A Power-order Semantics for Nonmonotonic Logic I & II

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Abstract: Given a system of classical logic and a set Σ of axioms, consider the unary concept 'is entailed' and the binary notion 'is entailed by'. Each may be defined in terms of the frame consisting of the models of Σ: let $\wp\, Mod(\Sigma)$ be the dual powerset Boolean algebra of sets of models of Σ ordered by $\supseteq$, then α is entailed (implicitly-by Σ) iff the set $Mod(\alpha)$, comprising those models of Σ which are models also of α, is the minimum element, and α is entailed by β (implicitly-in the context of the axioms Σ) iff $Mod(\alpha) \supseteq Mod(\beta)$.

In a system of nonmonotonic logic, one wishes to replace the two classical concepts by the unary notion 'is plausible' and the binary notion 'is at least as plausible as' respectively. For example, non-monotonic systems based on minimal model semantics achieve this roughly as follows. Assume that in addition to the set Σ of axioms we have a preorder $\leq$ on the models of Σ. Then α is plausible iff $Mod(\alpha) \supseteq X$, where X is the set of $\leq$-minimal models of Σ, and the relation 'is at least as plausible as' is the coarse relation that views any two plausible sentences as being each at least as plausible as the other.

We propose a power-order semantics for nonmonotonic logic that lifts the preorder $\leq$ on models to a power-order $\Vdash$ on sets of models. Given sentences α and β, we define α to be at least as plausible as β iff $Mod(\alpha) \Vdash Mod(\alpha)$, where in general for sets X and Y of models $X \Vdash Y$ iff

$$(\forall y \in Y)(\exists x \in X)(x \leq y).$$

And we define α to be plausible iff α is at least as plausible as Σ.

From these definitions it follows that

- If α is entailed (by Σ) then α is plausible (in the sense of power-order semantics).
- If α is entailed by β then α is at least as plausible as β.
- If α is plausible in the sense of minimal model semantics (and provided that models of Σ exist that are minimal with respect to $\leq$ then α is plausible in the sense of power-order semantics (relative to the power-order induced by $\leq$, of course).
- In terms of power-order semantics, α is plausible iff α is at least as plausible as $\neg\alpha$.

The value of the power-order semantics is that it unifies different forms of nonmonotonic reasoning, such as reasoning based on the minimal model semantics and diagnostic reasoning (in the style of Reiter's model-based diagnoses).

Keywords: artificial intelligence, knowledge representation, default reasoning, diagnostic reasoning, nonmonotonic logic, minimal model semantics, power-ordering, plausibility.

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