PROFILING MOMENTUM IN EQUITY MARKETS

by

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submitted in accordance with the requirements for the degree of

DOCTOR OF PHILOSOPHY

in the subject

MANAGEMENT STUDIES

at the

UNIVERSITY OF SOUTH AFRICA

SUPERVISOR: PROF J MARX

May 2023

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ABSTRACT AND KEY TERMS

Abstract

This study created a customised model and a custom index to profile momentum in equity markets. The customised model used a momentum term structure grouped into different entry zones to create visual profiles for individual equity shares or stocks. A momentum profile describes a particular equity market in terms of the composition of its momentum cycles. Profiling shifts the focus onto the holding period while differentiating between false, neutral, negative, and positive momentum cycles as determined by the eventual outcomes. The composition of the momentum cycles and average hold per cycle type provide a unique description of the momentum effect in a market. The customised model identifies the stocks with momentum cycles in progress while the custom index quantifies the collective outcome to show the progression of momentum in a market over the years. Therefore, each equity market has a different profile related to the composition of its momentum cycles and the performance of its custom index. These profiles can be compared in terms of the number of momentum cycles, composition, basic profiles (average holds, price ranges, sectors, and entry zones), average parameter scores, and performance. This study created momentum profiles for three different equity markets – the Johannesburg Stock Exchange (JSE), the Toronto Stock Exchange (TSX), and the TSX Venture Exchange. The settings of the model parameters (momentum, volatility, quality, and activity scores) were calibrated on data from the JSE to enable direct comparison between the three exchanges. These exchanges or markets are distinct in size, the number of qualifying listings, and the number of listings that generated momentum cycles. The composition or configuration of the momentum cycles is unique to each market. The overall outcomes, in terms of average hold and compound return per average hold, favoured the emerging market represented by the Johannesburg Stock Exchange (JSE). The developed market, represented by the Toronto Stock Exchange (TSX) generated the largest number of momentum cycles and outperformed the JSE based on positive cycles. The venture market, represented by the TSX Venture Exchange (TSXV), underperformed the other two markets overall but produced the best results in terms of positive cycles. The positive cycles ultimately determined the performance of the respective momentum indices with the TSXV Momentum Index (TSXV-MI) outperforming the other two indices, the JSE Momentum Index (JSE-MI) and the TSX Momentum Index (TSX-MI) over the 13-year period (2009-2021) of analysis.

Key terms

Custom index	Hold	Momentum profile
Customised model	Individual profile	Negative cycle
Entry zone	Market profile	Neutral cycle
False cycle	Momentum curve	Positive cycle
Formation	Momentum cycle	Term structure

TRANSLATION: Afrikaans

Uittreksel

Hierdie studie het 'n doelgemaakte model en 'n pasgemaakte indeks geskep om momentum in aandelemarkte te profileer. Die doelgemaakte model het 'n momentumtermynstruktuur gebruik wat in verskillende toetreesones ingedeel is om visuele profiele vir enkelaandele te skep. 'n Momentumprofiel beskryf 'n besondere aandelemark in terme van die samestelling van sy momentumsiklusse. Profilering verskuif die klem na die houtydperk terwyl daar onderskei word tussen vals, neutrale, negatiewe en positiewe momentumsiklusse soos bepaal deur die uiteindelike uitkomste. Die samestelling van die momentumsiklusse en gemiddelde houtyd per siklussoort verskaf 'n eiesoortige of unieke beskrywing van die momentumeffek in 'n mark. Die doelgemaakte model wys die aandele met ontwikkelende momentumsiklusse uit, terwyl die pasgemaakte indeks die gesamentlike uitkoms kwantifiseer om die vordering van momentum in 'n mark oor die jare weer te gee. Elke aandelemark het daarom 'n anderse profiel wat verband hou met die samestelling van sy momentumsiklusse en die vertoning van sy pasgemaakte indeks. Hierdie profiele kan vergelyk word in terme van die aantal momentumsiklusse, samestelling, basiese profiele (gemiddelde houtye, prysklasse, sektore en toetreesones), gemiddelde parametertellings en vertoning. Hierdie studie het momentumprofiele vir drie verskillende aandelemarkte geskep – die Johannesburg Aandelebeurs (JSE), die Toronto Aandelebeurs (TSX) en die TSX Waagkapitaalbeurs. Die stellings van die modelparameters (momentum, volatiliteit, kwaliteit en aktiwiteit tellings) is op data van die JSE ingestel om direkte vergelyking tussen die drie beurse moontlik te maak. Hierdie beurse of markte is verskillend in grootte, die aantal geskikte noterings en die aantal noterings wat momentumsiklusse ondergaan het. Die samestelling of konfigurasie van die momentumsiklusse is eiesoortig aan elke mark. Die algehele uitkomste, in terme van gemiddelde houtyd en saamgestelde opbrengs per gemiddelde houtyd, het die ontluikende mark soos deur die Johannesburg Aandelebeurs (JSE) verteenwoordig bevoordeel. Die ontwikkelde mark, verteenwoordig deur die Toronto Aandelebeurs (TSX), het die grootste aantal momentumsiklusse voortgebring en het beter as die JSE gevaar op grond van positiewe siklusse. Die waagmark, verteenwoordig deur die TSX Waagkapitaalbeurs (TSXV), het oor die algemeen swakker gevaar as die ander twee markte, maar het die beste uitslag gelewer in terme van positiewe siklusse. Die positiewe siklusse het uiteindelik die vertoning van die onderskeie momentumindekse bepaal met die TSXV Momentum Indeks (TSXV-MI) wat die ander twee indekse, die JSE Momentum Indeks (JSE-MI) en die TSX Momentum Indeks (TSX-MI), oor die 13-jaar tydperk (2009-2021) van ontleding oortref.

Sleutelterme

Pasgemaakte indeks	Houtyd	Momentumprofiel
Doelgemaakte model	Enkelprofiel	Negatiewe siklus
Toetreesone	Markprofiel	Neutrale siklus
Vals siklus	Momentumkurwe	Positiewe siklus
Vorming	Momentumsiklus	Termynstruktuur

TRANSLATION: isiZulu

Isifingqo

Lolu cwaningo ludale imodeli eyenziwe ngokwezifiso kanye nenkomba yangokwezifiso ukuze kuphrofayili umfutho ezimakethe zokulingana. Imodeli eyenziwe ngendlela oyifisayo isebenzise ukwakheka kwethemu lomfutho eliqoqwe ezindaweni zokungena ezihlukene ukuze kwakhe amaphrofayili abonakalayo wamasheya angawodwana okulingana noma amasheya. Iphrofayili yomfutho ichaza imakethe ethile yezabelomali ngokuya ngokwakheka kwemijikelezo yayo yomfutho. Ukwenza iphrofayela kushintsha ukugxila kunkathi yokubamba kuyilapho kuhlukanisa phakathi kwemijikelezo yamanga, engathathi hlangothi, engemihle, kanye nenhle njengoba kunqunywa imiphumela yokugcina. Ukwakheka kwemijikelezo yomfutho nokubamba okumaphakathi kohlobo ngalunye lomjikelezo kunikeza incazelo ehlukile yomthelela womfutho emakethe. Imodeli eyenziwe ngendlela oyifisayo ikhomba amasheya anemijikelezo yomfutho eqhubekayo kuyilapho inkomba yangokwezifiso ilinganisela umphumela ohlangene ukuze ubonise ukuqhubeka komfutho emakethe phakathi neminyaka. Ngakho-ke, imakethe yezabelomali ngayinye inephrofayili ehlukile ehlobene nokwakheka kwemijikelezo yayo yomfutho kanye nokusebenza kwenkomba yayo yangokwezifiso. Lawa maphrofayili angafaniswa ngokwenani lemijikelezo yomfutho, ukwakheka, amaphrofayili ayisisekelo (ukubanjwa okumaphakathi, ububanzi bentengo, imikhakha, nezindawo zokungena), isilinganiso semiphumela yepharamitha, kanye nokusebenza. Lolu cwaningo ludale umfutho ezimakethe ezintathu ezahlukene zamasheya – iJohannesburg Stock Exchange (JSE), iToronto Stock Exchange (TSX), kanye ne-TSX Venture Exchange. Izilungiselelo zamapharamitha wemodeli (umfutho, ukuguquguquka, ikhwalithi, namaphuzu omsebenzi) zilinganiswa ngedatha evela e-JSE ukuze kuvunyelwe ukuqhathanisa okuqondile phakathi kwalokhu kushintshana okuthathu. Lokhu kushintshana noma izimakethe zihlukile ngosayizi, inombolo yokufakwa kuhlu okufanelekayo, kanye nenani lokufakwa kuhlu okukhiqize imijikelezo yomfutho. Ukwakheka noma ukucushwa kwemijikelezo yomfutho bekuhlukile emakethe ngayinye. Isiyonke imiphumela, ngokwesilinganiso sokubamba kanye nembuyiselo ehlanganisiwe ngokwesilinganiso sokubamba, ivune izimakethe ezisafufusa ezimelwe yiJohannesburg Stock Exchange (JSE). Imakethe ethuthukisiwe, emelwe yi-Toronto Stock Exchange (TSX) ikhiqize inani elikhulu kakhulu lemijikelezo yomfutho futhi yadlula i-JSE ngokusekelwe emijikelezweni emihle. Imakethe yezohwebo, emelwe yi-TSX Venture Exchange (TSXV), yenza kabi ezinye izimakethe ezimbili zizonke kodwa yakhiqiza imiphumela engcono kakhulu ngokwemijikelezo emihle. Imijikelezo eqondile igcine inqume ukusebenza kwezinkomba zomfutho ngokulandelana kwazo ne-TSXV Momentum Index (TSXV-MI) idlula ezinye izinkomba ezimbili, i-JSE Momentum Index (JSE-MI) kanye ne-TSX Momentum Index (TSX-MI) phakathi neminyaka 13 unyaka inkathi (2009-2021) yokuhlaziya.

Imigomo ebalulekile

Inkomba yangokwezifiso	Ukubamba isikhathi	Umfutho iphrofayili
Imodeli engokwezifiso	Iphrofayili ngamunye	Umjikelezo ongamuhle
Indawo yokungena	Iphrofayili yemakethe	Umjikelezo ongachemile
Umjikelezo wamanga	Umfutho ijika	Umjikelezo omuhle
Ukwakheka	Umfutho umjikelezo	Isakhiwo sethemu

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ABBREVIATIONS AND TERMINOLOGY

Abbreviations

200dMA	200-day Moving Average
3MA	3-month Moving Average
ADD	Additions
ADd	Average Drawdown
ADD/T	Additions ratio (Additions/Total)
ADur	Average Duration
AH	Average Hold
AH-O	Average Hold – Overall
AH-P	Average Hold – Positive
AS	Activity Score
AVG	Average
CAGR	Compound Annual Growth Rate
CARpAH	Compound Annual Rate per Average Hold
CPGR	Compound Period Growth Rate
CRpAH	Compound Rate per Average Hold
CTGR	Compound Total Growth Rate
CV	Coefficient of Variation
C/I	Average number of cycles per ticker (Cycles/Identify)
DdR	Drawdown Ratio
DEL	Deletions
dMS	days-Momentum Score
Dur	Duration
I/Q	Momentum cycles ratio (Identify/Qualify)
JSE	Johannesburg Stock Exchange
MCap	Market Capitalisation
MDd	Maximum Drawdown
MDur	Maximum Duration
MEM	Members
MI	Momentum Index
MOM	Momentum
MS	Momentum Score
PbMA	Price below Moving Average
Per	Period
QS	Quality Score
Rec	Recovery
StdD	Standard Deviation
TSX	Toronto Stock Exchange
TSXV	TSX Venture Exchange
TTen	Top Ten
VS	Volatility Score

ABBREVIATIONS AND TERMINOLOGY

Terminology

Average hold	The average holding period per cycle type.
Calibration	Tuning a model to fit a particular equity market.
Custom index	An index using a specific methodology to update, assign weights, calculate levels, and set member numbers.
Customised model	A model with a unique set of parameters that can be calibrated to fit a particular market.
Entry zone	A group of successive term-structure periods exhibiting high momentum.
False cycle	A cycle holding shorter than 3 months irrespective of the size of the annualised gain or loss (failed outcome).
Formation period	A period of high momentum, ranging from 60 to 250 days, indicating a momentum cycle in progress.
Holding period	The period between entering and exiting a momentum cycle.
Individual profile	An evolving visual pattern that provides a graphic history of an equity share’s momentum cycles in terms of occurrence, duration, shape, and outcome.
Market profile	The composition of an equity market’s momentum cycles, the average hold per cycle type, price range and sector activity, as well as performance measured via an index.
Momentum curve	A graphical representation of the level of momentum over a range of formation periods (i.e., the slope of different momentum formations).
Momentum cycle	A full cycle comprising the formation and holding periods regardless of the outcome.
Momentum profile	A description of an equity market in terms of the composition of its momentum cycles (market profile); the visual pattern of a stock’s momentum (individual profile).
Negative cycle	A cycle holding at least 3 months and recording an annualised loss of more than 10% (unexpected outcome).
Neutral cycle	A cycle holding at least 3 months but recording an annualised gain or loss not exceeding 10% (no outcome).
Optimisation	Fine-tuning a model to exactly fit a particular market during a specific period.
Positive cycle	A cycle holding at least 3 months and recording an annualised gain of more than 10% (optimal outcome).
Positive-cycle rate	The proportion of positive cycles relative to all cycles, expressed as a percentage of the total.
Profiling	The analysis of behaviour and characteristics to make generalisations and stereotypical assumptions.
Term structure	A structure of several momentum terms or periods of increasing length grouped into an entry zone.

CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND

Momentum refers to price continuation based on past outperformance. Jegadeesh and Titman (1993) never mention the word momentum in their original study, even though their paper is considered to be a seminal work on momentum strategies. The term momentum was adopted after Mark Carhart published his University of Chicago thesis in The Journal of Finance (Gray & Vogel 2016:45). Carhart (1997) created a momentum factor, which essentially reflected the relative strength of the stock selection strategies outlined in Jegadeesh and Titman (1993).

Factor investing or customised indexing enables investors to capture the different risk premia available in the market as it provides explicit exposure to the underlying risk factors (Kula, Raab & Stahn 2017). The most common factors are size (small-cap stocks outperforming large-cap stocks), value (undervalued stocks outperforming higher-valued stocks), dividends (high-dividend stocks outperforming low-dividend stocks), volatility (low-beta or low-volatility stocks outperforming high-beta or high-volatility stocks), quality (low debt and stable earnings growth), and momentum (price continuation based on past outperformance). Smart beta (alternative or advanced beta) can be viewed as a subset of factor investing and uses mechanical index construction rules to capture the risk factors or investment styles without involving any human judgment or subjectivity once put in place (Zaher 2019). Smart beta investing combines passive and active investing by systematically incorporating momentum in a quantitative rule-based indexing approach. In this sense, the proposed study on profiling momentum in equity markets via a customised model and a custom index is related to a smart beta approach. The momentum profiles are created by mechanically entering momentum cycles, not making any discretionary or subjective decisions, and exiting on a fixed rule. The emphasis of this study, however, is on the momentum profiles that describe equity markets in terms of the composition (false, neutral, negative, and positive) of their momentum cycles. A basic profile includes the average hold per cycle type, price range and sector activity. The custom index quantifies the outcomes generated by the customised index to complete the market profiles. Individual equity shares or stocks have visual profiles of their momentum cycles in terms of occurrence, duration, shape, and outcome.

The upcoming sections provide an overview of past studies to motivate the problem statement and the research objectives of the study. This study is quantitative and observational in design, performing calculations based on historical stock price data and using descriptive statistics and performance metrics to evaluate the results. The potential contributions to research are stated before concluding with the outline or structure of the study.

1.2 RESEARCH OVERVIEW

Momentum investing has been a popular strategy for systematic and fundamental portfolio managers (Satchell & Grant 2021:103). Jegadeesh and Titman (1993) introduced the classic strategy of buying past winners and selling past losers on their relative strength. Stocks are ranked monthly in descending order based on performance over specific formation periods and divided into several portfolios. The two portfolios with the highest and lowest ranking stocks are compared after fixed holding periods. Periods vary from three to twelve months resulting in different formation/holding period combinations. A widening spread between the portfolio with high positive momentum and the portfolio with high negative momentum confirms the momentum effect.

The results reported by Jegadeesh and Titman (1993) were based on data from the United States market, but many subsequent studies followed this approach or some variation thereof to confirm the momentum effect in other equity markets. Initially, in addition to confirming the momentum effect, research focussed on explaining the sources of momentum. The momentum effect in a particular market is usually associated with a specific formation/holding (J/K) period combination.

Page, Britten and Auret (2016:44) reported that idiosyncratic risk (i.e., risk confined to a specific group of stocks) does not drive momentum profits and cannot explain its persistence on the Johannesburg Stock Exchange (JSE). Page and Auret (2019) added that the market risk factor as well as the size and value factors, do not explain or account for the momentum premium in the South African market. Momentum is a distinct pricing anomaly that consistently generates significant risk-adjusted returns that cannot be explained within a risk-based paradigm (Page & Auret 2019:15). The focus shifted to behavioural explanations for this anomaly because the magnitude and persistence of momentum returns are too strong to be explained by risk (Jegadeesh & Titman 2011:494).

Behavioural explanations offer two possible sources of momentum as the market responds with a delay to new information. Momentum results from either a delayed initial reaction (or underreaction) or a delayed overreaction that follows the initial underreaction (De Long, Shleifer, Summers & Waldmann 1990). If the underreaction and overreaction were elements of the same continuous process whereby prices build momentum, any underreaction would inevitably lead to a delayed overreaction that continues into the holding period (Alwathainani 2012). Stocks would lose momentum and start posting negative returns after 12 months. An underreaction confined to the formation period would gain momentum over a maximum period of 12 months and hold that momentum for up to 12 months with average returns after that (Jegadeesh & Titman 2001). Momentum driven by an underreaction would be preferred as it moves a stock towards its intrinsic value and does not reverse.

Alternative definitions of momentum were researched to improve on the basic measure (percentage change in price), attempting to secure a more persistent continuation in performance and retain the gains from the holding period or avoid reversal. Below are some examples of studies on alternative definitions of momentum.

Momentum strategies are predominantly cross-sectional in design, as performance is measured at a particular point in time and relative to other stocks (via ranking). In contrast, time-series momentum assigns stocks to long or short portfolios on their absolute or individual performance over time. Moskowitz, Ooi and Pedersen (2012) introduced time-series momentum as an alternative to cross-sectional momentum. Time-series momentum focuses solely on the past returns of individual stocks, buying stocks that generated positive returns and shorting those with negative returns over a particular formation period. Stocks with momentum under relative strength do not necessarily have momentum under absolute strength (Gulen & Petkova 2018). The time-series approach also introduces timing to momentum investing and avoids the short-term reversals reported with cross-sectional strategies (Goyal & Jegadeesh 2018). Moving-average momentum aligns with time-series or trend momentum but introduces even more timing into the buying and selling of stocks (Marshall, Nguyen & Visaltanachoti 2017).

Idiosyncratic momentum originates from the returns specific to each individual stock and not explained by any of the common factors (e.g., market risk, size, or value) included in a particular factor model. These stock-specific returns can be represented by either the error terms (residuals) or the alphas obtained from a regression (Hühn & Scholz 2018). Idiosyncratic momentum isolates stock-specific momentum and does not reverse strongly in the long term, consistent with an underreaction to stock-specific news (Blitz, Hanauer & Vidojevic 2020). A study by Page, McClelland and Auret (2020) provided evidence from the Johannesburg Stock Exchange (JSE) that idiosyncratic momentum subsumes or incorporates price momentum and better explains the cross-sectional variation in stock returns. However, in the South African market, gains from idiosyncratic momentum are as likely to reverse as those from price momentum, suggesting an overreaction to stock-specific news. This study is another example of the ongoing search for a more persistent momentum in stocks that does not inevitably reverse in the long run.

Focusing on price levels rather than past returns, a stock price at or near its 52-week-high level is a better indicator of momentum in price than extreme returns measured over some fixed formation period (George & Hwang 2004). The 52-week high serves as a reference point or anchor, and anchoring results in an underreaction that builds momentum without the eventual reversal experienced with a delayed overreaction to news (Liu, Liu & Ma 2011). Momentum based on the 52-week high of a stock does not rely on extreme returns. Therefore, the 52-week-high alternative identifies momentum in the absence of extreme returns (Bhootra & Hur 2013).

The momentum effect from a particular market is usually described or classified in reference to its formation (J) and holding (K) periods:

La Grange and Krige (2015) compared the returns of long-only momentum strategies in the South African market based on various formation and holding periods. Stock selection was restricted to the top 100 companies according to market capitalisation, screened on trading value. A delay of one month between the formation and holding periods accounted for short-term corrections. The best-performing momentum portfolio had a 4-month formation period with a 1-month holding period. Accounting for transaction costs, the best-performing portfolio had a 5-month/3-month formation/holding period combination without the 1-month delay.

O’Keeffe (2013) studied four medium-sized European markets (Ireland, Greece, Norway, and Denmark) but only observed significant price momentum in the Irish market. Average monthly returns were maximised via a relatively long 9-month formation period and an unusually short 2-month holding period. Following a similar approach, Murphy (2017) observed significant momentum in the United Kingdom. Largely inconsistent with other studies on momentum, finding that short (3 months) formation with long (24 months) holding periods delivered the best results.

Pavlova and Parhizgari (2011), with data from the United States, screened to exclude low-priced stocks, used a genetic algorithm to maximise the return from a momentum strategy with different iterations of formation (J) and holding (K) periods ranging between 1 and 18 months. The algorithm matched a formation period of 6 months (6J) with a holding period of 9 months (9K), whereas a formation period of 8 months (8J) matched optimally with a holding period of 6 months (6K). Dividing the set of data into two subsets, the optimal combination was respectively 12J/3K and 8J/3K. The full dataset delivered 9J/4K as the outperforming combination.

Bird, Gao and Yeung (2017) studied the formation and holding periods from 24 different markets, including Australia, Canada, the United States, and the United Kingdom. The best results were from relatively long formation periods, either 9 or 12 months. Performance gradually decreased as formation periods shortened, with the shortest formation period (3 months) performing worst in most markets. The short 3-month holding period balanced with the longer formation periods performed best in most markets. As a result, 9J/3K and 12J/3K combinations outperformed overall.

Apart from defining new measures of momentum to improve on the results obtained by the most basic measure, the percentage change in price over monthly periods ranging from 3 to 12 months (termed medium-term momentum), studies on price momentum generally try to capture the effect in terms of the formation and holding periods unique to a particular equity market. The notion of some optimal formation (J) and holding (K) period combination per equity market enabled comparison between different markets, also following certain market events, states, and stages.

1.3 PROBLEM STATEMENT

It was evident from the literature review that past research focused on the classic J-month/K-month (formation/holding period) approach to identify momentum and find the optimal J/K combination in different equity markets. Buying the best-performing stocks (top quantile) and selling the worst-performing stocks (bottom quantile) on their performance over the past 3 to 12 months at every update. A widening spread between the performance of the two groups would confirm the presence of momentum in that market. The long-only version ranks stocks on some definition of momentum, buying the top-ranked stocks (cross-sectional design) or stocks with high momentum (time-series design) and replacing individual stocks when a ranking or momentum falls below certain thresholds. Standard formation and holding periods are generally used (typically 3, 6, 9 and 12 months) to find the optimal combination for a particular equity market, perhaps iterating through different combinations with 1-month increments for a more exact calibration. Regarding momentum, equity markets are simply classified on their optimal J/K combinations. Past studies made no attempt to describe a particular equity market in terms of the composition of the momentum cycles from that market.

This study will introduce the concept of momentum profiling. A momentum profile describes a particular equity market in terms of the composition of its momentum cycles. Profiling shifts the focus onto the holding period while differentiating between false, neutral, negative, and positive momentum cycles as determined by the eventual outcomes. Formation periods are substituted with entry zones, ensuring variability in formation. These entry zones also create visual profiles for individual stocks. A performance analysis via a custom index completes the momentum profile for a particular equity market.

1.4 RESEARCH OBJECTIVES

The objectives of this study are to:

- Customise a model to profile momentum in equity markets.
- Construct a custom momentum index to quantify and present the outcomes.
- Create and compare the momentum profiles of three different equity markets.

The focus of this study is on positive momentum and long-only investing. A momentum cycle comprises both a formation and a holding period. There is a distinction between positive or negative momentum based on a change in value during the formation period and positive or negative momentum cycles based on the eventual outcome at the end of the holding period. This study differentiates between false, neutral, negative, and positive momentum cycles. The composition of the momentum cycles and average hold per cycle type provide a unique description of the momentum effect in a particular equity market. A custom index quantifies the collective outcome to show the progression of momentum in a market over the years.

1.5 RESEARCH DESIGN

This study is observational in design, based on the distinction between observational and experimental when doing quantitative research related to equity investing. Descriptive statistics and various performance metrics will evaluate the momentum model via the custom index that quantifies the outcomes to present the actual results over time. All calculations will be based on historical stock price data downloaded from Bloomberg (Bloomberg 2022).

The customised momentum model with its four parameters (momentum, volatility, quality, and activity) will be calibrated on the Johannesburg Stock Exchange (JSE). The same parameter settings will be applied to the Toronto Stock Exchange (TSX) and the TSX Venture Exchange (TSXV) to allow direct comparison between the equity markets. These exchanges were chosen to respectively represent an emerging market (JSE), a developed market (TSX), and a venture market (TSXV).

The model uses a momentum term structure that displays as stepped visual profiles for individual stocks. In this instance, the term structure refers to six momentum terms or periods of increasing length grouped into four different entry zones. The concept behind this model is to identify stocks relatively early in their respective momentum cycles via three successive term-structure periods of high momentum (i.e., an entry zone). The model exits on the momentum parameter.

The custom index is constructed as equal weighted in that new members enter at the average weight of the current members. The index is updated monthly, and the number of members is variable. The individual weights of the remaining members are adjusted for any additions to or deletions from the index.

1.6 POTENTIAL CONTRIBUTIONS

The potential contributions of this study are the following:

- Creating momentum profiles for equity markets by describing each market in terms of the composition of its momentum cycles.
- Creating graphic (visual) momentum profiles for individual companies.
- Introducing the concept of a momentum term structure, several formation periods, to enter momentum cycles early and exit as late as possible.
- Customising a momentum model that makes the pre-sorting on price, market capitalisation (size), sector, trading volume, or volatility redundant.
- Customising a momentum model that can be calibrated for a particular market but does not require optimisation.
- Constructing a custom momentum index to quantify and present the outcomes of a mechanical or systematic approach to momentum investing.
- Providing retail and institutional investors with information on the likely performance of momentum investing in a particular market.

1.7 OUTLINE

The remainder of the study will be structured as follows:

Chapter 2 – PRICE-BASED MOMENTUM

A literature review of the explanations for price momentum in equity markets, different definitions of momentum, and the quantitative approaches for identifying sustainable or more persistent momentum.

Chapter 3 – RESEARCH DESIGN

This chapter introduces the research design for this study by identifying the research paradigm (positivism), methodology or approach (quantitative), design (observational), and methods (descriptive statistics and performance metrics) of the study. It includes information on the methodology (i.e., construction, weighting, calculation, and review) of the custom momentum index.

Chapter 4 – MOMENTUM MODEL

The model specifications, parameter descriptions and settings, as well as the momentum profiles of selected companies illustrating the various outcomes, are included in this chapter dedicated to the customised momentum model.

Chapter 5 – MOMENTUM PROFILE: JOHANNESBURG STOCK EXCHANGE

The momentum profile for an emerging equity market, the Johannesburg Stock Exchange (JSE) in South Africa, is created by applying the customised model mechanically to generate a set of false, neutral, negative, and positive cycles unique to this market. A custom momentum index quantifies the performance of the model.

Chapter 6 – MOMENTUM PROFILE: TORONTO STOCK EXCHANGE

The momentum profile for a developed equity market, the Toronto Stock Exchange (TSX) in Canada, is created by applying the customised model mechanically to generate a set of false, neutral, negative, and positive cycles unique to this market. A custom momentum index quantifies the performance of the model.

Chapter 7 – MOMENTUM PROFILE: TSX VENTURE EXCHANGE

The momentum profile for a venture equity market, the TSX Venture Exchange (TSXV) in Canada, is created by applying the customised model mechanically to generate a set of false, neutral, negative, and positive cycles unique to this market. A custom momentum index quantifies the performance of the model.

Chapter 8 – EQUITY MARKET PROFILES

The momentum profiles of the three equity markets are compared by focussing on the positive cycles. The custom momentum indices allow a direct comparison between the different equity markets.

Chapter 9 – CONCLUSION

This concluding chapter confirms the objectives and contributions of the study, summarising the results and making suggestions for future research.

REFERENCES

Alwathainani, A.M. 2012. Consistent winners and losers. *International Review of Economics & Finance*, 21(1):210–220. DOI: 10.1016/j.iref.2011.05.009.

Bhootra, A. & Hur, J. 2013. The timing of 52-week high price and momentum. *Journal of Banking & Finance*, 37(10):3773–3782. DOI: 10.1016/j.jbankfin.2013.05.025.

Bird, R., Gao, X. & Yeung, D. 2017. Time-series and cross-sectional momentum strategies under alternative implementation strategies. *Australian Journal of Management*, 42(2):230–251. DOI: 10.1177/0312896215619965.

Blitz, D., Hanauer, M.X. & Vidojevic, M. 2020. The idiosyncratic momentum anomaly. *International Review of Economics & Finance*, 69:932–957. DOI: 10.1016/j.iref.2020.05.008.

Bloomberg. 2022. Bloomberg Terminal – Bloomberg Professional Services subscription. New York: Bloomberg Finance L.P.

Carhart, M.M. 1997. On persistence in mutual fund performance. *The Journal of Finance*, 52(1):57–82. DOI: 10.1111/j.1540-6261.1997.tb03808.x.

De Long, J.B., Shleifer, A., Summers, L.H. & Waldmann, R.J. 1990. Positive feedback investment strategies and destabilizing rational speculation. *The Journal of Finance*, 45(2):379–395. DOI: 10.2307/2328662.

George, T.J. & Hwang, C.-Y. 2004. The 52-week high and momentum investing. *The Journal of Finance*, 59(5):2145–2176. DOI: 10.1111/j.1540-6261.2004.00695.x.

Goyal, A. & Jegadeesh, N. 2018. Cross-sectional and time-series tests of return predictability: what is the difference? *The Review of Financial Studies*, 31(5):1784–1824. DOI: 10.1093/rfs/hhx131.

Gray, W.R. & Vogel, J.R. 2016. *Quantitative momentum: a practitioner's guide to building a momentum-based stock selection system*. Hoboken, New Jersey: John Wiley & Sons, Inc (Wiley Finance).

Gulen, H. & Petkova, R. 2018. Absolute strength: exploring momentum in stock returns. *SSRN Electronic Journal*, Online. DOI: 10.2139/ssrn.2638004.

Hühn, H.L. & Scholz, H. 2018. Alpha momentum and price momentum. *International Journal of Financial Studies*, 6(2):49. DOI: 10.3390/ijfs6020049.

Jegadeesh, N. & Titman, S. 1993. Returns to buying winners and selling losers: implications for stock market efficiency. *The Journal of Finance*, 48(1):65–91. DOI: 10.1111/j.1540-6261.1993.tb04702.x.

Jegadeesh, N. & Titman, S. 2001. Profitability of momentum strategies: an evaluation of alternative explanations. *The Journal of Finance*, 56(2):699–720. DOI: 10.1111/0022-1082.00342.

Jegadeesh, N. & Titman, S. 2011. Momentum. *Annual Review of Financial Economics*, 3(1):493–509. DOI: 10.1146/annurev-financial-102710-144850.

Kula, G., Raab, M. & Stahn, S. 2017. *Beyond smart beta: index investment strategies for active portfolio management*. Chichester, West Sussex: John Wiley & Sons, Inc (Wiley Finance). DOI: 10.1002/9781119395263.

La Grange, P.L. & Krige, J.D. 2015. Profitability of momentum strategies on the JSE. *Studies in Economics and Econometrics*, 39(3):49–65.

Liu, M., Liu, Q. & Ma, T. 2011. The 52-week high momentum strategy in international stock markets. *Journal of International Money and Finance*, 30(1):180–204. DOI: 10.1016/j.jimonfin.2010.08.004.

Marshall, B.R., Nguyen, N.H. & Visaltanachoti, N. 2017. Time series momentum and moving average trading rules. *Quantitative Finance*, 17(3):405–421. DOI: 10.1080/14697688.2016.1205209.

Moskowitz, T.J., Ooi, Y.H. & Pedersen, L.H. 2012. Time series momentum. *Journal of Financial Economics*, 104(2):228–250. DOI: 10.1016/j.jfineco.2011.11.003.

Murphy, Á. 2017. An investigation into momentum in the UK stock market and the behaviour of brokers and analysts. PhD thesis. Waterford Institute of Technology.

O'Keeffe, C. 2013. An investigation into the winner-loser and momentum anomalies in four medium-sized European markets. PhD thesis. Dublin City University.

Page, M.D. & Auret, C.J. 2019. Can non-momentum factor premiums explain the momentum anomaly on the JSE? An in-depth portfolio attribution analysis. *Investment Analysts Journal*, 48(1):1–17. DOI: 10.1080/10293523.2018.1483792.

Page, M.D., Britten, J.H.C. & Auret, C.J. 2016. Idiosyncratic risk and anomaly persistence on the Johannesburg Stock Exchange (JSE). *Investment Analysts Journal*, 45(1):31–46. DOI: 10.1080/10293523.2015.1125060.

Page, M.D., McClelland, D. & Auret, C.J. 2020. Idiosyncratic momentum on the JSE. *Investment Analysts Journal*, 49(3):180–198. DOI: 10.1080/10293523.2020.1783864.

Pavlova, I. & Parhizgari, A.M. 2011. In search of momentum profits: are they illusory? *Applied Financial Economics*, 21(21):1617–1639. DOI: 10.1080/09603107.2011.589804.

Satchell, S. & Grant, A. 2021. *Market momentum: theory and practice*. Chichester, West Sussex: John Wiley & Sons, Inc (Wiley Finance Series). DOI: 10.1002/9781119599364.

Zaher, F. 2019. *Index fund management: a practical guide to smart beta, factor investing, and risk premia*. Cham, Switzerland: Palgrave Macmillan. DOI: 10.1007/978-3-030-19400-0.

PRICE-BASED MOMENTUM

2.1 INTRODUCTION

For over 30 years, extensive research found evidence that there is price continuation based on past outperformance. This momentum effect is robust in different markets and across various asset classes, presenting one of the strongest contradictions of the efficient market hypothesis (Wiest 2023). According to Asness, Frazzini, Israel and Moskowitz (2014), some of the myths about momentum are that momentum cannot be captured by long-only investors; momentum is stronger among small-cap stocks than large-cap stocks; momentum is too volatile to rely on; and there is no theory behind momentum. Another myth about momentum is that it is not a stable process and possibly the result of data mining since different measures of momentum can give different results over a given period. Whatever the facts or the myths, momentum features prominently in academic research.

There are two main streams of research on momentum. The first stream concentrates on testing the profitability of traditional (cross-sectional and time-series) momentum strategies across different equity markets, explanations, and sources. The second, more recent stream concentrates on developing alternative measures or definitions of momentum, such as idiosyncratic (residual and alpha) momentum to improve the performance of traditional strategies. This stream includes studies combining different definitions of momentum, attempting to outperform standalone momentum strategies (Singh & Walia 2022).

According to Joshipura and Wats (2023:266-271), the research on momentum has evolved in several directions: empirical studies on momentum returns and the drivers of momentum returns; theories explaining momentum returns and the implications for market efficiency; behavioural (under or overreaction) and risk-based explanations for momentum; and momentum in alternative asset classes. Joshipura and Wats (2023:273) identified the following areas for future research: machine learning techniques identifying optimal formation and holding periods for different asset classes and markets; momentum in alternative asset classes such as cryptocurrencies; and the interaction of momentum with other factors.

The momentum factor also gained traction in the market for corporate bonds, while basis-momentum is a variant of momentum in the commodities market. One of the first comprehensive studies on momentum in corporate bond returns by Jostova, Nikolova, Philipov and Stahel (2013) documented significant price momentum in US corporate bonds. Boons and Prado (2019) introduced basis-momentum, which is related to the slope and curvature of the commodity futures curve or term structure.

The literature review to follow covers price-based momentum in the equities market as basis and justification for the study on profiling momentum in equity markets.

2.2 LITERATURE REVIEW

The literature review will show that research focused on the classic J-month/K-month (formation/holding period) approach to identify momentum and find the optimal J/K combination in different equity markets. Buying the best-performing stocks (top quantile) and selling the worst-performing stocks (bottom quantile) on their performance over the past 3 to 12 months at every update. A widening spread between the performance of the two groups would confirm the presence of momentum in that market. The long-only version ranks stocks on some definition of momentum, buying the top-ranked stocks (cross-sectional design) or stocks with high momentum (time-series design) and replacing individual stocks when a ranking or momentum falls below certain thresholds. The momentum in a market is classified on its J/K combination. Past studies made no attempt to describe a particular equity market in terms of the composition of the momentum cycles from that market.

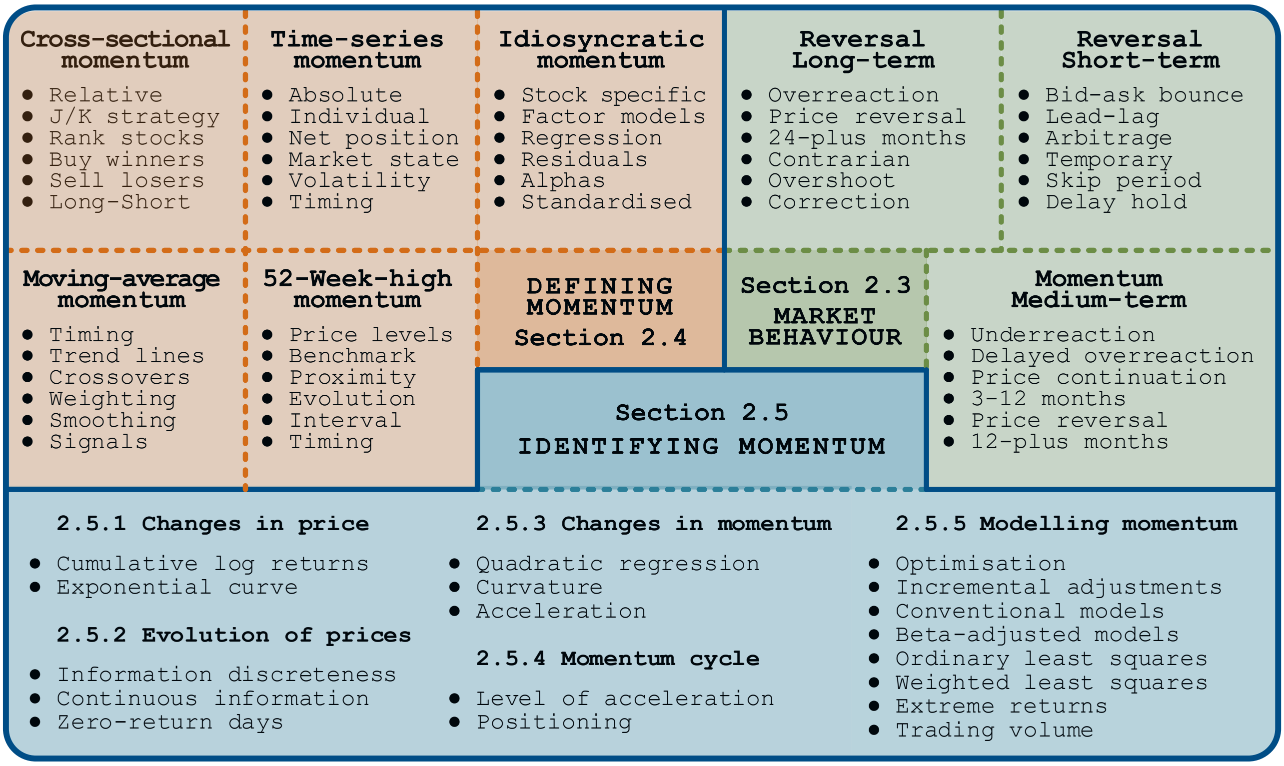

Figure 2.1 Sections

The structure of the literature review is presented in Figure 2.1 above. The behaviour of participants in the stock market may explain the continuation in performance or the momentum effect over the medium term, with reversal over the long term; and the source of momentum is either an underreaction or a delayed overreaction to new information by the market (refer to Section 2.3). Momentum strategies are predominantly cross-sectional in design, ranking stocks relative to other stocks but there are several alternative definitions of momentum (refer to Section 2.4). Various interrelated concepts are used to identify momentum (Section 2.5.1), persistence in momentum (Section 2.5.2), volatility (Section 2.5.3); to position momentum (Section 2.5.4) and model momentum (Section 2.5.5).

2.3 MARKET BEHAVIOUR

Jegadeesh (1990) presented evidence on the predictability of individual stock returns and reported highly significant negative first-order autocorrelation (i.e., interchanging positive and negative errors) in monthly returns, implying short-term reversals in performance, as well as significant higher-order positive autocorrelation (i.e., successive positive or negative errors) that points to longer-term continuations in performance. The 12-month autocorrelation was particularly strong (Jegadeesh 1990:881). Following that, Jegadeesh and Titman (1993) authored a seminal study showing that stock prices form and hold momentum during intermediate periods ranging from 3 to 12 months, and introduced their classic J-month/K-month strategy of buying past winners and selling past losers on their relative strength. This zero-cost or self-financing (winner-minus-loser) strategy confirmed the existence of cross-sectional or relative momentum in the United States stock market. Price-based momentum, in general, refers to this cross-sectional ranking of stocks on past returns, comparing the performance of the winner portfolio to that of the loser portfolio over time.

An often-referenced study by Conrad and Kaul (1998) maintained that profits from a momentum strategy originate mainly from the cross-sectional dispersion in the mean returns (assuming constant expected returns) of stocks in a portfolio, and not from any time-series predictability in individual stock returns. The assertion is that stocks with relatively higher expected returns during the formation period on average will outperform during any subsequent holding period. This claim was challenged by Jegadeesh and Titman (2001:719) who reported that portfolio returns reversed (became negative) when the holding period increased beyond 12 months, concluding that the cross-sectional differences in expected returns alone cannot explain the momentum effect. Also, momentum strategies rank stocks on their realised returns over the past 3 to 12 months, providing little evidence of their unconditional expected returns (Jegadeesh & Titman 2002:156).

Page, Britten and Auret (2016:44) reported that idiosyncratic risk (i.e., risk confined to a specific group of stocks) does not drive momentum profits and cannot explain its persistence on the Johannesburg Stock Exchange (JSE). Page and Auret (2019) added that the market risk factor as well as the size and value factors do not explain or account for the momentum premium in the South African market. Momentum is a distinct pricing anomaly that consistently generates significant risk-adjusted returns that cannot be explained within a risk-based paradigm (Page & Auret 2019:15). The magnitude and persistence of momentum returns are too strong to be explained by risk, so the focus is on behavioural explanations for this anomaly (Jegadeesh & Titman 2011:494).

The next section describes the momentum effect in terms of delayed reactions to new information. Depending on whether an initial underreaction or the eventual overreaction triggered momentum, gains may reverse in the long term.

2.3.1 Underreaction or overreaction

Barberis, Shleifer and Vishny (1998) presented their model of investor sentiment to explain the apparent medium-term underreaction and longer-term overreaction of stock prices to different types of information. An underreaction to news, such as earnings announcements or similar events, would result in the gradual assimilation of news with stock prices displaying positive autocorrelation. They equated this underreaction with investor conservatism, which is the tendency of people to hold on to prior views or forecasts longer and only gradually take on new information. An overreaction by investors to a consistent pattern or series of good or bad news over extended periods would result in stocks becoming either overvalued or undervalued with prices, on average, reverting to the mean afterwards. The representativeness heuristic supports the overreaction theory, with investors convinced that they identified patterns in historical data that represent certain outcomes. Their trading drives prices up or down and when they are disappointed or surprised by the actual outcome, the unwinding of positions results in stocks reversing earlier gains or losses (Barberis, Shleifer & Vishny 1998:316).

Self-attribution is the tendency of individuals to attribute successes to personal skills and failures to factors beyond their control, underlying and reinforcing investor overconfidence, which is another psychological bias (Hoffmann & Post 2014:23). Daniel, Hirshleifer and Subrahmanyam (1998) related the reactions of investors to these two psychological biases and developed a theory based on investor overconfidence resulting from the biased self-attribution of investment outcomes, showing that medium-term momentum can be consistent with long-term reversals. Their theory suggests that investors overreact to private information and underreact to public information. Overconfidence creates negative long-lag autocorrelations (long-term reversal), while biased self-attribution contributes to positive medium-lag autocorrelations (medium-term momentum). However, a public event that follows on pre-event or private information can trigger a continuing overreaction. Therefore, instead of associating positive return autocorrelations (momentum) with an underreaction and negative return autocorrelations (reversals) with an overreaction to news, they noted that positive return autocorrelations could also be the result of a continuing overreaction that carries into the holding period, followed by the eventual post-holding period correction or reversal.

Also attempting to reconcile both medium-term momentum and long-term reversal in stock returns, Hong and Stein (1999) proposed their gradual-information-diffusion as a unified theory of underreaction, momentum trading, and overreaction in asset markets. Assuming that information diffuses gradually and prices thereby initially underreact, positive-feedback trading (or trend-chasing) by investors will inevitably lead to an overreaction at longer horizons with price reversals to follow the unwinding of positions. Any medium-term underreaction eventually leads to a longer-term overreaction (Hong & Stein 1999:2169).

These studies on behavioural biases agreed that the momentum and reversal effects are part of the same phenomena, in that momentum creates the longer-term reversal in price. However, McLean (2010) maintained that a different underlying process generates each effect. Idiosyncratic risk plays an important role in preventing arbitrage in relatively large mispricing where the excess return most likely exceeds transaction costs. Arbitrage costs, however, are important in limiting arbitrage in smaller mispricing. Reversal represents larger mispricing than momentum and is prevalent in stocks with high idiosyncratic risk, suggesting that idiosyncratic risk limits arbitrage in price reversal but not in momentum. Momentum is not associated with higher idiosyncratic risk and generates a smaller excess return than reversal, so transaction costs are sufficient to prevent arbitrageurs from eliminating momentum mispricing (McLean 2010:903).

2.3.2 Continuation and reversal

Contrarian strategies or long-term reversals form over more than 24 months and can be associated with a sustained overreaction to new information. De Bondt and Thaler (1985) advanced the overreaction theory to explain the predictability of long-term reversals in stock prices. When stock prices systematically overshoot consequent to investors overreacting, a reversal in price is predictable from past return data only (De Bondt & Thaler 1985:795). More than thirty years later, Blackburn and Cakici (2017) published international evidence on overreaction and long-term reversals. More than enough time had passed for arbitrageurs to take full advantage of this predictability in stock returns. Still, their results confirmed that both momentum and long-term reversals coexisted in global stock markets (Blackburn & Cakici 2017:14).

Page and Way (1992) published early evidence on the long-term reversal of stock prices and the overreaction of investors in the South African market, reporting that past losers outperformed past winners based on the 24- and 36-month formation and holding periods. Muller (1999) provided additional evidence of investor overreaction on the Johannesburg Stock Exchange (JSE). Momentum strategies with short holding periods of 3 months and contrarian strategies with holding periods greater than 12 months posted excess returns. Overreacting investors take prices above their intrinsic values, and short-term gains turn into losses after 20-month holding periods when prices revert to their means (Muller 1999:16). A more recent study by Britten, Page and Auret (2016) investigated the interaction between long-term-reversal and value on the Johannesburg Stock Exchange (JSE). Confirming that the profits of historical winner portfolios decline as previous loser portfolios begin to outperform over holding periods that exceed 12 months, results showed sufficient evidence in support of investor overreaction. A weak association between undervalued stocks and loser stocks led them to conclude that value and overreaction are independent factors in the South African market.

Some stocks gain momentum due to investors overreacting to news, forcing prices away from their intrinsic values. Others gain momentum due to investors underreacting, causing prices to move gradually closer towards their intrinsic values. Momentum from an underreaction would be preferable to momentum from an overreaction, as it is not susceptible to price reversals. The notion of a delayed overreaction was put forward by De Long, Shleifer, Summers and Waldmann (1990), who suggested that early buying by rational speculators triggers positive feedback trading by other investors. An initial reaction followed by a delayed and continued overreaction due to positive feedback trading could cause stock prices to gain momentum, overshooting but ultimately reverting to their intrinsic values.

Positive-feedback traders react to price changes and chase trends, causing stock prices to diverge from their intrinsic values. Information uncertainty strengthens day-to-day positive feedback trading, inducing autocorrelation in returns and contributing to price momentum (Shi, Chiang & Liang 2012:527). According to Charteris and Rupande (2017), evidence of positive feedback trading in the South African market is a cause for concern as it creates volatility. Positive feedback trading, associated with momentum, perpetuates a trend and is destabilising. In contrast, negative feedback trading, associated with price reversal, is in the opposite direction of a trend and stabilising.

A delayed overreaction extends beyond the medium-term formation period to also include the holding period, pushing prices away from their intrinsic values before eventually reversing (Jegadeesh & Titman 2001:710). A pure underreaction, on the other hand, would result in momentum that forms over 3 to 12 months and holds for up to 12 months, all the while pushing prices to their intrinsic values. A study by Chan, Jegadeesh and Lakonishok (1996) was one of the first to relate the underreaction of investors to medium-term momentum in stock prices. Stocks selected under a momentum strategy carry along a different set of insights and expectations from stocks selected under a contrarian strategy (Chan, Jegadeesh & Lakonishok 1996:1711). A gradual adjustment to stock-specific news should result in stock prices building and maintaining momentum before levelling out to record more average returns.

Lee and Swaminathan (2000) used past trading volume to link medium-term underreaction and price continuation with long-term overreaction and price reversal. Accepting that price continuation eventually reverses, both the timing and magnitude of this reversal are predictable by past trading volume. With stock prices steadily converging toward their intrinsic values, medium-term underreaction and long-term overreaction are simply elements of the same continuous process whereby prices gradually adjust to new information. The longer the formation period, the shorter the continuation in price, and vice versa (Lee & Swaminathan 2000:2026). Drew, Veeraraghavan and Ye (2007), similarly, applied trading volume to predict the timing and magnitude of the reversal for momentum stocks listed on the Australian Stock Exchange (ASX).

The price continuation in Australia is longer than in the United States. Still, they confirmed that the speed of the reversal depends on the length of the formation period, with extended formation periods prompting quicker reversals (Drew, Veeraraghavan & Ye 2007:786).

Alwathainani (2012) considered momentum and reversal as two elements of the same continuous process in which stock prices overreact with a delay and only gradually adjust to news as the market concedes its biased expectations. Price momentum and price reversal are most likely driven by the same investor phycology and behavioural biases (Alwathainani 2012:210). However, Conrad and Yavuz (2017) saw momentum and reversal as two distinct and separate effects. Stocks that contribute to the momentum portfolio during the holding period do not experience any significant reversals in the post-holding period. Only those stocks that do not contribute much over the medium term, also experience strong reversals in the long term. Momentum and reversal patterns only appear to be linked as momentum portfolios typically comprise both these subgroups (Conrad & Yavuz 2017:578).

Lin and Rassenti (2012) suggested a novel theory termed price inertia to explain the familiar pattern of price continuation followed by a reversal, and thereby reconcile underreactions with overreactions. Prices generally underreact to news, and these underreacting continuations outnumber any overreacting reversals substantially (Lin & Rassenti 2012:39). Both the continuation and reversal phases are sluggish adjustments in price and mainly due to investors holding on to prior valuations. When information arrives sequentially over time, there is a slow convergence towards intrinsic value. A series of positive news events manifest in underreacting continuations as stock values fall behind their updated intrinsic values. Should negative news follow, stock values again react too slow to catch up with their newly updated intrinsic values, displaying as consecutive reversals supposedly due to overreactions. Both medium-term continuations (slow adjustments) and long-term reversals (slow readjustments to changing intrinsic values) can be explained by the inertia inherent in stock prices (Lin & Rassenti 2012:59).

Mun, Vasconcellos and Kish (2000) reported that investors in the Canadian market overreacted relatively quickly to new information and that this overreaction dissipated over time with one-year portfolios outperforming two-year portfolios, which in turn outperformed three-year portfolios. However, only the one-year winners, as well as the two-year winners and losers, showed significant excess returns. Abukari and Otchere (2017) reasoned that in an era of internet technology and fast-flowing information, stock prices assimilate relevant news more quickly. Therefore, reversals or corrections resulting from overreactions should generally occur sooner. A hybrid strategy ranking as contrarian (long term) but holding as momentum (medium term) outperformed conventional momentum and contrarian strategies in the Canadian market. Contrarian and hybrid returns, unlike momentum, do not reverse (Abukari & Otchere 2017:37).

Lin, Hung and Huang (2018) detected a significant contrarian effect in the Chinese stock market (represented by Shanghai and Shenzhen), but no momentum effect. The world’s second-largest stock market has a considerable price overreaction due to the dominance of retail investors. Listed companies in China do not have complete transparency as these companies do not have the regulatory requirement to disclose their actual financial statements. Reliable financial information has little effect on the trends in stock prices in the Shanghai and Shenzhen markets. Retail investors, therefore, lack credible information about listed companies and rely solely on market rumours (Lin, Hung & Huang 2018:281).

Hillert, Jacobs and Müller (2014) claimed that media coverage in the United States reinforces certain behavioural biases by instilling overconfidence in investors and affecting the way they collect, process, interpret and react to news. There is a systematic link between the extent of media coverage and the magnitude of medium-term momentum and eventual long-term reversal in stock returns (Hillert, Jacobs & Müller 2014:3497). Hou and McKnight (2004) found that momentum in the Canadian market is negatively related to analyst coverage and the book-to-market ratio. Low book-to-market stocks generally referred to as growth stocks, and especially stocks initially overlooked by analysts gained momentum via a delayed overreaction (Hou & McKnight 2004:342). In the Japanese market, Teplova and Mikova (2015) observed that the payoffs from momentum strategies depended on the method used to measure momentum, portfolio design, company fundamentals and the state of the economy as well as past stock market volatility (Teplova & Mikova 2015:104).

Novy-Marx (2012) argued that momentum is not really momentum, but rather an echo of a stock’s past performance over intermediate horizons. Momentum is generated by a stock’s performance 7 to 12 months prior to constructing a portfolio and not by prices trending up or down. The returns of the more recent months are mostly irrelevant, and the performance of a portfolio should increase if there is a delay of 6 months between the formation and holding periods. This study evaluated the returns to cross-sectional momentum strategies while varying the length of the formation period and the time between the formation period and the holding period. Winner-minus-loser portfolios formed on the predictive power of a single month, from 1 month to 15 months before holding the portfolios, revealed the full term-structure of momentum (Novy-Marx 2012:431). The return curve is upward sloping with spreads that increase with the time between forming and holding the portfolio. Return predictability abruptly falls off after 12 months (Novy-Marx 2012:451).

Goyal and Wahal (2015) tested whether this echo, rather than a continuation, in returns is also present outside of the United States. They did not find convincing evidence of an echo outside the United States, and within the United States an examination of the full term-structure of predictability suggested that the weak continuation in returns from the more recent months was caused by a carryover of short-term reversals into the month before last (Goyal & Wahal 2015:1257).

This observation was shared by Gong, Liu and Liu (2015), identifying the inclusion of month 12 as another estimation bias. A strong continuation in formation-month-12 returns (due to seasonality) and the short-term reversals carried over into formation-month-2 returns, respectively overestimates intermediate-horizon momentum and underestimates more recent momentum (Gong, Liu & Liu 2015:181).

Bandarchuk and Hilscher (2013) have the last word on this topic, stating that past returns contain all the information needed to profit from momentum. Attempting to maximise profits by sorting and ranking on specific stock-level characteristics (e.g., size, value, turnover and analyst coverage) simply results in selecting stocks with more extreme past returns. Therefore, any explanation of momentum needs only to consider the link between past returns, volatility and profits. The relationship between past returns and momentum profits determines the interaction between different stock characteristics, often used to support behavioural explanations of momentum (Bandarchuk & Hilscher 2013:838).

2.3.3 Short-term reversal

Schmitz and Cleary (2000) used a multivariate approach to determine the impact of different factors on future stock returns, ranking these factors in terms of their predictive ability. Two of the most statistically significant and stable predictors of stock returns in Canada proved to be 12-month momentum and 1-month reversals. Assoé and Sy (2003) examined the profitability of a short-term contrarian strategy in the Canadian market, buying the losers and selling the winners of the previous month. Describing the strategy as trading-intensive, they reported that the abnormal returns generated by a short-term contrarian strategy did not exceed the estimated transaction costs.

Lo and MacKinlay (1990) attributed the majority of short-term (weekly) contrarian profits to the lead-lag relation between the returns of outperforming (winner) and underperforming (loser) portfolios. Stock returns are often positively cross-autocorrelated, reconciling the negative autocorrelation in individual stocks with the positive autocorrelation in market indices (Lo & MacKinlay 1990:201). These cross effects display a lead-lag structure with larger stocks reacting more quickly to news than smaller stocks. Therefore, the returns of larger stocks lead during the formation period and the returns of smaller stocks lag or follow during the holding period, with cross-autocorrelations measuring the contribution of this size-related lead-lag effect to contrarian profits.

Lehmann (1990) reported that the outperformers and underperformers in one week experienced sizeable price reversals the next week, stating that short-term reversals probably demonstrate the temporary nature of arbitrage opportunities related to imbalances in the market for short-run liquidity. Jegadeesh and Titman (1995) argued that most of the short-term reversals might be explained by market makers setting bid and ask prices to account for inventory imbalances while providing liquidity. Short-term contrarian profits are compensation for bearing inventory risk and cannot be realised by investors transacting at the bid and ask prices (Jegadeesh & Titman 1995:130).

The bid-ask spread may introduce measurement errors as stock prices bounce around within this spread, and the significance of this bid-ask bounce is determined by the size of the spread (Rhee & Wang 1997:251). Researchers can avoid a bid-ask bounce by skipping the most recent formation period (day, week or month) or delay trading. Alternatively, returns can be calculated using bid prices only or midpoint prices (average between the lowest ask and highest bid) rather than transaction prices, which alternate between the bid and ask prices. Conrad, Gültekin and Kaul (1997) argued that contrarian profits from short-term reversals are primarily generated by this bid-ask bounce, and not by any overreaction from the market. Accounting for this bounce by using bid prices eliminated all profits from short-term reversals (Conrad, Gültekin & Kaul 1997:379).

Hühn and Scholz (2019) published a recent study on the relationship between short-term reversal and medium-term momentum, covering similar topics and generally coming to similar conclusions to past studies. They also found weekly stock returns to display short-term reversal and medium-term momentum patterns. Only medium-term momentum can be linked to behavioural biases, and a short-term reversal is neither due to any reaction to stock-specific news nor is it mainly driven by illiquidity (Hühn & Scholz 2019:273). While short-term contrarian strategies can be explained by high turnover and any profits accounted for by transaction costs, medium-term momentum strategies remain profitable even after accounting for transaction costs (Hühn & Scholz 2019:292).

Results obtained by Jiang and Zhu (2017) extended momentum to much shorter holding periods than the conventional 3 to 12 months. They observed momentum via intraday jumps (i.e., infrequent large changes in stock prices) and recorded positive returns over holding periods of up to 3 months. Overnight jumps can predict momentum over periods as short as one week with investors paying limited attention and underreacting to news over short periods (Jiang & Zhu 2017:61). However, it remains generally accepted theory that short-term price reversal, medium-term momentum, and long-term price reversal straddle different formation and holding periods (Hameed & Wu 2019; Heyman, Lescrauwaet & Stieperaere 2019; Zaremba, Kizys & Raza 2020). Long-term contrarian strategies typically anticipate reversals after stocks either outperformed or underperformed for extended periods ranging from 24 to 60 months. Momentum forms and holds over periods of between 3 and 12 months, with a full momentum cycle generally completing within 18 months. Short-term reversals involve adjoining daily, weekly or monthly formation and holding periods. Delaying the holding period or skipping the most recent formation period accommodates these observed short-term reversals in momentum strategies.

2.3.4 Summary

The behavioural biases and trading activities of investors induce autocorrelation in time-series data (Shi, Chiang & Liang 2012). As a result, stock prices are predictable to some extent and exhibit different patterns of continuation and reversal over time. The reaction of the market to new information shapes these patterns. A long-term price reversal is due to a sustained overreaction to news, resulting in stocks trading above or below their intrinsic values for extended periods (De Bondt & Thaler 1985). These stocks eventually revert to their intrinsic values over an equivalent period, allowing a long-term contrarian strategy of buying underperforming and selling outperforming stocks.

Refer to Figure 2.2: Accepting that the market responds with a delay to news, momentum results from either a delayed initial reaction (or underreaction) only, or it results from a delayed overreaction that follows on the initial underreaction (De Long, Shleifer, Summers & Waldmann 1990). If the underreaction and overreaction were elements of the same continuous process whereby prices build momentum, any underreaction would inevitably lead to a delayed overreaction that continues into the holding period (Alwathainani 2012). Stocks would lose momentum and start posting negative returns after 12 months. An underreaction confined to the formation period would gain momentum over a maximum period of 12 months and hold that momentum for up to 12 months with average returns after that (Jegadeesh & Titman 2001). Momentum driven by an underreaction would be preferred as it moves a stock towards its intrinsic value and does not reverse. A bid-ask bounce reportedly causes the observed short-term reversal (Rhee & Wang 1997).

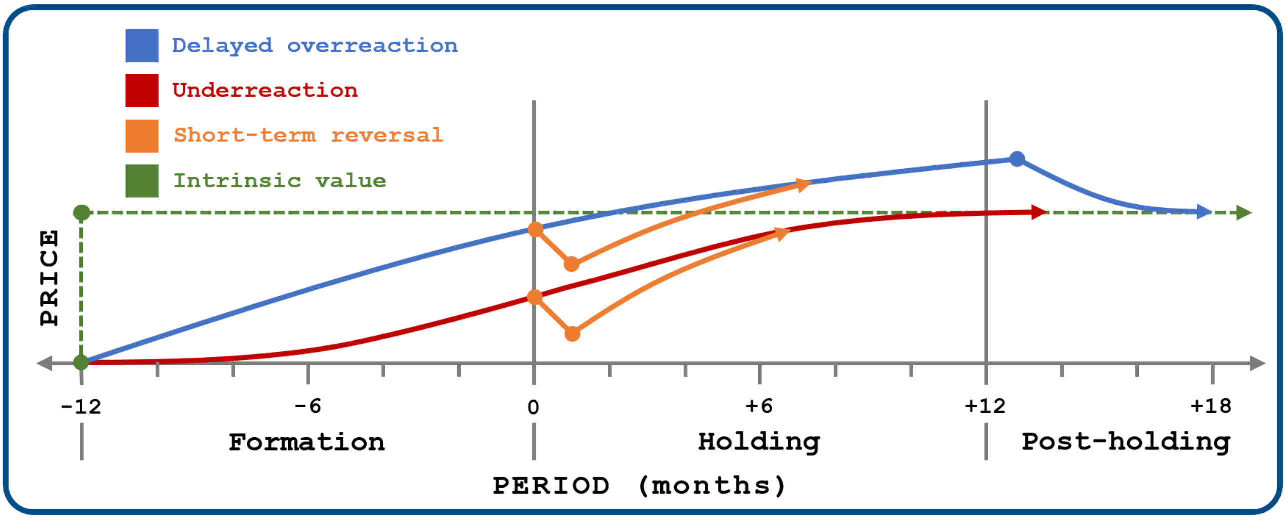

Figure 2.2 Medium-term momentum

Momentum in price returns includes a component due to common factors that triggers a delayed overreaction, while momentum in stock-specific returns originates from an underreaction (refer to Section 2.3.5). Ideally, one should isolate the momentum in stock-specific returns to profit from a true underreaction. Regardless, stock prices contain all the information needed to identify and measure momentum.

2.4 DEFINING MOMENTUM

Time-series momentum measures the performance of a stock over time, while cross-sectional momentum measures the performance of a stock relative to other stocks at a particular point in time. The most basic definition of momentum is a change in price over some fixed-length period that ranges between 3 and 12 months. Conventional momentum is based on actual returns calculated from market prices. Idiosyncratic momentum is based on stock-specific returns and excludes the returns required by a specific factor model. Fifty-two-week-high momentum compares the current price to its highest level over the past 12 months or its proximity (time since a previous high) to that level. Returns can be volatility-adjusted to add yet another definition of momentum to the mix. Regardless of the definition, most strategies are cross-sectional and long-short in design, meaning that stocks are sorted on performance, buying x-number of the top-ranked stocks (long portfolio), and selling x-number of the bottom-ranked stocks (short portfolio). The specific definition of momentum would determine a stock’s ranking and thereby the composition of the portfolios. Stocks are re-sorted at regular intervals.

2.4.1 Cross-sectional momentum

Cross-sectional momentum measures the relative performance of stocks over various formation periods. This relative performance extends to the difference or spread between the returns of the winner and loser portfolios during the holding period. The basic relative strength strategy put forward by Jegadeesh and Titman (1993), ranks and assigns stocks to quantiles (e.g., ten deciles or five quintiles) based on past performance during specific formation periods. The spread between the highest positive-momentum quantile (winners) returns and the highest negative-momentum quantile (losers) returns measures cross-sectional momentum. Different combinations of quarterly (3, 6, 9 or 12 months) formation and holding periods identify sets of optimal parameters. Equal-weighted portfolios are reconstructed each month, resulting in overlapping holding periods with a pre-set fraction (1/3, 1/6, 1/9 or 1/12) of the entire portfolio closed out in any given month, and the balance (2/3, 5/6, 8/9 or 11/12) carried over from the previous month. A gap or skip period based on the data frequency (daily, weekly, or monthly), between the formation and holding periods, can be included to account for any short-term reversal in price. In general, the notation J/S/K denotes a momentum strategy with a formation period of J months, a holding period of K months and a delay (if any) of S months between the formation and holding periods (Jegadeesh & Titman 1993:68). This zero-cost or self-financing (winner-minus-loser) strategy serves to detect or confirm the momentum effect in a particular market and facilitate analysis. All strategies include portfolios with overlapping holding periods to increase the power of statistical testing. Each market-neutral portfolio holds equal amounts in long and short positions and involves the simulated trading of a large number of stocks with transaction costs either estimated or ignored.

Refer to Figure 2.3: A cross-sectional strategy ranks and evaluates stocks on momentum using different combinations of formation (J) and holding (K) periods. The spread between the returns of the highest positive-momentum quantile and the highest negative-momentum quantile measures cross-sectional or relative momentum.

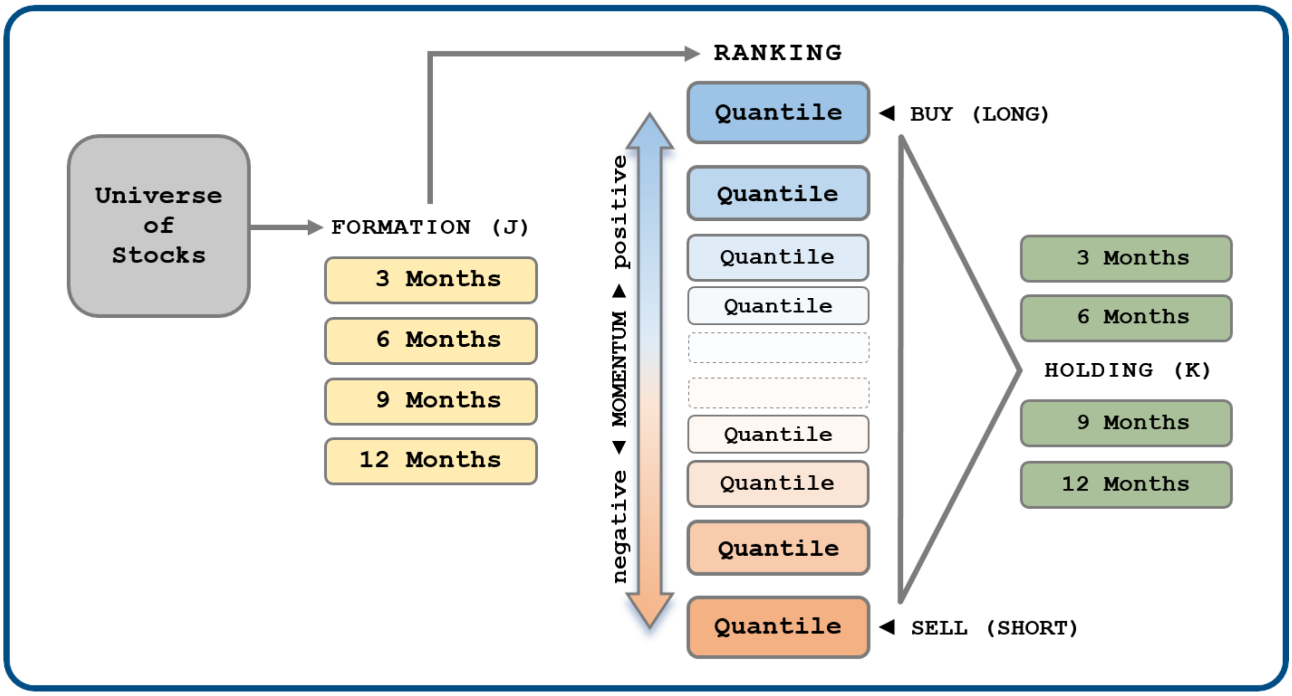

Figure 2.3 Cross-sectional momentum

Page, Britten and Auret (2013) took the lead in South Africa and tested multiple cross-sectional momentum strategies or J/K combinations to determine whether investors can exploit relative momentum on the Johannesburg Stock Exchange (JSE). Results favoured medium formation and holding periods of between 6 and 9 months. High and medium turnover strategies outperformed low turnover portfolios achieving significant average excess returns (Page, Britten & Auret 2013:71). Page (2016) investigated whether momentum is significant, independent of other non-momentum factors or investment styles and priced – that is, contributing to the variation in stock returns. Results delivered significant positive excess returns across the formation and holding periods of between 3 and 12 months. Originating from this PhD thesis, Page and Auret (2017) published the results from different formation and holding period combinations, allowing for a possible short-term reversal (bid-ask bounce), using both equal-weighted and value-weighted weighting schemes, accounting for transaction costs and liquidity. A classic 6J/1S/9K strategy, favouring equal-weighted portfolios and skipping a month between formation and holding periods, showed the best results. Transaction costs affected momentum profits significantly, and higher liquidity (turnover) resulted in earlier long-term reversals (Page & Auret 2017:163). Page and Auret (2019) published additional research on the different portfolio weighting schemes and the composition of the momentum premium. Excess returns obtained from the momentum-rank weighting scheme (weights based on momentum ranking) exceeded those from value- and equal-weighted portfolios (Page & Auret 2019:15).

Van Heerden (2014) examined the impact of stock-specific factors on the cross-sectional variation in stock returns on the Johannesburg Stock Exchange (JSE). A momentum effect based on 1-month holding periods and formed on lagged returns of between 6 and 12 months was confirmed. This momentum effect became sensitive to time (sample period) and liquidity (market capitalisation) when the holding period increased. A price reversal effect appeared to exist over both short (prior month) and long (prior 60 months or five years) formation periods, but also proved to be highly sensitive to time, liquidity and holding period. Originating from this PhD thesis, Van Heerden and Van Rensburg (2015) published the cross-sectional regression results that included the observed momentum and reversal effects. In two subsequent papers, Van Heerden and Van Rensburg (2016) reported on the sensitivity of the momentum effect to time and sample-period by filtering stocks on market capitalisation and liquidity; while Van Heerden and Van Rensburg (2017) determined that the portfolios of winner and loser stocks formed via an extreme performer approach, respectively out and underperformed a benchmark portfolio. Extreme winners and losers are stocks that either gained 100+ per cent or lost 50+ per cent in value over the past 12 months and, by definition, exhibit high levels of volatility. The volatility of extreme-loser stocks is higher compared to that of extreme-winner stocks (Van Heerden & Van Rensburg 2017:46).

Dittberner (2016) focused on fundamental momentum in earnings – that is, the rate of change in a fundamental variable, represented by company earnings or an underlying component of earnings. This PhD study examined whether fundamental momentum subsumes price momentum to thereby provide a viable alternative trading strategy. This study found price momentum to be profitable on the Johannesburg Stock Exchange (JSE) with a 6K/3J strategy showing the best results. All tests indicated that neither a value factor nor a size factor contributed to the price momentum results. Even though statistical evidence was not enough to conclude that fundamental momentum and price momentum do not capture the same effect, it did indicate that they are not subsumed by one other (Dittberner 2016:280).

Assogbavi and Leonard (2008) looked at some of the largest stocks listed on the Toronto Stock Exchange (TSX) to assess the optimal J/K combinations in the Canadian market when incorporating seasonality. Each formation period started in a particular quarter (January, April, July, or October) and the optimal formation period shortened as the quarters progressed from the fourth (12 months) to the first (9 months), second (6 months) and finally the third quarter (3 months), each ending in September. Momentum profits reportedly originated from the 9-month holding period, assuming the formation period ends in September, regardless of when it started. Using the same set of data, Assogbavi, Giguere and Sedzro (2011) tested these different J/K combinations using both price and trading volume. The optimal formation period for a high-volume winner portfolio was nine months, starting in April and in combination with a 3-month holding period.

This suggested that investors might benefit from holding momentum stocks for shorter periods when incorporating past trading volume. As with the previous study, the optimal formation varied greatly with the seasonal start of the period. The most noteworthy finding was that high-volume portfolios consistently outperformed low-volume portfolios (Assogbavi, Giguere & Sedzro 2011:11).

Chai, Limkriangkrai and Ji (2017) reported that there is momentum in weekly returns, using data from the Australian Securities Exchange (ASX). Reversals immediately followed on extreme one-week gains by stocks. Still, the average returns on these stocks over longer holding periods of up to 52 weeks (12 months) trended in the same direction as during the formation week. A similar return pattern with much stronger momentum formed over 26-week (6 months) periods, verifying that medium-term momentum dominates short-term momentum. Momentum formed on 13-week returns also generated more substantial profits than 4-week formation periods, implying that different ranking periods contain different information on future returns. Profits declined as the holding period increased. Ejaz and Polak (2018) compared short (weekly) to medium (monthly) momentum in the Australian market. Even though the monthly strategies significantly outperformed the weekly strategies, a similar pattern emerged with the shorter formation and longer holding periods proving to be more optimal (Ejaz & Polak 2018:230-231).

An issue with cross-sectional or relative momentum, particularly for retail investors (as opposed to institutional investors), is the requirement to sell the underperforming stocks (losers) in addition to buying the outperforming stocks (winners) to benefit from the widening spread.

Retail investors, unlike institutional investors, are not in the position to buy and short hundreds of stocks as suggested by most studies on momentum, according to Siganos (2010). Schneider and Gaunt (2012), in turn, concluded that the momentum effect is due substantially to the underperforming or short side. Therefore, the barriers (cost and opportunity) to shorting underperformers, combined with the liquidity demanded by momentum investing, cast some doubt on the practicality of a cross-sectional momentum strategy (Brailsford & O’Brien 2008:482).

In addition, a cross-sectional strategy can experience infrequent but persistent periods of large negative returns. These momentum crashes are somewhat predictable as they occur in panic states, following market declines and during periods of high market volatility (Daniel & Moskowitz 2016:221). A momentum crash is solely due to rebounding loser stocks. At the bottom of a prolonged market downturn, a loser portfolio would mainly be composed of highly volatile and leveraged stocks, poised to rebound sharply having lost most of their value during the downturn (Bohl, Czaja & Kaufmann 2016:139). Barroso and Santa-Clara (2015) agreed that momentum crashes are predictable and suggested volatility scaling to virtually eliminate these crashes associated with the short leg of the strategy.

However, long-only strategies would avoid momentum crashes altogether and suit retail investors not in the position to short stocks. Ross, Moskowitz, Israel and Serban (2017) concluded that long-only momentum investing is implementable in practice as a stand-alone strategy. Results published by Page and Auret (2019) actually identified the winner stocks or long portfolio as the primary contributor to the momentum premium on the Johannesburg Stock Exchange (JSE). Investors can gain adequate exposure to the momentum effect solely through a long position in winner stocks (Page & Auret 2019:15). An earlier study by La Grange and Krige (2015) confirmed that long-only momentum strategies are profitable in the South African market. They constructed equal-weighted portfolios of ten stocks per holding period without any rebalancing to reduce transaction costs. A delay of 1 month between the formation and holding periods accounted for the observed short-term reversals. The best-performing momentum portfolio had a 4-month formation period, followed by a 1-month delay and a 1-month holding period after that – a 4J/1S/1K strategy. With transaction costs taken into account, the best-performing portfolio had a 5-month/3-month formation/holding period combination and no skip period or delay – a 5J/0S/3K strategy (La Grange & Krige 2015:63).

2.4.2 Time-series momentum

Momentum strategies are predominantly cross-sectional in design as performance is measured at a particular point in time and relative to other stocks (via ranking). In contrast, time-series momentum assigns stocks to long or short portfolios on their absolute or individual performance over time. Moskowitz, Ooi and Pedersen (2012) introduced time-series momentum as an alternative to cross-sectional momentum. Time-series momentum focuses solely on the past returns of individual stocks, buying stocks that generated positive returns and shorting those with negative returns over a particular look-back or formation period. Distinct from cross-sectional momentum but related in that both result from autocorrelation (or auto-covariance) in the returns of individual stocks.

Returns from time-series momentum strategies exceed those of cross-sectional strategies because of the active position taken, being net long or short depending on the condition or state (up or down) of the market (Cheema, Nartea & Szulczyk 2018:2600). The time-series strategy outperforms the cross-sectional strategy in market continuations but underperforms in market transitions due to its opposite active position (Cheema, Nartea & Man 2018:713). The outperformance of time-series momentum results from holdings being more in tune with market conditions and can be attributed to the timing element, absent in cross-sectional momentum, embedded in the stock selection process (Bird, Gao & Yeung 2017:231).

The excess returns of cross-sectional and time-series strategies are largely equal after adjusting for these active positions. In essence, the only substantive difference is that the time-series approach avoids the reported short-term reversals (Goyal & Jegadeesh 2018:1822).

With time-series momentum, all stocks that realised a positive past return are identified as winners and those with a negative return as losers. Alternatively, the cut-off for splitting winners and losers can be a given level of return, say three per cent, or the market return over the formation period. However, these rules would classify every stock in a particular universe of stocks as either a winner or a loser. Therefore, taking a position in every one of these stocks, which may number in the hundreds, makes this strategy impractical to implement. Addressing this issue, Lim, Wang and Yao (2018) proposed two alternatives – namely: revised time-series momentum and dual momentum, both reducing the number of positions. Revised (or standardised) time-series momentum only takes positions in stocks whose prior-year returns are greater than one standard deviation, and dual-momentum combines time-series and cross-sectional factors. The dual momentum strategy double-sorts stocks by allocating each to a time-series group with either positive or negative returns and ranking the high-momentum stocks within each group into quantiles – buying the top quantile in the positive-return group (winners) and shorting the bottom quantile in the negative-return group (losers). Compared to ordinary time-series momentum, the revised strategy both requires fewer positions and generates higher returns; higher dual-momentum returns come with higher volatility and are driven almost entirely by the winner portfolio, with the loser portfolio generating a near-zero return (Lim, Wang & Yao 2018:291).

Similarly, rather than using past realised returns, Dudler, Gmür and Malamud (2015) constructed portfolios based on past risk-adjusted returns – that is, realised returns standardised by some measure of realised volatility. Changes in volatility drive the variation in ordinary time-series momentum and lead to excessive trading. Standardising a stock’s past return by its realised volatility removes that portion of its variation in return driven exclusively by changes in volatility and not by any changes in intrinsic value. Daily updating also incorporates the most recent information into trading positions. Still, for ordinary time-series momentum strategies, the gains from frequent rebalancing are more than offset by the transaction costs. However, this is not the case for risk-adjusted time-series momentum as standardising momentum reduces turnover by 30% to 50%, depending on the momentum period. Such a substantial reduction in turnover has important implications for both the efficiency of a strategy and the cost to implement it, allowing for more frequent updating. Volatility or risk-adjusted time-series momentum outperforms ordinary time-series momentum for most combinations of formation and holding periods (Dudler, Gmür & Malamud 2015:100).

Georgopoulou and Wang (2017) compared time-series momentum in developed markets to that in emerging markets and found a stronger momentum effect in the emerging markets that lasted for shorter periods. They questioned whether a time-series momentum strategy is suitable for retail investors, as it requires frequent trading that generates transaction costs.

This study also explained the logic behind trading momentum on a time-series basis. During a trending market, a time-series momentum strategy takes long and short positions in stocks depending on whether their returns were positive or negative over the formation period. A time-series momentum portfolio will be net long if the market increases over this period, and vice versa. Should the market transition or change direction, a time-series momentum portfolio will initially suffer significant losses due to it being net long or short at that stage. After the market transitioned, a time-series momentum strategy will profit from the reversal and post substantial gains as it would have adjusted its net position to the existing direction of the market. Time-series momentum strategies are most profitable when the reversals sustain for long periods. A time-series strategy is useful for managing risk, as the momentum portfolio will be net short during an extended market downturn and record substantial gains in that period. The payoff from a time-series momentum strategy is similar to that of an options straddle strategy with its V-shaped profile, as it records its largest gains during extreme market upturns and downturns (Georgopoulou & Wang 2017:1590).

Momentum investing, in general, is predicated on large changes in prices but occasional large spikes or drops in return can push a stock into one of the high momentum (positive or negative) portfolios because of a few extreme returns. When extreme stocks are included in a momentum portfolio, the return on this portfolio tends to be highly volatile (Gupta, Locke & Scrimgeour 2013:226). Excluding stocks with extreme absolute strength from either cross-sectional or time-series momentum strategies can improve performance (Yang & Zhang 2019:71). Stocks with extreme absolute strength exhibits high volatility, are less likely to maintain momentum and more likely to experience reversals. The same does not hold for removing stocks with extreme relative strength. Most of the improvement in performance resulted from avoiding losses in periods when a conventional strategy would have crashed. Removing stocks with extreme absolute strength can increase the average return of a momentum strategy while reducing its volatility (Yang & Zhang 2019:77).

Brush (2001) also advised that stocks with extreme absolute strength should be excluded, at least in the short term. Stocks that experienced extreme changes in price over a short period should not be treated the same as other stocks ranked in the top quantile. The initial ranking needs adjusting to exclude these stocks from the top quantile until the effect of the extreme price change passed.

It should be noted that Van Heerden and Van Rensburg (2017) reported that momentum portfolios comprising extreme performers, respectively out and underperformed benchmark portfolios. Extreme winners and losers were defined as stocks that either gained 100+ per cent or lost 50+ per cent in value over the past 12 months (relatively long term), thereby exhibiting high volatility. The volatility of extreme-loser stocks generally exceeds that of extreme-winner stocks.

2.4.3 Moving-average momentum

Moving averages introduces even more timing to stock trading than does time-series momentum. Moving-average trading rules generate buy or sell signals when a stock's price moves above or below its average historical price over some predefined period. A time-series momentum strategy trades when the return on a stock over some past period changes from positive to negative and vice versa. Both are different from a cross-sectional momentum strategy that trades stocks on their relative performance or ranking over some period at a particular point in time.

Marshall, Nguyen and Visaltanachoti (2017) found that time-series momentum and moving-average rules are closely related, showing differences in the timing of trading signals and risk-adjusted returns. Moving-average rules generate signals earlier and enter or exit positions sooner, potentially capturing larger returns. Moving average rules only require prices to change and cross some moving average to generate entry or exit signals, which is more likely than the moving average changing direction. Since trading on time-series momentum occurs when a moving average changes direction, these rules tend to be slower in generating buy or sell signals. Compared to cross-sectional momentum, both moving-average rules and time-series momentum are less susceptible to suffering large losses, normally exiting positions before the stock market drops or rebounds significantly. As moving-average rules are even better than time-series momentum at avoiding severe losses, there is no indication that the larger returns from moving-average rules simply compensate for higher risk (Marshall, Nguyen & Visaltanachoti 2017:417). A trading strategy of holding positions for fixed periods by ignoring any signals that occur before the end of these periods aligns moving-average and time-series momentum rules with a conventional momentum strategy. A matching signal on an existing position at the end of the initial holding period maintains that position.

Another application of moving averages, moving-average crossovers, can be related to time-series momentum, as explained by Levine and Pedersen (2016) in summary:

A time-series momentum strategy buys stocks that recorded positive returns, and short those stocks with negative past returns. The simplest time-series momentum signal is a stock's return over 12 months, measured either as the ratio of two prices or as the difference between two (log) prices. A more refined signal calculates returns over shorter periods within the total period, using monthly or even daily returns and allocating different weights to each. Smoothing the prices used to calculate the returns is an alternative to weighting the returns. Smoothing reduces random noise in data but also delays the signal. Back-end smoothing uses an average of multiple past prices instead of any single past price. With front-end smoothing, recent price changes are smoothed out, only gradually affecting and, thereby, delaying the trading signal (Levine & Pedersen 2016:52).

The moving average crossover strategy calculates two moving averages of past prices: a faster or short-term moving average, and a slower or longer-term moving average. The trading signal is the moving average crossover or difference between these two moving averages. Calculated as a weighted average of past prices, the faster moving-average puts more weight on recent prices, whereas the slower moving-average puts more weight on older prices. Therefore, the moving average crossover measures whether recent prices, captured by the faster moving-average, are above or below older prices, captured by the slower moving-average. A positive moving average crossover signals that recent prices are higher than older prices, confirming a rising trend (Levine & Pedersen 2016:54).

As stated, a simple time-series momentum signal can be calculated as the difference between the current (log) price and the lagged price (12 months ago), showing that a time-series momentum strategy can be viewed as a moving average crossover. The fastest moving-average is simply the current price if the weighting scheme puts all the weight on the most recent price and the slowest moving-average is the lagged price if all the weight is on the oldest price. Another approach would use front-end smoothing where the fast moving-average becomes an average of recent prices, as well as back-end smoothing where the slow moving-average becomes an average of lagged prices. Therefore, a moving-average crossover can be viewed as a time-series strategy based on averages (Levine & Pedersen 2016:56).

The most general forms of time-series momentum and moving-average crossovers are equivalent, and also capture all other linear filters – for example: the Hodrick-Prescott (HP) filter, the Kalman filter, and an ordinary least squares (OLS) trend regression. The HP filter identifies trends by removing cyclical fluctuations from time-series data and is based on the premise that stock prices have both a growth and a cyclical component. The growth component is a moving average of past prices, and the trend is the change in this growth or the difference between two moving averages. The Kalman filter detects the hidden variables of dynamic linear systems with noisy observations. In the context of trends, the Kalman filter determines the underlying or hidden trend variable driving stock returns. An ordinary least squares (OLS) regression fits a straight line through a price series to determine the trend over a particular period. This process is equivalent to a generalised time-series momentum signal, presented as a linear combination of weighted prices or a weighted combination of past price changes (Levine & Pedersen 2016:57-59). Results suggested that the filtering methodology is secondary to factors that may be useful in determining the quality of a trend. Differences in the performance of these signals materialise from the specific parameters and settings, possible nonlinear transformations, and practical issues related to transaction costs, active trading, size of positions and portfolio construction as well as risk management (Levine & Pedersen 2016:64).

Han, Zhou and Zhu (2016) devised a trend factor based on moving averages of various lengths to capture all three price effects – namely: short-term reversal, medium-term momentum and long-term reversal. This trend factor aggregates daily stock price data across multiple investment periods or lags that range from three days to a thousand trading days (four years). Moving averages are calculated and divided (standardised or normalised) by the closing price on the last trading day. A two-step process predicts the expected stock returns: Firstly, in each month, a cross-sectional regression of stock returns on these normalised moving-average signals obtains the time-series of the coefficients on the signals. These signals indicate the daily, weekly, monthly, quarterly, one-year, two-year, three-year and four-year price trends of the underlying stock. The second step predicts the expected return on a stock for the following month. This prediction is based on the coefficient of each trend signal with a particular lag, which is the average of the estimated loadings on the trend signals over the past 12 months. Then, as in most studies on relative strength, this strategy buys those stocks with the highest expected returns in the top quantile, and short those in the bottom quantile with the lowest. The return on this strategy is the spread between the returns on these two extreme quantiles. This trend factor approach is highly correlated with a conventional momentum strategy; however, substantially outperforming individual strategies based on the separate price reversals and continuation effects (Han, Zhou & Zhu 2016:353).

A filter based on a trend-following rule results in a momentum portfolio with less volatility and a reduced maximum drawdown, according to Clare, Seaton, Smith and Thomas (2016). Cross-sectional momentum and trend following differ in that the former is a relative and the latter an absolute concept like time-series momentum. It is possible to have a momentum portfolio of relative, down-trending winners. A relative-momentum portfolio may experience large drawdowns, and one way to overcome this is to combine or overlay it with a trend-following strategy based on the long-term moving average of each stock (Clare, Seaton, Smith & Thomas 2016:79). Combining relative momentum with trend following identifies a stock as an up-trending winner. By not holding any down-trending stocks and assuming that no loser stocks were sold short, it ensures minimal exposure to possible momentum crashes. The trend-following rule (price exceeding its x-month moving average) is applied to each stock in the momentum portfolio. Stocks must be winners in both relative and absolute terms to be included in a momentum portfolio.

Regular and trailing stop losses are effective exit strategies to achieve higher profits and lower volatility (Foltice & Langer 2015:102). However, exiting via stop-losses to protect gains or limit losses is not the only option. Gray and Vogel (2016) recommended simple trend-following (moving average) rules to manage the risk of momentum portfolios. These rules can be applied to long-only strategies and avoid the complexity and commitment required from assessing momentum portfolios on a daily basis (Gray & Vogel 2016:172).

2.4.4 Idiosyncratic momentum

Idiosyncratic momentum originates from returns that are specific to each individual stock and not explained by any of the common factors (e.g., market risk, size, or value) included in a particular factor model. Focussing on stock-specific returns seeks to avoid the concentrated exposure to common factors that may characterise conventional momentum portfolios. These returns are commonly represented by the residuals (error terms) from factor-model regressions, or in some studies by the alphas (intercept terms). Idiosyncratic momentum encompasses both residual momentum and alpha momentum. Blitz, Hanauer and Vidojevic (2018) presented evidence that idiosyncratic momentum is distinct from conventional momentum. Momentum portfolios formed on stock-specific returns generate comparable average returns at half the volatility of the conventional strategy.

Idiosyncratic momentum identifies high-momentum stocks whose future returns would likely not reverse. Idiosyncratic momentum and conventional momentum are not linked to the same underlying factors and tend to complement rather than substitute for one another. Refer to Equations 2.1 to 2.4: The alphas and residuals from capital asset pricing model (CAPM) and Fama-French 3-Factor (FF3F) model regressions represent the stock-specific returns used to calculate idiosyncratic momentum.

CAPM: $$r_{i,t} = r_{rf,t} + b(R_{M,t} - r_{rf,t}) \qquad (2.1)$$

FF3F: $$r_{i,t} = r_{rf,t} + b_1(R_{M,t} - r_{rf,t}) + b_2 SMB_t + b_3 HML_t \qquad (2.2)$$

Where: $r_{i,t}$ is the rate of return on stock i at time t
$r_{rf,t}$ is the risk-free rate of return at time t
$R_{M,t}$ is the market rate of return at time t

$$R_{i,t} = a_{CAPM,i} + b_{MKT,i} MKT_t + e_{CAPM,i,t} \qquad (2.3)$$

$$R_{i,t} = a_{FF3F,i} + b_{MKT,i} MKT_t + b_{SMB,i} SMB_t + b_{HML,i} HML_t + e_{FF3F,i,t} \qquad (2.4)$$

Where: $R_{i,t}$ is the equity risk premium $(r_{i,t} - r_{rf,t})$
MKT_t is the market risk premium $(R_{M,t} - r_{rf,t})$
SMB_t is the size premium **(Small Minus Big)**
HML_t is the value premium **(High Minus Low)**
$a_{CAPM,i}$; $a_{FF3F,i}$ are the alphas or intercept terms
$e_{CAPM,i,t}$; $e_{FF3F,i,t}$ are the residuals or error terms
$b_{MKT,i}$; $b_{SMB,i}$; $b_{HML,i}$ are the betas or factor coefficients

The original study on idiosyncratic momentum by Gutierrez and Pirinsky (2007) looked at the momentum in residual returns from a capital asset pricing model (CAPM) regression. The alphas (intercepts) from the estimation periods were excluded from the calculation of abnormal returns, only serving as a general control for model misspecification (Gutierrez & Pirinsky 2007:52). This study cumulated the monthly residuals of each stock and the variances of these residuals over the formation period. The residual returns were standardised to measure the extent to which information is news, as opposed to noise.

Abnormal returns are those residual returns more than one standard deviation from zero. The variation in residual returns determined what is abnormal. A one, one-and-a-half or two standard-deviation threshold regulated the number of stocks in the winner and loser portfolios in lieu of a relative ranking. Residual momentum and conventional momentum perform similarly over the first 12 months, but differently beyond that period. Conventional momentum returns reverse strongly in months 13 to 60 after portfolio formation. This longer-term reversal is consistent with a delayed overreaction to information regarding common factors. Residual momentum returns do not reverse, consistent with an underreaction to stock-specific news (Gutierrez & Pirinsky 2007:58).

Blitz, Huij and Martens (2011) extended the research by Gutierrez and Pirinsky (2007) by comparing risk-adjusted returns, using the FF3F model, and by ranking stocks on their standardised residuals representing abnormal returns. This study confirmed that the exposure of conventional momentum strategies to market, size and value factors could be reduced by ranking stocks on abnormal returns instead of actual returns, thereby isolating the stock-specific component of momentum. Residual momentum displayed consistent results across different market states, and its risk-adjusted returns exceeded those of conventional momentum. Apart from separating common-factor momentum from stock-specific momentum, this study showed that the risk-adjusted performance of residual momentum is also superior during the first 12 months after portfolio construction (Blitz, Huij & Martens 2011:507).

Chaves (2016) claimed that all of the improvements in performance could be obtained by merely reducing exposure to the market risk factor. Momentum strategies tend to experience short periods of poor performance when markets rebound after significant downturns, referred to as momentum crashes. These crashes are the result of shorting underperformers during market downturns, resulting in sizeable losses during the rebounds to follow. Ranking underperformers on idiosyncratic momentum should avoid those stocks that underperformed due mainly to high market betas. As the market rebounds, the shorted portfolio would experience less severe crashes and, consequently, lower volatility and drawdown (Chaves 2016:65).

The previous studies ranked stocks on their residuals only and excluded alphas, with parameters estimated over periods of up to 60 months using monthly data that largely predate the maximum formation period of 12 months. A novel strategy that ranks stocks on their FF3F-model alphas, estimated on returns in the formation period only, was introduced by Hühn and Scholz (2018), relying on shorter time frames using daily price data with monthly rebalancing. By regressing daily returns during the formation period, both the sum and mean of the residuals are zero. Therefore, alpha represents the return not explained by the FF3F model. This particular version of alpha momentum does not exhibit significant reversals and aligns with an underreaction to stock-specific news (Hühn & Scholz 2018:2).

Zaremba, Umutlu and Maydybura (2018) studied, what they referred to as, volatility-adjusted residual momentum in several individual country stock indices from both developed and emerging markets, as well as sector indices from some countries. Standardised conventional momentum is calculated as a stock's mean monthly return over the previous 12 months divided by its 12-month realised volatility – its mean return per unit of volatility. Using either the capital asset pricing model (CAPM) or Fama-French 3-Factor (FF3F) model over a rolling 60-month window to calculate monthly residuals for each stock, unadjusted residual momentum is based on the mean residual returns over a trailing 12-month period. Volatility-adjusted residual momentum, described as a pure momentum strategy in which the remaining unsystematic risk is isolated by standardising each residual return by its idiosyncratic volatility, outperforms and subsumes conventional (adjusted and unadjusted) momentum.

Zaremba, Umutlu and Karathanasopoulos (2019) used the same set of country and sector indices to also look at the relationship between past alphas from CAPM regressions and future returns, relying on monthly data and rebalancing. This study employed two specific alpha-based variables – namely: alpha momentum measured as the volatility-adjusted alpha estimated over 12 months; and alpha reversal as the volatility-adjusted alpha estimated over 60 months, skipping the 12 most recent months. Alphas were adjusted or standardised by the volatility of returns. This volatility adjustment resulted in a slight improvement, but the unadjusted alphas frequently generated marginally larger raw and risk-adjusted returns on long-short portfolios. Alpha momentum entirely subsumes its conventional return-based counterpart and is robust to alternative factor models, trading costs and different weighting schemes. The return-based reversal effect becomes insignificant when controlling for the alpha reversal.

In general, conventional momentum strategies do not deliver any significant profits in Japan, according to Hanauer (2014) citing several past studies. Momentum returns are higher when the market trends or continues in the same direction and does not transition to a different state. Market transitions occur more frequently in Japan compared to the United States, explaining why average momentum returns have historically been low in Japan. Different market dynamics cause the overall low momentum returns in Japan (Hanauer 2014:157). However, Chang, Ko, Nakano and Rhee (2018) reported on the profitability of residual momentum in Japan. This study used the Fama-French 3-Factor (FF3F) model to run monthly regressions over rolling 36-month windows. The average residual returns over the past 12 and 6 months were standardised by their respective volatilities over the same formation periods. Residual momentum proved profitable in Japan for holding periods ranging between 3 and 12 months. Unlike conventional momentum, returns do not reverse in the subsequent two to five years, consistent with investors underreacting to stock-specific news (Chang, Ko, Nakano & Rhee 2018:298).

Residual momentum also exists in four Asian stock markets (Hong Kong, Singapore, Taiwan and Thailand) where there is little evidence of conventional momentum, according to Chiao, Hsiao, Chen and An (2018). This study calculated monthly residual returns on the betas (alphas excluded) estimated from 60-month rolling-window capital asset pricing model (CAPM) regressions and standardised these residual returns by their volatilities over the formation period. Residual momentum strategies significantly outperformed the conventional strategies based on actual returns, generating higher and more consistent profits.

Viljoen (2016) confirmed residual momentum to be a viable investing style on the Johannesburg Stock Exchange (JSE), with risk-adjusted returns exceeding those of conventional momentum. Like most studies, betas were estimated over 60-month rolling windows and required a minimum of 24 months' historical price data. This study employed a Fama-French 3-Factor (FF3F) type asset pricing model that differentiated between resource and non-resource stocks, in addition to size (small, medium and large-cap), and value or growth stocks. While the residual momentum strategy underperformed conventional momentum, it exhibited much lower volatility and significantly less drawdown (Viljoen 2016:80).

A study by Page, McClelland and Auret (2020) provided evidence from the Johannesburg Stock Exchange (JSE) that idiosyncratic momentum subsumes or incorporates price momentum and better explains the cross-sectional variation in stock returns. However, in the South African market, gains from idiosyncratic momentum are as likely to reverse as those from price momentum, suggesting an overreaction to stock-specific news. Each sort was limited to the top 100 stocks based on market capitalisation, and momentum was based on a 12J/1S/1K configuration.

Kim (2022) applied the concept of residual returns, successful with cross-sectional momentum, to time-series momentum. This study used the Fama-French 3-Factor (FF3F) model to generate residual returns for individual stocks. A time-series residual momentum strategy buys all stocks with positive residual returns. The residual return represents the relative strength of a stock's return after removing its factor exposures (Kim 2022:586). It is considered relative because its mean is zero over the estimation period. Given two stocks with positive absolute returns in the formation period, the stock with a positive residual or relative return would trend stronger than the one with a negative relative return. Substituting the conventional time-series strategy with its residual momentum version delivered larger gains than switching from conventional cross-sectional momentum to cross-sectional residual momentum (Kim 2022:592).

The next section deals with an approach that does not rely on extreme returns but on relative price levels during the past 12 months. The 52-week high serves as a reference point, and 52-week-high (52WH) momentum compares a stock's current price and proximity to that reference point.

2.4.5 Fifty-two-week-high momentum

A 52-week high is the highest level at which a stock traded during the last 12 months. George and Hwang (2004) claimed that momentum can be measured by the proximity of a stock's current price to its 52-week high, ranking stocks on their P52WH ratio and buying those with the highest while selling those with the lowest ratios. Price levels are more important than past returns, and a stock price at or near its 52-week high level is a better predictor of future returns than extreme past returns over some static formation period (George & Hwang 2004:2146).

These results from the United States were confirmed by Marshall and Cahan (2005) in an out-of-sample test using Australian stock data, finding that the 52-week-high momentum strategy outperformed its return-based counterpart. Also, momentum via the 52-week high does not reverse in the long run, providing more evidence that medium-term momentum and long-term reversals are largely separate phenomena. Liu, Liu and Ma (2011) suggested that investors use the 52-week high as a reference point or anchor when evaluating the potential impact of news. Anchoring leads to an underreaction that generates momentum over the medium term without the eventual reversal over the longer term when generated by a delayed overreaction (Liu, Liu & Ma 2011:203). The proximity to the 52-week high measures the degree of this underreaction to news. The closer a stock price is to its 52-week high, the higher the likelihood that this stock underreacted to recent good news.

Li and Yu (2012) added the historical high to improve on the 52-week-high momentum strategy. While proximity to the 52-week high tracks underreaction and predicts a continuation in price, proximity to the historical high tracks prolonged periods of overreaction that should lead to reversals in price. Stocks are double-sorted into sets by proximity to their 52-week highs in descending order of highest to lowest ratios, and within each set by their proximity to the historical high ascending from lowest to highest ratio. Buying those stocks with current prices closest to their 52-week highs but farthest from their historical highs, and selling vice-versa, captures stronger relative momentum. Controlling for a second anchor, the historical high, enhances the momentum effect (Li & Yu 2012:418).

The 52-week-high level of a stock is a significant, widely reported and readily available piece of data. Accepting that investors put more weight on recent news, Bhootra and Hur (2013) suggested that the underreaction would be stronger when the 52-week high occurred more recently, near the end of the formation period. If the 52-week high occurred early on during the last 12 months, most of any underreaction would supposedly have completed. Therefore, this study focused on the timing of a stock's 52-week high level and not on its proximity to the current price, taking long positions in stocks with a recent 52-week high and shorting stocks with a distant 52-week high. Conditioning on the recentness of the 52-week high increases the profitability of a 52-week-high momentum strategy (Bhootra & Hur 2013:3782).

Stocks that outperform other stocks over the medium term tend to be priced close to their 52-week highs, and have higher moving average ratios – that is, the ratio of a short-term (e.g., 50 days) moving average to a longer-term (e.g., 200 days) moving average. Park (2010) combined the moving-average ratio with the 52-week-high rule to capture momentum. This study identified the marginal effect of being a high-momentum stock under a given strategy while controlling for also being a high-momentum stock under other strategies by using a Fama-MacBeth style two-step cross-sectional regression. Investors use moving averages and the 52-week high as reference points to estimate intrinsic values. The momentum generated by this type of benchmarking does not reverse in the long run (Park 2010:415).

Chen and Yang (2016) reported that the 52-week-high momentum strategy also exhibits the Novy-Marx style echo effect – referring to evidence that momentum is formed 7 to 12 months prior to holding the portfolio and that recent returns are mostly irrelevant. A skip period of 3 to 6 months between momentum formation and portfolio construction increased performance significantly. The span of this skip-period is shorter than that of the echo found by Novy-Marx (2012) in return-based momentum, supporting the notion that 52-week-high momentum is a distinct form of momentum with unique (echo) features (Chen & Yang 2016:46).

Chang (2019) looked at the evolution of the 52-week high, calculated over a rolling 52-week window and variable over the whole period. Mean reversion is the tendency of a stock price to return to its long-term mean. When a 52-week high adjusts downward, it means that a stock price has not set a new high for at least a year. As a result, future prices will tend to mean-revert and record increasing returns. An upward-adjusted 52-week high implies that a stock price crossed its previous 52-week high, usually coinciding with a large return within the current period, attracting more attention. A downward-adjusted 52-week high implies that the stock price has not set a higher 52-week high for an extended period, drawing less attention. The longer the interval between successive upward-adjusting 52-week highs, the more likely a stock will outperform and the higher its potential return.

2.4.6 Summary

The relative-strength strategy put forward by Jegadeesh and Titman (1993) serves to detect and confirm the momentum effect in a particular market and facilitate analysis. Dubbed cross-sectional momentum (CSMOM), this strategy ranks stocks on momentum, taking a long position in high positive-momentum stocks and a short position in high negative-momentum stocks. Cross-sectional momentum tends to reverse, consistent with a delayed overreaction to common factors and conditional on the formation period, with longer formations imposing shorter holding periods on investors (Drew, Veeraraghavan & Ye 2007). Market-neutral portfolios, holding equal amounts in long and short positions, are vulnerable to momentum crashes that may result from rebounding bear markets (Daniel & Moskowitz 2016).

Time-series momentum (TSMOM) outperforms cross-sectional momentum in market continuations but underperforms in transitions due to its active, net long or short, position (Cheema, Nartea & Szulczyk 2018). A time-series approach introduces timing to momentum investing and avoids the short-term reversals reported with cross-sectional strategies (Goyal & Jegadeesh 2018). Even though a time-series strategy may generate excessive trading and transaction costs, standardising returns by some measure of volatility reduces the need for frequent rebalancing (Dudler, Gmür & Malamud 2015). Moving-average momentum (MAMOM) aligns with time-series (or trend) momentum but may enter or exit positions more quickly via superior timing (Marshall, Nguyen & Visaltanachoti 2017).

Idiosyncratic momentum (ISMOM) largely avoids exposure to common market factors and does not reverse strongly in the long term, consistent with an underreaction to stock-specific news (Blitz, Hanauer & Vidojevic 2020). A short position in high negative-momentum stocks sorted on their stock-specific returns should experience less severe momentum crashes (Chaves 2016). As a result of having significantly lower exposures to common factors than conventional momentum, an idiosyncratic momentum strategy also exhibits substantially lower volatility (Blitz, Huij & Martens 2011).

Price levels are more important than past returns and a stock price at or near its 52-week-high level is a better indicator of momentum in price than extreme returns measured over some fixed formation period (George & Hwang 2004). The 52-week high serves as a reference point or anchor, and anchoring results in an underreaction that builds momentum without the eventual reversal experienced with a delayed overreaction to news (Liu, Liu & Ma 2011). Momentum via the 52-week high (52WHM) does not rely on extreme returns, which normally occur during periods of high volatility (Bhootra & Hur 2013).

The alternative definitions of momentum all attempt to identify a more persistent continuation in price and limit the reversal of gains at the end of a cycle. Several techniques aid these objectives by analysing the evolution of prices during the formation period, detecting changes in momentum, and possibly locating different stages in a momentum cycle.

2.5 IDENTIFYING MOMENTUM

Basic positive, medium-term momentum is an increase in price during formation periods ranging from 60 to 250 trading days (3 to 12 months). This increase comprises daily changes in price, either many small increases outnumbering the decreases or relatively few increases overriding the majority decreases. A momentum cycle consists of both a formation and a holding period. Accepting that a cycle generally completes within 24 months, these two periods must be in balance at some level. Changes in momentum, accelerating or decelerating changes in price, may help to position momentum in its cycle. Modelling moves the objective from identifying momentum to isolating momentum and involves optimisation.

2.5.1 Changes in price

Momentum is measured as the f-period cumulative daily logarithmic returns of a stock (Equation 2.5). Logarithmic returns are additive and aggregate across time. The change in price (momentum) is the sum of the log returns in the formation period.

$$\mathbf{m}_{i,t}(\mathbf{f}) = \sum_{d=1}^{f} \mathbf{LN}\left(\frac{\mathbf{p}_{i,d}}{\mathbf{p}_{i,d-1}}\right) \tag{2.5}$$

Where: $\mathbf{m}_{i,t}(\mathbf{f})$ is the momentum of stock i at time t in formation period f
$\mathbf{p}_{i,d}$ is the price of stock i on day d
$\mathbf{p}_{i,d-1}$ is the price of stock i on day d-1

Momentum can be measured by fitting an exponential curve (Equation 2.6) to daily stock prices and obtaining the slope (Equation 2.7) or the average daily percentage change in price (Clenow 2015). The average daily percentage is annualised and multiplied by the goodness of fit or R-squared of the regression (Equation 2.8) to moderate the percentage and arrive at a momentum score (Equation 2.9).

$$\mathbf{p}_{i,d} = \mathbf{a}_i \mathbf{b}_i^{d} \; ; \qquad \ln(\mathbf{p}_{i,d}) = \ln(\mathbf{a}_i) + \mathbf{b}_i(\mathbf{d}) \tag{2.6}$$

$$\mathbf{b}_i = \frac{\sum (d - \bar{d}) \times \left[\ln(p_{i,d}) - \overline{\ln(p_{i,d})}\right]}{\sum (d - \bar{d})^2} \tag{2.7}$$

$$\mathbf{R}_i^2 = \left(\frac{\sum (d - \bar{d}) \times \left[\ln(p_{i,d}) - \overline{\ln(p_{i,d})}\right]}{\sqrt{\sum (d - \bar{d})^2 \times \sum \left[\ln(p_{i,d}) - \overline{\ln(p_{i,d})}\right]^2}}\right)^2 \tag{2.8}$$

$$\mathbf{M}_i = \mathbf{b}_i^{TD} \times \mathbf{R}_i^2 \tag{2.9}$$

Where: $\mathbf{p}_{i,d}$ is the price of stock i on day d
$\mathbf{a}_i$ is the intercept term of an exponential regression
$\mathbf{b}_i$ is the average daily percentage change in the price of stock i
$\mathbf{R}_i^2$ is the coefficient of determination or goodness of fit
TD is the number of trading days in a year
$\mathbf{M}_i$ is the momentum score of stock i

2.5.2 Evolution of prices

A study by Da, Gurun and Warachka (2014), stating that investors underreact to smaller bits of information arriving continuously, the so-called frog-in-the-pan hypothesis, presented a proxy for information discreteness that measures the relative frequency of small signals. Information discreteness captures time-series variation in the daily returns that comprise the formation-period return of a high-momentum stock. Continuous information induces stronger and more persistent return continuation than discrete information and does not reverse in the long run (Da, Gurun & Warachka 2014:2174). The shape and distribution of returns over the formation period affect the shape and distribution of returns expected over the holding period (Vanstone & Hahn 2017:283).

Da, Gurun and Warachka (2014) used sequential double-sorts to condition on formation-period returns, followed by their Information Discreteness (ID) measure that captures the relative frequency of small signals. A high percentage of positive daily returns relative to negative daily returns implies that a stock's high cumulative formation-period return comprises many small positive returns. A positive ID value signifies discrete information, while a negative ID value denotes continuous information (refer to Equation 2.10). The ID value varies between -1 (all positive) and +1 (all negative). Continuous information is believed to induce strong and persistent momentum (Da, Gurun & Warachka 2014:2171).

$$ID_U = \text{sign}(FPR) \times [\%neg - \%pos] \tag{2.10}$$

$$ID_W = -\frac{1}{N}\,\text{sign}(FPR) \times \sum_{d=1}^{N} \text{sign}(r_{i,d}) \times w_d \tag{2.11}$$

$$ID_Z = \text{sign}(FPR) \times \frac{[\%neg - \%pos]}{[\%neg + \%pos]} \tag{2.12}$$

Where:
- **FPR** is the cumulative return in the formation period
- $r_{i,d}$ is a daily return of stock i
- w_d is the weight assigned to a daily return
- **N** is the number of trading days in the formation period
- **%pos** is the percentage positive-return days in the formation period
- **%neg** is the percentage negative-return days in the formation period
- ID_U is the unweighted proxy for Information Discreteness
- ID_W incorporates the magnitude or weight of returns
- ID_Z incorporates zero-return days

Equation 2.11 calculates an average of the signed (positive or negative) daily returns in a period, assigning weights to each return to overweight either smaller or more recent returns and detect a particular pattern or consistency in returns. Equation 2.12 implicitly accounts for zero-return days (an indication of illiquidity) and when there are no zero-return days it reduces to Equation 2.10 where the ID value is simply the difference between the percentage negative-return days and percentage positive-return days.

The results showed that these alternative proxies for information discreteness that either assign larger weights to smaller or more recent daily returns, or account for zero-return days are of limited use. Da, Gurun and Warachka (2014:2187) provided justification and intuition for using the basic unweighted ID measure (Equation 2.10) as the primary proxy for information discreteness.

Even though Da, Gurun and Warachka (2014) concluded that zero-return days do not affect the usefulness of the Information Discreteness (ID), it can be argued that the ratio of zero-return days to the total number of trading days proxies for liquidity (refer to Equation 2.13). Illiquid stocks face more difficulty to trade, increasing the probability of these stocks having days with zero returns. The proportion of days with zero returns in a period also serves as a simple proxy for transaction costs on the premise that a stock with low transaction costs will have more frequent changes in price and fewer zero returns, compared to a stock with high transaction costs (Le & Gregoriou 2020:1175).

Importantly, according to Lee (2006:13), the liquidity measure based on zero returns (refer to Equation 2.13) must include zero-volume days as well as positive-volume days since a zero-return day with positive volume is a day when noise or uninformed trading induced trading volume. Noise trading by uninformed traders increases liquidity but delays the reaction and slows the adjustment of prices to new information (Bloomfield, O'Hara & Saar 2009:2300). The underreaction to news and the delayed overreaction to news are accepted behavioural explanations for momentum in stock returns (refer to Section 2.2.1).

$$L_{ZR} = \left(1 - \frac{zrd}{TD}\right) = 1 - ZR = \frac{rd}{TD} \qquad (2.13)$$

$$L_{ZV} = \left(1 - \frac{zvd}{TD}\right) = 1 - ZV = \frac{vd}{TD} \qquad (2.14)$$

Where: **(z)rd** is the number of (zero-)return days in a period
(z)vd is the number of (zero-)volume days in a period
TD is the number of trading days in a period

Kang and Zhang (2014) proposed an equivalent proxy, the ratio of zero-volume days, to measure liquidity more directly than zero returns. The zero-volume measure of liquidity (refer to Equation 2.14) does not perform well in liquid markets such as the New York Stock Exchange, but it is a straightforward and reliable measure for less liquid markets (Armitage, Brzeszczyński & Serdyuk 2014:191). Page and Auret (2017) used a combination of historical average turnover and zero volume as liquidity filters, setting a maximum number of zero-volume days and a minimum threshold for turnover to exclude stocks from a momentum analysis.

The volatility of stock returns – that is, the standard deviation of the daily returns over a given period, is another proxy for liquidity with lower volatility indicating higher liquidity. Both the percentage of zero-return days and return volatility are inversely related to liquidity. Stocks that are more liquid have relatively few zero-return days and tend to move in smaller increments resulting in lower volatility (Chai, Faff & Gharghori 2010:184). The trading pattern of a stock becomes more regular when the percentage of zero returns falls and trading actually results in prices changing (Lesmond 2005:423).

Gray and Vogel (2016) included the Information Discreteness (ID) measure from Da, Gurun and Warachka (2014) in their stock selection system. Labelled quantitative momentum, it is described as an active investing strategy that is all about individual stock selection or stock picking, and not based on stock valuation or market timing (Gray & Vogel 2016:11). Their quantitative approach focuses on the time-series behaviour of a momentum stock by tracking the evolution of its historical prices during the formation period. The path to momentum matters and momentum is stronger when past returns consist of a series of frequent gradual changes rather than infrequent dramatic changes, according to Gray and Vogel (2016:100), referencing the study by Da, Gurun and Warachka (2014). The consistency and persistency of momentum ultimately govern the quality and strength of the momentum effect (Grinblatt & Moskowitz 2004; Chen, Chou & Hsieh 2018).

The quantitative approach put forward by Gray and Vogel (2016) ranks individual stocks on their momentum over equivalent formation periods at a particular point in time. Stocks with the highest momentum are re-ranked or double-sorted on quality (ID values), essentially selecting those with the largest number of positive daily returns during the formation period. The ID measure captures the extent of frequent and gradual increases in price, in effect favouring momentum stocks exhibiting low volatility. Stocks experiencing momentum at low volatility are more likely to maintain momentum and less likely to experience reversals (Yang & Zhang 2019:71).

Any quantifiable metric aimed at identifying sustainable momentum in price can be part of a quantitative approach. Momentum is a well-known concept in physics, and Choi (2014) suggested a physics approach to price momentum by quantifying it in terms of mass and velocity (daily log return). Financial mass is supposed to capture the distinct properties of each stock and amplify the rate of change in price (velocity). Trading volume, transaction value, and volatility are viable candidates for representing financial mass. Daily price changes are more significant at larger trading volumes and higher transaction values. Financial mass is inversely proportional to volatility and a less volatile stock price, therefore, has more mass. A short-term (six weeks) contrarian strategy with physical momentum as the ranking criterion and volume, value and volatility as the respective proxies for mass, delivered promising results in two different markets (Choi 2014:71).

2.5.3 Changes in momentum

A change in momentum, when prices are accelerating or decelerating and returns are increasing or decreasing, is a sensitive measure representing the remaining component (plus noise) of a time series after isolating or removing its trend and speed (Kaufman 2013:414). A series of daily stock prices can evolve as a convex (accelerating) or concave (decelerating) function of time during the formation period. Conditioning or sorting stocks on momentum before double sorting on the k-values of quadratic regressions (Equation 2.15) captures the curvature of a trending time series, revealing acceleration (if positive) or deceleration (if negative).

$$\mathbf{p_i(t) = a_i + b_i t + k_i t^2} \qquad \mathbf{(2.15)}$$

Where: $\mathbf{p_i(t)}$ is a time series of stock i prices
$\mathbf{a_i}$ is the intercept term of a quadratic regression
$\mathbf{b_i}$ is the trend in a time series of stock i prices
$\mathbf{k_i}$ is the curvature in a time series of stock i prices

Chen, Yu and Wang (2018) published a study on price acceleration and deceleration in the momentum-formation period, based on a subset of the winner and loser stocks with a specific evolution in historical prices. Stocks are double-sorted – first into quantiles according to performance over a particular J-month formation period, and then further within each return group into quantiles after running quadratic regressions on the daily stock prices in that formation period.

The coefficient of time squared (t^2) reveals the curvature of a trend, representing acceleration (if positive) or deceleration (if negative). When positive (negative), the evolution of the historical prices is a convex (concave) function of time during the formation period. Accelerating winners with convex-shaped historical prices are stocks at the top of the winner return group, and decelerating winners with concave-shaped prices are stocks at the bottom of the winner group. Conversely, decelerating losers (convex-shaped) are stocks at the top of the loser return group, and accelerating losers (concave-shaped) are stocks at the bottom of the loser group. A conventional momentum strategy would require an investor to buy winners and short losers. In contrast, an accelerating momentum strategy would see investors buying accelerating-winners and selling accelerating-losers short. Accelerating winners and losers outperform and underperform both their conventional and decelerating counterparts, at the cost of higher turnover and the risk of sudden large reversals (Chen, Yu & Wang 2018:134).

A study by Ardila-Alvarez, Forrò and Sornette (2020) confirmed the relevance of the formation process. It provided evidence that changes in momentum, defined as the first difference of successive returns (series of changes in log prices from one period to the next), better identify persistence in returns than momentum.

However, Xiong and Ibbotson (2015) also showed that an accelerating increase in price is not sustainable over the short term and that this acceleration contributes to the well-documented short-term reversal that follows the formation of medium-term momentum (Xiong & Ibbotson 2015:86).

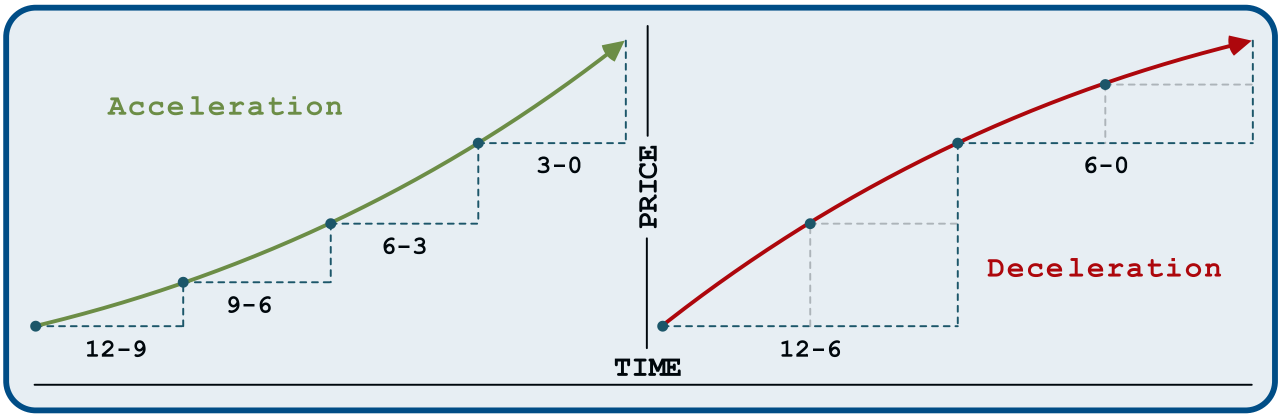

Figure 2.4 Curvature

Ardila-Alvarez, Forro and Sornette (2020) substituted changes in price (momentum) for changes in momentum (acceleration) and identified it as an important source of momentum profits. Gamma (G) quantifies acceleration as the first difference of successive returns that measures the increase in momentum over equivalent periods (Equation 2.16). Acceleration represents the unsustainable portion of momentum, driven by positive feedback trading and resulting in long-term reversals (Ardila-Alvarez, Forro & Sornette 2020:27). Accelerating returns are also not sustainable over the short term and contribute to the short-lived reversal that immediately follows the formation of medium-term momentum (Xiong & Ibbotson 2015:86).

$$\mathbf{G_{i,t}(f) = m_{i,t}(f) - m_{i,t-f}(f)} \qquad \mathbf{(2.16)}$$

Where: $\mathbf{G_{i,t}(f)}$ is the acceleration in momentum
$\mathbf{m_{i,t}(f)}$ is the momentum of stock i at time t in period f
$\mathbf{m_{i,t-f}(f)}$ is the momentum of stock i at time t-f in period f

As shown in Equation 2.16, a simple method to calculate acceleration is to take the first difference of successive returns of a time series. The volatility of acceleration reflects downside risk as well as upside risk – the risk of uncertain positive returns. Using the first difference of daily returns as proxy, the volatility of acceleration captures the stability in a stock’s momentum. Furthermore, changes in the direction of momentum amplify the volatility of acceleration, which provides an early warning signal for risk and tends to lead the volatility of returns (Varadi 2014).

A full momentum cycle has a limited lifespan and early entry extends the holding period, thereby increasing the potential for gains. The length or duration of the holding period largely determines the eventual outcome.

2.5.4 Momentum cycle

The change in price or return of a stock over the medium term (3 to 12 months) represents high positive momentum when it increased significantly in this period. Once a stock begins to gain momentum, it usually maintains it for 15 months before faltering, reversing some gains after 18 months (Bukowski 2018). The formation period plus the holding period, the momentum cycle, is a maximum of 18 months – a somewhat arbitrary number that makes little intuitive, statistical or economic sense (Hoffstein 2018). The optimal holding period, therefore, is a function of the formation period – the longer the formation period, the shorter the holding period and vice versa. The level of acceleration as represented by the H-ratio (Equation 2.17), gives some indication of where a stock positions in its momentum cycle by gaging shorter-term performance relative to longer-term performance (Bird & Casavecchia 2006:109). Stocks with fast acceleration are likely to be in the early stages of continuation, at the beginning of a cycle. Stocks with slow acceleration are likely to be in the late stages of continuation, approaching the end of a momentum cycle (Bird & Casavecchia 2007:232).

$$H_{i,t}(f) = \frac{m_{i,t}(f)}{m_{i,t}(2f)} \qquad (2.17)$$

Where: $H_{i,t}(f)$ is the level of acceleration in momentum
$m_{i,t}(f)$ is the momentum of stock i at time t in period f
$m_{i,t}(2f)$ is the momentum of stock i at time t in period 2×f

Refer to Figure 2.5: Longer-term underperformance (green) in combination with medium-term outperformance (blue), with the 24-month price (AC) above both the 12-month starting price (S) and the current price (C), positions a momentum stock at the beginning of a momentum cycle – fast acceleration. Longer-term average performance (orange) in combination with medium-term outperformance, with the 24-month price (AS) above the 12-month starting price (S), positions a momentum stock in the middle or nearer the end of a momentum cycle – slow acceleration.

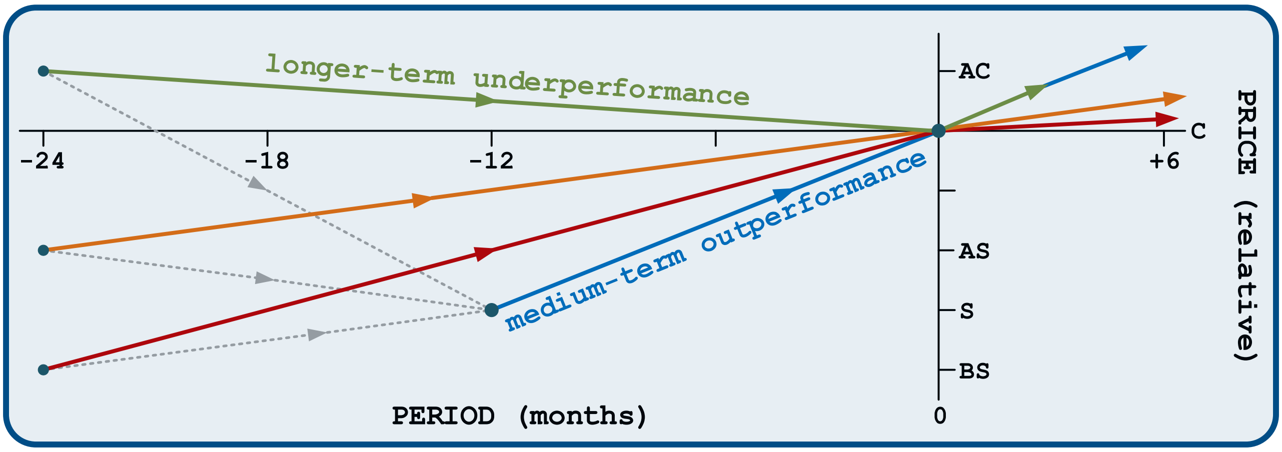

Figure 2.5 Positioning

Outperformance over both the longer term (red) and the medium term, with the 24-month price (BS) below the 12-month starting price (S), positions a momentum stock at the end of a momentum cycle – deceleration after the formation period.

This concept is similar to a strategy proposed by Chen, Kadan and Kose (2009) that invests in fresh winners and fresh losers only, by double-sorting stocks on medium-term price continuation (momentum) and long-term price reversal (contrarian). Double-sorted portfolios result where momentum and contrarian portfolios intersect. Fresh winners (early in the momentum cycle) comprise momentum winners and contrarian losers – that is, stocks that outperformed over the medium term but underperformed over the long term. Stale winners (late in the cycle) comprise both momentum and contrarian winners – that is, stocks that outperformed over the medium term as well as over the long term. This fresh-momentum strategy is based on the premise that when the momentum and contrarian effects coincide and work together, the medium-term momentum of a portfolio will be reinforced. For stale winners and losers, the momentum and contrarian effects work against each other.

In summary: The change in price or return of a stock over the medium term (3 to 12 months) represents high positive momentum when its price increased significantly in a particular formation period. The evolution of prices within this formation period may indicate which stocks are most likely to sustain their momentum during an extended holding period. Momentum tends to form and hold for up to 24 months, with cycles peaking at 18 months (Bukowski 2018; Hoffstein 2018). All things being equal, shortening the formation period lengthens the holding period, within limits. Stocks may have little scope to appreciate when waiting too long before entering a cycle, or momentum may not have formed fully when entering the cycle too early. The acceleration and deceleration in momentum may help to position a stock in its momentum cycle.

Modelling moves the objective from identifying momentum to isolating momentum and involves optimisation. The next section provides a relatively detailed overview of certain types of models and the approach to isolating momentum via modelling, which requires optimisation and continuous adjustment or refinement. Optimisation may confine a model to a particular equity market during a specific period.

2.5.5 Modelling momentum

Brush and Boles (1983) combined relative strength analysis with the capital asset pricing model (CAPM) and tested several conventional and beta-adjusted price momentum approaches. These models were categorised as: conventional non-beta adjusted; 60-month equal-weighted beta adjusted, and time-weighted beta adjusted (refer to Table 2.1). The alphas (intercepts) of the two-parameter (alpha/beta) regressions represented the beta-adjusted or non-market returns. This study assigned every S&P 500 issue with continuous monthly price data beginning in 1962 or earlier to two alternate datasets. Each dataset contained 168 stocks with similar industry representation. The first or development dataset was used to refine the models in each of the three categories. Each model was optimised by examining its forecasting success over 26 consecutive 6-month test or holding periods beginning in 1967. Incremental adjustments to model parameters were made until there was no improvement in the criteria. The best model in each category was tested on the second or reserve dataset, retaining the optimised parameters.

Model T, the most successful non-beta model in its category, ranked stocks on their returns over 3-month formation periods and evaluated their performance after 6-month holding periods. Formation periods ranging from 1 to 9 months as well as more complex past-return weighting systems were tested, but this simple model offered the best performance in the non-beta category (Brush & Boles 1983:21).

Using a conventional ordinary least squares (OLS) equal-weighted regression over the preceding 60 months of data, alphas were calculated for each stock at each of the 26 test points. All models considered in the first category were retested on these historical alpha coefficients with the S&P 500 Index as the benchmark. They also tested these models by applying various penalties to past price changes for the level of beta; the standard error of the estimate; and the residual standard error of the regression. Model ROA, in effect Model T with a 60-month historic beta adjustment, ranked stocks on their 3-month alphas. Model K, a Kalman-filter approach where repeated passes through past data determine a predictor or corrector rule applicable to all stocks, applied this intricate smoothing procedure to detect price trends in the beta-adjusted returns (Brush & Boles 1983:22).

The final category of models tested whether alphas estimated over truncated periods would improve performance by using a weighted least squares (WLS) regression with exponentially-decaying weights applied to observations. Model AG explored a wide range of decay factors in addition to various penalties and corrections to the resulting alphas. Using a WLS regression to estimate alpha and beta for each stock simultaneously, it was evident that data beyond the last 12 months have little impact on the results (Brush & Boles 1983:22). Alpha from this model represented the portion of a recent price change not explained by a changing beta and adjusted for the volatility in past prices.

Table 2.1 Models (Brush & Boles 1983)

MODEL	DESCRIPTION
Conventional non-beta model	
Model T	Ranks stocks on their 3-month returns and evaluated performance after 6-month holding periods – a 3J/6K strategy.
Ordinary least squares (OLS) regressions – equal weighted	
Model RAO	Model T using 60-month beta-adjusted returns – a 3J/6K strategy accounting for the changing betas from OLS regressions.
Model K	A Kalman-filter smoothing procedure detected price trends in the 60-month beta-adjusted returns.
Weighted least squares (WLS) regression – exponentially decaying weighted	
Model AG	Ranks stocks on their alphas from weighted least squares (WLS) regressions with exponentially-decaying weights over 12 months.

Refer to Table 2.1: Model T showed some ability to predict future returns, and when beta-adjusted returns (alphas) were used, as in Model RAO and Model K, there was only a small improvement in performance. Model K, a more complex approach to trend capturing, performed about the same as the simpler Model RAO. There is a considerable jump in conceptual complexity between Model RAO (an OLS regression) and Model AG (a WLS regression), the best-performing and most sophisticated of the models tested by Brush and Boles (1983:22).

Three years later, Brush (1986) extended the previous study and compared eight relative strength approaches that were close approximations of models in active use by portfolio managers at that time (refer to Table 2.2). These models ranged from ranking stocks on simple percentage changes (over 1, 3 or 6 months); ranking each stock based on its weighted-average returns over a prior four-quarter period, double-weighting the most recent quarter; ranking stocks on their excess returns (alphas) as determined by the capital asset pricing model (CAPM) applying a weighted linear regression to past data (Model AG in the 1983 study). The performance of each model was evaluated after 1, 3, 6 and 12-month holding periods.

Conventional price momentum models ranked stocks on their returns over the past month (Model 1), the past three months (Model 3), and the past six months (Model 6), respectively. Model 2A ranked stocks on their 7-month returns after subtracting half of the most recent month's return to incorporate a short-term reversal. Model 2D ranked stocks on their ratios of current price to the average price over the past seven months. Model Q ranked stocks on the weighted average of their returns over the past four quarters, double-weighting the most recent quarter. Model B (Model AG) ranked stocks on the alphas from weighted least squares (WLS) regressions with exponentially-decaying weights. Model E, similarly, ranked stocks on the alphas from WLS regressions – in effect Model B boosted with the short-term reversal effect, which comes from a slight reduction of the weight applied to the most recent month's return. The small adjustment in weight penalises stocks that have spiked upward and accommodates those stocks that showed temporary weakness in the last month (Brush 1986:23).

Table 2.2 Models (Brush 1986)

MODEL	DESCRIPTION
Conventional non-beta models	
Model 1	Ranks stocks on their 1-month returns and evaluated performance after 1, 3, 6 and 12-month holding periods.
Model 2	Ranks stocks on their 3-month returns and evaluated performance after 1, 3, 6 and 12-month holding periods.
Model 6	Ranks stocks on their 6-month returns and evaluated performance after 1, 3, 6 and 12-month holding periods.
Model 2A	Ranks stocks on their 7-month returns after subtracting half of the most recent month's return to incorporate a short-term reversal.
Model 2D	Ranks stocks on their ratios of current price to average price over the past seven months.
Model Q	Ranks stocks on the weighted average of returns over the past four quarters, double weighting the most recent quarter.
Weighted least squares (WLS) regressions – exponentially decaying weighted	
Model B	Ranks stocks on their alphas from weighted least squares (WLS) regressions with exponentially-decaying weights over 12 months.
Model E	Model B with a slight reduction of the weight applied to the most recent return to incorporate the short-term reversal effect.

Refer to Table 2.2: Results showed significant differences in performance among models 3, 6, 2A, 2D, and Q at the short to medium holding periods, with performance converging at 12-month rebalancing intervals. Model 1, ranking stocks on past 1-month returns, experienced the anticipated short-term reversals over the following month. Model 2A benefited from a similar most-recent month adjustment, successful in improving Model B to Model E, exploiting the anticipated short-term reversals. Beta-adjusted returns (alphas) estimated by exponentially-decaying weighted least squares (WLS) regressions improved significantly on the performance of simpler price-momentum models. Model Q was the best of the non-alpha/beta models and, according to Brush (1986:26), had been in use for many years at that time.

The first study in Canada on price momentum actually used Model Q to sort stocks listed on the Toronto Stock Exchange (TSX) from 1977 to 1992, and included in the TSE 100 Index. Foerster, Prihar and Schmitz (1994) ranked each stock on its weighted-average total return over a prior four-quarter period (double-weighting the most recent quarter) and selected the top ten outperformers and the bottom ten underperformers, updating and rebalancing these two portfolios each quarter. The returns on the positive-momentum portfolio, adjusted for risk and transaction costs, exceeded the returns of the benchmark TSE 300 Total Return Index in 14 of 15 years as well as those of the negative-momentum portfolio. However, this strategy was tested on a subset of 92 large capitalisation stocks for which data until the end of the sample period were available. Therefore, companies that may have delisted or gone into bankruptcy during this period were excluded from the sample, possibly introducing a survivorship bias. Stocks must be selected from those that were available at the time of the trade when evaluating a strategy.

Controlling for any survivorship bias by accounting for changes in the composition of the TSE 100 Index, Kan and Kirikos (1996) concluded that this specific strategy did not outperform the market in backtesting. Foerster (1996) argued that even though their methodology did involve a small survivorship bias, the differences in results can be attributed to the different sample periods (1977-1992 versus 1975 to mid-1991). The strategy produced reliable results for both 1991 and 1992 when any survivorship bias would be at its smallest, as the sample most closely resembled the TSE 100 Index.

Cleary and Inglis (1998) replicating Model Q, formed ten different portfolios in ascending order of momentum and compared the returns. The classic long (buy the strongest portfolio) and short (sell the weakest portfolio) relative strength strategy generated excess returns over the sample period (1978-1990). Confirming that Canadian stocks do exhibit momentum, this study also found that transaction costs eliminated any excess returns for the typical retail investor. Their database consisted of 238 listed companies whose market capitalisation exceeded a certain threshold at the beginning of the sample period, limiting any survivorship bias.

Table 2.3 Models (Brush 2001)

MODEL	DESCRIPTION
Simple models	
Model 1	Ranks stocks on their 1-month returns and evaluated performance after 1, 6 and 12-month holding periods.
Model 3	Ranks stocks on their 3-month returns and evaluated performance after 1, 6 and 12-month holding periods.
Model 6	Ranks stocks on their 6-month returns and evaluated performance after 1, 6 and 12-month holding periods.
Model 6-1	Ranks stocks on their 6-month returns after subtracting the most recent month's return to incorporate short-term reversal effect.
Model T	Ranks stocks on their 12-month returns and evaluated performance after 1, 6 and 12-month holding periods.
Model T-3	Ranks stocks on the combined total of their 12-month and 3-month returns – Model T plus the 3-month returns of these stocks.
Model T-1	Ranks stocks on their returns over the 11 months that end one month before the holding period, thereby excluding the most recent month.
Complex models	
Model W	Ranks stocks based on the sum of their weighted monthly returns over the past 12 months with less weight applied to recent months.
Model B	Ranks stocks on their alphas from weighted least squares (WLS) regressions over 12 months using the Model W weighting structure.
Model CA	Model B with an adjustment for extreme price changes, and an adjustment for a particular pattern of change in trading volume.

Refer to Table 2.3: Simple or elementary price momentum models merely calculate the changes in stock prices over a particular period. Brush (2001) stated that the key to enhancing elementary price momentum models is to eliminate those volatile stocks that record substantial returns without displaying any persistent momentum.

By weighting the monthly returns of each stock, Model W excludes stocks with high monthly volatility in price posing as momentum. Reducing the weights of the most recent months incorporates the short-term reversal effect. Model B ranks stocks on the alphas obtained from weighted least squares (WLS) regressions using the monthly returns of each stock and the market. This model corrects for the market's distorting effect on stock returns by using current betas and applies the same weighting structure to the monthly returns as Model W. The method for calculating beta is critical, as traditional 36-month or 60-month ordinary least squares (OLS) betas are not as useful as weighted least squares (WLS) betas calculated over shortened 12-month periods (Brush 2001:4).

Despite the reduced weighting of the most recent month's return, stocks ranked in the top quantile of Model B may still suffer from the short-term reversal effect if their latest price changes are substantial. Model CA has the same design as Model B but with two non-linear improvements that incorporate adjustments for extreme price changes in the most recent month, and for a particular pattern in trading volume (refer to Table 2.3). There is a point where the short-term strength in a stock becomes extreme. Stocks that experienced extreme changes in price over a short period should not be treated the same as other stocks ranked in the top quantile. The initial ranking needs adjusting to exclude these stocks from the top quantile until the effect of the extreme price change passed. The extreme-return adjustment is highly dependent on a percentage-change threshold, requiring optimisation (Brush 2001:5).

Short-term increases in trading volume improve short-term performance, confirming rising prices on rising volume as a positive signal. However, even though an increase in trading volume does have some ability to improve price momentum, the effect is small and short-lived. Longer-term increases in volume work in the opposite direction from short-term increases and its effect is comparatively persistent. Brush (2001:6) noted that excessive trading for longer periods normally results in stocks underperforming for up to 36 months. Model CA identifies and excludes stocks with significant percentage increases in volume over the past 12 months relative to previous years from the top quantile.

In summary: The basic concept of ranking stocks on returns and evaluating performance after certain holding periods still applied, but past returns were penalised or corrected based on ordinary least squares (OLS) betas and standard errors. Weighted least squares (WLS) regression applied different weights to observations. Recent returns were underweighted or overweighted, and smoothing techniques were used to detect price trends in beta-adjusted returns. Another model adjusted for extreme price changes and observable patterns in trading volume. While these models differ in complexity and sophistication, ultimately the objective was to isolate momentum in stocks and obtain more predictable outcomes.

2.6 CONCLUSION

The literature review showed that research focused on the classic J-month/K-month (formation/holding period) approach to identify momentum and find the optimal J/K combination in different equity markets. The long-only version ranks stocks on some definition of momentum, buying the top-ranked stocks (cross-sectional design) or stocks with high momentum (time-series design) and replacing individual stocks when a ranking or momentum falls below certain thresholds. Secondary sorts may introduce additional parameters to select between those stocks identified by the primary sort on momentum. The modelling of momentum takes this process a step further by testing on historical data to optimise settings for a specific period and market. Modelling moves the objective from identifying momentum to isolating momentum, which requires optimisation and continuous adjustment or refinement.

Standard formation and holding periods are generally used (typically 3, 6, 9 and 12 months) to find the optimal combination for a particular equity market, perhaps iterating through different combinations with 1-month increments for a more exact calibration. The momentum in a market is classified on its J/K combination.

Apart from the optimal J/K combination, whether momentum supposedly originates from an underreaction or a delayed overreaction to new information features prominently in research. In addition, performance is assumed to depend on more refined definitions of momentum, not the basic concept of momentum.

Past studies made no attempt to describe a particular equity market in terms of the composition of the momentum cycles generated by that market. This study will introduce the concept of momentum profiling. A momentum profile describes a particular equity market in terms of the composition of its momentum cycles. Profiling shifts the focus onto the holding period while differentiating between false, neutral, negative, and positive momentum cycles as determined by the eventual outcomes. Price range and sector activity add to the market profiles. Formation periods are substituted with entry zones, ensuring variability in formation. These entry zones also create profiles for individual stocks. A performance analysis via a custom momentum index completes each market profile.

Following this chapter, Chapter 3 describes the design of the study, provides an overview of the data and the techniques used, and summarises the reasoning for doing additional research on the topic of momentum in equity prices. Chapter 4 explains the momentum model, while the model is applied to three different markets in chapters 5 to 7. Chapter 8 compares the momentum profiles of these markets.

REFERENCES

Abukari, K. & Otchere, I. 2017. Dominance of hybrid contratum strategies over momentum and contrarian strategies: half a century of evidence. in *FMA Annual Meeting 2017 Conference*. Tampa, Florida: Financial Management Association International.

Alwathainani, A.M. 2012. Consistent winners and losers. *International Review of Economics & Finance*, 21(1):210–220. DOI: 10.1016/j.iref.2011.05.009.

Ardila-Alvarez, D., Forrò, Z. & Sornette, D. 2021. The acceleration effect and gamma factor in asset pricing. *Physica A: Statistical Mechanics and its Applications*, 569(125367):1–23. DOI: 10.1016/j.physa.2020.125367.

Armitage, S., Brzeszczyński, J. & Serdyuk, A. 2014. Liquidity measures and cost of trading in an illiquid market. *Journal of Emerging Market Finance*, 13(2):155–196. DOI: 10.1177/0972652714541340.

Asness, C.S., Frazzini, A., Israel, R. & Moskowitz, T.J. 2014. Fact, fiction, and momentum investing. *The Journal of Portfolio Management*, 40(5):75–92. DOI: 10.3905/jpm.2014.40.5.075.

Assoé, K. & Sy, O. 2003. Profitability of the short-run contrarian strategy in Canadian stock markets. *Canadian Journal of Administrative Sciences*, 20(4):311–319. DOI: 10.1111/j.1936-4490.2003.tb00707.x.

Assogbavi, T., Giguere, M. & Sedzro, K. 2011. The impact of trading volume on portfolios effective time formation/holding periods based on momentum investment strategies. *International Business & Economics Research Journal*, 10(7):1–12. DOI: 10.19030/iber.v10i7.4662.

Assogbavi, T. & Leonard, B. 2008. Portfolios effective time formation/holding period based on momentum investment strategy. *International Business & Economics Research Journal*, 7(5):51–58. DOI: 10.19030/iber.v7i5.3254.

Bandarchuk, P. & Hilscher, J. 2013. Sources of momentum profits: evidence on the irrelevance of characteristics. *Review of Finance*, 17(2):809–845. DOI: 10.1093/rof/rfr036.

Barberis, N., Shleifer, A. & Vishny, R. 1998. A model of investor sentiment. *Journal of Financial Economics*, 49(3):307–343. DOI: 10.1016/S0304-405X(98)00027-0.

Barroso, P. & Santa-Clara, P. 2015. Momentum has its moments. *Journal of Financial Economics*, 116(1):111–120. DOI: 10.1016/j.jfineco.2014.11.010.

Bhootra, A. & Hur, J. 2013. The timing of 52-week high price and momentum. *Journal of Banking & Finance*, 37(10):3773–3782. DOI: 10.1016/j.jbankfin.2013.05.025.

Bird, R. & Casavecchia, L. 2006. Insights into the momentum life cycle for European stocks. *The Journal of Investing*, 15(3):105–118. DOI: 10.3905/joi.2006.650150.

Bird, R. & Casavecchia, L. 2007. Value enhancement using momentum indicators: the European experience. *International Journal of Managerial Finance*, 3(3):229–262. DOI: 10.1108/17439130710756907.

Bird, R., Gao, X. & Yeung, D. 2017. Time-series and cross-sectional momentum strategies under alternative implementation strategies. *Australian Journal of Management*, 42(2):230–251. DOI: 10.1177/0312896215619965.

Blackburn, D.W. & Cakici, N. 2017. Overreaction and the cross-section of returns: international evidence. *Journal of Empirical Finance*, 42:1–14. DOI: 10.1016/j.jempfin.2017.02.001.

Blitz, D., Hanauer, M.X. & Vidojevic, M. 2020. The idiosyncratic momentum anomaly. *International Review of Economics & Finance*, 69:932–957. DOI: 10.1016/j.iref.2020.05.008.

Blitz, D., Huij, J. & Martens, M.P.E. 2011. Residual momentum. *Journal of Empirical Finance*, 18(3):506–521. DOI: 10.1016/J.JEMPFIN.2011.01.003.

Bloomfield, R.J., O’Hara, M. & Saar, G. 2009. How noise trading affects markets: an experimental analysis. *Review of Financial Studies*, 22(6):2275–2302. DOI: 10.1093/rfs/hhn102.

Bohl, M.T., Czaja, M.-G. & Kaufmann, P. 2016. Momentum profits, market cycles, and rebounds: evidence from Germany. *The Quarterly Review of Economics and Finance*, 61:139–159. DOI: 10.1016/j.qref.2016.01.003.

Boons, M. & Prado, M.P. 2019. Basis-momentum. *The Journal of Finance*, 74(1):239–279. DOI: 10.1111/jofi.12738.

Brailsford, T. & O’Brien, M.A. 2008. Disentangling size from momentum in Australian stock returns. *Australian Journal of Management*, 32(3):463–484. DOI: 10.1177/031289620803200305.

Britten, J.H.C., Page, M.D. & Auret, C.J. 2016. Investigating the interaction between long-term reversal and value on the JSE. *Studies in Economics and Econometrics*, 40(2):1–24.

Brush, J.S. 1986. Eight relative strength models compared. *The Journal of Portfolio Management*, 13(1):21–28. DOI: 10.3905/jpm.1986.409080.

Brush, J.S. 2001. *Price momentum: a twenty year research effort*. *Columbine Newsletter*. Colorado Springs, Colorado: Columbine Capital Services, Inc.

Brush, J.S. & Boles, K.E. 1983. The predictive power in relative strength & CAPM. *The Journal of Portfolio Management*, 9(4):20–23. DOI: 10.3905/jpm.1983.20.

Bukowski, P. 2018. *Quantitative insights*. *HIMCO Monthly Insights [Online]*. Hartford Investment Managament Company. Available at: http://www.himco.com/sites/himco/files/1444642061445.pdf (Accessed: 26 December 2020).

Chai, D., Faff, R.W. & Gharghori, P. 2010. New evidence on the relation between stock liquidity and measures of trading activity. *International Review of Financial Analysis*, 19(3):181–192. DOI: 10.1016/j.irfa.2010.02.005.

Chai, D., Limkriangkrai, M. & Ji, P.I. 2017. Momentum in weekly returns: the role of intermediate-horizon past performance. *Accounting & Finance*, 57(S1):45–68. DOI: 10.1111/acfi.12144.

Chan, L.K.C., Jegadeesh, N. & Lakonishok, J. 1996. Momentum strategies. *The Journal of Finance*, 51(5):1681–1713. DOI: 10.2307/2329534.

Chang, R.P., Ko, K.-C., Nakano, S. & Rhee, S.G. 2018. Residual momentum in Japan. *Journal of Empirical Finance*, 45:283–299. DOI: 10.1016/j.jempfin.2017.11.005.

Chang, T.-P. 2019. Inside of the 52-week high momentum: does price change or updating effect drive momentum profit? in *FMA Annual Meeting 2019 Conference*. New Orleans, Louisiana: Financial Management Association International.

Charteris, A. & Rupande, L. 2017. Feedback trading on the JSE. *Investment Analysts Journal*, 46(3):235–248. DOI: 10.1080/10293523.2017.1342319.

Chaves, D.B. 2016. Idiosyncratic momentum: U.S. and international evidence. *The Journal of Investing*, 25(2):64–76. DOI: 10.3905/joi.2016.25.2.064.

Cheema, M.A., Nartea, G. V. & Man, Y. 2018. Cross-sectional and time series momentum returns and market states. *International Review of Finance*, 18(4):705–715. DOI: 10.1111/irfi.12148.

Cheema, M.A., Nartea, G. V. & Szulczyk, K.R. 2018. Cross-sectional and time-series momentum returns and market dynamics: evidence from Japan. *Applied Economics*, 50(23):2600–2612. DOI: 10.1080/00036846.2017.1403560.

Chen, A.-S. & Yang, W. 2016. Echo effects and the returns from 52-week high strategies. *Finance Research Letters*, 16:38–46. DOI: 10.1016/j.frl.2015.10.015.

Chen, H.-Y., Chou, P.-H. & Hsieh, C.-H. 2018. Persistency of the momentum effect. *European Financial Management*, 24(5):856–892. DOI: 10.1111/eufm.12140.

Chen, L.-W., Yu, H.-Y. & Wang, W.-K. 2018. Evolution of historical prices in momentum investing. *Journal of Financial Markets*, 37:120–135. DOI: 10.1016/J.FINMAR.2017.07.001.

Chen, L., Kadan, O. & Kose, E. 2009. *Fresh momentum*. Working Paper. St Louis, Missouri: Washington University in St Louis.

Chiao, C.-S., Hsiao, Y.-J., Chen, J.-C. & An, N.M. 2018. Residual momentum versus price momentum: evidence from four Asian markets. *Asia-Pacific Journal of Accounting & Economics*, Online:1–10. DOI: 10.1080/16081625.2018.1474772.

Choi, J. 2014. Physical approach to price momentum and its application to momentum strategy. *Physica A: Statistical Mechanics and its Applications*, 415:61–72. DOI: 10.1016/j.physa.2014.07.075.

Clare, A., Seaton, J., Smith, P.N. & Thomas, S. 2016. The trend is our friend: risk parity, momentum and trend following in global asset allocation. *Journal of Behavioral and Experimental Finance*, 9:63–80. DOI: 10.1016/j.jbef.2016.01.002.

Cleary, S. & Inglis, M. 1998. Momentum in Canadian stock returns. *Canadian Journal of Administrative Sciences*, 15(3):279–291. DOI: 111/j.1936-4490.1998.tb00168.x.

Clenow, A.F. 2015. *Stocks on the move: beating the market with hedge fund momentum strategies*. Scotts Valley, California: CreateSpace.

Conrad, J.S., Gültekin, M.N. & Kaul, G. 1997. Profitability of short-term contrarian strategies: implications for market efficiency. *Journal of Business & Economic Statistics*, 15(3):379–386. DOI: 10.2307/1392341.

Conrad, J.S. & Kaul, G. 1998. An anatomy of trading strategies. *The Review of Financial Studies*, 11(3):489–519. DOI: 10.1093/rfs/11.3.489.

Conrad, J.S. & Yavuz, M.D. 2017. Momentum and reversal: does what goes up always come down? *Review of Finance*, 21(2):555–581. DOI: 10.1093/rof/rfw006.

Da, Z., Gurun, U.G. & Warachka, M. 2014. Frog in the pan: continuous information and momentum. *The Review of Financial Studies*, 27(7):2171–2218. DOI: 10.1093/rfs/hhu003.

Daniel, K.D., Hirshleifer, D.A. & Subrahmanyam, A. 1998. Investor psychology and security market under- and overreactions. *The Journal of Finance*, 53(6):1839–1885. DOI: 10.1111/0022-1082.00077.

Daniel, K.D. & Moskowitz, T.J. 2016. Momentum crashes. *Journal of Financial Economics*, 122(2):221–247. DOI: 10.1016/j.jfineco.2015.12.002.

De Bondt, W.F.M. & Thaler, R.H. 1985. Does the stock market overreact? *The Journal of Finance*, 40(3):793–805. DOI: 10.2307/2327804.

De Long, J.B., Shleifer, A., Summers, L.H. & Waldmann, R.J. 1990. Positive feedback investment strategies and destabilizing rational speculation. *The Journal of Finance*, 45(2):379–395. DOI: 10.2307/2328662.

Dittberner, A.G. 2016. Fundamental momentum: a new approach to investment analysis. PhD thesis. University of Pretoria.

Drew, M.E., Veeraraghavan, M. & Ye, M. 2007. Do momentum strategies work? Australian evidence. *Managerial Finance*, 33(10):772–787. DOI: 10.1108/03074350710779223.

Dudler, M., Gmür, B. & Malamud, S. 2015. Momentum and risk adjustment. *The Journal of Alternative Investments*, 18(2):91–103. DOI: 10.3905/jai.2015.18.2.091.

Ejaz, A. & Polak, P. 2018. Australian Stock Exchange and sub-variants of price momentum strategies. *Investment Management and Financial Innovations*, 15(1):224–235. DOI: 10.21511/imfi.15(1).2018.19.

Foerster, S. 1996. Back to the future, again: in defence of momentum-based trading strategies [and now you see them, then you don’t]. *Canadian Investment Review*, 9(2):n/a.

Foerster, S., Prihar, A. & Schmitz, J.J. 1994. Back to the future [Price momentum models]. *Canadian Investment Review*, 7(4):9–13.

Foltice, B. & Langer, T. 2015. Profitable momentum trading strategies for individual investors. *Financial Markets and Portfolio Management*, 29(2):85–113. DOI: 10.1007/s11408-015-0246-4.

George, T.J. & Hwang, C.-Y. 2004. The 52-week high and momentum investing. *The Journal of Finance*, 59(5):2145–2176. DOI: 10.1111/j.1540-6261.2004.00695.x.

Georgopoulou, A. & Wang, J. 2017. The trend is your friend: time-series momentum strategies across equity and commodity markets. *Review of Finance*, 21(4):1557–1592. DOI: 10.1093/rof/rfw048.

Gong, Q., Liu, M. & Liu, Q. 2015. Momentum is really short-term momentum. *Journal of Banking & Finance*, 50:169–182. DOI: 10.1016/j.jbankfin.2014.10.002.

Goyal, A. & Jegadeesh, N. 2018. Cross-sectional and time-series tests of return predictability: what is the difference? *The Review of Financial Studies*, 31(5):1784–1824. DOI: 10.1093/rfs/hhx131.

Goyal, A. & Wahal, S. 2015. Is momentum an echo? *Journal of Financial and Quantitative Analysis*, 50(6):1237–1267. DOI: 10.1017/S0022109015000575.

Gray, W.R. & Vogel, J.R. 2016. *Quantitative momentum: a practitioner's guide to building a momentum-based stock selection system*. Hoboken, New Jersey: John Wiley & Sons, Inc (Wiley Finance).

Grinblatt, M. & Moskowitz, T.J. 2004. Predicting stock price movements from past returns: the role of consistency and tax-loss selling. *Journal of Financial Economics*, 71(3):541–579. DOI: 10.1016/S0304-405X(03)00176-4.

Gupta, K., Locke, S. & Scrimgeour, F. 2013. Profitability of momentum returns under alternative approaches. *International Journal of Managerial Finance*, 9(3):219–246. DOI: 10.1108/IJMF-03-2012-0028.

Gutierrez, R.C. & Pirinsky, C.A. 2007. Momentum, reversal, and the trading behaviors of institutions. *Journal of Financial Markets*, 10(1):48–75. DOI: 10.1016/j.finmar.2006.09.002.

Hameed, A. & Wu, H. 2019. Decomposing momentum. *SSRN Electronic Journal*, Online. DOI: 10.2139/ssrn.3401656.

Han, Y., Zhou, G. & Zhu, Y. 2016. A trend factor: any economic gains from using information over investment horizons? *Journal of Financial Economics*, 122(2):352–375. DOI: 10.1016/j.jfineco.2016.01.029.

Hanauer, M.X. 2014. Is Japan different? Evidence on momentum and market dynamics. *International Review of Finance*, 14(1):141–160. DOI: 10.1111/irfi.12024.

Heyman, D., Lescrauwaet, M. & Stieperaere, H. 2019. Investor attention and short-term return reversals. *Finance Research Letters*, 29:1–6. DOI: 10.1016/j.frl.2019.03.003.

Hillert, A., Jacobs, H. & Müller, S. 2014. Media makes momentum. *The Review of Financial Studies*, 27(12):3467–3501. DOI: 10.1093/rfs/hhu061.

Hoffmann, A.O.I. & Post, T. 2014. Self-attribution bias in consumer financial decision-making: how investment returns affect individuals' belief in skill. *Journal of Behavioral and Experimental Economics*, 52:23–28. DOI: 10.1016/j.socec.2014.05.005.

Hoffstein, C. 2018. *Momentum's magic number*. *Flirting with Models [Online]*. Newfound Research LLC. Available at: https://blog.thinknewfound.com/2018/07/momentums-magic-number/ (Accessed: 26 December 2021).

Hong, H. & Stein, J.C. 1999. A unified theory of underreaction, momentum trading, and overreaction in asset markets. *The Journal of Finance*, 54(6):2143–2184. DOI: 10.1111/0022-1082.00184.

Hou, T.C.-T. & McKnight, P.J. 2004. An explanation of momentum in Canadian stocks. *Canadian Journal of Administrative Sciences*, 21(4):334–343. DOI: 10.1111/j.1936-4490.2004.tb00349.x.

Hühn, H.L. & Scholz, H. 2018. Alpha momentum and price momentum. *International Journal of Financial Studies*, 6(2):49. DOI: 10.3390/ijfs6020049.

Hühn, H.L. & Scholz, H. 2019. Reversal and momentum patterns in weekly stock returns: European evidence. *Review of Financial Economics*, 37(2):272–296. DOI: 10.1002/rfe.1037.

Jegadeesh, N. 1990. Evidence of predictable behavior of security returns. *The Journal of Finance*, 45(3):881–898. DOI: 10.1111/j.1540-6261.1990.tb05110.x.

Jegadeesh, N. & Titman, S. 1993. Returns to buying winners and selling losers: implications for stock market efficiency. *The Journal of Finance*, 48(1):65–91. DOI: 10.1111/j.1540-6261.1993.tb04702.x.

Jegadeesh, N. & Titman, S. 1995. Short-horizon return reversals and the bid-ask spread. *Journal of Financial Intermediation*, 4(2):116–132. DOI: 10.1006/jfin.1995.1006.

Jegadeesh, N. & Titman, S. 2001. Profitability of momentum strategies: an evaluation of alternative explanations. *The Journal of Finance*, 56(2):699–720. DOI: 10.1111/0022-1082.00342.

Jegadeesh, N. & Titman, S. 2002. Cross-sectional and time-series determinants of momentum returns. *The Review of Financial Studies*, 15(1):143–157. DOI: 10.1093/rfs/15.1.143.

Jegadeesh, N. & Titman, S. 2011. Momentum. *Annual Review of Financial Economics*, 3(1):493–509. DOI: 10.1146/annurev-financial-102710-144850.

Jiang, G.J. & Zhu, K.X. 2017. Information shocks and short-term market underreaction. *Journal of Financial Economics*, 124(1):43–64. DOI: 10.1016/j.jfineco.2016.06.006.

Joshipura, M. & Wats, S. 2023. Decoding momentum returns: an integrated bibliometric and content analysis approach. *Qualitative Research in Financial Markets*, 15(2):254–277. DOI: 10.1108/QRFM-12-2021-0211.

Jostova, G., Nikolova, S., Philipov, A. & Stahel, C.W. 2013. Momentum in corporate bond returns. *Review of Financial Studies*, 26(7):1649–1693. DOI: 10.1093/rfs/hht022.

Kan, R. & Kirikos, G. 1996. Now you see them, then you don’t: the phenomenal returns a trading strategy can produce when tested using historical data often vanish once theory is put into practice. *Canadian Investment Review*, 9(2):n/a.

Kang, W. & Zhang, H. 2014. Measuring liquidity in emerging markets. *Pacific-Basin Finance Journal*, 27:49–71. DOI: 10.1016/j.pacfin.2014.02.001.

Kaufman, P.J. 2013. *Trading systems and methods*. 5th edition. Hoboken, New Jersey: John Wiley & Sons, Inc (Wiley Trading). DOI: 10.1002/9781119202561.

Kim, S. 2022. Time-series residual momentum strategies. *Applied Economics*, 54(5):580–594. DOI: 10.1080/00036846.2021.1967862.

La Grange, P.L. & Krige, J.D. 2015. Profitability of momentum strategies on the JSE. *Studies in Economics and Econometrics*, 39(3):49–65.

Le, H. & Gregoriou, A. 2020. How do you capture liquidity? A review of the literature on low-frequency stock liquidity. *Journal of Economic Surveys*, 34(5):1170–1186. DOI: 10.1111/joes.12385.

Lee, C.M.C. & Swaminathan, B. 2000. Price momentum and trading volume. *The Journal of Finance*, 55(5):2017–2069. DOI: 10.1111/0022-1082.00280.

Lee, K.-H. 2006. Liquidity risk and asset pricing. PhD thesis. The Ohio State University.

Lehmann, B.N. 1990. Fads, martingales, and market efficiency. *The Quarterly Journal of Economics*, 105(1):1–28. DOI: 10.2307/2937816.

Lesmond, D.A. 2005. Liquidity of emerging markets. *Journal of Financial Economics*, 77(2):411–452. DOI: 10.1016/j.jfineco.2004.01.005.

Levine, A. & Pedersen, L.H. 2016. Which trend is your friend? *Financial Analysts Journal*, 72(3):51–66. DOI: 10.2469/faj.v72.n3.3.

Li, J. & Yu, J. 2012. Investor attention, psychological anchors, and stock return predictability. *Journal of Financial Economics*, 104(2):401–419. DOI: 10.1016/j.jfineco.2011.04.003.

Lim, B.Y., Wang, J. & Yao, Y. 2018. Time-series momentum in nearly 100 years of stock returns. *Journal of Banking & Finance*, 97:283–296. DOI: 10.1016/j.jbankfin.2018.10.010.

Lin, H.-W., Hung, M.-W. & Huang, J.-B. 2018. Artificial momentum, native contrarian, and transparency in China. *Computational Economics*, 51(2):263–294. DOI: 10.1007/s10614-017-9699-z.

Lin, S. & Rassenti, S. 2012. Are under- and over-reaction the same matter? Experimental evidence. *Journal of Economic Behavior & Organization*, 84(1):39–61. DOI: 10.1016/j.jebo.2012.07.004.

Liu, M., Liu, Q. & Ma, T. 2011. The 52-week high momentum strategy in international stock markets. *Journal of International Money and Finance*, 30(1):180–204. DOI: 10.1016/j.jimonfin.2010.08.004.

Lo, A.W. & MacKinlay, A.C. 1990. When are contrarian profits due to stock market overreaction? *The Review of Financial Studies*, 3(2):175–205. DOI: 10.1093/rfs/3.2.175.

Marshall, B.R. & Cahan, R.M. 2005. Is the 52-week high momentum strategy profitable outside the US? *Applied Financial Economics*, 15(18):1259–1267. DOI: 10.1080/09603100500386008.

Marshall, B.R., Nguyen, N.H. & Visaltanachoti, N. 2017. Time series momentum and moving average trading rules. *Quantitative Finance*, 17(3):405–421. DOI: 10.1080/14697688.2016.1205209.

McLean, R.D. 2010. Idiosyncratic risk, long-term reversal, and momentum. *Journal of Financial and Quantitative Analysis*, 45(4):883–906. DOI: 10.1017/S0022109010000311.

Moskowitz, T.J., Ooi, Y.H. & Pedersen, L.H. 2012. Time series momentum. *Journal of Financial Economics*, 104(2):228–250. DOI: 10.1016/j.jfineco.2011.11.003.

Muller, C. 1999. Investor overreaction on the Johannesburg Stock Exchange. *Investment Analysts Journal*, 28(49):5–17. DOI: 10.1080/10293523.1999.11082392.

Mun, J.C., Vasconcellos, G.M. & Kish, R. 2000. The contrarian/overreaction hypothesis: an analysis of the US and Canadian stock markets. *Global Finance Journal*, 11(1–2):53–72. DOI: 10.1016/S1044-0283(00)00011-9.

Novy-Marx, R. 2012. Is momentum really momentum? *Journal of Financial Economics*, 103(3):429–453. DOI: 10.1016/j.jfineco.2011.05.003.

Page, M.D. 2016. An in-depth validation of momentum as a dominant explanatory factor on the Johannesburg Stock Exchange. PhD thesis. University of the Witwatersrand.

Page, M.D. & Auret, C.J. 2017. Univariate tests of momentum on the JSE. *Investment Analysts Journal*, 46(3):149–164. DOI: 10.1080/10293523.2017.1319162.

Page, M.D. & Auret, C.J. 2019. Can non-momentum factor premiums explain the momentum anomaly on the JSE? An in-depth portfolio attribution analysis. *Investment Analysts Journal*, 48(1):1–17. DOI: 10.1080/10293523.2018.1483792.

Page, M.D., Britten, J.H.C. & Auret, C.J. 2013. Momentum and liquidity on the Johannesburg Stock Exchange. *International Journal of Economics and Finance Studies*, 5(1):56–73.

Page, M.D., Britten, J.H.C. & Auret, C.J. 2016. Idiosyncratic risk and anomaly persistence on the Johannesburg Stock Exchange (JSE). *Investment Analysts Journal*, 45(1):31–46. DOI: 10.1080/10293523.2015.1125060.

Page, M.D., McClelland, D. & Auret, C.J. 2020. Idiosyncratic momentum on the JSE. *Investment Analysts Journal*, 49(3):180–198. DOI: 10.1080/10293523.2020.1783864.

Page, M.J. & Way, C.V. 1992. Stock market over-reaction: the South African evidence. *Investment Analysts Journal*, 21(36):35–49. DOI: 10.1080/10293523.1992.11082314.

Park, S.-C. 2010. The moving average ratio and momentum. *The Financial Review*, 45(2):415–447. DOI: 10.1111/j.1540-6288.2010.00254.x.

Rhee, S.G. & Wang, C.-J. 1997. The bid-ask bounce effect and the spread size effect: evidence from the Taiwan stock market. *Pacific-Basin Finance Journal*, 5(2):231–258. DOI: 10.1016/S0927-538X(97)00014-0.

Ross, A., Moskowitz, T.J., Israel, R. & Serban, L. 2017. Implementing momentum: what have we learned? *SSRN Electronic Journal*, Online. DOI: 10.2139/ssrn.3081165.

Schmitz, J.J. & Cleary, S. 2000. What has worked on Bay Street [Several factors appear to be powerful predictors of future stock returns]. *Canadian Investment Review*, 13(4):n/a.

Schneider, P. & Gaunt, C. 2012. Price and earnings momentum in Australian stock returns. *Accounting & Finance*, 52(2):495–517. DOI: 10.1111/j.1467-629X.2010.00395.x.

Shi, J., Chiang, T.C. & Liang, X. 2012. Positive-feedback trading activity and momentum profits. *Managerial Finance*, 38(5):508–529. DOI: 10.1108/03074351211217832.

Siganos, A. 2010. Can small investors exploit the momentum effect? *Financial Markets and Portfolio Management*, 24(2):171–192. DOI: 10.1007/s11408-009-0120-3.

Singh, S. & Walia, N. 2022. Momentum investing: a systematic literature review and bibliometric analysis. *Management Review Quarterly*, 72(1):87–113. DOI: 10.1007/s11301-020-00205-6.

Teplova, T. & Mikova, E. 2015. New evidence on determinants of price momentum in the Japanese stock market. *Research in International Business and Finance*, 34:84–109. DOI: 10.1016/j.ribaf.2014.12.001.

Van Heerden, J.D. 2014. The impact of firm-specific factors on the cross-sectional variation in Johannesburg Security Exchange listed equity returns. PhD thesis. University of Cape Town.

Van Heerden, J.D. & Van Rensburg, P. 2015. The cross-section of Johannesburg Securities Exchange listed equity returns (1994-2011). *Studies in Economics and Finance*, 32(4):422–444. DOI: 10.1108/SEF-09-2014-0181.

Van Heerden, J.D. & Van Rensburg, P. 2016. The impact of liquidity on the cross section of equity returns on the Johannesburg Securities Exchange. *Economics, Management, and Financial Markets*, 11(2):59–86.

Van Heerden, J.D. & Van Rensburg, P. 2017. Common firm-specific characteristics of extreme performers on the Johannesburg Securities Exchange. *Economics, Management, and Financial Markets*, 12(3):25–50. DOI: 10.22381/EMFM12320172.

Vanstone, B.J. & Hahn, T. 2017. Australian momentum: performance, capacity and the GFC effect. *Accounting & Finance*, 57(1):261–287. DOI: 10.1111/acfi.12140.

Varadi, D. 2014. *A new (better?) measure of risk and uncertainty: the volatility of acceleration*. *New Concepts in Quantitative Research [Online]*. CSS Analytics. Available at: https://cssanalytics.wordpress.com/2014/11/28/a-new-better-measure-of-risk-and-uncertainty-the-volatility-of-acceleration/ (Accessed: 31 January 2021).

Viljoen, L.E. 2016. Residual momentum and investor sentiment on the Johannesburg Stock Exchange (JSE). MBA dissertation. University of Pretoria.

Wiest, T. 2023. Momentum: what do we know 30 years after Jegadeesh and Titman's seminal paper? *Financial Markets and Portfolio Management*, 37(1):95–114. DOI: 10.1007/s11408-022-00417-8.

Xiong, J.X. & Ibbotson, R.G. 2015. Momentum, acceleration, and reversal. *Journal of Investment Management*, 13(1):84–95.

Yang, X. & Zhang, H. 2019. Extreme absolute strength of stocks and performance of momentum strategies. *Journal of Financial Markets*, 44:71–90. DOI: 10.1016/j.finmar.2019.01.001.

Zaremba, A., Kizys, R. & Raza, M.W. 2020. The long-run reversal in the long run: insights from two centuries of international equity returns. *Journal of Empirical Finance*, 55:177–199. DOI: 10.1016/j.jempfin.2019.11.007.

Zaremba, A., Umutlu, M. & Karathanasopoulos, A. 2019. Alpha momentum and alpha reversal in country and industry equity indexes. *Journal of Empirical Finance*, 53:144–161. DOI: 10.1016/j.jempfin.2019.07.003.

Zaremba, A., Umutlu, M. & Maydybura, A. 2018. Less pain, more gain: volatility-adjusted residual momentum in international equity markets. *Investment Analysts Journal*, 47(2):165–191. DOI: 10.1080/10293523.2018.1469290.

CHAPTER THREE

RESEARCH DESIGN

3.1 INTRODUCTION

This chapter introduces the research design for this study, identifying the approach and methods for constructing a custom momentum stock index and evaluating its performance. Apart from giving an overview of the data and techniques used in this study, it summarises the reasoning for doing additional research on the well-researched topic of momentum in equity prices.

3.2 PURPOSE OF RESEARCH

Research to date focused on the classic J-month/K-month (formation/holding period) approach to identify momentum and find the optimal J/K combination in different equity markets. Buying the best-performing stocks (top quantile) and selling the worst-performing stocks (bottom quantile) on their performance over the past 3 to 12 months at every update. A widening spread between the performance of the two groups would confirm the presence of momentum in that market. The long-only version ranks stocks on some definition of momentum, buying the top-ranked stocks (cross-sectional design) or stocks with high momentum (time-series design) and replacing individual stocks when a ranking or momentum falls below certain thresholds.

Standard formation and holding periods are generally used (typically 3, 6, 9 and 12 months) to find the optimal combination for a particular equity market, perhaps iterating through different combinations with 1-month increments for a more exact calibration. The momentum in a market is classified on its J/K combination. Apart from the optimal J/K combination, whether momentum supposedly originates from an underreaction or a delayed overreaction to new information features prominently in research. In addition, performance is assumed to depend on more refined definitions of momentum, not the basic concept of momentum.

This study will introduce the concept of momentum profiling. Profiling shifts the focus onto the holding period while differentiating between false, neutral, negative, and positive momentum cycles as determined by the eventual outcomes. Formation periods are substituted with entry zones, ensuring variability in formation. These entry zones also create profiles for individual stocks. A performance analysis via a custom momentum index completes each market profile. The term momentum-profiling has a double meaning in that individual stocks are profiled as well as a particular equity market. Individual profiling may enable the selective targeting of stocks that have distinct visual profiles and past behaviour associated with momentum. The composition of the momentum cycles and average hold per cycle type provide a unique description of the momentum effect in a particular equity market. A custom index quantifies the collective outcome to show the progression of momentum in a market over the years.

3.3 RESEARCH DESIGN

Positivism can be described as an objective and deductive research paradigm or philosophical framework driven by theory and evidence, directing research to be scientific and systematic. Therefore, it is objective as opposed to subjective; deductive as opposed to inductive; and theory-driven as opposed to theory-building. Deductive – drawing specific conclusions from general premises, and not inductive – coming to general conclusions from specific observations. It is based on facts instead of opinion and therefore does not require interpretation by either the researcher (pragmatism) or the participant (constructivism). Positivists collect predominantly quantitative data and typically apply descriptive and inferential statistical techniques to test hypotheses formed by reviewing theories (Strang 2015:17-23).

Positivism imposes the strict application of a scientific methodology to reveal universal facts or truths by quantifying and controlling the variables or factors that may affect the findings of a research study. This rigid control allows the findings and conclusions from a research study conducted on a sample population to be extended to the population at large. The strict application of scientific methods and highly controlled procedures validate the findings and conclusions, making it possible for the study to be replicated by other researchers (Mukherji & Albon 2018).

Positivists claim that only scientific knowledge can be fully objective, valid, certain, and accurate (Mertens 2020:11). However, the constraints imposed by pure positivism in terms of rigid control may limit research, according to Strang (2015:22). Post-positivism presents a more refined version of positivism, not claiming to reveal absolute truths but putting forward a deterministic view where causes probably determine effects or outcomes. Research variables or factors are limited to what can be practically identified and controlled while no attempt is made to quantify uncertainty or articulate any unknowns (Strang 2015:23). Post-positivism is based on probability testing and building evidence to reject or support hypotheses without conclusively proving them (Leavy 2017:92).

The positivist paradigm demands a scientific, systematic approach to research and as such underpins the use of quantitative methodology and methods to produce numerical data, thereby allowing for statistical analysis (Mukherji & Albon 2018). A quantitative approach to research lends itself to some form of either experimental or observational design.

An experimental research design allows a researcher to control and manipulate parameters or factors to generalise outcomes of cause-and-effect. The researcher modifies a model or a process by adjusting or introducing new factors to record the impact. Factors that are adjusted or introduced (i.e., controlled) are independent variables, while those factors changed by the impact of independent variables are dependent variables (Novikov & Novikov 2013:57).

Descriptive observational research classifies, compares, and measures data to describe some phenomenon or anomaly in terms of what and where as well as when and possibly how. It is an appropriate research design when identifying the characteristics of the phenomenon or anomaly by determining frequencies, trends, and categories. Analytical observational research, on the other hand, would test some causal hypothesis or relationship between variables to determine why the phenomenon or anomaly occurs, focusing on cause and effect without controlling or manipulating the variables (Rezigalla 2020).

Experimental and observational research for quantitative investing purposes can be viewed as two directions of travel on a continuous scale. Greater control of the factors combined with the ability or capacity to repeat the process or rerun a model to generate data, move research towards the experimental end of the scale. Datasets become smaller in size and more prone to selection bias (sample not representing the population) with actual information harder to distinguish from irrelevant or inconsistent information (noise) when moving in the opposite direction. High-frequency trading utilises vast amounts of data available at short intervals, enabling it to operate experimentally. Conversely, fundamental equity analysis is generally restricted to only a few hundred data points, making it more observational in character (Winton 2022).

Descriptive statistical measures analyse data to reveal patterns by summarising and graphically presenting the information contained in a set of data. Descriptive statistics provides tabulated and graphical descriptions of data for statistical commentary and a discussion of the results. These descriptive measures are applied to populations and the properties of a population, referred to as parameters, represent a full set of data (Boslaugh 2013:83-84). Inferential statistical techniques are used to ensure that the properties of sample populations, referred to as statistics, accurately (but not perfectly, due to sampling errors) represent populations. Inferential statistics, as the term suggests, makes inferences (decisions, estimates, predictions, or generalisations) about a population based on the information contained in a subset or sample of that population (Boslaugh 2013:45-46).

Table 3.1 Research design

Research paradigm	Positivism
Research methodology	Quantitative
Research design	Observational
Research methods	Descriptive statistics and performance metrics
Data source	Secondary data - historical stock prices

Table 3.1 above summarises the research design of this study. It is quantitative and observational, making use of descriptive statistics and performance metrics based on secondary stock price data obtained from Bloomberg (Bloomberg 2022).

3.4 DATA COLLECTION

Historical stock price data that covers the 13-year period from January 2009 to December 2021 obtained via a Bloomberg Professional Services subscription are analysed. Data were collected from the end of 2006 (15 years). The 250-day analysis required a two-year lead period (2007 and 2008) to calculate the Volatility Score (volatility of changes in momentum, as opposed to changes in price). A full set of results that includes the 250-day scores, therefore, was available from 2009 onwards. The initial 15-year period was selected to obtain a sufficient but manageable amount of data.

The price data include all common stocks (ordinary shares) listed on the Johannesburg Stock Exchange (JSE), Toronto Stock Exchange (TSX) and the TSX Venture Exchange (TSXV) during this period with a minimum trading history of 24 months. These exchanges were chosen to respectively represent an emerging market (JSE), a developed market (TSX), and a venture market (TSXV). All delisted stocks during this period were eligible for analysis, thereby controlling for survivorship bias affecting the results. A delisted stock remains in the dataset and, if included in a momentum index, exits at the end of the delisting month at its final closing price.

The customised model was calibrated (as opposed to optimised) on the South African market (in-sample data) and applied to the two other markets (out-of-sample data) with the same parameter settings. Apart from allowing a direct comparison between the three markets, the model was validated on the out-of-sample data.

3.4.1 Delisted stocks

Breaching the listing requirements of an exchange may result in a suspension and the subsequent delisting of a company. Taking a company private also results in a stock delisting from a public exchange. However, the main reasons for companies delisting are mergers or acquisitions and financial distress. Companies that delist due to mergers (or acquisitions) typically experience positive momentum in the pre-merger period. Companies that delist due to financial distress (bankruptcy) may experience a period of negative momentum. Eisdorfer (2008) showed that momentum strategies suffer from delisting drifts and delisting returns. The delisting effect is largely attributed to bankruptcies during the holding period, while mergers have a minor effect on momentum profits (Eisdorfer 2008:177). Huynh and Smith (2017:157) confirmed the delisting effect in the Australian market.

Comprehensive delisting data may be missing or difficult to process and there is no agreed-on method to calculate the returns for delisted stocks, according to Li, Wang, Huang and Hoi (2018:1419). O'Keeffe and Gallagher (2017) recorded stocks that delisted during an inclusion period at a zero price at the time of delisting if due to financial distress or at the acquired price when delisting due to a merger or acquisition. Any analysis may not truly reflect the actual decisions faced by investors when managing a portfolio (O'Keeffe & Gallagher 2017:4719).

3.4.2 Adjustments

Several corporate actions affect the recorded share price of a listed company at different points in time. In between these regular or occasional actions, a series of prices may require adjustment to align and span across multiple periods. The modelling of momentum requires the use of adjusted data to maintain consistency in price per share over time. Therefore, the set of data used in an analysis must contain a uniform series of historical prices for each stock across different periods. Table 3.2 summarises the possible adjustments to historical time-series data used in backtesting and the analysis of different investing strategies.

Table 3.2 Data adjustments

Unadjusted data	Historical stock price and volume data as recorded on the actual trading day in the past.
Adjusted data	Historical data adjusted to reflect stock splits or consolidations (reverse splits), stock dividends or bonus shares and rights issues/offerings – corporate actions that alter the number of outstanding shares of a listed company. An unbundling (spin-off or spin-out) is handled in the same manner as the stock price of a company issuing a cash dividend in that its value falls by the value of the spin off.
Dividend-adjusted data	Historical prices adjusted retrospectively with any cash dividends paid to shareholders since listing on an exchange, in addition to all other corporate actions.

The overall effect of cash dividends on a stock price series will depend on the frequency and size of the dividends as well as the timeframe and duration of backtesting (Harris 2018). The choice of adjusted versus dividend-adjusted data may, therefore, have a limited effect on backtest results. However, to maintain consistency with actual market prices, adjusted data are preferred to dividend-adjusted data when modelling momentum – refer to Table 3.2. This study used adjusted data and did not adjust for cash dividends to exclude a possible dividend-induced upward drift in the analysis, as described below.

Adjusting for dividends could result in a dividend-induced upward drift in a stock price series, effectively creating artificial momentum. Therefore, a positive cumulative return on a dividend-adjusted price series may only reflect the upward drift from incorporating future price changes retrospectively while the actual unadjusted series may not exhibit a strong uptrend. This dividend-induced drift distorts reality and may have a spoiling effect on momentum models with past results depending on future dividend adjustments (Harris 2015). When a drift is constantly introduced in a price series it changes the actual levels where momentum cycles could have been entered in the past. Adjusting for stock splits, on the other hand, removes any gaps in historical time-series data to maintain consistency and ensure a uniform series of historical prices for each stock across different periods. Dividend-adjusted data are useful for calculating the total returns of investment portfolios (Harris 2011). It is not the goal of this study to construct investment portfolios and account for dividends or trading costs. This study attempts to isolate the momentum in price, explicitly excluding cash dividends.

3.5 RETURN CALCULATIONS

All return calculations and results are based on natural log returns and converted to geometric returns where required, as shown in the following set of equations.

$$\bar{r} = \frac{1}{n}\sum_{t=1}^{n} LN\left(\frac{P_t}{P_{t-1}}\right) \tag{3.1}$$

$$CPGR = (e^{\bar{r}} - 1) \times 100 \tag{3.2}$$

$$CTGR = (e^{\bar{r} \times n} - 1) \times 100 \tag{3.3}$$

$$CAGR = (e^{\bar{r} \times n \div a} - 1) \times 100 \tag{3.4}$$

Where: $\bar{r}$ is the average log return per period
n is the number of periods
t is a point in time
a is the total period in years
n ÷ a is 250 days, 50 weeks, 12 months, or 1 year
P_t is the price at time t
P_{t-1} is the price at time t-1
CPGR is the compound period growth rate
CTGR is the compound total growth rate
CAGR is the compound annual growth rate

The advantage of using logarithmic (continuously compounded) returns is that they are additive. The sum of independent normally distributed random variables is normal. Assuming that log returns are independent and normally distributed, then the logarithm of the compounding return is normally distributed (Dunbar 2019).

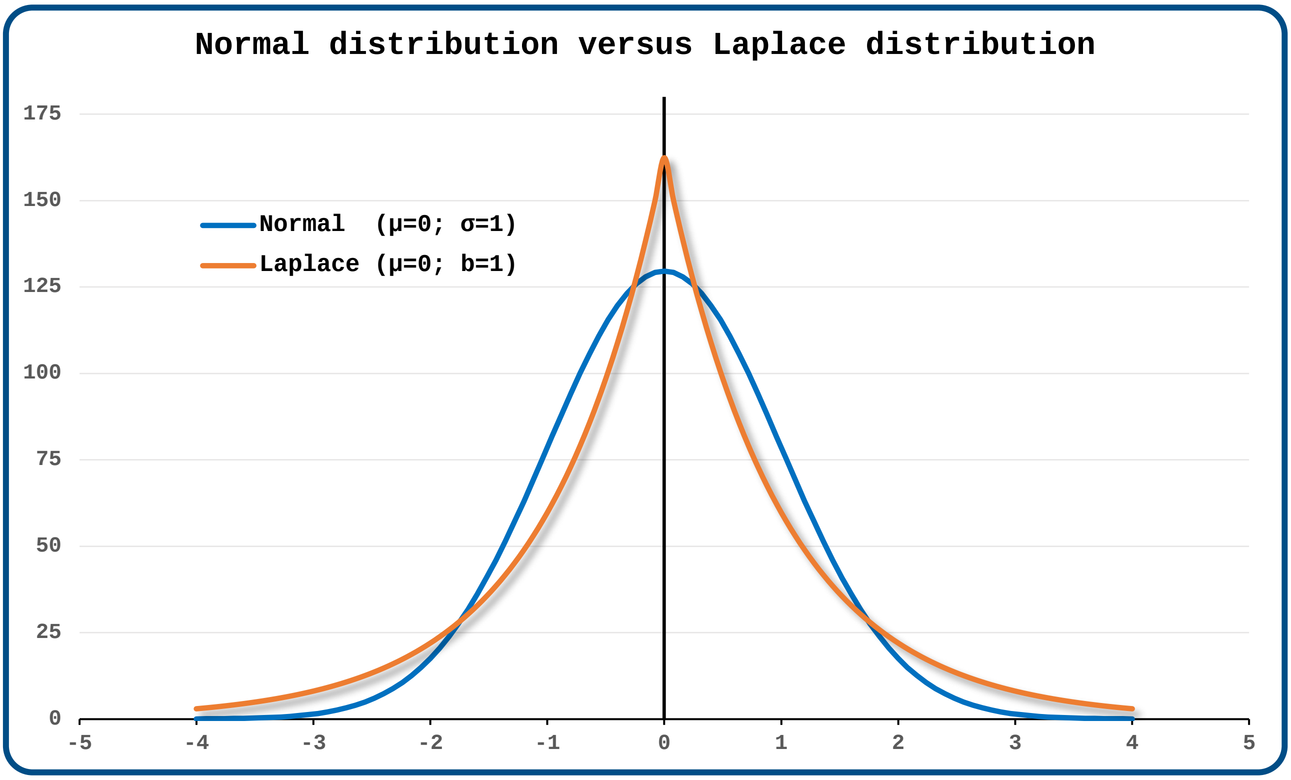

Figure 3.1 Log-return distributions

The assumption that the log returns of stock prices are normally distributed, is convenient when performing statistical analysis (refer to Section 3.9.4). Mota (2012:56), however, showed that the assumption of normality fails as sample sizes increase with the frequency (from monthly to weekly to daily) of observations. The Laplace distribution, shown in Figure 3.1, with its high central peak, narrow upper shoulders and heavy tails provides a better fit for log returns than the normal distribution (Harckbart 2019; Toth & Jones 2019).

The Laplace distribution is symmetric about its location parameter (median) with the scale parameter (beta) determining its profile. The normal distribution is completely defined by its mean and standard deviation. Kotz, Kozubowski and Podgórski (2001) introduced a generalisation of the symmetric Laplace distribution to capture the peakedness and fat-tailedness (high kurtosis) as well as skewness observed with stock price data, the asymmetric Laplace. However, for log prices it was found that the lognormal process (referred to as a geometric or exponential Brownian motion) is strongly rejected in favour of the symmetric Laplace motion while the asymmetric Laplace (AL) motion makes no significant improvement in fit over the symmetric version (Kotz, Kozubowski & Podgórski 2001:296).

3.6 MOMENTUM MODEL

The customised momentum model has four parameters: Momentum Score (MS); Volatility Score (VS); Quality Score (QS); and Activity Score (AS). It is based on the principle of entering momentum cycles early and exiting as late as possible. The primary parameter measures momentum by assigning a momentum score to each stock across the term structure. The volatility score measures the standard deviation of changes in momentum (acceleration and deceleration) to exclude stocks with volatile changes in momentum. Momentum is considered to have quality when positive changes in price account for the majority of non-zero trading days in a period, measured with the quality score. Related to the quality score, the activity score measures activity as the ratio of positive changes in price to trading days (non-zero days plus zero-return days) in a period to ensure a minimum level of trading activity or liquidity. Stocks are filtered on these parameters and classified as high momentum stocks when they score above the minimum (momentum, quality, and activity) and below the maximum (volatility) parameter settings. Stocks are not sorted or ranked on any of the parameter scores. These parameters can be calibrated to suit a particular equity market. In this study, the model parameters were calibrated on price data from the Johannesburg Stock Exchange (JSE).

The model was customised from concepts found during the literature review – refer to Chapter 2, Section 2.4.1 (exponential curve fitting), Section 2.4.2 (evolution of prices), and Section 2.4.3 (changes in momentum). The momentum model and its assumptions are covered in detail in Chapter 4. The custom momentum index, described in the next section, quantifies the actual performance of the model.

3.7 INDEX CONSTRUCTION

The momentum index is constructed as equal-weighted in that new members enter at the average weight of the current members (Equation 3.6). The index is updated monthly, and the number of members is variable. The individual weights of the remaining members are adjusted for the number of additions, and the total weight of any deletions is distributed equally between members (Equation 3.5). Remaining members are allowed to retain the gains or losses from previous changes in price.

$$\mathbf{AdjW_{rm}} = \mathbf{W_{rm}} \times \left(\frac{\#\mathbf{rm}}{\#\mathbf{cm}}\right) + \left(\frac{\sum \mathbf{W_{xm}}}{\#\mathbf{cm}}\right) = \left(\mathbf{W_{rm}} + \frac{\sum \mathbf{W_{xm}}}{\#\mathbf{rm}}\right) \times \left(\frac{\#\mathbf{rm}}{\#\mathbf{cm}}\right) \quad (3.5)$$

$$\mathbf{W_{nm}} = \frac{\sum \mathbf{AdjW_{rm}}}{\#\mathbf{rm}} = \frac{\sum \bar{\mathbf{W}}_{\mathbf{cm}}}{\#\mathbf{cm}} \quad (3.6)$$

Where: $\mathbf{AdjW_{rm}}$ is the adjusted weight of a remaining member
$\mathbf{W_{rm}}$ is the weight of a remaining member
$\mathbf{W_{xm}}$ is the weight of an exiting member
$\mathbf{W_{nm}}$ is the weight of a new member
$\bar{\mathbf{W}}_{\mathbf{cm}}$ is the average weight of the current members
#rm is the number of remaining members
#cm is the number of current members

The custom momentum index maintains a relatively active position over a true equal-weighted or unweighted design, which would normally reset all the member weights to the average weight when updated (Taljaard & Maré 2019).

3.8 DRAWDOWN ANALYSIS

A drawdown analysis focuses on the potential for sudden large losses in value and the likely time to recovery. It records the size and speed of previous declines in index levels, as well as the time it required to return to past highs.

Table 3.3 Drawdown analysis

Drawdown	Percentage decline from high (peak) to low (valley)
Maximum drawdown (MDd)	Largest percentage decline from peak to valley
Maximum drawdown period	Number of days from peak to valley (largest decline)
Maximum drawdown recovery	Number of days back to original peak (valley to peak)
Average drawdown	Average percentage decline (peak to valley)
Maximum duration	Maximum duration of a drawdown (peak to peak)
Average duration	Average duration of drawdowns (peak to peak)
Drawdown ratio (CAGR/MDd)	Compound annual growth rate relative to maximum drawdown

Choi (2021) showed that maximum drawdown and its subsequent recovery are important drivers for the profitability of momentum strategies. Maximum drawdown is closely related to price momentum, affecting its direction and magnitude. Maximum drawdown is part of the mean-reversion process in stock prices, which alternates between momentum and mean reversion depending on the size of the drawdown (Choi 2021).

3.9 STATISTICAL ANALYSIS

Statistics are quantitative measures derived from data and when classified by function, there are descriptive statistics and inferential statistics. Descriptive statistics make use of summary statistics to describe and analyse sets of data. Inferential statistics generalise or extend findings based on subsets (samples) to full sets of data (populations) and make comparisons between subsets of data.

3.9.1 Descriptive statistics

A summary statistic provides a single score to represent a set of observations. Summary statistics identify typical values (central tendency) for the observations and the size of possible deviations (variability) from those values. Descriptive statistics is the process of using and analysing summary statistics (Lee 2020). Table 3.4 below shows the summary statistics included in this study.

Table 3.4 Summary statistics

Mean	Mean or average of all values in the dataset, sensitive to extreme values or outliers.
Standard error	Indication of the reliability of the mean when drawing a sample from the population.
Median	Middle observation or value when arranging data in ascending or descending order.
Standard Deviation	Square root of the variance, a standardised measure commonly referred to as volatility.
Sample variance	Average of the squared deviations between each individual value and the mean of a sample.
Kurtosis	Provides information on the tails (extremes or outliers) of a distribution in reference to a normal distribution.
Skewness	Measures the degree of symmetry (or asymmetry) of a distribution based on the concentration of its values.
Range	Spread between the highest (maximum) and lowest (minimum) value in the distribution.
Coefficient of variation	Relative standard deviation indicating the extent of variability in relation to the mean.

The mean and standard deviation can describe most sets of data sufficiently. However, skewness and kurtosis provide detail about the distribution of data. Skewness indicates whether a distribution is symmetrical or skewed to either the lower values or the higher values. A distribution with more values smaller than the mean is positively skewed with a longer right tail. A distribution with more values greater than the mean is negatively skewed with a longer left tail. Kurtosis is a measure of the degree to which values cluster around the mean and in the tail of a distribution – that is, its peakedness and tailedness. Datasets with high kurtosis tend to have a distinct peak near the mean and heavy or fat tails with many outliers. Values cluster around the mean and in the tails. Datasets with low kurtosis tend to have a flat peak with thin tails. Values are more evenly dispersed with fewer values near the mean and in the tails (Lee 2020).

3.9.2 Inferential statistics

There are two basic types of statistical inference – namely, estimation and hypothesis testing. Each inferential statistic is associated with a probability distribution, its sampling distribution. The shape of a sampling distribution is determined by its sample size (degrees of freedom) but can be approximated by well-known distributions such as the standard normal or chi-square distributions.

A point estimator is a statistic that estimates the value of an unknown population parameter. However, the exact location of any particular statistic within its sampling distribution is unknown and interval estimation calculates a range of possible values with a specific probability (confidence interval) of capturing the actual value of a population parameter (Scott 2020).

Hypothesis testing draws inferences or conclusions about the values of population parameters based on the sample statistics estimating those parameters. These tests either compare population parameters or find some relationship between variables. The null hypothesis usually states that no significant difference or relationship exists. The decision either to reject or not to reject the null hypothesis is reached by comparing the test statistic (or p-value) to the critical value (or alpha) based on a specific alpha or level of significance, which is the probability of rejecting the null hypothesis when it is true (Scott 2020).

3.9.3 Analysis of variance

A one-way or one-factor analysis-of-variance (ANOVA) design splits a set of observations from a single factor into different groups based on certain outcomes. The differences in means between these groups are assessed using an F-test to compare the mean squares from the analysis. The total variation in the observations is divided into a part due to differences between group means (between-groups sum of squares) and a part due to the differences between observations in the same group (within-groups or residual sum of squares). The between-groups and within-groups mean squares will be the same if the means of the different groups are the same, yielding an F-statistic (ratio of between-groups to within-groups mean squares) near one. The F-test assumes that the different groups or samples have normal distributions and share a common variance (RealStats 2022).

The F-statistic (ratio of two variances) is relatively robust to violations of normality if the sample sizes are equal and sufficiently large, provided their distributions are symmetrical or at least similar in shape (e.g., negatively skewed). However, it is not so robust to violations of homogeneity of variances. Generally, the F-test will be valid if the sample sizes are equal and the ratio of the largest to smallest variance is less than four. Smaller differences in variances can invalidate the F-test if the sample sizes are unequal. Therefore, more attention needs to be paid to unequal variances than to the non-normality of data (RealStats 2022). The presence of outliers can also cause problems (see Section 3.9.6).

The Tukey HSD (honestly significant difference) test is the follow-up or post-hoc test to the one-way ANOVA test when the F-test indicates the existence of a significant difference between the means of some groups. The one-way ANOVA only detects that at least two groups are different, but not which ones. Tukey HSD compares the difference between each pair of means and adjusts the p-value for these multiple comparisons (NCSS 2022). The q-statistic (essentially a modified t-statistic that corrects for multiple comparisons) for each pairing is compared to the Studentised Range critical value for q as determined by the number of groups (k), degrees of freedom (df), and alpha (α). A large q-statistic that exceeds the q critical value (or p-value < alpha) rejects the null hypothesis of no significant difference between the means or averages of a particular pairing. The Tukey HSD/Kramer version is performed when the number of observations in the different groups is unequal (RealStats 2022).

When dealing with groups where the variances are heterogeneous or unequal, apart from possibly performing log or square root transformations of the data, Welch’s test of means (a modified ANOVA test) is often suggested. The Games-Howell post-hoc test for identifying which pairings are different follows Welch’s ANOVA when group variances are heterogenous, especially when group sizes are not equal. Welch’s test adjusts the denominator (within-groups variance) of the F-ratio, to have the same expectation (i.e., mean square) as the numerator (between-groups variance) when the null hypothesis of no significant differences is true, despite the unequal within-group variances (XLSTAT 2022).

The Games-Howell test uses a different pooled variance for each pair instead of the common pooled variance from the Tukey-Kramer test. The Studentised Range q-critical values are determined by the degrees of freedom associated with each pairing as defined by a two-sample t-test with unequal variances. A q-statistic that exceeds its q-critical value (or p-value < alpha) rejects the null hypothesis of no significant difference between the means or averages of that pairing. When the group variances are similar, there is not much of a difference between the results from the Games-Howell and Tukey-Kramer tests (RealStats 2022). Unless the standard deviations of the different groups are very similar, Welch’s ANOVA is preferred to the one-way ANOVA test. It is less powerful for homoscedastic data, but it is more accurate for unbalanced heteroscedastic data (GraphPad 2022).

The Kruskal-Wallis H test (with its post-hoc Nemenyi test) is a non-parametric alternative to the ANOVA tests when normality does not hold, making no assumptions about the shape of the underlying distribution. It only requires that the distribution of each group can be arranged in a particular order and that these distributions are identical except for location (central value or position), thereby also assuming homogeneous variances. Kruskal-Wallis compares the medians (not means or averages) of the different groups, and its H-statistic must be corrected for repeated values or ties (NCSS 2022).

3.9.4 Normality and symmetry

Several statistical tests rely on the assumption of normality, but the violation of this assumption should not cause major issues with large sets of data, suggesting that parametric tests can be used even when data are not normally distributed. As stated by Ghasemi and Zahediasl (2012:486), the distribution of observations can be ignored with a large set of data because the distribution tends to be normal regardless of its shape. It is also noted that as the number of observations increases, normality parameters become more restrictive, making it harder to statistically find that the data are normally distributed. Therefore, for large sets of data, normality testing becomes less important. However, it remains insightful to know to what extent a set of data deviates from normality. Lack of symmetry (skewness), and peakedness or tailedness (kurtosis) are the two main ways in which a distribution can deviate from normal (Ghasemi & Zahediasl 2012:487). Even though normality implies symmetry, data can be symmetric without being normally distributed (RealStats 2022). Apart from reviewing the distribution graphically via histograms, boxplots, and quantile-quantile (Q-Q) plots, normality tests such as the D'Agostino-Pearson test and the Shapiro-Wilk test can indicate whether data are normally distributed.

The D'Agostino-Pearson Omnibus (K-squared) test combines its skewness and kurtosis tests to produce a single universal or omnibus statistic. This test calculates skewness and kurtosis to quantify how far the distribution is from normal in terms of asymmetry and shape. It squares and sums the statistics from these two tests to produce a single DA-statistic (K-squared) and p-value. The distribution of this test is approximately chi-square (right-tailed, shaped by the chosen alpha level and degrees of freedom) with two degrees of freedom under the null hypothesis that the dataset is normally distributed. A large DA statistic that exceeds the Chi-square critical value (or p-value < alpha) rejects the null hypothesis of normality (NCSS 2022).

The Shapiro-Wilk (SW) test is the ratio of two estimates for the variance of a normal distribution based on a random sample of observations. The numerator is proportional to the square of the best linear estimator of the standard deviation, and the denominator is the sum of squares of the observations about the sample mean. The closer the W-statistic is to one (p-value > alpha), the more normal the sample. The original SW-test is limited to 50 observations, but the expanded test or Royston version uses approximations, accommodating an unlimited number of observations (NCSS 2022).

As mentioned, the F-statistic (ratio of two variances) is relatively robust to violations of normality if the sample sizes are equal and sufficiently large. Relatively small differences in variances can invalidate the F-test if the sample sizes are unequal. Therefore, more attention needs to be paid to heterogeneous or unequal variances than to the non-normality of data (RealStats 2022).

3.9.5 Homogenous variances

The Levene test for equality of variances (homogeneity) does not assume that all populations are normally distributed. If the p-value exceeds the level of significance or alpha (i.e., the probability of rejecting the null hypothesis when it is true), the null hypothesis cannot be rejected, and it is concluded that there is not a significant difference between the variances. Levene’s test calculates the p-value for the means, medians, and trimmed means. These three alternatives determine the robustness and power of the test. Robustness refers to the ability of the test to not falsely detect unequal variances when the underlying data are not normally distributed. Power refers to the ability of the test to detect unequal variances when the variances are in fact unequal. The trimmed mean is suggested when the underlying data have a heavy-tailed distribution and the median when the underlying data have a skewed distribution. The mean provides the best power for symmetric, moderate-tailed distributions. While the optimal choice ultimately depends on knowledge of the underlying distribution of the data, the median provides robustness against many types of non-normal data while retaining good power (RealStats 2022).

3.9.6 Outliers

Outliers can result from data input errors, or just be true outliers (extreme values) that contain important information about the full set of data. Any input errors must be corrected but removing outliers or replacing them with either the mean (retaining the original mean of the set), median or mode generally results in additional outliers due to the smaller standard deviation of the post-adjustments dataset. Care must be taken when using regression models for forecasting, which requires generalisation, as outliers may degrade these models. True outliers, however, are not removed or adjusted when simply describing datasets. Outliers provide deeper insights into data when resulting from the same processes or methods as the central values (Aggarwal 2017).

Grubbs’ test is used to find a single outlier, either the minimum or maximum value, in a normally distributed set of data (except possibly for the outlier). The Extreme Studentised Deviate (ESD) test is a generalisation of Grubbs’ test for finding more than one outlier based on an upper bound of potential outliers. The Grubbs/ESD test assumes normality and, therefore, requires a sufficiently large set of data that follows an approximately normal distribution (RealStats 2022).

Ratio G is calculated as the difference between the outlier and the mean divided by the standard deviation from all values, including the outlier. If the calculated G value exceeds the critical G, the value is considered an outlier at a certain level of significance. The critical value for G is calculated from the critical value of the t-distribution with $(n-2)$ degrees of freedom and a level of significance (alpha) adjusted for the number of observations (n).

3.9.7 Correlation

The Pearson (product-moment) correlation coefficient is the most common correlation measure. Correlation is a unitless measure, which shows the linear association between two time-series and ranges between negative-one and positive-one. The Spearman rank correlation and Kendall's Tau are non-parametric alternatives when data is not normally distributed or when the presence of outliers gives a distorted picture of the association. Correlation coefficients are often reported alone but can also be used with hypothesis tests and confidence intervals (NCSS 2022).

Correlation can be quantified. Cointegration, to follow, can only be identified but its magnitude cannot be quantified. Working with financial time-series data, log returns (not price levels) are used for measuring correlation, while cointegration is based on price levels (log prices). Correlation is a shorter-term concept while cointegration describes a long-run association between time series (RealStats 2022).

3.9.8 Cointegration

Cointegration would indicate that, although two series move independently, the average spread or difference between them should remain relatively constant or evolve gradually over time. The price series will correct for any short-term deviations to revert to the mean spread. Although correlation and cointegration both describe some underlying association between time series, the two properties are not synonymous. It is possible for two series to have a strong correlation but no cointegration and vice versa. Cointegration does not say anything about the correlation between the time series. If two time-series are cointegrated, there exists some stationary linear combination of both series (RealStats 2022).

Cointegration relates to the concepts of unit root and stationarity. A unit root refers to a stochastic or unpredictable component in a time series. A time series with a unit root is non-stationary with a changing mean and variance. These non-stationary processes can either be with or without a drift (constant change) and with a trend (variable change), which causes the statistics of a time series to change over time and not revert to long-term averages (XLSTAT 2022).

Two times series are potentially cointegrated when neither price series is stationary, but their first differences are stationary. Cointegration requires that the time series consisting of the residuals from the linear regression of one time series on the other is stationary. The Engle-Granger method is a three-step process that uses the Augmented Dickey-Fuller (ADF) test for stationarity in log prices as well as log returns (first differences), and a modified version of the ADF test (different table of critical values) to test for stationarity of the residuals. The decision rule is based on the tau-statistic and its corresponding tau-critical value, or the p-value and its alpha (RealStats 2022).

3.10 SUMMARY

This study is observational in design, based on the distinction between observational and experimental when doing quantitative research related to equity investing. Descriptive statistics and various performance metrics will evaluate the momentum model via the custom index.

The statistical tests for normality, symmetry, homogeneity of variances, and outliers will be used to validate the sets of time-series data. An analysis of variance will determine whether there are statistically significant differences between the average parameter scores of the three markets and the different cycle types. The different market and momentum indices will be analysed in terms of correlation and possible cointegration. Refer to annexures A to D.

Details on the parameters and the assumptions of the momentum model are to follow in Chapter 4. By recording the outcomes from mechanically entering and exiting the momentum cycles identified by the customised model, a mix of false, neutral, negative, and positive cycles will be generated to profile the momentum in a particular market.

STATISTICAL EQUATIONS

Welch's test of means allowing for unequal group variances (Welch's ANOVA)

$$W^* = \frac{\sum w_i(\bar{Y}_i - \hat{\mu})^2 / (K - 1)}{1 + \left[2(K - 2)/(K^2 - 1)\right] \sum h_i}$$

Where:

$$w_i = \frac{n_i}{s_i^2} \qquad W = \sum w_i \qquad \hat{\mu} = \sum w_i \bar{Y}_i / W$$

$$h_i = \frac{(1 - w_i / W)^2}{(n_i - 1)} \qquad f = \frac{K^2 - 1}{3\sum h_i} \qquad s_i^2 = \frac{1}{n_i - 1}\sum_{j=1}^{n_i}(Y_{ij} - \bar{Y}_i)^2$$

The value of W* is compared to an F distribution with K-1 and f degrees of freedom.

Games-Howell multiple comparison procedure (MCP) or post-hoc test

$$\frac{|\bar{y}_i - \bar{y}_j|}{\sqrt{\frac{1}{2}\left(\frac{s_i^2}{n_i} + \frac{s_j^2}{n_j}\right)}} \geq q_{\alpha,k,v} \qquad \text{with} \quad df = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_Y^2}{n_Y}\right)^2}{\frac{\left(\frac{s_x^2}{n_x}\right)^2}{n_x - 1} + \frac{\left(\frac{s_Y^2}{n_Y}\right)^2}{n_Y - 1}}$$

Levene test of homogeneity (equal variance)

$$w = \frac{(N - K)\sum_{i=1}^{K} n_i(Z_i - \bar{Z})^2}{(K - 1)\left(\sum_{i=1}^{K}\sum_{j=1}^{n_i}(Z_{ij} - \bar{Z}_i)^2\right)}$$

Where:

$$Z_{ij} = |Y_{ij} - \bar{Y}_i| \qquad \bar{Z}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} Z_{ij}$$

$$\bar{Z} = \frac{1}{N}\sum_{i=1}^{K}\sum_{j=1}^{n_i} Z_{ij} \qquad \bar{Y}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} Y_{ij}$$

Follows the F distribution with K-1 and N-K degrees of freedom.

Grubbs' test for a single outlier

$$G = \max_{i=1...n}\frac{|X_i - \bar{X}|}{s}$$

$$G_{crit} = \frac{(n - 1)t_{crit}}{\sqrt{n(n - 2 + t_{crit}^2}}$$

Skewness normality test

$$z_s = b \times LN(u + \sqrt{u^2 + 1})$$

Where:

$$c = \frac{3(n^2 + 27n - 70)(n + 1)(n + 3)}{(n - 2)(n + 5)(n + 7)(n + 9)} \qquad w^2 = -1 + \sqrt{2(c - 1)}$$

$$a = \sqrt{\frac{w^2 - 1}{2}} \qquad b = \frac{1}{\sqrt{LN(w)}}$$

$$u = a \times skewp\sqrt{\frac{(n + 1)(n + 3)}{6(n - 2)}}$$

Kurtosis normality test

$$z_k = \frac{1 - r - v^{1/3}}{\sqrt{r}}$$

Where:

$$d = \sqrt{\frac{(n + 1)^2(n + 3)(n + 5)}{24n(n - 2)(n - 3)}}$$

$$e = \frac{6(n^2 - 5n + 2)}{(n + 7)(n + 9)} \times \sqrt{\frac{6(n + 3)(n + 5)}{n(n - 2)(n - 3)}}$$

$$f = 6 + \frac{8}{e}\left(\frac{2}{e} + \sqrt{1 + \frac{4}{e^2}}\right) \qquad g = d\left(kurtp - \frac{3(n - 1)}{n + 1}\right) \times \sqrt{\frac{2}{f - 4}}$$

$$v = \frac{1 - \frac{2}{f}}{1 + g} \qquad r = \frac{2}{9f}$$

D’Agostino-Pearson Omnibus (K-squared) test

$$K^2 = z_s^2 + z_k^2$$

Follows a chi-square distribution with 2 degrees of freedom.

Pearson correlation coefficient

$$r = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2\sum_{i=1}^{n}(Y_i - \bar{Y})^2}}$$

REFERENCES

Aggarwal, C.C. 2017. *Outlier analysis*. 2nd edition. Cham, Switzerland: Springer. DOI: 10.1007/978-3-319-47578-3.

Bloomberg. 2022. Bloomberg Terminal – Bloomberg Professional Services subscription. New York: Bloomberg Finance L.P.

Boslaugh, S. 2013. *Statistics in a nutshell: a desktop quick reference*. 2nd edition. Sebastopol, California: O’Reilly Media, Inc.

Choi, J. 2021. Maximum drawdown, recovery, and momentum. *Journal of Risk and Financial Management*, 14(11):1–25. DOI: 10.3390/jrfm14110542.

Dunbar, S.R. 2019. *Mathematical modeling in economics and finance: probability, stochastic processes, differential equations*. Providence, Rhode Island: MAA Press, American Mathematical Society (Volume 49).

Eisdorfer, A. 2008. Delisted firms and momentum profits. *Journal of Financial Markets*, 11(2):160–179. DOI: 10.1016/j.finmar.2007.12.001.

Ghasemi, A. & Zahediasl, S. 2012. Normality tests for statistical analysis: a guide for non-statisticians. *International Journal of Endocrinology and Metabolism*, 10(2):486–489. DOI: 10.5812/ijem.3505.

GraphPad. 2022. Prism 9 statistics guide [Website]. Dotmatics. Available at: https://www.graphpad.com.

Harckbart, G. 2019. Laplace versus the normal distribution for daily stock market returns. *SSRN Electronic Journal*, Online. DOI: 10.2139/ssrn.3479681.

Harris, M. 2011. *Chaos in technical analysis and backtesting: close vs adjusted close*. *Price Action Lab Blog [Online]*. Michael Harris. Available at: https://www.priceactionlab.com/Blog/2011/03/chaos-in-technical-analysis-and-backtesting-part-i-close-vs-adjusted-close/ (Accessed: 7 January 2021).

Harris, M. 2015. *Fooled by dividend-induced upward drift*. *Price Action Lab Blog [Online]*. Michael Harris. Available at: https://www.priceactionlab.com/Blog/2015/08/dividend-induced-upward-drift/ (Accessed: 7 January 2021).

Harris, M. 2018. *Effect of dividend adjustments on portfolio backtests*. *Price Action Lab Blog [Online]*. Michael Harris. Available at: https://www.priceactionlab.com/Blog/2018/07/effect-of-dividend-adjustments-on-portfolio-backtests/ (Accessed: 8 January 2021).

Huynh, T.D. & Smith, D.R. 2017. Delisted stocks and momentum: evidence from a new Australian dataset. *Australian Journal of Management*, 42(1):140–160. DOI: 10.1177/0312896214565118.

Kotz, S., Kozubowski, T.J. & Podgórski, K. 2001. *The Laplace distribution and generalizations: a revisit with applications to communications, economics, engineering, and finance*. Boston, Massachusetts: Birkhäuser Boston. DOI: 10.1007/978-1-4612-0173-1.

Leavy, P. 2017. *Research design: quantitative, qualitative, mixed methods, arts-based, and community-based participatory research approaches*. New York City, New York: The Guilford Press.

Lee, J. 2020. Statistics, descriptive. in Kobayashi, A. (ed.) *International Encyclopedia of Human Geography Volume 13*. 2nd edition. Amsterdam, Netherlands: Elsevier. DOI: 10.1016/B978-0-08-102295-5.10428-7.

Li, B., Wang, J., Huang, D. & Hoi, S.C.H. 2018. Transaction cost optimization for online portfolio selection. *Quantitative Finance*, 18(8):1411–1424. DOI: 10.1080/14697688.2017.1357831.

Mertens, D.M. 2020. *Research and evaluation in education and psychology: integrating diversity with quantitative, qualitative, and mixed methods*. 5th edition. Los Angeles, California: SAGE Publications Inc.

Mota, P.P. 2012. Normality assumption for the log-return of the stock prices. *Discussiones Mathematicae Probability and Statistics*, 32(1–2):47–58. DOI: 10.7151/dmps.1143.

Mukherji, P. & Albon, D. 2018. *Research methods in early childhood: an introductory guide*. 3rd edition. London: SAGE Publications Ltd.

NCSS. 2022. Data analysis and graphics documentation. NCSS Statistical Software. Available at: https://www.ncss.com.

Novikov, A.M. & Novikov, D.A. 2013. *Research methodology: from philosophy of science to research design*. Boca Raton, Florida: Chapman & Hall / CRC Press (Communications in Cybernetics, Systems Science and Engineering). DOI: 10.1201/b14562.

O’Keeffe, C. & Gallagher, L.A. 2017. The winner-loser anomaly: recent evidence from Greece. *Applied Economics*, 49(47):4718–4728. DOI: 10.1080/00036846.2017.1293786.

RealStats. 2022. Real statistics using Excel [Website]. Charles Zaiontz. Available at: https://www.real-statistics.com.

Rezigalla, A.A. 2020. Observational study designs: synopsis for selecting an appropriate study design. *Cureus*, Online. DOI: 10.7759/cureus.6692.

Scott, D.M. 2020. Statistics, inferential. in Kobayashi, A. (ed.) *International Encyclopedia of Human Geography Volume 13*. 2nd edition. Amsterdam, Netherlands: Elsevier. DOI: 10.1016/B978-0-08-102295-5.10429-9.

Strang, K.D. (ed.) 2015. *The Palgrave handbook of research design in business and management*. New York City, New York: Palgrave Macmillan. DOI: 10.1057/9781137484956.

Taljaard, B. & Maré, E. 2019. Considering the use of an equal-weighted index as a benchmark for South African equity investors. *South African Actuarial Journal*, 19(1):53–70. DOI: 10.4314/saaj.v19i1.3.

Toth, D. & Jones, B. 2019. Against the norm: modeling daily stock returns with the Laplace distribution. *Cornell University Open Access*, Online. Available at: http://arxiv.org/abs/1906.10325.

Winton. 2022. *Experiment and observation in quantitative investment*. *Winton Research [Online]*. Winton Capital Management Limited. Available at: https://www.winton.com/research/experiment-and-observation-in-quantitative-investment-management (Accessed: 3 June 2022).

XLSTAT. 2022. A complete statistical add-in for Microsoft Excel [Website]. Addinsoft. Available at: https://www.xlstat.com.

CHAPTER FOUR

MOMENTUM MODEL

4.1 INTRODUCTION

The momentum profiles in this chapter originate from a customised model that uses a momentum term structure, displaying as a stepped visual profile for individual stocks. The term structure, in this instance, refers to six momentum terms of increasing length (measured in days) and comprises 60-day, 90-day, 125-day, 180-day, 210-day and 250-day momentum terms grouped into four different entry zones. The concept behind this model is to identify stocks relatively early in their respective momentum cycles via three successive term-structure periods of high momentum (i.e., an entry zone). A momentum cycle is an extended period of sustained momentum with clear entry and exit points. The model exits on the 250-day momentum parameter. The customised momentum model aims to enter momentum cycles early and exit as late as possible.

The model has four parameters – namely, a Momentum Score (MS), Volatility Score (VS), Quality Score (QS), and Activity Score (AS). Each parameter either has a maximum (VS) or a minimum (MS, QS and AS) setting. No attempt was made to optimise these parameters. Clenow (2015) advised against optimisation and simply apply the concept of momentum. Substantial differences in results from different parameter values would indicate that the overall concept of momentum is not stable. Optimisation requires continuous adjustments and refinements, confining a model to a particular equity market and period. Instead, the model was calibrated on the Johannesburg Stock Exchange (JSE) with the same parameter settings applied to the other two exchanges. The model was customised from concepts found during the literature review – refer to Chapter 2, Section 2.4.1 (exponential curve fitting), Section 2.4.2 (evolution of prices), and Section 2.4.3 (changes in momentum).

All the stocks listed on a particular exchange are eligible for selection. The investment universe is not predefined, and companies are not filtered or scanned on price, market capitalisation (size), liquidity or sector. The stocks identified by the model are not ranked or sorted on any of the parameter scores.

The identification and selection of stocks and the compilation of momentum profiles were performed using the Python Programming Language (Python 2022), the Python Data Analysis Library (Pandas 2022), and Microsoft Excel 365 (Excel 2022).

The next section provides the model specifications, followed by a section describing the four parameters of the model in more detail. Section 4.4 contains subsections on positive, negative, neutral, and false cycles. These subsections include the momentum profiles of selected companies to illustrate the different types, and the alternative outcomes of different exit rules with positive cycles.

4.2 MODEL SPECIFICATIONS

Momentum tends to follow a stepped pattern with shorter periods leading longer periods as illustrated in Table 4.1, which shows the ideal profile for entering the momentum cycle early and exiting it as late as possible.

Table 4.1 Generic momentum profile

Dates	MOM250	MOM210	MOM180	MOM125	MOM090	MOM060	Price
YYYY-MM-DD	< min	< min	< min	< min	< min	< min	cccc
YYYY-MM-DD	< min	< min	< min	< min	< min	≥ min	cccc
YYYY-MM-DD	< min	< min	< min	< min	≥ min	≥ min	cccc
YYYY-MM-DD	< min	< min	< min	≥ min	≥ min	≥ min	entry@z1
YYYY-MM-DD	< min	< min	≥ min	≥ min	≥ min	≥ min	entry@z2
YYYY-MM-DD	< min	≥ min	≥ min	≥ min	≥ min	≥ min	entry@z3
YYYY-MM-DD	≥ min	≥ min	≥ min	≥ min	≥ min	≥ min	entry@z4
YYYY-MM-DD	≥ min	≥ min	≥ min	≥ min	≥ min	≥ min	cccc
YYYY-MM-DD	≥ min	≥ min	≥ min	≥ min	≥ min	≥ min	cccc
YYYY-MM-DD	≥ min	≥ min	≥ min	≥ min	≥ min	≥ min	cccc
YYYY-MM-DD	≥ min	≥ min	≥ min	≥ min	≥ min	≥ min	cccc
YYYY-MM-DD	≥ min	≥ min	≥ min	≥ min	≥ min	< min	cccc
YYYY-MM-DD	≥ min	≥ min	≥ min	≥ min	< min	< min	cccc
YYYY-MM-DD	≥ min	≥ min	≥ min	< min	< min	< min	cccc
YYYY-MM-DD	≥ min	≥ min	< min	< min	< min	< min	exit@180
YYYY-MM-DD	≥ min	< min	< min	< min	< min	< min	exit@210
YYYY-MM-DD	< min	< min	< min	< min	< min	< min	exit@250
YYYY-MM-DD	< min	< min	< min	< min	< min	< min	cccc

The earliest entry would occur in Zone 1 (refer to Figure 4.1) when the model requires high momentum in three successive periods to confirm the formation of a momentum cycle. Zones 2 to 4 allow for later entries and irregular formation patterns.

Zone 1

MOM125				MOM090				MOM060			
MS	VS	QS	AS	MS	VS	QS	AS	MS	VS	QS	AS
20	1.5	48	35	20	1.5	48	35	20	1.5	48	35

Zone 2

MOM180				MOM125				MOM090			
MS	VS	QS	AS	MS	VS	QS	AS	MS	VS	QS	AS
20	1.5	48	35	20	1.5	48	35	20	1.5	48	35

Zone 3

MOM210				MOM180				MOM125			
MS	VS	QS	AS	MS	VS	QS	AS	MS	VS	QS	AS
20	1.5	48	35	20	1.5	48	35	20	1.5	48	35

MOM250				MOM210				MOM180			
MS	VS	QS	AS	MS	VS	QS	AS	MS	VS	QS	AS
20	1.5	48	35	20	1.5	48	35	20	1.5	48	35

Zone 4

Figure 4.1 Entry zones

The parameters are set uniformly for all periods (shown in Figure 4.1), but individual settings can vary by period. Shorter periods, in general, accommodate higher minimum quality (QS) and activity (AS) score settings. The volatility score (VS) maximum can be lowered for longer periods.

4.3 MODEL PARAMETERS

The primary parameter measures momentum by assigning a momentum score (MS) to each stock across the term structure. The volatility score (VS) measures the standard deviation of changes in momentum (acceleration and deceleration) to exclude stocks with volatile changes in momentum. Momentum is considered to have quality when positive changes in price account for the majority of non-zero trading days in a period, measured with the quality score (QS). Related to the quality score, the activity score (AS) measures activity as the ratio of positive changes in price to trading days (non-zero days plus zero-return days) in a period to ensure a minimum level of trading activity or liquidity. Stocks are filtered on these parameters and classified as high momentum stocks when they score above the minimum (momentum, quality, and activity) and below the maximum (volatility) parameter settings. Stocks are not sorted or ranked on the parameter scores.

These parameters were derived and customised from some of the concepts discussed in Chapter 2 (Price-based Momentum) – namely, the exponential regression slope (Momentum Score), volatility of acceleration (Volatility Score), the evolution of prices (Quality Score), and a proxy for liquidity (Activity Score).

4.3.1 Momentum score

Momentum is quantified by fitting an exponential curve (Equation 4.1) to a time series of daily stock prices and obtaining the slope (Equation 4.2) or the average daily percentage change in the price of a particular stock over some given period. This average daily percentage change in price is not annualised but rather adjusted to the relevant period. The goodness of fit or R-squared of each regression (Equation 4.3) moderates the momentum score (Equation 4.4).

$$\mathbf{p}_{i,d} = \mathbf{a}_i \mathbf{b}_i^{d} \; ; \qquad \ln(\mathbf{p}_{i,d}) = \ln(\mathbf{a}_i) + \mathbf{b}_i(\mathbf{d}) \tag{4.1}$$

$$\mathbf{b}_i = \frac{\sum (d - \bar{d}) \times \left\lfloor \ln(p_{i,d}) - \overline{\ln(p_{i,d})} \right\rfloor}{\sum (d - \bar{d})^2} \tag{4.2}$$

$$\mathbf{R}_i^2 = \left(\frac{\sum (d - \bar{d}) \times \left[\ln(p_{i,d}) - \overline{\ln(p_{i,d})} \right]}{\sqrt{\sum (d - \bar{d})^2 \times \sum \left[\ln(p_{i,d}) - \overline{\ln(p_{i,d})} \right]^2}} \right)^2 \tag{4.3}$$

$$\mathbf{MS}_i = \mathbf{b}_i^{TD} \times \mathbf{R}_i^2 \tag{4.4}$$

Where: $\mathbf{p}_{i,d}$ is the price of stock i on day d
$\mathbf{a}_i$ is the intercept term of an exponential regression
$\mathbf{b}_i$ is the average daily percentage change in the price of stock i
$\mathbf{R}_i^2$ is the coefficient of determination or goodness of fit
TD is the number of trading days in a period
$\mathbf{MS}_i$ is the Momentum Score of stock i

The momentum score parameter is a minimum-level filter (cut-off percentage) used to identify high-momentum stocks – the primary parameter.

4.3.2 Volatility score

Using the first differences of successive daily momentum scores as proxy (Equation 4.5), the volatility of changes in these scores, acceleration, and deceleration, (Equation 4.6) captures the stability in a stock’s momentum.

$$G_{i,t} = MS_{i,t} - MS_{i,t-1} \tag{4.5}$$

$$VS_i = \sqrt{\frac{1}{TD-1}\sum_{t=1}^{TD}(G_{i,t} - \overline{G_i})^2} \tag{4.6}$$

Where: $G_{i,t}$ is the change in the Momentum Score of stock i at time t
$MS_{i,t}$ is the Momentum Score of stock i at time t
$MS_{i,t-1}$ is the Momentum Score of stock i at time t-1
TD is the number of trading days in period
VS_i is the Volatility Score of stock i

The volatility score parameter is a maximum-level filter for high-momentum stocks.

4.3.3 Quality score

Numerous smaller positive returns are preferred to a few large increases in price. A high momentum stock with a quality score (Equation 4.7) substantially below 50 would indicate that momentum was generated by a few large positive returns relative to the negative returns making up the non-zero returns in a period.

$$QS_i = \frac{prd}{prd + nrd} \times 100 = \frac{prd}{nzd} \times 100 \tag{4.7}$$

$$AS_i = \frac{prd}{nzd + zrd} \times 100 = \frac{prd}{TD} \times 100 \tag{4.8}$$

Where: **prd** is the number of positive-return days in a period
nrd is the number of negative-return days in a period
zrd is the number of zero-return days in a period
nzd is the number of non-zero days in a period
TD is the number of trading days in a period
QS_i is the Quality Score of stock i
AS_i is the Activity Score of stock i

The quality score parameter is a minimum-level filter for high-momentum stocks.

4.3.4 Activity score

The activity score extends the quality of momentum concept by calculating the percentage of positive-return days to the total number of trading days in a period including zero-return days. A large drop from quality score to activity score would indicate a lack of active trading and low liquidity. The activity score parameter is a minimum-level filter for high-momentum stocks.

4.4 MOMENTUM PROFILES

The companies, all current or previous listings on the Johannesburg Stock Exchange (JSE), in this section were selected to emphasise certain concepts or to illustrate specific patterns. Profiles are unique to each company during a specific period of momentum. Companies may have experienced several momentum cycles of different types over the 13-year research period. Possible momentum cycles identified by the model may show to be positive, negative, neutral, or false cycles.

A positive cycle would last at least 3 months and record an annualised gain of more than 10%. Negative cycles would record annualised losses exceeding 10%, also lasting at least 3 months. Neutral cycles exit after 3 months at annualised returns not exceeding 10%. False cycles exit before 3 months.

Section 4.4.1 (General) includes an example of a momentum cycle with the ideal stepped pattern, matching the generic profile. Also, an example of extreme momentum with volatile acceleration, disqualified by the volatility parameter. Not pre-screening for liquidity or market capitalisation, this section shows a low-priced stock with momentum that qualified for selection.

Section 4.4.2 (Positive cycles) includes three examples of long-term positive cycles showing alternative outcomes at 180dMS, 210dMS and 250dMS exits as well as the optional backup exit (PbMA) when the price falls below the 200dMA. It shows entries in zones 3 and 4 to account for more irregular patterns. In addition, it illustrates the advantage of specifying a 250dMS exit, thereby largely avoiding mechanically exiting well-established cycles prematurely.

Section 4.4.3 (Negative cycles) offers a plausible explanation for negative cycles. Industry or company-specific events that interrupt cycles that have been building momentum for several months. Sudden declines in price force exits before cycles complete naturally. Large losses can be limited with backup exits (PbMA) or avoided with discretionary exits based on new information.

Section 4.4.4 (Neutral cycles) presents two companies with cycles that lasted for several months without gaining or maintaining momentum before eventually exiting. The entry and exit levels of neutral cycles are similar, posting small gains or losses after relatively extended periods. The concept of momentum assumes cycles of between 3 and 12 months on average (60 to 250 trading days) and a high return.

Section 4.4.5 (False cycles) describes a cycle that completed before 3 months. With monthly updating, false cycles exit after one or two months not breaking the minimum 3-month threshold for momentum. False cycles can record relatively large gains or losses but do not comply with one of the basic assumptions of momentum, the minimum holding period.

4.4.1 General

PSG Group serves as an example of a stock displaying the typical stepped pattern, holding momentum for 14 months and gaining 70.89% (CAGR:58.30%) from its momentum cycle. Entering early in Zone 1 and exiting late when the 250-day momentum score (250dMS) drops below 20%.

Table 4.2 PSG Group Limited (PSG:SJ)

Dates	MOM250	MOM210	MOM180	MOM125	MOM090	MOM060	MOM020	Price	200dMA
2020-05-29	-18	-15	-25	-31	-17	0	3	3729	4622
2020-06-30	-18	-19	-27	-14	0	12	-12	3635	4550
2020-07-31	-17	-25	-22	-1	11	0	1	3541	4403
2020-08-31	-20	-21	-12	1	0	0	26	4500	4236
2020-09-30	-14	-6	0	14	4	19	0	4598	4131
2020-10-30	-3	0	6	26	45	45	20	5222	4096
2020-11-30	0	9	65	50	65	37	3	5686	4149
2020-12-31	5	63	69	77	42	16	3	5988	4286
2021-01-29	35	90	77	68	30	10	14	6451	4587
2021-02-26	101	100	103	58	26	23	-1	6651	4932
2021-03-31	123	106	98	40	24	8	1	7066	5256
2021-04-30	120	110	78	31	15	0	3	7487	5572
2021-05-31	125	101	67	34	13	17	1	7874	6003
2021-06-30	128	82	56	31	22	15	0	7959	6412
2021-07-30	102	61	42	14	8	0	-5	7599	6768
2021-08-31	70	40	25	6	0	-7	0	7564	6996
2021-09-30	44	23	9	0	-8	-3	0	7398	7175
2021-10-29	31	13	4	-1	0	2	0	7648	7380
2021-11-30	22	7	3	0	4	8	0	8042	7528
2021-12-31	19	9	2	7	17	11	8	8924	7714

Microsoft Excel 365

Source: Price data downloaded from Bloomberg (2022)

The stock price did not fall below its 200-day moving average (200dMA), an optional backup exit to protect unrealised gains or limit losses, during this period. The 200dMA is set with a lower band or buffer to limit premature exits – that is, avoidable exits between entry and ultimate exit. Entry at R52.22 with the 20dMS at 20% was relatively expensive (refer to Table 4.2).

Referring to Table 4.3 (page 4-7) and Table 4.5 (page 4-8), Efora Energy is an example of a stock disqualified under a moderate volatility score setting of 1.5 with quality scores as low as 36 and as high as 63 (minimum cut-off at 48). Active trading, as proxied by the activity scores, measured between 9 and 57 (35 minimum).

Referring to Table 4.4 (page 4-7) and Table 4.5 (page 4-8), Jubilee Metals is a so-called penny stock with sustainable momentum. Table 4.5 shows two entries on 2019-06-28 (Zone 2) and 2020-09-30 (Zone 4) with scores falling within the maximum and minimum ranges. Penny stocks may be volatile and lack adequate liquidity, but the scores did exceed the maximum and minimum settings at high momentum on those dates. At the Zone 4 entry on 2020-09-30, the activity scores tracked the quality scores quite closely. Jubilee Metals gained 173.17% (CAGR:136.64%) over this 14-month Zone 4 cycle starting at R1.23 and ending at R3.36 (see Table 4.4).

Table 4.3 Efora Energy Limited (EEL:SJ)

Dates	MOM250	MOM210	MOM180	MOM125	MOM090	MOM060	MOM020	Price	200dMA
2010-06-30	-1	-9	-1	0	0	2	25	300	201
2010-07-30	2	8	30	63	157	348	44	650	243
2010-08-31	28	87	144	262	471	274	40	800	285
2010-09-30	142	292	378	643	474	19	1	890	358
2010-10-29	432	637	743	814	181	137	112	1510	465
2010-11-30	1024	1256	1449	726	268	168	4	1890	646
2010-12-31	1552	1685	1579	257	133	19	-22	1490	805
2011-01-31	1716	1547	892	157	19	-16	-6	1500	950
2011-02-28	1715	1063	389	55	0	0	28	1960	1108
2011-03-31	1529	580	203	23	7	36	-1	2200	1326
2011-04-29	971	244	119	2	14	1	-11	1810	1463
2011-05-31	351	76	11	0	-2	-42	-38	1120	1530
2011-06-30	32	0	-5	-18	-56	-64	-2	870	1541
2011-07-29	0	-9	-34	-54	-71	-49	-20	590	1521

Microsoft Excel 365

Source: Price data downloaded from Bloomberg (2022)

The Efora Energy profile above shows extreme momentum from a low base (R6.50) and volatile acceleration at relatively low liquidity. The actual gains would largely depend on discretionary exits, trading activity and the quoted bid prices.

Table 4.4 Jubilee Metals Group PLC (JBL:SJ)

Dates	MOM250	MOM210	MOM180	MOM125	MOM090	MOM060	MOM020	Price	200dMA
2019-05-31	8	9	10	16	33	20	-14	54	48
2019-06-28	16	14	26	24	25	0	0	56	49
2019-07-31	17	24	24	20	0	-2	0	55	50
2019-08-30	16	22	12	1	-5	-6	-4	55	51
2019-09-30	21	14	11	-1	0	0	18	63	53
2019-10-31	31	22	14	1	12	38	31	80	55
2019-11-29	42	40	20	27	59	59	-6	81	59
2019-12-31	49	33	18	43	39	1	-4	75	62
2020-01-31	58	29	39	45	7	0	0	75	65
2020-02-28	44	31	39	15	-1	0	0	65	67
2020-03-31	7	7	4	-3	-13	-26	-17	58	67
2020-04-30	4	2	0	-15	-12	-4	32	71	67
2020-05-29	4	1	0	-4	-1	9	0	69	68
2020-06-30	6	0	-1	0	10	43	5	83	71
2020-07-31	8	0	0	18	81	42	58	114	75
2020-08-31	16	6	20	122	99	78	3	137	80
2020-09-30	23	37	60	137	82	25	-5	123	85
2020-10-30	41	77	136	102	38	0	9	133	92
2020-11-30	99	158	213	85	18	32	31	180	100
2020-12-31	186	287	222	106	82	106	19	265	116
2021-01-29	330	343	261	124	163	89	-6	250	137
2021-02-26	475	357	270	188	128	28	30	342	159
2021-03-31	520	368	262	200	76	24	-1	320	188
2021-04-30	487	334	241	122	31	8	0	340	212
2021-05-31	446	284	245	67	30	4	-1	363	239
2021-06-30	371	255	165	35	6	4	0	362	264
2021-07-30	272	177	84	16	3	-1	0	361	291
2021-08-31	189	78	25	0	-3	-7	0	342	311
2021-09-30	105	25	5	-1	-6	-4	-1	334	326
2021-10-29	47	6	0	-5	-1	1	7	350	336
2021-11-30	15	1	0	-2	0	1	1	336	344
2021-12-31	4	0	-1	0	1	0	-2	356	347

Microsoft Excel 365

Source: Price data downloaded from Bloomberg (2022)

Table 4.5 Parameter values: Efora Energy & Jubilee Metals

Efora Energy Limited (EEL:SJ)																								
Date	MS060	MS090	MS125	MS180	MS210	MS250	VS060	QS060	AS060	VS090	QS090	AS090	VS125	QS125	AS125	VS180	QS180	AS180	VS210	QS210	AS210	VS250	QS250	AS250	Price
2010-06-30	2	0	0	-1	-9	-1	1.02	36	13	0.57	38	11	0.9	43	10	0.58	40	9	0.63	40	9	0.45	44	11	300
2010-07-30	348	157	63	30	8	2	9.23	43	22	4.27	42	19	1.59	46	17	0.8	43	13	0.67	43	12	0.39	45	13	650
2010-08-31	274	471	262	144	87	28	11.8	52	37	6.93	47	29	3.84	47	23	1.92	48	18	1.29	46	17	0.5	46	15	800
2010-09-30	19	474	643	378	292	142	14.82	60	52	7.89	57	38	7	53	32	3.8	53	24	3.06	52	21	1.55	51	20	890
2010-10-29	137	181	814	743	637	432	9.49	63	57	11.78	58	50	7.03	56	39	6.43	53	30	5.52	54	27	4.12	52	24	1510
2010-11-30	168	268	726	1449	1256	1024	8.05	62	57	12.33	60	54	8.91	58	47	11.01	55	36	9.36	56	33	8.19	55	28	1890
2010-12-31	19	133	257	1579	1685	1552	6.57	53	48	11.06	59	52	14.03	56	50	12.13	53	39	10.37	53	34	10.03	53	30	1490
2011-01-31	-16	19	157	892	1547	1716	4.58	47	42	9.02	52	47	14.39	56	50	18.16	53	41	11.51	52	38	10.12	52	34	1500
2011-02-28	0	0	55	389	1063	1715	4.29	49	40	7.27	50	43	13.8	56	49	20.57	55	46	14.8	54	41	10.17	53	36	1960
2011-03-31	36	7	23	203	580	1529	1.65	58	52	6.31	54	47	10.46	56	50	20.95	56	49	16.99	56	47	11.16	54	42	2200
2011-04-29	1	14	2	119	244	971	2.16	58	50	4.13	55	47	9.01	53	46	21.01	57	51	18.07	55	49	14.81	54	43	1810
2011-05-31	-42	-2	0	11	76	351	2.44	50	47	1.77	52	47	8.48	49	43	20.92	54	48	18.3	54	48	17.64	54	45	1120
2011-06-30	-64	-56	-18	-5	0	32	1.6	42	37	1.32	49	44	3.94	49	43	20.4	51	46	18.33	53	47	18.33	53	47	870
2011-07-29	-49	-71	-54	-34	-9	0	1.55	36	32	1.3	40	36	2.15	46	41	18.2	46	40	18.05	48	43	18.34	50	45	590
Jubilee Metals Group PLC (JBL:SJ)																								
Date	MS060	MS090	MS125	MS180	MS210	MS250	VS060	QS060	AS060	VS090	QS090	AS090	VS125	QS125	AS125	VS180	QS180	AS180	VS210	QS210	AS210	VS250	QS250	AS250	Price
2019-05-31	20	33	16	10	9	8	1.79	49	38	0.68	51	41	0.28	51	39	0.22	49	37	0.26	50	37	0.31	48	35	54
2019-06-28	0	25	24	26	14	16	1.97	50	42	0.93	51	41	0.34	50	40	0.29	50	38	0.26	49	37	0.3	48	35	56
2019-07-31	-2	0	20	24	24	17	1.27	47	33	1.15	50	38	0.42	51	39	0.32	51	38	0.21	49	36	0.24	49	36	55
2019-08-30	-6	-5	1	12	22	16	0.57	48	35	1.16	49	38	0.6	50	38	0.41	50	39	0.24	51	38	0.24	49	36	55
2019-09-30	0	0	-1	11	14	21	0.49	54	43	0.84	52	40	0.6	52	42	0.42	51	41	0.29	52	40	0.24	50	38	63
2019-10-31	38	12	1	14	22	31	1.32	56	52	0.81	54	44	0.57	53	43	0.43	53	43	0.33	51	41	0.24	53	42	80
2019-11-29	59	59	27	20	40	42	2.07	63	55	1.06	59	49	0.73	55	45	0.43	54	43	0.41	54	44	0.26	53	42	81
2019-12-31	1	39	43	18	33	49	2.99	56	50	1.35	57	51	0.81	55	46	0.46	53	44	0.44	53	44	0.26	52	42	75
2020-01-31	0	7	45	39	29	58	2.39	51	42	1.61	56	48	0.8	55	47	0.56	52	42	0.46	53	44	0.28	53	44	75
2020-02-28	0	-1	15	39	31	44	1.53	45	37	1.63	49	40	0.97	53	45	0.56	51	42	0.47	51	42	0.38	51	42	65
2020-03-31	-26	-13	-3	4	7	7	0.89	45	37	1.03	47	40	1.06	51	44	0.86	52	44	0.71	51	43	0.7	51	43	58
2020-04-30	-5	-12	-15	0	2	4	1.5	49	45	0.82	51	44	1.11	51	44	0.85	53	46	0.7	53	44	0.7	52	44	71
2020-05-29	9	-1	-4	0	1	4	1.58	53	48	0.62	47	40	1	50	42	0.83	54	47	0.68	53	45	0.69	52	44	69
2020-06-30	43	10	0	-1	0	6	2.66	59	50	0.6	54	48	0.86	52	44	0.83	53	46	0.67	54	47	0.68	53	45	83
2020-07-31	42	81	18	0	0	8	2.93	64	57	1.64	62	56	1.02	56	50	0.82	54	47	0.66	55	49	0.68	55	48	114
2020-08-31	78	99	122	20	6	16	2.91	61	52	1.54	61	52	2.16	59	52	0.9	55	47	0.65	54	47	0.69	56	48	137
2020-09-30	25	82	137	60	37	23	2.73	61	55	2.05	59	52	2.73	59	52	1.08	55	47	0.75	54	47	0.69	55	48	123
2020-10-30	0	38	102	136	77	41	2.59	48	40	2.53	54	47	2.92	56	48	1.52	54	47	0.93	53	45	0.73	53	46	133
2020-11-30	32	18	85	213	158	99	2.53	57	50	1.84	56	48	3.01	58	50	1.79	58	51	1.47	55	48	1.06	54	47	180
2020-12-31	106	82	106	222	287	186	2.07	60	53	2.47	56	50	3.01	60	54	1.55	58	51	2.22	57	51	1.53	55	48	265
2021-01-29	89	163	124	261	343	330	3.6	59	55	3.25	56	50	2.99	54	47	1.58	58	51	2.44	57	50	2.39	55	48	250
2021-02-26	28	128	188	270	357	475	4.25	57	55	3.1	59	56	2.57	56	51	1.6	57	52	2.44	58	51	2.87	57	52	342
2021-03-31	24	76	200	262	368	520	3.33	52	50	3.34	54	52	2.1	55	51	1.74	56	51	2.45	57	51	2.96	57	51	320
2021-04-30	8	31	122	241	334	487	1.82	50	48	3.53	50	48	2.56	55	52	1.9	52	47	2.63	55	50	3.08	55	50	340
2021-05-31	4	30	67	245	284	446	0.93	48	47	1.86	49	48	2.75	51	50	2.02	53	50	2.87	53	49	3.26	55	50	363
2021-06-30	4	6	35	165	255	371	0.89	43	42	1.34	47	46	2.74	47	46	2.55	51	48	2.99	51	48	3.57	53	49	362
2021-07-30	-1	3	16	84	177	272	0.44	46	45	1.25	45	43	2.65	49	48	2.57	52	50	3.33	52	49	3.96	51	48	361
2021-08-31	-7	-3	0	25	78	189	0.53	44	43	0.69	47	46	1.87	46	45	2.32	49	47	3.68	51	50	4.21	51	48	342
2021-09-30	-4	-6	-1	5	25	105	0.47	50	48	0.53	46	44	1.44	46	44	2.1	47	46	3.52	49	47	4.41	50	48	334
2021-10-29	1	-1	-5	0	6	47	0.5	49	47	0.47	51	49	0.92	48	46	1.94	49	47	2.74	49	47	4.49	51	49	350
2021-11-30	1	0	-2	0	1	15	0.3	53	50	0.41	50	48	0.66	49	47	1.77	47	46	1.93	50	48	4.43	50	48	336
2021-12-31	0	1	0	-1	0	4	0.29	56	53	0.43	54	51	0.52	52	50	1.81	49	47	1.78	49	47	4.21	50	48	356

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Source: Price data downloaded from Bloomberg (2022)

4.4.2 Positive cycles

The model traced five distinct momentum cycles in Capitec's historical price series from 2009 to 2021. The first cycle started on 2009-05-29 at R38.72 with the fifth cycle in progress at the end of 2021 based on the 210dMS and 250dMS exits. The 180dMS-exit cycle ended on 2021-12-31 at R2039.80, generating a 57.45% compound total growth rate (CTGR) over 13 months.

Table 4.6 Momentum cycles: Capitec Bank

ENTRY			EXIT			OUTCOME		
Zone	Date	Price	dMS	Date	Price	Cycle	CAGR	CTGR
1	2009-05-29	3872	**180**	2011-08-31	18249	27	**99.18**	371.31
			210	2011-08-31	18249	27	**99.18**	371.31
			250	2011-11-30	17818	30	**84.15**	360.18
			PbMA	2011-12-30	17237	31	78.26	345.17
1	2014-11-28	31626	**180**	2015-09-30	50100	10	**73.68**	58.41
			210	2015-10-30	59850	11	**100.54**	89.24
			250	2016-01-29	48100	14	**43.25**	52.09
			PbMA	2016-01-29	48100	14	43.25	52.09
4	2017-02-28	72500	PbMA	2018-01-31	80060	11	11.43	10.43
			180	2018-02-28	83246	12	**14.82**	14.82
			210	2018-02-28	83246	12	**14.82**	14.82
			250	2018-03-29	87024	13	**18.36**	20.03
3	2018-11-30	110000	**180**	2019-07-31	118000	8	**11.11**	7.27
			PbMa	2019-07-31	118000	8	11.11	7.27
			210	2019-08-30	109490	9	**-0.62**	-0.46
			250	2019-08-30	109490	9	**-0.62**	-0.46
1	2020-11-30	129552	**180**	2021-12-31	203980	13	**52.05**	57.45
			210	2021-12-31	203980	13	**52.05**	57.45
			250	2021-12-31	203980	13	**52.05**	57.45
			PbMA	-	-	-	-	-

Source: Price data downloaded from Bloomberg (2022)

The most profitable cycles developed from Zone 1 in stepped patterns, while zones 3 and 4 captured two irregular patterns (refer to Table 4.8). The 9-month cycle, which starts on 2018-11-30 and enters in Zone 3, records a negative growth rate when exiting at 250dMS. Any delayed discretionary exit between 2019-09-30 and 2020-01-31 would result in an annualised return that exceeds 20%. Table 4.6 demonstrates the outcomes from mechanically exiting on fixed rules, which includes the optional backup exit when the stock price falls below the 200dMA. Table 4.7 compares the monthly, annual, and total growth rates across all five momentum cycles and dMS-exits with the buy-and-hold data.

Table 4.7 Buy and hold: Capitec Bank

BEGIN		END		OUTCOME				
Date	Price	Date	Price	dMS	Months	CMGR	CAGR	CTGR
2009-05-29	**3872**	2021-12-31	**203980**	180	70	3.89	58.12	1347.96
				210	72	3.93	58.82	1505.00
				250	79	3.32	47.93	1216.59
				HOLD	**151**	**2.66**	**37.03**	**5168.08**

Source: Price data downloaded from Bloomberg (2022)

The profile for Capitec Bank below shows two irregular patterns with momentum not building across the term structure. Zones 3 and 4 allow for alternative entry points into momentum cycles with different or irregular patterns.

Table 4.8 Capitec Bank Holdings (CPI:SJ)

Dates	MOM250	MOM210	MOM180	MOM125	MOM090	MOM060	MOM020	Price	200dMA
2015-12-31	25	4	3	24	8	-1	0	53856	51861
2016-01-29	5	0	2	0	-3	-23	-12	48100	51569
2016-02-29	0	0	1	-1	-20	-7	0	47400	50895
2016-03-31	0	2	1	-4	-1	11	12	57303	51276
2016-04-29	2	4	1	0	11	32	1	59106	52562
2016-05-31	6	2	0	3	23	1	7	59499	53614
2016-06-30	7	1	0	18	5	0	0	59500	54816
2016-07-29	6	1	3	14	0	5	7	61550	55765
2016-08-31	4	3	17	5	3	2	-4	58258	56023
2016-09-30	3	13	21	2	2	0	2	64237	56279
2016-10-31	9	27	17	13	9	8	0	68500	57791
2016-11-30	25	24	12	12	6	4	-1	65029	59922
2016-12-30	32	18	14	10	8	0	6	69500	61489
2017-01-31	31	19	20	13	5	7	2	70201	62883
2017-02-28	26	25	21	13	6	7	5	72500	64126
2017-03-31	29	29	23	13	18	10	6	76137	66233
2017-04-28	35	30	26	17	12	8	0	76254	67793
2017-05-31	36	29	24	18	8	0	3	77878	69587
2017-06-30	35	29	21	11	3	2	4	83000	71466
2017-07-31	36	26	25	12	3	8	9	85979	73533
2017-08-31	38	31	25	12	15	15	3	90050	75709
2017-09-29	37	33	25	19	16	8	0	85907	78105
2017-10-31	41	32	26	22	13	5	2	93984	80692
2017-11-30	43	34	27	23	12	8	5	98479	83454
2017-12-29	42	34	33	21	12	8	8	109796	86044
2018-01-31	43	36	31	15	9	1	-6	80060	88735
2018-02-28	21	14	6	0	-1	-10	0	83246	89476
2018-03-29	12	5	0	-1	-7	-6	1	87024	90544
2018-04-30	5	0	0	-5	-11	1	0	88912	91173
2018-05-31	0	0	-2	-9	-1	-1	0	87444	91272
2018-06-29	0	-1	-5	-4	0	0	-3	86800	91190
2018-07-31	0	-2	-3	2	2	5	8	95153	91440
2018-08-31	0	-1	0	5	12	14	6	100275	91770
2018-09-28	0	0	1	14	12	5	0	102424	91816
2018-10-31	0	1	14	16	8	1	0	99067	91334
2018-11-30	1	22	22	21	11	12	2	110000	93081
2018-12-31	8	26	29	18	9	8	0	111800	95133
2019-01-31	33	35	33	19	14	3	3	116617	97875
2019-02-28	40	42	37	26	18	18	15	130621	101495
2019-03-29	52	47	39	31	23	20	4	134999	105672
2019-04-30	61	52	46	34	31	16	0	133669	110742
2019-05-31	60	49	45	27	11	0	-4	131921	114804
2019-06-28	53	41	33	11	0	-7	0	129874	117768
2019-07-31	38	26	13	0	-7	-6	-3	118000	120892
2019-08-30	15	3	0	-9	-17	-16	-2	109490	121964
2019-09-30	7	1	0	-9	-2	0	14	128744	123355
2019-10-31	4	0	0	0	2	30	3	137298	126310
2019-11-29	5	0	0	5	31	18	0	141727	129311
2019-12-31	4	0	1	19	24	4	3	144618	130917
2020-01-31	2	1	7	24	3	0	-5	134615	131802
2020-02-28	2	5	11	5	0	-1	0	129999	131992
2020-03-31	0	0	0	-6	-14	-21	-27	88000	129852

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Source: Price data downloaded from Bloomberg (2022)

Table 4.9 Coronation Fund Managers Limited (CML:SJ)

Dates	MOM250	MOM210	MOM180	MOM125	MOM090	MOM060	MOM020	Price	200dMA
2009-04-30	-2	0	-1	1	5	28	19	570	485
2009-05-29	0	0	0	10	26	17	-5	527	490
2009-06-30	1	1	8	26	33	5	5	611	498
2009-07-31	5	14	35	53	28	29	5	680	510
2009-08-31	17	43	60	58	35	29	10	760	532
2009-09-30	44	74	83	52	45	21	0	770	562
2009-10-30	76	88	86	50	23	4	6	807	594
2009-11-30	101	102	77	43	20	12	4	835	634
2009-12-31	114	95	68	35	18	17	1	875	679
2010-01-29	118	81	66	29	20	6	0	890	720
2010-02-26	99	68	46	18	5	0	0	885	751
Deleted interim period									
2010-11-30	97	90	69	58	43	31	3	1688	1221
2010-12-31	112	93	73	61	48	27	13	1875	1314
2011-01-31	119	89	84	55	25	4	-2	1746	1402
2011-02-28	101	76	63	23	1	-2	-3	1655	1459
2011-03-31	80	61	38	4	-1	-2	1	1780	1527
2011-04-29	74	49	30	1	0	7	4	1900	1592
2011-05-31	67	39	19	1	8	14	3	1965	1657
2011-06-30	52	26	11	3	13	1	0	1930	1723
2011-07-29	43	20	9	18	11	5	5	2002	1786
2011-08-31	32	14	10	15	4	3	0	1990	1837
2011-09-30	23	13	16	6	2	0	0	2000	1869
2011-10-31	18	16	19	6	1	4	13	2255	1895
2011-11-30	23	29	21	14	12	20	0	2270	1956
2011-12-30	29	29	20	13	17	6	-1	2270	2012
2012-01-31	39	28	24	23	16	2	12	2535	2080
2012-02-29	45	37	35	35	19	26	2	2745	2172
2012-03-30	49	47	43	36	27	16	1	2850	2264
2012-04-30	55	51	53	31	27	5	0	2920	2350
2012-05-31	55	50	42	18	1	0	-12	2632	2428
2012-06-29	43	34	19	1	-3	-7	7	2767	2493
2012-07-31	44	27	14	0	0	4	0	2944	2593
2012-08-31	40	20	12	1	4	13	0	2955	2680
2012-09-28	34	19	7	3	19	5	2	3100	2750
2012-10-31	30	17	8	22	15	10	3	3345	2860
2012-11-30	34	18	21	34	25	24	13	3751	2971
2012-12-31	37	30	37	38	39	31	10	3966	3075
2013-01-31	43	50	68	54	47	27	10	4398	3232
2013-02-28	62	78	76	64	45	25	7	4719	3403
Deleted interim period									
2013-11-29	113	82	62	32	26	29	0	8300	6342
2013-12-31	102	73	50	26	23	4	0	7996	6656
2014-01-31	84	55	36	18	4	-3	0	7964	6997
2014-02-28	74	45	33	16	0	3	8	8801	7299
2014-03-31	69	47	36	14	11	24	9	9900	7624
2014-04-30	65	48	41	17	30	25	0	10149	7971
2014-05-30	65	52	42	31	28	6	-5	10126	8316
2014-06-30	58	46	30	25	5	-1	-1	9551	8612
2014-07-31	50	33	23	8	0	-1	5	9795	8947
2014-08-29	41	22	19	0	-1	0	2	10000	9148
2014-09-30	31	21	13	0	1	2	-2	9665	9354
2014-10-31	16	8	0	0	0	-2	0	9550	9507
2014-11-28	13	3	0	0	0	0	14	11056	9716

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Source: Price data downloaded from Bloomberg (2022)

Table 4.10 Alternative outcomes: Coronation Fund Managers

ENTRY			EXIT			OUTCOME		
Zone	Date	Price	dMS	Date	Price	Cycle	CAGR	CTGR
1	2009-07-31	680	180	2011-05-31	1965	22	78.39	188.97
4	2011-11-30	2270		2012-06-29	2767	7	40.41	21.89
1	2012-11-30	3751		2014-08-29	10000	21	75.12	166.60
				Combined outcome		**50**	**71.18**	**839.05**
1	2009-07-31	680	210	2011-08-31	1990	25	67.43	192.65
4	2011-11-30	2270		2012-09-28	3100	10	45.35	36.56
1	2012-11-30	3751		2014-10-31	9550	23	62.84	154.60
				Combined outcome		**58**	**61.61**	**917.50**
1	2009-07-31	680	250	2011-10-30	2255	27	70.37	231.62
4	2011-11-30	2270		2014-10-31	9550	35	63.66	320.70
				Combined outcome		**62**	**66.55**	**1295.13**
1	2009-07-31	680	**250**	2014-10-31	9550	63	65.41	1304.41

Source: Price data downloaded from Bloomberg (2022)

The profile of Coronation shows the result from mechanically exiting a cycle and not allowing for possible discretionary exits based on the term structure. Note the large increase of 13% in the 20dMS on 2011-10-31 and the increasing 180dMS as well as 210dMS values on this date with all momentum scores positive.

It may be more profitable to exit at 180dMS or 210dMS at times but exiting on the 250dMS extends the momentum cycle and largely avoids premature exits. EOH Holdings (refer to Table 4.12) maintained the longest momentum cycle (62 months) and posted the highest total return of 871.22% (CAGR:54.27%) during the 2009-2021 research period while avoiding premature exits. The initial entry on 2010-10-29 at R13.90 occurs once the parameter scores in Zone 1 satisfy all the set minimum and maximum cut-offs. The ideal exit would be on 2015-07-31 at R172.34, but the 250dMS only drops below 20% on 2015-12-31 (R134.00). The optional backup exit on 2015-11-30 at R152.09 improves the total return to 994.17% (CAGR:60.11%) over a shorter 61-month cycle. Table 4.13 shows the outcomes when exiting on the 180-day (180dMS), 210-day (210dMS) and 250-day (250dMS) scores.

Table 4.11 Alternative outcomes: EOH Holdings

ENTRY			EXIT			OUTCOME		
Zone	Date	Price	dMS	Date	Price	Cycle	CAGR	CTGR
1	2010-10-29	1390	180	2012-11-30	3680	25	59.57	164.75
1	2013-03-28	4967		2014-05-30	8430	14	57.37	69.72
2	2014-11-28	11500		2015-09-30	14853	10	35.94	29.16
				Combined outcome		**49**	**53.82**	**480.34**
1	2010-10-29	1390	210	2012-11-30	3680	25	59.57	164.75
1	2013-03-28	4967		2014-06-30	9025	15	61.24	81.70
2	2014-11-28	11500		2015-10-30	15300	11	36.54	33.04
				Combined outcome		**51**	**54.77**	**540.00**
1	2010-10-29	1390	PbMA	2015-11-30	15209	61	60.11	994.17
1	2010-10-29	1390	**250**	2015-12-31	13500	**62**	**55.27**	**871.22**

Source: Price data downloaded from Bloomberg (2022)

Table 4.12 EOH Holdings Limited (EOH:SJ)

Dates	MOM250	MOM210	MOM180	MOM125	MOM090	MOM060	MOM020	Price	200dMA
2009-07-31	-13	-7	-1	2	8	2	2	640	626
2009-08-31	-3	0	0	23	9	16	4	715	625
2009-09-30	0	5	16	37	42	49	23	900	641
2009-10-30	8	29	54	56	57	26	-1	869	668
2009-11-30	28	57	71	57	28	0	6	940	693
2009-12-31	57	85	70	50	13	6	1	995	733
2010-01-29	88	91	82	38	13	23	1	1030	782
2010-02-26	108	89	74	24	22	8	1	1042	824
2010-03-31	107	87	61	28	18	5	12	1150	878
2010-04-30	106	79	50	34	17	16	0	1160	929
2010-05-31	94	57	35	18	7	0	-5	1090	979
2010-06-30	70	37	28	5	0	-5	0	1100	1018
2010-07-30	46	26	16	1	-2	0	5	1130	1042
2010-08-31	31	19	7	0	0	1	-1	1128	1069
2010-09-30	32	17	9	3	14	19	15	1385	1109
2010-10-29	36	20	16	23	34	30	0	1390	1153
2010-11-30	37	29	23	43	38	16	16	1565	1202
2010-12-31	44	39	45	57	37	21	8	1790	1263
2011-01-31	56	55	70	56	29	18	-4	1670	1322
2011-02-28	63	72	73	38	21	3	2	1770	1377
Deleted interim period									
2012-10-31	49	29	20	12	9	4	-2	3698	3428
2012-11-30	33	19	9	4	0	-2	0	3680	3505
2012-12-31	25	12	9	2	0	-1	1	3785	3570
2013-01-31	20	12	8	1	0	8	11	4110	3625
2013-02-28	21	16	12	7	17	27	15	5060	3735
2013-03-28	27	23	22	23	41	37	0	4967	3871
2013-04-30	36	33	31	46	39	13	0	4945	4021
2013-05-31	46	45	44	54	28	11	0	5295	4219
Deleted interim period									
2014-02-28	96	76	62	20	2	0	5	8600	7076
2014-03-31	90	68	44	10	2	3	-3	8250	7410
2014-04-30	72	46	24	1	0	0	1	8400	7692
2014-05-30	57	28	12	0	0	0	0	8430	7966
2014-06-30	40	16	5	1	0	3	4	9025	8181
2014-07-31	29	12	6	4	9	10	2	9420	8456
2014-08-29	23	10	9	8	13	9	0	9400	8624
2014-09-30	20	14	13	20	14	3	0	9470	8792
2014-10-31	20	20	18	21	12	10	5	10762	9015
2014-11-28	27	28	31	27	21	19	8	11500	9312
2014-12-31	34	34	38	24	20	8	0	10857	9552
2015-01-30	40	45	39	26	15	2	6	12043	9869
2015-02-27	51	51	43	31	18	19	8	13549	10311
2015-03-31	70	63	55	43	37	43	13	15917	10998
2015-04-30	83	73	71	55	57	35	0	16150	11678
2015-05-29	93	84	76	60	38	7	0	15832	12341
2015-06-30	88	76	61	33	3	-4	1	15654	12936
2015-07-31	88	68	51	13	0	0	3	17234	13673
2015-08-31	79	55	40	3	0	3	-10	15850	14278
2015-09-30	60	36	17	0	0	-1	-3	14853	14745
2015-10-30	38	16	2	0	-2	-4	-3	15300	15216
2015-11-30	20	2	0	0	-6	-2	4	15209	15537
2015-12-31	3	0	-3	-10	-7	-8	-7	13500	15584
2016-01-29	0	-9	-10	-20	-17	-17	0	13404	15272
Microsoft Excel 365									

Source: Price data downloaded from Bloomberg (2022)

4.4.3 Negative cycles

Uranium One was listed in the Metals and Mining industry, according to the Global Industry Classification Standard (GICS 2018). Companies in this industry, which include nuclear energy metals, are responsive to changes in the global supply and demand for metals (Harper, Diao, Panousi, Nuss, Eckelman & Graedel 2015).

Table 4.13 Uranium One Inc (UUU:SJ)

Dates	MOM250	MOM210	MOM180	MOM125	MOM090	MOM060	MOM020	Price	200dMA
2010-07-30	0	-7	-11	-4	0	19	0	2050	2075
2010-08-31	0	-3	-2	1	22	28	16	2450	2077
2010-09-30	0	0	0	19	39	12	-3	2350	2084
2010-10-29	0	0	8	57	32	14	16	2825	2124
2010-11-30	7	22	59	81	61	60	1	3584	2231
2010-12-31	23	69	101	75	50	17	-8	3230	2355
2011-01-31	66	118	134	77	45	2	44	4359	2540
2011-02-28	135	176	147	93	37	37	-10	4558	2811
2011-03-31	135	112	74	12	0	-3	-30	2700	3010
2011-04-29	93	44	17	0	-4	-42	0	2730	3084
2011-05-31	40	9	1	-8	-36	-15	-10	2527	3153
2011-06-30	2	0	-6	-35	-45	-28	-20	1798	3122
2011-07-29	0	-6	-25	-47	-16	-6	26	2400	3113
Microsoft Excel 365									

Source: Price data downloaded from Bloomberg (2022)

Uranium One had its primary listing on the Toronto Stock Exchange (TSX) and a secondary listing on the Johannesburg Stock Exchange (JSE), delisting from both exchanges in October 2013 after Russia's Rosatom State Atomic Energy Corporation took full control of the mining company (SENS_S336740 2013). Demand for uranium, used mainly as fuel for nuclear power plants, came under pressure after the March 2011 earthquake and tsunami near Japan triggered a meltdown at the country's Fukushima Daiichi Nuclear Power Plant (Hayashi & Hughes 2013:105). The earthquake occurred on Friday 11 March 2011 and Uranium One (UUU:SJ) closed at R41.56 on that day, falling to R31.60 the following Monday. The momentum profile for Uranium One above shows the effect of that event, after entering the momentum cycle on R34.84 (2010-11-30) and exiting on R17.98 (250dMS < 20%), losing almost half its value during the seven-month period and recording the largest percentage loss in the Johannesburg Stock Exchange Momentum Index (JSE-MI). The optional backup exit at R27.00 would limit the loss to less than a third of the entry value.

Negative cycles generally occur when, after building momentum for several months, prices suddenly fall, and momentum is halted due to industry or company-specific events. The momentum profile of Royal Bafokeng Platinum (Table 4.14) reveals strong momentum building during the eleven months from 2019-03-29 (R33.00) to 2020-02-28 (R49.99) with the price growing at a compound monthly rate of 3.85%. At the following monthly review on 2020-03-31, the price was at R24.71 and momentum had faded away. The 180/210/250 dMS-periods and the 200dMA backup all converged in an abrupt exit, 25% below the initial entry price.

Table 4.14 Royal Bafokeng Platinum (RBP:SJ)

Dates	MOM250	MOM210	MOM180	MOM125	MOM090	MOM060	MOM020	Price	200dMA
MONTHLY representation of momentum (entry and exit)									
2019-01-31	0	10	19	13	1	3	0	2701	2350
2019-02-28	5	27	25	13	10	16	12	3053	2412
2019-03-29	28	38	30	21	33	18	17	3300	2520
2019-04-30	53	50	50	44	41	31	1	3358	2662
2019-05-31	52	44	33	26	4	0	-15	2850	2740
2019-06-28	49	38	27	8	0	-3	13	3460	2838
2019-07-31	53	37	33	6	0	12	0	3412	2971
2019-08-30	39	27	15	0	0	0	4	3695	3037
2019-09-30	38	27	11	1	8	0	9	3900	3157
2019-10-31	47	28	15	26	15	45	1	4399	3353
2019-11-29	52	31	19	32	42	16	0	4379	3521
2019-12-31	53	36	40	41	38	12	12	4962	3695
2020-01-31	61	54	73	66	30	23	0	5199	3890
2020-02-28	68	84	75	52	32	14	0	4999	4097
2020-03-31	9	5	1	-2	-10	-35	-30	2471	4104
2020-04-30	2	0	0	-11	-27	-24	31	3268	4083
WEEKLY representation of momentum (potential exits)									
2020-02-21	66	77	76	57	34	24	17	5998	4044
2020-02-28	68	84	75	52	32	14	0	4999	4097
2020-03-06	66	83	67	39	17	1	-11	4276	4137
2020-03-13	52	63	43	10	1	-1	-43	2903	4152
2020-03-20	22	22	9	0	-1	-14	-62	2500	4131
2020-03-27	12	8	2	-1	-7	-29	-45	2200	4112
2020-04-03	6	2	0	-3	-16	-41	-6	2499	4094
2020-04-09	4	1	0	-6	-22	-43	13	3200	4087
DAILY representation of momentum (potential exits)									
2020-02-27	68	84	76	53	33	17	2	5214	4088
2020-02-28	68	84	75	52	32	14	0	4999	4097
2020-03-02	68	85	74	51	30	11	0	5000	4107
2020-03-03	68	85	73	48	27	7	0	4781	4115
2020-03-04	68	84	72	45	24	4	-2	4700	4123
2020-03-05	67	84	70	42	21	2	-6	4543	4131
2020-03-06	66	83	67	39	17	1	-11	4276	4137
2020-03-09	64	81	64	34	13	0	-17	4000	4142
2020-03-10	62	79	60	28	9	0	-23	3826	4147
2020-03-11	60	75	56	23	5	0	-31	3650	4151
2020-03-12	56	69	50	15	2	-1	-38	3000	4152
2020-03-13	52	63	43	10	1	-1	-43	2903	4152
2020-03-16	45	54	34	5	0	-3	-48	2290	4149
2020-03-17	39	45	26	2	0	-5	-54	2142	4146
2020-03-18	32	34	18	0	0	-8	-59	1700	4140
2020-03-19	25	26	12	0	-1	-11	-63	1565	4133
2020-03-20	22	22	9	0	-1	-14	-62	2500	4131
2020-03-23	20	18	7	0	-2	-17	-61	2375	4128
2020-03-24	17	14	5	0	-3	-21	-61	2051	4121
2020-03-25	15	12	4	0	-4	-24	-57	2350	4118
2020-03-26	14	10	3	0	-6	-26	-49	2700	4116
2020-03-27	12	8	2	-1	-7	-29	-45	2200	4112
2020-03-30	10	6	2	-1	-9	-32	-38	2319	4107
2020-03-31	9	5	1	-2	-10	-35	-30	2471	4104
2020-04-01	8	4	1	-2	-12	-37	-22	2377	4099

Microsoft Excel 365

Source: Price data downloaded from Bloomberg (2022)

Note: PbMA is an acronym for Price below Moving Average.

Referring to Table 4.14 on the previous page, the weekly data slow the progression down to reveal earlier potential exits at R29.03 (PbMA), R24.00 (180dMS < 20%) and R22.00 (210dMS/250dMS < 20%). The daily data expand the pattern even more to show the progression from the end of February (R49.99) to the end of March (R24.71). On Monday (2020-03-09) the price drops below the 200dMA, after Anglo American Platinum (Amplats) announced the temporary shutdown of the Anglo Converter Plant (ACP) on Friday (SENS_S427702 2020) via the Stock Exchange News Service (SENS) of the Johannesburg Stock Exchange (JSE). Amplats declared force majeure (i.e., contract void due to unforeseeable circumstances), stating that it would be unable to receive any platinum concentrate until repair work has been completed in approximately 80 days. Royal Bafokeng Platinum, who sells all of its concentrate to Rustenburg Platinum Mines (a wholly owned subsidiary of Amplats) acknowledged notification on 2020-03-10 (SENS_S427824 2020). On 2020-03-18 Royal Bafokeng Platinum (RBPlat) announced that an agreement was reached with Amplats to resume delivery of the concentrate on the same terms but with delayed payments (SENS_S428273 2020). Soon afterwards, the closing price of RBPlat recovered from R17.00 (2020-03-18) to R24.71 (2020-03-31) and R32.68 (2020-04-30). The timeline shows the effect of unexpected events on price momentum and the need for investors to stay informed and exit positions when prompted by major news.

4.4.4 Neutral cycles

Neutral cycles are defined as those lasting a minimum of 3 months without gaining or maintaining much momentum between entry and exit. The threshold for momentum (positive or negative) is a compound annual growth rate (CAGR) of 10% (gain or loss). Therefore, momentum requires continuation of the large increases in price over the last 60 (3 months) to 250 (12 months) trading days.

Table 4.15 Discovery Limited (DSY:SJ)

Dates	MOM250	MOM210	MOM180	MOM125	MOM090	MOM060	MOM020	Price	200dMA
2017-06-30	3	11	17	8	0	0	1	12792	12126
2017-07-31	8	20	17	4	0	1	9	14031	12327
2017-08-31	21	24	20	5	8	19	3	15060	12686
2017-09-29	29	25	18	10	14	4	-1	14066	12997
2017-10-31	29	21	12	11	5	0	2	14655	13312
2017-11-30	31	21	14	16	4	10	13	16471	13745
2017-12-29	33	23	25	18	14	25	12	18600	14183
2018-01-31	37	35	37	23	31	15	0	16885	14690
2018-02-28	36	38	34	24	12	0	8	17878	15080
2018-03-29	43	41	30	21	2	0	-2	17050	15600
2018-04-30	40	31	21	5	0	0	0	17325	16001
2018-05-31	28	14	9	0	-2	-11	-9	15415	16268
2018-06-29	8	2	0	-11	-19	-17	-1	14750	16281
2018-07-31	1	0	-2	-8	-6	0	9	17000	16443
2018-08-31	1	0	-4	-2	0	21	3	17521	16687
2018-09-28	0	-1	-1	0	16	11	0	17000	16810
2018-10-31	0	-2	-1	1	0	-5	-4	15793	16588

Microsoft Excel 365

Source: Price data downloaded from Bloomberg (2022)

Referring to Discovery (Table 4.15) on the previous page and Cashbuild (Table 4.16), cycles lasted between 8 and 13 months without holding much momentum before exiting within 10% (annualised) of the entry prices. An earlier discretionary exit after 6 months on 2018-02-28 at R178.78 would have resulted in a gain for the neutral Discovery cycle. A later exit after 12 months on 2018-08-31 at R175.21 would also have avoided the loss of the 10-month exit.

Table 4.16 Cashbuild Limited (CSB:SJ)

Dates	MOM250	MOM210	MOM180	MOM125	MOM090	MOM060	MOM020	Price	200dMA
2010-09-30	0	0	0	0	0	0	10	7800	7259
2010-10-29	1	1	1	6	8	35	15	9450	7380
2010-11-30	4	7	10	22	38	28	0	8985	7566
2010-12-31	13	18	26	39	38	10	0	9500	7788
2011-01-31	22	30	38	42	13	1	-4	9044	8008
2011-02-28	25	32	29	11	0	-7	-5	8379	8161
2011-03-31	24	22	17	0	-4	-5	7	9350	8329
2011-04-29	28	21	13	0	0	3	-1	9300	8521
2011-05-31	24	15	3	0	0	3	1	9700	8714
2011-06-30	20	7	0	0	6	0	0	9480	8985
2011-07-29	14	1	0	3	0	0	0	9150	9153
2011-08-31	4	0	0	1	0	-4	0	9250	9178
2011-09-30	1	0	1	0	0	3	7	10294	9227
2011-10-31	1	2	6	3	5	11	3	10300	9263
2011-11-30	4	11	8	10	19	12	0	11190	9427
2011-12-30	11	18	15	25	25	19	5	11800	9691
2012-01-31	24	19	25	32	20	8	1	11745	9949
2012-02-29	28	28	29	21	7	0	-4	11120	10175
2012-03-30	29	32	34	16	4	2	13	12600	10448
2012-04-30	39	40	36	14	7	12	2	13098	10742
2012-05-31	44	43	32	15	14	8	-1	13000	11156
2012-06-29	52	42	31	20	19	7	3	13700	11610
2012-07-31	58	43	33	31	19	19	4	15100	12153
2012-08-31	61	47	42	37	31	26	8	16600	12794
2012-09-28	61	49	47	31	21	3	-9	15500	13259
2012-10-31	52	44	37	14	1	-6	0	14994	13690
2012-11-30	43	34	20	2	-2	-1	-6	14800	14086
2012-12-31	36	22	10	0	-4	0	4	15400	14415
2013-01-31	23	8	2	-5	-1	-1	-6	13090	14683
2013-02-28	6	0	-1	-9	-8	-10	-1	13000	14721
2013-03-28	0	-1	-12	-14	-16	-18	-3	12520	14685

Microsoft Excel 365

Source: Price data downloaded from Bloomberg (2022)

Referring to Cashbuild, several earlier discretionary exits would have avoided the negative outcome from the mechanical exit after 13 months on 2013-02-28. These results highlight a drawback of a purely mechanical system.

4.4.5 False cycles

The Vodacom profile on the next page, expanded with weekly and daily data, is an example of a false cycle – high momentum in three successive periods that does not continue to build or settle into the stepped pattern of a genuine momentum cycle. Exploring weekly or daily data to possibly extend the momentum cycle by locating earlier entries at lower prices confirms the false cycle (refer to Table 4.17).

Table 4.17 Vodacom Group Limited (VOD:SJ)

Dates	MOM250	MOM210	MOM180	MOM125	MOM090	MOM060	MOM020	Price	200dMA
MONTHLY representation of momentum (false entry and quick exit)									
2017-07-31	3	10	15	15	15	9	9	17807	15404
2017-08-31	12	21	21	24	20	13	0	18159	15787
2017-09-29	16	17	14	9	0	-2	-6	16114	15998
2017-10-31	9	5	3	0	-5	-17	0	15360	16076
WEEKLY representation of momentum (earlier potential entry)									
2017-08-04	4	13	16	17	19	9	8	18256	15474
2017-08-11	6	15	17	19	22	9	4	18105	15545
2017-08-18	8	17	19	21	23	11	0	18084	15628
2017-08-25	10	20	20	23	22	13	0	18421	15719
2017-09-01	12	22	21	24	20	13	0	18200	15804
2017-09-08	13	22	20	20	13	6	-3	16499	15874
2017-09-15	14	21	17	16	5	0	-11	16070	15921
2017-09-22	15	18	15	11	1	0	-13	16136	15959
DAILY representation of momentum (earliest potential entry)									
2017-08-04	4	13	16	17	19	9	8	18256	15474
2017-08-07	4	14	17	17	20	9	7	18551	15493
2017-08-08	5	14	17	18	21	9	7	18414	15511
2017-08-10	5	15	17	18	21	9	6	18309	15529
2017-08-11	6	15	17	19	22	9	4	18105	15545
2017-08-14	6	16	18	19	22	10	3	18229	15562
2017-08-15	7	16	18	20	23	10	3	18229	15579
2017-08-16	7	17	18	20	23	10	2	18212	15596
2017-08-17	7	17	19	21	23	11	1	18037	15612
2017-08-18	8	17	19	21	23	11	0	18084	15628
2017-08-21	8	18	19	21	22	11	0	18060	15646
2017-08-22	9	18	20	22	22	12	0	18310	15664
2017-08-23	9	19	20	22	22	12	0	18270	15682
2017-08-24	10	19	20	23	22	12	0	18399	15701
2017-08-25	10	20	20	23	22	13	0	18421	15719
2017-08-28	10	20	21	23	22	13	0	18385	15738
2017-08-29	11	20	21	23	21	13	0	18156	15755
2017-08-30	11	21	21	24	21	13	0	18150	15771
2017-08-31	12	21	21	24	20	13	0	18159	15787
2017-09-01	12	22	21	24	20	13	0	18200	15804
2017-09-04	12	22	21	24	19	13	0	17985	15822
2017-09-05	13	22	21	23	19	12	0	17830	15841
2017-09-06	13	22	21	22	16	9	-1	16428	15852
2017-09-07	13	22	20	21	15	7	-2	16570	15864
2017-09-08	13	22	20	20	13	6	-3	16499	15874
2017-09-11	14	22	19	19	11	4	-5	16550	15884
2017-09-12	14	21	19	19	10	3	-6	16590	15895
2017-09-13	14	21	19	18	8	2	-8	16500	15905
2017-09-14	14	21	18	17	7	1	-9	16365	15914
2017-09-15	14	21	17	16	5	0	-11	16070	15921
2017-09-18	14	20	17	15	4	0	-13	16103	15929
2017-09-19	15	20	16	14	3	0	-14	15850	15935
2017-09-20	15	19	16	13	2	0	-15	16017	15941
Microsoft Excel 365									

Source: Price data downloaded from Bloomberg (2022)

False cycles are those that last less than 3 months, regardless of the outcome. The Vodacom cycle lost 11.26% in value, exiting after one month even though the parameter scores on 2017-08-31 in Zone 2 were within the maximum and minimum ranges. An assumption of momentum is a holding period of at least 3 months.

4.5 SUMMARY

The model developed and customised for this study makes use of a momentum term structure (i.e., a range of gradually increasing momentum periods) and four parameters respectively measuring momentum, the volatility of changes in momentum, the quality of momentum, and activity. Stocks are filtered on these parameters and classified as high momentum stocks when they score above the minimum (momentum, quality, and activity) and below the maximum (volatility) parameter settings. Stocks are not sorted or ranked on any of the parameter scores.

There are four entry zones and the concept behind the customised model is to enter momentum cycles early, preferably in the first entry zone (060-090-125 grouping) and exit as late as possible on the longest momentum period (250 days). Exiting late generally avoids premature exits but exiting earlier (210-day or 180-day periods) shortens the holding period and may result in higher annualised returns. Cycle entries and exits are strictly mechanical according to the parameter settings and the exit rule. These entries (additions) and exits (deletions) will be used to construct comparable momentum indices for the different equity markets.

The eventual outcome classifies a momentum cycle as either positive, negative, neutral, or false. Examples of each type were presented graphically using the profiles of companies listed on the Johannesburg Stock exchange (JSE). The next chapter defines the different types in terms of hold and minimum return based on the theory underlying price momentum.

The next three chapters will apply the model to stock exchanges in an emerging market (South Africa), a developed market (Canada) and a venture exchange (Canada):

Chapter 5 creates a momentum profile for the Johannesburg Stock Exchange (JSE) and constructs a custom momentum index (JSE-MI).

Chapter 6 creates a momentum profile for the Toronto Stock Exchange (TSX) and constructs a custom momentum index (TSX-MI).

Chapter 7 creates a momentum profile for the TSX Venture Exchange (TSXV) and constructs a custom momentum index (TSXV-MI).

REFERENCES

Bloomberg. 2022. Bloomberg Terminal – Bloomberg Professional Services subscription. New York: Bloomberg Finance L.P.

Clenow, A.F. 2015. *Stocks on the move: beating the market with hedge fund momentum strategies*. Scotts Valley, California: CreateSpace.

Excel. 2022. Microsoft Excel 365 [Website]. Microsoft Corporation. Available at: https://www.microsoft.com/en-za/microsoft-365/excel.

GICS. 2018. *Global Industry Classification Standard*. *Guide to the GICS Methodology [Online]*. S&P Global Market Intelligence. Available at: https://www.spglobal.com/marketintelligence/en/documents/112727-gics-mapbook_2018_v3_letter_digitalspreads.pdf (Accessed: 15 March 2022).

Harper, E.M., Diao, Z., Panousi, S., Nuss, P., Eckelman, M.J. & Graedel, T.E. 2015. The criticality of four nuclear energy metals. *Resources, Conservation and Recycling*, 95:193–201. DOI: 10.1016/j.resconrec.2014.12.009.

Hayashi, M. & Hughes, L. 2013. The Fukushima nuclear accident and its effect on global energy security. *Energy Policy*, 59:102–111. DOI: 10.1016/j.enpol.2012.11.046.

Pandas. 2022. Python data analysis [Website]. The pandas development team. Available at: https://pandas.pydata.org/.

Python. 2022. Python programming language [Website]. Python Software Foundation. Available at: https://www.python.org/.

SENS_S336740. 2013. Uranium One. Thursday 26 September. Available at: https://www.profiledata.co.za/BrokerSites/BusinessLive/SENS.aspx?id=220347.

SENS_S427702. 2020. Anglo American Platinum. Friday 06 March 2020. Available at: https://www.profiledata.co.za/BrokerSites/BusinessLive/SENS.aspx?id=354691.

SENS_S427824. 2020. Royal Bafokeng Platinum. Tuesday 10 March 2020. Available at: https://www.profiledata.co.za/BrokerSites/BusinessLive/SENS.aspx?id=354862.

SENS_S428273. 2020. Royal Bafokeng Platinum. Wednesday 18 March 2020. Available at: https://www.profiledata.co.za/BrokerSites/BusinessLive/SENS.aspx?id=355613.

CHAPTER FIVE

MOMENTUM PROFILE: JOHANNESBURG STOCK EXCHANGE

5.1 INTRODUCTION

The customised momentum model with its assumptions was introduced and explained in Chapter 4. In summary, the concept behind the model is to identify stocks relatively early in their respective momentum cycles via three successive momentum formation periods (i.e., an entry zone). Zone 1 (60, 90, and 125 days) presents the earliest and Zone 4 (180, 210, and 250 days) the latest possible entry into a cycle due to the stepped pattern of momentum when based on formation periods of increasing length. The parameter settings of the model determine the entry points of cycles. A momentum profile for an equity market is created by entering these cycles mechanically (i.e., not making any discretionary decisions) and exiting on a fixed rule. The results from applying the model mechanically provide a set of positive, negative, neutral, and false cycles unique to a particular market – the different types of cycles are defined in the next section.

All the stocks listed on a particular exchange are eligible for selection. The investment universe is not predefined, and companies are not pre-sorted on price, market capitalisation (size), index, or sector. Also, the stocks identified by the model are not ranked or sorted on any of the parameters. This model identified 701 stocks listed on the Johannesburg Stock Exchange (JSE) with momentum cycles in progress, meaning that momentum has been forming for up to 250 trading days (12 months), depending on the entry zone. When entering a cycle, it is expected that momentum will hold for at least another 60 trading days (3 months). The results will show that the momentum identified by this customised model generally must hold longer than 6 months to exit as a positive cycle.

Every company or stock selected by the model is also included in a custom momentum index – refer to Chapter 3. This index has a variable number of members that are equally weighted when added to the index, which is updated monthly. The momentum index maintains a relatively active position over a true equal-weighted or unweighted design by allowing the existing members to retain the momentum gained. The custom index quantifies and represents the collective outcome of mechanically entering and exiting the momentum cycles identified by the model. Therefore, the index quantifies the actual performance of the momentum model and enables comparison with benchmark indices. The relative performance of the index, also in terms of correlation, drawdown, and descriptive statistics completes the momentum profile of an equity market. In this instance, the Johannesburg Stock Exchange (JSE).

The section to follow presents the momentum profile for an emerging equity market in terms of average hold, price range activity, sector activity, outcomes per entry zone (refer to Chapter 4), and the average parameter scores per cycle type.

5.2 MOMENTUM MODEL OUTCOMES

The momentum cycles identified by the model are classified as either false, neutral, negative, or positive depending on the outcome. In this study, it is assumed that a positive cycle (optimal outcome) would hold at least 3 months and record an annualised gain of more than 10%. A negative cycle (unexpected outcome) would record an annualised loss of more than 10% while also holding at least 3 months. A neutral cycle (no outcome) is assumed to hold a minimum of 3 months but gain or lose a maximum of 10% annualised. A false cycle (failed outcome) holds shorter than 3 months. These assumptions are based on the theory of price momentum, which states that momentum formed over 3 to 12 months should hold for 3 to 12 months (60 to 250 trading days) – refer to Chapter 2. At a momentum score setting of 20% per period, the 10% annualised cut-off was chosen as minimum evidence of some momentum between entry and exit.

A full momentum cycle comprises both a formation and a holding period. The change in price between entry and exit (in effect the holding period) classifies a cycle as either positive, negative, or neutral. False cycles are assumed to hold shorter than 3 months, based on the concept of medium-term momentum.

In the following five subsections, a momentum profile for this equity market will be created by analysing the different cycles in terms of duration, price range activity, sector activity, outcomes per momentum zone (refer to Chapter 4), and the average parameter (momentum, volatility, quality, and activity) scores per cycle type.

5.2.1 Holding periods

The results per average holding period or Average Hold (AH), in Table 5.1 on the next page, show that the different cycles are distinct in average hold period. Each type tends to dominate a particular range. False cycles are confined to shorter than 3 months by definition and account for almost 8% (55 from 701) of all cycles, posting a high negative annual return due to the short average hold of 1.58 months. The majority (92 from 121 or 76%) of neutral cycles clustered in the 6-11-month range with small returns, both negative (6-8) and positive (9-11), at a relatively long average hold before ultimately exiting without much change in value.

Negative cycles (212 from 701 or 30%) are shorter in average hold than neutral cycles, dominating the 3-8-month range and falling by more than 15% on average. Positive cycles (313 from 701 or 45%) are predominant in the 9-17-month range while several cycles (53 from 313 or 17%) also hold longer than 18 months to record annualised returns exceeding 40% on average.

It can be concluded that momentum cycles that hold beyond 9 months generally record high positive returns. Negative cycles have a shorter average hold of 5 months with only 3% (6 from 212) extending beyond 9 months.

Table 5.1 Average hold

HOLD	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
1-2	55	55	---	---	---	1.58	-7.50	-44.66
	AH	1.58	---	---	---			
	CRpAH	-7.50	---	---	---			
	CARpAH	-44.66	---	---	---			
3-5	168	---	25	123	20	3.95	-9.78	-26.86
	AH	---	4.36	3.86	3.95			
	CRpAH	---	-0.58	-14.77	13.40			
	CARpAH	---	-1.60	-39.13	46.54			
6-8	177	---	50	83	44	7.05	-3.32	-5.59
	AH	---	7.32	6.82	7.16			
	CRpAH	---	-0.03	-16.26	22.06			
	CARpAH	---	-0.04	-26.83	39.67			
9-11	150	---	42	5	103	9.90	15.99	19.69
	AH	---	9.52	9.00	10.10			
	CRpAH	---	0.87	-11.92	24.44			
	CARpAH	---	1.09	-15.57	29.67			
12-17	98	---	4	1	93	13.91	43.10	36.24
	AH	---	12.25	12.00	14.00			
	CRpAH	---	4.83	-25.12	46.05			
	CARpAH	---	4.73	-25.12	38.35			
18-23	31	---	---	---	31	20.03	83.89	44.04
	AH	---	---	---	20.03			
	CRpAH	---	---	---	83.89			
	CARpAH	---	---	---	44.04			
24+	22	---	---	---	22	32.00	211.84	53.19
	AH	---	---	---	32.00			
	CRpAH	---	---	---	211.84			
	CARpAH	---	---	---	53.19			
JSE	701	55	121	212	313	8.80	11.09	15.42
	AH	1.58	7.64	5.18	12.97			
	CRpAH	-7.50	0.32	-15.34	43.45			
	CARpAH	-44.66	0.51	-32.02	39.61			

Source: Price data downloaded from Bloomberg (2022)

Overall results show 55 false (8%), 121 neutral (17%), 212 negative (30%), and 313 positive (45%) cycles. Referring to Table 5.1, note the increasingly higher returns when positive cycles move into the 12-17-month range and beyond in contrast to the shorter negative cycles. The average hold of positive cycles is 13 months, with the average hold of negative cycles much shorter at 5 months. The false and neutral cycles did either not hold (< 3 months) or build (CAGR ≤ 10%) any momentum.

5.2.2 Price ranges

Based on the results below, low-priced stocks are more likely to complete full momentum cycles with the below-R5 and R10-R25 price ranges the most promising.

Table 5.2 Price range activity

ZAR	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
< 5	127	10	21	30	66	8.98	14.62	20.01
	AH	1.40	8.71	5.70	11.70			
	CRpAH	-11.91	-0.75	-14.87	42.95			
	CARpAH	-66.27	-1.03	-28.74	44.28			
≥ 5 < 10	70	6	12	24	28	8.39	11.24	16.47
	AH	1.83	8.08	4.63	13.14			
	CRpAH	-1.50	0.56	-16.77	52.89			
	CARpAH	-9.39	0.83	-37.90	47.35			
≥ 10 < 25	112	9	15	29	59	10.80	21.77	24.45
	AH	1.89	6.47	5.45	15.90			
	CRpAH	-5.60	-0.17	-15.25	59.11			
	CARpAH	-30.65	-0.32	-30.54	41.98			
≥ 25 < 50	107	13	17	28	49	8.88	12.48	17.23
	AH	1.38	7.35	5.25	13.47			
	CRpAH	-3.97	0.10	-17.01	45.32			
	CARpAH	-29.61	0.17	-34.70	39.52			
≥ 50 < 100	102	5	27	24	46	9.06	13.46	18.21
	AH	1.60	7.67	4.88	12.87			
	CRpAH	-10.01	1.31	-13.93	43.64			
	CARpAH	-54.65	2.05	-30.87	40.17			
≥ 100 < 200	100	5	18	38	39	7.37	2.99	4.91
	AH	1.80	7.28	4.74	10.69			
	CRpAH	-8.35	0.78	-13.05	24.52			
	CARpAH	-44.08	1.30	-29.83	27.90			
≥ 200 < 500	63	6	7	31	19	7.68	-1.05	-1.63
	AH	1.50	8.57	5.45	12.95			
	CRpAH	-14.03	1.26	-15.49	32.70			
	CARpAH	-70.17	1.77	-30.97	29.98			
≥ 500	20	1	4	8	7	6.90	-4.23	-7.24
	AH	1.00	6.00	5.63	9.71			
	CRpAH	-1.52	-2.19	-21.34	18.01			
	CARpAH	-16.77	-4.32	-40.07	22.70			
JSE	701	55	121	212	313	8.80	11.09	15.42
	AH	1.58	7.64	5.18	12.97			
	CRpAH	-7.50	0.32	-15.34	43.45			
	CARpAH	-44.66	0.51	-32.02	39.61			

Source: Price data downloaded from Bloomberg (2022)

Referring to Table 5.2 on the previous page, almost 40% (125 from 313) of the positive cycles fall within the below-R5 and R10-R25 ranges. The upper threshold for stock prices appears to be R100 with the R50-R100 range still recording comparable results. The number of neutral cycles (38) equals the number of positive cycles (39) in the R100-R200 price range, which recorded a compound return of 2.99% at an average hold of 7.37 months. Neutral cycles (31) exceed positive cycles (19) in the R200-R500 price range, which recorded small negative returns of -1.05% at an average hold of 7.68 months.

Note that stocks priced at less than R5 account for 21% (66 from 313) of all the positive cycles. Only the R10-R25 stocks outperformed these below-R5 penny stocks. Overall, almost 80% (248 from 313) of the positive cycles entered at prices below R100. The negative cycles are evenly divided between the different price ranges (excluding the R500+ range). Many of the neutral cycles (27 from 121 or 22%) occurred in the R50-R100 range. False cycles are overrepresented in the below-R5 (10 from 55 or 18%) and the R25-R50 (13 from 55 or 24%) price ranges.

5.2.3 Sectors

Consumer discretionary stocks tend to do well when the economy is strong and expanding, while consumer staples are always in demand regardless of the state of the economy (De Longis, Zanin & Ellis 2022). The cyclicality of the Consumer Discretionary sector seems to align with momentum in this equity market, outperforming all the other active sectors with 80-plus cycles.

Table 5.3 Sector activity

SECTOR	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
CO	29	2	3	10	14	9.14	15.34	20.62
	AH	1.50	6.67	5.00	13.71	COMMUNICATIONS		
	CRpAH	-10.84	1.15	-11.86	49.15			
	CARpAH	-60.06	2.08	-26.13	41.88			
CD	102	5	18	27	52	9.61	16.63	21.18
	AH	1.60	7.72	4.74	13.56	CONSUMER DISCRETIONARY		
	CRpAH	-15.81	0.98	-11.86	46.29			
	CARpAH	-72.50	1.53	-27.34	40.03			
CS	102	7	22	25	48	8.50	10.59	15.28
	AH	1.57	7.45	4.44	12.10	CONSUMER STAPLES		
	CRpAH	-4.25	1.22	-11.83	32.36			
	CARpAH	-28.25	1.97	-28.83	32.04			
EN	5	1	---	1	3	7.00	22.04	40.70
	AH	2.00	---	4.00	9.67	ENERGY		
	CRpAH	-8.16	---	-13.29	50.36			
	CARpAH	-39.99	---	-34.82	65.92			

Table 5.3 Sector activity (continued)

SECTOR	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
FI	114	6	29	29	50	9.29	13.39	17.62
	AH	1.17	7.45	5.00	13.82	FINANCIALS		
	CRpAH	-3.62	0.24	-14.17	45.95			
	CARpAH	-31.57	0.39	-30.70	38.86			
HC	26	4	2	4	16	9.50	14.39	18.50
	AH	1.75	5.50	6.50	12.69	HEALTH CARE		
	CRpAH	-10.88	-1.58	-13.22	32.93			
	CARpAH	-54.59	-3.41	-23.03	30.89			
IN	86	8	11	31	36	7.99	6.88	10.51
	AH	1.75	7.91	5.45	11.61	INDUSTRIALS		
	CRpAH	-5.51	0.54	-16.58	38.53			
	CARpAH	-32.22	0.83	-33.06	40.05			
MA	180	19	25	63	73	8.27	6.63	9.77
	AH	1.58	8.48	5.30	12.49	MATERIALS		
	CRpAH	-7.55	-0.16	-19.11	43.67			
	CARpAH	-44.92	-0.23	-38.12	41.63			
RE	22	2	3	9	8	8.23	6.79	10.05
	AH	1.50	5.00	5.89	13.75	REAL ESTATE		
	CRpAH	-16.94	1.72	-13.04	45.90			
	CARpAH	-77.35	4.19	-24.78	39.05			
TE	35	1	8	13	13	10.31	19.00	22.43
	AH	2.00	7.50	6.08	16.92	TECHNOLOGY		
	CRpAH	20.73	-2.38	-15.03	88.03			
	CARpAH	209.69	-3.78	-27.50	56.48			
JSE	701	55	121	212	313	8.80	11.09	15.42
	AH	1.58	7.64	5.18	12.97	JOHANNESBURG STOCK EXCHANGE		
	CRpAH	-7.50	0.32	-15.34	43.45			
	CARpAH	-44.66	0.51	-32.02	39.61			

Source: Price data downloaded from Bloomberg (2022)

Consumer Staples recorded lower returns at a shorter average hold compared to Consumer Discretionary. Materials, which includes the Metals and Mining industries, recorded the highest rate of false cycles (63 from 180 or 35%). Mining companies are cyclical in nature and heavily influenced by the demand for metals in domestic and international markets during upswings and downswings. Strike actions by labour unions are also quite common in the mining industries and contribute to the volatility in this sector (Humphreys 2020). Financials outperformed both Industrials and Materials. Among the less active sectors, Technology and Health Care outperformed. Technology maintained the longest average hold, while Health Care registered the highest rate of positive cycles (16 from 26 or 62%). Communications and Real Estate, comparable in number of cycles, delivered contrasting results.

5.2.4 Entry zones

An entry zone, three successive formation periods, identifies and confirms a momentum cycle in progress. The earliest entry (i.e., shortest formation) with potentially the longest hold should occur in Zone 1. The stepped pattern of a regular momentum profile exits each cycle as late as possible. Zones 2 to 4 allow for later entries and more irregular patterns or individual profiles.

Table 5.4 Results per entry zone

ZONE	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
1	269	17	53	77	122	9.39	12.52	16.26
	AH	1.53	8.34	5.86	13.18			
	CRpAH	-7.26	0.10	-15.29	45.47			
	CARpAH	-44.62	0.14	-28.82	40.67			
2	142	11	22	45	64	9.14	10.17	13.56
	AH	1.55	8.00	4.91	13.81			
	CRpAH	-11.61	1.48	-18.45	45.42			
	CARpAH	-61.64	2.23	-39.25	38.45			
3	135	11	23	41	60	8.36	10.42	15.29
	AH	1.73	6.61	4.98	12.57			
	CRpAH	-10.24	0.93	-16.28	43.43			
	CARpAH	-52.80	1.70	-34.85	41.12			
4	155	16	23	49	67	7.85	10.07	15.81
	AH	1.56	6.70	4.53	12.16			
	CRpAH	-2.85	-0.85	-11.64	38.03			
	CARpAH	-19.91	-1.52	-27.95	37.44			
JSE	701	55	121	212	313	8.80	11.09	15.42
	AH	1.58	7.64	5.18	12.97			
	CRpAH	-7.50	0.32	-15.34	43.45			
	CARpAH	-44.66	0.51	-32.02	39.61			

Source: Price data downloaded from Bloomberg (2022)

Table 5.4 above shows the outcomes from momentum cycles entered at these four different zones. Zone 1, as expected, generated the greatest number of entries (269 from 701 or 38%) at the longest average hold per zone. The average hold decreases from Zone 1 to Zone 4. The longest average hold for positive cycles is in Zone 2, but the negative cycles (-18.45%) and the false cycles (-11.61%) dragged the overall performance of this cycle down. Zone 2 recorded the worst compound annual return per average hold (CARpAH) for this market.

False cycles recorded negative returns in every zone with the smallest impact on the overall result of Zone 4. Neutral cycles generally recorded small positive returns, except for Zone 4, at relatively long average holds that exceed those of negative cycles. The Zone 1 entries in this equity market outperformed in general.

5.2.5 Parameter scores

The model identified 701 individual cycles with the [20|1.5|48|35] parameter setting combination. The average parameter scores for each period – which resulted in false, neutral, negative, or positive cycles – are included in Table 5.5 below.

Table 5.5 Average parameter scores

MOMENTUM	MS060	MS090	MS125	MS180	MS210	MS250	
False	20.24	29.76	29.95	27.53	25.53	21.18	25.70
Neutral	20.51	26.05	31.51	29.31	27.11	23.59	26.35
Negative	19.71	28.94	31.47	31.10	26.58	19.92	26.29
Positive	22.56	31.83	36.61	32.75	28.59	21.36	28.95
	21.16	29.80	33.65	31.25	27.49	21.30	27.44
VOLATILITY	**VS060**	**VS090**	**VS125**	**VS180**	**VS210**	**VS250**	
False	1.00	0.77	0.62	0.44	0.39	0.37	0.60
Neutral	0.91	0.71	0.57	0.45	0.42	0.39	0.57
Negative	0.94	0.78	0.62	0.47	0.43	0.40	0.61
Positive	0.97	0.79	0.66	0.48	0.44	0.39	0.62
	0.95	0.77	0.63	0.47	0.43	0.39	0.61
QUALITY	**QS060**	**QS090**	**QS125**	**QS180**	**QS210**	**QS250**	
False	54.64	54.73	53.73	52.16	51.55	51.13	52.99
Neutral	56.83	55.90	55.00	53.45	52.79	52.04	54.34
Negative	55.89	55.40	54.38	52.99	52.34	51.78	53.80
Positive	56.38	55.58	54.59	52.99	52.24	51.53	53.89
	56.17	55.51	54.53	53.01	52.31	51.66	53.87
ACTIVITY	**AS060**	**AS090**	**AS125**	**AS180**	**AS210**	**AS250**	
False	50.38	50.05	49.22	47.65	47.05	46.47	48.47
Neutral	51.50	50.39	49.47	47.67	46.83	46.26	48.69
Negative	51.04	50.29	49.07	47.57	46.93	46.36	48.54
Positive	51.05	49.89	48.77	47.16	46.39	45.67	48.15
	51.07	50.11	49.02	47.41	46.68	46.04	48.39

Source: Price data downloaded from Bloomberg (2022)

A one-factor ANOVA (Welch's test) analysis was performed to differentiate between the parameter scores that eventually ended up recording either positive, negative, neutral, or false cycles (see Annexure A). It attempts to determine whether the behaviour of stocks post-selection depends on the size of the scores at selection.

The momentum score (MS) averages for the positive cycles across most momentum periods are higher than those for the other cycles. Positive cycles have the highest and false cycles the lowest overall scores on average. Zone 2 (090-125-180) has the highest average momentum scores overall. The results from Welch's ANOVA show that the difference between the average momentum scores for positive (28.95) and negative (26.29) cycles is statistically significant at a 5% level.

The results from the volatility score (VS) averages show that positive cycles have higher scores across most periods and the highest score on average. Neutral cycles have some of the lowest scores per period and the lowest average score overall. Zone 1 (060-90-125) has the highest and Zone 4 (180-210-250) has the lowest average volatility scores overall. Scores decline as the momentum periods increase. None of the overall differences but most of the per-period differences (except VS180/VS210 and VS210/VS250) are statistically significant at a 5% level.

The quality score (QS) averages of neutral cycles (per period and overall) are the highest, followed by positive cycles. False cycles, on the other hand, have the lowest average scores per period and overall. Scores decline as the momentum periods increase. Overall, the results from Welch's ANOVA show that the difference between the average quality scores for positive (53.89) and false (52.99) cycles are statistically significant at a 5% level. Also, all the per-period pairings (except QS060/QS090) are statistically significant at a 5% level.

The activity score (AS) averages for positive cycles are generally lower than those of the other cycles. Neutral and negative cycles have the highest activity scores on average. Scores decline as the momentum periods increase. None of the overall differences but most of the per-period differences (except AS060/AS090, AS180/AS210, and AS210/AS250) are statistically significant at a 5% level.

Table 5.6 Generalised outcomes

Parameters Cycles	MOMENTUM		VOLATILITY		QUALITY		ACTIVITY	
	High	Low	High	Low	High	Low	High	Low
False		X		X		X		X
Neutral	X			X	X		X	
Negative		X	X			X	X	
Positive	X		X		X			X

In summary, the results show that there is some indication that, in this equity market and on average, cycles with higher momentum, higher volatility, and higher quality scores combined with lower activity scores tend to be positive. Negative cycles, in comparison, have lower momentum and quality scores combined with higher activity. False cycles, on average, recorded some of the lowest scores in every category. Neutral cycles recorded lower volatility and higher activity scores on average compared to positive cycles. Note that even though some average scores are statistically different, the same combinations may not produce equivalent outcomes for individual cycles.

In the previous five subsections an analysis of the average hold, price range activity, sector activity, outcomes per entry zone, and the average parameter scores per cycle type provided a momentum profile for the Johannesburg Stock Exchange (JSE). In the next section, a custom momentum index evaluates the actual performance of the momentum model. The results are presented graphically and compared to benchmark indices as to performance, correlation, drawdown, and descriptive statistics.

Table 5.7 Statistically significant results

Momentum Score (MS)										
group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
POS	NEG	2.6627	0.6155	4.3258	3007.633	3.6330	0.4265	4.8990	0.0120	2.2363
Volatility Score (VS)										
group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
None										
Quality Score (QS)										
group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
FAL	NEU	1.3482	0.1926	7.0009	671.826	3.6330	0.6486	2.0478	0.0000	0.6996
FAL	POS	0.8982	0.1721	5.2181	476.077	3.6461	0.2706	1.5258	0.0014	0.6276
FAL	NEG	0.8085	0.1792	4.5123	549.689	3.6330	0.1576	1.4595	0.0082	0.6509
NEU	NEG	0.5397	0.1418	3.8053	1542.983	3.6330	0.0244	1.0550	0.0362	0.5153
Activity Score (AS)										
group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
None										
MS060-MS250										
group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
MS060	MS090	8.6334	0.8614	10.0229	1357.899	4.0300	5.1621	12.1047	0.0000	3.4713
MS060	MS125	12.4879	0.8915	14.0077	1329.173	4.0300	8.8951	16.0806	0.0000	3.5927
MS060	MS180	10.0870	0.8385	12.0304	1376.578	4.0300	6.7080	13.4660	0.0000	3.3790
MS060	MS210	6.3238	0.8575	7.3749	1361.303	4.0300	2.8682	9.7795	0.0000	3.4557
MS090	MS250	8.4979	0.9772	8.6964	1389.683	4.0300	4.5598	12.4359	0.0000	3.9380
MS125	MS210	6.1641	0.9585	6.4309	1394.154	4.0300	2.3013	10.0269	0.0001	3.8628
MS125	MS250	12.3524	1.0038	12.3051	1398.805	4.0300	8.3069	16.3978	0.0000	4.0455
MS180	MS210	3.7632	0.9094	4.1382	1397.872	4.0300	0.0984	7.4280	0.0407	3.6648
MS180	MS250	9.9515	0.9570	10.3982	1375.902	4.0300	6.0946	13.8084	0.0000	3.8569
MS210	MS250	6.1883	0.9738	6.3551	1387.788	4.0300	2.2641	10.1125	0.0001	3.9242
VS060-VS250										
group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
VS060	VS090	0.1785	0.0259	6.8859	1350.331	4.0300	0.0740	0.2829	0.0000	0.1045
VS060	VS125	0.3202	0.0233	13.7541	1140.097	4.0300	0.2264	0.4140	0.0000	0.0938
VS060	VS180	0.4838	0.0216	22.4403	919.521	4.0300	0.3969	0.5707	0.0000	0.0869
VS060	VS210	0.5231	0.0214	24.4358	898.114	4.0300	0.4368	0.6093	0.0000	0.0863
VS060	VS250	0.5595	0.0216	25.9287	922.291	4.0300	0.4725	0.6465	0.0000	0.0870
VS090	VS125	0.1417	0.0203	6.9722	1274.015	4.0300	0.0598	0.2236	0.0000	0.0819
VS090	VS180	0.3053	0.0183	16.6572	1014.393	4.0300	0.2314	0.3792	0.0000	0.0739
VS090	VS210	0.3446	0.0181	18.9872	985.289	4.0300	0.2714	0.4177	0.0000	0.0731
VS090	VS250	0.3810	0.0184	20.7622	1018.121	4.0300	0.3071	0.4550	0.0000	0.0740
VS125	VS180	0.1636	0.0144	11.3971	1227.604	4.0300	0.1057	0.2214	0.0000	0.0578
VS125	VS210	0.2029	0.0141	14.3650	1190.080	4.0300	0.1460	0.2598	0.0000	0.0569
VS125	VS250	0.2393	0.0144	16.6382	1232.179	4.0300	0.1813	0.2973	0.0000	0.0580
VS180	VS250	0.0757	0.0114	6.6498	1399.939	4.0300	0.0298	0.1216	0.0000	0.0459
QS060-QS250										
group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
QS060	QS125	1.6419	0.1843	8.9083	1184.641	4.0300	0.8991	2.3847	0.0000	0.7428
QS060	QS180	3.1655	0.1774	17.8461	1083.765	4.0300	2.4507	3.8803	0.0000	0.7148
QS060	QS210	3.8631	0.1763	21.9110	1066.758	4.0300	3.1525	4.5736	0.0000	0.7105
QS060	QS250	4.5078	0.1744	25.8422	1036.212	4.0300	3.8049	5.2108	0.0000	0.7030
QS090	QS125	0.9815	0.1566	6.2662	1343.207	4.0300	0.3502	1.6127	0.0001	0.6312
QS090	QS180	2.5050	0.1484	16.8803	1252.586	4.0300	1.9070	3.1030	0.0000	0.5980
QS090	QS210	3.2026	0.1471	21.7688	1234.198	4.0300	2.6097	3.7955	0.0000	0.5929
QS090	QS250	3.8474	0.1449	26.5571	1199.181	4.0300	3.2635	4.4312	0.0000	0.5838
QS125	QS180	1.5235	0.1303	11.6928	1370.046	4.0300	0.9984	2.0486	0.0000	0.5251
QS125	QS210	2.2211	0.1288	17.2396	1358.853	4.0300	1.7019	2.7403	0.0000	0.5192
QS125	QS250	2.8659	0.1263	22.6972	1334.071	4.0300	2.3570	3.3748	0.0000	0.5089
QS180	QS210	0.6976	0.1187	5.8770	1398.992	4.0300	0.2192	1.1759	0.0005	0.4783
QS180	QS250	1.3424	0.1159	11.5820	1391.754	4.0300	0.8753	1.8094	0.0000	0.4671
QS210	QS250	0.6448	0.1143	5.6433	1396.476	4.0300	0.1843	1.1053	0.0010	0.4605
AS060-AS250										
group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
AS060	AS125	2.0556	0.2587	7.9452	1388.846	4.0300	1.0130	3.0983	0.0000	1.0427
AS060	AS180	3.6648	0.2590	14.1502	1389.385	4.0300	2.6210	4.7085	0.0000	1.0437
AS060	AS210	4.3880	0.2594	16.9163	1390.187	4.0300	3.3427	5.4334	0.0000	1.0454
AS060	AS250	5.0300	0.2615	19.2351	1393.809	4.0300	3.9761	6.0838	0.0000	1.0538
AS090	AS125	1.0927	0.2491	4.3870	1399.559	4.0300	0.0889	2.0965	0.0240	1.0038
AS090	AS180	2.7019	0.2494	10.8353	1399.663	4.0300	1.6970	3.7068	0.0000	1.0049
AS090	AS210	3.4251	0.2498	13.7127	1399.795	4.0300	2.4185	4.4317	0.0000	1.0066
AS090	AS250	4.0670	0.2520	16.1416	1399.960	4.0300	3.0516	5.0824	0.0000	1.0154
AS125	AS180	1.6091	0.2471	6.5111	1399.993	4.0300	0.6132	2.6051	0.0001	0.9960
AS125	AS210	2.3324	0.2476	9.4214	1399.955	4.0300	1.3347	3.3301	0.0000	0.9977
AS125	AS250	2.9743	0.2498	11.9085	1399.253	4.0300	1.9678	3.9809	0.0000	1.0066
AS180	AS250	1.3652	0.2500	5.4599	1399.390	4.0300	0.3575	2.3728	0.0016	1.0077

5.3 MOMENTUM INDEX

All stocks or tickers identified by the customised model are included in the custom momentum index. The index is updated monthly when newly identified tickers (if any) are added (i.e., cycles entered), while current members with dMS250 scores below the set minimum (if any) are deleted from the index (i.e., cycles exited). The base date for the index is 31 December 2008, and the base or starting value is 100. The number of members is variable, and the index maintains a relatively active position over a true equal-weighted design, which resets all the weights to the average weight when updating. However, any new members are assigned the average weight of the current members, adjusted for the number of additions and the total weight of any deletions, equally distributed among all members.

5.3.1 Levels and members

The JSE Momentum Index (JSE-MI) can serve as a benchmark for momentum on the Johannesburg Stock Exchange (JSE). Figure 5.1 below contrasts the performance of the custom JSE Momentum Index to the FTSE/JSE All Share Index (ALSH) with its base date adjusted to 31 December 2008 and its base value to 100. The JSE-MI moved clear of ALSH in 2011. Starting with one member, Mr Price Group (MRP:SJ) in the Consumer Discretionary sector, on 31 December 2008 and ending 2009 with 80 members in the index. The MRP cycle lasted 32 months until 31 August 2011, with the price increasing from R24.75 to R73.75 during this period at a compound annual growth rate (CAGR) of 50.60%. The methodology of the momentum index may explain or account for the increasing outperformance since 2019.

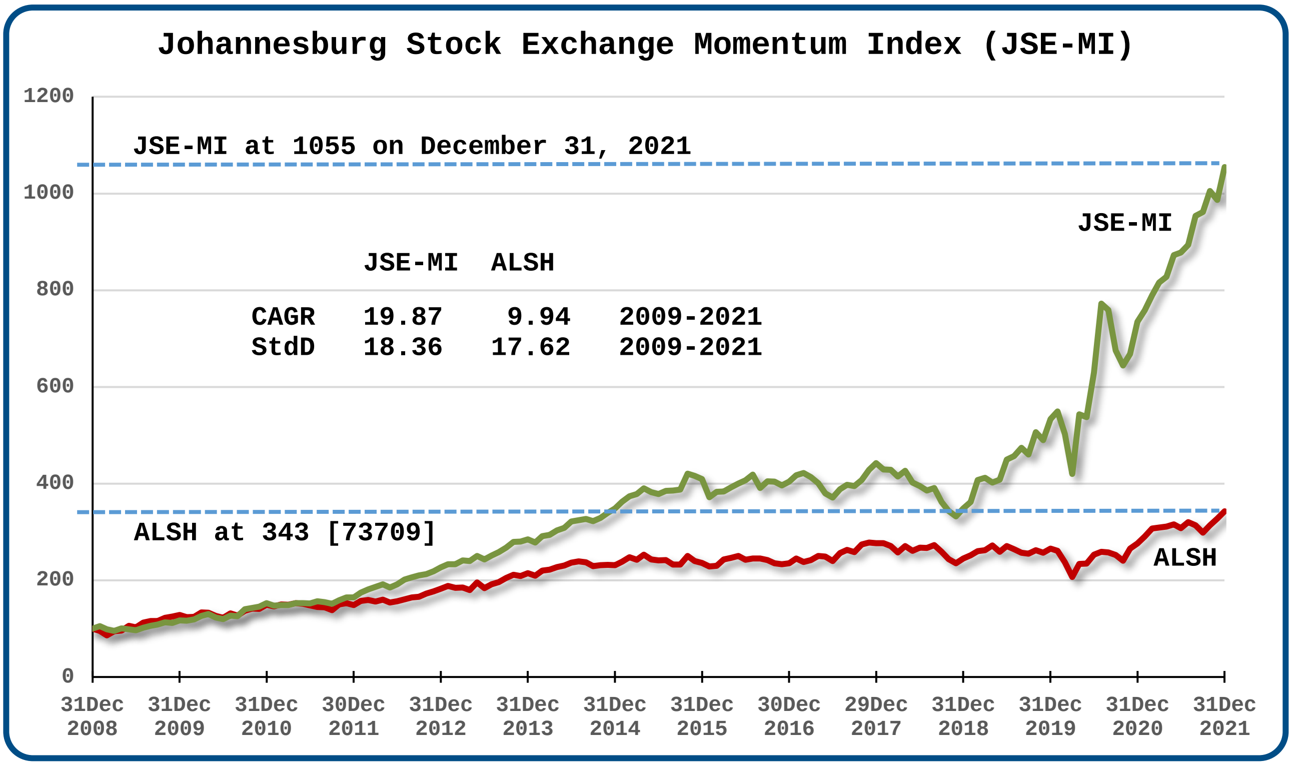

Figure 5.1 JSE Momentum Index (Source of price data: Bloomberg 2022)

The year 2020 was the most volatile period for the JSE Momentum Index (refer to Table 5.11 on page 5-16). Figure 5.2 below displays the volatility of the index during this period graphically. The three-year period beginning 2019 and ending 2021 recorded a compound annual growth rate (CAGR) of 45.62% with a standard deviation (StdD) of 29.89%. The individual statistics for years 2019 (CAGR:52.96% & StdD:18.40%), 2020 (CAGR:37.82% & StdD:47.23%) and 2021 (CAGR:43.49% & StdD:11.69%) confirms the increased volatility during this period. The index level dropped to 420 at the end of March 2020 and rebounded to 773 within four months, ending the year at 735. Equities outperformed during 2021 with the momentum index ending at 1055.60, an all-time high, posting the second largest year-on-year increase after 2019.

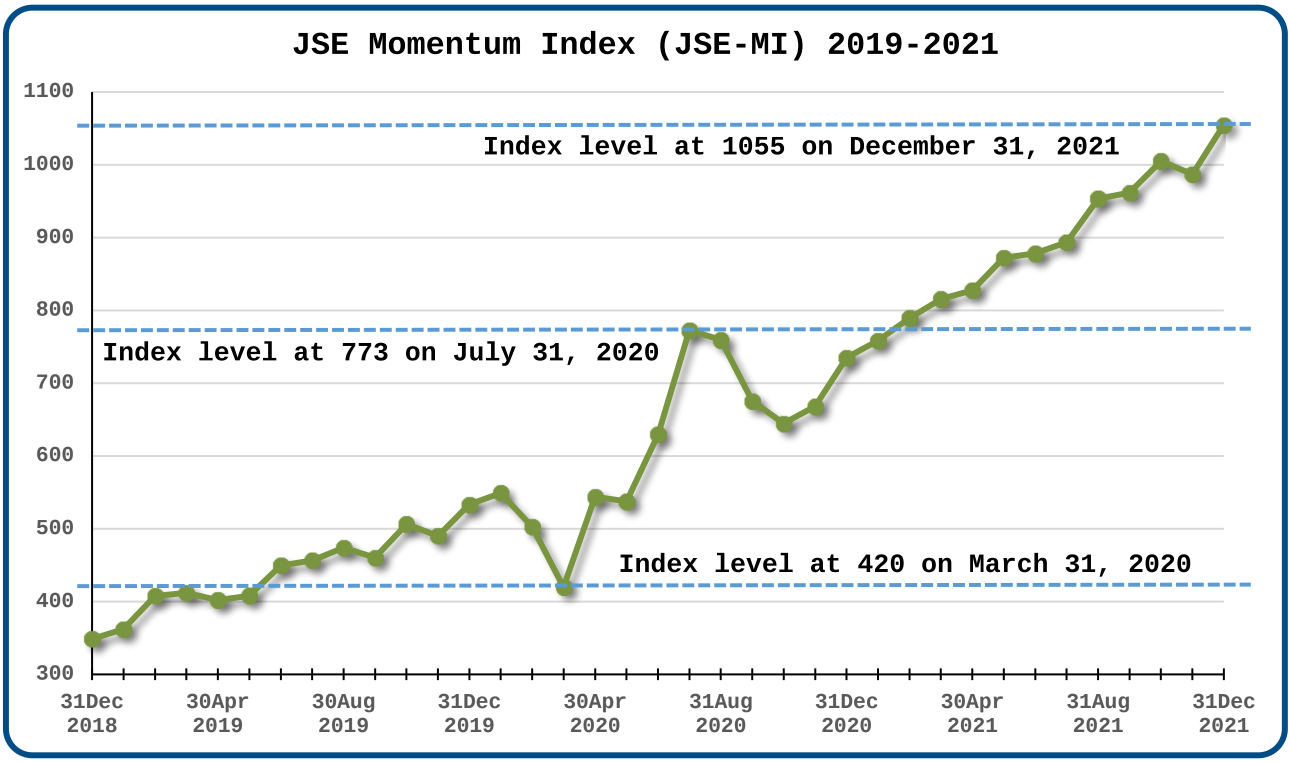

Figure 5.2 JSE-MI 2019-2021 (Source of price data: Bloomberg 2022)

Table 5.8, on the next page, describes the activity during the 2019-2021 period starting with the index at 349 and comprising only 7 members. The index lost 21.18% in value during the previous year, 2018, dropping 32 members and adding 13 (net 19 deletions). By the end of 2019, the index netted 15 additions and its value increased by 52.96%. The net amount of 15 additions is made up of 33 cycles entered and 18 cycles exited. Index members numbered 22 at the end of 2019. These are cumulative or annual returns – refer to Table 5.9 (page 5-14).

During 2020, the index gained 37.82% and netted 9 additions by entering 31 new cycles and exiting 22 cycles. Index members numbered 31 at the end of 2020.

Compare 2019 and 2020 to 2021, when the index gained 43.49% and netted 45 additions by entering 94 cycles and exiting 49 (30 gains and 19 losses). Index members numbered 76 at the end of 2021. Entering many new cycles and exiting the majority of completed cycles at a profit in a year when the index reached a high.

Table 5.8 Updating 2019-2021

Date	LEVEL	GROWTH	MEM	3MA	ADD	DEL	ADD/T
Dec 2018	348.64	4.99 %	7	7	1	1	0.50
Jan 2019	362.01	3.83 %	9	8	4	2	0.67
Feb 2019	407.63	12.60 %	14	10	5	0	1.00
Mar 2019	411.96	1.06 %	16	13	2	0	1.00
Apr 2019	401.99	-2.42 %	18	16	3	1	0.75
May 2019	408.15	1.53 %	18	17	1	1	0.50
Jun 2019	449.51	10.13 %	20	19	2	0	1.00
Jul 2019	456.64	1.59 %	22	20	2	0	1.00
Aug 2019	473.82	3.76 %	22	21	2	2	0.50
Sep 2019	460.16	-2.88 %	22	22	2	2	0.50
Oct 2019	506.31	10.03 %	20	21	2	4	0.33
Nov 2019	490.20	-3.18 %	21	21	5	4	0.56
Dec 2019	533.27	8.79 %	22	21	3	2	0.60
Jan 2020	549.13	2.97 %	22	22	2	2	0.50
Feb 2020	502.39	-8.51 %	20	21	1	3	0.25
Mar 2020	419.87	-16.43 %	14	19	0	6	0.00
Apr 2020	543.70	29.49 %	13	16	0	1	0.00
May 2020	537.62	-1.12 %	9	12	1	5	0.17
Jun 2020	629.59	17.11 %	7	10	0	2	0.00
Jul 2020	772.51	22.70 %	7	8	1	1	0.50
Aug 2020	759.36	-1.70 %	8	7	1	0	1.00
Sep 2020	675.22	-11.08 %	15	10	7	0	1.00
Oct 2020	644.57	-4.54 %	19	14	4	0	1.00
Nov 2020	668.14	3.66 %	26	20	7	0	1.00
Dec 2020	734.94	10.00 %	31	25	7	2	0.78
Jan 2021	758.56	3.21 %	40	32	12	3	0.80
Feb 2021	789.33	4.06 %	57	43	17	0	1.00
Mar 2021	815.63	3.33 %	71	56	16	2	0.89
Apr 2021	827.71	1.48 %	80	69	10	1	0.91
May 2021	872.45	5.41 %	82	78	6	4	0.60
Jun 2021	878.22	0.66 %	89	84	9	2	0.82
Jul 2021	893.75	1.77 %	87	86	3	5	0.38
Aug 2021	953.69	6.71 %	86	87	2	3	0.40
Sep 2021	961.60	0.83 %	81	85	3	8	0.27
Oct 2021	1005.11	4.52 %	82	83	4	3	0.57
Nov 2021	986.92	-1.81 %	80	81	7	9	0.44
Dec 2021	1054.60	6.86 %	76	79	5	9	0.36

Source: Price data downloaded from Bloomberg (2022)

Index activity may give some indication of the sentiment and volatility in the market when looking at the number of cycles entered versus exited. The range between additions and deletions in 2020 was relatively narrow and resulted in a volatile period. The progressively increasing number of members during 2019, 2020 and 2021 shows that the equity market trended upward after undergoing a slump in 2018. A simple gain versus loss comparison of completed cycles does not account for the much shorter negative cycles when matched with positive cycles.

Figure 5.3 overlays a line chart with changing index levels on a bar chart showing the variation in index members. There was a steady increase in value since the base date on 31 December 2008, building from a single member and peaking at 88 members within a year. From 2017 onwards the index members appear to synchronise with the index levels to some degree, surging and receding with the availability of momentum stocks in the market. After exiting many positions during a downswing, the index level surges as the number of member stocks grows.

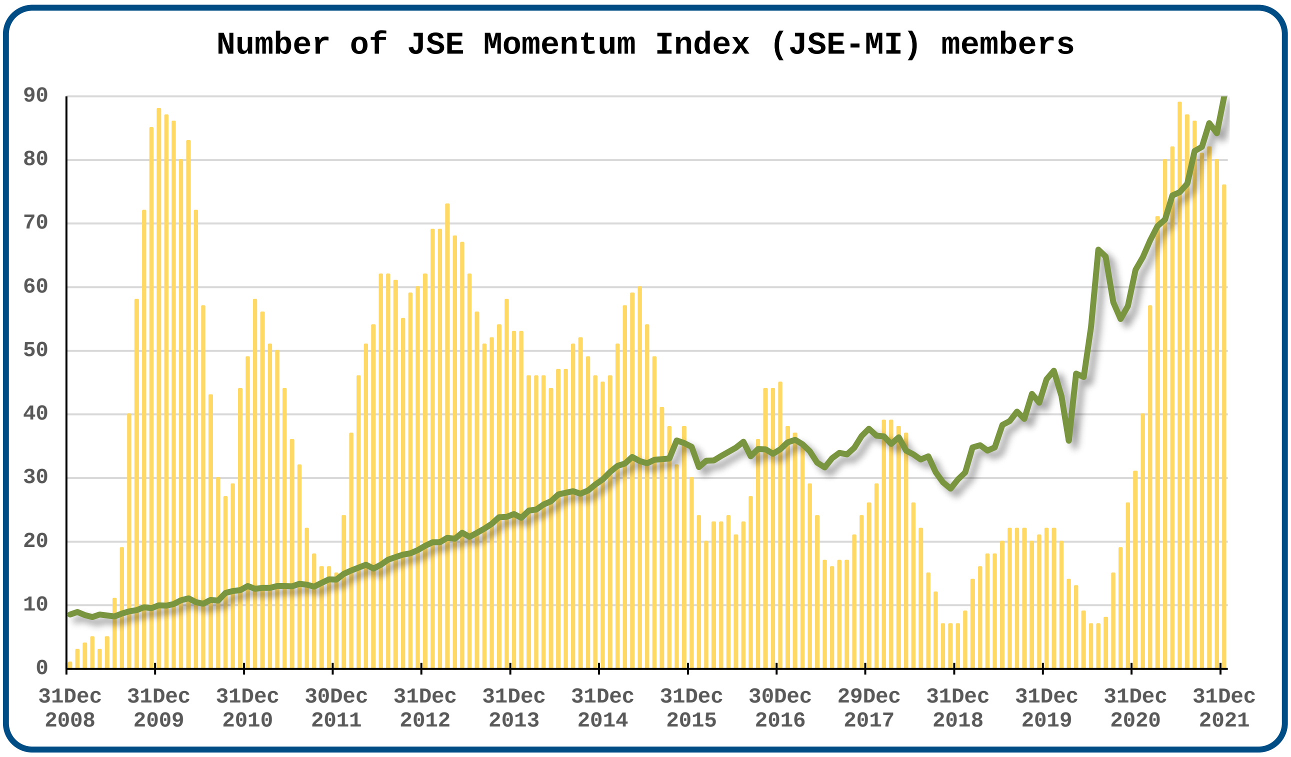

Figure 5.3 JSE-MI member numbers (Source of price data: Bloomberg 2022)

The table below summarises the annual results for the full 13-year period. The end-of-year members against the average members reflects the state of the market at year-end.

Table 5.9 Annual results 2009-2021

Year	LEVEL	GROWTH	MEM	AVG	ADD	DEL	ADD/T
2009	117.30	17.30 %	88	33	95	8	0.92
2010	152.82	30.28 %	49	57	44	83	0.35
2011	164.36	7.55 %	15	35	30	64	0.32
2012	226.63	37.89 %	62	53	75	28	0.73
2013	285.04	25.77 %	53	61	64	73	0.47
2014	348.84	22.38 %	45	48	52	60	0.46
2015	409.27	17.33 %	30	46	51	66	0.44
2016	404.33	-1.21 %	45	30	61	46	0.57
2017	442.31	9.39 %	26	25	40	59	0.40
2018	348.64	-21.18 %	7	23	35	54	0.39
2019	533.27	52.96 %	22	19	33	18	0.65
2020	734.94	37.82 %	31	16	31	22	0.58
2021	1054.60	43.49 %	76	76	94	49	0.66

Source: Price data downloaded from Bloomberg (2022)

5.3.2 Relative performance

A comparison between the performance of the custom JSE Momentum Index and indices from the FTSE/JSE Africa Index Series focuses on the relative performance of the model. The series of seven headline indices (ALSH, TOPI, LARM, LARG, MIDC, SMLS, FLED) segments the ordinary equity market into companies of various sizes or market capitalisations. A headline variant series (ETOP) replicates the Top 40 Index (TOPI) without weighting the individual member stocks, aligning it with the JSE Momentum Index methodology to some degree. Refer to Table 5.10 below for information on the different benchmarks.

Table 5.10 Benchmark information

FTSE/JSE All Share Index	
ALSH 2002-06-24	Represents 99% of the full market capitalisation value of all ordinary stocks listed on the Main Board of the JSE, only excluding the fledgling stocks (1%).
FTSE/JSE Top 40 Index	
TOPI 2002-06-24	Consists of the 40 largest and most investable ordinary stocks listed on the Main Board of the JSE. Unlike the other indices, the number of member stocks is fixed.
FTSE/JSE Top 40 Equally-Weighted Index	
ETOP 2010-07-01	Replicates the capitalisation-weighted Top 40 Index (TOPI) without weighting the member stocks, thereby allowing each stock to contribute equally to the value of the index.
FTSE/JSE Large & Mid Cap Index	
LARM 2016-10-19	Represents up to 96% of the full market capitalisation value of all ordinary stocks listed on the Main Board of the JSE, excluding the small-cap (3%) and fledgling (1%) stocks.
FTSE/JSE Large Cap Index	
LARG 2016-10-19	Represents up to 85% of the full market capitalisation value of all ordinary stocks listed on the Main Board of the JSE, excluding the mid-cap (11%), small-cap (3%) and fledgling (1%) stocks.
FTSE/JSE Mid Cap Index	
MIDC 2002-06-24	Represents approximately 11% of the full market capitalisation of all ordinary stocks listed on the Main Board of the JSE, excluding the large-cap (85%), small-cap (3%) and fledgling (1%) stocks.
FTSE/JSE Small Cap Index	
SMLC 2002-06-24	Represents approximately 3% of the full market capitalisation value of all ordinary stocks listed on the Main Board of the JSE, excluding the large-cap (85%), mid-cap (11%) and fledgling (1%) stocks.
FTSE/JSE Fledgling Index	
FLED 2002-06-24	Represents the lowest 1% of the full market capitalisation value of all ordinary stocks listed on the Main Board of the JSE, which are too small to be included in the All Share Index (ALSH).
S&P Momentum South Africa	
SPMZ 2014-11-20	This index comprises JSE-listed stocks with high price momentum and makes use of a rule-based methodology to provide exposure to the momentum factor. Capitalisation-weighted and rebalanced semi-annually in March and September.
SATRIX Momentum Index Fund	
STXM 2013-10-21	An open-end fund based on a proprietary SATRIX momentum index and tilted towards stocks with positive momentum and away from stocks with negative momentum. This fund is rebalanced every 6 weeks (8 times per annum).

Sources: FTSEI (2021); SATRIX (2022); SPDJM (2022)

Table 5.11, on the next page, shows the progression and relative performance of the JSE Momentum Index (JSE-MI) over time from its 2009 base year to the end of 2021. Note its performance in 2011 relative to the different benchmarks.

Table 5.11 JSE-MI results versus benchmarks (2009-2021)

Year	Metric	JSE-MI	ALSH	TOPI	ETOP	MIDC	SMLC	FLED	LARG	LARM	SPMZ	STXM
2009	CAGR	17.30	28.63	28.56	29.12	30.27	22.79	20.06	---	---	---	---
	StdD	20.91	24.57	26.92	23.17	15.80	9.80	10.58	---	---	---	---
2010	CAGR	30.28	16.09	14.57	16.90	25.77	20.93	18.73	---	---	---	---
	StdD	13.22	16.94	18.56	16.44	10.99	8.27	7.31	---	---	---	---
2011	CAGR	7.55	-0.41	-0.59	1.63	0.65	-2.28	0.52	---	---	---	---
	StdD	11.39	18.71	20.86	18.94	10.16	7.21	9.09	---	---	---	---
2012	CAGR	37.89	22.71	22.22	26.12	25.24	24.35	23.34	---	---	---	---
	StdD	7.61	11.39	12.74	13.80	7.93	5.34	5.99	---	---	---	---
2013	CAGR	25.77	17.85	19.22	8.22	9.41	21.80	7.38	---	---	---	---
	StdD	9.49	14.74	16.13	16.27	11.27	7.37	7.77	---	---	---	---
2014	CAGR	22.38	7.60	6.00	7.31	15.97	15.80	5.83	---	---	---	17.35
	StdD	9.33	12.84	14.09	13.67	9.58	6.99	7.57	---	---	---	13.80
2015	CAGR	17.33	1.85	4.16	-1.06	-10.10	-8.04	-5.09	---	---	3.96	9.83
	StdD	13.28	16.92	18.35	17.96	14.56	13.32	8.84	---	---	18.47	17.94
2016	CAGR	-1.21	-0.08	-4.14	-1.32	23.42	15.92	23.80	---	---	-15.44	-5.94
	StdD	13.08	16.96	18.23	19.35	16.76	10.82	10.39	---	---	17.73	18.53
2017	CAGR	9.39	17.47	19.66	8.12	3.68	-1.04	-4.12	20.49	18.18	7.62	29.34
	StdD	9.62	10.13	11.19	9.64	11.76	8.11	7.53	11.15	10.42	12.64	12.14
2018	CAGR	-21.18	-11.37	-11.05	-13.88	-12.96	-18.10	-11.23	-10.97	-11.14	-11.72	-19.55
	StdD	17.15	17.19	18.77	16.80	14.65	10.07	7.72	18.80	17.62	20.51	19.85
2019	CAGR	52.96	8.24	8.75	2.90	11.00	-7.95	-13.14	8.11	8.79	23.11	12.91
	StdD	18.40	12.75	13.79	13.72	12.19	7.93	11.80	13.97	13.04	14.50	13.96
2020	CAGR	37.82	4.07	7.01	2.43	-17.13	-3.28	-5.42	8.76	4.25	10.73	-7.10
	StdD	47.23	30.21	30.77	34.22	33.30	30.04	17.71	30.73	30.41	48.76	32.73
2021	CAGR	43.49	24.07	23.30	26.84	24.05	51.85	56.74	22.85	23.08	7.89	23.27
	StdD	11.69	15.84	16.95	15.07	14.72	13.32	13.18	18.11	16.24	21.02	20.71
FULL 2009 2021	CTGR	954.60	242.69	244.84	173.06	199.40	197.25	158.62	---	---	---	---
	CAGR	19.87	9.94	9.99	8.03	8.80	8.74	7.58	---	---	---	---
	StdD	18.36	17.62	18.97	18.50	15.36	12.25	10.14	---	---	---	---
10Y 2012 2021	CTGR	541.66	130.45	135.52	78.01	81.56	104.87	80.49	---	---	---	---
	CAGR	20.43	8.71	8.94	5.94	6.15	7.44	6.08	---	---	---	---
	StdD	19.09	16.73	17.83	18.12	16.10	13.17	10.44	---	---	---	---
5Y 2017 2021	CTGR	160.83	45.52	52.73	24.47	2.98	9.57	9.60	54.97	46.57	39.73	34.55
	CAGR	21.14	7.79	8.84	4.47	0.59	1.84	1.85	9.16	7.95	6.92	6.12
	StdD	24.78	18.53	19.46	19.75	19.07	16.18	12.24	19.68	18.81	26.74	21.12
3Y 2019 2021	CTGR	202.49	39.77	43.50	33.68	14.11	35.19	28.77	44.45	39.58	47.07	29.31
	CAGR	44.62	11.81	12.79	10.16	4.50	10.57	8.79	13.04	11.76	13.72	8.95
	StdD	29.89	20.98	21.74	22.93	22.12	19.50	14.53	22.07	21.23	31.64	23.72
1Y 2021	CTGR	43.49	24.07	23.30	26.84	24.05	51.85	56.74	22.85	23.08	7.89	23.27
	CAGR	43.49	24.07	23.30	26.84	24.05	51.85	56.74	22.85	23.08	7.89	23.27
	StdD	11.69	15.84	16.95	15.07	14.72	13.32	13.18	18.11	16.24	21.02	20.71

Source: Price data downloaded from Bloomberg (2022)

Apart from the 2009 base year and 2018, which recorded the worst result over the evaluation period, the JSE Momentum Index (JSE-MI) statistics measure well against the benchmarks. The 10-Year Compound Annual Growth Rate (CAGR), the 5-Year CAGR, and the 3-Year CAGR represent an improving performance and a consistent outperformance by the JSE-MI of the benchmarks.

The methodology of the momentum index may explain the increasing outperformance since 2019. Recall that the number of members is variable and that the index maintains a relatively active position when updated and rebalanced monthly.

5.3.3 Correlation analysis

Correlation measures the degree of co-movement or size of the linear association between two time-series. Correlation-squared (R-squared) indicates how closely an index tracks the performance of a particular benchmark. It also points to the reliability of the alpha (excess return) and beta (volatility) coefficients from a linear regression. Table 5.12 below shows the changes in correlation with the different benchmarks from year to year. The lowest correlation with other indices occurred during 2019 when the momentum index led the market in recovering from the downturn in 2018.

Table 5.12 Correlations: JSE-MI versus benchmarks

Year	ALSH	TOPI	ETOP	MIDC	SMLC	FLED	LARG	LARM	SPMZ	STXM
2009	0.62	0.60	0.65	0.72	0.47	0.34	---	---	---	---
2010	0.89	0.87	0.90	0.88	0.73	0.40	---	---	---	---
2011	0.82	0.80	0.81	0.82	0.70	0.32	---	---	---	---
2012	0.69	0.65	0.61	0.78	0.61	0.27	---	---	---	---
2013	0.84	0.82	0.84	0.82	0.75	0.36	---	---	---	---
2014	0.79	0.77	0.77	0.76	0.66	0.44	---	---	---	0.83
2015	0.75	0.71	0.82	0.79	0.60	0.62	---	---	0.88	0.83
2016	0.52	0.48	0.46	0.60	0.51	0.43	---	---	0.50	0.53
2017	0.50	0.44	0.48	0.51	0.50	0.45	0.45	0.49	0.54	0.53
2018	0.60	0.58	0.60	0.62	0.53	0.25	0.57	0.60	0.67	0.66
2019	0.25	0.22	0.31	0.53	0.22	-0.04	0.19	0.25	0.60	0.42
2020	0.71	0.71	0.70	0.63	0.63	0.43	0.71	0.71	0.90	0.73
2021	0.68	0.64	0.75	0.74	0.78	0.45	0.59	0.67	0.65	0.64
AVG	0.67	0.64	0.67	0.71	0.59	0.36	0.50	0.54	0.68	0.65
5Y	0.62	0.60	0.63	0.61	0.59	0.32	0.58	0.61	0.81	0.64
3Y	0.63	0.62	0.64	0.62	0.60	0.32	0.60	0.63	0.84	0.65
1Y	0.68	0.64	0.75	0.74	0.78	0.45	0.59	0.67	0.65	0.64

Source: Price data downloaded from Bloomberg (2022)

Results show the JSE-MI index mainly aligns with the Mid Cap index (MIDC) since 2009 based on yearly data and the average correlation coefficient. During 2021 it aligned most closely with the Small Cap index (SMLC). Measured over longer periods, the 3-year and 5-year correlations showed the highest co-movement occurring between the JSE Momentum Index and the S&P Momentum South Africa Index (SPMZ).

5.3.4 Drawdown analysis

A drawdown analysis highlights the potential for sudden large (20%-plus) losses in value and the likely time to recover (Wilmington 2018). It records the size and speed of maximum drawdowns and the time to return to former highs. Referring to Table 5.13, the JSE Momentum Index (JSE-MI) experienced its maximum drawdown in March 2020, the same as all the other indices except the Fledgling Index (FLED). It occurred over a much shorter period (19 days), also recovering within a comparatively short period (62 days) to its original high. The other indices had much longer maximum drawdown periods and recoveries. Unlike the other indices, the maximum drawdown for JSE-MI fell outside its maximum drawdown duration period of 362 days.

Table 5.13 Drawdown analysis (2009-2021)

Metric	JSE-MI	ALSH	TOPI	ETOP	MIDC	SMLC
Maximum drawdown	40.39%	38.46%	37.82%	46.95%	45.02%	57.06%
Date	2020-03-19	2020-03-19	2020-03-19	2020-03-23	2020-03-23	2020-03-19
Period	19 days	537 days	581 days	1226 days	903 days	749 days
Recovery	62 days	198 days	186 days	286 days	444+ days	409 days
Average drawdown	5.63%	5.59%	5.89%	7.58%	8.41%	10.88%
Maximum duration	362 days	735 days	767 days	1512 days	1347+ days	1158 days
From:	2018-01-10	2018-01-26	2017-11-22	2015-04-29	2016-08-15	2017-03-22
To:	2019-06-24	2021-01-05	2020-12-15	2021-05-17	2021-12-31	2021-11-08
Average duration	16 days	22 days	24 days	28 days	26 days	25 days
Annualised return	19.87%	9.94%	9.99%	8.03%	8.80%	8.74%
Drawdown ratio	0.49	0.26	0.26	0.17	0.20	0.15
Metric	**JSE-MI**	**FLED**	**LARG**	**LARM**	**SPMZ**	**STXM**
Maximum drawdown	40.39%	52.84%	37.67%	37.87%	51.66%	44.03%
Date	2020-03-19	2020-05-14	2020-03-19	2020-03-19	2020-03-19	2020-03-19
Period	19 days	806 days	581 days	537 days	1235 days	537 days
Recovery	62 days	359 days	178 days	198 days	86 days	443 days
Average drawdown	5.63%	10.92%	6.79%	6.92%	15.90%	10.50%
Maximum duration	362 days	1165 days	759 days	735 days	1321 days	980 days
From:	2018-01-10	2017-02-20	2017-11-22	2018-01-26	2015-04-13	2018-01-26
To:	2019-06-24	2021-10-18	2020-12-03	2021-01-05	2020-07-24	2021-12-28
Average duration	16 days	24 days	37 days	34 days	83 days	42 days
Annualised return	19.87%	7.58%	8.34%	7.28%	3.10%	6.60%
Drawdown ratio	0.49	0.14	0.22	0.19	0.06	0.15

Source: Price data downloaded from Bloomberg (2022)

On average, the size of a JSE-MI drawdown is 5.63%, lasting 16 days (peak to peak). It is apparent from Table 5.13 that the JSE-MI recovers more quickly from drawdowns than the other indices. The Mid Cap Index (MIDC) and the Small Cap Index (SMLC) experienced average drawdowns of 8.41% (lasting 26 days) and 10.88% (lasting 25 days) respectively. A higher drawdown ratio (annualised return to maximum drawdown) points to higher returns for an index on a risk-adjusted basis over the specified timeframe. The timeframe can be shortened to 3 or 5 years.

5.3.5 Descriptive statistics

Descriptive statistics, the process of describing data and presenting it graphically, provides the individual summary statistics listed in the table below. Summary statistics include the mean return of each index with its accompanying standard deviation. The coefficient of variation (CV), the size of the standard deviation about its mean, shows that the relative variability of the JSE-MI is comparatively low.

Table 5.14 Summary statistics (2009-2021)

Metric	JSE-MI	ALSH	TOPI	ETOP	MIDC	SMLC
Mean	0.0725 %	0.0379 %	0.0381 %	0.0309 %	0.0338 %	0.0335 %
Standard Error	0.0203 %	0.0194 %	0.0209 %	0.0204 %	0.0170 %	0.0135 %
Median	0.1096 %	0.0624 %	0.0753 %	0.0492 %	0.0610 %	0.0466 %
Standard Deviation	1.1545 %	1.1083 %	1.1927 %	1.1632 %	0.9667 %	0.7719 %
Sample Variance	1.3328	1.2282	1.4226	1.3530	0.9344	0.5958
Kurtosis	21.1831	6.2297	5.1139	6.2593	16.0612	41.4286
Skewness	-0.3633	-0.4809	-0.3548	-0.5219	-1.3906	-1.3271
Range	25.56 %	17.49 %	18.36 %	16.57 %	16.86 %	21.59 %
Maximum	12.49 %	7.26 %	7.91 %	6.44 %	5.65 %	10.29 %
Minimum	-13.07 %	-10.23 %	-10.45 %	-10.13 %	-11.21 %	-11.30 %
Sum	235.57 %	123.17 %	123.79 %	100.45 %	109.66 %	108.94 %
Count	3249	3249	3249	3249	3249	3249
CV	15.92	29.24	31.30	37.62	28.64	23.02

Metric	JSE-MI	FLED	LARG	LARM	SPMZ	STXM
Mean	0.0725 %	0.0292 %	0.0320 %	0.0281 %	0.0122 %	0.0256 %
Standard Error	0.0203 %	0.0112 %	0.0341 %	0.0326 %	0.0366 %	0.0271 %
Median	0.1096 %	0.0368 %	0.0607 %	0.0505 %	0.0415 %	0.0708 %
Standard Deviation	1.1545 %	0.6396 %	1.2294 %	1.1758 %	1.5406 %	1.2269 %
Sample Variance	1.3328	0.4091	1.5115	1.3824	2.3734	1.5052
Kurtosis	21.1831	10.8615	8.9279	10.8416	17.9250	8.7507
Skewness	-0.3633	-0.4182	-0.6622	-0.9335	-0.8145	-0.8238
Range	25.56 %	11.77 %	18.37 %	17.73 %	26.69 %	17.18 %
Maximum	12.49 %	5.70 %	8.29 %	7.45 %	10.70 %	6.75 %
Minimum	-13.07 %	-6.07 %	-10.09 %	-10.28 %	-15.99 %	-10.43 %
Sum	235.57 %	95.02 %	41.61 %	36.53 %	21.68 %	52.34 %
Count	3249	3249	1299	1299	1776	2047
CV	15.92	21.87	38.39	41.81	126.18	47.98

Source: Price data downloaded from Bloomberg (2022)

Some of these sets of data are not symmetric but negatively or left skewed with the means (averages) smaller than the medians (middle values). A left-skewed distribution has more values in the right tail, but the left tail is longer indicating many smaller positive returns and a few large negative returns. Data are moderately left-skewed with values between -1 and -0.5 (ETOP, LARG, LARM, SPMZ and STXM) and highly left-skewed when values are lower than -1 (MIDC and SMLC). The distributions of JSE-MI, the FTSE/JSE All Share Index (ALSH) and the FTSE/JSE Top 40 Index (TOPI) are approximately symmetric with skewness measuring between -0.5 and 0.0 for these indices.

Referring to Table 5.14 on the previous page, the kurtosis values point to heavy-tailed distributions with outliers or extreme positive and negative returns. Extreme returns can be defined as returns that exceed the 90th percentile, the top and bottom 10% of returns (Sankaran, Nguyen & Harikumar 2012). Compared to a normal distribution, described as mesokurtic, these distributions can be described as leptokurtic with excess kurtosis. Negatively skewed, heavy-tailed distributions are common in stock market data (Samunderu & Murahwa 2021).

Figure 5.4 below shows the dispersion of JSE-MI returns with most returns clustering around the mean. The histogram confirms the large kurtosis value with extreme positive and negative returns as outliers. The JSE-MI has the highest kurtosis value of all the indices and is therefore more likely to record extreme returns. A high kurtosis in combination with negative skewness may favour extreme negative returns, but with a skewness measuring between -0.5 and 0.5 the distribution of JSE-MI is almost symmetrical. The daily standard deviation and the sizeable range between the maximum and minimum daily returns also point to high variability in returns for the custom momentum index.

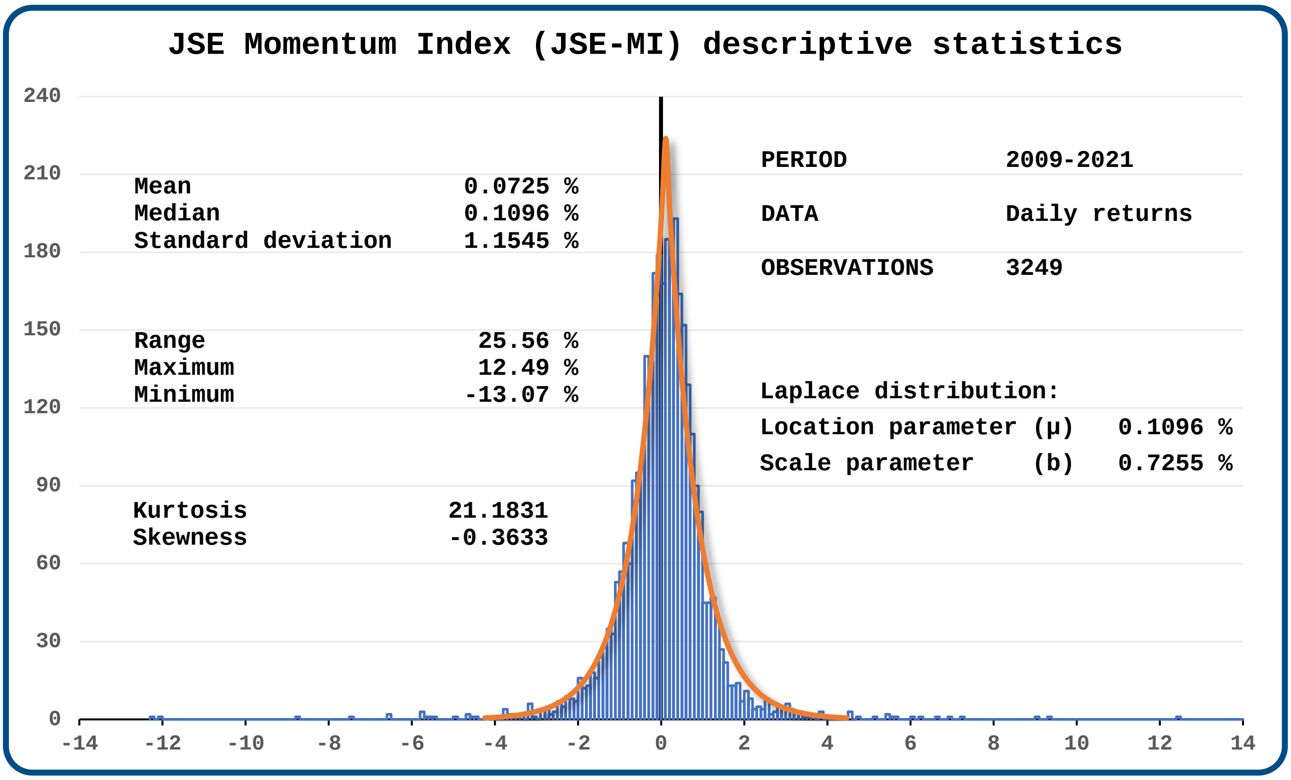

Figure 5.4 JSE-MI descriptive statistics (Source of price data: Bloomberg 2022)

As stated in Chapter 3, the assumption that the log returns of stock prices are normally distributed is convenient when performing statistical analysis. However, as evident from Figure 5.4, the Laplace distribution with its high central peak, narrow upper shoulders and heavy tails provides a better fit for log returns than the normal distribution. The Laplace distribution is symmetric about its location parameter (median) with the scale parameter (beta) determining its profile while the normal distribution is completely defined by its mean and standard deviation.

5.4 CONCLUSION

This chapter created a momentum profile for the Johannesburg Stock Exchange (JSE) by mechanically entering and exiting momentum cycles identified by the customised momentum model based on its four parameters and an exit rule. The results from applying the model mechanically provided a set of positive, negative, neutral, and false cycles unique to this equity market.

Momentum cycles with holds that extend beyond 9 months generally record positive returns. Positive cycles at an average hold of 13 months gained 43% in value. Negative cycles, in comparison, lost 15% in value at an average hold of 5 months. False cycles, holding shorter than 2 months on average lost 8% in value. Neutral cycles at an average hold of 8 months gained less than half a per cent in value. The average hold for this market is close to 9 months.

Stocks priced at less than R5 account for 21% of all the positive cycles. Only the R10-R25 stocks outperformed the below-R5 penny stocks. Overall, almost 80% of the positive cycles entered at prices below R100. The Consumer Discretionary sector outperformed all the other active sectors with 80-plus cycles. The outcomes show that a company listed in the Consumer Discretionary sector at a price ranging from R10 to R25 is likely to record a positive cycle. Among the less active sectors, Technology recorded the longest average hold while Health Care generated the highest rate of positive cycles.

Zone 1, presenting the earliest entry into any cycle, outperformed in general and generated the greatest number of entries at the longest average hold per zone. False cycles recorded negative returns in every zone while neutral cycles generally recorded small positive returns at relatively long average holds. Zone 2 recorded the worst compound annual return per average hold (CARpAH), largely due to the outcomes of the false and negative cycles in this zone.

A custom momentum index was used to evaluate the model by quantifying the process of entering the cycles at certain prices and exiting at either a gain or a loss. The performance of the momentum index compared favourably with the benchmark indices, generally tracking the mid-cap index most closely. A drawdown analysis showed that the custom index recovered more quickly from drawdowns and outperformed the other indices on a risk-adjusted basis.

Chapter 6 to follow evaluates the performance of the customised model for stocks listed on the Toronto Stock Exchange (TSX), similarly constructing a custom index, the TSX Momentum Index (TSX-MI).

Chapter 7 evaluates the customised model when applied to the TSX Venture Exchange (TSXV) by constructing a custom index (TSXV-MI) from small, less liquid stocks with momentum.

REFERENCES

Bloomberg. 2022. Bloomberg Terminal – Bloomberg Professional Services subscription. New York: Bloomberg Finance L.P.

De Longis, A., Zanin, D. & Ellis, D. 2022. Measuring sector cyclicality: a factor-based approach. *The Journal of Beta Investment Strategies*, 13(4):147–162. DOI: 10.3905/jbis.2022.1.014.

FTSEI. 2021. *FTSE/JSE Africa Index Series*. *Ground rules: FTSE/JSE Africa Index Series [Online]*. London Stock Exchange Group. Available at: https://research.ftserussell.com/products/downloads/FTSE_JSE_Africa_Index_Series_new.pdf (Accessed: 15 April 2022).

Humphreys, D. 2020. Mining productivity and the fourth industrial revolution. *Mineral Economics*, 33(1–2):115–125. DOI: 10.1007/s13563-019-00172-9.

Samunderu, E. & Murahwa, Y.T. 2021. Return based risk measures for non-normally distributed returns: an alternative modelling approach. *Journal of Risk and Financial Management*, 14(11):540. DOI: 10.3390/jrfm14110540.

Sankaran, H., Nguyen, A. & Harikumar, J. 2012. Extreme return correlation and volatility: a two-threshold approach. *American Journal of Business*, 27(2):154–173. DOI: 10.1108/19355181211274451.

SATRIX. 2022. *Satrix Momentum Index Fund*. *Fund Fact Sheet [Online]*. Satrix Managers. Available at: https://satrix.co.za/media/62531?inline=true (Accessed: 15 April 2022).

SPDJM. 2022. *S&P Momentum indices methodology*. *S&P Dow Jones Indices [Online]*. S&P Global. Available at: https://www.spglobal.com/spdji/en/documents/methodologies/methodology-sp-momentum-indices.pdf (Accessed: 15 June 2022).

Wilmington. 2018. *Understanding investment risk through drawdown analysis*. *Investment Insights [Online]*. Wilmington Trust Corporation. Available at: https://www.wilmingtontrust.com/repositories/wtc_sitecontent/PDF/Understanding-investment-risk-through-drawdown-analysis.pdf (Accessed: 27 April 2021).

CHAPTER SIX

MOMENTUM PROFILE: TORONTO STOCK EXCHANGE

6.1 INTRODUCTION

The customised momentum model, described in Chapter 4, was used to identify the momentum cycles of stocks listed on the Toronto Stock Exchange (TSX). In total, 2335 cycles-in-progress were identified with entry points determined by the parameter settings. A momentum profile for this equity market will be created by entering these cycles mechanically and exiting on a fixed exit rule. The results from applying the model mechanically (i.e., without taking any discretionary actions) will provide a set of positive, negative, neutral, and false cycles unique to this market – the different types of cycles are defined in the next section.

A custom momentum index will be used to evaluate the model by quantifying the process of entering cycles at certain prices and exiting at either a gain or a loss. The index level follows and accumulates the prices of the incumbent member stocks, with cycles overlapping as stocks are added to and deleted from the index when updated. The construction of the index (refer to Chapter 3), equally weighting new members but allowing existing members to retain their momentum, should maintain a relatively active position in the market. In addition, the changing number of members should indicate the availability of momentum stocks (as identified by this specific model) in this market at a particular point in time.

The section to follow uses the outcomes generated by the customised model to create a momentum profile for this equity market.

6.2 MOMENTUM MODEL OUTCOMES

The momentum cycles generated by the model are classified as either false, neutral, negative, or positive depending on the outcome. In this study, it is assumed that a positive cycle (optimal outcome) would hold at least 3 months and record an annualised gain of more than 10%. A negative cycle (unexpected outcome) would record an annualised loss of more than 10% while also holding at least 3 months. A neutral cycle (no outcome) is assumed to hold a minimum of 3 months but gain or lose a maximum of 10% annualised. A false cycle (failed outcome) holds shorter than 3 months. These assumptions are based on the theory of price momentum, which states that momentum formed over 3 to 12 months should hold for 3 to 12 months (60 to 250 trading days) – refer to Chapter 2. At a momentum score setting of 20% per period, the 10% annualised cut-off was chosen as minimum evidence of some momentum between entry and exit.

In the following five subsections, a momentum profile for this equity market will be created by analysing the different cycles in terms of average hold, price range activity, sector activity, outcomes per entry zone (refer to Chapter 4), and the average parameter (momentum, volatility, quality, and activity) scores per cycle type.

6.2.1 Holding periods

The results per average hold period or Average Hold (AH), in Table 6.1 below, show that the different cycles are distinct in average hold period. Each type tends to dominate a particular range. False cycles are confined to shorter than 3 months by definition and account for almost 9% (208 from 2335) of all cycles, posting a high negative annual return due to the short average hold of 1.51 months.

Table 6.1 Average hold

HOLD	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
1-2	208	208	---	---	---	1.51	-12.60	-65.71
	AH	1.51	---	---	---			
	CRpAH	-12.60	---	---	---			
	CARpAH	-65.71	---	---	---			
3-5	492	---	62	368	62	4.05	-13.83	-35.63
	AH	---	4.32	3.99	4.13			
	CRpAH	---	-0.47	-20.25	18.18			
	CARpAH	---	-1.29	-49.33	62.50			
6-8	649	---	220	318	111	7.05	-8.04	-13.30
	AH	---	7.27	6.81	7.30			
	CRpAH	---	-0.35	-21.16	21.89			
	CARpAH	---	-0.57	-34.24	38.48			
9-11	500	---	149	67	284	9.83	11.48	14.19
	AH	---	9.62	9.25	10.07			
	CRpAH	---	1.13	-23.62	28.28			
	CARpAH	---	1.41	-29.48	34.55			
12-17	312	---	29	5	278	13.91	47.79	40.07
	AH	---	13.45	12.80	13.98			
	CRpAH	---	3.92	-28.17	55.32			
	CARpAH	---	3.49	-26.67	45.93			
18-23	113	---	1	---	112	19.92	89.67	47.05
	AH	---	23.00	---	19.89			
	CRpAH	---	19.54	---	90.45			
	CARpAH	---	9.76	---	47.49			
24+	61	---	---	---	61	28.72	184.27	54.73
	AH	---	---	---	28.72			
	CRpAH	---	---	---	184.27			
	CARpAH	---	---	---	54.73			
TSX	2335	208	461	758	908	8.62	6.94	9.78
	AH	1.51	8.06	5.70	12.99			
	CRpAH	-12.60	0.42	-20.99	48.88			
	CARpAH	-65.71	0.62	-39.12	44.44			

Source: Price data downloaded from Bloomberg (2022)

The majority (369 from 461 or 80%) of neutral cycles cluster in the 6-11-month range with small returns, both negative (6-8) and positive (9-11), at a relatively long average hold before ultimately exiting without much change in value. Note that neutral cycles record positive returns at holds longer than 9 months. Negative cycles (758 from 2335 or 32%) are shorter in average hold than neutral cycles and dominate the 3-8-month range, falling by more than 20% per average hold of 5.30 months. Positive cycles (908 from 2335 or 39%) are predominant in the 9-17-month range (562 from 908 or 62%) while several cycles (173 from 908 or 19%) also hold longer than 18 months to record annualised returns of 50% on average. Note that the 6-8-month range recorded a negative return with both the neutral and negative cycles outnumbering the positive cycles.

It can be concluded that momentum cycles that hold beyond 9 months generally record high positive returns. Negative cycles have a shorter average hold at 5.70 months with only 9% (72 from 758) extending beyond 9 months.

Overall results show 208 false (9%), 461 neutral (20%), 758 negative (32%), and 908 positive (39%) cycles. Referring to Table 6.1 on the previous page, note the increasingly higher compound returns when positive cycles move into the 12-17-month range and beyond in contrast to the shorter negative cycles. The average hold of positive cycles is 13 months, with the average hold of negative cycles half as long at shorter than 6 months. The false and neutral cycles did either not hold (< 3 months) or build (CAGR ≤ 10%) any momentum.

6.2.2 Price ranges

Based on the results per price range, stocks trading between $1 and $2 recorded the highest compound return (13.56%) and compound annual return (18.43%) per average hold of 9 months. However, referring to Table 6.2 on the next page, stocks below $1 with positive cycles outperformed all the other positive-cycle price ranges. Also note that the number of negative cycles (74 from 174 or 43%) in this range exceeds the number of positive cycles, shortening the average hold and dragging the overall performance of this range down.

Most of the positive cycles (674 from 908 or 74%) fall within the $2 to $50 range with 36% (328 from 908) falling within the $2 to $10 range and 38% (346 from 908) within the $10 to $50 range. Note the fall in both the average hold and annualised returns of the positive cycles when stocks trade at progressively higher prices. The false, neutral, and negative cycles cluster in the same $2 to $50 range. The neutral cycles generally recorded small positive returns, but at an average hold approaching that of positive cycles – duration without continuation. False cycles align more with negative cycles but at a far shorter average hold – reversal without duration. False cycles present a larger problem than negative cycles, which in many instances can be explained by external events.

Table 6.2 Price range activity

CAD	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
< 1	174	21	19	74	60	8.30	6.99	10.27
	AH	1.43	7.89	5.77	13.95			
	CRpAH	-15.04	-0.74	-27.36	91.50			
	CARpAH	-74.58	-1.12	-48.57	74.88			
≥ 1 < 2	187	15	24	64	84	9.02	13.56	18.43
	AH	1.53	8.42	5.81	12.98			
	CRpAH	-23.42	0.35	-25.02	73.17			
	CARpAH	-87.61	0.50	-44.82	66.17			
≥ 2 < 5	376	30	61	121	164	9.25	9.77	12.85
	AH	1.50	8.77	6.17	13.12			
	CRpAH	-18.86	0.92	-24.21	57.30			
	CARpAH	-81.21	1.26	-41.65	51.35			
≥ 5 < 10	395	39	76	116	164	9.32	9.21	12.00
	AH	1.49	8.74	5.90	13.88			
	CRpAH	-8.47	0.91	-22.68	50.82			
	CARpAH	-51.04	1.25	-40.75	42.64			
≥ 10 < 20	491	40	122	153	176	8.58	5.99	8.48
	AH	1.55	7.57	5.51	13.54			
	CRpAH	-10.30	-0.12	-17.64	42.84			
	CARpAH	-56.89	-0.19	-34.47	37.16			
≥ 20 < 50	456	39	103	144	170	8.01	3.62	5.47
	AH	1.46	8.00	5.48	11.66			
	CRpAH	-7.45	0.78	-16.74	30.17			
	CARpAH	-47.07	1.17	-33.06	31.18			
≥ 50 < 100	162	16	36	54	56	7.86	2.98	4.58
	AH	1.56	7.58	5.20	12.41			
	CRpAH	-11.52	0.46	-18.19	36.40			
	CARpAH	-60.95	0.74	-37.06	35.01			
≥ 100	94	8	20	32	34	7.54	1.87	2.99
	AH	1.75	7.10	5.50	11.09			
	CRpAH	-17.40	-0.50	-17.65	32.58			
	CARpAH	-73.05	-0.85	-34.55	35.69			
TSX	2335	208	461	758	908	8.62	6.94	9.78
	AH	1.51	8.06	5.70	12.99			
	CRpAH	-12.60	0.42	-20.99	48.88			
	CARpAH	-65.71	0.62	-39.12	44.44			

Source: Price data downloaded from Bloomberg (2022)

Note the declining number of cycles in the two $50+ ranges, recording the shortest average holds and the worst compound returns. The results confirm that hold duration largely determines the outcome.

6.2.3 Sectors

Materials (28%), which includes the Metals and Mining industries, was the most active sector overall with Energy (16%) and Industrials (12%) lagging far behind. Real Estate (2%) was the least active sector overall followed by Utilities (3.5%) and Communications (4%). Activity per cycle type exhibits a similar pattern but with the positive cycles in Materials lower at 24% (222 from 908) and its negative cycles higher at 36% (273 from 758).

Table 6.3 Sector activity

SECTOR	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
CO	88	11	15	25	37	8.51	8.42	12.08
	AH	1.36	7.93	5.24	13.08	COMMUNICATIONS		
	CRpAH	-9.61	0.18	-19.67	44.73			
	CARpAH	-58.91	0.27	-39.44	40.37			
CD	186	18	40	48	80	9.45	11.32	14.59
	AH	1.50	8.08	5.23	14.46	CONSUMER DISCRETIONARY		
	CRpAH	-8.01	1.00	-17.27	45.79			
	CARpAH	-48.73	1.49	-35.28	36.72			
CS	135	8	41	28	58	9.16	12.57	16.77
	AH	1.75	7.49	5.39	13.19	CONSUMER STAPLES		
	CRpAH	-9.24	0.53	-13.94	43.00			
	CARpAH	-48.54	0.85	-28.39	38.46			
EN	373	29	60	127	157	8.58	8.38	11.91
	AH	1.59	8.57	5.69	12.23	ENERGY		
	CRpAH	-9.76	0.44	-21.78	50.26			
	CARpAH	-54.01	0.61	-40.46	49.12			
FI	199	14	46	58	81	8.18	3.34	4.94
	AH	1.36	8.00	5.17	11.60	FINANCIALS		
	CRpAH	-16.18	0.76	-17.11	27.30			
	CARpAH	-78.99	1.15	-35.30	28.35			
HC	133	13	26	49	45	8.92	2.74	3.70
	AH	1.62	7.62	6.49	14.42	HEALTH CARE		
	CRpAH	-31.91	1.95	-27.95	71.02			
	CARpAH	-94.24	3.09	-45.45	56.28			
IN	282	25	65	83	109	8.57	8.10	11.52
	AH	1.40	7.57	5.73	12.97	INDUSTRIALS		
	CRpAH	-5.73	0.16	-18.48	44.73			
	CARpAH	-39.70	0.25	-34.78	40.77			
MA	658	60	103	273	222	8.39	3.79	5.47
	AH	1.53	8.87	5.94	13.04	MATERIALS		
	CRpAH	-14.02	-0.13	-23.23	61.08			
	CARpAH	-69.34	-0.18	-41.38	55.09			

Table 6.3 Sector activity (continued)

SECTOR	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
RE	44	4	8	9	23	9.82	16.05	19.95
	AH	1.75	8.13	4.56	13.87	REAL ESTATE		
	CRpAH	-11.92	-1.61	-14.95	45.62			
	CARpAH	-58.13	-2.38	-34.72	38.43			
TE	155	14	41	36	64	9.41	13.88	18.04
	AH	1.36	7.05	5.61	14.81	TECHNOLOGY		
	CRpAH	-12.40	0.33	-18.03	57.38			
	CARpAH	-68.98	0.56	-34.64	44.40			
UT	82	12	16	22	32	6.73	0.35	0.63
	AH	1.58	7.81	4.82	9.44	UTILITIES		
	CRpAH	-10.05	1.15	-16.43	18.11			
	CARpAH	-55.19	1.77	-36.05	23.58			
TSX	2335	208	461	758	908	8.62	6.94	9.78
	AH	1.51	8.06	5.70	12.99	TORONTO STOCK EXCHANGE		
	CRpAH	-12.60	0.42	-20.99	48.88			
	CARpAH	-65.71	0.62	-39.12	44.44			

Source: Price data downloaded from Bloomberg (2022)

Positive cycles account for 39% (908 from 2335) of all cycles, with comparatively higher percentages in Consumer Discretionary (43%), Consumer Staples (43%) and Energy (42%) among the active sectors. Real Estate (52%) and Technology (41%) recorded comparatively higher percentages among the less active sectors. Materials (34%) and Health Care (34%) recorded comparatively lower percentages.

Negative cycles account for 32% (758 from 2335) of all cycles with comparatively higher percentages in Materials (41%), Health Care (37%) and Energy (34%). Technology (23%), Consumer Staples (21%), and Real Estate (20%) recorded lower percentages. Negative cycles seemed to drift from two strong positive-cycle sectors (Consumer Staples and Technology) to the Materials, Health Care and Energy sectors.

Neutral cycles cluster in Consumer Staples (30%), Technology (26%) and Financials (23%) relative to an overall representation of 20% (461 from 2335). Materials (16%), Energy (16%) and Communications (17%) are underrepresented. Communications is also one of the stronger false-cycle sectors at 12.5% (11 from 88) relative to the 9% (208 from 2335) overall representation by false cycles. Utilities recorded the highest rate of false cycles (12 from 82 or 15%).

The small Real Estate sector appears to favour positive outcomes when momentum cycles do form. Materials as the most active but also the largest sector produced average results but also generated the most negative cycles, outnumbering its positive cycles. Utilities, Financials and Health Care were the worst-performing sectors on the Toronto Stock Exchange (TSX) overall.

6.2.4 Entry zones

An entry zone, three successive formation periods, identifies and confirms a momentum cycle in progress. The earliest entry (i.e., shortest formation) with potentially the longest hold should occur in Zone 1. The stepped pattern of a regular momentum profile exits each cycle as late as possible. Zones 2 to 4 allow for later entries and more irregular patterns or individual profiles.

Table 6.4 Results per entry zone

ZONE	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
1	837	85	173	259	320	9.13	6.88	9.14
	AH	1.42	8.71	6.12	13.84			
	CRpAH	-9.72	0.42	-21.36	48.18			
	CARpAH	-57.75	0.58	-37.57	40.64			
2	490	45	88	182	175	8.75	6.05	8.39
	AH	1.49	8.34	5.74	13.96			
	CRpAH	-12.93	-0.47	-20.59	55.61			
	CARpAH	-67.23	-0.68	-38.26	46.25			
3	497	33	103	155	206	8.43	7.36	10.63
	AH	1.48	7.73	5.46	12.13			
	CRpAH	-16.20	0.61	-20.30	44.37			
	CARpAH	-76.02	0.95	-39.27	43.80			
4	511	45	97	162	207	7.87	7.49	11.65
	AH	1.71	6.98	5.21	11.70			
	CRpAH	-14.89	1.01	-21.52	48.94			
	CARpAH	-67.73	1.74	-42.77	50.47			
TSX	2335	208	461	758	908	8.62	6.94	9.78
	AH	1.51	8.06	5.70	12.99			
	CRpAH	-12.60	0.42	-20.99	48.88			
	CARpAH	-65.71	0.62	-39.12	44.44			

Source: Price data downloaded from Bloomberg (2022)

Table 6.4 above shows the outcomes from momentum cycles entered at these four different zones. Zone 1, as expected, generated the greatest number of entries at the longest average hold. The remaining number of cycles is spread evenly among the other zones. Note that the negative cycles in Zone 2 outnumber the positive cycles, which resulted in the lowest compound returns from this zone despite having the second-longest average hold. The rate of positive cycles in Zone 3 (41.5%) and Zone 4 (40.5%) were higher than the overall average for positive cycles (39%).

The average hold decreases from Zone 1 to Zone 4, but the shorter average holds in zones 3 and 4 generated higher compound returns. Apart from Zone 2, neutral cycles posted small positive returns. False cycles generated large negative compound annual returns, and the high percentage (85 from 208 or 41%) of false cycles in Zone 1 impacted its performance.

6.2.5 Parameter scores

The model identified 701 individual cycles with the [20|1.5|48|35] parameter setting combination. The average parameter scores for each period – which resulted in false, neutral, negative, or positive cycles – are included in Table 6.5 below.

Table 6.5 Average parameter scores

MOMENTUM	MS060	MS090	MS125	MS180	MS210	MS250	
False	20.24	29.99	32.53	24.99	19.45	14.11	23.55
Neutral	22.70	32.77	37.06	31.51	26.68	18.23	28.16
Negative	30.34	39.92	42.34	36.81	30.30	22.30	33.67
Positive	27.84	38.60	40.98	37.13	31.21	23.95	33.29
	26.96	37.11	39.89	34.83	28.97	21.41	31.35
VOLATILITY	**VS060**	**VS090**	**VS125**	**VS180**	**VS210**	**VS250**	
False	1.20	0.92	0.72	0.61	0.59	0.56	0.77
Neutral	1.08	0.89	0.71	0.55	0.52	0.52	0.71
Negative	1.39	1.09	0.81	0.60	0.57	0.57	0.84
Positive	1.26	1.04	0.84	0.61	0.58	0.53	0.81
	1.26	1.02	0.79	0.59	0.57	0.55	0.80
QUALITY	**QS060**	**QS090**	**QS125**	**QS180**	**QS210**	**QS250**	
False	54.15	54.06	53.10	51.82	50.93	50.50	52.43
Neutral	56.31	55.55	54.47	52.93	52.19	51.56	53.83
Negative	55.82	54.98	53.90	52.46	51.77	51.18	53.35
Positive	55.96	54.87	53.88	52.51	51.84	51.16	53.37
	55.82	54.97	53.93	52.52	51.80	51.19	53.37
ACTIVITY	**AS060**	**AS090**	**AS125**	**AS180**	**AS210**	**AS250**	
False	49.12	48.85	47.96	46.81	46.04	45.70	47.41
Neutral	52.35	51.25	50.02	48.39	47.69	47.05	49.46
Negative	51.28	50.22	48.92	47.30	46.61	46.01	48.39
Positive	51.16	49.78	48.57	47.10	46.42	45.76	48.13
	51.25	50.13	48.92	47.39	46.70	46.09	48.41

Source: Price data downloaded from Bloomberg (2022)

One-factor ANOVA (Welch's test) analyses were performed to possibly differentiate between the average parameter scores of the four different groups – positive (POS), negative (NEG), neutral (NEU), and false (FAL) cycles. In several instances, the differences between the averages of these four groups, and the per-period averages for each parameter were found to be statistically significant – refer to Annexure B.

The momentum score (MS) averages for the positive and negative cycles across all momentum periods are higher than those for the false and neutral cycles. All the overall differences (except POS/NEG) are significant at a 5% level. Negative cycles have the highest and false cycles have the lowest overall scores on average. Zone 2 (090-125-180) has the highest average momentum scores overall.

In general, the average volatility scores for the positive cycles are high in every period with the negative cycle scores the highest on average. Scores decline as the momentum periods increase. Based on the overall averages, the FAL/NEG, NEU/POS, and NEU/NEG pairings are significantly different at a 5% level. Most of the per-period differences (except VS180/MS210, VS180/VS250, and VS210/VS250) are statistically significant at a 5% level (refer to Annexure B).

The quality score (QS) averages for neutral and positive cycles are higher than those for false and negative cycles. In general and on average, neutral cycles have the highest and false cycles have the lowest quality scores. Scores decline as the momentum periods increase. Most of the differences between the overall averages (except POS/NEG), and all the per-period differences are statically significant at a 5% level (refer to Annexure B).

The activity score (AS) averages for neutral and negative cycles are higher than those for false and positive cycles. Neutral cycles have the highest average score overall, with false cycles the lowest. Scores decline as the momentum periods increase. Most of the differences between the overall averages (except POS/NEG), and all the per-period differences are statically significant at a 5% level.

Table 6.6 Generalised outcomes

Parameters Cycles	MOMENTUM		VOLATILITY		QUALITY		ACTIVITY	
	High	Low	High	Low	High	Low	High	Low
False		X		X		X		X
Neutral		X		X	X		X	
Negative	X		X			X	X	
Positive	X		X		X			X

In summary, the results show that there is some indication that, in this developed market and on average, cycles with higher momentum, higher volatility, and higher quality scores combined with lower activity scores tend to be positive. Negative cycles, in general and on average, have the highest momentum, volatility, and activity scores with lower quality scores compared to positive cycles. False cycles, on average, recorded the lowest scores in every category but volatility. Neutral cycles delivered higher quality and volatility scores in combination with lower momentum and volatility. Even though some scores are statistically different, the behaviour of individual stocks post-selection may not depend on the size of their scores at selection. These results only point to likely outcomes in general and on average.

In the previous five subsections an analysis of the average hold, price range activity, sector activity, outcomes per entry zone, and the average parameter scores per cycle type provided a momentum profile for the Toronto Stock Exchange (TSX). In the next section, a custom momentum index evaluates the actual performance of the momentum model. The results are presented graphically and compared to benchmark indices as to performance, correlation, drawdown, and descriptive statistics.

Table 6.7 Statistically significant results

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
Momentum Score (MS)										
FAL	NEU	4.6066	0.5717	8.0584	2957.901	3.6330	2.5298	6.6834	0.0000	2.0768
FAL	POS	9.7344	0.5356	18.1760	2639.075	3.6330	7.7887	11.6802	0.0000	1.9457
FAL	NEG	10.1146	0.5659	17.8727	3094.620	3.6330	8.0586	12.1706	0.0000	2.0560
NEU	POS	5.1278	0.4795	10.6946	6414.215	3.6330	3.3859	6.8698	0.0000	1.7419
NEU	NEG	5.5080	0.5132	10.7334	6791.613	3.6330	3.6436	7.3723	0.0000	1.8643
Volatility Score (VS)										
group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
FAL	NEG	0.0715	0.0197	3.6338	1991.8500	3.6330	0.0000	0.1431	0.0502	0.0715
NEU	POS	0.1010	0.0126	7.9974	5755.2122	3.6330	0.0551	0.1469	0.0000	0.0459
NEU	NEG	0.1261	0.0137	9.2198	6491.4347	3.6330	0.0764	0.1758	0.0000	0.0497
Quality Score (QS)										
group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
FAL	NEU	1.4093	0.0974	14.4651	2473.131	3.6330	1.0553	1.7633	0.0000	0.3540
FAL	POS	0.9444	0.0896	10.5405	1924.369	3.6330	0.6189	1.2699	0.0000	0.3255
FAL	NEG	0.9257	0.0907	10.2053	2006.721	3.6330	0.5961	1.2552	0.0000	0.3295
NEU	POS	0.4649	0.0685	6.7903	5635.727	3.6330	0.2162	0.7137	0.0000	0.2487
NEU	NEG	0.4836	0.0699	6.9177	5764.592	3.6330	0.2296	0.7376	0.0000	0.2540
Activity Score (AS)										
group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
FAL	NEU	2.0451	0.1489	13.7348	2441.593	3.6330	1.5042	2.5861	0.0000	0.5410
FAL	POS	0.7197	0.1378	5.2230	1937.670	3.6330	0.2191	1.2204	0.0013	0.5006
FAL	NEG	0.9778	0.1415	6.9085	2126.830	3.6330	0.4636	1.4919	0.0000	0.5142
NEU	POS	1.3254	0.1045	12.6836	5752.634	3.6330	0.9457	1.7050	0.0000	0.3796
NEU	NEG	1.0674	0.1094	9.7596	6144.906	3.6330	0.6700	1.4647	0.0000	0.3973
MS060-MS250										
group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
MS060	MS090	10.1537	0.6913	14.6889	4546.572	4.0300	7.3680	12.9395	0.0000	2.7857
MS060	MS125	12.9340	0.6742	19.1841	4601.088	4.0300	10.2170	15.6511	0.0000	2.7171
MS060	MS180	7.8758	0.6013	13.0986	4616.419	4.0300	5.4527	10.2989	0.0000	2.4231
MS060	MS250	5.5495	0.6060	9.1577	4631.678	4.0300	3.1073	7.9916	0.0000	2.4421
MS090	MS210	8.1388	0.6533	12.4585	4276.339	4.0300	5.5061	10.7714	0.0000	2.6327
MS090	MS250	15.7032	0.6673	23.5320	4396.741	4.0300	13.0139	18.3925	0.0000	2.6893
MS125	MS180	5.0582	0.6452	7.8393	4446.013	4.0300	2.4579	7.6586	0.0000	2.6003
MS125	MS210	10.9191	0.6352	17.1897	4367.270	4.0300	8.3592	13.4789	0.0000	2.5599
MS125	MS250	18.4835	0.6496	28.4518	4476.309	4.0300	15.8655	21.1016	0.0000	2.6181
MS180	MS210	5.8608	0.5572	10.5185	4660.017	4.0300	3.6153	8.1063	0.0000	2.2455
MS180	MS250	13.4253	0.5736	23.4057	4666.602	4.0300	11.1137	15.7368	0.0000	2.3116
MS210	MS250	7.5645	0.5623	13.4533	4651.993	4.0300	5.2985	9.8304	0.0000	2.2660
VS060-VS250										
group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
VS060	VS090	0.2434	0.0214	11.3632	4434.734	4.0300	0.1571	0.3298	0.0000	0.0863
VS060	VS125	0.4684	0.0192	24.4500	3622.595	4.0300	0.3912	0.5456	0.0000	0.0772
VS060	VS180	0.6666	0.0182	36.5371	3148.076	4.0300	0.5931	0.7402	0.0000	0.0735
VS060	VS210	0.6949	0.0189	36.8428	3475.306	4.0300	0.6189	0.7710	0.0000	0.0760
VS060	VS250	0.7145	0.0196	36.4328	3832.532	4.0300	0.6355	0.7936	0.0000	0.0790
VS090	VS125	0.2250	0.0162	13.9055	4155.701	4.0300	0.1598	0.2902	0.0000	0.0652
VS090	VS180	0.4232	0.0151	28.0495	3572.543	4.0300	0.3624	0.4840	0.0000	0.0608
VS090	VS210	0.4515	0.0158	28.5256	3991.157	4.0300	0.3877	0.5153	0.0000	0.0638
VS090	VS250	0.4711	0.0167	28.1844	4359.451	4.0300	0.4037	0.5384	0.0000	0.0674
VS125	VS180	0.1982	0.0117	17.0142	4390.215	4.0300	0.1513	0.2452	0.0000	0.0470
VS125	VS210	0.2265	0.0126	17.9860	4644.612	4.0300	0.1758	0.2773	0.0000	0.0508
VS125	VS250	0.2461	0.0137	17.9746	4627.259	4.0300	0.1909	0.3013	0.0000	0.0552
QS060-QS250										
group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
QS060	QS090	0.8574	0.1008	8.5020	4428.305	4.0300	0.4510	1.2638	0.0000	0.4064
QS060	QS125	1.8921	0.0945	20.0289	4011.032	4.0300	1.5114	2.2728	0.0000	0.3807
QS060	QS180	3.3079	0.0909	36.4027	3680.062	4.0300	2.9417	3.6741	0.0000	0.3662
QS060	QS210	4.0206	0.0898	44.7649	3571.726	4.0300	3.6586	4.3825	0.0000	0.3620
QS060	QS250	4.6373	0.0895	51.8393	3533.713	4.0300	4.2768	4.9978	0.0000	0.3605
QS090	QS125	1.0347	0.0810	12.7768	4505.455	4.0300	0.7083	1.3610	0.0000	0.3264
QS090	QS180	2.4505	0.0768	31.9266	4223.031	4.0300	2.1412	2.7599	0.0000	0.3093
QS090	QS210	3.1632	0.0755	41.8947	4108.814	4.0300	2.8589	3.4674	0.0000	0.3043
QS090	QS250	3.7799	0.0751	50.3487	4066.493	4.0300	3.4773	4.0824	0.0000	0.3025
QS125	QS180	1.4158	0.0682	20.7725	4573.800	4.0300	1.1412	1.6905	0.0000	0.2747
QS125	QS210	2.1285	0.0667	31.8892	4501.278	4.0300	1.8595	2.3975	0.0000	0.2690
QS125	QS250	2.7452	0.0663	41.4302	4470.833	4.0300	2.4782	3.0122	0.0000	0.2670
QS180	QS210	0.7126	0.0615	11.5782	4656.204	4.0300	0.4646	0.9607	0.0000	0.2480
QS180	QS250	1.3293	0.0610	21.7844	4646.163	4.0300	1.0834	1.5753	0.0000	0.2459
QS210	QS250	0.6167	0.0594	10.3753	4666.440	4.0300	0.3772	0.8562	0.0000	0.2395
AS060-AS250										
group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
AS060	AS090	1.1229	0.1391	8.0729	4618.311	4.0300	0.5624	1.6835	0.0000	0.5606
AS060	AS125	2.3379	0.1363	17.1542	4565.669	4.0300	1.7887	2.8871	0.0000	0.5492
AS060	AS180	3.8630	0.1352	28.5621	4540.679	4.0300	3.3179	4.4080	0.0000	0.5450
AS060	AS210	4.5563	0.1347	33.8179	4527.090	4.0300	4.0134	5.0993	0.0000	0.5430
AS060	AS250	5.1636	0.1351	38.2144	4537.444	4.0300	4.6191	5.7081	0.0000	0.5445
AS090	AS125	1.2150	0.1287	9.4395	4657.838	4.0300	0.6963	1.7337	0.0000	0.5187
AS090	AS180	2.7400	0.1276	21.4716	4648.450	4.0300	2.2258	3.2543	0.0000	0.5143
AS090	AS210	3.4334	0.1271	27.0209	4642.540	4.0300	2.9213	3.9455	0.0000	0.5121
AS090	AS250	4.0407	0.1275	31.6968	4647.089	4.0300	3.5269	4.5544	0.0000	0.5137
AS125	AS180	1.5251	0.1245	12.2451	4666.455	4.0300	1.0231	2.0270	0.0000	0.5019
AS125	AS210	2.2184	0.1240	17.8930	4664.488	4.0300	1.7188	2.7181	0.0000	0.4996
AS125	AS250	2.8257	0.1244	22.7133	4666.052	4.0300	2.3243	3.3271	0.0000	0.5014
AS180	AS210	0.6934	0.1228	5.6445	4667.601	4.0300	0.1983	1.1884	0.0010	0.4950
AS180	AS250	1.3006	0.1233	10.5514	4667.977	4.0300	0.8039	1.7974	0.0000	0.4968
AS210	AS250	0.6073	0.1227	4.9493	4667.771	4.0300	0.1128	1.1018	0.0063	0.4945

6.3 MOMENTUM INDEX

All stocks or tickers identified by the customised model are included in the custom momentum index. The index is updated monthly when newly identified tickers (if any) are added (i.e., cycles entered), while current members with dMS250 scores below the set minimum (if any) are deleted from the index (i.e., cycles exited). The base date for the index is 31 December 2008, and the base or starting value is 100. The number of members is variable, and the index maintains a relatively active position over a true equal-weighted design, which resets all the weights to the average weight when updating. However, any new members are assigned the average weight of the current members, adjusted for the number of additions and the total weight of any deletions, equally distributed among all members.

6.3.1 Levels and members

The TSX Momentum Index (TSX-MI) can serve as a benchmark for momentum on the Toronto Stock Exchange (TSX). Figure 6.1 below contrasts the performance of the custom TSX Momentum Index to the S&P/TSX Composite Index (TXCX) with its base date adjusted to 31 December 2008 and its base value to 100. Starting with four members on 31 December 2008, Empire Company [5m;-2.40%CTGR;NEU], Forsys Metals [4m;-12.41%CTGR;NEG], Metro Incorporated [8m;-0.16%CTGR;NEU], and Green River Gold Corporation [7m;-25.87%CTGR;NEG]. The momentum index ended 2009 at 139.93 with 242 members (refer to Table 6.9 on page 6-14) and moved clear of the composite index during 2010. The methodology of the momentum index, retaining the momentum of the remaining members, may explain the increasing outperformance of TSX-MI over time.

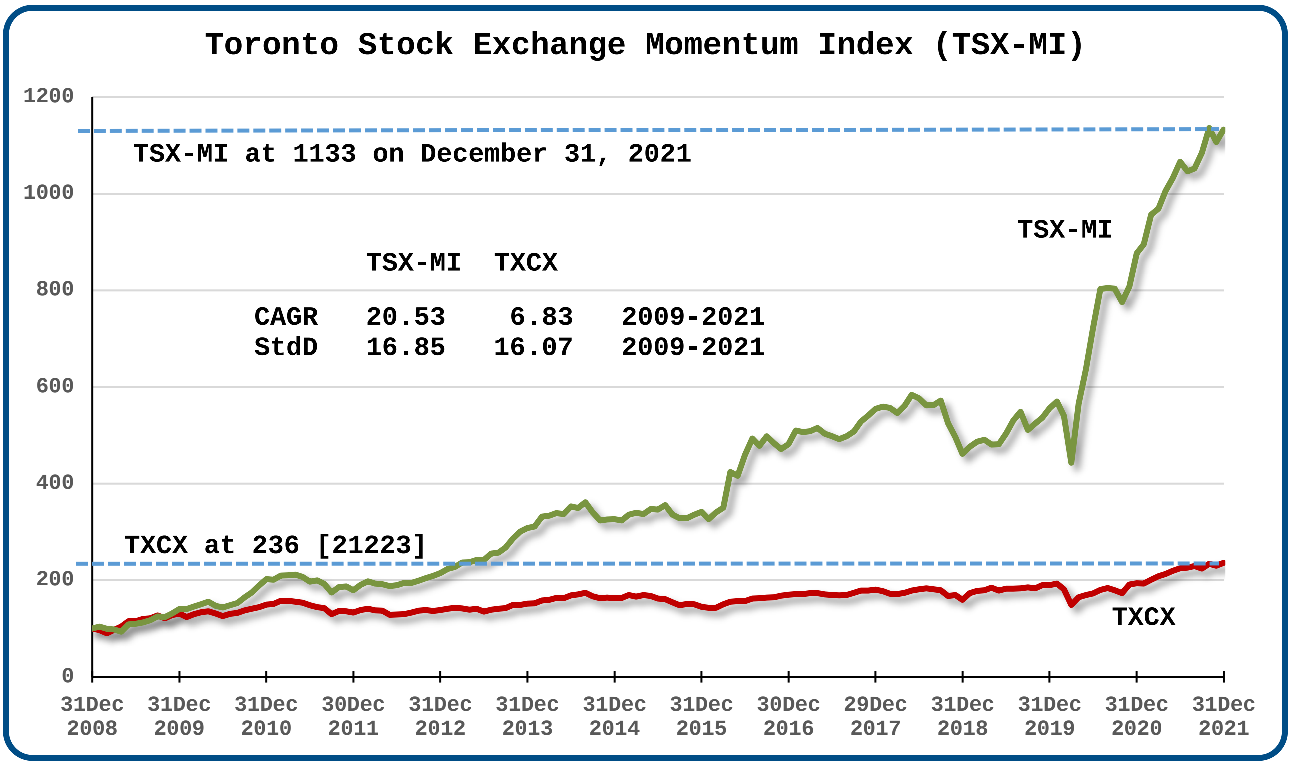

Figure 6.1 TSX Momentum Index (Source of price data: Bloomberg 2022)

The year 2020 was the most volatile period for the TSX Momentum Index (refer to Table 6.11 on page 6-16). Figure 6.2 below displays the volatility of the index during this period graphically. The three-year period beginning 2019 and ending 2021 recorded a compound annual growth rate (CAGR) of 34.87% with a standard deviation (StdD) of 22.97%. The individual statistics for years 2019 (CAGR:20.42% & StdD:10.92%), 2020 (CAGR:57.67% & StdD:34.76%) and 2021 (CAGR:29.21% & StdD:16.29%) confirms the increased volatility during this period. The index level dropped to 443 at the end of March 2020 and rebounded to 803 within four months, ending the year at 877. Equities outperformed during 2021 with the momentum index ending at 1132.81, down a little from the all-time high of 1135.65 reached at the end of October 2021.

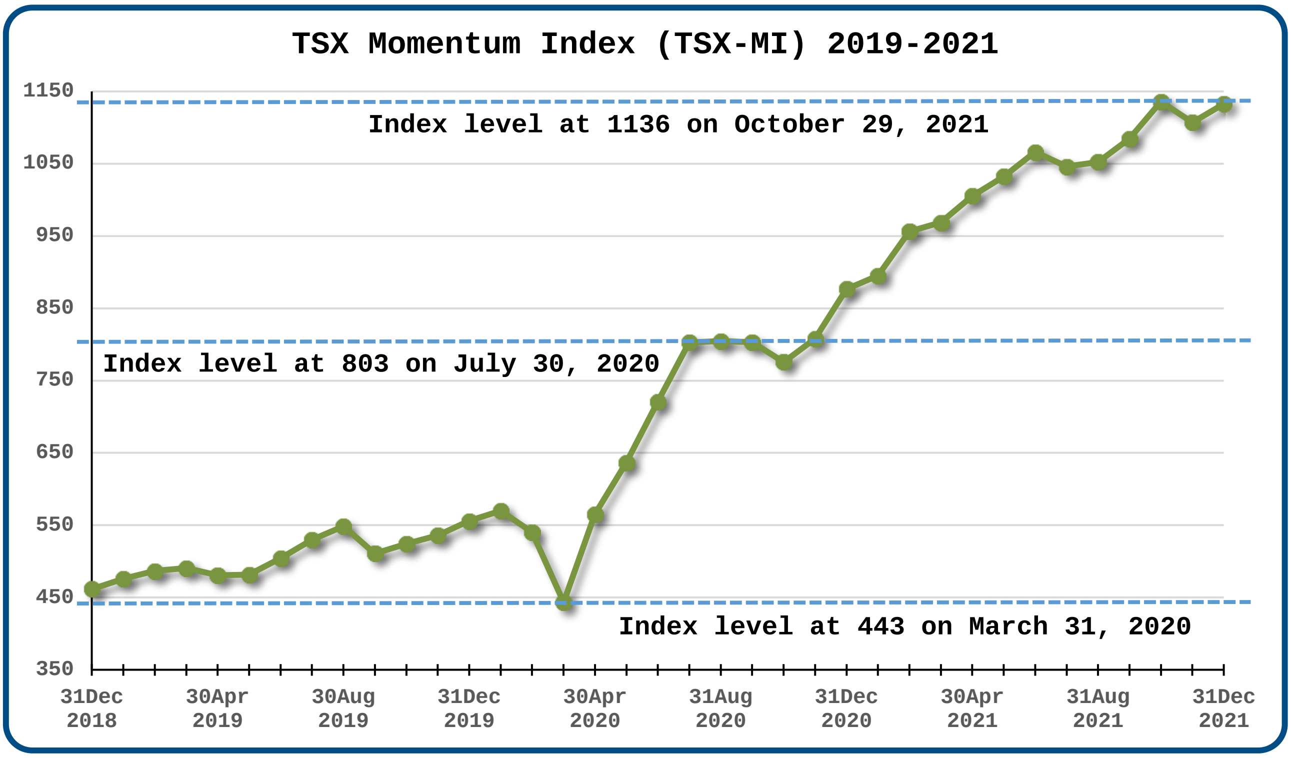

Figure 6.2 TSX-MI 2019-2021 (Source of price data: Bloomberg 2022)

Table 6.8, on the next page, describes the activity during the 2019-2021 period starting with the index at 462 comprising 29 members. The index lost 16.79% in value during the previous year, 2018, dropping 147 members and adding 98 (net 49 deletions). By the end of 2019, the index netted 71 additions and its value increased by 20.42%. The net amount of 71 additions is made up of 157 cycles entered and 86 cycles exited (28 at a gain versus 58 at a loss). During 2020, the index gained 57.67% and netted 68 additions by entering 177 new cycles and exiting 109 cycles (39 gains versus 70 losses). [Returns are cumulative or annual]

Compare 2019 and 2020 to 2021, when the index gained 29.21% and netted 36 deletions by entering 187 cycles and exiting 223 (124 gains and 99 losses). Index members numbered 132 at the end of 2021. Entering many new cycles and exiting the majority of completed cycles at a profit in a year when the index reached a high. Negative and annual returns correspond to large net deletions when many cycles were exited at a loss. Loss-making cycles also include false and neutral cycles.

Table 6.8 Updating 2019-2021

Date	LEVEL	GROWTH	MEM	3MA	ADD	DEL	ADD/T
Dec 2018	461.75	-6.83 %	29	41	2	9	0.18
Jan 2019	475.95	3.07 %	24	30	3	8	0.27
Feb 2019	486.85	2.29 %	34	29	13	3	0.81
Mar 2019	490.76	0.80 %	41	33	13	6	0.68
Apr 2019	480.82	-2.03 %	51	42	15	5	0.75
May 2019	481.51	0.14 %	57	50	10	4	0.71
Jun 2019	504.10	4.69 %	67	58	15	5	0.75
Jul 2019	530.04	5.15 %	94	73	29	2	0.94
Aug 2019	548.48	3.48 %	101	87	17	10	0.63
Sep 2019	511.01	-6.83 %	105	100	10	6	0.63
Oct 2019	524.19	2.58 %	100	102	4	9	0.31
Nov 2019	536.14	2.28 %	94	100	9	15	0.38
Dec 2019	556.03	3.71 %	100	98	19	13	0.59
Jan 2020	569.96	2.51 %	99	98	11	12	0.48
Feb 2020	540.21	-5.22 %	104	101	13	8	0.62
Mar 2020	443.34	-17.93 %	55	86	0	49	0.00
Apr 2020	565.07	27.46 %	43	67	0	12	0.00
May 2020	636.85	12.70 %	41	46	3	5	0.38
Jun 2020	721.25	13.25 %	49	44	9	1	0.90
Jul 2020	802.81	11.31 %	61	50	14	2	0.88
Aug 2020	804.64	0.23 %	82	64	21	0	1.00
Sep 2020	803.19	-0.18 %	107	83	28	3	0.90
Oct 2020	775.68	-3.42 %	119	103	15	3	0.83
Nov 2020	807.96	4.16 %	136	121	20	3	0.87
Dec 2020	876.72	8.51 %	168	141	43	11	0.80
Jan 2021	895.16	2.10 %	197	167	41	12	0.77
Feb 2021	956.28	6.83 %	217	194	30	10	0.75
Mar 2021	968.76	1.30 %	228	214	29	18	0.62
Apr 2021	1005.37	3.78 %	240	228	24	12	0.67
May 2021	1032.30	2.68 %	250	239	24	14	0.63
Jun 2021	1066.03	3.27 %	257	249	14	7	0.67
Jul 2021	1046.03	-1.88 %	248	252	6	15	0.29
Aug 2021	1052.34	0.60 %	231	245	5	22	0.19
Sep 2021	1084.48	3.05 %	220	233	6	17	0.26
Oct 2021	1135.65	4.72 %	183	211	4	41	0.09
Nov 2021	1107.12	-2.51 %	150	184	2	35	0.05
Dec 2021	1132.81	2.32 %	132	155	2	20	0.09

Source: Price data downloaded from Bloomberg (2022)

Index activity may give some indication of the sentiment and volatility in the market when looking at the number of cycles entered versus exited. The turnover of members, net additions or deletions, and the results when exiting cycles correspond to large decreases and increases in the index value. A progressively increasing or decreasing number of members during a particular period shows the equity market trending upwards or downwards. A simple gain versus loss comparison of completed cycles does not account for the much shorter negative cycles and false cycles when matched with positive cycles.

Figure 6.3 overlays a line chart with changing index levels on a bar chart showing the variation in index members. There was a steady increase in value since the base date on 31 December 2008, building from four members and peaking at 242 members within a year. From 2017 onwards the index members appear to synchronise with the index levels to some degree, surging and receding with the availability of momentum stocks in the market. After exiting many positions during a downswing, the index level surges as the number of member stocks grows.

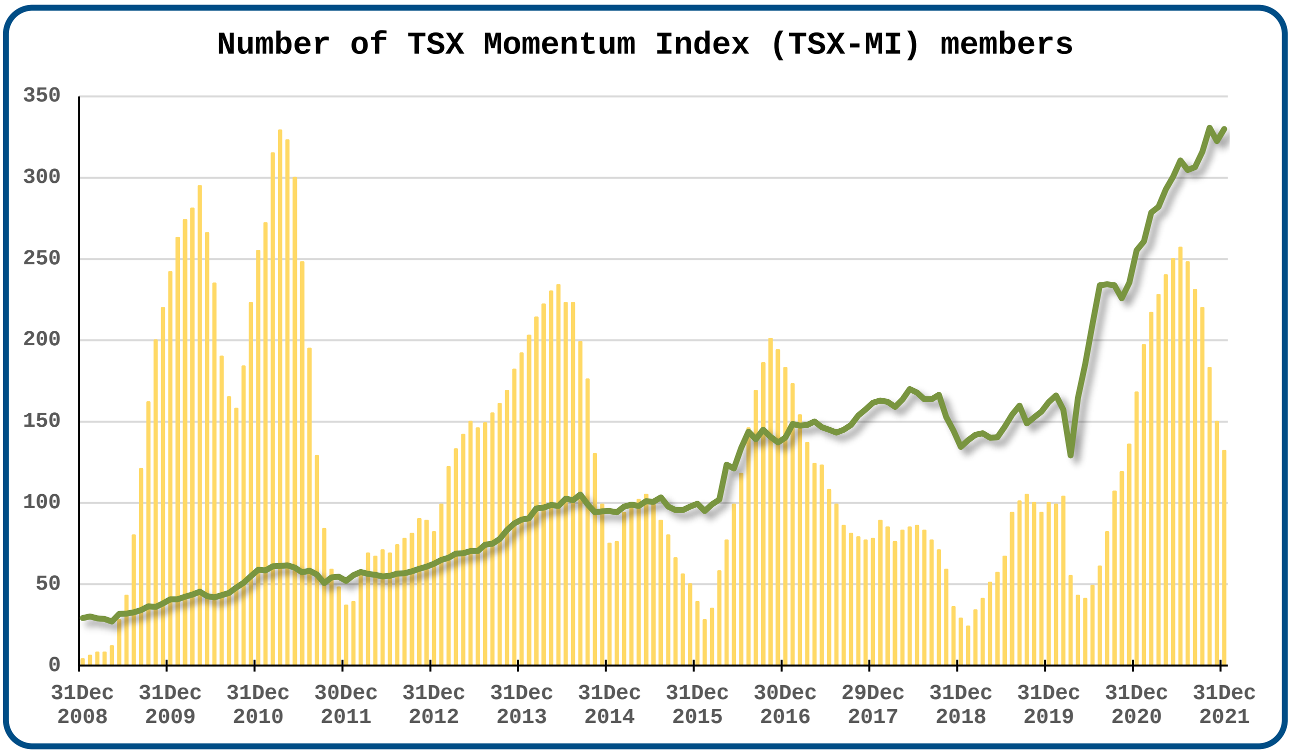

Figure 6.3 TSX-MI member numbers (Source of price data: Bloomberg 2022)

The table below summarises the annual results for the full 13-year period. The end-of-year members against the average reflects the state of the market at year-end.

Table 6.9 Annual results 2009-2021

Year	LEVEL	GROWTH	MEM	AVG	ADD	DEL	ADD/T
2009	139.93	39.93 %	242	94	262	24	0.92
2010	202.23	44.52 %	255	232	282	269	0.51
2011	179.07	-11.45 %	37	195	151	369	0.29
2012	214.86	19.98 %	82	72	145	100	0.59
2013	308.10	43.40 %	192	150	228	118	0.66
2014	326.20	5.87 %	75	186	184	301	0.38
2015	341.38	4.65 %	39	80	100	136	0.42
2016	481.40	41.02 %	183	125	241	97	0.71
2017	554.90	15.27 %	78	110	121	226	0.35
2018	461.75	-16.79 %	29	72	98	147	0.40
2019	556.03	20.42 %	100	72	157	86	0.65
2020	876.72	57.67 %	168	89	177	109	0.62
2021	1132.81	29.21 %	132	213	187	223	0.46

Source: Price data downloaded from Bloomberg (2022)

6.3.2 Relative performance

A comparison between the performance of the custom TSX Momentum Index and indices from the S&P Dow Jones Toronto Stock Exchange (TSX) series focuses on the relative performance of the model. The S&P/TSX Composite Index (TXCX) is the headline index for the stock exchange. Three indices segment the ordinary equity market into large (TXLC), mid (TXMC), and small (TXSC) sized companies based on market capitalisation or value. Two equal-weighted indices replicate the large-cap index (TXEW) and the composite index (TXCE) without weighting the individual member stocks. Refer to Table 6.10 below for information on the different benchmarks.

Table 6.10 Benchmark information

S&P/TSX Composite Index	
TXCX **1977-01-03**	A broad capitalisation-weighted market index containing 230 to 250 of the largest companies listed on the Toronto Stock Exchange. These companies represent approximately 95% of the equities and 70% of the entire market in terms of market capitalisation.
S&P/TSX 60 Index	
TXLC **1998-12-31**	A capitalisation-weighted index representing the 60 largest, most liquid and heavily traded companies (large-cap stocks) listed on the Toronto Stock Exchange.
S&P/TSX 60 Equal Weight Index	
TXEW **1999-09-17**	Replicates the capitalisation-weighted S&P/TSX 60 Index (TXLC) without weighting the member companies. Stocks are allocated equal weights at each quarterly rebalancing.
S&P/TSX Completion Index	
TXMC **1999-05-17**	A capitalisation-weighted index for mid-cap stocks, the remainder of the S&P Composite Index (TXCX) companies not included in the S&P/TSX 60 Index (TXLC).
S&P/TSX Small Cap Index	
TXSC **1999-05-17**	A capitalisation-weighted index containing about 230 companies representing the market for small-cap stocks in Canada.
S&P/TSX Equity Index	
TXEQ **1990-01-02**	A capitalisation-weighted index that does not contain investment trusts and only measures the performance of equity stocks listed on the Toronto Stock Exchange.
S&P/TSX Composite Equal Weight Index	
TXCE **2011-10-24**	An equal-weighted version of the S&P/TSX Composite Index (TXCX). Stocks are allocated equal weights at each quarterly rebalancing.
S&P/TSX Composite Momentum Index	
TXMM **2018-10-26**	A momentum-weighted index for stocks included in the S&P/TSX Composite Index with persistent medium-term (3-12 months) outperformance. Rebalanced semi-annually.

Sources: SPTSX (2021); SPTSX (2022)

Table 6.11, on the next page, shows the progression and relative performance of the TSX Momentum Index (TSX-MI) over time from its 2009 base year to the end of 2021. Note its performance in 2012 relative to the different benchmarks, rebounding after the market performed poorly in 2011. Apart from 2018, which recorded the worst result over the evaluation period, the TSX Momentum Index (TSX-MI) statistics measure well against the benchmarks. Note the large rebound in 2020 in contrast to the other indices. The methodology of the momentum index may explain these rebounds with the index maintaining a relatively active position when updated monthly. The 10-Year Compound Annual Growth Rate (CAGR), the 5-Year CAGR, and the 3-Year CAGR represent an improving performance and a consistent outperformance by the TSX-MI of the benchmarks.

Table 6.11 TSX-MI results versus benchmarks (2009-2021)

Year	Metric	TSX-MI	TXCX	TXLC	TXEW	TXMC	TXSC	TXEQ	TXCE	TXMM
2009	CAGR	39.93	30.69	27.94	35.41	41.70	56.16	30.81	---	---
	StdD	18.91	26.30	27.56	27.18	23.15	23.39	26.35	---	---
2010	CAGR	44.52	14.45	10.88	17.50	26.47	31.32	14.20	---	---
	StdD	12.90	12.96	13.23	12.89	13.43	15.68	13.17	---	---
2011	CAGR	-11.45	-11.07	-11.42	-11.50	-10.26	-18.39	-11.83	---	---
	StdD	19.29	18.41	18.85	18.30	18.54	22.12	18.63	---	---
2012	CAGR	19.98	4.00	4.82	6.13	1.72	-4.89	3.79	1.79	---
	StdD	9.97	11.93	12.35	13.22	11.89	14.91	12.18	13.67	---
2013	CAGR	43.40	9.55	9.81	5.53	8.79	4.35	10.10	3.39	---
	StdD	8.62	9.95	10.38	11.52	10.10	12.87	10.09	12.27	---
2014	CAGR	5.87	7.42	9.07	5.51	2.79	-5.19	7.42	-0.72	---
	StdD	12.18	10.31	10.33	11.19	11.48	15.03	10.50	12.71	---
2015	CAGR	4.65	-11.09	-10.56	-13.31	-12.68	-15.84	-11.27	-16.51	---
	StdD	11.53	14.39	14.88	16.89	14.58	16.96	14.63	17.71	---
2016	CAGR	41.02	17.51	17.72	27.76	17.09	35.15	17.80	25.47	---
	StdD	21.50	12.52	12.80	14.68	12.99	19.13	12.73	15.64	---
2017	CAGR	15.27	6.03	6.63	6.51	4.24	0.28	6.09	5.01	---
	StdD	10.21	7.30	7.51	8.50	8.10	12.20	7.45	9.23	---
2018	CAGR	-16.79	-11.64	-10.46	-9.28	-15.23	-20.10	-11.82	-14.43	---
	StdD	13.78	10.54	10.99	11.59	10.95	12.59	10.75	10.96	---
2019	CAGR	20.42	19.13	18.11	16.47	22.67	12.84	18.93	18.25	24.60
	StdD	10.92	7.34	7.80	8.11	7.21	8.93	7.51	7.67	8.66
2020	CAGR	57.67	2.17	1.96	5.36	3.22	10.01	2.59	9.61	8.83
	StdD	34.76	33.72	34.20	33.54	33.96	37.08	33.68	35.03	31.52
2021	CAGR	29.21	21.74	24.37	24.88	12.56	18.16	21.72	18.33	7.30
	StdD	16.29	10.54	10.47	10.76	12.68	18.45	10.71	13.03	17.29
FULL 2009 2021	CTGR	1032.81	136.13	137.53	172.89	135.58	110.55	134.82	---	---
	CAGR	20.53	6.83	6.88	8.03	6.81	5.89	6.79	---	---
	StdD	16.85	16.07	16.50	16.80	16.06	18.90	16.20	---	---
10Y 2012 2021	CTGR	532.59	77.52	89.02	93.81	46.48	25.81	78.29	50.82	---
	CAGR	20.26	5.91	6.57	6.84	3.89	2.32	5.95	4.19	---
	StdD	16.72	14.71	15.01	15.60	15.14	18.32	14.83	16.44	---
5Y 2017 2021	CTGR	135.31	38.82	42.99	48.09	25.95	17.52	38.94	37.80	---
	CAGR	18.67	6.78	7.41	8.17	4.72	3.28	6.80	6.62	---
	StdD	19.39	17.06	17.35	17.33	17.55	20.44	17.12	18.15	---
3Y 2019 2021	CTGR	145.33	48.17	49.77	53.25	42.53	46.68	48.51	53.36	45.49
	CAGR	34.87	14.01	14.41	15.29	12.54	13.62	14.09	15.32	13.31
	StdD	22.97	20.75	21.05	20.79	21.25	24.37	20.78	21.93	21.29
1Y 2021	CTGR	29.21	21.74	24.37	24.88	12.56	18.16	21.72	18.33	7.30
	CAGR	29.21	21.74	24.37	24.88	12.56	18.16	21.72	18.33	7.30
	StdD	16.29	10.54	10.47	10.76	12.68	18.45	10.71	13.03	17.29

Source: Price data downloaded from Bloomberg (2022)

6.3.3 Correlation analysis

Correlation measures the degree of co-movement or size of the linear association between two time-series. Correlation-squared (R-squared) indicates how closely an index tracks the performance of a particular benchmark. It also points to the reliability of the alpha (excess return) and beta (volatility) coefficients from a linear regression. Table 6.12 below shows the changes in correlation with the different benchmarks from year to year. The lowest correlation with other indices occurred during 2019 when the momentum index led the market in recovering from the downturn in 2018.

Table 6.12 Correlations: TSX-MI versus benchmarks

Year	TXCX	TXLC	TXEW	TXMC	TXSC	TXEQ	TXCE	TXMM
2009	0.51	0.49	0.49	0.58	0.63	0.51	---	---
2010	0.84	0.79	0.82	0.92	0.93	0.84	---	---
2011	0.88	0.83	0.84	0.95	0.96	0.87	---	---
2012	0.77	0.73	0.73	0.82	0.81	0.77	0.80	---
2013	0.76	0.74	0.70	0.74	0.70	0.76	0.69	---
2014	0.87	0.82	0.83	0.91	0.88	0.87	0.89	---
2015	0.74	0.72	0.64	0.71	0.63	0.74	0.63	---
2016	0.45	0.38	0.56	0.62	0.79	0.44	0.62	---
2017	0.75	0.68	0.75	0.82	0.80	0.75	0.82	---
2018	0.78	0.71	0.76	0.85	0.83	0.78	0.85	---
2019	0.18	0.11	0.21	0.42	0.53	0.18	0.40	0.40
2020	0.76	0.73	0.74	0.83	0.85	0.76	0.79	0.86
2021	0.85	0.80	0.84	0.90	0.93	0.85	0.92	0.77
AVG	0.70	0.66	0.69	0.77	0.79	0.70	0.74	0.68
5Y	0.73	0.70	0.72	0.81	0.84	0.74	0.79	---
3Y	0.73	0.70	0.71	0.81	0.84	0.73	0.78	0.81
1Y	0.85	0.80	0.84	0.90	0.93	0.85	0.92	0.77

Source: Price data downloaded from Bloomberg (2022)

Results show a strong association between the TSX Momentum Index (TSX-MI) and the Small Cap index (TXSC) as well as the Completion index (TXMC) since 2009 based on yearly data and the average correlation coefficient. During 2021 it aligned most closely with the Small Cap index (TXSC) and the Equity index (TXEQ). Measured over longer periods, the 3-year and 5-year correlations showed the strongest association between TSX-MI and TXSC.

Note that the correlations between TSX-MI and the equal-weighted equivalents of the composite index (TXCE), and the large-cap index (TXEW) are generally higher than those for the capitalisation-weighted versions. As stated previously, the methodology of the momentum index, retaining the momentum of the remaining members, may account for the outperformance of TSX-MI to some degree. A variable number of members in combination with more frequent updating allows for a relatively active approach to indexing or benchmarking momentum in an equity market.

6.3.4 Drawdown analysis

A drawdown analysis highlights the potential for sudden large (20%-plus) losses in value and the likely time to recover (Wilmington 2018). It records the size and speed of maximum drawdowns and the time to return to former highs. Referring to Table 6.13, the TSX Momentum Index (TSX-MI) experienced its maximum drawdown in March 2020, the same as all the other indices. It occurred over a short period (18 days) and recovered comparatively quickly, after only 40 days, to its original high. Some indices (TXCX, TXLC, TXEW, TXMC, and TXEQ) had similar maximum drawdown periods but took longer to recover. The S&P/TSX Composite Momentum Index (TXMM) experienced similar maximum drawdown and recovery periods to the custom TSX Momentum Index. The maximum drawdown for TSX-MI fell outside its maximum drawdown duration period of 431 days.

Table 6.13 Drawdown analysis (2009-2021)

Metric	TSX-MI	TXCX	TXLC	TXEW	TXMC
Maximum drawdown	36.00%	37.43%	35.73%	36.25%	43.71%
Date	2020-03-18	2020-03-23	2020-03-23	2020-03-23	2020-03-23
Period	18 days	22 days	22 days	23 days	22 days
Recovery	40 days	197 days	197 days	163 days	195 days
Average drawdown	6.29%	6.92%	6.68%	6.74%	8.99%
Maximum duration	431 days	726 days	752 days	726 days	1343 days
From:	2011-04-11	2011-04-06	2011-03-07	2011-04-06	2014-08-27
To:	2012-12-31	2014-03-03	2014-03-07	2014-03-03	2020-01-13
Average duration	16 days	27 days	25 days	27 days	38 days
Annualised return	20.53%	6.83%	6.88%	8.03%	6.81%
Drawdown ratio	0.57	0.18	0.19	0.22	0.16

Metric	TSX-MI	TXSC	TXEQ	TXCE	TXMM
Maximum drawdown	36.00%	60.06%	36.92%	43.92%	29.71%
Date	2020-03-18	2020-03-23	2020-03-23	2020-03-23	2020-03-23
Period	18 days	2237 days	22 days	1392 days	23 days
Recovery	40 days	393 days	196 days	168 days	68 days
Average drawdown	6.29%	20.89%	7.05%	10.31%	5.67%
Maximum duration	431 days	2630 days	727 days	1560 days	335+ days
From:	2011-04-11	2011-04-11	2011-04-06	2014-08-27	2020-08-27
To:	2012-12-31	2021-10-19	2014-03-04	2020-11-23	2021-12-31
Average duration	16 days	57 days	26 days	57 days	18 days
Annualised return	20.53%	5.89%	6.79%	4.02%	10.00%
Drawdown ratio	0.57	0.10	0.18	0.09	0.34

Source: Price data downloaded from Bloomberg (2022)

On average, the size of a TSX-MI drawdown is 6.29%, lasting 16 days (peak to peak). It is apparent from Table 6.13 that the TSX-MI recovers more quickly from drawdowns than the other indices. The mid-cap (TXMC) and small-cap (SMLC) indices experienced average drawdowns of 8.99% (lasting 38 days) and 20.89% (lasting 57 days) respectively. TSX-MI has a higher drawdown ratio (annualised return to maximum drawdown), pointing to comparatively higher returns on a risk-adjusted basis.

6.3.5 Descriptive statistics

Descriptive statistics, the process of describing data and presenting it graphically, provides the individual summary statistics listed in the table below. It includes the mean return of each index with its accompanying standard deviation. The coefficient of variation (CV), the size of the standard deviation about its mean, shows that the relative variability of the TSX-MI is comparatively low.

Table 6.14 Summary statistics (2009-2021)

Metric	TSX-MI	TXCX	TXLC	TXEW	TXMC
Mean	0.0747 %	0.0265 %	0.0266 %	0.0309 %	0.0264 %
Standard Error	0.0186 %	0.0178 %	0.0182 %	0.0186 %	0.0177 %
Median	0.1396 %	0.0777 %	0.0734 %	0.0716 %	0.0701 %
Standard Deviation	1.0606 %	1.0117 %	1.0384 %	1.0573 %	1.0110 %
Sample Variance	1.1249	1.0236	1.0782	1.1179	1.0221
Kurtosis	14.0274	26.2762	26.2706	19.8047	22.2948
Skewness	-1.1402	-1.1870	-0.9957	-1.0539	-1.5608
Range	21.49 %	24.47 %	25.05 %	24.13 %	22.11 %
Maximum	8.85 %	11.29 %	11.68 %	10.83 %	9.65 %
Minimum	-12.64 %	-13.18 %	-13.37 %	-13.30 %	-12.46 %
Sum	242.73 %	85.92 %	86.51 %	100.39 %	85.69 %
Count	3248	3248	3248	3248	3248
CV	14.19	38.24	38.98	34.21	38.32
Metric	**TSX-MI**	**TXSC**	**TXEQ**	**TXCE**	**TXMM**
Mean	0.0747 %	0.0229 %	0.0263 %	0.0158 %	0.0381 %
Standard Error	0.0186 %	0.0209 %	0.0179 %	0.0206 %	0.0471 %
Median	0.1396 %	0.0896 %	0.0756 %	0.0716 %	0.0744 %
Standard Deviation	1.0606 %	1.1884 %	1.0194 %	1.0389 %	1.3268 %
Sample Variance	1.1249	1.4123	1.0391	1.0794	1.7604
Kurtosis	14.0274	14.8056	25.6962	25.9464	13.9199
Skewness	-1.1402	-1.4162	-1.1394	-1.7213	-1.0344
Range	21.49 %	22.41 %	24.53 %	22.91 %	20.54 %
Maximum	8.85 %	8.65 %	11.28 %	9.69 %	9.67 %
Minimum	-12.64 %	-13.76 %	-13.26 %	-13.22 %	-10.87 %
Sum	242.73 %	74.46 %	85.37 %	40.10 %	30.22 %
Count	3248	3248	3248	2545	793
CV	14.19	51.84	38.78	65.94	34.82

Source: Price data downloaded from Bloomberg (2022)

Some of these sets of data are not symmetric but negatively or left skewed with the means (averages) smaller than the medians (middle values). A left-skewed distribution has more values in the right tail, but the left tail is longer indicating many smaller positive returns and a few large negative returns. Data are moderately left-skewed with values between -1 and -0.5 (refer to TXLC). The other distributions are highly left-skewed with values lower than -1, referring to TXCE (-1.76), TXMC (-1.56), and TXSC (-1.42) in particular. The distribution of TSX-MI is highly left-skewed (-1.14), the same as TXEQ (-1.14) and similar to TXCX (-1.19).

Referring to Table 6.14 on the previous page, the kurtosis values point to heavy-tailed distributions with outliers or extreme positive and negative returns. Extreme returns can be defined as returns that exceed the 90th percentile, the top and bottom 10% of returns (Sankaran, Nguyen & Harikumar 2012). Compared to a normal distribution, described as mesokurtic, these distributions can be described as leptokurtic with excess kurtosis. Negatively skewed, heavy-tailed distributions are common in stock market data (Samunderu & Murahwa 2021).

Figure 6.4 below shows the dispersion of TSX-MI returns with most returns clustering around the mean. The histogram confirms the comparatively moderate kurtosis value with some extreme positive and negative returns as outliers. Highly left-skewed distributions in combination with high kurtosis favour extreme negative returns. The TSX-MI has one of the lowest kurtosis values (14.0) of all the indices and is therefore less likely to record extreme returns compared to TXCX (26.3) and TXLC (26.3). The daily standard deviation of TSX-MI and the range between the maximum and minimum daily returns compare well against the other indices.

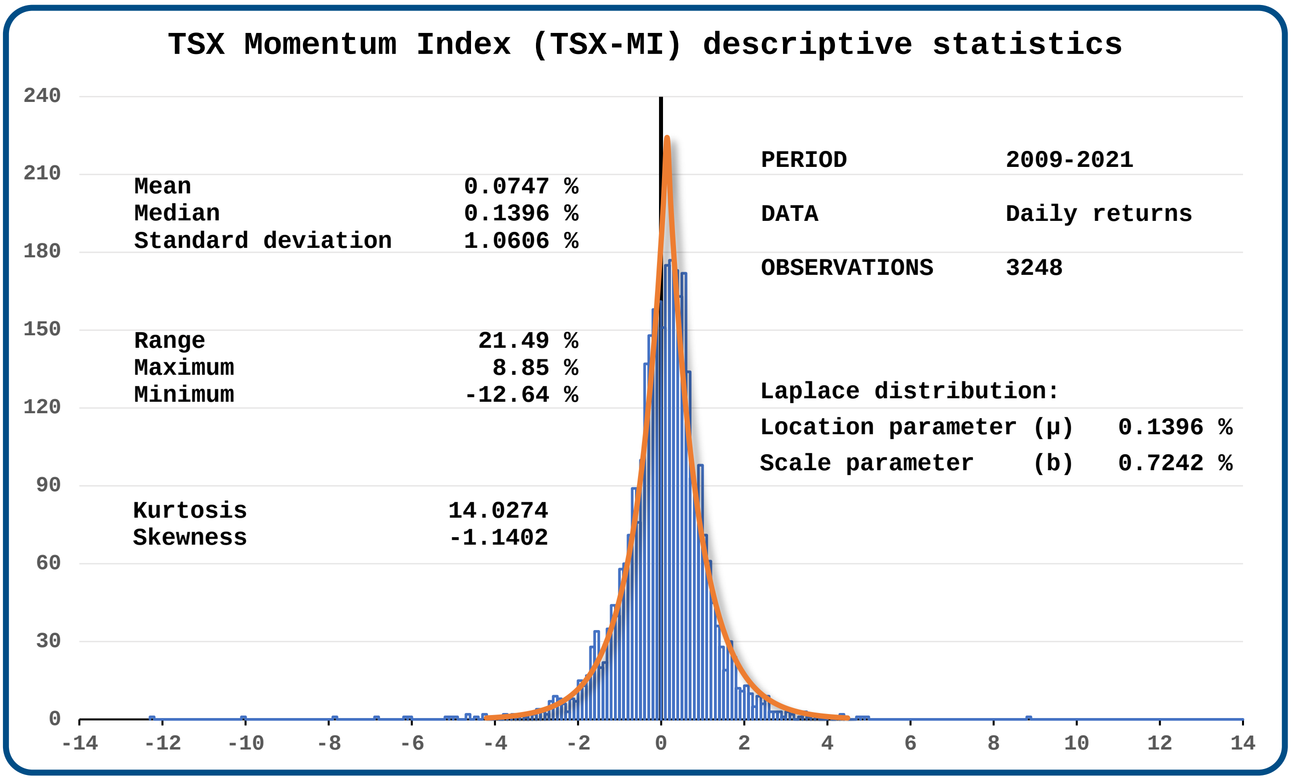

Figure 6.4 TSX-MI descriptive statistics (Source of price data: Bloomberg 2022)

As stated in Chapter 3, the assumption that the log returns of stock prices are normally distributed is convenient when performing statistical analysis. However, as evident from Figure 6.4, the Laplace distribution with its high central peak, narrow upper shoulders and heavy tails provides a better fit for log returns than the normal distribution. The Laplace distribution is symmetric about its location parameter (median) with the scale parameter (beta) determining its profile while the normal distribution is completely defined by its mean and standard deviation.

6.4 CONCLUSION

This chapter created a momentum profile for the Toronto Stock Exchange (TSX) by mechanically entering and exiting momentum cycles identified by the customised momentum model based on its four parameters and an exit rule. The results from applying the model mechanically provided a set of positive, negative, neutral, and false cycles unique to this equity market.

Momentum cycles with holds that extend beyond 9 months generally record positive returns. Positive cycles at an average hold of 13 months gained 49% in value. Negative cycles lost 21% in value at an average hold shorter than 6 months. False cycles at an average hold of 1.5 months lost 13% in value. Neutral cycles at an average hold of 8 months only gained half a per cent in value.

Even though stocks in the $10-$20 price range were the most actively traded in this market, the stocks trading between $1 and $2 recorded the highest compound returns per average hold. However, stocks below $1 with positive cycles outperformed all the other positive-cycle price ranges. The small Real Estate sector favoured positive outcomes. Technology and Consumer Staples along with Real Estate were the best-performing sectors overall. Utilities, Financials and Health Care were the worst-performing sectors on the Toronto Stock Exchange (TSX) overall.

The average hold decreases going from Zone 1 to Zone 4, but the shorter average holds in zones 3 and 4 generated higher compound returns. Apart from Zone 2, neutral cycles posted small positive compound returns. False cycles generated large negative compound annual returns in each zone and overall.

A custom momentum index was used to evaluate the model by quantifying the process of entering the cycles at certain prices and exiting at either a gain or a loss. The performance of the custom momentum index compared favourably with the benchmark indices, generally tracking the small-cap and mid-cap indices most closely. A drawdown analysis showed that the custom index recovered more quickly from drawdowns and outperformed the other indices on a risk-adjusted basis.

Chapter 7 to follow evaluates the performance of the customised model for stocks listed on the TSX Venture Exchange (TSXV), similarly constructing a custom index, the TSXV Momentum Index (TSXV-MI).

Chapter 8 contrasts the results obtained in three different markets – an emerging market exchange, the Johannesburg Stock Exchange (JSE); a developed market exchange, the Toronto Stock Exchange (TSX); and a venture market exchange, the TSX Venture Exchange (TSXV).

REFERENCES

Bloomberg. 2022. Bloomberg Terminal – Bloomberg Professional Services subscription. New York: Bloomberg Finance L.P.

Samunderu, E. & Murahwa, Y.T. 2021. Return based risk measures for non-normally distributed returns: an alternative modelling approach. *Journal of Risk and Financial Management*, 14(11):540. DOI: 10.3390/jrfm14110540.

Sankaran, H., Nguyen, A. & Harikumar, J. 2012. Extreme return correlation and volatility: a two-threshold approach. *American Journal of Business*, 27(2):154–173. DOI: 10.1108/19355181211274451.

SPTSX. 2021. *S&P/TSX Composite single factor indices methodology*. *S&P Dow Jones Indices [Online]*. S&P Global. Available at: https://www.spglobal.com/spdji/en/documents/methodologies/methodology-sp-tsx-composite-single-factor-indices.pdf (Accessed: 15 June 2022).

SPTSX. 2022. *S&P/TSX Canadian indices methodology*. *S&P Dow Jones Indices [Online]*. S&P Global. Available at: https://www.spglobal.com/spdji/en/documents/methodologies/methodology-sp-tsx-canadian-indices.pdf (Accessed: 15 June 2022).

Wilmington. 2018. *Understanding investment risk through drawdown analysis*. *Investment Insights [Online]*. Wilmington Trust Corporation. Available at: https://www.wilmingtontrust.com/repositories/wtc_sitecontent/PDF/Understanding-investment-risk-through-drawdown-analysis.pdf (Accessed: 27 April 2021).

CHAPTER SEVEN

MOMENTUM PROFILE: TSX VENTURE EXCHANGE

7.1 INTRODUCTION

The customised momentum model, described in Chapter 4, was used to identify the momentum cycles of stocks listed on the TSX Venture Exchange (TSXV). In total, 581 cycles-in-progress were identified with entry points determined by the parameter settings. A momentum profile for this equity market will be created by entering these cycles mechanically and exiting on a fixed exit rule. The results from applying the model mechanically (i.e., without taking any discretionary actions) will provide a set of positive, negative, neutral, and false cycles unique to this market – the different types of cycles are defined in the next section.

A custom momentum index will be used to evaluate the model by quantifying the process of entering cycles at certain prices and exiting at either a gain or a loss. The index level follows and accumulates the prices of the incumbent member stocks, with cycles overlapping as stocks are added to and deleted from the index when updated. The construction of the index (refer to Chapter 3), equally weighting new members but allowing existing members to retain their momentum, should maintain a relatively active position in the market. In addition, the changing number of members should indicate the availability of momentum stocks (as identified by this specific model) in this market at a particular point in time.

The section to follow uses the outcomes generated by the customised model to create a momentum profile for this equity market.

7.2 MOMENTUM MODEL OUTCOMES

The momentum cycles identified by the model are classified as either false, neutral, negative, or positive depending on the outcome. In this study, it is assumed that a positive cycle (optimal outcome) would hold at least 3 months and record an annualised gain of more than 10%. A negative cycle (unexpected outcome) would record an annualised loss of more than 10% while also holding at least 3 months. A neutral cycle (no outcome) is assumed to hold a minimum of 3 months but gain or lose a maximum of 10% annualised. A false cycle (failed outcome) holds shorter than 3 months. These assumptions are based on the theory of price momentum, which states that momentum formed over 3 to 12 months should hold for 3 to 12 months (60 to 250 trading days) – refer to Chapter 2. At a momentum score setting of 20% per period, the 10% annualised cut-off was chosen as minimum evidence of some momentum between entry and exit.

In the following five subsections, a momentum profile for this venture market will be created by analysing the different cycles in terms of average hold, price range activity, sector activity, outcomes per entry zone (refer to Chapter 4), and the average parameter (momentum, volatility, quality, and activity) scores per cycle type.

7.2.1 Holding periods

The results per average hold period or Average Hold (AH), in Table 7.1 below, show that the different cycles are distinct in average hold period. Each type tends to dominate a particular range. False cycles account for 13% (78 from 581) of all cycles, outnumbering the neutral cycles (60 from 581 or 10%). False cycles recorded a high negative annual return due to the short average hold of 1.55 months.

Table 7.1 Average hold

HOLD	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
1-2	78	78	---	---	---	1.55	-15.24	-72.18
	AH	1.55	---	---	---			
	CRpAH	-15.24	---	---	---			
	CARpAH	-72.18	---	---	---			
3-5	165	---	9	131	25	4.05	-21.40	-50.97
	AH	---	3.89	4.10	3.88			
	CRpAH	---	-0.20	-30.97	42.41			
	CARpAH	---	-0.61	-66.21	198.46			
6-8	157	---	16	112	29	6.98	-16.80	-27.11
	AH	---	7.31	6.91	7.07			
	CRpAH	---	-1.75	-29.85	46.71			
	CARpAH	---	-2.86	-45.98	91.68			
9-11	109	---	27	26	56	9.91	10.51	12.87
	AH	---	9.96	9.38	10.13			
	CRpAH	---	0.36	-22.94	36.87			
	CARpAH	---	0.43	-28.34	45.06			
12-17	50	---	7	---	43	14.04	91.33	74.12
	AH	---	14.14	---	14.02			
	CRpAH	---	6.01	---	110.63			
	CARpAH	---	5.08	---	89.17			
18-23	17	---	1	---	16	20.06	145.27	71.04
	AH	---	18.00	---	20.19			
	CRpAH	---	-0.73	---	159.54			
	CARpAH	---	-0.49	---	76.28			
24+	5	---	---	---	5	28.20	378.64	94.70
	AH	---	---	---	28.20			
	CRpAH	---	---	---	378.64			
	CARpAH	---	---	---	94.70			
TSXV	581	78	60	269	174	7.14	-2.56	-4.27
	AH	1.55	8.97	5.78	11.13			
	CRpAH	-15.24	0.33	-29.76	70.31			
	CARpAH	-72.18	0.44	-51.98	77.58			

Source: Price data downloaded from Bloomberg (2022)

The majority (43 from 60 or 72%) of neutral cycles cluster in the 6-11-month range with small returns, both negative (6-8) and positive (9-11), at a relatively long average hold before ultimately exiting without much change in value. Negative cycles are shorter in average hold than the neutral cycles, dominating the 3-5-month (131 from 165 or 79%) and 6-8-month (112 from 157 or 71%) ranges. Positive cycles are predominant in the 9-17-month range (99 from 174 or 57%) while several cycles (21 from 174 or 13%) also hold longer than 18 months to record annualised returns of 80% on average. Note that negative cycles (269 from 581 or 46%) outnumber positive cycles (174 from 581 or 30%) in this market. The overall result shows a compound return of -2.56% at an average hold of 7.14 months. The compound return per average hold turns positive in the 9-11-month range, at an increasing rate as the average hold extends beyond 9 months.

The 181 momentum cycles with an average hold extending beyond 9 months generally (155 from 181 or 86%) record positive returns. Most of these cycles (120 from 155 or 77%) are classified as positive cycles. Negative cycles hold shorter on average with only 14% (26 from 181) holding beyond 9 months. Overall results show 78 false (14%), 60 neutral (10%), 269 negative (46%), and 174 positive (30%) cycles. The average positive cycle holds 11 months while the average negative cycle holds shorter than 6 months. A relatively large number of cycles (138 from 581 or 24%) did either not hold (false cycles) or build (neutral cycles) momentum.

7.2.2 Price ranges

Based on the results per price range, stocks trading below $0.50 recorded the highest compound return (28.39%) and compound annual return (45.48%) per average hold even though this range only represents about 5% (28 from 581) of all cycles. Stocks priced at less than $5 account for 86% (150 from 174) of all positive cycles with the $0.50 to $1.00 range recording the greatest number of cycles (37) at an annualised return of more than 86%. Note that the number of negative cycles exceeds the number of positive cycles in each range, shortening the average hold per range to between 6 and 8 months.

The negative cycles account for between 43% (< $0.50) and 59% ($5-$10) of the cycles in the different ranges, averaging 46% (269 from 581) overall. The neutral cycles at a low of 4% (< $0.50) and a high of 17% ($3-$5) contributed the smallest number of cycles (60 from 581 or 10%) to the overall total. Neutral cycles recorded small positive returns (less than 0.5%) at an average hold of 9 months (versus 11 months for positive cycles), obtaining duration without continuation. False cycles, on the other hand, show reversal without duration as this category aligns with negative cycles at a much shorter average hold (1.55 versus 5.78 months). False cycles only recorded positive returns (18.48% annualised) in the $5-$10 range.

Table 7.2 Price range activity

CAD	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
< 0.5	28	5	1	12	10	8.00	28.39	45.48
	AH	1.60	4.00	5.17	15.00			
	CRpAH	-19.39	2.70	-26.87	225.55			
	CARpAH	-80.14	8.33	-51.65	157.09			
≥ 0.5 < 1	122	16	11	58	37	7.13	0.13	0.23
	AH	1.56	9.09	5.62	11.32			
	CRpAH	-14.19	0.28	-28.13	79.97			
	CARpAH	-69.12	0.37	-50.60	86.39			
≥ 1 < 1.5	89	12	6	41	30	7.79	3.08	4.78
	AH	1.42	11.50	5.66	12.50			
	CRpAH	-6.24	1.43	-30.23	83.08			
	CARpAH	-42.04	1.50	-53.39	78.70			
≥ 1.5 < 2	70	9	9	31	21	7.13	-6.05	-9.97
	AH	1.44	9.89	6.10	9.90			
	CRpAH	-17.53	-1.54	-33.62	62.61			
	CARpAH	-79.83	-1.87	-55.36	80.22			
≥ 2 < 3	74	7	9	35	23	7.30	-2.21	-3.60
	AH	1.57	7.22	6.37	10.48			
	CRpAH	-17.48	-1.14	-26.38	57.96			
	CARpAH	-76.94	-1.88	-43.83	68.81			
≥ 3 < 5	84	14	10	31	29	6.73	0.73	1.31
	AH	1.57	9.60	5.87	9.14			
	CRpAH	-20.50	1.73	-27.59	60.17			
	CARpAH	-82.65	2.17	-48.31	85.63			
≥ 5 < 10	63	3	6	37	17	7.16	-10.98	-17.71
	AH	1.67	8.00	5.59	11.24			
	CRpAH	2.38	2.37	-28.07	31.46			
	CARpAH	18.48	3.58	-50.67	33.93			
≥ 10	51	12	8	24	7	6.04	-21.31	-37.88
	AH	1.67	8.38	5.58	12.43			
	CRpAH	-18.01	-0.18	-38.71	31.68			
	CARpAH	-76.06	-0.26	-65.08	30.43			
TSXV	581	78	60	269	174	7.14	-2.56	-4.27
	AH	1.55	8.97	5.78	11.13			
	CRpAH	-15.24	0.33	-29.76	70.31			
	CARpAH	-72.18	0.44	-51.98	77.58			

Source: Price data downloaded from Bloomberg (2022)

Note that the $0.50-$1.00 range was the most actively traded (122 from 581 or 21%) but that the small number of stocks priced at less than $0.50 (28 from 581 or 5%) delivered the best positive-cycle and overall outcomes.

7.2.3 Sectors

Materials (51%), which includes the Metals and Mining industries, was the most active sector, dominating all the other sectors. However, with negative cycles outnumbering positive cycles overall, it is not surprising that this sector also recorded negative returns per average hold. Sectors such as Communications, Consumer Discretionary, Consumer Staples, Financials, and Real Estate only contributed a combined 10% (57 from 581) to the total number of cycles generated by this venture market.

Table 7.3 Sector activity

SECTOR	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
CO	11	---	1	7	3	8.18	-24.61	-33.92
	AH	---	9.00	6.14	12.67	COMMUNICATIONS		
	CRpAH	---	0.00	-40.36	18.58			
	CARpAH	---	0.00	-63.57	17.52			
CD	10	2	3	1	4	8.00	23.00	36.41
	AH	1.50	8.33	6.00	11.50	CONSUMER DISCRETIONARY		
	CRpAH	5.79	-0.84	-7.50	67.38			
	CARpAH	56.93	-1.21	-14.44	71.18			
CS	13	1	1	4	7	8.23	21.52	32.86
	AH	2.00	18.00	5.50	9.29	CONSUMER STAPLES		
	CRpAH	-13.37	-0.73	-41.59	99.52			
	CARpAH	-57.73	-0.49	-69.06	144.15			
EN	93	14	10	50	19	6.58	-10.76	-18.75
	AH	1.50	10.50	5.88	10.11	ENERGY		
	CRpAH	-20.01	3.16	-27.85	56.85			
	CARpAH	-83.24	3.61	-48.64	70.66			
FI	13	1	2	5	5	6.85	2.30	4.07
	AH	1.00	6.00	5.40	9.80	FINANCIALS		
	CRpAH	-17.20	1.34	-22.81	44.92			
	CARpAH	-89.62	2.70	-43.75	57.50			
HC	32	5	4	10	13	7.97	14.70	22.93
	AH	2.00	10.50	6.60	10.54	HEALTH CARE		
	CRpAH	-9.52	0.52	-18.38	70.01			
	CARpAH	-45.12	0.59	-30.88	82.99			
IN	47	6	7	26	8	7.15	-1.76	-2.94
	AH	1.33	8.86	5.85	14.25	INDUSTRIALS		
	CRpAH	-11.56	-1.08	-21.65	120.33			
	CARpAH	-66.88	-1.46	-39.40	94.49			
MA	298	42	26	144	86	6.96	-4.34	-7.36
	AH	1.50	8.31	5.65	11.42	MATERIALS		
	CRpAH	-14.64	0.08	-32.76	80.04			
	CARpAH	-71.82	0.12	-56.94	85.52			

Table 7.3 Sector activity (continued)

SECTOR	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
RE	10	1	---	3	6	7.00	10.02	17.79
	AH	2.00	---	6.67	8.00	REAL ESTATE		
	CRpAH	-0.34	---	-22.81	33.54			
	CARpAH	-2.03	---	-37.25	54.31			
TE	52	5	6	18	23	8.23	5.61	8.28
	AH	2.00	8.17	5.78	11.52	TECHNOLOGY		
	CRpAH	-28.53	-1.19	-23.08	49.91			
	CARpAH	-86.67	-1.75	-42.01	52.46			
UT	2	1	---	1	---	4.00	-23.25	-54.79
	AH	1.00	---	7.00	---	UTILITIES		
	CRpAH	1.35	---	-41.88	---			
	CARpAH	17.48	---	-60.56	---			
TSXV	581	78	60	269	174	7.14	-2.56	-4.27
	AH	1.55	8.97	5.78	11.13	TSX VENTURE EXCHANGE		
	CRpAH	-15.24	0.33	-29.76	70.31			
	CARpAH	-72.18	0.44	-51.98	77.58			

Source: Price data downloaded from Bloomberg (2022)

Negative cycles account for 46% (269 from 581) of all cycles, with comparatively greater numbers in Materials (144 from 298 or 48%) and Energy (50 from 93 or 54%). The negative cycles in Technology, a relatively active sector, generated less than 35% (18 from 52) of all cycles in that sector.

Positive cycles account for 30% (174 from 581) of all cycles, with comparatively greater numbers in Technology (23 from 52 or 44%) and Health Care (13 from 32 or 41%) from relatively active sectors. Among the less active sectors, Consumer Staples (7 from 13 or 54%) and Consumer Discretionary (4 from 10 or 40%) delivered positive cycles that contributed to the outperformance of these sectors. Positive cycles performed comparatively poorly in the Energy (19 from 93 or 20%) and Industrials (8 from 47 or 17%) sectors.

Neutral cycles have an overall representation of 10% (60 from 581) with Industrials overrepresented at 15% (7 from 47) and Materials somewhat underrepresented at 8.72% (26 from 298). Real Estate and Utilities did not record any neutral cycles. False cycles, at 13% overall (78 from 581), made similar contributions to the negative results from the Energy (14 from 93 or 15%) and Materials (42 from 298 or 14%) sectors. Communications did not experience any false cycles.

Technology, Health Care, Consumer Discretionary, and Consumer Staples appear to favour positive outcomes. However, the two most active sectors (Materials and Energy) generated 194 (72%) of the 269 negative cycles, resulting in the overall negative compound return per average hold (CRpAH) of -2.56% per 7.14 months.

7.2.4 Entry zones

An entry zone, three successive formation periods, identifies and confirms a momentum cycle in progress. The earliest entry (i.e., shortest formation) with potentially the longest hold should occur in Zone 1. The stepped pattern of a regular momentum profile exits each cycle as late as possible. Zones 2 to 4 allow for later entries and more irregular patterns or individual profiles.

Table 7.4 Results per entry zone

ZONE	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
1	182	37	21	72	52	6.78	-1.26	-2.23
	AH	1.51	10.10	5.63	10.79			
	CRpAH	-9.85	0.26	-25.52	54.67			
	CARpAH	-56.06	0.31	-46.66	62.44			
2	121	12	13	58	38	7.53	-2.14	-3.39
	AH	1.67	9.31	5.78	11.45			
	CRpAH	-19.22	2.18	-28.62	65.85			
	CARpAH	-78.49	2.82	-50.37	69.95			
3	129	14	9	68	38	7.50	-3.63	-5.74
	AH	1.50	9.11	5.87	12.26			
	CRpAH	-18.95	-1.53	-32.21	91.82			
	CARpAH	-81.38	-2.01	-54.85	89.15			
4	149	15	17	71	46	6.96	-3.56	-6.06
	AH	1.60	7.24	5.86	10.30			
	CRpAH	-21.13	0.01	-32.44	75.96			
	CARpAH	-83.14	0.02	-55.20	93.11			
TSXV	581	78	60	269	174	7.14	-2.56	-4.27
	AH	1.55	8.97	5.78	11.13			
	CRpAH	-15.24	0.33	-29.76	70.31			
	CARpAH	-72.18	0.44	-51.98	77.58			

Source: Price data downloaded from Bloomberg (2022)

Table 7.4 above shows the outcomes from momentum cycles entered at these four different zones. Zone 1 generated the most entries but, surprisingly, at the shortest average hold. A large number of false cycles (20% versus 10% overall) impacted the performance of this zone. Zones 1 and 3, respectively, recorded the smallest (72 from 182 or 40%) and greatest (68 from 129 or 53%) number of negative cycles measured against the overall negative-cycle average of 46% (269 from 581).

Note that Zone 1 recorded the best overall result (lowest negative return), Zone 2 the longest average hold, and Zone 3 the highest return per average hold for positive cycles. Neutral cycles posted small negative returns in Zone 3. However, the main observation relates to false cycles, generating large negative annual returns in each zone to impact overall performance negatively.

7.2.5 Parameter scores

The model identified 581 individual cycles with the [20|1.5|48|35] parameter setting combination. The average parameter scores for each period – which resulted in false, neutral, negative, or positive cycles – are included in Table 7.5 below.

Table 7.5 Average parameter scores

MOMENTUM	MS060	MS090	MS125	MS180	MS210	MS250	
False	28.74	43.56	45.51	36.00	27.92	20.10	33.64
Neutral	31.40	41.88	40.70	41.30	39.82	36.38	38.58
Negative	36.00	55.24	62.78	56.17	49.07	38.83	49.68
Positive	38.26	52.04	53.39	48.20	45.34	40.70	46.32
	35.23	51.33	55.37	49.54	44.16	36.62	45.37
VOLATILITY	VS060	VS090	VS125	VS180	VS210	VS250	
False	1.90	1.51	1.08	0.79	0.78	1.08	1.19
Neutral	1.65	1.32	1.09	1.02	1.42	1.60	1.35
Negative	2.12	1.60	1.14	0.91	0.95	1.01	1.29
Positive	1.84	1.51	1.15	0.89	0.90	1.07	1.23
	1.96	1.53	1.13	0.90	0.96	1.10	1.26
QUALITY	QS060	QS090	QS125	QS180	QS210	QS250	
False	53.90	53.17	51.99	50.56	49.81	49.14	51.43
Neutral	55.40	54.37	53.48	52.12	51.67	50.93	52.99
Negative	54.88	54.15	53.21	51.90	51.32	50.60	52.68
Positive	54.97	54.28	53.25	52.06	51.45	50.93	52.82
	54.83	54.08	53.09	51.79	51.19	50.54	52.58
ACTIVITY	AS060	AS090	AS125	AS180	AS210	AS250	
False	46.18	44.36	42.79	41.19	40.44	39.73	42.45
Neutral	45.80	43.88	42.48	40.73	39.92	38.95	41.96
Negative	45.24	43.57	41.89	39.99	39.19	38.24	41.35
Positive	45.64	43.73	42.24	40.34	39.44	38.57	41.66
	45.54	43.75	42.18	40.33	39.51	38.61	41.65

Source: Price data downloaded from Bloomberg (2022)

One-factor ANOVA (Welch's test) analyses were performed to possibly differentiate between the average parameter scores of the four different groups – positive (POS), negative (NEG), neutral (NEU), and false (FAL) cycles. In several instances, the differences between the averages of these four groups, and the per-period averages for each parameter were found to be statistically significant – refer to Annexure C.

The momentum score (MS) averages for the positive and negative cycles across all momentum periods are higher than those for the false and neutral cycles. The FAL/POS, FAL/NEG, NEU/POS, and NEU/NEG pairings are all significantly different at a 5% level. Negative cycles have the highest and false cycles have the lowest overall scores on average. Zone 2 (090-125-180) has the highest average momentum scores overall.

The more varied results from the volatility score (VS) averages show that negative cycles generally have high scores and the second-highest score on average after neutral cycles. False cycles recorded the lowest average score overall. Zone 1 (060-090-125) has the highest and Zone 4 (180-210-250) has the lowest average volatility scores per zone. None of the overall differences but most of the per-period differences (except VS125/VS210, VS125/VS250, VS180/VS210, VS180/VS250, VS210/VS250) are statistically significant at a 5% level (refer to Annexure C).

The quality score (QS) averages for neutral and positive cycles are higher than those for false and negative cycles. Neutral cycles have the highest and false cycles have the lowest overall scores on average. Scores decline as the momentum periods increase. Based on overall averages, the FAL/NEU, FAL/POS, and FAL/NEG pairings are significantly different at a 5% level. All the per-period pairings (except QSA060/QS090) are statically different at a 5% level.

The activity score (AS) averages for negative and positive cycles are lower than those for false and neutral cycles. False cycles have the highest and negative cycles the lowest overall scores on average with this difference statistically significant at a 5% level. Scores decline as the momentum periods increase. All the per-period pairings are statically different at a 5% level.

Table 7.6 Generalised outcomes

Parameters Cycles	MOMENTUM		VOLATILITY		QUALITY		ACTIVITY	
	High	Low	High	Low	High	Low	High	Low
False		X		X		X	X	
Neutral		X	X		X		X	
Negative	X		X			X		X
Positive	X			X	X			X

In summary, the results show that there is some indication that, on average and in this equity market, cycles with higher momentum and quality scores in combination with lower volatility and activity scores tend to be positive. Negative cycles have the highest average momentum score overall and higher volatility with lower quality scores relative to positive cycles. False cycles, on average, recorded the lowest scores in every category but activity. Neutral cycles recorded high volatility, quality, and activity scores on average. Note that even though several average scores are statistically different, the same combinations may not produce equivalent outcomes for individual cycles.

In the previous five subsections, a momentum profile for the TSX Venture Exchange (TSXV) was created via an analysis of the different cycles in terms of average hold, price range activity, sector activity, outcomes per entry zone, and the average parameter scores per cycle type. In the section to follow, a custom momentum index evaluates the actual performance of the momentum model. The results are presented graphically and compared to a market index in terms of performance, correlation, drawdown, and descriptive statistics.

Table 7.7 Statistically significant results

Momentum Score (MS)										
group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
FAL	POS	12.6808	1.5973	7.9391	1051.027	3.6330	6.8780	18.4836	0.0000	5.8028
FAL	NEG	16.0393	1.4751	10.8730	861.672	3.6330	10.6801	21.3985	0.0000	5.3592
NEU	POS	7.7413	1.6492	4.6940	792.943	3.6330	1.7498	13.7327	0.0052	5.9914
NEU	NEG	11.0997	1.5312	7.2490	636.881	3.6330	5.5369	16.6626	0.0000	5.5629
Volatility Score (VS)										
group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
None										
Quality Score (QS)										
group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
FAL	NEU	1.5671	0.1911	8.2022	788.842	3.6330	0.8730	2.2612	0.0000	0.6941
FAL	POS	1.3945	0.1556	8.9632	898.848	3.6330	0.8293	1.9597	0.0000	0.5652
FAL	NEG	1.2486	0.1451	8.6052	727.045	3.6330	0.7215	1.7758	0.0000	0.5271
Activity Score (AS)										
group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
FAL	NEG	1.0962	0.2043	5.3657	722.721	3.6330	0.3540	1.8384	0.0009	0.7422
MS060-MS250										
group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
MS060	MS090	16.1084	1.7599	9.1529	1135.588	4.0300	9.0159	23.2010	0.0000	7.0925
MS060	MS125	20.1429	1.7651	11.4121	1133.952	4.0300	13.0297	27.2560	0.0000	7.1132
MS060	MS180	14.3150	1.5936	8.9828	1158.070	4.0300	7.8928	20.7372	0.0000	6.4222
MS060	MS210	8.9363	1.6707	5.3487	1156.743	4.0300	2.2033	15.6694	0.0023	6.7331
MS090	MS250	14.7143	1.9054	7.7226	1159.452	4.0300	7.0357	22.3929	0.0000	7.6786
MS125	MS210	11.2065	1.8065	6.2034	1148.675	4.0300	3.9263	18.4868	0.0002	7.2803
MS125	MS250	18.7487	1.9101	9.8156	1159.677	4.0300	11.0510	26.4464	0.0000	7.6977
MS180	MS250	12.9208	1.7529	7.3712	1112.238	4.0300	5.8567	19.9849	0.0000	7.0641
MS210	MS250	7.5422	1.8233	4.1366	1144.654	4.0300	0.1943	14.8900	0.0409	7.3479
VS060-VS250										
group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
VS060	VS090	0.4246	0.0544	7.7994	1089.338	4.0300	0.2052	0.6440	0.0000	0.2194
VS060	VS125	0.8254	0.0481	17.1485	848.965	4.0300	0.6314	1.0194	0.0000	0.1940
VS060	VS180	1.0571	0.0486	21.7329	874.174	4.0300	0.8611	1.2532	0.0000	0.1960
VS060	VS210	0.9911	0.0584	16.9698	1150.601	4.0300	0.7557	1.2264	0.0000	0.2354
VS060	VS250	0.8569	0.0777	11.0325	1011.238	4.0300	0.5439	1.1699	0.0000	0.3130
VS090	VS125	0.4008	0.0395	10.1411	989.926	4.0300	0.2415	0.5600	0.0000	0.1593
VS090	VS180	0.6325	0.0401	15.7579	1019.537	4.0300	0.4708	0.7943	0.0000	0.1618
VS090	VS210	0.5664	0.0515	10.9913	1128.092	4.0300	0.3588	0.7741	0.0000	0.2077
VS090	VS250	0.4323	0.0727	5.9503	866.918	4.0300	0.1395	0.7251	0.0004	0.2928
VS125	VS180	0.2317	0.0310	7.4646	1156.968	4.0300	0.1066	0.3569	0.0000	0.1251
QS060-QS250										
group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
QS060	QS125	1.7418	0.1814	9.5997	957.071	4.0300	1.0106	2.4731	0.0000	0.7312
QS060	QS180	3.0379	0.1743	17.4332	865.404	4.0300	2.3356	3.7401	0.0000	0.7023
QS060	QS210	3.6368	0.1733	20.9891	851.816	4.0300	2.9385	4.3351	0.0000	0.6983
QS060	QS250	4.2926	0.1733	24.7683	852.336	4.0300	3.5942	4.9910	0.0000	0.6984
QS090	QS125	0.9931	0.1505	6.5973	1108.235	4.0300	0.3865	1.5998	0.0001	0.6066
QS090	QS180	2.2892	0.1418	16.1452	1019.782	4.0300	1.7178	2.8606	0.0000	0.5714
QS090	QS210	2.8881	0.1406	20.5452	1003.753	4.0300	2.3216	3.4546	0.0000	0.5665
QS090	QS250	3.5439	0.1406	25.2019	1004.379	4.0300	2.9772	4.1106	0.0000	0.5667
QS125	QS180	1.2960	0.1233	10.5103	1128.100	4.0300	0.7991	1.7930	0.0000	0.4969
QS125	QS210	1.8950	0.1219	15.5435	1117.486	4.0300	1.4037	2.3863	0.0000	0.4913
QS125	QS250	2.5508	0.1220	20.9132	1117.922	4.0300	2.0592	3.0423	0.0000	0.4915
QS180	QS210	0.5990	0.1109	5.3993	1159.104	4.0300	0.1519	1.0460	0.0020	0.4471
QS180	QS250	1.2547	0.1110	11.3046	1159.172	4.0300	0.8074	1.7020	0.0000	0.4473
QS210	QS250	0.6558	0.1094	5.9919	1159.999	4.0300	0.2147	1.0968	0.0004	0.4410
AS060-AS250										
group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
AS060	AS090	1.7900	0.2285	7.8332	1124.399	4.0300	0.8691	2.7109	0.0000	0.9209
AS060	AS125	3.3666	0.2179	15.4495	1066.906	4.0300	2.4884	4.2448	0.0000	0.8782
AS060	AS180	5.2100	0.2173	23.9777	1062.538	4.0300	4.3343	6.0856	0.0000	0.8757
AS060	AS210	6.0379	0.2184	27.6414	1070.487	4.0300	5.1576	6.9182	0.0000	0.8803
AS060	AS250	6.9294	0.2210	31.3554	1086.832	4.0300	6.0388	7.8200	0.0000	0.8906
AS090	AS125	1.5766	0.1954	8.0673	1142.442	4.0300	0.7890	2.3642	0.0000	0.7876
AS090	AS180	3.4200	0.1947	17.5624	1140.122	4.0300	2.6352	4.2047	0.0000	0.7848
AS090	AS210	4.2478	0.1960	21.6709	1144.265	4.0300	3.4579	5.0378	0.0000	0.7899
AS090	AS250	5.1394	0.1989	25.8437	1151.587	4.0300	4.3380	5.9408	0.0000	0.8014
AS125	AS180	1.8434	0.1822	10.1190	1159.922	4.0300	1.1092	2.5775	0.0000	0.7341
AS125	AS210	2.6713	0.1835	14.5540	1159.946	4.0300	1.9316	3.4109	0.0000	0.7397
AS125	AS250	3.5628	0.1866	19.0952	1158.247	4.0300	2.8109	4.3147	0.0000	0.7519
AS180	AS210	0.8279	0.1828	4.5289	1159.739	4.0300	0.0912	1.5646	0.0175	0.7367
AS180	AS250	1.7194	0.1859	9.2518	1157.432	4.0300	0.9705	2.4684	0.0000	0.7490
AS210	AS250	0.8916	0.1872	4.7628	1158.805	4.0300	0.1372	1.6460	0.0101	0.7544

7.3 MOMENTUM INDEX

All stocks or tickers identified by the customised model are included in the custom momentum index. The index is updated monthly when newly identified tickers (if any) are added (i.e., cycles entered), while current members with dMS250 scores below the set minimum (if any) are deleted from the index (i.e., cycles exited). The base date for the index is 31 December 2008, and the base or starting value is 100. The number of members is variable, and the index maintains a relatively active position over a true equal-weighted design, which resets all the weights to the average weight when updating. However, any new members are assigned the average weight of the current members, adjusted for the number of additions and the total weight of any deletions, equally distributed among all members.

7.3.1 Levels and members

The TSXV Momentum Index (TSXV-MI) started with the first qualifying member, International Tower Hill Mines (ITH:CV), included on 31 March 2009. The ITH cycle lasted 8 months with the price increasing from \$2.84 to \$6.98 during this period at a compound total growth rate (CTGR) of 145.77%. The index ended 2009 with 33 members (refer to Table 7.9 on page 7-14) and moved clear of the TSX Venture Composite Index (TXVC) in 2013. The custom index can serve as a benchmark for momentum on the TSX Venture Exchange (TSXV) as it is updated monthly and has a variable number of members. Figure 7.1 below contrasts the performance of the custom TSXV Momentum Index to the S&P/TSX Venture Composite Index (TXVC) with its base date adjusted to 31 December 2008 and its base value to 100.

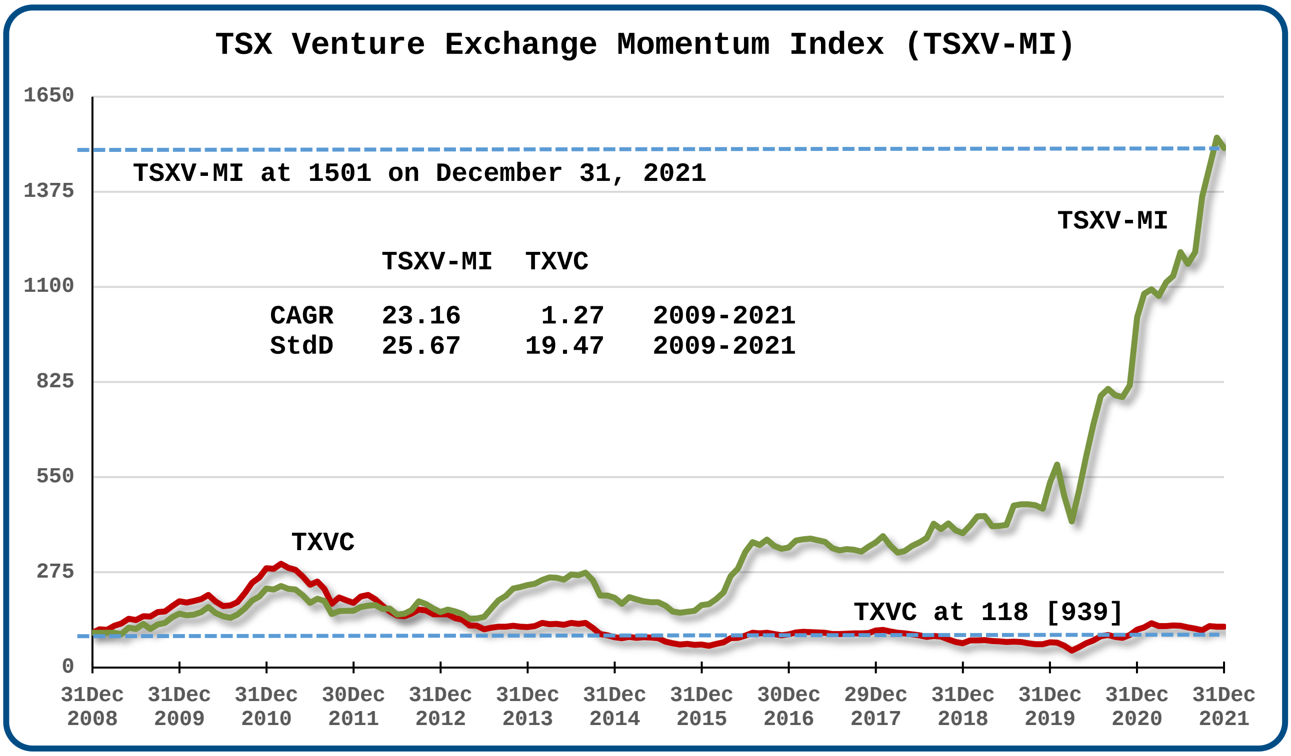

Figure 7.1 TSXV Momentum Index (Source of price data: Bloomberg 2022)

The year 2020 was the most volatile period for the TSXV Momentum Index (refer to Table 7.11 on page 7-15). Figure 7.2 below displays the volatility of the index during this period graphically. The three-year period beginning 2019 and ending 2021 recorded a compound annual growth rate (CAGR) of 57.04% with a standard deviation (StdD) of 29.28%. The individual statistics for years 2019 (CAGR:38.06% & StdD:20.04%), 2020 (CAGR:89.20% & StdD:41.10%) and 2021 (CAGR:48.27% & StdD:22.23%) confirms the increased volatility during this period. The index level dropped to 422 at the end of March 2020 and rebounded to 805 within five months, ending the year at 1012. Equities outperformed during 2021 with the momentum index ending at 1500.74, down a little from the all-time high of 1531.08 reached at the end of November 2021.

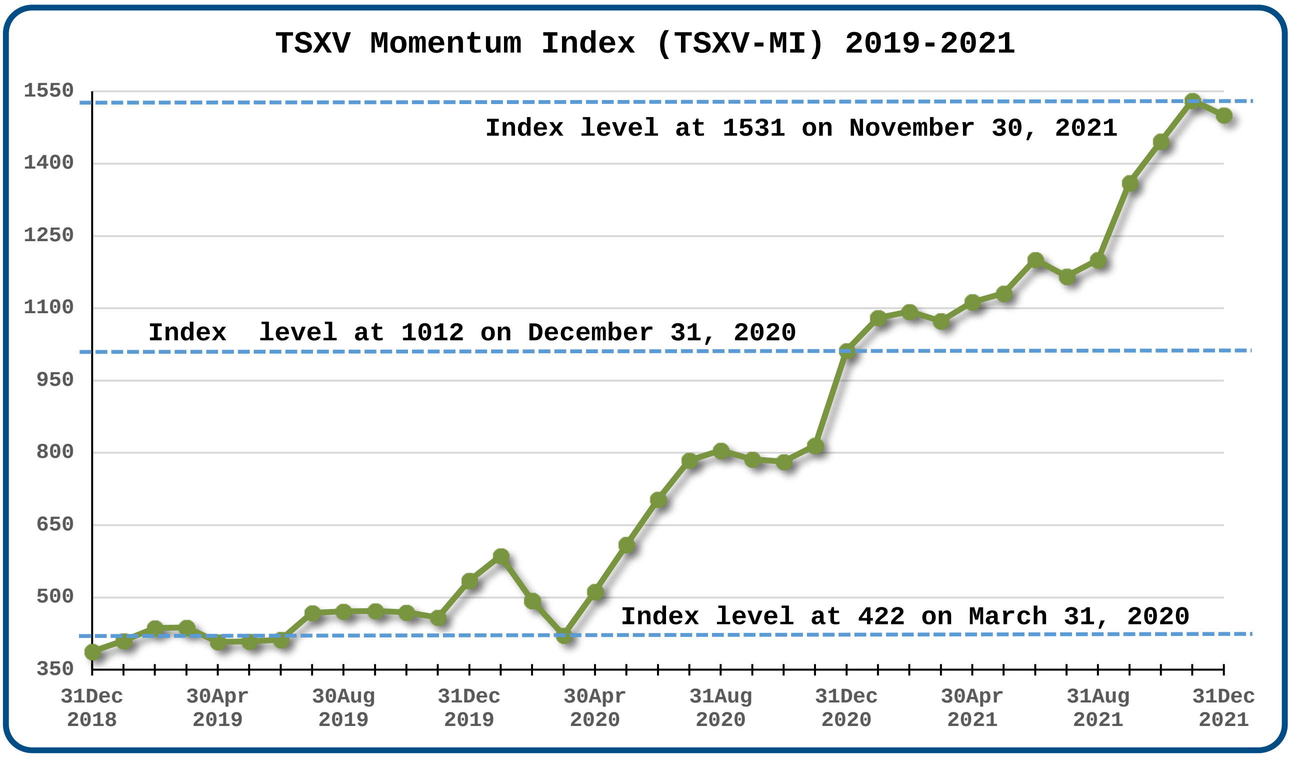

Figure 7.2 TSXV-MI 2019-2021 (Source of price data: Bloomberg 2022)

Table 7.8, on the next page, describes the activity during the 2019-2021 period starting with the index at 388 comprising 16 members. The index only gained 7.14% in value during the previous year, 2018, dropping 42 members and adding 32 (net 10 deletions). By the end of 2019, the index netted 9 additions and its value increased by 38.06%. The net amount of 9 additions is made up of 42 cycles entered and 33 cycles exited (9 at a gain versus 24 at a loss). During 2020, the index gained 89.20% and netted 22 additions by entering 55 new cycles and exiting 33 cycles (12 gains versus 21 losses). [Returns are cumulative or annual]

Compare 2019 and 2020 to 2021, when the index gained 48.27% and netted 22 deletions by entering 46 cycles and exiting 68 (39 gains and 29 losses). Index members numbered 25 at the end of 2021. Entering many new cycles and exiting the majority of completed cycles at a profit in a year when the index reached a high. Negative and annual returns correspond to large net deletions when many cycles were exited at a loss. Loss-making cycles also include false and neutral cycles.

Table 7.8 Updating 2019-2021

Date	LEVEL	GROWTH	MEM	3MA	ADD	DEL	ADD/T
Dec 2018	387.51	-2.20 %	16	19	1	4	0.20
Jan 2019	409.90	5.78 %	15	17	1	2	0.33
Feb 2019	436.62	6.52 %	14	15	2	3	0.40
Mar 2019	437.47	0.19 %	18	16	6	2	0.75
Apr 2019	407.80	-6.78 %	20	17	3	1	0.75
May 2019	408.66	0.21 %	23	20	6	3	0.67
Jun 2019	412.16	0.86 %	21	21	0	2	0.00
Jul 2019	467.83	13.51 %	17	20	0	4	0.00
Aug 2019	471.15	0.71 %	22	20	10	5	0.67
Sep 2019	471.83	0.14 %	22	20	3	3	0.50
Oct 2019	468.94	-0.61 %	26	23	5	1	0.83
Nov 2019	458.44	-2.24 %	26	25	3	3	0.50
Dec 2019	534.98	16.69 %	25	26	3	4	0.43
Jan 2020	586.41	9.61 %	28	26	5	2	0.71
Feb 2020	493.24	-15.89 %	29	27	2	1	0.67
Mar 2020	421.50	-14.54 %	24	27	0	5	0.00
Apr 2020	511.99	21.47 %	20	24	1	5	0.17
May 2020	609.51	19.05 %	22	22	3	1	0.75
Jun 2020	703.12	15.36 %	28	23	6	0	1.00
Jul 2020	784.69	11.60 %	34	28	10	4	0.71
Aug 2020	804.99	2.59 %	38	33	6	2	0.75
Sep 2020	786.47	-2.30 %	45	39	8	1	0.89
Oct 2020	781.69	-0.61 %	51	45	6	0	1.00
Nov 2020	815.23	4.29 %	47	48	3	7	0.30
Dec 2020	1012.18	24.16 %	47	48	5	5	0.50
Jan 2021	1080.19	6.72 %	47	47	6	6	0.50
Feb 2021	1093.12	1.20 %	52	49	8	3	0.73
Mar 2021	1073.53	-1.79 %	48	49	4	8	0.33
Apr 2021	1112.88	3.67 %	46	49	6	8	0.43
May 2021	1131.22	1.65 %	47	47	4	3	0.57
Jun 2021	1201.02	6.17 %	39	44	2	10	0.17
Jul 2021	1166.52	-2.87 %	33	40	1	7	0.13
Aug 2021	1200.42	2.91 %	29	34	1	5	0.17
Sep 2021	1360.39	13.33 %	28	30	1	2	0.33
Oct 2021	1446.16	6.30 %	30	29	6	4	0.60
Nov 2021	1531.08	5.87 %	30	29	4	4	0.50
Dec 2021	1500.74	-1.98 %	25	28	3	8	0.27

Source: Price data downloaded from Bloomberg (2022)

Index activity may give some indication of the sentiment and volatility in the market when looking at the number of cycles entered versus exited. The turnover of members, net additions or deletions, and the results when exiting cycles correspond to large decreases and increases in the index value. A progressively increasing or decreasing number of members during a particular period shows the equity market trending upwards or downwards. A simple gain versus loss comparison of completed cycles does not account for the much shorter negative cycles and false cycles when matched with positive cycles.

Figure 7.3 overlays a line chart with changing index levels on a bar chart showing the variation in index members. There was a steady increase in value since the first member was included on 31 March 2009, building from this single member to peak at 33 members within a year. The index members appear to synchronise with the index levels to some degree, surging and receding with the availability of momentum stocks in the market. After exiting many positions during a downswing, the index level surges as the number of member stocks grows.

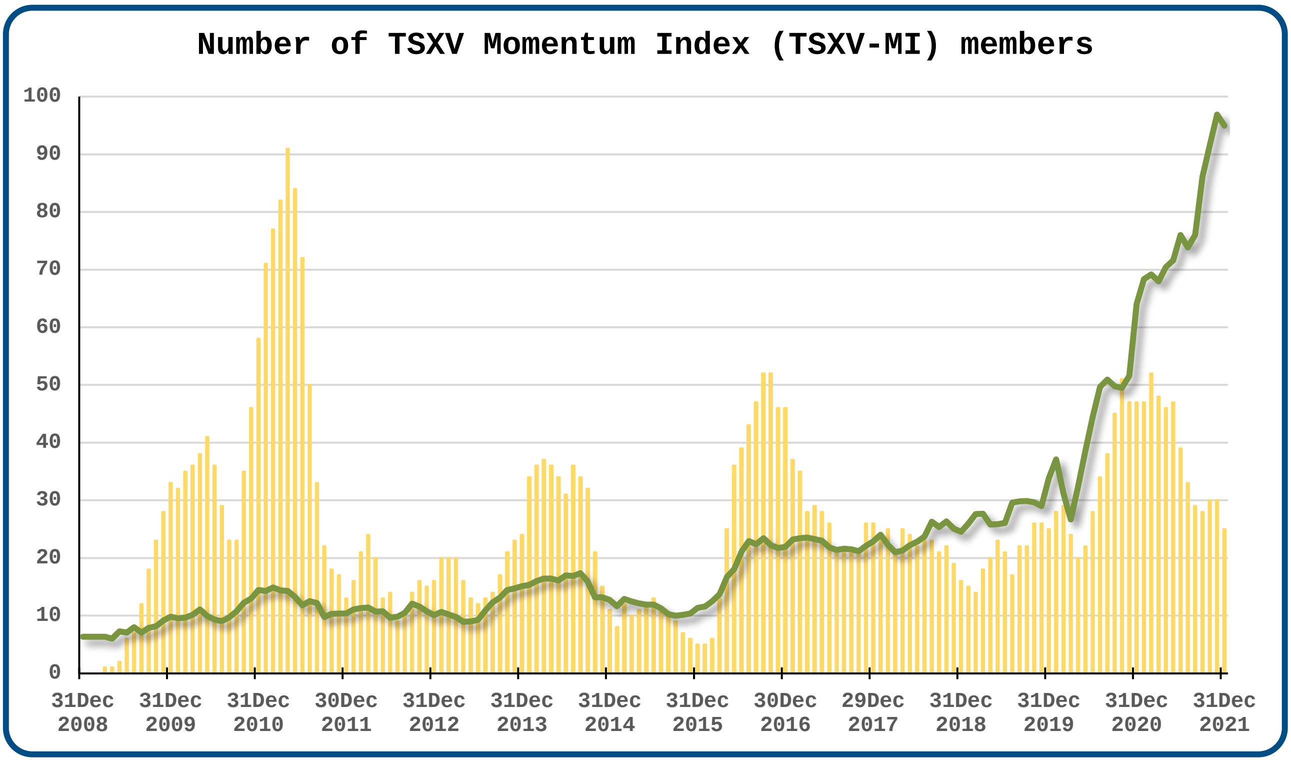

Figure 7.3 TSXV-MI member numbers (Source of price data: Bloomberg 2022)

The table below summarises the annual results for the full 13-year period. The end-of-year members against the average reflects the state of the market at year-end.

Table 7.9 Annual results 2009-2021

Year	LEVEL	GROWTH	MEM	AVG	ADD	DEL	ADD/T
2009	155.12	55.12 %	33	11	38	5	0.88
2010	228.41	47.25 %	58	36	75	50	0.60
2011	163.48	-28.43 %	13	53	64	109	0.37
2012	159.85	-2.22 %	16	16	39	36	0.52
2013	238.47	49.19 %	24	18	31	23	0.57
2014	201.11	-15.67 %	11	30	40	53	0.43
2015	179.78	-10.61 %	5	10	18	24	0.43
2016	346.45	92.71 %	46	34	65	24	0.73
2017	361.68	4.40 %	26	27	39	59	0.40
2018	387.51	7.14 %	16	22	32	42	0.43
2019	534.98	38.06 %	25	21	42	33	0.56
2020	1012.18	89.20 %	47	34	55	33	0.63
2021	1500.74	48.27 %	25	38	46	68	0.40

Source: Price data downloaded from Bloomberg (2022)

7.3.2 Relative performance

A comparison between the performance of the custom momentum index and the S&P/TSX Venture Composite Index (refer to Table 7.10), the headline index for the TSX Venture Exchange, focuses on the relative performance of the model.

Table 7.10 Benchmark information

S&P/TSX Venture Composite Index	
TXVC 2001-12-14	A broad market indicator of Canadian micro cap securities listed on the TSX Venture Exchange. It is a capitalisation-weighted market index containing 150 securities, and rebalanced quarterly.

Source: SPTXV (2022)

Table 7.11 below shows the progression and relative performance of the TSXV Momentum Index (TSXV-MI) over time from its 2009 base year to the end of 2021. Note its performance in 2013 relative to the benchmark, rebounding after the market performed poorly in both 2011 and 2012. Apart from 2011 and 2012 (which recorded the worst result over the evaluation period), the custom index also recorded negative growth in 2014 and 2015. Two successive years of decline followed by a rebound (92.71% in 2016). The methodology of the index may explain its outperformance as it retains the momentum of members while maintaining a relatively active position. The growth over 10, 5 and 3 years confirms the consistent outperformance by the TSXV-MI of its benchmark.

Table 7.11 TSXV-MI results versus benchmark (2009-2021)

Year	Metric	TSXV-MI	TXVC	Year	Metric	TSXV-MI	TXVC
2009	CAGR	55.12	90.80	2010	CAGR	47.25	50.45
	StdD	37.62	22.05		StdD	19.20	17.20
2011	CAGR	-28.43	-35.11	2012	CAGR	-2.22	-17.74
	StdD	27.55	26.92		StdD	25.97	18.33
2013	CAGR	49.19	-23.69	2014	CAGR	-15.67	-25.37
	StdD	20.62	15.48		StdD	19.38	14.63
2015	CAGR	-10.61	-24.42	2016	CAGR	92.71	45.03
	StdD	22.63	13.61		StdD	26.40	16.32
2017	CAGR	4.40	11.59	2018	CAGR	7.14	-34.50
	StdD	14.69	10.24		StdD	23.22	17.08
2019	CAGR	38.06	3.65	2020	CAGR	89.20	51.57
	StdD	20.04	10.90		StdD	41.10	32.42
2021	CAGR	48.27	7.29	1Y	CAGR	48.27	7.29
	StdD	22.23	24.06		StdD	22.23	24.06
FULL 2009 2021	CTGR	1400.74	17.84	10Y 2012 2021	CTGR	818.02	-36.74
	CAGR	23.16	1.27		CAGR	24.82	-4.48
	StdD	25.67	19.47		StdD	24.54	18.43
5Y 2017 2021	CTGR	333.18	23.19	3Y 2019 2021	CTGR	287.28	68.55
	CAGR	34.07	4.26		CAGR	57.04	19.01
	StdD	25.80	20.72		StdD	29.28	24.11

Source: Price data downloaded from Bloomberg (2022)

7.3.3 Correlation analysis

Correlation measures the degree of co-movement or size of the linear association between two time-series. Correlation-squared (R-squared) indicates how closely an index tracks the performance of a particular benchmark. It also points to the reliability of the alpha (excess return) and beta (volatility) coefficients from a linear regression.

Table 7.12 Correlations: TSXV-MI versus benchmark

Year	TXVC	Year	TXVC	Year	TXVC	Year	TXVC
2009	0.42	**2013**	0.56	**2017**	0.57	**1Y**	0.74
2010	0.81	**2014**	0.79	**2018**	0.56	**3Y**	0.81
2011	0.91	**2015**	0.53	**2019**	0.55	**5Y**	0.76
2012	0.73	**2016**	0.74	**2020**	0.91	**AVG**	0.68

Source: Price data downloaded from Bloomberg (2022)

Table 7.12 above shows the changes in correlation with the benchmark from year to year, averaging 0.68 per year. The weakest associations, apart from the 2009 base year, occurred during 2013, 2015, and 2017 to 2019 when the momentum index led the market in recovering from downturns.

Results show the strongest association between the custom momentum index and the venture composite index during 2010, 2011, 2014, and 2020. Measured over longer periods, the 3-year and 5-year correlations confirm the strong correlation between the custom index and the composite index. As stated previously, the methodology of the momentum index, retaining the momentum of the remaining members, may account for the outperformance of TSXV-MI to some degree. A variable number of members in combination with more frequent updating allows for a relatively active approach to indexing or benchmarking momentum in an equity market.

7.3.4 Drawdown analysis

A drawdown analysis highlights the potential for sudden large (20%-plus) losses in value and the likely time to recover (Wilmington 2018). It records the size and speed of maximum drawdowns and the time to return to former highs. Referring to Table 7.13 on the next page, the TSXV Momentum Index (TSXV-MI) experienced its maximum drawdown at the end of August 2015 after declining for 243 consecutive days and taking another 181 days to recover to previous levels (424 days from peak to peak). Another large drawdown occurred in March 2020 with the custom index declining sharply, dropping 44% in value within 18 days and recovering after 57 days (75 days peak to peak). The TSX Venture Composite did in fact experience its maximum drawdown in March 2020 when it declined by 86.11% within 2259 days while taking 446+ days to recover (2705+ days peak to peak). An average TSXV-MI drawdown is 14.76% and lasts 27 days (peak to peak) compared to a TXVC drawdown averaging 55.08% and lasting 62 days.

Table 7.13 Drawdown analysis (2009-2021)

Metric	TSXV-MI	TXVC
Maximum drawdown	45.95%	86.11%
Date	2015-08-24	2020-03-18
Period	243 days	2259 days
Recovery	181 days	446+ days
Average drawdown	14.76%	55.08%
Maximum duration	709 days	2705+ days
From:	2011-03-08	2011-03-07
To:	2014-01-07	2021-12-31
Average duration	27 days	62 days
Annualised return	23.16%	1.27%
Drawdown ratio	0.50	0.01

Source: Price data downloaded from Bloomberg (2022)

A higher annualised return (23.16% versus 1.27%) relative to a lower maximum drawdown (45.95% versus 86.11%) confirmed the risk-adjusted outperformance of the custom momentum index (TSXV-MI) as reflected in its higher drawdown ratio (i.e., annualised return to maximum drawdown) of 0.50 (versus the 0.01 of TXVC).

7.3.5 Descriptive statistics

Descriptive statistics, the process of describing data and presenting it graphically, provides the individual summary statistics listed in the table below. It includes the mean returns for both indices with their accompanying standard deviations. The coefficient of variation (CV), the size of the standard deviation about its mean, shows that the relative variability of the custom momentum index (TSXV-MI) is low compared to the S&P/TSX Venture Composite Index (TXVC). The respective standard deviations and ranges indicate a higher variability in general for the custom index.

Table 7.14 Summary statistics (2009-2021)

Metric	TSXV-MI	TXVC
Mean	0.0834 %	0.0051 %
Standard Error	0.0283 %	0.0215 %
Median	0.0842 %	0.0770 %
Standard Deviation	1.6109 %	1.2244 %
Sample Variance	2.5951	1.4991
Kurtosis	6.2724	7.7977
Skewness	-0.3459	-0.9870
Range	24.08 %	19.21 %
Maximum	10.97 %	8.08 %
Minimum	-13.12 %	-11.13 %
Sum	270.85 %	16.41 %
Count	3248	3248
CV	19.32	242.30

Source: Price data downloaded from Bloomberg (2022)

The two sets of data are not fully symmetric but negatively or left skewed with the means (averages) smaller than the medians (middle values). A left-skewed distribution has more values in the right tail, but the left tail is longer indicating many smaller positive returns and a few large negative returns. The distribution of TSXV-MI is approximately symmetric with its skewness measuring between -0.5 and 0. Data are moderately left-skewed with values between -1 and -0.5 as with TXCV.

High kurtosis values would point to heavy-tailed distributions with outliers or extreme positive and negative returns. Extreme returns can be defined as returns that exceed the 90th percentile, the top and bottom 10% of returns (Sankaran, Nguyen & Harikumar 2012). Compared to a normal distribution, described as mesokurtic, these distributions can be described as leptokurtic with excess kurtosis. Negatively skewed, heavy-tailed distributions are common in stock market data (Samunderu & Murahwa 2021). Figure 7.4 below shows the dispersion of TSXV-MI returns with most returns clustering around the mean. The histogram confirms its relatively low kurtosis value with some extreme positive and negative returns as outliers. The momentum index (TSXV-MI), being more symmetric and with a lower kurtosis, is less likely than the composite index (TXVC) to record extreme negative returns.

As evident from Figure 7.4, the Laplace distribution with its high central peak, narrow upper shoulders and heavy tails provides a more reasonable fit for log returns than the normal distribution. The Laplace distribution is symmetric about its location parameter (median) with the scale parameter (beta) determining its profile while the normal distribution is completely defined by its mean and standard deviation (Kotz, Kozubowski & Podgórski 2001).

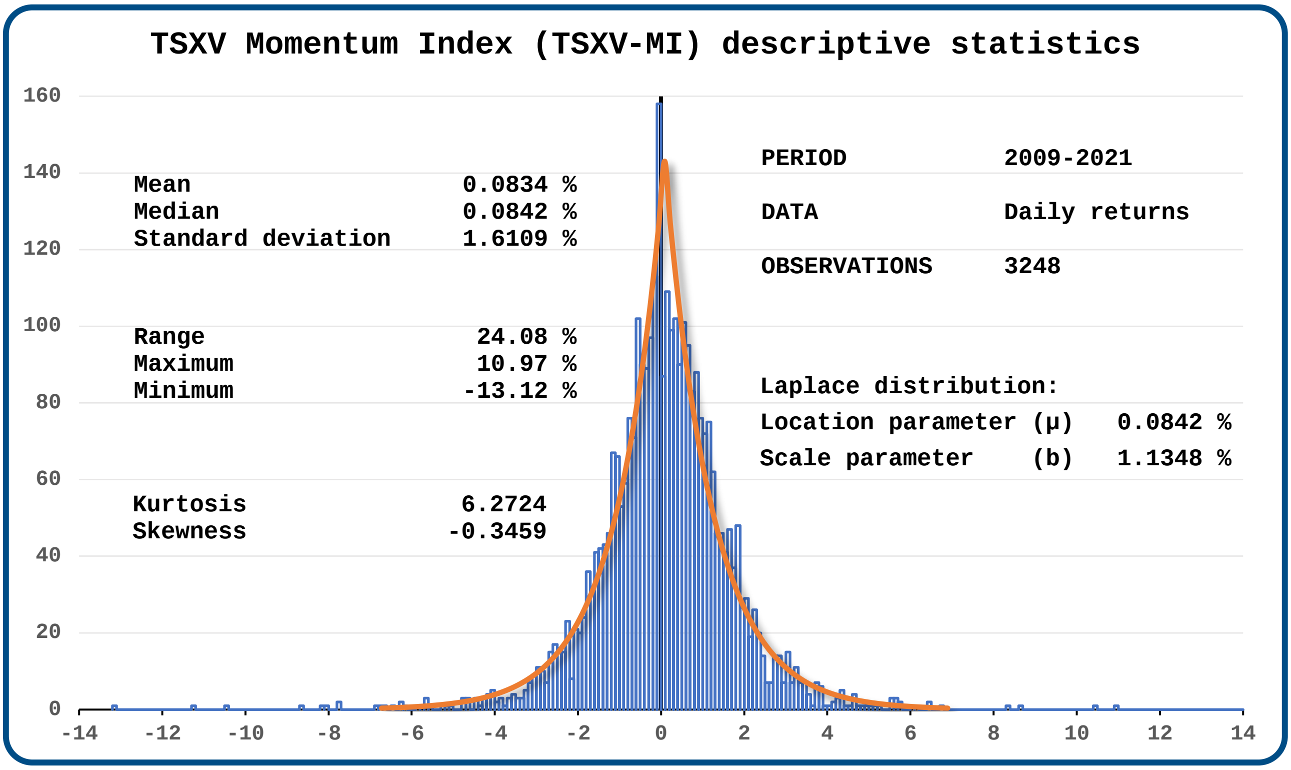

Figure 7.4 TSXV-MI descriptive statistics (Source of price data: Bloomberg 2022)

7.4 CONCLUSION

This chapter created a momentum profile for the TSX Venture Exchange (TSXV) by mechanically entering and exiting momentum cycles identified by the customised momentum model based on its four parameters and an exit rule. The results from applying the model mechanically provided a set of positive, negative, neutral, and false cycles unique to this equity market.

Momentum cycles with holds that extend beyond 9 months generally record positive returns. Positive cycles at an average hold of 11 months gained 70% in value. Negative cycles lost 30% in value at an average hold shorter than 6 months. False cycles, holding shorter than 2 months on average while losing 15% in value, outnumbered the neutral cycles with an average hold of 9 months that only gained half a per cent in value.

Even though stocks in the $0.50-$1.00 price range were the most actively traded in this market, the relatively small number of stocks priced at less than $0.50 delivered the best positive-cycle and overall outcomes. Most of the momentum cycles originated in the Materials sector (51%) and the Energy sector (16%). But cycles from these two sectors also account for 72% of all negative cycles. The Technology, Health Care, Consumer Discretionary, and Consumer Staples sectors generally favoured positive outcomes.

Zone 1 (060-090-125) recorded the best overall result, Zone 2 (090-125-180) the longest average hold, Zone 3 (125-180-210) the highest return per average positive cycle, and Zone 4 (180-210-250) the worst overall result. Neutral cycles generally posted small positive gains in all entry zones apart from Zone 3. False cycles, outnumbering neutral cycles in this venture market, generated large negative annualised returns in each zone.

A custom momentum index was used to evaluate the model by quantifying the process of entering the cycles at certain prices and exiting at either a gain or a loss. The performance of the custom momentum index compared favourably with the benchmark venture composite index and tracked it closely during certain years. A drawdown analysis showed that while both the custom index and the composite index recover relatively quickly from drawdowns, the momentum index outperformed on a risk-adjusted basis. While the overall return per average hold was negative, the cumulative change in prices over time, the compound annual growth rate (CAGR) of the index level, was positive.

Chapter 8 to follow contrasts the results obtained from the three different markets – an emerging market exchange, the Johannesburg Stock Exchange (JSE); a developed market exchange, the Toronto Stock Exchange (TSX); and a venture market exchange, the TSX Venture Exchange (TSXV).

REFERENCES

Bloomberg. 2022. Bloomberg Terminal – Bloomberg Professional Services subscription. New York: Bloomberg Finance L.P.

Kotz, S., Kozubowski, T.J. & Podgórski, K. 2001. *The Laplace distribution and generalizations: a revisit with applications to communications, economics, engineering, and finance*. Boston, Massachusetts: Birkhäuser Boston. DOI: 10.1007/978-1-4612-0173-1.

Samunderu, E. & Murahwa, Y.T. 2021. Return based risk measures for non-normally distributed returns: an alternative modelling approach. *Journal of Risk and Financial Management*, 14(11):540. DOI: 10.3390/jrfm14110540.

Sankaran, H., Nguyen, A. & Harikumar, J. 2012. Extreme return correlation and volatility: a two-threshold approach. *American Journal of Business*, 27(2):154–173. DOI: 10.1108/19355181211274451.

SPTXV. 2022. *S&P/TSX Venture Composite methodology*. *S&P Dow Jones Indices [Online]*. S&P Global. Available at: https://www.spglobal.com/spdji/en/documents/methodologies/methodology-sp-tsx-venture-composite.pdf (Accessed: 15 June 2022).

Wilmington. 2018. *Understanding investment risk through drawdown analysis*. *Investment Insights [Online]*. Wilmington Trust Corporation. Available at: https://www.wilmingtontrust.com/repositories/wtc_sitecontent/PDF/Understanding-investment-risk-through-drawdown-analysis.pdf (Accessed: 27 April 2021).

CHAPTER EIGHT

EQUITY MARKET PROFILES

8.1 INTRODUCTION

The three equity markets differ in size and number of listings. The Toronto Stock Exchange (TSX), the developed market, is the largest of the markets included in this study. The TSX Venture Exchange (TSXV), the venture market, has the most listings. The Johannesburg Stock Exchange (JSE), the emerging market, has a lower number of listings but a market capitalisation per listing comparable to that of the TSX.

The customised model generated sets of positive, negative, neutral, and false cycles unique to each market. The focus in this chapter is on the positive cycles as these cycles ultimately drive the performance of the momentum index. It will be shown that the Venture index (TSXV-MI) outperforms the other two momentum indices despite recording less favourable statistics per average hold overall. The Venture Exchange (TSXV) has a lower rate of positive cycles at a shorter average hold but with significantly higher compound returns.

The analysis will show that a cycle must generally hold for a minimum number of months to exit positive, and that performance declines as entry prices increase beyond certain levels. In general, momentum should favour lower-priced stocks as small absolute changes translate to large relative changes when working from low base values. Some sectors, regardless of size and activity, may prove more disposed to momentum than other sectors.

The different entry zones are expected to deliver contrasting results per exchange and category. The results would indicate if any zones dominated on a particular exchange or in general. In addition, a comparison of the average parameter scores may indicate if there are statistically significant differences between the three exchanges. The average parameter scores for each cycle type per market could identify a combination of high and low scores most likely to deliver positive cycles.

The custom indices quantify the actual performance of the customised model in each market and allow a direct comparison between them to complete the momentum profiles for these equity markets. The index levels and member numbers per update could indicate the state of momentum in a particular market and period. A correlation analysis may point to changes in the co-movement of the indices in up or down markets, while cointegration would confirm a longer-term association between indices.

A drawdown analysis highlights the potential for sudden large losses in value and the estimated time to recover from these losses, to also compare the momentum indices on a risk-adjusted basis. Summary statistics, besides providing basic statistical information on each index, describe and compare their respective distributions regarding symmetry and extreme returns or outliers.

8.2 EQUITY MARKETS

The Johannesburg Stock Exchange (JSE) proxies for an emerging market, South Africa. The Toronto Stock Exchange (TSX) is the main exchange in Canada, a developed market, and the TSX Venture Exchange (TSXV) represents a venture market. Table 8.1 below shows the number of ordinary shares listed per sector on the three stock exchanges at the end of 2021.

Table 8.1 Market size

31Dec 2021	JSE			TSX			TSXV		
	#	US$B	%	#	US$B	%	#	US$B	%
CO	10	282.77	23.1	31	166.0	5.4	42	16.2	20.3
CD	35	103.61	8.5	49	106.8	3.5	29	0.9	1.1
CS	31	251.44	20.5	58	184.2	6.0	41	1.4	1.7
EN	6	4.36	0.4	99	368.4	12.1	120	5.3	6.7
FI	53	132.31	10.8	72	865.6	28.4	134	1.1	1.5
HC	9	14.01	1.1	51	21.9	0.7	70	2.7	3.3
IN	30	10.95	0.9	65	313.6	10.3	61	2.3	2.9
MA	49	411.22	33.5	232	472.0	15.5	947	38.7	48.6
RE	14	11.17	0.9	15	26.6	0.9	21	0.9	1.1
TE	15	4.27	0.3	63	387.9	12.7	125	9.9	12.4
UT	2	0.03	0.0	24	136.2	4.5	7	0.3	0.4
MCap	254	1226.1	100.0	759	3049.2	100.0	1597	79.7	100.0
TTen	10	856.2	69.8	10	1004.7	32.9	10	27.0	33.9

Source: Price data downloaded from Bloomberg (2022)

The data in Table 8.1 include all common stocks (ordinary shares), with a limited number of depository receipts (issued by banks to represent common stocks), actively traded on the respective exchanges at the end of 2021. The momentum model used the same criteria (i.e., common stocks) to identify candidate listings.

The Toronto Stock Exchange (TSX) with 759 listings at a market capitalisation of 3.05 trillion US dollars, is the largest of the three exchanges. Financials is the largest sector in terms of market capitalisation (28%) while Materials have the most listing at 232 on the senior Canadian exchange.

The Johannesburg Stock Exchange (JSE) is a more concentrated market with the top 10 companies accounting for 70% of its total market capitalisation (1.23 trillion US dollars) with 254 stocks listed at the end of 2021. A small number of large and mega capitalisation companies dominate the South African market. Materials is the largest sector in terms of market capitalisation (34%) while Financials has the most listings at 53. A relatively small sector in listings, Communications, has the second largest market capitalisation (23%) on this exchange.

The TSX Venture Exchange (TSXV) with the most listings at 1597 had a total market capitalisation of 80 billion US dollars at the end of 2021. Materials is the largest sector in market capitalisation (49%) and listings (947).

8.3 MOMENTUM MODEL OUTCOMES

The outcomes per cycle type (positive, negative, neutral, and false) are presented in Table 8.2, which shows the average hold (AH), compound return per average hold (CRpAH), and the compound annual return per average hold (CARpAH) for each of the three equity markets. As defined: positive and negative cycles would hold at least 3 months while respectively gaining and losing more than 10% (annualised) in value. Neutral cycles also hold a minimum of 3 months but gain or lose a maximum of 10% (annualised) in value, while false cycles hold shorter than 3 months.

The Johannesburg Stock Exchange (JSE) recorded the highest rate of positive cycles (45%) at an average hold of almost 13 months and an annual return of close to 40%. The Toronto Stock Exchange (TSX) recorded a lower rate of positive cycles (39%) at a similar average hold but at a higher annual return of 44%. The TSX Venture Exchange (TSXV) registered the lowest rate of positive cycles (30%) at the shortest average hold (11 months) but at the highest annual return of almost 78%.

Table 8.2 Outcomes

JSE	Cycles	%	Tickers	AH	CRpAH	CARpAH
False	55	7.8	48	1.58	-7.50	-44.66
Neutral	121	17.3	91	7.64	0.32	0.51
Negative	212	30.2	140	5.18	-15.34	-32.02
Positive	313	44.7	182	12.97	43.45	39.61
ALL	701	100.0	247	8.80	11.09	15.42
TSX	**Cycles**	**%**	**Tickers**	**AH**	**CRpAH**	**CARpAH**
False	208	8.9	196	1.51	-12.60	-65.71
Neutral	461	19.7	351	8.06	0.42	0.62
Negative	758	32.5	524	5.70	-20.99	-39.12
Positive	908	38.9	604	12.99	48.88	44.44
ALL	2335	100.0	916	8.62	6.94	9.78
TSXV	**Cycles**	**%**	**Tickers**	**AH**	**CRpAH**	**CARpAH**
False	78	13.4	75	1.55	-15.24	-72.18
Neutral	60	10.3	58	8.97	0.33	0.44
Negative	269	46.3	228	5.78	-29.76	-51.98
Positive	174	30.0	150	11.13	70.31	77.58
ALL	581	100.0	412	7.14	-2.56	-4.27

Source: Price data downloaded from Bloomberg (2022)

Note that the senior exchange (TSX) in Canada recorded the highest rate of neutral cycles (20%), with its junior exchange (TSXV) the lowest (10%) while recording the highest rates of negative (46%) and false cycles (13%). The South African exchange (JSE) registered the lowest rate of negative cycles (30%) at the shortest average hold of 5 months. Overall, the Johannesburg Stock Exchange has the longest average hold (8.80 months) at the highest annual return (15.42%).

8.3.1 Holding periods

Most cycles fall in the 3-to-11-month holding range, but Table 8.3 shows that a cycle must generally hold longer than 9 months to exit as positive. Both the Johannesburg Stock Exchange (JSE) and the Toronto Stock Exchange (TSX) recorded an increasing annual return along with an increase in the average hold. The 12-to-17-month range for these two stock exchanges outperformed when the results are viewed jointly in terms of total cycles, the rate of positive cycles (90-95%), and the annual return per average hold (38-46%).

While generating relatively many positive cycles, the 9-to-11-month range for the TSX Venture Exchange (TSXV) underperformed the shorter ranges in annual return. The 12-to-17-month range outperformed when viewing its rate of positive cycles (86%) in combination with its annual return per average hold (89%).

Table 8.3 Average hold

JSE	Cycles	%	Positive	%	AH	CRpAH	CARpAH
1-2	55	7.8	---	---	---	---	---
3-5	168	24.0	20	11.9	3.95	13.40	46.54
6-8	177	25.3	44	24.9	7.16	22.06	39.67
9-11	150	21.4	103	68.7	10.10	24.44	29.67
12-17	98	14.0	93	94.9	14.00	46.05	38.35
18-23	31	4.4	31	100.0	20.03	83.89	44.04
24+	22	3.1	22	100.0	32.00	211.84	53.19
ALL	701	100.0	313	44.7	12.97	43.45	39.61
TSX	**Cycles**	**%**	**Positive**	**%**	**AH**	**CRpAH**	**CARpAH**
1-2	208	8.9	---	---	---	---	---
3-5	492	21.1	62	12.6	4.13	18.18	62.50
6-8	649	27.8	111	17.1	7.30	21.89	38.48
9-11	500	21.4	284	56.8	10.07	28.28	34.55
12-17	312	13.4	278	89.1	13.98	55.32	45.93
18-23	113	4.8	112	99.1	19.89	90.45	47.49
24+	61	2.6	61	100.0	28.72	184.27	54.73
ALL	2335	100.0	908	38.9	12.99	48.88	44.44
TSXV	**Cycles**	**%**	**Positive**	**%**	**AH**	**CRpAH**	**CARpAH**
1-2	78	13.4	---	---	---	---	---
3-5	165	28.4	25	15.2	3.88	42.41	198.46
6-8	157	27.0	29	18.5	7.07	46.71	91.68
9-11	109	18.8	56	51.4	10.13	36.87	45.06
12-17	50	8.6	43	86.0	14.02	110.63	89.17
18-23	17	2.9	16	94.1	20.19	159.54	76.28
24+	5	0.9	5	100.0	28.20	378.64	94.70
ALL	581	100.0	174	30.0	11.13	70.31	77.58

Source: Price data downloaded from Bloomberg (2022)

Overall, the Venture Exchange (TSXV) has a lower rate of positive cycles (30%) at a shorter average hold (11 months) but a higher compound annual return (78%).

8.3.2 Price ranges

Johannesburg Stock Exchange (JSE): The R10-R25 range outperformed in positive cycles (53%) with the highest compound return (59%) due to the longest average hold (16 months). The below-R5 range recorded a similar positive-cycle rate (52%) at a higher compound annual return (44%) due to the shorter average hold (12 months). The R5-R10 range generated a 40% positive-cycle rate and registered the highest compound annual return (47%) at an average hold of 13 months.

Table 8.4 Price range activity

JSE	Cycles	%	Positive	%	AH	CRpAH	CARpAH
R < 5	127	18.1	66	52.0	11.70	42.95	44.28
5 =< R < 10	70	10.0	28	40.0	13.14	52.89	47.35
10 =< R < 25	112	16.0	59	52.7	15.90	59.11	41.98
25 =< R < 50	107	15.3	49	45.8	13.47	45.32	39.52
50 =< R < 100	102	14.5	46	45.1	12.87	43.64	40.17
100 =< R < 200	100	14.3	39	39.0	10.69	24.52	27.90
200 =< R < 500	63	9.0	19	30.2	12.95	32.70	29.98
500 =< R	20	2.8	7	35.0	9.71	18.01	22.70
ALL	701	100.0	313	44.7	12.97	43.45	39.61
TSX	Cycles	%	Positive	%	AH	CRpAH	CARpAH
$ < 1	174	7.5	60	34.5	13.95	91.50	74.88
1 =< $ < 2	187	8.0	84	44.9	12.98	73.17	66.17
2 =< $ < 5	376	16.1	164	43.6	13.12	57.30	51.35
5 =< $ < 10	395	16.9	164	41.5	13.88	50.82	42.64
10 =< $ < 20	491	21.0	176	35.8	13.54	42.84	37.16
20 =< $ < 50	456	19.5	170	37.3	11.66	30.17	31.18
50 =< $ < 100	162	7.0	56	34.6	12.41	36.40	35.01
100 =< $	94	4.0	34	36.2	11.09	32.58	35.69
ALL	2335	100.0	908	38.9	12.99	48.88	44.44
TSXV	Cycles	%	Positive	%	AH	CRpAH	CARpAH
$ < 0.5	28	4.8	10	35.7	15.00	225.55	157.09
0.5 =< $ < 1	122	21.0	37	30.3	11.32	79.97	86.39
1 =< $ < 1.5	89	15.3	30	33.7	12.50	83.08	78.70
1.5 =< $ < 2	70	12.1	21	30.0	9.90	62.61	80.22
2 =< $ < 3	74	12.7	23	31.1	10.48	57.96	68.81
3 =< $ < 5	84	14.5	29	34.5	9.14	60.17	85.63
5 =< $ < 10	63	10.8	17	27.0	11.24	31.46	33.93
10 =< $	51	8.8	7	13.7	12.43	31.68	30.43
ALL	581	100.0	174	30.0	11.13	70.31	77.58

Source: Price data downloaded from Bloomberg (2022)

Toronto Stock Exchange (TSE): The \$1-\$2 range outperformed in positive cycles (45%), registering a compound return of 73% at an average hold of 13 months. The below-\$1 range recorded a lower positive-cycle rate (35%) at higher compound (92%) and compound annual (75%) returns from the longest average hold (14 months).

TSX Venture Exchange (TSXV): Most cycles fall in the $0.50-$1.00 range with a relatively low positive-cycle rate of 30% while recording the second-highest compound annual rate of 86% at an average hold of 11 months. The longest average hold is 15 months from the below-$0.50 range with the highest rate of positive cycles as well as the highest compound returns. However, the below-$0.50 range registered the lowest number of momentum cycles. Based on its positive-cycle rate (35%) and compound annual return of 86%, due to the shortest average hold of 9 months, the $3-$5 range performed comparatively well.

Performance declines as entry prices increase beyond certain levels, depending on the stock exchange. These levels appear to be R100 (JSE), $50 (TSX), and $5 (TSXV). In general, momentum favours lower-priced stocks as small absolute changes translate to large relative changes when working from low base values.

8.3.3 Sectors

The Materials, Financials, Consumer Discretionary, and Consumer Staples sectors were the most active in generating momentum cycles on the Johannesburg Stock Exchange (JSE). Among these active sectors, Consumer Discretionary outperformed with a positive-cycle rate of 51% and a compound annual return of 40%. Health Care registered the highest positive-cycle rate (62%) but underperformed in compound returns per average hold. Technology, a less active sector, with a 37% positive-cycle rate recorded the longest average hold (17 months) with the highest compound return (88%). Energy with only 5 cycles but a 60% positive-cycle rate recorded the highest compound annual return at the shortest average hold (10 months).

The Materials, Energy, and Industrials sectors were the most active in generating momentum cycles on the Toronto Stock Exchange (TSX). Among these sectors, Materials outperformed with the lowest positive-cycle rate (34%) at an average hold of 13 months. Consumer Discretionary and Consumer Staples share the highest rate for positive cycles (43%). Health Care, a relatively active sector, outperformed in compound (71%) and compound annual (56%) returns at an average hold of 14.5 months. Technology with a higher positive-cycle rate and longer average hold underperformed both Materials and Health Care in compound return per average hold.

The Materials sector dominated activity on the TSX Venture Exchange (TSXV) and performed well at a low positive-cycle rate of 29%. Industrials recorded the lowest positive-cycle rate (17%) but outperformed in average hold (14 months) and, as a result, compound return (120%). Consumer Staples outperformed at the lowest average hold (9 months) and, as a result, compound annual return (144%). Health Care and Technology were among the outperforming sectors while Communications recorded the lowest compound return at a relatively long average hold (13 months).

Table 8.5 Sector activity

JSE	Cycles	%	Positive	%	AH	CRpAH	CARpAH
Communications	29	4.1	14	48.3	13.71	49.15	41.88
C.Discretionary	102	14.5	52	51.0	13.56	46.29	40.03
C.Staples	102	14.5	48	47.1	12.10	32.36	32.04
Energy	5	0.7	3	60.0	9.67	50.36	65.92
Financials	114	16.3	50	43.9	13.82	45.95	38.86
Health Care	26	3.7	16	61.5	12.69	32.93	30.89
Industrials	86	12.3	36	41.9	11.61	38.53	40.05
Materials	180	25.7	73	40.6	12.49	43.67	41.63
Real Estate	22	3.2	8	36.4	13.75	45.90	39.05
Technology	35	5.0	13	37.1	16.92	88.03	56.48
Utilities	---	---	---	---	---	---	---
ALL	701	100.0	313	44.7	12.97	43.45	39.61
TSX	**Cycles**	**%**	**Positive**	**%**	**AH**	**CRpAH**	**CARpAH**
Communications	88	3.8	37	42.0	13.08	44.73	40.37
C.Discretionary	186	7.9	80	43.0	14.46	45.79	36.72
C.Staples	135	5.8	58	43.0	13.19	43.00	38.46
Energy	373	16.0	157	42.1	12.23	50.26	49.12
Financials	199	8.5	81	40.7	11.60	27.30	28.35
Health Care	133	5.7	45	33.8	14.42	71.02	56.28
Industrials	282	12.1	109	38.7	12.97	44.73	40.77
Materials	658	28.2	222	33.7	13.04	61.08	55.09
Real Estate	44	1.9	23	52.3	13.87	45.62	38.43
Technology	155	6.6	64	41.3	14.81	57.38	44.40
Utilities	82	3.5	32	39.0	9.44	18.11	23.58
ALL	2335	100.0	908	38.9	12.99	48.88	44.44
TSXV	**Cycles**	**%**	**Positive**	**%**	**AH**	**CRpAH**	**CARpAH**
Communications	11	1.9	3	27.3	12.67	18.58	17.52
C.Discretionary	10	1.7	4	40.0	11.50	67.38	71.18
C.Staples	13	2.3	7	53.8	9.29	99.52	144.15
Energy	93	16.0	19	20.4	10.11	56.85	70.66
Financials	13	2.3	5	38.5	9.80	44.92	57.50
Health Care	32	5.5	13	40.6	10.54	70.01	82.99
Industrials	47	8.1	8	17.0	14.25	120.33	94.49
Materials	298	51.3	86	28.9	11.42	80.04	85.52
Real Estate	10	1.7	6	60.0	8.00	33.54	54.31
Technology	52	8.9	23	44.2	11.52	49.91	52.46
Utilities	2	0.3	---	---	---	---	---
ALL	581	100.0	174	30.0	11.13	70.31	77.58

Source: Price data downloaded from Bloomberg (2022)

Note that the TSX Venture Exchange (TSX) registered shorter average holds for most sectors, and overall, but at higher compound annual returns per average holds. The worst-performing sectors in terms of compound return per average hold were Consumer Staples (JSE), Financials (TSX) and Communications (TSXV).

8.3.4 Entry zones

An entry zone, three successive formation periods, identifies and confirms a momentum cycle in progress. The earliest entry (i.e., shortest formation) with potentially the longest hold should occur in Zone 1. The stepped pattern of a regular momentum profile exits each cycle as late as possible. Zones 2 to 4 allow for later entries and more irregular patterns or individual profiles.

Table 8.6 Results per entry zone

JSE	Cycles	%	Positive	%	AH	CRpAH	CARpAH
Zone 1	269	38.4	122	45.4	13.18	45.47	40.67
Zone 2	142	20.2	64	45.1	13.81	45.42	38.45
Zone 3	135	19.3	60	44.4	12.57	43.43	41.12
Zone 4	155	22.1	67	43.2	12.16	38.03	37.44
ALL	701	100.0	313	44.7	12.97	43.45	39.61
TSX	Cycles	%	Positive	%	AH	CRpAH	CARpAH
Zone 1	837	35.8	320	38.2	13.84	48.18	40.64
Zone 2	490	21.0	175	35.7	13.96	55.61	46.25
Zone 3	497	21.3	206	41.4	12.13	44.37	43.80
Zone 4	511	21.9	207	40.5	11.70	48.94	50.47
ALL	2335	100.0	908	38.9	12.99	48.88	44.44
TSXV	Cycles	%	Positive	%	AH	CRpAH	CARpAH
Zone 1	182	31.3	52	28.6	10.79	54.67	62.44
Zone 2	121	20.8	38	31.4	11.45	65.85	69.95
Zone 3	129	22.2	38	29.5	12.26	91.82	89.15
Zone 4	149	25.7	46	30.9	10.30	75.96	93.11
ALL	581	100.0	174	30.0	11.13	70.31	77.58

Source: Price data downloaded from Bloomberg (2022)

Refer to Table 8.6: Most cycles were entered in Zone 1 and, as a result, this zone also generated the greatest number of positive cycles for each stock exchange. The highest positive-cycle rate per stock exchange is respectively from Zone 1 (JSE: 45%), Zone 3 (TSX: 41%), and Zone 2 (TSXV: 31%).

The longest average hold for positive cycles is 14 months from Zone 2 entries for both the JSE and the TSX while positive cycles on the TSXV hold the longest when entered in Zone 3, on average lasting 12 months. The average hold across all zones per stock exchange is 13 months for the JSE and the TSX, and 11 months for the TSXV.

Compound returns, which favour longer average holds, are highest in Zone 1 for the JSE (45.5%), Zone 2 for the TSX (55.6%), and Zone 3 for the TSXV (91.8%). Compound annual returns, which favour shorter average holds, are highest in Zone 3 for the JSE (41.1%) and Zone 4 for both the TSX (50.5%) and TSXV (93.1%).

8.3.5 Parameter scores

The customised model identified 701 (JSE), 2335 (TSX), and 581 (TSXV) individual momentum cycles with the [20|1.5|48|35] parameter setting combination. The average scores for each parameter per stock exchange – which resulted in false, neutral, negative, or positive cycles – are included in Table 8.7 below.

Table 8.7 Average parameter scores

CYCLES	Momentum scores			Volatility scores		
	JSE	TSX	TSXV	JSE	TSX	TSXV
False	25.70	23.55	33.64	0.60	0.77	1.19
Neutral	26.35	28.16	38.58	0.57	0.71	1.35
Negative	26.29	33.67	49.68	0.61	0.84	1.29
Positive	28.95	33.29	46.32	0.62	0.81	1.23
Average	27.44	31.35	45.37	0.61	0.80	1.26

CYCLES	Quality scores			Activity scores		
	JSE	TSX	TSXV	JSE	TSX	TSXV
False	52.99	52.43	51.43	48.47	47.41	42.45
Neutral	54.34	53.83	52.99	48.69	49.46	41.96
Negative	53.80	53.35	52.68	48.54	48.39	41.35
Positive	53.89	53.37	52.82	48.15	48.13	41.66
Average	53.87	53.37	52.58	48.39	48.41	41.65

Source: Price data downloaded from Bloomberg (2022)

Overall, the average momentum scores for the stock exchanges are statistically different at a 5% level (refer to Annexure D). The lowest average momentum score on entry is 27.4 (JSE) and the highest is 45.4 (TSXV). Within these overall scores, the highest scores were recorded by positive cycles (JSE) and negative cycles (TSX and TSXV) – refer to chapters 5 to 7 for a comparison of the different cycle types.

The average volatility scores for the stock exchanges are statistically different at a 5% level. The lowest average volatility score on entry is 0.6 (JSE) and the highest is 1.3 (TSXV). Within these overall scores, the highest scores were recorded by positive cycles (JSE), negative cycles (TSX), and neutral cycles (TSXV).

The average quality scores for the stock exchanges are statistically different at a 5% level. The lowest average quality score on entry is 52.6 (TSXV) and the highest is 53.9 (JSE). Within these overall scores, the highest average score on each of the stock exchanges was recorded by the neutral cycles.

The average activity scores for the JSE/TSXV and TSX/TSXV pairings are statistically different at a 5% level. The lowest average activity score on entry is 41.7 (TSXV) and the highest is 48.4 (JSE and TSX). Within these overall scores, the highest scores were recorded by neutral cycles (JSE and TSX) and false cycles (TSXV).

Generalising the outcomes from the average parameter scores, Table 8.8 shows some similarities between the stock exchanges. Positive cycles tend to have higher momentum scores on entry, while false cycles generally have lower scores. False cycles also tend to have lower volatility scores with negative cycles recording higher scores on average. Like the generalisation for momentum scores, positive cycles tend to have higher and false cycles lower quality scores. Neutral cycles have higher activity scores on average in contrast to the lower scores for positive cycles.

Table 8.8 Generalised outcomes

Parameters	Cycles	MOMENTUM		VOLATILITY		QUALITY		ACTIVITY	
		High	Low	High	Low	High	Low	High	Low
JSE	False		⊗		⊗		⊗		X
	Neutral	X			X	X		⊗	
	Negative		X	⊗			X	X	
	Positive	⊗		X		⊗			⊗
TSX	False		⊗		⊗		⊗		X
	Neutral		X		X	X		⊗	
	Negative	X		⊗			X	X	
	Positive	⊗		X		⊗			⊗
TSXV	False		⊗		⊗		⊗	X	
	Neutral		X	X		X		⊗	
	Negative	X		⊗			X		X
	Positive	⊗			X	⊗			⊗

Therefore, based on the average parameter scores of the customised model, cycles with higher momentum and quality in combination with lower activity are more likely to be positive. False cycles registered lower momentum, volatility, and quality scores on entry relative to the other cycle types.

Table 8.9 Summary of ANOVA results

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
Momentum Score (MS)										
MS-JSE	MS-TSX	4.0898	0.3305	12.3750	8544.597	3.3140	2.9946	5.1851	0.0000	1.0953
MS-JSE	MS-TSXV	17.9344	0.5864	30.5855	5331.728	3.3140	15.9911	19.8776	0.0000	1.9432
MS-TSX	MS-TSXV	13.8445	0.5520	25.0810	4431.158	3.3140	12.0152	15.6738	0.0000	1.8293
Volatility Score (VS)										
VS-JSE	VS-TSX	0.1899	0.0075	25.2613	10876.12	3.3140	0.1650	0.2148	0.0000	0.0249
VS-JSE	VS-TSXV	0.6566	0.0179	36.7114	4275.46	3.3140	0.5974	0.7159	0.0000	0.0593
VS-TSX	VS-TSXV	0.4667	0.0176	26.4555	4082.17	3.3140	0.4082	0.5252	0.0000	0.0585
Quality Score (QS)										
QS-JSE	QS-TSX	0.4947	0.0534	9.2633	6654.76	3.3140	0.3177	0.6717	0.0000	0.1770
QS-JSE	QS-TSXV	1.2812	0.0662	19.3438	7650.66	3.3140	1.0617	1.5007	0.0000	0.2195
QS-TSX	QS-TSXV	0.7865	0.0525	14.9866	5634.56	3.3140	0.6126	0.9605	0.0000	0.1739
Activity Score (AS)										
AS-JSE	AS-TSX	0.0237	0.0850	0.2790	6579.69	3.3140	-0.2580	0.3054	0.9788	0.2817
AS-JSE	AS-TSXV	6.7343	0.0995	67.6638	7661.39	3.3140	6.4045	7.0641	0.0000	0.3298
AS-TSX	AS-TSXV	6.7580	0.0755	89.5389	6247.11	3.3140	6.5079	7.0081	0.0000	0.2501

Table 8.9: All pairings, except AS-JSE/AS-TSX (JSE and TSX activity scores), shows statistically significant differences in averages parameter scores.

8.4 DISCUSSION

In the previous five subsections, the momentum profiles for the three equity markets were compared in terms of average hold, price range activity, sector activity, outcomes per momentum zone, and the average parameter scores per cycle type. A generalisation regarding the level (high or low) of each parameter score identified a combination of scores possibly favouring positive cycles. This section discusses possible or plausible justifications and the implications of the results.

8.4.1 Holding periods

The observations regarding holding periods are aligned in terms of cycle type. False cycles are confined to shorter than 3 months by definition. Neutral cycles cluster in the 6-11-month range while negative cycles are shorter in average hold, dominating the 3-8-month range. Positive cycles, on the other hand, are predominant in the 9-17-month range. It can be concluded that momentum cycles that hold beyond 9 months generally record high positive returns. The implication being that momentum investing is a longer-term strategy, which require investors to hold stocks for at least 9 months to record a profit. Although the three exchanges exhibit a similar pattern in hold-per-cycle-type, the composition (i.e., mix of false, neutral, negative, and positive cycles) of their momentum cycles differ (refer to Subsection 8.4.4 on the next page).

8.4.2 Price ranges

The results per price range suggest that low-priced stocks are more likely to complete full momentum cycles (i.e., record positive cycles) with stocks priced below R50 (JSE), $10 (TSX) and $5 (TSXV) recording the best results on the respective exchanges. The most obvious explanation relates to smaller absolute changes translating to larger relative changes in price (i.e., momentum) when working from a low base. Low-priced stocks tend to be volatile for the same reason. These stocks are more affordable, possibly attracting many novice investors or investors with limited resources and the opportunity to earn large profits. However, low-priced stocks may trade infrequently with large fluctuations in price. High-risk investing due to volatility and illiquidity. This is especially true for the venture market (TSXV). Assuming frequent trading due to affordability, low-priced but high-volume stocks may be less volatile. A large following of novice and less-informed investors may result in the underreaction or delayed overreaction to news, the increasing stock price, thereby extending the continuation in price according to many studies on momentum (refer to Chapter 2, Section 2.3.1). The implication is that investors should target less-volatile low-priced stocks with sufficient liquidity to construct momentum portfolios. The customised model of this study used volatility and activity parameters to identify liquid, low-volatility, low-priced stocks with momentum.

8.4.3 Sectors

The Consumer Discretionary sector outperformed all the other active sectors on the Johannesburg Stock Exchange (JSE). Among the less active sectors, Technology and Health Care outperformed. Health Care generated the highest rate of positive cycles, while Technology recorded the best results overall. Materials, which includes the Metals and Mining industries, recorded the highest rate of false cycles and the worst performance overall on the JSE. The large Technology sector and the small Real Estate sector outperformed on the Toronto Stock Exchange (TSX). Materials as the most active sector produced average results but also generated the most negative cycles, outnumbering its positive cycles. Utilities, Financials and Health Care were the worst-performing sectors on the TSX. On the TSX Venture Exchange (TSXV), most of the momentum cycles originated in the Materials sector and the Energy sector. Cycles from these two sectors also accounted for most of the negative cycles in this market. The Technology, Health Care, Consumer Discretionary, and Consumer Staples sectors generally favoured positive outcomes.

Technology outperformed on all three exchanges, while the Consumer Discretionary, Health Care, and Real Estate sectors recorded mixed result. Materials, being the most active sector on the exchanges, recorded average results overall. Activity, and therefore opportunity or availability, plays an obvious role in identifying momentum stocks. The results do show that investors can target certain sectors and avoid others when selecting between momentum stocks for their portfolios.

8.4.4 Stock exchanges

Table 8.2 (page 8-3) shows the composition of each market’s momentum cycles. The Johannesburg Stock Exchange (JSE) recorded the highest rate of positive cycles. The Toronto Stock Exchange (TSX) recorded a lower rate of positive cycles at a similar average hold but at a higher annual return. The TSX Venture Exchange (TSXV) registered the lowest rate of positive cycles at the shortest average hold but at the highest annual return. The TSXV generated the highest rate of false and negative cycles, possibly pointing to the volatility in this venture market. The parameter settings (calibrated on the JSE) can be changed to adapt to a particular market. The Toronto Stock Exchange (TSX) should be able to handle a lower volatility setting in combination with higher quality and activity score minimums to possibly reduce its high rate of false and neutral cycles. The results, however, do indicate that the size and the maturity of a market affect the composition of momentum cycles, average holds and returns per average hold.

In the next section, the relative performance of each stock exchange’s momentum index completes the profiles for these equity markets. The results are presented graphically and evaluated in terms of performance, correlation, cointegration, drawdown, and descriptive statistics.

8.5 MOMENTUM INDEX

All stocks (tickers) identified by the customised model are included in the momentum index. The index is updated monthly when any new members are added to the index and those at the end of their cycles are deleted from the index. The base date for the index is 31 December 2008, and the base or starting value is 100. Unlike a true unweighted or equal-weighted design, all weights do not reset to the average weight when updated. Any new members hold the average weight after updating but the current members largely maintain their weights (momentum), depending on the number of additions and the total weight of any deletions. The methodology of the index (refer to Chapter 3), retaining the momentum of the remaining members, may account for the outperformance of the momentum indices to some degree. A variable number of members in combination with more frequent updating allows for a relatively active approach to benchmarking momentum in an equity market.

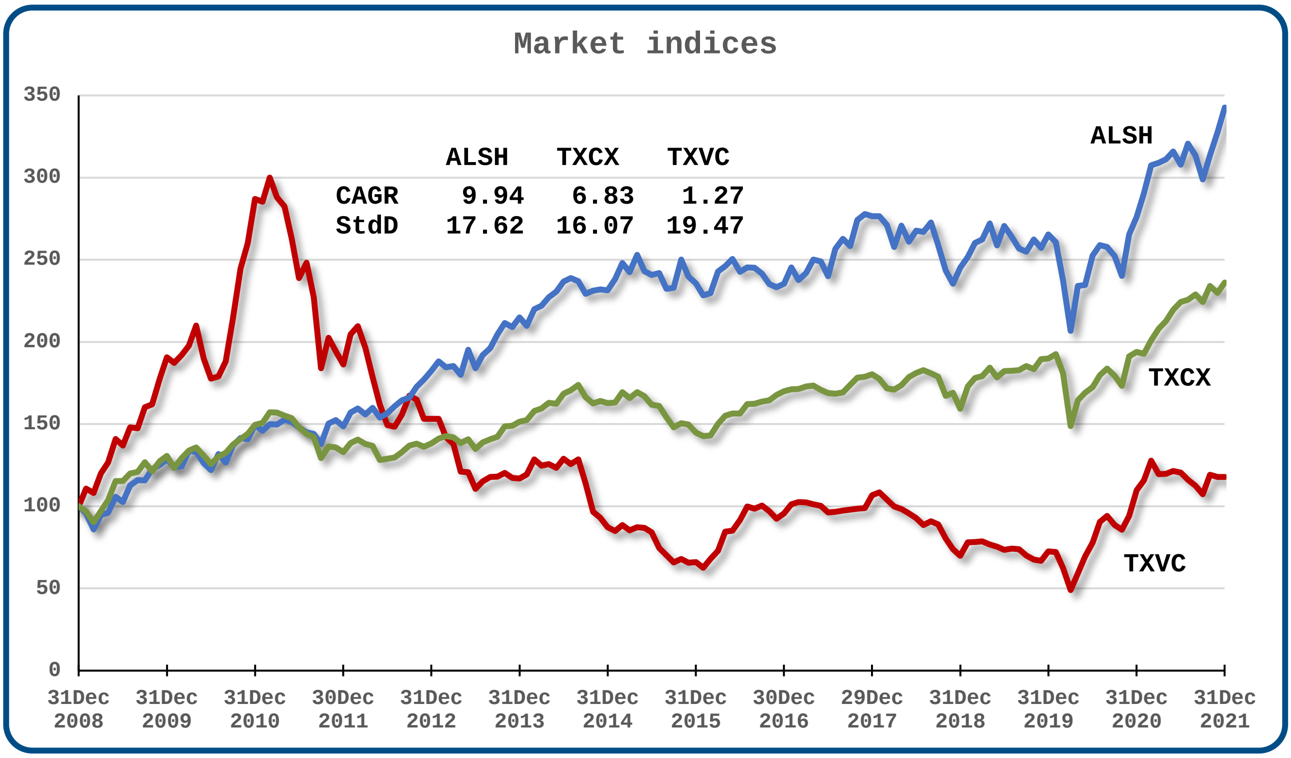

Figure 8.1 Market indices (Source of index level data: Bloomberg 2022)

Figure 8.1 shows the levels of the equity markets, represented by their respective market indices, over the 13-year period from 2009 to 2021. The FTSE/JSE All Share Index (ALSH) recorded a compound annual growth rate (CAGR) of 9.94% at a volatility (StdD) of 17.62% per annum over this period. The S&P/TSX Composite Index (TXCX) recorded a lower compound annual growth rate (CAGR) of 6.83% at a lower volatility (StdD) of 16.07% per annum. The S&P/TSX Venture Composite Index (TXVC) recorded the lowest compound annual growth rate (CAGR) of 1.27% at a higher volatility (StdD) of 19.47% per annum.

Compare the graphs for the market indices in Figure 8.1 above to the graphs of their corresponding momentum indices in Figure 8.2 on the next page. Note that the Venture market generated both the worst (market) and the best (momentum) rate of growth.

8.5.1 Levels and members

The momentum indices are compared graphically in Figure 8.2 below, showing parallel declines from January to March 2020, also recovering in unison as the markets rebounded. The three indices started from similar levels on 31 March 2020 with the Johannesburg index (JSE-MI) at 419.87, the Toronto index (TSX-MI) at 443.34, and the Venture index (TSXV-MI) at 421.50.

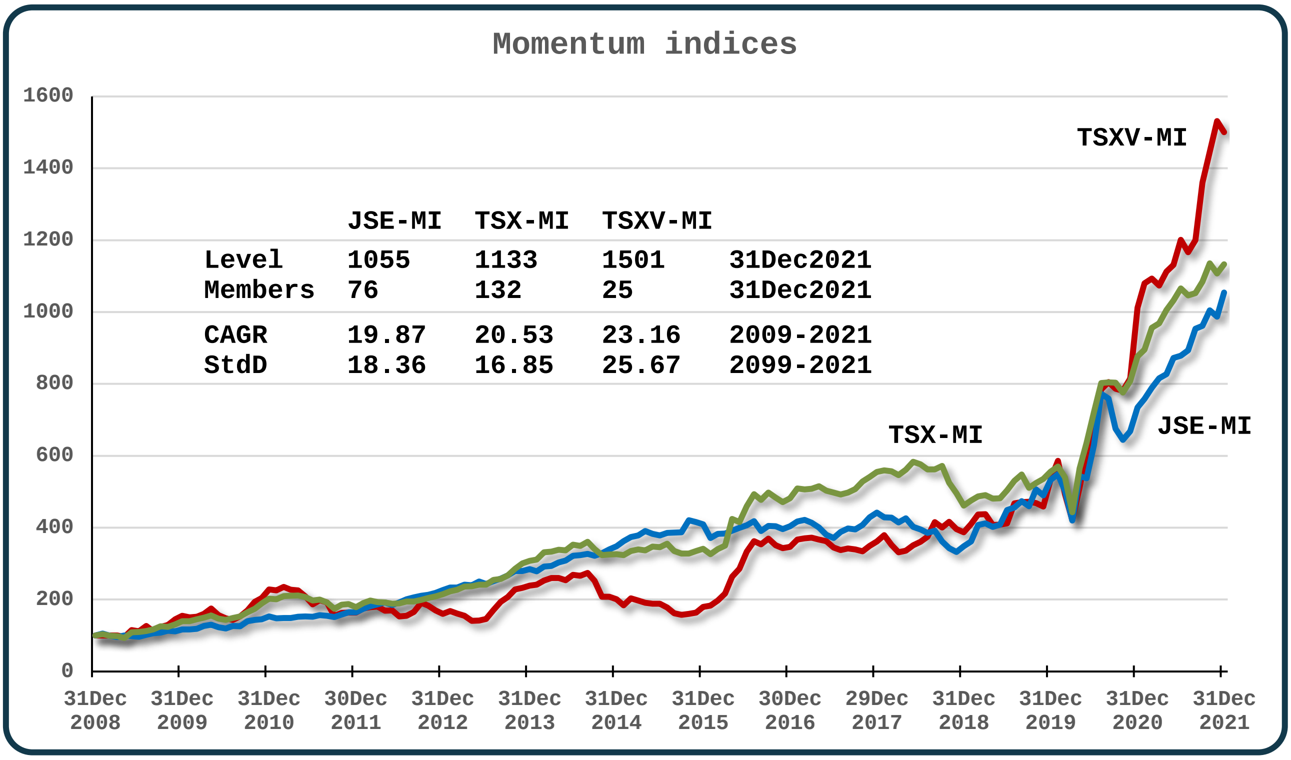

Figure 8.2 Momentum indices (Source of price data: Bloomberg 2022)

Table 8.10, on the next page, shows the yearly activity from 2009 to 2021. As stated, the number of members is variable with indices updated monthly when stocks are added and deleted based on the results from the momentum model. An increasing number from year to year normally coincides with increasing index levels. The end-of-year number relative to the average number of members for that year is indicative of the upward or downward trend of an index level at year-end. The ratio of additions to total activity (additions plus deletions) may give an indication of the sentiment in the market for that year, a sentiment that also reflects in the index level.

The short but steep decline in early 2020 followed the steady recovery during 2019. Referring to Table 8.10, during 2019 all three indices recorded large increases with year-end members exceeding the average members for that year. The additions ratio for each index also exceeded 0.5, indicating positive sentiments in all three markets at the end of 2019. This trend continued in 2020, apart from the sudden decline in levels from January to March. At the end of 2021, after achieving record levels, the situation changed somewhat with additions ratios below 0.5 and year-end members below their averages for the Toronto and Venture indices.

Table 8.10 Updating 2009-2021

Year	Index	LEVEL	GROWTH	MEM	AVG	ADD	DEL	ADD/T
2009	JSE-MI	117.30	17.30 %	88	33	95	8	0.92
	TSX-MI	139.93	39.93 %	242	94	262	24	0.92
	TSXV-MI	155.12	55.12 %	33	11	38	5	0.88
2010	JSE-MI	152.82	30.28 %	49	57	44	83	0.35
	TSX-MI	202.23	44.52 %	255	232	282	269	0.51
	TSXV-MI	228.41	47.25 %	58	36	75	50	0.60
2011	JSE-MI	164.36	7.55 %	15	35	30	64	0.32
	TSX-MI	179.07	-11.45 %	37	195	151	369	0.29
	TSXV-MI	163.48	-28.43 %	13	53	64	109	0.37
2012	JSE-MI	226.63	37.89 %	62	53	75	28	0.73
	TSX-MI	214.86	19.98 %	82	72	145	100	0.59
	TSXV-MI	159.85	-2.22 %	16	16	39	36	0.52
2013	JSE-MI	285.04	25.77 %	53	61	64	73	0.47
	TSX-MI	308.10	43.40 %	192	150	228	118	0.66
	TSXV-MI	238.47	49.19 %	24	18	31	23	0.57
2014	JSE-MI	348.84	22.38 %	45	48	52	60	0.46
	TSX-MI	326.20	5.87 %	75	186	184	301	0.38
	TSXV-MI	201.11	-15.67 %	11	30	40	53	0.43
2015	JSE-MI	409.27	17.33 %	30	46	51	66	0.44
	TSX-MI	341.38	4.65 %	39	80	100	136	0.42
	TSXV-MI	179.78	-10.61 %	5	10	18	24	0.43
2016	JSE-MI	404.33	-1.21 %	45	30	61	46	0.57
	TSX-MI	481.40	41.02 %	183	125	241	97	0.71
	TSXV-MI	346.45	92.71 %	46	34	65	24	0.73
2017	JSE-MI	442.31	9.39 %	26	25	40	59	0.40
	TSX-MI	554.90	15.27 %	78	110	121	226	0.35
	TSXV-MI	361.68	4.40 %	26	27	39	59	0.40
2018	JSE-MI	348.64	-21.18 %	7	23	35	54	0.39
	TSX-MI	461.75	-16.79 %	29	72	98	147	0.40
	TSXV-MI	387.51	7.14 %	16	22	32	42	0.43
2019	JSE-MI	533.27	52.96 %	22	19	33	18	0.65
	TSX-MI	556.03	20.42 %	100	72	157	86	0.65
	TSXV-MI	534.98	38.06 %	25	21	42	33	0.56
2020	JSE-MI	734.94	37.82 %	31	16	31	22	0.58
	TSX-MI	876.72	57.67 %	168	89	177	109	0.62
	TSXV-MI	1012.18	89.20 %	47	34	55	33	0.63
2021	JSE-MI	1054.60	43.49 %	76	76	94	49	0.66
	TSX-MI	1132.81	29.21 %	132	213	187	223	0.46
	TSXV-MI	1500.74	48.27 %	25	38	46	68	0.40

Source: Price data downloaded from Bloomberg (2022)

Note the rebounds after poorly performing years, specifically 2015 (TSXV) and 2018 (all indices). Also, note the year-end versus average members and additions ratios for 2021.

8.5.2 Relative performance

A comparison of the growth and volatility of the momentum indices focuses on the relative performance of the model in each equity market. The correlations per period and for each year show the changing associations between the indices along with the variations in performance over time.

Table 8.11 Relative performance per period

Period	Index	JSE-MI	TSX-MI	TSXV-MI	CTGR	CAGR	StdD
FULL 2009 2021	JSE-MI	1.00	0.40	0.31	954.60	19.87	18.36
	TSX-MI	0.40	1.00	0.62	1032.81	20.53	16.85
	TSXV-MI	0.31	0.62	1.00	1400.74	23.16	25.67
10Y 2012 2021	JSE-MI	1.00	0.42	0.32	541.66	20.43	19.09
	TSX-MI	0.42	1.00	0.63	532.59	20.26	16.72
	TSXV-MI	0.32	0.63	1.00	818.02	24.82	24.54
5Y 2017 2021	JSE-MI	1.00	0.47	0.37	160.83	21.14	24.78
	TSX-MI	0.47	1.00	0.68	135.31	18.67	19.39
	TSXV-MI	0.37	0.68	1.00	333.18	34.07	25.80
3Y 2019 2021	JSE-MI	1.00	0.51	0.43	202.49	44.62	29.89
	TSX-MI	0.51	1.00	0.76	145.33	34.87	22.97
	TSXV-MI	0.43	0.76	1.00	287.28	57.04	29.28
1Y 2021	JSE-MI	1.00	0.41	0.32	43.49	43.49	11.69
	TSX-MI	0.41	1.00	0.69	29.21	29.21	16.29
	TSXV-MI	0.32	0.69	1.00	48.27	48.27	22.23

Source: Price data downloaded from Bloomberg (2022)

Table 8.11 above shows the performance of each index over the 3-year period from 2019 to 2021 during a recovery phase of the markets. The Venture index (TSXV) outperformed with comparable volatility. The performance of the Venture index improves as the period is shortened from 13 to 3 years, also outperforming in 2021. Figure 8.2 on page 8-14 confirms the outperformance of the Venture index during this period when it surpassed the Johannesburg and Toronto indices.

The individual years in Table 8.12 on the next page confirm 2019, 2020, and 2021 as the best years for the indices, rebounding after 2018. The Venture Exchange recorded the largest rebounds, in 2016 (93%) after two successive years of decline, and in 2020 (89%) following the mini-collapse (28% lost in three months) that same year.

The correlation between the indices increases as the period shortens from 13 to 3 years. The Canadian indices maintained a strong co-movement during each of the extended periods and for many of the individual years, notably 2011 and 2020. The South African index, generally, has a weak correlation with the Canadian indices, specifically 2017 and 2018 with 2011 and 2020 the notable exceptions.

The next section, on page 8-16, reports on the 3-year correlations between the different markets, and between the different market and momentum indices.

Table 8.12 Relative performance per annum (2009-2021)

Year	Index	JSE-MI	TSX-MI	TSXV-MI	LEVEL	CAGR	StdD
2009	JSE-MI	1.00	0.14	0.14	117.30	17.30	20.91
	TSX-MI	0.14	1.00	0.40	139.93	39.93	18.91
	TSXV-MI	0.14	0.40	1.00	155.12	55.12	37.62
2010	JSE-MI	1.00	0.46	0.35	152.82	30.28	13.22
	TSX-MI	0.46	1.00	0.74	202.23	44.52	12.90
	TSXV-MI	0.35	0.74	1.00	228.41	47.25	19.20
2011	JSE-MI	1.00	0.59	0.57	164.36	7.55	11.39
	TSX-MI	0.59	1.00	0.86	179.07	-11.45	19.29
	TSXV-MI	0.57	0.86	1.00	163.48	-28.43	27.55
2012	JSE-MI	1.00	0.36	0.20	226.63	37.89	7.61
	TSX-MI	0.36	1.00	0.50	214.86	19.98	9.97
	TSXV-MI	0.20	0.50	1.00	159.85	-2.22	25.97
2013	JSE-MI	1.00	0.35	0.19	285.04	25.77	9.49
	TSX-MI	0.35	1.00	0.49	308.10	43.40	8.62
	TSXV-MI	0.19	0.49	1.00	238.47	49.19	20.62
2014	JSE-MI	1.00	0.29	0.25	348.84	22.38	9.33
	TSX-MI	0.29	1.00	0.70	326.20	5.87	12.18
	TSXV-MI	0.25	0.70	1.00	201.11	-15.67	19.38
2015	JSE-MI	1.00	0.33	0.25	409.27	17.33	13.28
	TSX-MI	0.33	1.00	0.40	341.38	4.65	11.53
	TSXV-MI	0.25	0.40	1.00	179.78	-10.61	22.63
2016	JSE-MI	1.00	0.32	0.33	404.33	-1.21	13.08
	TSX-MI	0.32	1.00	0.67	481.40	41.02	21.50
	TSXV-MI	0.33	0.67	1.00	346.45	92.71	26.40
2017	JSE-MI	1.00	0.14	0.15	442.31	9.39	9.62
	TSX-MI	0.14	1.00	0.29	554.90	15.27	10.21
	TSXV-MI	0.15	0.29	1.00	361.68	4.40	14.69
2018	JSE-MI	1.00	0.22	0.11	348.64	-21.18	17.15
	TSX-MI	0.22	1.00	0.42	461.75	-16.79	13.78
	TSXV-MI	0.11	0.42	1.00	387.51	7.14	23.22
2019	JSE-MI	1.00	0.32	0.24	533.27	52.96	18.40
	TSX-MI	0.32	1.00	0.30	556.03	20.42	10.92
	TSXV-MI	0.24	0.30	1.00	534.98	38.06	20.04
2020	JSE-MI	1.00	0.56	0.50	734.94	37.82	47.23
	TSX-MI	0.56	1.00	0.85	876.72	57.67	34.76
	TSXV-MI	0.50	0.85	1.00	1012.18	89.20	41.10
2021	JSE-MI	1.00	0.41	0.32	1054.60	43.49	11.69
	TSX-MI	0.41	1.00	0.69	1132.81	29.21	16.29
	TSXV-MI	0.32	0.69	1.00	1500.74	48.27	22.23

Source: Price data downloaded from Bloomberg (2022)

Note 2020 was one of the best-performing years but also the most volatile, experiencing a sudden decline from January to March before continuing the rebound from 2019.

8.5.3 Correlation and cointegration

Correlation measures the degree of co-movement or strength of the linear association between two time-series. Correlation-squared (R-squared) indicates how closely an index tracks the performance of a particular benchmark. It also points to the reliability of the alpha (excess return) and beta (volatility) coefficients from a linear regression. Table 8.13 below shows the changes in correlation between the different indices for the 3-year period from 2019 to 2021 when the momentum indices led the market indices in recovering from the downturn in 2018.

Table 8.13 Correlation matrix (2019-2021)

Indices	JSE-MI	TSX-MI	TSXV-MI	ALSH	TXCX	TXVC
JSE-MI	**1.0000**	0.5136	0.4278	0.6346	0.4156	0.4547
TSX-MI	0.5136	**1.0000**	0.7561	0.5012	0.7306	0.8203
TSXV-MI	0.4278	0.7561	**1.0000**	0.4236	0.5679	0.8109
ALSH	0.6346	0.5012	0.4236	**1.0000**	0.6257	0.5074
TXCX	0.4156	0.7306	0.5679	0.6257	**1.0000**	0.6732
TXVC	0.4547	0.8203	0.8109	0.5074	0.6732	**1.0000**

Source: Price data downloaded from Bloomberg (2022)

Arbitrary limits for the strength of this association label absolute values between 0 and 0.19 as very weak, 0.20 to 0.39 as weak, 0.40 to 0.59 as moderate, 0.60 to 0.79 as strong, and 0.80 to 1 as very strong correlation.

Markets: The association between the South African market (ALSH) and Canadian markets varies between moderate (venture market, TXVC) and strong (senior market, TXCX). The correlation between the two Canadian markets is strong.

Momentum and market: The results show a strong association between the Johannesburg momentum index and its market index (ALSH). Similarly, there is a strong association between the Toronto momentum index and its market index (TXCX) but, surprisingly, a very strong correlation with the venture market index (TXVC). The Venture momentum index, as expected, has a very strong association with its market index (TXVC) during this period. Refer to Annexure D for the statistical results.

Momentum: The Johannesburg index (JSE-MI) has a moderate association with both the Toronto index (TSX-MI) and the Venture index (TSXV-MI). There is a strong correlation between the Toronto and the Venture indices in Canada.

Correlation measures between very-weak and very-strong. Cointegration, on the other hand, either exists or does not. Its strength cannot be quantified or measured. The significance test states the confidence with which statements can be made about the presence or absence of cointegration. Only the Toronto momentum index (TSX-MI) and its market index (TXCX) appear to be cointegrated, for the full 13-year period and at a 10% level of significance (refer to Table 8.16).

8.5.4 Drawdown analysis

A drawdown analysis highlights the potential for sudden large (20%-plus) losses in value and the estimated time to recover from these losses (Wilmington 2018). It records the size and speed of maximum drawdowns and the time to return to former highs.

Refer to Table 8.14: The Johannesburg index (JSE-MI) experienced its maximum drawdown in March 2020. It occurred over a period of 19 days and the index recovered within 62 days to its original high (81 days from peak to peak).

The Toronto index (TSX-MI) also experienced its maximum drawdown in March 2020. It occurred over a shorter period of 18 days and the index recovered within 40 days to its original high (58 days from peak to peak).

The Venture index (TSXV-MI) experienced its maximum drawdown at the end of August 2015 after declining for 243 consecutive days and taking another 181 days to recover to previous levels (424 days from peak to peak). Another large drawdown occurred in March 2020, like the other two indices, declining sharply and dropping 44% in value within 18 days and recovering within 57 days (75 days peak to peak).

Table 8.14 Drawdown analysis (2009-2021)

Metric	JSE-MI	TSX-MI	TSXV-MI
Maximum drawdown	40.39%	36.00%	45.95%
Date	2020-03-19	2020-03-18	2015-08-24
Period	19 days	18 days	243 days
Recovery	62 days	40 days	181 days
Average drawdown	5.63%	6.29%	14.76%
Maximum duration	362 days	431 days	709 days
From:	2018-01-10	2011-04-11	2011-03-08
To:	2019-06-24	2012-12-31	2014-01-07
Average duration	16 days	16 days	27 days
Annualised return	19.87%	20.53%	23.16%
Drawdown ratio	0.49	0.57	0.50

Source: Price data downloaded from Bloomberg (2022)

The size of a JSE-MI drawdown is 5.63% on average, lasting 16 days (peak to peak). The average drawdown for the TSX-MI is 6.29%, also lasting 16 days. The average size of a TSXV-MI drawdown is higher (14.76%) and lasts longer (27 days).

The drawdown ratio (annualised return to maximum drawdown) adjusts returns for risk (in this instance, maximum drawdown). It, therefore, compares returns on a risk-adjusted basis over the specified timeframe.

It is apparent from Table 8.14 that the Toronto index (TSX-MI) recovers more quickly from drawdowns than the other two indices. The higher drawdown ratio for this index also points to higher returns on a risk-adjusted basis. The Venture Exchange (TSXV-MI) takes longer to recover from drawdowns on average and its high return is adjusted down to equal that of the Johannesburg index (JSE-MI).

8.5.5 Descriptive statistics

Descriptive statistics, the process of describing data and presenting it graphically, provides the individual summary statistics listed in the table below. It includes the mean returns for all indices with their accompanying standard deviations. The coefficient of variation (CV), the size of the standard deviation about its mean, shows the relative variability of each index. The respective standard deviations and ranges indicate higher variability for the Venture index (TSXV-MI). Refer to figures 8.3 to 8.5 on the next page for a visual comparison of volatility.

Table 8.15 Summary statistics (2009-2021)

Metric	JSE-MI	TSX-MI	TSXV-MI
Mean	0.0725 %	0.0747 %	0.0834 %
Standard Error	0.0203 %	0.0186 %	0.0283 %
Median	0.1096 %	0.1396 %	0.0842 %
Standard Deviation	1.1545 %	1.0606 %	1.6109 %
Sample Variance	1.3328	1.1249	2.5951
Kurtosis	21.1831	14.0274	6.2724
Skewness	-0.3633	-1.1402	-0.3459
Range	25.56 %	21.49 %	24.08 %
Maximum	12.49 %	8.85 %	10.97 %
Minimum	-13.07 %	-12.64 %	-13.12 %
Sum	235.57 %	242.73 %	270.85 %
Count	3249	3248	3248
CV	15.92	14.19	19.32

Source: Price data downloaded from Bloomberg (2022)

The three sets of data are not fully symmetric but negatively or left skewed with the means (averages) smaller than the medians (middle values). A left-skewed distribution has more values in the right tail, but the left tail is longer indicating many smaller positive returns versus fewer but larger negative returns. The distributions of JSE-MI and TSXV-MI are both approximately symmetric with skewness measuring between -0.5 and 0. The distribution of TSX-MI is highly left-skewed with its value below -1.

High kurtosis values would point to heavy-tailed distributions with outliers or extreme positive and negative returns. Extreme returns can be defined as returns that exceed the 90th percentile, the top and bottom 10% of returns (Sankaran, Nguyen & Harikumar 2012). Compared to a normal distribution, described as mesokurtic, these distributions can be described as leptokurtic with excess kurtosis. Negatively skewed, heavy-tailed distributions are common in stock market data (Samunderu & Murahwa 2021). The Venture index (TSXV-MI), being more symmetric and with the lowest kurtosis, is less likely than the two other indices to record outliers and extreme negative returns. The JSE-MI has the highest kurtosis value of the three indices and is therefore likely to record more returns as outliers. The TSX-MI has a lower kurtosis than the JSE-MI, but with its highly left-skewed distribution it is more likely to record extreme negative returns.

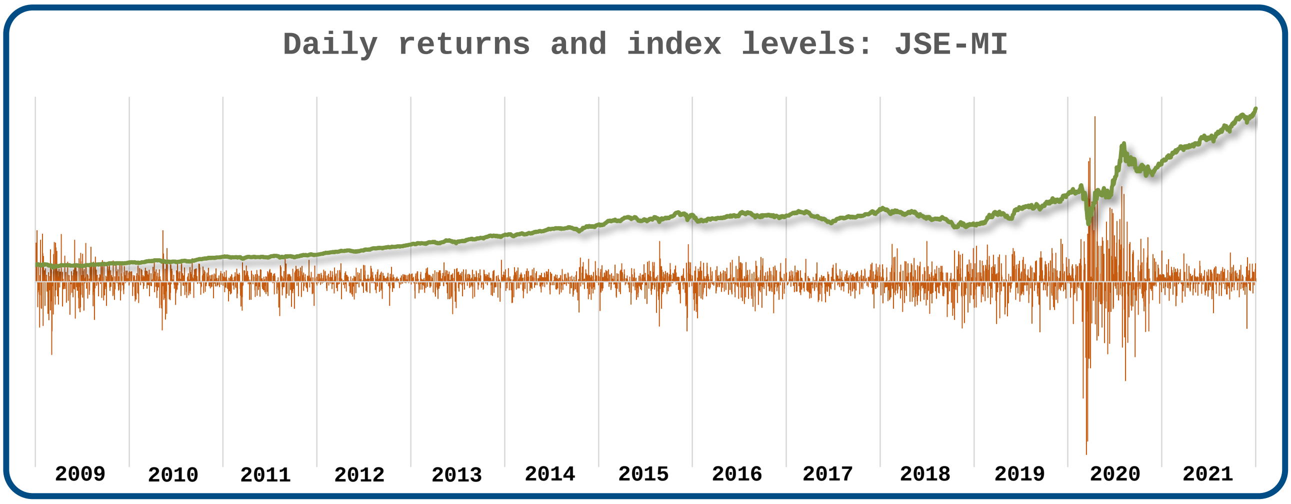

Figure 8.3 Daily returns: JSE-MI (Source of price data: Bloomberg 2022)

Note the increased volatility in 2020 for each of the indices.

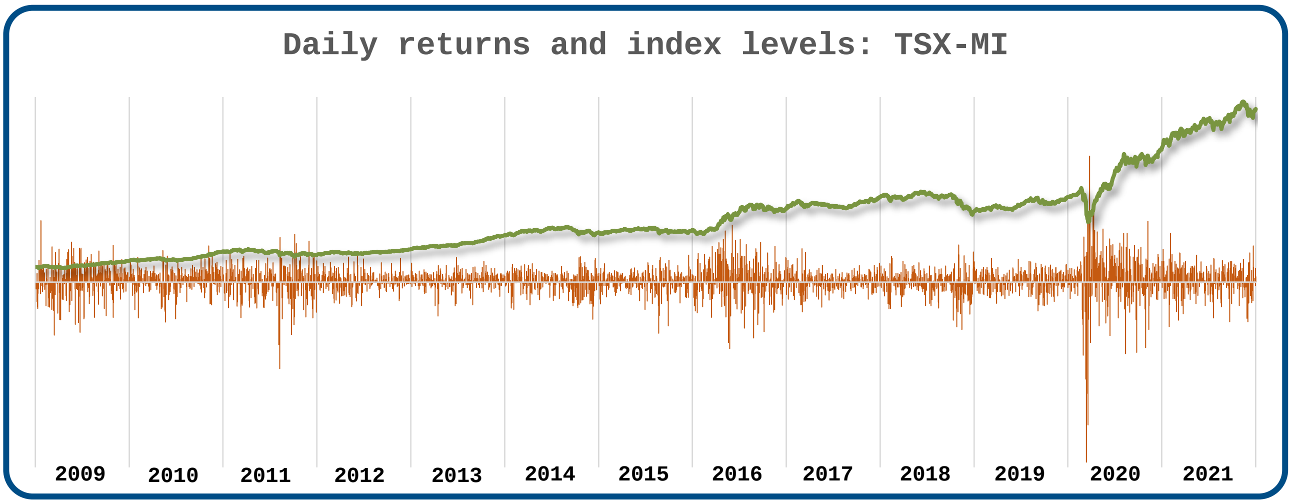

Figure 8.4 Daily returns: TSX-MI (Source of price data: Bloomberg 2022)

Note the amplified rebounds after the short but steep declines in early 2020.

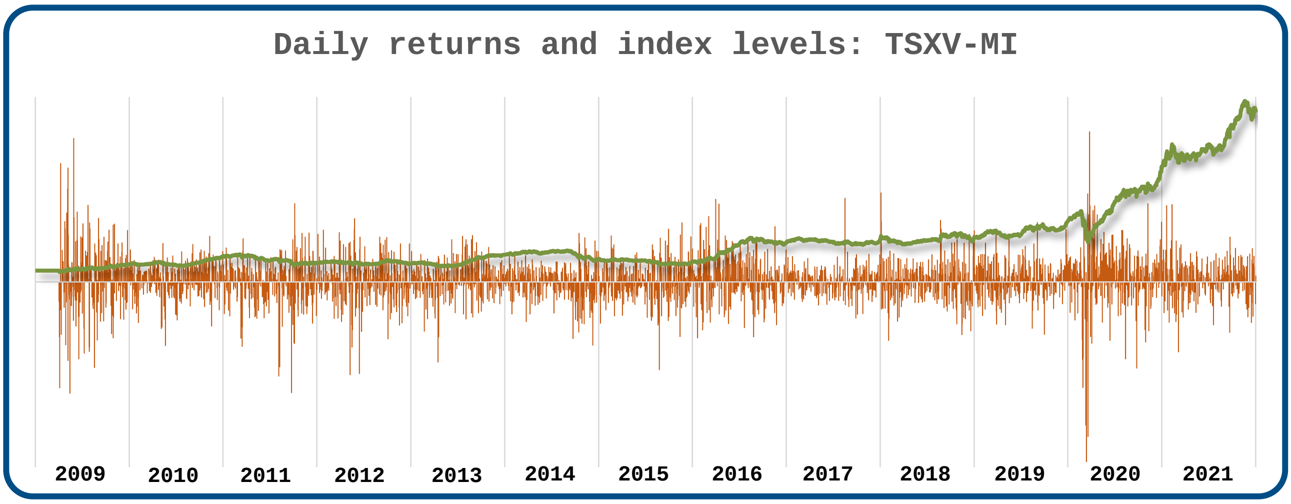

Figure 8.5 Daily returns: TSXV-MI (Source of price data: Bloomberg 2022)

The graph for the Venture index not only confirms its high volatility but also its approximately symmetric distribution with many positive and negative returns.

8.6 CONCLUSION

This chapter focussed on the positive cycles as these cycles ultimately drive the performance of the momentum index. It was shown that the Venture index outperformed the other two momentum indices despite recording a lower rate of positive cycles (30%) at a shorter average hold (11 months) but with a significantly higher compound return (70%) on average. With momentum cycles overlapping and the different types of cycles clustering in certain periods, the 3-year period of cumulative outperformance from 2019 to 2021 by the Venture index largely determined its eventual overall performance at the end of 2021.

The analysis showed that a cycle must generally hold longer than 9 months to exit positive, and that performance declines as entry prices increase beyond certain levels. Depending on the stock exchange, these levels maxed at R100 (Johannesburg), $50 (Toronto), and $5 (Venture). In general, momentum favoured lower-priced stocks as small absolute changes translate to large relative changes when working from low base values. The Consumer Discretionary and Technology sectors outperformed on the Johannesburg Exchange. Health Care and Technology outperformed on both the Toronto Exchange and the Venture Exchange. The worst-performing sectors were Consumer Staples (Johannesburg), Financials (Toronto) and Communications (Venture).

The different entry zones were expected to deliver contrasting results per exchange and category. Most cycles were entered in Zone 1 and, as a result, this zone also generated the greatest number of positive cycles for each stock exchange. Compound returns, which favour longer average holds, were highest in Zone 1 for the Johannesburg Exchange, Zone 2 for the Toronto Exchange, and Zone 3 for the Venture Exchange. An analysis of the average parameter scores confirmed statistically significant differences between the three exchanges. Generalising the outcomes, cycles with higher momentum and quality in combination with lower activity on entry are more likely to exit positive. It must be noted that this generalised outcome or combination may not hold for individual momentum cycles.

The custom indices quantified the actual performance of the customised model in each market and allowed a direct comparison between them to complete the momentum profiles for these equity markets. The number of members is variable with indices updated monthly when stocks are added and deleted based on the results from the momentum model. The index levels and member numbers per update indicated the state of momentum in a particular market and period. An increasing number of members from year to year normally coincided with increasing index levels. The end-of-year number relative to the average number of members for that year pointed to an upward or downward trend in momentum. The additions ratio, likewise, provided an indication of the sentiment in the market for that year. The number of year-end members, the average number of members, and the additions ratio generally reflected in the index level for that year.

Individually 2019, 2020, and 2021 were the best years for the indices as they rebounded after 2018. The performance of the Venture index improved when the period shortened to 3 years, outperforming from 2019 onwards when it surpassed the Johannesburg and Toronto indices. The correlation between the indices increased as the period shortened from 13 to 3 years. The Canadian indices maintained a strong co-movement during each of the extended periods and for many of the individual years while the South African index, generally, measured a weak correlation with the Canadian indices. Only the Toronto momentum index (TSX-MI) and its market index (TXCX) appear to be cointegrated, for the full 13-year period and at a 10% level of significance.

The Toronto index recovered more quickly from drawdowns than the other two indices. The higher drawdown ratio for this index also pointed to higher returns on a risk-adjusted basis during the period of analysis. The Venture index took longer to recover from drawdowns on average and its high return was, therefore, adjusted down to equal that of the Johannesburg index.

Based on the period of analysis (2009-2021), the Venture index being more symmetric and with the lowest kurtosis is less likely than the other two indices to record outliers and extreme negative returns. The Johannesburg index with the highest kurtosis value of the three indices is more likely to record extreme returns (negative and positive) or outliers. The Toronto index has a lower kurtosis than the Johannesburg index, but with its highly left-skewed distribution is more likely to record some extreme negative returns.

Refer to Annexure E for supplementary results and testing.

Table 8.16 Cointegration: Market/TSX-MI (2009-2021)

ADF Tests		TXCX/TSX-MI (FULL)			Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.1
tau-stat	-3.0427	-0.3610	-14.2629	-13.8040	type	2
tau-crit	-3.1278	-3.1278	-3.1278	-3.1278	max lags	15
stationary	no	no	yes	yes	criteria	none
aic	3.5490	6.1692	3.5507	6.1670		
bic	3.5820	6.2022	3.5837	6.2000	tau-stat	-3.6893
lags	15	15	15	15	tau-crit	-3.4984
coeff	-6.1E-03	-3.3E-04	-9.7E-01	-9.0E-01	cointegrated	yes
p-value	> .1	> .1	< .01	< .01	lags	15
					p-value	0.0665

Table 8.16: Note that the two series are not stationary, but that their first differences are stationary. The two original time series are now considered to be cointegrated provided the time series of the residuals is stationary, which is the case at a 10 per cent (-3.6893 < -3.4984) but not a 5 per cent (-3.6893 > -3.7834) level of significance (p-value = 0.0665). Refer to Annexure D for the full results.

REFERENCES

Bloomberg. 2022. Bloomberg Terminal – Bloomberg Professional Services subscription. New York: Bloomberg Finance L.P.

Samunderu, E. & Murahwa, Y.T. 2021. Return based risk measures for non-normally distributed returns: an alternative modelling approach. *Journal of Risk and Financial Management*, 14(11):540. DOI: 10.3390/jrfm14110540.

Sankaran, H., Nguyen, A. & Harikumar, J. 2012. Extreme return correlation and volatility: a two-threshold approach. *American Journal of Business*, 27(2):154–173. DOI: 10.1108/19355181211274451.

Wilmington. 2018. *Understanding investment risk through drawdown analysis*. *Investment Insights [Online]*. Wilmington Trust Corporation. Available at: https://www.wilmingtontrust.com/repositories/wtc_sitecontent/PDF/Understanding-investment-risk-through-drawdown-analysis.pdf (Accessed: 27 April 2021).

CHAPTER NINE

CONCLUSION

9.1 INTRODUCTION

This chapter confirms that the objectives of the study were obtained by summarising its contributions. The main objective was to describe an equity market in terms of the composition of its momentum cycles. This study created a customised momentum model and a custom index to profile momentum in equity markets. The customised model used a momentum term structure (several momentum terms of increasing length) grouped into different entry zones to create unique visual profiles for individual stocks. An example of such a unique profile is included to illustrate the stepped pattern with its entry zone and the ultimate exit. Each equity market has a different profile related to the composition of its momentum cycles and the performance of the custom index. A description of the momentum cycles generated by each market includes the overall outcomes, price range activity, sector activity, entry zones, and parameter scores. The profiles of the three markets are compared in terms of the number of momentum cycles, the composition of these cycles, basic profiles (average holds, price ranges, sectors, and entry zones), average parameter scores, and performance (via the custom indices). The chapter concludes with some general notes or observations about the model and suggestions for future research.

9.2 RESEARCH

The literature review (refer to Chapter 2) showed that research focused on the classic J-month/K-month (formation/holding period) approach to identify momentum and find the optimal J/K combination in different equity markets. Buying the best-performing stocks (top quantile) and selling the worst-performing stocks (bottom quantile) on their performance over the past 3 to 12 months at every update. A widening spread between the performance of the two groups would confirm the presence of momentum in that market. The long-only version ranks stocks on some definition of momentum, buying the top-ranked stocks (cross-sectional design) or stocks with high momentum (time-series design) and replacing individual stocks when a ranking or momentum falls below certain thresholds. Apart from the optimal J/K combination, whether momentum supposedly originates from an underreaction or a delayed overreaction to new information featured prominently in research. In addition, performance was assumed to depend on more refined definitions of momentum, not the basic concept of momentum.

Standard formation and holding periods were generally used (typically 3, 6, 9 and 12 months) to find the optimal combination for a particular equity market, perhaps iterating through different combinations with 1-month increments for a more exact calibration. Regarding momentum, equity markets were simply classified on their optimal J/K combinations. Past studies made no attempt to describe a particular equity market in terms of the composition of the momentum cycles from that market.

9.2.1 Objectives

The objectives of this study were to:

- Customise a model to profile momentum in equity markets.
- Construct a custom momentum index to quantify and present the outcomes.
- Create and compare the momentum profiles of three different equity markets.

This study was observational in design, based on the distinction between observational and experimental when doing quantitative research related to equity investing. Descriptive statistics and several performance metrics evaluated the effectiveness of the momentum model in each equity market via the custom index.

Using only historical stock price data, this study introduced the concept of momentum profiling. Profiling shifts the focus onto the holding period while differentiating between false, neutral, negative, and positive momentum cycles as determined by the eventual outcomes. Apart from classifying the momentum cycles, average holding periods, price ranges, sector activity, and the average parameter scores added additional information to the market profiles. Formation periods were substituted with entry zones to ensure variability in formation. These entry zones also created profiles for individual stocks. A performance analysis via a custom index completed the momentum profile for each equity market.

9.2.2 Contributions

The contributions of this study are the following:

- Creating momentum profiles for equity markets by describing each market in terms of its momentum cycles.
- Creating graphic (visual) momentum profiles for individual companies.
- Introducing the concept of a momentum term structure, several formation periods, to enter momentum cycles early and exit as late as possible.
- Customising a momentum model that makes the pre-sorting on price, market capitalisation (size), sector, trading volume, or volatility redundant.
- Customising a momentum model that can be calibrated for a particular market but does not require optimisation.
- Constructing a custom momentum index to quantify and present the outcomes of a mechanical or systematic approach to momentum investing.
- Providing retail and institutional investors with information on the likely performance of momentum investing in a particular market.

The term momentum-profiling has a double meaning in that individual stocks are profiled as well as a particular equity market. Individual profiling may enable the selective targeting of stocks that have distinct visual profiles and past behaviour associated with momentum. The composition of the momentum cycles and average hold per cycle type provide a unique description of the momentum effect in a particular equity market.

9.3 MOMENTUM MODEL

The momentum profiles originated from a customised model that used a momentum term structure, displaying as a stepped visual profile for individual stocks (refer to Chapter 4). The term structure, in this instance, refers to six momentum terms of increasing length (measured in days) and comprises 60-day, 90-day, 125-day, 180-day, 210-day and 250-day momentum terms grouped into four different entry zones. The concept behind this model is to identify stocks relatively early in their respective momentum cycles via three successive term-structure periods of high momentum (i.e., an entry zone). The model has four parameters – namely, a Momentum Score (MS), Volatility Score (VS), Quality Score (QS), and Activity Score (AS). Each parameter either has a maximum (VS) or a minimum (MS, QS and AS) setting and the settings were calibrated on the Johannesburg Stock Exchange (JSE). The same parameter settings were applied to the Toronto Stock Exchange (TSX) and the TSX Venture Exchange (TSXV). Stocks qualified on all four settings but were not sorted or ranked on their scores. All the stocks listed on a particular exchange were eligible for selection and the investment universe was not predefined by filtering companies on price, market capitalisation (size), liquidity or sector in advance.

The model exited cycles as late as possible on the 250-day momentum score parameter to extend the holding period and avoid premature exits (refer to Chapter 4). Cycle entries and exits were strictly mechanical according to the parameter settings and the exit rule. The eventual outcome classified momentum cycles as either positive, negative, neutral, or false. It was assumed that a positive cycle (optimal outcome) would hold at least 3 months and record an annualised gain of more than 10%. A negative cycle (unexpected outcome) would record an annualised loss of more than 10% while also holding at least 3 months. A neutral cycle (no outcome) is assumed to hold a minimum of 3 months but gain or lose a maximum of 10% annualised. A false cycle (failed outcome) holds shorter than 3 months.

These entries (additions) and exits (deletions) were used to construct comparable custom momentum indices for the three different equity markets.

9.4 MOMENTUM INDEX

The custom momentum index was constructed as equal-weighted in that new members entered at the average weight of the current members (refer to Chapter 3). The index was updated monthly, and the number of members fluctuated. The individual weights of the remaining members were adjusted for the number of additions, and the total weight of any deletions was distributed equally between members. The remaining members were allowed to retain the gains or losses from previous changes in price. The custom index, therefore, was designed to maintain a relatively active position over a true equal-weighted or unweighted design, which would normally reset all the member weights to the average weight when updated. Stocks with momentum were allowed to drift from their original weights.

9.5 INDIVIDUAL PROFILING

The customised model uses a momentum term structure, which creates unique visual profiles for individual stocks. Depending on the entry zone, the stepped pattern varies between regular (Zone 1) to more irregular (Zone 4). Below is an example, showing the momentum cycle of Shopify Inc, listed on the Toronto Stock Exchange in the Technology sector.

Table 9.1 Shopify Inc (SHOP:CT)

Dates	MOM250	MOM210	MOM180	MOM125	MOM090	MOM060	MOM020	Price	200dMA
2018-11-30	19	2	1	-5	0	-1	0	20224	18786
2018-12-31	8	0	0	-1	0	0	-9	18879	18899
2019-01-31	3	1	0	0	1	2	13	22118	19276
2019-02-28	4	1	0	6	24	21	11	24928	19932
2019-03-29	14	4	10	40	47	46	5	27586	20706
2019-04-30	17	22	43	65	69	30	17	32575	21501
2019-05-31	35	74	82	102	74	47	5	37184	23190
2019-06-28	79	114	146	122	77	52	0	39358	25366
2019-07-31	144	171	167	104	64	18	1	41941	27811
2019-08-30	193	208	191	101	50	24	16	51282	31066
2019-09-30	221	188	140	54	10	0	-20	41230	33610
2019-10-31	187	130	76	7	0	-13	-1	41300	36178
2019-11-29	142	74	33	0	-4	-1	10	44545	38163
2019-12-31	117	54	20	0	0	17	11	51630	40685
2020-01-31	100	48	19	6	48	63	15	61633	44111
2020-02-28	95	45	28	42	82	42	0	62322	47358
2020-03-31	70	34	26	47	14	0	0	58962	49615
2020-04-30	61	43	40	48	8	2	92	88278	52964
2020-05-29	86	80	114	68	43	122	7	104497	58996
2020-06-30	128	154	168	92	122	59	35	128977	65748
2020-07-31	192	238	200	140	114	28	0	136978	75746
2020-08-31	276	262	194	160	46	17	0	139323	85322
2020-09-30	292	207	153	57	7	-1	0	136169	93890
2020-10-30	269	171	129	18	0	0	-3	122823	102798
2020-11-30	193	118	72	2	0	0	1	139777	109216
2020-12-31	151	90	29	0	1	1	8	143732	117587
2021-01-29	118	48	15	2	5	15	0	139429	126791
2021-02-26	104	35	16	18	26	16	2	164873	134664
2021-03-31	47	15	7	4	1	-1	-2	138743	139053
2021-04-30	23	5	6	3	-1	-8	0	145090	141285
2021-05-31	9	2	2	0	-4	0	7	148032	141862
2021-06-30	7	7	3	0	2	9	32	181287	145439
2021-07-30	15	12	9	3	35	52	2	187300	151614
2021-08-31	26	22	11	40	43	10	0	192659	157772
2021-09-29	26	17	14	33	11	0	-6	171791	163265
2021-10-29	25	14	13	13	-2	-6	2	180702	166025
2021-11-30	23	18	31	3	0	2	3	194103	170630
2021-12-31	17	24	14	0	0	0	-2	174169	172569

An earlier entry in Zone 1 (2019-03-29) was possible, but the volatility score on that date exceeded the maximum setting. The parameter settings were calibrated on the Johannesburg Stock Exchange (JSE) and increasing or decreasing the individual settings may identify earlier entries. Entering the cycle in Zone 2 on 2019-04-30 ($325.75) and exiting mechanically on 2021-05-31 ($1480.32) when the 250-day momentum score dropped below the minimum level after 25 months (CAGR:106.82%). A discretionary exit on 2021-11-30 ($1941.03) would have delivered a better outcome.

9.6 EQUITY MARKET PROFILES

Each equity market has a different profile related to the composition of its momentum cycles and the performance of a custom index that quantifies the performance of the momentum model.

9.6.1 Emerging market

This section summarises the analysis from Chapter 5. It covers the overall outcomes, price ranges, sector activity, entry zones, and parameter scores of the Johannesburg Stock Exchange (JSE), and the performance of the JSE Momentum Index (JSE-MI).

Table 9.2 Overall outcomes: JSE

PERIOD	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
2009 2021	701	55	121	212	313	8.80	11.09	15.42
	AH	1.58	7.64	5.18	12.97			
	CRpAH	-7.50	0.32	-15.34	43.45			
	CARpAH	-44.66	0.51	-32.02	39.61			

Refer to Table 9.2: Positive cycles (313 from 701 or 45%) at an average hold of 13 months gained 43% in value. Negative cycles (212 from 701 or 30%), in comparison, lost 15% in value at an average hold of 5 months. False cycles (55 from 701 or 8%), holding shorter than 2 months on average lost 8% in value. Neutral cycles (121 from 701 or 17%) at an average hold of 8 months gained less than half a per cent in value. Overall, a momentum cycle in this equity market holds for 9 months on average while gaining 11% in value.

The false cycles from the 1-2-month range recorded a high negative compound annual return due to the short average hold. Negative cycles dominate holds of 3 to 8 months, with the 3-5 range generating the most negative cycles and the worst result overall. Positive cycles are predominant when holds are 9 months and longer, with most cycles in the 9-11 range. The overall performance of the 9-11-month range is impacted by the number of neutral cycles. With almost all cycles holding longer than 12 months exiting positive, returns increase along with an increase in the average hold (refer to Table 9.3).

Table 9.3 Average hold: JSE

HOLD	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
1-2	55	55	---	---	---	1.58	-7.50	-44.66
3-5	168	---	25	123	20	3.95	-9.78	-26.86
6-8	177	---	50	83	44	7.05	-3.32	-5.59
9-11	150	---	42	5	103	9.90	15.99	19.69
12-17	98	---	4	1	93	13.91	43.10	36.24
18-23	31	---	---	---	31	20.03	83.89	44.04
24+	22	---	---	---	22	32.00	211.84	53.19
ALL	701	55	121	212	313	8.80	11.09	15.42

The R10-R25 range outperformed, followed by the below-R5 range, which generated the most momentum cycles (refer to Table 9.4). The upper threshold for stock prices appears to be R100, with the R50-R100 range still recording comparable results. The compound returns per average hold are negative at an entry price above R200, with the negative cycles outnumbering the positive cycles.

Table 9.4 Price range activity: JSE

ZAR	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
< 5	127	10	21	30	66	8.98	14.62	20.01
5 =< 10	70	6	12	24	28	8.39	11.24	16.47
10 =< 25	112	9	15	29	59	10.80	21.77	24.45
25 =< 50	107	13	17	28	49	8.88	12.48	17.23
50 =< 100	102	5	27	24	46	9.06	13.46	18.21
100 =< 200	100	5	18	38	39	7.37	2.99	4.91
200 =< 500	63	6	7	31	19	7.68	-1.05	-1.63
500 =<	20	1	4	8	7	6.90	-4.23	-7.24
ALL	701	55	121	212	313	8.80	11.09	15.42

Refer to Table 9.5: The Consumer Discretionary sector outperformed all the other active sectors with 80-plus cycles. Among the less active sectors, Technology recorded the longest average hold, while Health Care mainly generated positive cycles. Financials outperformed Industrials and Materials but also recorded the highest rate of neutral cycles. Materials, apart from the inactive Energy sector and less-active Health Care sector, registered the highest rate of false cycles. The Industrials sector registered the highest rate of negative cycles.

Table 9.5 Sector activity: JSE

SECTOR	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
CO	29	2	3	10	14	9.14	15.34	20.62
CD	102	5	18	27	52	9.61	16.63	21.18
CS	102	7	22	25	48	8.50	10.59	15.28
EN	5	1	---	1	3	7.00	22.04	40.70
FI	114	6	29	29	50	9.29	13.39	17.62
HC	26	4	2	4	16	9.50	14.39	18.50
IN	86	8	11	31	36	7.99	6.88	10.51
MA	180	19	25	63	73	8.27	6.63	9.77
RE	22	2	3	9	8	8.23	6.79	10.05
TE	35	1	8	13	13	10.31	19.00	22.43
ALL	701	55	121	212	313	8.80	11.09	15.42

The outcomes show that a company listed in the Consumer Discretionary sector at a price ranging from R10 to R25 is likely to record a positive cycle. When cycles form in the Health Care sector, they generally exit as positive. Penny stocks, stocks in the below-R5 range, have the most potential for forming momentum as small absolute (price) changes result in large relative (percentage) changes.

Table 9.6 shows the composition of cycles for each zone. Zone 1, presenting the earliest entry into any cycle, generated the most entries at the longest average hold. Relative to its total number of cycles, this zone has the greatest number of positive and neutral cycles with the smallest number of false and negative cycles.

Table 9.6 Results per entry zone: JSE

ZONE	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
1	269	17	53	77	122	9.39	12.52	16.26
2	142	11	22	45	64	9.14	10.17	13.56
3	135	11	23	41	60	8.36	10.42	15.29
4	155	16	23	49	67	7.85	10.07	15.81
ALL	701	55	121	212	313	8.80	11.09	15.42

Zone 4 generated the second most cycles but at the shortest average hold due to it offering the latest possible entry into a cycle. Relative to its total number of cycles, this zone has the greatest number of false cycles (i.e., the highest rate of false cycles). Zone 2 recorded the highest rate of negative cycles.

Table 9.7 presents the average parameter scores per individual momentum period. It shows that the momentum score is highest in the 125-day period. Volatility, quality, and activity scores decrease as the momentum periods increase.

Table 9.7 Parameter scores per period: JSE

PARAMETER	060	090	125	180	210	250	AVG
Momentum	21.16	29.80	33.65	31.25	27.49	21.30	27.44
Volatility	0.95	0.77	0.63	0.47	0.43	0.39	0.61
Quality	56.17	55.51	54.53	53.01	52.31	51.66	53.87
Activity	51.07	50.11	49.02	47.41	46.68	46.04	48.39

Table 9.8 below shows the average parameter scores per cycle type and overall. It indicates that, on average, cycles with higher momentum, higher volatility, and higher quality scores combined with lower activity scores tend to be positive. Negative cycles, in comparison, have lower momentum and quality scores combined with higher activity. False cycles, on average, recorded some of the lowest scores in every category. Neutral cycles recorded lower volatility and higher activity scores on average compared to positive cycles.

Table 9.8 Average parameter scores: JSE

CYCLE	Momentum	Volatility	Quality	Activity
False	25.70	0.60	52.99	48.47
Neutral	26.35	0.57	54.34	48.69
Negative	26.29	0.61	53.80	48.54
Positive	28.95	0.62	53.89	48.15
AVERAGE	27.44	0.61	53.87	48.39

A custom momentum index evaluated the model by quantifying the process of entering the cycles at certain prices and exiting at either a gain or a loss. Table 9.9 below shows the performance of the Johannesburg Stock Exchange Momentum Index (JSE-MI) over the years. The variable number of members (M) reflects in the growth (G) and the level (L) of the index at year-end. The year 2018 shows a large decline from the previous year, with the index containing only 7 members. The index rebounded during 2019, recording its highest growth. The most volatile year proved to be 2020, recording the highest standard deviation (S) in the 13-year period.

Table 9.9 Performance per year: JSE-MI

Y	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
M	88	49	15	62	53	45	30	45	26	7	22	31	76
L	117.30	152.82	164.36	226.63	285.04	348.84	409.27	404.33	442.31	348.64	533.27	734.94	1054.60
G	17.30	30.28	7.55	37.89	25.77	22.38	17.33	-1.21	9.39	-21.18	52.96	37.82	43.49
S	20.91	13.22	11.39	7.61	9.49	9.33	13.28	13.08	9.62	17.15	18.40	47.23	11.69

Table 9.10 below shows the cumulative annual growth rates for different periods. The 10-Year, 5-Year, and 3-year rates confirm an improved performance during the latter periods. The three-year period from 2019 to 2021 was the main driver of the performance, generating an annualised rate of almost 45% for the index.

Table 9.10 Annualised performance: JSE-MI

Metric	FULL	10-Year	5-Year	3-Year	1-Year
CTGR	954.60	541.66	160.83	202.49	43.49
CAGR	19.87	20.43	21.14	44.62	43.49
StdD	18.36	19.09	24.78	29.89	11.69

The drawdown analysis in Table 9.11 below indicates the potential of the index to suffer sudden large losses in value and its ability to recover those losses. It records the size and speed of drawdowns and the time to return to former highs. The JSE Momentum Index (JSE-MI) experienced its maximum drawdown in March 2020, and it occurred over a relatively short period (19 days), also recovering within a comparatively short period (62 days) to its original high. The duration (peak to peak) of the maximum drawdown was 81 days. The longest drawdown lasted 362 days, but on average drawdowns for this index last 16 days while losing less than 6% in value. The JSE-MI recorded a drawdown ratio of 0.49 over the 13-year period.

Table 9.11 Drawdown analysis: JSE-MI

Maximum drawdown	40.39%	Maximum duration	362 days
Date	2020-03-19	From:	2018-01-10
Period	19 days	To:	2019-06-24
Recovery	62 days	Average duration	16 days
Duration	81 days	Annualised return	19.87%
Average drawdown	5.63%	Drawdown ratio	0.49

9.6.2 Developed market

The Toronto Stock Exchange (TSX), representing a developed equity market, produced a larger number of cycles in a different configuration compared to the emerging market. Summarising the analysis from Chapter 6, this section describes the composition of the momentum cycles unique to the Canadian senior market. It covers the overall outcomes, price range activity, sector activity, entry zones, parameter scores, and performance of the TSX Momentum Index (TSX-MI).

Table 9.12 Overall outcomes: TSX

PERIOD	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
2009 2021	2335	208	461	758	908	8.62	6.94	9.78
	AH	1.51	8.06	5.70	12.99			
	CRpAH	-12.60	0.42	-20.99	48.88			
	CARpAH	-65.71	0.62	-39.12	44.44			

Refer to Table 9.12: Positive cycles (908 from 2335 or 39%) at an average hold of 13 months gained 49% in value. Negative cycles (758 from 2335 or 32%), in comparison, lost 21% in value at an average hold of 6 months. False cycles (208 from 2335 or 9%), holding shorter than 2 months on average lost 13% in value. Neutral cycles (461 from 2335 or 20%) at an average hold of 8 months gained less than half a per cent in value. Overall, a momentum cycle in this equity market holds for 9 months on average while gaining 7% in value.

Refer to Table 9.13. The false cycles from the 1-2-month range recorded a high negative compound annual return due to the short average hold. Negative cycles dominate holds of 3 to 8 months, with the 3-5 range generating the most negative cycles and the worst result overall. Positive cycles are predominant when holds are 9 months and longer. The overall performance of the 9-11-month range is impacted by the number of neutral cycles. The 12-17 range with an equivalent number of positive cycles but fewer neutral and negative cycles outperformed the 9-11-month range. Most cycles that hold longer than 12 months exit positive and returns increase along with an increase in the average hold.

Table 9.13 Average hold: TSX

HOLD	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
1-2	208	208	---	---	---	1.51	-12.60	-65.71
3-5	492	---	62	368	62	4.05	-13.83	-35.63
6-8	649	---	220	318	111	7.05	-8.04	-13.30
9-11	500	---	149	67	284	9.83	11.48	14.19
12-17	312	---	29	5	278	13.91	47.79	40.07
18-23	113	---	1	---	112	19.92	89.67	47.05
24+	61	---	---	---	61	28.72	184.27	54.73
ALL	2335	208	461	758	908	8.62	6.94	9.78

The $1-$2 range outperformed, followed by the $2-$5 range (refer to Table 9.14). The $10-$20 range generated the most momentum cycles. The upper threshold for stock prices appears to be $20, with the $10-$20 range still recording comparable results. The positive cycles outnumber the negative cycles in each range except the below-$1 range. Relative to its total number of cycles, the below-$1 range has the greatest number of false and negative cycles with the smallest number of neutral and positive cycles. The $10-$20 range registered the highest rate of neutral cycles. The $1-$2 range recorded the highest rate of positive cycles.

Table 9.14 Price range activity: TSX

CAD	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
< 1	174	21	19	74	60	8.30	6.99	10.27
1 =< 2	187	15	24	64	84	9.02	13.56	18.43
2 =< 5	376	30	61	121	164	9.25	9.77	12.85
5 =< 10	395	39	76	116	164	9.32	9.21	12.00
10 =< 20	491	40	122	153	176	8.58	5.99	8.48
20 =< 50	456	39	103	144	170	8.01	3.62	5.47
50 =< 100	162	16	36	54	56	7.86	2.98	4.58
100 =<	94	8	20	32	34	7.54	1.87	2.99
ALL	2335	208	461	758	908	8.62	6.94	9.78

Refer to Table 9.15: The Technology sector outperformed all the other active sectors with 100-plus cycles. Among the less active sectors, Real Estate recorded the longest average hold at the highest compound returns with the highest rate of positive cycles. The Consumer Discretionary and Consumer Staples sectors recorded some of the best results in this market. Materials as the most active sector produced average results with its negative cycles outnumbering its positive cycles. Utilities, Financials and Health Care were the worst-performing sectors on the Toronto Stock Exchange (TSX) overall.

Table 9.15 Sector activity: TSX

SECTOR	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
CO	88	11	15	25	37	8.51	8.42	12.08
CD	186	18	40	48	80	9.45	11.32	14.59
CS	135	8	41	28	58	9.16	12.57	16.77
EN	373	29	60	127	157	8.58	8.38	11.91
FI	199	14	46	58	81	8.18	3.34	4.94
HC	133	13	26	49	45	8.92	2.74	3.70
IN	282	25	65	83	109	8.57	8.10	11.52
MA	658	60	103	273	222	8.39	3.79	5.47
RE	44	4	8	9	23	9.82	16.05	19.95
TE	155	14	41	36	64	9.41	13.88	18.04
UT	82	12	16	22	32	6.73	0.35	0.63
ALL	2335	208	461	758	908	8.62	6.94	9.78

Table 9.16 shows the composition of cycles for each zone. Zone 1, presenting the earliest entry into any cycle, generated the most entries at the longest average hold. Relative to its total number of cycles, this zone has the greatest number of false cycles with the smallest number of negative cycles.

Table 9.16 Results per entry zone: TSX

ZONE	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
1	837	85	173	259	320	9.13	6.88	9.14
2	490	45	88	182	175	8.75	6.05	8.39
3	497	33	103	155	206	8.43	7.36	10.63
4	511	45	97	162	207	7.87	7.49	11.65
ALL	2335	208	461	758	908	8.62	6.94	9.78

Zone 4 generated the second most cycles but at the shortest average hold due to it offering the latest possible entry into a cycle. Relative to its total number of cycles, Zone 3 has the greatest number of neutral and positive cycles. Zone 2 registered the highest rate of negative cycles.

Table 9.17 below presents the average parameter scores per individual momentum period. It shows that the momentum score is highest in the 125-day period. Volatility, quality, and activity scores decrease as the momentum periods increase.

Table 9.17 Parameter scores per period: TSX

PARAMETER	060	090	125	180	210	250	AVG
Momentum	26.96	37.11	39.89	34.83	28.97	21.41	31.35
Volatility	1.26	1.02	0.79	0.59	0.57	0.55	0.80
Quality	55.82	54.97	53.93	52.52	51.80	51.19	53.37
Activity	51.25	50.13	48.92	47.39	46.70	46.09	48.41

Table 9.18 below shows the average parameter scores per cycle type and overall. It indicates that, on average, cycles with higher momentum, volatility, and quality scores combined with lower activity scores tend to be positive. Negative cycles, in comparison, have the highest momentum and volatility scores, and high activity scores combined with lower quality scores. False cycles, on average, recorded the lowest scores in every category but volatility. Neutral cycles delivered higher quality and volatility scores in combination with lower momentum and volatility.

Table 9.18 Average parameter scores: TSX

CYCLE	Momentum	Volatility	Quality	Activity
False	23.55	0.77	52.43	47.41
Neutral	28.16	0.71	53.83	49.46
Negative	33.67	0.84	53.35	48.39
Positive	33.29	0.81	53.37	48.13
AVERAGE	31.35	0.80	53.37	48.41

A custom momentum index evaluated the model by quantifying the process of entering the cycles at certain prices and exiting at either a gain or a loss. Table 9.19 below shows the performance of the Toronto Stock Exchange Momentum Index (TSX-MI) over the years. The variable number of members (M) reflects in the growth (G) and the level (L) of the index at year-end. The year 2018 shows a large decline from the previous year, with the index containing only 29 members. The index rebounded during 2019 with members increasing to 100. The most volatile year proved to be 2020, with the highest compound return and standard deviation (S) of the 13-year period.

Table 9.19 Performance per year: TSX-MI

Y	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
M	242	255	37	82	192	75	39	183	78	29	100	168	132
L	139.93	202.23	179.07	214.86	308.10	326.20	341.38	481.40	554.90	461.75	556.03	876.72	1132.81
G	39.93	44.52	-11.45	19.98	43.40	5.87	4.65	41.02	15.27	-16.79	20.42	57.67	29.21
S	18.91	12.90	19.29	9.97	8.62	12.18	11.53	21.50	10.21	13.78	10.92	34.76	16.29

Table 9.20 below shows the cumulative annual growth rates for different periods. The 10-Year, 5-Year, and 3-year rates confirm an improved performance during the latter periods. The three-year period from 2019 to 2021 was the main driver of the performance, generating an annualised rate of almost 35% for the index.

Table 9.20 Annualised performance: TSX-MI

Metric	FULL	10-Year	5-Year	3-Year	1-Year
CTGR	1032.81	532.59	135.31	145.33	29.21
CAGR	20.53	20.26	18.67	34.87	29.21
StdD	16.85	16.72	19.39	22.97	16.29

The drawdown analysis in Table 9.21 below indicates the potential of the index to suffer sudden large losses in value and its ability to recover those losses. It records the size and speed of drawdowns and the time to return to former highs. The TSX Momentum Index (TSX-MI) experienced its maximum drawdown in March 2020, and it occurred over a relatively short period (18 days), also recovering within a comparatively short period (40 days) to its original high. The duration (peak to peak) of the maximum drawdown was 58 days. The longest drawdown lasted 431 days, but on average drawdowns for this index last 16 days while losing less than 7% in value. The TSX-MI recorded a drawdown ratio of 0.57 over the 13-year period.

Table 9.21 Drawdown analysis: TSX-MI

Maximum drawdown	36.00%	Maximum duration	431 days
Date	2020-03-18	From:	2011-04-11
Period	18 days	To:	2012-12-31
Recovery	40 days	Average duration	16 days
Duration	58 days	Annualised return	20.53%
Average drawdown	6.29%	Drawdown ratio	0.57

9.6.3 Venture market

The Toronto Venture Exchange (TSXV), representing an equity market for small fledgling companies, produced a smaller number of cycles in a different configuration compared to the emerging market and the developed market. Summarising the analysis from Chapter 7, this section describes the composition of the momentum cycles unique to the Canadian junior market. It covers the overall outcomes, price range activity, sector activity, entry zones, parameter scores, and performance of the TSXV Momentum Index (TSXV-MI).

Table 9.22 Overall outcomes: TSXV

PERIOD	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
2009 2021	581	78	60	269	174	7.14	-2.56	-4.27
	AH	1.55	8.97	5.78	11.13			
	CRpAH	-15.24	0.33	-29.76	70.31			
	CARpAH	-72.18	0.44	-51.98	77.58			

Refer to Table 9.22: Positive cycles (174 from 581 or 30%) at an average hold of 11 months gained 70% in value. Negative cycles (269 from 581 or 46%), in comparison, lost 30% in value at an average hold of 6 months. False cycles (78 from 581 or 14%), holding shorter than 2 months on average lost 15% in value. Neutral cycles (60 from 581 or 10%) at an average hold of 9 months gained less than half a per cent in value. Overall, a momentum cycle in this equity market holds for 7 months on average while losing 3% in value.

The false cycles from the 1-2-month range recorded a high negative compound annual return due to the short average hold. Negative cycles dominate holds of 3 to 8 months, with the 3-5 range generating the most negative cycles and the worst result overall. Positive cycles are predominant when holds are 9 months and longer, with most cycles in the 9-11 range. The overall performance of the 9-11-month range is impacted by the number of neutral cycles. With almost all cycles holding longer than 12 months exiting positive, returns increase along with an increase in the average hold (refer to Table 9.23).

Table 9.23 Average hold: TSXV

HOLD	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
1-2	78	78	---	---	---	1.55	-15.24	-72.18
3-5	165	---	9	131	25	4.05	-21.40	-50.97
6-8	157	---	16	112	29	6.98	-16.80	-27.11
9-11	109	---	27	26	56	9.91	10.51	12.87
12-17	50	---	7	---	43	14.04	91.33	74.12
18-23	17	---	1	---	16	20.06	145.27	71.04
24+	5	---	---	---	5	28.20	378.64	94.70
ALL	581	78	60	269	174	7.14	-2.56	-4.27

Even though stocks in the $0.50-$1.00 price range were the most actively traded in this market, the relatively small number of stocks priced at less than $0.50 delivered the best results (refer to Table 9.24). The upper threshold for stock prices appears to be $1.50, with the $1.00-$1.50 range still recording comparable results. The negative compound returns per average hold are highest at an entry price above $5. The $0.50-$1.00 and $3.00-$5.00 ranges are negatively impacted by the many false and neutral cycles.

Table 9.24 Price range activity: TSXV

CAD	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
< 0.5	28	5	1	12	10	8.00	28.39	45.48
0.5 =< 1	122	16	11	58	37	7.13	0.13	0.23
1 =< 1.5	89	12	6	41	30	7.79	3.08	4.78
1.5 =< 2	70	9	9	31	21	7.13	-6.05	-9.97
2 =< 3	74	7	9	35	23	7.30	-2.21	-3.60
3 =< 5	84	14	10	31	29	6.73	0.73	1.31
5 =< 10	63	3	6	37	17	7.16	-10.98	-17.71
10 =<	51	12	8	24	7	6.04	-21.31	-37.88
ALL	581	78	60	269	174	7.14	-2.56	-4.27

Refer to Table 9.25: The Technology sector outperformed all the other relatively active sectors with 30-plus cycles. The Consumer Discretionary and Consumer Staples sectors recorded the best results in this market but at low activity. Materials as the most active sector delivered negative results. Most of the momentum cycles originated in the Materials (51%) and the Energy (16%) sectors but these two sectors also account for 72% (194 from 269) of all the negative cycles. Real Estate registered the highest rate of positive cycles. In this venture market, the standout sectors are Technology, Health Care, Consumer Discretionary, and Consumer Staples.

Table 9.25 Sector activity: TSXV

SECTOR	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
CO	11	---	1	7	3	8.18	-24.61	-33.92
CD	10	2	3	1	4	8.00	23.00	36.41
CS	13	1	1	4	7	8.23	21.52	32.86
EN	93	14	10	50	19	6.58	-10.76	-18.75
FI	13	1	2	5	5	6.85	2.30	4.07
HC	32	5	4	10	13	7.97	14.70	22.93
IN	47	6	7	26	8	7.15	-1.76	-2.94
MA	298	42	26	144	86	6.96	-4.34	-7.36
RE	10	1	---	3	6	7.00	10.02	17.79
TE	52	5	6	18	23	8.23	5.61	8.28
UT	2	1	---	1	---	4.00	-23.25	-54.79
ALL	581	78	60	269	174	7.14	-2.56	-4.27

Table 9.26 shows the composition of cycles for each zone. Zone 1, presenting the earliest entry into any cycle, generated the most entries but, surprisingly, at the shortest average hold. Relative to its total number of cycles, this zone has the greatest number of false and neutral cycles with the smallest number of negative and positive cycles.

Table 9.26 Results per entry zone: TSXV

ZONE	Cycles	False	Neutral	Negative	Positive	AH	CRpAH	CARpAH
1	182	37	21	72	52	6.78	-1.26	-2.23
2	121	12	13	58	38	7.53	-2.14	-3.39
3	129	14	9	68	38	7.50	-3.63	-5.74
4	149	15	17	71	46	6.96	-3.56	-6.06
ALL	581	78	60	269	174	7.14	-2.56	-4.27

Zone 4, offering the latest possible entry into a cycle, generated the second most cycles at the second shortest average hold. Zone 2 recorded the highest rate of positive cycles. Zone 3 registered the highest rate of negative cycles and the lowest rate of neutral cycles.

Table 9.27 below presents the average parameter scores per individual momentum period. It shows that the momentum score is highest in the 125-day period. The quality and activity scores decrease as the momentum periods increase.

Table 9.27 Parameter scores per period: TSXV

PARAMETER	060	090	125	180	210	250	AVG
Momentum	35.23	51.33	55.37	49.54	44.16	36.62	45.37
Volatility	1.96	1.53	1.13	0.90	0.96	1.10	1.26
Quality	54.83	54.08	53.09	51.79	51.19	50.54	52.58
Activity	45.54	43.75	42.18	40.33	39.51	38.61	41.65

Table 9.28 below shows the average parameter scores per cycle type and overall. It indicates that, on average, cycles with higher momentum and quality scores in combination with lower volatility and activity scores tend to be positive. Negative cycles have the highest average momentum score overall and higher volatility with lower quality scores relative to the positive cycles. False cycles, on average, recorded the lowest scores in every category but activity. Neutral cycles recorded high volatility, quality, and activity scores on average.

Table 9.28 Average parameter scores: TSXV

CYCLE	Momentum	Volatility	Quality	Activity
False	33.64	1.19	51.43	42.45
Neutral	38.58	1.35	52.99	41.96
Negative	49.68	1.29	52.68	41.35
Positive	46.32	1.23	52.82	41.66
AVERAGE	45.37	1.26	52.58	41.65

A custom momentum index evaluated the model by quantifying the process of entering the cycles at certain prices and exiting at either a gain or a loss. Table 9.29 below shows the performance of the TSX Venture Exchange Momentum Index (TSXV-MI) over the years. The variable number of members (M) reflects in the growth (G) and the level (L) of the index at year-end. The year 2015 shows a continued decline from the previous year, with the index containing only 5 members. The index rebounded during 2016 with members increasing to 46. The most volatile year proved to be 2020, with the highest compound return and standard deviation (S) of the 13-year period.

Table 9.29 Performance per year: TSXV-MI

Y	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
M	33	58	13	16	24	11	5	46	26	16	25	47	25
L	155.12	228.41	163.48	159.85	238.47	201.11	179.78	346.45	361.68	387.51	534.98	1012.18	1500.74
G	55.12	47.25	-28.43	-2.22	49.19	-15.67	-10.61	92.71	4.40	7.14	38.06	89.20	48.27
S	37.62	19.20	27.55	25.97	20.62	19.38	22.63	26.40	14.69	23.22	20.04	41.10	22.23

Table 9.30 below shows the cumulative annual growth rates for different periods. The 10-Year, 5-Year, and 3-year rates confirm an improved performance during the latter periods. The three-year period from 2019 to 2021 was the main driver of the performance, generating an annualised rate of 57% for the index.

Table 9.30 Annualised performance: TSXV-MI

Metric	FULL	10-Year	5-Year	3-Year	1-Year
CTGR	1400.74	818.02	333.18	287.28	48.27
CAGR	23.16	24.82	34.07	57.04	48.27
StdD	25.67	24.54	25.80	29.28	22.23

The drawdown analysis in Table 9.31 below indicates the potential of the index to suffer sudden large losses in value and its ability to recover those losses. It records the size and speed of drawdowns and the time to return to former highs. The TSXV Momentum Index (TSXV-MI) experienced its maximum drawdown in August 2015. It occurred over a long period (243 days) and the index recovered within a relatively short period (181 days) to its original high. The duration (peak to peak) of the maximum drawdown was 424 days. The longest drawdown lasted 709 days, but on average drawdowns for this index last 27 days while losing less than 15% in value. The TSXV-MI recorded a drawdown ratio of 0.50 over the 13-year period.

Table 9.31 Drawdown analysis: TSXV-MI

Maximum drawdown	45.95%	Maximum duration	709 days
Date	2015-08-24	From:	2011-03-08
Period	243 days	To:	2014-01-07
Recovery	181 days	Average duration	27 days
Duration	424 days	Annualised return	23.16%
Average drawdown	14.76%	Drawdown ratio	0.50

9.6.4 Comparison

This section compares the profiles of the three markets. It is based on chapters 5 to 8, which provide more comprehensive analyses of the markets studied. It compares the number of momentum cycles generated by each market, the composition of these cycles, basic profiles (average holds, price ranges, sectors, and entry zones), average parameter scores, and performance (via the custom indices).

The number of commons stocks (ordinary shares) per market that qualified for selection during the 13-year period (2009-2013) of analysis is shown in Table 9.32 below. Referring to the South African market (JSE), 526 stocks were available for selection and the customised model identified 701 momentum cycles in progress from 247 different stocks or tickers. Therefore, 47% of the original 526 common stocks (247 tickers) experienced 701 momentum cycles, which converts to 2.8 cycles per ticker. In the senior Canadian market (TSX), 49% of the common stocks experienced momentum cycles at 2.5 cycles per ticker. In the junior Canadian market or venture market (TSXV), only 11% of the common stocks experienced momentum cycles at 1.4 cycles per ticker on average.

Table 9.32 Cycles per market

MARKET	Qualify	Identify	I/Q	Cycles	C/I
Johannesburg Stock Exchange (JSE)	526	247	47%	701	2.8
Toronto Stock Exchange (TSX)	1865	916	49%	2335	2.5
TSX Venture Exchange (TSXV)	3610	412	11%	581	1.4

Table 9.33 below shows the composition or configuration of the momentum cycles per market. The Johannesburg Stock Exchange (JSE) generated the highest percentage of positive cycles, while the configuration for the Toronto Stock Exchange (TSX) shows comparatively higher rates of negative, neutral, and false cycles. The configuration for the TSX Venture Exchange (TSXV) confirms the dominance of negative cycles, with more false cycles and fewer neutral cycles. The positive cycles ultimately drive the performance of the momentum index. The Venture index outperformed the other two momentum indices despite recording a lower rate of positive cycles (30%) at a shorter average hold (11 months) but with a significantly higher compound return (70%) on average. With momentum cycles overlapping and the different types of cycles clustering in certain periods, the 3-year period of cumulative outperformance from 2019 to 2021 by the Venture index largely determined its eventual overall performance at the end of 2021.

Table 9.33 Composition of cycles

Market	%FAL	%NEU	%NEG	%POS	AH-O	CRpAH	AH-P	CRpAH
JSE	8	17	30	45	8.80	11.09	12.97	43.45
TSX	9	20	32	39	8.62	6.94	12.99	48.88
TSXV	14	10	46	30	7.14	-2.56	11.13	70.31

Refer to Table 9.34: The basic profiles of the different markets indicate that cycles must generally hold longer than 9 months to exit positive. The outperforming price ranges confirm that momentum favours lower-priced stocks as small absolute changes translate to large relative changes when working from low base values. Considering both active and less-active sectors, the active Consumer Discretionary (CD) sector and the less-active Technology (TE) sector outperformed on the Johannesburg Exchange. The Technology (TE) and Real Estate (RE) combination outperformed on the Toronto Exchange, while Technology (TE) and Health Care (HC) were the outperforming sectors on the Venture Exchange. Zone 1, presenting the earliest possible entry into any cycle, should potentially generate the most entries at the longest average hold. This is true for the emerging market (JSE), and to a lesser extent for the venture market (TSXV). The developed market (TSX) favoured Zone 4 entries that allow for more irregular patterns and normally result in shorter holding periods.

Table 9.34 Basic profiles

Market	Hold	Price range	Sector	Zone
JSE	9+ months	10 =< R < 25	CD / TE	1:060-090-125
TSX	9+ months	1 =< $ < 2	TE / RE	4:180-210-250
TSXV	9+ months	$ < 0.5	TE / HC	1:060-090-125

An analysis of the average parameter scores confirmed statistically significant differences between the three exchanges (refer to Annexure D). Table 9.35 below shows that the emerging market (JSE) recorded the lowest average momentum and volatility scores with the highest average quality score. The developed market (TSX) recorded the highest average activity score, which proxies for liquidity. As expected, the venture market (TSXV) recorded the highest average momentum and volatility scores, and the lowest average quality and activity scores.

Table 9.35 Average parameter scores

Market	Momentum	Volatility	Quality	Activity
JSE	27.44	0.61	53.87	48.39
TSX	31.35	0.80	53.37	48.41
TSXV	45.37	1.26	52.58	41.65

The custom indices quantified the actual performance of the customised model in each market to allow a direct comparison in terms of relative performance. Table 9.36 on the next page summarises the risk and returns of each index over different periods. The Venture index (TSXV-MI) recorded the highest annualised returns at the highest volatility in every period, but the coefficient of variation (CV) does indicate that it has a better risk/return ratio (relative dispersion) for the 5-year and 3-year periods compared to the other indices. The Toronto index (TSX-MI) generally experienced less volatility and lower returns. The Johannesburg index (JSE-MI) posted the best results of the three indices in 2021 (1Y).

Table 9.36 Risk and return per period

Index Period	JSE-MI			TSX-MI			TSXV-MI		
	CAGR	StdD	CV	CAGR	StdD	CV	CAGR	StdD	CV
10Y	20.43	19.09	0.93	20.26	16.72	0.83	24.82	24.54	0.99
5Y	21.14	24.78	1.17	18.67	19.39	1.04	34.07	25.80	0.76
3Y	44.62	29.89	0.67	34.87	22.97	0.66	57.04	29.28	0.51
1Y	43.49	11.69	0.27	29.21	16.29	0.56	48.27	22.23	0.46

The drawdown analysis in Table 9.37 below indicates the potential of an index to suffer sudden large losses in value and its ability to recover those losses. The Toronto index recovered more quickly from drawdowns than the other two indices. The higher drawdown ratio for this index also pointed to higher returns on a risk-adjusted basis during the period of analysis. The Venture index took longer to recover from drawdowns on average and its high return was, therefore, adjusted down to equal that of the Johannesburg index.

Table 9.37 Drawdown analysis

Index	MDd	Per	Rec	Dur	ADd	MDur	ADur	CAGR	DdR
JSE-MI	40.39%	19d	62d	81d	5.63%	362d	16d	19.87%	0.49
TSX-MI	36.00%	18d	40d	58d	6.29%	431d	16d	20.53%	0.57
TSXV-MI	45.95%	243d	181d	424d	14.76%	709d	27d	23.16%	0.50

Using the Johannesburg index as an example, its maximum drawdown (MDd) occurred over a relatively short period (19 days), also recovering within a comparatively short period (62 days) to its original high. The duration (peak to peak) of the maximum drawdown was 81 days. The longest drawdown (MDur) lasted 362 days, but the average drawdown (ADur) for this index lasts 16 days while losing less than 6% in value on average (ADd). Its drawdown ratio (DdR) is the lowest of the three indices.

This section showed that the three different markets are distinct in size, the number of qualifying listings (common stocks), and the number of listings that experienced momentum cycles. The composition or configuration of the momentum cycles is unique to each market. The overall outcomes, in terms of average hold and compound return per average hold, favoured the emerging market represented by the Johannesburg Stock Exchange (JSE). However, the outcomes related to the positive cycles favoured the venture market, represented by the TSX Venture Exchange (TSXV). The positive cycles ultimately determined the performance of the respective momentum indices with the TSXV Venture Momentum Index (TSXV-MI) outperforming the other two indices over the 13-year period (2009-2021) of analysis.

The study is concluded with some general notes or observations about the momentum model and suggestions for future research.

9.7 GENERAL NOTES

General notes or observations are the following:

- It is not suggested that the customised momentum model would outperform any other model based on momentum. It is simply a model with different parameters that can be calibrated. The focus was on a mechanical and consistent approach to identifying stocks with momentum cycles in progress.
- The same parameter settings allowed a direct comparison between the three markets. However, the settings can be calibrated for a particular market. The volatility score setting may have been too high for the Venture Exchange (TSXV), thereby causing later entries and shortening the holding periods. The maximum setting for the volatility score (VS) can be increased while the quality (QS) and activity (AS) score minimums can be decreased to adapt to this type of equity market. The Toronto Stock Exchange (TSX), in turn, should be able to handle a lower volatility setting in combination with higher quality and activity score minimums (refer to Table 9.35 on page 9-18) to possibly reduce the number of false and neutral cycles.
- The individual profiles allow for discretionary as opposed to mechanical exits depending on their evolving visual patterns. Individual profiles provide a graphic history of a stock's momentum cycles in terms of occurrence, duration, shape, and outcome.
- The custom index with its variable members aligns with the time-series design or approach to momentum investing (no sorting or ranking of stocks).

9.8 FUTURE RESEARCH

Suggestions for future research are the following:

- Study shorter-term (as opposed to medium-term) momentum based on the term-structure concept.
- Study the informational value of constructing momentum curves, related to the term structure of momentum, for individual stocks.
- Study the possible correlation between index levels, number of members, additions, deletions, and the outcomes (loss or gain) on exit.
- Approach the study from a portfolio perspective by accounting for trading costs, total returns, and stock selection constrained by portfolio size.

RESULTS: STATISTICAL TESTS (JSE)

A.1 DESCRIPTIVE STATISTICS

The statistical analysis for this study was generated using the Real Statistics Resource Pack software for Excel (Release 8.3.1), Copyright (2013-2022) by Charles Zaiontz (RealStats 2022).

Since the skewness and kurtosis of the normal distribution are zero, these two parameters should be close to zero for data to follow a normal distribution. Rough measures of the standard errors of skewness and kurtosis are sqrt(6/n) and sqrt(24/n) respectively, where n is the sample size. The data are not symmetric (and therefore not normal) or normal if the absolute values of skewness and kurtosis are more than twice their standard errors.

Table A.1 Descriptive statistics: JSE Momentum Index (JSE-MI)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	JSE-MI		*JSE-MI*	alpha	0.05
Mean	0.0725	W-stat	0.8344		
Standard Error	0.0203	p-value	0		JSE-MI
Median	0.1096	alpha	0.05	outlier	-13.0657
Standard Deviation	1.1545	normal	no	G	11.3801
Sample Variance	1.3328			G-crit	4.1624
Kurtosis	21.1831	d'Agostino-Pearson		sig	yes
Skewness	-0.3633			ESD outliers	36
Range	25.5568	DA-stat	892.2956		
Maximum	12.4911	p-value	0		
Minimum	-13.0657	alpha	0.05		
Sum	235.5746	normal	no		
Count	3249				
CV	15.9225				

Table A.2 Descriptive statistics: FTSE/JSE All Share Index (ALSH)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	ALSH		*ALSH*	alpha	0.05
Mean	0.0379	W-stat	0.9518		
Standard Error	0.0194	p-value	0		ALSH
Median	0.0624	alpha	0.05	outlier	-10.2268
Standard Deviation	1.1083	normal	no	G	9.2620
Sample Variance	1.2282			G-crit	4.1624
Kurtosis	6.2297	d'Agostino-Pearson		sig	yes
Skewness	-0.4809			ESD outliers	12
Range	17.4883	DA-stat	540.3527		
Maximum	7.2615	p-value	0		
Minimum	-10.2268	alpha	0.05		
Sum	123.1649	normal	no		
Count	3249				
CV	29.2350				

Table A.3 Descriptive statistics: FTSE/JSE Top 40 Index (TOPI)

Descriptive Statistics	TOPI
Mean	0.0381
Standard Error	0.0209
Median	0.0753
Standard Deviation	1.1927
Sample Variance	1.4226
Kurtosis	5.1139
Skewness	-0.3548
Range	18.3575
Maximum	7.9071
Minimum	-10.4504
Sum	123.7915
Count	3249
CV	31.3041

Shapiro-Wilk Test	TOPI
W-stat	0.9589
p-value	0
alpha	0.05
normal	no
d'Agostino-Pearson	
DA-stat	433.8814
p-value	0
alpha	0.05
normal	no

Grubbs/ESD Test	
alpha	0.05
	TOPI
outlier	-10.4504
G	8.7937
G-crit	4.1624
sig	yes
ESD outliers	9

Table A.4 Descriptive statistics: FTSE/JSE Top 40 Equally-Weighted Index (ETOP)

Descriptive Statistics	ETOP
Mean	0.0309
Standard Error	0.0204
Median	0.0492
Standard Deviation	1.1632
Sample Variance	1.3530
Kurtosis	6.2593
Skewness	-0.5219
Range	16.5691
Maximum	6.4383
Minimum	-10.1308
Sum	100.4518
Count	3249
CV	37.6221

Shapiro-Wilk Test	ETOP
W-stat	0.9452
p-value	0
alpha	0.05
normal	no
d'Agostino-Pearson	
DA-stat	559.8034
p-value	0
alpha	0.05
normal	no

Grubbs/ESD Test	
alpha	0.05
	ETOP
outlier	-10.1308
G	8.7361
G-crit	4.1624
sig	yes
ESD outliers	16

Table A.5 Descriptive statistics: FTSE/JSE Mid Cap Index (MIDC)

Descriptive Statistics	MIDC
Mean	0.0338
Standard Error	0.0170
Median	0.0610
Standard Deviation	0.9667
Sample Variance	0.9344
Kurtosis	16.0612
Skewness	-1.3906
Range	16.8638
Maximum	5.6495
Minimum	-11.2143
Sum	109.6623
Count	3249
CV	28.6396

Shapiro-Wilk Test	MIDC
W-stat	0.9011
p-value	0
alpha	0.05
normal	no
d'Agostino-Pearson	
DA-stat	1348.9422
p-value	0
alpha	0.05
normal	no

Grubbs/ESD Test	
alpha	0.05
	MIDC
outlier	-11.2143
G	11.6360
G-crit	4.1624
sig	yes
ESD outliers	21

Table A.6 Descriptive statistics: FTSE/JSE Small Cap Index (SMLC)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	SMLC		*SMLC*	alpha	0.05
Mean	0.0335	W-stat	0.8035		
Standard Error	0.0135	p-value	0		SMLC
Median	0.0466	alpha	0.05	outlier	-11.2994
Standard Deviation	0.7719	normal	no	G	14.6825
Sample Variance	0.5958			G-crit	4.1624
Kurtosis	41.4286	d'Agostino-Pearson		sig	yes
Skewness	-1.3271			ESD outliers	35
Range	21.5912	DA-stat	1619.4544		
Maximum	10.2918	p-value	0		
Minimum	-11.2994	alpha	0.05		
Sum	108.9417	normal	no		
Count	3249				
CV	23.0196				

Table A.7 Descriptive statistics: FTSE/JSE Fledgling Index (FLED)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	FLED		*FLED*	alpha	0.05
Mean	0.0292	W-stat	0.9232		
Standard Error	0.0112	p-value	0		FLED
Median	0.0368	alpha	0.05	outlier	-6.0702
Standard Deviation	0.6396	normal	no	G	9.5367
Sample Variance	0.4091			G-crit	4.1624
Kurtosis	10.8615	d'Agostino-Pearson		sig	yes
Skewness	-0.4182			ESD outliers	19
Range	11.7725	DA-stat	690.5446		
Maximum	5.7023	p-value	0		
Minimum	-6.0702	alpha	0.05		
Sum	95.0196	normal	no		
Count	3249				
CV	21.8690				

Table A.8 Descriptive statistics: FTSE/JSE Large Cap Index (LARG)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	LARG		*LARG*	alpha	0.05
Mean	0.0320	W-stat	0.9226		
Standard Error	0.0341	p-value	0		LARG
Median	0.0607	alpha	0.05	outlier	-10.0881
Standard Deviation	1.2294	normal	no	G	8.2315
Sample Variance	1.5115			G-crit	3.9425
Kurtosis	8.9279	d'Agostino-Pearson		sig	yes
Skewness	-0.6622			ESD outliers	8
Range	18.3732	DA-stat	302.6149		
Maximum	8.2851	p-value	0		
Minimum	-10.0881	alpha	0.05		
Sum	41.6057	normal	no		
Count	1299				
CV	38.3850				

Table A.9 Descriptive statistics: FTSE/JSE Large & Mid Cap Index (LARM)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	LARM		*LARM*	alpha	0.05
Mean	0.0281	W-stat	0.9068		
Standard Error	0.0326	p-value	0		LARM
Median	0.0505	alpha	0.05	outlier	-10.2816
Standard Deviation	1.1758	normal	no	G	8.7685
Sample Variance	1.3824			G-crit	3.9425
Kurtosis	10.8416	d'Agostino-Pearson		sig	yes
Skewness	-0.9335			ESD outliers	11
Range	17.7340	DA-stat	388.7655		
Maximum	7.4524	p-value	0		
Minimum	-10.2816	alpha	0.05		
Sum	36.5272	normal	no		
Count	1299				
CV	41.8134				

Table A.10 Descriptive statistics: S&P Momentum South Africa (SPMZ)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	SPMZ		*SPMZ*	alpha	0.05
Mean	0.0122	W-stat	0.8535		
Standard Error	0.0366	p-value	0		SPMZ
Median	0.0415	alpha	0.05	outlier	-15.9864
Standard Deviation	1.5406	normal	no	G	10.3848
Sample Variance	2.3734			G-crit	4.0192
Kurtosis	17.9250	d'Agostino-Pearson		sig	yes
Skewness	-0.8145			ESD outliers	18
Range	26.6873	DA-stat	582.3052		
Maximum	10.7009	p-value	0		
Minimum	-15.9864	alpha	0.05		
Sum	21.6844	normal	no		
Count	1776				
CV	126.1772				

Table A.11 Descriptive statistics: SATRIX Momentum Index Fund (STXM)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	STXM		*STXM*	alpha	0.05
Mean	0.0256	W-stat	0.9238		
Standard Error	0.0271	p-value	0		STXM
Median	0.0708	alpha	0.05	outlier	-10.4264
Standard Deviation	1.2269	normal	no	G	8.5192
Sample Variance	1.5052			G-crit	4.0534
Kurtosis	8.7507	d'Agostino-Pearson		sig	yes
Skewness	-0.8238			ESD outliers	15
Range	17.1775	DA-stat	520.4443		
Maximum	6.7511	p-value	0		
Minimum	-10.4264	alpha	0.05		
Sum	52.3424	normal	no		
Count	2047				
CV	47.9805				

A.2 ANALYSIS OF VARIANCE

The single factor analysis-of-variance (ANOVA) tests for differences in averages.

Table A.12 Analysis of variance: Momentum Score (MS)

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	*Count*	*Sum*	*Mean*	*Variance*	*SS*	*Std Err*	*Lower*	*Upper*
FAL	330	8480	25.6970	681.1115	224085.7	1.3735	23.0041	28.3898
NEU	726	19128	26.3471	631.5069	457842.5	0.9260	24.5316	28.1626
POS	1878	54369	28.9505	697.7040	1309590.4	0.5758	27.8217	30.0793
NEG	1272	33438	26.2877	491.3522	624508.7	0.6996	24.9162	27.6593

ANOVA

Sources	*SS*	*df*	*MS*	*F*	*P value*	*Eta-sq*	*RMSSE*	*Omega Sq*
Between Groups	7843.33	3	2614.444	4.1995	0.0056	0.0030	0.0581	0.0023
Within Groups	2616027.3	4202	622.5672					
Total	2623870.6	4205	623.9883					

TUKEY HSD/KRAMER — alpha 0.05

group	*mean*	*n*	*ss*	*df*	*q-crit*
FAL	25.6970	330	224085.7		
NEU	26.3471	726	457842.5		
POS	28.9505	1878	1309590.4		
NEG	26.2877	1272	624508.7		
		4206	2616027.3	4202	3.633

Q TEST

group 1	*group 2*	*mean*	*std err*	*q-stat*	*lower*	*upper*	*p-value*	*mean-crit*	*Cohen d*
FAL	NEU	0.6501	1.1713	0.5550	-3.6054	4.9056	0.9795	4.2555	0.0261
FAL	POS	3.2535	1.0531	3.0894	-0.5724	7.0795	0.1278	3.8259	0.1304
FAL	NEG	0.5908	1.0900	0.5420	-3.3690	4.5506	0.9809	3.9598	0.0237
NEU	POS	2.6034	0.7710	3.3764	-0.1979	5.4046	0.0797	2.8012	0.1043
NEU	NEG	0.0594	0.8207	0.0723	-2.9221	3.0408	1.0000	2.9815	0.0024
POS	NEG	2.6627	0.6407	4.1561	0.3351	4.9903	0.0175	2.3276	0.1067

Shapiro-Wilk Test

	FAL	*NEU*	*POS*	*NEG*
W-stat	0.7320	0.6736	0.7883	0.8208
p-value	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	4.0867
df1	3
df2	1221.3248
p-value	0.0067
sig	yes

d'Agostino-Pearson

	FAL	NEU	POS	NEG
DA-stat	264.4214	747.9483	1179.5080	726.7129
p-value	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

GAMES HOWELL — alpha 0.05

group	*mean*	*size*	*variance*
FAL	25.6970	330	681.1115
NEU	26.3471	726	631.5069
POS	28.9505	1878	697.7040
NEG	26.2877	1272	491.3522

Levene's Tests

type	p-value	
means	0.0018	[< 0.05]
medians	0.0106	[< 0.05]
trimmed	0.0073	[< 0.05]

Grubbs/ESD Test

FAL	16
NEU	18
POS	37
NEG	25

Q TEST

group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
FAL	NEU	0.6501	1.2112	0.5368	615.1614	3.6330	-3.7500	5.0503	0.9814	4.4001
FAL	POS	3.2535	1.1035	2.9483	455.5124	3.6467	-0.7707	7.2777	0.1596	4.0242
FAL	NEG	0.5908	1.1069	0.5337	459.5057	3.6466	-3.4455	4.6270	0.9817	4.0362
NEU	POS	2.6034	0.7878	3.3045	1379.3648	3.6330	-0.2588	5.4656	0.0904	2.8622
NEU	NEG	0.0594	0.7925	0.0749	1359.0192	3.6330	-2.8198	2.9385	0.9999	2.8792
POS	NEG	2.6627	0.6155	4.3258	3007.6331	3.6330	0.4265	4.8990	0.0120	2.2363

Unequal variances: Yes

Normally distributed: No

Significantly different: Yes (BG;POS/NEG)

Table A.13 Analysis of variance: Volatility Score (VS)

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
FAL	330	197.34	0.5980	0.2804	92.27	0.0288	0.5415	0.6545
NEU	726	417.13	0.5746	0.2491	180.63	0.0194	0.5365	0.6127
POS	1878	1166.11	0.6209	0.2694	505.68	0.0121	0.5972	0.6446
NEG	1272	772.03	0.6069	0.2942	373.91	0.0147	0.5782	0.6357

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	1.16	3	0.3851	1.4040	0.2396	0.0010	0.0372	0.0003
Within Groups	1152.49	4202	0.2743					
Total	1153.65	4205	0.2744					

TUKEY HSD/KRAMER — alpha 0.05

group	mean	n	ss	df	q-crit
FAL	0.5980	330	92.27		
NEU	0.5746	726	180.63		
POS	0.6209	1878	505.68		
NEG	0.6069	1272	373.91		
		4206	1152.49	4202	3.633

Q TEST

group 1	group 2	mean	std err	q-stat	lower	upper	p-value	mean-crit	Cohen d
FAL	NEU	0.0234	0.0246	0.9534	-0.0659	0.1128	0.9069	0.0893	0.0448
FAL	POS	0.0229	0.0221	1.0375	-0.0574	0.1032	0.8836	0.0803	0.0438
FAL	NEG	0.0089	0.0229	0.3909	-0.0742	0.0921	0.9926	0.0831	0.0171
NEU	POS	0.0464	0.0162	2.8654	-0.0124	0.1052	0.1786	0.0588	0.0885
NEU	NEG	0.0324	0.0172	1.8800	-0.0302	0.0950	0.5442	0.0626	0.0618
POS	NEG	0.0140	0.0134	1.0403	-0.0349	0.0628	0.8827	0.0489	0.0267

Shapiro-Wilk Test

	FAL	NEU	POS	NEG
W-stat	0.6043	0.6818	0.6912	0.6351
p-value	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	1.5019
df1	3
df2	1230.0977
p-value	0.2124
sig	no

d'Agostino-Pearson

	FAL	NEU	POS	NEG
DA-stat	378.7826	595.8092	1553.7397	1244.6717
p-value	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

GAMES HOWELL — alpha 0.05

group	mean	size	variance
FAL	0.5980	330	0.2804
NEU	0.5746	726	0.2491
POS	0.6209	1878	0.2694
NEG	0.6069	1272	0.2942

Levene's Tests

type	p-value	
means	0.7096	[> 0.05]
medians	0.7346	[> 0.05]
trimmed	0.6715	[> 0.05]

Grubbs/ESD Test

FAL	13
NEU	21
POS	40+
NEG	38

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
FAL	NEU	0.0234	0.0244	0.9598	603.6827	3.6330	-0.0653	0.1122	0.9052	0.0887
FAL	POS	0.0229	0.0223	1.0290	447.2138	3.6470	-0.0583	0.1042	0.8860	0.0813
FAL	NEG	0.0089	0.0232	0.3846	522.4228	3.6330	-0.0755	0.0934	0.9930	0.0845
NEU	POS	0.0464	0.0156	2.9729	1365.6565	3.6330	-0.0103	0.1030	0.1529	0.0567
NEU	NEG	0.0324	0.0169	1.9107	1613.4891	3.6330	-0.0292	0.0940	0.5304	0.0616
POS	NEG	0.0140	0.0137	1.0220	2647.0991	3.6330	-0.0357	0.0637	0.8881	0.0497

Unequal variances: No

Normally distributed: No

Significantly different: No

Table A.14 Analysis of variance: Quality Score (QS)

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
FAL	330	17486	52.9879	16.1944	5327.95	0.2385	52.5202	53.4555
NEU	726	39448	54.3361	18.2207	13209.99	0.1608	54.0208	54.6514
POS	1878	101198	53.8860	19.1197	35887.61	0.1000	53.6900	54.0821
NEG	1272	68429	53.7964	19.2512	24468.26	0.1215	53.5582	54.0346

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	421.81	3	140.6050	7.4888	0.0001	0.0053	0.1295	0.0046
Within Groups	78893.82	4202	18.7753					
Total	79315.64	4205	18.8622					

TUKEY HSD/KRAMER — alpha 0.05

group	mean	n	ss	df	q-crit
FAL	52.9879	330	5327.95		
NEU	54.3361	726	13209.99		
POS	53.8860	1878	35887.61		
NEG	53.7964	1272	24468.26		
		4206	78893.82	4202	3.633

Q TEST

group 1	group 2	mean	std err	q-stat	lower	upper	p-value	mean-crit	Cohen d
FAL	NEU	1.3482	0.2034	6.6278	0.6092	2.0872	0.0000	0.7390	0.3111
FAL	POS	0.8982	0.1829	4.9112	0.2338	1.5626	0.0029	0.6644	0.2073
FAL	NEG	0.8085	0.1893	4.2714	0.1208	1.4962	0.0136	0.6877	0.1866
NEU	POS	0.4500	0.1339	3.3610	-0.0364	0.9365	0.0819	0.4865	0.1039
NEU	NEG	0.5397	0.1425	3.7870	0.0219	1.0575	0.0374	0.5178	0.1246
POS	NEG	0.0897	0.1113	0.8059	-0.3145	0.4939	0.9410	0.4042	0.0207

Shapiro-Wilk Test

	FAL	NEU	POS	NEG
W-stat	0.9469	0.9682	0.9656	0.9524
p-value	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	8.2789
df1	3
df2	1247.9959
p-value	0.0000
sig	yes

d'Agostino-Pearson

DA-stat	46.2696	75.5706	205.2552	141.5946
p-value	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

GAMES HOWELL — alpha 0.05

group	mean	size	variance
FAL	52.9879	330	16.1944
NEU	54.3361	726	18.2207
POS	53.8860	1878	19.1197
NEG	53.7964	1272	19.2512

Levene's Tests

type	p-value	
means	0.0798	[> 0.05]
medians	0.1446	[> 0.05]
trimmed	0.0903	[> 0.05]

Grubbs/ESD Test

FAL	0
NEU	3
POS	2
NEG	1

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
FAL	NEU	1.3482	0.1926	7.0009	671.8255	3.6330	0.6486	2.0478	0.0000	0.6996
FAL	POS	0.8982	0.1721	5.2181	476.0772	3.6461	0.2706	1.5258	0.0014	0.6276
FAL	NEG	0.8085	0.1792	4.5123	549.6886	3.6330	0.1576	1.4595	0.0082	0.6509
NEU	POS	0.4500	0.1328	3.3885	1346.8934	3.6330	-0.0325	0.9325	0.0783	0.4825
NEU	NEG	0.5397	0.1418	3.8053	1542.9826	3.6330	0.0244	1.0550	0.0362	0.5153
POS	NEG	0.0897	0.1125	0.7970	2722.0418	3.6330	-0.3191	0.4984	0.9428	0.4087

Unequal variances: No

Normally distributed: No

Significantly different: Yes (BG;FAL/NEU;FAL/POS;FAL/NEG;NEU/NEG)

Table A.15 Analysis of variance: Activity Score (AS)

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
FAL	330	15996	48.4727	41.9157	13790.25	0.3817	47.7244	49.2211
NEU	726	35346	48.6860	51.6171	37422.40	0.2574	48.1814	49.1905
POS	1878	90435	48.1550	50.9504	95633.91	0.1600	47.8412	48.4687
NEG	1272	61748	48.5440	43.4332	55203.53	0.1944	48.1628	48.9252

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	199.78	3	66.5917	1.3849	0.2454	0.0010	0.0324	0.0003
Within Groups	202050.09	4202	48.0843					
Total	202249.87	4205	48.0975					

TUKEY HSD/KRAMER — alpha 0.05

group	mean	n	ss	df	q-crit
FAL	48.4727	330	13790.25		
NEU	48.6860	726	37422.40		
POS	48.1550	1878	95633.91		
NEG	48.5440	1272	55203.53		
		4206	202050.09	4202	3.633

Q TEST

group 1	group 2	mean	std err	q-stat	lower	upper	p-value	mean-crit	Cohen d
FAL	NEU	0.2132	0.3255	0.6550	-0.9694	1.3959	0.9670	1.1827	0.0307
FAL	POS	0.3178	0.2927	1.0858	-0.7455	1.3811	0.8690	1.0633	0.0458
FAL	NEG	0.0713	0.3029	0.2354	-1.0292	1.1718	0.9984	1.1005	0.0103
NEU	POS	0.5310	0.2143	2.4780	-0.2475	1.3095	0.2970	0.7785	0.0766
NEU	NEG	0.1419	0.2281	0.6223	-0.6867	0.9705	0.9715	0.8286	0.0205
POS	NEG	0.3891	0.1781	2.1851	-0.2578	1.0359	0.4106	0.6469	0.0561

Shapiro-Wilk Test

	FAL	NEU	POS	NEG
W-stat	0.9831	0.9651	0.9802	0.9773
p-value	0.0006	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	1.3359
df1	3
df2	1242.1783
p-value	0.2612
sig	no

d'Agostino-Pearson

	FAL	NEU	POS	NEG
DA-stat	2.6861	33.7348	50.7363	32.8382
p-value	0.2611	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	yes	no	no	no

GAMES HOWELL — alpha 0.05

group	mean	size	variance
FAL	48.4727	330	41.9157
NEU	48.6860	726	51.6171
POS	48.1550	1878	50.9504
NEG	48.5440	1272	43.4332

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0001	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

FAL	0
NEU	0
POS	0
NEG	0

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
FAL	NEU	0.2132	0.3147	0.6775	700.7621	3.6330	-0.9302	1.3567	0.9637	1.1434
FAL	POS	0.3178	0.2776	1.1446	480.7107	3.6330	-0.6908	1.3264	0.8500	1.0086
FAL	NEG	0.0713	0.2839	0.2512	519.9375	3.6330	-0.9600	1.1026	0.9980	1.0313
NEU	POS	0.5310	0.2216	2.3960	1310.1823	3.6330	-0.2741	1.3361	0.3271	0.8051
NEU	NEG	0.1419	0.2294	0.6187	1403.8939	3.6330	-0.6915	0.9753	0.9720	0.8334
POS	NEG	0.3891	0.1750	2.2228	2867.3670	3.6330	-0.2468	1.0250	0.3950	0.6359

Unequal variances: Yes

Normally distributed: Yes (FAL)

Significantly different: No

Table A.16 Analysis of variance: MS060-MS250

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
MS060	701	14835	21.1626	428.528	299969	0.9268	19.3457	22.9796
MS090	701	20887	29.7960	611.691	428184	0.9268	27.9791	31.6129
MS125	701	23589	33.6505	685.745	480021	0.9268	31.8336	35.4674
MS180	701	21906	31.2496	557.093	389965	0.9268	29.4327	33.0666
MS210	701	19268	27.4864	602.333	421633	0.9268	25.6695	29.3034
MS250	701	14930	21.2981	727.038	508927	0.9268	19.4812	23.1151

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	95171	5	19034.17	31.6145	0.0000	0.0363	0.2124	0.0351
Within Groups	2528700	4200	602.07					
Total	2623871	4205	623.99					

Shapiro-Wilk Test

	MS060	MS090	MS125	MS180	MS210	MS250
W-stat	0.7894	0.8060	0.6891	0.8036	0.7839	0.6748
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

d'Agostino-Pearson

DA-stat	496.5503	425.8284	628.4598	322.2812	482.7768	656.0872
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	32.786
df1	5
df2	1957.413
p-value	0.0000
sig	yes

Levene's Tests

type	p-value	
means	0.2353	[> 0.05]
medians	0.3786	[> 0.05]
trimmed	0.3536	[> 0.05]

Grubbs/ESD Test

MS060	12
MS090	13
MS125	13
MS180	28
MS210	19
MS250	22

GAMES HOWELL — alpha 0.05

group	mean	size	variance
MS060	21.1626	701	428.528
MS090	29.7960	701	611.691
MS125	33.6505	701	685.745
MS180	31.2496	701	557.093
MS210	27.4864	701	602.333
MS250	21.2981	701	727.038

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
MS060	MS090	8.6334	0.8614	10.0229	1357.899	4.0300	5.1621	12.1047	0.0000	3.4713
MS060	MS125	12.4879	0.8915	14.0077	1329.173	4.0300	8.8951	16.0806	0.0000	3.5927
MS060	MS180	10.0870	0.8385	12.0304	1376.578	4.0300	6.7080	13.4660	0.0000	3.3790
MS060	MS210	6.3238	0.8575	7.3749	1361.303	4.0300	2.8682	9.7795	0.0000	3.4557
MS060	MS250	0.1355	0.9079	0.1493	1312.421	4.0300	-3.5232	3.7942	1.0000	3.6587
MS090	MS125	3.8545	0.9620	4.0068	1395.454	4.0300	-0.0223	7.7313	0.0529	3.8768
MS090	MS180	1.4536	0.9130	1.5921	1396.952	4.0300	-2.2259	5.1332	0.8708	3.6796
MS090	MS210	2.3096	0.9305	2.4819	1399.917	4.0300	-1.4406	6.0597	0.4955	3.7501
MS090	MS250	8.4979	0.9772	8.6964	1389.683	4.0300	4.5598	12.4359	0.0000	3.9380
MS125	MS180	2.4009	0.9415	2.5500	1385.158	4.0300	-1.3935	6.1952	0.4640	3.7944
MS125	MS210	6.1641	0.9585	6.4309	1394.154	4.0300	2.3013	10.0269	0.0001	3.8628
MS125	MS250	12.3524	1.0038	12.3051	1398.805	4.0300	8.3069	16.3978	0.0000	4.0455
MS180	MS210	3.7632	0.9094	4.1382	1397.872	4.0300	0.0984	7.4280	0.0407	3.6648
MS180	MS250	9.9515	0.9570	10.3982	1375.902	4.0300	6.0946	13.8084	0.0000	3.8569
MS210	MS250	6.1883	0.9738	6.3551	1387.788	4.0300	2.2641	10.1125	0.0001	3.9242

Unequal variances: No

Normally distributed: No

Significantly different: Yes (All pairings except MS060/MS250, MS090/MS180, MS090/MS125, MS090/MS210, MS125/MS180)

Table A.17 Analysis of variance: VS060-VS250

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
VS060	701	666.7	0.9511	0.561	393	0.0183	0.9153	0.9869
VS090	701	541.58	0.7726	0.381	266	0.0183	0.7368	0.8084
VS125	701	442.24	0.6309	0.199	139	0.0183	0.5951	0.6667
VS180	701	327.57	0.4673	0.090	63	0.0183	0.4315	0.5031
VS210	701	300.03	0.4280	0.081	57	0.0183	0.3922	0.4638
VS250	701	274.49	0.3916	0.091	64	0.0183	0.3558	0.4274

ANOVA

Sources	SS	df	VS	F	P value	Eta-sq	RVSSE	Omega Sq
Between Groups	171	5	34.26	146.4605	0.0000	0.1485	0.4571	0.1474
Within Groups	982	4200	0.23					
Total	1154	4205	0.27					

Shapiro-Wilk Test

	VS060	VS090	VS125	VS180	VS210	VS250
W-stat	0.6846	0.6718	0.7281	0.7748	0.7349	0.6703
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

d'Agostino-Pearson

DA-stat	549.3660	601.5140	448.7005	507.4404	616.4507	654.2806
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	118.546
df1	5
df2	1933.335
p-value	0.0000
sig	yes

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0000	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

VS060	27
VS090	26
VS125	22
VS180	5
VS210	9
VS250	14

GAMES HOWELL — alpha 0.05

group	mean	size	variance
VS060	0.9511	701	0.561
VS090	0.7726	701	0.381
VS125	0.6309	701	0.199
VS180	0.4673	701	0.090
VS210	0.4280	701	0.081
VS250	0.3916	701	0.091

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
VS060	VS090	0.1785	0.0259	6.8859	1350.331	4.0300	0.0740	0.2829	0.0000	0.1045
VS060	VS125	0.3202	0.0233	13.7541	1140.097	4.0300	0.2264	0.4140	0.0000	0.0938
VS060	VS180	0.4838	0.0216	22.4403	919.521	4.0300	0.3969	0.5707	0.0000	0.0869
VS060	VS210	0.5231	0.0214	24.4358	898.114	4.0300	0.4368	0.6093	0.0000	0.0863
VS060	VS250	0.5595	0.0216	25.9287	922.291	4.0300	0.4725	0.6465	0.0000	0.0870
VS090	VS125	0.1417	0.0203	6.9722	1274.015	4.0300	0.0598	0.2236	0.0000	0.0819
VS090	VS180	0.3053	0.0183	16.6572	1014.393	4.0300	0.2314	0.3792	0.0000	0.0739
VS090	VS210	0.3446	0.0181	18.9872	985.289	4.0300	0.2714	0.4177	0.0000	0.0731
VS090	VS250	0.3810	0.0184	20.7622	1018.121	4.0300	0.3071	0.4550	0.0000	0.0740
VS125	VS180	0.1636	0.0144	11.3971	1227.604	4.0300	0.1057	0.2214	0.0000	0.0578
VS125	VS210	0.2029	0.0141	14.3650	1190.080	4.0300	0.1460	0.2598	0.0000	0.0569
VS125	VS250	0.2393	0.0144	16.6382	1232.179	4.0300	0.1813	0.2973	0.0000	0.0580
VS180	VS210	0.0393	0.0111	3.5533	1395.975	4.0300	-0.0053	0.0838	0.1210	0.0446
VS180	VS250	0.0757	0.0114	6.6498	1399.939	4.0300	0.0298	0.1216	0.0000	0.0459
VS210	VS250	0.0364	0.0111	3.2838	1394.931	4.0300	-0.0083	0.0811	0.1858	0.0447

Unequal variances: Yes

Normally distributed: No

Significantly different: Yes (All pairings except VS180/VS210, VS210/VS250)

Table A.18 Analysis of variance: QS060-QS250

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	*Count*	*Sum*	*Mean*	*Variance*	*SS*	*Std Err*	*Lower*	*Upper*
QS060	701	39377	56.1726	33.969	23778	0.1517	55.8752	56.4700
QS090	701	38914	55.5121	20.733	14513	0.1517	55.2147	55.8096
QS125	701	38226	54.5307	13.661	9563	0.1517	54.2332	54.8281
QS180	701	37158	53.0071	10.141	7099	0.1517	52.7097	53.3046
QS210	701	36669	52.3096	9.611	6728	0.1517	52.0121	52.6070
QS250	701	36217	51.6648	8.692	6084	0.1517	51.3673	51.9622

ANOVA

Sources	*SS*	*df*	*QS*	*F*	*P value*	*Eta-sq*	*RQSSE*	*Omega Sq*
Between Groups	11551	5	2310.16	143.1812	0.0000	0.1456	0.4519	0.1446
Within Groups	67765	4200	16.13					
Total	79316	4205	18.86					

Shapiro-Wilk Test

	QS060	*QS090*	*QS125*	*QS180*	*QS210*	*QS250*
W-stat	0.9904	0.9897	0.9801	0.9741	0.9737	0.9829
p-value	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

d'Agostino-Pearson

DA-stat	11.6464	9.4635	22.2807	40.6497	52.2084	18.2808
p-value	0.0030	0.0088	0.0000	0.0000	0.0000	0.0001
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	140.679
df1	5
df2	1947.773
p-value	0.0000
sig	yes

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0000	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

QS060	2
QS090	0
QS125	1
QS180	2
QS210	2
QS250	2

GAMES HOWELL — alpha 0.05

group	*mean*	*size*	*variance*
QS060	56.1726	701	33.969
QS090	55.5121	701	20.733
QS125	54.5307	701	13.661
QS180	53.0071	701	10.141
QS210	52.3096	701	9.611
QS250	51.6648	701	8.692

Q TEST

group 1	*group 2*	*mean*	*std err*	*q-stat*	*df*	*q-crit*	*lower*	*upper*	*p-value*	*mean-crit*
QS060	QS090	0.6605	0.1975	3.3438	1322.570	4.0300	-0.1355	1.4565	0.1697	0.7960
QS060	QS125	1.6419	0.1843	8.9083	1184.641	4.0300	0.8991	2.3847	0.0000	0.7428
QS060	QS180	3.1655	0.1774	17.8461	1083.765	4.0300	2.4507	3.8803	0.0000	0.7148
QS060	QS210	3.8631	0.1763	21.9110	1066.758	4.0300	3.1525	4.5736	0.0000	0.7105
QS060	QS250	4.5078	0.1744	25.8422	1036.212	4.0300	3.8049	5.2108	0.0000	0.7030
QS090	QS125	0.9815	0.1566	6.2662	1343.207	4.0300	0.3502	1.6127	0.0001	0.6312
QS090	QS180	2.5050	0.1484	16.8803	1252.586	4.0300	1.9070	3.1030	0.0000	0.5980
QS090	QS210	3.2026	0.1471	21.7688	1234.198	4.0300	2.6097	3.7955	0.0000	0.5929
QS090	QS250	3.8474	0.1449	26.5571	1199.181	4.0300	3.2635	4.4312	0.0000	0.5838
QS125	QS180	1.5235	0.1303	11.6928	1370.046	4.0300	0.9984	2.0486	0.0000	0.5251
QS125	QS210	2.2211	0.1288	17.2396	1358.853	4.0300	1.7019	2.7403	0.0000	0.5192
QS125	QS250	2.8659	0.1263	22.6972	1334.071	4.0300	2.3570	3.3748	0.0000	0.5089
QS180	QS210	0.6976	0.1187	5.8770	1398.992	4.0300	0.2192	1.1759	0.0005	0.4783
QS180	QS250	1.3424	0.1159	11.5820	1391.754	4.0300	0.8753	1.8094	0.0000	0.4671
QS210	QS250	0.6448	0.1143	5.6433	1396.476	4.0300	0.1843	1.1053	0.0010	0.4605

Unequal variances: Yes

Normally distributed: Yes (QS060)

Significantly different: Yes (All pairings except QSA060/QS090)

Table A.19 Analysis of variance: AS060-AS250

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
AS060	701	35802	51.0728	51.130	35791	0.2529	50.5770	51.5685
AS090	701	35127	50.1098	44.264	30985	0.2529	49.6141	50.6056
AS125	701	34361	49.0171	42.720	29904	0.2529	48.5213	49.5129
AS180	701	33233	47.4080	42.910	30037	0.2529	46.9122	47.9038
AS210	701	32726	46.6847	43.205	30243	0.2529	46.1890	47.1805
AS250	701	32276	46.0428	44.741	31319	0.2529	45.5470	46.5386

ANOVA

Sources	SS	df	AS	F	P value	Eta-sq	RASSE	Omega Sq
Between Groups	13971	5	2794.18	62.3306	0.0000	0.0691	0.2982	0.0680
Within Groups	188279	4200	44.83					
Total	202250	4205	48.10					

Shapiro-Wilk Test

	AS060	AS090	AS125	AS180	AS210	AS250
W-stat	0.9881	0.9756	0.9602	0.9441	0.9430	0.9322
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

d'Agostino-Pearson

DA-stat	7.8245	48.2479	53.0944	67.8297	57.3274	68.5580
p-value	0.0200	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	60.082
df1	5
df2	1959.726
p-value	0.0000
sig	yes

Levene's Tests

type	p-value	
means	0.3223	[> 0.05]
medians	0.2123	[> 0.05]
trimmed	0.2869	[> 0.05]

Grubbs/ESD Test

AS060	0
AS090	0
AS125	0
AS180	0
AS210	0
AS250	0

GAMES HOWELL — alpha 0.05

group	mean	size	variance
AS060	51.0728	701	51.130
AS090	50.1098	701	44.264
AS125	49.0171	701	42.720
AS180	47.4080	701	42.910
AS210	46.6847	701	43.205
AS250	46.0428	701	44.741

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
AS060	AS090	0.9629	0.2608	3.6915	1392.783	4.0300	-0.0883	2.0141	0.0954	1.0512
AS060	AS125	2.0556	0.2587	7.9452	1388.846	4.0300	1.0130	3.0983	0.0000	1.0427
AS060	AS180	3.6648	0.2590	14.1502	1389.385	4.0300	2.6210	4.7085	0.0000	1.0437
AS060	AS210	4.3880	0.2594	16.9163	1390.187	4.0300	3.3427	5.4334	0.0000	1.0454
AS060	AS250	5.0300	0.2615	19.2351	1393.809	4.0300	3.9761	6.0838	0.0000	1.0538
AS090	AS125	1.0927	0.2491	4.3870	1399.559	4.0300	0.0889	2.0965	0.0240	1.0038
AS090	AS180	2.7019	0.2494	10.8353	1399.663	4.0300	1.6970	3.7068	0.0000	1.0049
AS090	AS210	3.4251	0.2498	13.7127	1399.795	4.0300	2.4185	4.4317	0.0000	1.0066
AS090	AS250	4.0670	0.2520	16.1416	1399.960	4.0300	3.0516	5.0824	0.0000	1.0154
AS125	AS180	1.6091	0.2471	6.5111	1399.993	4.0300	0.6132	2.6051	0.0001	0.9960
AS125	AS210	2.3324	0.2476	9.4214	1399.955	4.0300	1.3347	3.3301	0.0000	0.9977
AS125	AS250	2.9743	0.2498	11.9085	1399.253	4.0300	1.9678	3.9809	0.0000	1.0066
AS180	AS210	0.7233	0.2478	2.9183	1399.984	4.0300	-0.2755	1.7220	0.3072	0.9988
AS180	AS250	1.3652	0.2500	5.4599	1399.390	4.0300	0.3575	2.3728	0.0016	1.0077
AS210	AS250	0.6419	0.2505	2.5631	1399.573	4.0300	-0.3674	1.6513	0.4580	1.0093

Unequal variances: No

Normally distributed: No

Significantly different: Yes (All pairings except AS060/QS090, AS180/AS210, AS210/AS250)

A.3 CORRELATION COEFFICIENTS

When a set of data is not normally distributed or when the presence of outliers gives a distorted picture of the association between two random variables, Spearman's rank correlation is a non-parametric test that substitutes for Pearson's correlation.

The coefficient of determination or correlation-squared indicates how closely two time-series track each other. It also points to the reliability of the alpha (excess return) and beta (volatility) coefficients from a linear regression.

Table A.20 Correlation: JSE-MI/ALSH (2021)

Correlation Coefficients: JSE-MI/ALSH (2021)			
Pearson	0.6813		
Spearman	0.6352		
Kendall	0.4626		
Pearson's coeff (t test)		Pearson's coeff (Fisher)	
Alpha	0.05	Hyp rho	0
Tails	2	Alpha	0.05
		Tails	2
corr	0.6813		
std err	0.0465	corr	0.6813
t	14.6591	std err	0.0634
p-value	0	z	13.0700
lower	0.5898	p-value	0
upper	0.7729	lower	0.6087
		upper	0.7426

Table A.21 Correlation: JSE-MI/ALSH (2019-2021)

Correlation Coefficients: JSE-MI/ALSH (2019-2021)			
Pearson	0.6346		
Spearman	0.4541		
Kendall	0.3211		
Pearson's coeff (t test)		Pearson's coeff (Fisher)	
Alpha	0.05	Hyp rho	0
Tails	2	Alpha	0.05
		Tails	2
corr	0.6346		
std err	0.0282	corr	0.6346
t	22.4736	std err	0.0365
p-value	0	z	20.4879
lower	0.5792	p-value	2.8E-93
upper	0.6901	lower	0.5899
		upper	0.6755

Table A.22 Correlation: JSE-MI/ALSH (2017-2021)

Correlation Coefficients: JSE-MI/ALSH (2017-2021)				
Pearson	0.6187			
Spearman	0.4856			
Kendall	0.3452			
Pearson's coeff (t test)			Pearson's coeff (Fisher)	
Alpha	0.05		Hyp rho	0
Tails	2		Alpha	0.05
			Tails	2
corr	0.6187			
std err	0.0222		corr	0.6187
t	27.8189		std err	0.0283
p-value	0		z	25.5260
lower	0.5750		p-value	1.0E-143
upper	0.6623		lower	0.5832
			upper	0.6518

REFERENCE

RealStats. 2022. Real statistics using Excel [Website]. Charles Zaiontz. Available at: https://www.real-statistics.com.

ANNEXURE B

RESULTS: STATISTICAL TESTS (TSX)

B.1 DESCRIPTIVE STATISTICS

The statistical analysis for this study was generated using the Real Statistics Resource Pack software for Excel (Release 8.3.1), Copyright (2013-2022) by Charles Zaiontz (RealStats 2022).

Since the skewness and kurtosis of the normal distribution are zero, these two parameters should be close to zero for data to follow a normal distribution. Rough measures of the standard errors of skewness and kurtosis are sqrt(6/n) and sqrt(24/n) respectively, where n is the sample size. The data are not symmetric (and therefore not normal) or normal if the absolute values of skewness and kurtosis are more than twice their standard errors.

Table B.1 Descriptive statistics: TSX Momentum Index (TSX-MI)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	TSX-MI		*TSX-MI*	alpha	0.05
Mean	0.0747	W-stat	0.9040		
Standard Error	0.0186	p-value	0		TSX-MI
Median	0.1396	alpha	0.05	outlier	-12.6438
Standard Deviation	1.0606	normal	no	G	11.9916
Sample Variance	1.1249			G-crit	4.1623
Kurtosis	14.0274	d'Agostino-Pearson		sig	yes
Skewness	-1.1402			ESD outliers	23
Range	21.4896	DA-stat	1155.7004		
Maximum	8.8458	p-value	0		
Minimum	-12.6438	alpha	0.05		
Sum	242.7289	normal	no		
Count	3248				
CV	14.1923				

Table B.2 Descriptive statistics: S&P/TSX Composite Index (TXCX)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	TXCX		*TXCX*	alpha	0.05
Mean	0.0265	W-stat	0.8428		
Standard Error	0.0178	p-value	0		TXCX
Median	0.0777	alpha	0.05	outlier	-13.1761
Standard Deviation	1.0117	normal	no	G	13.0498
Sample Variance	1.0236			G-crit	4.1623
Kurtosis	26.2762	d'Agostino-Pearson		sig	yes
Skewness	-1.1870			ESD outliers	26
Range	24.4706	DA-stat	1391.3769		
Maximum	11.2945	p-value	0		
Minimum	-13.1761	alpha	0.05		
Sum	85.9221	normal	no		
Count	3248				
CV	38.2443				

Table B.3 Descriptive statistics: S&P/TSX 60 Index (TXLC)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	TXLC		*TXLC*	alpha	0.05
Mean	0.0266	W-stat	0.8441		
Standard Error	0.0182	p-value	0		TXLC
Median	0.0734	alpha	0.05	outlier	-13.3652
Standard Deviation	1.0384	normal	no	G	12.8969
Sample Variance	1.0782			G-crit	4.1623
Kurtosis	26.2706	d'Agostino-Pearson		sig	yes
Skewness	-0.9957			ESD outliers	25
Range	25.0454	DA-stat	1279.1202		
Maximum	11.6802	p-value	0		
Minimum	-13.3652	alpha	0.05		
Sum	86.5121	normal	no		
Count	3248				
CV	38.9845				

Table B.4 Descriptive statistics: S&P/TSX 60 Equal Weight Index (TXEW)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	TXEW		*TXEW*	alpha	0.05
Mean	0.0309	W-stat	0.8760		
Standard Error	0.0186	p-value	0		TXEW
Median	0.0716	alpha	0.05	outlier	-13.3012
Standard Deviation	1.0573	normal	no	G	12.6096
Sample Variance	1.1179			G-crit	4.1623
Kurtosis	19.8047	d'Agostino-Pearson		sig	yes
Skewness	-1.0539			ESD outliers	22
Range	24.1324	DA-stat	1219.9979		
Maximum	10.8312	p-value	0		
Minimum	-13.3012	alpha	0.05		
Sum	100.3906	normal	no		
Count	3248				
CV	34.2076				

Table B.5 Descriptive statistics: S&P/TSX Completion Index (TXMC)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	TXMC		*TXMC*	alpha	0.05
Mean	0.0264	W-stat	0.8611		
Standard Error	0.0177	p-value	0		TXMC
Median	0.0701	alpha	0.05	outlier	-12.4632
Standard Deviation	1.0110	normal	no	G	12.3536
Sample Variance	1.0221			G-crit	4.1623
Kurtosis	22.2948	d'Agostino-Pearson		sig	yes
Skewness	-1.5608			ESD outliers	20
Range	22.1120	DA-stat	1556.9148		
Maximum	9.6488	p-value	0		
Minimum	-12.4632	alpha	0.05		
Sum	85.6862	normal	no		
Count	3248				
CV	38.3230				

Table B.6 Descriptive statistics: S&P/TSX Small Cap Index (TXSC)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	TXSC		*TXSC*	alpha	0.05
Mean	0.0229	W-stat	0.9054		
Standard Error	0.0209	p-value	0		TXSC
Median	0.0896	alpha	0.05	outlier	-13.7581
Standard Deviation	1.1884	normal	no	G	11.5964
Sample Variance	1.4123			G-crit	4.1623
Kurtosis	14.8056	d'Agostino-Pearson		sig	yes
Skewness	-1.4162			ESD outliers	14
Range	22.4085	DA-stat	1336.3676		
Maximum	8.6503	p-value	0		
Minimum	-13.7581	alpha	0.05		
Sum	74.4559	normal	no		
Count	3248				
CV	51.8414				

Table B.7 Descriptive statistics: S&P/TSX Equity Index (TXEQ)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	TXEQ		*TXEQ*	alpha	0.05
Mean	0.0263	W-stat	0.8474		
Standard Error	0.0179	p-value	0		TXEQ
Median	0.0756	alpha	0.05	outlier	-13.2560
Standard Deviation	1.0194	normal	no	G	13.0299
Sample Variance	1.0391			G-crit	4.1623
Kurtosis	25.6962	d'Agostino-Pearson		sig	yes
Skewness	-1.1394			ESD outliers	24
Range	24.5340	DA-stat	1356.0251		
Maximum	11.2780	p-value	0		
Minimum	-13.2560	alpha	0.05		
Sum	85.3667	normal	no		
Count	3248				
CV	38.7847				

Table B.8 Descriptive statistics: S&P/TSX Composite Equal Weight Index (TXCE)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	TXCE		*TXCE*	alpha	0.05
Mean	0.0158	W-stat	0.8442		
Standard Error	0.0206	p-value	0		TXCE
Median	0.0716	alpha	0.05	outlier	-13.2172
Standard Deviation	1.0389	normal	no	G	12.7370
Sample Variance	1.0794			G-crit	4.1052
Kurtosis	25.9464	d'Agostino-Pearson		sig	yes
Skewness	-1.7213			ESD outliers	16
Range	22.9079	DA-stat	1336.7056		
Maximum	9.6908	p-value	0		
Minimum	-13.2172	alpha	0.05		
Sum	40.0979	normal	no		
Count	2545				
CV	65.9410				

Table B.9 Descriptive statistics: S&P/TSX Composite Momentum Index (TXMM)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	TXMM		*TXMM*	alpha	0.05
Mean	0.0381	W-stat	0.8542		
Standard Error	0.0471	p-value	0		TXMM
Median	0.0744	alpha	0.05	outlier	-10.8683
Standard Deviation	1.3268	normal	no	G	8.2201
Sample Variance	1.7604			G-crit	3.8174
Kurtosis	13.9199	d'Agostino-Pearson		sig	yes
Skewness	-1.0344			ESD outliers	12
Range	20.5417	DA-stat	279.28867		
Maximum	9.6734	p-value	0		
Minimum	-10.8683	alpha	0.05		
Sum	30.2206	normal	no		
Count	793				
CV	34.8156				

B.2 ANALYSIS OF VARIANCE

The single factor analysis-of-variance (ANOVA) tests for differences in averages.

Table B.10 Analysis of variance: Momentum Score (MS)

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
FAL	1248	29393	23.5521	478.8810	597164.6	0.8824	21.8226	25.2816
NEU	2766	77887	28.1587	746.4149	2063837.3	0.5927	26.9970	29.3205
POS	5448	181345	33.2865	1034.8213	5636671.7	0.4223	32.4587	34.1143
NEG	4548	153116	33.6667	1168.0063	5310924.7	0.4622	32.7607	34.5727

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	148441	3	49480.403	50.9253	0.0000	0.0108	0.1536	0.0106
Within Groups	13608598	14006	971.6263					
Total	13757040	14009	982.0144					

TUKEY HSD/KRAMER — alpha 0.05

group	mean	n	ss	df	q-crit
FAL	23.5521	1248	597165		
NEU	28.1587	2766	2063837		
POS	33.2865	5448	5636672		
NEG	33.6667	4548	5310925		
		14010	13608598	14006	3.633

Q TEST

group 1	group 2	mean	std err	q-stat	lower	upper	p-value	mean-crit	Cohen d
FAL	NEU	4.6066	0.7516	6.1291	1.8760	7.3372	0.0001	2.7306	0.1478
FAL	POS	9.7344	0.6917	14.0733	7.2215	12.2474	0.0000	2.5129	0.3123
FAL	NEG	10.1146	0.7043	14.3604	7.5557	12.6734	0.0000	2.5589	0.3245
NEU	POS	5.1278	0.5146	9.9647	3.2583	6.9973	0.0000	1.8695	0.1645
NEU	NEG	5.5080	0.5315	10.3637	3.5771	7.4388	0.0000	1.9308	0.1767
POS	NEG	0.3801	0.4427	0.8587	-1.2282	1.9885	0.9298	1.6084	0.0122

Welch's Test

Alpha	0.05
F-stat	75.3272
df1	3
df2	5172.2470
p-value	0.0000
sig	yes

Shapiro-Wilk Test

	FAL	NEU	POS	NEG
W-stat	0.8819	0.7608	N/A	0.7197
p-value	0.0000	0.0000	N/A	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	N/A	no

d'Agostino-Pearson

	FAL	NEU	POS	NEG
DA-stat	530.8703	2148.0886	2953.8693	4145.3681
p-value	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0000	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

FAL	18
NEU	40+
POS	40+
NEG	40+

GAMES HOWELL — alpha 0.05

group	mean	size	variance
FAL	23.5521	1248	478.8810
NEU	28.1587	2766	746.4149
POS	33.2865	5448	1034.8213
NEG	33.6667	4548	1168.0063

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
FAL	NEU	4.6066	0.5717	8.0584	2957.9008	3.6330	2.5298	6.6834	0.0000	2.0768
FAL	POS	9.7344	0.5356	18.1760	2639.0747	3.6330	7.7887	11.6802	0.0000	1.9457
FAL	NEG	10.1146	0.5659	17.8727	3094.6198	3.6330	8.0586	12.1706	0.0000	2.0560
NEU	POS	5.1278	0.4795	10.6946	6414.2152	3.6330	3.3859	6.8698	0.0000	1.7419
NEU	NEG	5.5080	0.5132	10.7334	6791.6130	3.6330	3.6436	7.3723	0.0000	1.8643
POS	NEG	0.3801	0.4726	0.8043	9446.6308	3.6330	-1.3369	2.0972	0.9414	1.7171

Unequal variances: Yes

Normally distributed: No

Significantly different: Yes (All pairings except POS/NEG)

Table B.11 Analysis of variance: Volatility Score (VS)

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
FAL	1248	956.35	0.7663	0.7574	944.5	0.0231	0.7210	0.8117
NEU	2766	1968.66	0.7117	0.5698	1575.5	0.0155	0.6813	0.7422
POS	5448	4427.92	0.8128	0.6164	3357.7	0.0111	0.7911	0.8345
NEG	4548	3810.51	0.8378	0.7648	3477.7	0.0121	0.8141	0.8616

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	30.22	3	10.075	15.0828	0.0000	0.0032	0.0679	0.0030
Within Groups	9355.5	14006	0.6680					
Total	9385.7	14009	0.6700					

TUKEY HSD/KRAMER — alpha 0.05

group	mean	n	ss	df	q-crit
FAL	0.7663	1248	944.5		
NEU	0.7117	2766	1575.5		
POS	0.8128	5448	3357.7		
NEG	0.8378	4548	3477.7		
		14010	9355.5	14006	3.633

Q TEST

group 1	group 2	mean	std err	q-stat	lower	upper	p-value	mean-crit	Cohen d
FAL	NEU	0.0546	0.0197	2.7691	-0.0170	0.1262	0.2044	0.0716	0.0668
FAL	POS	0.0465	0.0181	2.5614	-0.0194	0.1123	0.2682	0.0659	0.0568
FAL	NEG	0.0715	0.0185	3.8737	0.0044	0.1386	0.0315	0.0671	0.0875
NEU	POS	0.1010	0.0135	7.4875	0.0520	0.1500	0.0000	0.0490	0.1236
NEU	NEG	0.1261	0.0139	9.0498	0.0755	0.1767	0.0000	0.0506	0.1543
POS	NEG	0.0251	0.0116	2.1608	-0.0171	0.0673	0.4208	0.0422	0.0307

Shapiro-Wilk Test

	FAL	NEU	POS	NEG
W-stat	0.5059	0.6114	N/A	0.6175
p-value	0.0000	0.0000	N/A	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	N/A	no

Welch's Test

Alpha	0.05
F-stat	16.2086
df1	3
df2	4650.7155
p-value	0.0000
sig	yes

d'Agostino-Pearson

	FAL	NEU	POS	NEG
DA-stat	1662.5735	2480.2995	5658.5600	4414.5500
p-value	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

GAMES HOWELL — alpha 0.05

group	mean	size	variance
FAL	0.7663	1248	0.7574
NEU	0.7117	2766	0.5698
POS	0.8128	5448	0.6164
NEG	0.8378	4548	0.7648

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0003	[< 0.05]
trimmed	0.0001	[< 0.05]

Grubbs/ESD Test

FAL	38
NEU	40+
POS	40+
NEG	40+

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
FAL	NEU	0.0546	0.0202	2.7067	2126.6767	3.6330	-0.0187	0.1278	0.2224	0.0732
FAL	POS	0.0465	0.0190	2.4482	1741.4400	3.6330	-0.0225	0.1154	0.3077	0.0689
FAL	NEG	0.0715	0.0197	3.6338	1991.8500	3.6330	0.0000	0.1431	0.0502	0.0715
NEU	POS	0.1010	0.0126	7.9974	5755.2122	3.6330	0.0551	0.1469	0.0000	0.0459
NEU	NEG	0.1261	0.0137	9.2198	6491.4347	3.6330	0.0764	0.1758	0.0000	0.0497
POS	NEG	0.0251	0.0119	2.1149	9234.4206	3.6330	-0.0180	0.0682	0.4404	0.0431

Unequal variances: Yes

Normally distributed: No

Significantly different: Yes (BG;FAL/NEG;NEU/POS;NEU/NEG)

Table B.12 Analysis of variance: Quality Score (QS)

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
FAL	1248	65427	52.4255	16.0137	19969.1	0.1166	52.1970	52.6540
NEU	2766	148907	53.8348	17.0183	47055.5	0.0783	53.6813	53.9883
POS	5448	290759	53.3699	17.5601	95649.7	0.0558	53.2605	53.4792
NEG	4548	242641	53.3511	16.4773	74922.2	0.0611	53.2314	53.4709

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	1712.46	3	570.820	33.6491	0.0000	0.0072	0.1434	0.0069
Within Groups	237596.5	14006	16.9639					
Total	239309.0	14009	17.0825					

TUKEY HSD/KRAMER — alpha 0.05

group	mean	n	ss	df	q-crit
FAL	52.4255	1248	19969.1		
NEU	53.8348	2766	47055.5		
POS	53.3699	5448	95649.7		
NEG	53.3511	4548	74922.2		
		14010	237596.5	14006	3.633

Q TEST

group 1	group 2	mean	std err	q-stat	lower	upper	p-value	mean-crit	Cohen d
FAL	NEU	1.4093	0.0993	14.1906	1.0485	1.7701	0.0000	0.3608	0.3422
FAL	POS	0.9444	0.0914	10.3328	0.6123	1.2764	0.0000	0.3320	0.2293
FAL	NEG	0.9257	0.0931	9.9462	0.5876	1.2638	0.0000	0.3381	0.2247
NEU	POS	0.4649	0.0680	6.8375	0.2179	0.7119	0.0000	0.2470	0.1129
NEU	NEG	0.4836	0.0702	6.8870	0.2285	0.7388	0.0000	0.2551	0.1174
POS	NEG	0.0187	0.0585	0.3200	-0.1938	0.2312	0.9959	0.2125	0.0045

Shapiro-Wilk Test

	FAL	NEU	POS	NEG
W-stat	0.9556	0.9693	N/A	0.9728
p-value	0.0000	0.0000	N/A	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	N/A	no

Welch's Test

Alpha	0.05
F-stat	34.8970
df1	3
df2	4714.5804
p-value	0.0000
sig	yes

d'Agostino-Pearson

	FAL	NEU	POS	NEG
DA-stat	175.2715	239.7052	501.4627	251.7294
p-value	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

GAMES HOWELL — alpha 0.05

group	mean	size	variance
FAL	52.4255	1248	16.0137
NEU	53.8348	2766	17.0183
POS	53.3699	5448	17.5601
NEG	53.3511	4548	16.4773

Levene's Tests

type	p-value	
means	0.0381	[< 0.05]
medians	0.0414	[< 0.05]
trimmed	0.0429	[< 0.05]

Grubbs/ESD Test

FAL	5
NEU	3
POS	5
NEG	0

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
FAL	NEU	1.4093	0.0974	14.4651	2473.1308	3.6330	1.0553	1.7633	0.0000	0.3540
FAL	POS	0.9444	0.0896	10.5405	1924.3688	3.6330	0.6189	1.2699	0.0000	0.3255
FAL	NEG	0.9257	0.0907	10.2053	2006.7212	3.6330	0.5961	1.2552	0.0000	0.3295
NEU	POS	0.4649	0.0685	6.7903	5635.7272	3.6330	0.2162	0.7137	0.0000	0.2487
NEU	NEG	0.4836	0.0699	6.9177	5764.5922	3.6330	0.2296	0.7376	0.0000	0.2540
POS	NEG	0.0187	0.0585	0.3199	9776.7963	3.6330	-0.1938	0.2313	0.9959	0.2126

Unequal variances: Yes

Normally distributed: No

Significantly different: Yes (All pairings except POS/NEG)

Table B.13 Analysis of variance: Activity Score (AS)

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
FAL	1248	59170	47.4119	37.7420	47064.3	0.1833	47.0525	47.7712
NEU	2766	136798	49.4570	39.0034	107844.4	0.1232	49.2156	49.6984
POS	5448	262221	48.1316	42.1529	229606.6	0.0878	47.9596	48.3036
NEG	4548	220076	48.3896	44.6619	203077.6	0.0960	48.2014	48.5779

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	4699.35	3	1566.450	37.3383	0.0000	0.0079	0.1308	0.0077
Within Groups	587592.9	14006	41.9529					
Total	592292.3	14009	42.2794					

TUKEY HSD/KRAMER — alpha 0.05

group	mean	n	ss	df	q-crit
FAL	47.4119	1248	47064.3		
NEU	49.4570	2766	107844.4		
POS	48.1316	5448	229606.6		
NEG	48.3896	4548	203077.6		
		14010	587592.9	14006	3.633

Q TEST

group 1	group 2	mean	std err	q-stat	lower	upper	p-value	mean-crit	Cohen d
FAL	NEU	2.0451	0.1562	13.0947	1.4777	2.6125	0.0000	0.5674	0.3157
FAL	POS	0.7197	0.1437	5.0076	0.1976	1.2419	0.0023	0.5222	0.1111
FAL	NEG	0.9778	0.1464	6.6807	0.4460	1.5095	0.0000	0.5317	0.1510
NEU	POS	1.3254	0.1069	12.3948	0.9369	1.7138	0.0000	0.3885	0.2046
NEU	NEG	1.0674	0.1104	9.6650	0.6661	1.4686	0.0000	0.4012	0.1648
POS	NEG	0.2580	0.0920	2.8047	-0.0762	0.5922	0.1946	0.3342	0.0398

Shapiro-Wilk Test

	FAL	NEU	POS	NEG
W-stat	0.9933	0.9886	N/A	0.9879
p-value	0.0000	0.0000	N/A	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	N/A	no

Welch's Test

Alpha	0.05
F-stat	40.2250
df1	3
df2	4752.6104
p-value	0.0000
sig	yes

d'Agostino-Pearson

	FAL	NEU	POS	NEG
DA-stat	4.8829	34.2743	21.0380	67.1998
p-value	0.0870	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	yes	no	no	no

GAMES HOWELL — alpha 0.05

group	mean	size	variance
FAL	47.4119	1248	37.7420
NEU	49.4570	2766	39.0034
POS	48.1316	5448	42.1529
NEG	48.3896	4548	44.6619

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0000	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

FAL	0
NEU	0
POS	1
NEG	0

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
FAL	NEU	2.0451	0.1489	13.7348	2441.5933	3.6330	1.5042	2.5861	0.0000	0.5410
FAL	POS	0.7197	0.1378	5.2230	1937.6696	3.6330	0.2191	1.2204	0.0013	0.5006
FAL	NEG	0.9778	0.1415	6.9085	2126.8301	3.6330	0.4636	1.4919	0.0000	0.5142
NEU	POS	1.3254	0.1045	12.6836	5752.6336	3.6330	0.9457	1.7050	0.0000	0.3796
NEU	NEG	1.0674	0.1094	9.7596	6144.9064	3.6330	0.6700	1.4647	0.0000	0.3973
POS	NEG	0.2580	0.0937	2.7538	9573.6742	3.6330	-0.0824	0.5984	0.2087	0.3404

Unequal variances: Yes

Normally distributed: Yes (FAL)

Significantly different: Yes (All pairings except POS/NEG)

Table B.14 Analysis of variance: MS060-MS250

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
MS060	2335	62949	26.9589	933.390	2178532	0.6352	25.7139	28.2039
MS090	2335	86658	37.1126	1298.065	3029683	0.6352	35.8676	38.3577
MS125	2335	93150	39.8929	1189.381	2776015	0.6352	38.6479	41.1380
MS180	2335	81339	34.8347	754.927	1762000	0.6352	33.5897	36.0797
MS210	2335	67654	28.9739	694.918	1621939	0.6352	27.7289	30.2189
MS250	2335	49991	21.4094	781.525	1824079	0.6352	20.1644	22.6544

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	564791	5	112958.14	119.9087	0.0000	0.0411	0.2266	0.0407
Within Groups	13192249	14004	942.03					
Total	13757040	14009	982.01					

Shapiro-Wilk Test

	MS060	MS090	MS125	MS180	MS210	MS250
W-stat	0.7214	0.7009	0.6883	0.8452	0.8498	0.7502
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

d'Agostino-Pearson

	MS060	MS090	MS125	MS180	MS210	MS250
DA-stat	1873.7913	2283.8820	1827.4179	891.1349	941.6582	1638.4223
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	119.082
df1	5
df2	6522.339
p-value	0.0000
sig	yes

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0000	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

MS060	40+
MS090	40+
MS125	40+
MS180	33
MS210	26
MS250	40+

GAMES HOWELL — alpha 0.05

group	mean	size	variance
MS060	26.9589	2335	933.390
MS090	37.1126	2335	1298.065
MS125	39.8929	2335	1189.381
MS180	34.8347	2335	754.927
MS210	28.9739	2335	694.918
MS250	21.4094	2335	781.525

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
MS060	MS090	10.1537	0.6913	14.6889	4546.572	4.0300	7.3680	12.9395	0.0000	2.7857
MS060	MS125	12.9340	0.6742	19.1841	4601.088	4.0300	10.2170	15.6511	0.0000	2.7171
MS060	MS180	7.8758	0.6013	13.0986	4616.419	4.0300	5.4527	10.2989	0.0000	2.4231
MS060	MS210	2.0150	0.5905	3.4124	4569.980	4.0300	-0.3647	4.3946	0.1521	2.3797
MS060	MS250	5.5495	0.6060	9.1577	4631.678	4.0300	3.1073	7.9916	0.0000	2.4421
MS090	MS125	2.7803	0.7298	3.8095	4659.105	4.0300	-0.1609	5.7215	0.0768	2.9412
MS090	MS180	2.2779	0.6630	3.4356	4362.652	4.0300	-0.3941	4.9500	0.1465	2.6720
MS090	MS210	8.1388	0.6533	12.4585	4276.339	4.0300	5.5061	10.7714	0.0000	2.6327
MS090	MS250	15.7032	0.6673	23.5320	4396.741	4.0300	13.0139	18.3925	0.0000	2.6893
MS125	MS180	5.0582	0.6452	7.8393	4446.013	4.0300	2.4579	7.6586	0.0000	2.6003
MS125	MS210	10.9191	0.6352	17.1897	4367.270	4.0300	8.3592	13.4789	0.0000	2.5599
MS125	MS250	18.4835	0.6496	28.4518	4476.309	4.0300	15.8655	21.1016	0.0000	2.6181
MS180	MS210	5.8608	0.5572	10.5185	4660.017	4.0300	3.6153	8.1063	0.0000	2.2455
MS180	MS250	13.4253	0.5736	23.4057	4666.602	4.0300	11.1137	15.7368	0.0000	2.3116
MS210	MS250	7.5645	0.5623	13.4533	4651.993	4.0300	5.2985	9.8304	0.0000	2.2660

Unequal variances:	Yes
Normally distributed:	No
Significantly different:	Yes (All pairings except MS060/MS210, MS090/MS125, MS090/MS180)

Table B.15 Analysis of variance: VS060-VS250

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
VS060	2335	2945.55	1.2615	1.317	3075	0.0160	1.2301	1.2929
VS090	2335	2377.13	1.0180	0.826	1928	0.0160	0.9866	1.0495
VS125	2335	1851.79	0.7931	0.397	926	0.0160	0.7616	0.8245
VS180	2335	1388.95	0.5948	0.237	554	0.0160	0.5634	0.6263
VS210	2335	1322.87	0.5665	0.344	803	0.0160	0.5351	0.5980
VS250	2335	1277.15	0.5470	0.479	1117	0.0160	0.5155	0.5784

ANOVA

Sources	SS	df	VS	F	P value	Eta-sq	RVSSE	Omega Sq
Between Groups	983	5	196.66	327.7677	0.0000	0.1048	0.3747	0.1044
Within Groups	8402	14004	0.60					
Total	9386	14009	0.67					

Shapiro-Wilk Test

	VS060	VS090	VS125	VS180	VS210	VS250
W-stat	0.6789	0.6906	0.7455	0.6230	0.4971	0.4592
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

d'Agostino-Pearson

	VS060	VS090	VS125	VS180	VS210	VS250
DA-stat	2147.2867	1767.1210	1423.8222	3175.3066	3525.6326	3000.2667
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	247.459
df1	5
df2	6471.574
p-value	0.0000
sig	yes

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0000	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

VS060	40+
VS090	40+
VS125	40+
VS180	21
VS210	40+
VS250	40+

GAMES HOWELL — alpha 0.05

group	mean	size	variance
VS060	1.2615	2335	1.317
VS090	1.0180	2335	0.826
VS125	0.7931	2335	0.397
VS180	0.5948	2335	0.237
VS210	0.5665	2335	0.344
VS250	0.5470	2335	0.479

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
VS060	VS090	0.2434	0.0214	11.3632	4434.734	4.0300	0.1571	0.3298	0.0000	0.0863
VS060	VS125	0.4684	0.0192	24.4500	3622.595	4.0300	0.3912	0.5456	0.0000	0.0772
VS060	VS180	0.6666	0.0182	36.5371	3148.076	4.0300	0.5931	0.7402	0.0000	0.0735
VS060	VS210	0.6949	0.0189	36.8428	3475.306	4.0300	0.6189	0.7710	0.0000	0.0760
VS060	VS250	0.7145	0.0196	36.4328	3832.532	4.0300	0.6355	0.7936	0.0000	0.0790
VS090	VS125	0.2250	0.0162	13.9055	4155.701	4.0300	0.1598	0.2902	0.0000	0.0652
VS090	VS180	0.4232	0.0151	28.0495	3572.543	4.0300	0.3624	0.4840	0.0000	0.0608
VS090	VS210	0.4515	0.0158	28.5256	3991.157	4.0300	0.3877	0.5153	0.0000	0.0638
VS090	VS250	0.4711	0.0167	28.1844	4359.451	4.0300	0.4037	0.5384	0.0000	0.0674
VS125	VS180	0.1982	0.0117	17.0142	4390.215	4.0300	0.1513	0.2452	0.0000	0.0470
VS125	VS210	0.2265	0.0126	17.9860	4644.612	4.0300	0.1758	0.2773	0.0000	0.0508
VS125	VS250	0.2461	0.0137	17.9746	4627.259	4.0300	0.1909	0.3013	0.0000	0.0552
VS180	VS210	0.0283	0.0112	2.5366	4515.355	4.0300	-0.0167	0.0733	0.4700	0.0450
VS180	VS250	0.0479	0.0124	3.8669	4190.884	4.0300	-0.0020	0.0978	0.0690	0.0499
VS210	VS250	0.0196	0.0133	1.4751	4546.169	4.0300	-0.0339	0.0731	0.9033	0.0535

Unequal variances: Yes

Normally distributed: No

Significantly different: Yes (All pairings except VS180/MS210, VS180/VS250, VS210/VS250)

Table B.16 Analysis of variance: QS060-QS250

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
QS060	2335	130349	55.8240	29.271	68319	0.0782	55.6707	55.9773
QS090	2335	128347	54.9666	18.222	42529	0.0782	54.8133	55.1199
QS125	2335	125931	53.9319	12.405	28952	0.0782	53.7786	54.0852
QS180	2335	122625	52.5161	9.291	21685	0.0782	52.3628	52.6694
QS210	2335	120961	51.8034	8.401	19607	0.0782	51.6501	51.9567
QS250	2335	119521	51.1867	8.099	18903	0.0782	51.0334	51.3400

ANOVA

Sources	SS	df	QS	F	P value	Eta-sq	RQSSE	Omega Sq
Between Groups	39314	5	7862.85	550.5712	0.0000	0.1643	0.4856	0.1640
Within Groups	199995	14004	14.28					
Total	239309	14009	17.08					

Shapiro-Wilk Test

	QS060	QS090	QS125	QS180	QS210	QS250
W-stat	0.9962	0.9910	0.9760	0.9835	0.9802	0.9825
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

d'Agostino-Pearson

DA-stat	3.3234	25.7172	87.1159	38.6741	52.1324	35.4824
p-value	0.1898	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	yes	no	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	522.464
df1	5
df2	6496.847
p-value	0.0000
sig	yes

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0000	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

QS060	0
QS090	0
QS125	1
QS180	1
QS210	0
QS250	0

GAMES HOWELL — alpha 0.05

group	mean	size	variance
QS060	55.8240	2335	29.271
QS090	54.9666	2335	18.222
QS125	53.9319	2335	12.405
QS180	52.5161	2335	9.291
QS210	51.8034	2335	8.401
QS250	51.1867	2335	8.099

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
QS060	QS090	0.8574	0.1008	8.5020	4428.305	4.0300	0.4510	1.2638	0.0000	0.4064
QS060	QS125	1.8921	0.0945	20.0289	4011.032	4.0300	1.5114	2.2728	0.0000	0.3807
QS060	QS180	3.3079	0.0909	36.4027	3680.062	4.0300	2.9417	3.6741	0.0000	0.3662
QS060	QS210	4.0206	0.0898	44.7649	3571.726	4.0300	3.6586	4.3825	0.0000	0.3620
QS060	QS250	4.6373	0.0895	51.8393	3533.713	4.0300	4.2768	4.9978	0.0000	0.3605
QS090	QS125	1.0347	0.0810	12.7768	4505.455	4.0300	0.7083	1.3610	0.0000	0.3264
QS090	QS180	2.4505	0.0768	31.9266	4223.031	4.0300	2.1412	2.7599	0.0000	0.3093
QS090	QS210	3.1632	0.0755	41.8947	4108.814	4.0300	2.8589	3.4674	0.0000	0.3043
QS090	QS250	3.7799	0.0751	50.3487	4066.493	4.0300	3.4773	4.0824	0.0000	0.3025
QS125	QS180	1.4158	0.0682	20.7725	4573.800	4.0300	1.1412	1.6905	0.0000	0.2747
QS125	QS210	2.1285	0.0667	31.8892	4501.278	4.0300	1.8595	2.3975	0.0000	0.2690
QS125	QS250	2.7452	0.0663	41.4302	4470.833	4.0300	2.4782	3.0122	0.0000	0.2670
QS180	QS210	0.7126	0.0615	11.5782	4656.204	4.0300	0.4646	0.9607	0.0000	0.2480
QS180	QS250	1.3293	0.0610	21.7844	4646.163	4.0300	1.0834	1.5753	0.0000	0.2459
QS210	QS250	0.6167	0.0594	10.3753	4666.440	4.0300	0.3772	0.8562	0.0000	0.2395

Unequal variances: Yes

Normally distributed: Yes (QS060)

Significantly different: Yes (All pairings)

Table B.17 Analysis of variance: AS060-AS250

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
AS060	2335	119677	51.2535	49.864	116382	0.1290	51.0007	51.5064
AS090	2335	117055	50.1306	40.491	94507	0.1290	49.8778	50.3835
AS125	2335	114218	48.9156	36.878	86072	0.1290	48.6628	49.1685
AS180	2335	110657	47.3906	35.559	82996	0.1290	47.1377	47.6434
AS210	2335	109038	46.6972	34.908	81475	0.1290	46.4444	46.9501
AS250	2335	107620	46.0899	35.401	82625	0.1290	45.8371	46.3428

ANOVA

Sources	SS	df	AS	F	P value	Eta-sq	RASSE	Omega Sq
Between Groups	48235	5	9647.00	248.3130	0.0000	0.0814	0.3261	0.0811
Within Groups	544057	14004	38.85					
Total	592292	14009	42.28					

Shapiro-Wilk Test

	AS060	AS090	AS125	AS180	AS210	AS250
W-stat	0.9895	0.9888	0.9816	0.9714	0.9684	0.9680
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

d'Agostino-Pearson

DA-stat	18.6975	31.8075	60.9103	111.7106	106.7777	111.8458
p-value	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	234.229
df1	5
df2	6531.715
p-value	0.0000
sig	yes

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0000	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

AS060	0
AS090	0
AS125	0
AS180	0
AS210	0
AS250	0

GAMES HOWELL — alpha 0.05

group	mean	size	variance
AS060	51.2535	2335	49.864
AS090	50.1306	2335	40.491
AS125	48.9156	2335	36.878
AS180	47.3906	2335	35.559
AS210	46.6972	2335	34.908
AS250	46.0899	2335	35.401

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
AS060	AS090	1.1229	0.1391	8.0729	4618.311	4.0300	0.5624	1.6835	0.0000	0.5606
AS060	AS125	2.3379	0.1363	17.1542	4565.669	4.0300	1.7887	2.8871	0.0000	0.5492
AS060	AS180	3.8630	0.1352	28.5621	4540.679	4.0300	3.3179	4.4080	0.0000	0.5450
AS060	AS210	4.5563	0.1347	33.8179	4527.090	4.0300	4.0134	5.0993	0.0000	0.5430
AS060	AS250	5.1636	0.1351	38.2144	4537.444	4.0300	4.6191	5.7081	0.0000	0.5445
AS090	AS125	1.2150	0.1287	9.4395	4657.838	4.0300	0.6963	1.7337	0.0000	0.5187
AS090	AS180	2.7400	0.1276	21.4716	4648.450	4.0300	2.2258	3.2543	0.0000	0.5143
AS090	AS210	3.4334	0.1271	27.0209	4642.540	4.0300	2.9213	3.9455	0.0000	0.5121
AS090	AS250	4.0407	0.1275	31.6968	4647.089	4.0300	3.5269	4.5544	0.0000	0.5137
AS125	AS180	1.5251	0.1245	12.2451	4666.455	4.0300	1.0231	2.0270	0.0000	0.5019
AS125	AS210	2.2184	0.1240	17.8930	4664.488	4.0300	1.7188	2.7181	0.0000	0.4996
AS125	AS250	2.8257	0.1244	22.7133	4666.052	4.0300	2.3243	3.3271	0.0000	0.5014
AS180	AS210	0.6934	0.1228	5.6445	4667.601	4.0300	0.1983	1.1884	0.0010	0.4950
AS180	AS250	1.3006	0.1233	10.5514	4667.977	4.0300	0.8039	1.7974	0.0000	0.4968
AS210	AS250	0.6073	0.1227	4.9493	4667.771	4.0300	0.1128	1.1018	0.0063	0.4945

Unequal variances: Yes

Normally distributed: No

Significantly different: Yes (All pairings)

B.3 CORRELATION COEFFICIENTS

When a set of data is not normally distributed or when the presence of outliers gives a distorted picture of the association between two random variables, Spearman's rank correlation is a non-parametric test that substitutes for Pearson's correlation.

The coefficient of determination or correlation-squared indicates how closely two time-series track each other. It also points to the reliability of the alpha (excess return) and beta (volatility) coefficients from a linear regression.

Table B.18 Correlation: TSX-MI/TXCX (2021)

Correlation Coefficients: TSX-MI/TXCX (2021)			
Pearson	0.8517		
Spearman	0.8348		
Kendall	0.6474		
Pearson's coeff (t test)		Pearson's coeff (Fisher)	
Alpha	0.05	Hyp rho	0
Tails	2	Alpha	0.05
		Tails	2
corr	0.8517		
std err	0.0333	corr	0.8517
t	25.5391	std err	0.0635
p-value	0	z	19.7963
lower	0.7860	p-value	0
upper	0.9173	lower	0.8135
		upper	0.8825

Table B.19 Correlation: TSX-MI/TXCX (2019-2021)

Correlation Coefficients: TSX-MI/TXCX (2019-2021)			
Pearson	0.7306		
Spearman	0.5382		
Kendall	0.3959		
Pearson's coeff (t test)		Pearson's coeff (Fisher)	
Alpha	0.05	Hyp rho	0
Tails	2	Alpha	0.05
		Tails	2
corr	0.7306		
std err	0.0250	corr	0.7306
t	29.2646	std err	0.0365
p-value	0	z	25.4189
lower	0.6816	p-value	1.6E-142
upper	0.7796	lower	0.6954
		upper	0.7623

Table B.20 Correlation: TSX-MI/TXCX (2017-2021)

Correlation Coefficients: TSX-MI/TXCX (2017-2021)				
Pearson	0.7344			
Spearman	0.5986			
Kendall	0.4431			
Pearson's coeff (t test)			Pearson's coeff (Fisher)	
Alpha	0.05		Hyp rho	0
Tails	2		Alpha	0.05
			Tails	2
corr	0.7344			
std err	0.0192		corr	0.7344
t	38.2083		std err	0.0283
p-value	0		z	33.1166
lower	0.6967		p-value	1.7E-240
upper	0.7721		lower	0.7077
			upper	0.7589

REFERENCE

RealStats. 2022. Real statistics using Excel [Website]. Charles Zaiontz. Available at: https://www.real-statistics.com.

RESULTS: STATISTICAL TESTS (TSXV)

C.1 DESCRIPTIVE STATISTICS

The statistical analysis for this study was generated using the Real Statistics Resource Pack software for Excel (Release 8.3.1), Copyright (2013-2022) by Charles Zaiontz (RealStats 2022).

Since the skewness and kurtosis of the normal distribution are zero, these two parameters should be close to zero for data to follow a normal distribution. Rough measures of the standard errors of skewness and kurtosis are sqrt(6/n) and sqrt(24/n) respectively, where n is the sample size. The data are not symmetric (and therefore not normal) or normal if the absolute values of skewness and kurtosis are more than twice their standard errors.

Table C.1 Descriptive statistics: TSXV Momentum Index (TSXV-MI)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	TSXV-MI		*TSXV-MI*	alpha	0.05
Mean	0.000834	W-stat	0.9400		
Standard Error	0.000283	p-value	0		TSXV-MI
Median	0.000842	alpha	0.05	outlier	-0.1312
Standard Deviation	0.016109	normal	no	G	8.1949
Sample Variance	0.000260			G-crit	4.1623
Kurtosis	6.272372	d'Agostino-Pearson		sig	yes
Skewness	-0.345888			ESD outliers	19
Range	0.240845	DA-stat	490.3039		
Maximum	0.109666	p-value	0		
Minimum	-0.131179	alpha	0.05		
Sum	2.708545	normal	no		
Count	3248				
CV	19.3177				

Table C.2 Descriptive statistics: S&P/TSX Venture Composite Index (TXVC)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	TXVC		*TXVC*	alpha	0.05
Mean	0.000051	W-stat	0.9262		
Standard Error	0.000215	p-value	0		TXVC
Median	0.000770	alpha	0.05	outlier	-0.1113
Standard Deviation	0.012244	normal	no	G	9.0910
Sample Variance	0.000150			G-crit	4.1623
Kurtosis	7.797698	d'Agostino-Pearson		sig	yes
Skewness	-0.987050			ESD outliers	17
Range	0.192075	DA-stat	874.5887		
Maximum	0.080816	p-value	0		
Minimum	-0.111259	alpha	0.05		
Sum	0.164127	normal	no		
Count	3248				
CV	242.3013				

C.2 ANALYSIS OF VARIANCE

The single factor analysis-of-variance (ANOVA) tests for differences in averages.

Table C.3 Analysis of variance: Momentum Score (MS)

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
FAL	468	15744	33.6410	1467.8666	685493.7	1.9879	29.7434	37.5387
NEU	360	13889	38.5806	1250.4837	448923.7	2.2666	34.1366	43.0245
POS	1044	48360	46.3218	2052.4658	2140721.9	1.3310	43.7122	48.9314
NEG	1614	80184	49.6803	1962.0106	3164723.0	1.0705	47.5815	51.7791

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	111908.72	3	37302.906	20.1695	0.0000	0.0171	0.1694	0.0162
Within Groups	6439862.3	3482	1849.4722					
Total	6551771.0	3485	1879.9917					

TUKEY HSD/KRAMER — alpha 0.05

group	mean	n	ss	df	q-crit
FAL	33.6410	468	685493.7		
NEU	38.5806	360	448923.7		
POS	46.3218	1044	2140721.9		
NEG	49.6803	1614	3164723.0		
		3486	6439862.3	3482	3.633

Q TEST

group 1	group 2	mean	std err	q-stat	lower	upper	p-value	mean-crit	Cohen d
FAL	NEU	4.9395	2.1318	2.3171	-2.8054	12.6844	0.3572	7.7449	0.1149
FAL	POS	12.6808	1.6917	7.4961	6.5350	18.8266	0.0000	6.1458	0.2949
FAL	NEG	16.0393	1.5965	10.0464	10.2391	21.8394	0.0000	5.8002	0.3730
NEU	POS	7.7413	1.8586	4.1651	0.9889	14.4937	0.0172	6.7524	0.1800
NEU	NEG	11.0997	1.7725	6.2623	4.6603	17.5391	0.0001	6.4394	0.2581
POS	NEG	3.3585	1.2078	2.7807	-1.0294	7.7463	0.2011	4.3878	0.0781

Shapiro-Wilk Test

	FAL	NEU	POS	NEG
W-stat	0.8154	0.8596	0.8185	0.8519
p-value	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	23.7887
df1	3
df2	1178.9623
p-value	0.0000
sig	yes

d'Agostino-Pearson

	FAL	NEU	POS	NEG
DA-stat	235.8418	204.4291	559.6259	622.2681
p-value	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

GAMES HOWELL — alpha 0.05

group	mean	size	variance
FAL	33.6410	468	1467.8666
NEU	38.5806	360	1250.4837
POS	46.3218	1044	2052.4658
NEG	49.6803	1614	1962.0106

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0002	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

FAL	14
NEU	3
POS	14
NEG	12

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
FAL	NEU	4.9395	1.8180	2.7171	799.1431	3.6330	-1.6652	11.5442	0.2199	6.6047
FAL	POS	12.6808	1.5973	7.9391	1051.0269	3.6330	6.8780	18.4836	0.0000	5.8028
FAL	NEG	16.0393	1.4751	10.8730	861.6720	3.6330	10.6801	21.3985	0.0000	5.3592
NEU	POS	7.7413	1.6492	4.6940	792.9433	3.6330	1.7498	13.7327	0.0052	5.9914
NEU	NEG	11.0997	1.5312	7.2490	636.8812	3.6330	5.5369	16.6626	0.0000	5.5629
POS	NEG	3.3585	1.2613	2.6628	2190.1547	3.6330	-1.2237	7.9406	0.2357	4.5822

Unequal variances: Yes

Normally distributed: No

Significantly different: Yes (BG;FAL/POS;FAL/NEG;NEU/POS;NEU/NEG)

Table C.4 Analysis of variance: Volatility Score (VS)

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
FAL	468	556.94	1.1900	1.6724	780.99	0.0654	1.0618	1.3183
NEU	360	485.99	1.3500	3.4155	1226.18	0.0746	1.2037	1.4962
POS	1044	1283.08	1.2290	2.0444	2132.27	0.0438	1.1431	1.3149
NEG	1614	2078.66	1.2879	1.7570	2833.98	0.0352	1.2188	1.3570

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	7.42	3	2.4733	1.2350	0.2953	0.0011	0.0494	0.0002
Within Groups	6973.41	3482	2.0027					
Total	6980.83	3485	2.0031					

TUKEY HSD/KRAMER — alpha 0.05

group	mean	n	ss	df	q-crit
FAL	1.1900	468	780.99		
NEU	1.3500	360	1226.18		
POS	1.2290	1044	2132.27		
NEG	1.2879	1614	2833.98		
		3486	6973.41	3482	3.633

Q TEST

group 1	group 2	mean	std err	q-stat	lower	upper	p-value	mean-crit	Cohen d
FAL	NEU	0.1599	0.0702	2.2798	-0.0949	0.4148	0.3719	0.2549	0.1130
FAL	POS	0.0390	0.0557	0.6999	-0.1633	0.2412	0.9602	0.2022	0.0275
FAL	NEG	0.0979	0.0525	1.8625	-0.0930	0.2887	0.5521	0.1909	0.0691
NEU	POS	0.1210	0.0612	1.9779	-0.1012	0.3432	0.5003	0.2222	0.0855
NEU	NEG	0.0621	0.0583	1.0643	-0.1498	0.2740	0.8756	0.2119	0.0439
POS	NEG	0.0589	0.0397	1.4817	-0.0855	0.2033	0.7213	0.1444	0.0416

Shapiro-Wilk Test

	FAL	NEU	POS	NEG
W-stat	0.5732	0.4777	0.5308	0.5882
p-value	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	1.1413
df1	3
df2	1075.4362
p-value	0.3313
sig	no

d'Agostino-Pearson

DA-stat	578.6944	405.5460	1123.7252	1965.1456
p-value	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

GAMES HOWELL — alpha 0.05

group	mean	size	variance
FAL	1.1900	468	1.6724
NEU	1.3500	360	3.4155
POS	1.2290	1044	2.0444
NEG	1.2879	1614	1.7570

Levene's Tests

type	p-value	
means	0.0689	[> 0.05]
medians	0.3929	[> 0.05]
trimmed	0.3313	[> 0.05]

Grubbs/ESD Test

FAL	23
NEU	23
POS	40+
NEG	31

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
FAL	NEU	0.1599	0.0808	1.9790	613.4566	3.6330	-0.1337	0.4535	0.5002	0.2936
FAL	POS	0.0390	0.0526	0.7408	986.4300	3.6330	-0.1521	0.2300	0.9533	0.1911
FAL	NEG	0.0979	0.0483	2.0267	774.0663	3.6330	-0.0776	0.2733	0.4790	0.1754
NEU	POS	0.1210	0.0756	1.5991	514.9349	3.6330	-0.1539	0.3958	0.6707	0.2748
NEU	NEG	0.0621	0.0727	0.8537	444.8037	3.6470	-0.2031	0.3273	0.9309	0.2652
POS	NEG	0.0589	0.0390	1.5088	2104.4211	3.6330	-0.0829	0.2007	0.7098	0.1418

Unequal variances: No

Normally distributed: No

Significantly different: No

Table C.5 Analysis of variance: Quality Score (QS)

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
FAL	468	24068	51.4274	15.6414	7304.53	0.1775	51.0793	51.7754
NEU	360	19078	52.9944	14.2507	5115.99	0.2024	52.5976	53.3913
POS	1044	55146	52.8218	15.6480	16320.86	0.1189	52.5888	53.0549
NEG	1614	85019	52.6760	14.0183	22611.53	0.0956	52.4885	52.8634

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	759.46	3	253.1527	17.1651	0.0000	0.0146	0.1858	0.0137
Within Groups	51352.91	3482	14.7481					
Total	52112.37	3485	14.9533					

TUKEY HSD/KRAMER — alpha 0.05

group	mean	n	ss	df	q-crit
FAL	51.4274	468	7304.53		
NEU	52.9944	360	5115.99		
POS	52.8218	1044	16320.86		
NEG	52.6760	1614	22611.53		
		3486	51352.91	3482	3.633

Q TEST

group 1	group 2	mean	std err	q-stat	lower	upper	p-value	mean-crit	Cohen d
FAL	NEU	1.5671	0.1904	8.2319	0.8755	2.2587	0.0000	0.6916	0.4081
FAL	POS	1.3945	0.1511	9.2312	0.8457	1.9433	0.0000	0.5488	0.3631
FAL	NEG	1.2486	0.1426	8.7581	0.7307	1.7666	0.0000	0.5179	0.3251
NEU	POS	0.1726	0.1660	1.0400	-0.4304	0.7756	0.8829	0.6030	0.0449
NEU	NEG	0.3185	0.1583	2.0122	-0.2565	0.8935	0.4851	0.5750	0.0829
POS	NEG	0.1459	0.1079	1.3526	-0.2459	0.5377	0.7742	0.3918	0.0380

Shapiro-Wilk Test

	FAL	NEU	POS	NEG
W-stat	0.9492	0.9754	0.9642	0.9525
p-value	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	16.3176
df1	3
df2	1106.8693
p-value	0.0000
sig	yes

d'Agostino-Pearson

	FAL	NEU	POS	NEG
DA-stat	63.7355	14.4174	94.1683	224.2867
p-value	0.0000	0.0007	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

GAMES HOWELL — alpha 0.05

group	mean	size	variance
FAL	51.4274	468	15.6414
NEU	52.9944	360	14.2507
POS	52.8218	1044	15.6480
NEG	52.6760	1614	14.0183

Levene's Tests

type	p-value	
means	0.2917	[> 0.05]
medians	0.1979	[> 0.05]
trimmed	0.2522	[> 0.05]

Grubbs/ESD Test

FAL	1
NEU	0
POS	2
NEG	6

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
FAL	NEU	1.5671	0.1911	8.2022	788.8419	3.6330	0.8730	2.2612	0.0000	0.6941
FAL	POS	1.3945	0.1556	8.9632	898.8476	3.6330	0.8293	1.9597	0.0000	0.5652
FAL	NEG	1.2486	0.1451	8.6052	727.0450	3.6330	0.7215	1.7758	0.0000	0.5271
NEU	POS	0.1726	0.1652	1.0449	650.2443	3.6330	-0.4275	0.7727	0.8814	0.6001
NEU	NEG	0.3185	0.1554	2.0500	528.1611	3.6330	-0.2459	0.8829	0.4690	0.5644
POS	NEG	0.1459	0.1088	1.3408	2137.8270	3.6330	-0.2494	0.5411	0.7788	0.3953

Unequal variances: No

Normally distributed: No

Significantly different: Yes (BG;FAL/NEU;FAL/POS;FAL/NEG)

Table C.6 Analysis of variance: Activity Score (AS)

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
FAL	468	19866	42.4487	31.1087	14527.77	0.2493	41.9599	42.9375
NEU	360	15106	41.9611	29.5807	10619.46	0.2842	41.4038	42.5184
POS	1044	43494	41.6609	30.5561	31869.97	0.1669	41.3337	41.9882
NEG	1614	66743	41.3525	27.4373	44256.40	0.1342	41.0893	41.6157

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	476.26	3	158.7521	5.4582	0.0010	0.0047	0.0866	0.0038
Within Groups	101273.59	3482	29.0849					
Total	101749.85	3485	29.1965					

TUKEY HSD/KRAMER — alpha 0.05

group	mean	n	ss	df	q-crit
FAL	42.4487	468	14527.77		
NEU	41.9611	360	10619.46		
POS	41.6609	1044	31869.97		
NEG	41.3525	1614	44256.40		
		3486	101273.59	3482	3.633

Q TEST

group 1	group 2	mean	std err	q-stat	lower	upper	p-value	mean-crit	Cohen d
FAL	NEU	0.4876	0.2673	1.8239	-0.4836	1.4588	0.5696	0.9712	0.0904
FAL	POS	0.7878	0.2121	3.7136	0.0171	1.5585	0.0432	0.7707	0.1461
FAL	NEG	1.0962	0.2002	5.4752	0.3688	1.8235	0.0006	0.7274	0.2033
NEU	POS	0.3002	0.2331	1.2879	-0.5466	1.1470	0.7991	0.8468	0.0557
NEU	NEG	0.6086	0.2223	2.7379	-0.1990	1.4161	0.2132	0.8075	0.1128
POS	NEG	0.3084	0.1515	2.0361	-0.2419	0.8586	0.4746	0.5502	0.0572

Shapiro-Wilk Test

	FAL	NEU	POS	NEG
W-stat	0.9811	0.9874	0.9826	0.9695
p-value	0.0000	0.0033	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	no	no	no

d'Agostino-Pearson

	FAL	NEU	POS	NEG
DA-stat	20.5896	4.9799	37.2062	117.0649
p-value	0.0000	0.0829	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05
normal	no	yes	no	no

Welch's Test

Alpha	0.05
F-stat	5.3168
df1	3
df2	1100.8929
p-value	0.0012
sig	yes

Levene's Tests

type	p-value	
means	0.2418	[> 0.05]
medians	0.1973	[> 0.05]
trimmed	0.1911	[> 0.05]

Grubbs/ESD Test

FAL	0
NEU	0
POS	1
NEG	3

GAMES HOWELL — alpha 0.05

group	mean	size	variance
FAL	42.4487	468	31.1087
NEU	41.9611	360	29.5807
POS	41.6609	1044	30.5561
NEG	41.3525	1614	27.4373

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
FAL	NEU	0.4876	0.2726	1.7886	781.5791	3.6330	-0.5028	1.4780	0.5857	0.9904
FAL	POS	0.7878	0.2188	3.6007	891.4105	3.6330	-0.0071	1.5827	0.0538	0.7949
FAL	NEG	1.0962	0.2043	5.3657	722.7213	3.6330	0.3540	1.8384	0.0009	0.7422
NEU	POS	0.3002	0.2360	1.2717	632.6697	3.6330	-0.5574	1.1578	0.8052	0.8576
NEU	NEG	0.6086	0.2227	2.7330	517.9763	3.6330	-0.2004	1.4175	0.2157	0.8090
POS	NEG	0.3084	0.1521	2.0275	2139.6971	3.6330	-0.2442	0.8610	0.4784	0.5526

Unequal variances: No

Normally distributed: Yes (NEU)

Significantly different: Yes (BG;FAL/NEG)

Table C.7 Analysis of variance: MS060-MS250

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
MS060	581	20466	35.2255	1535.716	890715	1.7733	31.7487	38.7022
MS090	581	29825	51.3339	2063.419	1196783	1.7733	47.8571	54.8107
MS125	581	32169	55.3683	2084.381	1208941	1.7733	51.8916	58.8451
MS180	581	28783	49.5404	1415.249	820844	1.7733	46.0637	53.0172
MS210	581	25658	44.1618	1707.832	990543	1.7733	40.6850	47.6385
MS250	581	21276	36.6196	2155.115	1249967	1.7733	33.1429	40.0964

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	193977	5	38795.42	21.2350	0.0000	0.0296	0.1912	0.0282
Within Groups	6357794	3480	1826.95					
Total	6551771	3485	1879.99					

Shapiro-Wilk Test

	MS060	MS090	MS125	MS180	MS210	MS250
W-stat	0.7722	0.8316	0.8143	0.9004	0.8672	0.7835
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

d'Agostino-Pearson

DA-stat	415.5461	284.2992	298.3924	139.6456	199.4592	302.0597
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	20.965
df1	5
df2	1622.227
p-value	0.0000
sig	yes

Levene's Tests

type	p-value	
means	0.0006	[< 0.05]
medians	0.0239	[< 0.05]
trimmed	0.0035	[< 0.05]

Grubbs/ESD Test

MS060	12
MS090	9
MS125	11
MS180	2
MS210	7
MS250	13

GAMES HOWELL — alpha 0.05

group	mean	size	variance
MS060	35.2255	581	1535.716
MS090	51.3339	581	2063.419
MS125	55.3683	581	2084.381
MS180	49.5404	581	1415.249
MS210	44.1618	581	1707.832
MS250	36.6196	581	2155.115

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
MS060	MS090	16.1084	1.7599	9.1529	1135.588	4.0300	9.0159	23.2010	0.0000	7.0925
MS060	MS125	20.1429	1.7651	11.4121	1133.952	4.0300	13.0297	27.2560	0.0000	7.1132
MS060	MS180	14.3150	1.5936	8.9828	1158.070	4.0300	7.8928	20.7372	0.0000	6.4222
MS060	MS210	8.9363	1.6707	5.3487	1156.743	4.0300	2.2033	15.6694	0.0023	6.7331
MS060	MS250	1.3941	1.7822	0.7823	1128.225	4.0300	-5.7882	8.5765	0.9939	7.1823
MS090	MS125	4.0344	1.8893	2.1354	1159.970	4.0300	-3.5795	11.6484	0.6579	7.6140
MS090	MS180	1.7935	1.7302	1.0365	1121.079	4.0300	-5.1794	8.7663	0.9779	6.9728
MS090	MS210	7.1721	1.8015	3.9811	1149.778	4.0300	-0.0880	14.4323	0.0558	7.2601
MS090	MS250	14.7143	1.9054	7.7226	1159.452	4.0300	7.0357	22.3929	0.0000	7.6786
MS125	MS180	5.8279	1.7354	3.3582	1119.089	4.0300	-1.1659	12.8217	0.1661	6.9938
MS125	MS210	11.2065	1.8065	6.2034	1148.675	4.0300	3.9263	18.4868	0.0002	7.2803
MS125	MS250	18.7487	1.9101	9.8156	1159.677	4.0300	11.0510	26.4464	0.0000	7.6977
MS180	MS210	5.3787	1.6394	3.2808	1149.908	4.0300	-1.2282	11.9855	0.1868	6.6068
MS180	MS250	12.9208	1.7529	7.3712	1112.238	4.0300	5.8567	19.9849	0.0000	7.0641
MS210	MS250	7.5422	1.8233	4.1366	1144.654	4.0300	0.1943	14.8900	0.0409	7.3479

Unequal variances: Yes

Normally distributed: No

Significantly different: Yes (All pairings except MS060/MS250, MS090/MS125, MS090/MS180, MS090/MS210, MS125/MS180, MS180/MS210)

Table C.8 Analysis of variance: VS060-VS250

ANOVA: Single Factor

DESCRIPTION

Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
VS060	581	1136.47	1.9561	2.161	1253	0.0567	1.8448	2.0673
VS090	581	889.76	1.5314	1.284	744	0.0567	1.4202	1.6426
VS125	581	656.91	1.1307	0.531	308	0.0567	1.0194	1.2419
VS180	581	522.27	0.8989	0.589	341	0.0567	0.7877	1.0101
VS210	581	560.66	0.9650	1.803	1045	0.0567	0.8538	1.0762
VS250	581	638.6	1.0991	4.850	2813	0.0567	0.9879	1.2104

ANOVA

Sources	SS	df	VS	F	P value	Eta-sq	RVSSE	Omega Sq
Between Groups	475	5	95.06	50.8532	0.0000	0.0681	0.2958	0.0667
Within Groups	6506	3480	1.87					
Total	6981	3485	2.00					

Shapiro-Wilk Test

	VS060	VS090	VS125	VS180	VS210	VS250
W-stat	0.7894	0.7653	0.8022	0.5718	0.4072	0.3332
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

d'Agostino-Pearson

DA-stat	281.3281	325.1713	349.5218	626.6066	710.3207	728.6409
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	62.618
df1	5
df2	1594.501
p-value	0.0000
sig	yes

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0000	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

VS060	17
VS090	12
VS125	9
VS180	19
VS210	29
VS250	38

GAMES HOWELL alpha 0.05

group	mean	size	variance
VS060	1.9561	581	2.161
VS090	1.5314	581	1.284
VS125	1.1307	581	0.531
VS180	0.8989	581	0.589
VS210	0.9650	581	1.803
VS250	1.0991	581	4.850

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
VS060	VS090	0.4246	0.0544	7.7994	1089.338	4.0300	0.2052	0.6440	0.0000	0.2194
VS060	VS125	0.8254	0.0481	17.1485	848.965	4.0300	0.6314	1.0194	0.0000	0.1940
VS060	VS180	1.0571	0.0486	21.7329	874.174	4.0300	0.8611	1.2532	0.0000	0.1960
VS060	VS210	0.9911	0.0584	16.9698	1150.601	4.0300	0.7557	1.2264	0.0000	0.2354
VS060	VS250	0.8569	0.0777	11.0325	1011.238	4.0300	0.5439	1.1699	0.0000	0.3130
VS090	VS125	0.4008	0.0395	10.1411	989.926	4.0300	0.2415	0.5600	0.0000	0.1593
VS090	VS180	0.6325	0.0401	15.7579	1019.537	4.0300	0.4708	0.7943	0.0000	0.1618
VS090	VS210	0.5664	0.0515	10.9913	1128.092	4.0300	0.3588	0.7741	0.0000	0.2077
VS090	VS250	0.4323	0.0727	5.9503	866.918	4.0300	0.1395	0.7251	0.0004	0.2928
VS125	VS180	0.2317	0.0310	7.4646	1156.968	4.0300	0.1066	0.3569	0.0000	0.1251
VS125	VS210	0.1657	0.0448	3.6965	894.577	4.0300	-0.0149	0.3463	0.0949	0.1806
VS125	VS250	0.0315	0.0680	0.4631	705.578	4.0300	-0.2427	0.3058	0.9995	0.2742
VS180	VS210	0.0661	0.0454	1.4566	922.300	4.0300	-0.1167	0.2489	0.9079	0.1828
VS180	VS250	0.2002	0.0684	2.9268	718.754	4.0300	-0.0755	0.4759	0.3045	0.2757
VS210	VS250	0.1341	0.0757	1.7730	958.828	4.0300	-0.1708	0.4391	0.8100	0.3049

Unequal variances: Yes

Normally distributed: No

Significantly different: Yes (All pairings except VS125/VS210, VS125/VS250, VS180/VS210, VS180/VS250, VS210/VS250)

Table C.9 Analysis of variance: QS060-QS250

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
QS060	581	31855	54.8279	27.936	16203	0.1472	54.5393	55.1165
QS090	581	31420	54.0792	16.011	9286	0.1472	53.7906	54.3678
QS125	581	30843	53.0861	10.320	5986	0.1472	52.7975	53.3747
QS180	581	30090	51.7900	7.349	4262	0.1472	51.5014	52.0786
QS210	581	29742	51.1910	6.951	4032	0.1472	50.9024	51.4797
QS250	581	29361	50.5353	6.966	4041	0.1472	50.2467	50.8239

ANOVA

Sources	SS	df	QS	F	P value	Eta-sq	RQSSE	Omega Sq
Between Groups	8303	5	1660.56	131.9065	0.0000	0.1593	0.4765	0.1581
Within Groups	43810	3480	12.59					
Total	52112	3485	14.95					

Shapiro-Wilk Test

	QS060	QS090	QS125	QS180	QS210	QS250
W-stat	0.9941	0.9863	0.9821	0.9660	0.9741	0.9731
p-value	0.0245	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

d'Agostino-Pearson

DA-stat	2.1384	11.0539	12.2304	51.6841	31.3390	36.9925
p-value	0.3433	0.0040	0.0022	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	yes	no	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	125.066
df1	5
df2	1613.271
p-value	0.0000
sig	yes

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0000	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

QS060	0
QS090	0
QS125	0
QS180	2
QS210	1
QS250	1

GAMES HOWELL — alpha 0.05

group	mean	size	variance
QS060	54.8279	581	27.936
QS090	54.0792	581	16.011
QS125	53.0861	581	10.320
QS180	51.7900	581	7.349
QS210	51.1910	581	6.951
QS250	50.5353	581	6.966

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
QS060	QS090	0.7487	0.1945	3.8499	1080.447	4.0300	-0.0350	1.5324	0.0717	0.7837
QS060	QS125	1.7418	0.1814	9.5997	957.071	4.0300	1.0106	2.4731	0.0000	0.7312
QS060	QS180	3.0379	0.1743	17.4332	865.404	4.0300	2.3356	3.7401	0.0000	0.7023
QS060	QS210	3.6368	0.1733	20.9891	851.816	4.0300	2.9385	4.3351	0.0000	0.6983
QS060	QS250	4.2926	0.1733	24.7683	852.336	4.0300	3.5942	4.9910	0.0000	0.6984
QS090	QS125	0.9931	0.1505	6.5973	1108.235	4.0300	0.3865	1.5998	0.0001	0.6066
QS090	QS180	2.2892	0.1418	16.1452	1019.782	4.0300	1.7178	2.8606	0.0000	0.5714
QS090	QS210	2.8881	0.1406	20.5452	1003.753	4.0300	2.3216	3.4546	0.0000	0.5665
QS090	QS250	3.5439	0.1406	25.2019	1004.379	4.0300	2.9772	4.1106	0.0000	0.5667
QS125	QS180	1.2960	0.1233	10.5103	1128.100	4.0300	0.7991	1.7930	0.0000	0.4969
QS125	QS210	1.8950	0.1219	15.5435	1117.486	4.0300	1.4037	2.3863	0.0000	0.4913
QS125	QS250	2.5508	0.1220	20.9132	1117.922	4.0300	2.0592	3.0423	0.0000	0.4915
QS180	QS210	0.5990	0.1109	5.3993	1159.104	4.0300	0.1519	1.0460	0.0020	0.4471
QS180	QS250	1.2547	0.1110	11.3046	1159.172	4.0300	0.8074	1.7020	0.0000	0.4473
QS210	QS250	0.6558	0.1094	5.9919	1159.999	4.0300	0.2147	1.0968	0.0004	0.4410

Unequal variances: Yes

Normally distributed: Yes (QS060)

Significantly different: Yes (All pairings except QSA060/QS090)

Table C.10 Analysis of variance: AS060-AS250

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
AS060	581	26461	45.5439	35.738	20728	0.2004	45.1510	45.9367
AS090	581	25421	43.7539	24.941	14466	0.2004	43.3610	44.1467
AS125	581	24505	42.1773	19.439	11275	0.2004	41.7844	42.5701
AS180	581	23434	40.3339	19.123	11091	0.2004	39.9410	40.7268
AS210	581	22953	39.5060	19.706	11429	0.2004	39.1132	39.8989
AS250	581	22435	38.6145	21.013	12188	0.2004	38.2216	39.0073

ANOVA

Sources	SS	df	AS	F	P value	Eta-sq	RASSE	Omega Sq
Between Groups	20573	5	4114.62	176.3912	0.0000	0.2022	0.5510	0.2010
Within Groups	81177	3480	23.33					
Total	101750	3485	29.20					

Shapiro-Wilk Test

	AS060	AS090	AS125	AS180	AS210	AS250
W-stat	0.9855	0.9724	0.9652	0.9690	0.9791	0.9791
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

d'Agostino-Pearson

DA-stat	8.5472	24.0378	25.1064	22.9909	15.1832	10.4574
p-value	0.0139	0.0000	0.0000	0.0000	0.0005	0.0054
alpha	0.05	0.05	0.05	0.05	0.05	0.05
normal	no	no	no	no	no	no

Welch's Test

Alpha	0.05
F-stat	155.469
df1	5
df2	1621.610
p-value	0.0000
sig	yes

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0000	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

AS060	0
AS090	0
AS125	0
AS180	0
AS210	0
AS250	0

GAMES HOWELL — alpha 0.05

group	mean	size	variance
AS060	45.5439	581	35.738
AS090	43.7539	581	24.941
AS125	42.1773	581	19.439
AS180	40.3339	581	19.123
AS210	39.5060	581	19.706
AS250	38.6145	581	21.013

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
AS060	AS090	1.7900	0.2285	7.8332	1124.399	4.0300	0.8691	2.7109	0.0000	0.9209
AS060	AS125	3.3666	0.2179	15.4495	1066.906	4.0300	2.4884	4.2448	0.0000	0.8782
AS060	AS180	5.2100	0.2173	23.9777	1062.538	4.0300	4.3343	6.0856	0.0000	0.8757
AS060	AS210	6.0379	0.2184	27.6414	1070.487	4.0300	5.1576	6.9182	0.0000	0.8803
AS060	AS250	6.9294	0.2210	31.3554	1086.832	4.0300	6.0388	7.8200	0.0000	0.8906
AS090	AS125	1.5766	0.1954	8.0673	1142.442	4.0300	0.7890	2.3642	0.0000	0.7876
AS090	AS180	3.4200	0.1947	17.5624	1140.122	4.0300	2.6352	4.2047	0.0000	0.7848
AS090	AS210	4.2478	0.1960	21.6709	1144.265	4.0300	3.4579	5.0378	0.0000	0.7899
AS090	AS250	5.1394	0.1989	25.8437	1151.587	4.0300	4.3380	5.9408	0.0000	0.8014
AS125	AS180	1.8434	0.1822	10.1190	1159.922	4.0300	1.1092	2.5775	0.0000	0.7341
AS125	AS210	2.6713	0.1835	14.5540	1159.946	4.0300	1.9316	3.4109	0.0000	0.7397
AS125	AS250	3.5628	0.1866	19.0952	1158.247	4.0300	2.8109	4.3147	0.0000	0.7519
AS180	AS210	0.8279	0.1828	4.5289	1159.739	4.0300	0.0912	1.5646	0.0175	0.7367
AS180	AS250	1.7194	0.1859	9.2518	1157.432	4.0300	0.9705	2.4684	0.0000	0.7490
AS210	AS250	0.8916	0.1872	4.7628	1158.805	4.0300	0.1372	1.6460	0.0101	0.7544

Unequal variances: Yes

Normally distributed: No

Significantly different: Yes (All pairings)

C.3 CORRELATION COEFFICIENTS

When a set of data is not normally distributed or when the presence of outliers gives a distorted picture of the association between two random variables, Spearman's rank correlation is a non-parametric test that substitutes for Pearson's correlation.

The coefficient of determination or correlation-squared indicates how closely two time-series track each other. It also points to the reliability of the alpha (excess return) and beta (volatility) coefficients from a linear regression.

Table C.11 Correlation: TSXV-MI/TXVC (2021)

Correlation Coefficients: TSXV-MI/TXVC (2021)			
Pearson	0.7444		
Spearman	0.7004		
Kendall	0.5212		
Pearson's coeff (t test)		Pearson's coeff (Fisher)	
Alpha	0.05	Hyp rho	0
Tails	2	Alpha	0.05
		Tails	2
corr	0.7444		
std err	0.0425	corr	0.7444
t	17.5194	std err	0.0635
p-value	0	z	15.0605
lower	0.6607	p-value	0
upper	0.8281	lower	0.6833
		upper	0.7951

Table C.12 Correlation: TSXV-MI/TXVC (2019-2021)

Correlation Coefficients: TSXV-MI/TXVC (2019-2021)			
Pearson	0.8109		
Spearman	0.6965		
Kendall	0.5150		
Pearson's coeff (t test)		Pearson's coeff (Fisher)	
Alpha	0.05	Hyp rho	0
Tails	2	Alpha	0.05
		Tails	2
corr	0.8109		
std err	0.0214	corr	0.8109
t	37.8956	std err	0.0365
p-value	0	z	30.8730
lower	0.7689	p-value	2.8E-209
upper	0.8529	lower	0.7848
		upper	0.8340

Table C.13 Correlation: TSXV-MI/TXVC (2017-2021)

Correlation Coefficients: TSXV-MI/TXVC (2017-2021)			
Pearson	0.7583		
Spearman	0.6461		
Kendall	0.4703		
Pearson's coeff (t test)		Pearson's coeff (Fisher)	
Alpha	0.05	Hyp rho	0
Tails	2	Alpha	0.05
		Tails	2
corr	0.7583		
std err	0.0185	corr	0.7583
t	41.0823	std err	0.0283
p-value	0	z	35.0275
lower	0.7221	p-value	8.6E-269
upper	0.7946	lower	0.7337
		upper	0.7810

REFERENCE

RealStats. 2022. Real statistics using Excel [Website]. Charles Zaiontz. Available at: https://www.real-statistics.com.

RESULTS: STATISTICAL TESTS (Markets)

D.1 DESCRIPTIVE STATISTICS

The statistical analysis for this study was generated using the Real Statistics Resource Pack software for Excel (Release 8.3.1), Copyright (2013-2022) by Charles Zaiontz (RealStats 2022).

Since the skewness and kurtosis of the normal distribution are zero, these two parameters should be close to zero for data to follow a normal distribution. Rough measures of the standard errors of skewness and kurtosis are sqrt(6/n) and sqrt(24/n) respectively, where n is the sample size. The data are not symmetric (and therefore not normal) or normal if the absolute values of skewness and kurtosis are more than twice their standard errors.

Table D.1 Descriptive statistics: JSE Momentum Index (JSE-MI)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	JSE-MI		*JSE-MI*	alpha	0.05
Mean	0.0725	W-stat	0.8344		
Standard Error	0.0203	p-value	0		JSE-MI
Median	0.1096	alpha	0.05	outlier	-13.0657
Standard Deviation	1.1545	normal	no	G	11.3801
Sample Variance	1.3328			G-crit	4.1624
Kurtosis	21.1831	d'Agostino-Pearson		sig	yes
Skewness	-0.3633			ESD outliers	36
Range	25.5568	DA-stat	892.2956		
Maximum	12.4911	p-value	0		
Minimum	-13.0657	alpha	0.05		
Sum	235.5746	normal	no		
Count	3249				
CV	15.9225				

Table D.2 Descriptive statistics: TSX Momentum Index (TSX-MI)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	TSX-MI		*TSX-MI*	alpha	0.05
Mean	0.0747	W-stat	0.9040		
Standard Error	0.0186	p-value	0		TSX-MI
Median	0.1396	alpha	0.05	outlier	-12.6438
Standard Deviation	1.0606	normal	no	G	11.9916
Sample Variance	1.1249			G-crit	4.1623
Kurtosis	14.0274	d'Agostino-Pearson		sig	yes
Skewness	-1.1402			ESD outliers	23
Range	21.4896	DA-stat	1155.7004		
Maximum	8.8458	p-value	0		
Minimum	-12.6438	alpha	0.05		
Sum	242.7289	normal	no		
Count	3248				
CV	14.1923				

Table D.3 Descriptive statistics: TSXV Momentum Index (TSXV-MI)

Descriptive Statistics		Shapiro-Wilk Test		Grubbs/ESD Test	
	TSXV-MI		*TSXV-MI*	alpha	0.05
Mean	0.000834	W-stat	0.9400		
Standard Error	0.000283	p-value	0		TSXV-MI
Median	0.000842	alpha	0.05	outlier	-0.1312
Standard Deviation	0.016109	normal	no	G	8.1949
Sample Variance	0.000260			G-crit	4.1623
Kurtosis	6.272372	d'Agostino-Pearson		sig	yes
Skewness	-0.345888			ESD outliers	19
Range	0.240845	DA-stat	490.3039		
Maximum	0.109666	p-value	0		
Minimum	-0.131179	alpha	0.05		
Sum	2.708545	normal	no		
Count	3248				
CV	19.3177				

D.2 ANALYSIS OF VARIANCE

The single factor analysis-of-variance (ANOVA) tests for differences in averages.

Table D.4 Analysis of variance: Momentum Score (MS)

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
MS-JSE	4206	115415	27.4406	623.99	2623871	0.5013	26.4580	28.4231
MS-TSX	14010	441741	31.5304	982.01	13757040	0.2747	30.9921	32.0688
MS-TSXV	3486	158177	45.3749	1879.99	6551771	0.5506	44.2957	46.4542

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	694064	2	347031.9	328.3631	0.0000	0.0294	0.2891	0.0293
Within Groups	22932681	21699	1056.9					
Total	23626745	21701	1088.7					

TUKEY HSD/KRAMER — alpha 0.05

group	mean	n	ss	df	q-crit
MS-JSE	27.4406	4206	2623871		
MS-TSX	31.5304	14010	13757040		
MS-TSXV	45.3749	3486	6551771		
		21702	22932681	21699	3.314

Q TEST

group 1	group 2	mean	std err	q-stat	lower	upper	p-value	mean-crit	Cohen d
MS-JSE	MS-TSX	4.0898	0.4042	10.1191	2.7504	5.4293	0.0000	1.3394	0.1258
MS-JSE	MS-TSXV	17.9344	0.5265	34.0622	16.1895	19.6793	0.0000	1.7449	0.5517
MS-TSX	MS-TSXV	13.8445	0.4351	31.8199	12.4026	15.2864	0.0000	1.4419	0.4259

Shapiro-Wilk Test

	MS-JSE	MS-TSX	MS-TSXV
W-stat	0.7713	N/A	0.8388
p-value	0.0000	N/A	0.0000
alpha	0.05	0.05	0.05
normal	no	N/A	no

Welch's Test

Alpha	0.05
F-stat	233.98
df1	2
df2	7195.06
p-value	0.0000
sig	yes

d'Agostino-Pearson

	MS-JSE	MS-TSX	MS-TSXV
DA-stat	2953.10	10685.95	1577.59
p-value	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05
normal	no	no	no

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0000	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

MS-JSE	40+
MS-TSX	40+
MS-TSXV	27

GAMES HOWELL — alpha 0.05

group	mean	size	variance
MS-JSE	27.4406	4206	623.99
MS-TSX	31.5304	14010	982.01
MS-TSXV	45.3749	3486	1879.99

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
MS-JSE	MS-TSX	4.0898	0.3305	12.3750	8544.597	3.3140	2.9946	5.1851	0.0000	1.0953
MS-JSE	MS-TSXV	17.9344	0.5864	30.5855	5331.728	3.3140	15.9911	19.8776	0.0000	1.9432
MS-TSX	MS-TSXV	13.8445	0.5520	25.0810	4431.158	3.3140	12.0152	15.6738	0.0000	1.8293

Unequal variances: Yes

Normally distributed: No

Significantly different: Yes (All pairings)

Table D.5 Analysis of variance: Volatility Score (VS)

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
VS-JSE	4206	2552.61	0.6069	0.2744	1154	0.0139	0.5797	0.6341
VS-TSX	14010	11163.44	0.7968	0.6700	9386	0.0076	0.7819	0.8117
VS-TSXV	3486	4404.67	1.2635	2.0031	6981	0.0152	1.2337	1.2934

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	879	2	439.7	544.5931	0.0000	0.0478	0.3760	0.0477
Within Groups	17520	21699	0.8					
Total	18400	21701	0.8					

TUKEY HSD/KRAMER — alpha 0.05

group	mean	n	ss	df	q-crit
VS-JSE	0.6069	4206	1154		
VS-TSX	0.7968	14010	9386		
VS-TSXV	1.2635	3486	6981		
		21702	17520	21699	3.314

Q TEST

group 1	group 2	mean	std err	q-stat	lower	upper	p-value	mean-crit	Cohen d
VS-JSE	VS-TSX	0.1899	0.0112	17.0007	0.1529	0.2269	0.0000	0.0370	0.2114
VS-JSE	VS-TSXV	0.6566	0.0146	45.1198	0.6084	0.7049	0.0000	0.0482	0.7308
VS-TSX	VS-TSXV	0.4667	0.0120	38.8086	0.4269	0.5066	0.0000	0.0399	0.5194

Shapiro-Wilk Test

	VS-JSE	VS-TSX	VS-TSXV
W-stat	0.6656	N/A	0.5509
p-value	0.0000	N/A	0.0000
alpha	0.05	0.05	0.05
normal	no	N/A	no

Welch's Test

Alpha	0.05
F-stat	407.97
df1	2
df2	7402.25
p-value	0.0000
sig	yes

d'Agostino-Pearson

DA-stat	3706.98	14337.85	3940.73
p-value	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05
normal	no	no	no

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0000	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

VS-JSE	40+
VS-TSX	40+
VS-TSXV	40+

GAMES HOWELL — alpha 0.05

group	mean	size	variance
VS-JSE	0.6069	4206	0.2744
VS-TSX	0.7968	14010	0.6700
VS-TSXV	1.2635	3486	2.0031

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
VS-JSE	VS-TSX	0.1899	0.0075	25.2613	10876.12	3.3140	0.1650	0.2148	0.0000	0.0249
VS-JSE	VS-TSXV	0.6566	0.0179	36.7114	4275.46	3.3140	0.5974	0.7159	0.0000	0.0593
VS-TSX	VS-TSXV	0.4667	0.0176	26.4555	4082.17	3.3140	0.4082	0.5252	0.0000	0.0585

Unequal variances: Yes

Normally distributed: No

Significantly different: Yes (All pairings)

Table D.6 Analysis of variance: Quality Score (QS)

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
QS-JSE	4206	226561	53.8661	18.8622	79316	0.0637	53.7412	53.9911
QS-TSX	14010	747734	53.3714	17.0825	239309	0.0349	53.3030	53.4399
QS-TSXV	3486	183311	52.5849	14.9533	52112	0.0700	52.4477	52.7221

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	3166	2	1582.9	92.6445	0.0000	0.0085	0.1563	0.0084
Within Groups	370737	21699	17.1					
Total	373903	21701	17.2					

TUKEY HSD/KRAMER — alpha 0.05

group	mean	n	ss	df	q-crit
QS-JSE	53.8661	4206	79316		
QS-TSX	53.3714	14010	239309		
QS-TSXV	52.5849	3486	52112		
		21702	370737	21699	3.314

Q TEST

group 1	group 2	mean	std err	q-stat	lower	upper	p-value	mean-crit	Cohen d
QS-JSE	QS-TSX	0.4947	0.0514	9.6265	0.3244	0.6650	0.0000	0.1703	0.1197
QS-JSE	QS-TSXV	1.2812	0.0669	19.1385	1.0594	1.5031	0.0000	0.2219	0.3100
QS-TSX	QS-TSXV	0.7865	0.0553	14.2179	0.6032	0.9699	0.0000	0.1833	0.1903

Shapiro-Wilk Test

	QS-JSE	QS-TSX	QS-TSXV
W-stat	0.9631	N/A	0.9620
p-value	0.0000	N/A	0.0000
alpha	0.05	0.05	0.05
normal	no	N/A	no

Welch's Test

Alpha	0.05
F-stat	97.30
df1	2
df2	7350.70
p-value	0.0000
sig	yes

d'Agostino-Pearson

	QS-JSE	QS-TSX	QS-TSXV
DA-stat	453.495	1134.867	357.669
p-value	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05
normal	no	no	no

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0000	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

QS-JSE	4
QS-TSX	4
QS-TSXV	3

GAMES HOWELL — alpha 0.05

group	mean	size	variance
QS-JSE	53.8661	4206	18.8622
QS-TSX	53.3714	14010	17.0825
QS-TSXV	52.5849	3486	14.9533

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
QS-JSE	QS-TSX	0.4947	0.0534	9.2633	6654.76	3.3140	0.3177	0.6717	0.0000	0.1770
QS-JSE	QS-TSXV	1.2812	0.0662	19.3438	7650.66	3.3140	1.0617	1.5007	0.0000	0.2195
QS-TSX	QS-TSXV	0.7865	0.0525	14.9866	5634.56	3.3140	0.6126	0.9605	0.0000	0.1739

Unequal variances: Yes

Normally distributed: No

Significantly different: Yes (All pairings)

Table D.7 Analysis of variance: Activity Score (AS)

ANOVA: Single Factor

DESCRIPTION — Alpha 0.05

Group	Count	Sum	Mean	Variance	SS	Std Err	Lower	Upper
AS-JSE	4206	203525	48.3892	48.0975	202250	0.0991	48.1950	48.5834
AS-TSX	14010	678265	48.4129	42.2794	592292	0.0543	48.3065	48.5193
AS-TSXV	3486	145209	41.6549	29.1965	101750	0.1089	41.4415	41.8683

ANOVA

Sources	SS	df	MS	F	P value	Eta-sq	RMSSE	Omega Sq
Between Groups	133420	2	66710.0	1615.03	0.0000	0.1296	0.6060	0.1295
Within Groups	896292	21699	41.3					
Total	1029712	21701	47.4					

TUKEY HSD/KRAMER — alpha 0.05

group	mean	n	ss	df	q-crit
AS-JSE	48.3892	4206	202250		
AS-TSX	48.4129	14010	592292		
AS-TSXV	41.6549	3486	101750		
		21702	896292	21699	3.314

Q TEST

group 1	group 2	mean	std err	q-stat	lower	upper	p-value	mean-crit	Cohen d
AS-JSE	AS-TSX	0.0237	0.0799	0.2968	-0.2411	0.2885	0.9760	0.2648	0.0037
AS-JSE	AS-TSXV	6.7343	0.1041	64.6966	6.3893	7.0793	0.0000	0.3450	1.0478
AS-TSX	AS-TSXV	6.7580	0.0860	78.5674	6.4730	7.0431	0.0000	0.2851	1.0515

Shapiro-Wilk Test

	AS-JSE	AS-TSX	AS-TSXV
W-stat	0.9786	N/A	0.9784
p-value	0.0000	N/A	0.0000
alpha	0.05	0.05	0.05
normal	no	N/A	no

Welch's Test

Alpha	0.05
F-stat	2118.37
df1	2
df2	7594.99
p-value	0.0000
sig	yes

d'Agostino-Pearson

DA-stat	107.787	102.308	163.008
p-value	0.0000	0.0000	0.0000
alpha	0.05	0.05	0.05
normal	no	no	no

Levene's Tests

type	p-value	
means	0.0000	[< 0.05]
medians	0.0000	[< 0.05]
trimmed	0.0000	[< 0.05]

Grubbs/ESD Test

AS-JSE	0
AS-TSX	0
AS-TSXV	0

GAMES HOWELL — alpha 0.05

group	mean	size	variance
AS-JSE	48.3892	4206	48.0975
AS-TSX	48.4129	14010	42.2794
AS-TSXV	41.6549	3486	29.1965

Q TEST

group 1	group 2	mean	std err	q-stat	df	q-crit	lower	upper	p-value	mean-crit
AS-JSE	AS-TSX	0.0237	0.0850	0.2790	6579.69	3.3140	-0.2580	0.3054	0.9788	0.2817
AS-JSE	AS-TSXV	6.7343	0.0995	67.6638	7661.39	3.3140	6.4045	7.0641	0.0000	0.3298
AS-TSX	AS-TSXV	6.7580	0.0755	89.5389	6247.11	3.3140	6.5079	7.0081	0.0000	0.2501

Unequal variances: Yes

Normally distributed: No

Significantly different: Yes (All pairings except AS-JSE/AS-TSX)

D.3 CORRELATION COEFFICIENTS

When a set of data is not normally distributed or when the presence of outliers gives a distorted picture of the association between two random variables, Spearman’s rank correlation is a non-parametric test that substitutes for Pearson’s correlation.

The coefficient of determination or correlation-squared indicates how closely two time-series track each other. It also points to the reliability of the alpha (excess return) and beta (volatility) coefficients from a linear regression.

Table D.8 Correlation: JSE-MI/TSX-MI (2017-2021)

Correlation Coefficients: JSE-MI/TSX-MI (5Y)			
Pearson	0.4698		
Spearman	0.3035		
Kendall	0.2116		
Pearson's coeff (t test)		Pearson's coeff (Fisher)	
Alpha	0.05	Hyp rho	0
Tails	2	Alpha	0.05
		Tails	2
corr	0.4698		
std err	0.0246	corr	0.4698
t	19.0750	std err	0.0279
p-value	0	z	18.2662
lower	0.4214	p-value	0
upper	0.5181	lower	0.4261
		upper	0.5113

Table D.9 Correlation: JSE-MI/TSX-MI (2019-2021)

Correlation Coefficients: JSE-MI/TSX-MI (3Y)			
Pearson	0.5136		
Spearman	0.3620		
Kendall	0.2554		
Pearson's coeff (t test)		Pearson's coeff (Fisher)	
Alpha	0.05	Hyp rho	0
Tails	2	Alpha	0.05
		Tails	2
corr	0.5136		
std err	0.0309	corr	0.5136
t	16.6301	std err	0.0360
p-value	0	z	15.7595
lower	0.4529	p-value	0
upper	0.5742	lower	0.4597
		upper	0.5636

Table D.10 Correlation: JSE-MI/TSXV-MI (2017-2021)

Correlation Coefficients: JSE-MI/TSXV-MI (5Y)			
Pearson	0.3737		
Spearman	0.2517		
Kendall	0.1730		
Pearson's coeff (t test)		Pearson's coeff (Fisher)	
Alpha	0.05	Hyp rho	0
Tails	2	Alpha	0.05
		Tails	2
corr	0.3737		
std err	0.0259	corr	0.3737
t	14.4406	std err	0.0279
p-value	0	z	14.0706
lower	0.3229	p-value	5.8E-45
upper	0.4244	lower	0.3257
		upper	0.4197

Table D.11 Correlation: JSE-MI/TSXV-MI (2019-2021)

Correlation Coefficients: JSE-MI/TSXV-MI (3Y)			
Pearson	0.4278		
Spearman	0.3056		
Kendall	0.2108		
Pearson's coeff (t test)		Pearson's coeff (Fisher)	
Alpha	0.05	Hyp rho	0
Tails	2	Alpha	0.05
		Tails	2
corr	0.4278		
std err	0.0325	corr	0.4278
t	13.1497	std err	0.0360
p-value	0	z	12.6943
lower	0.3639	p-value	6.4E-37
upper	0.4916	lower	0.3684
		upper	0.4837

Table D.12 Correlation: TSX-MI/TSXV-MI (2017-2021)

Correlation Coefficients: TSX-MI/TSXV-MI (5Y)			
Pearson	0.6823		
Spearman	0.5121		
Kendall	0.3650		
Pearson's coeff (t test)		Pearson's coeff (Fisher)	
Alpha	0.05	Hyp rho	0
Tails	2	Alpha	0.05
		Tails	2
corr	0.6823		
std err	0.0204	corr	0.6823
t	33.4565	std err	0.0279
p-value	0	z	29.8641
lower	0.6423	p-value	5.8E-196
upper	0.7223	lower	0.6520
		upper	0.7105

Table D.13 Correlation: JSE-MI/TSXV-MI (2019-2021)

Correlation Coefficients: TSX-MI/TSXV-MI (3Y)			
Pearson	0.7561		
Spearman	0.5831		
Kendall	0.4226		
Pearson's coeff (t test)		Pearson's coeff (Fisher)	
Alpha	0.05	Hyp rho	0
Tails	2	Alpha	0.05
		Tails	2
corr	0.7561		
std err	0.0236	corr	0.7561
t	32.1020	std err	0.0360
p-value	0	z	27.4084
lower	0.7099	p-value	2.2E-165
upper	0.8024	lower	0.7242
		upper	0.7848

D.4 COINTEGRATION

Two time-series are cointegrated when neither time series is stationary but their first differences are stationary, provided the time series of the residuals from the linear regression of one of the time series on the other is also stationary. Therefore, both series are individually non-stationary but there exists a linear combination that is stationary, meaning that the average distance between them remains relatively constant even though they move independently.

The maximum number of lags for the tests is calculated as the cube root of the number of observations in the time series, raised to the next highest integer.

Table D.14 Cointegration: ALSH/TXCX/TXVC (2009-2021)

ADF Tests		ALSH/TXCX (FULL)			Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-3.1095	-3.0427	-14.9711	-14.2629	type	2
tau-crit	-3.4117	-3.4117	-3.4117	-3.4117	max lags	15
stationary	no	no	yes	yes	criteria	none
aic	4.4876	3.5490	4.4902	3.5507		
bic	4.5206	3.5820	4.5233	3.5837	tau-stat	-2.1043
lags	15	15	15	15	tau-crit	-3.7834
coeff	-6.3E-03	-6.1E-03	-1.1E+00	-9.7E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	15
					p-value	> .1

ADF Tests		ALSH/TXVC (FULL)			Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-3.1095	-2.0985	-14.9711	-13.1037	type	2
tau-crit	-3.4117	-3.4117	-3.4117	-3.4117	max lags	15
stationary	no	no	yes	yes	criteria	none
aic	4.4876	3.8400	4.4902	3.8406		
bic	4.5206	3.8731	4.5233	3.8736	tau-stat	-2.6262
lags	15	15	15	15	tau-crit	-3.7834
coeff	-6.3E-03	-1.5E-03	-1.1E+00	-7.0E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	15
					p-value	> .1

ADF Tests		TXCX/TXVC (FULL)			Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-3.0427	-2.0985	-14.2629	-13.1037	type	2
tau-crit	-3.4117	-3.4117	-3.4117	-3.4117	max lags	15
stationary	no	no	yes	yes	criteria	none
aic	3.5490	3.8400	3.5507	3.8406		
bic	3.5820	3.8731	3.5837	3.8736	tau-stat	-2.2011
lags	15	15	15	15	tau-crit	-3.7834
coeff	-0.0061	-0.0015	-0.9698	-0.7009	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	15
					p-value	> .1

Table D.15 Cointegration: ALSH/TXCX/TXVC (2012-2021)

ADF Tests	ALSH/TXCX (10Y)				Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-2.7927	-2.6657	-13.1889	-12.9309	type	2
tau-crit	-3.4121	-3.4121	-3.4121	-3.4121	max lags	14
stationary	no	no	yes	yes	criteria	none
aic	4.6449	3.5233	4.6478	3.5261		
bic	4.6838	3.5621	4.6866	3.5650	tau-stat	-2.9319
lags	14	14	14	14	tau-crit	-3.7843
coeff	-7.2E-03	-6.2E-03	-1.0E+00	-9.3E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	14
					p-value	> .1

ADF Tests	ALSH/TXVC (10Y)				Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-2.7927	-2.2712	-13.1889	-11.7712	type	2
tau-crit	-3.4121	-3.4121	-3.4121	-3.4121	max lags	14
stationary	no	no	yes	yes	criteria	none
aic	4.6449	3.1513	4.6478	3.1488		
bic	4.6838	3.1901	4.6866	3.1876	tau-stat	-2.1467
lags	14	14	14	14	tau-crit	-3.7843
coeff	-7.2E-03	-2.1E-03	-1.0E+00	-7.1E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	14
					p-value	> .1

ADF Tests	TXCX/TXVC (10Y)				Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-2.6657	-2.2712	-12.9309	-11.7712	type	2
tau-crit	-3.4121	-3.4121	-3.4121	-3.4121	max lags	14
stationary	no	no	yes	yes	criteria	none
aic	3.5233	3.1513	3.5261	3.1488		
bic	3.5621	3.1901	3.5650	3.1876	tau-stat	-2.1739
lags	14	14	14	14	tau-crit	-3.7843
coeff	-0.0062	-0.0021	-0.9293	-0.7113	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	14
					p-value	> .1

Referring to ALSH/TXCX (10y) in Table D.11 above, note that the two series are not stationary, but that their first differences are stationary. The maximum number of lags was calculated to be 14 (i.e., the cube root of the size of the time series, which in this instance is 2573, raised to the next highest integer). Type equals 2 as both time series have a drift and a trend. The two original time series are now considered to be cointegrated provided the time series of the residuals is stationary, which is not the case (-2.9319 > -3.7843) at a 5 per cent level of significance (p-value > 0.10).

Table D.16 Cointegration: ALSH/TXCX/TXVC (2017-2021)

ADF Tests		ALSH/TXCX (5Y)			Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-1.9480	-2.4121	-10.4461	-9.4596	type	2
tau-crit	-3.4136	-3.4136	-3.4136	-3.4136	max lags	11
stationary	no	no	yes	yes	criteria	none
aic	4.9435	3.8261	4.9473	3.8296		
bic	5.0001	3.8826	5.0039	3.8862	tau-stat	-3.4271
lags	11	11	11	11	tau-crit	-3.7880
coeff	-7.6E-03	-8.0E-03	-9.9E-01	-8.2E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	11
					p-value	> .1
ADF Tests		ALSH/TXVC (5Y)			Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-1.9480	-1.3280	-10.4461	-9.7032	type	2
tau-crit	-3.4136	-3.4136	-3.4136	-3.4136	max lags	11
stationary	no	no	yes	yes	criteria	none
aic	4.9435	3.1272	4.9473	3.1292		
bic	5.0001	3.1838	5.0039	3.1858	tau-stat	-2.1884
lags	11	11	11	11	tau-crit	-3.7880
coeff	-7.6E-03	-2.4E-03	-9.9E-01	-7.8E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	11
					p-value	> .1
ADF Tests		TXCX/TXVC (5Y)			Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-2.4121	-1.3280	-9.4596	-9.7032	type	2
tau-crit	-3.4136	-3.4136	-3.4136	-3.4136	max lags	11
stationary	no	no	yes	yes	criteria	none
aic	3.8261	3.1272	3.8296	3.1292		
bic	3.8826	3.1838	3.8862	3.1858	tau-stat	-1.5383
lags	11	11	11	11	tau-crit	-3.7880
coeff	-0.0080	-0.0024	-0.8215	-0.7820	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	11
					p-value	> .1

The maximum number of lags for the 5-year period is 11, calculated as the cube root of the size of the time series (1287) and raised to the next highest integer.

Table D.17 Cointegration: Market/Momentum Index (2009-2021)

ADF Tests	ALSH/JSE-MI (FULL)				Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-3.1095	0.0566	-14.9711	-13.1051	type	2
tau-crit	-3.4117	-3.4117	-3.4117	-3.4117	max lags	15
stationary	no	no	yes	yes	criteria	none
aic	4.4876	6.2104	4.4902	6.2105		
bic	4.5206	6.2434	4.5233	6.2435	tau-stat	-0.7186
lags	15	15	15	15	tau-crit	-3.7834
coeff	-6.3E-03	6.1E-05	-1.1E+00	-8.8E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	15
					p-value	> .1

ADF Tests	TXCX/TSX-MI (FULL)				Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.1
tau-stat	-3.0427	-0.3610	-14.2629	-13.8040	type	2
tau-crit	-3.1278	-3.1278	-3.1278	-3.1278	max lags	15
stationary	no	no	yes	yes	criteria	none
aic	3.5490	6.1692	3.5507	6.1670		
bic	3.5820	6.2022	3.5837	6.2000	tau-stat	-3.6893
lags	15	15	15	15	tau-crit	-3.4984
coeff	-6.1E-03	-3.3E-04	-9.7E-01	-9.0E-01	cointegrated	yes
p-value	> .1	> .1	< .01	< .01	lags	15
					p-value	0.0665

ADF Tests	TXVC/TSXV-MI (FULL)				Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-2.0985	1.2965	-13.1037	-12.0925	type	2
tau-crit	-3.4117	-3.4117	-3.4117	-3.4117	max lags	15
stationary	no	no	yes	yes	criteria	none
aic	3.8400	6.7082	3.8406	6.7089		
bic	3.8731	6.7413	3.8736	6.7420	tau-stat	0.8569
lags	15	15	15	15	tau-crit	-3.7834
coeff	-0.0015	0.0009	-0.7009	-0.7592	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	15
					p-value	> .1

Referring to TXCX/TSX-MI (13y) in Table D.13 above, note that the two series are not stationary, but that their first differences are stationary. The maximum number of lags was calculated to be 15 (i.e., the cube root of the size of the time series, which in this instance is 3347, raised to the next highest integer). Type equals 2 as both time series have a drift and a trend. The two original time series are now considered to be cointegrated provided the time series of the residuals is stationary, which is the case at a 10 per cent (-3.6893 < -3.4984) but not a 5 per cent (-3.6893 > -3.7834) level of significance (p-value = 0.0665).

Table D.18 Cointegration: Market/Momentum Index (2012-2021)

ADF Tests		ALSH/JSE-MI (10Y)			Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-2.7927	-0.2509	-13.1889	-11.6265	type	2
tau-crit	-3.4121	-3.4121	-3.4121	-3.4121	max lags	14
stationary	no	no	yes	yes	criteria	none
aic	4.6449	6.4682	4.6478	6.4670		
bic	4.6838	6.5071	4.6866	6.5059	tau-stat	-1.4987
lags	14	14	14	14	tau-crit	-3.7843
coeff	-7.2E-03	-3.2E-04	-1.0E+00	-8.7E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	14
					p-value	> .1

ADF Tests		TXCX/TSX-MI (10Y)			Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-2.6657	-0.6366	-12.9309	-11.8040	type	2
tau-crit	-3.4121	-3.4121	-3.4121	-3.4121	max lags	14
stationary	no	no	yes	yes	criteria	none
aic	3.5233	6.4124	3.5261	6.4126		
bic	3.5621	6.4512	3.5650	6.4515	tau-stat	-3.0658
lags	14	14	14	14	tau-crit	-3.7843
coeff	-6.2E-03	-7.3E-04	-9.3E-01	-8.6E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	14
					p-value	> .1

ADF Tests		TXVC/TSXV-MI (10Y)			Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-2.2712	0.7802	-11.7712	-10.7520	type	2
tau-crit	-3.4121	-3.4121	-3.4121	-3.4121	max lags	14
stationary	no	no	yes	yes	criteria	none
aic	3.1513	6.9432	3.1488	6.9395		
bic	3.1901	6.9820	3.1876	6.9784	tau-stat	0.5670
lags	14	14	14	14	tau-crit	-3.7843
coeff	-0.0021	0.0007	-0.7113	-0.7588	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	14
					p-value	> .1

Table D.19 Cointegration: Market/Momentum Index (2017-2021)

ADF Tests	ALSH/JSE-MI (5Y)				Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-1.9480	-1.3668	-10.4461	-9.1190	type	2
tau-crit	-3.4136	-3.4136	-3.4136	-3.4136	max lags	11
stationary	no	no	yes	yes	criteria	none
aic	4.9435	7.0966	4.9473	7.0963		
bic	5.0001	7.1532	5.0039	7.1529	tau-stat	-2.7127
lags	11	11	11	11	tau-crit	-3.7880
coeff	-7.6E-03	-3.3E-03	-9.9E-01	-8.9E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	11
					p-value	> .1

ADF Tests	TXCX/TSX-MI (5Y)				Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-2.4121	-1.1190	-9.4596	-9.4329	type	2
tau-crit	-3.4136	-3.4136	-3.4136	-3.4136	max lags	11
stationary	no	no	yes	yes	criteria	none
aic	3.8261	6.9626	3.8296	6.9642		
bic	3.8826	7.0192	3.8862	7.0208	tau-stat	-2.9531
lags	11	11	11	11	tau-crit	-3.7880
coeff	-8.0E-03	-2.0E-03	-8.2E-01	-8.8E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	11
					p-value	> .1

ADF Tests	TXVC/TSXV-MI (5Y)				Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-1.3280	-0.7399	-9.7032	-9.8957	type	2
tau-crit	-3.4136	-3.4136	-3.4136	-3.4136	max lags	11
stationary	no	no	yes	yes	criteria	none
aic	3.1272	7.5652	3.1292	7.5663		
bic	3.1838	7.6217	3.1858	7.6229	tau-stat	-2.1094
lags	11	11	11	11	tau-crit	-3.7880
coeff	-0.0024	-0.0013	-0.7820	-0.9196	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	11
					p-value	> .1

Table D.20 Cointegration: JSE-MI/TSX-MI/TSXV-MI (2009-2021)

ADF Tests		JSE-MI/TSX-MI (FULL)			Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	0.0566	-0.3610	-13.1051	-13.8040	type	2
tau-crit	-3.4117	-3.4117	-3.4117	-3.4117	max lags	15
stationary	no	no	yes	yes	criteria	none
aic	6.2104	6.1692	6.2105	6.1670		
bic	6.2434	6.2022	6.2435	6.2000	tau-stat	-2.2272
lags	15	15	15	15	tau-crit	-3.7834
coeff	6.1E-05	-3.3E-04	-8.8E-01	-9.0E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	15
					p-value	> .1

ADF Tests		JSE-MI/TSXV-MI (FULL)			Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	0.0566	1.2965	-13.1051	-12.0925	type	2
tau-crit	-3.4117	-3.4117	-3.4117	-3.4117	max lags	15
stationary	no	no	yes	yes	criteria	none
aic	6.2104	6.7082	6.2105	6.7089		
bic	6.2434	6.7413	6.2435	6.7420	tau-stat	-1.4959
lags	15	15	15	15	tau-crit	-3.7834
coeff	6.1E-05	8.6E-04	-8.8E-01	-7.6E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	15
					p-value	> .1

ADF Tests		TSX-MI/TSXV-MI (FULL)			Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-0.3610	1.2965	-13.8040	-12.0925	type	2
tau-crit	-3.4117	-3.4117	-3.4117	-3.4117	max lags	15
stationary	no	no	yes	yes	criteria	none
aic	6.1692	6.7082	6.1670	6.7089		
bic	6.2022	6.7413	6.2000	6.7420	tau-stat	0.6115
lags	15	15	15	15	tau-crit	-3.7834
coeff	-0.0003	0.0009	-0.9020	-0.7592	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	15
					p-value	> .1

Table D.21 Cointegration: JSE-MI/TSX-MI/TSXV-MI (2012-2021)

ADF Tests	JSE-MI/TSX-MI (10Y)				Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-0.2509	-0.6366	-11.6265	-11.8040	type	2
tau-crit	-3.4121	-3.4121	-3.4121	-3.4121	max lags	14
stationary	no	no	yes	yes	criteria	none
aic	6.4682	6.4124	6.4670	6.4126		
bic	6.5071	6.4512	6.5059	6.4515	tau-stat	-2.0573
lags	14	14	14	14	tau-crit	-3.7843
coeff	-3.2E-04	-7.3E-04	-8.7E-01	-8.6E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	14
					p-value	> .1

ADF Tests	JSE-MI/TSXV-MI (10Y)				Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-0.2509	0.7802	-11.6265	-10.7520	type	2
tau-crit	-3.4121	-3.4121	-3.4121	-3.4121	max lags	14
stationary	no	no	yes	yes	criteria	none
aic	6.4682	6.9432	6.4670	6.9395		
bic	6.5071	6.9820	6.5059	6.9784	tau-stat	-2.8814
lags	14	14	14	14	tau-crit	-3.7843
coeff	-3.2E-04	6.6E-04	-8.7E-01	-7.6E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	14
					p-value	> .1

ADF Tests	TSX-MI/TSXV-MI (10Y)				Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-0.6366	0.7802	-11.8040	-10.7520	type	2
tau-crit	-3.4121	-3.4121	-3.4121	-3.4121	max lags	14
stationary	no	no	yes	yes	criteria	none
aic	6.4124	6.9432	6.4126	6.9395		
bic	6.4512	6.9820	6.4515	6.9784	tau-stat	-0.3975
lags	14	14	14	14	tau-crit	-3.7843
coeff	-0.0007	0.0007	-0.8601	-0.7588	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	14
					p-value	> .1

Table D.22 Cointegration: JSE-MI/TSX-MI/TSXV-MI (2017-2021)

ADF Tests		JSE-MI/TSX-MI (5Y)			Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-1.3668	-1.1190	-9.1190	-9.4329	type	2
tau-crit	-3.4136	-3.4136	-3.4136	-3.4136	max lags	11
stationary	no	no	yes	yes	criteria	none
aic	7.0966	6.9626	7.0963	6.9642		
bic	7.1532	7.0192	7.1529	7.0208	tau-stat	-2.1777
lags	11	11	11	11	tau-crit	-3.7880
coeff	-3.3E-03	-2.0E-03	-8.9E-01	-8.8E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	11
					p-value	> .1

ADF Tests		JSE-MI/TSXV-MI (5Y)			Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-1.3668	-0.7399	-9.1190	-9.8957	type	2
tau-crit	-3.4136	-3.4136	-3.4136	-3.4136	max lags	11
stationary	no	no	yes	yes	criteria	none
aic	7.0966	7.5652	7.0963	7.5663		
bic	7.1532	7.6217	7.1529	7.6229	tau-stat	-3.3367
lags	11	11	11	11	tau-crit	-3.7880
coeff	-3.3E-03	-1.3E-03	-8.9E-01	-9.2E-01	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	11
					p-value	> .1

ADF Tests		TSX-MI/TSXV-MI (5Y)			Engle-Granger Test	
	X var	Y var	X diff	Y diff	alpha	0.05
tau-stat	-1.1190	-0.7399	-9.4329	-9.8957	type	2
tau-crit	-3.4136	-3.4136	-3.4136	-3.4136	max lags	11
stationary	no	no	yes	yes	criteria	none
aic	6.9626	7.5652	6.9642	7.5663		
bic	7.0192	7.6217	7.0208	7.6229	tau-stat	-2.2173
lags	11	11	11	11	tau-crit	-3.7880
coeff	-0.0020	-0.0013	-0.8820	-0.9196	cointegrated	no
p-value	> .1	> .1	< .01	< .01	lags	11
					p-value	> .1

REFERENCE

RealStats. 2022. Real statistics using Excel [Website]. Charles Zaiontz. Available at: https://www.real-statistics.com.

ANNEXURE E

SUPPLEMENTARY RESULTS AND TESTING

E.1 EQUAL-WEIGHTED BENCHMARK

The custom momentum index is constructed as equal-weighted in that new members enter at the average weight of the current members (refer to Chapter 3, Equation 3.6). The index is updated monthly, and the number of members is variable. The individual weights of the remaining members are adjusted for the number of additions, and the total weight of any deletions is distributed equally between members (refer to Chapter 3, Equation 3.5). Remaining members are allowed to retain the gains or losses from previous changes in price. The custom momentum index, therefore, maintains a relatively active position over a true equal-weighted or unweighted design, which would normally reset all the member weights to the average weight when updated (Taljaard & Maré 2019). A direct comparison between the custom momentum index and a true equal-weighted index, constructed from all the stocks available for selection (also variable) during the analysis period, highlights the contrasting results. Table E1 below shows the relative performance of the custom momentum index and its true equal-weighted counterpart.

Table E.1 Momentum index results versus equal-weighted benchmark

Year	Metric	JSE-MI	JSE-EWI	Year	Metric	JSE-MI	JSE-EWI
2009	CAGR	17.30	31.01	2010	CAGR	30.28	9.48
	StdD	20.91	25.81		StdD	13.22	16.17
2011	CAGR	7.55	-11.58	2012	CAGR	37.89	4.65
	StdD	11.39	15.20		StdD	7.61	11.57
2013	CAGR	25.77	5.52	2014	CAGR	22.38	-6.43
	StdD	9.49	14.41		StdD	9.33	12.03
2015	CAGR	17.33	-13.92	2016	CAGR	-1.21	18.29
	StdD	13.28	14.79		StdD	13.08	23.83
2017	CAGR	9.39	10.04	2018	CAGR	-21.18	-15.78
	StdD	9.62	8.46		StdD	17.15	14.17
2019	CAGR	52.96	14.64	2020	CAGR	37.82	4.60
	StdD	18.40	12.12		StdD	47.23	28.95
2021	CAGR	43.49	20.92	1Y	CAGR	43.49	20.92
	StdD	11.69	14.01		StdD	11.69	14.01
FULL 2009 2021	CTGR	954.60	79.33	10Y 2012 2021	CTGR	541.66	41.41
	CAGR	19.87	4.60		CAGR	20.43	3.53
	StdD	18.36	17.26		StdD	19.09	16.48
5Y 2017 2021	CTGR	160.83	34.40	3Y 2019 2021	CTGR	202.49	45.01
	CAGR	21.14	6.09		CAGR	44.62	13.19
	StdD	24.78	17.01		StdD	29.89	19.79

Source: Price data downloaded from Bloomberg (2022)

Figures E.1 (equal-weighted index) and E.2 (custom momentum index) depicts the contrasting results in terms of index levels and member numbers graphically.

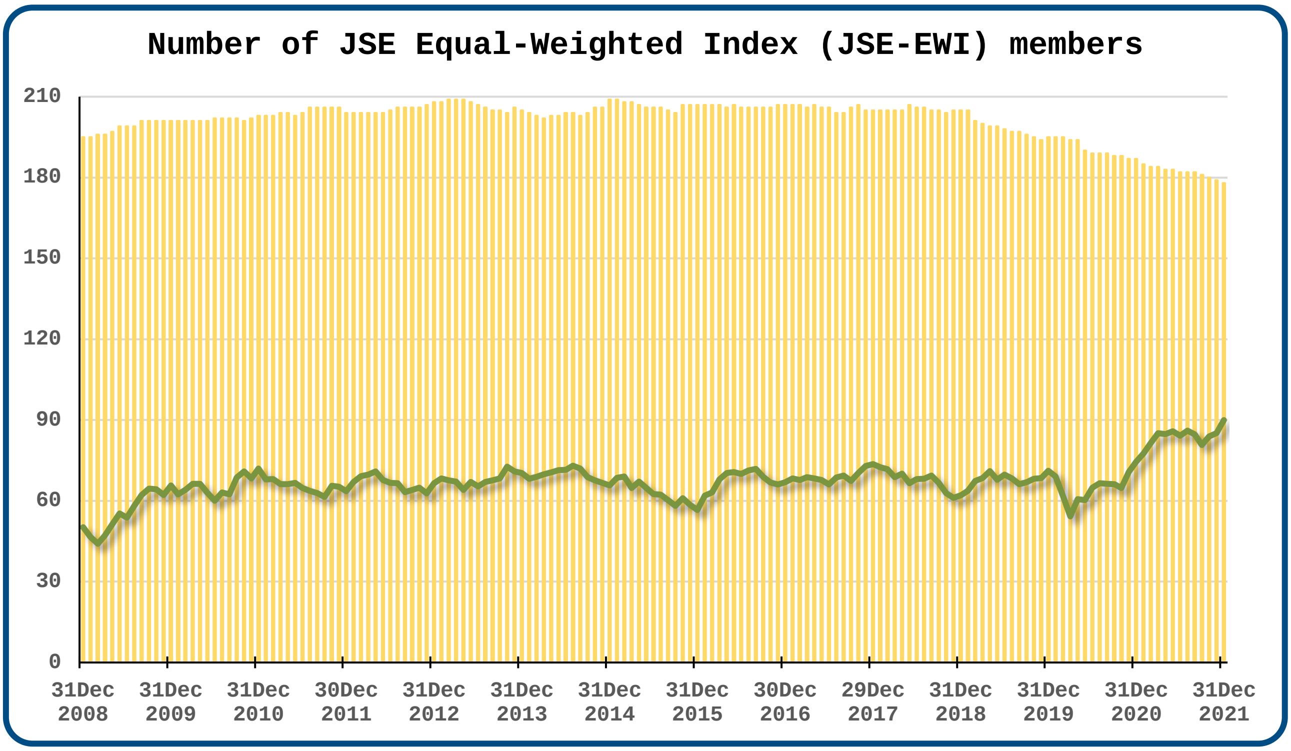

Figure E.1 JSE-EWI member numbers (Source of price data: Bloomberg 2022)

Note the relatively constant number of members (varying between 178 and 209) and the more restrained progression of the index levels for the equal-weighted index compared to the custom momentum index falling to 7 members (excluding the initial 6 months since inception) and peaking at 89 members during the analysis period.

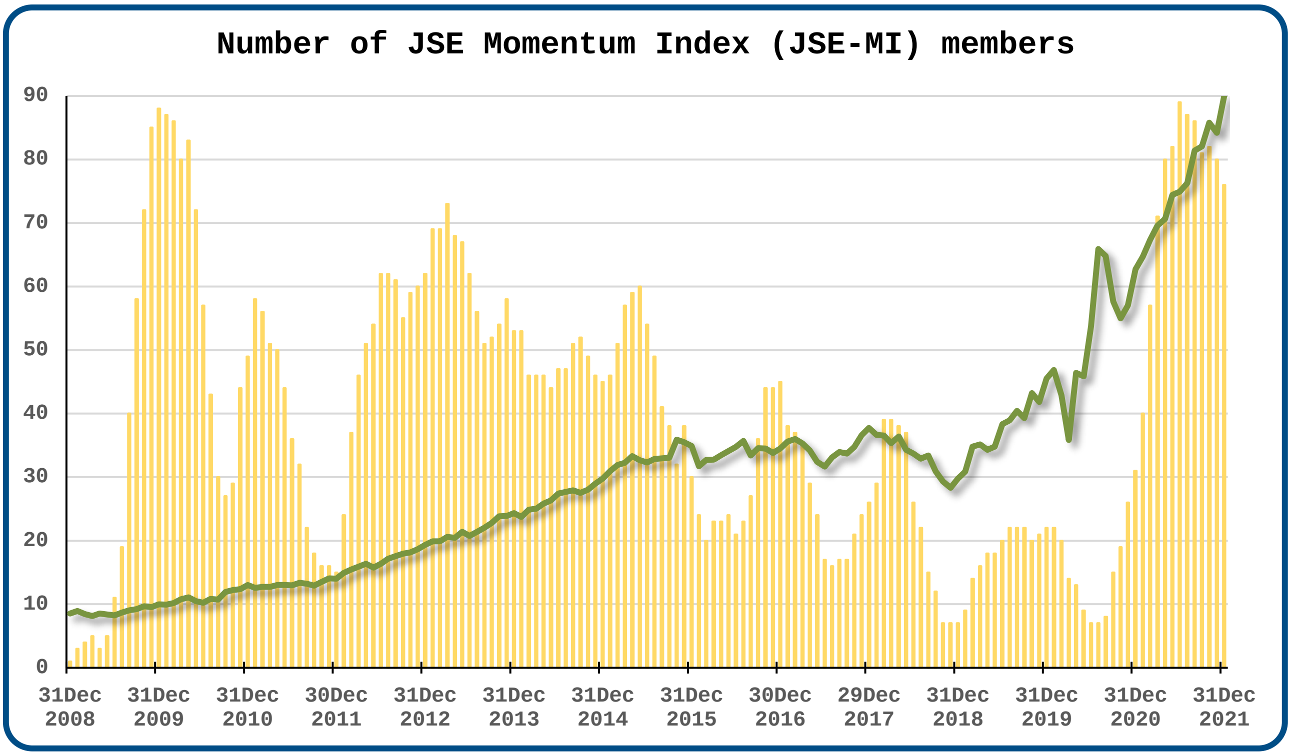

Figure E.2 JSE-MI member numbers (Source of price data: Bloomberg 2022)

The correlation between the momentum index and its equal-weighted counterpart, in general and on average, is strong (0.60 to 0.79) – refer to Table E.2 below.

Table E.2 Correlation between momentum index and equal-weighted benchmark

Year	JSE-EWI	Year	JSE-EWI	Year	JSE-EWI	Year	JSE-EWI
2009	0.58	2013	0.68	2017	0.62	1Y	0.72
2010	0.83	2014	0.72	2018	0.64	3Y	0.66
2011	0.74	2015	0.77	2019	0.39	5Y	0.65
2012	0.60	2016	0.28	2020	0.71	AVG	0.64

Source: Price data downloaded from Bloomberg (2022)

The momentum index outperforms in a drawdown analysis – refer to Table E.3 below.

Table E.3 Drawdown analysis (2009-2021)

Metric	JSE-MI	JSE-EWI
Maximum drawdown	40.39%	58.75%
Date	2020-03-19	2020-03-19
Period	19 days	537 days
Recovery	62 days	198 days
Average drawdown	5.63%	8.92%
Maximum duration	362 days	1038 days
From:	2018-01-10	2013-10-29
To:	2019-06-24	2018-01-08
Average duration	16 days	88 days
Annualised return	19.87%	4.60%
Drawdown ratio	0.49	0.12

Source: Price data downloaded from Bloomberg (2022)

Both distributions are approximately symmetric with the momentum index more likely to record outliers (higher kurtosis). The coefficient of variance (CV) indicating greater relative variability for the equal-weighted index (see Table E.4 below).

Table E.4 Summary statistics (2009-2021)

Metric	JSE-MI	JSE-EWI
Mean	0.0725 %	0.0180 %
Standard Error	0.0203 %	0.0190 %
Median	0.1096 %	0.0423 %
Standard Deviation	1.1545 %	1.0856 %
Sample Variance	1.3328	1.1786
Kurtosis	21.1831	13.8672
Skewness	-0.3633	-0.1742
Range	25.56 %	21.06 %
Maximum	12.49 %	10.87 %
Minimum	-13.07 %	-10.19 %
Sum	235.57 %	58.41 %
Count	3249	3249
CV	15.92	60.39

Source: Price data downloaded from Bloomberg (2022)

E.2 TWO-SAMPLE T-TEST

When the population variances are known, hypothesis testing can be done using a normal distribution, but population variances are not usually known. Instead, the sample variances are pooled, and testing is done using the t distribution (RealStats 2022). An independent samples t-test compares the means of two groups. There is not an assumption of normal distribution, but there is an assumption that the two standard deviations are equal. If the sample sizes are equal or very similar in size, even that assumption is not critical (Ross & Willson 2017).

Equal variance:

$$t_{df} = \frac{(\bar{x} - \bar{y}) - (\mu_x - \mu_Y)}{\sqrt{s^2\left(\frac{1}{n_x} + \frac{1}{n_Y}\right)}} \tag{E.1}$$

Where:

$$df = n_x + n_Y - 2 \quad \text{and} \quad s^2 = \frac{(n_x - 1)s_x^2 + (n_Y - 1)s_Y^2}{(n_x - 1) + (n_Y - 1)}$$

Alternatively, when the assumption of equal population variances is not met for the two-sample t-test with equal variances, a modified version of the t-test can be used (RealStats 2022).

Unequal variance:

$$t_{df} = \frac{(\bar{x} - \bar{y}) - (\mu_x - \mu_Y)}{\sqrt{\left(\frac{s_x^2}{n_x} + \frac{s_Y^2}{n_Y}\right)}} \tag{E.2}$$

Where:

$$df = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_Y^2}{n_Y}\right)^2}{\frac{\left(\frac{s_x^2}{n_x}\right)^2}{n_x - 1} + \frac{\left(\frac{s_Y^2}{n_Y}\right)^2}{n_Y - 1}}$$

To determine whether the average daily returns of the momentum indices exceed the average returns of their respective benchmark indices (representing different markets), the following hypotheses were be tested:

Null hypothesis (H0): $\mu_{MI} - \mu_B = 0$

Alternative hypothesis (Ha): $\mu_{MI} - \mu_B > 0$

Table E.5 T-Tests: JSE Momentum Index (JSE-MI)

T Test: Two Independent Samples (13Y: 2009-2021)

SUMMARY

Groups	Count	Mean	Variance	Cohen d
JSE-MI	3249	0.07251	1.33285	
ALSH	3249	0.03791	1.22824	
Pooled			1.28054	0.03057

T TEST: Equal Variances — Alpha 0.05

	std err	t-stat	df	p-value	t-crit	lower	upper	sig	effect r
One Tail	0.02808	1.23230	6496	0.10894	1.64509			no	0.01529
Two Tail	0.02808	1.23230	6496	0.21788	1.96033	-0.02044	0.08964	no	0.01529

T TEST: Unequal Variances — Alpha 0.05

	std err	t-stat	df	p-value	t-crit	lower	upper	sig	effect r
One Tail	0.02808	1.23230	6485.18	0.10894	1.64509			no	0.01530
Two Tail	0.02808	1.23230	6485.18	0.21788	1.96033	-0.02044	0.08964	no	0.01530

T Test: Two Independent Samples (10Y: 2012-2021)

SUMMARY

Groups	Count	Mean	Variance	Cohen d
JSE-MI	2499	0.07439	1.44023	
ALSH	2499	0.03341	1.10818	
Pooled			1.27421	0.03630

T TEST: Equal Variances — Alpha 0.05

	std err	t-stat	df	p-value	t-crit	lower	upper	sig	effect r
One Tail	0.03193	1.28322	4996	0.09974	1.64516			no	0.01815
Two Tail	0.03193	1.28322	4996	0.19948	1.96044	-0.02163	0.10358	no	0.01815

T TEST: Unequal Variances — Alpha 0.05

	std err	t-stat	df	p-value	t-crit	lower	upper	sig	effect r
One Tail	0.03193	1.28322	4912.60	0.09974	1.64516			no	0.01831
Two Tail	0.03193	1.28322	4912.60	0.19948	1.96045	-0.02163	0.10358	no	0.01831

T Test: Two Independent Samples (5Y: 2017-2021)

SUMMARY

Groups	Count	Mean	Variance	Cohen d
JSE-MI	1250	0.07670	2.41740	
ALSH	1250	0.03001	1.35760	
Pooled			1.88750	0.03398

T TEST: Equal Variances — Alpha 0.05

	std err	t-stat	df	p-value	t-crit	lower	upper	sig	effect r
One Tail	0.05495	0.84953	2498	0.19783	1.64546			no	0.01699
Two Tail	0.05495	0.84953	2498	0.39567	1.96091	-0.06108	0.15445	no	0.01699

T TEST: Unequal Variances — Alpha 0.05

	std err	t-stat	df	p-value	t-crit	lower	upper	sig	effect r
One Tail	0.05495	0.84953	2315.50	0.19784	1.64551			no	0.01765
Two Tail	0.05495	0.84953	2315.50	0.39568	1.96099	-0.06108	0.15445	no	0.01765

Table E.5 T-Tests: JSE Momentum Index (JSE-MI) continued

T Test: Two Independent Samples (3Y: 2019-2021)

SUMMARY

Groups	*Count*	*Mean*	*Variance*	*Cohen d*
JSE-MI	751	0.14739	3.50185	
ALSH	751	0.04458	1.73456	
Pooled			2.61820	0.06353

T TEST: Equal Variances — Alpha 0.05

	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.08350	1.23116	1500	0.10923	1.64587			no	0.03177
Two Tail	0.08350	1.23116	1500	0.21846	1.96155	-0.06099	0.26660	no	0.03177

T TEST: Unequal Variances — Alpha 0.05

	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.08350	1.23116	1346.61	0.10924	1.64599			no	0.03353
Two Tail	0.08350	1.23116	1346.61	0.21848	1.96173	-0.06100	0.26661	no	0.03353

T Test: Two Independent Samples (1Y: 2021)

SUMMARY

Groups	*Count*	*Mean*	*Variance*	*Cohen d*
JSE-MI	250	0.14445	0.54289	
ALSH	250	0.08628	0.99413	
Pooled			0.76851	0.06636

T TEST: Equal Variances — Alpha 0.05

	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.07841	0.74193	498	0.22924	1.64792			no	0.03323
Two Tail	0.07841	0.74193	498	0.45848	1.96474	-0.09588	0.21223	no	0.03323

T TEST: Unequal Variances — Alpha 0.05

	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.07841	0.74193	458.48	0.22925	1.64818			no	0.03463
Two Tail	0.07841	0.74193	458.48	0.45851	1.96515	-0.09591	0.21226	no	0.03463

The results from the t-tests for the JSE Momentum Index (JSE-MI), measured against the general market (represented by the JSE All Share Index, ALSH) show that the mean daily returns of the momentum index exceed those of the market during the 10-year period.

The positive difference in mean daily returns over the 10-year period is statistically significant at a 10% level of significance (1.283 > 1.282).

Table E.6 T-Tests: TSX Momentum Index (TSX-MI)

T Test: Two Independent Samples (13Y: 2009-2021)

SUMMARY

Groups	*Count*	*Mean*	*Variance*	*Cohen d*
TSX-MI	3248	0.07473	1.12490	
TXCX	3248	0.02645	1.02356	
Pooled			1.07423	0.04658

T TEST: Equal Variances — Alpha 0.05

	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.02572	1.87713	6494	0.03027	1.64509			yes	0.02329
Two Tail	0.02572	1.87713	6494	0.06055	1.96033	-0.00214	0.09870	no	0.02329

T TEST: Unequal Variances — Alpha 0.05

	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.02572	1.87713	6479.58	0.03027	1.64509			yes	0.02331
Two Tail	0.02572	1.87713	6479.58	0.06055	1.96033	-0.00214	0.09870	no	0.02331

T Test: Two Independent Samples (10Y: 2012-2021)

SUMMARY

Groups	*Count*	*Mean*	*Variance*	*Cohen d*
TSX-MI	2499	0.07382	1.10658	
TXCX	2499	0.02297	0.85784	
Pooled			0.98221	0.05131

T TEST: Equal Variances — Alpha 0.05

	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.02804	1.81366	4996	0.03489	1.64516			yes	0.02565
Two Tail	0.02804	1.81366	4996	0.06979	1.96044	-0.00412	0.10581	no	0.02565

T TEST: Unequal Variances — Alpha 0.05

	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.02804	1.81366	4917.16	0.03490	1.64516			yes	0.02586
Two Tail	0.02804	1.81366	4917.16	0.06979	1.96045	-0.00412	0.10582	no	0.02586

T Test: Two Independent Samples (5Y: 2017-2021)

SUMMARY

Groups	*Count*	*Mean*	*Variance*	*Cohen d*
TSX-MI	1249	0.06851	1.48741	
TXCX	1249	0.02626	1.15215	
Pooled			1.31978	0.03678

T TEST: Equal Variances — Alpha 0.05

	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.04597	0.91908	2496	0.179072	1.64546			no	0.01839
Two Tail	0.04597	0.91908	2496	0.358144	1.96091	-0.04789	0.13240	no	0.01839

T TEST: Unequal Variances — Alpha 0.05

	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.04597	0.91908	2456.37	0.17907	1.64547			no	0.01854
Two Tail	0.04597	0.91908	2456.37	0.35814	1.96093	-0.04790	0.13240	no	0.01854

Table E.6 T-Tests: TSX Momentum Index (TSX-MI) continued

T Test: Two Independent Samples (3Y: 2019-2021)

SUMMARY

Groups	Count	Mean	Variance	Cohen d
TSX-MI	750	0.11966	2.08105	
TXCX	750	0.05243	1.69989	
Pooled			1.89047	0.04890

T TEST: Equal Variances — Alpha 0.05

	std err	t-stat	df	p-value	t-crit	lower	upper	sig	effect r
One Tail	0.07100	0.94686	1498	0.17193	1.64587			no	0.02446
Two Tail	0.07100	0.94686	1498	0.34386	1.96155	-0.07204	0.20650	no	0.02446

T TEST: Unequal Variances — Alpha 0.05

	std err	t-stat	df	p-value	t-crit	lower	upper	sig	effect r
One Tail	0.07100	0.94686	1482.93	0.17193	1.64588			no	0.02458
Two Tail	0.07100	0.94686	1482.93	0.34386	1.96156	-0.07205	0.20650	no	0.02458

T Test: Two Independent Samples (1Y: 2021)

SUMMARY

Groups	Count	Mean	Variance	Cohen d
TSX-MI	249	0.10292	1.05496	
TXCX	249	0.07899	0.44330	
Pooled			0.74913	0.02765

T TEST: Equal Variances — Alpha 0.05

	std err	t-stat	df	p-value	t-crit	lower	upper	sig	effect r
One Tail	0.07757	0.30847	496	0.37893	1.64793			no	0.01385
Two Tail	0.07757	0.30847	496	0.75786	1.96476	-0.12848	0.17633	no	0.01385

T TEST: Unequal Variances — Alpha 0.05

	std err	t-stat	df	p-value	t-crit	lower	upper	sig	effect r
One Tail	0.07757	0.30847	425.14	0.37894	1.64845			no	0.01496
Two Tail	0.07757	0.30847	425.14	0.75788	1.96556	-0.12854	0.17640	no	0.01496

The results from the t-tests for the TSX Momentum Index (TSX-MI), measured against the general market (represented by the TSX Composite Index, TXCX) show that the mean daily returns of the momentum index exceed those of the market during the 13-year and 10-year periods.

The positive difference in mean daily returns over the 13-year period is statistically significant at a 5% level of significance (1.877 > 1.645).

The positive difference in mean daily returns over the 10-year period is statistically significant at a 5% level of significance (1.814 > 1.645).

Table E.7 T-Tests: TSXV Momentum Index (TSXV-MI)

T Test: Two Independent Samples (13Y: 2009-2021)

SUMMARY

Groups	*Count*	*Mean*	*Variance*	*Cohen d*
TSXV-MI	3248	0.08339	2.59507	
TXVC	3248	0.00505	1.49914	
Pooled			2.04710	0.05475

T TEST: Equal Variances — Alpha 0.05

	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.03550	2.20646	6494	0.01369	1.64509			yes	0.02737
Two Tail	0.03550	2.20646	6494	0.02739	1.96033	0.00874	0.14794	yes	0.02737

T TEST: Unequal Variances — Alpha 0.05

	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.03550	2.20646	6059.80	0.01369	1.64511			yes	0.02833
Two Tail	0.03550	2.20646	6059.80	0.02739	1.96036	0.00874	0.14794	yes	0.02833

T Test: Two Independent Samples (10Y: 2012-2021)

SUMMARY

Groups	*Count*	*Mean*	*Variance*	*Cohen d*
TSXV-MI	2499	0.08872	2.37329	
TXVC	2499	-0.01832	1.34388	
Pooled			1.85858	0.07852

T TEST: Equal Variances — Alpha 0.05

	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.03857	2.77544	4996	0.00277	1.64516			yes	0.03924
Two Tail	0.03857	2.77544	4996	0.00553	1.96044	0.03143	0.18265	yes	0.03924

T TEST: Unequal Variances — Alpha 0.05

	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.03857	2.77544	4640.14	0.00277	1.64518			yes	0.04071
Two Tail	0.03857	2.77544	4640.14	0.00553	1.96048	0.03143	0.18265	yes	0.04071

T Test: Two Independent Samples (5Y: 2017-2021)

SUMMARY

Groups	*Count*	*Mean*	*Variance*	*Cohen d*
TSXV-MI	1249	0.11737	2.62144	
TXVC	1249	0.01670	1.69636	
Pooled			2.15890	0.06852

T TEST: Equal Variances — Alpha 0.05

	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.05880	1.71225	2496	0.04349	1.64546			yes	0.03425
Two Tail	0.05880	1.71225	2496	0.08698	1.96091	-0.01462	0.21597	no	0.03425

T TEST: Unequal Variances — Alpha 0.05

	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.05880	1.71225	2386.46	0.04349	1.64549			yes	0.03503
Two Tail	0.05880	1.71225	2386.46	0.08698	1.96096	-0.01462	0.21597	no	0.03503

Table E.7 T-Tests: TSXV Momentum Index (TSXV-MI) continued

T Test: Two Independent Samples (3Y: 2019-2021)

SUMMARY				
Groups	*Count*	*Mean*	*Variance*	*Cohen d*
TSXV-MI	750	0.18053	3.36687	
TXVC	750	0.06961	2.28937	
Pooled			2.82812	0.06596

T TEST: Equal Variances			Alpha	0.05				
	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.08684	1.27724	1498	0.10086	1.64587			no	0.03298
Two Tail	0.08684	1.27724	1498	0.20172	1.96155	-0.05943	0.28126	no	0.03298

T TEST: Unequal Variances				Alpha	0.05				
	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.08684	1.27724	1445.54	0.10086	1.64591			no	0.03357
Two Tail	0.08684	1.27724	1445.54	0.20172	1.96161	-0.05943	0.28127	no	0.03357

T Test: Two Independent Samples (1Y: 2021)

SUMMARY				
Groups	*Count*	*Mean*	*Variance*	*Cohen d*
TSXV-MI	249	0.15818	1.95699	
TXVC	249	0.02826	2.28974	
Pooled			2.12337	0.08915

T TEST: Equal Variances				Alpha	0.05				
	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.13060	0.99478	496	0.16017	1.64793			no	0.04462
Two Tail	0.13060	0.99478	496	0.32033	1.96476	-0.12668	0.38650	no	0.04462

T TEST: Unequal Variances				Alpha	0.05				
	std err	*t-stat*	*df*	*p-value*	*t-crit*	*lower*	*upper*	*sig*	*effect r*
One Tail	0.13060	0.99478	492.97	0.16017	1.64795			no	0.04476
Two Tail	0.13060	0.99478	492.97	0.32033	1.96479	-0.12668	0.38651	no	0.04476

The results from the t-tests for the TSXV Momentum Index (TSX-MI), measured against the venture market (represented by the TSX Venture Composite Index, TXVC) show that the mean daily returns of the momentum index exceed those of the market during the 13-year, 10-year, and 5-year periods.

The positive difference in mean daily returns over the 13-year period is statistically significant at a 5% level of significance (2.206 > 1.645).

The positive difference in mean daily returns over the 10-year period is statistically significant at a 0.5% level of significance (2.775 > 2.576).

The positive difference in mean daily returns over the 5-year period is statistically significant at a 5% level of significance (1.712 > 1.645).

E.3 RISK-ADJUSTED PERFORMANCE

Jensen's Alpha is a risk-adjusted performance metric that measures the returns of an index or portfolio against those of a benchmark. The benchmark is usually a broad market index. Alpha (α) represents the return that is in excess over that of the market. If the alpha is not statistically different from zero, there is no excess return after adjusting for risk or is beta (β) with the market. The excess return or performance, therefore, is in line with that of the market or as expected based on the associated level of risk. A statistically significant positive alpha means that the index or portfolio has outperformed the market on a risk-adjusted basis.

$$R_t - r_f = \alpha + \beta(M_t - r_f) + \varepsilon_t \qquad \text{(E.3)}$$

Jensen's alpha is the intercept of the regression equation (refer to Equation E.3) in the capital asset pricing model (CAPM) and is in effect the excess return adjusted for systematic risk (Bacon 2013).

Null hypothesis (H0): $\alpha = 0$

Alternative hypothesis (Ha): $\alpha > 0$

Alpha (α) is calculated by regressing the daily log returns of each momentum index on the daily log returns of their respective benchmarks:

Table E.8 Jensen's Alpha: JSE Momentum Index (JSE-MI)

JSE-MI		13Y	10Y	5Y	3Y	1Y
Multiple R		0.63374	0.61805	0.61882	0.63466	0.68133
R Square		0.40162	0.38198	0.38294	0.40280	0.46421
Standard Error		0.89333	0.94380	1.22204	1.44716	0.54042
Observations		3249	2499	1250	751	250
alpha	α	0.03946	0.04405	0.04788	0.10522	0.09363
	std err	0.01567	0.01888	0.03457	0.05282	0.03427
	t stat	2.51778	2.33282	1.38513	1.99225	2.73248
	p-value	0.01186	0.01974	0.16626	0.04671	0.00674
beta	β	0.66020	0.70459	0.82573	0.90162	0.50350
	std err	0.01414	0.01794	0.02967	0.04011	0.03435
	t stat	46.68328	39.28518	27.82972	22.47621	14.65840
	p-value	0.00000	0.00000	0.00000	0.00000	0.00000

The daily excess-returns (alphas) of the JSE Momentum Index (JSE-MI) were benchmarked against the JSE All Share Index (ALSH):

13Y: 1 % level of significance (2.518 > 2.326)
10Y: 1 % level of significance (2.333 > 2.326)
5Y: 10 % level of significance (1.385 > 1.282)
3Y: 2.5 % level of significance (1.992 > 1.960)
1Y: 0.5 % level of significance (2.732 > 2.576)

Table E.9 Jensen's Alpha: TSX Momentum Index (TSX-MI)

TSX-MI		13Y	10Y	5Y	3Y	1Y
Multiple R		0.69005	0.70059	0.73444	0.73064	0.85166
R Square		0.47617	0.49083	0.53941	0.53384	0.72532
Standard Error		0.76781	0.75083	0.82813	0.98567	0.53939
Observations		3248	2499	1249	750	249
alpha	α	0.05476	0.05488	0.04604	0.07671	-0.00072
	std err	0.01348	0.01502	0.02344	0.03602	0.03442
	t stat	4.06318	3.65318	1.96420	2.12984	-0.02095
	p-value	0.00005	0.00026	0.04973	0.03351	0.98330
beta	β	0.72343	0.79574	0.83453	0.80845	1.31381
	std err	0.01332	0.01622	0.02184	0.02762	0.05144
	t stat	54.31951	49.06196	38.21477	29.26744	25.53902
	p-value	0.00000	0.00000	0.00000	0.00000	0.00000

The daily excess-returns (alphas) of the TSX Momentum Index (TSX-MI) were benchmarked against the TSX Composite Index (TXCX):

13Y: 0.05 % level of significance (4.063 > 3.291)
10Y: 0.05 % level of significance (3.653 > 3.291)
5Y: 2.5 % level of significance (1.964 > 1.960)
3Y: 2.5 % level of significance (2.130 > 1.960)
1Y: Negative alpha – not significant

Table E.10 Jensen's Alpha: TSXV Momentum Index (TSXV-MI)

TSXV-MI		13Y	10Y	5Y	3Y	1Y
Multiple R		0.70263	0.72222	0.75839	0.81091	0.74438
R Square		0.49368	0.52161	0.57515	0.65757	0.55410
Standard Error		1.14648	1.06579	1.05582	1.07453	0.93603
Observations		3248	2499	1249	750	249
alpha	α	0.07849	0.10617	0.10144	0.11203	0.13859
	std err	0.02012	0.02132	0.02988	0.03927	0.05933
	t stat	3.90174	4.97911	3.39519	2.85248	2.33593
	p-value	0.00010	0.00000	0.00071	0.00446	0.02030
beta	β	0.92438	0.95970	0.94268	0.98334	0.68816
	std err	0.01643	0.01839	0.02294	0.02595	0.03928
	t stat	56.25854	52.17821	41.08734	37.89956	17.51943
	p-value	0.00000	0.00000	0.00000	0.00000	0.00000

The daily excess-returns (alphas) of the TSXV Momentum Index (TSXV-MI) were benchmarked against the TSX Venture Composite Index (TXVC):

13Y: 0.05 % level of significance (3.902 > 3.291)
10Y: 0.05 % level of significance (4.979 > 3.291)
5Y: 0.05 % level of significance (3.395 > 3.291)
3Y: 0.5 % level of significance (2.852 > 2.576)
1Y: 1 % level of significance (2.336 > 2.326)

E.4 MULTIFACTOR MODEL

The multifactor regression model, refer to Equation E.4, includes the equity premium (R_t-r_f) as the dependent or explained variable along with the market premium (M_t-r_f), size (SMB), and momentum (WML) factors as the independent or explanatory variables. The North-American and Emerging markets Fama-French factors for the market premium, size (small minus big, SMB), and momentum, (winner minus loser, WML) were obtained from the Fama-French website (Fama & French 2023).

$$R_t - r_f = \alpha + \beta_{MKT}(M_t - r_f) + \beta_{SIZE}SMB_t + \beta_{MOM}WML_t + \varepsilon_t \qquad \text{(E.4)}$$

Testing for the normality of the residuals (Shapiro-Wilks or d'Agostino-Pearson tests), serial or autocorrelation (Durbin-Watson test), multicollinearity (Variance Inflation Factor, VIF), and heteroskedasticity (Breusch-Pagan or White tests) verifies the reliability of the estimated coefficients. A common solution for dealing with the possibility of heteroskedasticity (non-constant variance of the residuals) is the use of Heteroskedasticity-Consistent (robust) standard errors (RealStats 2022). One method to detect multicollinearity (correlation between independent variables) is to calculate the VIF-value for each independent variable. A VIF value greater than 1.5 would indicate evidence of multicollinearity while values exceeding 10 are viewed as problematic. Serial correlation (correlation between residuals or error terms) causes the estimated variances of the regression coefficients to be biased, leading to unreliable hypothesis testing (Asteriou & Hall 2021). The normality assumption is necessary to estimate unbiased standard errors, confidence intervals and p-values. However, in sample sizes where the number of observations per variable exceeds 10, violations of the normality assumption often do not markedly affect the results (Schmidt & Finan 2018).

The coefficient of determination or R-Squared and the Standard Error of the regression are two goodness-of-fit measures for regression analysis. R-Squared provides the relative measure of the percentage of the dependent variable variance explained by the model. The Standard Error of the regression is in the units of the dependent variable and provides the absolute measure of the typical distance that the data points fall from the regression line. The adjusted R-Squared accounts for the number of explanatory variables included in a model (Min 2019). A regression model may have significant variables (low p-values) but explains little of the variability (low R-squared). A significant coefficient would indicate that the explanatory variable (predictor) still provides information about the explained variable (response) even though data points fall further from the regression line. Therefore, even when R-squared is low, low p-values still confirm a real relationship between the explanatory and the explained variables. Even though the interpretations of the significant variables remains the same, low R-Squared values are problematic for making precise predictions (Frost 2014).

Table E.11 Multifactor model: JSE-MI (13Y: 2009-2021)

Regression Analysis: JSE-MI (13Y: 2009-2021)

OVERALL FIT

Multiple R	0.54733	AIC	461.61
R Square	0.29957	AICc	462.01
Adjusted R Square	0.28574	SBC	473.81
Standard Error	4.33560		
Observations	156		

ANOVA — Alpha 0.05

	df	*SS*	*MS*	*F*	*p-value*	*sig*
Regression	3	1221.986	407.329	21.669	0.00000	yes
Residual	152	2857.206	18.797			
Total	155	4079.192				

	coeff	*std err*	*t stat*	*p-value*	*lower*	*upper*	*vif*
α (alpha)	0.721121	0.359657	2.005021	0.046736	0.010548	1.431693	
Mt-rf	0.584531	0.116275	5.027150	0.000001	0.354807	0.814254	1.298034
SMB	0.627630	0.278506	2.253558	0.025654	0.077387	1.177873	1.040252
WML	0.387597	0.179078	2.164404	0.031995	0.033794	0.741400	1.254964

Shapiro-Wilk Test

W-stat	0.94133
p-value	0.00000
alpha	0.05
normal	no

d'Agostino-Pearson

DA-stat	39.61863
p-value	0.00000
alpha	0.05
normal	no

Durbin-Watson Test

Alpha	0.05
D-stat	2.10504
D-lower	1.69924
D-upper	1.77755
sig	no

Heteroskedascity Testing

Sample size	156
Independent variables	3

Breusch-Pagan		White Test	
LM stat	6.46782	LM stat	28.22745
df	3	df	2
p-value	0.09094	p-value	0.00000
F stat	2.19152	F stat	16.90034
df1	3	df1	2
df2	152	df2	153
p-value	0.09136	p-value	0.00000

Distribution of the error terms not normal; Serial correlation not significant; Multicollinearity not significant; Heteroscedasticity accounted for via robust standard errors (HC3 setting). Despite the significance of the variables, the adjusted R-squared is low, only explaining 29% of the variance in the dependant variable.

JSE-MI (13Y) outperformance (alpha) statistically significant at 2.5% (2.005 > 1.960).

Table E.12 Multifactor model: JSE-MI (10Y: 2012-2021)

Regression Analysis: JSE-MI (10Y: 2012-2021)

OVERALL FIT				
Multiple R	0.60210		AIC	362.19
R Square	0.36253		AICc	362.72
Adjusted R Square	0.34604		SBC	373.34
Standard Error	4.44937			
Observations	120			

ANOVA — Alpha 0.05

	df	*SS*	*MS*	*F*	*p-value*	*sig*
Regression	3	1305.968	435.323	21.989	0.00000	yes
Residual	116	2296.441	19.797			
Total	119	3602.408				

	coeff	*std err*	*t stat*	*p-value*	*lower*	*upper*	*vif*
α (alpha)	0.766156	0.403525	1.898659	0.060094	-0.033076	1.565388	
Mt-rf	0.782717	0.161829	4.836693	0.000004	0.462194	1.103240	1.217306
SMB	0.689984	0.312130	2.210569	0.029028	0.071772	1.308196	1.038122
WML	0.369835	0.222474	1.662374	0.099138	-0.070803	0.810473	1.190695

Shapiro-Wilk Test

W-stat	0.95690
p-value	0.00072
alpha	0.05
normal	no

d'Agostino-Pearson

DA-stat	23.22685
p-value	0.00001
alpha	0.05
normal	no

Durbin-Watson Test

Alpha	0.05
D-stat	2.12317
D-lower	1.65126
D-upper	1.75361
sig	no

Heteroskedascity Testing

Sample size	120
Independent variables	3

Breusch-Pagan		White Test	
LM stat	10.59787	LM stat	30.98144
df	3	df	2
p-value	0.01411	p-value	0.00000
F stat	3.74567	F stat	20.35996
df1	3	df1	2
df2	116	df2	117
p-value	0.01303	p-value	0.00000

Distribution of the error terms not normal; Serial correlation not significant; Multicollinearity not significant; Heteroscedasticity accounted for via robust standard errors (HC3 setting). Despite the significance of the variables, the adjusted R-Squared is quite low, explaining 35% of the variance in the dependant variable.

JSE-MI (10Y) outperformance (alpha) statistically significant at 5% (1.899 > 1.645).

Table E.13 Multifactor model: JSE-MI (5Y: 2017-2021)

Regression Analysis: JSE-MI (5Y: 2017-2021)

OVERALL FIT

Multiple R	0.67252	AIC	208.65
R Square	0.45228	AICc	209.76
Adjusted R Square	0.42294	SBC	217.03
Standard Error	5.51026		
Observations	60		

ANOVA — Alpha 0.05

	df	*SS*	*MS*	*F*	*p-value*	*sig*
Regression	3	1404.068	468.023	15.414	0.00000	yes
Residual	56	1700.326	30.363			
Total	59	3104.394				

	coeff	*std err*	*t stat*	*p-value*	*lower*	*upper*	*vif*
α (alpha)	0.302057	0.774586	0.389959	0.698046	-1.249625	1.853739	
Mt-rf	1.022593	0.220895	4.629315	0.000022	0.580087	1.465100	1.086776
SMB	1.118371	0.474769	2.355608	0.022020	0.167293	2.069448	1.008592
WML	0.559184	0.364182	1.535452	0.130304	-0.170360	1.288727	1.086306

Shapiro-Wilk Test

W-stat	0.97488
p-value	0.25112
alpha	0.05
normal	yes

d'Agostino-Pearson

DA-stat	2.90182
p-value	0.23436
alpha	0.05
normal	yes

Durbin-Watson Test

Alpha	0.05
D-stat	2.18617
D-lower	1.47965
D-upper	1.68891
sig	no

Heteroskedascity Testing

Sample size	60
Independent variables	3

Breusch-Pagan		White Test	
LM stat	7.97092	LM stat	16.94361
df	3	df	2
p-value	0.04662	p-value	0.00021
F stat	2.85975	F stat	11.21536
df1	3	df1	2
df2	56	df2	57
p-value	0.04494	p-value	0.00008

Distribution of the error terms is normal; Serial correlation not significant; Multicollinearity not significant; Heteroscedasticity accounted for via robust standard errors (HC3 setting). Despite some variables not being significant, the adjusted R-squared explains 42% of the variance in the dependant variable.

JSE-MI (5Y) outperformance (alpha) not statistically significant.

Table E.14 Multifactor model: TSX-MI (13Y: 2009-2021)

Regression Analysis: TSX-MI (13Y: 2009-2021)

OVERALL FIT

Multiple R	0.64413	AIC	430.44
R Square	0.41490	AICc	430.84
Adjusted R Square	0.40335	SBC	442.64
Standard Error	3.92345		
Observations	156		

ANOVA — Alpha 0.05

	df	*SS*	*MS*	*F*	*p-value*	*sig*
Regression	3	1659.166	553.055	35.928	0.00000	yes
Residual	152	2339.809	15.393			
Total	155	3998.975				

	coeff	*std err*	*t stat*	*p-value*	*lower*	*upper*	*vif*
α (alpha)	0.848822	0.337414	2.515670	0.012919	0.182196	1.515449	
Mt-rf	0.633909	0.104855	6.045562	0.000000	0.426747	0.841071	1.278109
SMB	0.501338	0.160198	3.129481	0.002100	0.184835	0.817841	1.204153
WML	0.303124	0.165470	1.831896	0.068924	-0.023794	0.630043	1.112514

Shapiro-Wilk Test

W-stat	0.88437
p-value	0.00000
alpha	0.05
normal	no

d'Agostino-Pearson

DA-stat	70.37151
p-value	0.00000
alpha	0.05
normal	no

Durbin-Watson Test

Alpha	0.05
D-stat	2.08383
D-lower	1.69924
D-upper	1.77755
sig	no

Heteroskedascity Testing

Sample size	156
Independent variables	3

Breusch-Pagan		White Test	
LM stat	9.94199	LM stat	8.20717
df	3	df	2
p-value	0.01907	p-value	0.01651
F stat	3.44882	F stat	4.24817
df1	3	df1	2
df2	152	df2	153
p-value	0.01821	p-value	0.01601

Distribution of the error terms not normal; Serial correlation not significant; Multicollinearity not significant; Heteroscedasticity accounted for via robust standard errors (HC3 setting). The adjusted R-squared explains 40% of the variance in the dependant variable (index return minus risk-free rate, Rt-rf).

TSX-MI (13Y) outperformance (alpha) statistically significant at 1% (2.516 > 2.326).

Table E.15 Multifactor model: TSX-MI (10Y: 2012-2021)

Regression Analysis: TSX-MI (10Y: 2012-2021)

OVERALL FIT			
Multiple R	0.67142	AIC	326.00
R Square	0.45080	AICc	326.52
Adjusted R Square	0.43660	SBC	337.15
Standard Error	3.82643		
Observations	120		

ANOVA — Alpha 0.05

	df	*SS*	*MS*	*F*	*p-value*	*sig*
Regression	3	1394.126	464.709	31.739	0.00000	yes
Residual	116	1698.419	14.642			
Total	119	3092.545				

	coeff	*std err*	*t stat*	*p-value*	*lower*	*upper*	*vif*
α (alpha)	0.671946	0.404077	1.662917	0.099029	-0.128379	1.472270	
Mt-rf	0.737539	0.142192	5.186930	0.000001	0.455911	1.019168	1.252169
SMB	0.572674	0.173681	3.297268	0.001296	0.228676	0.916671	1.172681
WML	0.378849	0.145962	2.595536	0.010664	0.089753	0.667945	1.141779

Shapiro-Wilk Test

W-stat	0.89302
p-value	0.00000
alpha	0.05
normal	no

d'Agostino-Pearson

DA-stat	57.45776
p-value	0.00000
alpha	0.05
normal	no

Durbin-Watson Test

Alpha	0.05
D-stat	2.10854
D-lower	1.65126
D-upper	1.75361
sig	no

Heteroskedascity Testing

Sample size	120
Independent variables	3

Breusch-Pagan		White Test	
LM stat	6.91713	LM stat	12.30416
df	3	df	2
p-value	0.07459	p-value	0.00213
F stat	2.36519	F stat	6.68358
df1	3	df1	2
df2	116	df2	117
p-value	0.07461	p-value	0.00178

Distribution of the error terms not normal; Serial correlation not significant; Multicollinearity not significant; Heteroscedasticity accounted for via robust standard errors (HC3 setting). The adjusted R-squared explains 44% of the variance in the dependant variable (index return minus risk-free rate, Rt-rf).

TSX-MI (10Y) outperformance (alpha) statistically significant at 5% (1.663 > 1.645).

Table E.16 Multifactor model: TSX-MI (5Y: 2017-2021)

Regression Analysis: TSX-MI (5Y: 2017-2021)

OVERALL FIT

Multiple R	0.77166	AIC	166.63
R Square	0.59547	AICc	167.74
Adjusted R Square	0.57379	SBC	175.01
Standard Error	3.88225		
Observations	60		

ANOVA — Alpha 0.05

	df	*SS*	*MS*	*F*	*p-value*	*sig*
Regression	3	1242.389	414.130	27.477	0.00000	yes
Residual	56	844.026	15.072			
Total	59	2086.416				

	coeff	*std err*	*t stat*	*p-value*	*lower*	*upper*	*vif*
α (alpha)	0.259493	0.511285	0.507531	0.613775	-0.764733	1.283719	
Mt-rf	0.849810	0.186404	4.558962	0.000028	0.476398	1.223223	1.276485
SMB	0.661557	0.230795	2.866430	0.005839	0.199220	1.123895	1.205684
WML	0.545817	0.215543	2.532296	0.014163	0.114034	0.977601	1.159265

Shapiro-Wilk Test

W-stat	0.96053
p-value	0.04992
alpha	0.05
normal	no

d'Agostino-Pearson

DA-stat	9.60782
p-value	0.00820
alpha	0.05
normal	no

Durbin-Watson Test

Alpha	0.05
D-stat	2.00499
D-lower	1.47965
D-upper	1.68891
sig	no

Heteroskedascity Testing

Sample size	60
Independent variables	3

Breusch-Pagan		White Test	
LM stat	11.35636	LM stat	30.23864
df	3	df	2
p-value	0.00995	p-value	0.00000
F stat	4.35792	F stat	28.95705
df1	3	df1	2
df2	56	df2	57
p-value	0.00791	p-value	0.00000

Distribution of the error terms not normal; Serial correlation not significant; Multicollinearity not significant; Heteroscedasticity accounted for via robust standard errors (HC3 setting). Despite some variables not being significant, the adjusted R-squared explains 57% of the variance in the dependant variable.

TSX-MI (5Y) outperformance (alpha) not statistically significant.

Table E.17 Multifactor model: TSXV-MI (13Y: 2009-2021)

Regression Analysis: TSXV-MI (13Y: 2009-2021)

OVERALL FIT

Multiple R	0.49126	AIC	598.55
R Square	0.24134	AICc	598.95
Adjusted R Square	0.22636	SBC	610.75
Standard Error	6.72463		
Observations	156		

ANOVA — Alpha 0.05

	df	*SS*	*MS*	*F*	*p-value*	*sig*
Regression	3	2186.522	728.841	16.117	0.00000	yes
Residual	152	6873.529	45.221			
Total	155	9060.051				

	coeff	*std err*	*t stat*	*p-value*	*lower*	*upper*	*vif*
α (alpha)	1.065926	0.601107	1.773271	0.078187	-0.121678	2.253529	
Mt-rf	0.733417	0.139403	5.261131	0.000000	0.457999	1.008834	1.278109
SMB	0.560904	0.245775	2.282183	0.023866	0.075327	1.046480	1.204153
WML	0.344111	0.196861	1.747992	0.082485	-0.044826	0.733047	1.112514

Shapiro-Wilk Test

W-stat	0.98057
p-value	0.02685
alpha	0.05
normal	no

d'Agostino-Pearson

DA-stat	6.39847
p-value	0.04079
alpha	0.05
normal	no

Durbin-Watson Test

Alpha	0.05
D-stat	1.75963
D-lower	1.69924
D-upper	1.77755
sig	unclear

Heteroskedascity Testing

Sample size	156
Independent variables	3

Breusch-Pagan		White Test	
LM stat	5.47320	LM stat	0.04295
df	3	df	2
p-value	0.14025	p-value	0.97875
F stat	1.84225	F stat	0.02107
df1	3	df1	2
df2	152	df2	153
p-value	0.14188	p-value	0.97916

Distribution of the error terms not normal; Serial correlation unclear; Multicollinearity not significant; Heteroscedasticity accounted for via robust standard errors (HC3 setting). Despite the significance of the variables, the adjusted R-squared is low, only explaining 23% of the variance in the dependant variable.

TSXV-MI (13Y) outperformance (alpha) statistically significant at 5% (1.773 > 1.645).

Table E.18 Multifactor model: TSXV-MI (10Y: 2012-2021)

Regression Analysis: TSXV-MI (10Y: 2012-2021)

OVERALL FIT

Multiple R	0.46526	AIC	459.19
R Square	0.21647	AICc	459.71
Adjusted R Square	0.19620	SBC	470.34
Standard Error	6.66516		
Observations	120		

ANOVA — Alpha 0.05

	df	*SS*	*MS*	*F*	*p-value*	*sig*
Regression	3	1423.694	474.565	10.683	0.00000	yes
Residual	116	5153.224	44.424			
Total	119	6576.918				

	coeff	*std err*	*t stat*	*p-value*	*lower*	*upper*	*vif*
α (alpha)	1.202311	0.741185	1.622148	0.107487	-0.265699	2.670321	
Mt-rf	0.707387	0.182577	3.874459	0.000177	0.345770	1.069003	1.252169
SMB	0.655962	0.278049	2.359158	0.019988	0.105250	1.206674	1.172681
WML	0.322927	0.189554	1.703619	0.091130	-0.052508	0.698363	1.141779

Shapiro-Wilk Test

W-stat	0.97054
p-value	0.00978
alpha	0.05
normal	no

d'Agostino-Pearson

DA-stat	5.56925
p-value	0.06175
alpha	0.05
normal	yes

Durbin-Watson Test

Alpha	0.05
D-stat	1.56568
D-lower	1.65126
D-upper	1.75361
sig	yes

Heteroskedascity Testing

Sample size	120
Independent variables	3

Breusch-Pagan

LM stat	8.09610
df	3
p-value	0.04407
F stat	2.79748
df1	3
df2	116
p-value	0.04326

White Test

LM stat	0.99843
df	2
p-value	0.60701
F stat	0.49082
df1	2
df2	117
p-value	0.61338

Distribution of the error terms is normal; Serial correlation significant; Multicollinearity not significant; Heteroscedasticity accounted for via robust standard errors (HC3 setting). Despite the significance of the variables, the adjusted R-squared is low, only explaining 20% of the variance in the dependant variable.

TSXV-MI (10Y) outperformance (alpha) statistically significant at 10% (1.622 > 1.282).

Table E.19 Multifactor model: TSXV-MI (5Y: 2017-2021)

Regression Analysis: TSXV-MI (5Y: 2017-2021)

OVERALL FIT

Multiple R	0.60498	AIC	221.71
R Square	0.36600	AICc	222.82
Adjusted R Square	0.33203	SBC	230.08
Standard Error	6.14346		
Observations	60		

ANOVA — Alpha 0.05

	df	*SS*	*MS*	*F*	*p-value*	*sig*
Regression	3	1220.108	406.703	10.776	0.00001	yes
Residual	56	2113.558	37.742			
Total	59	3333.666				

	coeff	*std err*	*t stat*	*p-value*	*lower*	*upper*	*vif*
α (alpha)	1.436998	1.051214	1.366990	0.177091	-0.668836	3.542833	
Mt-rf	0.791047	0.242401	3.263377	0.001879	0.305459	1.276635	1.276485
SMB	0.762551	0.363860	2.095726	0.040639	0.033652	1.491451	1.205684
WML	0.689330	0.228577	3.015746	0.003849	0.231436	1.147225	1.159265

Shapiro-Wilk Test

W-stat	0.95069
p-value	0.01679
alpha	0.05
normal	no

d'Agostino-Pearson

DA-stat	6.11958
p-value	0.04690
alpha	0.05
normal	no

Durbin-Watson Test

Alpha	0.05
D-stat	2.02581
D-lower	1.47965
D-upper	1.68891
sig	no

Heteroskedascity Testing

Sample size	60
Independent variables	3

Breusch-Pagan		White Test	
LM stat	6.00062	LM stat	2.44385
df	3	df	2
p-value	0.11158	p-value	0.29466
F stat	2.07431	F stat	1.21012
df1	3	df1	2
df2	56	df2	57
p-value	0.11394	p-value	0.30571

Distribution of the error terms not normal; Serial correlation not significant; Multicollinearity not significant; Heteroscedasticity accounted for via robust standard errors (HC3 setting). Despite the significance of the variables, the adjusted R-squared is quite low, explaining 33% of the variance in the dependant variable.

TSXV-MI (5Y) outperformance (alpha) statistically significant at 10% (1.367 > 1.282).

REFERENCES

Asteriou, D. & Hall, S.G. 2021. *Applied econometrics*. 4th edition. London: Bloomsbury Publishing.

Bacon, C.R. 2013. *Practical risk-adjusted performance measurement*. Chichester, West Sussex: John Wiley & Sons, Inc (Wiley Finance). DOI: 10.1002/9781118673621.

Bloomberg. 2022. Bloomberg Terminal – Bloomberg Professional Services subscription. New York: Bloomberg Finance L.P.

Fama, E.F. & French, K.R. 2023. Fama-French factors [Website]. Kenneth R. French. Available at: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html.

Frost, J. 2014. *How to interpret a regression model with low R-Squared and low p values*. *Minitab Blog [Online]*. Coventry, United Kingdom: Minitab, LLC. Available at: https://blog.minitab.com/en/adventures-in-statistics-2/how-to-interpret-a-regression-model-with-low-r-squared-and-low-p-values (Accessed: 4 May 2023).

Min, C. 2019. *Applied econometrics: a practical guide*. New York: Routledge. DOI: 10.4324/9780429024429.

RealStats. 2022. Real statistics using Excel [Website]. Charles Zaiontz. Available at: https://www.real-statistics.com.

Ross, A. & Willson, V.L. 2017. *Basic and advanced statistical tests*. Rotterdam: Sense Publishers. DOI: 10.1007/978-94-6351-086-8.

Schmidt, A.F. & Finan, C. 2018. Linear regression and the normality assumption. *Journal of Clinical Epidemiology*, 98:146–151. DOI: 10.1016/j.jclinepi.2017.12.006.

Taljaard, B. & Maré, E. 2019. Considering the use of an equal-weighted index as a benchmark for South African equity investors. *South African Actuarial Journal*, 19(1):53–70. DOI: 10.4314/saaj.v19i1.3.

The statistical analysis for this study was generated using the Real Statistics Resource Pack software for Excel (Release 8.3.1), Copyright (2013-2022) by Charles Zaiontz (RealStats 2022).

LIST OF REFERENCES

Abukari, K. & Otchere, I. 2017. Dominance of hybrid contratum strategies over momentum and contrarian strategies: half a century of evidence. in *FMA Annual Meeting 2017 Conference*. Tampa, Florida: Financial Management Association International.

Aggarwal, C.C. 2017. *Outlier analysis*. 2nd edition. Cham, Switzerland: Springer. DOI: 10.1007/978-3-319-47578-3.

Alwathainani, A.M. 2012. Consistent winners and losers. *International Review of Economics & Finance*, 21(1):210–220. DOI: 10.1016/j.iref.2011.05.009.

Ardila-Alvarez, D., Forrò, Z. & Sornette, D. 2021. The acceleration effect and gamma factor in asset pricing. *Physica A: Statistical Mechanics and its Applications*, 569(125367):1–23. DOI: 10.1016/j.physa.2020.125367.

Armitage, S., Brzeszczyński, J. & Serdyuk, A. 2014. Liquidity measures and cost of trading in an illiquid market. *Journal of Emerging Market Finance*, 13(2):155–196. DOI: 10.1177/0972652714541340.

Asness, C.S., Frazzini, A., Israel, R. & Moskowitz, T.J. 2014. Fact, fiction, and momentum investing. *The Journal of Portfolio Management*, 40(5):75–92. DOI: 10.3905/jpm.2014.40.5.075.

Assoé, K. & Sy, O. 2003. Profitability of the short-run contrarian strategy in Canadian stock markets. *Canadian Journal of Administrative Sciences*, 20(4):311–319. DOI: 10.1111/j.1936-4490.2003.tb00707.x.

Assogbavi, T., Giguere, M. & Sedzro, K. 2011. The impact of trading volume on portfolios effective time formation/holding periods based on momentum investment strategies. *International Business & Economics Research Journal*, 10(7):1–12. DOI: 10.19030/iber.v10i7.4662.

Assogbavi, T. & Leonard, B. 2008. Portfolios effective time formation/holding period based on momentum investment strategy. *International Business & Economics Research Journal*, 7(5):51–58. DOI: 10.19030/iber.v7i5.3254.

Asteriou, D. & Hall, S.G. 2021. *Applied econometrics*. 4th edition. London: Bloomsbury Publishing.

Bacon, C.R. 2013. *Practical risk-adjusted performance measurement*. Chichester, West Sussex: John Wiley & Sons, Inc (Wiley Finance). DOI: 10.1002/9781118673621.

Bandarchuk, P. & Hilscher, J. 2013. Sources of momentum profits: evidence on the irrelevance of characteristics. *Review of Finance*, 17(2):809–845. DOI: 10.1093/rof/rfr036.

Barberis, N., Shleifer, A. & Vishny, R. 1998. A model of investor sentiment. *Journal of Financial Economics*, 49(3):307–343. DOI: 10.1016/S0304-405X(98)00027-0.

Barroso, P. & Santa-Clara, P. 2015. Momentum has its moments. *Journal of Financial Economics*, 116(1):111–120. DOI: 10.1016/j.jfineco.2014.11.010.

Bhootra, A. & Hur, J. 2013. The timing of 52-week high price and momentum. *Journal of Banking & Finance*, 37(10):3773–3782. DOI: 10.1016/j.jbankfin.2013.05.025.

Bird, R. & Casavecchia, L. 2006. Insights into the momentum life cycle for European stocks. *The Journal of Investing*, 15(3):105–118. DOI: 10.3905/joi.2006.650150.

Bird, R. & Casavecchia, L. 2007. Value enhancement using momentum indicators: the European experience. *International Journal of Managerial Finance*, 3(3):229–262. DOI: 10.1108/17439130710756907.

Bird, R., Gao, X. & Yeung, D. 2017. Time-series and cross-sectional momentum strategies under alternative implementation strategies. *Australian Journal of Management*, 42(2):230–251. DOI: 10.1177/0312896215619965.

Blackburn, D.W. & Cakici, N. 2017. Overreaction and the cross-section of returns: international evidence. *Journal of Empirical Finance*, 42:1–14. DOI: 10.1016/j.jempfin.2017.02.001.

Blitz, D., Hanauer, M.X. & Vidojevic, M. 2020. The idiosyncratic momentum anomaly. *International Review of Economics & Finance*, 69:932–957. DOI: 10.1016/j.iref.2020.05.008.

Blitz, D., Huij, J. & Martens, M.P.E. 2011. Residual momentum. *Journal of Empirical Finance*, 18(3):506–521. DOI: 10.1016/J.JEMPFIN.2011.01.003.

Bloomberg. 2022. Bloomberg Terminal – Bloomberg Professional Services subscription. New York: Bloomberg Finance L.P.

Bloomfield, R.J., O'Hara, M. & Saar, G. 2009. How noise trading affects markets: an experimental analysis. *Review of Financial Studies*, 22(6):2275–2302. DOI: 10.1093/rfs/hhn102.

Bohl, M.T., Czaja, M.-G. & Kaufmann, P. 2016. Momentum profits, market cycles, and rebounds: evidence from Germany. *The Quarterly Review of Economics and Finance*, 61:139–159. DOI: 10.1016/j.qref.2016.01.003.

Boons, M. & Prado, M.P. 2019. Basis-momentum. *The Journal of Finance*, 74(1):239–279. DOI: 10.1111/jofi.12738.

Boslaugh, S. 2013. *Statistics in a nutshell: a desktop quick reference*. 2nd edition. Sebastopol, California: O'Reilly Media, Inc.

Brailsford, T. & O'Brien, M.A. 2008. Disentangling size from momentum in Australian stock returns. *Australian Journal of Management*, 32(3):463–484. DOI: 10.1177/031289620803200305.

Britten, J.H.C., Page, M.D. & Auret, C.J. 2016. Investigating the interaction between long-term reversal and value on the JSE. *Studies in Economics and Econometrics*, 40(2):1–24.

Brush, J.S. 1986. Eight relative strength models compared. *The Journal of Portfolio Management*, 13(1):21–28. DOI: 10.3905/jpm.1986.409080.

Brush, J.S. 2001. *Price momentum: a twenty year research effort*. *Columbine Newsletter*. Colorado Springs, Colorado: Columbine Capital Services, Inc.

Brush, J.S. & Boles, K.E. 1983. The predictive power in relative strength & CAPM. *The Journal of Portfolio Management*, 9(4):20–23. DOI: 10.3905/jpm.1983.20.

Bukowski, P. 2018. *Quantitative insights*. *HIMCO Monthly Insights [Online]*. Hartford Investment Managament Company. Available at: http://www.himco.com/sites/himco/files/1444642061445.pdf (Accessed: 26 December 2020).

Carhart, M.M. 1997. On persistence in mutual fund performance. *The Journal of Finance*, 52(1):57–82. DOI: 10.1111/j.1540-6261.1997.tb03808.x.

Chai, D., Faff, R.W. & Gharghori, P. 2010. New evidence on the relation between stock liquidity and measures of trading activity. *International Review of Financial Analysis*, 19(3):181–192. DOI: 10.1016/j.irfa.2010.02.005.

Chai, D., Limkriangkrai, M. & Ji, P.I. 2017. Momentum in weekly returns: the role of intermediate-horizon past performance. *Accounting & Finance*, 57(S1):45–68. DOI: 10.1111/acfi.12144.

Chan, L.K.C., Jegadeesh, N. & Lakonishok, J. 1996. Momentum strategies. *The Journal of Finance*, 51(5):1681–1713. DOI: 10.2307/2329534.

Chang, R.P., Ko, K.-C., Nakano, S. & Rhee, S.G. 2018. Residual momentum in Japan. *Journal of Empirical Finance*, 45:283–299. DOI: 10.1016/j.jempfin.2017.11.005.

Chang, T.-P. 2019. Inside of the 52-week high momentum: does price change or updating effect drive momentum profit? in *FMA Annual Meeting 2019 Conference*. New Orleans, Louisiana: Financial Management Association International.

LIST OF REFERENCES

Charteris, A. & Rupande, L. 2017. Feedback trading on the JSE. *Investment Analysts Journal*, 46(3):235–248. DOI: 10.1080/10293523.2017.1342319.

Chaves, D.B. 2016. Idiosyncratic momentum: U.S. and international evidence. *The Journal of Investing*, 25(2):64–76. DOI: 10.3905/joi.2016.25.2.064.

Cheema, M.A., Nartea, G. V. & Man, Y. 2018. Cross-sectional and time series momentum returns and market states. *International Review of Finance*, 18(4):705–715. DOI: 10.1111/irfi.12148.

Cheema, M.A., Nartea, G. V. & Szulczyk, K.R. 2018. Cross-sectional and time-series momentum returns and market dynamics: evidence from Japan. *Applied Economics*, 50(23):2600–2612. DOI: 10.1080/00036846.2017.1403560.

Chen, A.-S. & Yang, W. 2016. Echo effects and the returns from 52-week high strategies. *Finance Research Letters*, 16:38–46. DOI: 10.1016/j.frl.2015.10.015.

Chen, H.-Y., Chou, P.-H. & Hsieh, C.-H. 2018. Persistency of the momentum effect. *European Financial Management*, 24(5):856–892. DOI: 10.1111/eufm.12140.

Chen, L.-W., Yu, H.-Y. & Wang, W.-K. 2018. Evolution of historical prices in momentum investing. *Journal of Financial Markets*, 37:120–135. DOI: 10.1016/J.FINMAR.2017.07.001.

Chen, L., Kadan, O. & Kose, E. 2009. *Fresh momentum*. Working Paper. St Louis, Missouri: Washington University in St Louis.

Chiao, C.-S., Hsiao, Y.-J., Chen, J.-C. & An, N.M. 2018. Residual momentum versus price momentum: evidence from four Asian markets. *Asia-Pacific Journal of Accounting & Economics*, Online:1–10. DOI: 10.1080/16081625.2018.1474772.

Choi, J. 2014. Physical approach to price momentum and its application to momentum strategy. *Physica A: Statistical Mechanics and its Applications*, 415:61–72. DOI: 10.1016/j.physa.2014.07.075.

Choi, J. 2021. Maximum drawdown, recovery, and momentum. *Journal of Risk and Financial Management*, 14(11):1–25. DOI: 10.3390/jrfm14110542.

Clare, A., Seaton, J., Smith, P.N. & Thomas, S. 2016. The trend is our friend: risk parity, momentum and trend following in global asset allocation. *Journal of Behavioral and Experimental Finance*, 9:63–80. DOI: 10.1016/j.jbef.2016.01.002.

Cleary, S. & Inglis, M. 1998. Momentum in Canadian stock returns. *Canadian Journal of Administrative Sciences*, 15(3):279–291. DOI: 111/j.1936-4490.1998.tb00168.x.

Clenow, A.F. 2015. *Stocks on the move: beating the market with hedge fund momentum strategies*. Scotts Valley, California: CreateSpace.

Conrad, J.S., Gültekin, M.N. & Kaul, G. 1997. Profitability of short-term contrarian strategies: implications for market efficiency. *Journal of Business & Economic Statistics*, 15(3):379–386. DOI: 10.2307/1392341.

Conrad, J.S. & Kaul, G. 1998. An anatomy of trading strategies. *The Review of Financial Studies*, 11(3):489–519. DOI: 10.1093/rfs/11.3.489.

Conrad, J.S. & Yavuz, M.D. 2017. Momentum and reversal: does what goes up always come down? *Review of Finance*, 21(2):555–581. DOI: 10.1093/rof/rfw006.

Da, Z., Gurun, U.G. & Warachka, M. 2014. Frog in the pan: continuous information and momentum. *The Review of Financial Studies*, 27(7):2171–2218. DOI: 10.1093/rfs/hhu003.

Daniel, K.D., Hirshleifer, D.A. & Subrahmanyam, A. 1998. Investor psychology and security market under- and overreactions. *The Journal of Finance*, 53(6):1839–1885. DOI: 10.1111/0022-1082.00077.

Daniel, K.D. & Moskowitz, T.J. 2016. Momentum crashes. *Journal of Financial Economics*, 122(2):221–247. DOI: 10.1016/j.jfineco.2015.12.002.

De Bondt, W.F.M. & Thaler, R.H. 1985. Does the stock market overreact? *The Journal of Finance*, 40(3):793–805. DOI: 10.2307/2327804.

De Long, J.B., Shleifer, A., Summers, L.H. & Waldmann, R.J. 1990. Positive feedback investment strategies and destabilizing rational speculation. *The Journal of Finance*, 45(2):379–395. DOI: 10.2307/2328662.

De Longis, A., Zanin, D. & Ellis, D. 2022. Measuring sector cyclicality: a factor-based approach. *The Journal of Beta Investment Strategies*, 13(4):147–162. DOI: 10.3905/jbis.2022.1.014.

Dittberner, A.G. 2016. Fundamental momentum: a new approach to investment analysis. PhD thesis. University of Pretoria.

Drew, M.E., Veeraraghavan, M. & Ye, M. 2007. Do momentum strategies work? Australian evidence. *Managerial Finance*, 33(10):772–787. DOI: 10.1108/03074350710779223.

Dudler, M., Gmür, B. & Malamud, S. 2015. Momentum and risk adjustment. *The Journal of Alternative Investments*, 18(2):91–103. DOI: 10.3905/jai.2015.18.2.091.

Dunbar, S.R. 2019. *Mathematical modeling in economics and finance: probability, stochastic processes, differential equations*. Providence, Rhode Island: MAA Press, American Mathematical Society (Volume 49).

Eisdorfer, A. 2008. Delisted firms and momentum profits. *Journal of Financial Markets*, 11(2):160–179. DOI: 10.1016/j.finmar.2007.12.001.

Ejaz, A. & Polak, P. 2018. Australian Stock Exchange and sub-variants of price momentum strategies. *Investment Management and Financial Innovations*, 15(1):224–235. DOI: 10.21511/imfi.15(1).2018.19.

Excel. 2022. Microsoft Excel 365 [Website]. Microsoft Corporation. Available at: https://www.microsoft.com/en-za/microsoft-365/excel.

Fama, E.F. & French, K.R. 2023. Fama-French factors [Website]. Kenneth R. French. Available at: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html.

Foerster, S. 1996. Back to the future, again: in defence of momentum-based trading strategies [and now you see them, then you don't]. *Canadian Investment Review*, 9(2):n/a.

Foerster, S., Prihar, A. & Schmitz, J.J. 1994. Back to the future [Price momentum models]. *Canadian Investment Review*, 7(4):9–13.

Foltice, B. & Langer, T. 2015. Profitable momentum trading strategies for individual investors. *Financial Markets and Portfolio Management*, 29(2):85–113. DOI: 10.1007/s11408-015-0246-4.

Frost, J. 2014. *How to interpret a regression model with low R-Squared and low p values*. *Minitab Blog [Online]*. Coventry, United Kingdom: Minitab, LLC. Available at: https://blog.minitab.com/en/adventures-in-statistics-2/how-to-interpret-a-regression-model-with-low-r-squared-and-low-p-values (Accessed: 4 May 2023).

FTSEI. 2021. *FTSE/JSE Africa Index Series*. *Ground rules: FTSE/JSE Africa Index Series [Online]*. London Stock Exchange Group. Available at: https://research.ftserussell.com/products/downloads/FTSE_JSE_Africa_Index_Series_new.pdf (Accessed: 15 April 2022).

George, T.J. & Hwang, C.-Y. 2004. The 52-week high and momentum investing. *The Journal of Finance*, 59(5):2145–2176. DOI: 10.1111/j.1540-6261.2004.00695.x.

Georgopoulou, A. & Wang, J. 2017. The trend is your friend: time-series momentum strategies across equity and commodity markets. *Review of Finance*, 21(4):1557–1592. DOI: 10.1093/rof/rfw048.

Ghasemi, A. & Zahediasl, S. 2012. Normality tests for statistical analysis: a guide for non-statisticians. *International Journal of Endocrinology and Metabolism*, 10(2):486–489. DOI: 10.5812/ijem.3505.

GICS. 2018. *Global Industry Classification Standard*. *Guide to the GICS Methodology [Online]*. S&P Global Market Intelligence. Available at: https://www.spglobal.com/marketintelligence/en/documents/112727-gics-mapbook_2018_v3_letter_digitalspreads.pdf (Accessed: 15 March 2022).

Gong, Q., Liu, M. & Liu, Q. 2015. Momentum is really short-term momentum. *Journal of Banking & Finance*, 50:169–182. DOI: 10.1016/j.jbankfin.2014.10.002.

Goyal, A. & Jegadeesh, N. 2018. Cross-sectional and time-series tests of return predictability: what is the difference? *The Review of Financial Studies*, 31(5):1784–1824. DOI: 10.1093/rfs/hhx131.

Goyal, A. & Wahal, S. 2015. Is momentum an echo? *Journal of Financial and Quantitative Analysis*, 50(6):1237–1267. DOI: 10.1017/S0022109015000575.

GraphPad. 2022. Prism 9 statistics guide [Website]. Dotmatics. Available at: https://www.graphpad.com.

Gray, W.R. & Vogel, J.R. 2016. *Quantitative momentum: a practitioner's guide to building a momentum-based stock selection system*. Hoboken, New Jersey: John Wiley & Sons, Inc (Wiley Finance).

Grinblatt, M. & Moskowitz, T.J. 2004. Predicting stock price movements from past returns: the role of consistency and tax-loss selling. *Journal of Financial Economics*, 71(3):541–579. DOI: 10.1016/S0304-405X(03)00176-4.

Gulen, H. & Petkova, R. 2018. Absolute strength: exploring momentum in stock returns. *SSRN Electronic Journal*, Online. DOI: 10.2139/ssrn.2638004.

Gupta, K., Locke, S. & Scrimgeour, F. 2013. Profitability of momentum returns under alternative approaches. *International Journal of Managerial Finance*, 9(3):219–246. DOI: 10.1108/IJMF-03-2012-0028.

Gutierrez, R.C. & Pirinsky, C.A. 2007. Momentum, reversal, and the trading behaviors of institutions. *Journal of Financial Markets*, 10(1):48–75. DOI: 10.1016/j.finmar.2006.09.002.

Hameed, A. & Wu, H. 2019. Decomposing momentum. *SSRN Electronic Journal*, Online. DOI: 10.2139/ssrn.3401656.

Han, Y., Zhou, G. & Zhu, Y. 2016. A trend factor: any economic gains from using information over investment horizons? *Journal of Financial Economics*, 122(2):352–375. DOI: 10.1016/j.jfineco.2016.01.029.

Hanauer, M.X. 2014. Is Japan different? Evidence on momentum and market dynamics. *International Review of Finance*, 14(1):141–160. DOI: 10.1111/irfi.12024.

Harckbart, G. 2019. Laplace versus the normal distribution for daily stock market returns. *SSRN Electronic Journal*, Online. DOI: 10.2139/ssrn.3479681.

Harper, E.M., Diao, Z., Panousi, S., Nuss, P., Eckelman, M.J. & Graedel, T.E. 2015. The criticality of four nuclear energy metals. *Resources, Conservation and Recycling*, 95:193–201. DOI: 10.1016/j.resconrec.2014.12.009.

Harris, M. 2011. *Chaos in technical analysis and backtesting: close vs adjusted close*. *Price Action Lab Blog [Online]*. Michael Harris. Available at: https://www.priceactionlab.com/Blog/2011/03/chaos-in-technical-analysis-and-backtesting-part-i-close-vs-adjusted-close/ (Accessed: 7 January 2021).

Harris, M. 2015. *Fooled by dividend-induced upward drift*. *Price Action Lab Blog [Online]*. Michael Harris. Available at: https://www.priceactionlab.com/Blog/2015/08/dividend-induced-upward-drift/ (Accessed: 7 January 2021).

Harris, M. 2018. *Effect of dividend adjustments on portfolio backtests*. *Price Action Lab Blog [Online]*. Michael Harris. Available at: https://www.priceactionlab.com/Blog/2018/07/effect-of-dividend-adjustments-on-portfolio-backtests/ (Accessed: 8 January 2021).

Hayashi, M. & Hughes, L. 2013. The Fukushima nuclear accident and its effect on global energy security. *Energy Policy*, 59:102–111. DOI: 10.1016/j.enpol.2012.11.046.

Heyman, D., Lescrauwaet, M. & Stieperaere, H. 2019. Investor attention and short-term return reversals. *Finance Research Letters*, 29:1–6. DOI: 10.1016/j.frl.2019.03.003.

Hillert, A., Jacobs, H. & Müller, S. 2014. Media makes momentum. *The Review of Financial Studies*, 27(12):3467–3501. DOI: 10.1093/rfs/hhu061.

Hoffmann, A.O.I. & Post, T. 2014. Self-attribution bias in consumer financial decision-making: how investment returns affect individuals’ belief in skill. *Journal of Behavioral and Experimental Economics*, 52:23–28. DOI: 10.1016/j.socec.2014.05.005.

Hoffstein, C. 2018. *Momentum’s magic number*. *Flirting with Models [Online]*. Newfound Research LLC. Available at: https://blog.thinknewfound.com/2018/07/momentums-magic-number/ (Accessed: 26 December 2021).

Hong, H. & Stein, J.C. 1999. A unified theory of underreaction, momentum trading, and overreaction in asset markets. *The Journal of Finance*, 54(6):2143–2184. DOI: 10.1111/0022-1082.00184.

Hou, T.C.-T. & McKnight, P.J. 2004. An explanation of momentum in Canadian stocks. *Canadian Journal of Administrative Sciences*, 21(4):334–343. DOI: 10.1111/j.1936-4490.2004.tb00349.x.

Hühn, H.L. & Scholz, H. 2018. Alpha momentum and price momentum. *International Journal of Financial Studies*, 6(2):49. DOI: 10.3390/ijfs6020049.

Hühn, H.L. & Scholz, H. 2019. Reversal and momentum patterns in weekly stock returns: European evidence. *Review of Financial Economics*, 37(2):272–296. DOI: 10.1002/rfe.1037.

Humphreys, D. 2020. Mining productivity and the fourth industrial revolution. *Mineral Economics*, 33(1–2):115–125. DOI: 10.1007/s13563-019-00172-9.

Huynh, T.D. & Smith, D.R. 2017. Delisted stocks and momentum: evidence from a new Australian dataset. *Australian Journal of Management*, 42(1):140–160. DOI: 10.1177/0312896214565118.

Jegadeesh, N. 1990. Evidence of predictable behavior of security returns. *The Journal of Finance*, 45(3):881–898. DOI: 10.1111/j.1540-6261.1990.tb05110.x.

Jegadeesh, N. & Titman, S. 1993. Returns to buying winners and selling losers: implications for stock market efficiency. *The Journal of Finance*, 48(1):65–91. DOI: 10.1111/j.1540-6261.1993.tb04702.x.

Jegadeesh, N. & Titman, S. 1995. Short-horizon return reversals and the bid-ask spread. *Journal of Financial Intermediation*, 4(2):116–132. DOI: 10.1006/jfin.1995.1006.

Jegadeesh, N. & Titman, S. 2001. Profitability of momentum strategies: an evaluation of alternative explanations. *The Journal of Finance*, 56(2):699–720. DOI: 10.1111/0022-1082.00342.

Jegadeesh, N. & Titman, S. 2002. Cross-sectional and time-series determinants of momentum returns. *The Review of Financial Studies*, 15(1):143–157. DOI: 10.1093/rfs/15.1.143.

Jegadeesh, N. & Titman, S. 2011. Momentum. *Annual Review of Financial Economics*, 3(1):493–509. DOI: 10.1146/annurev-financial-102710-144850.

Jiang, G.J. & Zhu, K.X. 2017. Information shocks and short-term market underreaction. *Journal of Financial Economics*, 124(1):43–64. DOI: 10.1016/j.jfineco.2016.06.006.

Joshipura, M. & Wats, S. 2023. Decoding momentum returns: an integrated bibliometric and content analysis approach. *Qualitative Research in Financial Markets*, 15(2):254–277. DOI: 10.1108/QRFM-12-2021-0211.

Jostova, G., Nikolova, S., Philipov, A. & Stahel, C.W. 2013. Momentum in corporate bond returns. *Review of Financial Studies*, 26(7):1649–1693. DOI: 10.1093/rfs/hht022.

Kan, R. & Kirikos, G. 1996. Now you see them, then you don't: the phenomenal returns a trading strategy can produce when tested using historical data often vanish once theory is put into practice. *Canadian Investment Review*, 9(2):n/a.

Kang, W. & Zhang, H. 2014. Measuring liquidity in emerging markets. *Pacific-Basin Finance Journal*, 27:49–71. DOI: 10.1016/j.pacfin.2014.02.001.

Kaufman, P.J. 2013. *Trading systems and methods*. 5th edition. Hoboken, New Jersey: John Wiley & Sons, Inc (Wiley Trading). DOI: 10.1002/9781119202561.

Kim, S. 2022. Time-series residual momentum strategies. *Applied Economics*, 54(5):580–594. DOI: 10.1080/00036846.2021.1967862.

Kotz, S., Kozubowski, T.J. & Podgórski, K. 2001. *The Laplace distribution and generalizations: a revisit with applications to communications, economics, engineering, and finance*. Boston, Massachusetts: Birkhäuser Boston. DOI: 10.1007/978-1-4612-0173-1.

Kula, G., Raab, M. & Stahn, S. 2017. *Beyond smart beta: index investment strategies for active portfolio management*. Chichester, West Sussex: John Wiley & Sons, Inc (Wiley Finance). DOI: 10.1002/9781119395263.

La Grange, P.L. & Krige, J.D. 2015. Profitability of momentum strategies on the JSE. *Studies in Economics and Econometrics*, 39(3):49–65.

Le, H. & Gregoriou, A. 2020. How do you capture liquidity? A review of the literature on low-frequency stock liquidity. *Journal of Economic Surveys*, 34(5):1170–1186. DOI: 10.1111/joes.12385.

Leavy, P. 2017. *Research design: quantitative, qualitative, mixed methods, arts-based, and community-based participatory research approaches*. New York City, New York: The Guilford Press.

Lee, C.M.C. & Swaminathan, B. 2000. Price momentum and trading volume. *The Journal of Finance*, 55(5):2017–2069. DOI: 10.1111/0022-1082.00280.

Lee, J. 2020. Statistics, descriptive. in Kobayashi, A. (ed.) *International Encyclopedia of Human Geography Volume 13*. 2nd edition. Amsterdam, Netherlands: Elsevier. DOI: 10.1016/B978-0-08-102295-5.10428-7.

Lee, K.-H. 2006. Liquidity risk and asset pricing. PhD thesis. The Ohio State University.

Lehmann, B.N. 1990. Fads, martingales, and market efficiency. *The Quarterly Journal of Economics*, 105(1):1–28. DOI: 10.2307/2937816.

Lesmond, D.A. 2005. Liquidity of emerging markets. *Journal of Financial Economics*, 77(2):411–452. DOI: 10.1016/j.jfineco.2004.01.005.

Levine, A. & Pedersen, L.H. 2016. Which trend is your friend? *Financial Analysts Journal*, 72(3):51–66. DOI: 10.2469/faj.v72.n3.3.

Li, B., Wang, J., Huang, D. & Hoi, S.C.H. 2018. Transaction cost optimization for online portfolio selection. *Quantitative Finance*, 18(8):1411–1424. DOI: 10.1080/14697688.2017.1357831.

Li, J. & Yu, J. 2012. Investor attention, psychological anchors, and stock return predictability. *Journal of Financial Economics*, 104(2):401–419. DOI: 10.1016/j.jfineco.2011.04.003.

Lim, B.Y., Wang, J. & Yao, Y. 2018. Time-series momentum in nearly 100 years of stock returns. *Journal of Banking & Finance*, 97:283–296. DOI: 10.1016/j.jbankfin.2018.10.010.

Lin, H.-W., Hung, M.-W. & Huang, J.-B. 2018. Artificial momentum, native contrarian, and transparency in China. *Computational Economics*, 51(2):263–294. DOI: 10.1007/s10614-017-9699-z.

Lin, S. & Rassenti, S. 2012. Are under- and over-reaction the same matter? Experimental evidence. *Journal of Economic Behavior & Organization*, 84(1):39–61. DOI: 10.1016/j.jebo.2012.07.004.

Liu, M., Liu, Q. & Ma, T. 2011. The 52-week high momentum strategy in international stock markets. *Journal of International Money and Finance*, 30(1):180–204. DOI: 10.1016/j.jimonfin.2010.08.004.

Lo, A.W. & MacKinlay, A.C. 1990. When are contrarian profits due to stock market overreaction? *The Review of Financial Studies*, 3(2):175–205. DOI: 10.1093/rfs/3.2.175.

Marshall, B.R. & Cahan, R.M. 2005. Is the 52-week high momentum strategy profitable outside the US? *Applied Financial Economics*, 15(18):1259–1267. DOI: 10.1080/09603100500386008.

Marshall, B.R., Nguyen, N.H. & Visaltanachoti, N. 2017. Time series momentum and moving average trading rules. *Quantitative Finance*, 17(3):405–421. DOI: 10.1080/14697688.2016.1205209.

McLean, R.D. 2010. Idiosyncratic risk, long-term reversal, and momentum. *Journal of Financial and Quantitative Analysis*, 45(4):883–906. DOI: 10.1017/S0022109010000311.

Mertens, D.M. 2020. *Research and evaluation in education and psychology: integrating diversity with quantitative, qualitative, and mixed methods*. 5th edition. Los Angeles, California: SAGE Publications Inc.

Min, C. 2019. *Applied econometrics: a practical guide*. New York: Routledge. DOI: 10.4324/9780429024429.

Moskowitz, T.J., Ooi, Y.H. & Pedersen, L.H. 2012. Time series momentum. *Journal of Financial Economics*, 104(2):228–250. DOI: 10.1016/j.jfineco.2011.11.003.

Mota, P.P. 2012. Normality assumption for the log-return of the stock prices. *Discussiones Mathematicae Probability and Statistics*, 32(1–2):47–58. DOI: 10.7151/dmps.1143.

Mukherji, P. & Albon, D. 2018. *Research methods in early childhood: an introductory guide*. 3rd edition. London: SAGE Publications Ltd.

Muller, C. 1999. Investor overreaction on the Johannesburg Stock Exchange. *Investment Analysts Journal*, 28(49):5–17. DOI: 10.1080/10293523.1999.11082392.

Mun, J.C., Vasconcellos, G.M. & Kish, R. 2000. The contrarian/overreaction hypothesis: an analysis of the US and Canadian stock markets. *Global Finance Journal*, 11(1–2):53–72. DOI: 10.1016/S1044-0283(00)00011-9.

Murphy, Á. 2017. An investigation into momentum in the UK stock market and the behaviour of brokers and analysts. PhD thesis. Waterford Institute of Technology.

NCSS. 2022. Data analysis and graphics documentation. NCSS Statistical Software. Available at: https://www.ncss.com.

Novikov, A.M. & Novikov, D.A. 2013. *Research methodology: from philosophy of science to research design*. Boca Raton, Florida: Chapman & Hall / CRC Press (Communications in Cybernetics, Systems Science and Engineering). DOI: 10.1201/b14562.

Novy-Marx, R. 2012. Is momentum really momentum? *Journal of Financial Economics*, 103(3):429–453. DOI: 10.1016/j.jfineco.2011.05.003.

O'Keeffe, C. 2013. An investigation into the winner-loser and momentum anomalies in four medium-sized European markets. PhD thesis. Dublin City University.

O’Keeffe, C. & Gallagher, L.A. 2017. The winner-loser anomaly: recent evidence from Greece. *Applied Economics*, 49(47):4718–4728. DOI: 10.1080/00036846.2017.1293786.

Page, M.D. 2016. An in-depth validation of momentum as a dominant explanatory factor on the Johannesburg Stock Exchange. PhD thesis. University of the Witwatersrand.

Page, M.D. & Auret, C.J. 2017. Univariate tests of momentum on the JSE. *Investment Analysts Journal*, 46(3):149–164. DOI: 10.1080/10293523.2017.1319162.

Page, M.D. & Auret, C.J. 2019. Can non-momentum factor premiums explain the momentum anomaly on the JSE? An in-depth portfolio attribution analysis. *Investment Analysts Journal*, 48(1):1–17. DOI: 10.1080/10293523.2018.1483792.

Page, M.D., Britten, J.H.C. & Auret, C.J. 2013. Momentum and liquidity on the Johannesburg Stock Exchange. *International Journal of Economics and Finance Studies*, 5(1):56–73.

Page, M.D., Britten, J.H.C. & Auret, C.J. 2016. Idiosyncratic risk and anomaly persistence on the Johannesburg Stock Exchange (JSE). *Investment Analysts Journal*, 45(1):31–46. DOI: 10.1080/10293523.2015.1125060.

Page, M.D., McClelland, D. & Auret, C.J. 2020. Idiosyncratic momentum on the JSE. *Investment Analysts Journal*, 49(3):180–198. DOI: 10.1080/10293523.2020.1783864.

Page, M.J. & Way, C.V. 1992. Stock market over-reaction: the South African evidence. *Investment Analysts Journal*, 21(36):35–49. DOI: 10.1080/10293523.1992.11082314.

Pandas. 2022. Python data analysis [Website]. The pandas development team. Available at: https://pandas.pydata.org/.

Park, S.-C. 2010. The moving average ratio and momentum. *The Financial Review*, 45(2):415–447. DOI: 10.1111/j.1540-6288.2010.00254.x.

Pavlova, I. & Parhizgari, A.M. 2011. In search of momentum profits: are they illusory? *Applied Financial Economics*, 21(21):1617–1639. DOI: 10.1080/09603107.2011.589804.

Python. 2022. Python programming language [Website]. Python Software Foundation. Available at: https://www.python.org/.

RealStats. 2022. Real statistics using Excel [Website]. Charles Zaiontz. Available at: https://www.real-statistics.com.

Rezigalla, A.A. 2020. Observational study designs: synopsis for selecting an appropriate study design. *Cureus*, Online. DOI: 10.7759/cureus.6692.

Rhee, S.G. & Wang, C.-J. 1997. The bid-ask bounce effect and the spread size effect: evidence from the Taiwan stock market. *Pacific-Basin Finance Journal*, 5(2):231–258. DOI: 10.1016/S0927-538X(97)00014-0.

Ross, A., Moskowitz, T.J., Israel, R. & Serban, L. 2017. Implementing momentum: what have we learned? *SSRN Electronic Journal*, Online. DOI: 10.2139/ssrn.3081165.

Ross, A. & Willson, V.L. 2017. *Basic and advanced statistical tests*. Rotterdam: Sense Publishers. DOI: 10.1007/978-94-6351-086-8.

Samunderu, E. & Murahwa, Y.T. 2021. Return based risk measures for non-normally distributed returns: an alternative modelling approach. *Journal of Risk and Financial Management*, 14(11):540. DOI: 10.3390/jrfm14110540.

Sankaran, H., Nguyen, A. & Harikumar, J. 2012. Extreme return correlation and volatility: a two-threshold approach. *American Journal of Business*, 27(2):154–173. DOI: 10.1108/19355181211274451.

Satchell, S. & Grant, A. 2021. *Market momentum: theory and practice*. Chichester, West Sussex: John Wiley & Sons, Inc (Wiley Finance Series). DOI: 10.1002/9781119599364.

SATRIX. 2022. *Satrix Momentum Index Fund. Fund Fact Sheet [Online]*. Satrix Managers. Available at: https://satrix.co.za/media/62531?inline=true (Accessed: 15 April 2022).

Schmidt, A.F. & Finan, C. 2018. Linear regression and the normality assumption. *Journal of Clinical Epidemiology*, 98:146–151. DOI: 10.1016/j.jclinepi.2017.12.006.

Schmitz, J.J. & Cleary, S. 2000. What has worked on Bay Street [Several factors appear to be powerful predictors of future stock returns]. *Canadian Investment Review*, 13(4):n/a.

Schneider, P. & Gaunt, C. 2012. Price and earnings momentum in Australian stock returns. *Accounting & Finance*, 52(2):495–517. DOI: 10.1111/j.1467-629X.2010.00395.x.

Scott, D.M. 2020. Statistics, inferential. in Kobayashi, A. (ed.) *International Encyclopedia of Human Geography Volume 13*. 2nd edition. Amsterdam, Netherlands: Elsevier. DOI: 10.1016/B978-0-08-102295-5.10429-9.

SENS_S336740. 2013. Uranium One. Thursday 26 September. Available at: https://www.profiledata.co.za/BrokerSites/BusinessLive/SENS.aspx?id=220347.

SENS_S427702. 2020. Anglo American Platinum. Friday 06 March 2020. Available at: https://www.profiledata.co.za/BrokerSites/BusinessLive/SENS.aspx?id=354691.

SENS_S427824. 2020. Royal Bafokeng Platinum. Tuesday 10 March 2020. Available at: https://www.profiledata.co.za/BrokerSites/BusinessLive/SENS.aspx?id=354862.

SENS_S428273. 2020. Royal Bafokeng Platinum. Wednesday 18 March 2020. Available at: https://www.profiledata.co.za/BrokerSites/BusinessLive/SENS.aspx?id=355613.

Shi, J., Chiang, T.C. & Liang, X. 2012. Positive-feedback trading activity and momentum profits. *Managerial Finance*, 38(5):508–529. DOI: 10.1108/03074351211217832.

Siganos, A. 2010. Can small investors exploit the momentum effect? *Financial Markets and Portfolio Management*, 24(2):171–192. DOI: 10.1007/s11408-009-0120-3.

Singh, S. & Walia, N. 2022. Momentum investing: a systematic literature review and bibliometric analysis. *Management Review Quarterly*, 72(1):87–113. DOI: 10.1007/s11301-020-00205-6.

SPDJM. 2022. *S&P Momentum indices methodology*. *S&P Dow Jones Indices [Online]*. S&P Global. Available at: https://www.spglobal.com/spdji/en/documents/methodologies/methodology-sp-momentum-indices.pdf (Accessed: 15 June 2022).

SPTSX. 2021. *S&P/TSX Composite single factor indices methodology*. *S&P Dow Jones Indices [Online]*. S&P Global. Available at: https://www.spglobal.com/spdji/en/documents/methodologies/methodology-sp-tsx-composite-single-factor-indices.pdf (Accessed: 15 June 2022).

SPTSX. 2022. *S&P/TSX Canadian indices methodology*. *S&P Dow Jones Indices [Online]*. S&P Global. Available at: https://www.spglobal.com/spdji/en/documents/methodologies/methodology-sp-tsx-canadian-indices.pdf (Accessed: 15 June 2022).

SPTXV. 2022. *S&P/TSX Venture Composite methodology*. *S&P Dow Jones Indices [Online]*. S&P Global. Available at: https://www.spglobal.com/spdji/en/documents/methodologies/methodology-sp-tsx-venture-composite.pdf (Accessed: 15 June 2022).

Strang, K.D. (ed.) 2015. *The Palgrave handbook of research design in business and management*. New York City, New York: Palgrave Macmillan. DOI: 10.1057/9781137484956.

Taljaard, B. & Maré, E. 2019. Considering the use of an equal-weighted index as a benchmark for South African equity investors. *South African Actuarial Journal*, 19(1):53–70. DOI: 10.4314/saaj.v19i1.3.

Teplova, T. & Mikova, E. 2015. New evidence on determinants of price momentum in the Japanese stock market. *Research in International Business and Finance*, 34:84–109. DOI: 10.1016/j.ribaf.2014.12.001.

Toth, D. & Jones, B. 2019. Against the norm: modeling daily stock returns with the Laplace distribution. *Cornell University Open Access*, Online. Available at: http://arxiv.org/abs/1906.10325.

Van Heerden, J.D. 2014. The impact of firm-specific factors on the cross-sectional variation in Johannesburg Security Exchange listed equity returns. PhD thesis. University of Cape Town.

Van Heerden, J.D. & Van Rensburg, P. 2015. The cross-section of Johannesburg Securities Exchange listed equity returns (1994-2011). *Studies in Economics and Finance*, 32(4):422–444. DOI: 10.1108/SEF-09-2014-0181.

Van Heerden, J.D. & Van Rensburg, P. 2016. The impact of liquidity on the cross section of equity returns on the Johannesburg Securities Exchange. *Economics, Management, and Financial Markets*, 11(2):59–86.

Van Heerden, J.D. & Van Rensburg, P. 2017. Common firm-specific characteristics of extreme performers on the Johannesburg Securities Exchange. *Economics, Management, and Financial Markets*, 12(3):25–50. DOI: 10.22381/EMFM12320172.

Vanstone, B.J. & Hahn, T. 2017. Australian momentum: performance, capacity and the GFC effect. *Accounting & Finance*, 57(1):261–287. DOI: 10.1111/acfi.12140.

Varadi, D. 2014. *A new (better?) measure of risk and uncertainty: the volatility of acceleration*. *New Concepts in Quantitative Research [Online]*. CSS Analytics. Available at: https://cssanalytics.wordpress.com/2014/11/28/a-new-better-measure-of-risk-and-uncertainty-the-volatility-of-acceleration/ (Accessed: 31 January 2021).

Viljoen, L.E. 2016. Residual momentum and investor sentiment on the Johannesburg Stock Exchange (JSE). MBA dissertation. University of Pretoria.

Wiest, T. 2023. Momentum: what do we know 30 years after Jegadeesh and Titman's seminal paper? *Financial Markets and Portfolio Management*, 37(1):95–114. DOI: 10.1007/s11408-022-00417-8.

Wilmington. 2018. *Understanding investment risk through drawdown analysis*. *Investment Insights [Online]*. Wilmington Trust Corporation. Available at: https://www.wilmingtontrust.com/repositories/wtc_sitecontent/PDF/Understanding-investment-risk-through-drawdown-analysis.pdf (Accessed: 27 April 2021).

Winton. 2022. *Experiment and observation in quantitative investment*. *Winton Research [Online]*. Winton Capital Management Limited. Available at: https://www.winton.com/research/experiment-and-observation-in-quantitative-investment-management (Accessed: 3 June 2022).

Xiong, J.X. & Ibbotson, R.G. 2015. Momentum, acceleration, and reversal. *Journal of Investment Management*, 13(1):84–95.

XLSTAT. 2022. A complete statistical add-in for Microsoft Excel [Website]. Addinsoft. Available at: https://www.xlstat.com.

Yang, X. & Zhang, H. 2019. Extreme absolute strength of stocks and performance of momentum strategies. *Journal of Financial Markets*, 44:71–90. DOI: 10.1016/j.finmar.2019.01.001.

Zaher, F. 2019. *Index fund management: a practical guide to smart beta, factor investing, and risk premia*. Cham, Switzerland: Palgrave Macmillan. DOI: 10.1007/978-3-030-19400-0.

Zaremba, A., Kizys, R. & Raza, M.W. 2020. The long-run reversal in the long run: insights from two centuries of international equity returns. *Journal of Empirical Finance*, 55:177–199. DOI: 10.1016/j.jempfin.2019.11.007.

Zaremba, A., Umutlu, M. & Karathanasopoulos, A. 2019. Alpha momentum and alpha reversal in country and industry equity indexes. *Journal of Empirical Finance*, 53:144–161. DOI: 10.1016/j.jempfin.2019.07.003.

Zaremba, A., Umutlu, M. & Maydybura, A. 2018. Less pain, more gain: volatility-adjusted residual momentum in international equity markets. *Investment Analysts Journal*, 47(2):165–191. DOI: 10.1080/10293523.2018.1469290.