews, P., Taylor, R. S., & McEldoon, K. L. (2011). Assessing knowledge of mathematical equivalence. A construct-modelling approach. *Journal of Educational Psychology,* 103, 85-104.

Sfard, A. (1991). On the dual nature of mathematical conceptions: reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, 22(1), 1-36.

Tashakkori, A., & Teddlie, C. (2010). *Mixed Methods in Social Behavioral Research.* Thousand Oaks: Sage Publications.