Teaching of Difficult Concepts in Tertiary Mathematics

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Abstract

The woeful performance of students in external examinations at the Senior Secondary Schools in Mathematics and at the first year undergraduate mathematics courses that has become a recurring decimal of late in most of our schools is so enormous that no concerned mind can continue to fold arms and watch the system decay by the day. Most teachers find it difficult to match content with practice. This paper opined that what to teach and how to teach should be paramount in the mind of an effective teacher. The researchers puts forward the view that preparation of a good lesson/lecture notes with conscious learner-centred delivery would be a good panacea for the teaching of difficult concepts in Tertiary Mathematics

Introduction

The method of teaching and quality of delivery of teachers came first among the plethora of problems that was blighting the education sector in Nigeria (Guardian Newspapers, 2002). In Nigeria, 23.5% of the total number of candidates that sat for the examination obtained a credit pass in Mathematics and English Language at the West African (WASSCE) Senior School Certificate Examination in 2008 while 25.99% obtained it in 2009. In May/June 2011, 540,250 representing 38.93% out of 1, 587,630 candidates that sat for the examination obtained credits and above in Mathematics (The Guardian Nigeria Newspaper, 2011). It has been very difficult to match content with practice. The contentious issue of where to start a lecture, that is, starting lectures from a high cognitive to low cognitive level and vice-versa needs to be properly addressed. The authors are however in agreement with the submission that students achieved better learning, when a teacher starts a lesson from a low cognitive to a high cognitive level (Spierring & Ashby, 2008). Tertiary Mathematics education research is disciplined inquiry into the learning and teaching of Mathematics at the University level. Research could be based on how students come to understand aspects of Mathematics or how they develop effective mathematical practices, good problem-solving skills, or the ability to generate reasonable conjectures and to produce proofs (Sanni, 2007).

Mji (2003) remarked that lecturers and students expectations are at variance with each other in that, lecturers complain of students' unwillingness to ask questions and therefore deliver lectures that provide answers and leave little room for questions. Students on the other hand dedicate themselves to note taking when questioning was perceived as discouraged, leaving them with little enthusiasm for admitting uncertainty. There is urgent need for teachers to acquire new skills that will enhance their performance in class management, subject content delivery and overall supervision of their students (Punch Newspapers, 2011). As a further step towards improving teaching and learning of mathematics in tertiary Institutions, it is suggested that mathematics lecturers should acquire pedagogical skills necessary for teaching mathematics. This is indeed the intention of government as enunciated in the National Policy on Education that "all teachers in the

tertiary institutions shall be encouraged to undergo training in the methods and techniques of teaching" (Nigerian Educational Research and Development Council, 2004, p. 31). Encouraging lecturers in tertiary institutions to acquire teaching skills is a noticeable trend internationally (Arigbabu, 2004). In certain cases, this has taken introducing or incorporating elements of teacher education into the training of doctoral students whose intention is to be lecturers. For instance, in the UK a number of universities have organized series of short courses for doctoral graduates, new to university teaching. Citing the example of a university, which introduced a two-year part time education course that integrated 'learning about teaching' with 'learning about research', Harland (2001) has opined that this mode of addressing the problem by 'crash programmes' was probably not ideal as "… the kind of support necessary for the education of future academics would need to go far beyond the provision of short courses" (p. 270). What is apparent here is that efforts are currently geared towards the improvement of the quality of teaching and teacher education at all levels the world over. The acquisition of the teaching skills by lecturers in the tertiary institutions will develop their knowledge and consequently their global competitiveness in the sector. The World Bank Group asserted that better learning for all students worldwide is vital, because economic growth and better development depend on the knowledge and skills that students gain and not the years they spend in schools. The behaviour of lecturers and students as observed by Mji (2003) invariably result in undesirable teaching/learning outcomes. This paper was intended to demonstrate how perceived difficult topics in tertiary mathematics especially first year undergraduate mathematics courses could be taught.

The chief examiners' report (2007) of West African Examination Council (WAEC) observed from marks and attendance sheet that some questions at the West African Senior School Certificate Examination (WASSCE) are no go areas for students, an indication that the topics from which the question were set ,were either not well taught or sparingly taught by teachers. The prevailing situation calls for the need to take the global pedagogical debate on whether the subject content knowledge should be placed above the pedagogical content knowledge or vice-versa more seriously at the teacher preparation institutions. Shulman (1986) has however argued and supported by some other educators that for effective teaching of any subject at any level, all the three aspects of content knowledge- subject, pedagogical and curricula must be possessed by an effective teacher. Analysis of the marks and attendance sheets as observed and remarked by the chief examiners' report revealed that students have difficulties in topics in further-mathematics such as coordinate-geometry, partial-fractions, inequalities, vectors, dynamics, probability and statistics. There is need for teacher preparation institutions to review the curricula of the pre-service teachers and make it amenable to what the students will meet in the classroom after their graduation. Undergraduates' (both Honours and Education) specialising in mathematics should be made to offer all the required mathematics courses throughout their stay in the university. This will make them to be content- rich in mathematics.

Students' difficulties in mathematics at the tertiary level start from the first year. There is a wide gap between secondary school mathematics and the first year undergraduate mathematics courses. Majority of high school graduate students are meeting Further Mathematics topics for the first time at the tertiary level. Further Mathematics is the bridge between secondary mathematics and first year undergraduate mathematics courses. It is therefore strongly advocated that lecturers handling first year undergraduate mathematics

courses should be experienced teachers that could effectively match subject content knowledge with pedagogical content knowledge. They should do more of in-depth teaching than the lecture method approach.

Practical Demonstration
Find the roots of the quadratic equation $5x^2 + 8x + 3 = 0$

Step 1: Adoption of Problem-solving process
Stage I: Identify the problem – This requires (a) identifying the coefficients of x^2, x and constant (b) considering the nature of the curve (discriminants)
Stage II: Make assumptions – This requires ability to guess through 'trial and error' values of x that will satisfy the given equation i.e. values of x that will make left hand side of the equation to become zero
Stage III: Formulate a model – This requires ability to identify a valid approach among many alternatives. In solving the problem, three options are feasible namely factorisation, completing the square and the almighty formula
Stage IV: Use the model → $5x^2 + 8x + 3 = 0$, a = 5; b = 8 and c = 3
The discriminant $b^2 - 4ac = 3$ hence $b^2 > 4ac$ (the equation is solvable with 2 distinct real roots). Factorising the left hand side of the equation gives $5x^2 + 5x + 3x + 3 = 0$, 5x(x + 1) + 3(x + 1) = 0, (5x + 3)(x + 1) = 0, Equating each of the two factors to 0 gives x = -1 or -3/5. Some students might substitute the coefficients of a, b, c directly into the almighty formula to obtain the values of x. Note that almighty formula is obtainable through completing the square process
Stage V: Evaluate the model – substituting x = -1 or -3/5 into the given equation makes the left and right-hand side of the equation to be equal. Any other values apart from these two values will obviously not satisfy the equation. Hence, the whole process will be repeated.

Step 2: The teacher gives class work to each group while encouraging them to follow the heuristics of the model in their discussion.

Step 3: The teacher appoints any member of the group to present their findings to the entire class while acting as facilitator in a scaffolding manner and other groups engage the presenter in dialogue to arrive at a consensus.

Step 4: The teacher gives an overview of the whole lesson leaning on the model. Thereafter the teacher gives assignment to the class.
The under- listed procedure is suggested for a teacher introducing the concept of quadratic equation to the students for the first time at a learner-centred classroom:
   (i)   The teacher may ask the students to distinguish between linear and quadratic form of equation [The linear form, y = mx + c is expected to have been taught at the Junior secondary level]
   (ii)   The teacher may ask the students to give geometrical interpretations to the two forms of the equation (meaning of y = mx; y = mx + c; y = k and x = p
   (iii)   The teacher may ask the students in turns to identify the coefficients of x , x^2 and constant in a given quadratic equation of the form $ax^2 + bx + c$
   (iv)   The teacher may ask the students to explain the term 'discriminant'

(v)   The teacher may ask the students to write down the mathematical expression for discriminant in their notebooks (identify b^2 - 4ac as discriminant usually written as $D = b^2 - 4ac$)

(vi)   The teacher should let the students know that the discriminant could be used to describe the nature of curves. Its applications in Physical Sciences should also be stressed especially in diffusion and waves equations.

(vii)   The teacher may introduce the concept of equality and inequality to the students (two possibilities exists for inequality; greater than '>' or less than <'

(viii) The teacher may give the interpretation of each of the nature of the curves to the students as:

D > 0, existence of two different real roots

D < 0, existence of two complex roots

D = 0, existence of equal, repeated or coincident roots

Lesson Plan Preparation

Teaching involves decision-making. Decisions are made as you plan lessons. Considering what happened today, what will move the students forward? What is the best task to propose tomorrow? Decisions are made minute to minute in the classroom.

Some questions that may occupy the teacher's mind during lesson preparation and lesson presentation may include, how should I respond? Should they struggle some more, or should I intervene? Is progress being made? A teacher who keeps these ideas in mind can be said to be basing his or her instruction on a constructivist view of learning or a developmental approach (Van de Walle, 2007).

Nigeria like U.S. teachers typically spends a small portion of a lesson explaining or reviewing an idea and then go into "production mode," where students wade through a set of exercises. Lessons that are organized in this explain-then-practice pattern condition students to focus on procedures so that they can finish the exercises. Teachers find themselves going from task to desk re-teaching and explaining to individuals. This act is noted to be of significant contrast to a lesson built around a single problem, which is the typical approach for student-centred problem-based lessons.

It is noteworthy to think of a lesson consisting of three simple parts; before, during and after. For most lessons, these three lesson parts are built around a single problem or task for the students. An effective mathematics teacher should take full cognizance of the three components in order to prepare for and present a good mathematics lesson.

The before phase of a lesson:

There are three related agenda for the before phase of a lesson:

(1) The teacher should be sure that students understand the problem so that he or she will not need to clarify or explain to individuals later in the lesson.

(2)   Expectations of the teacher to students should be clarified before they begin working on the problem. This includes both how the students will be working (individually or in pairs or small groups) and what product you expect in addition to an answer.

(3)   Teachers should get students mentally prepared to work on the problem and think about the previous knowledge they have that will be most helpful.

The order in the above listed agenda of the before-phase lesson need not be strictly followed. For example, for some lessons, a short activity to activate students' prior knowledge for the problem might be necessary. The problem could be presented and expectations clarified by the teacher. Teacher actions will however vary in the 'Before Phase Lesson'

Example.
Having treated the analysis of Partial Fractions and Free falling bodies (Projectiles) to a Senior Secondary Three class/First year undergraduate mathematics course, a teacher intends to engage the students in problem-solving activities at the next period.   The problems to be solved are:

(i)   Resolve $(4x^2 + 3x - 4)/(6x^3 - 29x^2 + 46x - 24)$ into Partial fractions

(ii)   Express $(r^2 + 1)/(r^2 + r)$ in Partial Fractions

(iii)   Deriving time of flight, total time of flight, vertical distance, range and maximum range

Discuss the before-phase of the lesson: An insight to students difficulty areas in solving the problems should be anticipated by the teacher. The teacher is to prepare well ahead for students queries.

The during phase of a lesson
This is the portion of the lesson when students work alone or with classmate. There are four agenda that are helpful in the during phase of a lesson:

(i)   The teacher should give students a chance to work without any guidance or direction. Teacher should avoid stepping in front of their struggle.

(ii)   The teacher should actively listen to the students. This time is to be used by the teacher to find out how different students are thinking, what ideas they are using, and how they are approaching the problem. This is the time for observation, assessment and not teaching.

(iii) The teacher should cautiously provide appropriate hints that are based only on students' ideas and ways of thinking.

(iv) The teacher should provide profitable activity for students who finish quickly. This is to prevent such students from disturbing the peace of the class. They could often be challenged in some manner connected to the problem just solved.

(v)   The value of students solving a problem in more than one way cannot be overestimated. He shifts the value system in the classroom from answers to processes and thinking that enables students to make new and different connections.

Activity: Practical Demonstration of how the Partial fraction /Projectiles problems are to be presented in the lesson.

The after phase of a lesson
In the after phase of the lesson, certain time must be allocated for students to work as a community of learners, discussing, justifying and challenging various solutions to the problem all have just worked on. Much of the learning will occur in this section as students

reflect individually and collectively on the ideas they have been struggling with. The agenda for the after phase lessons are:

  (i) **Engage the class in productive discussion**, helping students work together as a community of learners.
  (ii) **Listen actively without evaluation**. This is another major opportunity for the teacher to find out how students are thinking and are approaching the problem.
  (iii) **Summarize main ideas** and identify problems for future exploration.

Activity: Reflect over the entire lesson and properly take note of all likely students anticipated queries.

Conclusion

Effective teachers of Mathematics are expected to possess high subject-content knowledge, pedagogical-content knowledge and curricula-knowledge. Good mastery of subject-content with poor content delivery has been identified as one of the factors of students' poor achievement in Mathematics and Further Mathematics. Effective application of learning trajectories like – dialogue, selection of instructional materials, equity and a Three-Part Lesson format if properly and logically applied will enhance preparation for and presentation of a good mathematics lesson. Encouraging dialogue to take place in the class with the teacher as a facilitator increases students' interest, help them develop positive attitude and motivates them to learning mathematics. The teaching of difficult concepts in tertiary mathematics requires among other things the selection and implementation of appropriate tasks that consequently improves learners' achievement and mathematical proficiency.

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