Students as partners in mathematics course design: an ethnographic case study

F. K. Duah & A. C. Croft
Loughborough University, United Kingdom
Email: f.k.duah@lboro.ac.uk, a.c.croft@lboro.ac.uk

Abstract

This paper reports some findings from an ethnographic case study of four second-year undergraduate mathematics students and eight academic staff who were involved in collaborative mathematics course design in the Department of Mathematical Sciences at Loughborough University, UK. Data on the four students' experiences were collected via diaries, self-reflection and evaluation reports produced by the students, observation, and fieldnotes. Findings from the study showed that the four students were socialised and drawn from the margins of 'legitimate peripheral participation' in academic practice into full participation in a community of practising mathematicians. They acted as 'brokers' and 'change agents', crossing the artificial boundary that exists between the student learning community and the community of practice into which they were drawn. The four students, who worked as summer interns, were able to play an important role as mathematics course designers, and gained a deeper understanding of the mathematics on which they worked. We conclude that students have a role to play in the mathematics course design process; they act as change agents and accrue a range of benefits through the process.

Keywords: undergraduate mathematics, communities of practice, engagement, course design

1. Introduction

In the UK, some students studying undergraduate mathematics as their major, have been found to experience difficulties with their studies beyond their first year. For example, Brown, Wiliam, Barnard, Rodd, & Macrae (2005) describe the experiences of failing students from three Russell Group universities. They cite lack of skills required for independent study in higher education as a contributing factor. However, there are likely to be students who have difficulties with their studies and these difficulties could be inherent in the nature of course design and delivery. Irrespective of the nature of attribution of the difficulties that students face with their learning of mathematics beyond their first year, such difficulties may contribute to a less enjoyable learning experience, lead to failure and reduction in retention rates. For those who stay on, it is conceivable that they may report dissatisfaction with their learning experience even when they are able to graduate. Interestingly, Macrae, Brown, & Bartholomew (2003, p.6) state: 'it is difficult to know what more the [universities] could do to support these struggling students'. We suggest that strenuous efforts made by

staff to engage students in the university learning community would not only enhance the students' learning experience but also lead them to engage with their study of mathematics.

In order to enhance students' learning experience in general, and to improve the academic performance of students, the 1994 Group of Universities in the UK have adopted a policy statement which aims to encourage its members to involve students in the planning, design and delivery of courses. The policy statement states that '*students know how they want to be taught and have ideas about how techniques can be improved. They play an important role as 'change agents', challenging the established modes of teaching and learning, and contributing to making it more exciting and relevant for themselves and future generations of students*' (Kay, Marshall, & Norton, 2007, p.4). Although this statement comes from one group of UK universities, there is a general call from the Higher Education Academy (HEA UK) to Higher Education Institutions (HEIs) to engage 'students as active partners in shaping their learning experiences' (HEA, 2012).

Notable examples of students as partners in the delivery of courses within the UK HEIs, and around the world, are Supplemental Instruction (SI), Peer Assisted Learning (PAL), and Peer Assisted Study Sessions (PASS) (Arendale, 2001; Capstick, & Fleming, 2002; Dobbie, & Joyce, 2009; Fayowski, & MacMillan, 2008; Harding, Englebrecht, & Verwey, 2011). Several studies have looked into the effectiveness of such schemes in improving students' final course grades (e.g. Arendale, 2001). Also, increasingly some institutions in the UK and US are directly involving students in the design of courses (Bovill, Bulley, & Morss, 2011). However, there are few examples of empirical research into the efficacy of students' involvement in course design and how that impacts on students' learning of the content of the courses they help design (e.g Bovill, Cook-Sather, & Felten, 2011). Although examples of active student involvement in course design and teaching approaches exist in the literature, those examples relate to courses in non-mathematical sciences disciplines. Hence little is known about how such collaborative approaches would have worked in the Department of Mathematical Sciences at Loughborough University. In particular, for upper level courses such as *Complex Variables* and *Vector Spaces*, mathematics lecturers might legitimately ask: what role can students play in course design? Mathematics education specialists might also be interested to know how student involvement in course design could impact on the students' learning of the courses they help to design.

In the light of the foregoing, this paper reports an ethnographic case study of four undergraduate mathematicians employed as summer interns to work with staff to redesign the two courses, *Complex Variables* and *Vector Spaces*. The paper aims to answer the following research questions:
1.   What role can students play in the mathematics course design process?
2.   What is the nature of the partnership between staff and students in the course design process?

3.   What tangible outputs are students able to produce?

4.   How does students' involvement in course design impact on their learning and personal development?

2. Background to the study

This study was conducted at Loughborough University, UK, as part of a curriculum development project that was aimed at enhancing the undergraduate student learning experience of mathematics. The curriculum development project was undertaken because, historically, some second-year students studying for a degree in mathematics as a major or minor subject have found the two upper level courses, *Complex Variables* and *Vector Spaces*, difficult. Although some students do very well in these courses, there are those whose experiences could be described as similar to the students that Brown, Wiliam, Barnard, Rodd, & Macrae (2005) described in their study. The lecturers who teach the two courses were willing to work with students to produce resources that could potentially engage future cohorts of second-year students more and hence they volunteered to collaborate with the four student interns in the course design process. Besides the two lecturers, six other staff also took interest in the students' efforts to produce resources for teaching and learning and offered to be available during the internship to provide support and consultancy to the students regarding matters such as mathematics content knowledge.

3. Methodology

In March 2011, all second-year undergraduate mathematicians who had enrolled on and studied *Complex Variables* and *Vector Spaces* were invited to apply for four positions as student interns. Eight students out of a cohort of about 100 applied for the positions. All eight students were interviewed by the staff who teach *Complex Variables* and *Vector Spaces* and an additional member of staff. Four students were successful and commenced their internship in March 2011. They worked part-time for two hours per week between March and June 2011, conducting focus groups to collect the views of their peers about the teaching and learning of *Complex Variables* and *Vector Spaces* in order to inform the course design process.

During July and August 2011, the students worked fulltime as student interns for six weeks. They worked closely with staff but with considerable autonomy to design teaching and learning resources. An ethnographic study was designed to understand the students' role, experiences, working relationship with staff, and the resources that they were able to produce. During the six weeks internship, the first author immersed himself amongst the student interns; sharing an open plan office with them, interacting with them and responding to questions they might have about the use of technology in producing resources. Data on the students' experiences were collected via diaries kept by the students, self-reflection and evaluation reports produced by the students, participant observation, and fieldnotes. Staff were also interviewed individually in order to collect data

to triangulate the students' accounts so as to increase the validity and reliability of the findings. However, in this paper, we report on only the data collected from the students.

While the student interns worked, Monday to Friday, the first author observed their activities and their interactions with staff and each other and took fieldnotes. The student interns also kept diaries, which they wrote up daily and sent to the first author at the end of each week. At the end of their internship, the student interns also wrote a self-reflection and evaluation report on their six weeks' experience. The qualitative data collected were subjected to thematic analysis (Braun & Clarke, 2006) using NVivo 8 to generate codes which were later categorized into themes, three of which we describe and discuss in section 5. We also drew on themes from the literature on student participation in course design in non-mathematical sciences disciplines (Bovill, Bulley, & Morss, 2011; Bovill, Cook-Sather, & Felten, 2011) to explore the benefits that accrued to the student interns and their relationship with staff.

4. Theoretical Framework

Community of Practice (Wenger, 1998; Wenger, McDermott, & Snyder, 2004) was used as an analytical lens to explore the relationship and interactions between staff and the four student interns. Community of Practice is defined as a group of "people who share a concern, a set of problems, or a passion about a topic, and who deepen their knowledge and expertise in this area by interacting on an on-going basis" (Wenger, McDermott, & Snyder, 2004, p.4). For a group of people to constitute a Community of Practice, the group must have a joint enterprise or domain, mutual engagement or community, and shared repertoire of resources or practice. These defining characteristics, according to Wenger, McDermott, & Snyder (2004), foster learning and knowledge creation. Within organizations such as Mathematics Departments, there may exist multiple communities of practice and staff and students may belong to one or more of such communities of practice. There exist "boundaries" between different communities of practice. Some members of each community may act as "brokers", crossing boundaries to foster continuity between different communities of practice (Wenger, 1998) and acting as change agents (Dunne & Zandstra, 2011).

5. Findings

In this section we use extracts from transcripts, diaries, self-reflection and evaluation reports, and fieldnotes to provide evidence in support of our answers to the research questions. Extracts attributable to the four student interns are identified as S1, S2, S3, and S4. Each of these identifiers is shown on the right of the related extract.

5.1 Role played by the students

Staff and the four student interns formed a community of practising mathematicians. They formed a community of practice because they had a joint enterprise; that is, to produce engaging teaching and learning resources to enhance the student learning experience.

Throughout their internship, the student interns interacted with staff, discussed the mathematical content of the resources they produced, and built equal but professional relationships with staff. Thus, there was a mutual engagement amongst staff and the student interns. During their internship, the four student interns had a one-hour tea break each working day when they met in the office of a member of staff who provided refreshments. Through their mutual engagement, staff and students engaged in mathematical discourse in ways that lectures and tutorials do not make possible and developed a shared repertoire of resources as the following quotations from the diaries of three of the student interns show:

> "Meeting up with some of the staff for tea and biscuits was a good opportunity to get to know people a bit more, and made me feel much more involved and valued as a member of the project." (S2)

> "Its good to be able to comfortably talk to lecturers about interesting points in mathematics, it's also interesting to hear what they do as mathematicians and how they work together or alone." (S3)

> "I feel Lecturer 1 is more approachable now." (S4)

During the one-hour afternoon tea break, not only were the students acculturated to academic practice as the above quotations indicate, but they also received feedback on the content of the resources they produced. The mathematical accuracy of the resources the four students produced was of paramount importance since one of the aims of the course redesign process was to make the resources available for use by other institutions. Hence, notwithstanding the autonomy the students had in their role as interns, they felt it was essential that the content of the resources they produced was reviewed by members of staff. Where such feedback was constructive it was often well received and led to revision of the resources as indicated by the following extracts from the diaries of two participants:

> "Lecturer 1 has reviewed all of the materials that I have produced and provided feedback for each of them, so I now have to amend these." (S1)

> "Got feedback which I found helpful and constructive." (S4)

From the observations and fieldnotes data, we found that the students played two essential roles during their internship; intermediaries and competent academic apprentices. These new terms will be discussed in a future publication elsewhere and the full research report. However, in this paper, we suggest that the student interns played the role of intermediaries between staff and the second-year students by soliciting the 'student voice' through focus groups and other informal communication channels. Thus they acted as 'change agents' and 'brokers'. The student voice sought for was more valuable than could be provided by the traditional feedback mechanism, which is perceived to have a different purpose; quality assessment rather than quality enhancement. The richness and depth of the students' views about the teaching and learning of *Complex Variables* and *Vector Spaces*

would not have been obtained with the traditional quantitative survey on course evaluations.

The internship provided the student interns with opportunities to work with the content of *Complex Variables* and *Vector Spaces* as competent academic apprentices. Again, although we have not discussed and defined this terminology in this paper, we suggest that the student interns were competent in the content of the mathematics they worked on by the virtue of having taken and passed the examinations. At the start of their internship, three of the student interns, while being competent, showed lack of understanding in some aspects of the content of the courses they were working on. Through the process of resource production and feedback, we observed the students receive informal training and advice akin to the 'apprenticeship model' in a work place. Hence our introduction and use of the term competent academic apprentices to describe the role played by the student interns.

Although the student interns were enthusiastic about their role and sought and received constructive feedback regularly, our observations and fieldnotes data indicated that when feedback was perceived to be overtly critical or unrelated to mathematics, such feedback had an unexpected impact on the way the students sought feedback thereafter. For example, one intern hesitated seeking feedback on a very well produced document with a novel approach to solving a problem on Orthogonal Projections because he did not want to receive what he perceived to be critical feedback. Another participant receiving feedback on the use of good grammatical structures of the English language was not amused. For these students, it was the enjoyment of the mathematics that sustained their interest in their role and anything else seen as not mathematically related was not welcomed. This was particularly evident in week 1 when two student interns, identified as S1 and S2, felt that much of what they were doing was administrative duty and not challenging as can be seen from the following two statements made by the two student interns and recorded in the fieldnotes:

"I am getting bored with this [creating LaTex files]." (S1)

"I created LaTex files [all day] which I found boring." (S3)

5.2 Tangible resources produced by the student interns
While working during the six week internship, the student interns had to liaise with the course leaders, produce teaching and learning resources, and seek feedback on the quality and mathematical accuracy of the content of the resources they produced. Samples of resources that the students produced for the *Vector Spaces* course are shown respectively in Figures 1 and 2 below. Screencast videos were produced for use by second-year students independently and out-of-lecture. The videos were produced to supplement lectures but not to replace them. Also supplementary help sheets in the form of notes or problems and solutions were also produced for either independent use or in Peer Assisted Learning (PAL)

sessions. These sessions were scheduled for second-year undergraduate mathematicians for the first time at the Department of Mathematical Sciences at Loughborough University in order to enhance the students' learning experiences during the 2011-2012 academic year.

Figure 1 Screenshot of screencast video on Systems of Equations over a finite field

Vector Spaces

Orthogonal Projections

1   Orthogonality

1.1   Basic concepts in $\mathbb{R}^n$

If we consider two vectors $u, v \in \mathbb{R}^n$, we say that u and v are orthogonal, or perpendicular, in $\mathbb{R}^n$ if $u \cdot v = 0$. This is making use of the standard inner product on $\mathbb{R}^n$, namely

$$\langle u, v \rangle = u_1 \cdot v_1 + u_2 \cdot v_2 + \cdots + u_n \cdot v_n.$$

Similarly, we can say that a vector u is orthogonal to a subspace V, if it is orthogonal to every vector $v \in V$. This can be simplified by considering a basis of V. As the vectors in a basis span V, every vector $v \in V$ can be written as a linear combination of the vectors in the basis. As u is orthogonal to both of the vectors in the basis, u is therefore orthogonal to V if it is orthogonal to the basis vectors of V.

We can represent this graphically, by considering a vector $u = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} \in \mathbb{R}^3$ and a plane T which lies within $\mathbb{R}^3$. A plane is a 2-dimensional object, and so can be spanned by two basis vectors, such as

$$v_1 = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} \quad \text{and} \quad v_2 = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}.$$

For u to be orthogonal to T, it must be orthogonal to both v_1 and v_2 (see Figure **??**). By direct calculation,

$$\langle u, v_1 \rangle = (0 \cdot 4) + (0 \cdot -1) + (4 \cdot 0) = 0 \quad \text{and}$$
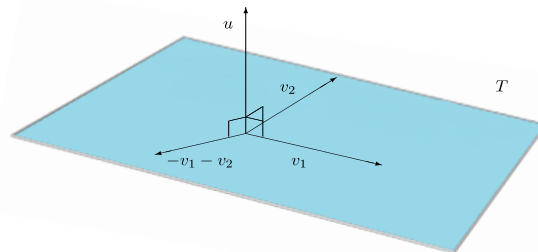$$\langle u, v_2 \rangle = (0 \cdot 3) + (0 \cdot 2) + (4 \cdot 0) = 0$$

Figure 1: An example of a vector u being orthogonal to a plane V, as it is orthogonal to the basis vectors v_1 and v_2. Note that u is also orthogonal to the linear combination $w \in W$, for example $w = -(v_1 + v_2)$.

1.2   Generalisation to vector spaces

These concepts can be generalised to any subspace of a vector space that has an inner product, which we will refer to as an inner product space. This means that, for a subspace W of an inner product space V with inner product $\langle \cdot, \cdot \rangle$, a vector u is orthogonal to V if, for a basis $\{w_1, w_2, \ldots, w_n\}$ of W,

$$\langle u, w_1 \rangle = 0, \langle u, w_2 \rangle = 0, \ldots, \langle u, w_n \rangle = 0.$$

This follows the same principle as in Figure **??**; it is orthogonal to each of the basis vectors, and therefore as they span the inner product space, it is orthogonal to every vector within W. This graphical intuition is limited however, as graphically representing a function such as "$1 + 2x$" in $\mathbb{R}_2[x]$ is not easily achievable.

Figure 2. A resource for a peer assisted learning session.

5.3 Impact on the student interns' mathematical knowledge

The internship and the course redesign process provided opportunities for the student interns to gain a much deeper understanding of the course they helped to redesign. Consequently, they gained increased confidence in their abilities as demonstrated through the following quotations:

"My knowledge of Vector Spaces is also improving, as I discovered an application for a Theorem that I had not previously realized was possible." (S1)

"I found that as I was creating videos my understanding of the topics is becoming much deeper and I hope these skills will be transferable to other courses I take in the future." (S2)

"I feel [that] my knowledge of the eigenvalue equation has improved a lot. My approach to learning will be very different after this internship." (S4)

Amongst the four student interns, the student identified as S1 was often positioned by the other three as the most able student. He is believed to be on track for a first class degree in Mathematics. However from the fieldnotes, we note that until the end of the six weeks internship, he did not have secured understanding in all areas of *Vector Spaces*, the course on which they worked. He was observed on three occasions using a board and a chalk to devise a solution to a problem on Orthogonal Projection using a geometric approach and then used his solution to produce a supplementary help sheet for student use. He notes in his diary that his solution to the problem on Orthogonal Projection is different from the way the lecturer had previously explained it in lectures and tutorials. The following extract from the diary of S1 is typical of how the student interns believe that the internship experience has impacted on their mathematical understanding:

"I have had to use the blackboard several times to work through a problem, so that I understand it completely and can convey my understanding through the solutions. This has helped me understand the topics within the course better though, which I believe is very helpful." (S1)

5.4 Staff Approach to Teaching

While our focus in this paper is on how the student interns benefited from their internship experience, we mention briefly the impact of the staff-student partnership on the two lecturers who normally teach *Complex Variables* and *Vector Spaces*.

First, we note that the changing relationship between the lecturers and the student interns provided impetus for the two lecturers to become more receptive to students' suggestions for changes to how the two courses had previously been designed. Suggestions for changes to the structure of lecture notes and the provision of additional resources such as mathematics screencasts were acted upon for both *Complex Variables* and *Vector Spaces*. While the lecturer responsible for *Complex Variables* responded to students' suggestions for lecture notes to have gaps ('gappy notes') for additional writing by students during lectures, the lecturer responsible for *Vector Spaces* did not do likewise. Furthermore, the staff member responsible for *Complex Variables* slightly changed the assessment policy in response to students' feedback from focus group discussions. Previously, there was neither a coursework nor a class test component as part of the *Complex Variables* course, but a class test has been introduced for the 2011/2012 academic year.

Second, as a consequence of the successful working relationship between the staff and the student interns, the two course leaders provided support to the student interns to produce learning resources for mathematics PAL sessions which ran in the 2011/2012 academic year. Also, in addition to their normal office hours, the two lecturers offered to provide additional office hours so that they could provide assistance to PAL facilitators in relation to the mathematical content they will work through during PAL sessions. We intend to report fully on the staff experiences of the student summer internship after analysing the data collected through interviews from staff.

6. Discussion

This study showed that students can make a contribution as partners in mathematics course design and that they benefit from the experience in several ways including a deeper understanding of the mathematics on which they work. The limitation of the current study, however, is that the four student interns constituted a convenience sample and hence we do not make generalization from the experiences of these four students. Nonetheless, this study appears to support the call for higher education institutions to involve students in shaping their own learning. The full findings of our study including the discussion of students as *intermediaries* and *competent academic apprentices* will be published in due course.

7. Acknowledgement

8. References

Arendale, D. (2001). *Review of research concerning the effectiveness of Supplemental Instruction from the University of Missouri-Kansas City and other institutions from across the United States.* Retrieved March 31, 2011 from http://www.tc.umn.edu/~arend011/SIresearchreview01.pdf

Bovill, C., Bulley, C. J., & Morss, K. (2011). Engaging and empowering first-year students through curriculum design: perspectives from the literature. *Teaching in Higher Education,* Vol.16 (No. 2): 197-209.

Bovill, C., Cook-Sather, A., & Felten, P. (2011). Students as co-creators of teaching approaches, course design, and curricula: implications for academic developers. *International Journal for Academic Development,* Vol. 16 (No. 2): 133-145.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology,* Vol. 3: 77-101.

Brown, M., Wiliam, D., Barnard, T., Rodd, M., & Macrae, S. (2005). *Student experiences of undergraduate mathematics.* Retrieved March 31, 2011 from http://www.esrc.ac.uk/my-esrc/grants/R000238564/read

Capstick, S., & Fleming, H. (2002). Peer assisted learning in an undergraduate hospitality course : Second year students supporting first year students in group learning. *Journal of Hospitality, Leisure, Sport and Tourism*, Vol.1 (No. 1): 69-75.

Dobbie, M., & Joyce, S. (2009). Does peer-assisted learning improve student marks in accounting? *Asian Social Science*, Vol. 5(No.10): 3-9.

Dunne, E., & Zandstra, R. (2011) . *Students as change agents.* Bristol: Escalate.

Fayowski, V., & MacMillan, P. D. (2008). An evaluation of the Supplemental Instruction programme in a first year calculus course. *International Journal of Mathematical Education in Science and Technology*, Vol. 39 (No. 1): 843-855.

Harding, A., Englebrecht, J., & Verwey, A. (2011). Implementing Supplemental Instruction for a large group in mathematics. *International Journal of Mathematical Education in Science and Technology*, Vol. 42 (No. 1): 847-856.

Kay, J., Marshall, P. M., & Norton, T. (2007). *Enhancing the student experience.* London: 1994 Group of Universities.

Higher Education Academy (2012). *Student engagement*. Retrieved March 31, 2012 from http://www.heacademy.ac.uk/student-engagement.

Macrae, S., Brown, M., & Bartholomew, H. (2003) The tale of the tail: an investigation of failing single honours mathematics students in one university. *British Society for Research into Learning Mathematics.* Vol. 23 (No. 2): 55-60.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge: Cambridge University Press.

Wenger, E., McDermott, R., & Snyder, W. M. (2004). *Cultivating communities of practice.* Boston, MA: Harvard Business School Press.