

Using pedagogical content knowledge (PCK) by competent mathematics teachers to deal with learners' conceptions in statistics teaching

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Abstract

PCK entails subject matter content knowledge and instructional skills and strategies, learners' conceptions and learning difficulties. This study focuses on how mathematics teachers apply PCK to identify and deal with learners' conceptions in statistics. These conceptions entail both learners' preconceptions and their misconceptions. Six mathematics teachers were initially selected on the basis of their schools' performance in the senior certificate examination in mathematics over two years, and from these four participants were selected based on the results of conceptual knowledge exercise in statistics to participate in the second phase of the research. Data about learners' conceptions (preconceptions and misconceptions) were collected by means of lesson observation, teacher interview, questionnaires, video recordings, teachers' written reports and documents analysis. Qualitative analysis involving the coding and categorisation of the data was applied. The results indicate that mathematics teachers deal with learners' conceptions in statistics by applying topic-specific instructional skills and strategies; using learners' responses to oral questioning; setting pre-activities; the checking and marking of learners' class work, homework and assignments; and analysing learners' responses to class work, homework and post-teaching discussions. The implications for a mathematics education programme are discussed.

Introduction

Data on mathematics enrolment and learner performance in the South African Senior Certificate examination covering a period of 16 years (1995–2010) show that learners generally underachieve in mathematics. Mathematics failure rates in the SC examination remain unacceptably high, and the number or percentage of learners leaving grade 12 with a higher grade pass in mathematics is unacceptably low. While the percentage of candidates who wrote mathematics examination over the period in question increased, the number of learners who passed mathematics was below 30% in the standard grade and below 10% in the higher grade (DoE, 2010). This clearly suggests a crisis regarding mathematics underachievement at secondary school level.

The chief examiner's report on learners' performance in both mathematics and mathematical literacy in the 2008 and 2009 SC examinations shows that learners generally

underperform in statistics (DoE, 2009). According to this report there was a steady increase in learners' enrolment in mathematics, compared to previous years, but approximately 60% of those who passed scored between 30% and 40% (according to the NCS, the pass mark for mathematics is 30%) (Jansen, 2012 & DoE, 2010). Furthermore, in questions relating to statistics learners scored below 35%. As a result of the poor performance in statistics, teachers' ability to teach this topic, the quality of SC (high school) products, and university enrolment in mathematics and statistics-related subjects have been called into question (Keeton, 2009).

In recent years, more attention has been paid to developing statistical thinking in learners across all levels of education (ICMI/IASE, 2011) – such that many countries have decided to include statistics in both primary and secondary school curriculums. At secondary school level, statistics is taught as part of the mathematics curriculum by teachers who may or may not be specifically trained to teach statistics. However, while most teachers acknowledge the practical importance of statistics and are willing to pay more attention to the teaching of statistics, many teachers do not consider themselves to be well prepared to teach statistics or to face the challenge of resolving learners' difficulties (ICMI/IASE, 2011). In addition, a variety of difficulties are experienced in the teaching and learning of statistics in respect of fundamental statistics (ICMI/IASE, 2011). Unfortunately to the knowledge of the researcher, little research has been done on the extent to which teachers possess the statistical PCK necessary to address learners' learning difficulties and conceptions (preconceptions and misconceptions). The little information at our disposal suggests that teachers' statistical PCK is too weak to enable them adequately to deal with learners' conceptions and learning difficulties.

Research reports (ICMI/IASE, 2007 & 2011; DoE, 2010) show that statistics was introduced very recently in some countries (eg South Africa, in 2006) and that few teachers have little or no formal knowledge of the topic. Hence they may or may not be able to deal with learners' preconceptions, misconceptions and learning difficulties in statistics teaching (ICMI/IASE, 2007 & 2011; DoE, 2010). Therefore, in this period of rapid development of statistics education, further research is needed on how to deal with learners' conceptions and learning difficulties in statistics teaching so as to enhance learners understanding of the topic (Godino, Ortiz, Roa, Wilhemi, 2011).

This study addresses the following research questions:

- a) What knowledge of learners' conceptions, if any, do competent mathematics teachers possess and demonstrate during classroom practice?
- b) How do competent mathematics teachers deal with learners' conceptions in statistics teaching?

Conceptual Framework

Detailed discussion of PCK guided this research and data collection. PCK was defined as covering topic-specific content knowledge and pedagogical knowledge (instructional skills and strategies), as well as knowledge regarding conceptions of the teaching and learning of statistics and of learners' learning difficulties. The framework is based on the assumption that PCK exists and can be measured and shared with other mathematics educators for use in their classrooms. However, in this research the focus has been on learners' conceptions in statistics teaching, from which the conceptual framework was derived. Learners' conceptions in statistics teaching (which forms part of knowledge for mathematics teaching) consist of preconceptions and misconceptions. A mathematical misconception is a mathematical belief or idea that is based on incorrect or erroneous information about a given mathematical concept (Olivier, 1989). According to Olivier (1989), most mathematical misconceptions arise as a result of pre-existing conceptions or preconceptions in the mind of the learner or of the teacher. Misconceptions can occur when an attempt is made to link preconceptions to new knowledge. Olivier (1989) argues that misconceptions play a key role in how well or how badly a new concept is understood. The role of the mathematics teacher in resolving mathematical misconceptions is usually to develop some form of teaching and learning approach, such as teacher-learner or learner-learner discussion, communication, reflection and meaningful negotiation that addresses the missing concept (Penso, 2002; Cazorla, 2006). Through these approaches the mathematics teacher may be able to get to the root of the misconception.

Cazorla (2006), for example, reports that misconceptions and the way in which mathematics lessons are taught are among the factors that cause learning difficulties. According to her, most statistics teachers do not have adequate knowledge of the school curriculum and the approaches needed to teach and learn statistics, which could result in poor content delivery in the classroom situation. Jong (2003) notes that, in order to identify and resolve misconceptions and learners' learning difficulties during classroom practice, a teacher could use convergent and inferential techniques. Convergent and inferential techniques are data-collection techniques that entail developing questions for a topic in short-answer and multiple-choice formats to probe the preconceptions and misconceptions of learners (Jong, 2003). Classroom observation, document analysis, teachers' written reports, interviews and questionnaires were used to determine whether teachers were adopting these techniques or any other related techniques to identify learners' preconceptions. Teachers' written reports and learners' notebooks may help to identify where learners' learning difficulties lie (Jong, 2003; Jong *et al.*, 2005, Penso, 2002; Van Driel *et al.*, 1998).

To deal with learners' conception in statistics teaching, the following needs to be determined:

- how to identify a preconception in statistics teaching
- how to address a preconception that does not contribute to the effective teaching and learning of statistics
- how to identify a misconception in statistics teaching

- how to address learners' misconceptions, if any, in statistics teaching

In this study, a few issues raised by Shulman (1986) on teachers' knowledge are in line with this framework.

Methodology

This study adopted a case study research design. A case study allows the researcher to focus on one or more issues that is/are fundamental to a system in order to develop rich and comprehensive understanding of the system under investigation. In fact a case study enables the researcher to gain greater insight and understanding of the dynamics of the situation by observing the phenomenon physically (Cresswell, 2008). In this study, the case study research design was considered adequate and conventional in the field of the authors' research interest in order to gain greater insight and comprehensive understanding of how the participating teachers dealt with learners' conceptions in statistics during classroom practice. The subjects for this study were four mathematics teachers who were teaching grades 11 and 12 and who were identified and selected on the basis of their schools' consistently good senior certificate results in mathematics over a period of at least two years. The research question that guided the study was: *How do mathematics teachers use pedagogical content knowledge to deal with learners' conceptions in statistics teaching*

This research question was derived from the conceptual framework.

The study used essentially a qualitative research approach to collect both qualitative and quantitative data. Teacher conceptual knowledge exercise (CKE) and concept mapping exercise (CME) were the instruments used to collect the quantitative data, whilst the primary sources of the qualitative data were in-depth, structured interviews with the teachers, artefacts of the interview process, field notes taken and video recordings made during lesson observations, completed teacher questionnaires, written documents in the form of written/reflective reports by the teachers, and document analyses. The validity and reliability index of all the instruments were determined accordingly. The reliability index of the conceptual knowledge exercise was 0.81 (Table 1.0).

Table 1.0: Summary of test characteristics of the conceptual knowledge exercise

| Test characteristic | Range of values for test characteristics | Results from pilot study |
|-----------------------|--|--------------------------|
| Reliability | 0,70–1,00 | 0,81 |
| Discrimination index | 0,3–1,0 | 0,7 |
| Index of difficulties | 0,4–1,0 | 0,7 |
| Content validity | 0,97 | 0,7 |

The study's methodology consisted of two phases. In the first phase, the six identified mathematics teachers undertook a conceptual knowledge written exercise. The results of

this exercise were used to select the four best performing teachers for the second phase of the study. The second phase consisted mainly of lesson observations; interviews; the analysis of written documents in the form of completed questionnaires, reflective diaries or reports; and document analysis designed to produce rich, detailed descriptions of participating teachers' PCK in the context of teaching statistics concepts at school level. The concept mapping exercise was used indirectly to assess participating teachers' content knowledge and their conceptions of the nature of statistics at school level and of how it is to be taught. The qualitative data obtained were analysed to determine individual teachers' content knowledge of school statistics, their related pedagogical knowledge, their learning difficulties and how they developed their PCK in statistics teaching. The analysis was based on the iterative coding and categorisation of responses and observations made to identify themes, patterns and gaps in statistics teaching at school level. Commonalities and differences, if any, in the PCK of the four participating teachers and in how they dealt with learners' conceptions in statistics teaching were also analysed and determined.

Data Collection

In this study, teacher conceptual knowledge exercises, a concept mapping exercise, lesson observation, teacher interviews, questionnaires, written reports and documents analysis were the instruments used to collect data to examine how the teachers deal with learners' conceptions in statistics teaching.

The conceptual knowledge exercise consisted of 20 multiple choice questions in statistics design in line with the National Curriculum Statement for mathematics for grades 10-12. The aim of the conceptual knowledge exercise was to select the participating teachers for the qualitative aspect of the research. The top four performed mathematics teachers were selected for the second phase of the research. The semi-structured interview schedule was based on several literature sources on PCK. Questions were developed to address the teachers' teaching experience, qualifications, educational background and professional development, knowledge of instructional strategies, and preconceptions in teaching and learning statistics. For example, one of the questions asked:

'What course/module did you study in the university/college?'

The purpose of the semi-structured interview was to gain some insight into mathematics teachers' educational background that may have enabled them to develop their content and topic specific PCK in statistics. The lesson observation schedules were standard ones recommended by the Provincial Department of Education for normal classroom practice (DoE, 2010). The schedule was therefore adopted for assessing instructional knowledge used in teaching statistics as well as dealing with learners' conceptions, which is the major focus of this study. The purpose of using the standard lesson observation schedule was to collect data from real-life situations and to assess how well the teachers prepared for lessons, as well as to check for consistency in their implementation of plans. The teacher questionnaire was designed by the researcher to assess teachers' PCK in terms of their knowledge of instructional skills and strategies, learners' conceptions in teaching and

learning statistics, and learning difficulties. The teacher questionnaire consisted of 16 questions designed to triangulate data collected during lesson observation. An example of the questions that could reveal the instructional skills and strategies is:

How did you identify the prior knowledge (preconceptions) which the learners bring to the class about statistical graphs?

The teachers' structured written reports, in which they recorded what made the lessons easy or difficult, were used to assess instructional strategies and learners' learning difficulties after a four-week period of teaching statistics. The purpose of the teachers' written reports was to determine what (for the teacher) made the lessons easy or difficult, and to triangulate other data related to how the teachers developed their PCK over time. The written reports were compiled from teachers' and learners' portfolios, as well as learners' workbooks. For instance, the participating teachers were asked,

'How did learners respond to classroom activities as well as homework or assignments?'

The documents analysed in terms of teachers' compliance with curricular recommendations for teaching and learning school statistics were the learners' class workbooks, learners' and teachers' portfolios, and the NCS for mathematics. The purpose of the analysis was to triangulate the data, using the teacher interviews, questionnaires, lesson observation and written reports on how teachers developed their PCK in statistics teaching. At the end of the four weeks' teaching, these documents were made available to the researcher.

The purpose of the video recording was to record the teachers' teaching (lessons), which would enable the researcher to triangulate the data collected from the lesson observations. The duration of the lessons observed ranged from 40 to 45 minutes for each of the eight lessons. The transcribed protocols were used to gain insight into teachers' content knowledge and how it was used, including the instructional strategies demonstrated in the lessons on statistical graphs.

Analysis, Results and Discussion

Teacher conceptual knowledge exercises, a concept mapping exercise, lesson observation, teacher interviews, questionnaires, written reports and documents analysis were used to collect data to examine how the teachers deal with learners' conceptions in statistics teaching. The results of the conceptual knowledge exercises show that teachers A, B, C and D scored 85%, 90%, 90 and 75% respectively. A concept mapping exercise aimed at enabling participants to reflect on their understanding of statistics in school mathematics demonstrated by teachers during classroom practice was used. The results of the concept map showed that the four teachers scored 100%, 92%, 100% and 80% respectively, which bore a close relationship with the results of the conceptual knowledge exercise. Lesson observations, were used to triangulate the results of the concept mapping exercise about the teachers' content knowledge in statistics teaching, and to examine how the teachers demonstrated their content knowledge in statistics teaching.

The data collected during lesson observations revealed many aspects of the teachers' pedagogical content knowledge (PCK). For instance, in their planning of a lesson on statistics based on the work schedule and the textbooks, the teachers discussed possible levels of treatment of the topic. They included analyses of simple and complex versions of statistical graphs they taught and of how they could be logically and sequentially taught as observed during lessons.

For teaching statistical graphs all the four teachers based their lessons on the learning outcome which required learners "to be able to collect, organise, construct, analyse and interpret statistical and probability models to solve related problems" (DoBE, 2011). The lesson observations showed that the teachers dealt with learners' conceptions in statistics teaching by identifying and addressing learners' preconceptions and misconceptions during statistics classroom practice which answered research question two. For example, teacher D adopted a procedural knowledge approach of explaining how to construct, analyse and interpret a bar graph. This teacher taught the lesson in a step-wise fashion, beginning with the identification of learners' existing knowledge about the construction of bar graphs. He then tried to set a pre-activity (instructional strategy) during which he instructed learners to prepare a frequency table of the number of cars in a car park according to make in order to identify learners' preconceptions in bar graph construction. Using the frequency table prepared by the learners, a bar graph was constructed by first drawing its horizontal and vertical axes. The data values were placed on the vertical axis (company), and the frequencies were labelled on the horizontal axis. The scale was chosen by the teacher. He mentioned that this was done by considering highest and lowest values of the frequencies and the companies and the dimension of the graph paper provided. The points were then plotted and the best line of fit was joined to produce the bar graph (Fig 1.0). This process entailed a rule-oriented, formal approach in specifically teaching the construction of a bar graph which his learners seemed to create some learning difficulties. Teachers A, B and C also used rule-oriented approaches and also tried to identify their learners' preconceptions in the teaching of histograms, ogives and scatter plots respectively. While teacher A tried to identify his learners' preconceptions regarding the construction of a histogram by means of oral questioning, during which learners were requested to explain the concepts of the mode, median and mean of a distribution of ungrouped data, teacher B used a pre-activity and oral questioning to determine the preconceptions about a bar graph and ogive by instructing learners to prepare a frequency table of data given and mentioning different types of statistical graphs they were familiar with. Teacher C used the checking and marking of learners' homework to determine learners' preconceptions about ogives. The responses of the learners to the diagnostic questions (oral questioning and pre-activities) showed that the four participating teachers have no knowledge of the learners' preconceptions in the topics they taught.

The procedural teaching approach used by teacher D could presumably have been developed over his many years of teaching mathematics (15 years at secondary school level). He also used the recommended lesson plan and work schedule of the Department of Education (DoE, 2010) to teach statistics. These documents specify that learners should be able to collect, organise, construct, analyse and interpret statistical and probability models to solve related problems. Teacher D followed this order in teaching the construction of a bar graph. Some of the factors that may have contributed to the stepwise way in which he taught the bar graph following a particular order or sequence could also be attributed to the formulation of the learning outcome for data handling in the mathematics curriculum (DoE 2010), the nature of the data values, and the procedure for constructing a given graph (Leinhardt et al 1990). In order to understand a graph, one should be able to construct, analyse, interpret and apply the knowledge of graphing to everyday, familiar situations. Teacher D followed the sequence suggested by Leinhardt et al (1990) for teaching the construction of a bar graph. Consequently it would appear that teacher D applied limited though adequate methods of presenting the construction of a bar graph in terms of his PCK, and therefore exhibited a less powerful representation of data handling using the rule-oriented approach which resulted to some learning difficulties encountered by the learners. Teachers A, B and C used the same rule-oriented approach in teaching histograms, ogives and scatter plots respectively.

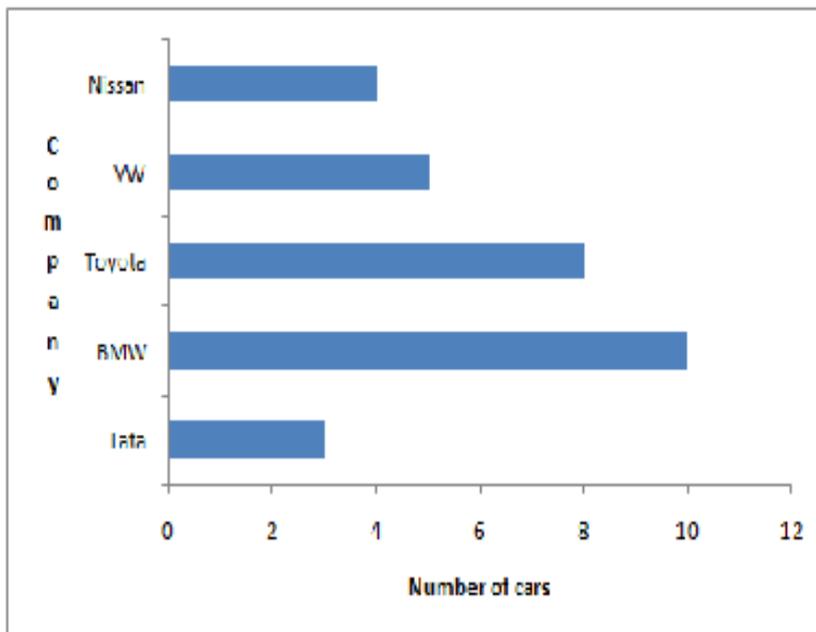


Figure 1.0: Bar graph showing the number of cars according to Company/Make in a car park

Teacher D did however on occasion and with regard to certain topics display evidence of a conceptual instructional approach to the teaching of histograms. When he used this approach, more learners showed evidence of understanding the topic better than when he

taught them according to the procedural approach or rule-oriented approach. In the lesson observed, teacher D explained in detail the meaning of a histogram and the relationships between concepts used in analysing and interpreting histograms (eg determining the mode of a histogram). In using the conceptual approach, the teacher seemed to have exhibited his PCK and taught the construction of a histogram in a way that enhanced the conceptual understanding of the topic (Fig 2.0). Teacher B adopted the rule-oriented approach throughout his lesson on bar graphs and ogives, however teachers A and C adopted a conceptual approach in their lessons on histograms and scatter plots respectively.

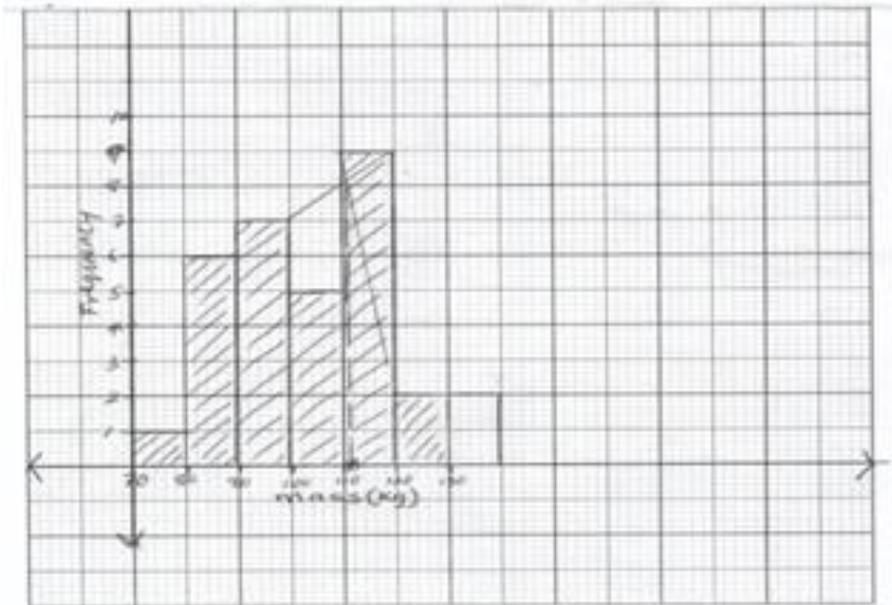


Figure 2.0: A histogram showing the distribution of the mass values of players in a 2003 South African rugby squad

During the lessons teacher D was able to identify learners' learning difficulties and alternative conceptions in the various statistical graphing exercises done during the lessons. For example, teacher D indicated in his written report and portfolio that learners had some misconceptions and learning difficulties regarding the construction of graphs on grouped data. One misconception existed in the learners labelling data values that started at 70 on the horizontal axis as if the data values started at zero. Similar evidence of the learners not being able to construct a histogram was obtained from their workbooks. The difficulties were addressed by means of extra tutoring, and by providing additional compulsory activities that relate to everyday familiar situations (instructional strategies) which also answers research question two. Teachers A, B and C also used an analysis of learners' class work, homework, assignments and post-teaching discussion to determine their misconceptions during their lesson on histograms, ogives and scatter plots. By analysis, teachers A, B and C have to review the learners' responses to the classwork, homework and assignments and determine how well the learners did their classwork, homework and assignments. If any difficulty or misconception exists, the teachers decide how to address the difficulty and the misconception. However, it was observed that all the four teachers

were deficient in their knowledge of learners' preconception of school statistics which may be attributed to the fact the topic was recently introduced in the mathematics curriculum. Misconceptions, for example, the drawing of a bar graph instead a histogram due to wrong scaling of the data axis, were resolved by means of familiar examples and by using the mother tongue to make meanings more accessible to learners. Post-teaching discussion, extra tutoring, extra problem solving activities, individualised teaching and repetition of lessons were also used to resolve misconceptions and learning difficulties about statistical graphs. For instance, while teacher A used extra tutoring and post-teaching discussion to resolve the misconceptions some learners had about the construction and interpretation of histograms, teacher B used extra tutoring and problem solving activities related to real life situations to address the misconceptions which the learners have about labelling of the data axis of an ogive which resulted in drawing a histogram instead of an ogive. Teacher C used individualised teaching and more problem solving activities related to real life to resolve learners' misconceptions regarding the construction and interpretation of ogives such as choosing scale of data that started from 70 marks, and post-teaching discussions about the construction and interpretation of scatter plots in which a negative correlated graph was misinterpreted as having no correlation due to an outlier.

Using PCK to deal with learners' conceptions in statistics teaching

As explained above, teachers used oral probing, questioning, post-activity discussions, on-the-spot checking and the marking of learners' classwork and homework assignments to try to identify learners' preconceptions about statistics. The four teachers could have been considered to have possessed the knowledge needed to enable them to apply the appropriate instructional skills and strategies to deal with learners' preconceptions about the statistical graphs they taught. But their strategies only yield previous knowledge that the learners' have depicting the fact that they were deficient in their knowledge of learners' preconceptions about statistical graphs. For example, teacher B tried to identify the preconception about the construction of ogives during the lesson by requesting learners to mention various ways of representing data. The learners mentioned the various ways of representing data such as bar graphs, frequency table and histograms based on what they have been taught. Learners' misconceptions and learning difficulties were identified by the various teachers through the analysis of learners' class work, homework, assignments and even post-teaching discussions about statistics. Instructional strategies such as post-teaching discussions, extra tutoring, extra problem-solving activities, individualised teaching and the repetition of lessons were used to address learners' misconceptions and learning difficulties regarding statistical graphs. For example, teacher B identified learners' inability to label the axis data by choosing appropriate scale which can be used to construct the ogive so that it can be accommodated on the graph paper provided. Other teachers individually identified and addressed their learners' learning difficulties with one or more of the strategies earlier mentioned (post-teaching discussions, extra tutoring, extra problem-solving activities, individualised teaching and the repetition of lessons). All four teachers

therefore had knowledge of learners' misconceptions and learning difficulties regarding statistical graphs, which were identified and addressed during classroom practice by means of instructional strategies they used for teaching statistics which answers research question one.

As explained under "Conceptual framework", if the participating teachers were aware of their learners' preconceptions and learning difficulties when they were teaching statistical graphs and they had the means to determine learners' preconceptions and learning difficulties, they would be able to design appropriate instructional skills and strategies for addressing the misconceptions and difficulties. Based on this explanation, PCK of knowledge of learners' preconceptions cannot be considered as a relevant construct for describing a teachers' topic-specific PCK in statistics teaching but previous knowledge and knowledge of learners' learning difficulties. It therefore becomes necessary to explore the PCK of mathematics teachers who demonstrates good content-specific knowledge (Godino *et al*, 2011) to determine how his or her PCK is applied while teaching difficult topics and dealing with learners' conceptions in statistics teaching.

Cazorla (2006), Jong (2003), Jong *et al.* (2005), Penso (2002) and Van Driel *et al* (1998) confirm the findings of this study in terms of identifying the preconceptions learners bring to the class by asking probing oral questions, checking and marking of homework and the use of pre-activities and pre-discussions, and identifying misconceptions by means of an analysis of learners' classwork, homework and assignments in statistical graphs. The misconceptions were dealt with by means of extra tutoring, additional problem solving activities related to familiar situation situations, individualised teaching as well as post-teaching discussions. Teachers may explore additional teaching strategies that will help them to present new topics in a manner that will help learners to understand them. Erickson (1999) supports this argument by pointing out that teaching a topic while emphasising learner understanding is one of the teaching strategies that holds the most promise which depicted an important implication of this study for mathematics education programmes.

In this study, several instructional skills and strategies to deal with learners' conceptions in statistics teaching were explored. The four teachers showed evidence of knowing how to identify and address learners' previous knowledge and learning difficulties regarding a specific statistics topic during classroom practice.

By implication, the findings of this study can be used to provide a knowledge base and process to be employed by mathematics teachers in dealing with learners' conceptions in statistics teaching for the continuous improvement of effective mathematics classroom practice. For instance, the teachers developed knowledge of learners' learning difficulties by analysing their responses to classwork, homework and assignments and during pre- and post-activity discussions. Regular examinations of learners' workbooks helped to reinforce

the teachers' familiarity with learners' conceptions and learning difficulties of statistics topics. Learning difficulties were generally addressed by the teacher engaging the learners on a one-to-one basis or collectively during or after school hours. Although, organising lessons outside normal school hours posed some challenges as learners were sometimes tired at the end of the school day and even extra-curricular activities at the schools occasionally affected the teaching programme, adjustments had to be made to ensure consistency and uniformity in all the statistics topics.

Conclusion

Dealing with learners' conceptions in the teaching of statistics is one of the components of PCK used in this study. Learners' conceptions regarding statistics include both their preconceptions and their misconceptions. Learners' preconceptions and misconceptions were used as the conceptual framework for this study. The participating teachers dealt with learners' preconceptions and misconceptions in statistics teaching by trying to identify and address them. While the participating teachers dealt with learners' preconceptions by trying to identify their pre-existing knowledge of statistics using probing oral questioning, pre-activities, and reviewing of learners' classwork, homework and assignments and pre-activities, misconceptions were identified by an analysis of learners' responses to classwork, homework and assignments. If practicing statistics teachers were able to deal with learners' conceptions in statistics teaching, they would be able to design good teaching and learning strategies that would enable them to present topics in such a way that the teaching and learning becomes comprehensible and accessible to learners. This may however; help to improve the teaching and learning of school statistics and consequently improve learners' achievements in the topic. It is hoped that the findings of this study will be incorporated into mathematics teachers' education programme with a view to the continuous improvement of teaching and learning and of school statistics.

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