CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

From the background information provided in Chapter 1 it is clear that increasing access to mathematics in South Africa in a distance-learning environment would involve a large number of factors. The literature review in this chapter provides a broad overview of several specific factors that are deemed to have been most salient to the creation of the Mathematics Access Module. These include factors that are relevant to learning mathematics in general, and to learning mathematics at tertiary level, in particular in South Africa. Factors particularly relevant to learning mathematics in a distance-learning context are discussed. To create the context for the creation of the UNISA Mathematics Access Module, Chapter 2 then focuses on three specific South African examples that existed at the time the module was being designed, namely the Science Foundation Programmes at the University of Cape Town and the University of Natal (Pietermaritzburg), and the College of Science at the University of the Witwatersrand. In addition to access programmes pressure of numbers in tertiary education led to the implementation of a number of placement assessment initiatives, internationally and locally. In considering testing in South Africa, Chapter 2 deals firstly with *what* should be tested, and then considers the tests that used in the Alternative Admissions Research Project, and the testing aspect of the Teach-Test-Teach Programme at the University of Natal (Durban).

Factors relevant to the effective study of mathematics that are discussed in this chapter include students' backgrounds, teaching approach and learning styles, the linguistic and contextual aspects of mathematics discourse, affective factors and meta-cognitive skills, and assessment practices. In the context of distance learning additional factors play a role, such as the media involved, language of instruction, provision of contact opportunities and support, and infra-structural aspects. For many South African students, the existence of factors conducive to learning mathematics cannot be guaranteed (for example there are large numbers of under-prepared students who have a poor self image regarding their ability to learn mathematics; also students who have difficulty reading in English), and this situation is not easily changed. Further discussion of other relevant theory will be given in the chapters that relate to the impact of reading skills on students' ability to deal with the discourse of mathematics (Chapters 5 and 6), and an investigation into an alternative form of assessment (Chapter 7).

2.2 Research in mathematics education

In recent years many books and articles have been published describing various research initiatives in mathematics education (consider for example the number of articles that appear annually in the different editions of *The Journal for Research in Mathematics Education, Educational Studies in Mathematics*, and many similar journals); the number of congresses devoted to the topic (for example the quadrennial meetings of The International Congress on Mathematics Education (ICME), held under the auspices of The International Congress on Mathematics Instruction (ICMI), and annual meetings of the Psychology of Mathematics Education (PME)). In an authoritative source of some of the most recent thinking on mathematics education research (the *Handbook of International Research in Mathematics Education*) we read that

> Research in mathematics education has undergone a number of major paradigm shifts, both in its theoretical perspectives (e.g., from behaviourism to cognitive psychology) and in its research methodologies (e.g., from a focus on quantitative experimental methodologies where controlled laboratory studies were the norm, to qualitative approaches where analyses of mathematical thinking and learning within complex social environments have been possible; … (English, 2002, pp. 10 - 11).

Lesh (2002) posed the following question in relation to research in mathematics education:

> What kinds of research designs have proven to be especially useful in mathematics education, and what principles exist for improving (and assessing) their usefulness, power, shareability, and cumulativeness (p. 27)?

The mathematics education research literature suggests directions for future research, or questions the nature of research projects (see for example Lesh, 2002). Lesh lists many questions that have been raised and concludes by stating that the kind of research that is needed most in mathematics education must aim to make a difference in theory or in practice. The quality of the research is related to the extent to which the research results and products are meaningful, useful, can be shared, and are cumulative. Following the research methodology described in the next chapter, the research results discussed in Chapters 5, 6, 7 and 9 appear to satisfy these criteria, in the UNISA context as well as in the broader educational environment.

2.3 Learning mathematics

We now consider some of the theoretical aspects of learning mathematics, and specific factors which, firstly, relate to studying mathematics at an access level in South Africa, and secondly, to studying mathematics at a distance. A few initiatives undertaken elsewhere are also noted, as information regarding certain aspects of these initiatives was valuable in the design and implementation of access to mathematics at UNISA.

2.3.1 Factors relevant to learning mathematics

Many students find mathematics challenging, and enjoy the challenges. Learning mathematics is then not perceived as a problem. However, there are also many students who are reluctantly engaged in studying mathematics, to obtain entry into university, or entry into specific courses. This applies specifically at the level of access, where past circumstances have limited students' chances of success, and students with little confidence in their ability to succeed are now faced with both the subject difficulties and their own hesitation in dealing with them. Learning and understanding mathematics is dependent on specific cognitive skills, such as application, critical analysis, synthesis, evaluation, the ability to reason logically, and to interpret abstract representations (see for example Beevers & Paterson, 2002). Additional cognitive skills required are the ability to make deductions and inferences (i.e. to identify cause and effect), to make appropriate assumptions, have a sound sense of number (at the level required for the content to be taught), to know why and not just how a problem is solved, and to 'translate' from everyday language to mathematics (again at the level required). Most standard mathematics textbooks suggest that mathematics students need both conceptual understanding and technical skill (e.g. Stewart et al., 2002), which presuppose these cognitive skills.

Studying mathematics effectively at tertiary level depends on a number of different factors. We now consider a number of relevant factors; in the subsequent sections (2.3.3 and 2.3.4, respectively) we consider these factors in the South African context and in relation to distance learning.

2.3.2 Factors influencing effective study of mathematics at tertiary level

Success is influenced by many factors related to content and cognitive issues; to the unique discourse of mathematics; to affective issues such as motivation, confidence, willingness to persevere and practise; and to individual learning styles, such as an inclination towards problem solving, and an acceptance of the importance of understanding as opposed to memorisation.

Certain factors exist prior to enrolment, such as the student’s background, prior knowledge of the subject and previous learning experiences. If prior knowledge is lacking (and, more importantly, if neither the student nor the lecturer is aware of the gaps) the foundation for new knowledge is unsound.

In addition to the influence of past learning experiences, additional factors impact on the difficulties students experience with mathematics once they have started studying. Some of these are inherent within the learning environment, such as the teaching approach and the design of the curriculum (Anthony, 2000). Factors such as lack of time, too great a volume of work, boredom and tiredness, and perceived lack of relevance emerge from the same study. If concepts and problems to be solved are seen to meaningful, students appear to be motivated to learn and apply the mathematics taught (Malloy, 2002). Central to the issues of teaching and learning mathematics is the idea that ‘mathematics has to be learned through actively engaging with it’ (Mason, 2002, p. v). Students need to recognise the importance of being *actively* involved in the learning process.

Students’ backgrounds

Apart from lack of prior knowledge that may have arisen from poor teaching at the secondary level, or from (in the case of adult learners returning to the academic environment) decreasing levels of accurate recall, students need ‘hooks to hang things on’, i.e. a mental schema into which new mathematical concepts can be integrated. Learning will be negatively affected if the prerequisite content knowledge or general knowledge is missing. Students’ background knowledge is also affected by their parents’ levels of education. In the USA research has shown that there is a strong correlation between the level of the parents’ education and the mathematical performance of their children (Tate & Rousseau, 2002). For example, a student who has never experienced motorcars will not benefit if second-order differentiation is illustrated by acceleration; neither will a student who lives without access to running water understand measurement of volume if it is illustrated in terms of a swimming pool. In order to solve problems, reasonable assumptions need to be made (such as ‘Let’s assume the height of the room is about 2,25 metres, because my height is 1,5 metres and the ceiling appears to be about half as far again above me.’), which depend on adequate background knowledge. Fischbein (1993) suggested that mathematical activities require the use of algorithms, formal knowledge and intuitive knowledge. Intuition is based on experience, and limited experience of real-life issues in which mathematics plays a role necessarily affects the ability of students to develop a ‘feel’ for how a problem should be identified or approached. As far back as 1945 Polya described four steps in solving problems, namely to understand the problem, design a plan to solve the problem, carry out the plan, and finally to look back to check the validity of the solution. Without adequate background knowledge, the first step is already too steep an ascent for many students.

Teaching approach

Much research on mathematics teaching and learning has been done at secondary level (i.e. in the classroom, involving teachers). Nevertheless, much of the experience gained is applicable to learning at the tertiary level as well, especially at the access level. During much of the third quarter of the 20^{th} century, lecturing and teaching were both defined in terms of 'imparting information'. This behaviourist perspective of education suggested that lecturers and teachers possessed the knowledge and were responsible for transmitting it to passive students, who would assimilate and remember the facts imparted. This transmission mode of teaching mathematics has excluded access to mathematics for many students (Ahmed et al., 2002). With evolving educational reform, this focus on universities as institutions where lecturers lecture, and schools as places where teachers teach, is no longer accepted (see for example Goldin, 2002). In the more recent and more popular constructivist perspective[1], which arose out of the work of Piaget (see Goldin, 2002), knowledge was seen to be gained by means of an active process of construction, involving conceptual growth and reorganisation, as well as growth in cognitive and meta-cognitive skills (see for example van Glaserfeld, 1990; Wheatley, 1992; Orton, 1994; van Glaserfeld, 1995).

Communication and interaction are central to constructivist practices, and in a study of seven different methods of teaching mathematics Ellerton & Clements (cited in Garaway, 1994) discovered that the common feature of the more successful approaches was 'quality, interactive communication' (p. 106). Interaction is also a key feature of Vygotsky's notion of a Zone of Proximal Development (ZPD) (Vygotsky, 1978), where students are capable of constructing a certain amount of mathematics on their own, but through the scaffolding of those who are more knowledgeable, they are able to reach higher levels of conceptual development.

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[1] 'Radical' constructivism, with its refusal to accept any objective knowledge, is no longer a popular mathematical learning theory (cf. Goldin, 2002); however, the idea of individual engagement with concepts in order to construct knowledge has remained central to effective mathematics teaching and learning.

Current teaching favours an approach in which students are actively involved in the learning process (Ahmed et al., 2002). It is recognised that teaching mathematics involves helping students learn to interpret the conventional mathematical representational systems, and use them to solve problems. Effective teachers continuously

> make inferences about student's internal representations [of these conventional, external systems], their mathematical conceptions and misconceptions, based on their interaction with or production of external representations (Goldin, 2002, p. 211).

In the relationship between student and teacher, while such inferences are being made and acted upon, scaffolding and 'fading' are of equal importance, so that students move from dependent to independent learning (see Mason, 2000, p. 178) and come to a realisation that lecturer dependence will not facilitate learning. The notion of scaffolding and fading has particular implications for distance learning, where it is virtually impossible for lecturers to make frequent or meaningful inferences regarding their students' understanding of different topics.

Learning styles

Success in studying mathematics depends to some extent on the way in which a student learns. If the focus at secondary level has favoured rote learning, this habit needs to be un-learnt before new learning can occur at tertiary level, and the necessity to do so must be explicated in advance of the learning that is to take place. Rote learning has an additional implication with respect to the discourse of mathematics: if the discourse is memorised rather than understood, students will not have learnt to read mathematics actively, and will be further disadvantaged in the future. For mathematics rote learning is singularly ineffective, and is not conducive to reflective practices, or the development of problem-solving strategies.

Autonomy in learning is also seen as an important determiner of success. In 1985 the mathematics educator Elizabeth Fennema highlighted the importance of students taking control of their own learning (Tobias, 1993).

Study techniques also play a significant role (cf. Manalo et al., 1996, in Anthony, 2000), and many students are either unaware of the appropriate skills, or do not apply them properly. For example, they study passively, and miss the fact that in mathematics, 'thinking' involves writing and doing (Tobias, 1987). Many students do not recognise that mathematics and hard work are equivalent. As Tobias (1987) puts it: 'What awaits you in college-level mathematics is an excitement *that is inseparable from hard work*' (p. 4, italics added).

Linguistic and contextual aspects of mathematics discourse

Mathematics is not a language-free discipline, defined by a discourse featuring only numbers and symbols, and it is thus not surprising that proficiency in English should feature prominently in cases where the medium of instruction and/or the assessment is English. According to Zevenbergen (2000), mathematics is a language within a language as it is dependent on another language, the language of instruction. Mathematics discourse generally contains items that have linguistic, cognitive and contextual dimensions (Gibbs & Orton, 1994). The linguistic dimension involves both the receptive level (e.g. reading) and the productive level (e.g. writing, discussing). The cognitive dimension reflects the level of complexity of the concepts. Cognitive skills such as logical reasoning, critical analysis and interpretation of abstract concepts are essential in order to study mathematics effectively. The contextual dimension[2] reflects the level of contextual support provided. Difficulty with any one of these aspects has serious implications for students.

The conceptual complexity and problem-solving nature of mathematics make extensive demands on the reasoning, interpretive and strategic skills of students, even more so when these activities are carried out in a language that is not the student's primary language. Many studies have investigated the role of *language* in mathematics (see for example Ellerton & Clements, 1991; Bartolini Bussi, 1998; Ellerton et al., 2000), and poorly developed language skills (in the language of instruction) undermine students' mathematical performance. Understanding and doing mathematics require reading and writing skills (Hackworth, 1992). Mathematics is taught and understood via the mathematics register (Frawley, 1992; Dale & Cuevas, 1987). In general the mathematics register is abstract, non-redundant (Prins, 1997) and conceptually dense. Mathematical symbols and graphic aspects, such as charts, tables and graphs, increase the conceptual density. Mathematical discourse features more complex and more compact relationships than normal discourse (Crandall et al., 1980). Furthermore, mathematics is characterised by precision, conciseness and lack of ambiguity. The reading of mathematics texts (an integral part of studying mathematics in any context) thus requires close attention to detail. For disadvantaged students the additional demands on the time needed to process large amounts of English text are often underestimated. Parts of mathematics texts tend to be procedural, in that they explain how to carry out a task or algorithm. Other activities aim to develop conceptual knowledge. Mathematics texts are also hierarchical and cumulative, in the sense that understanding each statement or proposition is necessary for understanding subsequent statements. Overlooking or misunderstanding a particular step in a method, procedure or

[2] In this case the context of the problem within the text is implied, not the external context of the student.

argument has severe consequences for overall comprehension. Studying mathematics thus requires integration of all the available information.

The language problem remains a complex issue: with time it has become increasingly clear that

> mastery of natural language in its logical, reflective, exploratory, and command functions emerges … as one of the crucial conditions in approaching more-or-less elementary theoretical aspects of mathematics. Indeed, only if students reach a sufficient level of familiarity with the use of natural language in the proposed mathematical activities can they perform in a satisfactory way and fully profit from these activities (Boero et al., 2002, p. 242).

However, nothing can be understood about specific mathematical concepts, such as the square root of a number, through natural language; it is only once a student has understood the particular mathematical notion of a power that he/she is able to comprehend roots; in other words mathematical concepts need to be acquired (albeit through reading and other use of natural language) in the context of mathematics (Sfard et al., 1998).

Apart from the importance of proficiency in English in order to engage with the mathematics itself, students also require a reasonable level of proficiency in order to communicate with their lecturers or tutors when they need to clarify mathematical problems.

Affective factors

A number of affective factors play a role in success in mathematics and need to be taken into account. The Fennema-Sherman scales rate several important affective factors relevant to mathematical success, namely attitude toward success, anxiety when doing mathematics, confidence in learning mathematics, motivation to learn mathematics, the perceived usefulness of mathematics, parental attitudes, and teacher attitude (Tobias, 1993). Anthony (2000) notes the importance of student motivation and the influence of cultural expectations on success. Student attributes such as confidence, willingness to persevere and to practise are fundamental to studying mathematics successfully. Poor attitude is seen as one of the main barriers to success in mathematics. Factors that contribute to negative attitudes are limited belief in one’s own ability to succeed in mathematics (what Garaway calls a ‘poor self-concept as a doer of mathematics’), negative perception of the real-life value of mathematics, and discouraging influences of significant others such as teachers and parents (Garaway, 1994, p. 105). These attitudes influence student inclination towards problem solving and choice of understanding as opposed to memorisation as a learning style. ‘Understanding’ mathematics does not necessarily convey the same meaning to all students, under all circumstances. Mason (2002) refers to Skemp’s distinction between relational and instrumental understanding of mathematical topics, and extends this to the notion that understanding should ideally be ‘a complex personal state of self-confidence, where you know from experience that you can trust ideas to come to mind when needed, and, when they do not, you feel able to admit it’ (Mason, 2002, p. 171). By implication, then, understanding implies being able to know what to do about a situation in which an answer or solution is not immediately obvious.

At entry level, particularly at the access and bridging levels, students often begin studying mathematics with negative attitudes based on past negative experiences. Low expectations have been seen to impact negatively on performance in mathematics (Malloy, 2002); conversely, high expectations appear to have a positive impact (see Drew, 1996, in Howie, 1997).

Meta-cognitive skills

Not all students have well developed meta-cognitive skills. A significant difference between good and weak mathematics students is the fact that weak students do not know whether they know and what they know (Hackworth, 1992). If students are unable to tell whether they understand what they have read, whether they have grasped a concept or not, whether they are studying effectively or just for a long time; unable to comprehend the demands of problems that are posed, or to recognise logical contradictions when these occur, they will not make effective progress with their studies. Students with weak meta-cognitive skills are also often over-optimistic about the amount of time they will need to devote to their studies.

In the process of academic maturation, students acquire different degrees of competency, executive and self-regulatory skills. These skills form the basis for effective learning, where learning activities can be planned, and problem solving can take place. Self-regulation involves analysing tasks, choosing strategies, monitoring the course of learning or problem solving, and adjusting or revising strategies if necessary (Borkowski, 1992, in Muthukrishna, 1994).

Assessment principles

Linking learning and assessment

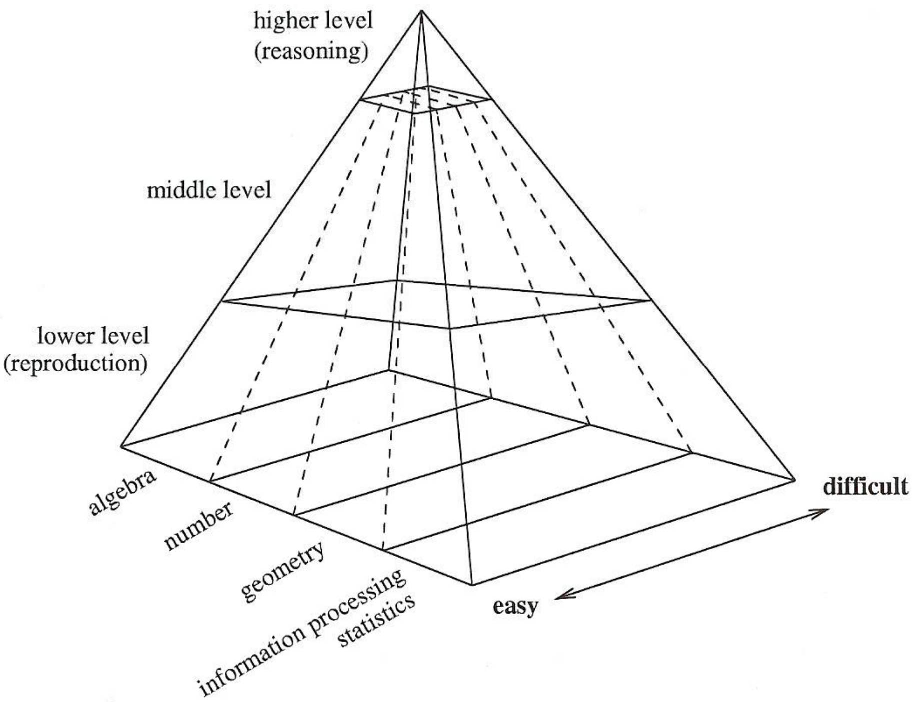
Educational goals and assessment goals are linked: learning involves the acquisition of skills and knowledge, and assessment identifies the level of knowledge or skill that has been acquired.

Assessment is a way of measuring what students know and of finding out what students should learn (MSEB, 1993). If assessment is to be meaningful, to the student and to the lecturer, it needs to advance learning and not simply record its status. Volmink (1994) noted that assessment should 'reveal value rather than merely identify deficiency' (p. 63). Unfortunately learning is generally driven by the extent and type of assessment involved (Kahn, 2002). As Rowntree (1997) put it, 'assessment is often the tail that wags the dog of learning' (p. 16).

Acceptable assessment practice suggests that assessment tasks should provide balanced problems which are meaningful, informative, set in recognisable contexts, and involve higher order thinking (van den Heuvel-Panhuizen, 1996). One possible categorisation of educational goals into lower, middle and higher levels is provided by de Lange (1994) in Verhage & de Lange (1997). This categorisation is illustrated in Figure 2.2, over the page. In terms of this categorisation, straightforward real-life assessment tasks may be at the lower level, while meaningless but difficult tasks are also lower level activities if they require no insight and simply the application of routine skills. Middle level tasks require that students relate two or more concepts or procedures, and although they may be easier to solve they are richer and more meaningful than more involved but meaningless routine tasks. The third level includes other aspects such as mathematical thinking, communication, critical attitude, creativity, interpretation, reflection, generalisation and 'mathematizing' (Verhage & de Lange, 1997, p. 16). Freudenthal coined the term 'mathematization' in the 1960s to 'signify the process of generating mathematical problems, concepts, and ideas from a real-world situation and using mathematics to attempt a solution to the problems so derived' (Perry & Dockett, 2002, p. 89). Although Freudenthal's intention was that this process should begin in early childhood, even if such opportunities may have been lacking during the developmental years they should not necessarily be excluded later. The important point is that students should discover and use mathematical tools which will enable them to organise and solve real-life problems.

Figure 2.2:

Levels of mathematical educational goals



Five assessment principles flow from an understanding of de Lange's different levels (de Lange, 1987, in Verhage & de Lange, 1997). The first and main purpose of assessment is to improve learning and teaching; in addition, methods of assessment should enable students to demonstrate what they know rather than what they do not know; assessment should integrate lower, middle and higher level goals of mathematical education (see Figure 2.2); the quality of an assessment task should not primarily be determined by its accessibility to objective scoring; finally, assessment tools should be practical.

The first three points above were emphasised by Greenes (1995), who believed that assessment should provide a comprehensive understanding of what students know, how they know it, and what appropriate instruction will enhance their understanding. In a context where deep rather than surface learning is an educational goal it is important to assess the extent to which students are achieving the deeper and higher order goals (English et al., 2002). It also follows that good assessment should aim for 'reconciliation of assessment and instruction, through the deliberate use of tasks which legitimately serve both purposes … assessment [should] both mirror and inform instructional practice' (Clarke & Jasper, 1995, in Clements & Ellerton, 1996, p. 156).

Good teaching accommodates student differences. In the same way, good assessment should accommodate differences in the ways that students think and demonstrate mathematical understanding (see for example MSEB, 1993). In other words, assessment should satisfy the following three important educational principles (MSEB, 1993). It should satisfy the content principle: assessment should reflect the mathematics that is most important. It should satisfy the learning principle: assessment should enhance learning by being itself part of the instructional process. It should satisfy the equity principle: assessment should provide an opportunity for all students to learn mathematics.

Types of assessment

The need to link learning with assessment influences the choice of assessment tasks. Assessment methods should be appropriate to the learning outcomes and knowledge base of a particular discipline. In a recent study involving mathematics, statistics and operations research at the University of Birmingham, one of the three aspects of assessment identified as being most important was the need to match appropriate assessment methods to the learning outcomes related to knowledge, basic skills, understanding and problem solving (Beevers & Paterson, 2002).

Different types of assessment, such as formative or summative assessment, are undertaken for different reasons and involve the use of a number of assessment tools. Formative assessment supports the teaching and learning process, since it is used to encourage, direct and reinforce learning. Assessment is only formative if suitable feedback is provided so that students can learn (new concepts, or new cognitive skills) from the assessment process (Beevers & Paterson, 2002). Summative assessment usually makes a contribution to the overall assessment of the student by establishing what students have achieved at the end of a programme or course, and measures achievement in all specified learning outcomes (Beevers & Paterson, 2002). Summative assessment thus also measures the student's ability to manage and integrate a large body of knowledge, but as such unfortunately usually ends up showing what students cannot do (Mason, 2002). In theory the marks obtained in any assessment are less important than the potential of the assessment task for 'collecting information to be directly used for instruction' (van den Heuvel-Panhuizen, 1996, p.159).

English et al. (2002) make the valid points that assessment should be free from bias (e.g. cultural, gender, etc.), it should yield useful information for decision makers (from students to policymakers), and it differs from evaluation.

> Whereas the goal of evaluation is to assign a value to the subjects being inspected, the goal of assessment is to provide information for decision makers, perhaps by describing the subject's location (and recent history of progress) in some landscape of ideas and abilities that it is desirable for them to learn (p. 801).

English et al. propose two types of assessment designs in which issues such as these can be taken into consideration, namely mechanistic and systemic assessment. These two perspectives are contrasted in Table 2.1 (English et al., 2002). The table is useful to keep in mind when considering student assessment, as it suggests that an overall picture of student competence needs to be derived from a number of different of sources, under various conditions. A further useful categorisation of mathematical skills, called the Mathematical Teaching Hierarchy Taxonomy (MATH Taxonomy), was developed by Smith, Wood, Coupland, Stephenson, Crawford & Ball (Smith et al., 1996, in Pountney et al., 2002). It divides mathematical skills into three levels, as shown in Table 2.2.

Table 2.1:

Mechanistic vs. systemic perspectives on assessment

Mechanistic perspectives	Systemic perspectives	Implications
Testing: creating an ordeal (barrier, or filter) for accepting or rejecting (but not helping)	Examining: Inspecting closely and generating high-fidelity descriptions	It is possible to examine students closely without having to expose them to non-productive ordeals.
Measuring: partitioning (fragmenting) into distinguishable pieces	Documenting: gathering tangible evidence which is credible to decision makers	It is possible to document achievements and abilities without reducing the relevant information to a numerical score or grade.
Evaluating: assigning a value without specifying conditions or purposes	Assessing: taking stock, orienting with respect to known landmarks and goals	It is possible to assess current competence and future direction without comparing students with one another along a ‘good-bad’ scale.
Summative information: focuses on decision-making issues of administrators	Formative information: focuses on decision-making issues of students and teachers	Decision-making needs of administrators can be addressed without neglecting decision-making needs of students.
Improving tests: goal is to make credible evaluations that are better aligned with instructional goals	Improving teachers’ assessments: goal is to make authentic assessments (such as those based on teachers’ judgements) more credible and reliable	Students cannot be considered ‘good’ without qualifying the label in terms of ‘good for what’ or ‘good under what conditions’.

Table 2.2:

The MATH Taxonomy

Group A	**Group B**	**Group C**
factual knowledge	information transfer	justifying and interpreting
comprehension	application in new situations	implications, conjectures and comparisons
routine use of procedures		evaluation

Table 2.2 may also be helpful in planning assessment tasks. Assessment tasks need to test different types of knowledge, conceptual understanding and cognitive skill. The mathematical skills suggested in the table are important learning outcomes. If most students enter tertiary education with mainly Group A skills, it may be useful to consider the skills assessed in any given assessment task from the perspective of skills in Groups B and C. Assessment always involves current knowledge and skill but should also include a measure of the student's ability to transfer knowledge to new contexts and to apply knowledge in specific situations.

A variety of assessment tools is required if all goal levels are to be involved. Assessment tasks can include for example projects, tests, examinations and self-assessment. An understanding of the different developmental levels of potential tasks is helpful in the selection of assessment tools for different purposes and disciplines. Assessment tasks can be categorised in various ways. The list below ranks assessment tasks in terms of the type of thinking required, from lowest to highest order (Grayson & Clarke, unpublished).

Memory of

(a) factual information (b) procedures or algorithms

Application of remembered information to

(a) known context (b) unknown context

Application of remembered procedures or algorithms to

(a) known context (b) unknown context

Application of factual information given in the question to

(a) known context (b) unknown context

Application of procedures or algorithms given in the question to

(a) known context (b) unknown context

Conceptual *understanding*

Proficiency with respect to individual *thinking / reasoning skills*, for example the use of analogy, classification, comparing and contrasting, translation between different types of representations, development of logical argument

Proficiency with respect to *higher order cognitive skills*, for example synthesis, integration of knowledge, appreciating relationships between knowledge acquired in different contexts

Proficiency with *strategies*, such as problem-solving approaches and techniques

Understanding of the *epistemology* of the discipline.

This list ranking assessment tasks is a generic one, but is nevertheless relevant to mathematics assessment at different levels.

Practical considerations of assessment tasks

Assessment practices ‘need to be practical and affordable, credible to the profession, and acceptable to the public’ (MSEB, 1993, p. 137). Credibility and acceptability suggest that assessment should achieve the goals for which it is purportedly designed. Assessment cannot be effective without adequate and appropriate follow up. Feedback should be as immediate as possible if it is to have any impact (Mason, 2002).

2.3.3 Factors influencing the study of tertiary-level mathematics in South Africa

Students’ backgrounds

One of the results of low general levels of education was that many students entering university in the 1990s were first generation learners, with no academic role models in the home. In 1999 many of the South African students who participated in the TIMSS-R were reported to come from homes where primary school was the highest level of education of the parents, and where there were very few books (Howie, 2001). This filters through into the students’ academic environment, and is evident in the limited general knowledge (this will be discussed in Chapter 7) and poor reading skills (see Chapters 5 and 6) of students. Both these aspects play a role in studying mathematics effectively.

Few students from previously disadvantaged communities have grown up in homes where mathematics is spoken about or regarded as important, as study opportunities were strictly limited. We have seen that parental and teacher attitudes and expectations play a role: where there was little encouragement for Africans to study mathematics for 40 years before the first democratic elections in 1994, it is perhaps not surprising that students do not find encouragement or understanding at home.

Teaching approach and learning style

Past authoritarian styles of teaching, stemming from repressive political practices which carried over into the classroom, and affected pupil and teacher perceptions of learning, encouraged an unquestioning passivity in learning, and did not encourage learners to take control of their own learning (Taylor & Vijnevold, 1999).

Before 1992 the education in South Africa operated in a fundamental pedagogics paradigm, based on Verwoerdian ideology. During the early 1990's and in the era after the advent of democracy the philosophy in mathematics education shifted from fundamental pedagogics to constructivism, in some cases involving an extreme shift to radical constructivism. The emphasis shifted to an outcomes-based education (OBE) philosophy in 1998, with government-led initiatives to focus on critical thinking and engagement with real-life problems, situated within a largely social constructivist environment (Setati, 2004, personal communication).

Outcomes-based education (OBE) has many features in common with what is generally accepted as good educational practice, but differs from the 'traditional' systems of education, which tended to focus less on the ends and more on the means of instruction. In the 1990s OBE became increasingly recognised as an educational reform, both internationally and nationally (Spady, 1994; Furman, 1994). According to the Department of Education,

> Outcomes are the results of a learning process whether formal, non-formal or informal. In outcomes-based education and training, curriculum developers work backwards from agreed desired outcomes, within a particular context, which clearly state what the learner should … demonstrate an understanding of and … apply appropriately. Programmes of learning are then designed to help the learners achieve these outcomes. (Ministerial Committee for development work on the NQF, 1996, in Brodie, 1997, p. 33)

OBE can be seen as a melding of several trends, including the principles of mastery learning[3], and the movement toward criterion-referenced assessment (Glatthorn, 1993). Outcomes thus relate to

[3] Mastery learning is an approach to instruction in which students are provided with structured, hierarchical, sequential units of material; and (theoretically) the time necessary to master individual units of learning before moving on to subsequent units (Towers, 1994). There have been criticisms of some of the ideas of mastery learning, particularly from those who reject its behaviourist perspective (see for example Clements & Ellerton, 1996).

particular student competencies. These competencies are categorised as foundational competence, practical competence, and reflexive competence. Outcomes relate to the application of knowledge in authentic tasks, i.e. the kinds of tasks that are required in real-life situations. There are culminating (i.e. exit-level) outcomes as well as enabling outcomes (i.e. building blocks). Different outcomes apply at different levels: there are whole-qualification outcomes, learning programme outcomes, course/module outcomes, or study unit outcomes.

OBE theory embraces contextual and cultural diversity, communication skills in all disciplines, self-empowerment, critical thinking skills, collaborative learning, and student-centredness. In OBE teaching we find integration of skills (language and communication, critical thinking, problem-solving, opportunities for reflection, meta-cognition) and contexts with, as far as possible, authentic learning situations. OBE, in theory, is process oriented, provides for interactive learning, applies skills across the curriculum, facilitates learning beyond the classroom, allows for bottom-up development, provides assessment-rich learning based on students' needs and curricula based on society's needs.

Educational institutions were thus expected to design study material with OBE principles in mind. The permanent nature of printed distance learning materials implies that it would be immediately obvious whether or not OBE principles had been followed. It was thus necessary to determine in advance of designing any study material what aspects of OBE could be included, and how this would be done. Designing a course of study with learning outcomes in mind has implications for structure, assessment and teaching support. Kahn (2002) makes the valid point that for mathematics courses to flourish it is essential that their design is driven by more than just the selected mathematical content.

In an OBE framework teachers were required to make decisions. Teachers trained in the fundamental pedagogics paradigm were ill equipped for these new demands. An added complexity was the fact that with the introduction of OBE there was a shift away from mathematical content to methods of teaching. Some of the problems that arose were recognised and addressed. A review of changing curricula (first Curriculum 2000, then Curriculum 2005, to reflect the intention that the first Grade 12 examinations on the new OBE curriculum would occur in 2005) will show that the latter once again becomes more prescriptive and direct, with a renewed focus on mathematical content.

Rote learning thrived in an environment in which teachers did not encourage dialogue and critical thinking. Mathematics and physical science suffered from the emphasis given to the production of pre-conceived correct answers and the reproduction of factual information encouraged by the examination system which resulted in ‘blind de-contextualised rote learning’ (EduSource, 1997, p.46).

Some of the practices and policies which predisposed teachers to favour rote learning were poor qualifications, as well as the limited participation by teachers in policy decisions regarding what should be taught; little insight into the relevance of the topics included; difficulty in coping with the demands of an academic, content-driven and de-contextualised syllabus; domination of the syllabus by examinations which stressed factual recall and the routine application of algorithms; and little confidence in communicating in the language of instruction (Delvare, 1995). Furthermore, it is also clear that the ‘combination of lower levels of competence and confidence in language, and the consequent reliance on rote-learning techniques, increases the adjustment problems of students entering university for the first time’ (Delvare, 1995, p. 40).

Linguistic and contextual aspects of mathematics discourse

In TIMSS-R (1999) more than 70% of the South Africa learners tested did not have the language of the test (viz. English) as their home language (Howie, 2001). This highlights the fact that many students are faced with the problem of dealing with three languages simultaneously: the mother tongue, English (or Afrikaans) and the discourse of mathematics. Many teachers make use of the practice of code switching, defined as a voluntary action by bilingual teachers and learners, and regarded as a useful way of elaborating on content. It is seen as one of the ways mathematics teachers can assist learners from a position of not understanding mathematics in the language of instruction (English), to the stage of being able to use English to talk about mathematics (Setati, 1998). However, code switching may also diminish learners’ exposure to spoken English, and further disadvantage them when they need to study through the medium of English (Cleghorn & Dube, 1998).

The limited English language proficiency of teachers and learners has significant implications for learning mathematics: teaching a subject that has its own discourse in a multilingual classroom, through the medium of a language that is not the primary language for anyone in the classroom *and* has not been mastered by the teacher, provides vast possibilities for confusion. In South Africa, teachers at African secondary schools are predominantly non-native speakers of English.

This results in the use of non-standard forms of English and a relatively low level of general English language competence and confidence.

At tertiary level, most of the mathematics textbooks used is written in English. Students need to become comfortable studying mathematics (i.e. reading textbooks, among other things) through the medium of English. Low levels of literacy thus have a significant impact. In a report published in 1996 by the Joint Education Trust (JET), illiteracy is defined as fewer than seven years of formal schooling or the equivalent. In terms of this definition the adult illiteracy rate was measured at 29% (South Africa Survey, 1996/7). However, many of those with more than seven years of formal schooling may not reach the required level of literacy. The Read, Educate and Develop (READ) Educational Trust Annual Report in 1999 stated that in rural areas, while the average chronological age of entry of Grade 8 learners was 14,4 years, the average reading levels were equivalent to children at age 7,6 years (READ, 1999). In its annual report for 1992 the DET stated that many matriculants were not fully literate in spite of having matriculated, and were thus ill prepared for the labour market, and not equipped for further training (Delvare, 1995). Many of these matriculants moved on to study at tertiary level at teaching colleges, technikons and universities, and because of their limited reading and English proficiency levels, were poorly equipped to cope with the demands that such study entails. For example, a study of Grade 12 English second language applicants to technikons in Gauteng over a period of eight consecutive years showed a steady decline in functional literacy levels, as shown in Table 2.3 (Horne, 2001).

Table 2.3:

Literacy levels at technikons in Gauteng

Year	**Number assessed**	**Functionally literate (Grade 8 or above) (%)**
1990	568	51
1991	490	35
1992	899	33
1993	770	31
1994	930	28
1995	1 314	25
1996	520	24
1997	434	22

It could be argued that since these figures related to technikons the problem did not apply to university students. However, the following findings were equally disturbing.

The results of a standardised English literacy test given to 766 Grade 12 school leavers who applied to and were accepted by a teacher training college in 1995 showed that 95% of the applicants were reading at below Grade 8 level, 3% were reading at Grade 8 level, while only 2% were reading above Grade 8 level. Yet these students qualified as teachers in 1998 and the majority went on to teach English and/or a content subject through the medium of English (Horne, 2001), with potentially serious consequences for the learners in their classrooms.

Every year there has been considerable public concern over the low marks and poor pass rates of learners writing the Senior Certificate examination, particularly those who had to write the examination in a language other than the mother tongue. In an attempt to address this issue, the South African Certification Council (SAFCERT) agreed in 1999 to ‘apply a compensatory measure for learners whose first language was neither English nor Afrikaans and who offered an African language as their first language’ (Umalusi Report, 2004, p. 20). As mentioned in Chapter 1, a compensation of 5% was given for non-language subjects, based on examination marks. This policy was regarded as an interim measure, to be reviewed at some future time, but was still in force at the time of writing. The compensation to some extent masks problems in both the language of instruction and in mathematics.

Reading speed is noted as a problem as well. In a study at the University of the Witwatersrand (Wits) students reported that they had to read texts, sometimes only small sections of texts, several times before understanding them. Even then, ‘they often found it difficult to know whether they were extracting the essential points of texts’ (Delvare, 1995, p.40). This also served to undermine student confidence, and had an impact on time management as well, in that it took students longer than expected to work through the study material.

Poor reading levels were also evident at other universities. At a National Association of Educators of Teachers of English (NAETE) conference in 1999, Webb reported that many of the first-year students at the University of Pretoria who were tested, whose mother tongue was not English, had reading levels of Grade 7 - 8 students (Webb, 1999). Similarly, using inferential questions as an index of reading ability, Pretorius found that first year Psychology and Sociology students at UNISA were reading at frustration level, i.e. well below their assumed reading level, with an average comprehension level of 53% (Pretorius, 2000a). The first-year Medical and Occupational Therapy students at Medunsa students fared marginally better, with a mean comprehension level of 57% (Pretorius, 2000b). Studies with mathematics students at UNISA revealed similar problems (see Chapter 5).

In 2000 the *Sunday Times* (16 July) proclaimed South African children to be the 'dunces of Africa'. The feature article reported on the findings of a comparative study of literacy and numeracy rates of primary school children from 12 countries in Africa, with South African children in general faring poorly in comparison to their African peers on both literacy and numeracy measures. Such results have not been confined to the primary school.

Affective factors

Mathematics is a cognitively complex subject, requiring perseverance and determination. Learners will only work hard at understanding difficult concepts if they are motivated. Since mathematics students at university have elected to study mathematics, it could perhaps be assumed that they have sufficient motivation to do so; however, at the access level, many are only studying mathematics as a path to other fields of study, with limited interest in the subject itself. The fact that they are required to begin at the access level implies that they have not had much past success in mathematics, suggesting that they may have limited confidence in their ability to succeed at university. Previous experiences of failure promote lack of confidence, which inhibits progress. Also, in homes and schools where very few learners have studied mathematics successfully, if at all, expectations of success are low. At the other end of the spectrum, though, there is also the problem that students who may previously have fared reasonably well in their own small circle over-estimate their abilities, and feel that they should not have to 'waste time' studying low level courses.

The absence of many of the factors conducive to effectively learning mathematics (which were discussed in 2.3.2) has been noted previously (in Chapter 1). The negative perception of the real-life value of mathematics and the discouraging influences of teachers and parents limit the extent to which students engage in a problem-solving approach rather than try to rely on memorisation to 'get by'.

Meta-cognitive skills

Meta-cognitive skills develop more readily in an environment in which active engagement with concepts or critical thinking is encouraged. As we have seen in Chapter 1, many South African students were not taught in an environment in which critical thinking or active engagement with concepts was modelled or encouraged. The existence and impact of weak meta-cognitive skills for access mathematics students will be discussed further in Chapter 7.

Assessment

OBE is largely an assessment-driven system. A South African Qualifications Authority (SAQA)[4] position paper points out that assessment in education and training has to do with making judgements about the results of learning so that decisions can be made. These decisions may relate to the learner's ability to function in a particular work environment, carry out a given task, or embark on a particular course of study. The decisions may also relate to other learning that the learner needs in order to become qualified. They may also relate to aspects of the learning programme, such as its quality, possible improvements, or necessary changes. (See Mokhobo-Nomvete, 1999.)

OBE requires that assessment criteria specify unambiguously the levels of complexity and understanding of knowledge students will be expected to have reached (Mokhobe-Nomvete, 1999). Assessment is thus linked to the stated outcomes. OBE principles affect assessment practices in several ways. They dictate that learning should not be assessed by means of examinations only, i.e. there should not only be 'once-off' and 'high-stakes' assessment. These principles reinforce the point that assessment should reflect different purposes and take on different forms, and should test theoretical and practical knowledge in an integrated way. Assessment should determine whether learners have understood, and not merely memorised, new concepts.

[4] In South Africa higher education provision falls under the control of the Council for Higher Education (CHE) and the South African Qualifications Authority (SAQA), which registers all programmes on the National Qualifications Framework (NQF). Quality assurance and the maintenance of standards fall within the domain of the Higher Education Quality Committee (HEQC). Quality criteria formulated for distance education were in fact adapted for all education and training provision by SAQA in the document *Criteria and Guidelines for Providers* issued in October 2001 (Council for Higher Education, 2004).

2.3.4 Additional factors involved in studying mathematics at a distance

Although UNISA is open to ‘qualified’ persons (see 1.6.2), relatively few disadvantaged learners could be regarded as ‘qualified’, which had implications for access. Matriculation exemption and mathematics passed with an E on HG (or D on SG) was a prerequisite for studying in the Science Faculty. Mathematics on HG with a D symbol was required to study mathematics as a subject. Few disadvantaged learners could meet these requirements. For those who did, success was limited. One of the reasons suggested was limited support: UNISA was criticised for the perceived lack of support it gave its students and its poor success rates, although in 1995 it was reported to be attempting to address these problems (Perraton, 2000).

Some of the significant aspects of distance learning have been pointed out, and some of the opportunities and constraints of distance learning have been noted (see 1.6.1). Within the UNISA context the constraints of distance learning have been significant for many students, particularly the more disadvantaged students. Many of the aspects of distance teaching and learning that have been noted may be problematic for teaching mathematics at an access level in the South African context. We now consider some of these aspects, and later review them in the context of access mathematics.

Media

Effective distance teaching utilises a variety of different media (see for example Laurillard, 1993; Singh, 1995; Perraton, 2000); however, ‘Routine applications of technology will not meet the order of magnitude of challenges we face in bringing much more mathematics learning to many more students of diverse backgrounds’ (Roschelle et al., 2000, p.72). These challenges include socio-economic problems, and problems associated with limited exposure to the available technology. Decisions regarding appropriate media need to be made with the needs of all students in mind, so as not to further limit access for disadvantaged students. Creation of material for distance-learning purposes requires that ‘close attention needs to be paid to programme design and appropriate pedagogic strategies for quality delivery that integrates the use of ICT[5]’ (Council on Higher Education, 2004, p. 29). In the UNISA context, limited use is made of audio and video technology, and very little use is made of computer technology for content delivery, even for courses attracting large numbers of students. These choices are influenced by the cost implications of creating material appropriate for these delivery media, cost implications of using the different media, as well as by socio-economic circumstances which impact upon the provision

[5] Information and communication technology

of electricity in the home, which determines students' ability to utilise audio, video or computer technology. Research in distance learning institutions in South Africa has suggested that only between 35% to 40% of students have access to the use of a computer (Council on Higher Education, 2004). Print is thus the most commonly used medium.

Interactivity

The teaching-learning process, if it is effective, is essentially interactive (Laurillard, 1993; Garrison, 1995). We have seen that in mathematics ideas are best developed by interaction between learners and teachers, and between learners and learners, mediated in a variety of ways. When such interaction is limited, limited learning takes place. In distance education, interaction can only take place through interactive media, such as face-to-face discussions, telephone conversations, inter-active video, and (asynchronous) computer-mediated conversation. Mathematical discourse is dependent to a large extent on its symbols and special notation. Ambiguity easily creeps in when the participants in a dialogue cannot 'see' what each one is saying. In the distance-learning environment, when students contact lecturers by telephone to discuss their problems, it is often difficult to know whether either party fully understands what the other is saying. For mathematics the most sophisticated form of interaction, namely computer-mediated communication, requires more than just word-processing capacity. Possession of specific mathematical software packages is necessary for all participants in the process to express the mathematics clearly, and thus have equal access to the topic under discussion. Expense of the requisite software packages and the possibly limited ability to use them effectively mean that not all students acquire the software (even if they have computers). In other words, in e-mails to lecturers and their replies to students there is often the need to resort to 'pidgen mathematics', thereby creating further opportunities for confusion.

For practical reasons in many instances print, at the other end of the interactivity continuum, remains the dominant medium (Perraton, 2000). Since print is relatively non-interactive it is least suited to driving the learning process even though instructional design devices, such as the use of open-ended questions, and the provision of activities which students should carry out, attempt to introduce an element of interactivity into printed study material.

Contact and support

An important feature of distance education is that a significant level of contact is still required (Singh, 1995). This is even more important in the case of developing countries, as we have seen from Table 1.4 in Chapter 1. Contact between staff and students is required, for example to respond effectively to student difficulties. Equally important is contact among the students themselves, where mathematics problems can be resolved through discussion and where mutual support provides motivation to persevere. Good distance teaching also depends on the provision of support, such as contact sessions with tutors, access to the library, etc. However, socio-economic factors limit the extent to which students can avail themselves of such support opportunities.

Time management

Although the students who register for access modules are chronologically adults, they may not yet have the required academic maturity to be successful in their studies. Table 1.4 in Chapter 1 points out that in the context of distance education provision in developing countries, it is not possible to assume academic maturity of students. In South Africa in 2001 roughly 80% of students enrolled in distance education institutions (technikons and universities) were over the age of 23 (Council on Higher Education, 2004). Apart from a loss of subject knowledge as a result of the lapse of time, they may also lack certain affective characteristics that are prerequisites for effective distance learning, such as the ability to work independently or manage their time. In the context of distance education time-management skills are even more necessary than in face-to-face teaching, since distance learning requires immense self-discipline and provides little peer support (Evans, 1994). Poor time-management skills impact negatively on academic success. Many students also underestimate the additional time required to understand mathematics in an environment where they cannot rely on a lecturer to make concepts clear.

Adequate infrastructure

Effective distance education is dependent on good national infrastructure, effective management and reliable information systems to trace the progress and assess the performance of individual students (ADEA, 1997). It has already been pointed out (see1.6.2) that the main form of communication between students and lecturers is often via the assignments that students submit, which are marked with varying degrees of comment by lecturers, tutors, etc., and returned to the students. In an environment where technology cannot play a major role, students depend on the postal system for their assignments (many of which have fixed submission dates) to reach the university on time, and to be sent back to them. An efficient assignment process is thus dependent on an efficient infrastructure. Table 1.4 in Chapter 1 points out that while the assumption that efficient infrastructure exists may be valid in economically developed countries, this may not be the case in developing countries, such as South Africa. In South Africa reliable postage in many remote areas, or even in several urban areas, cannot be guaranteed, weakening one of the fundamental pillars of the distance-learning process.

For a significant number of students, few of the benefits of the assignment system and interpersonal interaction are possible within the UNISA distance-learning environment, given their socio-economic circumstances, high transport and telecommunication costs, and poor transport and postal services.

Teaching approach and learning style

In a distance-learning context it is impossible to find out about the learning styles of all students within a particular course, and much needs to be assumed. There will thus be students who favour a specific learning style, for example activists, theorists, pragmatists and reflectors, who respond to different approaches to learning (Lockwood, 1997). Some students will use different approaches for different tasks, or the same approach for all learning tasks. The teaching approach adopted in the presentation of any course thus needs to provide new challenges for the activists, interesting ideas for the theorists to probe further, problems that relate to everyday issues with solutions that 'work' for the pragmatists, and opportunities for the reflectors to observe, or think things through.

Students need to utilise efficient study skills, such as reflection. In a study undertaken by Thorpe (1995) the role of reflection was emphasised as an important factor in academic success.

Feedback and assessment

Constructive feedback on assessment tasks is especially important in distance education, where students do not have the benefit of frequent interaction with academic staff, through which they can discover weak areas and take appropriate action (Chaudary, 1995). In a print-based distance-learning context, assessment activities should be designed to optimise student interaction with the text, and thus require the exercise of high-level cognitive abilities (Marland, Patching, Putt & Putt, 1990). However, the reality of distance education is that summative assessment often plays a greater role than formative assessment, and the final examination carries considerable weight, even though it is acknowledged that there should be reduced reliance on 'high-stakes' assessment (see for example Clements & Ellerton, 1996).

Frequent assessment is helpful in assisting in the meta-cognitive development of students, particularly in the sense that preparing for some assessment activity raises students' awareness of how much time it takes to fully understand concepts in order to use and apply them; feedback (oral or written) exposes for the students what they have not understood, and suggests remedial action. In a distance-learning context frequent, meaningful assessment opportunities are practically and logistically problematic. Mature students will usually be able to identify for themselves what they do not know and understand; students with under-developed meta-cognitive skills usually progress more slowly when infrequent assessment limits the opportunities for growth. Informal discussions with students frequently reveal the potential deficiencies of the UNISA assignment system, as against the more frequent tests that take place in contact-teaching contexts. The freedom to skip assignments, or to work with a friend to get answers rather than develop individual understanding, contribute to the limited benefit derived from whatever assessment system is used.

English as the language of instruction

The student demographics at UNISA have varied over the years. In 1999 UNISA recorded the composition of the student body[6] as 40% white, 4% coloured, 44% African and 12% Asian (UNISA Pocket Statistics, 1999). By 2002 this had changed to 36% white, 5% coloured, 47% African and 12% Asian (UNISA Annual Report, 2002). In 2004 the enrolment for the Mathematics Access Module (see Chapter 8) reflected a composition of approximately 24% white, 5% coloured, 62 % African and 9% Asian (Visser, UNISA Bureau of Management Information, 2004, personal communication). It is clear that many UNISA students will not study mathematics in their mother tongue.

The profile of an 'ideal' student, likely to be successful in coping with mainstream mathematics at UNISA would thus reflect someone whose home circumstances made time and space available for concerted effort and quiet reflection, who was highly disciplined with regard to time management and highly motivated to persevere in the face of difficult concepts to grasp or problems to solve; well organised and able to meet assignment deadlines; confident enough to contact lecturers or tutors to ask for help if necessary; with adequate academic and quantitative literacy skills to be able to interact meaningfully with the content of study guides, as well as sufficient competence in whatever other media are used; with adequate background knowledge of

[6] The UNISA 'Pocket Statistics' in 1999 carries the caveat that the distinction according to racial groupings was made to indicate the heterogeneous nature of the student body, and not for any other purpose.

mathematics to form a solid foundation for new concepts; and well-developed meta-cognitive skills in order to know whether he/she has or has not understood concepts properly. Clearly this is unrealistic, and alternative arrangements need to be made for students who cannot satisfy these requirements.

2.4 Theoretical considerations in the design of access programmes

Although we are considering the development of access to mathematics within a distance-learning context, it is important to consider this in relation to access initiatives developed elsewhere. Three of the earliest and better-known access initiatives were access *programmes* (as against access for specific subjects) developed by the Universities of Cape Town (UCT), Natal (Pietermaritzburg) and the Witwatersrand (Wits). These programmes all attempted to deal with content issues but included aspects of academic support as well. We consider briefly the theoretical aspects of these programmes.

2.4.1 The UCT Science Foundation Programme (SFP)

This programme, established in 1986, grew out of a variety of academic support programmes that had been in operation at UCT from the 1980s. Its key features were non-credit first semester courses (dealing with language proficiency and academic skills), and the following credit-bearing courses: introductory courses in chemistry and physics, an introductory course in mathematics, and a course called ‘Integrated Science’, which covered selected science topics together with the teaching of language and academic skills that are related to studying science (Delvare, 1996, p. 27). The most significant aspect of this programme was the emphasis on coordination of academic support across faculties, and the integration of aspects from other disciplines in the courses dealing with mathematics and science.

2.4.2 The Science Foundation Programme at the University of Natal (Pietermaritzburg)

A Science Foundation Programme was created at the University of Natal (Pietermaritzburg) in 1991, admitting a selected group of students without matriculation exemption to a programme consisting of courses in biology, chemistry, physics, mathematics, English language development and a counselling component (Grayson, 1992). Students were selected on the basis of performance after they had attended a ten-day programme modelled on the Teach-Test-Teach (T-T-T) Programme that had been developed at the University of Natal (Durban) even though it did not involve mathematics or other natural science subjects. This programme began in 1988, and aimed to open access to tertiary study in the arts and social sciences for disadvantaged students who would have been excluded on the basis of matriculation results.

> The underlying philosophy of the T-T-T programme is that
>
> Any attempt to identify academic potential must translate into an attempt to provoke individuals to realize abilities that are not manifest in their previous academic performance. The first ‘teach’ of the T-T-T approach is an attempt to provide students with the necessary tools to realize abilities that may not have been manifest precisely because they have been denied access to the kinds of tools required to achieve proficiency in typical university tasks. The second ‘teach’ component that occurs after testing students’ learning ability is necessary to consolidate whatever gains are made. The T-T-T paradigm represents a continuous process and requires that individuals be monitored through cycles of T-T-T (Griesel, 1991, in Delvare, 1996, p.11).

In the University of Natal Science Foundation Programme students dealt with limited content, in depth, to promote conceptual understanding. They were exposed to computers and laboratory apparatus, and attention was given to the thinking and reasoning skills important for a science degree, for example a sense of orders of magnitude, interpreting abstract representations such as graphs, and deductive reasoning (Delvare, 1996). In 1991/1992 prospective students were provided with workbooks (the teaching phase), and the test that followed (giving access to the university for successful students) was based on the workbook. Although the workbooks made it possible to reach dispersed students, community-based activities provided a measure of contact and support for students prior to taking the entrance examination.

2.4.3 The College of Science at the University of the Witwatersrand (Wits)

Wits established the College of Science in 1991 to address the needs of students who did not immediately qualify for entry on the basis of matriculation results. The College and its students have been an integral part of the university, facilitating interaction between mainstream and College (Delvare, 1996). Regular evaluation was important, with a specifically appointed evaluator to provide continuous, formative evaluation (Rutherford & Donald, 1993). After completing the first year of study at the College of Science, students could complete a second year in the College and then move into the second year of a mainstream B.Sc. (Delvare, 1996).

2.5 Placement assessment

The preceding sections have highlighted a number of aspects relevant to studying mathematics, at a distance, in South Africa. Effective learning and teaching depend on the readiness of the student to learn, and the provision of study material appropriate to the student's needs. Many students who attempt tertiary-level study with insufficient levels of skill or preparation need more support than can be provided within the subject itself, and awareness of these needs enables institutions to respond more meaningfully when problems arise.

In South Africa a Council on Higher Education (CHE)[7] discussion document in 2001, *A New Academic Policy for Programmes and Qualifications in Higher Education*, proposed an outcomes-based education model in which commitment to learner-centredness was central (Mokhobo-Nomvete, 1999). In a learner-centred approach the interface between learners and learning activities should be as smooth as possible, and this is clearly *not* possible when an institution has no clear sense of learner competence on admission. Entry-level assessment could provide information that would facilitate smoother articulation from school to tertiary study, by identifying existing levels of competence so that appropriate support could be made available for particular student needs. A great wastage of resources occurs as a result of incorrect placement in courses, either as a result of misguided student choice or lack of entry-level guidance (Rowntree, 1997).

2.5.1 International practice

Acceptance and placement testing is a familiar educational practice, but not necessarily in the context of open and distance learning, especially where attempts to broaden access are strongly encouraged and expected. There is a perception that assessments of academic ability, whether for selection or placement purposes, tend to parallel race and social class differences (Oakes, 1990). In the USA, it would appear that mathematics assessment used to place students in specific 'tracks' would be unlikely to withstand legal review unless educators can demonstrate that the additional support subsequently provided ensured greater opportunity for educational success (Pullin, 1993). However, such testing need not be negatively perceived, provided required levels of post-testing support are available. The provision of remedial activities subsequent to testing unfortunately often has a stigma attached to it, and the awareness of poor performance may lower

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[7] As noted earlier (see footnote 4) South African higher education provision falls under the control of the Council for Higher Education (CHE) and the South African Qualifications Authority (SAQA), which registers all programmes on the National Qualifications Framework (NQF). Quality assurance and the maintenance of standards fall within the domain of the Higher Education Quality Committee (HEQC). Quality criteria formulated for distance education were in fact adapted for all education and training provision by SAQA in the document *Criteria and Guidelines for Providers* issued in October 2001 (CHE, 2004).

students' expectations of themselves. Studies in the USA have shown how classroom expectations tend to play out in lower achievement of those who are exposed to remedial or special programmes (Tate & Rousseau, 2002). Diagnostic assessment is by definition a way of determining strengths and weaknesses. However, it is important to remember that a diagnosis is a value-laden judgement, creating an emotional response within students. The way in which the test results are made known will influence the way in which students respond (Mason, 2002). Students' attitudes and the institution's attitude and approach to support are important factors in determining the effectiveness of the support offered (Appelby & Cox, 2002).

Testing in various forms takes place at many institutions In the USA, Scholastic Achievement Tests (SATs) are regularly used (see for example Isaacs, 2001); Israeli universities use a psychometric test as an additional admissions requirement (see Beller, 2001). In Sweden admission to higher education for students under the age of 25 is based on school performance or the results of an entrance examination (Baumslag, 2000).

In the United Kingdom, Appelby & Cox (2002) suggest that effective teaching requires that lecturers know at what level their students are when they enter a course of study. This leads to the idea of a survey of incoming students, 'to audit their strengths and weaknesses' (Appelby & Cox, 2002, p. 12). It then becomes necessary to provide support for students to reach the required level, if necessary.

2.5.2 Placement testing in South Africa

In South Africa the disparity of educational opportunities and resulting unreliability of matriculation results as predictors of academic performance, particularly for so-called disadvantaged students, has resulted in the development of various alternative admission routes (Foxcroft, 1999). The government also recognises that institutions

> have the right to determine entry requirements as appropriate beyond the statutory minimum. However, ..., selection criteria should be sensitive to the educational backgrounds of potential students ... (Government Gazette, 18 April 1997, no. 17944).

There are several South African examples of entry-level testing. For example, the tests used in the University of Cape Town's Alternative Admissions Research Project tests (discussed below) which were first introduced as a pilot project in 1987, and later expanded to one of the currently most effective and widely used means of assessment (Cliff, Visser, Hanslo & Yeld, 2003). Also, the dynamic testing used in at the University of Natal (Durban) (also discussed below). The University of Port Elizabeth has since 1999 used a battery of placement tests (Foxcroft, 2004). The College of Science at the University of the Witwatersrand has used selection tests as a means of entry to programmes for educationally disadvantaged students, consisting of the first two years of four-, five-, or seven-year degrees (NARSET Report, 1997). A placement test specifically for mathematics students was introduced at the University of the Western Cape (UWC) 'positively as a means of inclusion rather than exclusion' (Amoah, 1998, p. 47). The validity of this test was investigated over a three-year period (1994 to 1996), and it was concluded that there was a positive relationship between the placement test and student performance in one of the first-year mainstream mathematics courses. At the University of Pretoria students are placed into mainstream studies in the sciences, or into the University of Pretoria Foundation Year (UPFY) programme based on results obtained from different tests. The UPFY students then write standardised aptitude tests, a reading skills test (based on the UNISA intervention described later, in Chapter 6), a basic mathematics test (developed internally, based on what incoming students, having passed Grade 12 mathematics, should be expected to know, and some problem-solving skills). These results are considered together with the students' matriculation results (in mathematics, English, and physical science/ biology), to determine appropriate support.

2.5.3 What should be tested?

If the focus is on mathematics, in a distance-learning context, what components should specifically be included in any form of placement assessment? Clearly, at an access level, there would be little content knowledge that could be assumed. However, since students would be studying mathematics, it would seem that they would need some level of competence in numerical reasoning skills (which we shall call quantitative literacy). Students would also need reasonable levels of academic literacy. In addition, in order to study through the medium of print, they would need good reading skills. Above all students would need to have the potential (in whatever way that is defined) to benefit from the teaching, i.e. the potential to learn. We consider these aspects further.

Quantitative literacy

Quantitative literacy may be considered as the ability to deal with everyday problems involving numerical or quantitative information, presented verbally or graphically (for example, making sense out of statistical information presented in a newspaper). Appropriate levels of numeracy, or quantitative literacy, enable students to deal with verbally or visually presented quantitative information, such as numerical relationships, statistical data, etc., in many different domains. (See for example Steen, 2001; Usiskin, 2001; Jablonka, 2003).

Academic literacy

An academically literate student is one who is able to perceive the grammatical and textual structure of the language of instruction, and be able to use it efficiently in the more abstract context of an academic setting (as against the everyday context of social interaction. (See for example Yeld, 2001; Cliff, 2003). Academically literate students are able to grasp the point of a particular argument, perceive the structure of and logical relations inherent in academic text, read the text critically in order to be able to make logical conclusions and transfer concepts to different domains.

The need to read in order to learn mathematics[8]

Although language proficiency is important, in a review of several studies Secada (1992) found that language proficiency did not completely explain mathematical achievement. A less well-researched phenomenon is the extent to which *reading ability* influences a student's ability to comprehend and do mathematics. At UNISA, where print is the primary resource, students need to be good readers in order to 'read to learn'. Limited English reading comprehension and communication skills lead to frustration in learning through the medium of print-based English study material.

Maffei (1994) pointed out that specialised reading techniques are so important in mathematics that reading teachers were more effective than mathematics teachers in teaching students to solve word problems (see Maffei, in Braselton & Decker, 1994). Dale & Cuevas (1987) recognised that proficiency in the language in which mathematics is taught, *especially reading proficiency*, was a prerequisite for mathematics achievement (Dale & Cuevas, 1987; italics added).

In mathematics texts there are more concepts per word, per sentence, and per paragraph than in any other subject; in addition, the complexity of the discourse (including words, numerals, letters, symbols and graphics) requires the reader to shift from one type of vocabulary to another, and the reading level of the reader is often below that of the study material appropriate for that level (Braselton & Decker, 1994). The ability to read well in the mother tongue predisposes students'

[8] For a more detailed discussion of the role of reading in studying mathematics, see Chapters 5 and 6.

ability to learn to read well in the second or third language (Matjila & Pretorius, in process). For many South African students, reading skills in the primary language are also limited, and mastery of English reading skills takes place with difficulty. When students have difficulty reading to learn, it is often assumed that their limited language proficiency is the primary cause. This reflects an underlying assumption that language proficiency and reading ability are equivalent, which is not the case. Research shows that improving language proficiency does not necessarily improve reading (e.g. Hacquebord, 1994). Language proficiency and reading are related; however, they are conceptually and cognitively uniquely specific skills that develop in distinct ways. This distinction is important because it has pedagogical implications. As Freitag (1997) pointed out, 'under-developed reading skills can keep our students from realizing their full potential and developing into the mathematical learners they are capable of becoming' (p.18). Students need to interact with the text, be alert and attentive, be sensitive to comprehension failure as soon as it happens, and be capable of applying repair strategies if necessary. Reading rate adjustment and multiple readings are thus often necessary.

2.5.4 Specific examples of admissions or placement testing

The *Directory of Science, Engineering and Technology Foundation Programmes* (2001) outlines many different forms of entry-level testing that were in place at that time, in addition to various requirements with respect to minimum scores and grades on which mathematics or physical science, etc. were passed, for students aiming to gain access to study in the Natural Sciences via some or other foundation programme. The University of Cape Town used the results of the AARP tests (where students were assessed on the basis of being placed in the upper deciles, compared with their peers); the University of Durban-Westville applied a faculty selection test; the University of Natal (Pietermaritzburg) used tests developed by Science Foundation Programme staff in mathematics, English and science 'designed to eliminate those without the necessary background knowledge' (*Directory of Science, Engineering and Technology Foundation Programmes*, 2001, p. 112). At the University of the North, students hoping to be admitted to the University of the North Foundation Year wrote selection tests specifically developed 'to identify students with potential in mathematics and science' (*Directory of Science, Engineering and Technology Foundation Programmes*, 2001, p. 119). The University of Port Elizabeth (UPE) applied the UPE Admissions and Placement Assessment battery; the University of Pretoria made use of students' average performance in a set of three admissions tests: two were aptitude tests that aimed to measure the potential of candidates to do mathematics and science, and the third was an English test. At the Rand Afrikaans University the Learning Centre assessed learners' potential over the course of four sessions that investigated students' ability to learn, and also used interviews, biographical information and psychometric tests. The University of Stellenbosch made use of Standardised Assessment Tests in English, biology, mathematics and physical science. At the University of the Witwatersrand eligible students (eligible according to Table 1.3, in Chapter 1) were required to 'write a number of selection tests, complete a biographical questionnaire and for certain candidates, attend an interview' (*Directory of Science, Engineering and Technology Foundation Programmes*, 2001, p. 174). There were thus many different forms of admissions or placement tests. From the columns in Table 1.3 indicating numbers of students and admission criteria, it is clear that all of the institutions mentioned could choose a relatively small number of students, which implied that only 'the best' applicants were selected. It is also clear that in several cases the selection procedures required a measure of physical presence, in some cases for more than just a limited period on one day. We consider two examples.

Alternative Admissions Research Project (AARP) (University of Cape Town (UCT))

These tests were used as an additional indicator, together with matriculation results, to determine whether or not students could be accepted at UCT (from 1987). They were later used by many other institutions as well[9]. Students who obtained matriculation exemption under any of the various departments in charge of African education could write an English placement test, a mathematics comprehension test and a mathematics achievement test[10] to be offered a provisional place in a particular science degree programme. These tests, which could be taken at many different centres across the country, provided students with an opportunity to show *potential* for university study in advance of the matriculation results. Pencil-and-paper tests were used, and interviews were not required, thereby reducing costs. Furthermore, the direct test scores were adjusted according to a formula which took school background into account.

While there was considerable controversy regarding the meaning of 'potential', and whether or how it could be measured (Agar, Hofmeyer and Moulder, 1990, in Delvare, 1996), UCT recognised that

> Although the danger … of confusing potential with actual performance … is acknowledged, the word potential is used here to refer merely … to the identification of

[9] Sixteen centers in 1992 (Delvare, 1996). Twenty-two centres in 1997 (Yeld, personal communication).
[10] The components of the admissions test used in the AARP are discussed in more detail in Chapter 9.

> likelihood of success in a given environment, given the support … available (Badsha & Yeld, 1991).

The option to write these tests gave students advance opportunity to obtain finance, accommodation, etc., without having to wait until the matriculation results were published. Since 1990 students selected via AARP have had the opportunity to attend a four-day ‘pre-orientation’ programme, held before the normal UCT orientation week.

Students from educationally disadvantaged backgrounds are not sufficiently well prepared to take tests that require them to understand the demands of the ‘target situation’, and developing tests based on these demands yields results which ‘predict the past’ (Yeld, personal communication, 1997). Although the AARP tests include a content-dependent mathematics achievement component, they also include a mathematics comprehension component. The mathematics comprehension test incorporates a combination of teaching and modelling, and gives learners opportunities to demonstrate the potential to learn from some instruction and practice. The philosophy behind the tests is that if appropriate mediation is provided (through the tasks in the tests) the performance of the candidates will be altered so that the more able candidates demonstrate their potential to learn from the tasks and perform significantly better than the less able candidates (Yeld, personal communication, 1997). Testing in this way represents a more just method than merely testing content knowledge.

The predictive validity of the tests has been established by taking into account the academic performance at UCT of the students selected on the basis of these tests. Apart from assisting student entry into mainstream courses in this way, the Project’s instruments are the basis for UCT’s senate-discretionary admission process, which enables applicants without matriculation exemption to be admitted to degree courses. At a meeting of university principals in 1991 the diagnostic nature of the AARP tests was emphasised, i.e. the ability of the testing procedure to correctly channel students into mainstream study or into programmes providing additional support (Delvare, 1995), such as the Science Foundation Programme (which later became known as the General Entry in Science Programme: see *Directory of Science, Engineering and Technology Foundation Programmes*, 2001). The tests thus made access possible for students who may otherwise have been excluded from studying the natural sciences at university.

The testing aspect of the 'Teach-Test-Teach' Programme at the University of Natal (Durban)

In this programme, testing was regarded as an opportunity for learning. The tests were 'dynamic' in the sense that tests included a learning component, as well as a component to determine whether learning had taken place.

> The fundamental assumption is that education provides learning opportunities that alter the very abilities that are assumed or treated as fixed in the construction of psychometric/edumetric tests. Although the T-T-T Programme can serve as a vehicle for selection, with the students who perform best at the time of testing being selected, it is primarily an educational intervention whose end is not testing but teaching (Griesel, 1991, in Delvare, 1996, p. 11).

The fact that the University of Natal (Durban) was also a face-to-face teaching institution made it possible to use testing for selection.

2.6 Summary

This chapter has touched on a number of theoretical issues that were seen to be relevant to creating access to mathematics. Factors relevant to studying mathematics effectively were discussed. Two specific factors, namely assessment practices and the linguistic demands of mathematics discourse, will be further addressed later, as they gave rise to the research described in Chapters 5, 6 and 7. Chapter 2 also considered some aspects of access programmes in the sciences, which necessarily include mathematics. Aspects of these programmes had some bearing on the creation of the UNISA Mathematics Access Module, and on the research described in this thesis.