APPENDICES

Appendix A:

An outline of OBE principles and characteristics

Some basic principles

OBE is an assessment-driven system that operates through the setting up of standards. *Qualification/ unit standards* are parameters that guide learners and the general public about qualities, values, attitudes, knowledge and skills expected of them as competent citizens, professionals. *Outcomes* are statements regarding what a student can do and what he/she understands, the contextually demonstrated end products of the learning process. Outcomes are the results of learning processes. It is important that the results of learning processes, viz. knowledge, skills, attitudes and values are seen to be demonstrated within a particular context so that knowledge is applied, skills are developed into competencies, and attitudes and values harmonise with those of society and the workplace. Various types of competencies were identified, with particular focus on *applied competence*.

Applied competence is 'the ability to put into practice in the relevant context the learning outcomes acquired in obtaining a qualification' (SAQA Regulations, March 1998). It is a combination of three types of competence:

- practical: knowing how to do things, ability to make decisions
- fundamental: understanding what you are doing and why
- reflective: learn and adapt through self-reflection; apply knowledge appropriately and responsibly.

The focus is thus on output (assessment of competence in terms of pre-determined criteria) rather than input (content), although this does not mean that the knowledge base of a discipline should be neglected. Rather it implies a balance between theoretical and practical relevance.

In OBE, each module or qualification should state the desired outcomes and assessment criteria clearly, so that students know in advance what they need to do in order to achieve the outcomes, and assessors understand the criteria by which the outcomes can be reliably and objectively assessed. It is thus necessary that the outcomes are explicit, transparent, distinct and separately considered. Students need to understand clearly what is being assessed and what they are expected to achieve. The assessment criteria need to specify unambiguously the levels of complexity and understanding of knowledge students will be expected to have reached (Mokhobe-Nomvete, 1999).

Characteristics of OBE

OBE is *assessment driven*: thus assessment criteria must indicate how to determine whether a student has achieved the outcome to a satisfactory standard and what makes the difference between acceptable and unacceptable performance of the outcome.

OBE assessment tools are *more learner centred* than traditional forms of assessment. The OBE philosophy envisages a successful learning experience for all learners. Two broad types of assessment are identified: *formative* and *summative*. Both are designed to improve the quality of students= learning experiences by focusing on significant knowledge and skills and to provide accurate estimates of current competence or potential in relation to desired outcomes to enable lecturers to make appropriate decisions. *Formative assessment* refers to assessment that takes place during the process of learning and teaching. Its purposes may be to diagnose learner strengths and weaknesses, or provide feedback to learners on their progress (or lack thereof). Formative assessment takes place on a continuous basis during the course of learning. *Summative assessment* takes place at the end of a module or course, and traditionally takes the form of an exam.

OBE is *criterion referenced*, i.e. it measures the learner=s achievement against a set of predetermined criteria and not in relation to the achievement of other learners.

OBE makes use of *clearly stated outcomes* and *assessment criteria*. Each module or qualification states outcomes (known as general, cross-field and specific outcomes) and associated assessment criteria clearly, so that students understand in advance what they have to do to achieve these outcomes and assessors can use the criteria to assess the outcomes with reasonable objectivity/ reliability.

OBE permits a *variety of assessment methods and instruments*. Because assessment assesses a range of elements (knowledge, performance, abilities, etc.) it allows for the use of a variety of assessment methods and instruments. It also allows for the collection of evidence from a variety of sources.

Appendix B:
Syllabus and outcomes for the Mathematics Access Module

The module covers certain topics from school mathematics and relevant applications to real-life situations. The outcomes of the module are that students should learn to

- take responsibility for their own learning
- interpret and evaluate mathematical information, represented numerically, algebraically, geometrically, statistically or graphically, in articles appearing in newspapers etc.
- critically read and interpret mathematical texts
- interpret and solve mathematical problems, including word problems
- determine and express mathematically correct solutions to mathematical problems
- integrate and apply knowledge acquired from different mathematical topics
- apply mathematical thought processes to real-life situations.

The study package for this module consists of six study guides (seven, until the end of 2003) and several tutorial letters; provision is made for a number of assignments students will submit. Originally the assignments contained only multiple-choice questions which were marked by the computer; from 2001 assignments were written and marked by lecturers and external markers.

Book 1: Introduction

This book includes sections on the outcomes of the module, the way in which the module is structured, the use of the two main types of calculators and some language issues.

Topic 1: **Welcome and the answers to some questions**

Topic 2: **Calculators**

Topic 3: **Language matters**

Book 2: **Number Skills and Algebra Tools**

Topic 1: **The Set of Real Numbers**

Section 1.1: Kinds of Numbers

Study Unit 1.1A: Terminology

Study Unit 1.1B: Decimal representation of real numbers

Study Unit 1.1C: Ordering on the real number line

Study Unit 1.1D: Intervals

Section 1.2: Sets

Study Unit 1.2A: Terminology and set operations

Topic 2: Operations on Real Numbers

Section 2.1: Combining Numbers

Study Unit 2.1A: What we know so far

Study Unit 2.1B: Factors, multiples and fractions

Section 2.2: Rules that Numbers Obey

Study Unit 2.2A: Operations involving integers

Study Unit 2.2B: Properties of real numbers with respect to arithmetic operations

Study Unit 2.2C: Rules that fractions obey

Study Unit 2.2D: Approximation and estimation

Topic 3: Ratio, Proportion and Percentage

Section 3.1: Ratio

Study Unit 3.1A: Ratios involving two quantities

Study Unit 3.1B: Ratios involving more than two quantities

Section 3.1: Proportion

Study Unit 3.2A: Definition of proportion

Study Unit 3.2B: Problem solving

Section 3.3: Percentage

Study Unit 3.3A: Why are percentages important?

Study Unit 3.3B: The meaning of percentage

Study Unit 3.3C: Calculations involving percentages

Topic 4: Integral Exponents, Scientific Notation and Roots

Section 4.1: Integral exponents and scientific notation

Study Unit 4.1A: Integral Exponents

Study Unit 4.1B: Scientific notation and estimation

Section 4.2: Roots and Surds

Study Unit 4.2A: Square roots

Study Unit 4.2B: n^{th} roots (radicals)

Study Unit 4.2C: Surds

Study Unit 2.1A: Terminology and notation

Study Unit 2.1B: Substitution

Section 2.2: Combining Functions

Study Unit 2.2A: Addition, subtraction, multiplication and division

Topic 3: Straight Lines

Section 3.1: Straight Lines

Study Unit 3.1A: Graphical representation of a linear equation using a table of values

Study Unit 3.1B: Linear functions and lines

Study Unit 3.1C: The *y*-intercept and slope of a line

Study Unit 3.1D: Using two points, or one point and the slope, to draw a line

Study Unit 3.1E: Related pairs of lines

Study Unit 3.1F: Graphs of linear functions with restricted domains

Section 3.2: Finding Equations of Lines

Study Unit 3.2A: Slope-intercept method, point-slope method and the two -point method

Study Unit 3.2B: Some special lines

Study Unit 3.2C: The general equation of a line

Section 3.3: Applications of Lines and Linear Functions

Study Unit 3.3A: The intersection of lines and the vertical distance between lines

Study Unit 3.3B: Graphs of linear inequalities in two unknowns

Study Unit 3.3C: Direct proportion and the use of lines in the analysis of experimental data

Topic 4: Parabolas

Section 4.1: Characteristics of Parabolas

Study Unit 4.1A: Parabolas defined by $y = ax^2$

Study Unit 4.1B: Parabolas defined by $y = ax^2 + k$

Study Unit 4.1C: Parabolas defined by $y = a(x - h)^2 + k$

Section 4.2: Sketching Parabolas

Study Unit 4.2A: Sketching parabolas defined by $y = a(x - h)^2 + k$

Study Unit 4.2B: Sketching parabolas defined by $y = ax^2 + bx + c$

Section 4.3: Finding the Equation of a Parabola

Study Unit 4.3A: Three forms of the equation of a parabola

Section 4.4: Using Parabolas and Quadratic Functions

Study Unit 4.4A: Solving quadratic and simple rational inequalities

Study Unit 4.4B: Finding maximum and minimum values

Topic 5: Hyperbolas

Section 5.1: Characteristics of hyperbola

Study Unit 5.1A: Hyperbolas defined by $xy = k$, $k > 0$

Study Unit 5.1B: Hyperbolas defined by $xy = k$, $k < 0$

Study Unit 5.1C: Finding equations of hyperbolas

Section 5.2: Inverse Proportion

Study Unit 5.2A: Inverse proportion

Topic 6: Combination of Graphs

Section 6.1: Graphs, Graphs and more Graphs

Study Unit 6.1A: Graphical representation of systems of linear and quadratic equations

Study Unit 6.1B: Interpreting combinations of graphs

Topic 7: Statistics

Section 7.1: Data Collection and Organisation

Study Unit 7.1A: Collecting data

Study Unit 7.1B: Organising data

Section 7.2: Using Graphs to Represent Data

Study Unit 7.2A: Pie graphs, histograms and frequency polygons

Study Unit 7.2B: Line graphs

Section 7.3: Some Statistical Measurements

Study Unit 7.3A: What is average?

Study Unit 7.3B: Arithmetic mean, median, mode

Section 7.4: Probability

Study Unit 7.4A: A quick look at elementary probability theory

Book 5: **Geometry and the Measurement of Areas and Volumes**

Topic 1: **Geometry**

Section 1.1: Lines and Angles

Study Unit 1.1A: What geometry means

Study Unit 1.1B: Lines and Angles

Section 1.2: Polygons

Study Unit 1.2A: Terminology

Study Unit 1.2B: Triangles

Study Unit 1.2C: Quadrilaterals

Section 1.3: Circles

Study Unit 1.3A: Some basic facts about circles

Topic 2: **Perimeter, Area and Volume**

Section 2.1: Measurements of Perimeter and Area

Study Unit 2.1A: Perimeter and area of two-dimensional objects

Section 2.2: Surface Area and Volume of Three-dimensional Objects

Study Unit 2.2A: Some three-dimensional objects

Study Unit 2.2B: Surface area

Study Unit 2.2C: Volume

Book 6: ***How to Learn Maths*** by **Richard Freeman**

This book was printed under licence from the National Extension College (UK) by UNISA, only available (as part of the study package) to students registered with UNISA. General topics such as:

Part 1: What is maths?

Maths is pattern

Maths is number

Maths is logic

Part 2: How is your maths? (Quiz)

Part 3: Helpful methods

Draw a diagram

APPENDIX C:
SAQA REQUIREMENTS

(Results of a workshop presentation: Alice Goodwin-Davey; UNISA SAQA Action Group, Bureau for University Teaching, September 2000)

Requirements are set out in the **SAQA Act (Act 58 of 1995)** and **NSB Regulations (Regulation 452, No. 18787: March 1998)**.

SAQA requirements specify that modules and courses must lead to identifiable outcomes within disciplines and fields, as well to cross-disciplinary outcomes. The outcomes-based education approach requires planners to describe the knowledge, skills and values that a well-educated person would display, having qualified in a given field. SAQA documentation refers to specific and general learning outcomes, range statements (the breadth and depth in which a particular topic is covered), assessment criteria (the means of determining whether stated outcomes have been achieved), and critical cross-field outcomes (i.e. outcomes that are generic to any academic environment, regardless of discipline). Interpretation of SAQA requirements for the Mathematics Access Module appears in Appendix C*, below.

The National Qualifications Framework (NQF) is an outcomes-based framework for education and training standards and qualifications. The objectives of the NQF are the following (SAQA, 1995).

- To create an integrated national framework for learning achievements.
- To facilitate access to, and mobility and progression within, training and career paths.
- To enhance the quality of education and training.
- To accelerate the redress of past unfair discrimination in education, training and employment opportunities, and thereby.
- To contribute to the full personal development of each learner and the social and economic development of the nation at large.

Outcomes-based Education (OBE) is an assessment-driven system that operates through the setting up of standards. *Qualification/ unit standards* are parameters that guide learners and the general public about qualities, values, attitudes, knowledge and skills expected of them as competent citizens, professionals, etc. SAQA regulations stipulate that standards contain information on purpose, specific learning outcomes and their associated assessment criteria, integrated assessment (including formative and summative), embedded knowledge, critical cross-field outcomes, range statements, accreditation process (including moderation), applied competence, assumptions of prior learning and recognition of experiential learning.

Outcomes are what a student can do and what he or she understands, the contextually demonstrated end products of the learning process. Outcomes are the results of learning processes – knowledge, skills, attitudes and values – within a particular context so that knowledge is applied, skills develop into competencies and attitudes and values harmonize with those of society and the workplace. An outcome can be demonstrated and measured. Outcomes can be expressed as ‘Learners can ...’. For example, ‘Learners can solve problems responsibly and creatively’. These outcomes can be assessed against predetermined criteria using a range of appropriate assessment methods.

Applied competence is ‘the ability to put into practice in the relevant context the learning outcomes acquired in obtaining a qualification’ (SAQA Regulations, March 1998). It is a combination of three types of competence:

- practical: knowing how to do things, ability to make decisions
- fundamental: understanding what you are doing and why
- reflective: learn and adapt through self-reflection; apply knowledge appropriately and responsibly.

Competence has to be described in terms of the specialist field and expectations of student achievement have to be realistic in terms of that description.

Exit-level outcomes provide criteria for measuring foundational competence (i.e. knowledge: the essential general and specific knowledge that learners must have at their command); practical competence (i.e. skills: specific skills for specific contexts, specific interpersonal skills and management skills as well); reflexive competence (i.e. values and attitudes – essential principles outcomes must embody); outcomes relate to people applying knowledge through authentic tasks, i.e. the kinds of tasks required of us in real life situations.

The focus is thus on output (assessment of competence in terms of preset criteria) rather than input (content), although this does not mean that the knowledge base of a discipline should be neglected. Rather it implies a balance in university studies between formative/ theoretical and practical relevance.

Assessment/ performance criteria specify how much learning has to be evidenced, at what level of complexity and responsibility and how well. Assessment criteria provide evidence that the learner has achieved the outcome. Assessment criteria provide guidelines to authentic tasks; they holistically unpack the outcome: steps in a holistic process; how to show competence; knowledge skills and attitudes. Assessment criteria show evidence of learning inn different ways. Assessment criteria complete the stem: 'Evidence must show that learners ...': for instance, 'Evidence must show that learners use words according to standard dictionary definitions and the demands of context'. Assessment criteria have to indicate how to determine whether a student has achieved the outcome to a satisfactory standard and what makes the difference between acceptable and unacceptable performance of the outcome.

The outcomes and their associated assessment criteria will be available to students and other stakeholders so the learning and assessment system will be transparent, reliable and accountable. Students will know what is expected of them and employers will know what a learner who holds a particular qualification has achieved.

Range statements provide the scope and depth of the performance task (in terms of level of complexity and the extent of performance expectations); determine conditions which apply during assessment; fall within NQF level descriptors; note that the same outcome at different levels will have very different ranges.

Critical cross-field outcomes stipulate that learners can

- solve problems and think creatively
- work with others (groups/teams)
- organise and manage themselves
- collect, analyse and evaluate information
- communicate effectively
- use science and technology
- demonstrate understanding of their world.

There are also developmental outcomes, which state that learners can

- reflect on and explore learning strategies
- participate as responsible citizens, locally and globally; be culturally and aesthetically sensitive; explore education and career opportunities;
- develop entrepreneurial opportunities.

OBE has led to a broader, more learner-centred model of assessment, which aims at success for all learners. Two broad types of assessment are identified: formative and summative.

Appendix C*:
Interpretation of SAQA requirements for the Mathematics Access Module

The **specific outcome** of the module is the following:

- Demonstration of mathematical literacy and proficiency at the stated levels.

The **purpose of the module** as stated in the required SAQA form (Form 3) for modules on the Senate/Senate Executive (Senex) and Experiential Learning database, is:

- To gain the necessary mathematical knowledge and skills to study mathematics at tertiary level, and to provide a sound base on which to build the study of life sciences and other natural sciences, at tertiary level.

On the same form, the **stated learning outcomes** of the module are the following:

- Learners can acquire the necessary numerical, algebraic, geometrical and statistical skills that are appropriate for entrance to university study.
- Learners can solve problems that require integration of knowledge acquired from different mathematical topics.
- Learners can interpret, analyse, organise and critically evaluate mathematical information, represented numerically, algebraically, geometrically or graphically.
- Learners can interpret and solve appropriate mathematical problems, presented in various forms.
- Learners can apply mathematical thought processes to real-life situations.

As required by SAQA, a **range statement** (giving an indication of scope and context), and **associated assessment criteria**, are provided for each learning outcome.

The **range statement** (which applies to all the outcomes) is:

- Appropriate mathematical skills will be demonstrated in contexts typical of the demands of first-year undergraduate study, with appropriate adaptation necessary for learners who require academic and linguistic support.

The **assessment criteria** state that learners will be assessed on their ability to

- use mathematical terminology and notation correctly so that calculations and solutions to problems are correctly presented
- use arithmetic, such as ratio, proportion, percentage, integral exponents, roots, scientific notation and estimation, and units with sufficient accuracy to facilitate calculation
- demonstrate understanding of the real number system through calculations with its various elements
- demonstrate understanding of algebraic concepts, such as algebraic expressions, equations, inequalities, functions (specifically linear, quadratic, hyperbolic, exponential and logarithmic functions)
- draw, use and interpret graphs of functions, specifically linear, quadratic, hyperbolic, exponential and logarithmic functions
- use and interpret graphs and tables accurately
- apply simple data-handling techniques to deal with elementary statistical problems
- recognise, and use principles relevant to, basic geometric shapes, specifically the triangle, square, rectangle, parallelogram, rhombus, kite, cylinder, sphere, cone, and perform calculations of perimeter, area and volume in relation to the geometric shapes mentioned.

Apart from the specific learning outcomes, learning programmes are required to satisfy various critical cross-field outcomes as well. The **critical cross-field outcomes** of the module are the following: Learners should be able to

- communicate effectively using mathematical notation and terminology in a written medium, apply these skills to the mathematical tasks in the study guide, assignments and examinations.
- take responsibility for their own learning.
- interpret, analyse, organise and critically evaluate information, represented numerically, algebraically, geometrically or graphically

- identify and solve simple problems.

With these above ideas in mind, the **main aims** of the Mathematics Access Module are that students should cover the essential aspects of mathematics that are required to

- handle data
- do calculations (such as those involving percentages, ratios, proportions, decimals, fractions)
- perform basic algebraic manipulations (such as solving equations and inequalities)
- understand and work with functions represented by equations or graphs (linear, quadratic, exponential, logarithmic)
- understand the basics of Euclidean geometry
- perform calculations involving area and volume of a variety of objects
- solve problems related to all these topics

Appendix D:

Articles (Bohlmann & Pretorius)

READING SKILLS AND MATHEMATICS

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This paper considers the relationship between poorly-developed reading skills and academic performance in mathematics. It discusses some aspects that underpin all successful reading and considers these in relation to the reading difficulties experienced by a group of foundation phase mathematics students. The project investigating these difficulties was divided into a testing phase and an intervention phase. This paper reports on the testing phase.

1. INTRODUCTION

The persistent problem of poor academic performance of many students at primary, secondary and tertiary level is disturbing, particularly in science and mathematics. The conceptual complexity and problem solving nature of these disciplines make extensive demands on the reasoning, interpretive and strategic skills of students, especially when carried out in a language that is not the student's primary language. It is well known that South African students have achieved the questionable distinction of being at the bottom of the list in the TIMSS report (Third International Mathematics and Science Study 1999). Although there has been considerable controversy surrounding these tests, such as bias towards Western (particularly North American) learners, the fact remains that, even in comparison with the rest of Africa, South African learners performed extremely poorly.

There are obviously many factors, both extrinsic and intrinsic to a learner, that contribute to poor academic performance. Many studies, for instance, have been undertaken on the role of *language* in mathematics, and we know a lot about the ways in which poorly developed language skills undermine students' mathematical performance. But to what extent does *reading ability* influence a student's ability to comprehend and do mathematics? This question is particularly important in the context of print-based distance education where students need to be good readers in order to 'read to learn'. As long ago as 1987 it was pointed out (Dale & Cuevas, 1987) that proficiency in the language in which mathematics is taught, especially reading proficiency, was a prerequisite for mathematics achievement. When students have difficulty reading to learn, it is often assumed that their comprehension problems stem from limited language proficiency. This reflects an underlying assumption that language proficiency and reading ability are basically 'the same thing'. If this were so, then all mother-tongue speakers should automatically be good readers in their mother tongue. This is patently not so. Furthermore, if language proficiency and reading ability were basically 'the same thing', then improving the language proficiency of students should improve their reading comprehension. Research shows that this does not readily happen (e.g. Hacquebord 1994). Reading is more than fluency in articulating what is written; it is also more than understanding the sum of the meanings of individual words. Language proficiency and reading are clearly related. However, they are, conceptually and cognitively, uniquely specific skills that develop in distinct ways and that rely on specific cognitive operations. It is important to recognise this distinction because it has pedagogical implications.

In this paper we report on a study undertaken jointly by the Departments of Mathematics and Linguistics at the University of South Africa (Unisa), in which the reading skills of mathematics students were tested and the relationship between reading ability and performance in mathematics was examined. Reading ability, in the context of this study, refers to *reading comprehension* ability. In reading research, a distinction is made between *decoding* and *comprehension*. *Decoding* involves those aspects of reading activity whereby written signs and symbols are translated into language. *Comprehension* refers to the overall understanding process whereby meaning is assigned to the whole text. The interaction between decoding and comprehending in skilled readers happens rapidly and simultaneously. Most researchers and practitioners of reading agree that comprehension cannot effectively occur until decoding skills have been mastered (e.g. Perfetti 1988). However, skill in decoding does not necessarily imply skill in comprehension. Many readers may readily decode text but still have difficulty understanding what has just been decoded (Daneman 1991; Yuill & Oakhill 1991). In this study, students were tested on comprehension skills and not on decoding skills, since it was assumed that, by tertiary level, decoding skills are well established and automatised.

2. MATHEMATICS DISCOURSE AND READING

Mathematics is taught and understood via the sub-language of mathematical discourse, or the *mathematics register* (Frawley 1992; Dale & Cuevas 1987). In general the mathematics register is abstract, non-redundant (Prins 1997) and conceptually dense. It features *more complex* and *more compact relationships* than normal discourse (Crandall, Dale, Rhodes and Spanos, 1980). Furthermore, mathematics is a discipline characterised by precision, conciseness and lack of ambiguity. The reading of mathematics texts thus requires close attention to detail. Parts of mathematics texts tend to be procedural in that they provide instructions and explanations on how to carry out a task or algorithm. Other activities aim to develop conceptual knowledge. Mathematics texts are also *hierarchical* and *cumulative*, in the sense that understanding each statement or proposition is necessary for understanding subsequent statements. If a particular step in a method, procedure or argument is misunderstood or overlooked, this has severe consequences for overall comprehension. Reading mathematics texts also requires *integration of all the information* in the text. A mathematics reader thus needs to interact with the text, be alert and attentive, be sensitive to comprehension failure as soon as it happens, and be capable of applying repair strategies when comprehension failure occurs. *Reading rate adjustment* and *multiple readings* are also necessary because of the conceptual density of, and the interpretative demands made by, mathematical symbols and graphic aspects, such as charts, tables and graphs.

2.1 Rationale for study

In the Department of Mathematics at Unisa there has been significant growth in the number of students enrolling for the mathematics Access Module, which is provided through the medium of English. These students generally have a school-leaving certificate, which does not permit them to study at a university; or they may have a university exemption matric certificate (which does give them access to university) but without the necessary mathematics symbol to study in the Science Faculty. The student population is extremely heterogeneous, in that some students may not have studied mathematics beyond Grade 9, while some may have studied it up to Grade 12 but obtained less than 40% on the Higher Grade or less than 50% on the Standard Grade.

Many of the students who enrol for this module are characterised by weak academic, linguistic and mathematical ability. In spite of our recognition of their difficulties and some student support activities (limited, due to the constraints of distance education and the financial resources of both the university and the students) over the two years of the existence of the current module, student performance has been unacceptably low.

It is well recognised that mathematical thought is best developed by interaction between learners and teachers, and between learners and learners, mediated in a variety of ways. In our particular context, where student access to technological resources cannot be guaranteed, and where distance limits the extent to which group learning opportunities can be utilised, text becomes the primary resource for learning. In the light of these factors it was decided to examine more closely the reading abilities of the students registered for the Mathematics Access Module. The two main research questions that inform the study are:

1. Is there a relationship between reading ability and academic performance in mathematics?
2. What specific kinds of comprehension problems do students experience during the reading of mathematics texts?

3. READING COMPREHENSION: ANALYTIC FRAMEWORK

This paper focuses on *reading as a way of constructing meaning.* In the study the reading process is assumed to involve the simultaneous interaction of several component skills. Readers are viewed as active participants who construct meaning while they read by building up a coherent mental representation of the text. They add to and modify this representation as they encounter new information in the text. This representation is not identical to the text; rather, it is a mental map of what the text is about. It is constructed from explicit information in the text and from the inferencing and integrative processes that readers engage in to link up items of information in the text to one another, and to link items of information in the text to items of information stored in long-term memory, the storehouse of one's background knowledge. If readers cannot create a coherent representation of what the text is about, then meaningful comprehension does not occur.

In order to assess the extent to which readers are constructing meaning as they read, it is important to examine those aspects of a text that typically engage readers in perceiving links between items of information. We now take a closer look at some of the processes that contribute to meaning construction during reading.

3.1 Anaphoric resolution

Anaphora basically involves repeated reference in discourse. In other words, a referent relating to a person, entity, event, state or idea that is introduced into a discourse is referred to again, either by means of repetition of the same linguistic item (e.g. noun or proper noun) or by means of another linguistic item (e.g. pronoun or synonym). Consider, for example, the repeated reference in the following mathematics text:

> Multiplication is denoted by the symbols x, . or ( ). The result of multiplying two numbers *a* and *b* is called the product of *a* and *b*, and we usually write *a x b* or *a.b*. Because *a* and *b* represent two different numbers (and are not two digits that make up a single number) there is no confusion if we write *ab* instead of *a* x *b*. It is easy to see that we cannot do this when numerals are used, because 23 does not mean 2 x 3. (Singleton & Bohlmann 2000:54)

The highlighted noun phrase mentions a specific convention in the text, and it is referred to again by means of the determiner *this*. This determiner takes its interpretation from the conditional clause referring to the notion of writing *ab* instead of *a* x *b.* The underlined item in the text is called an anaphor, and the shaded entity it refers back to is called the referent. Successful anaphoric resolution contributes to the construction of a coherent text representation because it enables the reader to identify and track topic continuity in the text, shifts in the topic, and to make connections between elements in the discourse. Skilled readers should be able to resolve anaphors with almost total accuracy (Webber 1980).

3.2 Text-semantic relations

Coherence in a text derives to a large extent from the semantic and rhetorical relations underlying units of discourse. These relations can be represented in "general conceptual terms, abstracting away from the context-specific content of the segments" (Sanders et al. 1992:2). These conceptual relations include four basic types, viz. additive (X *and* Y), temporal (X *then* Y), causal (X, *as a result* Y*)* and contrastive relations (X. *However,* Y). Although these relations have been variously referred to as *basic thought patterns* or *logical relations* (Brostoff 1981), *semantic relations* (Fahnestock 1983) and *relational propositions* (Mann & Thompson 1986), they all basically refer to the same concept and will henceforth collectively be called *text-semantic relations*. These relations underlie the way we perceive the world and the way we think.

Perception of these relations is fundamental to the construction of coherent mental representations of text. Research findings indicate that knowledge of such relations distinguishes skilled readers from their less skilled counterparts (e.g. Meyer, Brand & Bluth 1980; Geva & Ryan 1985). Skilled readers perceive these relations, albeit unconsciously, and this enables them to see the connectedness between items of information in the texts as they read.

In this study, attention was specifically focused on causal and contrastive relations since they are a common feature of mathematic discourse. In order for a semantic relation to exist between text units there must, minimally, be a binary relationship between two (or more) text units: 'binary' in that one unit is semantically linked to the other and in some way completes the meaning of the other. For instance, in causal relations, the one member functions as an antecedent [X] and the other as a consequent [Y].

3.2.1 Causal and conditional relations

There are several notions of causality, ranging from strict formal definitions of a deductive nature to intuitive but fuzzy lay notions. The concept of causality refers basically to the relationship between two events or states of affairs, such that the first one, the antecedent X, brings about or enables the second, the consequent Y. Although there are many different types of causal relation (Pretorius 1994), in this study causal relations were defined quite broadly into two categories, viz. *general causal relations* and *conditional causal* relations.

The first category included relations where the antecedent causes the consequent or the antecedent comprises a premise or reason for a consequent result. In other words, the contents expressed in a prior unit of text provide a basis from which a conclusion is drawn. In expository writing, the supporting statement(s) X can comprise an observation, causal statement, argument, or evidence, from which a deduction or conclusion Y is drawn. As Fahnestock points out, the sentences comprising a premise-conclusion link in everyday logic are not always expanded into valid syllogisms, 'but they are intended by the writer and meant to be taken in by the reader as pairs of supporting and supported statements' (Fahnestock 1983:404). Consider the following extended causal argument with the causal conjunctives in italics.

> Consider the logarithmic function $y = f(x) = \log_a x$. We know that $D_f = \{x: x > 0\}$. *Thus* for the function g defined by $g(x) = \log_a(2x + 1)$, we have $D_g = \{x: x > - 1/2\}$. *Hence* $x = -1$ is not an element of D_g.

The second category of causal relations included a specific type of causality involving *conditional causal* clauses. This distinction was made because of the high frequency of *if - then* clauses in mathematical texts. Traditionally, in syntax and logic, conditional clauses describe a situation in which it is claimed that "something is, or will be, the case, provided that, or on condition that, some other situation obtains" (Flew 1984:70). This kind of logical relation is regarded as a subcategory of the causal relation since there is a dependency relation between the

X and Y members, such that Y does not occur unless X occurs. One of the main differences between conditional causal clauses and ordinary causal clauses lies in the truth-conditional status of the former, where the truth of the consequent is guaranteed by the truth of the antecedent. Consider the two examples of conditional relations below:

> Consider $-1 < x \leq 2$. Both the statements $-1 < 2$ and $-1 \leq 2$ are true. The first inequality, $-1 < 2$, is true ***because*** -1 *is* less than 2. The second inequality will be true ***if either*** $-1 < 2$ ***or*** $-1 = 2$ is true. Obviously $-1 = 2$ is not true, but *since* $-1 < 2$ is true, we may write $-1 \leq 2$. (Singleton & Bohlmann 2000:23)

Research has established that the recognition of elements in a text that are causally linked to each other is an important determinant of text comprehension (cf. Trabasso et al. 1989; Pretorius 1996).

3.3 Contrastive relations

Another category of text-semantic relations to be examined was that of contrastive relations. In a contrastive semantic relation, the second statement in the 'pair' carries information that counters the information in the first part. It presents an opposing or unexpected point to what has been stated, a concession or qualification to a previous statement, or a denied implication. It provides a less expected alternative to what has already been stated. Fahnestock (1983:415) points out that such text-semantic relations reflect "the processes of distinguishing, making exceptions, conceding or contrasting by which thinking, and the prose which represents thinking, is carried on". Skilled reading naturally includes the ability to perceive and follow discontinuative turns in the text. In order to understand contrastives, one needs to understand the line of argument presented in the first pair part in order to recognise that the second pair part reflects an opposing, contrasting or qualifying point. For example:

> . When we work with natural numbers we find that adding two natural numbers results in a natural number *but* subtracting one natural number from another does not necessarily result in a natural number. For example, $3 + 5 = 8$, and 3, 5 and 8 are all natural numbers. *However*, $8 - 5 = 3$ and 3 0 ⌉, *but* $5 - 8$ is a calculation that cannot be performed in ⌉. (Singleton & Bohlmann 2000:70)

In the above example, the argument concerns the additive operation involving natural numbers, which is then qualified. The view that follows (signalled in italics by *but* and later by *however*) is a contrastive one in the sense that it qualifies the previous phrase by pointing out that the subtractive operation does not necessarily result in a natural number.

Research has shown that negative statements tend to be processed more slowly than positive statements, and the same seems to be true for contrastive text-semantic relations. Because they present a contrasting point of view to that just given, they are not understood as readily as their continuative counterparts (Fahnestock 1983:406). Research also indicates that in both L1 and L2 learners, contrastive relations are usually acquired later than additive, temporal and causal relations (McClure & Geva 1983).

3.4 Sequencing

Sequencing refers to the way in which propositions in a text follow one another. The most obvious type of sequencing is temporal sequencing, where sentences in a discourse are linked by means of a temporal sequence. Temporal sequences are also often tied up with causal relations, since many states/events/arguments are the result of prior causes or reasons and follow them temporally (although their actual order of presentation in a text may be reversed). The sequencing of events, states or arguments also includes causal and contrastive sequences, as well as the ordering of examples after explanations (i.e. exemplification sequence).

The concepts and arguments dealt with in mathematical texts are often presented within a developmental or logical perspective. It is important for a reader to keep track of sequence in order to understand the relationships between successive propositions. The ability to sequence ideas is an important reasoning skill and, as Lesiak & Bradley-Johnson (1983:212) point out, this skill “can affect performance in several academic areas”.

3.4 Graphics

Graphics refers to tables, graphs, schemas and other visual aids that occur in texts to represent and complement verbal information. Graphic information forms an integral part of mathematics texts. The ability to understand the information represented in these visual forms and to relate it to the information in the text forms a crucial part of mathematics reading comprehension.

3.5 Low-frequency vocabulary

There are statistical differences in the distribution of words that occur in oral and written contexts. A distinction is commonly made in vocabulary studies between three main categories of words, viz. *common* or *high-frequency* words, *academic* words and *technical* words. *High-frequency* words are words that occur in our everyday conversations and comprise a core of about 5 000 - 6 000 basic words. A much larger group of words make up the rest of the English vocabulary, but comprise low- frequency words. *Academic words* refer to a group of about 800 - 1 000 words that occur commonly in academic discourse. These are words that occur across discipline boundaries, e.g. words such as *hypothesis, proponent, assumption, paradigm, posit,* etc. These words seldom occur in everyday conversations, and are hence categorised as low-frequency words. The final group of low-frequency words, *technical* words, are discipline related words that occur with high frequency in a specific discipline and reflect the 'tools of the trade' within that discipline, but outside of the discipline are seldom used. Technical terms in mathematics include *integers, logarithm, operation, square root, exponents,* etc.

Research has shown that in the learning context, knowledge of low-frequency words is associated with academic success. Research indicates that students with smaller vocabularies typically know a higher percentage of high-frequency words (Corson 1983; Cooper 1996), words which occur predominantly in oral contexts. Students who do little reading in English have poor exposure to low-frequency words. From the few vocabulary studies that have been undertaken in South Africa, we see disconcerting findings on the low vocabulary levels, particularly of low-frequency words, of L2 students who study through the medium of English. For example, Cooper (1996) found a relationship between vocabulary levels and academic performance, with weak students having significantly fewer low-frequency words.

Vocabulary knowledge is claimed by many to be one of the best predictors of reading comprehension (e.g. Davis 1968). Intuitively this makes sense, for words form the building blocks of meaning. However, this does not explain why differences in vocabulary knowledge arise in the first place. A typical feature of skilled readers is their large vocabulary, while weak readers typically have low vocabulary levels. How do skilled readers come to acquire so many more words than their unskilled counterparts? In fact Daneman argues that differences in vocabulary size are "the *result* of differences in reading skill rather than the primary cause of such differences" (1991:525). In other words, students' scores on a vocabulary test containing both high- and low-frequency words can indirectly indicate the extent to which they are 'readers'.

4. METHODOLOGY

In this section we describe the subjects, materials and procedures undertaken during the pilot phase and we focus on the nature of the reading tests and the results that they yielded.

4.1 Framework of the research project

The study spanned a period of two years. During the first phase (2000) appropriate reading tests were designed specifically to test mathematical comprehension. These were first piloted on a small group of students (25) before the larger group (n = 960) was tested. The pilot study was undertaken to get a 'feel' for the reading and language proficiency levels of the Mathematics Access students and to trial the reading tests that were designed. During the second phase (2001) an experimental and control group were set up and an intervention programme was undertaken for the experimental group, based on the results of the reading test of 2000. This paper reports on the first phase of the project.

4.2 Subjects

The pilot study consisted of a group of 25 students who were registered for the Mathematics Access Module (MAT011-K) and who came to campus everyday. They completed a battery of tests comprising four main components (to be explained below). They also wrote a standardised language proficiency test and a general reading test based on non-expository texts (2 newspaper reports) but which included similar items to those of the mathematics reading test. The purpose of the pilot tests was to identify potential problems with the selected texts and the test items, and to test the reliability of the test items. Once these tests had been administered and refined, they were then sent to all 960 students registered for the Mathematics Access Module in 2000.

4.3 Material

In order to build up an in-depth profile of the students' reading ability, a series of tests was designed that focused on specific reading skills. The texts that were used were authentic mathematics texts taken from the six study guides that form the basis of the Access Module.

The final test comprised the following four sections:

Section A — A questionnaire that obtained biographic details about the student such as matric performance, attitudes towards reading, their perceptions of their own reading skills, and information concerning their reading practices.

Section B	This section focused specifically on anaphoric references. This comprised 21 separate paragraphs of mathematics texts in which a total of 26 anaphoric items were tested.
Section C	This section focused specifically on vocabulary assessment via a discrete, context-embedded test. It comprised 15 multiple-choice items, testing high-frequency, academic and technical terms respectively.
Section D	This component of the test, comprising 56 items, focused mainly on testing the comprehension of text-semantic relations, sequencing and graphics. Items that tested the readers' ability to infer sequencing consisted of the re-ordering of scrambled paragraphs. This method of testing comprehension has been shown to have high psychometric value (Pagé 1990:118).

The test was posted to all 960 students registered for the Access Module as part of an extra, voluntary but credit-bearing assignment. To encourage participation, five prizes of R200 each were offered, with the winners' student numbers being randomly drawn. In all, 402 responses were received. Due to the distance education context, there were no controls regarding the time it took the students to complete the tests. Although the students were asked to be as honest as possible in answering the tests and to do it on their own, there were also no controls over the sources they might have used to help them.

5. RESULTS

In this section we report on various results of the reading test and the reliability of the test.

5.1 Reliability test

The alpha (Cronbach) model reliability test, available on the SPSS programme, was applied to each component of the test. This is an analysis of internal consistency, based on the average inter-item correlations. It provides an overall index of the internal consistency of a test and identifies problem items that could be eliminated from the test. Reliability scores of between 0.60 and 0.70 are regarded as satisfactory, while scores above 0.80 are regarded as desirable. Section B, C and D of the final mathematics reading test had alpha reliability coefficients of 0.85, 0.70 and 0.80 respectively.

5.2 The relationship between reading skill and mathematical performance

In order to get some idea of the relationship between reading ability and performance in mathematics, the students were categorised into different groups, based on the following two criteria:

1. Their overall reading scores, i.e. the sum of their scores for Sections B, C and D of the reading test, converted into percentages; these groups are referred to as 'reading groups'.
2. Their results in the final mathematics examination, expressed as a percentage; these groups are referred to as 'academic groups'.

On the basis of the results in the overall reading scores, the following four reading categories were identified:

1. Students with a comprehension level below 45%. These students have major reading comprehension problems.
2. Students with comprehension levels of between 46% - 59%. These students are reading at frustration level and have a fragmented understanding of the texts they read.
3. Students with comprehension levels of between 60% - 74%. These are students with reading comprehension problems who cope to some extent and who would probably benefit from reading instruction.
4. Students who scored 75% or higher. These are students who are fairly skilled readers and usually understand most of what they read.

The descriptive statistics in Table 1 below provide an overview of the overall reading scores as well as the scores for each component of the reading test across the different reading ability groups.

Table 1: **Mean reading comprehension scores for the four reading ability groups**[a]

It is not surprising that the mean scores in each subcomponent of the test improve as reading ability improves. What is interesting, however, is that the mean mathematics scores also improve as reading ability improves. In other words, these figures show that the worse the reading ability of the student, the worse their performance in mathematics tends to be. Further analyses showed a wide range in mathematics performance of the skilled reading group (Group 4), and conversely, limited mathematics performance of the very poor reading group (Group1), with scores that tend to cluster in the low 20%. What these results suggest is that while high reading scores do not guarantee mathematical success, a low reading score does limit mathematical achievement. In other words, poor reading ability in this group seems to function as a barrier to effective mathematical performance.

So far we have examined mathematical performance as a function of reading ability. However, it is also interesting to consider reading scores as a function of mathematical performance. The mean percentage for the mathematics examination was 34.5%, with a median of 31%. What was striking about the results was that very few students scored between 60% and 70%. Percentages tended to reflect bad failures (20% - 29%), failures ranging between 30% - 50%, and passes above 70%. On the basis of these results, the following three academic groups were categorised, viz. Fail (0% - 29%), At Risk (30% - 59%) and Pass (60% -100%). The bar graph in Figure 2 below shows the mean reading score across the three mathematics groups.

Fig 2: **Mean mathematics and reading comprehension scores for the three academic groups**[a]

A one-way ANOVA was used to further explore the relationship between the three different academic groups with regard to overall reading scores. The ANOVA determines whether the differences *between* the specified groups are larger than the differences *within* the specified groups. The analysis yielded F (2, 305) = 20,002, p < 0.000. A Scheffe test showed significant differences between all three groups in the overall reading scores. Differences between the three different groups according to the different components of the reading test are reflected in Table 2 below.

Table 2: ANOVA results: Differences in reading skill between the three academic groups

As these results show, in all three components of the reading test, there were consistently significant differences between the students who failed badly (below 30%) and those who were at risk (30% - 59%). If one excludes the vocabulary component from the reading test and considers only Sections B and D, then we see significant differences between all three groups. Likewise, the overall scores on the reading test (i.e. Sections B, C and D) also show significant differences between all three groups. These results provide robust evidence for differences in reading ability in relation to academic performance - students who pass their mathematics exam are students with higher reading scores than students who are at risk or who fail. Students who get less than

30% for mathematics are typically students with poor reading skills. What these results show is that the stronger a student's reading ability, the better his/her chances of performing well in the mathematics exam may be. Students whose overall scores in the reading test were 45% or lower typically failed their mathematics examination hopelessly. Students whose overall scores in the reading test were 75% or above typically passed their mathematics examination with 60% or more.

5.3 Cluster Analysis

To further explore the relationship between different aspects of reading skill and academic performance, a cluster analysis was applied to the data. Cluster analysis is a multivariate statistical technique used to identify and classify groups (clusters) in terms of the characteristics they possess. Hair et al. (1995:423) describe each object in a cluster as being " ... very similar to others in the cluster with respect to some predetermined selection criterion. The resulting clusters of objects should then exhibit high internal (within-cluster) homogeneity and high external (between-cluster) heterogeneity."

The selection criteria for the cluster analysis were the different aspects of reading skill that the test assessed, viz.

1. Pronominal anaphors (e.g. *it* and *they*)
2. Determiner anaphors (e.g. *this* and *these*)
3. Paraphrase anaphors (e.g. *this process*)
4. Low-frequency words (typical of mathematical discourse)
5. Causal relationships expressed in a text (e.g. *Since X, Y; If X ... then Y*)
6. Contrastive relationships expressed in a text (e.g. *Although X, Y; X, however, Y*)
7. Understanding information in graphs
8. Performance in the mathematics examination

K-means cluster analysis was performed for four different cluster values. Three and five cluster solutions were also performed for these eight criteria but convergence was not consistently attained among these cluster assignments, with no clear patterns emerging from the clusters. The four-cluster option thus appeared optimal.

Table 3: Summary of cluster descriptors

5.4 Problem areas in the reading of mathematics texts

The second aim of the study was to identify some typical reading problems that students experience when reading mathematics texts. We shall look at all nine major subcomponents of the reading test to determine which aspects proved to be the most challenging for the students. It is interesting that a similar pattern is reflected in each subcomponent of the reading test as the students move up the academic/mathematics groups. As can be seen, the sequencing task was the most difficult across the groups, followed by contrastive and then conditional text-semantic relations. This information is presented visually in the stacked bar graph in Figure 3 below.

Fig. 3: Performance in subcomponents of reading test according to academic groups.

6. DISCUSSION

While the importance of reading in the language, social and human sciences seems undisputed, it has often been assumed that success in mathematics and science requires primarily logico-deductive and numerical skills, and consequently the role that reading plays in constructing and understanding complex concepts and in problem solving in these disciplines is often underestimated or overlooked. It is of course important not to over-simplify the problem. Many people with poor reading skills are good mathematicians; many people with excellent reading ability do not cope with mathematics. There are obviously many different variables involved, not least of which are the issues of motivation, patience, persistence and other cognitive aspects uniquely (perhaps) associated with mathematical argument. Yet the analysis of the reading test administered to the Mathematics Access Module students in 2000 shows a robust relationship between reading ability and academic performance in mathematics. Students who failed their mathematics exam had considerably poorer reading skills than those who passed. Reading ability does not of course guarantee performance in mathematics but the results do suggest that lack of reading ability functions as a barrier to effective mathematics performance. Weak readers are only achieving reading comprehension levels of 50% or less, which effectively means that half of what they read they don't properly understand, with dire consequences for their academic performance.

Skilled readers resolve anaphors with at least 95% - 100% accuracy. The results of the reading test indicate that many of these mathematics students have problems with anaphoric resolution, with academically weaker students being consistently less successful than academically stronger students. Difficulty in this area of reading implies a lack of specification of content matter; in other words, readers who do not resolve anaphors successfully keep ‘missing the point’ about the focus of a paragraph. This affects their comprehension, and hence their ability to read to learn.

Linguistic research has shown that text-semantic relations ‘knit’ ideas and arguments in a text together and give it coherence. The ability to perceive such relations while reading enables a reader to construct a rich and coherent representation of what the text is about. The Pass students were much better at perceiving such relations than the At Risk students, who in turn were better than the Fail students. It is interesting to note that the conditional and contrastive relations in particular seemed to pose problems. Given that conditional and contrastive propositions are a common characteristic of mathematics discourse, the poor level of comprehension of such relations amongst these mathematics students is cause for concern.

The results indicated that the sequencing items posed the greatest challenge to the students. Although such tasks are not typically part of what mathematics students do, their performance on such tasks assesses the extent to which they pay attention to semantic and logical clues in a text to help them construct meaning while they read. These results indicate that the weaker students regularly miss vital clues that aid in constructing and keeping track of meaning in a text. This kind of response indicates an immature level of reading behaviour. Given that the reading of mathematics texts requires precision, and demands comprehension of each successive unit of text, failure to attend to explicit semantic and logical clues can cause a reader to miss the point of an argument and hence construct an erroneous and fragmented representation of the text.

These findings suggest that special attention should be given to helping students develop a deeper understanding of causal, conditional and contrastive relations in mathematics discourse and the different linguistic markers that are used to explicitly mark such relations, so that they come to understand the relationships in a meaningful and productive manner. In this way they can build up a schema of each relationship in their minds. Such schemata become bridges between recognising the nature of the argument in the text and constructing a meaningful representation of it during the reading process. Drawing students’ attention to the way in which anaphors function in discourse could also be helpful, especially for the weaker readers.

Reading is important in the learning context because it affords readers independent access to information in an increasingly information-driven society. It is also a powerful learning tool - a means of constructing meaning and acquiring new knowledge, and consolidating, modifying and expanding knowledge bases. Students need to be good readers in order to be able to 'read to learn'. If students have not properly mastered this learning tool their potential for success in the learning context is handicapped. We make sense of our world by constructing meaning largely through language. In the learning context, particularly in an access course at tertiary level where much of the access to mathematical information occurs via the written word, it is important to take into account the reading skills of students. It is essential that they be given all possible assistance in improving their proficiency in reading, understanding and using English in the context of mathematics. It is perhaps necessary to consider administering tests designed for the reading of mathematical texts, not to function as a gate-keeping device, but to be able to advise potential students to first upgrade their reading skills before enrolling for an access mathematics module. For this purpose it is necessary to design a reading module/course that will specifically help to develop those kinds of reading skills that are needed for understanding the sense and complex language of mathematics discourse. As Freitag (1997:18) points out, "underdeveloped reading skills can keep our students from realizing their full potential and developing into the mathematical learners they are capable of becoming".

The reading skills needed to comprehend mathematics texts and word problems are the tools with which students access, learn and apply mathematical concepts and skills. If education is "designed to empower individuals to become active participants in a technologically based world economy, true empowerment meaning becoming academically competent participants" (Garaway 1994), then we need to focus on reading as a fundamental skill underlying academic competence. If students can be given opportunities to improve their reading in the context of mathematics, they should have a better chance of success.

REFERENCES

Black, J B & Bern, H 1981. Causal coherence and memory for events in narratives. *Journal of Verbal Learning and Verbal Behavior* 20: 267-275.

Brostoff, A 1981. Coherence: 'Next to' is not 'connected to'. *College Composition and Communication* 32: 278-294.

Cooper, PAT 1996. In search of sufficient vocabulary: Testing the vocabulary levels of undergraduate students. *South African Journal of Linguistics, Supplement* 26: 25-37.

Corson, D 1983. Social dialect, the semantic barrier and access to curricular knowledge. *Language in Society* 12: 213-223.

Crandall J, Dale, T C, Rhodes, N C & Spanos, G A 1980. The Language of Mathematics: the English Barrier, in *Issues in L2: Theory as Practice, Practice as Theory,* edited by LaBarca, A & Bailey, LM Norwood, NJ.

Dale, T C & and Cuevas, G J 1987. ESL through content area instruction, in *Integrating Language and Mathematics Learning*, edited by J Crandall, Engelwood Cliffs, New Jersey: Prentice Hall Regents.

Daneman, M 1991. Individual differences in reading skills, in *Handbook of Reading Research Volume II* edited by R Barr R. et al., London: Longman.

Davis, F B 1968. Research in comprehension in reading. *Reading Research Quarterly* 3: 499-545.

Fahnestock, J 1983. Semantic and lexical coherence. *College Composition and Communication* 34:400- 416.

Frawley, W 1992. Lexicography and Mathematics Learning: A Case Study of *Variable*. *Applied Linguistics* 13(4):385 - 402.

Freitag, M 1997. Reading and writing in the Mathematics classroom. *The Mathematics Educator* 8: 16-21.

Flew, A 1984. *A dictionary of philosophy.* London: Pan Books.

Garaway, G B 1994. Language, culture and attitude in mathematics and science learning: a review of the literature. *The Journal of Research and Development in Education* 27(2):102-111.

Geva, E & Ryan, E B 1985. Use of conjunctions in expository texts by skilled and unskilled readers. *Journal of Reading Behavior* 17: 331-346.

Hacquebord, H 1994. L2 reading in the content areas: text comprehension in secondary education in the Netherlands. *Journal of Research in Reading* 17:83-98.

Hair, J F, Anderson , R E, Tatham, R L & Black, W C 1995. *Multivariate Data Analysis*. Englewood Cliffs, NJ: Prentice Hall.

Lesiak, J. & Bradley-Johnson, S. 1983. *Reading Assessment for Placement and Programming.* Springfield: Charles C. Thomas.

Mann, W C & Thompson, S A 1986. Relational propositions in discourse. *Discourse Processes* 9: 57-90.

Meyer, B J F, Brandt, D M & Bluth, G J 1980. Use of top-level structure in text: Key for reading comprehension of ninth-grade students. *Reading Research Quarterly* 16: 72-103.

Pagé, M 1990. Methodological issues in testing comprehension of texts, in *Literacy acquisition*, edited by M Spoelders, Lier: Van In and C&C.

Perfetti, C A 1988. Verbal efficiency in reading ability, in *Reading Research: Advances in theory and practice, Volume 6*, edited by Daneman, M, Mackinnon, G E & Waller, T G. New York: Academic Press.

Pretorius, E J 1994. A text linguistic perspective on causality: towards a taxonomy of causal relations. *South African Journal of Linguistics, Supplement 22*: 81-122.

Pretorius, E J 1996. A profile of causal development amongst ten-year-olds: Implications for reading and writing. *Reading and Writing* 8: 385-406.

Prins, E D 1997. Linguistic and cultural factors that influence readability of mathematics questions, in *The cultural context in foreign language teaching*, Duisberg Papers on Research in Language and Culture, 32, Peter Lang.

Sanders, T J M, Spooren, W P M & Noordman, L G M 1992. Towards a taxonomy of coherence relations. *Discourse Processes* 15: 1-35.

Singleton, J & Bohlmann, C 2000. *Mathematics Access Module Book 2: Number skills and algebra tools.* Pretoria: University of South Africa.

TIMSS Report http://bc.edu/timss1999i/math_achievement_report.html

Trabasso, T, Van den Broek, P & Suh, SY 1989. Logical necessity and transitivity of causal relations in stories. *Discourse Processes* 12: 1-25.

Webber B L 1980. Syntax beyond the sentence: Anaphora, in *Theoretical issues in reading comprehension.* edited by Spiro, R J , Bruce, B C & Brewer, W F. Hillsdale: Lawrence Erlbaum.

Winter, E O 1978. A look at the role of certain words in information structure, in *Informatics 3*. Proceedings of a conference held by the Aslib Co-Ordinate Indexing Group, 1975, edited by Jones, K P & Harsnell, V. London: Aslib.

Yuill, N & Oakhill, J 1991. *Children's problems in text comprehension.* Cambridge: Cambridge University Press.

Table 1: **Mean reading comprehension scores for the four reading ability groups[a]**

	Reading Group1 n = 45	Reading Group 2 n = 78	Reading Group 3 n = 95	Reading Group 4 n = 90
B: Anaphoric references	28,6	52,5	68,2	80,8
C: Vocabulary	45,4	58,4	73,7	80,8
D: Logical relations, graphs	37,2	49,4	61,4	78,8
Mathematics exam	24,8	32,5	33,4	41,7

Table 2: ANOVA results: Differences in reading skill between the three academic groups

	Group 1 Fail (n =154)	Group 2 At Risk (n=131)	Group 3 Pass (n=24)	Sig. level*
Section B: Anaphoric references	1*	2		0.00
Section C: Vocabulary	1*	2		0.01
Section D: Semantic relations, graphs, sequencing	1*	2		0.00
Section B + D	1*	2	3	0.03
Overall score: Section B + C + D	1*	2*	3	0.02

* significant difference between groups

Fig 2: **Mean reading comprehension scores for the three academic groups[a]**

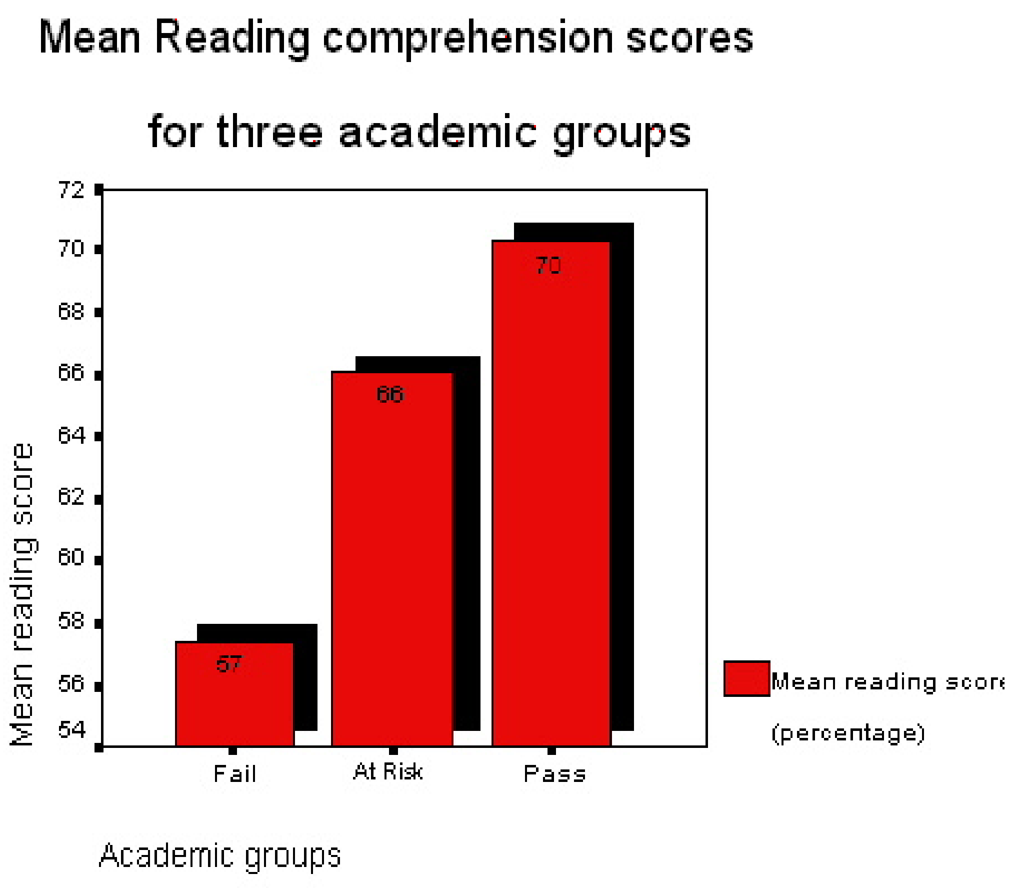

Table 3: Summary of cluster descriptors

	Cluster 1 n = 73	Cluster 2 n = 111	Cluster 3 n = 70	Cluster 4 n = 17
Pronominal anaphor	55,3	64,6	65,1	85,3
Determiner anaphor	57,8	65,6	68,2	77,6
Paraphrase anaphor	53,2	60,4	65	81,5
Low-frequency words	59,5	68,9	72,1	85,9
Causal relations	51,1	61	64,8	76,8
Contrastive relations	42,7	50	53,7	72,1
Graphs	57,4	71,7	77,4	89,7
Mathematics exam percentage	16,6	30,8	50,7	71,4

Fig 3: Performance in subcomponents of reading test according to academic groups.