JUMP-DIFFUSION BASED-SIMULATED EXPECTED SHORTFALL (SES) METHOD OF CORRECTING VALUE-AT-RISK (VaR) UNDER-PREDICTION TENDENCIES IN STRESSED ECONOMIC CLIMATE

by

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submitted in accordance with the requirements for the degree of

Master of Science

in the subject

Statistics

at the

University of South Africa

Supervisor: Prof. J.O. Olaomi

May 2014

Declaration

I declare that JUMP-DIFFUSION BASED-SIMULATED EXPECTED SHORTFALL (SES) METHOD OF CORRECTING VALUE-AT-RISK (VaR) UNDER-PREDICTION TENDENCIES IN STRESSED ECONOMIC CLIMATE is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

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SIGNATURE   DATE

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Abstract

Value-at-Risk (VaR) model fails to predict financial risk accurately especially during financial crises. This is mainly due to the model's inability to calibrate new market information and the fact that the risk measure is characterised by poor tail risk quantification. An alternative approach which comprises of the Expected Shortfall measure and the Lognormal Jump-Diffusion (LJD) model has been developed to address the aforementioned shortcomings of VaR. This model is called the Simulated-Expected-Shortfall (SES) model. The Maximum Likelihood Estimation (MLE) approach is used in determining the parameters of the LJD model since it's more reliable and authenticable when compared to other non-conventional parameters estimation approaches mentioned in other literature studies. These parameters are then plugged into the LJD model, which is simulated multiple times in generating the new loss dataset used in the developed model. This SES model is statistically conservative when compared to peers which means it's more reliable in predicting financial risk especially during a financial crisis.

Keywords: Historical-Simulation VaR model, Jump-Diffusion models, ES model, Coherence, Fat-tailed distribution, HES model and SES model

Acknowledgments

The research element of this qualification was challenging and required versatile skills in executing the research effectively. Based on this I would like to thank the Group Market Risk Monitoring (GMRM) team in Nedbank, especially Rajesh Rugunanan and my supervisor for assisting me when necessary. And most of all I would like to thank my daughter (Busisiwe) and wife (Nancy) for their constant encouragement and support.

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Chapter 1

INTRODUCTION

1.1 Background

According to Escanciano and Pei (2012), Value-at-Risk (VaR) models are standard tools used in measuring market risk/financial risk used by financial institutions and their regulators. van Greuning and Bratanovic (2009) defines market risk as losses due to unfair movements in market prices resulting from changes in the prices (volatility) of fixed-income instruments, equity instruments, commodities, currencies, and related off-balance-sheet instruments. These unfair movements are induced by financial crises encountered in the financial markets from time-to-time. Financial crises escalates losses encountered in normal day-to-day trading operations to peaky-proportions due to random jumps of various independent variables used to model the valuations of instruments modelled by the VaR measure. The main consequences of financial crises on the VaR model is that, due to the random jumps and deviation from the historical data's trend used in model, VaR measure tend to under-predict market risk. Model risk of VaR model translates in most cases for financial institutions using it as a risk measure into financial losses which is due to mismatches between the reserve (i.e. capital calculated by VaR) and actual losses encountered by the financial institutions.

This dissertation focuses on the consequences on the VaR model based on the 2008 subprime mortgage crisis and possible model interventions in reducing the model risk noted. The 2008 subprime mortgage crisis was caused by a saturation of subprime mortgages in the United State (US) housing sector prior to 2008. Subprime mortgage is a bond or home-loans issued to individuals who are not financially fit. From 2005 onwards, default rate began to spike as owners of these subprime mortgages were unable to make their monthly payments. Since these subprime mortgages were used as underlying assets for derivative known as home-loan bond, its demise spread systemically throughout the economy. Also owners of these "so-called" home-loan derivative with high returns bought Credit Default

Swaps (CDS)[1] to protect themselves from default risk of the subprime mortgagers. The involvement of the CDS and other exotic instruments in the subprime mortgage sage amplified its influences in the global economy (refer to appendix A from detailed description of the aforementioned crisis studied). However, during this crisis VaR models employed by various financial institutions under-predicted market risk "severely" causing some of the players to lose billions of dollars.

In order to respond to the challenges described above, there has been a growing development in attempts to develop tail sensitive and random jumps incorporative measures in market risk modelling literature. This is an emerging stochastic area in the field of quantitative risk modelling broadly referred to as Expected Shortfall (ES) or Conditional Value-at-Risk (CVaR). Artzner et al. (1997), Duffie and Pan (1997) and McNeil et al. (2005) investigated the ES/CVaR measures' mathematical properties in depth and as well justified reasons as why these approaches are candidates to replace VaR, especially after the subprime crisis.

1.2 Problem statement

VaR model under the historical simulation approach discussed in section 1.1 is unable to capture losses beyond the α percentile during financial crises because of extreme volatility in the markets causing unstableness. Boucher et al. (2014) submitted that the under-prediction observed in VaR during the subprime crisis and other crises before then are mainly attributed to the fact that, risk models like the VaR have high model risk levels during turbulent times. These high levels of model risk during financial crises because risk models to under forecast risk prior to the crisis events, to be slow to react as a crisis unfolds, and then slow to reduce risk levels post crisis.

Boucher et al. (2014) state that standard risk measures failed to forecast extreme risks and regulators required financial institutions to quantify the model risk issues seen during financial crisis. These under-prediction realisations of the risk models are causing regulators globally to question the efficiency of these models in predicting estimates of risk used for capital calculations. Therefore this dissertation studies ways of improving predictability of the risk model used to estimate market risk by improving the measure's ability in detecting irregularities in price movements induced by financial crisis.

1.3 Significance of the study

The current trending issue among the financial institutions (mainly in Banks) regarding risk management techniques since the subprime financial crisis in 2008 is about employing

[1] CDS is an insurance policy which protects its owners from defaulting risk.

Extreme Value Theory (EVT) techniques. The focal point of this dissertation is using EVT-based methodologies in solving the research problem presented in section 1.2. Findings derived from the dissertation might aid on whether EVT-based methodologies in measuring banking risks such as market risk are both feasible and practical when considering regulators' view when coming to adequately measuring the risks facing financial institutions especially during severe market turbulences induced by financial crisis.

1.4 Research objectives and hypotheses

1.4.1 Research objectives

The following are the main objectives that will be fulfilled by this research:

- Develop a risk measure where the dynamics of the risk factors follows a jump-diffusion process.
- Develop a risk measure more robust than other similar models studied in the literature to date, namely the historical-simulation VaR model and Expected Shortfall (ES) models under non-parametric approach.

1.4.2 Hypotheses of the research

Apart from other analysis conducted in this dissertation to test the validity of the Simulated Expected Shortfall (SES) model; the appropriateness of the jump-diffusion model will also be investigated.

1.5 Delimitations

The following are the elements not discussed or included in this research:

- Focus on market risk only.
- Reports and additional governance structures around the VaR modelling.
- The VaR modelling technique analysed only extends to financial institutions only.
- This study excludes all information and influences of financial crises before 2004.

1.6 Definitions of crucial terms in the dissertation

This section explains some of the crucial terms used throughout the dissertation.

Definition 1 *Backtesting – McNeil et al. (2005) defines a backtesting procedure as a monitoring approach which evaluates the performance of methods and compare their relative performance.*

Definition 2 *Basel Accord Capital (BAC) framework is minimum guidelines to identify and manage risk to an acceptable level.*

Definition 3 *Basel Committee of Banking Supervision (BCBS) is a committee established by the Central-Bank Governors of the Group of Ten (G-10) at end of 1974 (McNeil et al. (2005), p.8).*

Definition 4 *Capital – van Greuning and Bratanovic (2009) defines capital as reserve funds saved by the financial institutions with the Central Reserve Bank to safe-guide depositors money and sustainability of the industry by reducing systemic risk.*

Definition 5 *Coherency - A risk measure satisfying the four axioms of translation invariance, subadditivity, positive homogeneity, and monotonicity is called coherent. (Artzner et al. (1999), p. 210).*

- *Translation invariance: For all* $X \epsilon G$ *and all real numbers* α*, we have* $\rho\left(X + \alpha \times r\right) = \rho(X) - \alpha$.
- *Subadditivity: For all* X_1 *and* $X_2 \,\epsilon\, G$, $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$.
- *Positive homogeneity: For all* $\lambda \geq 0$ *and all* $X \epsilon G$, $\rho(\lambda X) = \lambda \rho(X)$.
- *For all* X *and* $Y \epsilon G$ *with* $X \geq Y$*, we have* $\rho(Y) \leq \rho(X)$.

All of the above axioms are extracted from (Artzner et al. (1999), p. 209 – 210).

Definition 6 *Expected Shortfall (ES) or Conditional Value-at-Risk (CVaR) – For a loss L with* $E(|L|) < \infty$ *and distribution function* F_L*; the ES at confidence level* α $(0, 1)$ *is defined as*

$$ES_\alpha = \left(\frac{1}{1-\alpha}\right) \int_\alpha^1 q_u(F_L)du, \tag{1.1}$$

where $q_u(F_L) = F_{\underline{L}}$ *is the quantile function of* F_L

Definition 7 *Financial crises are market turbulences which cause unruling fluctuations in the financial markets' risk factors which are used in the risk models predicting risks. Financial crisis in general cause losses for many companies since this unexpected losses are usually not calibrated in the quantitative methods used by the financial institutions.*

Definition 8 *Financial institutions are entities using depositor's funds to run their daily operations and generate profit.*

Definition 9 *Jump-diffusion model are defined by Feng and Linetsky (2008) as special class of the diffusion modelling framework by adding jumps to the diffusion dynamics of the state variable. Whereas Hull (2009) defines jump-diffusion as a risk measure that models where asset price has jumps superimposed on to a diffusion process such as Geometric Brownian Motion (GBM).*

Definition 10 *Leptokurtosis – A distribution is leptokurtic if it is more peaked in the center and thicker tailed than the normal distribution with the same mean and variance. Occasionally, leptokurtosis is also identified with a moment-based kurtosis measure larger than three (Haas and Pigorsch (2009), p.308).*

Definition 11 *Market risk – van Greuning and Bratanovic (2009) defines market risk as losses due to unfair movements in market prices resulting from changes in the prices (volatility) of fixed-income instruments, equity instruments, commodities, currencies, and related off-balance-sheet instruments."*

Definition 12 *Model risk is probability that model estimates from the risk models fails to assimilate the phenomenal being modelled accurately and therefore result in inaccurate decision-making.*

Definition 13 *Optimisation is defined by Kreyszig (1999) as an iterative numerical and computer-based process used to determine or solve given problem. Usually issues solved using optimisation techniques cannot traditionally be solved by finite-based problem solving approaches.*

Definition 14 *Regulators are referees in the financial industry ensuring depositors' funds are used appropriately by the Banks and as well relevant reserves are kept to safe guide the depositors' funds.*

Definition 15 *Return – Let S_t be the price of a financial asset at time t. Then the continuous return, r_t, is $r_t = \log\left(\frac{S_t}{S_{t-1}}\right)$. The discrete return R_t, is $R_t = \left(\frac{S_t}{S_{t-1}}\right) - 1$. Both are rather similar if -0.15 $< R_t <$ 0.15, because $r_t = \log(1 + R_t)$ (Haas and Pigorsch (2009), p.308).*

Definition 16 *Risk models - Boucher et al. (2014) cited that risk models are quantitative methodologies used to estimate and forecast risks based on historical data collected of the respective risk being measured.*

Definition 17 *Subprime financial crisis is the market turmoil which originated in the United States (US) subprime mortgage segment and quickly spread into other market segments and countries according to Ackermann (2008).*

Definition 18 *Subprime mortgage are homeloans credit contracts issued to borrowers who have poor credit record or no record.*

Definition 19 *Tail – The (upper) tail, denoted* $\overline{F}(x) = P\,(X>x)$*, characterises the probability that a random variable* X *exceeds a certain "large" threshold* x*. For analytical purposes, "large" is often translated with "as* $x \to \infty$*". For financial returns, a daily change of* 5% *is already infinitely large. A Gaussian model essential excludes such event (Haas and Pigorsch (2009), p.308).*

Definition 20 *Value-at-Risk (VaR) is the maximum expected loss on an investment over a specified horizon at a particular confidence level (Escanciano and Pei (2012), p. 2233).*

1.7 Structure of the dissertation

In addition to this chapter, there are four chapters. Chapter 2 (Literature review) presents overall background information and concepts relevant to the dissertation. It starts by providing a detailed definition with regard to VaR and as well explains reasons behind VaR not being able to measure market risk during financial crisis from literature findings outlined in the various studies. Then this chapter continues to discuss developments conducted over the years in fixing shortcomings about the VaR measure and expand on the significance of the dissertation using the literature studies. Finally it discusses various methods studied in literature to solve the problem outlined in section 1.2.

Chapter 3 (Methodology) discusses the current VaR model implemented in practice, that is, the Historical-simulation Value-at-Risk (HVaR) model. The aim of this discussion is then to provide a comparison between HVaR and the proposed method to highlight model benefits from the new approach. Remaining section in this chapter outlines detailed aspects about the proposed methodology and statistical tests employed to validate the principle of the model on whether or not is sound and robust. Chapter 4 (Data analysis and model results) details how the data gathering was carried out and sections thereafter discusses the empirical findings from tests and modelling processes defined in chapter 3.

Chapter 5 (Conclusion and recommendations) concludes this dissertation. It highlights what has been achieved by this research, assess the limitation that constrains the research problem and suggests future directions for further research.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

Chapter 2 outlines in detail a review of the literature and researches related to the use of quantitative risk models in estimating various risks impacting the financial industry, specifically focusing on market risk/ financial risk. The chapter is divided into sections that include:

- The historical developments of VaR, its appealing properties as a risk measure and shortcomings especially during financial crisis;
- Regulatory view and other banking related problems concerning VaR;
- Current interventions and discussions post 2008 subprime financial crisis;
- Academic response to VaR ineffectiveness in predicting market risk;
- Limitations of current literature on VaR; and
- Possible solutions in literature that would fix gaps in literature studies investigating VaR.

2.2 Originations of VaR and its appealing properties versus its shortcomings in measuring market risk during financial crisis

Holton (2002) stated that, VaR's origins can be traced back as far as 1922 to capital requirements the New York Stock Exchange imposed on member firms. Then the work of Markowitz (1952) and Roy (1952) regarding portfolio selection further expanded the use of VaR in the portfolio theory field. However, during 1922 to 1998 the use of VaR as a risk tool or capital model was somehow capped to fewer users. According to Holton (2002) during the early 1990s banks began to increase their market exposures in the financial markets which meant minimum guidelines stipulated in the 1988 Basel I Accord were no longer sufficient to measure market risk. Regulators at that time viewed quantification approaches as not being unique in the various banks because some banks were ahead in terms of measuring and managing risks when compared to others. Therefore, the regulators with an aim to promote effective risk management processes in the financial sector stated in their various regulations governing banks (i.e. based on Basel II Accord), that if banks can prove their processes are sound and efficient, they can use their own internal models to calibrate risk including market risk. After Basel II Accord was implemented in the financial sector, the use of VaR increased with great proportion prior to this accord. The VaR methodology stipulated in Basel II Accord is based on the RiskMetrics method developed and published online by JP Morgan prior 1998 which state that historical data can be used to calculate daily market risk. Today VaR is a standard risk management tool for market risk and regulatory capital figures. The popularity of VaR is due to the fact that it's easily understandable and according to Kerkhof and Melenberg (2004) VaR model is also popular with the regulators due to its backtesting abilities; refer to section 1.6 for definitions of key words used above.

However, VaR fails on many occasions to predict market risk especially during a financial crisis. Artzner et al. (1997, 1999) and Wirch (1999) cite that, VaR is not coherent. According to definition 5 in section 1.6 a measure is coherent if and only if the axioms of translation invariance, subadditivity, positive homogeneity and monotonicity hold. VaR model is not coherent because it violates the subadditivity axiom which implies the aggregated VaR of a set of portfolio is greater than the sum of the individual portfolios. According to McNeil et al. (2005) VaR model does not take into account diversification effect existing between portfolios, hence the alluded estimation. McNeil et al. (2005) also cited that, VaR subadditivity axiom holds if returns are normal or close to normality but Haas and Pigorsch (2009) state that financial returns are not normal but leptokurtic. Based on arguments provided by scholars such as Danielsson and de Vries (2000) and Haas and Pigorsch (2009), VaR model will continue to violate the subadditivity axiom since returns are non-normal. Then, apart from the coherency issue discussed above, Danielsson and de Vries (2000) state that VaR model is highly dependent on extreme returns which makes this risk model a poor tail estimating approach. Using Danielsson and de Vries (2000)'s argument, it can be concluded that VaR's model risk emanates from its tail risk problems since VaR discard any losses beyond the measuring percentile chosen in the quantile process defined in chapter 3. Finally, since the VaR model used in practice is based on historical simulations; VaR especially during financial crisis fails to capture the random jumps due to market turbulences since it assumes the historical trend will continue in the future. Somehow there is a delayed response of VaR to incorporate spikes or market jumps and this observation is also supported by Boucher et al. (2014)'s study.

Although VaR modelling process is able to summarise market risk exposures into a single understandable figure and the fact that it can be backtested. However the cons against VaR exceed the pros hence the continuous failing trend seen, especially during financial crisis.

2.3 Regulatory perspective on VaR's under-predicting tendencies during financial crisis

Section 2.2 has primarily focused on VaR risk measure as a quantification tool and its flaws. However, VaR modelling issues seen in the recent financial crisis are not purely a mathematical problem but a regulatory issue as well. The minimum guidelines used by VaR are extracted from banking-based regulations using Basel Accord as a point of reference.

The objective of financial regulators or central banks in various countries is to protect the financial system from systemic risks which can emanates from their processes and management of risk in general. Another key objective is to protect the depositor funds used by the financial institutions. These regulators achieve these objectives and other by instilling a culture of sound and efficient risk management practices by regulating the capital requirements needed for risk-taking issues existing in financial institutions. Since the inception of the BCBS in 1974 and the development of the Basel Accord in 1988; financial regulators have used this framework as a base for their own regulations in their respective countries. 1988 Basel I Accord framework suggested to financial institutions with exposures to credit, market and operational risks to take 8% as the pillar I capital reserve and there are other capital charges based on the financial environment of that country and the organisation itself. Then, the 1998 Basel II Accord stipulated that instead of using the 8% capital charge to calculate pillar I; internal model can be used provided the risk management practices and operations are sound and approved by the respective financial regulator. 1998 Basel II Accord was governing the financial institutions (i.e. banks) when the 2008 subprime crisis materialised.

Moosa (2010)'s views on the 1998 Basel II Accord failing the financial system is that financial regulators should have not allowed banks to use their own internal models because this allowance created inconsistencies in the regulatory sphere for financial institutions. Whereas Rossignolo et al. (2013)'s empirical study on Portugal, Ireland, Greece and Spain (PIGS) outlines that even the suggested interventions (inclusion of sVaR) in Basel III in terms of market risk are insufficient to capture tail extremity as seen in 2008 subprime crisis. As well Danielsson and de Vries (2000) cited that tail extremities from one crisis to another are unique and then using the inference, it can be concluded that using tail extremity of one crisis is not sufficient to forecast future tail extremities since market risk cannot be capped. Rossignolo et al. (2013)'s concise on sVaR being included is that, its inclusion does not improve significantly the market risk quantification process to estimate tail risk. Unfortunately during a financial crisis risks accounted for are usually situated at the tail extremity region of the distribution not accounted for by the risk measure such as VaR. Blundell-Wignall and Atkinson (2010) perspective is that, the 1998 Basel II Accord framework failed banks by placing heavy reliance on rating agencies.

The literature reviewed above are in agreement that, the 1998 Basel II Accord should have been more conservative in its allowance of internal models or it should have suggested more tail risk sensitive measure to calibrate market risk and others. Blundell-Wignall and Atkinson (2010)'s study cite that, 2010 Basel III Accord will work provided there are few modifications and the South African Reserve Bank (SARB) through working papers are suggesting that for market risk, Expected Shortfall (ES) is the best replacement for VaR. Scholars such as Kerkhof and Melenberg (2004) argue that central banks would be skeptical to use ES since it's not backtestable. At present, it is definitely unclear about the direction that the regulators will take in terms of the 2010 Basel III Accord to improve the predictability of tail risk, especially during financial crisis for market risk estimation process.

2.4 Current developments relating to VaR

Artzner et al. (1997) noted that Historical simulation Value-at-Risk (HVaR) model had mathematical inefficiencies in its modelling process as discussed in section 2.2. Then Artzner et al. (1999) and McNeil et al. (2005) responded to these inefficiencies by suggesting tail sensitive risk measure known as the Expected Shortfall (ES) model. These developments were made before the 2008 subprime financial crisis but were ignored on bases that VaR risk measure is still operative. The 2008 subprime crisis merely amplified the message outlined by Artzner et al. (1997, 1999) and McNeil et al. (2005). Post the 2008 subprime crisis, Rossignolo et al. (2013) indicated that regulators were aiming at reducing the model risk levels in the VaR modelling process by adding the sVaR component into the model. sVaR operates on the same methodology as VaR but its stressed period are severe than VaR's one. Moosa (2010) on the other hand, suggest in the study that 2008 subprime crisis highlighted the flaws of the 1998 Basel II Accord framework. Whereas, Blundell-Wignall and Atkinson (2010) discussions indicate that the main instigator in the recent crisis was due to the financial regulators allowance in the use of internal models to measure risk because this issue introduced inconsistencies into the governance of the stability of the financial industry as a whole. Boucher et al. (2014) in no so many words, supports Blundell-Wignall and Atkinson (2010) findings by stating that post the crisis, financial institutions needs to justify the validity of the risk models to the regulators due to high model risk level in an uncertain environment.

The fundamental message echoed prior and post the 2008 subprime crisis from a modelling perspective by various scholars such as McNeil et al. (2005) and Boucher et al. (2014) is the importance of using risk measures that a tail sensitive or based on Extreme Value Theory (EVT) methods. Danielsson and de Vries (2000) defines EVT methods as statistical studies which investigates the importance of the tail of the distribution in measuring fat-tail/leptokurtic distribution models. The theoretical developments of VaR are now aligned with practical developments since the financial regulators are fostering use of more tail sensitive measures as a risk measurement tool. Banks are slowing moving to the ES measure and VaR will be used for backtesting since VaR and ES are calculated from similar distribution. This is the same developments promoted by scholars such as McNeil et al. (2005).

It's clear that developments made in VaR in practice, regulatory and theoretically are beginning to be aligned in the sense that more tail sensitive risk measures are suggested consistently across the trio. Although regulatory perspective this time is more stringent since it suggest that standardised-based methodologies proposed in the 1988 Basel I Accord should form basis for the calculation of capital despite the allowance of using internal models.

2.5 Academic responses to VaR inefficiencies as risk management tool

Since the inception of VaR modelling process, various studies have been conducted to solve the limitation in the modelling process. This is mainly motivated by the fact that VaR modelling process is an intuitive approach summarising market risk into a single easily understandable figure. However, Acerbi and Tasche (2002) indicates in their study that by using ES risk measure fixes the VaR inefficiencies in tail risk estimation without necessarily impairing the intuitiveness of VaR. As cited in section 2.3 that, the ES measure is both coherent and tail sensitive since it includes all losses beyond the assigned percentile level α (i.e. usually, $\alpha = 0.01$). As a result of the model's intuitiveness, Yamai and Yoshiba (2005) cites that the VaR risk measure has become standard risk measure for financial risk management due to its mathematical competency when measuring tail risk. Since the ES measure does not severely impair the intuitiveness position held by VaR, Yamai and Yoshiba (2005) suggest that ES risk measure can replace VaR since it's a better estimate than VaR in terms of tail risk modelling. McNeil et al. (2005) has provided a detailed proof outlining the fact that the ES risk measure is coherent since it does not violate the subadditivity axiom; refer to the theorem below:

Theorem 1 *The translation invariance, positive homogeneity and monotonicity properties follow easily from the representation* $ES_\alpha = \frac{1}{1-\alpha}\int_\alpha^1 VaR_u(L)du$ *and the corresponding properties for quantiles. It remains to show subaddivity.*
Consider a generic sequence of rvs $L_1, \cdots, L_n$ *with associated order statistics* $L_{1,n} \geq \cdots \geq L_{n,n}$ *and note that for arbitrary m satisfying* $1 \leq m \leq n$ *we have*

$$\sum_{i=1}^{m} L_{i,n} = sup\{L_{i1} + \cdots L_{im} : 1 \leq i_1 < \cdots < i_m \leq m\}. \tag{2.1}$$

Now consider two random variables (rvs) L *and* L_α *with joint df* F *and a sequence of iid bivariate random vectors* $(L_1, L_{\alpha 1}), \cdots, (L_n, L_{\alpha n})$ *with the same df F. Writing* $(L + L_\alpha)_i := L_i + L_{\alpha i}$ *and* $(L + L_\alpha)_{i,n}$ *for an order statistic of* $(L + L_\alpha)_1, \cdots, (L + L_\alpha)_n$*, we observe that we must have*

$$\begin{aligned}\sum_{i=1}^{m}(L + L_\alpha)_{i,n} &= sup\{(L + L_\alpha)_{i1} + \cdots + (L + L_\alpha)_{im} : 1 \leq i_1 < \cdots < i_m \leq m\} \\ &\leq sup\{L_{i1} + \cdots + L_{im} : 1 \leq i_1 < \cdots < i_m \leq m\} + sup\{L_{\alpha 1} + \cdots + L_{\alpha m} : 1 \leq i_1 < \cdots < i_m \leq m\} \\ &= \sum_{i=1}^{m} L_{i,n} + \sum_{i=1}^{m} L_{\alpha i,n}.\end{aligned} \tag{2.2}$$

By setting m=[n(1-α)] and letting $n \to \infty$*, we infer from lemma 2.20 (i.e. McNeil et al. (2005), p.46) that* $ES_\alpha(L + L_\alpha) \leq ES_\alpha(L) + ES_\alpha(L_\alpha)$*.*

Despite the findings made by Artzner et al. (1997, 1999), Yamai and Yoshiba (2005) and McNeil et al. (2005) promoting the ES risk measure beyond the VaR model. Kerkhof and Melenberg (2004) states that, the ES measure is not backtestable. This might be a concern for the financial regulators since their objectives are to ensure risk measures' performance are maintained at high standards. According to Boucher et al. (2014) and other scholars, VaR is backtestable since it operates under the assumption that the daily comparison of the actual risk and the VaR estimate are independent Bernoulli random variable. This backtesting algorithm is based on Christoffersen (1998) methodology and from this it can be inference that backtesting over y period follows a Binomial distribution with parameters, T and α. T ($T = 250$) is the number of days considered in y period and α (i.e. $\alpha = 0.01$) is the percentile level of VaR model.

Furthermore as mentioned above that HVaR is used in practice to measure market risk but the HVaR risk measure tend to have a delayed response to any movements outside the incorporated historical data sample used in the model. Bali et al. (2008) in their paper cite that, the time volatility issue of HVaR can be fixed by using Autoregressive Conditional Hetroscedasticity (ARCH) model, initially presented by Engle (1982). The importance of methodologies such as the ARCH and ES are amplified by findings from Alexander et al. (2012) which state that financial losses suffered by financial institutions during 2007 - 2009 global financial crisis is due to VaR's high level model risk issues. Contrary to the ARCH and the ES risk measures, Alexander et al. (2012) developed conditional-based VaR using multiple constraints as the ES model to measure market risk and this model proven to be effective than the normal HVaR.

In summary, literatures studies have iterated VaR inefficiencies since its inception. Only after the 2008 subprime financial crisis, the financial regulators, financial institutions and other stakeholders begin to notice some of the scholars' views. Although the various academic responses have been adopted into the revised 2010 Basel III Accord; modifications are still required to align these methodologies to practical feasibility status.

2.6 Limitation in the current financial risk quantification process

The literature studies discussed in previous sections, however neglects the fact that, HVaR models' limitations regarding estimation errors are not accounted for in these interventions. HVaR risk measures are multivariate statistical models which imply that large sample size is required for the models' precision to be accurate. Yamai and Yoshiba (2005) is in support of this observation and explicitly indicates that the HVaR and ES risk measure are affected by estimation errors, such as limited sample size results in the sampling fluctuation. Few studies are focusing on treating this dependency of these models relating to the estimation error issues discussed above. Another element in financial risk modelling tools is that, these risk measures are heavily dependent on historical extremities when forecasted future risk. This assumes that previous transgressions will continue in the future, which is not the case. These limitations will be addressed by the proposed method in chapter 3. The next section outlines various model-based method that could be employed in minimising the estimation errors noted by Yamai and Yoshiba (2005).

2.7 Possible solutions to the estimation errors of VaR and ES risk measures

The financial risk quantification tool known as HVaR have been widely researched in literature and the ES model which supposed to replace HVaR according to the trending researches post the 2008 subprime crisis. However, the discussions in section 2.6 conveys that the estimation errors of HVaR and ES risk measure have been significantly neglected in literature since the focus is on the improvement of capturing tail-related losses, especially during financial crisis. Therefore, this section discusses in detail various employable methods that can be used to reduce the estimation error noted in Yamai and Yoshiba (2005)'s study. This section is sub-divided into the following subsections:

- Jump-diffusion models;
- Principal component;
- Factor Analysis; and
- Other stochastically-based models.

2.7.1 Jump-diffusion models

Historical developments of the jump-diffusion models

Jump diffusion models have been widely used in modern finance to model the discontinuities of underlying stock in an option derivative. Then an option derivative is defined by Hull (2009) as a derivative which gives it owners' the right to buy or sell an asset. Merton (1976) develops a Gaussian-based jump-diffusion model in calibrating option pricing when the returns of the underlying stock are discontinuous. Then Kou (2002) developed, the Double Exponential Jump-Diffusion (DEJD) model which is a higher-order model than Merton (1976). Kou and Wang (2004) further reemphasised the important of the DEJD models in calculating an option price accurately. According to Kou (2002) the discontinuity issue with the asset pricing of options was mainly due to the misalignment between the model assumptions of the Black-Scholes option pricing model used since the early 1970s and the empirical behaviour of such returns. The Black-Scholes model assumes that returns of underlying stocks for the options are normally distributed whereas empirical evidence indicates that these returns are asymmetric leptokurtic distributed.

Based on the literature discussions yield in the preceding paragraph it can be concluded that jump-diffusion models are ideal instruments to model discontinuities or sudden jumps in historical data trend. In the next sub-subsection it's outlined clearly that, the λ in the Poisson process embedded in the jump-diffusion models is responsible in calculating/assigning the discontinuity based on historical data trend. To conclude this section Moazeni et al. (2013) cited that, the jump-diffusion model could be used in obtaining robust price outputs under uncertain financial environment. Uncertain environment are similar circumstances presented by financial crisis.

Mathematical properties of Jump-Diffusion models

The following are the main properties regarding the jump diffusion modelling process:

- The jump-diffusion models are based on two robust stochastic processes namely the Poisson process and Brownian motion process; the mathematical benefits retrieved from these processes have been highlighted in detail in Swanepoel (2009) and Allison (2009).

- Jump-diffusion models due to the Poisson process, its estimation tends to follow a more heavy-tailed distribution which suits/match most of the empirical studies performed on financial asset pricing;
- All parameters used in the model can be derived using Maximum Likelihood Estimation (MLE) process and there are few papers supporting this statement, for example, Ramezani and Zeng (2007); and
- The sub-processes amalgamated in forming the jump-diffusion models are backed-up by extensive literature both in mathematical statistics and applied financial statistics which attests its robustness and accuracy in modelling phenomenal with embedded random market jumps.

Jump-diffusion models are extensively investigated in literature, however for improving model risk of the option pricing models where underlying assets/stock are discontinuous. Jump-diffusion application in reducing estimation errors discussed in section 2.6 in a risk management tool framework have not been used anywhere in literature as yet. The jump-diffusion model is dynamic and complex in modelling pricing than other models. In this dissertation this jump-diffusion modelling process defined in appendix C will be used reduce to estimate error in the proposed model. However in literature jump-diffusion models are not the models which could be used to induce stresses in prices or risk modelling of financial risk. The following section discusses other alternatives to the only jump-diffusion model aforementioned.

2.7.2 Principal component

Johnson and Wichern (2007) define Principal Component Analysis (PCA) as a statistical methodology which explains the variance-covariance structure of a set of variables through a few linear combinations of these variables. The overall objectives of PCA are namely, data reduction and interpretation. Johnson and Wichern (2007) on page 431 continues to define PCA formally as follows:

"*Let the random vector* $\bar{X}^T$ $=\{X_1, X_2, \cdots, X_p\}$ *have the covariance matrix* Σ *with eigenvalues* $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p \geq 0$.
Consider the linear combinations

$$\begin{aligned} Y_1 &= \bar{a}_1 \bar{X} = a_{11}X_1 + a_{12}X_2 + \cdots + a_{1p}X_p \\ Y_2 &= \bar{a}_2 \bar{X} = a_{21}X_1 + a_{22}X_2 + \cdots + a_{2p}X_p \\ &\vdots \end{aligned}$$

$$Y_p = \bar{a}_p\bar{X} = a_{p1}X_1 + a_{p2}X_2 + \cdots + a_{pp}X_p$$

Then, using equation (2-45)[1]*, directly taken from the source, we obtain*

$$\begin{aligned} Var(Y_i) &= a_i^T\Sigma a_i \forall i = 1, 2, \cdots, p \\ Cov(Y_i, Y_k) &= a_i^T\Sigma a_k \forall i, k = 1, 2, \cdots, p \end{aligned} \tag{2.3}$$

The principal components are those uncorrelated linear combinations $Y_1, Y_2, \cdots, Y_p$ whose variances in (2.3) are as large as possible. ∴ Cov(Y_i, Y_k) = 0 & VaR$\{Y_i\}$ is large."
Loretan (1997) used this PCA methodology described above to generate market risk factors for normal and severe market circumstances. Note that, market data used in market risk estimation is normally highly correlated and this cause problems in statistically models; PCA is a solution to this problem because it generates uncorrelated linear combinations. According to Loretan (1997), market risk factors are generated as follows:

- The observed data **X** is transformed into return format, that is,

$$\mathbf{X} = \begin{pmatrix} X_{11} & X_{21} & X_{31} & \cdots & X_{n,1} \\ X_{12} & X_{22} & X_{32} & \cdots & X_{n,2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ X_{1,p} & X_{2,p} & X_{3,p} & \cdots & X_{n,p} \end{pmatrix}, \therefore \text{trans}\{\mathbf{X}\} = \begin{pmatrix} \frac{X_{11}}{X_{12}} & \frac{X_{21}}{X_{22}} & \frac{X_{31}}{X_{32}} & \cdots & \frac{X_{n,1}}{X_{n,2}} \\ \frac{X_{12}}{X_{13}} & \frac{X_{22}}{X_{23}} & \frac{X_{32}}{X_{33}} & \cdots & \frac{X_{n,2}}{X_{n,3}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{X_{1,p-1}}{X_{1,p}} & \frac{X_{2,p-1}}{X_{2,p}} & \frac{X_{3,p-1}}{X_{3,p}} & \cdots & \frac{X_{n,p-1}}{X_{n,p}} \end{pmatrix}.$$

- The trans{**X**} matrix is then checked whether it is compatible with PCA; this is achieved by assessing the axis symmetry[2] of the variables in the matrix.

- Once it has been proven that the criteria holds for symmetric axes, the researcher can proceed with the PCA using statistical packages, like SAS, R and E-views.

- The risk factors retrieved from the above methodology is then used to infer on the current portfolio and the same methodology is applied to get the profit and loss vector; refer to appendix B on the methodology used in getting this vector.

- Lastly, since the Principal Component (PC) is random, we may pick tail-event quantiles of the empirical distribution of the PC to generate corresponding tail events of the observable series (Loretan (1997), p.30). Therefore, the PCA model-driven method can be used in generating robust scenarios to be implemented for estimating market risk. This is in line with the aim of this research that is, discovering a methodology of generating risk factors employable in market risk estimation model, which caters for large market turbulence which prevails during a financial crisis.

[1]The linear combinations, **Z=CX** have,
μ_z=E(**Z**)=E(**CX**)=**C**μ_x
Σ_z = Cov(**Z**) = Cov(**CX**) = **C**Σ_x**C** $\cdots$ (2.45) {Johnson and Wichern (2007), p.76}

[2]The joint distribution of the transformed data is assessed for symmetric properties in its axes. Loretan (1997) have implemented non-parametric density estimation in testing this because there is no formal statistical test developed for testing this phenomenal.

2.7.3 Factor analysis

Factors analysis can be considered as an extension of principal component analysis (Johnson and Wichern (2007), p.482). Therefore, in substituting the methodology above of determining risk factors or scenarios, the Factor Analysis Model (FAM) can be used because factor models provide more information than the PCA. PCA is used for finding optimal ways of the combining variables, while the FAM may be used to identify the structure underlying variables. The main aim is to create stressed risk factors using quantitative method to minimise subjective and since FAM is similar to PCA; the defined methodology in section 2.4.1 could be used. Johnson and Wichern (2007) defines the factor model in the following manner:

"*Let* $\boldsymbol{X}$ *be a random vector with* p *components;* $\boldsymbol{X}$ *has a mean* μ *and covariance matrix* Σ*. The factor model postulates that* $\boldsymbol{x}$ *is linearly dependent upon a few unobservable random variables* $F_1, F_2, \cdots, F_m$ *called common factors, and* p *additional sources of variation* $\epsilon_1, \epsilon_2, \cdots, \epsilon_p$*, called errors or sometimes, specific factors. In particular, the factor analysis model is*

$$\begin{aligned}
X_1 - \mu_1 &= l_{11}F_1 + l_{12}F_2 + \cdots + l_{1m}F_m + \epsilon_1 \\
X_2 - \mu_2 &= l_{21}F_1 + l_{22}F_2 + \cdots + l_{2m}F_m + \epsilon_2 \\
&\vdots \\
X_p - \mu_p &= l_{p1}F_1 + l_{p2}F_2 + \cdots + l_{pm}F_m + \epsilon_p
\end{aligned}$$

or, in matrix notation,

$$\boldsymbol{X} - \underline{\mu} = \boldsymbol{L}_{(p\times m)}\ \boldsymbol{F}_{(m\times 1)}\ + \epsilon_{(p\times 1)}$$

The coefficient l_{ij} *is called the loading of the* i^{th} *variable on the* j^{th} *factor, so the matrix* $\boldsymbol{L}$ *is the matrix of the factor loadings.*"

2.7.4 Other model-based methods

Other model-driven methods of generating scenarios are based on methodologies extracted from non-parametric statistics.

These non-parametric methods are advanced for the scope of this dissertation and therefore will not be discussed, they are namely:

- Non-linear regression; determining risk factors for complex derivative like an option which is not linear and not normal distributed. Therefore, scholars have developed non-parametric algorithms for computing these scenarios which involved density function estimation procedures applied usually in non-parametric statistics.

- Others are namely, the GARCH and ARCH models from the time-series area in statistics, geometric brownian motions, etc. These processes have been excluded in the literature because this dissertation focuses on jump models and its influences in improving the SES model's predictability.

The above methodologies determine market risk factors from functional form modeling structures because simulating market risk factors from historical data have shown weakness in the power of predicting risk accurately. One of the objectives for the research is to minimise the model's error rate brought by the historical simulation approach. Therefore the modeller will use one of these approaches discussed above to generate market risk factors for computing market risk exposures; since these approaches have a functional form the modeller can construct few scenarios in the model to compensate for stress market situations as seen in the 2008 subprime crisis.

2.7.5 Model-based' summary

Table 2.1: Rationale behind the chosen model-based risk scenario method

MODEL	MATHEMATICAL FORMULATION	USABLE OR NOT IN GENERATING RISK SCENARIOS FOR THE VaR MODEL POST 2008
Principal component	$$Y_i = a_i^T \Sigma a_i, \forall i = 1, 2, \cdots, p \quad (2.4)$$	This method allows the modeller to define the underlying structure of the data using uncorrelated data. Using this method will not create the artificial market jumps observed in the financial movements; however this method is mostly useful in summarizing the data.
Factor analysis	Similar to the principal component.	Same motivation as above.
Jump-diffusion model	See appendix C under section C.2 for the mathematical formulation of the jump-diffusion models.	Using jump-diffusion models in generating risk scenarios is useful since it mimics real financial movements which has market jumps randomly induced in the historical data. Such jumps are normally seen in stressed financial markets.

Continued on next page

Table 2.1 – *Continued from previous page*

MODEL	MATHEMATICAL FORMULATION	USABLE OR NOT IN GENERATING RISK SCENARIOS FOR THE VaR MODEL POST 2008
Other	High-order diffusion processes like the Cox-Ingersoll-Ross (CIR) defined as $$Dy_t = -k(y_t - \theta)dt + (\eta^2 y_t)^{\frac{1}{2}} dW_t \quad (2.5)$$ Or ARCH model defined as, $$X_t = \sigma_t Z_t \quad (2.6)$$ Where, $\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i}^2$.	These model could work in generating the required risk scenarios in the simulated model. The only problem is that they take into consideration the volatility which further complicates matters. The risk scenarios required by the simulated model is the model which consider price jumps meanwhile keeping volatility constant.

The financial market movements jump randomly up or down depending on the stress instilled in the market; these jumps are in most cases are due to or caused by mismatched data which cause VaR to under-predict as seen in 2008 crisis. Therefore from the pool of model-based methods listed and summarised in the table above; the selection of an appropriate model-based risk scenario method should fulfil the requirement stated in the preceding sentences. Based on this, the jump-diffusion model will be used in generating risk scenarios because it creates artificial jumps using the underlying data fed to the model; this will allow modelling data to mimic financial movements seen in the real financial markets, hence improving the mismatch mentioned in earlier sections.

Chapter 3

METHODOLOGY

3.1 Introduction

The chapter presents in detail the full mathematical construction of the proposed model to address the research problem defined in the previous chapters. It begins by describing the HVaR methodology in full as well as other VaR approaches investigated in literature but not used in practice. The aim of this section is to provide baseline when comparing the proposed model's benefits against HVaR. The remaindering sections focuses on outlining the proposed methodology and statistical tests employed to assess its validity. These sections are segmented as follows:

- Section 3.3 presents all work pertaining to the proposed model. It begins by explaining the algorithm proposed in addressing the research problem. Then, the following section outlines other analytical aspects of the proposed model not fully discussed under the proposed algorithm section;
- Section 3.4 discusses in brief detail other models also study in literature; and
- Section 3.5 then concludes the chapter by looking in a usable benchmarking methodology in assessing the proposed model efficiency and validity.

3.2 Historical-simulation Value-at-Risk (HVaR) model and other VaR approaches

The focus of discussions presented this far was on VaR and why it failed in the 2008 subprime financial crisis. However few discussions have been yield outlining; what is VaR? Which other approaches of VaR are investigated in literature and why they are not used? These questions are all answered in this section.

There are many definitions provided in literature about, what is VaR? However, McNeil et al. (2005)'s definition is more in aligned with the detailed description on how the HVaR method is calibrated. McNeil et al. (2005) defines VaR model in general for a given confidence level $\alpha \epsilon$ (0,1) as,

$$VaR_{\alpha}^{\Delta}(L) = inf\{\theta\epsilon\mathbb{R} : P_{\Delta}(\theta<L) \leq (1-\alpha)\} = inf\{\theta\epsilon\mathbb{R} : F_L(\theta) \geq \alpha\} \tag{3.1}$$

where,

- Δ is the holding period parameter (usually $\Delta = 10$ in practice);
- α is the confidence level, for example $\alpha = 0.99$; and
- $VaR_{\alpha}^{\Delta}(L)$ is the loss at α-quantile in the loss distribution constructed using returns of prices contained in a portfolio.

The VaR method defined in equation (3.1) above can be calibrated mainly under the following approaches, refer to the subsections below.

3.2.1 Variance-Covariance Method

The Variance-Covariance method assumes that the distribution of the returns in the portfolio is normally distributed over the sample period.

Variance-covariance VaR approach - Model assumptions

The following are the model assumptions:

- The risk factor changes $X_{t+1} \overset{iid}{\sim} N_d(\mu, \Sigma)$.
- The linearized loss in terms of the risk factors is a sufficiently accurate approximation of the actual loss and simplify the problem by considering the distribution of $L_{t+1}^{\Delta} = l_{[t]}^{\Delta}(X_{t+1}$ with $l_{[t]}^{\Delta}$[1](McNeil et al. (2005), p.48).

Variance-covariance VaR approach - Additional information

This approach is not employed in practise due to the following reasons:

- It assumes linearity which is not always applicable in the industry; and
- Exotic financial instruments and other non-linear financial product may not follow the normal distribution assumed in this approach.

[1] $l_{[t]}^{\Delta} = -(f_t(t, Z_t) + \sum_{i=1}^{d} f_{z_i}(t, Z_t)x_i)$, where Z_t is a d-dimensional time dependent vector used in the valuation, x_i are the various risk factors incorporated in the model and t is time.

A stylised fact of empirical finance suggests that the distribution of financial risk factors is leptokurtic and heavier-tailed than the Gaussian distribution. The implication is that an assumption of Gaussian risk factors will tend to underestimate the tail of the loss distribution and measures of risk, like VaR and Expected Shortfall (ES), which are based on this tail (McNeil, et al., (2005), p.49). Based on these rationales provided, the variance-covariance VaR model fails to model risk accurately in practise; hence this method is not used by financial companies to calibrate risk.

3.2.2 Historical Simulation Method

The Historical simulation methods assumes that the distribution of the returns in the portfolio is constant over the sample period, as cited by Danielsson and de Vries (2000).

Historical-simulation VaR approach - Model assumptions

The assumption is that the distribution of returns is constant over the sample period.

Historical-simulation VaR approach - Additional information

In the previous method described, L = $l_{[t]}(X_{t+1})$ was established under some explicit parametric model for X_{t+1} which is not the case; historical simulation method can be thought of as estimating the distribution of loss operator under the empirical distribution of data $X_{t-n+1}, \cdots, X_t$. This approach of VaR model is used under the prescription of the Basel regulatory framework. It allows the model to incorporate other dimensions of financial instruments in the calculation of risk without mathematical limitations seen in the first approach. Due to this the model was selected in 1998 as a risk measure for market risk and this dissertation will discuss alternative methods of enhancing or replacing this model because of its shortcoming observed in 2008 financial crisis; (see appendix A).

3.2.3 Monte-Carlo Method

The Monte-Carlo method assumes that the distribution of the returns in the portfolio is fat-tailed distributed in simulation algorithm over the sample period.

Monte-Carlo VaR approach - Model assumptions

The following are the assumptions:

- Parametric model for X_{t+1} is used to kick-start the simulation; and
- The linearity of risk factors is not assumed.

Monte-Carlo VaR approach - Additional information

This VaR approach is as good as the selected parametric model used in the simulation section of the modelling approach. In market risk, parametric models like the GARCH seem to be desirable candidates for calculating simulated risk factors due to its heavy-tailed attribute. However this method as well is not employable in practice due to its dependency to parametric model to run the VaR model effectively.

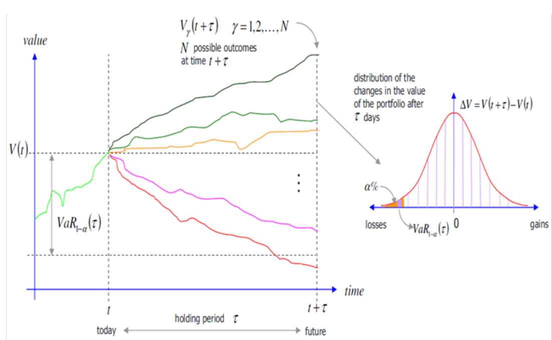
3.2.4 VaR approach used in practice

As highlighted in section 3.2.2 the historical-simulation method is easy to implement and reduces the risk-measure estimation issue into a one-dimensional problem. This attribute made this approach to be favorable among the financial regulators and institutions in capturing the dynamic risk levels in the financial markets; hence its inauguration in 1998 as a risk measure which estimate market risk.

3.2.5 An overview of the HVaR modelling process

Figure 3.1 outlines in short the historical-simulation VaR process as a simulation model using historical data in forecasting a portfolio's performance based on its progression currently.

Figure 3.1: VaR definition



The historical-simulation VaR model shown in Figure 3.1 is defined formally as follows:

Consider a general portfolio P which comprised of l positions $f_1(t), f_2(t), \cdots, f_l(t)$. The risk factors or returns are calculated as outlined in definition 15 in section 1.6. Which implies the vector of the risk factors or returns (i.e. denoted by $\mathbf{r}_t$) is an $(m \times 1)$ vector where $m = l - 1$. In the simplest case, these positions can be directly associated with a fundamental risk factor or it can be associated with a financial derivative which is a complex function of risk factors

$$f_i(t) = \begin{cases} r_j(t) & 1 \leq i \leq l \\ f_i(r_1(t), r_2(t), \ldots r_m(t)) & 1 \leq j \leq m \end{cases} \tag{3.2}$$

The value of the portfolio $V(t)$ is determined by the holding α_i in each position $f_i(t)$, $1 \leq i \leq l$

$$V(t) = \alpha_1 f_1(t) + \alpha_2 f_1(t) + \ldots + \alpha_l f_l(t) \tag{3.3}$$

Let us assume that the time series information associated with each risk factor $r_j(t)$, $1 \leq j \leq m$ has the following chronological order

$$\begin{bmatrix} r_j(t_1) \\ r_j(t_2) \\ \vdots \\ r_j(t_n) \end{bmatrix}_{current}^{historic} \quad \ldots 1 \leq j \leq m \tag{3.4}$$

where n represents the number of elements in the time series. This information is used to generate $n - 1$ historical scenarios using either the ratio method or the difference method, depending on the configuration for that risk factor,

$$dr_j(t_k) = \begin{cases} \frac{r_j(t_k)}{r_j(t_{k-1})} & 1 \leq j \leq m \\ \\ r_j(t_k) - r_j(t_{k-1}) & 2 \leq k \leq n \end{cases} \tag{3.5}$$

The base scenarios for these methods are defined as follows

$$dr_j(t_1) = \begin{cases} 1, & 1 \leq j \leq m \\ \\ 0, & Otherwise \end{cases} \tag{3.6}$$

The $n - 1$ scenarios can now be used to determine possible movements in the risk factors given that their current values $r_j(t_n)$ are known

$$r_{j,k}(t_{n+1}) = \begin{cases} r_j(t_n) dr_j(t_k) & 1 \leq j \leq m \\ r_j(t_n) + dr_j(t_k) & 2 \leq k \leq n \end{cases} \tag{3.7}$$

These movements are then used to recalculate the positions

$$f_{i,k}(t_{n+1}) = \begin{cases} r_{j,k}(t_{n+1}) & 1 \leq i \leq m \\ & 1 \leq j \leq m \\ f_i(r_{1,k}(t_{n+1}), r_{2,k}(t_{n+1}), \ldots, r_{m,k}(t_{n+1})) & 2 \leq k \leq n \end{cases} \tag{3.8}$$

which are used to revalue the portfolio

$$V_k(t_{n+1}) = \alpha_1 f_{1,k}(t_{n+1}) + \alpha_2 f_{2,k}(t_{n+1}) + \ldots + \alpha_l f_{l,k}(t_{n+1}) \quad 2 \leq k \leq n \tag{3.9}$$

The change in value of the portfolio is just the difference between these valuations and the current value

$$\triangle V_k = V_k(t_{n+1}) - V(t_n) \ldots 2 \leq k \leq n \tag{3.10}$$

Note that in the case of the base scenarios we have

$$\triangle V_1 = 0 \tag{3.11}$$

The next step involves formulating the vector $\triangle V$

$$\triangle V = \begin{bmatrix} \triangle V_2 \\ \triangle V_3 \\ \vdots \\ \triangle V_n \end{bmatrix} \tag{3.12}$$

These $n-1$ deviations represent either a positive or negative movement in the value of the portfolio which must be sorted in ascending order

$$\triangle V \uparrow = sort(\triangle V, ascending) \tag{3.13}$$

In the final step, the integer I is determined which indexes the 1% percentile of $\triangle V \uparrow$. The 1-day VaR is simply

$$I = round\{\frac{n-1}{100}\} \tag{3.14}$$

$$VaR_{1-\alpha}(1) = \triangle V \uparrow (I) \tag{3.15}$$

An empirical formulation of this VaR modelling process has been provided under appendix B.

3.3 Simulated Expected Shortfall (SES) model

Instead of using historical data as stipulated in the traditional VaR methodology explained in section 3.2.5; a jump-diffusion model will be implemented on the historical data to transform the data into a more receptive to volatile financial market environment (i.e. robust dataset).

3.3.1 Overall SES modelling process description

The following dynamics are proposed to model market risk accurately through minimising the shortcomings mentioned in the previous about the HVaR measure. This is achieved by combining the jump diffusion and HES models into a single modelling framework. Mastro (2013) developed an empirical method in modelling jump diffusion models under Merton (1976) conceptual. Let an $(m \times 1)$ vector $\mathbf{S}$ represent the historical data of portfolio price daily movements and S_t is the portfolio's position today. The vector $\mathbf{S}$ is transformed into prices' returns with discontinuous behaviour through incorporating Mastro (2013) jump-diffusion model and it follows for $i = (1, \cdots, m)$,

$$R_{S_{(jump-dif)}} = \begin{cases} R = \left(\frac{S_i}{S_{i-1}}\right) = e^{\left(\mu_D \times dt + \sigma \times RAND(n) \times \sqrt{dt} + Q\right)} \cdots, if(\lambda \times dt > UniDist(i)) \\ R = \left(\frac{S_i}{S_{i-1}}\right) = e^{\left(\mu_D \times dt + \sigma \times RAND(n) \times \sqrt{dt}\right)} \cdots, if(\lambda \times dt \leq UniDist(i)) \end{cases} \tag{3.16}$$

where $\mathbf{S}$ is the historical aggregated asset prices in a portfolio, R is the return calculated from changes in $\mathbf{S}$ and $R_{S_{(jump-dif)}}$ is the transformed returns which includes the discontinuity behaviour. In equation (3.16), there is a market jump if $(\lambda \times dt) > UniDist(i)$, where λ is the intensity parameter of the Poisson process in the jump diffusion, dt is the daily frequency and finally $UniDist(i)$ is the uniform distribution at i $\forall i = (1, 2, \cdots, m)$. Furthermore if $(\lambda \times dt) \leq UniDist(i)$ it implies that there is no jump. Note that, these functions denoted in equation (1) are discretised (i.e. introducing logs on both sides) version of Mastro (2013), which is defined as $\left(\frac{S_i}{S_{i-1}}\right) = e^{\left[\left(\alpha - \frac{\sigma^2}{2} - \lambda k\right)t + \sigma_t W_t\right]} \prod_{i=1}^{N_t} Y_i$. The following parameters denoted in equation (3.16) need to be estimated, namely, the drift parameter incorporating jumps (i.e. μ_D), negative market jump coefficient (i.e. q_1), positive market jump coefficient (i.e. q_2), volatility portfolio parameter (i.e. σ), standard normal random generator (i.e. $RAND(n)$), random generator (i.e. $rand$) and market jump coefficient $Q = (q_1 + (q_2 - q_1) \times rand)$.

Equation (3.16) is iterated 100 times with an aim of simulating a $(m - 1 \times 100)$ matrix of $R_{S_{(jump-dif)}}$ to improve prediction on the returns since jump-diffusion model's paths generated are not unique. Then these 100 simulated paths are then averaged into a $(m-1\times1)$

vector of "$ave\left(R_{S_{(jump-dif)}}\right)$" defined as follows:

$$ave\left(R_{S_{(jump-dif)}}\right) = \begin{bmatrix} r_j(t_1) \\ r_j(t_2) \\ \vdots \\ r_j(t_n) \end{bmatrix}_{current}^{historic} \quad \ldots 1 \leq j \leq m \tag{3.17}$$

where n represents the number of elements in the time series. This information is utilised to generate $n-1$ jump-diffusion historically based scenarios using either the ratio method or the difference method, depending on the configuration for that risk factor (i.e. return),

$$dr_j(t_k) = \begin{cases} \frac{r_j(t_k)}{r_j(t_{k-1})} & 1 \leq j \leq m \\ \\ r_j(t_k) - r_j(t_{k-1}) & 2 \leq k \leq n \end{cases} \tag{3.18}$$

The base scenarios for these methods are defined as follows

$$dr_j(t_1) = \begin{cases} 1, & 1 \leq j \leq m \\ \\ 0, & Otherwise \end{cases} \tag{3.19}$$

The $n-1$ scenarios can now be used to determine possible movements in the risk factors(i.e. returns) given that their current values $r_j(t_n)$ are known

$$r_{j,k}(t_{n+1}) = \begin{cases} r_j(t_n) dr_j(t_k) & 1 \leq j \leq m \\ r_j(t_n) + dr_j(t_k) & 2 \leq k \leq n \end{cases} \tag{3.20}$$

These movements are then used to recalculate the positions

$$S_{i,k}(t_{n+1}) = \begin{cases} r_{j,k}(t_{n+1}) & 1 \leq i \leq m \\ & 1 \leq j \leq m \\ S_i(r_{1,k}(t_{n+1}), r_{2,k}(t_{n+1}), \ldots, r_{m,k}(t_{n+1})) & 2 \leq k \leq n \end{cases} \tag{3.21}$$

which are used to revalue the portfolio

$$V_k(t_{n+1}) \quad = \quad \alpha_1 S_{1,k}(t_{n+1}) + \alpha_2 S_{2,k}(t_{n+1}) + \ldots + \alpha_l S_{l,k}(t_{n+1}) \quad 2 \leq k \leq n \tag{3.22}$$

The change in value of the portfolio is the difference between these valuations and the current value

$$\triangle V_k = V_k(t_{n+1}) - V(t_n) \ldots 2 \leq k \leq n \tag{3.23}$$

Note that in the case of the base scenarios we have

$$\triangle V_1 = 0 \tag{3.24}$$

The next step involves formulating the vector $\triangle V$

$$\triangle V = \begin{bmatrix} \triangle V_2 \\ \triangle V_3 \\ \vdots \\ \triangle V_n \end{bmatrix} \tag{3.25}$$

These $n-1$ deviations represent either a positive or negative movement in the value of the portfolio which must be sorted in ascending order

$$\triangle V \uparrow = sort(\triangle V, ascending) \tag{3.26}$$

In the final step, the integer I is determined by the index α percentile of $\triangle V \uparrow$. The 1-day VaR is simply,

$$\begin{aligned} I &= \left(\frac{n-1}{100}\right) \\ VaR_{\alpha}^{\Delta}(1) &= \triangle V \uparrow (I), \cdots (i.e.\triangle = 1) \end{aligned} \tag{3.27}$$

Then use $VaR_{\alpha}^{\Delta}(1)$ in equation (3.27) as an indicator for the calculation of market risk under the SES modelling framework. Let ω denote all $\triangle Vs$ in equation (3.26) that are greater than $VaR_{\alpha}^{\Delta}(1)$.

$$\begin{aligned} SES_{mean_{1-\alpha}^{\triangle}} &= \left\{ \frac{\left(\sum_j \omega_j\right)}{n} \right\} \times \tau, \cdots (\forall j = (m, \cdots, -\infty) \text{and m} > I); and \\ SES_{median_{1-\alpha}^{\triangle}} &= median\{\omega_j\} \times \tau, \cdots (\forall j = (m, \cdots, -\infty) \text{and m} > I) \end{aligned} \tag{3.28}$$

The τ factor in equation (3.28) is used to rectify the average method applied on the transformed returns. This τ is used mainly to regain the volatility introduced by the jump diffusion model which was minimised by the average method.

3.3.2 Other analytic aspects of the (SES) model

Parameter estimation method and its validity tests

In section 3.3.1, the mathematical detail of the Simulated Expected Shortfall (SES) model proposed was described in full. However, this algorithm defined above has not yet been tested. Therefore, the following sections discuss the validity of the parameters defined in equation (3.16). The model has five parameters mentioned in section 3.3.1, however this section before the validity test of the parameters, discusses how to derive these parameters from an empirical perspective; see the description in the table below.

Table 3.1: Detailed parameters explanation

Parameters	Formulation	Description
$\widehat{q_1}$	$\widehat{q_1} = \inf_l\{\tilde{f}_{\varphi_i}(t) : \varphi_i \epsilon \mathbb{Z}^+\}$	Estimated downward market jump coefficient
$\widehat{q_2}$	$\widehat{q_2} = \sup_l\{\tilde{f}_{\varphi_i}(t) : \varphi_i \epsilon \mathbb{Z}^+\}$	Estimated upward market jump coefficient
$\widehat{\mu_D}$	$\widehat{\mu_D} = \widehat{\mu_{\sigma D2}} + \frac{1}{2} \times \widehat{\sigma^2}$	Estimated drift parameter incorporating jump effect
$\widehat{\lambda}$	$\widehat{\lambda} = \frac{K}{Years}$	Estimated intensity parameter in the Poisson Process
$\widehat{\sigma}$	$\widehat{\sigma} = \frac{std}{\sqrt{dt}}$	Estimated volatility parameter

Where $\tilde{f}_{\varphi}(t)$ denotes the vector of continuous return as per definition 15 in section 1.6 which are calculated from the aggregated positions and (M_1, std, M_3 and M_4) are the moments of the returns as defined in Table 3.2. Then $dt = \frac{1}{252}$ is the "time frequency" parameter and 252 is used because there are approximately 252 trading/business days in a year where markets are operative and then $Years = n \times dt$. Other used formulations in Table 3.1 are $\mu_{Jump} = \left(\frac{q_1+q_2}{2}\right)$, $K = outlier_{param}$ and $\widehat{\mu_{\sigma D2}} = \frac{(M_1 - \mu_{Jump} \times \widehat{\lambda} \times dt)}{dt}$. Furthermore the moments of continuous returns vector (i.e. denoted by $\underline{\tilde{f}}_{\varphi}(t)$) for given m positions (i.e. vector $\mathbf{S}$) discussed in section 3.3.1 is defined as follows; see Table 3.2.

Table 3.2: Moments of return calculation

Parameters	Formulation	Description
M_1	$M_1 = \left(\frac{1}{l}\right) \times \sum_{i=1}^{l} \tilde{f}_{\varphi_i}(t)$	First moment
std	$std = \sqrt{\frac{\sum_{i=1}^{l}\left(\tilde{f}_{\varphi_i} - M_1\right)^2}{n-1}}$	Standard deviation
M_2	$M_2 = std^2$	Second moment
M_3	$M_3 = \left(\frac{1}{l}\right) \times \sum_{i=1}^{l}[\ (\tilde{f}_{\varphi_i}(t) - M_1)^3]$	Third moment
M_4	$M_4 = \left(\frac{1}{l}\right) \times \sum_{i=1}^{l}[\ (\tilde{f}_{\varphi_i}(t) - M_1)^4]$	Forth moment

Note that, parameters defined in Table 3.1 are the parameters whose validity will be assessed in this section and whereas Table 3.2 conveys supporting formulas needed to calculate the parameters in Table 3.1; see appendix D (i.e. section D.1.1 for the formulation of K defined above. Therefore, subsequent to estimating the parameters as defined in Table 3.1 required by the chosen model; Mastro (2013) developed a set of optimised variables which can be used in validating the validity of the modelling methods used to determine the parameters in the MATLAB algorithm. These optimised parameters are determined by running "fminsearch" optimising engine in the MATLAB and previously determined $\widehat{\lambda}$, $\widehat{q_1}$ and $\widehat{q_2}$ in Table 3.1 are used as input into the optimisation process. This optimisation process will be defined as follows,

$$\underline{\widehat{a}} = \begin{bmatrix} \widehat{\lambda} \\ \widehat{q_1} \\ \widehat{q_2} \end{bmatrix} \xrightarrow{\text{INPUT}} \boxed{\text{OPTIMIZATION ENGINE}} \xrightarrow{\text{OUTPUT}} \underline{\overbrace{a}} = \begin{bmatrix} \overbrace{\lambda} \\ \overbrace{q_1} \\ \overbrace{q_2} \end{bmatrix} \quad (3.29)$$

local minimum criteria[2] is used in selecting $\underline{\overbrace{a}}$ apart from other tolerance measures embedded in the "fminsearch" optimising engine. From equation (3.29) calculations, the following optimised parameters were determined which are defined in the table below.

Table 3.3: Detailed parameters explanation (optimised)

Parameters	**Formulation**	**Description**
$\overbrace{q_1}$	$\overbrace{q_1} = \inf_l\{\tilde{f}_{\varphi_i}(t) : \varphi_i \epsilon \mathbb{Z}^+\}$	Optimised downward market jump coefficient
$\overbrace{q_2}$	$\overbrace{q_2} = \sup_l\{\tilde{f}_{\varphi_i}(t) : \varphi_i \epsilon \mathbb{Z}^+\}$	Optimised upward market jump coefficient
$\overbrace{\mu_D}$	$\overbrace{\mu_D} = \widehat{\mu_{\sigma D2}} + \frac{1}{2} \times \widehat{\sigma^2}$	Optimised drift parameter incorporating jump effect
$\overbrace{\lambda}$	$\overbrace{\lambda} = \frac{K}{Years}$	Optimised intensity parameter in the Poisson Process
$\overbrace{\sigma}$	$\overbrace{\sigma} = \frac{std}{\sqrt{dt}}$	Optimised volatility parameter

In addition to the parameters defined in Table 3.1 and Table 3.3, Mastro (2013) added another algorithm which calculates the parameters but using self-assessed values. This have been included here to validate whether the empirical data extracted in this dissertation is similar to the data used by Mastro (2013) in modelling discontinuity of price for energy stock. The values used are shown in the table below.

[2]This criteria select optimising value which satisfies the following inequality: $f(\underline{\overbrace{a}}) \leq f(\underline{\widehat{a}}), \forall \widehat{a}$ in the interval.

Table 3.4: Detailed parameters explanation (theoretical)

Parameters	**Formulation**	**Description**
μ	$\mu = 0.11$	Drift parameter
σ	$\sigma = 0.25$	Volatility parameter
μ_D	$\mu_D = \mu - \frac{\sigma^2}{2}$	Drift parameter incorporating jump effect
λ	$\lambda = 5$	Rate of jumps per year = Intensity of Poisson Process
q_1	$q_1 = -0.14$	Downward market jump coefficient
q_2	$q_2 = 0.15$	Upward market jump coefficient

Then using the information from Table 3.1, Table 3.3 and Table 3.4; a parameter accuracy test can be developed using Table 3.4's input as basis for the test. It is essential to validate the parameters of Mastro (2013) since this method is untested under risk management tools. However if the error rate is between 2% to 5%, then the comparison is deemed accurate. Then, equation (3.30) denotes formulas used in calculating the error rate coefficients in validating these parameters (see section 4.3):

$$
\begin{aligned}
\Pi_1 &= \left[\frac{\left(\frac{((\theta_{est} - \theta_{theo}) + 100) \times \theta_{est}}{\theta_{theo}} \right)}{(\theta_{est} - \theta_{theo}) + 100} \right] \\
\Pi_2 &= \left[\frac{\left(\frac{((\theta_{optimise} - \theta_{theo}) + 100) \times \theta_{optimise}}{\theta_{theo}} \right)}{(\theta_{optimise} - \theta_{theo}) + 100} \right]
\end{aligned}
\tag{3.30}
$$

where, $\theta_{theo} = \begin{bmatrix} \mu_D \\ \sigma \\ \lambda \\ q_1 \\ q_2 \end{bmatrix}, \theta_{est} = \begin{bmatrix} \widehat{\mu_D} \\ \widehat{\sigma} \\ \widehat{\lambda} \\ \widehat{q_1} \\ \widehat{q_2} \end{bmatrix}, \theta_{optimise} = \begin{bmatrix} \overbrace{\mu_D} \\ \overbrace{\sigma} \\ \overbrace{\lambda} \\ \overbrace{q_1} \\ \overbrace{q_2} \end{bmatrix}$

and θ_{theo} is directly extracted from (Mastro (2013),p.45).

However, the parameter accuracy test is the only test that can be performed in understanding the empirical data modelled. Other tests involve the distribution properties' and goodness-of-fit tests. According to Haas and Pigorsch (2009), leptokurtic data modelled under financial scenario-based method are usually fat-tailed distributed and have a kurtosis of 3 or greater. This kurtosis of 3 is statistical significant since it is one of the indicators which shows that a distribution is leptokurtic and it is easily attained from empirical data.

The moments defined in Table 3.2 are used to calculate the skewness and kurtosis used in testing the "data's properties" and its definitions are as follows,

$$\begin{aligned} Skewness &= \left[\frac{M_3}{M_2^{1.5}}\right], \text{and} \\ Kurtosis &= \left[\frac{M_4}{M_2^2} - 3\right]. \end{aligned} \tag{3.31}$$

As mentioned above, another test regarding the data is to test whether empirical data used in the modelling framework fits the jump-diffusion model technique employed. In this dissertation the Kolmogorov-Simirnov test is used since it is flexible according to Mastro (2013) findings as it can compare two unknown distributions. Mastro (2013) cited that, the test finds the largest difference D_n in the Cumulative Distribution Function (CDF) of two distributions. Mastro (2013) further state that for jump-diffusion based scenario, the test compares the binned empirical CDF to the theoretical jump-diffusion CDF to give a maximum vertical distance as

$$D_n = \sup_x |\Phi^{JD}(x) - \Phi^{data}(x)| \tag{3.32}$$

where $\sup_x$ is the suprenum distance of the set of distances (i.e. CDF of the jump-diffusion and CDF of data respectively as shown in equation (3.32)). Finally Mastro (2013) defines the maximum vertical distance as follows,

$$d_n = \sqrt{n} D_n, \tag{3.33}$$

where n is the sample size. This D_n and d_n are then used as key input parameters in the formal test of Kolmogorov-Smirnov in detremining the p-values; refer to Table 4.2. Detailed description is noted in Mastro (2013)'s research. Finally, the graphical tests for the Kolmogorov-Smirnov are shown and discussed further under chapter 4.

Implementation of the jump-diffusion model in generating simulated positions

The parameter estimation method discussed above is subsequently used in simulating $\widehat{f_i(t)}$ $\forall$ i=$0, 1, 2, \cdots, l+1$, see equation (3.16). The simulations are ran 100 times per "m" instruments contained in portfolio P analysed by the SES model and this process is conducted as follows:

$$\Theta_{est_{n\times 5}} = \begin{bmatrix} \widehat{\mu_{D_1}} & \widehat{\sigma_1} & \widehat{\lambda_1} & \widehat{q_{1_1}} & \widehat{q_{2_1}} \\ \widehat{\mu_{D_2}} & \widehat{\sigma_2} & \widehat{\lambda_2} & \widehat{q_{1_2}} & \widehat{q_{2_2}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \widehat{\mu_{D_m}} & \widehat{\sigma_m} & \widehat{\lambda_m} & \widehat{q_{1_m}} & \widehat{q_{2_m}} \end{bmatrix} \xrightarrow{\text{INPUT}} \widehat{f_i(t)} = \begin{cases} \widehat{f_0(t)} \times e^{(\underline{\widehat{\mu_D}} \times dt + \underline{\widehat{\sigma}} \times \omega \times \sqrt{dt} + \eta)} \ldots if(\lambda \times dt > \beta) \\ \widehat{f_0(t)} \times e^{(\underline{\widehat{\mu_D}} \times dt + \underline{\widehat{\sigma}} \times \omega \times \sqrt{dt} + 0)} \ldots if(\lambda \times dt < \beta) \end{cases} \tag{3.34}$$

where β ia the $UniDist(i)$. The process shown in equation (3.34) is done 100 times and then averaged out to reduce variance brought by the simulation process. However, averaging the simulated $\widehat{f_i(t)}$ does cause the model to under-predict due to the flattening of the jumps by the averaging process defined in D.2 under appendix D. To conclude, the $\widehat{f_i(t)}$ were then plugged in equation (3.15) in section 3.3.1 in order to calibrate risk number under the defined simulated-model in this section.

3.4 Other models

Chapter 3 has focused on describing the current financial risk measure (i.e. HVaR) and the proposed model (i.e. SES). However there are other models which the proposed method will be compared against to assess its validity and this section discusses these methods in depth. The HVaR modelling process (i.e. chapter 3, section 3.2.5), shortcomings (i.e. chapter 2, section 2.2 - 2.5) and numerical example (appendix B) have been discussed throughout the dissertation.

3.4.1 Historical Expected Shortfall

McNeil et al. (2005) defines the Historical Expected Shortfall (HES) model as outlined in definition 6 in section 1.6 except the returns are calculated from the historical data collected. This method operates on the same notions defined for HVaR but instead of taking the risk measure as defined in equation (3.15), with HES model all losses beyond equation (3.15) are taken as the risk measure. This implies that HES risk measure using equation (3.15) is calculated as follows,

$$HES_{\alpha} = \text{function}\,(x_1, \cdots, x_m | (\text{such that}) \cdots (x_1, \cdots, x_m) > VaR_{1-\alpha}(1)) \tag{3.35}$$

where $(x_1, \cdots, x_m)$ are losses. To re-emphasis on the equation 3.35 using the Profit and Loss (P&L) vector defined in equation (B.5) in appendix B; if VaR is -0.23 and then HES will be the function of -0.53 and -0.69. In practice, HES is the mean of all losses beyond $VaR_{1-\alpha}$, which implies in the case of the example in appendix B; $\text{HES}_{\alpha}(1) = \frac{(-0.53-0.69)}{2}$=-0.61. This implies that for the given portfolio in appendix B, in one day, there is an expectation of 0.61 maximum losses expected based on historical movements.

3.4.2 Historical-simulation VaR (HVaR)

Prior 2008 subprime crisis the implemented model in predicting market risk exposures was the "historical-simulation VaR model", however, post the financial crisis it was decided that this model is unable to predict risk due to its inabilities to handle complex financial movements caused by stresses in the economy and as well the model does not test the severity of the constructed loss distribution, F_L. This modelling process has been fully outlined in section 3.2 and appendix B. This model will be used as a deciding factor testing the ultimate predictive-power of the proposed model (i.e. SES model) in this dissertation.

3.5 Benchmarking

The model discussed in section 3.2.5 and section 3.4 outputs various risk measures. Another way of assessing the model benefits of the proposed method is by checking against developed models in practice to see what contribution the new model has. This can be achieved by comparing the means of the risk estimates output by the models by using the following model outlined by Kutner et al. (2005),

$$Y_{ijk} = \mu_{ijk} + \epsilon_{ijkm} \tag{3.36}$$

where μ_{ijk} are parameters and ϵ_{ijkm} are independent $N(0, \sigma^2)$ for all, $i = 1, \cdots, a$, $j = 1, \cdots, b$, $k = 1, \cdots, c$, and $m = 1, \cdots, n$. However this methodology of Analysis of Variance (ANOVA) is linear-based method of determining these means' comparisons. Therefore, the model shown above can model the hypotheses defined below.

- **Hypothesis 1**

$$H_0 : \mu_{SES_{mean}} = \mu_{HES_{mean}} = \mu_{HES_{median}}$$
$$H_1 : \mu_{SES_{mean}} \neq \mu_{HES_{mean}} \neq \mu_{HES_{median}}$$

- **Hypothesis 2**

$$H_0 : \mu_{SES_{median}} = \mu_{HES_{mean}} = \mu_{HES_{median}}$$
$$H_1 : \mu_{SES_{median}} \neq \mu_{HES_{mean}} \neq \mu_{HES_{median}}$$

- **Hypothesis 3**

$$H_0 : \mu_{VaR} = \mu_{HES_{mean}} = \mu_{HES_{median}}$$
$$H_1 : \mu_{VaR} \neq \mu_{HES_{mean}} \neq \mu_{HES_{median}}$$

However, Haas and Pigorsch (2009) stated in their paper that financial data modelled by the financial risk measure such as VaR are leptokurtic distributed and not normal. Therefore, the ANOVA testing method since is based on assumption that test samples are normally distributed it will not be feasible to use this method here. However, Kruskal and Wallis (1952) developed a non-parametric test which is used for comparing two or more groups which are non-normal distributed. This test is more appropriate for testing comparison among more than two groups. The null hypothesis is that the samples come from the same population against the alternative hypothesis that they come from different population and these hypotheses are cited from Kruskal and Wallis (1952). Furthermore, this test is the distribution-free version of the F-test in a one-way ANOVA. Kruskal and Wallis (1952) cited that, the statistical attributes of the Kruskal-Wallis test shown below are the main differentiator when compared to other methods, such as the Wilcoxon and the Wilcoxon-Mann-Whitney tests.

- H is the test statistics and $H = \left[\frac{12}{N(N+1)} \times \Sigma \frac{T_c^2}{n_c}\right] - 3 \times (n+1)$; where N is the total number of subjects, T_c is the rank total for each group and n_c is the number of subjects in each group;
- *df* is the degrees of freedom which is $(N-1)$ and where N is the number of groups compared. The *df* of Chi-squared distribution at 5% significance level is used in deciding which hypothesis to conclude; and
- If H $>V \sim Chi\_Squared_{(df,0.05)}$ or p-value >0.05, reject the null hypothesis.

3.6 Summary

This chapter presents mathematical algorithms of the current VaR approach employed in practice to model market risk. This is done to motivate changes incorporated in the proposed model under section 3.3.1. This chapter also gives detail description of other VaR approaches and reasons behind for their abandonment as risk management tools. Then, section 3.3.2 outlined other statistical tests that need to be conducted to assess the validity of the proposed model in full totality. Section 3.4 discussed other risk measures which will be compared against the proposed method to assess its predictability. In addition to this, other benchmarking test such as the Kruskal-Wallis are defined and will be used to further re-emphasize the validity of the model. Unlike chapter 2 which presented survey of literature concerning the research problem; however chapter 3's discussions are focusing on methodologies and tests that will be used in the dissertation to solve the problem posed in section 1.2.

Chapter 4

DATA ANALYSIS AND MODEL RESULTS

4.1 Introduction

This chapter present all empirical evidences gathered from mathematical formulations defined in chapter 3.

- Section 4.2 discusses the data infrastructure and extraction process followed to gather the data used;
- Then section 4.3, further outlines various data quality tests performed to assess the data creditability and its overall fit into the methodology defined in section 3.3.1;
- Section 4.4 discusses the empirical findings from the parameter validity test and conclude on whether parameters determined under Mastro (2013)'s algorithm are accurate based on the theoretical-basis of jump-diffusion models;
- Simulation results attained from using section 3.3.1's methodology are discussed in full and some of the reservation about the simulative process; and
- Finally, section 4.6 discuss the validation or benchmarking process defined in section 3.5 and as well few literature observations based on tests used to assess the proposed model's performance.

4.2 Data and data extraction process

Prior to discussing the model results attained using the methodologies defined in chapter 3, a brief discussion on data extraction and data clean-up processes will be conducted. The data used in the model was extracted from "Thomson Reuters" application and only Mining sector shares were extracted due to the following reasons:

- The aim of the dissertation is to show that the SES model proposed mitigates all shortcomings seen in VaR modelling process during a financial crisis. Therefore instruments which were most vulnerable during the 2008-2009 subprime crisis in South Africa were selected and most of these instruments were from the mining sector.
- From the "Top40" share index, the shares with the highest volatility parameter was selected and this test as well mostly chosen the mining shares.
- The final attribute used in ascertaining which instruments to use in the model; equities/shares and options have been researched intensively in literature and the prices of these instruments are reactive to economic climate changes which fit the objective of the required data. However "option products" were excluded from this research since their historical data is limited to build a solid simulation model.

These were the main criteria used in selecting the set of instruments used in this empirical illustration of the modelling process defined clearly in chapter 3. The share/equity portfolio P comprises of the following shares:

Table 4.1: Modelling data description

Parameters	**Formulation**	**Description**
AGL	Anglo American PLC	Mining sector
BIL	BHP Billiton	Mining sector
AMS	Anglo American Plat Ltd	Mining sector
ANG	Anglogold Ashanti Limited	Mining sector
ARI	African Rainbow Minerals Limited	Mining sector
IMP	Impala Platinum Holdings Limited	Mining sector
OML	Old Mutual PLC	Financial & Insurance sector
GFI	Gold Field Limited	Mining sector

However the application used to retrieve the data excludes all holidays and weekends from the data since South African financial markets are non-functional during these events. The only data clean-up issue is that, the date variable is duplicated per instrument extracted from "Thomson Reuters" application; this is rectified by removing other duplicated date vectors from the raw dataset. The information extracted stretches from 31-August-2008 to 04-March-2014, which implies that, it also incorporated the subprime crisis period which forms the crux of this investigation.

4.3 Data fit tests

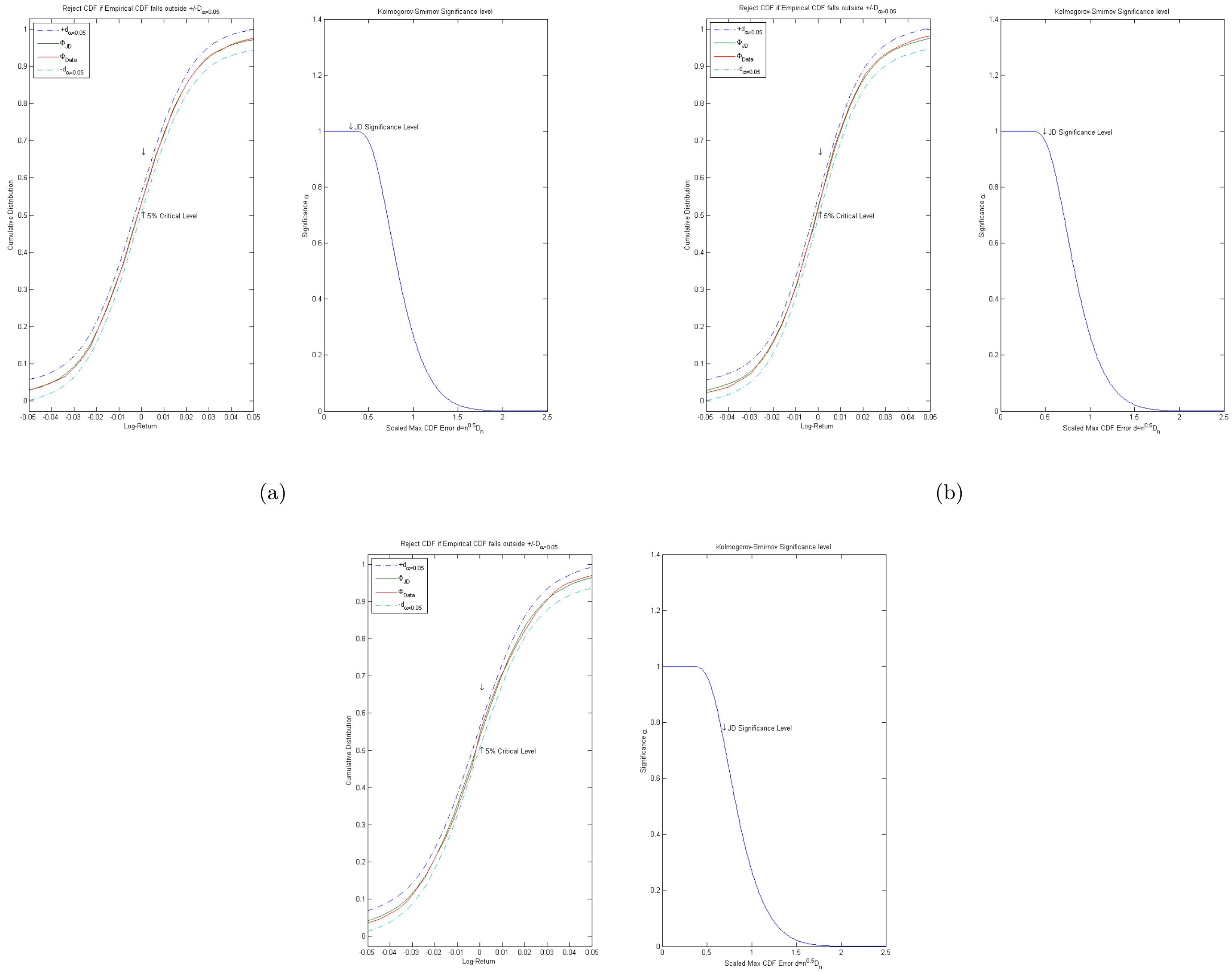
The initial step in any modelling process is checking whether the collected data mentioned in section 4.1 fits the modelling structure selected. If the modelling data does not fit the respective model, it implies that certain data transformation needs to be implemented so that it fit or select another model more appropriate for the data collected. The first test conducted is the Kolmogorov-Smirnov test, see the table below:

Table 4.2: Kolmogorov-Smirnov: Goodness-of-fit test

Share code	D_n	d_n	$D-\alpha$	**p-value**
AGL	0.005710	0.278935	0.027842	0.999999
BIL	0.009160	0.469408	0.027842	0.980233
AMS	0.013663	0.667383	0.027842	0.764616
ANG	0.008111	0.396199	0.027842	0.997557
ARI	0.026679	1.303199	0.027842	0.066968
IMP	0.014124	0.689923	0.027842	0.727939
OML	0.010321	0.504123	0.027842	0.961247
GFI	0.009788	0.478104	0.027842	0.976253

As stated in Mastro (2013) that significance level for this test is set at 5% and any p-value lesser than 5% it implies that H_0 is rejected. However, the tested null hypothesis is $F_{JD}(x) = F_{data}(x), \forall\, x$, where F denotes Cumulative Density Function (CDF). All p-values in Table 4.2 are above the significance level which suggests that null hypothesis is not rejected. This suggests that modelling data defined in section 4.1 fits the theoretical jump diffusion model. Therefore the model is deemed a proper fit to the data.

The second part of the Kolmogorov-Smirnov (K-S) test is to construct theoretical band under which data is expected to belong if it's fully represented by the jump-diffusion model. The construction of the graphical test uses all attributes denoted in Table 4.2. The graphs below are only showed for six of the eight shares analysed in this study. These graphs confirm that the model does fit the data properly.

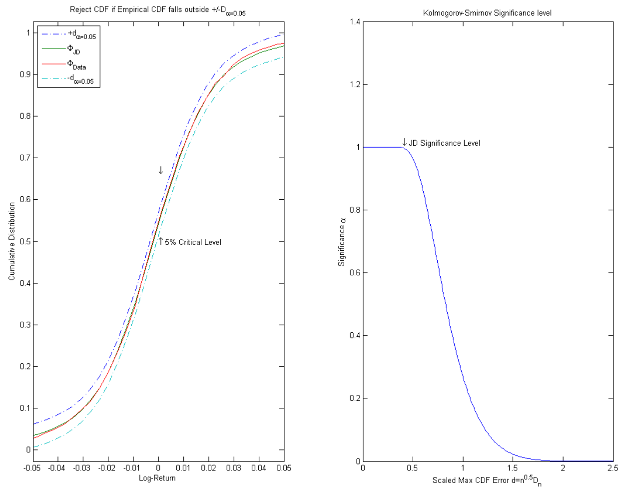


(a)

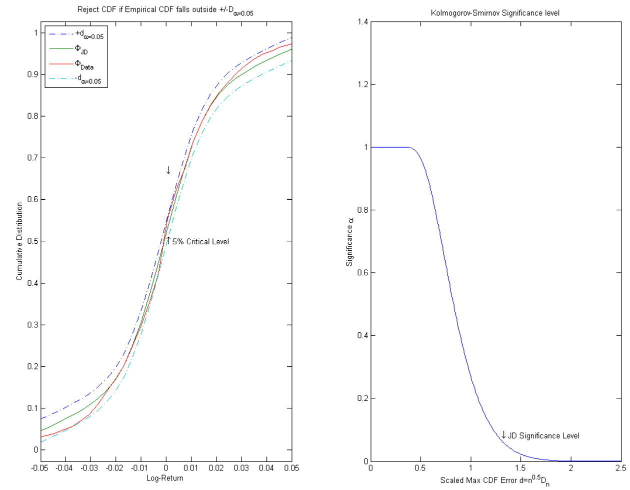
(b)

(c)

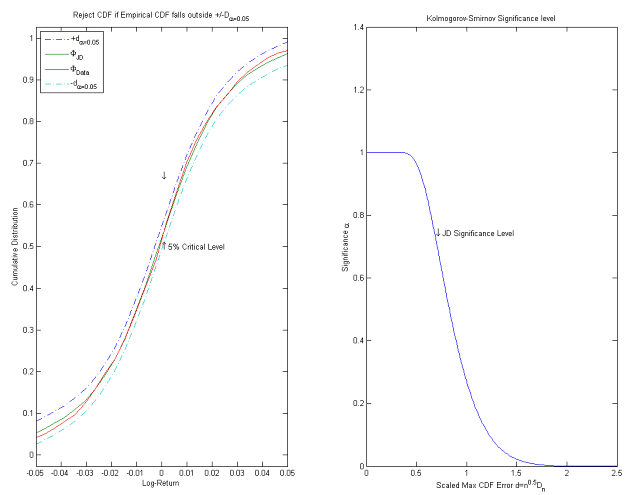
Figure 4.1: (a) AGL Kolmogorov-Smirnov test (b) BIL Kolmogorov-Smirnov test (c) AMS Kolmogorov-Smirnov test



(a)



(b)



(c)

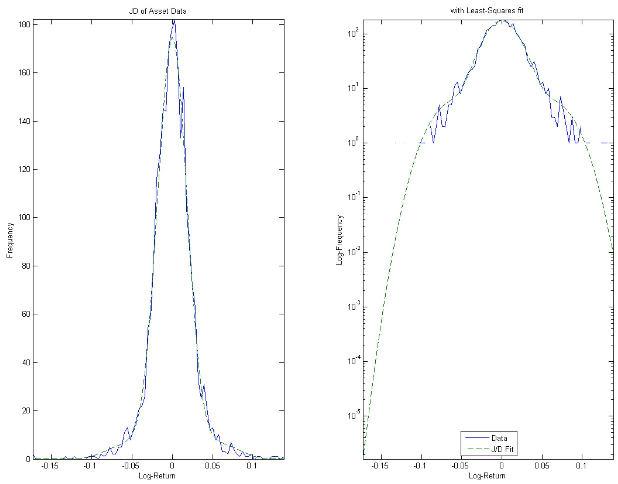
Figure 4.2: (a) ANG Kolmogorov-Smirnov test(b) ARI Kolmogorov-Smirnov test(c) IMP Kolmogorov-Smirnov test

Another important attributes mentioned in Mastro (2013) literature is that financial data used tend not to follow the traditional Gaussian distribution as prescribed in VaR model. VaR model is sub-additive if the log returns as outlined in definition 15 in section 1.6 are elliptically distributed and Mastro (2013) argues that the return will be far from this elliptic distribution assumption. Financial data in most cases tends to be modelled by heavy-tailed distributions; Mastro (2013) suggests that in proving that jump-diffusion model contains this attribute, the kurtosis and skewness of the data needs to be compared against the data. If jump-diffusion distribution is heavy-tailed then these values should be close to each other. The computation of skewness and kurtosis is sensitive to jumps and other outliers in the data, hence the implementation of the least-squares method. This method defined from equation (3.31) minimises least squares objective which show data as positive whereas the theoretical distribution is shown as a negative. The following table shows the skewness and kurtosis calculated under this methodology.

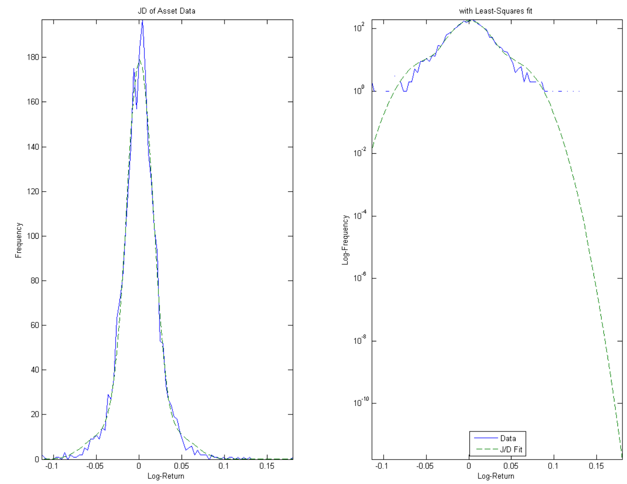
Table 4.3: Kurtosis and skewness: The fat-tailed analysis

SHARE CODE	$skewness_{data}$	$kurtosis_{data}$	$skewness_{estimated}$	$kurtosis_{estimated}$
AGL	-0.083915	4.659652	0.062636	5.349719
BIL	0.273800	4.634329	-0.057673	4.547337
AMS	-0.327286	2.921829	0.002345	3.911881
ANG	0.328662	3.264361	0.072149	4.138707
ARI	-0.404772	5.122530	-0.057438	4.635580
IMP	-0.287252	2.764744	-0.183598	3.214796
OML	-0.157283	7.343557	-0.094876	7.684796
GFI	0.172834	3.817916	0.222672	4.185137

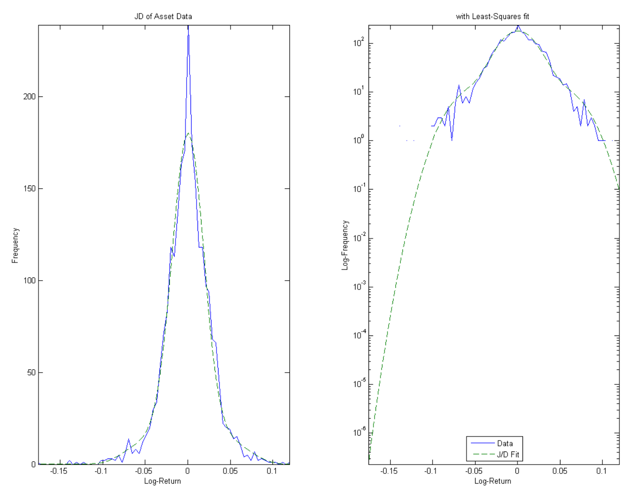
The data's distribution and jump-diffusion theoretical distribution have similar fat-tail trend as per definition under Mastro (2013), see the skew and kurtosis in Table 4.3. This concludes that the data is fitting the jump-diffusion modelling process. This means that jump-diffusion model can be implemented in estimating historical data with market jumps as shown in equation (3.16). The graphs below confirm the conclusion drawn from Table 4.3.



(a)

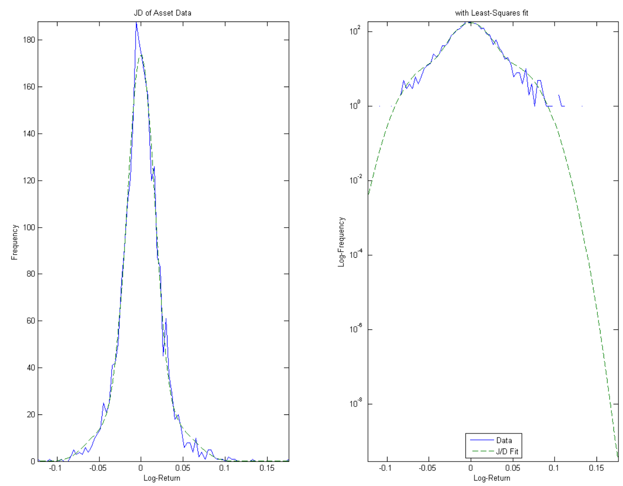


(b)

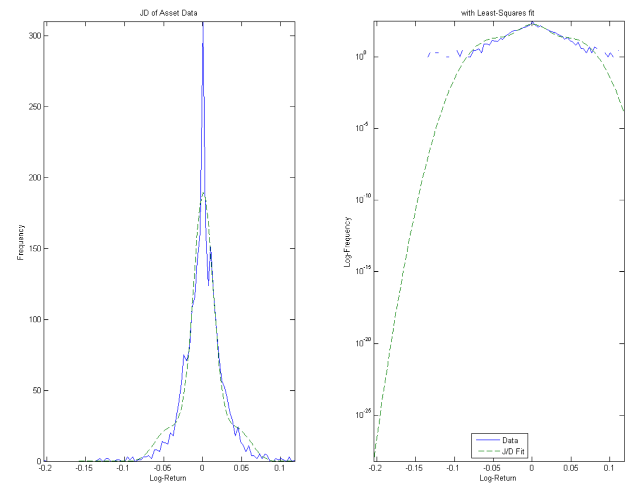


(c)

Figure 4.3: (a) AGL Least-Square Fit (b) BIL Least-Square Fit (c) AMS Least-Square Fit



(a)



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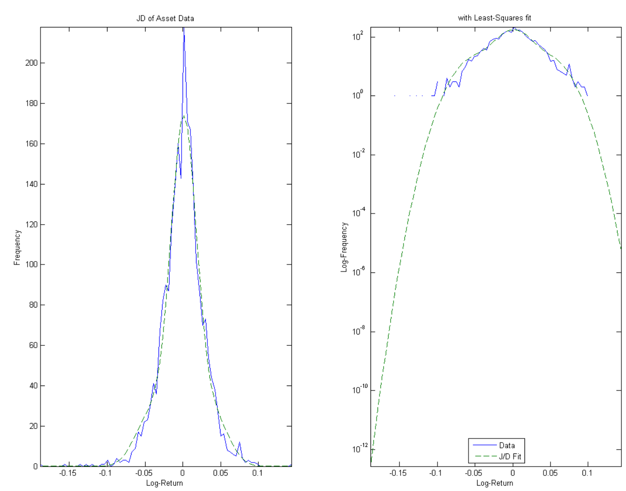


(c)

Figure 4.4: (a) ANG Least-Square Fit (b) ARI Least-Square Fit (c) IMP Least-Square Fit

4.4 Parameter-estimation method results

In this research there were three jump-diffusion models considered, namely, Pareto-Beta Jump-Diffusion (PBJD), Double Exponential Jump-Diffusion (DEJD) model and Lognormal Jump-Diffusion (LJD)/Merton (1976)) model, see appendix C for more description. In appendix C it has been mentioned that movements of the returns are used in calculating the parameters, however the detailed method followed is carved out under section 3.3.2. The complex jump-diffusion models like the PBJD and DEJD models require additional parameters to be able to illustrate the complex dimensions. In literature, non-conventional methods are used in estimating the density functions since their jump magnitudes are modelled by parametric distributions, for example under PBJD model up-jump is modelled by Pareto(η_u) and down-jump its modelled by Beta(η_d,1). Since these estimation methods have no defined statistical method in checking its validity without constructing a non-parametric method. On this basis both PBJD and DEJD models have been excluded from the dissertation. In Mastro (2013), he suggests that parameters can be validated by estimating these parameters from data and subsequently calculate another set of parameters from an optimising tool using the initially estimated parameters from data as initial values for the LJD model. Then if these outcomes are similar it implies that the parameter estimation method is valid.

Table 4.4: Estimated parameters for LJD model as defined in Table 3.1

SHARE CODE	**Initial Price ($f_0(t)$ in cents)**	$\widehat{\mu_D}$	$\widehat{\sigma}$	$\widehat{\lambda}$	$\widehat{q_1}$	$\widehat{q_2}$
AGL	15,530	0.202044	0.399417	4.01173	-0.173	0.138586
BIL	6,215	0.129938	0.365069	3.378299	-0.11421	0.179971
AMS	28,724	0.231226	0.428329	3.272727	-0.17589	0.119529
ANG	22,000	-0.02752	0.393059	3.695015	-0.12327	0.175643
ARI	3,500	0.445824	0.408387	4.117302	-0.20394	0.118198
IMP	6,981	0.208941	0.439604	2.533724	-0.18851	0.143432
OML	1,290	0.214252	0.353465	5.173021	-0.16511	0.146549
GFI	7,079	-0.02071	0.438176	3.695015	-0.15811	0.193927

Table 4.5: Optimisation's parameters for LJD model as defined in Table 3.3

SHARE CODE	**Initial Price ($f_0(t)$ in cents)**	$\widehat{\mu_D}$	$\widehat{\sigma}$	$\widehat{\lambda}$	$\widehat{q_1}$	$\widehat{q_2}$
AGL	15,530	0.05443	0.286119	32.98347	-0.08232	0.084486
BIL	6,215	0.244341	0.246614	48.48774	-0.06727	0.065639
AMS	28,724	0.088596	0.294777	52.6247	-0.07348	0.073564
ANG	22,000	-0.03535	0.259353	57.6989	-0.06591	0.068024
ARI	3,500	0.266567	0.19687	95.61985	-0.06367	0.06239
IMP	6,981	0.346976	0.260357	98.39988	-0.06398	0.05874
OML	1,290	0.169895	0.213611	38.96582	-0.07832	0.0761
GFI	7,079	-0.20919	0.295252	53.57953	-0.07235	0.079899

The parameters highlighted in Table 4.4 and Table 4.5 are the parameters calculated under Mastro (2013) modelling framework as specified in section 3.3.2. However, we need to assess whether these parameters are accurate, especially the parameters in Table 4.4; this is achieved by comparing these parameters to theoretical parameters extracted from Mastro (2013) which states: $\mu = 0.11$, $\sigma = 0.25$, $\lambda = 5.00$, $q_1 = -0.14$ and $q_2 = 0.15$. Equation (3.30) will be used in calculating Π_1 and Π_2 which are the respective error rates for the parameters estimated using empirical data and under optimisation technique as specified in equation (3.29).

Table 4.6: Error rates for estimated parameters - LJD model in absolute value notation

SHARE CODE	$\widehat{\Pi_{\mu_D}}$	$\widehat{\Pi_{\sigma}}$	$\widehat{\Pi_{\lambda}}$	$\widehat{\Pi_{q_1}}$	$\widehat{\Pi_{q_2}}$
AGL	1.84%	1.60%	0.80%	1.24%	0.92%
BIL	1.18%	1.46%	0.76%	0.82%	1.20%
AMS	2.10%	1.71%	0.65%	1.26%	0.80%
ANG	0.25%	1.57%	0.74%	0.88%	1.17%
ARI	4.05%	1.63%	0.82%	1.46%	0.79%
IMP	1.90%	1.76%	0.51%	1.35%	0.96%
OML	1.95%	1.41%	1.03%	1.18%	0.98%
GFI	0.19%	1.75%	0.74%	1.13%	1.29%

Table 4.7: Error rates for optimisation's parameters - LJD model in absolute value notation

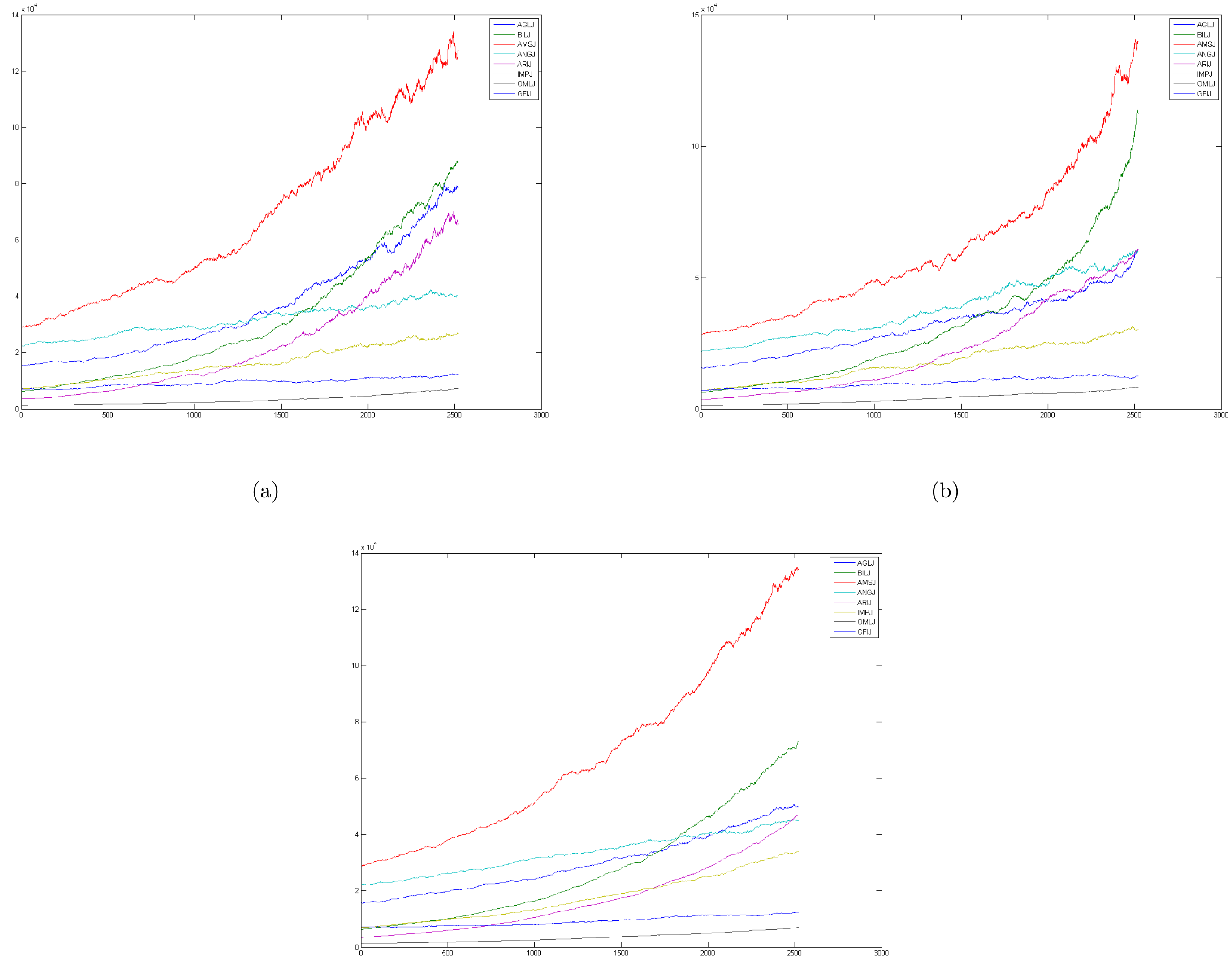
SHARE CODE	$\widehat{\Pi_{\mu_D}}$	$\widehat{\Pi_{\sigma}}$	$\widehat{\Pi_{\lambda}}$	$\widehat{\Pi_{q_1}}$	$\widehat{\Pi_{q_2}}$
AGL	0.49%	1.14%	6.60%	0.59%	0.56%
BIL	2.22%	0.99%	9.70%	0.48%	0.44%
AMS	0.81%	1.18%	10.52%	0.52%	0.49%
ANG	0.32%	1.04%	11.54%	0.47%	0.45%
ARI	2.42%	0.79%	19.12%	0.45%	0.42%
IMP	3.15%	1.04%	19.68%	0.46%	0.39%
OML	1.54%	0.85%	7.79%	0.56%	0.51%
GFI	1.90%	1.18%	10.72%	0.52%	0.53%

In section 3.3.2 it has been mentioned that significant level is set at a range of 2% to 5%, which means all error rates above this range are not accurate which suggest the parameters determined deviate significantly between the fitted-model's assumptions and theoretical parameters. According to Table 4.6 and Table 4.7 the parameters obtained under both regimes, that is, empirical estimation method and optimisation estimation method are mostly within the set range above. The error rates highlighted in red in Table 4.7 are due to the over-conservativeness of the optimisation technique used. In summary, these two tables suggest that parameters used in the SES model are accurate and consistent with the jump diffusion model employed. Lastly, results obtained here are in full agreement with the literature of Mastro (2013) since his using jump diffusion model in pricing financial instruments as studied in this dissertation.

4.5 Simulation results - data transformation as per equation (3.16)

The parameters calculated in Table 4.4 are plugged into simulations to improve the predictability of the transformed historical data with jumps as defined in equation (3.16). With simulations we have another problem of inconsistencies since each simulation is different from another. In order to solve this problem, we need to run the simulation multiple times and averaging the different paths. Graph (a) and graph (b) in Figure 4.5 denotes averaging paths of the modelled Mining shares calculated using empirically estimated parameters as shown in Table 4.4 whereas graph (c) in Figure 4.5 shows the averaging paths when optimisation's parameters are used. The reason behind the inclusion of graph (c) is to test whether the optimisation technique defined in equation (3.29) works optimally, see the graphs below.

Note that graph (b) and graph (c) in Figure 4.6 shows the level and the severity of the jumps per simulation conducted. These high volatilities are good in measuring risk but if implemented as it is the model will predict risk conservatively and unreliably per simulation. Therefore in order to restore the model's predictability these various simulations are averaged. The averaged paths denoted in graph (a) and graph (b) in Figure 4.5 are calculated using Table 4.4 whereas average paths denoted by graph (c) in Figure 4.5 and graph (a) in Figure 4.6 are calculated using Table 4.5. The optimisation technique's average paths graphs are less volatile than the average paths determined by Table 4.4 parameters. This implies that risk models using Table 4.4 will be more conservative than risk models using Table 4.5 parameters. Another observation made is that averaging in both cases restores the homoscedasticity but it reduces the vulnerability brought by the jump component.

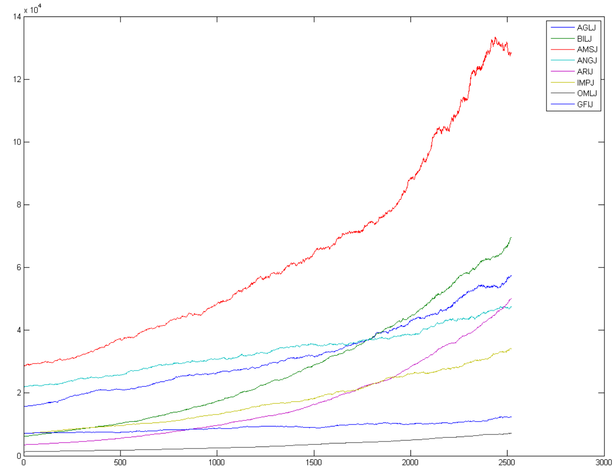


(a)

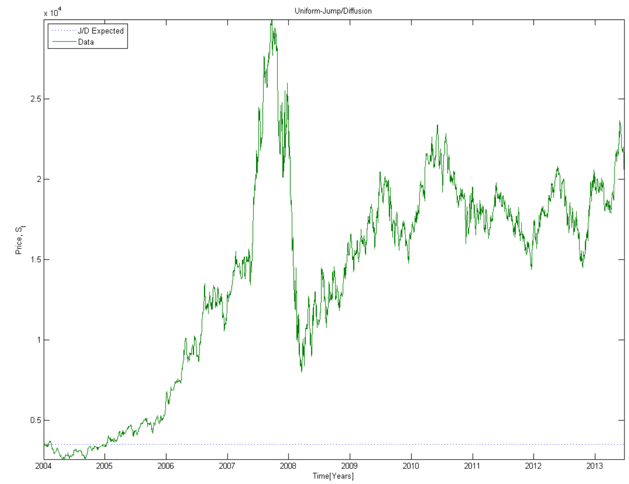
(b)

(c)

Figure 4.5: (a) Averaging paths for est. param simulation 1 (b) Averaging paths for est. param simulation 2 (c) Averaging paths for opt. param simulation 1



(a)



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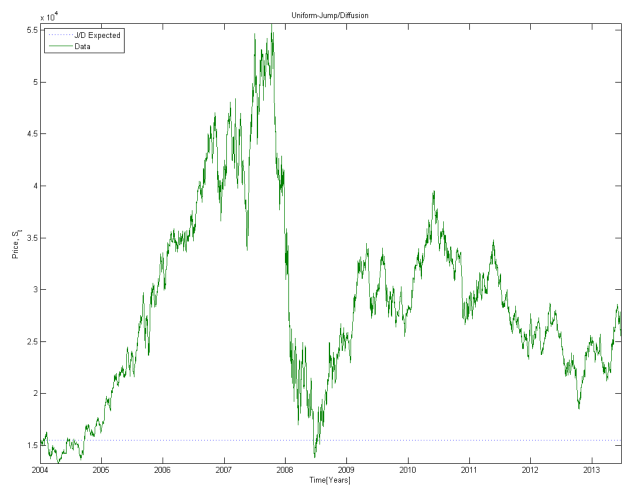


(c)

Figure 4.6: (a) Averaging paths for opt. param simulation 2 (b) Simulation 1 not averaged (c) Simulation 2 not averaged

4.6 SES modelling validation test results

In the previous section it was noted that averaging of the simulated paths might fix homoscedasticity of the SES model defined in section 3.3.1 but it worsens the predictability of the model in estimating losses. Therefore in order to restore this effect without losing the homoscedasticity retained by the averaging process; the overall risk numbers should be multiplied by a constant factor τ assumed to be 15 in this modelling process. A more model-based method should be investigated in follow-up research for calculating τ.

Furthermore, the SES model calculates risk numbers using mean and median approaches; the reason for this is to find the most appropriate-way of calculating the risk more accurately and fulfilling the objective of the research. The mean approach includes outlying elements in the constructed loss distribution which could be seen as a positive since risk or loss during a crisis occurs at the tail-end of the distribution. On the other hand the median approach excludes outlying elements in the loss distribution which might be a shortcoming if the worst possible loss is incurred. Several tests are conducted below in testing the predictability of the model under these approaches.

4.6.1 Kruskal-Wallis results

As mentioned in section 3.5 that Kruskal-Wallis test will be used to assess whether the risk measures output by the models discussed in section 3.2.5 and section 3.4 are independent or not. The various implications of the hypotheses have been fully discussed in section 3.5. The results of the Kruskal test are shown in the table below:

Table 4.8: Kruskal-Wallis results

Null Hypothesis	**Alternative Hypothesis**	**p-value**	**conclusion**
$H_0 : \mu_{SES_{mean}}, \mu_{HES_{mean}}$ and $\mu_{HES_{median}}$ are sampled from the same distribution.	$H_1 : \mu_{SES_{mean}}, \mu_{HES_{mean}}$ and $\mu_{HES_{median}}$ are not sampled from the same distribution.	p-value=0.971 (H=0.059,df=2)	Do not reject H_0
$H_0 : \mu_{SES_{median}}, \mu_{HES_{mean}}$ and $\mu_{HES_{median}}$ are sampled from the same distribution.	$H_1 : \mu_{SES_{median}}, \mu_{HES_{mean}}$ and $\mu_{HES_{median}}$ are not sampled from the same distribution.	p-value=0.971 (H=0.059,df=2)	Do not reject H_0
$H_0 : \mu_{VaR}, \mu_{HES_{mean}}$ and $\mu_{HES_{median}}$ are sampled from the same distribution.	$H_1 : \mu_{VaR}, \mu_{HES_{mean}}$ and $\mu_{HES_{median}}$ are not sampled from the same distribution.	p-value=$7.544e^{-10}$ (H=46.62,df=2)	Reject H_0

Continued on next page

Table 4.8 – *Continued from previous page*

Null Hypothesis	**Alternative Hypothesis**	**p-value**	**conclusion**
H_0 : $\mu_{VaR}, \mu_{SES_{mean}}$ and $\mu_{SES_{median}}$ are sampled from the same distribution.	H_1 : $\mu_{VaR}, \mu_{SES_{mean}}$ and $\mu_{SES_{median}}$ are not sampled from the same distribution.	p-value=$5.102e^{-13}$ (H=56.61,df=2)	Reject H_0

Table 4.8 aforementioned concludes the following:

- Hypothesis 1 and hypothesis 2 do not reject the H_0 which implies that the SES models and HES models are sampled from the same empirical distribution. This conclusion is mainly because both SES models and HES models formulations are based on the same modelling data (i.e. historical prices of key Mining shares) and the model outputs are sampled from similar percentile point of the empirical distribution for both approaches. In summary, these outputs noted of the aforementioned hypotheses suggest that statistically, the empirical distributions where SES models and HES models are sampled from were similar if not the same. However, further investigation should be conducted to assess whether the SES models or HES models when compared are there any statistical benefits seen since this Kruskal-Wallis test only states the sampling distribution among the tested groups are the same or not (see section 4.5.2); and

- Hypothesis 3 and hypothesis 4 rejects the H_0, which implies that both SES and HES model outputs are sampled from different empirical distributions when compared to historical-simulated VaR model outputs. However this finding should not be alarming as in McNeil et al. (2005) states that any form of Expected Shortfall (ES) model is simply a severe percentile reading of an empirical distribution than VaR. As stated previously that Kruskal-Wallis does not provide the causality and therefore at this stage no statistical meaning can be drawn from Table 4.9's hypotheses results.

4.6.2 Predictability tests

Once a market risk measurement model (i.e. SES model) has been constructed, there is a strong requirement to verify and quantify its accuracy. This is achieved through back testing which compares the forecasts from the SES model to the actual portfolio returns. An exception occurs when the actual loss exceeds the SES (i.e. SES_{median} or SES_{mean} risk numbers) by checking the frequency of the exceptions, the quality of the SES model can be determined.

However having a model with absence of any exceptions is highly desirable since it highlights the model's conservativeness required in a risk measure for a theoretical perspective.

These modelling outputs however have business implications since these risk estimates feed into the regulatory capital process. Regulatory capital process is a process whereby financial institutions calculates minimum reserve it needs to keep with the financial regulator given its risk exposures. If risk estimates from models and other process is generally high it will result in the reserve kept by a particular financial institution being relatively high compared to its peers. High capital charge/reserve halt business growth since the company or organisation will have less money to run its function due to high capital charge. Therefore an appropriate risk appetite regarding exceptions permissible in the model should be determined.

Therefore to test the accuracy of the SES forecast one has to determine whether the frequency of the exceptions is equal or close to the desired confidence level. This is achieved by defining a time series

$$\{I_k\}_{k=1}^{N} \tag{4.1}$$

for each of the N days over which the back testing is run.

The variable I_k is actually an indicator function that takes on the value one on the day that an exception occurred and is otherwise zero.

$$I_k = \begin{cases} 1 & Actual_{losses} > SES_\alpha \\ \\ 0 & Actual_{losses} < SES_\alpha \end{cases} \tag{4.2}$$

If the SES model is solid in terms of prediction, its sequence of exceptions noted should be independently distributed with the same probability of occurrence. Statistically, one can apply the binomial distribution to determine the probability of observing a certain number of exception in a given sample size.

Assume that the time horizon is $\gamma = 1$ and the confidence interval is $(1-\alpha) = 99\%$. If the SES model is accurate, then the probability of the SES being exceeded on any given day is

$$p = \alpha. \tag{4.3}$$

The expected number of exceptions is therefore defined to be

$$X = pN. \tag{4.4}$$

Graph (c) of Figure 4.8 shows the overall back testing graph for the analysed period which stretches from 31 August 2004 to 04 March 2014 (i.e. 2388 observations per instruments extracted). Using equation (4.4), it follows that desirable exceptions expected from the SES model is $0.01 \times 2388 \approx 24$, the table below tabulates the exceptions for the modelling period as shown in graph (c) of Figure 4.8.

Table 4.9: Error rates for optimisation's parameters - LJD model in absolute value notation

MODEL	DISERABLE EXCEPTIONS	MODEL EXCEPTIONS	DIFFERENCES
SES_{median}	24	34	10
SES_{mean}	24	30	6
HES_{median}	24	28	4
HES_{mean}	24	26	2
VaR	24	34	10

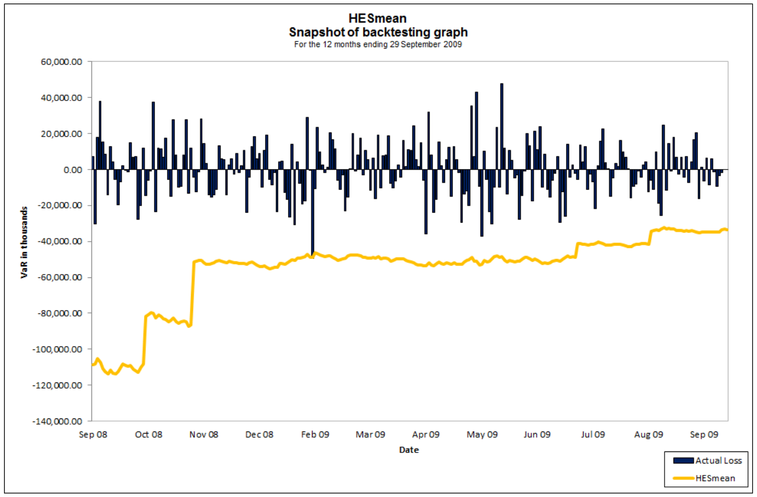
Despite the fact that Table 4.8 concludes that HES model's outputs and SES model's outputs are sampled from the same/similar empirical distribution; Table 4.9 that is, model exceptions table using the entire modelling dataset suggests that HES modelling processes is more conservative than SES modelling processes. However the aim of this dissertation is to answer the question which test whether or not would have SES modelling processes modelled the subprime risk better than VaR if implemented prior 2008? The SES modelling processes would have improved the under-prediction issue seen when VaR model was used. Table 4.10 shows the exception analysis during the subprime crisis (i.e. 25 September 2008 to 31 August 2010 = 483 observation per instrument extracted).

Table 4.10: Back testing results for the 2008 subprime crisis

MODEL	DISERABLE EXCEPTIONS	MODEL EXCEPTIONS	DIFFERENCES
SES_{median}	5	1	-4
SES_{mean}	5	0	-5
HES_{median}	5	2	-3
HES_{mean}	5	2	-3
VaR	5	3	-2

Although there are conflicting conclusions drawn from the Kruskal-Wallis test and the modelling exception test conducted in Table 4.9 and Table 4.10. An important aspect that should be drawn from this is that, overall both HES models and SES models would have soften the blow if used instead of VaR in prior 2008 subprime crisis. Another conclusion that could be drawn is that, SES modelling process due to the jump-diffusion component is better equipped in calibrating financial risk under stressed economic climate, hence the few model exceptions noted in Table 4.10. Tying these conclusions back to the literature study in chapter 2, it follows that:

- McNeil et al. (2005) states that any Expected Shortfall (ES) model is simply a severe percentile reading of the same empirical distribution used to sample VaR; Table 4.9 and Table 4.10 suggest that HES and SES models which are special cases for expected shortfall models are better at predicting financial risk since their estimation are around the actual losses confidence interval, especially in stressed circumstances.
- The conservativeness in the estimation process is in agreement with Acerbi and Tasche (2002), Yamai and Yoshiba (2005) conclusions concluded in their respective papers.
- Lastly, the backtesting graphs from Figure 4.7 and Figure 4.8 are in agreement with conclusions aforementioned.



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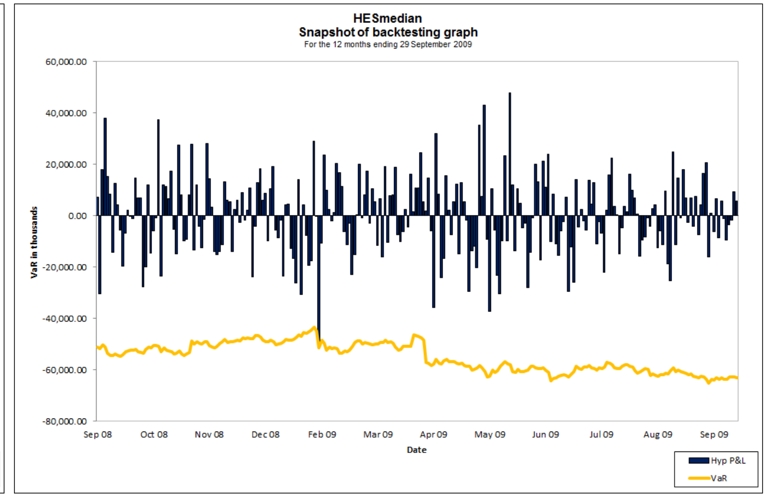

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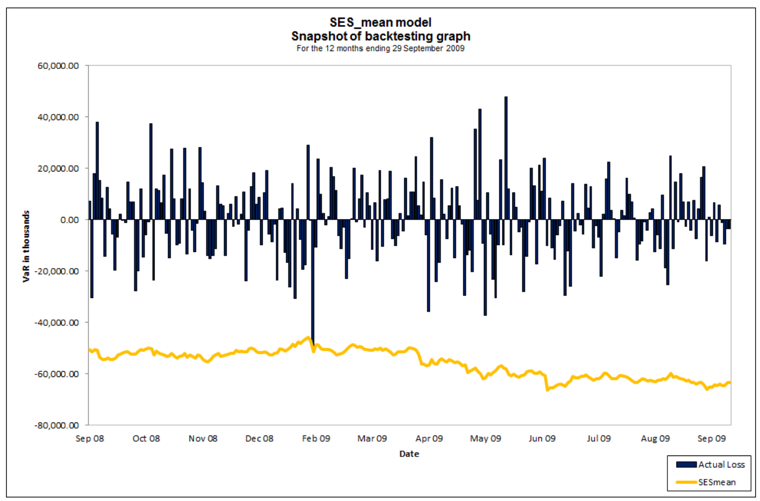
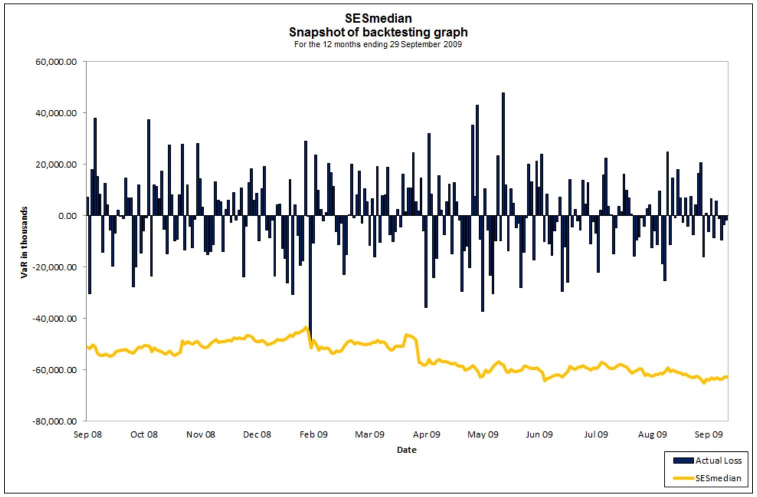
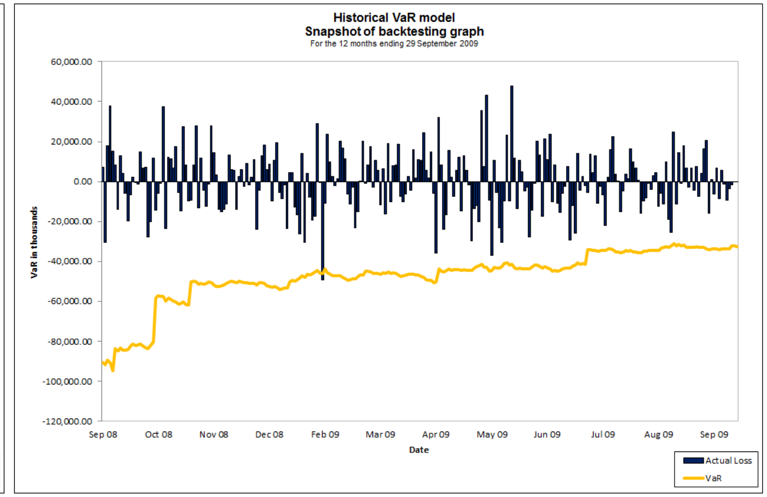


(c)

Figure 4.7: (a) Backtesting graph of HES_{mean} (b) Backtesting graph of HES_{median}(c) Backtesting graph of SES_{mean}



(a)



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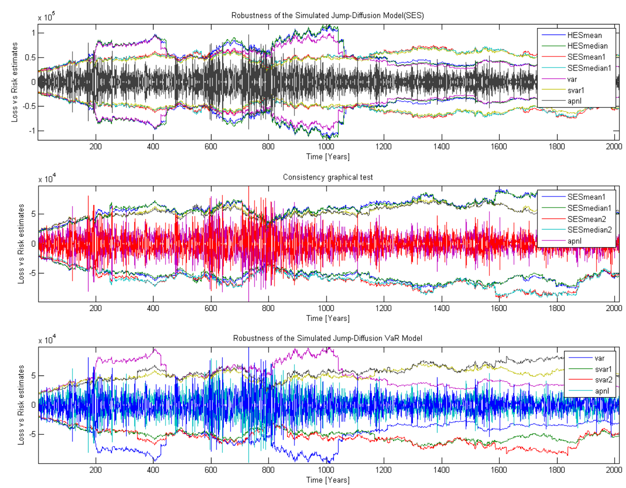

(c)

Figure 4.8: (a) Backtesting graph of SES_{median} (b) Backtesting graph of VaR (c) Overall backtesting graphs for all models

4.6.3 Other backtesting perspective

Based on Table 4.10, the SES model would have been a statistically-better model than the traditional HVaR model during the subprime crisis. Table 4.10 proves that SES model is capable in solving some of the HVaR problems defined in section 2.2 and section 2.6. However, if we look at the differences between the other models used in this dissertation is small. According to Kerkhof and Melenberg (2004), it might be that, the SES model is not backtestable just like the HES model as shown in McNeil et al. (2005). Again using Ziggel et al. (2014)'s argument, it might be argued that the reason there are no backtesting test for ES-based model is because there are limited literature. Although Ziggel et al. (2014)'s methodology uses the same philosophy as defined by Christoffersen (1998). It is more conservative in the sense that it includes stresses in its performances test of checking VaR. That being said there is a scarcity of literature addressing backtesting methodologies for an Expected Shortfall type of model, hence the use of the methodology defined in section 4.5.2.

However, Kerkhof and Melenberg (2004) argues that it is difficult to backtest expected shortfall models due to misalignment between actual and estimated losses (i.e. estimated losses are a function of losses whereas actual losses are not a combination calculated through mathematical formulae). But through the Hadamard differentiable derivations, the backtesting of such measures is possible and this is supported by the work of Kerkhof and Melenberg (2004). Therefore, based on Hadamard differentiable derivation, backtesting can be performed through the standard normal test statistic under null hypothesis and which is defined as $S_T = \sqrt{T}\frac{(\gamma(Q_T)-\gamma(Q))}{\sqrt{V}}$. This is where $\gamma(Q)$ is the estimated risk, $\gamma(Q_T)$ is the true risk and V is the variance movements of the "*actual risk*" in the observed period. The SES modelling framework is a stressed version of the HES model and therefore it's also Hadamard differentiable. For this test, the following significant level α will be used in accordance to Kerkhof and Melenberg (2004) findings which show that, median shortfall with level 2α (excluded not comparable) and the mean shortfall with level 2.5α corresponds to the HVaR with level α. Table 11 shows the Type I error results in backtesting the various models discussed in this paper $\alpha = 0.01$ and for $T = \{250, 500\}$.

Table 4.11: Type I error (in percentage) regarding backtesting model estimates

T	$VaR_{0.01}$	$HES_{0.025}$	$SES_{mean_{0.025}}$
250	4.81	5.14	5.38
500	2.91	9.38	9.33

In concluding remarks by using Kerkhof and Melenberg (2004) methodology and their results for $VaR_{0.01}$ and $HES_{0.025}$ in Table 11, it's clear that $SES_{mean_{0.025}}$ model have large Type I error. This implies 5.38% (for $T = 250$), the model will reject null hypothesis when it's true (i.e. null hypothesis - no model exception), see Table 4.11.

4.7 Summary

The overall conclusion which can be drawn from empirical findings outlined in chapter 4 is that, the SES model developed in this dissertation is generally conservative than the models defined in section 3.4. Despite, these conservativeness suggested by Table 4.9 and Table 4.10 some of the scholars have few reservations on the use of expected shortfall measures. Kerkhof and Melenberg (2004) stated in section 4.6.3 that, these reservations are mainly due to the fact that expected shortfall measures are usually hard to backtest.

To overcome the limitation of the ES or ES-based measures not being backtestable, Acerbi and Szekely (2014) stated that in October 2013 the Basel Committee opted to replace HVaR with ES but still keep HVaR measure for backtesting purposes. The financial institutions and financial regulators are arguing that, this is feasible since both HVaR and ES risk measures are derived from the same empirical non-parametric distributions at different percentile levels. Also in section 4.6.3, scholars such as Ziggel et al. (2014) argued that, the ES is not backtestable due to limited literature addressing this concept. As reiterated above that SES model is statistical-better than other models noted in section 3.4 and financial data selected in Table 4.1 fits the jump-diffusion modelling framework with minimal deviations; see Figure 4.2 and Figure 4.3. That being said, SES model requires an extensive follow-up work looking into the limitation outlined throughout the dissertation.

Chapter 5

CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

The Historical-simulation Value-at-Risk (HVaR) modelling approach under-predicts market risk due to inefficiencies in the model as indicated in section 1.2. This observation was corroborated by the mathematical inefficiencies of HVaR model outlined in section 2.2 and section 2.3. The aim of this dissertation is to establish tail-sensitive risk measure that calculates market risk using the designated sample size without inflating the estimation error. This chapter concludes this dissertation by highlighting what has been achieved by this dissertation, assessing the limitation that constrain the dissertation and suggesting future directions for further research.

5.2 Conclusion

Financial crises, especially the 2008 subprime mortgage meltdown showed HVaR model's shortcomings, which were:

- Lacks ability to evaluate tail risk;
- HVaR have poor aggregation properties; and
- Unable to incorporate new information into the model as it materialises.

Artzner et al. (1997, 1999) and McNeil et al. (2005) suggested that the ES measure is a better risk measure than VaR since it minimises the aggregation property's risk and the tail risk issues. But Yamai and Yoshiba (2005) stated that the HVaR and ES measures uses minimal sample size and therefore are unable to model the fluctuations required in such models. Furthermore, it has been uncovered that ES is not backtestable, see Kerkhof and Melenberg (2004) findings in their paper.

Therefore this study proposed an SES measure which takes into account the mishaps of using limited sample size in modelling market risk. This was achieved by adding a jump-diffusion methodology into the developed ES measure in Artzner et al. (1997, 1999) and McNeil et al. (2005) 's papers. Jump-diffusion models used stochastic processes, such as the Geometric Brownian Motion (GBM) and the Poisson process to calibrate expected jumps or discontinuities as suggested by Kou (2002) given historical movements of that asset or portfolio. These jumps helps to cater for the lack of fluctuations due to the limited sample size when modelling market risk under the ES model methodology, see section 3.3.1.

The results attained and discussed in chapter 4 suggested that the data extracted from REUTERS platform fits the jump-diffusion modelling process. This outcome was concluded by the Kolmogorov-Smirnov tests both formal and graphical tests. These results, attained under the methodology described in section 3.3.1 also shown that ES measure is not backtestable since the number of exceptions obtained in Table 4.10 does not meet logical sense as expected. Remember ES-based measure account for tail risk which in most cases causes model exceptions. However, based on the backtesting methodology described in section 4.6.2; differences between VaR and ES-based or ES measure's model exceptions were expected to be huge especially during the 2008 subprime crises were shares peaked due to the meltdown. The SES measure is conservative but the deviation between the model exception for VaR and ES is significantly small (i.e. 2 according to Table 4.10). These empirical results in Table 4.10 were backed by Kerkhof and Melenberg (2004)'s findings in their paper which suggested that, the ES measure is not backtestable. Therefore SES models since they were based on ES methodology described by Artzner et al. (1997, 1999) and others are not backtestable. However, based on graphical tests outlined in Figure 4.7 and Figure 4.8 in general SES measures are conservative but future researches are required to elevate this model to risk sensitive status approved by the financial regulators.

5.3 Limitation and constraints

Although this dissertation has achieved its aim, there are some limitations and constraints which need to be considered. These are:

- Backtestability of the SES; and
- Calibration of τ.

5.3.1 Backtestability of the SES

Backtesting is defined as a procedure which validates the performance of a risk measure over given observation period. This is important to financial regulator to monitor the estimation patterns of the risk measures employed by various financial institutions. Financial regulators want models which are extremely conservative during market turbulences and mildly conservative during normal market fluctuations. Section 4.6.3 stated that, ES-base models such as ES-methodology defined by McNeil et al. (2005) and SES defined in this dissertation in section 3.3.1 are not backtestable. Ziggel et al. (2014) continued to motivate it is because there are limited literature studies addressing the matter in great detail. Backtestability is an important aspect to financial regulators since it provides a consistent measure to assess the predictability of the model across various economic cycles the model is functioning in. Therefore introducing a consistent manner for regulators to monitor and verify the prediction power of the risk models employed by various financial institutions to determine whether these processes are sound. Despite these observations mainly outlined in chapter 2 and chapter 4 of this dissertation, the SES risk measure, especially under the mean approach is robust, see Table 4.9 and Table 4.10. However, future studies need to explore other backtesting methods than the proposed one in section 2.4 for real-world use. This future study will elevate the SES risk measure to the status of HVaR in terms of backtesting and since SES model is already conservative; financial regulators might buy-into the concept if this risk model is backtestable as well.

5.3.2 Calibration of τ

The averaging of the jump-diffusion returns done in equation (3.17) fixes the hetroscedasticity introduced by the simulation process in equation (3.16). But this reduce the jump-effect of the model estimates which constraint the predictability. Therefore, the tau (i.e. τ) coefficient was developed to regain the predictability of the SES model and as well attaining the simulated data showed in Table E.1 in Appendix E. In the present SES modelling process this element is calculated numerically or by a model, which might be argues that as employed currently it introduce biasness into the modelling process. Therefore future studies need to explore linking the current SES model with the tau coefficient so that, any modification done on the returns is re-calibrated by tau in the modelling approach.

5.4 Future work

This dissertation has established a reference point exploring other research questions. Using this dissertation as a starting point work on a thesis investigating backtesting methodologies for an Expected Shortfall (ES) based methods such as HES and SES models in South African Financial institutions with more diverse portfolio than the Equity portfolio used in this research; see Table 4.1. This will assist in emphasising the model benefits of using

ES-based method in predicting risk present in the financial industry.

The modelling framework outlined in section 3.3.1 can be extended to input calibration or modelling of the τ-factor. By incorporating these elements in the modelling process, it allows alignment between theoretical findings and practical application of these works seen in literature outlined in chapter 2. This as well, will answer some of the dilemmatic questions posed to financial institutions by the development of the 2010 Basel III Accord framework.

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Appendix A

Subprime Crisis and its influences on VaR model

A.1 Build-up to the 2008 financial crisis

A bond is defined as an interest-bearing instrument which guarantees regular interest payments on borrowed money during the lifespan of the instrument and principal (i.e. loaned amount) is due at maturity. These instruments are usually issued by government or corporations (i.e. issuer of bond = creditor & recipient of the bond = borrower). For generations, financial markets have traded bonds. The bond market grew significantly since the 1980s, Wall Street realised late in the 80s that "bond-like" financial products could be created from other debt-based income streams like credit cards, student loans and home mortgages. This financial innovation resulted in the inception of the "mortgage-bond" financial instruments and these instruments were sold vastly by various Wall Street investment banks, namely, Goldman Sachs, Merrill Lynch, Bears Sterns, JP Morgan and Morgan Stanley.

These mortgage bonds collects thousands of home mortgages, purchased from lenders, and packages their associated income streams (i.e. monthly mortgages payments) into financial products, which can be bought and sold like bonds. However, these mortgage bonds had embedded unique weaknesses, namely:

- Home owners often refinance their debt periods of low interest rates; and
- Premature repayment of the principal amount (i.e. mortgage loan amount).

Wall Street addressed these inefficiencies by structuring mortgage bond in stacked layers called "tranches" - the lowest layer (risky tranche) representing the first N mortgages settled early and the highest layer (less risky tranche) being the last N mortgages. Since investors in the financial markets are seeking for high returns on their monies, they will opt for the risky tranche.

In the 1990s, Wall Street firm began to issue even high risk mortgage bonds than the ones defined above; the underlying asset used in creating these mortgage bond was subprime mortgages[1]. The structural "tranches of the mortgage bonds built from subprime mortgages, at this point, represented not only pre-payment, but also outright default. With low-interest rate environment seen in the United State (US) during these times increased the demand in Wall Street to buy subprime mortgages which escalated the subprime mortgage exposure to trillions of dollars in the US house market. This increase seen on the subprime mortgage market created a real estate bubble which was not noticed by most of the financial market participants.

Post the DotCom crisis in the early 2000s global economy's interest rate was lowered with intention of re-stimulating the economic growth suffered from the crisis. This action however, stimulated most of the home owners to refinance (i.e. take an additional mortgage loan) their mortgages in all classes and because of lower interest rate mortgages became of lower quality. This caused Wall Street mortgage bonds to be inherently riskier; this should have slow downed or reduced the liquidity of the bonds because rating agencies positively rated subprime mortgages and since rating agencies input is regarded as a key decision-making attribute caused the market to ignore the riskiness of these loans post DotCom. Furthermore, there's an inherent conflict of interest between Wall Street and the rating agencies, since its Wall Street who pays the agencies to rate their products. Likely due to this conflict, the rating agencies assigned surprisingly high ratings for these ever-riskier mortgage bonds.

In early 2000s all signs highlighted a creation of financial bubble over the past two decades discussed in sections above. Wall Street still wanted to generate more profit; this led to further growth of the lower tranches in the mortgage bonds (i.e. these tranches mostly contained subprime mortgages). With lower mortgage bond tranches growth seen in 2003, Mike Burry noted in his study that a massive default will result in collapse of the real estate. Therefore, Credit Default Swaps (CDS) were created and CDS is an insurance policy protecting its holders from adverse movements. Investors wanted to mitigate their risk exposures from this high return mortgage bond (i.e. lower tranche) by purchasing and paying monthly premium on these CDSs. Since the financial market did not expect financial meltdown; the issuing of these CDSs grew exponentially since major and minor insurance or partly-insurance companies were issuing these instruments at a fast pace. Finally in 2008, defaults were incurred in the mortgage markets due to real estate price drops and as defaults increased especially in the subprime mortgage class caused all companies exposed to the mortgage bond (especially the riskiest tranche) in cashing-back on their purchased CDSs. This caused highly financial-leveraged companies to default since they failed to fully

[1] Subprime mortgages are non-confirming mortgages which do not meet the standards for confirming loans by a substantial margin (Schmudde, 2009, p.719)

all insurance claims from CDSs and this caused credit to dry-up which led to the 2008 subprime crisis.

A.2 Impact on emerging countries

Naude (2009) highlights that emerging countries relies heavily in financial-assistance from developed countries to sustain and cultivate economic growth. Emerging countries for this reason gets impacted if the developed countries like the US constrict its spending. In 2008 subprime crisis, the credit dry-up mentioned in A.1 caused the US to minimise its lending to emerging countries like South Africa. This constriction reduced growth in the emerging countries but did not cause the market volatilities seen in the financial market. Since developed countries were under severe stress, their various currencies, imports, exports experienced severe volatility and all of these volatilities from the key market indicators in developed countries translated into Foreign Exchange (FX) volatility, credit spread risk, reduction in import and exports by developed countries. These fluctuations contributed severely to the random market jumps which were not modelable by the historical VaR-Simulation model.

A.3 Regulatory changes in VaR modelling framework post 2008 subprime crisis

Insufficient present of regulatory body in the creation of the complex financial instruments is one of many reasons which led to the crisis briefly discussed under A.1. Financial regulatory bodies global responded to this crisis by amending Basel II Capital Adequacy Framework (CAF); these amendments' were directed to addressing modelling risk and transparency around financial risk modelling (i.e. liquidity and Value-at-Risk (VaR) modelling processes). However, this dissertation focuses more on regulatory changes done on VaR model. These changes regulatory were,

- Stressed Value-at-Risk (SVaR) model was incorporated into the overall regulatory capital, this operates in the same way as the VaR model in section 3.1 but the historical data used in under stressed economic climate;
- The add-on factors were increased to amplify robustness of the model. Add-ons are factors which the financial regulatory enforce the various banks to multiple their final capital/risk numbers and this is purely based on the risk management practise and capital modelling management implemented by the various banks;
- $Regulatory_{capital} = [\max\{VaR \times \sqrt{10}, 60 - day_{average} VaR \times add - on_{factor} \times \sqrt{10}\} + \max\{SVaR \times \sqrt{10}, 60 - day_{average} SVaR \times add - on_{factor} \times \sqrt{10}\} + \rho_2 + \rho_1]$, this is

revised market risk number; only the modelling section of the input is studied in the dissertation; ρ_1 and ρ_2 are specific and incremental risks respectively; and

- Construction of Basel III CAF which was implemented on 01 January 2013 but these changes have been excluded.

A.4 Impact on risk measurements like VaR

As defined under the background section in chapter 1, VaR model used to estimate market risk is a quantile model which returns minimal expected monetary loss based on the historical portfolio movements. During a financial crisis portfolio movements trends tend to deviate significantly from the historical pattern which causes a mismatch between model data and the actual financial/portfolio movements. This mismatch causes the VaR model to under predict risk and which implies companies using this risk measure take on additional risks because the is no indication from the model to constraint risky operations. Eventually it leads to monetary losses as companies realises they were delusion by the model about the true risk exposure in the modelled period.

The VaR model is unable in a timely manner alter the modeller about change in risk as financial market start to undergo stress circumstances brought by financial crises like the subprime crisis seen in 2008. This is purely because historical data used haven't included the market jumps/movements and because its occurrence is random in nature, modellers cannot anticipate these moves. The second properties which caused VaR model predictability or goodness-of-fit test of the phenomenal its model to be heavily impaired is the fact that the empirical distributions employed due to non-conventional products tended to be heavy-tailed distribution. The actual losses tend to be beyond the designated quantile chosen as VaR which implies VaR model lacks tail analysis hence its under-prediction if the financial environment changes severely when compared to historical data collected. The market conditions brought by the subprime crisis stimulated the visibility of the VaR shortcomings due to it is inherit modelling structure as mentioned in the document.

Appendix B

Empirical illustration of the historical Value-at-Risk model as currently implemented

In the empirical illustration a hypothetical bond portfolio has been created in showing the inner-workings of the VaR modelling process. In equation (1.1) VaR is defined as,

$$VaR_{\alpha}^{\Delta}(L) = inf\{\theta\epsilon\mathbb{R} : P_{\Delta}(\theta<L) \leq (1-\alpha)\} = inf\{\theta\epsilon\mathbb{R} : F_L(\theta) \geq \alpha\} \tag{B.1}$$

for this example $\Delta = 10$, $\alpha = 0.01$ and let P=$\underline{\mathbf{f}_t}$ is the portfolio.

$$\underline{\mathbf{f}_t} = \begin{bmatrix} Date & R157 & R203 & R227C & SZ18 & R186 \\ 12/08/13 & 6.06 & 6.79 & 6.12 & 6.62 & 8.16 \\ 13/08/13 & 6.10 & 6.87 & 6.16 & 6.66 & 8.29 \\ 14/08/13 & 6.06 & 6.82 & 6.12 & 6.62 & 8.26 \\ 15/08/13 & 6.14 & 6.91 & 6.20 & 6.70 & 8.39 \\ 16/08/13 & 6.17 & 6.92 & 6.23 & 6.73 & 8.38 \\ 19/08/13 & 6.37 & 7.15 & 6.43 & 6.93 & 8.58 \\ 20/08/13 & 6.32 & 7.13 & 6.38 & 6.88 & 8.52 \\ 21/08/13 & 6.39 & 7.23 & 6.45 & 6.95 & 8.58 \\ 22/08/13 & 6.48 & 7.36 & 6.54 & 7.04 & 8.65 \\ 23/08/13 & 6.34 & 7.23 & 6.40 & 6.90 & 8.49 \\ 26/08/13 & 6.37 & 7.26 & 6.43 & 6.93 & 8.52 \\ 27/08/13 & 6.47 & 7.38 & 6.53 & 7.03 & 8.50 \\ 28/08/13 & 6.58 & 7.45 & 6.64 & 7.14 & 8.51 \\ 29/08/13 & 6.58 & 7.44 & 6.64 & 7.14 & 8.55 \\ 30/08/13 & 6.47 & 7.32 & 6.53 & 7.03 & 8.45 \end{bmatrix} \tag{B.2}$$

. And then from the historical data, price movements or risk factors are calculated as follows,

$$
\underset{\sim}{\widetilde{f}}_t = \left[\begin{array}{llllll}
Date & R157 & R203 & R227C & SZ18 & R186 \\
13/08/13 & \left(\frac{6.06}{6.10}\right) = 0.99262 & \left(\frac{6.79}{6.87}\right) = 0.98908 & \left(\frac{6.12}{6.16}\right) = 0.99269 & \left(\frac{6.62}{6.66}\right) = 1.00302 & \left(\frac{8.16}{8.29}\right) = 0.99939 \\
14/08/13 & \left(\frac{6.10}{6.06}\right) = 1.00660 & \left(\frac{6.87}{6.82}\right) = 1.00660 & \left(\frac{6.16}{6.12}\right) = 1.00654 & \left(\frac{6.66}{6.62}\right) = 1.00604 & \left(\frac{8.29}{8.26}\right) = 1.00303 \\
15/08/13 & \left(\frac{6.06}{6.14}\right) = 0.98697 & \left(\frac{6.82}{6.91}\right) = 0.98769 & \left(\frac{6.12}{6.20}\right) = 0.98710 & \left(\frac{6.62}{6.70}\right) = 0.98806 & \left(\frac{8.26}{8.39}\right) = 0.98451 \\
16/08/13 & \left(\frac{6.14}{6.17}\right) = 0.99514 & \left(\frac{6.91}{6.92}\right) = 0.99783 & \left(\frac{6.20}{6.23}\right) = 0.99518 & \left(\frac{6.70}{6.73}\right) = 0.99554 & \left(\frac{8.39}{8.38}\right) = 1.0119 \\
19/08/13 & \left(\frac{6.17}{6.37}\right) = 0.96860 & \left(\frac{6.92}{7.15}\right) = 0.96783 & \left(\frac{6.23}{6.43}\right) = 0.96890 & \left(\frac{6.73}{6.93}\right) = 0.97114 & \left(\frac{8.38}{8.58}\right) = 097669 \\
20/08/13 & \left(\frac{6.37}{6.32}\right) = 1.00791 & \left(\frac{7.15}{7.13}\right) = 1.00281 & \left(\frac{6.43}{6.38}\right) = 1.00784 & \left(\frac{6.93}{6.88}\right) = 1.00727 & \left(\frac{8.58}{8.52}\right) = 1.00704 \\
21/08/13 & \left(\frac{6.32}{6.39}\right) = 0.98982 & \left(\frac{7.13}{7.23}\right) = 0.98617 & \left(\frac{6.38}{6.45}\right) = 0.98991 & \left(\frac{6.88}{6.95}\right) = 0.99064 & \left(\frac{8.52}{8.58}\right) = 0.99301 \\
22/08/13 & \left(\frac{6.39}{6.48}\right) = 0.98610 & \left(\frac{7.23}{7.36}\right) = 0.98300 & \left(\frac{6.45}{6.54}\right) = 0.98623 & \left(\frac{6.95}{7.04}\right) = 0.98721 & \left(\frac{8.58}{8.65}\right) = 0.99248 \\
23/08/13 & \left(\frac{6.48}{6.34}\right) = 1.02129 & \left(\frac{7.36}{7.23}\right) = 1.01729 & \left(\frac{6.54}{6.40}\right) = 1.02109 & \left(\frac{7.04}{6.90}\right) = 1.01957 & \left(\frac{8.65}{8.49}\right) = 1.01826 \\
26/08/13 & \left(\frac{6.34}{6.37}\right) = 0.99529 & \left(\frac{7.23}{7.26}\right) = 0.99587 & \left(\frac{6.40}{6.43}\right) = 0.99533 & \left(\frac{6.90}{6.93}\right) = 0.99567 & \left(\frac{8.49}{8.52}\right) = 0.99648 \\
27/08/13 & \left(\frac{6.37}{6.47}\right) = 0.98531 & \left(\frac{7.26}{7.38}\right) = 0.98374 & \left(\frac{6.43}{6.53}\right) = 0.98544 & \left(\frac{6.93}{7.03}\right) = 0.98548 & \left(\frac{8.52}{8.50}\right) = 1.00235 \\
28/08/13 & \left(\frac{6.47}{6.58}\right) = 0.98327 & \left(\frac{7.38}{7.45}\right) = 0.99060 & \left(\frac{6.53}{6.64}\right) = 0.98342 & \left(\frac{7.03}{7.14}\right) = 0.98458 & \left(\frac{8.50}{8.51}\right) = 0.99882 \\
29/08/13 & \left(\frac{6.58}{6.58}\right) = 1.00000 & \left(\frac{7.45}{7.44}\right) = 1.00134 & \left(\frac{6.64}{6.64}\right) = 1.00000 & \left(\frac{7.14}{7.14}\right) = 1.00000 & \left(\frac{8.51}{8.55}\right) = 0.99532 \\
30/08/13 & \left(\frac{6.58}{6.47}\right) = 1.01623 & \left(\frac{7.44}{7.32}\right) = 1.01639 & \left(\frac{6.64}{6.53}\right) = 1.01608 & \left(\frac{7.14}{7.03}\right) = 1.01494 & \left(\frac{8.55}{8.45}\right) = 1.01183
\end{array}\right] \tag{B.3}
$$

The historical VaR model is based on the 02/09/13 and the following simulation matrix **H** is determined by

$$\mathbf{H} = \sum_{i=1}^{5} Price\_Today_i \times \Phi$$

$$= 6.43 \times \begin{bmatrix} 0.99262 \\ 1.00660 \\ 0.98697 \\ 0.99514 \\ 0.96860 \\ 1.00791 \\ 0.98982 \\ 0.98610 \\ 1.02129 \\ 0.99529 \\ 0.98531 \\ 0.98327 \\ 1.00000 \\ 1.01623 \end{bmatrix} + 7.28 \times \begin{bmatrix} 0.98908 \\ 1.00660 \\ 0.98769 \\ 0.99783 \\ 0.96783 \\ 1.00281 \\ 0.98617 \\ 0.98300 \\ 1.01729 \\ 0.99587 \\ 0.98374 \\ 0.99060 \\ 1.00134 \\ 1.01639 \end{bmatrix} + 6.49 \times \begin{bmatrix} 0.99269 \\ 1.00654 \\ 0.98710 \\ 0.99518 \\ 0.96890 \\ 1.00784 \\ 0.98991 \\ 0.98623 \\ 1.02109 \\ 0.99533 \\ 0.98544 \\ 0.98342 \\ 1.00000 \\ 1.01639 \end{bmatrix} + 6.99 \times \begin{bmatrix} 0.99324 \\ 1.00604 \\ 0.98806 \\ 0.99554 \\ 0.97114 \\ 1.00727 \\ 0.99064 \\ 0.98721 \\ 1.01957 \\ 0.99567 \\ 0.98648 \\ 0.98458 \\ 1.00000 \\ 1.01494 \end{bmatrix} + 8.43 \times \begin{bmatrix} 0.98491 \\ 1.00303 \\ 0.98451 \\ 1.00119 \\ 0.97669 \\ 1.00704 \\ 0.99301 \\ 0.99248 \\ 1.01826 \\ 0.99648 \\ 1.00235 \\ 0.99882 \\ 0.99532 \\ 1.01183 \end{bmatrix}$$

$$= \begin{bmatrix} 6.38 \\ 6.47 \\ 6.35 \\ 6.40 \\ 6.23 \\ 6.48 \\ 6.36 \\ 6.34 \\ 6.57 \\ 6.40 \\ 6.34 \\ 6.32 \\ 6.43 \\ 6.53 \end{bmatrix} + \begin{bmatrix} 7.20 \\ 7.33 \\ 7.19 \\ 7.26 \\ 7.05 \\ 7.30 \\ 7.18 \\ 7.16 \\ 7.41 \\ 7.25 \\ 7.16 \\ 7.21 \\ 7.29 \\ 7.40 \end{bmatrix} + \begin{bmatrix} 6.44 \\ 6.53 \\ 6.41 \\ 6.46 \\ 6.29 \\ 6.54 \\ 6.42 \\ 6.40 \\ 6.63 \\ 6.46 \\ 6.40 \\ 6.38 \\ 6.49 \\ 6.59 \end{bmatrix} + \begin{bmatrix} 6.94 \\ 7.03 \\ 6.91 \\ 6.96 \\ 6.79 \\ 7.04 \\ 6.92 \\ 6.90 \\ 7.13 \\ 6.96 \\ 6.90 \\ 6.88 \\ 6.99 \\ 7.09 \end{bmatrix} + \begin{bmatrix} 8.30 \\ 8.46 \\ 8.30 \\ 8.44 \\ 8.23 \\ 8.49 \\ 8.37 \\ 8.37 \\ 8.58 \\ 8.40 \\ 8.45 \\ 8.42 \\ 8.39 \\ 8.53 \end{bmatrix} = \begin{bmatrix} 35.27 \\ 35.82 \\ 35.15 \\ 35.52 \\ 34.58 \\ 35.85 \\ 35.26 \\ 35.16 \\ 36.31 \\ 35.47 \\ 35.24 \\ 35.22 \\ 35.59 \\ 36.15 \end{bmatrix} \tag{B.4}$$

where Φ is the corresponding risk factor calculated under equation (B.3), the portfolio value today (i.e. 2/9/13) is 35.62 and L defined in equation (B.1) for this example it has been formulated as follows,

$$
\begin{aligned}
L &= 35.62 - \mathbf{H} \\
&= \begin{bmatrix} 0.35 \\ -0.20 \\ 0.47 \\ 0.10 \\ 1.04 \\ -0.23 \\ 0.36 \\ 0.46 \\ -0.69 \\ 0.15 \\ 0.38 \\ 0.40 \\ 0.03 \\ -0.53 \end{bmatrix}
\end{aligned} \tag{B.5}
$$

These $n-1$ deviations represent either a positive or negative movement in the value of the portfolio which must be sorted in ascending order

$$\triangle\widehat{L} \uparrow = sort(\triangle\widehat{L}, ascending) \tag{B.6}$$

and it follows that the integer I is determined which indexes the 1% percentile of $\triangle L \uparrow$. The 10-day (since Δ is assumed to be 10) VaR is simply

$$I = round\left\{\frac{n-1}{100}\right\} \tag{B.7}$$

$$VaR_{1-\alpha}(10) = \triangle L \uparrow (I) \times \sqrt{10}. \tag{B.8}$$

$\therefore VaR_{\{}1-\alpha\}(10) = -0.6692 \times \sqrt{10} = -2.1162$, which implies that for the hypothetical bond portfolio modelled in this example a minimal loss of 2.12 is expected to be incurred if the instruments are kept for 10 working days under 0.01 confidence level. It's clear from this empirical that historical VaR model does not disclose the severity of the losses under F_L (empirical loss distribution), which is one of the problem resulting in under-prediction of the model.

Appendix C

Mathematical competency of the jump-diffusion model

Jump-diffusion models uses stochastic processes in predicting random financial market jumps; although these models in the literature available have been implemented on futures, options and equity, it could be extended into other more complex financial derivatives. These models are effective in generating random jumps due to its sub-processes embedded in the overall model and this appendix will discuss in detail the mathematical competency of each sub-processes found under a general jump diffusion model.

C.1 Formulation and model assumptions

Jump-diffusion models are defined as,

$$\Omega_t = \Omega_{t-1} \times exp\{\mu - 0.5 \times \Sigma \times W_t\} \prod_{m=1}^{N_T} \Delta_m \tag{C.1}$$

where,

- Ω_t and Ω_{t-1} are initial and estimated price matrices respectively;
- μ is the drift matrix which shows the direction of the price movement, for example if drift of a particular financial product is negative, the price movements will tend to show a downward movement;
- Σ is the volatility matrix which shows normal price jumps from the observed period;
- W_t is a Brownian Motion (BM) process which is discussed in section C.3;
- N_T is the Poisson Process (PP) which is the number of financial market jumps in the observed period based on the data. This is discussed in section C.4; and

- Δ_m is the upward and downward jump amplitudes, this is modelled differently in the various jump-diffusion models. In the Merton (1976)/Lognormal Jump-Diffusion (LJD) model, Δ_m is determined from the empirical whereas in the Double Exponential Jump-Diffusion (DEJD) and Pareto-Beta Jump-Diffusion (PBJD) models the Δ_m is determined using parametric distributions, see section C.2.

The above defined model in equation (C.1) has inherited model assumptions which needs to hold for the model to be functional, namely:

- Basic and central moment of the return data needs to exist, see equation (3.32) for the return calculation and these returns needs to be asymptotical elliptic distributed or close;

- Output is based on daily prices and parameter "dt" can be adjusted to calculate either monthly, weekly, yearly or quarterly prices from the model defined above;

- The model jumps, that is, Δ_m follows a parameter distribution, for example $\Delta_{m_1} \sim exp(\eta_1)$ (i.e. upward jumps) and $\Delta_{m_2} \sim exp(\eta_2)$ (i.e. downward jumps) in the DEJD model;

- Returns follow a Poisson Process (PP);

- Price modelled is nonlinear but the model formulation it can be adjusted using logarithm to transform equation (C.1) such that,

$$\Omega_t = \Omega_{t-1} + [\mu - (0.5 \times \Sigma \times W_t)] + \sum_{m=1}^{N_T} \Delta_m; \tag{C.2}$$

- Increments in the embedded sub-processes (i.e. for Poisson and Brownian motion processes) are stationary and independent; and

- It requires large data sample.

C.2 Parameters estimation methodology for the jump-diffusion model

The moments of the returns is used in calculating the parameters of the jump-diffusion models discussed below. The detailed processes have been specified in chapter 3. Note that, the more complex the jump-diffusion method/model, the more unknowns which needs to be solved; since financial risk is modelled, heavy-tailed distribution is expected and therefore there is a requirement to assess the kurtosis and the skewness of the data to ensure this property holds.

C.2.1 PBJD model

The PBJD model assumes that upward and downward jumps are generated by two independent Poisson processes, namely $N(\lambda^j t)\forall, j = u, d$. However the respective jump magnitudes are modelled by Pareto and Beta distributions; these findings are stated in the work of Ramezani and Zeng (2007).

Ramezani and Zeng (2007) defines the PBJD model as,

$$f_t = f_0 e^{(\mu - 0.5\sigma^2)t + \sigma \times W_t} \prod_{j=u,d} v^j(N(\lambda^j t)) \tag{C.3}$$

where

$$\prod_{j=u,d} v^j(N(\lambda^j t)) = \begin{cases} 1, \text{if}, N(\lambda^j t) = 0 \\ \prod_{i=1}^{N(\lambda^j t)} v_i^j, \text{if}, N(\lambda^j t) = 1, 2, 3, \cdots \end{cases} \tag{C.4}$$

This equation it can be further simplified and written in return format as defined in equation (3.33),

$$\widetilde{f_\varphi} = log\left(\frac{f_0}{f_t}\right) = (\mu - 0.5\sigma^2)t + \sigma \times W_t + \sum_{j=u,d} v^j(N(\lambda^j t)) \tag{C.5}$$

where

$$\sum_{j=u,d} v^j(N(\lambda^j t)) = \begin{cases} 1, \text{if}, N(\lambda^j t) = 0 \\ \prod_{i=1}^{N(\lambda^j t)} v_i^j, \text{if}, N(\lambda^j t) = 1, 2, 3, \cdots \end{cases} \tag{C.6}$$

and the Pareto and Beta density functions for the up-jump and down-jump are assumed to follow: $f_{V^u}(x) = \eta_u \left(\frac{1}{x}\right)^{\eta_u + 1}$ with $V^u \geq 1$ and E(V^u)=$\frac{\eta_u}{\eta_u - 1}$ & $\sigma^2_{V^u}$=$\frac{\eta_u}{(\eta_u - 2)(\eta_u - 1)^2}$,similarly, $f_{V^d}(x) = \eta_d\,(\eta_d(x))^{\eta_d - 1}$ with $V^u \geq 1$ and E(V^u)=$\frac{\eta_d}{\eta_d + 1}$ & $\sigma^2_{V^d}$=$\frac{\eta_d}{(\eta_d + 2)(\eta_d + 1)^2}$. $v^d \sim Beta(\eta_d, 1)$ & $v^u \sim Pareto(\eta_u)$.

C.2.2 DEJD model

The DEJD model follows on the same premises as the PBJD model defined above. In the DEJD model the two jumps are generated using a single Poisson model, that $N(\lambda t_{DEJD}) = \sum_{j=u,d}^{2} N(\lambda^j t)$. On the other hand the jump magnitude are calculated by independent exponential distributions with parameter η_u and η_d respectively.

In Ramezani and Zeng (2007) its mentioned that DEJD model is one dimension less complex than the PBJD model in the since that a single Poisson Process $N(\lambda T)$ is used in generating the jump magnitudes instead of two independent Poisson processes. It follows in return data notation DEJD model is defined as:

$$\widetilde{f_\varphi} = log\left(\frac{f_0}{f_t}\right) = (\mu - 0.5\sigma^2)t + \sigma \times W_t + \sum_{j=u,d}^{N(\lambda t)} v^j(N(\lambda t)) \tag{C.7}$$

where

$$\sum_{j=u,d}^{N(\lambda t)} v^j(N(\lambda t)) = \begin{cases} 1, \text{if}, N(\lambda t) = 0 \\ \prod_{i=1}^{N(\lambda t)} v_i^j, \text{if}, N(\lambda t) = 1, 2, 3, \cdots \end{cases} \tag{C.8}$$

C.2.3 LJD model

Merton (1976) model is a less complex jump diffusion model when compared to the two jump diffusion model discussed above. It's less complex because the jump magnitude V^d and V^u (using Ramezani and Zeng (2007) notation) are empirically distributed and single Poisson process is used in generating the jumps. In essence this model is a non-parametric version of DEJD and PBJD models which relies heavily on parametric distributions.

For the Merton (1976) model, it follows that a single jump component with magnitude V is distributed $\overset{iid}{\sim}$ lognormal (α, β^2) and Poisson (λ) arrival rate/intensity parameter. However Mastro (2013) has developed this model which caters for upward and downward jumps. In return notation the model is written as,

$$\widetilde{f_\varphi} = log\left(\frac{f_0}{f_t}\right) = (\mu - 0.5\sigma^2)t + \sigma \times W_t + \sum_{j=q_1,q_2}^{N(t)} e^{Y_j} \tag{C.9}$$

where

$$Y_j \overset{iid}{\sim} lognormal(e^{\mu+0.5\delta^2}, e^{2\mu+\delta^2}(e^{\delta^2} - 1), \text{is the jump magnitude.} \tag{C.10}$$

C.3 Brownian motion model

In jump diffusion models, W_t is a standard Brownian motion process, here are few justifications why this method is efficient. Brownian motion is a continuous stochastic process discovered by a botanist Robert Brown in an experiment. This process is also known as the Wiener process; Wiener was the first mathematician to describe this process in mathematical format. According to Allison (2009), Wiener defined the brownian motion as, a continuous-time stochastic process{B(t), T≥0}, with state spaces={$-\infty, +\infty$} is said to be a Brownian motion process {or Wiener process} if it has the following properties

- B(0) = 0;
- B(t) has stationary and independent increment, i.e. the distribution B(t) - B(s) depends only on the length of the interval t-s, and B(t) - B(s) is independent {B(r):r≤s}; and
- For every t >0. B(t) is normally distributed with E{B(t)}=0 and var{B(t)}.

The properties defined above makes Brownian motion process stand out when compared to other stochastic processes and its ability to mimics market movements, hence its incorporation in the jump diffusion models.

C.4 Poisson process

Jump diffusion models are made up of various models and the last component constituting these models is the Poisson process. This section will highlight briefly the mathematical proficiency of this model. This Poisson process which is within the jump diffusion models calibrates the number of observable jumps from a series of financial prices. This process form the crux of the jump diffusion models and a process is Poisson if and only if the following assumptions hold:

- N(0) = 0;
- $\forall$ t_0= 0 <t_1 <$\cdots$ <t_n, the increments N(t_1) - N(t_0),N(t_2) - N(t_1), $\cdots$, N(t_n) - N(t_{n-1}) are independent random variables;
- for t$\geq$0, s >0 and non-negative integers k, the increments have the Poisson distribution,

$$Pr(N(t+s) - N(s) = k) = \frac{(\lambda^j t)^k e^{-\lambda^j t}}{k!}. \tag{C.11}$$

These mathematical properties highlights that the Poisson process used in the jump diffusion models are based on sound stochastic calculus and the above definition was extracted from Swanepoel (2009)'s course note.

C.5 Conclusion

The above proofs and theories have shown that all processes and models incorporated in jump diffusion models are based on sound mathematical principles. Therefore the researcher must ensure that all underlying assumptions of models used in the selected jump diffusion model holds before deeming the modeling results accurate.

Appendix D

Matlab code

D.1 Introduction

All code denoted below is extracted from Mastro (2013) apart from the ANOVA testing section.

D.2 Main script - Jump-Diffusion Model Execution

This is the main function which is used in calculating the parameters and "goodness-of-fit" tests as specified in chapter 3.

Comment: These lines clears and close all active windows within Matlab prior to executing the function or Matlab codes.

```
clear all;
close all;
clc;
format long;
```

Comment:Importing data into Matlab platform.

```
[sNUM,sTXT,sRAW]=xlsread('PriceData');
```

Comment:Redefining the uploaded data.

```
SData=sNUM;
```

Comment: The Matlab code below formats and create a text document which documentaries the parameters calculated in the loop below.

diary results.txt

diary on

for i=1:size(SData,2)
S = SData(:,i);
ModelJumpDiffusion(S);
end

diary off

D.2.1 Modeling Process

This function is called in the main script defined above in calculating parameters. This function uses other functions in calculating parameters and validating the results.

Comment: "S" is the data uploaded from Reuters and it's an input in this function.
function ModelJumpDiffusion(S)

These global variables are usable throughout the code.

global M1 M2
global fjd fData Centers lengthLD dt
global LSflag

Comment: If LSflag = 1 use Least-Square (LS) estimation or if LSflag = 2 then use Multinomial Estimation (MME).
LSflag = 1;

Comment: Assume daily prices but could add as an input in the function.
dt=1/252
SQRTdt=sqrt(dt);

Comment: This section of the code uses the actual data imported to calculates time variables required later in estimating the parameters from data. As well log of the transposed actual data is calculated.
if(nargin == 1), steps = length(S);

Years=steps*dt;
TimeLength=dt*steps; time=linspace(0,TimeLength,steps);
$[C, R]$ = size(S);
if(C>R)
S=S';
end
Lns=log(S);
end

Comment: Note that in chapter 3, it was mentioned that theoretically-based estimates are calculated as a comparison element in the parameter validity test, for more information refer to Law & Kelton (1982) or refer to the code below.
if(nargin == 0),
Szero=50;
mu=0.11; *Comment: drift parameter which is denoted by μ*
vol=0.25; *Comment: volatility parameter which is denoted by σ*
musig2=mu-0.5*vol^ 2;
lambda=5; *Comment: rate of the jumps per year = Intensity of Poisson Process which is denoted by λ*
q1=-0.14; q2=0.15; *Comment: The amplitude of the down and up jump respectively.*
Comment: nuMean = Average Jump size measured relative to previous stock prices & logNuP1=log(nuMean+1) (i.e. Drift of ln(jumps))
nuMean=(exp(q2)-exp(q1))/(q2-q1) - 1;
Years=7.5; steps= Years*252;
lambdadt=lambda*dt;
TimeLength=dt*steps;
time=dt*steps;
time=linspace(0,TimeLength,steps); *Comment: Measured in "years"*
S(1)=Szero; *Comment: LnS(1)=log(Szero); where Szero is the initial value or $f_0(t)$ in equation 3.15*
rand('state',0); randn('state',0);
UniDist=rand(1,steps);

Comment: Hanson suggests using center to avoid end bias in distribution and calculated as; jumpleft=$\frac{(1-\lambda\times dt)}{2}$ and jumpright=(1-jumpleft).
The following loop below calculates simulated price from actual data, that is, parameters used in the data transformation section.

for i=2:steps
if(lambdadt>UniDist(i))

Comment: if((UniDist(i)≥jumpleft)&(UniDist(i)≤jumpright))
Q=q1+(q2-q1)*rand; *Comment: Distributed-Size Jump*

Comment: $LnS_i = LnS_{i-1} + (musig2 \times dt + \sigma \times randn \times \sqrt{dt} + Q)$;

S(i)=S(i-1)*exp(musig2*dt+vol*randn*SQRTdt+Q);

else *Comment: No Jumps only Drift-Diffusion*

Comment: $LnS_i = LnS_{i-1} + (musig2 \times dt + \sigma \times randn \times \sqrt{dt})$;

S(i)=S(i-1)*exp(musig2*dt+vol*randn*SQRTdt);
end

end
Comment: $S = e^{(LnS)}$;
end

LnS=log(S); *Comment: calculate as if logarithmic returns log(St/S0) which are normally distributed.*

LogDelta=log(S(2:end))-log(S(1:end-1)); lengthLD=length(LogDelta);

M1=mean(LogDelta); *Comment: Technically this is the first* raw moment
StanDev = std(LogDelta);
M2=StanDevˆ 2; *Comment:* 2^{nd} *Central Moments*
M3=mean((LogDelta-M1).ˆ 3); *Comment:* 3^{rd} *Central Moments*
M4=mean((LogDelta-M1).ˆ 4); *Comment:* 4^{th} *Central Moments*

Skew=M3/(M2ˆ 1.5);
Kurtosis=M4/(M2ˆ 2)-3;

xmin=min(LogDelta); estQ1=xmin;
xmax=max(LogDelta); estQ2=xmax;

sorted=sort(LogDelta);

q25=sorted(floor(0.25*lengthLD)); q75=sorted(floor(0.75*lengthLD));

```
estMuJump =(estQ1 + estQ2)/2;
estNuMean = (exp(estQ2)-exp(estQ1))/(estQ2-estQ1)-1;
```

Comment: Count returns +/- 3 standard deviations as number of jumps and divide by years of data to estimate lambda and recalculates diffusion volatility without outliers. This modified approach is discussed in L.Clewlow, C. Strickland, V.Kaminski, "Extending Mean-Reversion Jump Diffusion".

```
outliersBottom=0; outliersTop=0;
neg3sd= M1-3*StanDev;
pos3sd= M1+3*StanDev;
bottom = 1; Comment: step through sorted array to find outliers
while (sorted(bottom)<neg3sd)
outliersBottom=outliersBottom+1; bottom=bottom+1;
end

top = lengthLD; Comment: Could also use Matlab 'find' function
while (sorted(top)>pos3sd)
outliersTop=outliersTop+1; top=top-1;
end
StanDev = std(LogDelta(bottom:top));
estVol = StanDev/sqrt(dt); Comment: Estimated annualized volatility
estLambda = (outliersTop+outliersBottom)/Years;
estMuDsig2 = (M1-estMuJump*estLambda*dt)/dt;
estMuD = estMuDsig2+0.5*estVol^ 2;
```

Comment: bin data based on process developed in D. Synowiez, Computers and Mathematics with application, 56, 2120 (2008)

```
k=round((((xmax-xmin)*lengthLD^(1/3)/(2.64*q75-q25)))+1);
db=(xmax-xmin)/k;
Edges=zeros(1,k+2); Centers=zeros(1,k+1);

for i=1:(k+2)
Edges(i)=xmin+(i-1.5)*db; Comment:-0.5*db ...(k+0.5)*db
end

for i=1:(k+1)
```

Centers(i)=xmin+(i-1)*db; *Comment: +0*db...k*db*
end

Comment: fData= Actual frequency in each bin
[*fData*]=histc(LogDelta,Edges); fData=fData(1:end-1);

Comment: Estimation more stable with proper pre-estimation (above) of estLambda,estQ1, estQ2

param=zeros(1,3); *Comment: Lambda, q1,q2 are independent variables*
param(1)=estLambda; param(2)=estQ1; param(3)=estQ2;

[pnew,likelihood]=fminsearch('LikeEval',param);
Comment: likehood=-likelihood;
Comment: fprintf(1,'Likelihood=%6.4 f \n',likelihood);

calcLam=pnew(1); a=pnew(2); b=pnew(3);
muJump=(a+b)/2;
sigJ2=(b-a)ˆ 2/12;
calcMuDsig2=(M1-muJump*calcLam*dt)/dt;
calcSig=sqrt((M2-(sigJ2+muJumpˆ 2)*calcLam*dt)/dt);
calcMuD=calcMuDsig2+0.5*calcSigˆ 2;
calcNuMean=((exp(b)-exp(a))/(b-a))-1;

fJD=((1-calcLam*dt)/(calcSig*sqrt(dt)))*myNormPDF((Centers-...
(calcMuDsig2)*dt)/(calcSig*sqrt(dt)))+(calcLam*dt/(b-a))*...
(myNormCDF((Centers-a-(calcMuDsig2)*dt)/(calcSig*sqrt(dt)))-...
myNormCDF((Centers-b-(calcMuDsig2)*dt)/(calcSig*sqrt(dt))));

fJD=lengthLD*fJD/sum(fJD);

fprintf(1, '\t\t \t mu \t vol \t\t lambda \t q1 \t q2 \n');
if(nargin == 0),fprintf(1,...
'Simulated \t %6.6f \t %6.6f \t %6.6f \t %6.6f \t %6.6f \n',...
mu, vol, lambda, q1, q2);
end

fprintf(1,...
'Estimated \t %6.6f \t %6.6f

```
textbackslash t %6.6f \t %6.6f \t %6.6f \n',...
estMuD, estVol, estLambda, estQ1, estQ2);
fprintf(1,...
'Calculated \t %6.6f \t %6.6f \t %6.6f \t %6.6f \t %6.6f \n',...
calcMuD, calcSig, pnew);

estM1=(calcMuDsig2+calcLam*muJump)*dt;
estM2=(calcSig^ 2+calcLam*(sigJ2+muJump^ 2))*dt;
estM3=(3*sigJ2+muJump^ 2)*muJump*calcLam*dt;
estM4=(muJump^ 4+3*sigJ2^ 2+6*muJump^ 2*sigJ2)*calcLam*dt...
+3*(calcSig^ 2+calcLam*(sigJ2+muJump^ 2))^ 2*dt^ 2;
estSkew=estM3/(estM2^ 1.5);
estKurtosis=estM4/(estM2^ 2)-3;

fprintf(1, '\n\t\t\t Skew \t Kurtosis \n');
fprintf(1, 'Data \t \t %6.6f \t %6.6f \n',Skew,Kurtosis);
fprintf(1, 'Estimated \t %6.6f \t %6.6f \n\n',estSkew,estKurtosis);

GoodFit(fJD,fData,lengthLD,Centers) Comment: Goodness of fit statistics

Comment: fDataANDfJD=[fData;fJD]';
Comment:figure; bar(Centers,fDataANDfJD,'group');

figure;
subplot(1,2,1);
plot(Centers,fData,Centers,fJD,'–');
xlabel('Log-Return'); ylabel('Frequency'); axis tight;
if(nargin == 0) Comment: self-simulation
title('JD Self-Simulated');
else
title('JD of Asset Data')
end
subplot(1,2,2); semilogy(Centers,fData,Centers,fJD,'–');
xlabel('Log-Return'); ylabel('Log-Frequency'); axis tight;
legend('Data','J/D Fit','location','South')
if(LSflag == 1) Comment: self-simulation
title('with Least-Squares fit');
else
title('with Multinomial Estimation');
end
```

ES= zeros(1,steps); ES(1)=S(1); *Comment: Elns(1)=log(S(1));*
for i=2:steps *Comment: Calculate expectation*
Comment: $Elns_i = Elns_{(i-1} + ((calcMuD + calcNuMean \times calcLam) \times dt)$*;*
ES(i)=ES(i-1)+((calcMuD+calcNuMean*calcLam)*dt);
end

Comment: ES(exp(Elns);

time=time+2003; *Comment: start in Year 2003*

figure
plot(time,ES,':',time,S)
legend('J/D Expected','Data','Location','NorthWest')
title('Uniform-Jump/Diffusion')
xlabel('$Time[Years]$'); ylabel('Price, S_t'); axis tight;
end

LikeEval function

function LH = LikeEval(parameter)

Comment: LikeEval returns negative of CHI SQUARE(λ,a,b) fit to data or negative of Multinomial Maximum Likelihood

global M1 M2 *Comment: mean and variance*
global fData Centers lengthLD dt
global LSflag *Comment: 1 for LS;2 for MME*

Comment: λ, a, b are independent variables

lambda=parameter(1);
a=parameter(2);
b=parameter(3);

Comment: μ and σ of dependent on (λ,a,b) via M1(mean) and M2(variance), see F.Hanson, J.J. Westman, Jump-Diffusion Stock Models in Finance: Stochastic Process Density with Uniform Jump Amplitude

```
muJump=(a+b)/2;
sigJ2=(b-a) ^ 2/12;
muSig2=(M1-muJump*lambda*dt)/dt;
sig=sqrt((M2-(sigJ2+muJump^ 2)*lambda*dt)/dt);
mu=muSig2+0.5*sig ^ 2;
```

Comment: uniform jump-diffusion log return probability density equation derived by D. Synowiec, Comp.Math w/ App. 56, 2120 (2008), uses a normalized PDF and CDF which calculated with myNormPDF and myNormCDF. PDF and CDF functions are not in standard Matlab package

```
fjd=((1-lambda*dt)/(sig*sqrt(dt)))*myNormPDF((Centers-...
(mu-0.5*sig*sig)*dt)/(sig*sqrt(dt)))+(lambda*dt/(b-a))*...
(myNormCDF((Centers-a-(mu-0.5*sig*sig)*dt)/(sig*sqrt(dt)))...
-myNormCDF((Centers-b-(mu-0.5*sig*sig)*dt)/(sig*sqrt(dt))));
```

Comment: Normalize so total frequency(fjd)=length(Energy or Stock Prices)

```
if(LSflag ==1)
fjd=lengthLD*fjd/sum(fjd);
```
Comment: Unweighted Chi Square approach: Fairly Stable and give reasonable results
```
LH=sum((fjd-fData). ^ 2);
```

Comment: Multinomial Maximum Likelihood Derived in Floyd B. Hanson, John J. Westman and Zongwu Zhu, "Maximum Multinomial Likelihood Estimation of Market Parameters for Stock Jump-Diffusion Models, in Mathematics of Finance"

```
LH=-sum((fData.*log(fjd)));
end
end

else
```

GoodFit function

```
function GoodFit(f1,f2,n,binCenters)
```

Comment: GootFit function provides goodness of Fit measure assume two CFD vector are *the same length*

CDF1=cumsum(f1)/n;
CDF2=cumsum(f2)/n;
Comment: Kolmogorov-Smirnov statistic

Dn1=max(abs(CDF1-CDF2));
dn1=Dn1*sqrt(n);

Comment: dn=scaled distances between two CDFs
j=1:1000; *Comment: assume 1000 ∼ infinity for this calculation*
Pd1=signif(dn1); *Comment: Pd1=significance level of scales values of dn*
fprintf(1,‘Kolmogorov-Smirnov \n’);
fprintf(1,‘Dn = %6.6f Scaled dn = %6.6f Significance = %6.6f \n’,...
Dn1, dn1, Pd1);

d=0.01:0.01:2.5;
pd=signif(d);

d05band=1.36/sqrt(n);
fprintf(1,‘D-alpha = %6.6f \n’, d05band);

d05up=CDF1+d05band;
d05down=CDF1-d05band;
figure
subplot(1,2,1);plot(binCenters,d05up,’-.’,binCenters,CDF1,binCenters,...
CDF2,binCenters,d05down,‘-.’)
legend(‘+d_*alpha* = 0.05’,‘ \ Phi_{*JD*}’,‘ \ Phi_{*Data*}’,‘-d_{ \ alpha=0.05}’,...
‘location’,‘NorthWest’)
axis tight
xlim([-0.05,0.05])
title(‘Reject CDF if Empirical CDF falls outside +/-D_{ \alpha=0.05}’)
text(0,0.67,‘ \downarrow’);
text(0,0.5, ‘ \uparrow 5% Critical Level’);
xlabel(‘Log-Return’); ylabel(‘Cumulative Distribution’)

subplot(1,2,2); plot(d,pd)
xlabel(‘Scaled Max CDF Error d=n ˆ 0.5D_n’);
ylabel(‘Significance \alpha’)

```
text(dn1,Pd1,' \downarrow JD Significance Level','VerticalAlignment',...
'bottom')
title('Kolmogorov-Smirnov Significance level')

Comment:H0 CFD1=CFD2 confidence
Comment: (no basis toreject at significance level alpha) if(dn¡q(1-alpha))
end

function Pd= signif(d);
ld=length(d);
j=1:1000; Comment: assume 1000 ~ infinity for calculation
for i=1:ld
Pd(i)=2*sum((-1). ^ (j-1).*exp(-2.*j. ^2.*d(i). ^2));

end
end
```

myNormCDF function

```
function ncdf = myNormCDF(x)

ncdf = 0.5*(1+erf(x/sqrt(2)));
ncdf = 0.5*erfc(-x/sqrt(2));
end
```

myNormPDF function

```
function npdf=myNormPDF(x)
npdf= (1/sqrt(2*pi))*exp(-(x.*x)/2);
end
```

D.3 Simulation/Data transformation section

Comment: After testing the validity of the parameters and data-fits test; the modeller should implement the jump diffusion model so that $\widehat{f_i(t)}$ *is determined as defined under equation (3.15).*

Comment: These lines clears and close all active windows within Matlab prior to executing the function or Matlab codes.
clear all;
close all;
clc;
warning off;
format long;

$[sNUM, sTXT, sRAW]$=xlsread('Estimates'); *Comment: Import estimates for jump-diffusion*
Est=sNUM; *Comment: Redefining the uploaded data*

S0 = Est(:,1);
Mu = Est(:,2);
Vol = Est(:,3);
Lambda = Est(:,4);
Q1 = Est(:,5);
Q2 = Est(:,6);

dt = 1/252; *Comment: dialy*
time = 10; *Comment: in years*
steps = time/dt;

N = 100;
M = size(sNUM,1);

SVec = zeros([steps,M]);

for i=1:M

MCVec = zeros([steps,N]);

for j=1:N

MCVec(:,j) = SimJD2(S0(i),Mu(i),Vol(i),Lambda(i),Q1(i),Q2(i),dt,time,steps);

end
SVec(:,i) = mean(MCVec,2);

end

Comment: plots the average path and from this path its visible as to why τ element incorporated in section (3.3) is necessary. The averaging part stabilises the path, therefore minimising the variance but this also flattens the jump effect; hence the need to τ.

plot(SVec);hold on;zoom on;
legend('AGLJ','BILJ','AMSJ','ANGJ','ARIJ','IMPJ','OMLJ','GFIJ');
Comment: This line of code copy the data onto an Excel platform
xlswrite('Simulated2.xls', SVec, 'sheet1','A2')

D.3.1 Simulation code

This is the code used to generate the simulated $\widehat{f_i(t)}$ mentioned in equation 3.15. This is a supporting function to the script mentioned above, which averages these simulations produced by this Matlab code.

function Jump1Diff=SimJD2(S0,mu,vol,lambda,q1,q2,dt,time,steps)

Comment: input explanation
Comment: S0 - initial values of the shares
Comment: mu - individual drift parameters for each share
Comment: vol - Volatility for each share
Comment: lambda - Intensity parameter for each share
Comment: q1 - is the amplitude of the downward jump
Comment: q2 - amplitude of the upward jump

Comment: Data used in the parameter estimation method stretches from Aug 2004 to Aug 2013 and based on this, I have to simulate 9 years' worth of data.

musig2=mu-0.5*vol ^2;
nuMean=(exp(q2)-exp(q1))/(q2-q1) - 1;
logNuP1=log(nuMean+1); *Comment: Drift of $ln(jumps)$*

SQRTdt=sqrt(dt); Jump1Diff(1) = S0; Jump1(1)=0; *Comment: Diffusion (1)=Szero;*
JumpDistDiff(1) = S0; JumpDist(1)=0;
Jump1DiffExpect(1) = S0; JumpDistDiffExpect(1)=S0;

for i=2:steps
Jump1DiffExpect(i)=Jump1DiffExpect(i-1)*exp((mu+nuMean*lambda)*dt);

JumpDistDiffExpect(i)=JumpDistDiffExpect(i-1)*exp((mu+nuMean*lambda)*dt);

Comment: we set nu(1Jump)=nuMean(distributed) -<E[1Jump]=E[Distributed Jump]

if(lambda*dt>rand)
Jump1Diff(i)=Jump1Diff(i-1)*exp(musig2*dt+vol*randn*SQRTdt+logNuP1);
Jump1(i)=Jump1Diff(i-1)*(exp(logNuP1)-1); *Comment: one-size jump*
Q=q1+(q2-q1)*rand;
JumpDistDiff(i)=JumpDistDiff(i-1)*exp(musig2*dt+vol*randn*SQRTdt+Q);
JumpDist(i)=JumpDistDiff(i-1)*exp((Q)-1); *Comment: Distributed-Size Jump*
else *Comment: No Jumps only Drift-Diffusion*
Jump1Diff(i)=Jump1Diff(i-1)*exp(musig2*dt+vol*randn*SQRTdt);
JumpDistDiff(i)=JumpDistDiff(i-1)*exp(musig2*dt+vol*randn*SQRTdt);
Jump1(i)=0; JumpDist(i)=0;

end
end
end

D.4 Risk Modelling

Comment: This code calculates the various model compared and studied in this dissertation. For the simulated model, the outcome determined in D2 is used as input in this section.

Comment: These lines clears and close all active windows within Matlab prior to executing the function or Matlab codes.
clear all; close all; clc;
Comment: Import the actual data.

$[NUM, TXT, RAW]$=xlsread(‘PriceData’);
shares = NUM;
shares(:,2:end) = shares(:,2:end)/100;
n = length(shares);

Comment: 8 years or more
*Comment: data=shares(n-2*tau:n-1,:);*

data=shares;

Comment: Annualized parameters

dt = 1;
tau = 252;

Comment: Allocation of the shares

portval = 1e6;
allocation = portval*(1/8);
currprice = shares(end,2:end);
N = floor(allocation./currprice);

Comment: import simulated data1
[$sNUM, sTXT, sRAW$]=xlsread('Simulated');
sshares1 = sNUM;
sshares1(:,2:end) = sshares1(:,2:end)/100;

Comment: 8 years or more
*Comment: sdata = sshares(n-2*tau:n-1,:);*
sdata1 = sshares1;
Comment: import simulated data1
[$sNUM, sTXT, sRAW$]=xlsread('Simulated2');
sshares2 = sNUM;
sshares2(:,2:end) = sshares2(:,2:end)/100;

Comment: 8 years or more
*Comment: sdata = sshares(n-2*tau:n-1,:);*
sdata2 = sshares2;

Comment: VaR, SES(median), SES(mean), HES(mean) & HES(median)- All of the methodologies are prescribed under Chapter 3.

for i=1:8*tau-1

Comment: actual share price
hdata = data(i:tau+(i-1),:);
cprice = data(tau+i,2:end);

```
tprice = data(tau+i+1,2:end);

cport = sum(cprice.*N);
tport = sum(tprice.*N);

rdata = Returns(hdata(:,2:end),dt);

Comment: simulated share price1
hsdata1 = sdata1(i:tau+(i-1),:);
rsdata1 = Returns(hsdata1(:,2:end),dt);

Comment: simulated share price2
hsdata2 = sdata2(i:tau+(i-1),:);
rsdata2 = Returns(hsdata2(:,2:end),dt);

for j=1:length(cprice)
eprice(:,j) = N(j)*cprice(j)*(rdata(:,j));
esprice1(:,j) = N(j)*cprice(j)*(rsdata1(:,j))*1;
esprice2(:,j) = N(j)*cprice(j)*(rsdata2(:,j))*1;
end

eport = sum(eprice,2);
esport1 = sum(esprice1,2);
esport2 = sum(esprice2,2);

epnl = eport-cport;
espnl1 = esport1-cport;
espnl2 = esport2-cport;

Comment: Historical Expected Shortfall(HES) model
a =quantile(epnl,0.01);
X =epnl(epnl'¡a);
HESmean(i) = mean(X);
HESmedian(i) = median(X);

Comment: Simulated Model 1 - Expected Shortfall (SES) model
b =quantile(espnl1,0.01);
Y =espnl1(espnl1'¡b);
SESmean1(i) = mean(Y)*15;
```

SESmedian1(i) = median(Y)*15;

Comment: Simulated Model 2 - Expected Shortfall (SES) model
c =quantile(espnl2,0.01);
Z =espnl2(espnl2'¡c);
SESmean2(i) = mean(Z)*15;
SESmedian2(i) = median(Z)*15;

Comment: Historical VaR model
var(i) = quantile(epnl,0.01);

Comment: Simulated 1 VaR model
svar1(i) = quantile(espnl1,0.01)*15;

Comment: Simulated 2 VaR model
svar2(i) = quantile(espnl2,0.01)*15;

Comment: actual PnL
apnl(i)= tport-cport;

end

Comment: defining matrices for the graphical test
R=[];
R=[HESmean',HESmedian', SESmean1',SESmedian1',var',svar1',apnl'];

H = [];
H = [SESmean1',SESmedian1',SESmean2',SESmedian2',apnl'];

G = [];
G = [var',svar1',svar2',apnl'];
Comment: defining matrices for the ANOVA testing conducted below
D = [];
D = [HESmedian',HESmean',var',SESmean1'];

E = [];
E = [HESmedian',HESmean',var',SESmedian1'];

F = [];

```
F = [var',SESmean1',SESmedian1'];

W = [];
W = [var',SESmedian1'];

Z = [];
Z = [SESmean1',SESmedian1'];
```

Comment: Graphical tests for the predictability of the SES models versus other benchmarking models

```
figure
subplot(3,1,1);
plot([R -R])
legend('HESmean','HESmedian', 'SESmean1','SESmedian1','var','svar1','apnl')
title('Robustness of the Simulated Jump-Diffusion Model(SES)');
xlabel ('Time [Years]');ylabel('Loss vs Risk estimates');
axis tight;

subplot (3,1,2);
plot([H -H])
legend('SESmean1','SESmedian1','SESmean2','SESmedian2','apnl')
title('Consistency graphical test');
xlabel ('Time [Years]');ylabel('Loss vs Risk estimates');
axis tight;

subplot (3,1,3);
plot([G -G])
legend('var','svar1','svar2','apnl')
title('Robustness of the Simulated Jump-Diffusion VaR Model');
xlabel ('Time [Years]');ylabel('Loss vs Risk estimates');
axis tight;
```

Comment: Graphical tests about the representation of the simulated data against the backdrop of the historical data

```
figure
subplot(2,1,1)
hist([rdata rsdata1])
legend('Returns of historical data','Returns of simulated data1')
```

title(‘Consistency in simulated data in representing historical data’);
xlabel (‘Time [Years]’);ylabel(‘Returns’);
axis tight;

subplot(2,1,2)
hist([rdata rsdata2])
legend(‘Returns of historical data’,’Returns of simulated data2’)
title(‘Consistency in simulated data in representing historical data’);
xlabel (‘Time [Years]’);ylabel(‘Returns’);
axis tight;

Comment: Respective ANOVA tests conducted
diary ANOVA.txt

diary on

display(’—start—’);
$[P, ANOVATab, Stats]$ = anova1(D)
title(‘Historical Expected Shortfall (HES) mean model versus Simulated Expected Shortfall (SES) mean model’);
display(‘—end—’);

display(‘—start—’);
$[P, ANOVATab, Stats]$ = anova1(E)
title(‘Historical Expected Shortfall (HES) median model versus Simulated Expected Shortfall (SES) median model’);
display(‘—end—’);

display(‘—start—’);
$[P, ANOVATab, Stats]$ = anova1(F)
title(‘Historical VaR model versus Simulated Expected Shortfall (SES) mean model’);
display(‘—end—’);

display(‘—start—’);
$[P, ANOVATab, Stats]$ = anova1(W)
title(‘Historical VaR model versus Simulated Expected Shortfall (SES) median model’);
display(‘—end—’);

display(‘—start—’);

$[P, ANOVATab, Stats]$ = anova1(Z)
title('Simulated Expected Shortfall (SES) mean model versus Simulated Expected Shortfall (SES) median model');
display('—end—');
diary off

D.4.1 Supporting code to Risk model

Comment: This function calculates the returns used in the comparison conduct in the main script as defined under section D.3.

```
function RetData=Returns(data,dt)

m=size(data,1);
k=size(data,2);
n=m-1;

for j=1:k

for i=2:m
Sin1 = data(i-1,j);
Si = data(i,j);

RSi = Si/Sin1;

CalcData(i-1,:)=[RSi];

end

RetData(:,j) = CalcData;

end
```

Appendix E

Simulated data

AGL	BIL	AMS	ANG	ARI	IMP	OML	GFI
15,530.61	6,215.00	28,724.08	22,000.07	3,500.00	6,981.25	1,289.52	7,078.98
15,568.61	6,223.80	28,592.84	22,015.91	3,506.57	7,007.61	1,293.03	7,093.12
15,547.43	6,238.71	28,617.20	22,035.16	3,511.22	7,039.12	1,297.20	7,124.81
15,582.48	6,254.07	28,581.41	21,983.99	3,514.14	7,046.83	1,294.50	7,123.27
15,677.30	6,247.19	28,560.48	21,942.27	3,515.38	7,075.80	1,300.56	7,098.63
15,625.85	6,234.16	28,502.57	21,942.08	3,519.46	7,092.67	1,300.60	7,105.96
15,568.90	6,240.29	28,452.61	21,938.53	3,524.20	7,090.49	1,297.43	7,133.72
15,511.15	6,252.47	28,336.47	22,029.45	3,530.04	7,071.09	1,297.23	7,118.08
15,542.67	6,238.65	28,362.84	22,041.47	3,525.70	7,039.36	1,291.74	7,152.18
15,587.99	6,238.52	28,384.05	22,135.02	3,530.77	7,048.72	1,292.58	7,149.33
15,581.09	6,246.65	28,393.88	22,083.67	3,521.16	7,037.81	1,289.82	7,178.04
15,606.81	6,243.35	28,426.61	22,066.91	3,518.14	7,039.74	1,292.52	7,174.27
15,542.03	6,249.92	28,470.70	22,164.66	3,527.38	7,041.04	1,292.48	7,154.14
15,532.98	6,243.13	28,603.56	22,173.18	3,527.93	7,021.59	1,290.25	7,124.94
15,558.77	6,239.03	28,623.59	22,149.04	3,539.83	7,039.89	1,288.08	7,124.46
15,542.86	6,221.03	28,593.20	22,208.99	3,558.66	7,028.74	1,294.16	7,119.55
15,593.91	6,228.27	28,623.49	22,156.30	3,566.41	7,037.20	1,299.90	7,096.89
15,599.30	6,258.73	28,553.99	22,208.02	3,589.17	7,042.64	1,297.61	7,061.70
15,616.19	6,255.84	28,628.89	22,133.48	3,594.70	7,087.47	1,297.28	7,106.36
15,623.66	6,268.59	28,698.98	22,093.46	3,607.96	7,127.04	1,294.72	7,128.86
15,615.98	6,274.27	28,694.98	22,051.29	3,618.64	7,140.58	1,293.37	7,126.97
15,646.99	6,280.57	28,759.53	22,018.17	3,630.66	7,118.44	1,293.09	7,118.79

Table E.1: Sample of the simulated data