AN INVESTIGATION INTO PROCESS-BASED INSTRUCTION IN THE TEACHING OF
GRADE 8 AND 9 EUCLIDEAN GEOMETRY

by

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I, MULIMISI ERDMANN KUTAMA, declare that AN INVESTIGATION INTO PROCESS-BASED INSTRUCTION IN THE TEACHING OF GRADE 8 AND 9 EUCLIDEAN GEOMETRY is my work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

SIGNATURE

02/04/03

DATE

MR ME KUTAMA
FOREWORD

I wish to acknowledge the following people and the institution:

1. Professor DCJ Wessels and Dr HM van Niekerk who tirelessly guided me to the perfection of my work.
2. Mr RA Jefferies who also tirelessly proofread several drafts of my dissertation.
3. I dedicate this piece of work to my son and two daughters, Muvhuya, Mulambilu and Divhani.
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SUMMARY

The teaching and learning of Euclidean geometry in black South African schools is characterised by teachers' poor knowledge of Euclidean geometry and the use of traditional teaching methods that encourage rote learning. In the light of this, the study investigated the extent to which learners perform in Process-Based Instruction.

Process-Based Instruction is characterised by the gradual transfer of instruction from the teacher to the learner. In Process-Based Instruction learners are expected to communicate thought, form concepts and master theorems by drawing, 'showed with and used hands', talking and writing. Learners' performance is assessed by rubrics and is analysed using graphs.

The findings of the empirical investigation revealed that learners of both grades 8 and 9 cannot communicate thought by any media (talking, writing 'showed with and used hands' and drawing). A few learners formed a few concepts and mastered theorems. On the basis of the findings and the shortcomings of the study recommendations are made.

For the afrikaans version please turn over.
OPSOMMING

Die onderrig van Euclediaanse Meetkunde in swart Suid-Afrikaanse skole word gekenmerk aan onderwysers se swak kennis van die vak, asook die gebruik van onderrigmetodes wat op die leerder se geheue staatmaak. Hierdie studie ondersoek die sukses van leerders se prestatie in proses-gebasseerde onderrig.

Proses-gebasseerde onderrig word gekenmerk aan die geleidelike oordra van kennis, van die onderwyser na die leerder. Daar word van leerders verwag om gedagtes oor te dra, konsepte (begrippe) te verstaan en stellings onder die knie te kry deur middel van ketse, die gebruik van gebare, verbale verduideling en skriflike werk.

Leerders se prestatie word geëvalueer deur die gebruik van klassifisering in kolomme (rubrics) en word ge-analiseer deur grafieke.

Die bevinding van die empiriese ondersoek het getoon dat leerders van sowel graad 8 as graad 9 nie gedagtes kan oordra deur enige van die gemelde metodes nie. ’n Paar leerders het wel enkele begrippe verstaan en kon stellings bemeester. Na aanleiding van die bevindings van die studie en die tekortkominge wat daardeur opgespoor is, word sekere voorstelle gemaak.
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CHAPTER ONE

INTRODUCTION AND OVERVIEW OF THE STUDY

1.1 Factors that lead to the study

This study came about as a result of several attempts to teach Euclidean geometry to grade 8 and 9 learners, with little success. It needs to be pointed out that the learners in question were from different schools attending lessons during the holiday. The researcher taught learners Euclidean geometry during Easter and winter holidays. Lessons were both remedial and supplementary in the sense that if learners experience difficulty with grade eight and nine geometry syllabus they were first taught senior primary school geometry. The process was carried out until the researcher was satisfied that they have understood the subject content. The researcher noticed that they were good at geometry problems involving calculations (measurement) but weak in deductive proof.

It has been a struggle to make learners understand properties of quadrilaterals, parallelogram in particular, using a variety of activities involving movements of a traced parallelogram on the acetate (transparency) sheet. The researcher again noticed that learners experience less difficulty in solving problems involving calculations. Solving problems involving deduction was like climbing a mountain for learners. The so-called best learners experienced a similar situation.

It appeared to me that there was a 'wide gap' between the grade 9 and 10 geometry syllabi. There was 'big jump' from the inductive approach of teaching grade 8 and 9 to the deductive approach of teaching grade 10 geometry. The gap that has been referred to above can clearly be observed in the way in which geometry content has been randomly introduced in grade 8 and 9 (Vorster 1998:191).
The study will also investigate whether teaching concepts and theorems using the principle of process-based teaching will improve disadvantaged learners' performance in Euclidean geometry.

Factors, which have lead to the study, are the following:

1.1.1 Poor teaching of Euclidean geometry

According to Hoffer (1981:11) most learners consider geometry to be difficult. If one asks secondary school learners in South African about which branch of mathematics they find difficult to learn, they often mention geometry (Scholefield 1997:153). The situation is attributed to poor teaching of geometry whereby learners are encouraged to learn geometry by memorising theorems and are expected to reproduce them in a test or examination (Luthuli 1996:17).

According to Morris (1985:92) teachers tend to give low priority to geometry as against algebra. He further stressed that if geometry is to be taught at all, only theorems are drilled into the minds of learners with little understanding of them. This is a typical situation in most South African mathematics classrooms whereby learners are taught mainly algebra, and during the last few weeks of the year they are hurriedly taught geometry.

According to Morris (1985:92), the root cause of poor geometry teaching lies in the mismatch between teacher education courses and the needs of the learners in the school. This shows that teachers have not been trained to be innovative in their teaching of geometry. In other words, the curricula for colleges do not properly prepare student teachers for challenges at the schools. Eventually, they find themselves poorly prepared to teach Euclidean geometry (Luthuli 1996:20).

It is important that teachers use teaching methods that take the learner's level of understanding geometry into account. They should encourage learners to reason.
Learners should ask themselves questions: What am I doing? How must I do it? Why am I doing it?

1.1.2 The nature of Euclidean geometry

It is difficult to teach theorem proving since it cannot be accomplished using an algorithm (Bell 1978:304). This is one of the aspects that make Euclidean geometry different from other branches of mathematics. There is no specific method to be followed to arrive at a solution. One must have, among other things, accumulated knowledge of theorems, being at Van Hiele level 3, and have accumulated deductive skills, in order to solve problems in Euclidean geometry. At Van Hiele level 3 a learner should be able to write proofs of theorems by using deduction.

According to Bell (1978:304) Euclidean geometry is not only difficult to learners, but it is also difficult to teachers. This is because of its emphasis on deductive proof. Success in proof writing is determined by having acquired deductive skills, knowledge of concepts and theorems. Most learners are unable to cope with deductive proof, because they lack knowledge of concepts, theorems and deductive skills.

There is a school of thought that advocates that proof by deduction should be done away with since it fails to help learners to learn Euclidean geometry. According to Stalling-Roberts (1994:403), the deductive approach alone has been found to have had limited success as far as the teaching of geometry is concerned. Hoffmann (1986:254) agrees in the sense that Euclidean geometry is too sophisticated for learners, because of its emphasis on proof by deduction. Nevertheless Van Hiele (1984:251) argues that teaching deductive systems requires patience. It is now clear why some learners consider Euclidean geometry a terrifying discipline of mathematics.
1.1.3 Teaching Euclidean geometry at a higher level than that of the learners

According to Van Hiele (1986:51) teaching geometry at another level is not possible without mastering the preceding level. This suggests that teachers are often either misunderstood or not understood at all by learners. This is due to the fact that learners have not yet reached the level that the teacher assumes they are at. They tend to teach Euclidean geometry at a level higher than that of learners. As a result, learners find it difficult to understand the teacher.

1.1.4 Lack of spatial knowledge

According to Sherard (1981:19) Euclidean geometry should be made a living discipline of mathematics by teaching or associating it with concrete objects. It is important to relate geometry to the learner's experiences. Euclidean geometry is often taught abstractly as if it is something that is only found in the textbooks and examination papers.

It is necessary that learners be exposed to learn geometry through concrete objects at an early age (Van Hiele-Geldof 1984:19). This will help them to develop spatial knowledge. Giving them the opportunity to cut shapes from paper, to draw pictures/shapes, to fold paper to make shapes and to construct real-life models, can develop their spatial knowledge. This knowledge will be needed when they reach secondary phase. It could place them in a better position to solve Euclidean geometry theorems.

1.1.5 Language problems that both learners and teachers encounter when they learn and teach Euclidean geometry

Learning theorems become more difficult for learners since the language of instruction is a second language. Luthuli (1992:27) has this to say:
In a teaching-learning situation, learners should be able to learn what the teacher intends them to learn without the language itself getting in the way.

To most black learners English is a medium through which they learn mathematics, Euclidean geometry in particular. As a result they are unable to understand theorems and concepts. This eventually leads to the breakdown in communication between the teacher and learners. The situation is even worse if the teacher also does not have a command of the language of instruction. Kambule (1987) argues:

We are fully aware of the role language plays in understanding and conceiving mathematical ideas. But our position is made adverse by the fact the language used is not everyone's language, the majority have English as a second language, and thus renders our task difficult...

Learners find it difficult to communicate their ideas with the teacher and fellow learners, if their command of the language of instruction is poor. Consequently, they are unable to develop reasoning strategies since language is a vehicle for thought. If a learner does not have a command of the language of instruction, then he or she will be unable to learn Euclidean geometry.

Another obstacle to the learning of Euclidean geometry is the confusion in the usage between normal English terms and the technical terms that are used in mathematics. A word in English may have a different meaning altogether from that in mathematics. For example, the word 'root' in normal English refers to the part of a plant or of a tree, which grows underground. In mathematics it refers to values on the x-axes or a square root and a cube root (Luthuli 1992:27). Von Ludwig (1991:39) points out that learners seldom understand the meaning of the following phrases: 'the line is produced', 'line produced its own length', 'is a point' and 'touches at'. Teachers will not easily notice the inability of learners to understand the technical language used in Euclidean geometry, unless they pay attention to problems encountered by an individual learner. They are not entirely to blame,
since teachers also pay very little attention to Euclidean geometry vocabulary in their classrooms. Van Niekerk (1998:31) attributed this, to lack of Euclidean geometry vocabulary and to the incompetence of black teachers in English.

Language as a vehicle of thought enables learners to progress through Van Hiele levels of thinking. If the teacher uses a language at a higher Van Hiele level, learners will not understand him. It is important that the language at appropriate levels should be used.

1.1.6 Lack of reasoning strategies

Geometry problems demand reasoning strategies, due to the fact that there are no specific rules for solving them (Bell 1978:306). Mudaly (1998:40) is of the opinion that poor performance in Euclidean geometry riders and theorems might be attributed to inadequate traditional teaching strategies that are employed when the proof is taught and the fact that there had been insufficient emphasis on proof heuristics. In Euclidean geometry they should be able to think both inductively and deductively, and be able to reflect upon their solutions.

The pre-requisite for learners to use reasoning strategies in solving problems is that they should, among other things, have a sound knowledge of theorems and concepts. It is not surprising that majority of learners do not possess them because they do not have knowledge of theorems and concepts. Rote learning also contributes to poor reasoning strategies (Bell 1978:305). Secondly, their level of reasoning should be at Van Hiele level of deduction (theoretical level). Very few learners' level of reasoning reaches this level (Senk 1985:454). Learners who are at Van Hiele level 3 are able to apply reasoning strategies and they are able to organise arguments logically.

According to Van Hiele (1984:251) teaching deductive system requires patience.
It is worth noting that reasoning strategies cannot be developed and acquired overnight. One acquires them after having solved a lot of problems. Only learners who can reflect on their work are able to develop reasoning strategies.

1.2 Formal statement of the problem

According to Williams (1992:336) Euclidean geometry is not only difficult to learn, but it is also difficult to teach. This is one of the reasons why most South African teachers prefer to teach other sections of mathematics rather than Euclidean geometry. This may be attributed, amongst other reasons, to:

- Teachers’ poor knowledge of Euclidean geometry.
- Learners’ inability to cope with deductive proof.
- Use of teaching methods that encourage rote learning.

It has already been stated that Euclidean geometry has been found to be inaccessible to most learners due to the fact it places more emphasis on proof by deduction (see section 1.1.2). The study does not minimise the role of the deductive approach in Euclidean geometry learning. However, it has been found to be unsuccessful, if it is used as the only method of teaching Euclidean geometry. Freudenthal (1973:406) supports the use of the innovative approaches in teaching and learning Euclidean geometry by saying:

...discoveries made by one's own eyes and hands are more convincing and surprising

The research problem concerns the following issues:

- Learners’ lack of spatial knowledge.
- Euclidean geometry is being taught at higher Van Hiele levels of thinking than that of the learners.
- Domination of traditional approach in the teaching and learning of Euclidean geometry.
- Poor command of the language of instruction by learners.
- The learners' inability to write proofs.

The study will answer the following research questions:

To what extent will the process-based instruction enable learners to:
- communicate thought,
- form concepts,
- and master theorems by execution media (talking, writing, 'showed with and used hands', drawing)?

The purpose of the study will be to investigate the effects of process-based instruction on the learners' performance in Euclidean geometry.

1.3 Analysis of the problem

1.3.1 Memorisation of theorems

Learners tend to memorise theorems without understanding. As a result they cannot use them to prove other theorems. This further leads to the fact that they are unable to realise that theorems they have learnt are reasons for certain statements of the problem that they ought to solve. According to Hoffer (1981:13) some learners 'get by' with Euclidean geometry by memorising proofs. Breen (1992:314) further argues that memorisation of theorems is partly attributed to the dominant use of the deductive approach. This is a clear indication that learners are unable to learn the deductive proof as it is presently taught. Memorisation of theorems has become an escape route out of frustration by learners.

1.3.2 Inability to establish relationships and logical arguments

Lack of understanding of theorems may lead to learners' failure to recognise
relationships between shapes. This is an area where learners sometimes write incorrect reasons for a correct statement. They seem to know that the relationship exists, but they do not know what it is. They seem to be full of theory that they cannot apply. Morris (1985:93) argues that attempts must be made to avoid making Euclidean geometry a random walk. Van Hiele (1984:243) has this to say about theorems:

Understanding mathematics comes down to this: knowing the relationships between the theorems that one studies. As soon as one understands the meaning of these theorems, one knows their relationships at the same time.

This suggests that emphasis should be placed on assisting learners to be able to write logical arguments. The statement seems to acknowledge that learners write proofs with little effort at being logical and discriminatory. This is an area where most learners consider Euclidean geometry to be difficult. Even the so-called 'able learners' share the same experience.

1.3.3 Methods of teaching Euclidean geometry

The approach that emphasis 

emphasis on the mathematics syllabus and textbooks in South Africa is content and teacher-centred (Nieuwoudt 1996:180). It does not give room for teachers to be innovative and creative. The methods of teaching are examination-driven since emphasis is on passing the examination. 'Chalk and talk' approach is still dominating South African classrooms. Learning Euclidean geometry by facilitation is missing (Brodie 1991:18).

However, one must acknowledge changes in the curriculum with the introduction of the Outcomes-Based Education (OBE) in grade one as from 1998. According to DNE (1997a:9) the new education system is learner-centred. It is based on the fact that the learner must be able to demonstrate what he/she knows or what he/she is able to do.
Dreyfus & Hadas (1987:48) argue that there is a need in Euclidean geometry to move away from the deductive approach to a more acceptable method in Euclidean geometry. They further argue that the problem is with the organisation of thoughts and construction of logical arguments rather than content. One tends to ask oneself whether the problem in Euclidean geometry lies with the overemphasis of the deductive approach or content. Freudenthal (1971:418) argues that Euclidean geometry failed because its deductivity could not be re-invented by learners, but it was imposed on them. This suggests that the deductive approach is not being presented to learners in a meaningful way.

On the other hand, the problem may not be lying with the deductive proof as such but with the fact that learners had not mastered subject content at lower Van Hiele levels than deduction. Therefore, mathematics teachers are presenting the subject content at a higher Van Hiele level than of the learners. Therefore, teaching methods that are used in our classrooms are not appropriate to learners' level of understanding. Teaching methods that require learners to thinking creatively should be used to teach Euclidean geometry.

### 1.3.4 Lack of drawing skills

One of the skills, associated with language (for example, communication), is drawing. Learners' understanding of Euclidean geometry is increased when they are given the opportunity to draw diagrams by themselves. They are able to notice some relationships on the diagram that they would not have noticed if they had not redrawn it by themselves.

In other cases learners fail to interpret a Euclidean geometry problem consisting of verbal description accompanied by a diagram. This can be noticed when learners fail to solve a Euclidean geometry rider because information that has been given in words or in terms of symbols (for example $AB = BC$) has not been indicated on the diagram. Drawing of inaccurate diagrams also impedes the ability of learners to
the purpose of explaining and writing proofs.

1.4 Aim and research objectives of the study

The main aim of the study is to investigate the effects of the process-based instruction on the performance of learners in the teaching of Euclidean geometry. The research objectives investigate:

- The geometric learning process-based models and other factors that influence teaching and learning of Euclidean geometry.
- Learners' poor command of English.
- The role of cognitive and meta-cognitive processes in communicating thought, formation of concepts and mastering theorems.
- The meaning of process-based instruction.
- Analysing individual learners' performance in communicating thought, forming concepts and mastering theorems by execution media for all activities.
- Analysing each group's (grade's) performance in communicating thought, forming concepts and mastering theorems by execution media for all activities.
- Comparison of competences (communicating thought, concept formation and mastering theorems) among individual learners for all activities.
- Comparison of competences among grade 8 and 9 learners for all activities.
- Comparison of competences between grades 8 and 9 for all activities.
- Comparison of competences between grades 8 and 9.
- Comparison of execution media between grades 8 and 9.
- The evaluation of the progress of learners in qualitative and quantitative ways.

The above-mentioned objectives have not been arranged in a particular order.

1.5 Demarcation of study

The study was conducted at Ngweni Secondary School as from the 2 August 1999
(see Appendix C). The school is situated at Dzanani Township (Makhado Town) in the Far North region of the Limpopo Province. The school population is mainly black. There are about 250 and 300 learners in grade 8 and 9 respectively. The study involved ten grade 8 and 9 learners who were selected randomly. Five learners were selected from each grade.

1.6 Research methodology

The researcher used both qualitative and quantitative methods in the study. The qualitative method involved the verbal description of data (Charles 1998:30). The quantitative method involved measuring learners’ performance. Average scores of an individual learner or a group of learners for all activities were calculated in respect of each execution medium for every three competences. In this way, it was possible to determine whether learners can communicate thought, form concepts and master theorems by four execution media. Since two grades are involved in the study, the stratified random sampling method is used in sampling. The researcher randomly selects a particular class from each grade. Five learners were selected at random from each grade.

Grade 8 learners are dropped as soon as their Euclidean geometry content is completed. Reasons for the researcher to proceed with grade 9 learners were twofold: To enable them to

- gain sufficient background knowledge of Euclidean geometry from the previous grade.
- establish a sound basis for learners to proceed with Euclidean geometry to grade 10.

Learners’ interviews were recorded on the audiocassettes. Every activity is prepared on the basis of how learners have performed in the previous one. Thereafter, lessons are transcribed and analysed together with learners’
worksheets. They receive process-based instruction in Euclidean geometry concepts and theorems. Initially, both groups are taught grade 8-Euclidean geometry content. The content involves geometric concepts and theorems. Activities are subject content based. This is done in order to establish whether both groups are able to improve their performance. Their performance is based on their ability to use execution media (talking, writing, 'showed with and used hands' and drawing) to communicate thought, to learn concepts and to master theorems. Learners' performance is assessed using a four-point scale assessment grid or rubrics. They are assessed according to the description of the level of performance in the rubrics. Their performance is presented in graphic form. Findings of the study are based on the information from the learners' worksheets, researcher’s observation notebook, audiocassettes and analysis of the results of the study.

1.7 Definition of concepts and terms

1.7.1 Theorem

A theorem is a conclusion that has been proved in the course of an argument upon the basis of certain given assumptions (Gibson 1981:184).

1.7.2 Proof

A proof is a sequence of related statements that form a logical argument or a convincing explanation.

1.7.3 Grade 8, 9 and 10

Grade 8, 9 and 10 refers to the eighth, ninth and tenth-year of study at a school respectively.
1.7.4 Euclidean geometry

A system of geometry described by the Greek mathematician Euclid in his book Elements. It is characterised by the parallel postulate. It states that if a point lies outside a straight line, only one straight line can be drawn through it parallel to the other line (Gibson 1981:71). It deals with the study of plane and solid or space geometry.

1.7.5 Process Based Instruction (PBI)

Process Based Instruction places emphasis on learners, their learning and thinking activities. It characterised by the gradual transfer of instruction from the teacher to the learner.

1.7.6 Spatial knowledge

Spatial knowledge will refer to all knowledge acquired as a result of visualisation, orientation in space and mental ability to manipulate images of objects on the learner.

1.8 Programme of study

Chapter two: Geometry learning process-based models and other factors that influence teaching and learning of Euclidean geometry.

This chapter will deal with the meaning of geometry, nature of Euclidean geometry and non-Euclidean geometry. Since process-based instruction has elements of constructivism, it is necessary to discuss it. Constructivism is characterised by the following factors: It is learner-centred, learning takes place through activities wherein problems are solved and the teacher plays the role of a facilitator. This
Chapter will also discuss two learning process-based models, namely Van Niekerk and Van Hiele models. The discussion on proof is necessary since the study is dealing with theorems. The chapter will also discuss problems that learners encounter when they learn geometry in English. A synopsis of the South African geometry curriculum and curricula for other countries will be discussed.

Chapter three: The role of process-based instruction in the teaching and learning of Euclidean geometry.

This chapter will look at current dominant teaching practices in the classroom. In other words, it will discuss the traditional teaching approach. A discussion on the meaning and the role of process-based approach will be made. This will be followed by the discussion on geometry problem-solving strategies. Process-based instruction is characterised by cognitive and meta-cognitive processes. They will be discussed bearing in mind their applicability in Euclidean geometry.

Chapter four: Research methodology

The study will use both the qualitative and quantitative methods. The stratified random sampling technique will be used for sampling subjects.

Chapter five: The influence of the execution media on the teaching of Euclidean geometry in a process-based instruction

The researcher will analyse data collected in chapter four. This discussion will be based on the analysis of the extent to which learners used the execution media ('showed with and used hands', writing, verbal and drawing) to communicate thought, to form concepts and discover theorems.

Chapter six: Summary, findings and recommendations
analysis of data. Recommendations will be made on the basis of findings.

1.9 Conclusion

This chapter outlined factors that led to the study. It also outlined the research problem. The main aim and the research objectives were also stated. The research methodology was described briefly. The programme of study was outlined.

The second and third subsequent chapters address the first four research objectives, whereas chapter four and five will address the remaining research objectives (see section 1.4).
CHAPTER TWO

GEOMETRY LEARNING PROCESS-BASED MODELS AND OTHER FACTORS THAT INFLUENCE TEACHING AND LEARNING OF EUCLIDEAN GEOMETRY

2.1 Introduction.

There is no fixed meaning of geometry. There are different views on what constitute geometry, details of which will be discussed in section 2.2. Geometry is comprised of Euclidean and non-Euclidean geometries. Euclidean geometry is characterised by axioms such as:

- The sum of interior angles of a triangle is equal to 180°.
- The sum of interior angles of a triangle is the same for all triangles.

Examples of Euclidean geometry are: Vector, analytical and solid geometry. Problems experienced by learners when they learn Euclidean geometry through a second language will be discussed. The retention of Euclidean geometry in the mathematics will also be discussed. Possible solutions to overcome obstacles in learning Euclidean geometry are also suggested.

On the other hand, non-Euclidean geometry is defined as a geometry that negates any statement of Euclidean geometry. Examples of non-Euclidean geometry are: transformation, hyperbolic, affine and projective geometry. Only transformation geometry will be discussed. Emphasis will be placed on what transformation geometry can offer to minimise problems regarding teaching and learning Euclidean geometry.

Using a combination or a variety of teaching and learning approaches is essential in order to attain success in teaching Euclidean geometry. In other words, no single teaching approach can guarantee success with all problems in geometry. Since process based approach has elements of constructivism, a discussion on constructivism becomes necessary.
This chapter also deals with process-based models involved in teaching and learning geometry. The process-based models are: Van Hiele levels of mental development and Van Niekerk's model on development of spatial knowledge of young children. Since the study concerns process-based approach, it becomes necessary to discuss process-based models (Van Niekerk's and Van Hiele's).

The following topics will also be discussed under proof: nature of proof, proof processes, research on proof writing, the role and function of proof in geometry. Since learners learn Euclidean geometry through a second language, it becomes necessary to discuss problems involved here.

The inclusion of geometry topics in a mathematics curriculum should have a purpose in education. Geometry curricula of different countries will be discussed. The countries are: Netherlands, United States of America, Britain, Singapore, Australia, Thailand and South Africa. The extent to which geometry curricula in these countries use the inductive approach will also be discussed.

2.2 What is geometry?

To most people geometry consists of theorems, proofs and diagrams (Giles 1982:30). The geometry being referred to here is Euclidean. This suggests that Euclidean geometry has been reduced to a geometry that does not address itself to the real physical world. The meaning of geometry is not as straightforward as one may think (Hansen 1993:9). Originally, the Greeks called it 'earth measure', the science of measuring land.

Freudenthal (1973: 403) defines geometry as a science of physical space. In other words, the study of the real physical space around the learner and the concrete objects constitute geometry. Freudenthal (1973:278) further argues that geometry does not start from formulating definitions and theorems, but it starts from organising spatial experiences that lead to the formulation of definitions and theorems. In other
words, geometry is about real-life situations. Theorems and definitions arise from real situations. This suggests that the study of space around the objects, the positions of the objects, movements of objects constitute geometry. Geometry that involves the study of positions of objects, movements of objects and the space around the objects improves learners' spatial imagination. Freudenthal (1973:409) further argues that learners with satisfactory performance in deductive geometry often perform poorly in space. They are said that they lack spatial knowledge.

2.3 The nature of Euclidean geometry

In addition to the axioms mentioned in section 2.1 (Wylie (1964:208) defines Euclidean geometry as follows:

- There exists a triangle similar, but not congruent, to any given triangle.
- There exists a rectangle.
- There exists a circle passing through any three non-collinear points.
- Through any point in the interior of an angle there is at least one line that intersects both sides of the angle.
- If a line lying in the plane of two parallel lines intersects one of the lines, it intersects the other.
- There is a unique line that passes through a given point not on a given line and is parallel to the given line.

Examples for Euclidean geometry are: Vector geometry and analytical geometry. In the subsequent section we will discuss Euclidean geometry.

2.3.1 Euclidean geometry

Euclidean geometry is sometimes referred to as deductive geometry, because of its emphasis on proof by deduction. Emphasis on the teaching of Euclidean geometry by means of deduction had been unsuccessful with learners. Learners are unable to write
proofs in terms of deduction. It should be noted that there are a variety of opinions concerning its methods of teaching and subject content. One opinion is that it should be removed from the mathematics curriculum and be replaced with a combination of Euclidean and transformation geometry (Scott 1983:26). The other opinion is that it should be retained and be taught through both the inductive and deductive approaches. The inductive approach must be used to learn concepts and discover theorems (Freudenthal 1971:416).

Learners have difficulties with the way Euclidean geometry is presently taught. Their problem does not lie with the subject content rather with the method of teaching it. It also lies with the learners’ inability to organise and construct logical arguments (Dreyfus & Hadas 1987:48).

Toumasis (1991:229-230) has identified the following reasons for learners’ difficulty with Euclidean geometry:

- The content requires a rigorous logical hierarchy that is not easily perceived by the learners.
- The concept of proof is the source of most difficulties for understanding the content.
- Application of the previously proved theorems, axioms and definitions by the learners to solve geometry problems.
- The isolation of the subject from the rest of mathematics and the removal of content from the learners’ activities.
- Most learners’ are not ready to cope with deductive reasoning.
- Absence of some rules or algorithms which learners can follow to solve problems.

The six points mentioned above imply that:

- The nature of Euclidean geometry is hierarchical since it requires logic.
• There is need for Euclidean geometry to be integrated with other mathematical disciplines.

• Emphasis should be on teaching Euclidean geometry through the inductive approach.

• Learners should be assisted through proper instruction to attain the Van Hiele level of deduction.

This suggests that an alternative method of teaching Euclidean geometry must be found. The teaching of Euclidean geometry must lie between the two extremes 'intuitive approach' and deductive approach (ICMI 1995:97). The alternative method must enable learners to progress through Van Hiele levels up to level 3. This level will enable them to solve problems involving deduction. Mathematical reasoning and proof are considered as the most powerful tool to develop and express insight about a wide range of phenomena (NCTM 2000:55). The ability to reason and to do proofs cannot be achieved by doing them in geometry only. It should be done in all areas of mathematics and at all levels of schooling. Reasoning mathematically must be made a habit (NCTM 2000:56).

Synthesis of researchers' findings involving the Van Hiele model reveals that learners have very little or no experience in inductive geometrical activity at the primary school phase and they are subsequently introduced to deductive geometry at high school, when they do not have background knowledge of properties of shapes (Kynigos 1993:179). This shows that they are not being taught geometry according to Van Hiele levels of thinking. The instruction is offered at a higher Van Hiele level of thinking than a learner is capable of. Van Hiele levels 1 and 2 are characterised by the inductive approach.

Learners are first taught through the inductive approach from primary school phase before they are introduced to the deductive geometry. Research shows that learners are given little opportunity to engage in the inductive geometrical thinking in their schools, even though it appears to be within their grasp (Kynigos 1993:179).
Freudenthal (Kynigos 1993:179) is of the opinion that Euclidean geometry has a high potential as a field for which learners can practice inductive inferences from personal experience, while simultaneously being a field inviting engagement in deductive thinking. This suggests that deductive thinking must be based on the practice of inductive thinking. In this way, learners are able to learn Euclidean geometry. Freudenthal (1973:417) argues:

Betraying a secret that could be discovered by the child itself is bad pedagogy.

Learners should not be deprived of the privilege to discover things by themselves.

Peard (1976:22) further maintains that Euclidean geometry should be in the school mathematics curriculum. In support of the retention of Euclidean geometry Scott (1983:23) claims that Euclidean geometry has an advantage over other geometries because it has a fixed body of material that constitutes it.

On the other hand, Cox (1985:405) proposes that the solution to geometry problems is in giving learners a geometry curriculum with options. Capable learners should have their own geometry curriculum, while less capable ones should do informal geometry.

Usiskin (1987:19-26) proposes the following points, amongst others, in order to improve learners' performance in Euclidean geometry:

- A detailed primary school geometry core-curriculum should be developed.
- Learners should not be discouraged from doing geometry because of poor performance in other branches of mathematics.
- Teachers should demand a greater degree of competence in geometry from all learners.
- Prospective mathematics teachers should study geometry at tertiary level.
Finally, there is a widespread agreement among researchers that Euclidean geometry should be retained in the school curriculum (Scott 1983:38). The following has been suggested to improve the performance of learners’ in geometry:

- Emphasis should be on the use of the inductive approach in teaching of Euclidean geometry. The inductive approach should be used to discover theorems.

Learners should be taught geometry in terms of the Van Hiele theory (De Villiers 1997:43).

2.4 The nature of non-Euclidean geometry

According to Golos (1968:170) a geometry that negates any statement of Euclidean geometry may be called non-Euclidean. The challenge made by the emergence of non-Euclidean geometry came about as a result of inadequate capacity of the Euclidean geometry to describe the complexity of the nature of shape. Mandelbrot (Hansen 1993:11) argues against the role that non-Euclidean geometry plays in describing the physical world:

Clouds are not spheres, mountains are not cones and lightning does not travel in a straight line.

This is an indication that there are geometries other than Euclidean geometry that describe the physical world. So, learners should also be exposed to non-Euclidean geometry in order to understand the physical world. The aim of teaching non-Euclidean geometry at school is not to make learners experts in the subject, but to provide them with another ‘way of seeing’ (Fish 1996:7). Different examples of non-Euclidean geometries have already been mentioned in section 2.1. Only transformation geometry will be discussed in this chapter. The knowledge of non-Euclidean geometry may provide them with alternative problem-solving strategies. A broader study of geometries will further enable learners to have deeper insight into spatial relations and geometric concepts of mathematics (McGreevy 1974:258).
2.4.1 Transformation geometry

In an attempt to find the meaning of transformation geometry we shall first consider its broad picture. Usiskin (1978:40) defines transformation geometry as one-to-one mapping of points or objects from one set to the other. There are different kinds of geometric transformations. According to Mortenson (1996) they all preserve certain properties of the objects.

According to Shilgalis (1978:9) similarity transformation maps of the plane are multiplied by a constant. Examples of similarity transformations are photographs and magnified figures. Affine transformations are those transformations that preserve distance between points and on the same line and parallel lines. Projective transformations do not preserve parallel lines, but they remain straight. On the other hand topology allows transformations that bend, stretch and form distortion of figure as long as the quantity that constitutes the object remain the same (Mortenson 1996).

According to Shilgalis (1978:9) transformation (isometry) geometry deals with mappings or transformations of the Euclidean plane and properties of figures that remain unchanged after being transformed. In the light of this definition, transformations that will be discussed hereunder are Euclidean. Examples of isometries are translation (sliding), reflection (flip) and rotation (turn). This section will only deal with the transformations concerning isometries and dilation.

Boyle and Griffiths (1979:22) argue that transformation geometry helps in investigating concepts and properties of geometric shapes. This shows that concepts in transformation geometry can be learnt inductively. Inductive teaching seems to be the predominant method of learning transformation geometry. According to Shilgalis (1978:9) the reason for studying geometric transformations is that they provide alternative forms of proofs of theorems.
Usiskin (1978:41-44) gives the following reasons for the inclusion of transformation geometry in the curriculum: Transformations

- Enable one to deal with a greater variety of figures than in Euclidean geometry.
- Bring Euclidean geometry much closer to the intuition of the learner.
- Make Euclidean geometry more accessible to slower learners.
- Enable Euclidean to be more easily applied to the real physical world and vastly increase the number of its applications.
- Provide assistance for future work in mathematics.
- Give a unifying concept to the geometry course that is geometric in nature.

It is a known fact that in Euclidean geometry concepts such as similarity and congruency are limited to triangles. In transformation geometry the same concepts can be dealt with in all types of polygons. This suggests that transformation geometry offers learners a broader understanding of concepts than Euclidean geometry.

Moalem (1981:38) cautions that the introduction of transformation geometry in the curriculum should not be done at the expense of Euclidean geometry. He proposes that Euclidean geometry be supplemented with analytic geometry, vector geometry and matrix geometry. Scott (1983:28) gives the following reasons for the retention of Euclidean geometry in the school curriculum:

- Transformation geometry lacks mathematical virtues.
- It is not clear as to what constitutes its body of materials.
- Transformation geometry has little deduction.

Transformation geometry can be used only for learning concepts because of its lack of mathematical rigour. According to Okolica & Macrina (1992:719) and Reddy (1987:109) transformation geometry should be introduced prior to deductive geometry and gradually integrated into it.
2.5. Constructivism

Nowadays, the emphasis is on teaching and learning mathematics by problem solving and a constructivist approach. These approaches appear to be similar or close in terms of meaning. On the other hand, problem solving involves learners solving both routine and non-routine problems. Some problems might have pre-determined solutions. Meanwhile the constructivist approach involves discovering new knowledge by solving problems.

2.5.1 The role of the learner

It is assumed in constructivism that the learner possesses some knowledge before he enters the classroom. Any new knowledge should be linked to the existing one. In terms of the constructivist approach the learner is responsible for constructing his own knowledge rather than it being transmitted to him. Constructivism is a theory about how the learner constructs knowledge as an active participant rather than a passive recipient (Anthony 1996:349). Knowledge acquired by the learner should be meaningful to him. It follows that meaning cannot be communicated, but it can only be shared within a group. Meaning is considered as a product of exploratory discussions within a group. This cooperation involves much more than arriving at a joint solution. However, it involves developing explanations that are meaningful and making sense to others (Yackel, Cobb, Merkel, Clements & Battista 1990:35).

The constructivist approach encourages the learner to be aware of the fact that he lives in the world constituted by his own experiences. In other words, new knowledge should be built on what the learner already knows (Wheatley 1991:12). The learner is considered as being able to use his/her own ways of solving problems. The learner's insight is respected.
2.5.2 The role of the teacher

The teacher's role in the constructivist approach is that of a facilitator. He is also responsible for creating an environment conducive to constructivism. The constructivist approach affords the teacher the opportunity to learn a lot from his learners. According to Wheeler (Breen 1992:170) the teacher's role in the constructivist approach is as follows:

- He sets the situation, giving essential information, but beyond that he tells children nothing;
- He obtains as much information from learners as possible, by observing, asking questions and asking for particular actions;
- He works with his feedback immediately;
- Except on rare occasion he does not indicate whether a response is right or not, though he often asks whether it is;
- He accepts errors as important feedback telling him more than correct responses, and by directing learners' attention back to the problem; he urges them to use what they know to correct themselves.
- He helps to plan the situation and allows ideas to emerge.

Although the learner is responsible for his own learning, the teacher intervention is still necessary to guide the discussion. A teacher who wants to create a constructivist environment should take note of the following four tenets of belief as critical (Pirie & Kieren 1992:507-508):

- Although a teacher may have the intention to move learners towards particular mathematics learning goals, he will be aware that such progress may not be achieved by some of the learners and may not be achieved as expected by others.
• In creating an environment or providing opportunities for learners to modify their mathematical understanding, the teacher will act upon the belief that there are different pathways to similar mathematical understanding.

• The teacher will be aware that different people will hold different mathematical understanding.

• The teacher will know that for any topic there are different levels of understanding, but that these are never achieved 'once and for all'.

2.5.3 Classroom organization

Learners are given a task to work on in small groups. These groups will later convene to share their work. Each group presents the solution to the class, not to the teacher. The teacher gives learners the opportunity to negotiate and reach consensus. In this way, learners are able to refine their thinking and deepen their understanding of the problem.

There is a belief among teachers that in order for an instructional environment to be constructivist per se, there should be group discussion. According to Pirie & Kieren (1992:505-506) although verbal interaction is necessary for learners to learn mathematics, teachers can still create an environment that will help the individual learner to construct knowledge without verbal interaction.

Pirie & Kieren (1992:506) are of the opinion that teachers based on their beliefs create these environments and knowledge put into action. This shows that the creation of an instructional environment depends on the belief and knowledge of teachers.

2.5.4 Learning activities

The constructivist approach emphasises that learning should take place through activities, which the learner will be part of. Learning activities should take into account that learners have different levels of understanding.
Thinking in terms of activities rather than content may be more useful in establishing effective learning environment (Wheatley 1991:13).

The constructivist approach activities are problem-centred. In this case, learning takes place by investigating, discussing, discovering and reflecting on mathematics problems. Krulik & Rudnick (1989:3) define a problem as a situation that confronts an individual or a group of individuals that requires a resolution and doesn’t have an obvious path to the solution. Learners are only exposed to mathematics problems within their scope. According to Tricket & Sulke (Daniels & Anghileri 1995:67) the following criteria should be used to select a problem for an activity:

- It should be accessible to everyone at start;
- It needs to allow further challenges and be extendable;
- It should involve learners in speculating, hypothesis making and testing, proving or explaining, reflecting and interpreting;
- It should not restrict learners from searching in other directions;
- It should promote discussion and communication;
- It should encourage originality/invention;
- It should bring an element of surprise;
- It should be enjoyable;
- It should encourage ‘what if’ and ‘what if not’ questions.

In the light of the above discussion the constructivist approach is a philosophy that encourages learners to solve problems through discovery. It follows that it has elements of inductive approach. Unlike in discovery learning, in constructivism learners may come up with different correct solutions.
2.6 The development of spatial knowledge of young children

Van Nierkerk is one of the experts on the development of spatial knowledge of young children. This section is based on her work. Her work has a bearing on this study because it deals with the following issues: How learners

- Learn geometry in a multicultural classroom.
- Develop spatial skills.
- Learn geometry through the first and second language.
- Undergo various cognitive processes.
- Learn geometry in terms of Van Hiele theory.

Her model is aimed at learners developing spatial competence through the following skills: verbal skills, mental skills, visual skills and tactile skills. These will happen when the learner works from three dimensions to two dimensions and vice versa (Van Nierkerk 1996:29-30).

2.6.1 Theoretical description of the model

Van Nierkerk's (1997:7-9) model is aimed at, among other things, describing conditions under which teaching and learning of spatial knowledge should take place and to investigate the following issues:

(a) The teaching philosophy employed by the researcher as well as the class teacher. In this model learners should be able to construct knowledge by themselves. Emphasis is on a problem centred approach. In other words, learners have the opportunity to find multiple solutions to the problem. The teacher acts as a facilitator. It has been acknowledged in this model that few black teachers are exposed to problem centred approach (Van Nierkerk 1996:30).
(b) The influence of the worldview and culture, demography and socio-economic background of learners on the learning of spatial knowledge.

Language and culture are vital in the development of spatial knowledge. As a result, language is an expression of culture. Therefore, culture is embedded in the language.

People who subscribe to this view also believe that mathematics is a particular formal expression and extension of different relationships, transformations and interconnections that exist within language (Van Niekerk 1999:4).

Any mathematics teacher should take into account the cultural demography of his learners. The fact that learners come from different cultural groups, is an indication that they have different worldviews. In other words, geometric activities and teaching materials selected should take into account these different worldviews.

Learners with a western worldview may gain spatial knowledge faster than learners from the black disadvantaged communities due to the fact that they are exposed to spatial concepts at an early age. For example, they are surrounded by buildings and streets of different shapes and dimensions. They also possess items, such as set of mathematical instruments to learn spatial concepts. The average black learner has little access to them due to his poor financial situation (Van Niekerk 1996:31).

(c) The nature of the classroom culture in a multicultural society.

In this model learners are given tasks and negotiate meaning within their groups. Learners from black communities do not discuss with their elders because of their tradition. They have to accept any word from them (Van Niekerk 1996:31). However, a multicultural group does not have the same problem since it comes from a tradition that encourages communication. In addition, they use English as first language.

To ensure that black learners grasp spatial concepts with insight, they are taught geometry both in their mother tongue and English. However, it is difficult for them to be able to
understand certain concepts since some spatial terms in English do not have equivalents in black languages.

In some of the black languages the same spatial term is used to indicate different actions which can only be distinguished if it is put into context (Van Niekerk 1996:31)

Black teachers should be competent in both English and black languages to enable learners to gain spatial knowledge with insight. Most black teachers are proficient in English (Van Niekerk 1996:31).

(d) The design and development of the appropriate materials and activities for the learner to be able to orientate himself and develop insight in space.

In this model learners not only familiarise themselves with space, but also gain insight into it. The teacher should able to design appropriate follow-up activities to help the learner to develop spatial concepts by identifying learning difficulties that they experience (Van Niekerk 1996:32). This model enables the teacher to design geometric activities that enable the learner to think inductively and deductively (Van Niekerk 1996:31). It enables the teacher to take into account the specific world that the learner grew up in when designing his activities. Thus, selection of concrete materials should take into account the world that learner grew up in (Van Niekerk 1999:33). In other words, activities should be designed and developed in a particular context.

(e) The utilisation of different execution media (writing, talking and drawing) in the teaching and learning of spatial knowledge.

The type of the task determines how the execution media will be utilised. This point will be discussed at a later stage. The diagram below shows the interrelationship between different variables in Van Niekerk’s model (figure 3.2).
FIGURE 3.2. THE INTERRELATIONSHIP BETWEEN THE DIFFERENT VARIABLES IN THE TEACHING AND LEARNING SITUATION (Van Niekerk 1997:94)
2.6.2 Factors that have an influence on the competence of a task

Van Niekerk (1997:280-281) has found that the following factors have an influence on the competence of a task or skill:

(a) The execution media (language, drawing, writing and construction)

The medium through which the task is done plays an important role in the development of skills, as well as to indicate some form of competence with a skill. Learners are found to be more competent with a skill when they do constructions on their own.

(b) The context of the task

The cultural background of the children influences the context within which the task is presented. In other words, the task that is to be presented should take into account whether learners have a western or non-western worldview. Van Niekerk (1997:94) argues that the specific way in which mathematics is viewed in a particular community determines the way it should be taught and understood. This is due to the fact learners have different cultural and socio-economic background.

(c) The order of activities

The order in which activities are presented is influenced by the way in which learners will execute the task. This model suggests that learning of geometry should be hierarchical. This further suggests that learners learn mathematical knowledge better when activities are arranged hierarchically in terms of the Van Hiele theory.

(d) The type of task

The movement of the object, as well as the movement of the viewer that is implied through the setting of the question can influence the type of task that is given to learners.
Movement of the viewer is either rotational or perspective, while that of the object results in different transformations (Euclidean, projective and topological). Figure 3.3 shows the rotational and the representational movements of blocks.

**Figure 3.3.** Two types of movements that the object can undergo (Van Niekerk 1997:100)
(e) The type of object

The type of object used in the activity can influence the difficulty of the task. To most learners, certain solids are more difficult to construct than others.

(f) The dimension of the stimulus materials

The dimension of the stimulus materials can be varied between two-dimensional (picture/written) and a three-dimensional representation (physical structure or undefined representation) to ensure that a range of skills is developed. The model suggests that learners learn shapes better when they are first exposed to three-dimensional ones and later to two-dimensional objects.

(g) The type of mathematical knowledge

The type of mathematical knowledge involved in the development of spatial knowledge influences the way learners learn. That is, they respond differently to different mathematical content that require the development of spatial knowledge. In turn, the teacher as a facilitator will have to try alternative strategies.

Finally, the model enables learners to orientate in space and develop insight into spatial knowledge. It is not only visual skills that learners acquire, drawing and constructions of shapes enable them to acquire the following skills: mental, verbal and tactile. Construction of solids from two-dimensional shapes and drawing of nets of solids improves their analytical and synthetic skills.

2.7 Van Hiele model of mental development

Van Hiele (1986:39-40) theory came about as a result of the frustration he experienced while teaching geometry to secondary school learners. He couldn't understand why learners still could not understand geometry even though it has been explained to them
several times. It dawned on him that geometry learning takes place according to levels of thinking. Van Hiele (1984:245) identified five different levels of thinking:

Level 0: Learners recognise shapes by their appearance. For example, a learner will draw a geometric shape if he/she is requested to describe it. He/she may point to a door to show rectangle.

Level 1: Learners recognise geometric shapes by their properties. They are able to recognise that a rectangle has four right angles and that its opposite sides/angles are equal to each other. However, they cannot establish relationship between shapes.

Level 2: Learners can now order shapes according to their properties. They can order shapes. For example, they can relate a rectangle to a square.

Level 3: Learners can think deductively. They understand the meaning of theorems. According to Van Hiele-Geldof (1984:190) as soon as learners are able to draw correct conclusions of others, they have the third level.

Although, Van Hiele (1996) identified five different levels of thinking, he confined himself to the first four. This was due to the fact that they were within grasp of secondary learners. The fourth level is concerned with learners who can deal with the axiomatic geometry other than Euclidean geometry. Learners are able to solve problems in non-Euclidean geometry (Van Hiele-Geldof 1984:192). Secondary school learners rarely reach this level (Hoffer 1981:14). Van Hiele (1996) later reduced levels of thinking to three, namely:

Level 1: Visual

Learners recognise shapes as a whole. However, they cannot identify them in terms of their properties.
Level 2: Descriptive

Learners recognise shapes by their properties. At this level geometry activities should be developed to enable learners to discover properties of shapes. They are also able to discover relationships between shapes using their properties. Learners should use language properly to describe properties of shapes. However, at this level they cannot think deductively.

Level 3: Theoretical

Learners are able to do formal proof and understand the processes involved. They can apply theorems to solve problems.

A learner can only progress to a higher level of thinking through proper instruction. In order to progress to a higher Van Hiele level of thinking a learner goes through the following instructional phases (Van Hiele 1986:53-54):

1. Information
Learners are acquainted with the content at their appropriate level. Their prior experiences are taken into account when the content is selected.

2. Guided orientation
Learners explore the topic of study through carefully guided, structured activities

3. Explication
Learners become conscious of relations of network that has to be formed. They also learn technical language that is appropriate at that level.

4. Free orientation
Learners learn by doing more complex tasks. They do more open-ended activities with different types of solutions (Teppo 1991:212).
5. **Integration**

The teacher helps learners to gain the overview of the activities that are being investigated and summarise the content.

These instructional phases help the teacher to sequence the activities. The sketch below shows how phases of learning help the learner to progress to a higher level:

**Theoretical (level 3): Use of deductive reasoning to prove geometric relationships.**

```
Learning  Instructional phases
Period 2   Integration
          Free orientation
          Explication
          Guided orientation
          Information
```

**Descriptive (level 2): Describe objects by their geometric properties and compare them.**

```
Learning  Instructional phases
Period 1   Integration
          Free orientation
          Explication
          Guided orientation
          Information
```

**Visual (level 1): Recognise geometric objects globally.**

(Teppo 1991:210)

Figure 3.1.

The above-mentioned levels of thinking are not situated in the content, but in the thinking of man (Van Hiele 1986:41). It is the thinking of man that places him at different Van Hiele levels of thinking. Appropriate geometry activities are developed according to instructional phases during a specific period to help learners progress to a higher level.

The transition from one level to the following is not a natural process; it takes place under influence of a teaching-learning program (Van Hiele 1986:50).
The activities should take into account the technical language at the learner's existing level. The learner will not progress to the next level if the language of a higher level is used.

Any geometry instruction should take into account the following properties of the Van Hiele levels of thinking (Crowley 1987:4):

- A learner cannot be at Van Hiele level \( n \) without having gone through level \( n-1 \). For the learner to understand knowledge at the existing level, the learner must have acquired knowledge of the preceding levels. No method of instruction allows a learner to skip a level.
- Progress from one level to the next depends on the content and method of instruction rather than age. The method of instruction must be adapted to the degree of difficulty of the content.
- Inherent objects of study at one level become an object of study in the next level. For example, at level 1 learners only recognise figures by their shapes, but at level 2, they recognise them in terms of their properties.
- Each level has its own linguistic symbols and its own network of connecting them. At one level a figure can have more than one name (for example a square is a rectangle). Instruction must take into account the language used at a specific level. A learner who is at level 1 will not progress to level two if the language used is at level 2. He will not understand the language because he has not yet reached that level.

The discussion on the Van Hiele theory implies:

- Geometry activities should be developed according to instructional phases.
- The method of instruction should be appropriate to the Van Hiele level of thinking.
- The language used at a Van Hiele level of thinking should be appropriate to that level.
- It should be noted that learners do not attain levels of thinking at the same time.
- Geometry learning is hierarchical. In other words, knowledge gained in the previous level is used to gain new knowledge at a higher level.
- The descriptive level emphasises learning geometry by discovery.
Van Hiele (1996) is of the opinion that primary school geometry should be at a visual level. It should help to facilitate the introduction of the secondary school learners to the descriptive level. In other words, geometry should be taught in such a way that learners are able to construct knowledge by themselves. Finally, the Van Hiele model places emphasis on the fact that learners should be assisted to learn geometry through the attainment of the levels of thinking. Whether a learner attains a particular level or not depends on the content and method of instruction. To ensure that a learner progresses to a higher level, the degree of difficulty of the content must be linked to the learner’s appropriate level. Furthermore, geometry activities should be developed and presented according to instructional phases.

2.8 Proof

2.8.1 The nature of proof

Bell (1978:291) defines a proof as an argument or a statement that convinces someone. He identified examples of proof as, personal experience, acceptance of authority, observation of instances, lack of counter-examples, usefulness of results and deductive arguments. Hanna (1983:29) argues that there is no consensus among mathematician as to what constitutes an acceptable proof. Hanna (1996:48) further argues that only learners who have learnt to follow arguments or modes of logical thinking have the ability and confidence to evaluate and construct proof. In agreement Clements and Battista (1992:442) have this to say:

Studies that have attempted to involve pupils in the crucial elements of mathematical discovery and discourse-conjecturing, careful reasoning and the building of valid arguments that can be scrutinised by others have shown more positive effects.

Mudaly (1998:112) defines proof as a logical explanation of why the results are true. That is, proof should serve as a means of explanation. It should satisfy a learner curiosity. In
other words, proof should not only help learners to reach conviction but it should help them to gain insight. Proof is about building up a logical argument or statements.

Volmink (1988:86) argues that proof operates at all Van Hiele levels. He further argues that a geometry proof does not only lead to verification of results but it also leads to organising and explaining one’s thought. It is also bound up with recognition, analysis and ordering of shapes. This suggests that learners at different Van Hiele levels of thinking are capable of doing proof appropriate to their respective levels. This further suggests that proof should not only be associated with deduction. However, geometry proof is often associated with Van Hiele level 3. According to Senk (1983:226) learners are unable to write a proof because they are at lower Van Hiele levels of thinking than that of deduction. In other words, in order to be able to write proofs one should be able to think deductively. This statement disputes an earlier assertion that proof occurs at all Van Hiele levels of thinking.

2.8.2 Research on proof writing

A study conducted in the United States by Senk (1983:225) has shown that 30% of the learners taking geometry at high school were found to have 75% mastery of proof writing. About 30% had virtually no competence in proof writing, about 20% could do trivial proofs and another 20% could do some proofs of greater complexity. In the same study Senk (1983:217) found that poor performance in geometry could be attributed, amongst other things, to the fact that most learners were at lower Van Hiele levels of thinking than that which they could be able to write proofs, namely deduction. She has also pointed out that new teaching methods should be found that would enable learners to write proofs.

Stover (1989:82-84) found there is a significant relationship between proof writing and inductive reasoning, deductive reasoning and Van Hiele levels of thinking. This is an indication that learners are unable to cope with proof because they lack inductive and deductive reasoning skills and the fact that the instruction does not take into consideration their Van Hiele levels of thinking. Both Senk (1983:228) and Stover (1989:89) suggest that
exploratory geometry should be taught from primary school phase to junior secondary phase.

In a recent study Mudaly (1998:106) showed that, with proper guidance, grade 9 learners could construct proof. Learners showed in this study that they always need explanation for the results obtained even though they were satisfied they were true. They still wanted to know why they were true. An explanation has provided them with insight as to why the results were true (Mudaly 1998:106). In this case proof has been described as an explanation of results. Mudaly (1998:113) suggests that learners should learn proof by exploring, conjecturing and explaining.

2.8.3 Proof processes

A study conducted by Chazan (1993:380) showed that learners preferred an argument based on inductive approach to deductive proof. In a situation where both types of argumentation were used, learners were able to determine the merits and value of both types of argumentation. Chazan (1993:385) argues that the extensive use of the inductive approach do not hinder the learner’s appreciation of the mathematical proof. He further argues:

> Though the extensive use of measurement of examples in geometry classes can bring to the fore difficulties students have in appreciating accepted views of the role of mathematical proof in verifying mathematical truth, it raises important and valuable questions that might not be raised otherwise.

In other words, the argumentation based of the inductive approach may be used to diagnose learners’ difficulties in geometry. It may further help learners to gain insight into the deductive proof. An argument based on the inductive approach cannot be considered as a proof by itself, since there may be other cases that were not considered. There may be counter-examples that may dispute the ‘proof’. This shows that both types of proofs have their roles in learning geometry.
Mudaly (1998:110) found in his experiment that grade 9 learners undergo the following processes in solving problems: They

- Observe instances during the activity.
- Formulate conjectures from observations.
- Examine and re-examine a conjecture.
- Test whether a conjecture is true.
- Refine conjectures and construct a proof.
- Seek explanations for their own observations.

Chazan (1993:383) is of the opinion that learners should first be introduced to arguments based on the inductive approach, and later to explanatory aspect of proof before deductive (axiomatic) proof. In situations which learners are given the opportunity to construct knowledge on their own, this type of proof may flourish. According to Chazan (1993:383) learners prefer an explanatory aspect of proof to the deductive proof that emphasises conclusion.

### 2.8.4 The role and function of proof in geometry

De Villiers (1990:18-21) has identified five functions of proof as an expansion of the three functions (verification, illumination and systematisation) identified by Bell (1976:24). They are as follows:

(a) Proof as a means of conviction/verification/justification
This type of proof is used to remove personal doubt. Doubt may be removed by giving several examples as a way to convince oneself. According to Bell (1976:24) there are means of reaching conviction other than logical proof. The inductive approach may also be used as another means of reaching conviction.

(b) Proof as a means of explanation/illumination
Several examples can be given to establish the truth of the theorem. However, they cannot provide the explanation as to why the theorem is true. According to Bell (1976:240) a good proof must convey insight into why a theorem is true. This enables a learner to have a better understanding of a theorem.

(c) Proof as a means of systematisation

In this case, proof is being associated with deductive or logical reasoning. Proof can serve as a means of organising individual statements as logical argument.

(d) Proof as a means of discovery

It is common knowledge that theorems may be arrived at through empirical methods before they are verified by proofs. Mudaly (1998:8) states that empirical results may ensure a high level of conviction but it cannot guarantee the truth of the statement. It provides no grounds to accept evidence. A proof may be used as a means of exploration, discovery and invention of theorems. Proofs may also be used to discover new knowledge by means of discovering new proofs of old theorems (Mudaly 1998:10).

(e) Proof as a means of communication

The validity of a proof depends on its meeting the demands of a body of accepted rules of inferences or established truth. A proof may be accepted as true or valid if axioms, definitions and theorems have been used logically. This body of knowledge or rules enable ones’ proof to be understood or accepted by others. According to Davis (De Villiers 1990:22) a proof is a unique way of communicating mathematical results between professional mathematicians and amongst learners themselves.

Finally, on the basis of information given by researchers regarding proof, one can deduce that proof writing will still remain an integral part of geometry now and in the near future. Its functions outweigh its criticism. According to Coe & Ruthven (1994:43) proof is the heart of
mathematics learning. Ways should be found to make it accessible to most learners. It has already been mentioned proof can be used to explain why the theorem is true. In this way learners gain insight into the theorem. Obviously this explanation will be done through a particular medium of instruction. In situations where the medium of instruction is the second language, learners are likely to encounter difficulty in gaining insight. Problems that learners encounter when they learn mathematics through the second language will be discussed in the subsequent paragraph.

2.9 Problems encountered by learners when they learn Euclidean Geometry using a second language

This section will deal with the second language problems that learners encounter when they learn mathematics, Euclidean geometry in particular. The use of English as a language of instruction in South African classrooms in black schools is found to be one of the reasons for poor performance in mathematics, Euclidean geometry in a particular. There is a school of thought that believes that using both first language and English may help learners to have a better understanding of Euclidean geometry. The issue concerning second language problems has a bearing on learning Euclidean geometry by inductive approach, since it involves reading and understanding instructions. Competence in the language of instruction is a pre-requisite for the learner’s ability to describe properties of shapes. It helps learners to attain level 2, which requires them to describe properties of shapes.

The use of English in black South African schools as a language of instruction from grade 5 was a political decision influenced by the Soweto uprising of 1976 (Rakgokong 1994:14). The decision was also based on the fact that learners will be able to communicate in English if they learn it in the early grades. This approach has not been successful since learners still could not handle the mathematics learning content in English (Rakgokong 1994:14).
The difficulties that learners experience when they learn mathematics through a second language are:

- Lack of fluency in the language of instruction.
- Use of first language (language of the learner) culture and the second language culture with regard to mathematics concepts, procedures and values (Hunter 1990:34).

Hunter (1990:34) is of the opinion that language-related obstacles that hinder maximum performance in mathematics might be divided as follows:

- Communicative difficulties.
- Ordinary English and the language of mathematics.

It follows that learners who learn mathematics through a second language will require more time than first language-speaking learners. This will also depend on whether the mathematics teacher required teaching language would be a suitable candidate. It will further depend on whether he/she will find time to do it. However, it is rare to find such a teacher (Hunter 1990:35). It is also not feasible to use either a mathematics or English second language teacher since each of them is handicapped to either subject. In Canada a team-teaching project is being conducted where a mathematics teacher and an English teacher team-teach mathematics through a second language. It is hoped that they will share their experiences with their colleagues. Hunter (1990:35) argues:

When a student is confronted with an obstacle to performance of a mathematical task, the immediate obstacle to getting help is linguistic: how to say it is one needs. But the underlying this is a meta-cognitive obstacle: knowing precisely what it is one needs.

For any geometry problem to be successfully solved the language used in the problem should first be understood by the learner, otherwise it will even be difficult to understand the problem itself.
Nowadays, mathematics teachers use the constructivist approach to teach mathematics. This approach will have implications for the use of the language of instruction in the learning of Euclidean geometry, since it involves learners communicating with one another. In order for them to be actively involved in the discussion they should be able to communicate with one another.

The relationship between a constructivist approach and bilingualism needs to be explored (Rakgokong 1994:15).

This implies that learners should use the language they feel comfortable to communicate with one another. The use of mother tongue and second language in the teaching of mathematics is becoming prevalent. Using one language in the teaching of mathematics has been found to hamper meaning making and progress in problem solving (Rakgokong 1994:16). That is the reason why some mathematics teachers advocate the use of bilingualism in the teaching and learning of mathematics.

Research has shown that bilingual learners who are competent in both mother tongue and second language perform better in mathematics than monolingual learners (Clarkson 1992:427-428). According to Rakgokong (1994:16) the education policy regarding language should recognise and appreciate language experiences that learners bring with them to school. The use of bilingualism may help learners to understand problems given to them and to acquire skills to develop concepts. It will further enable learners to progress from one level of thinking to the other.

Both parents and teachers in South Africa support the use of English as a language of instruction. They believe that English should receive first preference over other languages since it is an international language and a language of business (Rakgokong 1994:17). However, it is acknowledged that some teachers use both mother tongue and English in teaching mathematics.
2.10 Evaluation of the geometry curricula in South Africa and abroad

The inclusion of certain geometry topics in the school mathematics curriculum depends on their purpose in education and mathematics in particular (Giles 1982:31). The choice of geometry content depends on what we want to do with it in real-life situations. The geometry curriculum should reflect the actual and the potential needs of the society (Van Niekerk 1997:282). This section will deal with the geometry curriculum in countries such as the Netherlands, the United States, the United Kingdom, Thailand, Australia, Singapore and South Africa. Emphasis will be on the extent to which geometry curricula include process-based approaches and the extent to which they accommodate the Van Hiele theory.

2.10.1 The Netherlands

According to Freudenthal (Treffers 1993:94) there will be no mathematics in the 21st century, because mathematics in the Netherlands is so integrated with other subjects to extent that it will cease to exist on its own. This describes what is called Realistic Mathematics Education in the Netherlands. In Realistic Mathematics Education, mathematics taught is derived from the everyday reality around us and this reality serves as a source for learning mathematics (Verhage & De Lange 1997:14).

During the seventies Realistic Mathematics Education was characterised by the horizontal and vertical components of mathematising. The horizontal component places emphasis on application, practice and relating of knowledge and skills in context solutions. The vertical component places emphasis on practising of knowledge, skills and concepts. During the evaluation of the Wiskobas project (primary school mathematics education programme) it was found that it places more emphasis on the reality to be revealed to the learner and less emphasis on the subject content. In the eighties it placed emphasis on subject content rather than on the reality around the learner (Treffers 1993:94). Realistic Mathematics Education encourages the use of pictures, newspaper articles and other concrete objects in learning mathematics. The
process of constructing knowledge and principles by learners themselves receives
more emphasis in Realistic Mathematics Education (Wubbels, Korthagen & Broekman
1997:2). De Lange (1993:145) says:

Learning is no passive process of absorbing information and the compartmentalised storage
in the mind. Learning is something that must be done. For example, students are able to
construct mathematical concepts partly on their own, on the basis of ‘real’ problems. That
gives opportunity for own strategies, own levels, interaction and reflection on the followed
process.

In Realistic Mathematics Education not every contextualised problem is a realistic
problem. For the problem to be realistic, it is likely that learners experience it as real
and personally interesting (Wubbels, Korthagen & Broekman 1997:3). This is often the
case for problems involving hands-on activities. This implies that not all of realistic
problems are derived from learners’ everyday reality. The general objectives of
Realistic Mathematics Education are to:

- Become intelligent citizens and mathematically literate.
- Prepare for the workplace and for future education.
- Understand mathematics as a discipline.

(Verhage & De Lange 1997:14)

The committee for the Development of Mathematics Education (COW) reported on
issues concerning mathematics, geometry in particular (De Lange 1993:146). It found
that geometry is not flourishing. The introduction of transformation geometry did not
help either. Replacements of some sorts were found and they are called the ‘geometry
of looking’. De Lange (1993:147) says:

The ‘what’ and ‘how’ of looking are interesting questions by which to explore and understand
the surrounding world.

In this way the Realistic Mathematics Education encourages learners to reason both
inductively and deductively. According to De Lange (1993:147) the results of the
geometry programme are challenging due to positive reactions from the field. The criticism levelled at Realistic Mathematics Education is that it lacks mathematical structure. It is also said that it is hidden under context (De Lange 1993:149). It is further argued that it does not have clear objectives. Although, Realistic Mathematics Education involves nice activities the following questions need answers:

- Where do activities lead us?
- Why do activities?

The open-endedness of investigations also place teachers in difficulties, when learners come up with different correct answers. This may lead to uncertainties on the part of the teacher. Contexts of problems may also cause uncertainties on the part of teachers, since learners may come up with bright solutions that they would have not thought of.

From the above discussion, it follows that Realistic Mathematics Education is based on constructivism since it emphasises building knowledge on the learner’s own construction of knowledge. Like constructivism, it involves learning by discovery. It is also problem-centred. The realistic problems are not only based on everyday reality of the learners but also on hands-on activities. Thus, the Dutch curriculum has evidence of many elements of the process-based approach.

2.10.2 The United States of America

The United States of America mathematics curriculum places emphasis on the learners' knowing and being able to apply mathematics. The importance of mathematics in the United States of America (USA) is set to achieve the following objectives;

- Learners should be mathematically literate in order to cope with everyday life.
- Mathematics should empower learners to solve problems at workplace.
Mathematics should prepare the most able learners to be mathematicians, scientists, statistician and engineers.

Mathematics should be viewed and taught as part of culture (NCTM 2000:4).

In the USA, geometry is taught from pre-primary school to grade 12 (NCTM 2000:4). The primary school geometry curriculum involves the study of two- and three-dimensional shapes (NCTM 2000:48). Reasoning and proof receive the utmost importance at all levels. However, the rigour increases as learners' progress through grades. NCTM (2000:55) considers mathematical reasoning and proof as powerful tools to develop and express insights about a wide range of phenomena. Reasoning can be developed through exploring phenomena, justifying results and using conjectures. According to NCTM (2000:55) reasoning cannot be developed through geometry alone. However, it should be developed through all disciplines of mathematics in a variety of contexts at all grades. Reasoning should be made a habit (NCTM 2000:56).

According to NCTM (2000:57) primary school learners can learn to make, refine, and test conjectures. It was also found that computer software programmes could enable learners to make conjectures since they offer multiple opportunities to learn properties. Conjectures themselves are arrived at inductively. This gives learners the opportunity to learn geometry concepts and to discover theorems inductively.

The American curriculum allows the primary and junior secondary schools' learners to learn geometry inductively. The high school learners (grade 9-12) learn geometry deductively (NCTM 1989:12). Even though high school learners learn geometry using logical deduction, they also learn to investigate conjectures using concrete materials like all other lower grades (NCTM 2000:57).

It is not only the method of teaching that can help learners to understand geometry. The teacher's questioning techniques and language directed at the learner's thinking is critical in developing understanding of geometric relationship (NCTM 1989:113).
It was shown that the USA mathematics curriculum places emphasis on teaching geometry through the inductive approach from primary school level through to grade eight. Reasoning and proof is also taught inductively. Although it makes provision for the expansion of logical capabilities for grade 5-8 learners, without making it formal, it does not suggest strategies of doing it (NCTM 1989:112). As a result, there is not a strong link between the inductive and deductive approaches.

2.10.3 Australia

From the preceding paragraph, it is clear that the introduction of geometry (involving three-dimensional objects) in the primary school mathematics curriculum is becoming prevalent in many countries. Geometry has also been introduced in primary grades in Australia (McPhail & McPhail 1976:19). The high school mathematics in Australia offers topics in transformation geometry. Euclidean geometry has been thrown out of school for pedagogical and not for mathematical reasons (Scott 1983:25).

It is obvious that Australian teachers could not find methods of teaching that could assist learners to master Euclidean geometry. It also shows that the introduction of transformation geometry in primary schools could not overcome the problem. The introduction of transformation geometry was an attempt to find a suitable approach to teach Euclidean geometry. However, Scott (1983:25) supports the retention of Euclidean geometry because transformation geometry lacks mathematical virtues.

2.10.4 Britain

The geometry curriculum in Britain has been characterised to a large extent by transformation geometry in one form or another (Gower 1983:153). Transformation geometry was introduced to give a consistent view of mathematics to the learner. Euclidean geometry has not received general acceptance. The removal of deductive
geometry has left a gap that has not been filled (Kapadia 1981:327). This was due to the fact that transformation geometry was found to lack mathematical rigour.

The British curriculum that includes subjects such as topology, analytical, vectors, matrices and transformations lacks coherence and depth (Lang & Ruane 1981:128). The introduction of the geometric transformations in the British curriculum has reduced geometry to a collection of isolated facts and procedure (Gower 1983:153).

2.10.5 Singapore

The Singaporean mathematics syllabus has changed three times since the 1970's from syllabus B to syllabus D (Yee-ping 1989:7). Most changes have been in geometry. Emphasis is placed on teaching mathematics through investigations.

Syllabus B is comprised of traditional deductive geometry. Syllabus C is based on transformation geometry that places emphasis on analytical geometry, matrix algebra and vectors in develop geometric concepts. With further reviews syllabus D is introduced to include transformation geometry as in syllabus C, the latter consists of formal proof.

Transformation geometry has been introduced in the curriculum because it encourages learners to discover, to conjecture and to reflect. Aims of introducing transformation geometry were not achieved because of too much emphasis on drill practice by teachers (Yee-ping 1989:124-126). Teachers have realised that transformation geometry has failed because of using bad teaching methods.

They now advocate the creation of an environment wherein learners are able to construct knowledge that is meaningful to them (Yee-ping 1989:126).
The fact that the Singaporean geometry curriculum encourages learners to conjecture, to discover and to reflect is an indication it is inductive. Introduction of transformation geometry reinforces the idea that it places emphasis on the inductive approach.

2.10.6 Thailand

In 1978 the Institute for Promotion in Teaching Science and Technology (IPST) initiated change in the Thai mathematics syllabus (Chaiyasang 1987:12).

Geometry is taught together with other mathematical topics because mathematics is an integrated program. Euclidean geometry is taught in grades 7, 8 and 9, with emphasis on informal conclusions rather than on formal proof. In grade 9, there are two tracks in the mathematics programme: one for learners who intend to go further in mathematics, while the second is for those who do not. In grades 10, 11 and 12 Euclidean geometry is replaced by analytic geometry, vector geometry and introduction to symbolic logic.

Indications are that the Thai geometry curriculum has shifted away from formal proof in grades 7-9 to a curriculum that requires learners to investigate. It follows that the curriculum pursues the inductive approach.

2.10.7 South Africa

The interim syllabus for grades 7-9 based on the learning mathematics by discovery. Its objectives are:

- To develop insight into spatial relationships and measurement.
- To enable learners to discover mathematical concepts and patterns by experimentation, discovery and conjecture.
- To develop the ability to reason logically, to generalise, specialise, organise, draw analogies and prove.
The syllabus for grades 10 to 12 emphasise teaching Euclidean geometry deductively. Proofs of some theorems have indicated as being important for examination. Research conducted in KwaZulu-Natal shows that 45% of grade 12 learners could not master Van Hiele level 2, whereas the examination was assumed at Van Hiele level 3 and above (De Villiers 1997:42). De Villiers (1997:42-43) argues that transition from level 1 to level 2 is difficult for second language learners because it involves the use of technical mathematical language to describe and explore properties of shapes. He further argues that the primary school geometry curriculum should be revisited along Van Hiele theory if the secondary school geometry curriculum is to succeed.

The introduction of outcomes-based education in grade 1 as from 1998 may bring a new culture of teaching and learning geometry. The new policy document shows that mathematics should be learnt through investigations and discovery (DNE 1997b:20). It places emphasis on formulating, testing and justifying conjectures and solving problems. Geometry has been defined in terms of space, shape and motion. The new curriculum places emphasis on teaching geometry through integration of both Euclidean and non-Euclidean geometries, for example, in the use of co-ordinates in learning enlargements of shapes (DNE 1997b:10). These aspects are learnt inductively. This shows that the new curriculum is in line with geometry curricula in other countries.

At the time of conducting this study the policy document for Further Education and Training (FET) had not yet been published. It has already been pointed out in the above paragraph that geometry problems (theorems) may be taught both inductively and deductively. In other words, the inductive approach should play a major role in discovering theorems while the deductive approach should test and justify them. A foundation will have been laid in Senior phase classes for learners to cope with the deductive approach in the Further Education and Training phase.
The fact that emphasis is on teaching geometry using concrete objects at primary school phase it is an indication that the geometry curricula in all countries discussed is in accordance with Van Hiele level 1 (visual). Teaching transformation geometry at junior secondary phase is an indication that geometry curricula give learners the opportunity to explain themselves. The question is whether teachers will implement what is in the curriculum.

Both geometry curricula (primary and junior secondary phase) place emphasis on learning by discovery. They give learners the opportunity to experiment, to conjecture and to explain. This is an indication that the inductive approach has a role to play in teaching geometry. Transformation geometry has been used as a vehicle for the inductive approach.

2.11 Conclusion.

Geometry was defined as the study of physical space around objects, position that object occupies and movement of the object within space. However, it should be noted that geometry is also defined as measure of earth.

Euclidean geometry is characterised by proof by deduction. The manner, in which the current mathematics textbooks are written, is such that they have no room for the inductive approach. It was found that the problem that causes learners to perform poorly in Euclidean geometry is its emphasis on proof by deduction. Secondly, learners are being taught Euclidean geometry at inappropriate Van Hiele levels of thinking. The following statements are considered possible solutions to the abovementioned problems:

- Learners should be taught through the inductive approach from the primary school phase.
- Learners should be guided to attain appropriate Van Hiele levels of thinking.
• Deductive thinking should also be taught in other branches of mathematics throughout the learner’s school life. It should be made a habit. However, Van Hiele (1996) believes that the deduction method should be left for university learners.

Transformation geometry was found to be able to deal with a greater variety of shapes than Euclidean geometry.

The Van Hiele model of geometric thought is included in this chapter because it deals with the way children learn geometry. It also helps the teacher on how to structure instruction. The following components of the theory were discussed:

• Levels of thinking.
• Instructional phases.
• Properties of thinking.

Although, he initially introduced five levels of thinking in the theory, he later reduced them to three Van Hiele (1996). The new levels are: visual level, descriptive level and theoretical level. He is of the opinion that the primary and the junior secondary school geometry should be taught at the visual and descriptive levels respectively. At the visual level, learners recognise shape as a whole. At the descriptive level they recognise shapes in terms of their properties. They are also able classify them (class inclusion) and be able to explain their reasoning.

Instructional phases enable the teacher to structure and to order geometry activities. These activities take into account the learner’s prior geometry experiences. The transition period between one level to the higher level is called a learning period. In terms of the Van Hiele theory, geometry instruction should take into account properties of the levels of thinking. According to the theory no learner should skip a level. It should also be noted that these levels are assigned to the learner’s thinking rather than to the subject content. The learner may only progress from one level to next if he receives proper instruction. It is learner's knowledge of the subject content that places him to a level rather than age.
Most learners have been found to be performing poorly in geometry partly because they lack spatial knowledge. The inclusion of Van Niekerk’s model on the development spatial knowledge of young learners became necessary. The following should be taken into account when instruction is prepared and presented: the type of task, nature of object, the dimension of the stimulus, the context in which it will be presented and the execution media. In terms of Van Niekerk’s model when the teacher designs geometry activities, he should take into account the learner’s worldview and culture, socio-economic status and demography of the classroom. This will help him shape his teaching philosophy. To ensure that learners grasp spatial concepts with understanding, the model encourages the use of bilingualism. It encourages the use of problem-centred approach to teach spatial concepts.

It is accepted that the deductive approach alone will not help to improve learners’ performance in Euclidean geometry. The role of the inductive approach is that of learning concepts and discovering theorems. It is suggested that geometry should be taught inductively from primary school phase through to high school.

Different people have defined proof differently. There are no consensuses as to what constitute a proof. However, proof has remained an integral part of geometry. Learners have found it to be the most difficult part of geometry. This is due to the fact that it places emphasis on deduction. Processes involved in proof writing were discussed at length. The inductive approach was found to be suitable for learners to learn proof. It was also stressed that an aspect of explanatory proof should precede deductive proof. The following issues concerning proof were also discussed: research proof writing and role and function of proof. In this study proof would mean application of theorems to solve problems. This will be fully discussed under cognitive processes.

The discussion on second language problems in this study was necessary since it is based on black learners who use English as a language of instruction. It was found that use of a single language also hampers their effective learning. It was also found
that second language learners who are competent in two languages (one of which is a language of instruction) perform better than those who use a single language.

The discussion on the study of geometry curricula of different countries was to find the extent to which their curricula use the inductive approach. All geometry curricula emphasised that geometry be taught and learnt through discovery. Transformation geometry was most liked of all the geometries as an alternative to Euclidean geometry. Similarly, the inductive approach was prevalent in most countries. Most countries appear to favour a geometry curriculum that will enable learners to formulate conjectures, to discover and to reflect. However, transformation geometry was not successful in Singapore and the Netherlands. It failed in Singapore due to too much emphasis on rote learning. Critics of the transformation geometry are of the opinion that it will lead to the disappearance of mathematics.

The subsequent chapter will focus on the discussion on the traditional approach, process-based instruction, cognitive and meta-cognitive processes. The discussion on the traditional approach, cognitive processes and meta-cognitive processes becomes necessary since they are elements of the process-based instruction.
CHAPTER THREE

THE ROLE OF PROCESS-BASED INSTRUCTION IN TEACHING AND LEARNING EUCLIDEAN GEOMETRY

3.1. Introduction

Before any discussion on process-based instruction, it is necessary to outline the present practices in the classrooms. Most teachers in South Africa are still clinging to the traditional teaching approach. The traditional approach is characterised by transmission of knowledge. In the traditional teaching approach the success of teaching is measured by the assimilation and reproduction of the content. Learners are considered as passive recipients of knowledge. They are not given the opportunity to discover things by themselves (Nieuwoudt 1998:11-14).

At the beginning of the process-based instruction, like in the traditional approach, knowledge is considered external. It is due to the fact that, initially learners use the teacher's plan. In other words, the lesson is first explained to them and they follow the teacher's procedures to solve problems. Process-based instruction is characterised by gradual ceding or transfer of control of plans from the teacher to the learner. It is aimed at learning and thinking activities of the learners. It is further characterised by planning and monitoring procedures. Like the traditional approach, tests and examinations are used as instruments to measure success in process-based instruction. It also places emphasis on construction of knowledge by learners. This section will also discuss phases of process-based instruction.

Unlike algebra, Euclidean geometry has problem-solving strategies that are unique. For every problem a learner solves, a skill is acquired and experience is gained. A discussion will be made as to when should learners be taught problem solving as a skill. A discussion will also be made about schemas of high and low achieving learners concerning problem-solving strategies. Polya's stages for solving problems will also be
discussed. It is acknowledged that problem solving strategies cannot be learnt overnight. It will only take place over time. That is, problem solving is a ‘back and forth’ process.

It has already been stated that the process-based instruction is aimed at the learning and cognitive activities of learners. It has also been stated that process-based instruction is characterised by planning and monitoring procedures. Therefore, the discussion on cognitive and meta-cognitive processes becomes necessary. Cognitive processes are: inductive and deductive thinking, conceptualising, intuitive thinking, interpretation, application, analysis, integration/synthesis, conjecturing and generalisation.

3.2. Traditional approach to teaching and learning Euclidean geometry

South African mathematics classrooms are still dominated by the ‘chalk and talk’ (traditional) teaching approach (Nieuwoudt & Baxter 1996:191). It involves reproducing proofs of theorems from the textbook on the chalkboard and book by teacher and the learner respectively. According to Goffree (Nieuwoudt 1998:11) the sequence of topics in the textbooks influences the teacher’s presentation. In terms of the traditional approach, learners are expected to reproduce the proofs of theorems in tests and examinations. It is common to find the syllabus having to prescribe proofs of theorems that would be required for examination (DNE 1992).

The teacher’s role in the traditional approach is that of transmitting knowledge to learners on how to write proofs. In this case, the teacher thinks on behalf of learners (Van Hiele-Geldof 1984:19). Learners do not get the opportunity to think for themselves. They are expected to follow the teacher’s method to solve problems. According to Nickson (Nieuwoudt 1998:13) in the traditional approach learners are regarded as passive recipients of knowledge.
The traditional approach is characterised by the lecture method. Nieuwoudt & Baxter (1996:190) has this to say about the traditional approach:

Teaching and learning time in class were mostly spent on the explanation, discussion and drilling of basic facts, rules and standard procedures, followed by the time spent on the teacher's guiding of students in the execution of routine tasks to get the desired answers.

In terms of this approach the lesson presentation is aimed at the whole class. According to Van Hiele-Geldof (1984:19) geometry should be taught from the concrete situation to an abstract situation. The traditional approach does not give learners the opportunity to learn mathematics from the life outside the classroom (Nieuwoudt 1998:13). In other words, it is rigid. More often, concepts are learnt through memorisation of definitions.

The traditional approach is synonymous to algorithmic way of teaching and learning mathematics. It involves memorisation of procedures and their application thereof (Nieuwoudt 1998:12). Euclidean geometry problems cannot be solved by an algorithmic method, since they are no specific procedures to solve them. Finally, the traditional approach is product-based approach, since it relies more on desired results (achievements) (Nieuwoudt 1998:11). Nevertheless, traditional approach comprises a small component of the process based instruction (PBI). In the initial stages of the process-based instruction the teacher dominates the instruction until he transfers control to the learner. The subsequent paragraph will deal with the role of the process-based instruction (PBI) in teaching and learning geometry.

3.3. Process based instruction

Process based instruction is characterised by both the elements of the traditional approach and those of the constructivist approach, namely: discovering, discussing and reflecting. According to Vermunt (1994:15) the theory of process-based instruction is located within constructivism. Its emphasis is on learners, their learning and thinking activities. It is further characterised by the gradual transfer of instruction from the teacher to the learner. In other words, learners are eventually expected to take over responsibility of their own
learning. In process-based instruction learners are taught to adopt what was previously the teacher’s plan to make it their own. Later on, they adopt their own plans and problem solving strategies. Conway & Ashman (1989:229) define a plan as a step-by-step procedure that would lead to the completion of a task. Process-based instruction is characterised by the following steps:

- Starting point,
- A sequence of steps and
- Thinking or meta-cognitive steps. (Ashman, Wright & Conway 1994:201)

This shows that process-based instruction is concerned with reflective thinking. The first step implies that learners should know where and how to start with the task. The second step implies that each step of the solution should lead to the next. Lastly, learners should monitor their own procedures of solving problems. Successful completion of the task will depend on the extent to which learners follow the above three points. It is worth noting that process-based instruction also emphasises the construction of knowledge and its application by the learners. It is aimed at training individual learners to become independent and active problem solvers (Ashman, Wright & Conway 1994:199).

According to Ashman, Wright & Conway (1994:199) process-based instruction methods should satisfy the following criteria:

They should:

- Operate in classrooms with learners of all abilities.
- Integrate in an existing curriculum or classroom programme.
- Enable teachers and learners to apply newly developed learning problem-solving skills inside and outside the classroom.
- Permit learners to move away from teachers’ sequenced activities to their own personal plans.

The first criterion implies that the classroom should be comprised of both less gifted and gifted learners. The process-based instruction creates a situation in which group and
individual learning can occur (Ashman, Wright & Conway 1994:200). Learners may work as individuals within groups. They may also discuss one another's solutions of different tasks. This situation can be described around four activities:

- To show one's solution,
- To explain one's solution,
- To justify one's solution,
- To describe one's solution. (Dekker & Elshout-Mohr 1998:305)

The first activity ensures that learners show one another solutions. In this way they become aware of each other's work. They later explain to one another how they arrived at solutions. In the event of doubt an individual learner will justify his/her solution. If the learner fails to justify or convince other learners, then he/she will reconstruct the solution. In this way learners' level of mathematics is being raised (Dekker & Elshout-Mohr 1998:305). Learners have now gained insight into the problem.

The second criterion encourages teachers to retain what they regard as their well-established and successful methods. In other words, the introduction of process-based instruction does not imply that teachers should abandon their existing curriculum. They should adapt it to the existing content. They are always frictions when new methods of teaching are introduced. These frictions can be avoided by tailoring the process-based instruction to the learning and thinking activities learners mastered and used, their learning styles and their prior knowledge (Vermunt 1994:17). In this way they can adapt to the new approach.

Process-based instruction is not only aimed at teaching learners learning and thinking strategies, but it also helps them to apply problem solving skills acquired in mathematics to solve problems in other spheres of life. According to Charles and Lester (1984:15) mathematical problem solving improves learners' abilities to understand the problem and to plan solution strategies to obtain results. They further argue that the slower rate to obtain results is due to inability of learners to coordinate problem-solving skills with other
procedures. They also argue that learning to perform a skill is one thing and knowing how and when to use it is different matter.

Ashman, Wright & Conway (1994:199) argue that learners should be taught how to manage and refine their thought processes to achieve success in academic and non-academic tasks. The inclusion of meta-cognitive strategies makes process-based instruction different from all forms of plans. In a nutshell process-based instruction is characterised by planning and monitoring procedures.

In process-based instruction, it is the responsibility of the teacher to select the curriculum content, the sequence of activities and the methods of teaching. Learners are then given the opportunity to take the responsibility of their own learning, once it has been realised that they can do so in two ways:

- when plans and problem solving activities are given in learners’ own words.
- when they are encouraged to use peer-tutoring and cooperative strategies.

According to Ashman, Wright & Conway (1994:200) a process-based instruction has four phases; namely; (a) introduction, (b) establishment, (c) consolidation and (d) incorporation. In an introductory phase a teacher may introduce a plan by asking learners to attempt a task. Through discussion learners will amend the teacher’s plan to suit their learning style. In an establishment phase, once learners are able to apply plans to a specific curriculum, they may begin to use them in a wide range of situations within the same curriculum. In the consolidation phase, learners are given the opportunity to use plans in mathematics to solve problems in other curriculum areas. In the incorporation phase, teachers reflect on the effectiveness of the plans and ability of learners to develop plans for new or novel tasks.

Although, learners are expected to help one another to raise their mathematical level, guidance from the teacher is still essential. Constant monitoring of the learners’ performance helps the teacher to gain insight into the learner’s learning and thinking strategies.
According to Vermunt (1994:22) tests and examinations are strong regulators in process-based instruction. They measure the quality of the thinking activities that were taught. They do not only evaluate learners' knowledge of content, they also evaluate their ability to solve problems in mathematics.

Finally, process-based instruction is based on the fact that learners should turn the external regulation into self-regulation. It is further based on the fact that it adapts itself to the existing curriculum. It also harmonises cognitive, affective and meta-cognitive strategies. The subsequent paragraph will deal with problem solving strategies from process-based instruction perspective.

3.4. Geometry problem solving strategies

Problem solving is defined as a process by which the learner combines learned knowledge, rules, techniques, skills and concepts to provide a solution to a novel situation (Orton 1987:52).

If a learner solves a problem, he has learned something. Each problem a learner solves becomes a rule to solve other problems. In this way, a learner acquires skills and experience to solve similar problems. This does not make a learner a better problem solver, since each problem has its own content and context. Unlike in algebra, geometry problems require different approaches. In other words, a solution for one problem may not guarantee a solution for the subsequent problem. This does not necessarily mean that there are no similar approaches among geometric proofs (Posamentier & Stepelman1981:117).

Problem solving is a skill that should be taught to learners throughout their school experience (Krulik & Rudnick 1989:6). By the time they reach senior primary phase they should be skilled in recognising appropriate strategies and they should be able to decide when and how to use them. At secondary school level they should have access
to a variety of strategies and be able to determine when and how to use them and if possible invent new ones (NCTM 2000:54). These strategies should be appropriate to learners' levels of thinking. Learners should also be able to monitor and reflect upon their problem solving processes.

It should be noted that learners are not at the same level of sophistication in terms of solving problems. Problem solving approaches that are applied by high achieving learners involve the use of a wide range of geometric schemas than it is the case with low-achieving learners. High-achieving learners are found to be capable of activating schemas more often than low-achieving ones. Skemp (Ramnarain 1999:24) describes a schema as a collection of closely related concepts that have something in common and which together possess some measure of problem solving capacity. A schema is a knowledge structure from which learners tap problem solving strategies. They can also be accessed to geometric schemas that are more sophisticated and varied than those of low-achieving ones (Chinnapan 1998:212-213).

Polya (1948:6-4) has identified the following stages in solving any mathematical problem:

- Understanding the problem
- Devising a plan
- Carrying out a plan
- Looking back

The first stage involves whether the problem provides sufficient information to the learner. It also involves interpreting the problem. A variety of different strategies such as drawing a diagram, looking at a pattern and considering a related problem may be explored. The use of different strategies will enable the learner to have insight into the problem. The second and third stages are difficult. They demand creativity and inventiveness on the part of the learner. The last stage involves reflecting on the solution. A case of generalising results and looking at an alternative economical
solution is considered. In reality, it is not always the case that the learner obtains the solution of the problem at first attempt. The problem solving process involves circularity (see figure 2.1).

![Flowchart for stages on problem solving](image)

Figure 2.1. Flowchart for stages on problem solving (Orton 1987:93)

Problem solving in Euclidean geometry involves proving a theorem using the deductive approach and determining the measure of some parts of a geometric shape. Posamentier & Stepelman (1981:117) suggests the following strategies to prove theorems:
• Analyse the hypothesis (theorem)

A neat and relatively accurate diagram is drawn and marked; indicating segments, and angles that are known to be equal. Parallel lines should be marked. A well-drawn shape suggests a productive line of reasoning.

• Analyse the conclusion (proof)

Consider the conclusion and how it may be reached. It is advisable for learners to consider how conclusions have been arrived at in the past. They should be certain that they understand the desired conclusion before they proceed.

• Find the connection between the hypothesis and conclusion

In order to find connections, the learner has to have at his fingertips all appropriate definitions, previously proved theorems and common techniques. The experience the learner has acquired when solving other problems, intuition and imagination should all be put together in order to prove a theorem.

It has already been pointed out that problem-solving processes might involve circularity (back-and-forth approach). If the learner does not succeed in solving a problem at first attempt, then he should try again.

The best way to learn how to solve is by solving more problems. Experience is often the teacher (Posamentier & Stepelman 1981:120). Experience in problem solving builds one’s confidence to solve more problems (Cangelosi 1996:52). Successful solution of the problem is dependent on the learner not only having the knowledge and skills required but by being able to tap into them and establish a network or structure. The more problems the learner solves the more he develops insight into problems. Sometimes a flash of insight might occur. In addition to experience Ernest (1988:81)
proposes that in order for learners to be successful in problem solving, the following factors should be considered:

- Affective factors - for example, interest, motivation, pressure and anxiety.

- Experience factors - for example, age, previous mathematical background and familiarity with solution strategies.

- Cognitive factors - ability to use intuition to guide an attempted solution, spatial ability and logical ability.

It should be noted that the problem solving approach is a process. Once the learner solves a problem, he has discovered new knowledge to be applied in solving other problems. Thus, the process of solving problems leads to new discovery. The subsequent paragraph will discuss the cognitive and meta-cognitive processes that learners undergo during the process-based instruction.

3.5. Cognitive and meta-cognitive processes

In this study emphasis will be on cognitive processes that learners undergo when they solve geometry problems. They undergo these processes while learning concepts and discovering theorems. In the context of this study cognitive processes will refer to processes that learners employ in reasoning (English & Halford 1995:29). Meta-cognitive process refers to selecting, sifting and testing (planning, monitoring and evaluating) their work in search of a solution (Stoker 1993:9). Reflective thinking is an example of a meta-cognitive process. It should be noted that some cognitive processes are subsumed in others. This section will discuss:

(a) The meaning of each cognitive and meta-cognitive process.
(b) The role of each cognitive process in the teaching and learning of geometry in process-based instruction.
3.5.1. Conceptualising

Concept acquisition or formation will be defined in the context of geometry. A concept is characterised by the fact that it expresses an idea, a general, ideal representation of a class of objects, based on their common features (Fischbein 1993:139). For example, a triangle can be defined in terms of its properties. In addition, Cangelosi (1996:83) defines a concept attribute as a characteristic common to all examples of a particular concept.

In the light of the above definitions, one can deduce that concepts are arrived at or learned inductively. The process by which one groups specifics to construct a mental category is referred to as conceptualising (Cangelosi 1996:81).

Triadafilidis (1995:22) has this to say about geometric concepts:

The formation of a geometric concept has to be based on a number of critical attributes of the concept. These critical attributes are features and the properties that characterise and differentiate the particular concept from all others.

In other words, concept attributes should enable the concept to be differentiated from all other concepts in terms of their properties and features. There are three ways for introducing concepts in a traditional instruction, namely (Bender & Scheiber 1980:59):

- By definition,
- By giving examples,
- By drawing diagrams.

Bender & Scheiber (1980:59) further argue that learners should learn concepts by exposing them to the real world. That is, concepts should be linked to the real operations by which the real world is shaped. This takes us to the fact that concepts should be learnt through concrete objects.
Researchers agree on the role of images and visualisation on concept formation in geometry. It is not possible to introduce a geometric concept without giving an example. According to Mariotti (1995:191) it is not possible to form an image of a concept without visualising its elements.

This is an attempt to give learners the opportunity to construct knowledge by themselves. The inductive approach is a suitable approach to introduce concepts. However, Triadafilidis (1995:225) regards definitions as a link to higher levels of deductive thinking.

3.5.2. Analysis

For the learner to be able to understand geometry, he needs to break down the given information. This enables him to understand the problem. It further enables the learner to establish whether there is pattern or a relationship. He will now be able to sort out useless information.

3.5.3. Interpretation

Interpretation involves the ability to illustrate and explain mathematical concepts, notation, terms and principles. It further involves recognition of major ideas and interrelationship between them (Moodley 1992:104). In other words, the learner should be able to make inferences based on a geometry problem. The ability to establish relationships is essential in geometry problem solving. Thus, interpretative skills are also essential to solve geometry problems.

3.5.4. Conjecturing

Conjecturing is based on analysis and synthesis of the problem. The learner determines whether a pattern or relationships exists in several situations. If it exists then he formulates a rule. This rule is called a conjecture. Conjecturing plays an important role for learners to
be able to learn geometry inductively. According to NCTM (2000:57) in order for learners to be able to formulate a conjecture they need multiple opportunities.

Teachers should accept correct and incorrect conjectures, since a lot of things can be learnt even from incorrect ones. Almeida (1997:29) argues:

A student's conjecture is the result of 'thinking aside' and it will reap rich rewards even if it is incorrect because it directs the attention both of the other students and of the teacher, in an area that was not previously considered.

3.5.5. Generalisation

Once a pattern has been established or recognised a statement in the form of a generalisation is made. However, this generalised statement need to be tested in order to form a convincing argument.

In geometry a convincing argument is called a proof (Ramnarain 1999:18). If an argument has been found to be convincing, the statement will be called a theorem.

3.5.6. Intuitive thinking

Intuitive thinking is a process of thinking that does not subject itself to analytical thinking. That is, knowledge that comes as a result of intuitive thinking cannot be justified. Fischbein (1987:4) defines intuition as a global guess for which an individual cannot give justification.

It should be noted that intuitive thinking is a personal matter since what can be accepted as true by one person might not be true by the other. Intuitive thinking can neither be taught nor learnt, it can be acquired by mastering the art of the subject (discipline). In other words, the more experience one has in a subject, the more the learner can think intuitively. Experience is vital for the development of mathematical intuition (Fischbein 1987:21).
Intuition is the product of concept images of the individual. The more educated in logical thinking, the more likely the individual’s concept imagery will resonate with a logical response (Tall 1991:14).

An important factor about intuitive thinking is about sustained attention to one’s work (Schwartz 1988:36). The more one becomes focussed on ones work, the more luck you have of thinking intuitively. Terms related to intuition are insight, revelation and common sense. As a matter of fact it comes as a result of experience, whereas revelation can be considered as a ‘flash’ in an individual’s mind when one immediately finds a way out to solve a problem.

Intuition leads to discovery of certain mathematical truth without being able to give justification to it. Thus, it enables the learner to conjecture. It further links the unknown with the known. The importance of intuitive thinking in geometry learning is that

- Intuitive models can be used in introducing geometric concepts during the early ages of the learner,
- It generates ideas,
- It is an instrument and guide for action.

(Volmink 1988:40)

Finally, intuitive thinking does not lend itself to formal proof.

3.5.7. Inductive and deductive reasoning

Both the inductive and deductive reasoning are indispensable to teaching and learning geometry. Wylie (1964:2) argues that neither by itself is sufficient to support genuine scientific progress. Serra (1997:40) defines inductive reasoning as a process of observing data, recognising patterns and making generalisations from observations. Inductive reasoning involves exploring, experimenting and formulating conjectures. A generalisation obtained from observations is called a conjecture. It is only considered a theorem once it has been proved deductively.
The role of inductive reasoning in geometry is that of discovering concepts and theorems. It enables learners to understand the origin of theorems and the theorems themselves. This shows that inductive reasoning should be introduced during the early stages of schooling through learning of patterns. Brown (1982:445) argues that instead of announcing theorems in the classroom, they should be allowed to evolve inductively from discussion. Learners should be given the opportunity to discover them by giving them geometry activities. Stallings-Roberts (1994:408) claims that even though the amount of preparation increases, exploratory approach is more advantageous.

In view of the fact that learning Euclidean geometry through the deductive approach alone has been found to be unsuccessful, the following suggestions are made:

- Geometry should be taught inductively from primary school grades to junior secondary school grades (Reddy 1987:110). NCTM (2000:56) further suggest that it be taught inductively even at high school.
- Primary school geometry curriculum should help learners to develop spatial abilities. It should include topics such as measurement and estimation, movement of geometric shapes, varying parts of geometric shapes and visual studies of usual and unusual geometric shapes. Topics may involve paper folding activities, paper cutting activities, experimentation using computer graphics, construction with straightedge and compass and other endeavours that actively engage learners (Posamentier & Stepelman 1981:697).
- Secondary school geometry curriculum should allow learners to make visual justification and empirical thinking (Battista & Clements 1995:51).
- Geometry curriculum for all abilities at all levels of schooling should be developed.

The above suggestions mean that Euclidean geometry should be taught inductively at all levels of schooling.
The inductive approach encourages learners to explore, to conjecture, to refute a conjecture, to formulate a conjecture and explain it. Conjecturing itself leads to discovery. Conjecturing has been found to be one of the causes for learners’ difficulties with proof writing (Senk 1985:445). Peard (1976:19) proposes that learners should be made to arrive at a conjecture through activities and if there is no counter-example to it, it may be proved deductively.

Recent developments on dynamic geometry have shown that Euclidean geometry can be taught inductively using Geometer’s Sketchpad. Geometer’s Sketchpad is a computer package of geometry software programmes that can be used to do determine properties of geometric shapes.

In contrast to paper-and-pencil construction, dynamic geometry is accurate and it is extremely quick and easy to carry out complex constructions, and vary them afterwards (De Villiers 1997:50).

In dynamic geometry the size of the geometric shape can be changed while keeping its form. This change of shape is known as geometric transformation. Hence, in dynamic geometry learners use the inductive approach to learn Euclidean geometry concepts. Pedersen (1983) is of the opinion that learners need concrete experiences with geometry and a certain level of mathematical maturity before they are exposed to Euclidean geometry.

Wylie (1964:2) has identified two weaknesses from the inductive approach. Firstly, no matter the number of correct observations made no generalisation can be made since further investigations may contradict it. Unless every instance has been examined. Secondly, the assumption that observations have been made with perfect accuracy is often false, so that in many cases there is no exact information to base generalisation. This implies that the inductive reasoning is fallible. It is at this point that the deductive reasoning takes over from the inductive reasoning.
Serra (1997:680) defines deductive reasoning as a process of demonstrating that if certain statements are accepted as true, then all other statements can be shown to follow from them. In this case, the role of the reasoning is that of verification or justification of results/statements arrived at through the inductive reasoning. Hoffer (1998:24) argues that the role of deductive reasoning should not only be confined to verification of the results, but it must be applied to resolve practical problems and to explore and to discover a valid statement arising out of observable data. He further pointed out that deduction is not only a concluding activity, but it occurs all along the way.

Euclidean geometry failed because its deductivity could not be re-invented by the learner (Freudenthal 1971:416). He is of the opinion that it has been imposed on the learner. Over emphasis on the use of the deductive reasoning has been blamed as an obstacle to teaching and learning Euclidean geometry. According to Freudenthal (1971:426) Euclidean geometry can be made easier by using learning materials that takes into account the learner prior experiences. He further stated that it must be taught as a field that will make learners active. Peard (1976:20) is of the opinion that in order for learners to appreciate the deductive reasoning the following objectives must be met:

- Develop an awareness of the need for assumptions in a deductive system.
- Be able to distinguish between a conjecture arrived at inductively and a theorem proved deductively.
- Be familiar with the use of counter-examples to disprove a conjecture.
- Be able to arrive at conjectures through meaningful activities.

Learners whose level of thinking is at Van Hiele level 3 (theoretical) can meet the first objective. The rest of the objectives can be met by learners whose Van Hiele level 2 is descriptive. The above objectives show that geometry can be taught both inductively and deductively. It is an indication that there can be a harmonising relationship between the two cognitive processes.
3.5.8. Integration

Integration involves putting together of information to form a coherent whole (Ramnarain 1999:25). The learner should put together relevant information after analysing the problem. Integration enables the learner to establish whether relationships or a pattern exist. It further helps the learner to find appropriate strategies to solve a problem.

3.5.9. Application

Application involves use of previously acquired knowledge to solve a new problem (unfamiliar situation) (Ramnarain 1999:26). Application requires the transfer of knowledge from one situation to another. It further involves correct selection of methods and theorems to solve a problem (Moodley 1999:105).

3.5.10. Visualisation

Visualisation is characterised by the formation of mental images and being able to view things from different angles. It comes as a result of a stimulus (Bishop 1989:10). Fishbein (1993:139) defines an image as sensorial representation of an object. The object referred here is called a figure or a diagram.

Many learners are found to lack visualising skills (Dreyfus 1995:4). They have little interest in the use of diagrams and they are unable to make efficient use of them in solving problems in disciplines like algebra. Dreyfus (1995:14) has attributed this to the classroom culture that learners are exposed to. The role visual imagery is linked to the interpretation of information on a diagram or a geometric figure. It plays an important role in problem solving in geometry.

Like any cognitive process visualisation has limitations. Visual limitations are due to the fact that visualisation is a personal matter. In other words, it is not something which can be
imposed from outside. In the light of this, individuals should be given the opportunity to
develop their own images since it is not possible for them to develop the same images.

Mason (1988:297) and Bishop (1989:12) are of the opinion that manipulatives such as
cubes can help learners to improve learners' visual skills. Liedtke (1993:63) and Læson
(1994:8) also support the fact that geometry activities involving three-dimensional stimulus
can improve learners' spatial sense.

Fennema & Sherman (Clements & Battista 1992:443) found that there is a positive
 correlation between spatial ability and mathematics achievement at all grade levels. That
is, learners with a developed spatial ability perform better in geometry than those who are
not on the same level of development. Thomas (1983:19) and Sherard (1981:21) found
that transformation geometry is suited to the development of spatial abilities of primary
school learners. Learners whose spatial abilities are developed can be able to read two-
dimensional representations of three-dimensional objects (Sherard 1981:21).

According to Eisenberg & Dreyfus (1989:2) most mathematics teachers agree that the role
of visualisation in the curriculum is significant, but it plays a minor role in an average
secondary school classroom. This is debatable since the argument assumes that the role
of visualisation is of less value at secondary school phase. Bishop (1989:14) argues that
visual representation is of value to all learners and in all aspects of the mathematics
curriculum.

Finally, visualisation is trainable provided learners are involved in activities that will allow
them to use senses other than vision, for example, touching, manipulating, constructing
and drawing.

3.5.11. Reflective thinking

Reflective thinking is a meta-cognitive process. Meta-cognition processes that are
characteristic of problem solving include self-monitoring, self-regulating and self-
assessment (Ramnarain 1999:31). Reflective thinking is simply a process whereby one keeps on checking on oneself, if one is solving a problem correctly or not. In other words, reflection does not only take place after one has resolved a problem, but it also takes place during the process of solving a problem. Persons who reflect on their activities or solutions have control over their thinking and they can be able to pursue several paths to the solution rather than being in the action (Wheatley 1991:535).

Unlike in intuitive thinking where one is unaware of ones mental processes, in reflective thinking one is conscious of what is doing. Gagatsis & Patronis (1990:33) argues that if one is aware of the processes involved in solving a problem one is able to describe and explain to others. It follows that in order to be successful in doing proof, one should be able monitor and evaluate ones thinking. It enables learners to develop insight into geometry. Learners of all abilities should be encouraged to use meta-cognitive strategies.

Initial stages of reflective thinking are observation, noticing things and asking questions. Reflective thinking is often associated with clever learners in the classroom. Luthuli (1996:20) argues that not only 'able' learners should be involved in reflective thinking—all learners should be involved.

3.6 Conclusion

The discussion on the traditional approach was around the fact that the teacher based it on the transmission of knowledge. It was further indicated that it is characterised by the assimilation and reproduction of the content. The traditional approach does not give learners the opportunity to discover things on their own. The content in the textbooks that are written from the traditional approach is rigid and formal. The traditional approach places the teacher in a position that made him rely more on the textbook.

The process-based instruction, unlike the traditional approach, is teacher-centred to a less extent. It is characterised by a gradual transfer of control of the activities from the teacher to the learner. In this way, the learner is able to introduce his own problem solving
strategies. The discussion was on some few principles of the process-based instruction, namely, the instruction is based on the learning and cognitive activities of the learner. Phases of process-based instruction were also discussed. They were: introduction phase, establishment phase, consolidation phase and incorporating phase. It was also stated process-based instruction was characterised by planning and monitoring when learners solve problems using their own strategies.

Problem solving is a component of the process-based instruction. It is a skill that learners should be taught throughout their school experience. However, it is not a skill that can be acquired overnight. The more problems a learner solves, the more experience he gains. The learner becomes better equipped with problem solving strategies to solve difficult problems. This is considered as part of a learning process. In order for one to solve a problem successfully, one should go through the following Polya's problem solving stages: understanding the problem, devising a plan, carrying out a plan and looking back. Solving a geometry problem depends on the selection of appropriate problem-solving strategies. The following strategies for solving theorems were suggested: Analyse a hypothesis, analyse a conclusion and finding the connection between the hypothesis and conclusion.

It has been stated that process-based instruction is aimed at learning and cognitive activities of learners. The fact that it is also characterised by planning and monitoring procedures, it became necessary to discuss about cognitive and meta-cognitive processes. Learners undergo certain cognitive processes when they solve problems. There are certain cognitive processes that only help the learner to understand the problem better. The following cognitive processes and a meta-cognitive process were discussed at length: conceptualising, intuitive thinking, visualisation, interpretation, reflective thinking, analysis, integration, conjecturing, inductive and deductive reasoning and generalisation. These may not be the only cognitive processes in geometry learning, they are the only ones that the researcher was able to identify. The subsequent chapter will deal with the processes involved in collecting data.
CHAPTER FOUR

RESEARCH METHODOLOGY

4.1 Introduction

This chapter will first mention the aim and the objectives of the empirical investigation. A discussion on reasons for the choice of the school will also be made. A description on the location of the school will be given. This will be followed up by the description of the set up at the school. A discussion will be made on how the school timetable impacted on the investigation.

The table will be given that includes, the description of the activities, dates on which activities were conducted, execution media used and their objectives (see section 4.5).

The empirical investigation will describe the extent to which the process-based approach has influenced learners’ performance in geometry. In this regard, both the qualitative and quantitative methods were used. There will be a discussion on the reasons why they were chosen.

A discussion will be made on the instruments that were used to collect data (see 4.7)

4.2 Aim and research objectives of the empirical investigation

The main aim of the empirical investigation is to investigate the effects of process-based instruction on the performance of learners in Euclidean geometry. The study is conducted to achieve research objectives as outlined in section 1.4.

4.3 Background Information

The empirical investigation was conducted at Ngweni Secondary School, in the Far
North region of the Limpopo Province (see appendix C). The school was mainly comprised of African learners. It is situated at Dzanani Township (Makhado Town).

It is not an overstatement to indicate that none of the classrooms at the school had less than fifty learners. The problem of overcrowded classrooms impacts on the method of instruction and the performance of learners. Effective teaching and learning was not taking place since the teachers were not able to monitor learners' attendance and performance. This results in discipline problems such as truancy and lack of interest in schoolwork. Under these conditions process-based teaching approaches were difficult to apply. As a result teachers resorted to lecture method to teach geometry. During the empirical investigation the noise emanating from classrooms that had been left unattended by teachers disturbed the researcher.

The school has an acute shortage of classrooms. The researcher was provided with the classroom that belonged to the technical drawing subjects. It had no furniture of its own. Learners always fetched furniture from other classrooms at the beginning of every activity. As a result the empirical investigation was delayed. Sometimes the researcher was left with 45 minutes to conduct the empirical investigation. The classroom was only available from 12h30 to 13h30. In order to complete the empirical investigation the researcher agreed with learners to knock off at 13h45. Meanwhile, the researcher had to provide transport to take learners who were staying far from the school to their respective homes. This was done to compensate for the time lost when learners were fetching furniture from other classrooms. Sometimes learners in these classrooms would refuse to give furniture to learners who were participating in the empirical investigation because they would have been left without furniture. They would have been left without a table to write on, if tables were taken away. In most cases learners shared the same table.

There were days when the researcher had to postpone the empirical investigation to the following day because of the unavailability of the classroom. Even though the school knew it was going to make things difficult for itself, it has done the researcher a
favour by providing a classroom for the empirical investigation.

Learners who belonged to the same grade were grouped together. Each learner had a worksheet and a set of mathematical instruments to work with. Learners discussed the activity either in pairs or groups of three. However, the researcher addressed questions to individual learners during the interviews. This was done to enable the researcher to understand the reasoning behind a learner's actions. As the learner explains his own actions the researcher gained insight into the learner's reasoning. In other words, the researcher wanted to get closer to the individual learner.

Their poor background knowledge of Euclidean geometry in respect of the above was taken into account when worksheets and the presentation of activities were prepared. One deduced that learners have a serious backlog with regard to geometry. Taking into account the fact that most disadvantaged learners lack informal knowledge and the fact that they have not been exposed to an environment that could enable them to invent their own problem solving strategies. The researcher did more than assigning a problem to learners (Ramnarain 1999:7). The content of the worksheets was first explained to them. The researcher was not only confined to teaching geometry through the process-based approach, but he took into account, amongst other things, the fact that learners lack drawing and communication skills.

4.4 The school timetable

The empirical investigation began on the 4th August 1999 instead of the 2nd August 1999 and continued until 23rd September 1999. Initially, the researcher was granted permission to conduct empirical investigation from 12h30 to 13h30. The school day began at 7h30 and ended at 13h30. These arrangements lasted for only two weeks after some learners complained to the researcher that they were missing lessons in other subjects by attending the empirical investigation. After the principal had sought permission from their parents, it was agreed that empirical investigation would be conducted as from 13h30 to 14h30.
4.5 Examples of activities and their objectives

The table below shows the dates on which activities were performed, their objectives, as well as the execution media used. This means that in certain activities fewer execution media were used.

The researcher conducted the empirical investigation as follows:

For all activities, learners were given work sheets together with the acetate sheet or transparent papers or mirrors. The type of manipulative given to learners depended on the nature of the activity. The researcher explained to the learners what the activity was all about after he had realised that they could not interpret the worksheet. Learners were told what they were expected to do. They later performed the activities as individuals within a group. They showed and explained their results to one another.
<table>
<thead>
<tr>
<th>DATE</th>
<th>MEDIA</th>
<th>ACTIVITY</th>
<th>OBJECTIVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>04-08-1999</td>
<td>• 'Showed with and used</td>
<td>Activity 1: SORTING SHAPES</td>
<td>• The activity assessed learners’ familiarity with the concepts of similarity and congruency of shapes intuitively.</td>
</tr>
<tr>
<td></td>
<td>hands’</td>
<td></td>
<td>• To extend the concept of similarity and congruency to 3-dimensional shapes.</td>
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<tr>
<td></td>
<td>• Writing</td>
<td></td>
<td>• To introduce learners to a culture of working in groups and to be able construct knowledge by themselves.</td>
</tr>
<tr>
<td></td>
<td>• Talking</td>
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<tr>
<td>Wednesday</td>
<td>12.30-13.30</td>
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</tr>
<tr>
<td>05-08-1999</td>
<td>• Talking</td>
<td>Activity 2: DRAWING AND NAMING SHAPES</td>
<td></td>
</tr>
<tr>
<td>Thursday</td>
<td>• Writing</td>
<td></td>
<td>• To recognise geometric shapes.</td>
</tr>
<tr>
<td>12.30-13.30</td>
<td>• Drawing</td>
<td></td>
<td>• To draw shapes.</td>
</tr>
<tr>
<td></td>
<td>• ‘Showed with and used</td>
<td></td>
<td>• To write names of shapes.</td>
</tr>
<tr>
<td></td>
<td>hands’</td>
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<tr>
<td></td>
<td>• Writing</td>
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<tr>
<td></td>
<td>• Talking</td>
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<td></td>
<td>12.30-13.30</td>
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</tr>
<tr>
<td>06-08-1999</td>
<td>• ‘Showed with and used</td>
<td>Activity 3: CONSTRUCTION OF ANGLES</td>
<td>• To form and recognise different types of angles.</td>
</tr>
<tr>
<td>Friday</td>
<td>hands’</td>
<td></td>
<td>• To draw and mark different types of angles.</td>
</tr>
<tr>
<td>12.30-13.30</td>
<td>• Talking</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Writing</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>• Drawing</td>
<td></td>
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<tr>
<td></td>
<td>12.30-13.30</td>
<td></td>
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<tr>
<td>DATE</td>
<td>MEDIA</td>
<td>ACTIVITY</td>
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<tr>
<td>11-08-1999</td>
<td>12.45-13.45</td>
<td>ACTIVITY 4: MEASUREMENT OF ANGLES</td>
<td>They were assessed as to whether they could measure different types of angles using a protractor.</td>
</tr>
<tr>
<td>Wednesday</td>
<td></td>
<td>Learners were expected to measure angles they drew in activity 3 by means of a protractor.</td>
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<tr>
<td></td>
<td>'Showed with and used hands'</td>
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<tr>
<td></td>
<td>Writing</td>
<td></td>
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<tr>
<td>12-08-1999</td>
<td>12.45-13.45</td>
<td>ACTIVITY 5: RECOGNITION OF ANGLES AND SUPPLEMENTARY/ADJACENT ANGLES</td>
<td>The activity assessed</td>
</tr>
<tr>
<td>Thursday</td>
<td></td>
<td>Learners were given a Parallelogram Tiles Worksheet. They were asked to recognize different types of angles from it.</td>
<td>• Learners' knowledge of different types of angles they gained in activity 4.</td>
</tr>
<tr>
<td></td>
<td>'Showed with and used hands'</td>
<td></td>
<td>• Measurement of adjacent angles.</td>
</tr>
<tr>
<td></td>
<td>Talking</td>
<td></td>
<td>• supplementary angles.</td>
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<td></td>
<td>Writing</td>
<td></td>
<td>• Learners could calculate an unknown angles if some were given.</td>
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<tr>
<td>13-08-1999</td>
<td>12.45-13.45</td>
<td>ACTIVITY 6: BISECTED ANGLES</td>
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<tr>
<td>Friday</td>
<td></td>
<td>Learners were to draw an acute angle and obtuse angle on transparent paper. They were also to fold the two angles so that the lines that constitute angles coincide. They opened folded angles and traced the folded line by a pen or pencil. The angles that were formed as a result of the folded were marked and compared. Comparing angles was done by folding them and exposed the paper to sunlight to check if they were equal.</td>
<td>• Learners should notice that the folded line divides an acute or obtuse angle equally.</td>
</tr>
<tr>
<td></td>
<td>'Showed with and used hands'</td>
<td></td>
<td>• Drawing bisected angles accurately.</td>
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<td>Writing</td>
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<td></td>
<td>Drawing</td>
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</tbody>
</table>
| 16-08-1999 | • Showed with and used hands'  
             | • Talking                   | ACTIVITY 7: PERPENDICULAR LINES | Learners should construct, and draw perpendicular lines. |
| Monday     | • Writing                  | Learners were asked to draw a line on the transparent paper and fold it in such a way that one end of the line meets the other. When one looked through the paper one saw a single angle. Learners were asked to tell what the fold did to the drawn line. They were also asked to mention measurement of angles formed by the drawn line and by the folded line. They were further asked to trace the folded line by means of a pencil. They were also to draw perpendicular lines on the work sheet. | |
| 13.30-14.45| • Drawing                  |                               |                                               |
| 17-08-1999 | • Showed and used with hands  
<pre><code>         | • Talking                   | ACTIVITY 8: PARALLEL LINES                  | Learners were to, construct parallel lines, draw parallel lines. |
</code></pre>
<p>| Tuesday    | • Writing                  | This activity was based on activity 7. The transparent paper with perpendicular lines was once again used. The activity begun by folding one of the perpendicular lines. The new folded line was to be traced. All three lines were given names according to letters of the alphabets. Learners were requested to draw parallel lines on the worksheets. | |
| 13.30-14.45| • Drawing                  |                               |                                               |</p>
<table>
<thead>
<tr>
<th>DATE</th>
<th>MEDIA</th>
<th>ACTIVITY</th>
<th>OBJECTIVE</th>
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</thead>
<tbody>
<tr>
<td>17-08-1999</td>
<td>13.30-14.45</td>
<td><strong>ACTIVITY 9: ASSESSMENT TASK- PARALLEL AND PERPENDICULAR LINES</strong></td>
<td>Learners should recognise parallel and perpendicular lines (both skewed and not skewed). They should also be able to write lines that were perpendicular and parallel using letters of alphabets and symbols.</td>
</tr>
<tr>
<td>Tuesday</td>
<td>Talking</td>
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<tr>
<td></td>
<td>Writing</td>
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<td></td>
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<td></td>
<td>Drawing</td>
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<tr>
<td>18-08-1999</td>
<td>13.30-14.45</td>
<td><strong>ACTIVITY 10: VERTICAL OPPOSITE ANGLES</strong></td>
<td>Learners should be able to</td>
</tr>
<tr>
<td>Wednesday</td>
<td>‘Showed with and used hands’</td>
<td></td>
<td>• Tracing vertical opposite angles.</td>
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<tr>
<td></td>
<td>Talking</td>
<td></td>
<td>• Rotate intersecting lines.</td>
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<tr>
<td></td>
<td>Writing</td>
<td></td>
<td>• Demonstrate or explain which were equal</td>
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<td></td>
<td>Drawing</td>
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<tr>
<td>23-08-1999</td>
<td>13.30-14.45</td>
<td><strong>ACTIVITY 11: CORRESPONDING ANGLES</strong></td>
<td>Learners should be able to construct corresponding angles. They should know that corresponding come as a result of parallel lines.</td>
</tr>
<tr>
<td>Monday</td>
<td>‘Showed with and used hands’</td>
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<tr>
<td></td>
<td>Writing</td>
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</tr>
<tr>
<td>24-08-1999 Tuesday</td>
<td>Writing</td>
<td>ACTIVITY 12: CO-INTERIOR ANGLES</td>
<td>Learners should discover that angles between parallel lines and above/below the transversal line add to 180°.</td>
</tr>
<tr>
<td>13.30-14.45</td>
<td>'Showed with and used hands'</td>
<td>Learners were to trace supplementary angles (marked differently) from the worksheet on the acetate sheet. Learners were asked to determine the sum of two angles that were adjacent to each other. The supplementary angles on the acetate sheet (also marked in the same way as those on the worksheet) were translated along the horizontal line on the worksheet. They were asked to determine the sum of angles between the two parallel lines. They were further asked to give reasons for their answers.</td>
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</tr>
<tr>
<td></td>
<td>writing</td>
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</tr>
<tr>
<td>25-08-1999 Wednesday</td>
<td>'showed with and used hands'</td>
<td>ACTIVITY 13: ALTERNATE ANGLES</td>
<td>- Learners should be able to demonstrate how they found alternate angles to be equal.</td>
</tr>
<tr>
<td>13.30-14.45</td>
<td>writing</td>
<td>Learners were given a worksheet with an acute angle. They were to trace the acute angle on the acetate sheet. A pin was placed on one of the lines that constitute the acute angle. The angle on the acetate sheet was rotated about the pin through 180°. Learners were asked to comment on the relationship between lines.</td>
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</tr>
<tr>
<td>30-08-1999 Monday</td>
<td>Writing</td>
<td>ACTIVITY 14: ASSESSMENT TASK</td>
<td>Learners should be able to apply knowledge of alternate angles corresponding angles and co-interior angles to solve problems.</td>
</tr>
<tr>
<td>13.30-14.45</td>
<td></td>
<td>Learners were given assessment task to work on. It dealt with the application of knowledge of alternate angles, corresponding angles and co-interior angles.</td>
<td></td>
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<tr>
<td>DATE</td>
<td>MEDIA</td>
<td>ACTIVITY</td>
<td>OBJECTIVE</td>
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</tr>
<tr>
<td>31-08-1999</td>
<td>'Showing and used with hands’</td>
<td>ACTIVITY 15: ISOSCELES TRIANGLE</td>
<td>Learners should find angles and sides of the isosceles triangle that are equal to each other by folding it.</td>
</tr>
<tr>
<td>Tuesday</td>
<td>Talking</td>
<td>An isosceles triangle was cut from a folded rectangle A4 paper. It came out of the rectangle paper as two right-angled triangles. Learners were asked to find out sides and angles of the triangle that were equal to each other by it. They were also requested to draw the isosceles triangle.</td>
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<tr>
<td>13.30-14.45</td>
<td>Writing</td>
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<td></td>
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<tr>
<td>13.30-14.45</td>
<td>Talking</td>
<td></td>
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<tr>
<td>13.30-14.45</td>
<td>Writing</td>
<td></td>
<td></td>
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<tr>
<td>13.30-14.45</td>
<td>Drawing</td>
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</tr>
<tr>
<td>08-09-1999</td>
<td>'Showed with and used hands’</td>
<td>ACTIVITY 16: EQUILATERAL TRIANGLE</td>
<td>Learners should find angles and sides of the equilateral triangle that are equal to each other by folding it.</td>
</tr>
<tr>
<td>Wednesday</td>
<td>Talking</td>
<td>The equilateral triangle was also cut from a folded rectangle A4 paper. It also came out as two right-angled triangles. However, the hypotenuse was double one of the two sides. Learners were asked to determine sides and angles of the triangle that were equal paper by folding it. They were also requested to draw the equilateral triangle.</td>
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<tr>
<td>13.30-14.45</td>
<td>Writing</td>
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<td>13.30-14.45</td>
<td>Talking</td>
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<td>13.30-14.45</td>
<td>Writing</td>
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<tr>
<td>13.30-14.45</td>
<td>Drawing</td>
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<tr>
<td>09-09-1999</td>
<td>'Showed with and used hands’</td>
<td>ACTIVITY 17: ASSESSMENT TASK-ISOSCELES TRIANGLE</td>
<td>Learners should describe properties of the isosceles triangle.</td>
</tr>
<tr>
<td>Thursday</td>
<td>Talking</td>
<td>Learners were given mirrors and work sheets with a right-angled triangle. The hypotenuse was marked with double stroke sign. A mirror was placed on the side the hypotenuse of the triangle. Learners were asked to mention what they have seen in the mirror. They were further asked to mention the name of triangle that has been formed by the triangle and mirror image.</td>
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<tr>
<td>13.30-14.30</td>
<td>Writing</td>
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<td>13.30-14.30</td>
<td>Talking</td>
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<tr>
<td>13.30-14.30</td>
<td>Writing</td>
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</tr>
<tr>
<td>10-09-1999</td>
<td>Showed with and used hands’</td>
<td>ACTIVITY 18: SUM OF ANGLES OF A TRIANGLE</td>
<td>Learners were to discover that the sum of angles of any triangle was equal to 180°. They were also expected to apply the theorem to solve problems.</td>
</tr>
<tr>
<td>Friday</td>
<td>Talking</td>
<td>Learners were asked to cut triangle ABC that they had drawn. Angles of the triangle were traced around the point on the straight-line angle. The traced angles were given the same names as those the triangle. Learners were given assessment task that involved calculating an unknown angle.</td>
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<tr>
<td>13.30-14.45</td>
<td>Writing</td>
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<tr>
<td>13.30-14.45</td>
<td>Talking</td>
<td></td>
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<tr>
<td>13.30-14.45</td>
<td>Writing</td>
<td></td>
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<td>DATE</td>
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<tr>
<td>13-09-1999</td>
<td>'Showed with and used hands'</td>
<td><strong>ACTIVITY 19: ANY EXTERIOR ANGLE OF A TRIANGLE IS EQUAL TO THE SUM OF ITS TWO OPPOSITE INTERIOR ANGLES</strong></td>
<td>Learners were asked to cut any shape of a triangle and label its angles 1, 2 and 3. A piece of the triangle was traced on the worksheet with the angles labelled like the original one. The horizontal side of the triangle on the worksheet was produced to form an exterior angle. The exterior angle was labelled 4. Learners were to trace any of the two of three interior angles in the exterior angle. Learners were to determine which two interior angles of the triangle made the exterior angle. Learners were also given an assessment task to work on. It was mainly comprised of calculation problems. Learners should investigate which two of three interior angles of a triangle were equal to exterior angle. They should also be able to calculate unknown angles. They were also expected how they had arrived their conclusion.</td>
</tr>
<tr>
<td>Monday 13.30-</td>
<td>Talking</td>
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<tr>
<td>14.45</td>
<td>Writing</td>
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<td>Drawing</td>
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<tr>
<td>14-08-1999</td>
<td>'Showed with and used hands'</td>
<td><strong>ACTIVITY 20: CONGRUENCY OF TRIANGLES</strong></td>
<td>Part 1: Learners cut an A4 paper across its pair of opposite angles. They compared size of pair triangles. Part 2: Learners placed a mirror on one side of the triangle on the worksheet. They were three types of triangles on the worksheet. Two of their sides were marked differently. They compared each triangle with its mirror image in terms of their sizes. To determine conditions under which triangles can be congruent. To compare triangles and their respective parts.</td>
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<tr>
<td>Tuesday 13.30-</td>
<td>Talking</td>
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<tr>
<td>14.45</td>
<td>Writing</td>
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</tbody>
</table>
| 15-09-1999 Wednesday 13.30-14.45 | • 'Showed with and used hands'  
• Talking  
• Writing  
• Drawing | ACTIVITY 21: OPPOSITE SIDES/ANGLES OF A PARALLELOGRAM ARE EQUAL TO EACH OTHER.  
Learners were given a work sheet with a quadrilateral. They were asked to trace it on the acetate sheet. They were to show that it was a parallelogram by rotating the acetate sheet through 180°. Learners were to compare the length of the sides of the parallelogram on the work sheet to those on the acetate sheet. They were also to compare angles. They were expected to record their observation about length sides and angles of the parallelogram. Learners were also given an assessment task. It assessed them on properties of the parallelogram. | Learners should investigate whether opposite sides/angles of the parallelogram were equal through rotation. They should demonstrate knowledge of the parallelogram by solving problems. |
| 16-09-1999 Thursday 13.30-14.45 | • 'Showed and used proof  
• Talking  
• Writing  
• Drawing | ACTIVITY 22: DIAGONALS OF A PARALLELOGRAM BISECT EACH OTHER  
Learners were given a work sheet with a parallelogram PQRS. It has diagonals PQ and QS intersecting at T. They were asked to trace the parallelogram together with PT and QT on the acetate sheet. The parallelogram on the acetate sheet was rotated through 180°. Learners were asked to compare the length of PT and length of TR on the work sheet. They were also to compare lengths of QT (on the acetate sheet) and TS (on the work sheet). Learners were expected to record their observation on the worksheet.  
They were further asked to mark the segments of the diagonals that were equal. They were also evaluated on their knowledge of the diagonals of the parallelogram by doing calculation problems. | Learners should be able to investigate if the diagonals of the parallelogram bisect each other. They should be able to write proof |
<table>
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<th>DATE</th>
<th>MEDIA</th>
<th>ACTIVITY</th>
<th>OBJECTIVE</th>
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</thead>
<tbody>
<tr>
<td>17-09-1999 Friday 12:30-13:30</td>
<td>Writing</td>
<td>ACTIVITY 23: ASSESSMENT TASK ON PARALLELOGRAM Learners were given a worksheet containing problems involving properties of a parallelogram. Problems involved calculation and proof writing.</td>
<td>Learners should be able to investigate if the diagonals of the parallelogram bisect each other. They should be able to write proofs.</td>
</tr>
</tbody>
</table>

**TABLE 4.1. PROGRAMME OF DATES, EXECUTION MEDIA, ACTIVITIES AND THEIR OBJECTIVES**

The researcher provided transport to ferry learners (who came far from the school) to their respective homes after completion of the activities. There were learners whose transport leaves the school at 13h30. Consequently they could not afford to pay for any other transport because they had already bought a monthly ticket for their own transport.

This delayed the empirical investigation by an average of 10 minutes, due to the fact that learners had to sweep classrooms at 13h30. As a result learners knocked off 15 minutes late after they had completed the activity and the interview of the day. Sometimes the empirical investigation was disrupted by school activities such as excursions, extramural activities and religious gatherings, which the researcher did not know beforehand. Empirical investigation was conducted for one and half months.

### 4.6 Research methods.

The researcher used the qualitative and quantitative methods to collect and analyse data. The qualitative method was found suitable to the empirical investigation because it enables the researcher to describe the learners' behaviour at first-hand. It further enables the researcher to identify learners' learning problems. It does not solely rely on the written information and questionnaire, but it also enables the researcher to understand learners' reasoning through interviews (UNISA 1997). Through this method the researcher was not only able to know how learners performed in geometry but he was also able to know reasons for learners' poor or better performance. This method
enabled the researcher to observe learners closely as they worked on their worksheets. Probing questions from the researcher enabled learners to identify their own mistakes or to justify their own actions. On the other hand, the quantitative method enabled the researcher to determine an average achievement of an individual learner using various execution media (talking, 'showed with and used hands', writing and drawing) to communicate thought, to form concepts and to master theorems (see Appendix B). This method further enabled the researcher to determine the extent to which learners were able to communicate thought, to form concepts and master theorems using the four execution media in each activity. It also enabled the researcher to determine whether an individual learner was good at communicating thought or concept formation or mastering theorems. They were also interviewed during and towards the end of each session. Individual learners were asked to give reasons for their actions during each activity.

4.6.1. Sampling techniques

The researcher used the stratified random sampling technique to obtain the sample (Leedy 1989: 209). The technique was chosen due to the fact that two grades were involved. The researcher selected five learners from each grade; namely grade 8C and 9B. There were 57 and 77 learners respectively. Every eleventh and fifteenth learner was selected from grades 8C and 9B respectively. Their performance in the examination did not influence the selection of the sample. There were four grade 8 and five grade 9 classes at the school.

The stratified random sampling ensured that all grades were equally represented in the sample (Leedy 1989: 209). This level of sampling technique is called equalisation. A model for the stratified sampling technique is as follows:
Ten learners from the two grades constituted the total sample. The choice of a small sample was due to the fact that the researcher wanted a group of learners that would be manageable within an hour. The duration for interviews and activities was also taken into account when this decision was made.

The average age of learners in both grades was 15. All learners in grade 8C were over age, except one. In grade 9B there were three learners who were over age. Their ages are as they appear in the table below. There was one girl and four boys from grade 8C. Grade 9B had two girls and three boys (see Table 4.2)

<table>
<thead>
<tr>
<th>GRADE</th>
<th>LEARNER'S NAME</th>
<th>GENDER</th>
<th>AGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>8C</td>
<td>A</td>
<td>Male</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>Female</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>Male</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>Male</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>Male</td>
<td>13</td>
</tr>
<tr>
<td>9B</td>
<td>F</td>
<td>Male</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>Male</td>
<td>17</td>
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<td></td>
<td>H</td>
<td>Female</td>
<td>14</td>
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<tr>
<td></td>
<td>J</td>
<td>Female</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>Male</td>
<td>16</td>
</tr>
</tbody>
</table>

TABLE 4.2. COMPOSITION OF THE SAMPLE

This sample did not take into account the question of gender after equalisation of grades simply because of the application of the random sampling technique. Hence, there are more male than female learners.
It should be noted that all learners were subjected to the grade 8-geometry syllabus. Grade 8 learners were dropped after the completion of their syllabus. The researcher continued with grade 9 learners. They did activities involving congruency and parallelograms. In this case the researcher wanted to help learners to have a smooth transition into grade 10-geometry syllabus. Hence, the emphasis was on congruent triangles and parallelograms.

4.7 Instruments used to collect data and analyse information

4.7.1 Worksheets

The researcher designed twenty-three worksheets. They were designed in such a way that learners wrote their observations on them. It was also taken into account that they were meant for second language speakers. The first two activities were used as ‘warmers’. Not all activities were based on transformations. Some activities were based on geometry riders that learners were expected to solve. They were to evaluate learners’ knowledge of theorems, concept formation and the ability to communicate thought. Every worksheet was designed on the basis of learners’ previous performance. In some cases the same activity was repeated in another form, if it had produced poor responses.

All activities were based on the discovery of the following concepts and theorems: construction and measurement of angles; bisection of angle; parallel lines; perpendicular lines; sum of angles of a triangle is 180°; exterior angle of a triangle is equal to the sum of its two opposite interior angles; congruency axioms; opposite sides of a parallelogram are equal and parallel; opposite angles of a parallelogram are equal and the diagonals of a parallelogram bisect each other. The following resource materials were used in different activities: scissors, papers, acetate sheet, radio cassette, transparency pens and mirrors. Learners went through the following stages when they were taking part in the activities.
The progression structure of the worksheets was as follows:

- Recognition of shapes. (Van Hiele level 1)
- Investigation of properties of shapes through drawing, paper folding, reflection, rotation and translation. (Van Hiele level 2)
- Description of the relationships between shapes and writing of proofs. (Van Hiele level 3)

Learners recorded their observations on the worksheet.

Worksheets were also organise such that activities deals with comparing three-dimensional shapes and activity 2 deals with drawing of two and three-dimensional shapes. The rest of the activities deal with measurements.

4.7.2 Notebook

The researcher observed learners' work and their actions as they perform activities. In order to get clarity concerning their actions the researcher intervened by asking probing questions. The researcher took note of what learners were able to do or not able to do during the activities and formal interviews. These notes were used to improve the activity for the following day and to analyse learners' performance. Records of learners' activity were kept in the researcher's notebook.

4.7.3 Interviews.

Interviews were based on three premises, namely,

- Prepared questions
- Observations during the activities
- Learners' responses to questions in the worksheets.
Learners were asked questions during an activity either to get clarity or to make them aware of their mistakes. These questions helped them to express their conviction or to rectify their own mistakes. Formal interviews took place after the actual activity had been completed. All learners were interviewed in each activity. This excluded the activities where leaders of the two groups were interviewed. Typical questions in the interview were:

- What did you notice in the experiment?
- Show how you found the theorem.
- Why do you think the theorem is true?

Questions were rephrased in view of the difficulty experienced by the researcher when he communicated with learners in English. In other words, they were not asked questions purely as they appear above. The researcher had difficulties in using the audiocassette. As a result certain information was lost. Interviews of activities 11, 12, 13 and 14 were mistakenly erased because interviews of other activities were recorded on them. Fortunately, the researcher had already recorded information from the tape in the notebook before it was erased. However the researcher failed to transcribe the activities at a later stage. The duration for the actual activity varied from 30 to 45 minutes, while the interviews varied from 5 to 10 minutes. Interviews were recorded on the audiocassette. Noise from the siren, cars and learners outside the classroom interfered with the recording.

4.7.4 The role of the researcher

The researcher played the role of a facilitator during the empirical investigation. The researcher avoided helping learners by giving them answers when they experienced problems with their work. Learners were asked probing questions in order that they should be able to determine whether they were on the right track.

Initially, the researcher expected learners to read and understand instructions on the
worksheet on their own; instead, he had to explain to them what the activities were all about. At no stage were learners able to understand instructions on the worksheets on their own. Out of ten learners only two could express themselves in English. They were better than the rest of the group. The researcher was convinced that learners did not understand instructions because they did not have command of English. In order to insure that they are able to carry out activities verbal instructions was accompanied by demonstrations. There were two instances where the researcher used the learners' mother tongue to clarify certain instructions after the researcher repeatedly explained in English without success. It is worth noting that the fact that English is a second language to the researcher and learners might have had a bearing on the outcome of the empirical investigation.

Attempts were made to demonstrate activities to learners without telling them the results. They were expected to imitate and record observations.

4.7.5 Rubrics and score grid (card)

Four-point rating scale rubrics were developed according to the aim and the research objectives. They were meant to analyse learners' performance in terms of three competences (communicating thought, concept formation and mastering theorems). Learners' performance was analysed on how they communicated thought, formed concepts and mastered theorems by execution media ('showed with and used hands', talking, drawing and writing).

Two sets of rubrics were used to analyse learners' performance in Euclidean geometry. One was meant for grade 8 learners and the other for grade 9 learners. They differed in that the grade 9 rubrics assessed one and two more concepts and theorems respectively. They were developed to assess learners' performance in all 23 activities. Learners' scores were entered into a separate score grid (card) with columns of 23 activities. Letters of the alphabet were used to represent learners' names. Learners of each grade appear in a single score grid. The score grid was used to enter
scores in respect of a learner's ability to communicate thought, to form concepts and to master theorems. The researcher notebook, learners' worksheets and audiocassettes were used to score learners' performance. An individual learner was assessed and scores were awarded (based on the rubrics) to him/her for every activity. The score grid was structured in such a way that learners were assessed on how well they performed by using execution media to communicate thought, to form concepts and to master theorems. An average score for each execution medium of an individual learner or a group (grade 8 or 9 learners) was calculated in terms of three competences (see appendix B) for all activities. The raw data from the score grid was processed by computer to produce results of the learners' performance in terms of line and bar graphs.

4.7.6 The reliability of data

Reliability in data collection was ensured by both the researcher and the supervisor when they jointly scored learners' performance. The supervisor and the researcher jointly ensured learners' performance using the rubrics and scorecard. They both obtained similar scores.

The reliability of the quantitative data is established through consistency. In other words, if similar data is found by different, but equally qualified researchers, the data is considered to be reliable (Charles 1998:151).

The reliability of qualitative data is difficult to establish objectively since it involves internal criticism. This is to ensure that data has credibility (Charles 1998:150-151).

4.8 Conclusion

The empirical part of the investigation was conducted at Ngweni Secondary School. It is situated in the rural township of Dzanani (Makhado Town) in the Limpopo Province. It has an acute shortage of classrooms. Each classroom accommodates no less than fifty learners. Its set-up was not conducive to the application of process-oriented
approaches such as constructivism. As a result, teachers use the lecture method to teach mathematics and other subjects.

The researcher used the qualitative quantitative methods to collect and analyse data. The stratified random sampling technique was used to select grades. The simple random sampling was further used to sample learners from each grade.

It was also pointed out that all learners were Africans. They live in surrounding villages. They receive mathematics instruction in English. Most of them do not have a command of English. This has been found to be an obstacle to learners to follow instructions on the worksheets. The role of the researcher was that of a facilitator.

The researcher outlined difficulties experienced while conducting empirical investigation at the school. Empirical investigation had to be rescheduled as indicated above. There was a time that the researcher felt pressure from the school that the empirical investigation should be completed quickly. Although, it was not mentioned the researcher was no longer receiving cooperation as before.

Twenty-three worksheets were used during the empirical investigation. Interviews were recorded on audiocassette. Observations by the researcher during activities, and interviews formed part of the information collected. Learners were assessed using rubrics and score grid on how they could communicate thought, form concepts and master theorems using four execution media. The subsequent chapter will deal with the analysis of how learners learned Euclidean geometry through process-based approach. The chapter will deal with the analysis of learners’ results of how they communicated thought, formed concepts and mastered theorems using the execution media: ‘showed with and used hands’, drawing, writing and talking.
CHAPTER FIVE

AN ANALYSIS OF THE EMPIRICAL INVESTIGATION OF TEACHING AND LEARNING EUCLIDEAN GEOMETRY IN PROCESS-BASED INSTRUCTION

5.1 Introduction

This chapter will mainly deal with the results of the performance of grade 8 and 9 learners. Analysis will be made of how learners performed in communicating thought, concept formation and mastering of theorems through the execution media (writing, drawing, 'showed with and used hands' and talking). The results of the performance of the individual learners in each grade, in respect of competencies, using four execution media will be discussed.

This chapter will attempt to answer the following research questions.

To what extent will the process-based instruction (PBI) enable learners to communicate thought, to form concepts and to master theorems by
• Writing
• Drawing
• 'showed with and used hands'
• Talking?

This chapter will further analyse the average performance of each group of learners (grade 8 and 9) in respect of the three competencies by using the execution media for all activities. The average performance of each group in respect of three competencies in each activity will be discussed. The emphasis will be on the extent to which each group developed the three competencies by using four execution media. A four point rating scale was used to measure learners' performance.

An analysis will also be made of the extent to which an individual learner or a group of learners (grade 8 or 9 learners) was competent in communicating thought, forming
concepts and mastering theorems. The results of the grade 8 and 9 learners will be compared in respect of: communicating thought, concept formation and mastering theorems for all activities. Graphs representing learners' performance were calibrated using decimals; their performance was calculated in terms of an average. As a result their performance should be interpreted as follows:

0-1: No communication of thought took place; no concepts were formed; no theorem was mastered.
1,1- 2: Communicating thought took place after receiving assistance; a few concepts were formed; theorems partly mastered.
2,1- 3: Communicating thought but meaning can only be sifted; a substantial number of concepts were formed; theorems mastered with less difficulty
3,1- 4: Communicating thought fully; all concepts were formed; all theorems mastered with ease.

5.2 Communicating thought

Communicating thought in this study refers to the learner's ability to express himself by written or spoken word, using parts of the body and drawing. Both execution media may be used to communicate thought depending on the nature of the activity. In other words, two or more of these execution media may be used in a single activity. This section deals with the extent to which meaning was conveyed using the four execution media. The researcher will first analyse the results of empirical investigation of each individual learner for all activities. Emphasis will also be on the extent to which meaning is conveyed by each execution medium to communicate thought.

Lastly, focus will be on analysing the extent to which each group conveyed meaning per activity by one or more execution media.
5.2.1 Communicating thought by execution media for an individual grade 8 learner for all activities (see chart 5.1)

According to chart 5.1, learner A could communicate thought by drawing. In other words, a substantial number of diagrams were recognisable by the markings on them. She could not communicate thought by 'showed with and used hands'. That is, she was partly assisted to trace, cut, fold, translate, rotate, compare and sort shapes. Initially, she could not handle a pair of scissors to cut a smooth-edged triangle. Her main barrier to communicating thought was her inability to understand (speak or write) English. Hence, she performed below average in writing and talking (see chart 5.1). She wrote meaningless statements.

Learner B could communicate thought by drawing. In other words, he could draw shapes that were recognisable. He could also use his hands well to demonstrate his knowledge of geometric concepts. There were instances in which he could not
describe what parallel lines were. Instead he stretched his arms like a railway line and said: "I will say they are lines that are opposite each other".

He wrote statements whose meaning could only be sifted. This could be attributed to poor command of English. Moreover, he could only write the first mathematical statement in a solution. He could not go beyond this. However chart 5.1 shows that he performed slightly above average in talking. For example, he reported back to his group as follows:

Researcher: What makes shapes look alike?
B: As a team I used the colours. Again I used shape and size.
Researcher: How did you find out that shapes have equal size?
B: Like this (putting shapes together by their sides) I matched them.

Although he spoke a slightly broken English, the message was meaningful. The interviews below support this statement.

Researcher: What makes you say that AB and CD are parallel?
B: (Looking at the parallelogram Tile worksheet) Because———
Researcher: Ok, if somebody asked you, what parallel lines are, what would you say?
B: (Stretching his arms like railway lines) I will say they are lines that are opposite to each other.

This shows that he performed better in talking than in writing. Writing poor mathematical statements disadvantaged him.

Leaner C could communicate thought by 'showed with and used hands'. In other words, he was partly assisted in tracing, cutting, folding, translation, reflection, comparing, rotating and sorting of shapes. He was able to write mathematical statements with appropriate symbols. However his command of English was very poor. He could not write a correct full sentence. Whatever he wrote lacked coherence. He
was able to draw a few shapes. In other words, a few shapes were recognisable. He did not mark sides or angles of most shapes to show that they were equal or to show the direction of measurements. The discrepancy between his performances in communicating thought by talking as opposed to writing was due to the fact that most mathematical statements that he wrote were correct.

Learner D could communicate thought by ‘showed with and used hands’. In other words, he was able to trace, cut, fold, translate, reflect, compare, rotate and sort shapes. There were a substantial number of diagrams that were recognisable by the markings on them. However he performed below average (see chart 5.1) in communicating thought by writing and talking. He wrote meaningless sentences. For example, he was expected to respond in writing to a question that says: “What makes shapes look alike?” He responded as follows:

‘Because they look the same, are the same on colour’

The responses lacked coherence and meaning. He could have performed well in the medium of writing if he had written mathematical statements with reasons. It was clear from what he wrote that he knew what he was writing.

Learner E could communicate thought by either execution medium. This means that he could trace, reflect, fold, rotate, translate, compare and sort shapes after receiving assistance. A few diagrams could easily be recognised by the markings on them. However he could not communicate thought either through writing or talking. He wrote incoherent mathematical statements and sentences.
5.2.2 Communicating thought by execution media for the individual grade 9 learners for all activities (see chart 5.2)

Learner F could communicate thought by drawing and 'showed with and used hands'. This means that a substantial number of diagrams were recognisable by the markings on them. He could draw and trace most diagrams. He was the only learner who was able to draw a three-dimensional shape. He could cut, fold, trace, translate, rotate, reflect, compare and sort shapes. His written sentences were short. He did not write full sentences where it was expected of him to do so. However most of his mathematical statements were logical. They were also accompanied by appropriate reasons. He struggled to add like terms, when solving algebraic problems in geometry. He could not properly express himself verbally in English. In most cases, he was just providing a one-word answer or a number (measurement).

Learner G could not communicate thought by either execution medium. A few shapes (diagrams) were recognisable by the markings on them. In the construction and measurement of angles, she marked angles correctly. However she wrote a wrong
measurement (see chart 5.2). Obtuse and reflex angles were a case in point. She could communicate thought by ‘showed with and used hands’. This was only done after receiving assistance from the researcher. She received a lot of assistance in using a protractor to measure angles. She was also assisted in bisecting a straight-line angle and constructing perpendicular lines. She often wrote wrong names of shapes or measurement of angles. Most of her statements were characterised by short sentences. However there were a few good sentences. Hence, she performed below average. Her verbal statements were characterised by either short sentences or by a one-word answer. She did write some full sentences.

Learner H could communicate thought by drawing. He drew a substantial number of diagrams that were recognisable. However he failed to draw a three-dimensional shape (ball). His diagrams were neat and accurate. He could also communicate thought by ‘showed with and used hands’ after assistance was given by the researcher. He received assistance in measuring angles, translating acetate sheet (corresponding angles), congruency of triangles and exterior-interior angle theorem. He often wrote symbols inappropriately, for example, angle AB=CD. Nevertheless he wrote mathematical statements that were meaningful. They were always accompanied by an appropriate reason. Sometimes he would not respond to questions. He would just stare at the researcher. He would respond after the researcher spoke in the mother tongue. His command of English improved in subsequent interviews.

Learner J could not communicate thought by either execution medium (see chart 5.2). That is, she was assisted in using a protractor to measure all types of angles. She was assisted in constructing perpendicular lines and in placing a second angle of a triangle into its exterior angle. However she was able to use ‘showed with and used hands’ to communicate thought and meaning in most activities. Shapes that she drew were recognisable. She could not trace the triangle and its exterior properly. Angles of a triangle were curved. She was not accurate in tracing a folded line bisecting the angle. She failed to draw a triangle (isosceles). In general, her diagrams lacked accuracy. She excelled in writing mathematical statements. Most of the statements were
meaningful. In those cases that she was expected to explain or describe the results of an activity, she simply used a two-word sentence for example, ‘are equal’. One has to sift meaning in those cases where she wrote full sentences. However she was very confident when she was talking in English. This did not mean that she spoke good English. Her English was not good enough, but one could easily understand what she wanted to say. In comparison to other grade 9 learners, she was the best in talking.

Learner K could communicate thought by drawing diagrams (see chart 5.2). This means a substantial number of diagrams were recognisable by the markings on them. However he could not draw a three-dimensional shape (a ball). He drew it as a circle. He ‘demonstrated’ how angles could be measured by using a protractor after receiving assistance. He could trace, cut, fold, translate, rotate and sort shapes after receiving assistance. Paper folding could not help him recognise properties of an equilateral triangle. He wrote sentences that were full of grammatical errors. For example,

1. “All sum of the angle are 180° degree, because there are straight lines”.
2. “The size that it is inside are equal in angle that on out site”.

Although the above sentences represented most of his statements, there were also four positive statements. For example,

1. “I can see that there is another triangle”
2. “Two vertical lines are parallel”

The two statements were meaningful.

The fact that he was able to write meaningful mathematical statements when solving problems overshadowed the negative side of his written work. He wrote mathematical statements in such a way that he showed insight into the problem. There was logical arrangement of statements.
In the first place learner K stammered. Secondly, he could not construct a full meaningful sentence. Hence, he performed below average in communicating thought by talking (see figure 5.2).

5.2.3 Communicating thought by execution media for the group of grade 8 learners for all activities (see chart 5.3)

Activity 1

The group could not communicate thought by either execution medium. It was partly assisted in sorting and comparing shapes. It was able to compare sizes of both three-dimensional (rectangle boxes and cylinders) and two-dimensional shapes (squares, rectangles and parallelogram). However it had difficulty in comparing similar shapes. It could not demonstrate that shapes were similar. It could not communicate thought by talking proper English. Their sentence construction was poor. A typical example is that of learner B (representing the group). It constructed meaningless sentences.
Activity 2

The group could not communicate thought by either execution medium (see chart 5.3). This means that it wrote meaningless statements. In other words, it was difficult to follow what they were trying to say. It did not write geometric names of shapes that it drew. It drew inaccurate diagrams or wrong ones. This resulted in diagrams not being recognisable. Although there was an exceptional case, namely learner B, the rest of the learners could not express themselves in English.

Activity 3

The group could communicate thought by ‘showed with and used hands’. All learners excelled in constructing all types of angles by rotating an acetate sheet. A substantial number of angles were recognisable by the markings on them. This could be attributed to the fact that they were familiar with them from grade 7. The group wrote correct names of angles of all types of angles. However there were some spelling mistakes. For example, ‘niaght angle’ and ‘revolusion’. Measurements of angles did not correspond to the markings on them (see illustration 5.1). For example,

[Diagram showing angles 124° and 40°]

ILLUSTRATION 5.1. MEASUREMENTS DO NOT CORRESPOND TO THE SIZE OF THE ANGLE

All the learners, except one, wrote a correct or an approximate measurement of the reflex angle. Once again, it could not explain how it had constructed angles.
Activity 4

The group could not communicate thought by either execution medium. Most learners wrote measurement of the reflex angle as less than $180^\circ$. Although it was correctly marked the measurements conveyed a contradictory message to the viewer. The group also wrote correct measurements in the majority of angles except the reflex angle. It still needed assistance in this aspect. It was assisted on how to use a protractor to measure them, starting with those angles facing the right hand side. Later it struggled to measure angles greater than $90^\circ$ due to the fact that it could not read from the protractor. The same problem arose when it measured angles facing the left hand side and the reflex angle.

Activity 5

The group could not communicate thought by either execution medium. All learners could to a certain extent use a protractor to measure supplementary angles. It was difficult to measure three supplementary angles, but it was easier to measure the other two angles that shared the horizontal line. It was also difficult to measure an angle between them. Besides all these obstacles, it could place a protractor appropriately to measure angles. It could not be accurate, unless the two lines that constituted an angle were extended. A few learners measured a substantial number of angles. Initially, the group conducted activity that required them to calculate unknown angles as part of supplementary angles. It fared dismally. It was later given an improved activity to first measure all supplementary angles. Because the group could not accurately measure angles, it wrote inaccurate measurements. It could not properly read from the protractor. It struggled to talk about how they arrived at their measurements. This was usually the case when the group measured angles less than $90^\circ$ facing the left hand side and mentioned an angle greater than $90^\circ$. 
Activity 6

The group could communicate thought by 'showed with and used hands'. It bisected angles by the paper-folding method with ease. On the basis of its experience in bisecting the acute angle, it was easier for it to bisect the obtuse angle. It was also able to fold and demonstrate that a straight-line angle could be bisected. In all cases the folded line divided the angles equally. The group wrote that the folded line bisected the three angles. The group traced the folded line in such a way that the resulting pair of angles was unequal. Hence, a few shapes were recognisable.

Activity 7

The group could communicate thought by 'showed with and used hands' and drawing. Due to the experience that the group gained while bisecting a straight-line angle, it was easier for it to construct perpendicular lines by means of the paper-folding method. The two 90° angles could easily be recognised. Tracing and folding lines deepened its understanding of the concept. The group wrote statements that lacked coherence. For example,

1. “It change to be A bisect and makes four angles”.
2. “It used bisected the two lines perpendicular”.
3. “It was called A one line when I folded a paper”.

The group drew perpendicular lines with the 90°- sign between the lines. The sign made lines recognisable as perpendicular. In other words, they are recognisable. Nevertheless, a substantial number of shapes were recognisable (see chart 5.3). The group could not express itself verbally on how the two lines were found to be perpendicular. Their statements were meaningless.
Activity 8

The group could communicate thought by drawing and ‘showed with and used hands’. It was also able to redraw parallel lines on the worksheet. All diagrams were recognisable by the markings (arrows) on both pairs of lines (see chart 5.3). It would fold one of the perpendicular lines to obtain a new folded line parallel to one of the two lines with less difficulty. The group wrote meaningless statements. Only one learner could explain what parallel lines were.

Activity 9

The group could communicate thought by writing. It wrote names of parallel lines from the Parallelogram Tiles Worksheet correctly. It also wrote an appropriate symbol between names of parallel lines. The group was able to write at least one pair of lines that were perpendicular, accompanied by an appropriate symbol. In some cases, group would write lines that are even perpendicular as parallel lines. Only one learner in the group drew perpendicular lines. The rest did not even attempt it. The group could verbally mention lines that were parallel. Although it could tell which of the lines were perpendicular, it could not explain why they were perpendicular.

Activity 10

The group could communicate thought by drawing. It was able to find that a pair of vertical opposite angles was equal to each other after rotating two intersecting lines up to 180°. However the group was assisted in rotating the two intersecting lines up to 180°. Vertical opposite angles were recognisable by the markings on them. It could not verbally explain without assistance, except demonstrating how they arrived at the pair of angles being equal. Verbal and written explanations were meaningless.
Activity 11

Initially, the group was given a worksheet, which failed to prepare it to form a concept of corresponding angles. The language and format used on the worksheet were difficult to follow. The worksheet was later refined. The group found that after sliding an angle traced on the acetate sheet on the worksheet, angles above and behind horizontal/vertical lines were equal to each other. However, the group was assisted by the researcher to translate the acetate sheet. The group convinced itself on that angles were equal to each other by shifting or sliding the traced angle back to the original position on the worksheet in order to compare them. The group wrote meaningless statements.

Activity 12

The group performed dismally in both execution media (‘showed with and used hands’ and writing). It was easier for the group to slide the traced angles on the acetate sheet horizontally. What the group did not realise was that angles between parallel lines were the same angles that constituted the straight-line angle. It could not establish the relationship between parallel lines and co-interior angles. In other words, the concept of co-interior angle and the relationship between them was not formed at all. The group wrote meaningless statements about their relationship. In its written mathematical statements, it confused co-interior angles with corresponding angles. It also wrote that angles between parallel lines were equal to each other.

Activity 13

The group could communicate thought by ‘showed with and used hands’. Rotating an acute angle traced on the acetate sheet from the worksheet resulted in forming alternate angles. It convinced itself that alternate angles were equal to each other by rotating the acute angle back to the original position. The group wrote statements that lacked coherence about the relationship between parallel lines and the alternate
angles. Although it could establish the relationship between alternate angles it could not conclude statements.

Activity 14

This activity was meant to evaluate the group’s ability to apply knowledge of the previous activities to solve problems. Emphasis was on whether it was able to write meaningful mathematical statements. The group could not write the relationship between angles. It could recognise corresponding angles in problems involving measurements. It would write only the first statement in the solution of the problem. It couldn’t go beyond this statement. It wrote meaningless statements in the rest of the solution.

Activity 15

The group could not communicate thought by either execution medium. The group was able to cut out an isosceles triangle from paper with the researcher’s assistance. It was also assisted in folding the triangle to determine which sides or angles were equal. The group wrote meaningless statements about the properties of an isosceles triangle. They could not compare a pair of angles and sides. Hence the concept was not formed. It was also able to draw an isosceles triangle with a pair of sides and angles marked. The most difficult thing was to explain verbally which sides were equal. The researcher had assisted them in naming a side using two letters of the alphabet. It could hardly mention names of sides of the triangle.

Activity 16

The group could communicate thought by drawing. It drew and marked all the three sides as being equal to one another. It was able to demonstrate that it could compare and find that all angles of an equilateral triangle were equal to one another, after receiving assistance. None of the learners wrote that all sides of the equilateral
triangle were equal to one another. Confusion about how to mention names of the
sides of a triangle was still prevalent and was an obstacle for it to explain verbally the
properties of an equilateral triangle.

Activity 17

The group could not communicate thought by either execution medium. It was able to
place the mirror at 90° on the solid line/side of the triangle after receiving assistance
on how to place it. Initially, it placed the mirror in such a way that one could not see the
mirror image. The group wrote telegraphic sentences that lacked cohesion. On being
asked probing questions, it was able to explain verbally the properties of a triangle.

Activity 18

The group was unable to communicate thought by either execution medium. It best
communicated thought by 'showed and with used hands' and drawing. It had to repeat
the activity after it realised by itself that not all angles of the triangle fitted on the
straight-line angle. It was assisted by the researcher in placing the first angle on the
straight-line after it traced only one angle of the triangle and constructed the other two
by means of a ruler. It could demonstrate how it had found that the sum of angles of
the triangle was equal to 180° without receiving assistance from the researcher. The
traced angle looked very untidy and crooked. A few diagrams were recognisable. It
was even doubtful if they could add to 180°. The group wrote sentences that lacked
cohesion. A few mathematical statements were meaningful. Most of them were either
meaningless or they were not accompanied by reasons. The group made an attempt
to explain verbally that the sum of angles of the triangle add to 180° on the straight-
line angle. Verbal explanations lacked coherence.
Activity 19

The group could not communicate thought by either medium (see chart 5.3). It was easier for the group to trace one angle of the triangle into the exterior angle of the triangle drawn on the worksheet. This angle brought confusion to the group since it saw it as an exterior angle. It was also difficult for the group to fit the second angle into the exterior angle. The confusion was cleared after the researcher assisted them in differentiating the bigger exterior angle and the other two angles that constituted it. The group was assisted in drawing a triangle and its exterior angle (together with the angles that constituted it). The group wrote the relationship between the exterior angle of a triangle and its two opposite interior angles in symbols instead of explaining in words. The group wrote meaningful mathematical statements. Initially, the group could not verbally explain the difference between the exterior angle and its constituent parts. However it could explain the relationship between the exterior angle and the two interior opposite angles in symbols instead of words.

The group was partly given assistance in rotating, reflecting measuring, bisecting, comparing, translating, tracing, folding and sorting shapes. Mathematical statements and sentences lacked cohesion. Verbal explanation became meaningful after clarification. A few shapes were recognisable.

5.2.4 Communicating thought by execution media for the group of grade 9 learners for all activities (see chart 5.4)

Activity 1

This group could communicate thought by ‘showed with and used hands’. It could compare the sizes of all shapes available. These included three-dimensional shapes. It compared shapes with assistance from the researcher.
CHART 5.4. COMMUNICATING THOUGHT BY EXECUTION MEDIA FOR GRADE 9 LEARNERS FOR ALL ACTIVITIES

The group wrote that shapes look alike because they have equal 'comers'. It also wrote that shapes were equal by matching them. It did not remark on three-dimensional shapes. They wrote meaningless sentences. Due to poor command of English, the group could not verbally explain its observation.

Activity 2

The group was unable to communicate thought by either medium. It could only accurately draw a triangle. A few shapes could be recognised. The group lacked words to express itself verbally. It used one-word sentences to answer questions. It wrote a ball as a circle. Rectangles were called squares. The following shapes had proper names written beside them: reflex angle, triangle and hexagon. Hence their performance in writing was slightly above average.
Activity 3

The group could communicate thought by drawing and ‘showed with and used hands’. All their drawings could be recognised by the marking on them. The group could construct all types of angles by rotating the acetate sheet such that it could not differentiate between obtuse and reflex angles. All angles had correct names written beside them. However measurements of angles contradicted the size of the angle. For example, the acute angle and obtuse angles had measurements of 80° and 40° respectively. In some cases angles were marked as reflex but their measurements were less than 180°. Its verbal explanation was characterised by one-word sentences.

Activity 4

The group could not communicate thought by either medium. It was assisted in the use of a protractor in measuring angles. It could still not be able to measure angles greater than 180°. The measurement of revolution was not derived from a protractor, but recalled from previous experiences. Its measurements were characterised by inaccuracies, especially with right angles. The group wrote a measurement of 90°, even though it was less than 90°. In general, drawings and names written beside them were confusing.

Activity 5

The group could not realise that two or more angles about the point add to 180° on the straight-line angle. It was given a refined activity, which involved the use of a protractor to measure angles and adding them up to 180°. Due to shorter lines that constituted an angle, the group could not measure angles accurately. The group required assistance in varying degrees. The group could at least write the first statement of the problem that involved adding angles on a straight-line angle to 180°. However it could not apply additive inverse and addition of like terms. The group could not verbally explain how it arrived at the fact that the sum of angles on the straight –
line angle was 180°. The word *sum* was foreign to them. It could only use ‘the angles add to 180°.’

**Activity 6**

The group could communicate thought by ‘showed with and used hands’. Initially, it struggled to show how to bisect the acute angle by paper folding method. It could not put the two lines that constituted the acute angle together. In the other words, the lines could not coincide. Nonetheless, the group was later able to fold both the acute and obtuse angles. A few drawings were recognisable by the markings on them. The group wrote that the folded line bisected angles. Written statements lacked cohesion.

**Activity 7**

The group could not communicate effectively by either medium. It folded the line with assistance from the researcher. It also drew perpendicular lines without the 90° sign. Therefore, one may not able to recognise them as being perpendicular lines. They wrote statements that lacked cohesion. It did not write that the folded line was perpendicular with the drawn line. Its response bordered on a meaningless statement.

**Activity 8**

The group could communicate thought by drawing. A substantial number of lines or shapes were recognisable. Parallel lines were marked with arrows. The group used the very same tracing paper and the two perpendicular lines to construct parallel lines. Initially, it folded along the drawn straight-line. It was partly assisted to bend or to fold the first folded line away from the drawn line. It either wrote sentences in a wrong space on the worksheet or they were meaningless. The group could verbally explain by means of alphabetical letters, with regard to the relationship between lines. However it could not express itself in words.
Activity 9

The group could not communicate thought by either medium. It did not draw perpendicular lines based on Parallelogram Tile Worksheet. It also recognised and wrote correct names of lines that were parallel. It also wrote a correct symbol for names of lines that were parallel in an inappropriate space on the worksheet. It also correctly wrote names of angles that were perpendicular in an inappropriate space on the worksheet. The group could not verbally express themselves in words. Instead, it preferred to use letters of the alphabet to mention names of lines.

Activity 10

The group could communicate thought by drawing and ‘showed with and used hands’. A pair of vertical opposite angles were properly marked and traced on the acetate sheet. The group struggled to determine the extent to which the two intersecting lines on the acetate sheet should be rotated. It did not know the point at which it should have reached $180^\circ$, until the researcher showed it the measurement $180^\circ$. This enabled them to trace a pair of angles that were equal to each other. All learners were able to write meaningful mathematical statements. The group wrote mathematical statements that lacked cohesion. It preferred to demonstrate how it found vertical opposite angles equal instead of explaining it verbally.

Activity 11

The group could communicate thought by ‘showed with and used hands’. Initially, the group rotated, instead of, translating the angle traced on the acetate sheet from the worksheet. It was later able to rectify its own mistakes after the researcher has clarified it on the meaning of translation. The group wrote in a full sentence that lines were equal to each other. However it did not write that they were parallel to each other. It wrote:
1. We have corresponding angle when there is a cross line on the parallel line.
2. Corresponding – when we have two lines parallel, which are parallel.
3. We have corresponding line when we have two parallel lines.

This was an attempt to establish the relationship between corresponding angles and parallel lines. It wrote meaningless statements.

Activity 12

The group could not communicate thought by either medium. It could slide two supplementary angles traced on the acetate sheet from the worksheet. However it could neither recognise co-interior angles nor notice the relationship between them. It also wrote meaningless statements.

Activity 13

The group could communicate thought by ‘showed with and used hands’. It could rotate the acute angle on the acetate sheet to form alternate angles on the worksheet with ease. The group could not write about the relationship between alternate angles and parallel lines. It could write meaningful mathematical statements without giving reasons.

Activity 14

The group could not communicate thought by writing. It could write a correct relationship about alternate and corresponding angles but gave the wrong names. It could also correctly write the relationship between vertical opposite angles and called them alternate angles. The group wrote and solved some mathematical problems.
Activity 15

The group could communicate thought by drawing. A substantial number of diagrams (isosceles triangle) were recognisable. The group was able to cut an accurate shape of an isosceles triangle from paper with less difficulty. The group was able to compare the remaining pairs of sides and angles of the triangle after receiving assistance on how to fold it.

The group wrote meaningless statements. There was confusion about the use of the letters of the alphabet concerning names of sides of a triangle. A symbol (\(\angle\)) for an angle was used for the sides. The group could not verbally explain properties of an isosceles triangle in words. It preferred to use letters of alphabet to explain its properties. The group struggled to name a side of a triangle. It was very difficult to communicate with it in English.

Activity 16

The group could not communicate thought effectively by either medium. Initially, the group cut an isosceles triangle instead of an equilateral triangle. It managed to cut an equilateral triangle after it was instructed by the researcher to study instructions on the worksheet. The group was able to show that all the sides of the equilateral triangle were equal to each other. The group wrote meaningless statements in wrong spaces. A few shapes were recognisable by the marking of all the sides on them. The group was able to explain properties of the equilateral triangle after receiving assistance on how to mention names of sides.

Activity 17

The group could communicate thought by writing. It wrote telegraphic sentences whose meaning could be obtained through sifting. It also partly received assistance
from the researcher on how to place a mirror on one side of the triangle. However they could verbally explain properties of an isosceles triangle.

Activity 18

The group could communicate thought by drawing. It could trace all angles of the triangle around a point on the straight-line angle after receiving assistance from the researcher on how to place the second angle of a triangle. Although the sequence of the angles of a triangle drawn around a point on the straight-line was different, it could add to 180°. The group wrote a meaningful sentence about the sum of angles of a triangle being equal to 180° and the fact that it was equal to 180° because they constituted a straight-line angle. However meaning could be sifted from the sentence. The group could only write the first mathematical statement of each problem and the appropriate reasons: it could not add like terms. The group struggled to respond verbally to the researcher’s question about the sum of angles of the triangle. It responded after questions were rephrased. The group could not explain its observations and the reason why angles of the triangle added to 180° on their own without receiving assistance from the researcher.

Activity 19

The group could not communicate thought effectively by either execution medium. The drawing conveyed more meaning than other execution media. A few shapes were recognisable. Some drawings were inaccurate. Learners were partly assisted on tracing interior angles of a triangle into the exterior angle. The group was not sure of what to demonstrate. It could not write its responses in words, instead it used symbols such as, ‘∠1 and ∠2 fit into ∠4’. The group wrote and solved four out of five problems. It was unable to solve two problems involving writing of proof. It preferred to use symbols instead of words in talking about its observation. Explanations were only clear after clarification was sought.
Activity 20

The group could not communicate thought effectively by either medium. It was assisted by the researcher on how to compare shapes. It wrote meaningless statements about the relationship between the triangle and its mirror images. However it wrote meaningful statements using symbols. A few learners wrote reasons for their statements in writing proofs. As a result mathematical statements lacked cohesion. Verbal explanations were clear after seeking clarification on the naming of sides.

Activity 21

The group could not communicate thought effectively by either medium. It could write statements about opposite angles or sides of a parallelogram in symbols and words. It could also write correct statements with wrong reasons. It could not write deductive proof without receiving assistance from the researcher on how to write statements logically. Most statements lacked cohesion. Most learners could rotate the parallelogram on the acetate sheet at 180° and compare its angles and sides to that on the worksheet. There was a tendency among learners to describe properties of a parallelogram by saying that all angles are equal. However they pointed at a pair of opposite angles. Verbal explanation did not clarify what they meant.

Activity 22

The group could not communicate thought effectively by either execution media. Even though it was eventually able to mark correct pairs of line segments of the diagonals that were equal to each other, it could not write statements that showed what it had observed about diagonals of a parallelogram. Its conclusion lacked cohesion. The group was able to write mathematical statements accompanied by appropriate reasons in writing proofs after receiving assistance in writing logical statements. It was partly assisted in tracing segments of the diagonals. It was able to show line segments of the
diagonals of a parallelogram that were equal to each other. It could explain the relationship between diagonals of a parallelogram. Verbal explanations were only meaningful after clarification was sought from the group.

Activity 23

The group could not communicate in writing. The group wrote meaningless mathematical statements. These statements were characterised by reference to opposite angles as opposite sides. Sometimes opposite angles were referred to as alternate angles. There was also lack of transferring knowledge gained from previous statements to subsequent ones. There was also a tendency among learners in the group to write statements at random without systematising them.

5.3 Concept formation

Concept formation is defined here as in section 3.5.1. A concept is differentiated from all others by its features and properties. Concepts can be expressed by means of definition, examples, drawing diagrams and a practical demonstration.

In the subsequent paragraphs we shall discuss the extent to which individual learner in each grade form concepts by using the four execution media. In other words we would like to determine the execution medium or media by which an individual learners could form concepts. A discussion will also be made on whether individual learners were competent in forming concepts. It will be determined whether a group of learners could form concepts by a particular execution medium or not, for all activities.
5.3.1 Concept formation by execution media for the individual grade 8 learners for all activities (see chart 5.5).

![Chart 5.5: Concept Formation by Execution Media for Grade 8 Learners](chart)

Learner A could form a concept by drawing. This means that a substantial number of concepts were formed. This was followed by 'showed with and used hands' by which a few concepts were formed. In this case, shapes were either given geometric names or they were given wrong names. Verbal explanation lacked coherence and led to a few concepts being formed.

Learner B could form concepts by drawing and 'showed with and used hands'. Once again a substantial number of concepts were formed since drawings were recognisable. This was followed by the 'showed with and used hands' by which a substantial number of concepts were formed. On the other hand, sentences and mathematical statements in writing lacked coherence. As a result, few concepts were formed. Verbal explanation about features and properties of shapes lacked coherence. Hence few concepts were formed.
Learner C formed a few concepts even after receiving assistance from the researcher. Drawing led to the formation of a few concepts. He wrote meaningless statements. He was unable to explain features and properties of shapes.

Learner D could form concepts by drawing diagrams. This means he drew a substantial number of diagrams. This led to the formation of a substantial number of concepts. He further used 'showed with and used hands'. This resulted in a few concepts being formed. Verbal explanation of the features and properties of shapes lacked cohesion.

Incorrect geometric names of shapes were written or not written at all. As a result a few concepts were formed. For example, in activity 2 diagrams (namely, a hexagon and trapezium) did not have their names written. However he was to write the concepts, parallelism, corresponding angles, perpendicular, and alternate angles. He was also not able to write the properties of co-interior angles.

Learner E formed a few concepts by drawing and 'showed with and used hands'. However he formed more concepts by drawing than by 'showed with and used hands'. No concept was formed by writing and talking about features and properties of shapes.
5.3.2 Concept formation by execution media for the individual grade 9 learners (see chart 5.6)

![Chart 5.6: Concept Formation by Media for Individual Grade 9 Learners]

Learner F formed a substantial number of concepts by drawing diagrams. Drawing was followed by 'showed with and use of hands'. He also formed a substantial number of concepts by 'showed with and used hands' rather than by writing and talking. He formed a few concepts in writing and talking. The medium of talking was the least effective.

Learner G could form concepts by drawing. In this way she formed a substantial number of concepts. However she formed a few concepts by talking. She also formed a few concepts by 'showed with and used hands' and writing.

Learner H could form concepts by drawing. That is, he formed a substantial number of concepts by drawing rather than by other execution media. He also formed a substantial number of concepts by 'showed with and used hands'. He formed a few concepts by talking and writing.
Learner J could form concepts by 'showed with and used hands'. She formed a few concepts by talking. She could speak English better than other learners. She also formed a few concepts by drawing, but fewer than by talking. Her diagrams were characterised by inaccuracies and untidiness. Some of them were incomplete. Once again, she formed fewer concepts by writing than by other execution media. She often used two-word or incomplete sentences. However, she wrote logical mathematical statements.

Learner K formed a few concepts by writing. However, he was very close to forming a substantial number of concepts. His sentence construction was very poor. In most cases, he would write meaningless statements. However, he was very good in writing mathematical statements that helped him to develop concepts; this was followed by drawing accurate diagrams. He was close to forming a substantial number of concepts by drawing. It was just that some drawings were not recognisable. Once again, he formed a few concepts by 'showed with and used hands'. He became successful in some concepts after he was given assistance by the researcher. He failed dismally in forming a concept of an equilateral triangle by paper folding. He also formed fewer concepts by talking than by other execution media. He could not rephrase sentences.

5.3.3 Concept formation by the execution media for the group of grade 8 learners for all activities (see chart 5.7)

Activity 1

A few concepts were formed by 'showed with and used hands'. Concepts such as congruency of two-dimensional shapes were formed. The group could not find the similarities of two- and three-dimensional shapes. Neither by writing nor talking it could form any concepts. It could neither write nor speak meaningful sentences.
CHART 5.7. CONCEPT FORMATION BY EXECUTION MEDIA FOR GRADE 8 LEARNERS FOR ALL ACTIVITIES

Activity 2

The group formed a few concepts by drawing and talking. It could not write names of shapes correctly.

Activity 3

All concepts were formed by ‘showed with and used hands’ with ease. The group constructed different types of angles by rotating a line traced on an acetate sheet from a worksheet. It drew a substantial number of concepts. Due to poor command of English, a few concepts were formed by talking. The group could mention the names of angles. Moreover, it could explain how angles were constructed. No concept was formed by writing.
Activities 4

A few concepts were formed by 'showed with and used hands'. The group was able to measure angles after it was assisted in using a protractor. Writing of measurements of angles also formed a few concepts. In certain cases, the group wrote measurements that greatly differ from the size of an angle. For example, acute and reflex angles were given measurements of 140° and 110° respectively.

Activity 5

A few concepts were formed by both 'showed with and used hands' and writing. A protractor was used to measure a few supplementary angles. In some cases, the group did not measure angles accurately. As a result, angles could not be supplementary. It was difficult to measure the middle angle, where three angles were to be measured. The group wrote correct mathematical statements in solving problems without indicating what their reasons were. No concept was formed by talking. The group could not verbally explain why the sum of angles on the straight-line is equal to 180°.

Activity 6

A concept was formed by 'showed with and used hands'. The group folded most angles with less difficulty. It could fold and compare angles. The group formed the concept by drawing and writing after receiving assistance from the researcher on how to bisect angles. Diagrams lacked accuracy. The line that bisected angles was not accurately drawn. Written statements lacked coherence.

Activity 7

The concept was formed with ease by 'showed with and used hands'. The group showed insight into the concepts. It was able to construct a perpendicular line by
paper folding on its first attempt. Angles between the intersecting lines were marked by the 90°-sign. It also had less difficulty in drawing perpendicular lines on the tracing paper. It was further able to form the concept of perpendicularity by talking after receiving assistance from the researcher on how to write meaningful sentences on the worksheet. The group wrote meaningless statements.

Activity 8

The concept of parallelism was formed by drawing and ‘showed with and used hands’. The group drew two parallel lines and a transversal line with ease. Since this activity was an extension of activity 7, the group constructed parallel lines with less difficulty by paper folding method (‘showed with and used hands’). The group formed the concepts by writing after receiving assistance from the researcher on how to write names of line segments. It also found it somewhat difficult to explain verbally what parallel lines were without being assisted by the researcher. There was lack of coherence in the verbal explanation.

Activity 9

The group formed the concept by talking and writing after receiving assistance in mentioning lines that lines were perpendicular. It could mention lines on the parallelogram Tile Worksheets that were parallel. It could write names of a substantial number of parallel lines. However, it wrote names of perpendicular lines, together with non-perpendicular lines. This shows that it was still confusing perpendicular lines with other pairs of intersecting lines. The concept of perpendicularity was not formed by drawing.

Activity 10

The concept was formed by drawing. That is, the group had insight into the concept. Drawing deepened the group’s understanding of the concept. The group also
experienced less difficulty in forming the concept by ‘showed with and used hands’. That is, the relationship between vertical opposite angles was established by rotating two intersecting lines on the acetate sheet with less difficulty. The group could also form the concept by talking. In most cases, it was assisted by the researcher on how it should express itself verbally. The concept was not formed by writing. The group could not write logical statements. It was unable to conclude mathematical statements.

Activity 11

The concept was partly formed by both ‘showed with and used hands’ and writing after receiving assistance from the researcher. Corresponding angles were partly formed by both execution media with less difficulty. The group could translate the angle traced on the acetate sheet from the worksheet to form corresponding angles. It could not write meaningful sentences that showed how the concept was formed. However it could write meaningful mathematical statements. That is, it could show which of the angles were corresponding angles.

Activity 12

No concept was formed by both writing and ‘showed with and used hands’. Translating two supplementary angles traced on the acetate sheet did not lead to the group noticing that there were supplementary angles other than those that were adjacent to each other. The group did not write meaningful sentences and mathematical statements. For example (see illustration 5.2),

1. ‘Angles between vertical lines are equal is to 180°’
2. $\chi = 50^\circ$ [corresponding angles are equal]
There was a tendency to confuse co-interior angles with corresponding angles. This was an indication that the concept was not formed.

Activity 13

The concept was formed with ease by ‘showed with and used hands’. The alternate angles were formed with ease after the rotation of the angle up to 180°. The concept was partly formed by writing. The group was able to write that lines were parallel and the angles were of the same size. However it could not relate angles and parallel lines. A few learners could write a mathematical conclusion.

Activity 14

The concepts were not formed by writing. In most instances the group could recognise certain angles, but it could not find the relationship between them. It could only recognise adjacent angles, vertical opposite angles and corresponding angles.

Activity 15

The concept was partly formed by drawing, writing, ‘showed with and used hands’, and talking. The group received assistance on how to mark sides or angles that were equal to each other. It wrote statements that were not cohesive. The group would write that a pair of sides was equal to each other on the space for a pair of angles. Initially, the group struggled to form the concept (finding properties) by folding the triangle. It succeeded after receiving assistance from the researcher on how to fold the triangle. The group could only verbally explain the relationship between sides or angles of the triangle by using letters of the alphabet.
Activity 16

The concept was formed by drawing. All the sides of the triangles were marked the same way. Thus the triangle was recognisable. The group could only express itself verbally with assistance from the researcher. However the process of folding the triangle was explained in terms of letters of the alphabet. Most members of the group were able to compare sides of triangle by paper folding method. Although all members of the group eventually succeeded in finding that all sides were equal to one another, individual learners received assistance of varying degree, from the researcher. Although all sides of the triangle were marked equal to each other, the group did not write a statement relating to what they did to the side of the triangle.

Activity 17

The concept was formed by ‘showed with and used hands’ and writing. The group was able to place the mirror on one side of the triangle with less difficulty. Initially, the group did not know the name of the triangle, but it could verbally explain the properties of the triangle. It was able to find the name of the triangle after it was previously given the wrong one. The concept could not be formed by writing. Sentences written were meaningless.
5.3.4 Concept formation by the execution media for the group of grade 9 learners for all activities (see chart 5.8).

![Chart 5.8: Conception Formation by Execution Media for Grade 9 Learners for All Activities](chart)

**Activity 1**

A substantial number of concepts were formed by ‘showed with and used hands’, talking and writing. Although meaning could only be sifted from statements, the group had a better understanding of similarity and congruency concepts. It indicated that shapes looked alike because they have equal corners (angles). It found that shapes were equal by matching them. Two- and three- dimensional shapes were sorted according to their size with less difficulty.

**Activity 2**

Few concepts were formed by drawing, writing and talking. However a few shapes were recognisable. The group wrote correct names of the following shapes: triangle, hexagon, and reflex angle. However a rectangle and a ball were written as square and circle respectively. The group formed a few concepts by talking; verbal explanation about features and properties of shapes lacked coherence.
Activity 3

All concepts were formed by drawing. All the angles were recognisable. Drawing deepened the group’s understanding of angles. The group also formed a substantial number of concepts by ‘showed with and used hands’. It could rotate the line on the acetate sheet to form different angles. It also wrote names of a substantial number of angles. Thus, a substantial number of concepts were formed. A few concepts were formed by talking. The group could not verbally explain how they formed different angles. Its verbal explanation lacked coherence.

Activity 4

A few concepts were formed by writing and ‘showed and used hands’ after assistance was received from the researcher. The group wrote correct names of all types of angles. It also wrote correct measurements of most angles, except the reflex and right angles. The group wrote the reflex angles’ measurement as being less than 180°. It further misspelled names of certain angles. For example, right angle and revolution. The group was only able to use the protractor to measure angles after the researcher had assisted it. It still struggled to measure the reflex angle.

Activity 5

The group wrote a substantial number of statements supported by appropriate reasons. There were a few statements which were not supported by reasons. Thus, the concept was formed with less difficulty. The group received assistance from the researcher on how to measure angles on the straight-line angle. The concept was partly formed by ‘showed with and used hands’. The concept could not be formed by talking. The group was unable to explain verbally the features of a straight-line angle.
Activity 6

The concept was formed by drawing and 'showed with and used hands' because shapes were recognisable. A pair of equal angles was obtained by folding the angle with less difficulty. The concept was partly formed by talking after the group received assistance from the researcher. The group was unable to express itself in writing.

Activity 7

The drawing was recognisable by the marking on it. The group was able to form the concept with less difficulty. The concept was partly formed by 'showed with and used hands' after receiving assistance from the researcher on how to fold the line. The concept-perpendicularity of lines was also partly formed by writing, after receiving assistance from the researcher on how to construct meaningful and cohesive statements.

Activity 8

The concept was formed by drawing due to the fact that the diagram or drawing was recognisable by arrows on the lines. The concept was partly formed by 'showed with and used hands', writing and talking respectively, after receiving assistance from the researcher on how to fold, to write and speak. In other words, movements of shapes were made after receiving assistance from the researcher. The group's written and verbal statements lacked cohesion.

Activity 9

The concept was partly formed by writing and talking after assistance was received from the researcher. The group understood the concept "parallelism" better than "perpendicularity". The group wrote lines that were parallel with an appropriate symbol. It wrote some lines that were perpendicular with an appropriate symbol. The group got
clarity on which lines were parallel or verbally explained why it thought they were parallel or perpendicular. The concept was not formed by drawing since the group drew an inaccurate diagram or there was no diagram at all.

Activity 10

The concept was formed by drawing and 'showed with and used hands'. All vertical opposite angles were marked the same way. They were recognisable. This helped deepened the group's understanding of the concept. The group was able to rotate the acetate sheet with two intersecting lines. It wrote mathematical statements that lacked cohesion. A few learners wrote logical statements. Verbal explanation about vertical opposite angles also lacked cohesion.

Activity 11

The concept was formed by both 'showed with and used hands' and writing. Corresponding angles were formed with ease by sliding an angle on the acetate sheet. Most members wrote meaningful sentences about the relationship between corresponding angles. The group also wrote meaningful mathematical statements. A few learners were still confused about naming of angles especially when the drawing was facing down (see illustration 5.3).

\[ \begin{array}{ccc|ccc} A & B & C & F & G \\ \hline & 1 & 2 & 1 & 2 \\ & & & & \end{array} \]

ILLUSTRATION 5.3. CONFUSION AMONG LEARNERS IN RESPECT OF NAMING OF ANGLES

\[ \angle A_1 = \angle B_2 \text{ [corresponding angles are equal]} \quad \angle F_1 = \angle G_2 \text{ [corresponding angle are equal]} \]
Confusion arose when learners compare angles. They could not write proper names of angles, for example, $\angle A$ was incorrectly written as $\angle A_1$ and $\angle G$ as $\angle G_2$.

Activity 12

The concept was partly formed by ‘showed with and used hands’ and writing. The group was able to form co-interior angles with the assistance from the researcher. However it did not establish the relationship between them. It wrote sentences that lacked cohesion. It was still confusing co-interior angles with corresponding angles.

Activity 13

The concept was formed by ‘showed with and used hands’. The group was able to rotate an acetate sheet with an acute angle to form alternate angles with ease. It wrote sentences that lacked cohesion. It also wrote meaningful mathematical statements, but it lacked the ability to conclude them. However the concept was not formed by writing.

Activity 14

Concepts were not formed by writing. Giving angles wrong names was prevalent. The group wrote mathematical statements that are not cohesive. In some cases, it wrote correct statements without reasons.

Activity 15

The concept was formed by drawing. The diagram was recognisable by the markings on it. Drawing was followed by talking. The group was able to give verbal explanation about the properties of the isosceles triangle after receiving assistance from the researcher. However the explanation lacked coherence. The concept was not formed by writing, since the group wrote meaningless statements.
Activity 16

The concept was partly formed by drawing, 'showed with and used hands' and talking. A few shapes were recognisable by the marking on them. Verbal explanation lacked cohesion. The relationship between sides of the equilateral triangle was established after the assistance was received by the researcher on how to compare them. The concept was not formed by writing, because the group wrote meaningless statements.

Activity 17

The concept was formed by writing and 'showed with and used hands'. It wrote meaningful sentences about the features of an isosceles triangle. It placed the mirror property on one side of the triangle. However the concept was formed by talking after receiving assistance from the researcher on how to formulate a meaningful sentence. There was too much guesswork when mentioning the name of a triangle that came about as a result of the right-angled triangle and its mirror image.

Activity 20

The concept was best formed by talking, 'showed with and used hands' and writing. The group was able to give a verbal explanation as to under which conditions were triangles congruent. Although the explanation was telegraphic, one was able to follow what was being said. The concept was formed when the group was able to compare shapes by putting one on top of the other with less difficulty. All congruency axioms written by the group were correct. The group also wrote meaningful statements.

Activity 21

Concepts were not formed by writing. Meaningless statements about the properties of parallelograms were written. A few calculation problems were solved. They were
characterised by angles being given wrong names. The group lacked the ability to transfer knowledge gained from a set of statements to another statement. It further lacked the ability to conclude statements. It could not write proof.

Activity 22

A few concepts were formed by writing. Although the group was able find that line segments of each diagonals were equal to each other, it could not realise that the diagonals bisect each other. The group wrote meaningless statements in this regard. Solutions of problems involving measurements showed that the group realised that a pair of line segments of each diagonal were equal to each other. The group could not write proof without the assistance of the researcher on how to write logical statements.

Activity 23

Concepts were not formed by writing. Meaningless statements were written about the relationship between angles. Statements were characterised by calling angles by wrong names. There was also lack of ability to transfer knowledge by deduction.

5.4 Mastering theorems

The individual learner and the group has to show that they have mastered theorems by talking, showed with and used hand’, writing and drawing. In other words they must be able to show that they were able to use one of the execution media or both to master theorems.
5.4.1 Mastering theorems by execution media for the individual grade 8 learners for all activities (see chart 5.9).

![Chart 5.9. Mastering Theorems by Execution Media for Individual Grade 8 Learners for All Activities](image)

Learner A mastered theorems by drawing and talking. She drew accurate diagrams. Diagrams deepened her understanding of the theorems. She was able to give a verbal explanation about the result of the activity that was convincing. Fitting two interior angles of a triangle into its exterior was done after receiving assistance from the researcher. The learner could not master theorems by writing. This was due to the fact that diagrams were not properly traced.

Learner B showed mastering of more theorems by talking than by other execution media. In other words, a verbal explanation about the results of the activity convinced the researcher and the learners. Learner B was also able to master theorems by 'showed with and used hands'. The triangle was cut with less difficulty. It was easy placing the first and second angles of the triangle, but it was difficult to place the third angle on the straight-line angle. Fitting two angles of a triangle in the exterior angle was done with assistance of the researcher. Theorems were mastered by writing and drawing after assistance from the researcher. Angles of a triangle on the straight-line angle were not traced accurately. Learner B further showed lack of insight into the theorem when writing proof.
Learner C mastered theorems by 'showed with and used hands'. It was easy for him to trace the first angle of the triangle and fit two of its angles on the straight-line angle. However it was difficult for him to trace the third angle. Fitting the two interior angles of a triangle into the exterior angle succeeded after several attempts. Neither talking, drawing nor writing helped him to master theorems. Verbal explanation about the theorem lacked coherence. Drawing or tracing of angles of a triangle succeeded after the second attempt. Written mathematical statements were meaningless.

Learner D mastered theorems by 'showed with and used hands' and talking. That is, he was able to fit all three angles of a triangle on a straight-line angle with ease. Two angles of a triangle were also fitted with ease into its exterior angle. He was able to explain verbally about the results of the activity with confidence. Verbal explanation assisted him in gaining insight into the theorem. He did not master theorems by writing. Although he appeared to write meaningful sentences, he wrote mathematical statements that lacked cohesion. He could not master theorems by drawing. He drew a crooked line that resulted in an inaccurate diagram. Inaccurate diagrams led to poor performance in mastering theorems.

Learner E did not master any theorems by either medium. Diagrams were forged. Triangles were crooked. He could not cut a triangle with smooth sides. He could neither trace angles of a triangle on the straight-line angle nor in the exterior angle. He partly mastered theorems by drawing. He made meaningless written and verbal statements.
5.4.2. Mastering of theorems by execution media for the individual grade 9 learners (see chart 5.10)

CHART 5.10. MASTERING THEOREMS BY EXECUTION MEDIA FOR INDIVIDUAL GRADE 9 LEARNERS FOR ALL ACTIVITIES

Learner F mastered theorems by writing, ‘showed with and used hands’, drawing and talking. The fact that a substantial number of drawings were recognisable helped him to gain insight into the theorems. Although he expressed himself verbally in a telegraphic manner, his explanation helped him to have a better understanding of theorems. A substantial number of problems involving measurements were solved with less difficulty. He struggled to write proof. He could write proof with assistance from the researcher on how to write logical statements.

He was able to fit angles of a triangle on the straight-line angle and into its interior angle with less difficulty. Tracing the parallelogram with its diagrams was done with less difficulty. The relationship between angles, sides or line segments of the diagonal was established.
Learner G mastered theorems best by talking and ‘showed with and used hands’. That is, she was able to explain the results of the activity meaningfully. She could fit angles of a triangle on the straight-line angle and into its exterior angle with less difficulty. She was able to trace the parallelogram with its diagonals with less difficulty. The relationship between angles, sides or line segments of diagonals was established. She partly mastered theorems by drawing and writing respectively. She drew inaccurate diagrams, which led to a wrong conclusion. She wrote statements that lacked cohesion. She could write correct solutions to problems involving measurements. However she assigned wrong names to angles. She could not transfer knowledge gained in one situation to solve others. She could not write proof unaided.

Learner H mastered theorems best by drawing and talking. That is, a substantial number of drawings or diagrams were recognisable. These deepened his understanding of the theorems. Even though he used telegraphic sentences to explain meaning could be sifted from his statements. He struggled to fit angles of a triangle on the straight-line angle and into its exterior angle. Although he could rotate the parallelogram and its diagonals, he could not establish the relationship between angles, sides or diagonals. He partly mastered theorems by writing. He wrote sentences that lacked cohesion. He could also write meaningful mathematical statements. Due to the fact that certain concepts were not formed, he was unable to solve certain problems involving measurements. He could only write proof with the researcher’s assistance on how to organise statements.

Learner J mastered theorems by talking and ‘showed with and used hands’. She could verbally explain the results of the activity-involving theorem meaningfully. This enabled her to have a deeper understanding of the theorems. However she spoke English telegraphically. She rotated, while other learners translated, the triangle to fit its angles on the straight-line angle with ease. However she struggled to place the second angle of a triangle into the exterior angle. She rotated the parallelogram with its diagonals with ease. However she could only establish the relationship between a pair of sides and angles. She partly mastered theorems by drawing. All her diagrams or drawings
were inaccurate. Therefore, they had not enabled her to gain insight into theorems. She wrote statements that were not coherent. She was extremely good in writing mathematical statements that involved measurements. However she sometimes wrote angles with wrong names. She was extremely poor in writing proof.

Learner K mastered theorems best by drawing and talking. Accurate diagrams helped him to have better understanding of theorems. Although his English was poor, he could verbally explain how he obtained the results of the activity involving a theorem. He partly mastered theorems by writing and ‘showed with and used hands’. He could write problems involving measurements by showing insight into theorems involving a triangle. However he had no insight into theorems involving the parallelogram. He often wrote a wrong name for an angle. He also wrote meaningless sentences on the worksheet. He could not fit angles of a triangle on a straight-line angle and into its exterior angle even with the assistance of the researcher. However he did not know what to do with the parallelogram. The assistance from the researcher on how to rotate the parallelogram and diagonals did not enable him to gain insight into the theorems.

5.4.3 Mastering theorems by the execution media for the group of grade 8 learners for all activities (see chart 5.11.)

![Chart 5.11. Mastering Theorems by Execution Media for Grade 8 Learners for All Activities](chart5_11.png)
Activity 18

The group mastered theorems by talking. Although verbal explanation was given in telegraphic sentences, the researcher was able to sift meaning out of them. Talking enabled them to gain insight into the theorem. The group partly mastered theorems by ‘showed with and used hands’ and drawing. This means that the group was able to fit angles of a triangle on a straight-line angle with the assistance from the researcher. Thus, it could not trace or draw angles of a triangle on the straight-line angle. Furthermore, the group wrote meaningless sentences. Three learners could write only the first mathematical statement of each problem. They could not add like terms that were in variable form.

Activity 19

The group mastered theorems by drawing. Most (4) members of the group drew an accurate diagram. The diagram enabled the group to gain insight into the theorem. The group partly mastered theorems by ‘showed with and used hands’, talking and writing. It did not find meaning in fitting angles of a triangle into the exterior angle. Fitting the two angles succeeded after several attempts. Talking about the relationship between exterior angle and interior angle did not help the group gain insight into the theorem. Verbal explanation lacked coherence. Problems were solved in a way that showed lack of insight into the theorem.
5.4.4 Mastering theorems by execution media for the group of grade 9 learners for all activities (see chart 5.12)

Activity 18

The group mastered theorems by talking and 'showed with and used hands'. Meaning could be sifted from the group's verbal explanation even though it has poor command of English. It could only fit the first and second angles of the triangle on the straight-line angle. The group succeeded in fitting the third angle after several attempts. It partly mastered theorems by drawing and writing. It did not trace angles of a triangle accurately. It wrote sentences that lacked cohesion. However it was capable of writing correct solutions to three problems (involving measurement) out of five. It could only write the first statement and the appropriate reason. It could also write proof with the assistance of the researcher.

Activity 19

The group mastered theorems by talking. Verbal explanation was given by using names of angles (letters of the alphabet) instead of words. Although the group has poor command of English, their explanation was clear. The explanation has enabled them to have insight into the theorem. The group partly mastered theorems by
showed with and used hands', writing and drawing. It wrote meaningless sentences or statements about the relationship between the exterior angle of a triangle and its interior angles. Its inability to write meaningful sentences was compensated for by their ability to write meaningful mathematical statements together with appropriate reasons.

The statements enabled them to gain insight into the theorem. However the group could not write proof. Diagrams were not accurate. They were not properly traced. They made mastering of the theorems difficult. Nevertheless, the group could trace the first angle of the triangle into exterior angle. It could fit in the second angle after receiving assisted from the researcher on how to turn the angle around and fit it in.

Activity 21

The group mastered the theorems by talking and drawing. It was confident in explaining the properties of the parallelogram. Verbal explanation enabled the group to gain insight into the theorem. The group was also able to rotate the parallelogram on the acetate sheet with less difficulty and established the relationship between opposite sides or opposite angles. It was also able to trace or to draw the parallelogram on the acetate sheet. Angles and sides that were equal or parallel to each other were marked appropriately with less difficulty. It also wrote sentences whose meaning could only be sifted. This hindered their mastery of the theorem. This confusion was backed up by the manner in which they wrote mathematical statements. For example, co-interior angles were written as opposite angles of a parallelogram. They were also written as being equal.

The exercise was characterised by writing/calling angles by wrong names. The group partly mastered theorems by writing.
Activity 22

The group mastered the theorem by talking. It was easier in explaining the properties of the parallelogram with regard to its diagonals since it used letters of the alphabet to explain their observation. Their ability to describe the properties of the parallelogram in a telegraphic manner enabled the group to master theorems. Rotating the parallelogram up to 180° with its diagonals succeeded after their relationship was clarified. The relationship between diagonals was not established. As a result, the group did not master the theorem by ‘showed with and used hands’. Neither did it do so by drawing and writing. It was able to trace or to draw the parallelogram together with its diagonals. There was confusion as to where to mark on the diagonals, to show that their line segments were equal to each other. Marking of the diagonals succeeded after the group was assisted by the researcher. Marking did not convey meaning to them. The group could not write about the relationship between the diagonals of the parallelogram. However it could write meaningful mathematical statements in problems involving measurements. The group could only write proof after receiving assistance on how to write statements logically.

Activity 23

The group partly mastered theorems. It was still writing or calling angles by the wrong names. The written statements were correct, but reasons were not appropriate. The group could not transfer knowledge from one statement to another. It also lacked knowledge about co-interior angles. It could also not write proof.

5.5 Competences among individual grade 8 and 9 learners for all activities

In this section the discussion will be on the individual learner’s ability to do what was expected of him. It will be on the extent to which a learner was able to communicate thought, form concepts and master theorems in respect of the execution media.
5.5.1 Competences among individual grade 8 learners for all activities (see chart 5.13)

![Graph showing competences among grade 8 learners]

Learner A performed badly in all three competences. Her worst performance was in communicating thought. She could not express herself verbally in English. She wrote meaningless sentences, even though her mathematical statements were meaningful. She received assistance from the researcher in cutting, tracing, folding and sorting shapes. She drew recognisable shapes. However a few concepts were formed. This could be attributed to her inability to (verbally and in writing) express herself in English. She partly mastered theorems.

Learner B showed that he was able to master theorems and form concepts. He mastered theorems with less difficulty. He also formed a substantial number of concepts. His worst performance was in communicating thought. His performance in mastering theorems could be attributed to his ability to explain himself verbally and to demonstrate how he mastered them. His worst performance in communicating thought could be attributed to his inability to write meaningful sentences and statements. He could not verbally explain himself without the researcher’s assistance.
Learner C performed badly in all three competences. It was worse in concept formation than in communicating thought and mastering theorems. He formed a few concepts. Diagrams were not recognisable. He was able to communicate thought after assistance was given. He was assisted in tracing, cutting, folding, translation, reflecting, comparing and sorting shapes. Written statements were meaningless. He could only master theorems after receiving assistance from the researcher.

Learner D had an equal performance in both concept formation and mastering of theorems. He had the worst performance in communicating thought. He could not communicate thought, form concepts and master theorems without receiving assistance from the researcher. A few shapes that he drew were recognisable. Verbal and written explanation lacked coherence. They became meaningful after clarification had been sought by the researcher. He formed concepts by drawing. A substantial number of concepts were formed. A few shapes that he drew were recognisable. Verbal and written explanation lacked coherence. They become meaningful after clarification had been sought by the researcher.

Learner E performed badly in all three competences. He did not master a single theorem. This was attributed to inaccurate diagrams and inability to rotate, translate and reflect shapes. A few concepts were formed after receiving assistance from the researcher. He could communicate thought after receiving assistance from the researcher. This was attributed to poor command of English.
5.5.2 Competences among individual grade 9 learners for all activities (see chart 5.14).

![chart showing competences among grade 9 learners]

**CHART 5.14. COMPETENCES AMONG GRADE 9 LEARNERS**

Learner F mastered theorems with less difficulty. This was due to accurate diagrams and meaningful sentences. He was able to form a substantial number of concepts. He could communicate thought in writing after receiving assistance from the researcher.

Learner G mastered theorems with less difficulty. She was able to form a few concepts. They were formed after receiving assistance from the researcher. She could communicate thought after receiving assistance from researcher. This, once again, was attributed to poor command of English.

Learner H mastered theorems with less difficulty. This due to the fact that movements of shapes were done with less difficulty. His performance in concept formation was poor. As a result, a few concepts were formed. He could communicate thought after he had received assistance from the researcher.
Learner J could only master theorems with the assistance of the researcher. She partly
able to form concepts. Her worst performance was in communicating thought. She
could communicate thought after receiving assistance from the researcher. She wrote
meaningless statements.

Learner K mastered theorems with less difficulty. He drew all shapes such that they
were recognisable. Apart from poor command of English, he could verbally explain by
using names angles, sides and triangles how he mastered theorems. The researcher
could only sift meaning from his verbal explanation. He was partly able to form
concepts. He could form a few concepts by all execution media after receiving
assistance from the researcher. He could communicate after receiving assistance from
the researcher.

5.6 Competence among a group of grade 8 and 9 learners for all activities.

This section will discuss learner’s competence for all activities. The discussion will not
take into account a specific execution medium.

5.6.1 Competences among a group of grade 8 learners for all activities (see
chart 5.15)

![Chart 5.15. Competences among Grade 8 Learners for All Activities](chart5.15.png)

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According to chart 5.15 learners could only communicate thought in one activity. In the rest of the activities they were able to communicate thought either after receiving assistance from the researcher or they were not able to do so at all. According to chart 5.15 learners formed a substantial number of concepts in two activities. They formed a few concepts in twelve activities. No concept was formed in the other three activities.

According to chart 5.15 learners partly mastered theorems in the two activities.

5.6.2 Competences among a group of grade 9 learners for all activities (see chart 5.16).

![Chart 5.16: Competences among Grade 9 Learners for All Activities](chart516.png)

All learners experienced difficulty in communicating thought (see chart 5.16). Learners could only communicate thought in 19 activities after receiving assistance from the researcher. They could not communicate thought in four activities. Written and verbal explanation lacked cohesion.

Learners formed a substantial number of concepts in eight activities. A few concepts were formed in ten activities. Learners could not form concepts in one activity.
Learners were able to master one theorem with less difficulty. They partly mastered theorems in the three remaining activities.

5.7 Comparison of competences between grade 8 and 9 learners for all activities (see chart 5.17).

In this section there will be a comparison of competences between grades 8 and 9 learners. The discussion will be on the extent to which each grade performed in respect of the three competences for all activities.

According to chart 5.17, grade 9 learners showed the ability to communicate thought in more activities than grade 8 learners. However both groups could only communicate thought after they received assistance from the researcher (see appendix B).
According to chart 5.18, grade 9 learners formed more concepts in most activities than grade 8 learners. Nevertheless, both groups formed a few concepts.

According to chart 5.19, grade 9 learners mastered more theorems than grade 8 learners. In both activities, the average performance of grade 9 was higher than grade 8. Nevertheless, grade 9 learners mastered theorems with less difficulty (see appendix B). This could be attributed to the ability to draw accurate diagrams, to compare shapes and to write logical mathematical statements.
Both groups of learners could not communicate thought without assistance from the researcher. They formed a few concepts. Although both groups could form a few concepts and communicate thought after receiving assistance from the researcher, grade 9 learners performed better than grade 8 learners. Only grade 9 mastered theorems with less difficulty. Grade 8 learners partly mastered theorems (see chart 5.20).

5.8 Comparison of execution media between grade 8 and 9

The discussion in the section will be two fold. Firstly, it will be on comparing the two grades in respect of the four execution media. Secondly, the discussion will determine execution media, which produced better performances in each grade.
According to chart 5.21, grade 8 learners could use drawing, talking and ‘showed with and used hands’ after receiving assistance from the researcher. They could not use writing for both three competences. On the other hand, grade 9 learners could use drawing without difficulty to communicate thought, to form concepts and to master theorems. They could also use writing, ‘showed with and used hands’, and talking with the assistance from the researcher.

5.9 Conclusion

The discussion in this chapter was mainly based on three competences, namely; communicating thought, concept formation and mastering theorems. Each competence was discussed in respect of the four execution media (writing, drawing, ‘showed with and used hands’ and talking). Individual learner’s performances in communicating thought, concept formation and mastering theorems by four-execution media were discussed. The discussion also took into account individual learners’ performances in each grade for all activities. Learners A, B, and D could communicate thought by only one execution medium. Learners A and B communicated thought by drawing and learner D by ‘showed with and used hands’. Learners C and E could not communicate thought by either medium. Meanwhile, learners F, H and K could communicate thought only by drawing. Learner G and J could not communicate thought by either medium.
Learners A, C, D and G formed a substantial number of concepts by drawing alone, whereas learners B, F and H here formed a substantial number of concepts by drawing and 'showed with and used hands'. Learner J formed a substantial number of concepts only by 'showed with and used hands'. Learner C and E, except learner K, could not form concepts in writing and talking.

Learner F mastered theorems by all four-execution media. Learners A, H and K mastered theorems by drawing and talking. On the other hand, learners B, D, G and J by 'showed with and used hands'. Learner C mastered theorems only by 'showed with and used hands'. Learner E did not master any theorem

No grade 8 learner, except learner B, was competent in any of the competences. This learner was competent in mastering and concept formation (see chart 5.13). However he was weak in communicating thought. All grade 9 learners, except learner J, were able to master theorems with less difficulty (see chart 5.14). None of them, except learner F, could form a substantial number of concepts in all activities. All of them wrote and made statements that lacked cohesion.

Grade 8 learners could not communicate thought in most activities. They were not able to form concepts in most activities. Theorems were partly mastered (see chart 5.15). Grade 9 learners could not communicate thought without assistance from the researcher in all activities (see chart 5.16). They were able to form concepts in half of the activities. They also mastered one theorem and partly mastered the remaining ones.

Grade 9 learners communicated thought with the assistance of the researcher better than grade 8 learners in most activities (see chart 5.17). However both groups wrote statements that lacked cohesion. They also formed a substantially greater number of concepts than grade 8 learners in most activities (see chart 5.18). Furthermore, they
also mastered theorems more than did grade 8 learners in all the activities (see chart 5.19).

Both groups of learners wrote and statements that lacked cohesion. Once again they both formed a few concepts. Only grade 9 learners mastered theorems with less difficulty (see chart 5.20).

Grade 8 learners could only use all execution media, except writing, with the assistance of the researcher (see chart 5.21). They wrote meaningless sentences. On the other hand, grade 9 learners could only use drawing effectively to communicate thought, to form concepts and to master theorems. They could use the rest of the execution media with the researcher’s assistance.
CHAPTER SIX

SUMMARY, FINDINGS AND RECOMMENDATIONS

6.1. Introduction

This chapter will, amongst other things, include a summary of the whole study. In the summary a discussion of the statement of the problem and aims of the study will be made.

A brief discussion of the effects of process-based instruction on the teaching and learning of Euclidean geometry will be made. The meaning and the nature of Euclidean geometry and non-Euclidean geometry, constructivism, Van Niekerk’s model, Van Hiele’s model, proof, problems concerning second language and evaluation of Euclidean geometry curricula in South Africa and abroad will also be discussed.

A brief discussion of the role of process-based instruction on the teaching of Euclidean geometry will be made. The discussion will also be of the traditional teaching approach, process-based instruction, problem-solving strategies, cognitive processes and meta-cognitive processes.

A summary on the research methodology will also be given. The discussion will be based on how to achieve the aim and objectives of the study. Background information about the school and the timetable will also be given.

A summary of the analysis of data will also be given. It will include analysis of the effects of process-based instruction on learners’:

- communicating thought,
- forming concepts
• and mastering theorems by execution media.

Findings of the study will be based on the analysis of data. Shortcomings in the study will also be discussed. Recommendations will be based on the findings. Directions for further research will also be given.

6.2. Summary

The purpose for the study is to investigate the effects of process-based instruction in communicating thought, forming concepts and mastering theorems by the execution media. The research problem concerns the following issues:

• Learners’ lack of spatial knowledge.
• Euclidean geometry being taught at a higher level than that of the learners.
• Learners’ lack of knowledge and insight into concepts and theorems.
• Lack of command of the language of instruction (English).

The aim of the study was to investigate effects of the process-based instruction on learners’ performance in Euclidean geometry. The research objectives investigate the issues as stated in section 1.4.

There are different meanings associated with geometry. It is sometimes defined as either a science of measuring land or a science of the physical space. The latter definition implies that the space around the objects constitute geometry.

It has been indicated that the way in which Euclidean geometry is taught, is not accessible to learners. It does not afford learners the opportunity to organise and construct logical arguments. The present Euclidean geometry curriculum (1992 interim syllabus) is characterised by deduction. It is presented in grade 10, 11 and 12 mathematics textbooks at Van Hiele level 3. However most learners are at the lower Van Hiele levels of thinking. As a result, they perform poorly.
Euclidean geometry was found to be inadequate to shed light on some of the concepts. In the light of this, a discussion on non-Euclidean geometry was necessary, for example, transformation geometry. Transformation geometry, like dynamic geometry, helps learners to investigate concepts and properties of geometry shapes. It is inductive in its approach to teaching and learning Euclidean geometry. It encourages learners to discover concepts and properties of shapes on their own. Unlike Euclidean geometry, transformation geometry deals with a greater variety of shapes. However it lacks depth and mathematical virtues. Transformation geometry gives learners the opportunity to construct knowledge by themselves.

Constructivism as a method of teaching mathematics is about construction of knowledge by the learner. The learner is regarded as an active participant rather than a passive recipient. The teacher plays the role of a facilitator. In constructivism learning is problem-centred. However the study problem solving approach. It should be noted that this study only made use of the elements of constructivism. Learners also work on tasks in small groups. Although group work is necessary teachers can still create an environment in which individual learners can construct knowledge without verbal interaction. Constructivism is suitable for introducing concepts and discovering theorems.

The study looked at whether process-based instruction will enable learners to develop spatial knowledge. In the light of this, the discussion of Van Niekerk's model was necessary. Van Niekerk's model is aimed at the development of spatial knowledge of young learners through block building and use of solids. It further enables learners to develop reasoning using various execution media (talking, writing, 'showed with and used hands', drawing). The model encourages learners to explain their reasoning. The model is meant to develop learners in the following skills: verbal, tactile, visual and mental. It investigates how learners develop spatial knowledge and the following issues:
• The teaching philosophy employed by the researcher as well as the class teacher;
• The influence of the worldview and culture;
• Demography and socio-economic background of learners;
• The nature of the classroom culture in a multicultural society;
• The design and development of the appropriate materials;
• Utilisation of different execution media.

According to Van Niekerk's model the following factors influence the competence of a task or a skill:

• The execution media (talking, drawing and construction).
• The context of the task.
• The order of the activities.
• The type of task.
• The type of object.
• The dimension of the stimulus.
• The type of knowledge.

Van Niekerk's model has a bearing on the Van Hiele model, in the sense that it states that the type of content and the way it is sequenced has a bearing on the attainment of higher levels of thinking.

According to Van Hiele theory learning is discontinuous. In other words, learning takes place according to levels of thinking. Initially, Van Hiele (1984: 245-248) identified five levels of thinking. They were later reduced to three, namely, the visual level, descriptive level and the theoretical level. In order for learners to progress to a higher level of thinking, geometry activities should be developed according to instructional phases in a specific period. According to Van Hiele's theory no learner can skip a level. In other words, a learner cannot be at level n without having gone through level n-1.
There are different opinions as to what constitutes a proof. It is sometimes defined as observation of instances, lack of counter-examples, acceptance of authority, personal experience and deductive arguments. On the other hand, it is also defined as a logical explanation of results. The deductive proof implies that only learners whose level of thinking is at level 3 (theoretical) can do proof. Proof as a means of explanation requires that learners should be at descriptive level (level 2). It helps the learner to gain insight into why the results are true.

A study conducted in the United States of America (USA) found that 30% of learners can do proof. This showed that very few learners are able to do proof. This was attributed to the fact that most learners were at a lower Van Hiele level of thinking than that which would have enabled them to write proof. In another study it was found that there was a significant relationship between proof-writing and inductive reasoning, deductive reasoning and Van Hiele levels of thinking. This suggests that learners should first be taught geometry inductively before they are taught deductively.

A study conducted in KwaZulu-Natal (South Africa) found that 45% grade 12 black learners have attained Van Hiele level 2 or lower. However they wrote examination set at Van Hiele level 3 (theoretical). They were found to be incapable of writing proof. This study confirms the results of studies conducted in the USA as mentioned above. Another study conducted in South Africa found that if learners are given proper guidance they could construct proof. Learners were found to need further explanation of the results. They wanted to know why the results were true. In terms of the study any logical explanation of results that will help the learner to gain insight into them is considered as proof. The study advocates the use of inductive approach in teaching and learning Euclidean geometry through dynamic geometry. The proof-processes involved in teaching and learning Euclidean geometry inductively are:

- Observe instances in the activity.
- Formulate conjectures from observations.
- Examine and re-examine a conjecture
- Test whether a conjecture is true.
- Refine a conjecture and construct proof.
- Seek explanation for own observation.

These processes can only occur in conditions within which learners are given the opportunities to construct knowledge by themselves. They are integral part Polya’s stages of problem-solving.

The roles and functions of proof are as follows:

- Proof as a means of conviction/verification/justification.
- Proof as a means of explanation/illumination.
- Proof as a means of systematisation.
- Proof as a means of discovery.
- Proof as a means of communication.

Proof still has an important role to play in geometry. Its functions outweigh its criticism. It needs to be made accessible to most learners.

Competence in the language of instruction by learners improves their performance in geometry. It further helps them to have insight into the problem they solve. It is also a pre-requisite for learners’ ability to describe properties of shapes. This enables the learner to attain Van Hiele level 2. Learners experience the following difficulties when they learn mathematics through the second language:

- Communicative difficulties.
- Differences between ordinary English and the language of mathematics.

Research has shown that bilingual learners who are competent in both the first and second language perform better than monolingual learners. If learners are to be successful in conjecturing, they must be competent in both the mother tongue and the
second language.

It has been observed that curriculum documents often show principles that encourage learners to conduct investigations. However teachers may fail to apply them in practice. This was clear from the discussion on Realistic Mathematics Education in the Netherlands and the curricula in Singapore. This has also been the case with the interim syllabus in South Africa. Teachers failed to implement the theory from the curriculum documents. This was also attributed to insufficient training. Replacements of Euclidean geometry by transformation geometry in Australia, Britain, Singapore and the Netherlands did not either. Criticism was levelled at its lack of rigour and the inadequate training received by teachers. In terms of process-based instruction teachers are not supposed to abandon the existing curriculum content. What happened in the above-mentioned countries has violated this principle. However the United States of America (USA)’s curriculum looks coherent. However it fails to link inductive reasoning and deductive reasoning. Studies on geometry curricula in countries such as the Netherlands, United States, Australia, Britain, Singapore, Thailand and South Africa show that Euclidean geometry is taught through the use of concrete objects from the primary school to secondary school.

The teaching practice that is still dominant in South Africa classrooms is that of transmission of knowledge to learners by the teacher. The learner is expected to reproduce proofs of theorems during tests and examinations. In the traditional approach learners are expected to follow the teacher’s method of solving problems. In other words, learners are regarded as passive recipients of knowledge. The traditional approach comprises a small component of process-based instruction in the initial stages.

Process-based instruction is characterised by the elements of the traditional approach and constructivism. It places emphasis on the learner, their learning and cognitive activities. It is further characterised by teacher-dominance at the beginning and the gradual transfer of instruction to learners. Eventually, learners will construct knowledge.
by themselves. Process-based instruction is suitable for classes with learners of all abilities. It can easily be integrated into the existing curriculum. In process-based education, learners may also work on different tasks. This situation can be described as follows: Learners

- Show each one's solution.
- Explain each one's solution.
- Justify each one's solution.
- Describe each one's solution. (Dekker & Eshout-Mohr 1998:305)

What makes process-based instruction different from other forms of teaching is inclusion of meta-cognitive strategies. It helps learners to choose solution strategies that they may use and apply to solve a problem.

Like any discipline of mathematics, Euclidean geometry is characterised by problem solving. Problem solving in Euclidean geometry involves application of previously learned concepts, skills and theorems to solve a given problem or to prove a new theorem. It is argued that problem solving is a skill that must be taught learners throughout their school experience. Unlike in algebra, in Euclidean geometry problems require different approaches. However this does not necessarily mean that there are no similar approaches among geometric proofs. Polya has identified the following stages in solving a mathematical problem:

- Understanding a problem.
- Devising a plan.
- Carrying out a plan
- Looking back.

It must be noted that even though the above stages must be followed, problem solving involves circularity (back-and-forth). This also applies in geometry. The best way to master skills for solving problems is by solving more problems. Experience in problem solving builds the learner's confidence to solve more problems. This in turn helps the
learner to gain insight into problems. Consequently, problem solving leads to learning by discovery.

Various researchers propose that Euclidean geometry be taught inductively at all levels of schooling. It is believed that the inductive approach helps learners to formulate conjectures. Research has shown that most learners cannot write proofs because they do not know how to formulate conjectures. This suggests that the inductive approach should precede the deductive approach in teaching and learning Euclidean geometry. Dynamic geometry has been found to be one of the suitable approaches in teaching Euclidean geometry inductively.

It has already been mentioned that process-based instruction places emphasis on the learner, learning and cognitive activities. As learners apply solution strategies to a problem, they undergo cognitive processes, namely, conceptualisation, intuitive thinking, inductive and deductive thinking, analysis, interpretation, conjecturing, generalisation, integration, application and visualisation. It should be noted that process-based instruction involves learners in planning and monitoring their own procedures. In other words, it is characterised by meta-cognitive strategies, for example, reflective thinking.

Five learners from each grade (grades 8 and 9) were the subjects of the study. They were selected randomly from the June 1999 examination schedule. The criterion used to select the subjects was not based on their performance in the June 1999 mathematics examinations. Both the qualitative and quantitative methods were used. The study used the qualitative method for collecting data and interpreting data. The stratified sampling technique was used for sampling subjects. The quantitative method was used to quantify learners' performance. Four-point scale rubrics were used to assess learners' performance in respect of competences by execution media. Scores were recorded in a score grid. Graphs were used to organise data. Learners received process-based instruction in concepts and theorems. The empirical study was based on the learners' performance in communicating thought, concept formation and
mastering theorems by execution media (talking, writing, drawing, 'showed with and used hands'). The data was collected by: observing learners' responses to activities and questions asked during interviews, analysis of learners' work sheets, listening to audiocassettes and using observation notes.

Data analysis was conducted on the basis of assessing learners' performance in respect of communicating thought, concept formation and mastering theorems by execution media. In this study communicating thought was defined as a means of conveying meaning by written or spoken word, using parts of the body, using movements of shapes and drawing. An analysis was made of individual learners' performance in communicating thought by execution media. Comparison was made between execution media with regard to individual learners' communication of thought. Analysis was also made of each group of learners' (grade 8 and 9) performance in communicating thought. Execution media were compared so as to determine whether the group performed better in communicating thought using one or more of them. Similarly, analysis was made in concept formation and mastering of theorems.

Comparison between competences was made with regard to the average performance of an individual learner for all activities. A comparison between competences was also made with regard to all activities based on each group's average performance. A comparison between the two grades (8 and 9) with regard to communicating thought, concept formation and mastering theorems was made. The two grades were again compared with regard to the execution media.

In the light of this analysis the researcher made the findings set out below.

6.3. Findings

The data analysis has resulted in the researcher making the following findings:
1. A few individual grade 8 and 9 learners had the ability to communicate thought by drawing and 'showed with and used hands'. Groups of grade 8 and 9 learners had the ability to communicate thought by drawing and 'showed with and used hands' for a few activities. Groups of grade 8 and 9 learners could not communicate thought by writing and talking for most or all activities. In other words few shapes were recognisable, sentences and mathematical statements were meaningless or lacked cohesion, verbal explanation became meaningful after clarification was sought and assistance was given in tracing, folding, rotating, comparing, reflecting, measuring and sorting shapes.

2. Most learners of both grades 8 and 9 formed a substantial number of concepts by drawing rather than by other execution media. None of the learners could form concepts by writing and talking. In other words, they did not write meaningful sentences and statements that might have led to the formation of concepts. They were also not able to explain what they had observed during the activity which would have lead to the formation of concepts. As a result, few concepts were formed. A few learners formed concepts by 'showed with and used hands'.

3. A group of grade 8 learners formed more concepts by drawing and 'showed with and used hands' for a few activities. It did not form concepts by writing and talking. A group of grade 9 learners formed concepts by writing, drawing and 'showed with and used hands' for most of the activities. Concepts were formed by talking in few activities.

4. No individual grade 8 learner mastered theorems by three execution media. Most learners mastered theorems by 'showed with and used hands' and talking.

5. Most grade 9 learners mastered theorems with less difficulty. All grade 9 learners mastered theorems by talking. This was due to the

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fact that they were confident in explaining their observations, despite the fact that they had poor command of English. Three learners mastered theorems by drawing and ‘showed with and used hands’. Only one learner mastered theorems by writing.

6. A group of grade 8 learners did not master theorems for all activities. It only mastered theorems by talking and drawing for two different activities. On the other hand, a group of grade 9 learners partly mastered theorems for most of the activities. It mastered theorems by talking for all activities. It had also mastered theorems by ‘showed with and used hands’ for most of the activities. It could not master theorems by writing and drawing.

7. Only one grade 8 learner could master theorems with less difficulty and form a substantial number of concepts. Most learners could only form concepts with assistance from the researcher. None of the grade 8 learners could communicate thought.

8. Most grade 9 learners performed better in theorems than in other competences. However most learners mastered theorems with less difficulty. Only one learner could form a substantial number of concepts. None of the learners could communicate thought unaided.

9. A group of grade 8 learners could not communicate thought and form concepts for most of the activities. It could also not master theorems for all the activities. Poor command of English by learners contributed to poor performance in all competences.

10. A group of grade 9 learners formed concepts for a few of the activities. It did not master theorems for most of the activities. It could not communicate thought for all activities. Even though the group had poor command of English, it was better than grade 8 learners. Thus, they formed more concepts than grade 8 learners.

11. In most of the activities none of the two groups (grade 8 and 9) could communicate thought effectively and form concepts. Grade 9 learners mastered on theorems with less difficulty than grade 8 learners.
12. Only grade 9 learners could use drawing in communicating thought, forming concepts and mastering theorems effectively. Meanwhile, both groups (grade 8 and 9) of learners could only use the other three execution media with the researcher’s assistance.

13. All learners showed that they have not attained Van Hiele’s first (visual) level of thinking. They could not recognise most two-dimensional shapes. They are still confusing a rectangle with a square, and a side with an angle. Most of them could not draw three-dimensional shapes. A ball was drawn as a circle. Drawing diagrams derived from the medium ‘showed with and used hands’ reinforced knowledge of some shapes (isosceles and equilateral triangles). Drawing of an incomplete or wrong diagram was a result of poor imagery or visualisation. They could also not recognise shapes after they were rotated. Their spatial knowledge development was poor.

Despite the researchers’ findings above, the study has its own shortcomings. Some of them were beyond the researcher’s control.

6.4. Shortcomings in the study

This study is based on a small sample of ten learners. Its findings may not be applicable to a larger sample. On the basis of the verbal information from the school co-ordinator (the teacher whom the researcher was given by the school as contact person) the group the researcher sampled was considered to be poor in mathematics. The following factors impacted negatively on the performance of learners:

1. Too much emphasis was placed on the use of English to teach and learn Euclidean geometry. It took learners a long time to follow instruction both from the researcher and the work sheet. The use of English, as well as, mother tongue could have improved learners’ performance.

2. The absence of the Overhead Projector impacted on the effectiveness of the
activities. Movements of shapes could have been properly shown on the Overhead Projector. The message could have been effectively communicated to learners. The researcher relied on verbal communication.

3. Withdrawal of learners from investigations by teachers in order for them to attend their lessons affected their performance. As a result, it was difficult for them to catch up with the rest of the group.

4. Use of a variety of shapes in an activity could have helped learners to formulate conjectures. Learners had limited opportunities to explore because of a limited number of shapes that were made available to them.

5. The results could have been different if less content was covered. Too many activities were covered within a short space of time. This compromised the effectiveness of process-based instruction.

In the light of the above findings and shortcomings the recommendations are as follows:

6.5. Recommendations

In the light of the findings mentioned above the researcher recommends the following:

1. Geometry worksheets should be developed in such a way that they take into account each learner's Van Hiele level of thinking. The language used in the worksheets should be simple to all learners. In situations where Euclidean geometry is offered in the second language, learners should be encouraged to communicate their ideas in the mother tongue and the language of instruction interchangeably. This will enable learners' to switch over to a language with which they are comfortable to communicate their thoughts. Learners should be encouraged to talk about their experiences in their groups. The more they talk about each other experiences, the more they become confident they become in communicating thought. Consequently, this may facilitate progression to higher Van Hiele levels of thinking.
2. Knowledge of concepts and theorems alone is not sufficient to equip them with skills to solve problems. The more problems they practice, the more skills they acquire. They should be given simple problems at the beginning in order to boost their confidence. Once they are confident they will be able to attempt to solve more difficult problems.

3. A variety of resource materials should be used to teach communication, concepts and theorems. A particular resource material may help one learner, but it may fail to help the other. The resource materials would give learners the opportunity to explore.

4. Teachers should be encouraged to find out about what and how learners learn and think when they communicate thought, form concepts and master theorems. This enables them to gain insight into the learner’s learning difficulties and immediately find a way to guide them. If learners are given the opportunity to explain what they have discovered, then teachers will be able to identify their problems.

5. Assessment and teaching of geometry should be integrated. Teachers should not only assess learner’s solutions; they should assess (together with the individual learner) the processes involved in arriving at the solution.

6.6. Directions for further study.

In the light of the findings, shortcomings and recommendations, the following directions for further study were arrived at:

An investigation into
- the effects of process-based instruction in the use of bilingualism to teach and learn grades 8 and 9 geometry concepts.
- the effects of process-based instruction in the use of mother tongue in communicating thought and forming of grade 8 and 9 geometry concepts.
- the effects of the inductive approach in communicating thought and forming grades 8 and 9 geometry concepts.
• the development of assessment strategies in a process-based instruction that will enhance learners' ability to master theorems both inductively and deductively.

This does not mean that the above list is complete. The researcher might have overlooked some areas for further study.

6.7. Conclusion

This chapter dealt with the summary of the study, findings, shortcomings and recommendations. In the summary the following were discussed: the research problem, the aim and objectives of the study, the influence of the inductive approach on teaching and learning Euclidean geometry, the processes involved in the teaching and learning of Euclidean geometry and the research methodology.

The study achieved the objectives as stated in section 1.4.

In the light of the findings as outlined in section 6.3, the researcher recommends the following:

• Appropriate use of the mother tongue in teaching and learning Euclidean geometry should be encouraged.
• Geometry worksheets should take into account learners' Van Hiele levels of thinking.
• A variety of resource materials should be used to learn concepts and to master theorems.
• Teachers should be encouraged to gain insight into the learner's thinking.
• Learners should be given as many problems as possible.
• Assessment and teaching of Euclidean geometry should be integrated.

Finally, this study enabled the researcher to gain insight into how learners are introduced to learning geometry concepts and theorems inductively. It further enabled
the researcher to understand the extent to which geometry learning is impeded by using English as a language of instruction. The literature suggests use of the mother tongue together with the language of instruction in teaching and learning Euclidean geometry.
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APPENDIX A

ACTIVITY SHEETS
ACTIVITY ONE

You have been given shapes. Sort them as follows:

1. Put shapes, which look the same together.
2. What make shapes look alike?

3. In each group sort out shapes of equal size.
4. What did you do to find out that shapes have equal size?
ACTIVITY ONE

You have been given shapes. You must sort them as follows:

1. Put shapes, which look the same together.
2. What make shapes look alike?

3. In each group sort out shapes of equal size.

4. What did you do to find out that shapes have equal size?
ACTIVITY TWO

You must identify geometric shapes from pictures and drawings given to you.

What is in the pictures or drawings?

1. Draw shapes that you have identified
2. Wherever possible write names of these shapes.
ACTIVITY THREE

Draw a line on paper and then trace it on the acetate sheet with a water soluble transparency pen.

Place a pin on the end of the line on the acetate sheet and the paper.

Use your right hand to rotate the acetate sheet about the pin.

Which shape has been formed by the line on the acetate sheet and the line on paper?

Draw as many different shapes as you can, by rotating the acetate sheet.
ACTIVITY FOUR

Label (name) angles in activity three using three capital letters, e.g. A, B, C.

Measure all angles that you have drawn in activity three by means of a protractor.

Write down their measurements (in degrees).

Arrange their measurements as follows:

Angles between; 0° and 90°, 90° and 180°, 180° and 360°
ACTIVITY FIVE

Identify the following types of angles from a Parallelogram Tiles worksheet:

Acute angle
Obtuse angle
Right angle
Straight line angle
Reflex angle

Assessment
1. Use a protractor to find the measurement of the adjacent angles below:

(a) \[ \text{1} \quad \text{2} \]
(b) \[ \text{1} \quad \text{2} \]
(c) \[ \text{1} \quad \text{2} \]
(d) \[ \text{1} \quad \text{2} \quad \text{3} \]
ACTIVITY FIVE (CONTINUED)

2. Find $x$:

(a) \[110^\circ \quad x\]

(b) \[60^\circ \quad x\]

(c) \[x \quad 2x\]

(d) \[x + 13^\circ \quad 57^\circ\]

(e) \[42^\circ \quad 68^\circ\]

(f) \[37^\circ \quad 113^\circ \quad x\]
ACTIVITY SIX

Draw an acute angle on a transparent white paper.
Fold the angle in such a way the two lines that constitute it lie on top of each other.
Unfold the paper.
What does the folded line do to the angle?

Draw the acute angle above and show the folded line on paper.
ACTIVITY SEVEN

Draw a line on a white transparent paper.
Fold the line as if you are bending it, such that one end of the line meets the other end.
Unfold the paper.
What does the folded line do to the drawn line?

Draw two lines as shown on paper and give them names by using capital letters.
ACTIVITY EIGHT

Continue with the activity above by bending the folded line away from the drawn line. Unfold the paper.
What does the new folded line do to the previously folded line?

What do you notice between the second folded line and the drawn line?

Draw them below and show with arrows that they are parallel.

Draw all the lines as shown on the paper you have folded and give them names by using capital letters.
ACTIVITY NINE

Study the drawings before you. (Parallelogram Tile worksheet).

Write down, using letters of the alphabet, lines on Parallelogram Tile worksheet which are parallel to each other.

Which of the drawings and pictures contains perpendicular lines?

Draw perpendicular lines below and show with a sign by a small square.

Write down in symbols (letters), to show lines which are perpendicular to each other.
ACTIVITY TEN

Draw two intersecting lines and mark angles on a white paper clockwise from 1 to 4. Trace the two intersecting lines, together with numbered angles, on the acetate sheet. Rotate the acetate sheet about 180° with the pin or pen at the point of intersection. Compare angles on the acetate sheet and those that are on paper. Which angles on paper are equal to those on the acetate sheet after rotation?

Assessment
1. Colour angles that are equal with the same colour.

2. Mark vertical opposite angles that are equal to each other on the Parallelogram Tile worksheet.

3(a)

Calculate $\angle A_4$

(b) Which other angles are equal to $\angle A_1$ and $\angle A_2$ respectively?
ACTIVITY ELEVEN
Mark the angle on paper below.
Trace the angle and its mark below on the acetate sheet.
Shift the acetate sheet along one of the lines on paper.
Show which of the lines are parallel to each other.

What is the relationship between the two horizontal lines?

What do you notice between the two angles above the horizontal lines?

What is the relationship between the horizontal lines and the angles above each of them?
ACTIVITY ELEVEN (CONTINUED)

3. Which angles are equal on the diagram?
   (a) \[ \text{Diagram (a)} \]
   (b) \[ \text{Diagram (b)} \]
   (c) \[ \text{Diagram (c)} \]
   (d) \[ \text{Diagram (d)} \]
   (e) \[ \text{Diagram (e)} \]

4. Find \( x \)
   (a) \[ \text{Diagram (a)} \]
   (b) \[ \text{Diagram (b)} \]
   (c) \[ \text{Diagram (c)} \]

Show that \( z = x \) and say why this is the case.
ACTIVITY TWELVE
Mark the angles below with different signs
What is the sum of angles you marked?

Trace the angles and their signs below on the acetate sheet.
Slide the acetate sheet to the right.

What do you notice about the two vertical lines?

What do you notice about the angles between two vertical lines?

What is the relationship between the vertical lines and the angles between them?

1. Mark and write down co-interior angles that add to 180°.

(a) \(A \quad 1 \quad 2\)

(b) \(P \quad Q\)

(c) \(S\)

(d) \(M \quad 1 \quad 2\)
ACTIVITY TWELVE (CONTINUED)
2. Find \( x \)

(a) 
[Diagram of a line segment ST with an angle of 50° and an angle of \( x \) at point T.]

(b) A

[Diagram of a line segment AB with an angle of 140° and an angle of \( x \) at point B.]

(c) 

[Diagram of a line segment PQ with an angle of 60° and an angle of \( x \) at point Q.]
ACTIVITY THIRTEEN
Mark the angle below.
Trace it on the acetate sheet.
Place a pin at one end of one line of the angle.
Rotate the acetate sheet at 180°.

What do you call two the horizontal lines facing the opposite direction?

What did you notice about the two angles?

What is the relationship between alternate angles and parallel lines?

1. On the diagrams below mark and write down alternate angles on diagrams below that are equal:

(a) \[ \triangle \]

(b) \[ \triangle \]

(c) \[ \triangle \]

(d) \[ \triangle \]

(e) \[ \triangle \]
ACTIVITY THIRTEEN (CONTINUED)

2. Find $x$

(a) \[ \begin{array}{c}
\text{\text{53\textdegree}} \\
\end{array} \]

(b) \[ \begin{array}{c}
\text{\text{47\textdegree}} \\
\end{array} \]

(c) Show that $a = c$
ACTIVITY FOURTEEN
Assessment
1. Write down the following angles:
   (a) Corresponding angles
   (b) Alternate angles
   (c) Sum of co-interior angles
   (d) Vertical opposite angles
   (e) Adjacent angles

2. Find $x$, $y$, and $z$
   (a) 
   (b) 
   (c) 

3. (a) Show that $\angle A = \angle C$
   (b) Show that $\angle P = \angle R$
ACTIVITY FIFTEEN

Fold an A4 paper and simultaneously cut it horizontally and at an inclined angle (see diagram below).

The horizontal and the incline cut must meet to produce a folded right-angled triangle. Unfold the right-angled triangle to produce a bigger triangle.

Determine sides and angles that are equal by folding the triangle (mark them with the same colour).

What did you observe?

________________________________________________________

________________________________________________________

________________________________________________________

________________________________________________________

Draw the triangle and mark equal sides and angles with a similar sign.
ACTIVITY SIXTEEN

Fold an A4 paper and simultaneously cut it horizontally and at an inclined angle of lengths of your own choice. (see diagram below).

The length of the horizontal line must be half of the length of the line at an inclined angle.

Unfold the right-angled triangle to form a bigger triangle. Determine sides and angles that are equal by folding the triangle (Mark them with the same colour).

What did you observe?

________________________________________________________________________
________________________________________________________________________

- Draw the triangle and mark equal sides and similar angles with the same sign.
ACTIVITY SEVENTEEN

Place the mirror on the solid line.
What do you see inside the mirror?

What can you say about the sizes of the two triangles?

Which triangle has been formed by two triangles put together (one triangle is inside and another one is outside). Why?
ACTIVITY EIGHTEEN

Draw any triangle that is big enough and cut it out.
Mark all angles of the triangle with different colours.
Put a dot anywhere on the straight line.
Place angle C on the dot and trace it.

Slide the triangle to the right such that angle B is on the dot and trace it.
Try to fit in angle A on the third angle.
What did you notice?

What is the sum of the angles of a triangle? Why?

Assessment

Find x

(a) 

(b) 

48° 77°

54°
(c) \[\triangle \text{with angle } 36^\circ, x \text{ and } x\]

(d) \[\triangle \text{with angle } 48^\circ, 48^\circ, 48^\circ\]

(e) \[\triangle \text{with angles } 3x, 2x, 3x\]
ACTIVITY NINETEEN

Draw any big triangle and cut it from the blank sheet. Mark its angles in terms of 1, 2 and 3.

Trace the triangle in the space of the work sheet provided below and number its angles like the one above:

Extend the horizontal side of the triangle on the work sheet to the right. Mark the outside of the triangle with 4.

Which two angles of the triangle on the piece of paper fit in angle 4?

Which two angles inside the triangle are equal to angle 4? Write down their relationship below.

Write down in your own words the relationship of the angle outside the triangle and those inside it.
Assessment

1. Find $x$
   
   (a)
   \[ \angle 47^\circ \quad \angle 63^\circ \quad x \]

   (b)
   \[ \angle 35^\circ \quad \angle 75^\circ \quad x \]

   (c)
   \[ \angle 50^\circ \quad \angle 125^\circ \quad x \]

   (d)
   \[ \angle 48^\circ \quad \angle 108^\circ \quad x \]

   (e)
   \[ \angle 110^\circ \quad x \]

2. (a)
   \[ \angle 1 \quad \angle 2 \quad \angle 3 \quad \angle 4 \]
   Show that $\angle 3 + \angle 4 = \angle 1 + \angle 2$

   (b)
   \[ \angle 1 \quad \angle 2 \quad \angle 3 \quad \angle 4 \]
   Show that $\angle 4 = \angle 1 + \angle 2$
ACTIVITY TWENTY

PART ONE

Cut the rectangular A4 paper across, from one corner to the other.

What can you say about the sizes of the two triangles?

Mark one triangle with letters ABC and the other with letters DEF.

Write down sides of triangles ABC and DEF that are equal.

Write down angles in triangles ABC and DEF that are equal.

Write down two sides and one angle from triangles ABC and DEF that are equal.

Write down two angles and one side from triangles ABC and DEF that are equal.

PART TWO

Place the mirror on the solid line.

What do you notice about the size of the triangles inside.
Assessment
1. Determine whether a pair of triangles is congruent? Why?

(a)

(b)

2. (a) Show that \( \triangle ABC \cong \triangle ADC \)

(b) Show that \( \triangle PQS \cong \triangle QRS \)
ACTIVITY TWENTY ONE

Write down all lines that are parallel lines on the parallelogram below.

Trace the parallelogram on the acetate sheet together with the letters of the alphabet.

Rotate the acetate sheet such that the parallelogram on the acetate sheet fits on the one paper.
Record the lines that lie on top of each other and indicate whether they are equal.

Mark sides that are equal on paper on the parallelogram, as observed during rotation.
Record angles that lie on top of each other.

Mark angles that are equal on paper on the parallelogram, as observed during rotation.

What did you notice about opposite sides of a parallelogram?

What did you notice about opposite angles of a parallelogram?
ACTIVITY TWENTY ONE (CONTINUED)

1. Find $x$, $y$ and $z$:

(a) \[ \begin{array}{c}
\text{40mm} \\
\text{32mm}
\end{array} \]

(b) \[ \begin{array}{c}
x \\
\text{65°} \\
\text{115°}
\end{array} \]

(b) \[ \begin{array}{c}
\text{126°} \\
\text{30°} \\
\text{120°}
\end{array} \]

2. Show that $\triangle ABD \cong \triangle BCD$
ACTIVITY TWENTY TWO

What is the name of the shape above? ____________________________.
Trace figure PQRS, PT and QT.
Rotate the shape to 360°.
Which line segment of the diagonals on the acetate sheet, is going to lie on top?

______________________________________________________________

What did you notice about PT on the acetate sheet and the line segment below it?
______________________________________________________________

What did you notice about QT acetate sheet and the line segment below it?
______________________________________________________________

Mark lines that are equal inside the shape on paper.
What did you notice about PR and QS?

______________________________________________________________

Assessment
1. Find x and y:

   20 cm

   15 cm


2. Show that AC and BD bisect each other.
ACTIVITY 23

1. Find $x$, $y$ and $z$.
   (a) 
   \[ \begin{align*}
   &\text{70°} \\
   &\text{110°}
   \end{align*} \]
   \[ \begin{align*}
   &\text{x} \\
   &\text{y}
   \end{align*} \]

(b) 
\[ \begin{align*}
   &\text{120°} \\
   &\text{z}
   \end{align*} \]

2. 
(a) 
\[ \begin{align*}
   &\text{p} \\
   &\text{Q} \\
   &\text{R} \\
   &\text{S}
   \end{align*} \]
Show that $PQRS$ is a parallelogram.
(b) $\triangle ABC$ and $\triangle ACD$ are isosceles.
(c) $KM$ and $LN$ are diagonals which intersect at $O$.

Show that $ABCD$ is parallelogram

Show that $KLMN$ is a parallelogram.
APPENDIX B

CALCULATED AVERAGE ACHIEVEMENTS
<table>
<thead>
<tr>
<th>COM 8</th>
<th>COM 9</th>
<th>COMMC</th>
<th>CONCE</th>
<th>MASTERING THEOREMS</th>
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<td>2.4</td>
<td>GR8</td>
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APPENDIX C

LETTER FROM THE SCHOOL
Nngweni Sec. School
P.O.Box 142
Dzanani
0955
26 July 1999

Makhado College
P.O.Box
Dzanani
0955

Dear Mr Kutama

**PERMISSION TO CONDUCT RESEARCH.**

With reference to your letter dated the 19th of June 1999 permission is granted to you to conduct research at Nngweni Secondary School.

Hoping that you will learn a lot about the problems that the learners are experiencing in Mathematics and how they can be solved.

Wishing you all of the best in your research.

Yours in service
Muvhango M.J (Principal)