

## The effectiveness of problem-based learning in the further mathematics classroom

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### Abstract

The study investigated the effectiveness of Problem-based learning (PBL) in the Further Mathematics classrooms in Nigeria using a pre-test post-test control group quasi-experimental design. In all, 144 students participated in the study. Six research instruments: Researcher-Designed Test (RDT), Teacher-Made Test (TMT), Observation Schedule for Note-taking (OSN), Interview Schedule for Teachers (IST), Structured Questionnaire for Students (SQS) and Structured Questionnaire for Teachers (SQT) were administered; and data collected were analysed using percentages and t-test statistic. Results showed that there was statistically significant difference in the comprehension of students in the Control and Experimental groups. Furthermore, the achievement of the experimental group was found to be significantly better than that of the control group as reflected on the post-tests on RDT and TMT. Teachers in the experimental group reported that PBL approach exposed students to many findings and made them well prepared for the class; revealed students character in terms of being introverts and extroverts; and helped teachers to identify and help such students. They also noted that some students who were not seen to be brilliant as the others came up to argue logically even as much as those perceived to be brilliant. They however noted that PBL approach was time consuming. Based on the findings, the study recommended that PBL should not only be added to learner-centered, minds-on strategies for teaching Further Mathematics at the secondary schools but more importantly should be integrated into the curriculum of Pre-service teachers in the tertiary institutions.

**Key words:** Problem-based learning, Further Mathematics, Additional Mathematics, Subject Content Knowledge, Pedagogical Content Knowledge

### Background to the study

Traditional method of teaching, especially in mathematics, has been found to be very defective and full of many inadequacies that do not allow students to actively construct their own knowledge of mathematics (Dubinsky in Tall 1991; Mji, 2003). The pronounced ineffective teaching and learning of mathematics from research findings was because of the traditional method of teaching being adopted (Van de Walle, 2007). Much of the failure in school mathematics is due to a tradition of teaching that is inappropriate to the way most students learn (National Research Council (NRC) 1989:6). It has adversely affected effective learning at the different levels of education.

Clarke (1997) remarked that the call for reform draws its impetus from two main areas: (i) the changing needs of citizens for effective participation in an increasingly technological and global society, and (ii) increased research knowledge about the teaching and learning of mathematics. The perspective on what is meant by “understanding mathematics” derives its meaning from the view that learning is primarily a process of concept construction and active interpretation as opposed to the absorption and accumulation of received items of information - it is referred to as “constructivism” (Schifter & Fosnot, 1993). The recognition

of the need for reform in mathematics curriculum and instruction is broad and deep, ranging from professional organizations to government agencies (Douglas, 1986). The call for reform is remarkably consistent across the grade levels K-College. It has also stemmed from several sources, including the underachievement in mathematics by USA students (NRC, 1989; Feiler, Heritage & Gallimore, 2000) and changing perspectives on the nature and learning of mathematics.

Currently, the dominant method of teaching mathematics involves the rote learning of algorithms for solving a limited range of exercises. The textbooks that nurture this method are repetitive and uninspiring in their content and the students who are its victims are generally unable to transfer their skills from the textbook exercises to problems of the real world. Enormous reactions emanated from the above approach of teaching mathematics and the reaction focuses attention on its major weaknesses, urging the development of relevance, application, modeling and problem solving. The focus of PBL investigation has not only been learner-centered but also problem-centered. Students in a problem-centered classroom frequently participate in either teacher-led whole-group activities or small-group work. The Professional Standards for Teaching Mathematics assert that teachers must shift from a teacher-centered to a child-centered approach in their instruction (Van de Walle, 2007:5).

The path towards the shift and reform is the adoption of modern methods of teaching whose focus is on students sharpening their problem-solving abilities, as well as their abilities to reason, communicate, connect ideas, and shift among representations of mathematical concepts and ideas (Dossey et al, 2002). Adler (1997) described participatory-inquiry approach as one of the alternative modern methods to the traditional method of teaching. Participatory-inquiry is a structured learner-centred strategy in which multiple perspectives are sought through a process of group inquiry within the context of helping learners organise their thinking in solving problems. Clarke (2004) described another modern method of teaching and called it Kikan-Shido, meaning “walking between desks instruction” in Japanese language. Kikan-Shido is a classroom strategy that organizes mathematics instructions around problem solving activities and affords students more opportunities to think critically, present their creative ideas and communicate with peers mathematically (Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier, & Weane, 1996). PBL possesses some of the features in the participatory-inquiry and walking between desks instruction approaches as discussed by both Adler and Clarke and in addition has the learning trajectory that made it unique among other modern methods (Kyeong Ha, 2003). Duch, Gror & Deborah (2001) defined PBL as a powerful classroom process, which uses real word problems to motivate students to identify and apply research concepts and information work collaboratively and communicate effectively. PBL is also a classroom strategy that organizes mathematics instructions around problem solving activities and affords students more opportunities to think critically, present their creative ideas and communicate with peers mathematically (Krulic & Rudnick; 1999; Lewellen & Mikusa, 1999; Hiebert et al; 1996, 1997, Kyeong Ha; 2003). Major (2001) defined PBL as an educational approach in which complex problems serve as the context and the stimulus for learning. The common denominator to the varieties of PBL definitions is that students actively construct their own knowledge of mathematics.

The ineffective teaching and learning due to the traditional method of teaching that has dominated the classroom worldwide has been associated with the dismal performances of

students in mathematics (Pandor in Van de Walt 2007:223, Kifer in Dossey, McCrone, Giordano & Weir 2002:37). The erroneous believe is that everybody can teach. Teachers who attended teachers' preparation institutions can only understand teaching methods.

In-service teachers that do not have the opportunity of being specially trained with the fundamentals of modern teaching strategies such as PBL and have the belief and mindset in them cannot operate effectively in a modern Further Mathematics classroom. The question is: Which one comes first: how to teach or what to teach? This is a predicament, because it is a question about teacher's knowledge.

Shulman (1987:8) formulated seven types of teacher's knowledge of which 'subject content knowledge' is one. Recent researches like Brown and Borko (1992), Hallam and Ireson (2005) have established that the others, like general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics are some of the attributes expected of a teacher in an ideal PBL classroom. Others include knowledge of educational contexts, knowledge of educational ends, purposes, values and their philosophical and historical grounds. However, Shulman (1986:9) had distinguished among three categories of content knowledge, namely subject matter knowledge, pedagogical content knowledge, and curricular knowledge, and concluded that the three are inseparable for effective teaching and learning. Other studies like Wilson in (Szetela & Nicol 1992:43) confirmed the impact of teachers' content knowledge on student learning.

Research on educational effectiveness often investigates the importance of what is going on in the classroom with respect to cognitive and non-cognitive outcomes. Factors such as the quality of teaching, time on task, opportunity to learn (content covered), effective learning time, classroom management, classroom climate, and relationships within the classroom have not only often been included as promising explanatory variables in models about learning and educational effectiveness, but their relevance has also regularly been much in educational effectiveness research (Opdenaker 2006:1). This is in agreement with NCTM principles and standards on reform and shift. Educational researchers like Sungur and Tekkaya (2006:311), Hallam and Ireson (2005:3) seem to agree with the idea that, among other factors, the teacher's teaching style has some impact on student learning and the perceptions students develop about science learning and the work of scientists.

Efforts have been concentrated on students' performances in mathematics for some years; there is however little or no research carried out on the effectiveness of PBL in Further Mathematics (a bridge between mathematics offered at senior secondary schools and mathematics courses at the first year undergraduate level) in Nigeria. This study is against the backdrop of increased higher annual percentage of students that fail mathematics and Further Mathematics in Nigeria at the West African Senior School Certificate Examination (WASSCE). In addition, quite a large number of prospective undergraduates could not gain admission to tertiary institutions due to poor marks scored in mathematics at the Unified Tertiary Matriculation Examination (UTME). The questions set in mathematics paper at the UTME are drawn from mathematics and Further Mathematics syllabi. Topics such as calculus, coordinate geometry, matrices and determinants, permutation and combination, sum and products of quadratic equations, partial fractions, to mention a few, are only found in Further Mathematics syllabus. Prospective science and science-oriented undergraduates

that have never attended Further Mathematics lessons at the secondary school level have low probability of scoring high mark (or even pass mark) in the UTME mathematics.

Based on the above, this study sought to investigate the effectiveness of PBL in Further Mathematics classroom in Nigeria. The results of this study would provide the practical empirical basis for evaluating the effects of PBL on students' achievements in and beliefs about Further Mathematics at the senior secondary school level in Nigeria.

### **Research questions**

- (i) Will there be any significant difference between the pre-test (TMT) achievement scores of students exposed to the PBL and those exposed to the TM?
- (ii) Will there be any significant difference between the post-test (TMT) achievement scores of students exposed to the PBL and those exposed to the TM?
- (iii) Will there be any significant difference between the pre-test (RDT) achievement scores of students exposed to the PBL and those exposed to the TM?
- (iv) Will there be any significant difference between the post-test (RDT) achievement scores of students exposed to the PBL and those exposed to the TM?
- (v) Will there be any significant difference between the pre-SQS (beliefs about FM) scores of students exposed to the PBL and those exposed to the TM?
- (vi) Will there be any significant difference between the post-SQS (beliefs about FM) scores of students exposed to the PBL and those exposed to the TM?

### **Method**

#### ***Participants***

The researchers, for the purpose of this study, adopted the quasi-experimental design using pre-test post-test non-equivalent control group. The researcher found out at the Zonal Office of the State Ministry of Education, that there were thirty senior secondary schools in the division selected for the study. Of this only eight (27%) were offering Further Mathematics. All the SS1 science students offering Further Mathematics at the thirty senior secondary schools in this division of the state constituted the accessible population. The researcher selected three schools (38%), out of the eight, using purposive sampling technique based on the following criteria: high enrolment of students in Further Mathematics at the SS1 class, availability of mathematics graduate teachers teaching the subject, and schools that have students offering the subject at SS2 and SS3. One of the three schools was randomly assigned as the experimental group while the other two were used as control group.

#### ***Instrumentation***

The instruments for the study were: A Researcher-Designed Test (RDT) based on problem-based learning approach involving mathematical tasks and a Teacher-Made Test (TMT) (used as pre- and post-tests). Other instruments include an Observation Schedule for Note taking (OSN) at the control schools, an Interview Schedule for Teachers (IST) at both control and experimental schools and two structured questionnaires (one for the students (SQS) and the other for the teachers (SQT)). To construct the RDT, the researcher originally drew up 10 questions based on problem-based learning approach. The questions were drawn

from topics like Indices, Logarithms, Exponential, Quadratic equations, Sequences and Series. The ten questions were in word problems that required students' thorough comprehension. Two Mathematics educators in the tertiary institution subjected the questions to face validity. Based on experts' advice, the ten questions were finally reduced to four questions. The face validated four questions were administered to 40 students in the pilot study school. The responses of the students were used for item analysis. Both discriminating power and difficulty index were calculated. Each of the four questions showed discrimination power of more than 0.40 and difficulty index of 0.40 – 0.60. A Cronbach alpha formula was used to determine the internal consistency and reliability measure of the test and a reliability coefficient of 0.87 was obtained. The four questions then constituted the RDT. Each item on the RDT instrument attracted a score of 25marks. This gave a total of 100marks. Hence, a maximum score that could be obtained was 100marks. This then constituted the RDT.

The TMT is an essay test of 10 questions based on topics like Indices, Logarithms, Exponential, Quadratic equations, Sequences and Series, which constitute the course contents for the study. Test contents were organized in accordance to Bloom's Taxonomy (Okpala, Onocha & Oyedeji, 1993) of cognitive domain. Three Further Mathematics teachers at the three schools selected for the study were asked to prepare 20 essay questions each based on the course content for the study. The set of questions were then given to mathematics graduate teachers in other schools different from the sampled schools for their critique. Based on their advice seventeen of the questions featured in the selection of one or more of the graduate teachers were taken. These were then given to two Mathematics educators in the tertiary institution. Their recommendations led to further pruning of the questions to ten. The face validated ten questions were administered to 40 students in the pilot study school. The responses of the students were used for item analysis. Both discriminating power and difficulty index were calculated. Each of the ten questions showed discrimination power of more than 0.40 and difficulty index of 0.40 – 0.60. A Cronbach alpha formula was used to determine the internal consistency and reliability measure of the test and a reliability coefficient of 0.88 was obtained. The ten questions then constituted the TMT. Each item on the TMT instrument attracted a score of 10marks. This gave a total of 100marks. Hence, a maximum score that could be obtained was 100marks. Items on the TMT covered five out of the six cognitive levels namely knowledge, comprehension, application, analysis, and evaluation.

The Teacher-Made Test (TMT) was administered to the students at both the control and experimental schools as Pre and Post-tests. The Pre-test was conducted at the beginning of the study while the Post-test was at the end of the intervention period. Researcher-Designed Test (RDT) based on PBL approach involving mathematical tasks was also administered to the students at the control and experimental schools. Observation Schedule for Note-taking (OSN) prepared by the researcher was used to record comments while observing teachers at the control schools. Two structured questionnaires (one for the teachers - SQT and the other for the students - SQS) were administered to the participants at the control and experimental schools. The post SQT was administered to the control schoolteachers at the end of the intervention while it was deferred to the end of second term for the teachers at the experimental class. The participating teachers were asked to express their opinion in the SQT questionnaire at the end of second term having taught for

another twelve weeks using PBL approach. An Interview Schedule for Teachers (IST) was conducted for teachers at both the control and experimental schools.

## Results

Table 1 shows the mean and standard deviation of the pretest performance on the RDT for the experimental ( $M=1.20$ ,  $S.D=0.66$ ) and control ( $M=1.18$ ,  $S.D=1.07$ ) groups. The table also shows the mean and standard deviation of the pretest performance on the TMT for the experimental ( $M=32.64$ ,  $S.D=8.13$ ) and control ( $M=31.43$ ,  $S.D=8.60$ ) groups. The same table includes the mean and standard deviation of the pretest performance on the SQS for the experimental ( $M=71.57$ ,  $S.D=12.69$ ) and the control ( $M=78.22$ ,  $S.D=9.89$ ) groups. The difference in means was found to be statistically significant ( $t=3.03$ ,  $p<0.05$ ). The result shows that the mean of the pretest performance on the SQS for the control group was greater than that of the experimental group. If the reverse were the case the experimental group would have been said to be favoured at the beginning of the study. In addition, the effect size of 0.49 was small.

**Table 1** Means, standard deviations, t-test values and effect sizes for Experimental and Control groups on RDT, TMT and SQS

		Experimental			Control			T	D
		N	M	SD	N	M	SD		
Pretest	RDT	42	1.20	0.66	102	1.18	1.07	.15	
	TMT	42	32.64	8.13	102	31.43	8.60	.78	
	SQS	42	71.57	12.69	102	78.22	9.89	3.03*	0.27
Posttest	RDT	42	1.94	0.16	102	1.67	0.90	2.90*	0.20
	TMT	42	39.83	7.67	102	33.26	9.04	4.13*	0.36
	SQS	42	96.38	10.02	102	83.38	12.79	5.88*	0.49

\*significant at .05 level

Means, standard deviations and t-test values for posttest performance on the RDT for the experimental and control groups are presented in Table 1. For the experimental group, ( $M=1.94$ ,  $S.D=0.16$ ) while for the control group, ( $M=1.67$ ,  $S.D=0.90$ ). The difference in means was found to be statistically significant ( $t=2.90$ ;  $p<0.05$ ) and the effect size of 0.2 was small. Similarly, the means, standard deviations and t-test values for posttest performance on the TMT for the experimental and control groups are presented on the same table. For the experimental group, ( $M=39.83$ ,  $S.D=7.67$ ) while for the control group, ( $M=33.26$ ,  $S.D=9.04$ ). The difference in means was found to be statistically significant ( $t=4.13$ ,  $p<0.05$ ), though the effect size (.36) was small. Means, standard deviations and t-test values for post-test performance on the SQS for the experimental and control groups are similarly presented in Table 1. For the experimental group, ( $M=96.38$ ,  $S.D=10.02$ ) while for the control group, ( $M=83.38$ ,  $S.D=12.79$ ). The difference in means was found to be statistically significant ( $t=5.88$ ,  $p<0.05$ ) and the effect size of 0.49 was small. The significant mean differences obtained might be due to the intervention in the experimental class. The PBL approach adopted in the experimental class probably accounted for the improved performance of the experimental group.

Table 2 shows the means and standard deviations of the pretest ( $M=1.59$ ,  $S.D=1.20$ ) and the posttest ( $M=1.63$ ,  $S.D=0.63$ ) performances on RDT for the control group. For the pretest

( $M=31.43$ ,  $S.D=8.60$ ) and the posttest ( $M=33.26$ ,  $S.D=9.04$ ) performances on TMT for the control group. Also, for the pretest ( $M=78.93$ ,  $S.D=10.30$ ) and the posttest ( $M=77.15$ ,  $S.D=13.59$ ) performances on SQS for the control group. The same table 2 also shows the means and standard deviations of the pretest ( $M=1.21$ ,  $S.D=0.66$ ) and the posttest ( $M=2.42$ ,  $S.D=0.57$ ) performances on RDT for the experimental group. The difference in means was found to be statistically significant ( $t=8.89$ ,  $p<0.05$ ).

**Table 2** Means, standard deviations and t-test values for Pretest and Posttest on RDT, TMT, and SQS for experimental, control

		Pretest			Posttest			T
		N	M	SD	N	M	SD	
Control	RDT	102	1.59	1.20	102	1.63	0.63	0.29
	TMT	102	31.43	8.60	102	33.26	9.04	1.48
	SQS	102	78.93	10.30	102	77.15	13.59	1.06
Experimental	RDT	42	1.21	0.66	42	2.42	0.57	8.89*
	TMT	42	32.64	8.13	42	39.83	7.67	4.17*
	SQS	42	71.57	12.69	42	96.38	10.02	9.94*

\*significant at .05 level

For the pretest performance of the experimental group on TMT ( $M=32.64$ ,  $S.D=8.13$ ) and the posttest ( $M=39.83$ ,  $S.D=7.67$ ) and the difference in means was found to be statistically significant ( $t=4.17$ ,  $p<0.05$ ). Similarly, for the pretest performance of the experimental group on SQS ( $M=71.57$ ,  $S.D=12.69$ ) and the posttest ( $M=96.38$ ,  $S.D=10.08$ ) and the difference in means was found to be statistically significant ( $t=9.94$ ,  $p<0.05$ ).

There was no statistically significant difference for the pre- and posttests for the control group with respect to RDT, TMT and SQS. On the other hand, statistically significant differences in the pretests and posttests performances of the experimental group on RDT, TMT, and SQS were obtained. These differences in means might be a consequence upon the intervention in the experimental class. Using Tables 1 and 2 it can be observed that experimental students significantly performed better than control groups in RDT, TMT and SQS. The achievements of students taught with PBL approach could be said to be better than those taught with traditional expository method. The PBL approach used in the experimental class probably affected students' performances of the experimental group.

The participating teachers at the experimental class called the attention of the researcher in the discourse that ensued during the lessons to some students who were not seen to be brilliant as others came out to argue logically better than the perceived brilliant ones. PBL approach probably assisted the low able students to bring out the potentials in them. In the course of teaching and interaction, some students who were introverts were identified and assisted and were encouraged to put shyness behind them.

The participating teachers at the experimental school remarked that the PBL method really stimulated students' level and ways of thinking. The method allowed students to make decisions on their own. It helps students to develop their ability to frame and ask questions. PBL method makes students to be bold and convinced when a solution is appropriate or not. It agitates the minds of the students via their experience to be able to defend their

discoveries; hence, the method stimulates their reasoning capability. PBL approach makes learning permanent in the students because of their personal discoveries. The method encourages discussion between and among the students. It promotes interpersonal relationships among the students. It reveals the students' character in terms of being either introvert or extrovert and avails teachers the opportunity to identify and help such students. Within the group, students learn and gain various ideas. It encouraged teamwork among students. Criticism is allowed which makes students to understand better. The timid students have confidence in themselves. Students through PBL approach were made to defend their findings. The students through their task presentation made a lot of work and discoveries. The mission to get the students worked in-group was accomplished.

On the other hand, they reported that the method was time consuming for the few periods allocated to the subject on the timetable; and that the approach made them cover fewer topics in the scheme of work for the term.

### **Conclusion and Recommendations**

The adoption of PBL approach entails more than acquiring basic skills or bits of received knowledge as was obtained in other modern methods of teaching. The use of PBL also involves developing identity and affiliation, critical epistemic stance, and dispositions as learners participate in the discourse and actions of a collective social field. The use of PBL could increase reasonably the enrolment of students for Further Mathematics at all the external examinations. Based on the findings, the study recommended that PBL should not only be added to learner-centered, minds-on strategies for teaching Further Mathematics at the secondary schools but more importantly should be integrated into the curriculum of Pre-service teachers in the tertiary institutions.

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