

**IMMIGRANT LEARNERS LEARNING LINEAR
PROGRAMMING IN MULTILINGUAL CLASSROOMS IN
SOUTH AFRICA**

IMMIGRANT LEARNERS LEARNING LINEAR PROGRAMMING IN MULTILINGUAL
CLASSROOMS IN SOUTH AFRICA

by

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I declare that **IMMIGRANT LEARNERS LEARNING LINEAR PROGRAMMING IN MULTILINGUAL CLASSROOMS IN SOUTH AFRICA** is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

Tnkambule.....

DATE 25 / March / 2013

SIGNATURE

(MRS T NKAMBULE)

DEDICATION

My husband

Khakhalazi Jackie Nkambule

My children

Ndumiso, Phinda, Thandokuhle and Setsabile

My parents

Siphitha Ginindza who is late and Thandie Ginindza (nee Dlamini)

ABSTRACT

This study used discourse analysis (Gee, 2011; 2005; 1999) in order to explore a socio-situated view of how teachers created learning opportunities for the participation of immigrant learners when learning linear programming in a Grade 11 mathematics classroom in South Africa. The aim was to explore that which mathematics teachers do in classrooms with immigrant learners that they will not do if there were no immigrants. A discourse analysis approach was used in order to view the opportunities created through language use not as a tool for communication only but also as a tool for building reality.

The study reported in this thesis was conducted in three different settings which are in; urban, township and rural environments. The urban environment focuses on immigrant learners who were born in the Democratic Republic of Congo (DRC), and started schooling there, in the township and rural environment it focuses on immigrant learners born in South Africa with parents born in the Republic of Mozambique or Angola. Three different mathematics classrooms were observed in their natural environment during lessons focusing on linear programming. Data was collected through a learner questionnaire issued before lesson observations. The aim of the learner questionnaire was to understand the language background of the learners in the mathematics classrooms selected for the study. The second method included lesson observation for at most five consecutive days at each setting. It involved observing teachers and immigrant learners during teaching sessions of linear programming activities. The activities included reading, writing, speaking and participating in mathematical activities. These activities were then analysed to understand how teachers created learning opportunities for the immigrant learners.

The study contextualised the results from lesson observations by conducting clinical interviews with three immigrant learners, one from each site, to provide insights into the explanations on immigrant learners approaches when solving a linear programming task. The main conclusion in this study is that immigrant learners were successful in linear programming when teachers' created learning opportunities by using code switching to support them.

The main contribution of this study is that it focuses on multilingual mathematics classrooms of immigrant learners in South Africa – a context that has not yet been researched in South African

mathematics education. Exploring language practices in multilingual mathematics classrooms of immigrant learners provides a different gaze into teaching and learning mathematics in multilingual classrooms in South Africa. Equally important is the extent to which immigrant learners are distinct to multilingual learners in the teaching and learning of linear programming.

Key words: Immigrant learners, linear programming, constraint inequalities, optimizing, search line, discourse analysis and discourse models.

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CHAPTER 1

1.1 Introduction

The past few years have seen tension and violence directed at immigrants in South Africa. According to Crush & Williams (2003), this tension and violence is a demonstration of frustration by poor and unemployed citizens who accuse foreigners of stealing their jobs as well as an increase in competition for scarce resources such as housing. The accusations are mainly because as black South Africans claim space within their country's urban areas which were previously forbidden them¹, they confront non-nationals also seeking safety, employment and economic opportunities in the same urban areas. Unfortunately, most South African citizens are ignorant about immigrant's rights (Matsinhe, 2011). This led to ill-treatment of some immigrants including teachers which were witnessed by some children especially in 2008. My interest in this study was sparked by a concern that immigrant learners² in some South African mathematics classrooms may have to deal with hostility and a reduced participation in classroom activities as a result of the tension and violence against immigrants in the country.

The study explored learning opportunities for participating immigrant learners when learning linear programming³ in a Grade 11 mathematics classroom. Like many South African learners, immigrant learners use English as a Language of Learning and Teaching (LoLT), which is an additional language for them. The study reported here focused on three groups of immigrant learners at three different settings in South Africa. The first group whose home language is Lingala migrated from the Democratic Republic of Congo (DRC) where the language of learning and teaching (LoLT) is French. This group is located in an urban environment in the neighbourhoods of central Johannesburg. The second and third group of immigrants is from the Republic of Mozambique and their home language is Xitsonga. Xitsonga is one of the eleven official languages in South Africa. The second group is located in a township environment in the

¹ During apartheid-era South African migration policy promoted permanent white immigration and temporary black migration. The post-apartheid period is characterised by a mix of circular, permanent and transit migration (Landau & Kwabe-Segatti, 2009).

² In this study I refer to an immigrant learner as a learner who first enrolled at and entered a South African school in Grade 7 or a more senior grade (National policy pertaining to the programme and promotion requirement of the National Curriculum Statement Grades R-12 (January 2012).

³ Linear programming is a technique for the optimization of a linear objective function (such as maximum profit or lowest cost) subject to linear equality and linear inequality constraints in a given problem (mathematical model) (Dossey, McCrone, Giordano & Weir, 2002).

Western part of Johannesburg. The third group is located in a rural environment in the Eastern part of the Limpopo province.

The study was guided by the following questions:

1. How do teachers create opportunities for the participation of immigrant learners when teaching linear programming?
2. How are languages within the immediate environment of immigrant learners used during the teaching and learning linear programming?
3. How do immigrant learners take up these learning opportunities when solving a linear programming task?

1.2 Establishing a focus

This study was considered with the view that constructing inequalities⁴ from a given linear programming task creates some of the challenges that obstruct immigrant learner's correct solution to the task. These seemed to be linked with the interpretations and understandings that are involved in language use (Moschkovich, 2007; 1999a; Pirie, 1998; Pimm, 1991; Rowland, 1995). One of the challenges with immigrant learners might be that they receive instruction in an additional language, or they might have poor reading and comprehension skills in English, which is the language of learning and teaching (LoLT). For example, if they are from a French or Portuguese background and linear programming tasks are presented in English.

While this challenge is not specific to immigrant learners it is exacerbated by the fact that some of the immigrant learners do not have fluency in the local languages and they use their native languages for communication at home. Furthermore, immigrant learners do not have a contextual advantage; they do not share a home language with the teachers and also may not share the LoLT of their country of origin.

The revised South African curriculum places great emphasis on "the establishment of proper connections between mathematics as a discipline and the application of mathematics in the real-

⁴ In mathematics an inequality is an algebraic expression composed of the sum or difference of variables which are less or equal to (denoted by the symbol $x \leq k$) or greater or equal to a certain number (denoted by the symbol $x \geq k$)

world contexts” (NCS, 2003: 10) and highlights problem-solving, investigative learning approach as a means to achieving this. Furthermore, the curriculum states that “Mathematical problem solving enables us to understand the world and make use of that understanding in our daily lives” (DoE, 2005: 8). This suggests that tasks within this curriculum that have some real-life context appropriately mirror the goals of the revised curriculum thus making it somehow more applicable for immigrant learners to use that understanding in their daily lives. However, when incorporating the real world in linear programming tasks into a discourse of school mathematics, there is a need to keep the focus on the mathematics. This might lead to an improvement of information which might remove the complexity that real life mathematics usually presents (Mukhopadhyay & Greer, 2001).

Learning linear programming is viewed in terms of a relationship between an individual’s (immigrant learner’s) activities and their environment in which they have to think, feel, act and interact (Gee, 2003b). Any environment, in which an individual (immigrant learner) finds himself or herself in, is filled with possible actions determined by features in the environment. In order to act accordingly, an individual (immigrant learner) must have the capacity to transform the actions into actual and effective actions (Gee, 2003b). Furthermore, Gee argues that an individual (immigrant learner) can take an advantage of what is offered by the features in the environment by using objects, tools or technology to enhance performance. The use of objects, tools or technology might require an individual (immigrant learner) to interact with other immigrants or local learners in assisting each other in learning. This suggests that knowledge is differentially distributed across people. Obviously, people and their actions are part of the environment.

Vygotsky (1987) argued that human beings do not react directly to or interact directly with the environment. Rather, human reactions and interactions are mediated by signs (language and other symbol systems) and tools. He went on to argue that people learn how to use these mediating devices primarily through social interaction. Vygotsky’s (1981: 163) fundamental view is expressed in the following quotation:

Any function in the child’s cultural development appears twice, or on two planes. First it appears on the social plane, and then on the psychological plane. First it appears between people as an inter-psychological category, and then within the child as an intra-

psychological category. This is equally true with regard to voluntary attention, logical memory, the formation of concepts, and the development of volition.

According to Gee (2005a), people internalize the workings of their language, various symbols, norms, values and ways of acting and interacting through participation in common activities with already proficient others. The use of languages in school and the types of interaction teachers use can either advantage or disadvantage immigrant learners, particularly those from a French or Portuguese background, in an environment where English is the LoLT. Unfortunately, this mismatch of languages may be a shortcoming to immigrant learner's achievements in linear programming. Therefore, in such a context the learning of linear programming may be viewed as an introduction into the linear programming discourse. For that reason, participation in social activities allow us to ask how we can position immigrant learners in the interaction so as to allow them to be active generators of knowledge (Greeno, 1998), then one can explore the actions and constraints of different forms of participation. Consequently, language serves as a crucial window for researchers into the processes of teaching, learning and doing linear programming. These processes are considered as socially organized, taking place not only within a social environment but also structured by that environment (Morgan, 2002). This study therefore considered studying language in use in order to take into account both the immediate situation in which linear programming content is being exchanged (the context of the situation) and the background within which the participants are embedded (the context of their background).

1.3 Selecting a suitable theoretical framework

Gee (2005a, 2011) put forward the study of language in use, the notion of discourse analysis, with a theoretical perspective which places understanding in the public domain. Furthermore, he also provided tools (questions to ask) for doing a discourse analysis. Thus, according to Gee (2005a), oral or written texts are produced and interpreted within the social context. Gee's notion of Discourse (with a Capital D) is useful to interpret the opportunities for the participation of immigrant learners when learning linear programming.

Gee (1999: 7) states that a lowercase "d" discourse refers to how language is used "on site" to enact activities and identities. However, an uppercase "D" Discourse involves much more than sequential speech or writing:

A Discourse is a socially accepted association among ways of using languages, other symbolic expressions, and 'artifacts', of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or social network', or to signal (that one is playing) a socially meaningful 'role'. (Gee, 1996: 131)

According to the definition offered by Gee, Discourse is more than just language because it involves using languages and symbolic expressions in interaction with people belonging to specific communities. This means that Discourses are collective and not individual. Discourses give meaning to every human activity and thus well-established within Discourses are practices and knowledge from which people draw when engaging in the various roles that they play. Gee emphasises that the key to Discourse is recognition, which suggests that whatever you have done must be similar enough to other performances to be recognizable, if it is not recognizable, then, you are not in the Discourse (Gee, 2005a).

In a social setting like a mathematics classroom, immigrant learners use language, behaviours, actions, tools, etc to recognise themselves and others as belonging to a group learning linear programming or set of practices or Discourse. At the same time immigrant learners give meaning to that Discourse by reproducing it or transforming it. In other words, the immigrant learners can be recognized in the Discourses, but at the same time the Discourses make us recognize immigrant learners since Discourses are not individual but collective. Therefore, giving insight into the social setting in which the opportunity for the participation was made, as well as the opportunity that comes to be understood upon that social setting. This theoretical framework will be elaborated in Chapter two.

1.4 Migration to South Africa

Migration is about people reconsidering their place in the world, making decisions about their future, moving along and staying behind (Landau & Kabwe-Segatti, 2009; Landau, 2007). These scholars observe that migration is also about people sharing space via the acquisition of new neighbors in their cities without moving at all. Migration is a phenomenon that affects not only South Africa but also other countries around the globe, for example Europe, Australia, Canada and United States of America. Human beings have always moved from one place to another. Nowadays, immigration has been a key feature of countries in Africa with large waves of people

moving between countries (Crush & McDonald, 2002). According to Trumbauer (2005), people migrate from their home country to settle in a new country for two underlying reasons. The first is that negative conditions like poverty, political oppression or war in their native country push them to leave (push factors) (Landau, 2007). The second reason is that positive conditions in the new country pull them from their native country (pull factors). These people tend to seek opportunities that do not exist in their native country.

Political changes in South Africa have led to the escalation in the number of immigrants. Since 1994 she has been one of the major recipients of immigrants from Africa, Europe and Asia while prior to 1994; most authorized migrants came from Europe and neighboring countries (Crush, 2008). Crush argues that before 1994 immigration policy was an obvious means of racial control. Until 1991, the official definition of an immigrant was that he or she had to be able to fit into the white population. This suggests that Africans (blacks) were not considered immigrants. After 1994, political changes led to the South African government formulating a policy appropriate to the country's new role in a changing regional, continental and global migration regime (Crush & Williams, 2001). This policy on international migration replaced division amongst its citizens (blacks and whites) with shared roles of social engagement. However, discrimination against non-citizens threatens further social marginalisation for them.

South Africa integrated with the Southern African Development Community (SADC)⁵ and reconnected with the global economy and such opened up to forms of migration associated with globalization (Crush & McDonald, 2002). It is a reality that some countries of the SADC region are still dealing with the legacy of mass displacement and forced migration. Hence, migration to South Africa may be due to an increase in political instability, poverty and lack of employment in some of these countries. For example, people migrated from Angola due to civil war in that country, and because of conflict in the DRC, Burundi, Rwanda and Somalia. According to Harris (2002) some unskilled immigrants come to South Africa to escape the poverty and unemployment conditions in their own countries. As a result South Africa's major cities became

⁵ SADC member states are: Angola, Botswana, Democratic Republic of Congo (DRC), Lesotho, Madagascar, Malawi, Mauritius, Mozambique, Namibia, Sychelles, South Africa, Swaziland, united Republic of Tanzania, Zambia and Zimbabwe.

primary destinations for immigrants from around the continent, thereby transforming the country's population. This is a fact that aroused xenophobia among its citizens witnessed in the shameful events of May 2008 (Crush, 2008).

Despite the numerous problems that face the majority of blacks⁶ in South Africa like unemployment, for Africans from other parts of the continent, the country is perceived as being the land of increased economic opportunities and hope. Sassen (1996) argues that the structure, character and state of the South African economy, compared to other African countries is the driving mechanism. Hence as long as the widespread poverty, political instability and high levels of inequality prevail on the continent, South Africa will continue to attract migrants who bring with them diverse languages as they cross borders.

The demographic change is an eye opener and there is evidence in the 2001 census conducted by Statistics South Africa which shows that about 2.3 % of the population was immigrant people. It further shows that migrant people included 687 679 from SADC and 228 314 from Europe, 41 817 from the rest of Africa and Asia 40 886 (p. 15).⁷ This includes countries whose language of learning and teaching is not English. The 2001 census further identifies other languages spoken by these immigrants other than the eleven official languages. Regardless of their reasons for immigrating, some immigrants might include a significant number of children and teenagers. Some of these children might have been attending school and need to continue with their education in South Africa.

As immigrants flow into the country, they affect the composition of the population in several ways that result in changes across different generations of South Africans, for example, the languages people speak at home, in the neighbourhood or at school. The South African Government agreed to support and assist immigrants, refugees and asylum seekers when it signed the 1951 Refugee conventions⁸ and its protocol as well as the Organization of African

⁶ I use the word black South Africans to attach the identity to Africa

⁷ <http://www.statssa.gov.za/census01/html/RSAPrimary.pdf>

⁸ Website: <http://www.aresta.org.za/refugeeact.pdf>

Unity's (OAU) 1969 convention⁹. According to these conventions, immigrants, refugees and asylum seekers are afforded virtually the same rights as South African citizens. To be precise, immigrant children are entitled to the same basic primary education which the inhabitants of South Africa receive. Therefore, immigrant learners may be found in South African schools, consequently in mathematics classrooms where the majority learns mathematics in English, a language that is not their home language (Setati, 2005a). Therefore, immigrant learners from French, Portuguese or Spanish background will learn mathematics through the medium of English. Needless to say, some teachers in these classrooms find themselves faced with immigrant learners who come from a wide range of cultural and linguistic backgrounds.

Civil (2008b) whose work focuses on immigrant learners in the USA argues that in most cases education from another country is often not directly translated into the same educational level as in the receiving country. Individuals who have migrated from one country to another may continue to incorporate daily routines, activities and institutional affiliations that connect them to their country of origin even as they actively engage in their everyday lives in their destination country (Levitt & Schiller, 2004). It is against this background that this study sought to explore the learning opportunities made available to immigrant learners in South Africa.

The next section presents my experience as a mathematics teacher in Swaziland as well as a developing researcher in multilingual classrooms in South Africa.

1.5 My experience

Teaching mathematics in English at school had always seemed a normal thing with me during my teaching career in Swaziland. This normality was due to the fact that, while my home language was Siswati during my school years, English was the language of learning and teaching mathematics. In 2006 I enrolled at the University of Witwatersrand for a Bachelor of Science with Honours (Science Education) where I registered for a course entitled “expressing

⁹ Website: <http://www.aresta.org.za/oaconvention.pdf>- convention governing the specific aspects of refugees in Africa: This convention is specific to refugees in Africa. It contains a broader definition of a refugee and rules on asylum and voluntary repatriation among others.

mathematics” focusing on language issues in mathematics. The course was an eye opener and made me realize that learning mathematics is about acquiring fluency in the mathematics register (Halliday, 1975), which includes speaking, reading and writing mathematically.

The course exposed me to research that highlights the difficulties learners face when learning the mathematics register (Pimm, 1991) and challenges that learners who learn mathematics in a second language face when moving from ordinary language to mathematical language (Setati, 2002). These studies made me realize that both ordinary and mathematical languages are important for mathematics teaching and learning especially linear programming.

Having experienced teaching mathematics up to Grade Twelve, I can be of evidence to the fact that teaching mathematics is a series of obstacles and challenges, an activity met with continued failure. These days many learners do not succeed in mathematics, and research argues that the linguistic backgrounds of learners might have a positive influence on learners’ learning of mathematics and as well as on the teaching of mathematics (for example Gorgoriò & Planas, 2001).

From my experience as a developing researcher, I have learnt that the learners’ home languages may be used as a resource especially where the teacher shares a home language with the majority of learners (Nkambule, Setati & Duma, 2010; Nkambule, 2009; Nkambule & Setati, 2007). Furthermore, findings from recent studies in South Africa and elsewhere point to pedagogy that would allow learners to use their home languages as a support during teaching and learning to enable learners to engage in meaningful mathematical activities (Moschkovich, 2007; Adler, 2001; Setati, Molefe & Langa, 2008; Webb & Webb, 2008; Vorster, 2008; Brijlall, 2008). Although migration has been conceived as a global problem due to famine and civil war, research has rarely examined practices that immigrant learners in South Africa use to develop their mathematical understanding. Hence, the importance of this study reported.

Mathematics as a school subject has always had an important place in our society as well as in immigrant societies. From my observations, the experiences in these societies are very diverse; especially due to diverse languages so that equal opportunities are created, hence this is a

challenge facing the education system. Researchers in this area suggest that immigrant communities need a culture-sensitive (Gorgorio & Planas, 2001) education that can accommodate their special cultural needs and prepare them for life in an increasingly globalised world.

From my experience, the mathematics classroom is a social and educational situation; it has its social dynamics and its social interactions. The various moments of those dynamics have different meanings for the different participants in them. Mathematics classrooms with immigrant learners do not exist in a vacuum, they are socially, culturally, politically and historically located (Gee, 2000; Luke, 2003). Choices at the political level, about selecting the language to be taught, and at the institutional level, about staffing, curriculum content, resources, pedagogy, are all essentially related to social beliefs. These choices have significant implications for the learners' educational individuality. According to Planas & Civil (2004), the mathematics classrooms with immigrant learners are key sites of inclusion, exclusion, privilege and disadvantage settings where learners learn to be particular kinds of people.

In these mathematics classrooms, one might argue that everyday classroom interaction may support a community through particular understandings of individual learners, their unique experiences and educational uniqueness. However, it may be difficult in some classrooms. Research has shown that in some of these classrooms, immigrant learners are usually not provided with the opportunities to incorporate their rich cultural identity and life experiences into their formal schooling (Stathopoula, 2006; Gorgoriò & Planas, 2001; Civil & Planas, 2004; Civil, 2008b; Planas & Gorgoriò, 2004). I acknowledge that immigrant learners take upon themselves negotiations as they adapt to schools, learn a new language of learning and teaching as well as the official languages in South Africa. It is obvious that immigrant learners have less control over the cultural and social resources in the mathematics classroom context. However, they need to develop mathematics discourses that will enable them to succeed. The important question to ask is how do teachers support immigrant learners in South African mathematics classrooms when learning linear programming? Furthermore, are immigrant learners provided with opportunities to draw on languages they are fluent in while learning linear programming?

1.6 Language policy in South Africa

South Africa celebrated its change to democracy after the end of apartheid in 1994 and chose a multilingual approach as its language policy. The South African constitution recognizes eleven official languages. The majority of white South Africans have Afrikaans and English, as their first languages. The nine African languages are Sepedi, Sesotho, Setswana, siSwati, Tshivenda, Xitsonga; isiNdebele; isiXhosa and isiZulu. In recognising eleven languages, South Africa acted in line with the 1986 “organisation of African Unity” (OAU) Language Plan of Action for Africa (Maartens, 1996) which encouraged member states to recognize all languages within their boundaries. Webb (1999) contends that recognizing eleven official languages does form a logical part of the whole idea which underlines the constitution and its bill of rights that of national integration and unity within diversity.

Needless to say, in South Africa language differences had continued to be preserved thus regarding them as separate. However, most South Africans everyday speech use resources of other languages and treat the languages as “interconnected hybridized forms” (Makoni, 2003: 145). Makoni argues that given the hybridity that emerges in urban popular culture it is more useful to talk about linguistic resources than language. His argument is based on the fact that the notion that languages are discrete, bounded entities and countable institutions is a social construct. Makoni & Pennycook (2007: 22) suggest the “disinvention of language which critically unpacks the discursive processes involved in the classification, naming and invention of a language”.

The presence of communities in South Africa, like Greek community, shows that home languages as well as cultural practices are preserved. In this regard, teachers could create opportunities for learning linear programming through talk and actions during teaching and learning of linear programming. At issue here is the role of and use of languages immigrant learners are fluent in during the teaching and learning of linear programming. For example, opportunities can be created by including activities involving reading and writing in either English or French where spelling of a word is the same but differ in pronunciation.

Nowadays in South Africa, there are signs of support for multilingualism; for example, the official languages are used widely on national television and radio, some official documents are available in languages other than English or Afrikaans. Furthermore, language options are available in Automatic Teller Machines (ATM). This is evidence that the use of the official languages is advancing as most black South Africans are comfortably multilingual while the majority of whites are bilingual with proficiency in English and Afrikaans. This shows that language is one of the most mysterious possessions and a perfect example of our humanity. It is a main factor that enables individuals to become fully members of the group into which they are born while at the same time providing an important link between the individual and his/her social environment. That being the case, how are migrants supported in such an environment especially in mathematics when learning linear programming?

1.7 Language in education policy (LiEP) in South Africa

The South African Constitution (Section 29) stipulates that:

Everyone has the right to receive education in the official language or languages of their choice in public educational institutions where that education is reasonably practicable. In order to ensure the effective access to, and implementation of, this right, the state must consider all reasonable educational alternatives, including single medium institutions, taking into account

- a. equity;
- b. practicability; and
- c. the need to redress the results of past racially discriminatory laws and practices.

<http://www.info.gov.za/documents/constitution/1996/96cons2.htm>

While the Schools act determines that:

The provinces must formulate their own language-in-education policies (subject to national policy). Further: The governing body of a (public) school determines the school's language policy (subject to any relevant provincial acts).

According to the LiEP, multilingualism is encouraged and together with practices like code switching. However, research shows that local communities are not keen on the use of African languages as the LoLT even though research argues for the use of home languages as it is advantageous especially when learning mathematics (Setati, 2008; Setati, Molefe & Langa, 2008). The trend has been to reject the use of African languages as it was seen in the past to be a means of oppression by keeping the education standard inferior to that of the English speakers (Hartshorne, 1992). In support of Hartshorne's argument, Bamgbose (2008) argues that South African languages are surrounded in the political history of the country.

In fact, Bamgbose (2008) states that during apartheid all of the languages acquired socio-political meanings; English is highly prestigious, Afrikaans generally labelled as socially undesirable as it was regarded as the language of the oppressor and African languages are seen as of little economic or educational value. However, the Education Department declares itself committed to the promotion of multilingualism which means developing the eleven official languages and establishing respect for all the languages of the country. Therefore, it is not surprising to find that teachers in the mathematics classroom allow learners to use their languages as a resource when learning mathematics. But how about immigrant learners, are they provided with opportunities to draw on the languages they are fluent in while learning linear programming?

1.8 Multilingualism in the global era

Multilingualism is a broad notion and while its definition is often contested, it is acknowledged and recognized by many. According to the Oxford Advanced learners dictionary, multilingual means "speaking or using several different languages". Setati & Barwell, (2006: 27) whose research is in mathematics education, define multilingual classroom as one in which, "any of the participants (learners, teachers or other) is potentially able to draw on more than one language as they go about their work".

In linguistics, there are authors who prefer to talk about plural monolingualism and others prefer to use the phrase hybridized multilingualism (Makoni, 2003). Plural monolingualism is when multiple languages are used discretely and distinctly, even where individuals may be speakers of several languages. On the other hand, hybridized multilingualism refers to the form which

“contain lexical items which are an embodiment of linguistic information drawn from different languages” (Makoni, 2003; 139). According to Makoni, South Africa may be referred to as plural monolingualism. In this study, I use the phrase multilingual classroom to refer to classrooms in which participants (teachers and learners) have the ability to draw on one or more than one language as they go about their work in a mathematics classroom. The reality is that the concentration of these languages is mainly in industrialized areas like Johannesburg because people migrate to these places in search for jobs. Below I discuss types of school settings where the study was conducted.

1.8.1 Multilingual classroom settings in South Africa

Adler (2001) identifies three types of school settings found in South Africa. The types of school settings are rural, urban and township schools. According to Adler, most teachers and learners in the rural schools share the home language. It can be assumed that one primary African language is brought to the class by the teacher and learners. She argues that learners in these schools speak, read or write in English in the formal school context.

The majority of the teachers in township schools are multilingual. Many speak at least two African languages in addition to English and Afrikaans. Learners in these schools are also likely to speak more than one African language and will have ranging levels of English language proficiency.

The study focused on the following multilingual classrooms settings in South Africa in order to explore how immigrants’ learners are supported when learning linear programming:

- Urban school (School A), located in central Johannesburg, with immigrant teachers and learners from the DRC, Zimbabwe and Malawi. The mathematics teacher whose class the study focused on is from the DRC. The home languages of the learners are Lingala, Shona and Chichewa. The study focused on learners from the DRC who share a home language (Lingala) and also share the language of learning and teaching (French) in their country of origin with the teacher.

- Township school (School B) located in the west of Johannesburg, with two immigrant learners from Mozambique and had Xitsonga as their home language. One learner was born in Mozambique while the other was born in South Africa.
- Rural school (School C), located in the Limpopo Province, with immigrant learners from Mozambique and their home language is Xitsonga. While these immigrant learners shared a home language with their South African counterparts the LoLT in their home country is Portuguese. The teacher is from Zimbabwe and her home language is Shona.

1.9 Conclusion

This chapter located the study in South Africa by presenting a brief argument about migration to South Africa since 1994. It has been mentioned that these immigrants' stay in South Africa is supported by international policy documents which South Africa is signatory to. The immigrants migrate due to a number of reasons which might be pull or push factors. As a result, they settle in different locations in South Africa which might be urban, township or rural. The country's language policy, the LiEP has been presented and encourages multilingualism in schools. As a result, practices like code switching are evident in some mathematics classrooms like in the township. It has been highlighted that these policies also encourage the respect of other languages other than the eleven official languages in South Africa. The chapter also presents my experiences as a mathematics teacher in Swaziland and I argue that mathematics is one of the most valued subjects in our society, but our efforts as teachers are met with failure. In addition, my experience as a developing researcher in multilingual classrooms in South Africa suggests that the home languages may be used as a support while learners are learning English. The use of home languages as a support would enable second language learners understand the mathematical concepts presented in English. However, the issue of immigrant learners who might share or might not share the home languages as well as the LoLT with local learners in South Africa is still missing in the literature.

1.10 Structure of the report

The next chapter, Chapter 2, presents the literature that was reviewed to locate this study. There is not much in the literature on how immigrants in multilingual classrooms in South Africa are

supported when learning mathematics. Hence most of the literature relating to teaching and learning of immigrants is actually centred on research in Europe, Australia, United States of America and Canada.

The literature reviewed informed the study on the issues related to everyday mathematics, community and school mathematics hence different forms of mathematics. It also addresses teachers' attitude and knowledge of immigrant learners, educational policy, language and mathematics and immigrants learners. The researchers observe that the approach by teachers involve the use of the LoLT of the host countries to immigrants learners. Much of the research also points to the political role of English where English is the LoLT. The chapter also briefly looks at the window the researchers are using. Relevant to this study is the socio-cultural perspective. This chapter provides an insightful overview of some key constructs that are present in the articles reviewed namely power, identity, participation, discourse. For example of importance is a better understanding of immigrant learners' everyday discursive practices.

Although there is not much in the literature on how immigrants in multilingual classrooms in South Africa learn mathematics (linear programming), the literature reviewed informed the study on teaching approaches in multilingual classrooms in South Africa. Of relevance to this study is the approach of using learners' home languages deliberately.

The Chapter also presents the theoretical framework adopted for this study. I have relied on Gee's conception of discourse analysis (2011, 2005a, and 1999). The other section is on linear programming content studied in mathematics at high school level, which details linear programming content in the curriculum. This section is meant to provide information on the material to be covered at high school level. A historical account of the founders of linear programming is also discussed and shows that linear programming is a powerful tool.

Chapter three is the research design and methodological approach used in the study. It outlines the methods of data collection which are video recording of lesson observation, interview and clinical interview with the immigrant learners. It further discusses validity and reliability issues

that were considered to ensure that the study is valid. The chapter further discusses ethical issues that were considered before data was collected.

Chapter four discusses the process of gaining access to research participants. It argues that gaining access is a process which depends on establishing a relationship with research participants. The Chapter also provides a background of issues surrounding immigrants in South Africa. It further presents the location of the Schools where data was collected. Chapter four also presents an analysis of a learner questionnaire which aimed at illustrating the language backgrounds of the learners in each of the mathematics classroom that were observed at Schools A, B and C.

Chapter five, six and seven, present data analysis, of how the mathematics teachers created learning opportunities for the participation of immigrant learners in the mathematics multilingual classrooms during the teaching of linear programming. These chapters draw on the observation data in order to address research questions one and two which are mentioned earlier. The aim is to show how immigrant learners were supported linguistically or not when learning linear programming. The Chapters show that the teachers created opportunities for immigrant learners by drawing upon the language practices in the immediate environment of the learners. The chapter argues that separating the different contexts for language practices would not have provided the learners with a learning opportunity. These chapters establishes that the process of creating opportunities for immigrant learners varied depending on the language profile of the mathematics class and the teacher and on the location of the school (urban, township and rural).

Chapter eight draws on clinical interviews with three immigrant learners, one from each site and also on a microanalysis of selected work sample. The aim is to link the immigrant learners discourse with that of the lesson observations.

Chapter nine concludes with a discussion of the overview of the journey travelled to produce this thesis. The chapter concludes the study by arguing that teachers in the urban and township settings explored were fluent in the immigrant learners' languages and were therefore able to support learners by code switching.

CHAPTER 2

LITERATURE REVIEW AND THE IMPORTANT THEORETICAL ASSUMPTION

2.1 Introduction

In Chapter one, I mentioned that South Africa has become home to migrants from all over Africa. Some of the African migrants are from former French, Spanish and Portuguese colonies and so speak those languages as well as their home languages. As migrants travel to South Africa with an intention to stay either temporarily or permanently they bring with them their school going children. As a result immigrant learners are found in many multilingual classrooms in South Africa contributing to what is now known as the Rainbow Nation. However, immigrants are not well received by some South African citizens (Crush, 2008). Consequently, the past few years have seen tension and violence directed at immigrants. The study examined how immigrant learners are linguistically supported or not during the teaching and learning of linear programming in multilingual classrooms in South Africa.

This chapter presents and discusses the theory that informed explanations and interpretations in this study. Theory serves as a lens through which I viewed some observed episodes in the data. The data was collected in order to describe and explain how teachers created learning opportunities for the participation of immigrant learners in multilingual classrooms in South Africa to enable them access linear programming content. I start by presenting a review of literature in an attempt to explain why I had to choose this particular study for my thesis. The aim is to provide information and possibly solutions to educational issues that might affect immigrant learners in South Africa.

Since the study sought to explore how teachers created opportunities for the participation of immigrant learners in multilingual classrooms in South Africa, I had to consider a theory that explains how language can be used in different settings not only for successful exchange of ideas but also to perform certain identities or social activities. My assumption was that there is a need for an exchange of ideas in the multilingual settings that would enable immigrant learners understand discussions and be heard. If they cannot understand discussions because they do not understand English, the language of learning and teaching in South Africa, then how do they act

in response to linear programming tasks presented in English? The theoretical constructs that were used in this study are presented and discussed in this chapter in the learning theories assigned to Gee (1999, 2005, and 2011).

2.2.0 Literature review

This section presents an account of the literature concerning mathematics teaching and learning in multilingual classrooms with or without immigrant learners. The literature pays attention to the following:

- i) Importance of language when teaching mathematics as a tool for thinking and communicating.
- ii) Immigrant learners' use of home languages when solving mathematics tasks
- iii) Strategies used by teachers during teaching and learning of mathematics in multilingual classrooms
- iv) Challenges that teachers meet as they teach mathematics in multilingual classrooms with immigrant learners and
- v) The role of policy in the teaching and learning of mathematics.

This section therefore, presents a review of literature that explores language issues essential in teaching and learning mathematics in a school setting.

In mathematics classrooms, teachers use language and also the learning materials presented in a language which conveys knowledge while at the same time positioning learners in relation to that knowledge, directing their attention as well as their actions. This might lead to different kinds of participation resulting from different kinds of learning. The manner in which concepts are presented to learners during interaction or when using learning materials while exploring how they represent the mathematics they are learning is one way of examining what learners are being provided with access to and how they are achieving it. To do this, teachers use language(s) assumed to be understood by all learners in the mathematics classroom. Therefore, the discussion that follows describes the importance of formal mathematical language and the implications over the teaching and learning of mathematics in multilingual classrooms.

2.2.1 Importance of language during teaching and learning of mathematics

Pimm (1987) argue that learners do not openly hear or read much mathematics outside the classroom situation and so the mathematical language that they bring to a mathematics class is informal. Informal mathematical language is the language that learners in this case use to express mathematical ideas in their everyday language. In a school setting, it is the formal mathematical language that is valued and is presented in the Language of learning and teaching (LoLT) in most cases.

For that reason, learners learn formal mathematical language in a mathematics classroom through their mathematics teachers. In this case, it is the mathematics teacher in a mathematics classroom who acts as a model of how to speak mathematically for the learners. Hence one thing that a learner does in a mathematics classroom is to learn a range of accepted ways in which mathematics is to be communicated and discussed. This presents mathematics teachers with a challenge to help learners' move from the use of informal to formal mathematical language which might be in a language that is not their language of learning and teaching. Therefore, learners in this study needed to learn the distinction between the informal and formal ways of expressing mathematical ideas as well as oral or written responses in the LoLT. Even though, their former LoLT in their country of origin is not English, the LoLT in South Africa. Therefore, how do teachers create opportunities for their learning of the formal way of expressing mathematics in this context?

Pimm (1981) contends that part of learning mathematics is acquiring control over the mathematics register so that one is able to talk like a mathematician. Halliday defines a mathematics register as:

...a set of meanings that belong to the language of mathematics (the mathematical use of natural language) and that a language must express if it is used for mathematical purposes. We should not think of a mathematical register as constituting solely terminology, or of the development of a register as simply a process of adding new words (Halliday, 1975: 65).

Halliday's definition means that mathematics register is more than just vocabulary and technical terms. It also contains words, phrases, rules, syntax and methods of arguing within a given situation (Pimm, 1987). The words or phrases include everyday vocabulary that takes on a

different meaning in mathematics; for example, words like prime, difference, cancel. Learners are expected to know and become familiar with this type of language, which they have to learn from their mathematics teachers. The kind of language that is used to present mathematics to learners communicates to them what mathematics is. Therefore, appropriate use of key mathematical terminology indicates accuracy and complexity of understanding which according to Sfard (2001) relies on symbols as its communication tools which are regulated by rules. Sfard argues that teachers have to elicit these special elements using language as a tool for learning mathematics so that learners have a better chance of participating in the discourses and are able to act accordingly. The next section is a discussion on language when teaching mathematics as a tool for thinking and communicating.

2.2.2 Language as a tool for learning mathematics

Psychologists and educators in many parts of the world have acknowledged the issue of language as a tool for learning (Mercer, 1995; Orton, 1992; Vygotsky, 1986). Mercer (1995) argues that people use language to get things done and to engage in their activities. Getting things done through language sometimes depends on how well one is able to communicate his ideas to other people and how those people understand his ideas. In most cases instructions are communicated through talking or writing using a language that both parties understand. Language is seen as situated, jointly produced and organized by content to construct meaning through interaction in the mathematics classroom.

The meaning of concepts in a mathematics classroom is negotiated by teachers to learners in a language that they both understand. In the same mathematics classrooms, learners use language to communicate amongst themselves and with the teacher in a language that they understand. Orton (1992) argues that language is not only important for communication but also facilitates thinking and he explains that the language used for thinking is likely to be the home language. In Orton's view we can assume that in a classroom situation, a learner has to translate the given mathematical statement presented in the LoLT to his/her home language before communicating his/her understanding. This process of translation requires that learners have to understand the language in which the problem is given to make sense of the mathematics established. This

means that the linguistic choices that speakers in interaction and authors make when preparing material are following for the kind of learning that is offered and enabled.

For that reason, language shapes our social world as different in wording may construct different kinds of meanings and interaction. This may result in creating opportunities or not for participation which is crucial in achieving success. This suggests that speakers make choices as they use language and they construe the context in particular ways. Therefore, immigrant learners in South Africa will make a choice of a language they are fluent in during the teaching and learning process. This study will then explore the opportunities of a choice of language that will enable participation in the mathematics classroom. A number of researchers show evidence of learners using language(s) in a number of ways, for example using their home languages interchangeably with the LoLT when solving mathematics tasks. The discussion on the literature relating to learners switching languages follows.

2.2.3 Learners use home languages when solving tasks

Researchers in many parts of the world have shown that learners switch languages when solving mathematical tasks (for example Clarkson, 2007; Parvanehnezhad & Clarkson, 2008 in Australia; Planas & Setati 2009 in Barcelona). In Australia, Vietnamese and Iranian migrant learners have been shown to use their home languages when working with mathematical ideas as one of their solution strategies (Clarkson 2007; Parvanehnezhad & Clarkson 2008). Clarkson (2007) reports on how four high performing Vietnamese bilingual learners used their home language, Vietnamese, as they solve mathematical tasks. He noted that, for these learners, some mathematical processes were easier to complete in Vietnamese. He then maintains that if learners' experience in the first language and LoLT are improved, such learners would improve their mathematical proficiency. Clarkson bases his argument on a study of Grade 4 Vietnamese immigrant students in Australia, who were learning English the language of teaching and learning as a second language. He observed that when immigrant learners were engaged in a mathematical task, they switched between languages. The immigrant learners switched between Vietnamese, their home language and English, the LoLT. The language switching by these immigrant learners was spontaneous and similar to the manner in which they use languages outside school. The switching fits into a normal conversation and it is possible to interact in both

languages during the process of solving the tasks and when explaining. How about in cases where learners cannot use the home languages?

Clarkson interviewed the Vietnamese learners and further revealed that their parents sometimes assisted them in doing their mathematics at home and obviously used Vietnamese. The learners' understanding of the mathematics explained by parents using their home language, Vietnamese, was transferred to English (Cummins, 2000), the language of learning and teaching in the mathematics classrooms in Australia. So immigrant learners developed conceptual understanding that enabled them to engage in academic rather than everyday mathematical meanings. This is because for learners to take part in the classroom activities, they must come to follow and understand particular social rules which are often unspoken (Cazden, 2001) and not universal in their cultural basis. When learners take part in the classroom activities they become more proficient in mathematics. Therefore, the languages that the learners use to do mathematics, either individually, with knowledgeable other(s) or in whole classroom discussions play a significant role in their performance in the classroom participation. For that reason, it becomes crucial that immigrant learners in South Africa do mathematics in a language that they are comfortable with.

However, other researchers have shown that the context in which mathematics learning takes place influences the language(s) used (Planas & Setati, 2009). Learners may engage or not engage with the task due to the language in which they are expected to present their ideas in the public domain. The learners in Planas and Setati's study made choices as they used Spanish, their home language, during group discussions and Catalan, the LoLT, during public domain discussions when reporting their findings. In such a situation, the choices come out of the contexts in which speakers interact and in turn interpret the contexts in particular. So immigrant learners in South Africa may discuss linear programming in French but present their approaches in English, a language they are still learning.

The language in which the task is presented can be a challenge for the immigrant learners (Clarkson, 2007). This brings to light the fact that when immigrant learners in South Africa are presented tasks in English (LoLT) for instance, they might translate it to their home language or

French their second language if they are from former French colonies, for thinking purposes and then think of presenting their ideas in English. This suggests that the thinking process goes from English to French or to home language then to English (LoLT) in an attempt to communicate their understanding. Clarkson suggested that in switching between their languages, learners might well be developing the implications of mathematical notions and hence gaining a deeper understanding of mathematical ideas and processes. He argues from his results that learners need support in their home language because it plays a role in their learning of mathematics.

Furthermore, Clarkson (2007) maintains that learners' proficiency in both their languages makes a difference to their performance in some mathematical tasks. He observed from his results that learners who are highly proficient in two or more languages are likely to do better than average in mathematics. In case of learners who are strong in at least one language, one way of enhancing their level of attainment in mathematics might be to support the development of their proficiency in the other language. The question is how do teachers support immigrant learners when learning mathematics especially when they do not share a home language with them?

2.3 Strategies to support multilingual learners when learning mathematics

Reform initiatives replace traditional classrooms with learners talking to each other and group discussions where learners need to verbalise their opinions. Researchers focusing on how to engage learners in higher-level mathematical thinking have reported the cognitive advantages to learners through their participation in mathematical discussion (for example Moschkovich 1996, 1999; Barwell, 2003a, 2005c, 2009 and Khisty & Chval 2002). To realize these goals, teachers must take advantage of learners' home languages and experiences rather than disregard them. These researchers argue that when learners express their ideas, they are able to make their mathematical reasoning visible and open for reflection. These ideas provide a resource for teachers informing them about what learners already know and what they need to learn. These ideas are addressed by Moschkovich (1996, 1999) in an analysis of a lesson in a third Grade classroom.

2.3.1 Listening to the mathematical content in learners' utterances

Moschkovich (1996, 1999) illustrate how a teacher supported Spanish-speaking third grades in their mathematical discussions in English within the public domain. The teacher in Moschkovich's study listened to the mathematical content in the learners' discussion. The focus stayed on mathematics not on how the mathematical ideas were expressed in English. This suggests that it is important to pay careful attention to how learners express their mathematical ideas rather than on the correctness of their grammar because the home language and the language of learning and teaching might be simultaneously used by the learner during the thinking processes. The strategies the teacher used in Moschkovich's study included "re-voicing learner contribution and building on what learners say and probing what learners mean" (Moschkovich, 1999: 18). She further argues that mathematics teaching in bilingual classrooms should focus on mathematical discourse rather than on errors in English vocabulary or grammar. Such strategies of supporting the learners in participating in mathematical discussions can be applied to multilingual settings with immigrant learners. Immigrant learners can present their ideas in their home languages or French, and in different ways including non-verbal. From those forms, the teacher can facilitate interpretation into formal mathematical language.

Similarly, Khisty & Chval (2002) who investigated the way in which a teacher in their study, Ms Martinez interacted with her fifth-grade Latino learners (and with English-language learners) in the USA. The authors report that the teacher's focus on mathematical talk and meaning enabled the learners to develop mathematical reasoning in significant ways. The teacher facilitated learning through questioning that was concerned more with the views held by her learners in their home language.

In this approach there was access to resources which supported learners' mathematical meaning making. Khisty & Chval (2002) claim that, effective teachers are able to set up an environment in which mathematical language transfers from the teacher to the learners. Hence, the meanings that learners construct eventually exist from those captured through the kind of language the teacher uses. However, mathematical language presents difficulties for learners in general and presents challenges in multilingual classrooms especially where there are immigrant learners. Acknowledgment of learners' responses, while at the same time supporting them to build on their

mathematical language during mathematical discussions, can lead to learners' using a mathematically rich language or the mathematical English. This can be a sign of improved learning of mathematics. Further encouraging and supporting immigrant learners to draw on their home language or the LoLT in their home country as a resource while engaging in mathematics tasks, enables them to relate to a familiar language in which they have confidence in.

However, it should be said that teacher's ability to put into practice an approach that creates opportunities for the participation and task rigor to benefit all learners is not evenly distributed among school personnel (Khisty and Chval, 2002, Lim and Presmeg, 2010). Khisty and Chval's (2002) scrutiny of two teachers' classroom discourse practices provides insights into how a veteran teacher was able to effectively help learners develop mathematical language, while an inexperienced teacher was less successful in helping her learners develop mathematics language learning, even though both teachers received professional development regarding discourse practices. Although the inexperienced teacher had set up some of the processes needed to guide instruction using mathematics discourse strategies, she dominated classroom discussion and failed to develop appropriate mathematics language in her classroom. This study illustrated how less experienced and underqualified mathematics teachers may have difficulties providing learners opportunities to learn mathematics through engaging discourse practices. Such difficulties might also be experienced by teachers of immigrant learners who might not understand their home languages. However, learners may assist one another in their learning when they are allowed to work in groups.

2.3.2 Heterogeneously grouping learners

Creating opportunities for the participation of all learners has a great potential to address the dual needs to improve mathematics learning and expand access to high quality mathematics. Teachers' practices are critical for the ways discourse is structured in the mathematics classrooms. Teaching approaches that pay attention to learners' opportunities to acquire academic forms of language involves "learning not just the same content but also offering equal affordances for action, participation and learning" (Gee, 2008: 104). Therefore understanding the interplay between learners and their learning contexts determine whether they can assimilate the content being taught. In such contexts the opportunities include how resources like home

languages and or languages that learners understand, and learning tasks where learners engage in to demonstrate their understanding, translate into learning opportunities in the classroom. These learning opportunities include mathematics instruction with practices that promote more active roles for learners and might lead to increased achievement in mathematics.

In this respect, learners may be grouped in order to give them opportunities for exploring ideas through discussions amongst themselves. The groups may address differences in language proficiency (Barwell, 2003a, 2005c, 2009) and the gap in mathematics achievement (Boaler, 2006b and Boaler and Staples, 2008). Research by Barwell, Boaler and Staples shows that mathematics instruction with practices that promote more active intellectual roles for learners lead to increased achievement and learner's ideas also become a resource for themselves, challenging, stimulating and extending their own thinking, thus enabling learners to grasp formal mathematical language.

Barwell (2003a, 2009, 2005c) has investigated how elementary immigrant learners made sense of mathematics through the interaction with their monolingual peers in England. His analysis of how learners' engaged with arithmetic word problems shows three ways in which the task promoted learners' sense making. First the task allowed learners to bring personal experiences into their thinking. Secondly, the task led learners to pay attention to the nature of the word problem genre and thirdly the task allowed for a productive, mutually supportive interaction between language learning and mathematical thinking (Barwell, 2009). The study conducted by Barwell (2003a) shows that learners with different language proficiencies can interact in order to improve the group's mathematical communication. Varying the group's composition can further provide learners the chance to gain possible insights from many different learners' mathematical points of view in a language they understand.

Boaler and Staples (2008) in California classrooms indicate that heterogeneously grouped learners guided by a problem based curriculum were more enthusiastic about mathematics and showed significant gains in outcomes compared with learners in mathematics classroom using a traditional instruction. These researchers found out that the highest achieving learners in classrooms using more interactive and problem centered approach to curriculum made the

greatest gains in mathematics outcomes in the second year of their study compared with learners in traditional classroom. In problem based instruction the mathematics gaps among White, Latino and African American learners disappeared, though Asian Americans continued to achieve higher than other groups. However, in classrooms using traditional mathematics instruction they observed that racial gaps persisted. This evidence suggests that mathematics classrooms using a traditional mathematics approach limit learners' opportunity to learn.

2.3.3 Language switching as a resource

Language switching is considered to be a tool which can provide spontaneous and reactive discussion of concepts by learners and teachers in their home languages. However, it is often not obvious in immigrant contexts in other parts of the world (for example UK, Australia). Setati and Adler (2000) discussed the language practices of teachers in some primary schools in South Africa where the official teaching language is English. The authors suggest that code switching may be used as a resource, though there are challenges that cannot be overlooked. Of course exactly when and why a teacher should code-switch is not a straight forward choice and Adler (2001) refers to it as a dilemma that involves finding the balance between using the first language to aid understanding. Teachers who attempt to code switch encounter obstacles such as contending with the tension between access to English and access to meaning (Adler, 2001).

However, it is a major challenge for some teachers to use code switching in the classrooms as a strategy of teaching and learning (Setati & Adler 2000; Setati, 2005a; Lim & Presmeg, 2010). The researchers report that teachers switched between the language of instruction and the learners' home language in order to explain and clarify the concepts to learners. Setati & Adler (2000) as well as Lim & Presmeg (2010) contend that the use of code switching in the teaching and learning of mathematics makes sense; however, there is the dilemma in that the learners must also understand correct mathematical language. In some cases, what might be the home language for communication by the majority of the class might not necessarily be the teacher's or some other learners' main language. In this regard, it can be argued that how and when a teacher uses code switching is not a straight forward matter. In this context, Setati (2005a: 464) states that "to fully describe and explain the use of languages in multilingual mathematics

classrooms we need to go beyond the pedagogic and cognitive aspects and consider the political role of language”.

According to Setati (2005a), different forms of knowledge are associated with different forms of power: power through knowledge of the English language, epistemological power through mathematical knowledge gained through the medium of a familiar language. In this regard, it is important that learners choose a language they are fluent in during the teaching and learning of mathematics. In the process of choosing the language learners are enabled to negotiate their own paths thus creating opportunities for participation. However, language has different social, economic and political power beyond the classrooms. Like for example if the LoLT is English and the learner speaks French with its developed mathematics register, English will determine the success of the learner. Therefore, questions about what language is used in the classroom are thus not simply a matter of how effective communication may be achieved but also impact on the language outside the school like tertiary institutions.

Code switching during teaching and learning is often not planned but comes as a necessity to enable the learners and teachers to express their mathematical ideas. To advance the use of the learners’ home languages, other researchers are exploring how the learners’ home languages may be used simultaneously with the language of teaching and learning to overcome the dilemma in code switching. They are exploring how the learners’ home languages may be used deliberately in the process of teaching and learning mathematics in multilingual classrooms.

2.3.4 Creating space for the use of learners’ home languages deliberately

Recent studies in South Africa have looked at how the learners’ home languages can be used to improve access and success in mathematics learning (Setati, Molefe & Langa, 2008; Setati & Duma, 2009; Webb & Webb, 2008). The approach in all of these studies focused on what Setati *et al* (2008: 16) refer to as the “deliberate, proactive and strategic use of learners’ home languages” simultaneously with English. These studies recognise the fact that while learners communicate in their home languages, they develop their mathematical meanings and in this context ideas can be accepted, questioned or negotiated or even modified.

Setati *et al's* (2008) argue that the deliberate, proactive and strategic use of the learners' home languages ensures that language functions as a transparent resource and facilitates the learners' focus on the mathematics task under discussion rather than the language(s) used. In these studies learners are given tasks in two language versions: their home languages as well as in English. They report that despite grouping learners according to their home languages, robust discussions were observed taking place between and within language groups. They also observed that learners were observed using both language versions as they were tackling the tasks. They therefore concluded that not only did the learners remain focused on the mathematics of the task but the use of languages in this manner also facilitated active participation by all learners.

The teacher interacted with the learners in either home languages or English; the learners used both English and the home language in all their interactions. This suggested that learners connected languages, home languages and English, with mathematics in communicating their ideas and it enabled them to draw on their experiences when engaged in the mathematical tasks. Further linguistic support can be used for learners still learning English, to improve their participation in mathematical discussions, and hence build on the mathematical language brought from their country of origin.

Other South African researchers employed Setati *et al's* (2008) approach in a variety of ways (for example Webb & Webb, 2008). In these studies, there is a clear shift from viewing mathematics learning as acquisition to understanding mathematics learning as contribution in the classroom discussion. Webb & Webb (2008) illustrate how in-service teachers who were exposed to the tenets of exploratory talk used their home language when struggling with a problem solving situation. Teachers then applied their experience in their mathematics lessons in order to encourage understanding and discussion among learners. Webb & Webb (2008), report that the teachers observed that discussions were meaningful to the learners when they were allowed to use their home language.

These studies discussed above show that in multilingual classrooms teachers establish space for learners to use their home languages. Creating such space depends a great deal on a sociable environment that makes it possible to reason, reflect on, critique ideas and communicate in any

language. The researchers in these studies recognise the fact that while communicating in their home languages, learners develop their mathematical meanings and in this context ideas can be accepted, questioned, negotiated or even modified. However, the choice that learners make will still differentiate those who become able to produce legitimate texts both mathematically and linguistically from those who, for example, engage mathematically through the medium of French or their home language, but consequently do not develop their proficiency in English and hence are unable to achieve political power. This leads me to the next section showing that teaching and learning of mathematics in multilingual classrooms with immigrant learners can sometimes be a challenge.

2.4 Challenges in multilingual classrooms with immigrant learners

Researchers who work in this area (Gorgoriò and Planas, 2001; Planas & Gorgoriò, 2004; Barwell, 2003a, 2005b, 2005c; Civil & Planas 2004; Planas & Setati, 2009; Stathopoulou and Kalabasis, 2007) report on differential access to productive engagement in the mathematics classroom discourse. These researchers argue that in order to encourage participation of immigrant learners in the mathematics classroom, the teachers should encourage the use of their home languages as a support. These researchers have reported the cognitive advantages of learners through their participation in mathematical discussion. They have shown that when learners express their ideas, they are able to make their mathematical reasoning visible and open for reflection. The learners' ideas also provide information to the teachers about what learners already know and what they need to learn.

However, Gorgorió and Planas (2001) argue that it is hard to separate the social, cultural and linguistic aspects of mathematics teaching and learning. The authors report on a study conducted in Catalonia where the language of teaching is Catalan. They suggest that it is better to think of broader communication within the classroom than a narrow linguistic one, although language aspects cannot be ignored. The authors noted that in particular, the informal mathematical talk can often be broken communication especially for the teacher since it occurs in the learners' first languages. Therefore helping learners to move to the formal mathematics talking and writing can be wide-ranging with unknown linguistic set-backs.

Teachers in multilingual classrooms should note that the opportunity for learning is influenced by what learners are helped to make available through discussions (Planas & Gorgoriò 2004). It can be argued that in an environment where ideas are shared, learners' own ideas become resources for their own learning. Moreover, their explanations stimulate, challenge and extend other learners' thinking. However, in an environment where learners are not encouraged to participate in classroom discussions their mathematical learning is constrained (Civil & Planas 2004).

Planas & Gorgoriò (2004) investigated social interactions at the beginning of secondary school year in a mathematics classroom in Spain with a high percentage of immigrant learners. The teacher's actions in the study resulted in immigrant learners not to have personal experience of how participation could help clarify and modify thinking. The researchers observed the teacher's slight, repeated refusal of immigrants' attempts to explain and justify their strategies for solving problems. Yet local learners participated in mathematical argumentation. As Planas and Gorgoriò reported, the reduced social commitments and lesser cognitive demands placed on immigrant learners had the effect of excluding them from full engagement in mathematics and hence constrained their development of a mathematical disposition.

In support Civil and Planas (2004) report that immigrant learners from minority groups experience difficulties when trying to participate in contexts of mathematical practices where they do not feel themselves represented, when others do not recognize them or when they have to cope with actions and behaviors that are different from those they would expect. The authors argue that the accumulation of learning obstacles turns into failure at school. Failure at school, in turn, characterizes classrooms as sites for inclusion and exclusion. In this regard, the effect of classroom discursive practices on the participation of some immigrant learners is rather negative.

2.5 The role of policy

Researchers from different countries are critical of educational policies that put emphasis on multilingual learners, including immigrant learners' acquisition of the language of learning and teaching as the priority (Stathopoulou & Kalabas, 2006 in Greece; Gorgorio & Planas in Spain, 2001; Kazima, 2006 in Malawi; Civil and Planas, 2004 in USA and Spain). These researchers

argue that instead of promoting diversity as a resource for learning, these policies convey a deficit view of multilingual learners' language and background.

Kazima (2006) shows evidence of the dangers of imposing a colonial language as the language of instruction. In an investigation into secondary school learners' meaning of some probability words prior to instruction, Kazima has shown that learners pre-assign meanings which are linked to their home language, Chichewa. From a cognitive perspective she argues that meanings of probability words cannot be considered apart from the way they were constructed.

However, the teacher's response to diversity is informed by and informs what is valued as knowledge in the classroom. Stathoupoulo and Kalabasis (2006) in Greece address issues related to community mathematics and school mathematics. The authors report on how Romany learners use mathematics in everyday context, in particular they noted that their computation was grounded on the children's experiences with their involvement in their families business. The authors point to the fact that Greek schools and teachers seem to show little interest in what knowledge Romany learners bring with them. Stathoupoulo and Kalabasis argue for the need for schools to recognize and build on the oral tradition and experiences Romany children have from their participation in their community's everyday activities. Further these learners are from homes that speak a language different from the language of learning and teaching. Obviously, the oral traditions come into contact with the dominance of literate practices in the school which may be used as a support in the teaching and learning of mathematics.

Classrooms are social settings where in some situations, social activities come into play during teaching and learning. There is evidence that policies can influence learners to become members of a particular class and how to participate in the learning practices of the mathematics classroom. For example, Civil and Planas (2004) illustrate how an educational system in Arizona and Barcelona has an effect in the participation of the learners. These researchers found that learners' participation in the mathematics classroom was influenced by larger social organisational structures. These structures, including gifted programs and special education, set up differential power and status positions within the classroom and they state that:

The value of what is said and of who says it is established according to the place where each participant is located. Learners respond according to where they are supposed to belong. Learners placed in the high-status system may have easier access to the mathematical discourse, whereas learners placed in the low-status system are still supposed to prove their value. Roles influence the learners so that they learn to act and behave in ways that agree with social order of the educational arrangements (Civil & Planas, 2004: 11).

This suggests that the forming groups of learners influences learners' self-perceptions regarding their capacity to do mathematics. The social stigma associated with low-level track placement may limit learners' willingness to participate in mathematics classes using discourse practices. Civil and Planas pointed out that the learners most negatively affected by the social order created through institutional grouping practices are members of particular ethnic and language groups or those of lower socioeconomic status.

In their study of Grade 5 learners in Arizona, learners who participated were those who were popular because of being involved in social activities such as sports, hence their ideas were valued in the mathematics classroom. The authors provide evidence as; in one activity which involved a basket ball game presented by a popular basket ball player, learners did not focus on the mathematics intended but on the game. Further ideas from learners belonging to Gifted and Talented Education (GATE) program were valued and tended to widen the gap between learners.

In another case in Barcelona, the educational policy offer different learning opportunities for the learners in that a parallel system of special needs class is created within the regular system in a secondary school that is highly multiethnic. The selection criterion is not based on mathematical ability but on linguistic competency, and discipline. The aim is to improve their mathematics ability as well as their knowledge of the language of learning and teaching, Catalan. The learners attend mathematics lessons with another teacher for three out of four periods while the other learners remained in class with the regular class. Then the other period they join the rest of the class. The authors argue that the social and organizational structures in place have an effect in the participation of the learners, hence on their learning of mathematics. The above discussion shows that it is not only language issues that come into the fore during the teaching and learning of mathematics, social activities also play a role and might benefit or not benefit some immigrant learners.

The teaching of mathematics in classrooms with immigrant learners is complex. The literature discussed above has argued that it is necessary to create an atmosphere where learners would feel comfortable using their home language, first additional language or code switching (regardless of their English proficiency levels) and negotiating that practice with other learners who do not speak their language. In this way teachers would be building upon the resources that the learners already possessed (Moschkovich, 2007), but would also see the importance of learners communicating their arguments in English. Therefore it becomes obvious that teachers and learners have to overcome multiple hurdles. It is therefore not the mere presence of these components as resources for teachers and learners that matter; but also the meanings that emerge for teachers and learners as these resources are put into use in local contexts (Adler, 2001). Teaching mathematics in multilingual classrooms must take into consideration not only language issues but also other factors like policy.

In summary of the above discussion, given the fact that learning mathematical language can only be acquired in school and not through conversational interaction, a number of problems could present themselves. Some of which are firstly a limited understanding of the everyday language that is used as a base for understanding of the formal mathematical language. Secondly imperfect transference of the mathematics knowledge from one language or another and lastly teachers whose knowledge of the mathematics register in the learner's home language is limited. Therefore, it becomes crucial that teachers understand the linguistic characteristics of classroom language and also have mastery of techniques that will enable students in connecting everyday language with the language of mathematics.

2.6 Underlying principle for the study

Most of the research conducted in multilingual mathematics classrooms in South Africa has focused on improving access and enhance performance in mathematics learning. These studies suggest strategies that can be used to enhance performance. Some of the strategies are; code switching and using home languages deliberately to support learners while learning mathematics (e.g. Setati, Molefe & Langa 2008; Webb & Webb, 2008; Vorster, 2008). There is no study that focuses on how immigrant learners in these classrooms who do not speak nor understand these languages are supported. In fact most of the research on teaching and learning mathematics in

multilingual classrooms in South Africa creates an impression that there are no immigrant learners in these classrooms. Hence this study focuses on how teachers created the opportunities for the participation of immigrant learners. This was ensured by conducting classroom observations with immigrant learners to explore how teachers in these classrooms draw on language(s) to support immigrant learners when teaching linear programming.

This study is unique in that it focuses on African migrants some of whom are from former French, Spanish and Portuguese colonies and so speak those languages. The effort by this study to look at how teachers create opportunities for the participation of immigrant learners when learning linear programming is crucial to mathematics education in South Africa especially during this period with an ever increasing number of immigrants moving into the country. An understanding of the various forms in which other languages are used by learners as resources to ensure success in linear programming is therefore crucial. Such an understanding can enable mathematics teachers in multilingual classrooms to look into diverse languages including languages not indigenous to South Africa, and their developed mathematics register like French or Portuguese. The understanding might also include practices like code switching to function as a means of communicating success in the learning of linear programming. The responsibility of the mathematics teacher in South Africa in particular is to teach immigrant learners in a way that gives them a future in South Africa which is bound with the power of English which will define their adherence to South African values.

This study will add to the existing literature on mathematics teaching and learning in multilingual classrooms in South Africa and in other countries. There are no studies that have been conducted in South Africa to document learning opportunities for the participation of immigrant learners. For that reason, this study will fill the gap in the literature. The information provided in this report will go a long way in informing teachers about some of the strategies they can use to ensure effective communication in their classrooms. Effective communication might call upon teachers or learners to draw from immediate discursive resources such as switching between English and French to make clear some mathematical concepts and encourage immigrant learners to participate in discussions. This study will also inform policy, curriculum

developers and material producers about the need for material to support immigrant learners in other languages.

2.7 Theoretical framework

Studies have shown that one of the factors that obstruct second language learners' success in mathematics learning is limited language proficiency in the language of learning and teaching (LoLT). However, a number of researchers have shown that learners may express themselves and understand mathematics better in a language that they are fluent in rather than in a language that they are still learning (Kazima, 2006; Moshckovich, 2002b). This study is broadly informed by a socio-cultural perspective (Gee, 1996, 1999, and 2005), which argues that language is a tool to communicate information and also to support social activities, social identities and affiliations within cultures and institutions. So by using a socio-cultural perspective it is possible to view the language and cultural backgrounds of the teachers and immigrant learners as valued resources for teaching and learning linear programming. Language is then looked upon as it occurs in context in order to understand how immigrant learners learn linear programming. Looking at language in context was to make sure that I do not lose sight of the macro-dynamics behind micro-processes and power and social structures in these interactions. Therefore, it was crucial to understand the local environment where learning takes place and know how culture plays out in these settings. But cannot lose sight of how settings make up and are intertwined by broader societal forces.

Gee's theory is about how language gets used to perform or act out certain identities or social activities. This theory was useful in providing the tools of enquiry and in interpreting the opportunities created by teachers for the participation of immigrant learners when learning linear programming. Furthermore, how immigrant learners interacted with a linear programming task presented in English. Gee defines identities as the different ways of being involved in social groups like churches or in institutions. In any situation people pull off or try to pull off certain identities. To do this they use language and activities. They speak, act and dress in ways that portray certain identities. The speaking, the acting and the writing are all embedded in what Gee refers to as Discourse with a "capital D".

2.7.1 The meaning of Discourse

According to Gee (1999: 7) using a lowercase “d” discourse refers to how language is used “on site” to enact activities and identities. However, “Discourse” using an uppercase “D” involves much more than sequential speech or writing:

A Discourse is a socially accepted association among ways of using languages, other symbolic expressions, and ‘artifacts’, of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or social network’, or to signal (that one is playing) a socially meaningful ‘role’. (Gee, 1996: 131)

According to the definition offered by Gee, Discourse is more than just language because it involves using languages and symbolic expressions in interaction with people belonging to specific communities. This means that Discourses are collective and not individual. Discourses give meaning to every human activity and thus well-established within Discourses are practices and knowledge from which people draw when engaging in the various roles that they play in their lives. Gee emphasises that the key to Discourse is recognition. We are recognised by others as particular types of people who are engaged in a particular type of activity because we know how we can perform in different situations by attuning our words and actions. This suggests that whatever you have done must be similar enough to other performances to be recognizable, if it is not recognizable, then, you are not in the Discourse (Gee, 2005a). For example, a linear programming lesson involves more than just technical language but also ways of expressing inequalities graphically, ways of expressing inequalities using symbols and ways of expressing inequalities verbally.

So when the teacher and learners speak or write they use language in context to project themselves as mathematics teachers or mathematics learners engaged in linear programming activities. For example engaging in activities like drawing graphs of inequalities to represent a feasible region, finding coordinates of the maximum points in the feasible region. Discourses are social practices, mental entities and material realities, they have no clear boundaries, and they can be transformed, created and sustained. New Discourses emerge as old ones die out. The emergence of new Discourses is influenced by the social group of the people who enact them. Gee (2005) argues that the way we speak in social groups, we use particular *social languages* such that language in these socially meaningful groups has a “magical property” because the

words or phrases uttered by the speaker reflect the context within which they are spoken and at the same time shape the context. Gee claims that when we speak or write we can design and build what we have to say as it is suitable to a particular situation. This suggests that we create the situation. When people speak, or write we often relate to ideas that are the focus of a social group. These ideas play an important role in how language is interpreted.

2.7.2 Tools of enquiry

Social languages

Gee (2011) defines social languages as styles or different varieties of a language in Discourses and are associated with a particular social identity. For example, talking and writing as a mathematics learner and enacting different socially meaningful activities like solving a linear programming task in the mathematics Discourse. When learners solve a linear programming task or when they discuss how to solve a linear programming task during a linear programming lesson they use a specific language that projects them as mathematics learners learning linear programming. A different concept like surds has its *social language* like the use of square root sign ($\sqrt{2}$) that the same learners use when talking about how they solve surds tasks. Yet in another context, like a group discussion, learners may project themselves as members of the group or speak like the teacher when asking other members of the group to calculate values of x and y by substituting in an equation to find the intercepts in order to draw a line. In other words the ways with words are connected to different social identities and social activities. These ways of using *social languages* are acquired within specific socially shaped practices representing the values and interests of distinctive groups of people. For learners to know any specific social language is to know how its characteristic lexical and grammatical resources are combined to enact specific socially situated identities, that is being a linear programming learner. Therefore, to know a particular social language is either to be able to do a particular activity or to be able to recognize such an activity when we cannot actively participate. But how do these *social languages* differ?

Gee (2005a: 41) states that “each *social language* has its grammar and rules by which grammatical units like nouns and verbs, phrases and clauses are used to create patterns...” From these patterns listeners of our speech can attribute situated identities and specific activities to us

and our utterances. So in a linear programming lesson the teacher designs his/her oral or written language to have patterns in them which learners can recognize that it is about linear programming and then situate their problem solving processes using appropriate linear programming language in relation to his/her utterances. For example when representing a feasible region, learners may recognize that they have to represent it by drawing inequalities on graph paper using ruler and pencil, use dual intercept method when finding the coordinates of the intercepts of the boundary region, plot points on the graph in order to draw the line, shade the appropriate region above or below the boundary line to represent the region and mark the points in the feasible region; use languages like French or Lingala, Chichewa when explaining their ideas; or even translate some of the English mathematics register to French which has a developed register. All these are the patterns that will make them be recognized as learners learning linear programming during a mathematics lesson. Hence *social languages* trigger specific *situated meanings* in Discourses.

Situated meanings

Gee (2005a: 94) states that “a situated meaning is an image or pattern that we assemble ‘on the spot’ as we communicate in a given context, based on our construal of that context and our past experiences”. He further cites other scholars like Barsalou (1992) and Clark (1992) who emphasise that these meanings are assembled out of diverse features, as we speak, listen and act. For example in the context of inequalities, the mathematical symbolism ‘ $x < 3$ ’ will call upon a pattern of all numbers less than three including negative numbers like 2, 1, 0, -1, -2, -3...yet when solving a linear programming task, x less than three will not include negative numbers but will refer to 2, 1 and 0 showing that a different context can invoke different responses. Therefore, *situated meanings* are adapted each time to the specific contexts they are used in and are open to transformation from new experiences.

For example, the situated meaning of the phrase ‘feasible region’ in mathematics is associated with linear programming. The meaning learners’ make of ‘feasible region’ will be influenced by their relationship with mathematics lesson, therefore the patterns they will assemble will be acceptable to the mathematics community. Of course there are many specific meanings the phrase can take in other contexts. Gee (2005a: 59) points out that such patterns are really a

matter of unconscious recognition rather than of conscious thought. That is why some words have “more general meanings that are apparent in the sorts of situated meanings”. What guides the process of assembling feature then since the world is full of potentially meaningful patterns in any domain? According Gee (2005a) *social languages* and *situated meanings* reveal Discourse models.

Discourse models

Gee (2005a: 61) states that “Discourse models are theories (story lines, images, explanatory framework) that people hold often unconsciously and make sense of the world and their experiences in it”. This suggests that theories are descriptions of representations which give insight and understanding complicated realities by focusing on some things we think are normal for that situation. In other words Discourse models suggest a particular view and set of understanding.

Gee (2005a) maintains that Discourse models are learnt from experiences, shaped and normed by the social and cultural group to which you belong. However, they are not an individual experience but different bits of Discourse models are distributed across different people in a social group and are connected to specific socially and culturally distinctive identities people can take in society. For example, when learners are given a linear programming statement where they have to extract inequalities, their understanding of the problem might be influenced by the Discourse models they hold. This might give an understanding of their experiences in the community where they live. In other words Discourse models can also be used to make identities recognizable and visible. Making our identities recognizable and visible requires more than language. It requires also that we act, think, value and interact in ways that render our identities recognizable to others and ourselves (Gee, 2005a). Discourse models are thus not only understood from what people say, but also from how they act, think, value and interact with their Discourses.

Strauss (1992, in Gee 2005a: 83) distinguishes Discourse models into three types based on how they are put to use and on the effects they have on us: espoused, evaluative and models-in-(inter)-action.

Espoused Discourse models are consciously adopted models, which may be associated with our influenced experiences in society, perhaps with parents, or teachers and may be with peers. Such an influence contributes to what the teacher or learner will consider as appropriate when teaching linear programming. Does a teacher give precise instructions on how to find the maximum profit using a search line method because he wants learners to pass an examination or does he want learners to use the procedure in an out of school context. Surely revealing espoused Discourse models from the text will enlighten us about the manner the teacher provides opportunities to immigrant learners.

Evaluative Discourse models are used to judge ourselves or others. In a linear programming lesson evaluative Discourse models are used to judge learners about their capabilities of solving a linear programming task. An example might be giving them guidelines when constructing inequalities from a given task, because from the perspective of the teacher learners might be viewed as lacking necessary knowledge. Evaluative Discourse models may explain why teachers resort to telling instead of probing learners to elicit their views. Telling reflects the teacher's view in judging the students' capability.

Interactive Discourse models are used to guide our actions and interactions in the world. These may be viewed as the appropriate ways of acting, interacting, participating, beliefs and values that we think we are expected to enact in response to our environment. For example, teachers may listen to the mathematics in what learners say and not how they express themselves in English. In short the teacher's experiences in teaching mathematics may influence his actions and interactions with learners from diverse language background.

Linear programming is about using a particular *social language* which triggers *situated meanings* guided by Discourse models teachers and learners draw upon in order to respond appropriately in the world. Situated meanings are always relative to the speaker's socioculturally defined experiences in the world and more or less routinized through *Discourse models* and various social practices of the Discourse to which the speaker belong, like mathematics teaching or learning. Gee advises us that it is important to note that *Discourse models* do not just exist in

people's heads (e.g. mathematics learners or teachers), but are often shared across people (e.g. teachers, learners), books (prescribed textbook or teachers worksheet), other media and various social practices. So too *situated meanings* don't just reside in individual minds, very often they are negotiated between people in and through communicative social interaction through language which according to Gee has a magical property.

Magical property of language

Gee (2005a: 10), states that “when we speak or write we design what we have to say to fit the situation in which we are communicating. But at the same time, how we speak or write creates that very situation”. So in a linear programming lesson, the particular *social languages* which involved feasible region, phrases like ‘at most’ and ‘at least’, ‘maximum profit’ using a ‘search line’ etcetera, involved the teachers as well as the learners’ way of speaking, writing, valuing, acting, thinking and believing to relate them to the content under discussion. This means that by studying language-in-use of the teachers and immigrant learners in their classrooms settings, I got an insight into how they build their socially situated identities in relation to linear programming through language simultaneously with actions, interactions, non-linguistic symbols system, objects, tools, ways of valuing and feelings, etc.

Gee further points to the fact that our spoken and written language create or build the world of activities (for example mathematics classrooms), identities (teacher or learner) and institutions (classrooms) around us. Gee (2005a: 105) contends that “each social language uses somewhat different and characteristic grammatical resources to carry out seven building tasks” that helps in understanding a situation. Therefore, the teachers’ or immigrant learners’ written or spoken text can be analysed to find out how they use language to build their world. A description of the seven building tasks follows (Gee, 2005a: 97-104).

i. Building significance

How and what significance is given to things? What meanings and values are attached to what is being discussed? How are things made significant or insignificant through the choice of language, emphasis, gestures, etc?

ii. Building *activities*

What activities do the participants put forward through their use and choice of discourse and how is language used to show what activity one is involved in?

iii. Building *identities*

What identities (roles, positions) are participants enacting and describing? How is language used to make the identity in the situation identifiable and consequential?

iv. Building *relationships*

What relationships do the teacher and immigrant learner see as existing in the situations that they describe? How is language used to show what relationships the participants recognise as being in place? How is language used to put forward and negotiate relationships between participants?

v. Building *politics*

What social goods are perceived by the participants? How is language used to express these social goods in terms of how they affect the participants? In what way might these social goods benefit, advantage or inhibit the participants or those around them?

vi. Building *connections*

In what way are things connected or disconnected to each other? How is language used to create links or disassociations between things, events, and circumstances?

vii. Building significance for *sign systems and knowledge*

What sign-systems or ways of knowing do the participants refer to? What language is used to show how participants know or come to know, and are able to talk about events, objects and circumstances?

In the above discussion I have presented the theoretical framework that also provided me with tools to analyse the data in order to understand how teachers create opportunities for the participation of immigrant learners when learning linear programming in multilingual classrooms in South Africa. Furthermore, how languages within the immediate environment of these learners

are used as a support during the teaching and learning of linear programming. In summary, Discourses are ways of talking which are embedded in the practices of social groups. The role of language in Discourses is defined by *social languages*. *Social languages* trigger *situated meanings* through accomplishing the seven building tasks which in turn reveal or activate certain *Discourse models* which enabled me to explain how immigrant learners learn linear programming. *Discourse models* allow people to enact and recognize different Discourses at work.

Gee's theory has provided the framework with which data was analysed throughout the thesis. With this framework I was able to interpret the teachers' actions as well as the immigrant learners' actions, writing, talk and interactions and deduced the various identities they assumed when making utterances. From my experience, every classroom gives rise to a complex web of relationships among learners, their teacher and powerful traditions within the school, community and culture.

In conclusion, the theoretical stance for this study draws on a discursive paradigm and in particular advocated by Gee (2011, 2005a, 1999, 1992) which offered the opportunity to understand why people act in the way in which they did. It was possible to access the actions and interactions of immigrant learners and their teachers within the mathematics classrooms. This was possible through identifying discourse models that were actually at play within those actions and interactions and understanding the possible Discourses from which they become apparent, it gave an impression that we can come to gain insight into the reasons for certain types of actions that are sometimes hard to understand. In the next section I present linear programming content identified for the study.

2.8 Linear programming

This section presents a brief argument for why this study focuses on linear programming. It aims at providing information immigrant learners have to know in order to be familiar with the discourse. I will start by defining linear programming and look at the founders of this powerful skill in mathematics and all the fields where it has become useful. The chapter will proceed and look at how linear programming is attended at school. I will do that by presenting the learning

outcomes and assessment standards as they are presented in the curriculum documents in South Africa.

Linear programming is a mathematical method for determining a way to achieve the best outcome (such as maximum profit or lowest cost) in a given problem (mathematical model) for some list of requirements which can be represented as linear relationships. Linear programming arose as a mathematical model developed during the Second World War to plan expenditures and returns in order to reduce costs to the army and increase losses to the enemy.

The founders of the subject are Leonid Kantorovich, a Russian mathematician who developed linear programming problems in 1939, George B. Dantzig, who published the simplex method in 1947, and John von Neumann, who developed the theory of the duality in the same year (<http://en.wikipedia.org> 02/03/2011). Many practical problems in a field called operations research can be expressed as linear programming problems. Certain special cases of linear programming, such as *network flow* problems are considered important enough to have generated much research on specialized algorithms for their solution. Linear programming is used in microeconomics and company management, such as planning, production, transportation, technology and other issues. Most companies would like to maximize profits or minimize costs with limited resources. Therefore, many issues can be characterized as linear programming problems.

Linear programming is a mathematical modeling technique useful for guiding quantitative decisions in business planning, industrial engineering and, to a lesser extent, in the social and physical sciences. It is the maximization or minimization of a specific performance index, usually of an economic nature like profit, subject to a set of linear constraints. For this exercise to qualify as linear programming the performance index should also be linear.

Linear Programming was developed by George Dantzig in 1947. He was studying proposed training and logistics for the United States military training program as a mathematical adviser to the United States Air Force Controller in the Pentagon. Dantzig was also an expert on planning methods using desk calculators. His colleagues at The Pentagon, Hitchcock and Wood, asked

him to find a method that would rapidly compute a time-staged operation, training and logistical supply program (Dantzig, 2002).

Dantzig was influenced by the work of Leontief, who proposed a large but simple matrix structure called the 'inter-industry input-output model' (Leontief, 1933) of the American Economy. In this model there was a one-to-one correspondence between the production processes and the items being produced by these processes. Dantzig also used ideas from a paper by von Neumann (1928) on game theory together with his ideas on steady economic growth to formulate a highly dynamic model that could change overtime. He realized that The Air Force needed a model with alternate activities and which had to be computable; hence he invented the simplex method.

The simplex method can be described as a dynamic linear program with a staircase matrix structure. The method generates a sequence of feasible iterates by repeatedly moving from one vertex of the feasible region set to an adjacent vertex with a lower value of the objective function. When it is not possible to find an adjoining vertex with a lower value, the current vertex must be optimal and termination occurs. The simplex method is a tool for practical planning of large complex system (Dantzig, 2002)

Linear programming is about using the simplex method to formulate real-world problems in mathematical terms, which are solved using computers and software nowadays. The standard form of describing a linear programming problem consists of the following:

- a linear function to be maximized or minimized, e.g. $2x + 3y \leq 30$
- subject to linear constraints, e.g. $x + 6y \geq 24$ and $5x + 8y \leq 40$
- and non negative values, $x \geq 0$ and $y \geq 0$

(http://mathworld.wolfram.com/linear_programming.html 02/03/2011)

2.8.1 How is Linear Programming attended to in the School Mathematics Curriculum?

According to the South African National curriculum statement (NCS, 2003:31), linear programming is introduced at Grade 11 level. Learners continue to study linear programming at

Grade 12 level. Below is what the curriculum statement stipulates as content that needs to be covered as well as the assessment standards.

Table 2. 1: Linear Programming Content

The proposed content under linear programming for Grade 11	Assessment standards are as follows
<ul style="list-style-type: none"> ▪ The learner must be able to optimise a function in two variables subject to one or more linear constraints, by the numerical search along the boundary of the feasible region ▪ The learner solves a system of linear equations to find the coordinates of the vertices of the feasible region. (DoE, 2003: 53) 	<p>We know this when the learner is able to:</p> <ul style="list-style-type: none"> a) Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, by numerical search along the boundary of the feasible region. b) Solve a system of linear equations to find the co-ordinates of the vertices of the feasible region. (DoE, 2003: 31)

The curriculum assumes that it is crucial that learners become aware of and learn linear programming in the context of its current use. Linear programming content is introduced as needed in problem-solving situations and the document makes clear that it is crucial that learners can use the mathematics in an out of school experience. So when teaching linear programming teachers should assist learners to move from everyday language to the more formal register of mathematics. That is to say teachers should create learning opportunities during the teaching and learning process.

2.9 Conclusion

The chapter presented a review of literature in this area of study and provided the framework for this study. This study investigated how teachers created learning opportunities for the participation of immigrant learners when learning linear programming. The literature discussed

strategies teachers use to support learners who learn mathematics in a second language. Strategies like code switching, revoicing learner contribution and using home languages deliberately during teaching and learning of mathematics enable learners engage with mathematics. Researchers emphasise the role of the learners' home languages when learning mathematics.

The chapter also discussed the analytic framework employed. Gee (2005a) provided the language to describe, discuss and explain the approaches by these teachers. Gee's theory provided tools that can be used to analyse the language used in the mathematics classrooms. The theory has been chosen on the premise that language is important in analyzing learners' activities and the kind of identities they seek to enact by those activities. The activities and identities in different contexts were best analysed by studying the language they used and the other stuff that accompanied the social languages. The seven building tasks and the other tools of enquiry such as *social language*, *situated meanings*, *discourse models* were therefore used to analyse and explain the different purposes for which language was used.

The next chapter presents the research design and the methodology used in this study.

CHAPTER 3

RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

This chapter provides an outline of the research design, methods used for data collection and the methodical frame that guided data analysis. The chapter further discusses issues pertaining to rigour in research to ensure that factors affecting validity and reliability were minimized. The chapter concludes by discussing ethical issues that were considered before data collection. The context of the practical arrangements made with research participants and the practical decisions made regarding interpreting and synthesizing the data are also described. The classroom arrangements and interpretive decisions made created an opportunity to see mathematics classrooms in a way not seen before. A strong feature of the approach used is the decision for me as a researcher to work closely and listen to the mathematics teachers and immigrant learners.

The study was guided by the following research questions:

1. How do teachers create opportunities for the participation of immigrant learners when learning linear programming?
2. How are languages within the immediate environment of immigrant learners used during teaching and learning linear programming?
3. How do immigrant learners take up these learning opportunities when solving a linear programming task?

3.2 Research approach

This was an empirical case study which used qualitative methods of data collection and analysis. Merriam (1998: 19) defines a case study as “an intensive description and analysis ...of a bounded system such as an individual, a program...” She offers examples of such bounded phenomena in education thus, “a program, an event, a person, a process, an institution or a social group”. Hitchcock and Hughes (1995) argue that in a case study an individual’s or social group’s constructions are elicited by interactive dialogue between the researcher and the participants. Opie (2004) elaborates this point further by arguing that a case study can be viewed as an in-depth study of interactions of a single instance in an enclosed system.

The main focus of a case study is to organize social data for the purpose of viewing social reality in order to gain a wealth of detailed information on a small sample size (Patton, 2002). Therefore, the purpose of the in-depth study of interactions reported in this thesis was to gain insight into the opportunities created by teachers for the participation of immigrant learners when learning linear programming in a Grade eleven mathematics classroom. As will be reported, the concern was the meaning in context of how teachers created opportunities as well as how immigrant learners take up these opportunities in a country that has been reported to be hostile to foreigners.

3.2.1 Different types of case studies

Yin (1994) identifies three types of case studies, all of which seem to form important components of my study. He identifies them in terms of their end product as exploratory, descriptive and explanatory. According to these identifications, exploratory case studies have been used as pilots, mainly to generate further research questions or try out data collection methods (Yin, 2009). Descriptive case studies are aimed at giving a narrative account of social situation (Yin, 2009). The approach is able to generate a lot of detail; however it is low in theory building opportunities making it less suitable for some research studies. The explanatory case study tends to be used either to generate a new theory or test an existing one.

Another variation in case studies also comes from Stake (1994) who classifies case studies into intrinsic and instrumental. Intrinsic case studies are considered suitable when intentions are to understand the particular case in question whilst the latter is used when the intentions are to examine a particular case to gain insights into a certain issue. Stake (1994) also gave a third category which involves a study of multiple cases. But all these classifications highlight some overlap and the implication being that, drawing from more than one model would be an advantage.

3.2.2 Difference in case studies

A case study can follow a determined study of a single case or be a multiple case study project, which is a collection of single case studies (Yin, 2009; Stake, 1994). In the study described in this thesis, multiple case studies have been considered a more relevant methodology than a single

case study. This was mainly because I considered the diversity of school situations due to their locations. Multiple case study designs have distinct advantages suitable for such situations. This is so because it allows for investigations of what is particular to individual persons, to individual classrooms or individual schools. Differing views are permitted as they lead to multiple realities that become visible in each of the case studies. In this study, I identified, explored and described issues and existing practices with the intent of providing a link between current literatures on teaching mathematics to immigrant learners. The evidence from multiple cases is often considered more persuasive and considerable (Yin, 2009) than from single cases.

3.2.3 Why use a case study?

The case study approach was used because it is flexible and adaptable. It involves the researcher participating openly in people's daily lives for an extended period of time, gathering evidence that may be available to explain the issues under study. The data may be gathered by watching what happens, listening to what is said and / or asking questions through informal and formal interviews (Hammersley & Atkinson, 2007). The basic premise for the method is that people (e.g. teachers and immigrant learners) do not act in isolation. Their behavior and actions are shaped by social groups, cultures and institutions (Gee, 2005a) and therefore may be studied in their natural settings.

Therefore, the case study offered opportunities to take account of life experiences of teachers and immigrant learners while focusing on their schools and classroom processes. The method allows people to tell stories about their situations to others in order to make sense of the worlds they inhabit (Yin, 2009). Furthermore, it is more focused and intensive as it is conducted in clearly bounded environments (Denzin & Lincoln, 2000, 2006).

According to Denzin & Lincoln (2006), a case may be simple and straight forward or may be complex and extended, but its centrality lies in the focus and emphasis upon the clearly bounded and unique nature of a setting having some kind of internal coherence. The core for its identity rests in its notion of unity or overall that bounds the system like a school and it is a part of this unity that is studied. According to Opie (2004: 74):

...its aim then is to provide a picture of a certain feature of social behavior or activity in a particular setting and the factors influencing this situation. In this way the interactions of events, human relationships and other factors are studied in a unique location.

It is clear here that the interest in case studies is on the activities of the case and not on generalising the results across teachers and immigrant learners in South Africa. Hitchcock & Hughes (1995) claim that a case study is handy in that it not only gives an opportunity to look at the common, widespread and general, but it also allows for attempts to identify anything unique or particular to an individual teacher or immigrant learner. It is not a problem accomplishing these in case studies as it allows for employing a variety of data from a variety of sources.

These features allowed me to ask important questions during clinical interview with immigrant learners as well as when interviewing the teachers. They also allowed intensive analysis (Cohen & Manion, 1994) of data in that they provided an understanding of the opportunities created for the participation of immigrant learners when learning linear programming.

3.2.4 Identifying boundaries of the study

Identification of the case meant locating the boundaries, schools, teachers and immigrant learners learning linear programming in a Grade eleven mathematics classroom. This involved a purposeful sampling strategy which I discuss in the next section.

3.2.4.1 Sampling

Patton, (2002: 70) defines purposeful sampling as “a strategy in which particular settings, persons, or events are selected deliberately in order to provide important information that can’t be gotten anywhere from other choices”. In support, Merriam (1998) emphasizes that this purposive sampling strategy is used when the researcher wants to discover, understand, and gain insight into a phenomenon (for example immigrant learners learning linear programming). The advantage of purposeful sampling is that a researcher handpicks the cases to be included on the basis of his or her judgment of their typicality (Cohen & Manion 1994). Patton (1990: 70) develops the argument as follows, “the logic and power of purposeful sampling lies in selecting the information rich cases for study in depth. Information rich cases are those from which one can learn a great deal about the issues of central importance”.

In this study, purposeful sampling strategy was employed because of the kind of data that was to be collected, the data collection instruments used, and also due to the data sources which did not lend themselves to random sampling. So in line with my original interest in how teachers create opportunities for the participation of immigrant learners, it was necessary to observe mathematics classrooms where this occurred. Thus, the main consideration centered on choosing schools which enroll immigrants.

3.2.4.2 The selected cases

Merriam (1998) argues that in order to use purposive sampling one must decide the selection criteria. Cohen, Manion & Morrison (2000) broaden this argument further by pointing out that the quality of any research writing may be affected by the precision of sampling strategy employed. I designed a selection criterion that was to assist me in hand picking the ‘information rich case’.

The criteria entailed were: schools enrolling immigrant learners in an urban, township and rural environments; teachers teaching linear programming to Grade eleven learners and teachers willing to participate in the study. Grade eleven was included because; according to the Department of Education document (2003) linear programming is introduced at this level.

Three schools were involved in the study. Each was carefully selected so that it could predict similar results often called a literal replication or produces contrasting results but for expected reasons. I purposely selected the three schools drawn from three different multilingual contexts, three mathematics teachers from these schools and their Grade eleven classrooms with immigrant learners learning linear programming. The number of teachers and schools participating in this study is small. I wanted to select a manageable group of cases such that each case would illustrate the diversity of the different context in which the schools are located and yet show the need to examine the complex nature of creating learning opportunities for the participation of immigrant learners in multilingual classrooms in South Africa. These schools were identified through a process which is discussed in the next chapter (Chapter 4).

The sample is opportunistic and purposive in that I was looking for immigrant learners who might have first enrolled at their country of origin and entered school in any of the grades in South Africa. My interest was to gain insight into the opportunities created by teachers for the participation of immigrant learners in a country whose citizens have been reported to be hostile to foreigners. Reading from literatures (Landau & Kabwe-Segatti, 2009; Crush & Williams, 2003; Landau, 2007), I have become aware that some of these immigrant learners or teachers are from French, Portuguese and Spanish speaking countries. Furthermore, they cannot speak nor understand some of the official languages of South Africa. So I wanted to explore the resources the teachers recruit when engaged in the teaching of linear programming to this particular group of immigrant learners. However, the main purpose of the study was not to evaluate the teaching but to learn from it by making analytic judgments of teachers creating learning opportunities for the participation of immigrant learners when teaching linear programming.

The three selected schools were such that: one is located in an urban environment in central Johannesburg neighborhoods, which I will refer to as School A, the other school in a township environment located west of Johannesburg, School B and the third school located in a rural environment in the Limpopo province, School C in this thesis. The mathematics teachers from these schools are experienced and were willing to participate in the study. I was fortunate to work with teachers at these schools who are highly qualified and competent. They all have more than eight years teaching mathematics at secondary school level. Two are immigrants, one from the DRC teaching at School A and has been teaching in South Africa for more than ten years. The other teacher teaching at School C is from Zimbabwe, she started teaching in South Africa in 2010 while the third teacher is a South African citizen teaching at School B.

The location of the three participating schools is indicated in the map of South Africa, figure 3.1 shown in the next page:



<http://www.places.co.za/maps/mapcity.php>

Figure 3. 1: Map showing the location of the schools in South Africa

While showing the locations of the sites for case studies, the exact locations have been disguised to maintain anonymity.

3.3 Qualitative methods of data collection

Qualitative method of data collection involves a detailed study of a small sample (McMillan & Schumacher, 2006). These scholars argue that the method of data collection that focuses on a small sample and that elicits quality information is the qualitative method. The second reason is that a qualitative method is appropriate because it is used when one investigates the quality of relationships, actions, situations. Lastly, the procedure that I employed in data gathering and analysis fit the distinct features (Fraenkel & Wallen, 1990; Lincoln & Guba, 1985) described in qualitative research. These characteristics are:

1. Researchers collect data within the natural setting of the information they seek. I was the key instrument. I was involved in the entire data collection process at the research sites. This involvement helped me to obtain information directly from the sources,

which in this case were the immigrant learners and their teachers. Lincoln & Guba (1985: 334) state that humans are preferred as instruments because of “their greater insightfulness, their ability to utilize tacit knowledge and their ability to process and ascribe meaning to data simultaneously with their acquisition”.

2. I was concerned with the process of teaching and learning linear programming and when immigrant learners solve a linear programming task. I had to look at the activities within the total context of mathematics classrooms and the school in order to describe the ongoing interactions occurring during the teaching process. My main concern was how teachers created learning opportunities for the participation of immigrant learners as well as how these learners take up these opportunities in particular when responding to a linear programming task. This approach therefore helped to analyze the opportunities during teaching and when learners were solving a linear programming task.
3. I wanted to know what the participants do to support each other during teaching and learning of linear programming.

3.3.1 Methods of data collection

A characteristic feature of a case study is that data is gathered from a wide variety of sources. Data for this study was collected through a learner questionnaire, videotaped lesson, individual learner clinical interview and a reflective teacher interview. The use of multiple data collection techniques permits the verification and validation of qualitative data.

3.3.2 Learner questionnaires

A learner questionnaire was issued before classroom observation to elicit information on languages represented in the mathematics classrooms. Furthermore, learners were to indicate their citizenship. On the issue of languages, learners were to indicate their home languages, languages they speak at home, languages they understand and languages they can read and write. So it was to understand the language background (Setati, Adler, Reed & Bapoo, 2002: 129) of learners in the mathematics classrooms selected for the study.

3.3.3 Classroom observations

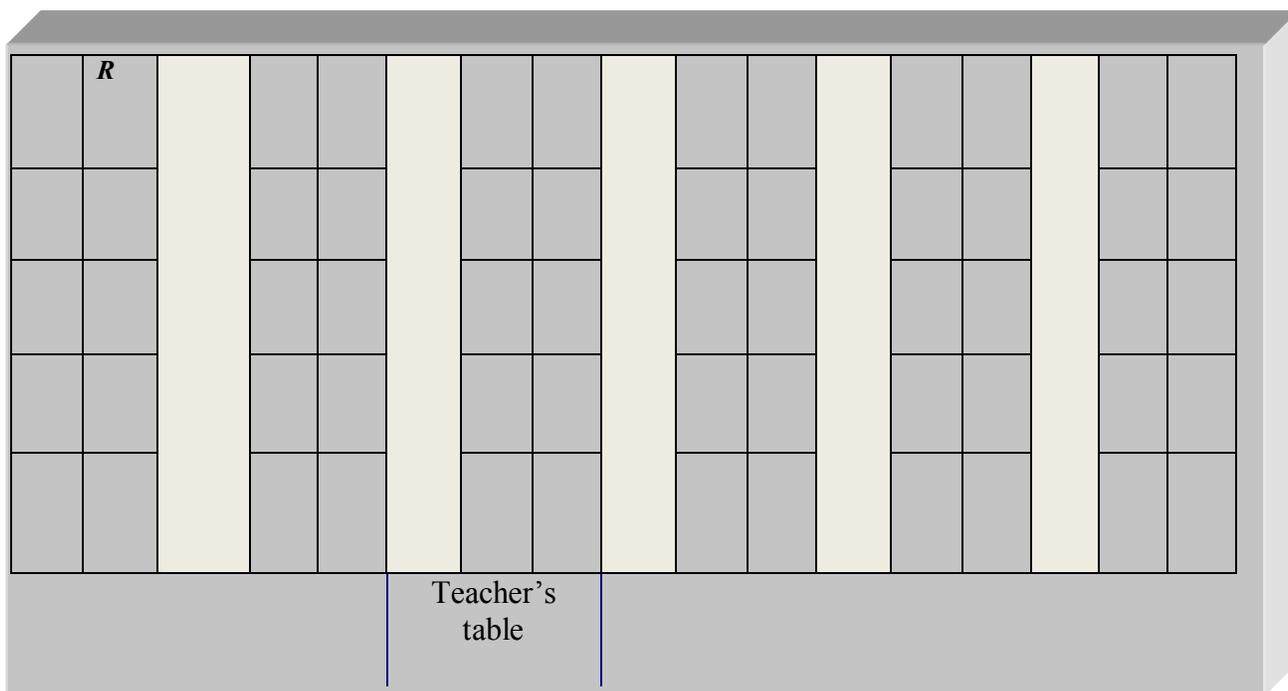
The study reported in this thesis investigates learning opportunities created for the participation of immigrant learners with specific focus to the teaching of linear programming in multilingual classrooms in South Africa. Since knowledge is situated in practice and is shaped by human interactions and context (Gee, 2005a), I had to observe the teachers while teaching linear programming to Grade eleven mathematics learners. I thought that this would afford me an opportunity to gain more insight into what teachers do to provide opportunities for immigrant learners that they would not ordinarily do when there are no immigrants. Patton (2002) argues that observation affords the researcher the opportunity to look at what is taking place in situ rather than at second hand. He further points out that observation enables researchers to understand the content of the situation, to see things that might otherwise be unconsciously missed and to discover things that participants might not freely talk about in interviews.

Observation was supported by video recording of the lessons. Cohen *et al.* (2000: 305) indicate that the kinds of observations that can be carried out by a researcher lie in a continuum ranging from “unstructured observation to structured observation”.

In order to carry out the observation successfully I had to assume a certain position during data collection. Cohen *et al.* (2000: 305) suggest three roles that might be taken by the researcher. They argue that these roles lie in a continuum from “complete participant ... to complete observer”. Opie (2004) classified these roles into two main categories namely; participatory and non-participatory roles. I was a non-participatory observer. I found this to be the best role for me because firstly, the school environments were unfamiliar to me as I was there as a researcher only and secondly; I wanted to be as less intrusive as possible. I had “no interaction with the participants during data collection” (Opie, 2004:128). But I was able to collect any handouts that the teacher distributed to learners. These handouts were necessary as the teachers had carefully selected the tasks for teaching ([Appendix M, O, P](#)) and being able to view them as the lesson progressed helped me follow lessons with ease. The non-participatory role further gave me the opportunity to jot down incidences (linear programming tasks, procedures and resources) as they occurred during the lesson.

My seating position was at the back of the mathematics classroom (indicated by R in table 3.1 below) and it helped me avoid being engaged in the class activities. Moreover, I did not show interest in what was going on during the lessons. Most importantly, I did not want to communicate any feelings through body/facial means as this could interfere with the teacher's disposition during the lesson. So I avoided eye contact with either teachers or learners. When I was talked to I smiled and politely referred the learner to the teacher.

Table 3. 1: Learners seating arrangement



Apart from observation I video recorded all the lessons observed (at most five lessons at each school). In support of video recording, Opie (2004) argues that video recording offers the researcher the opportunity to make sense of the non-verbal behavior. Cohen *et al.* (2000: 313) point out that comprehensive audio-visual such as video-recording can overcome the partialness of the observer's view of single event and can overcome the tendency towards only recording the frequently occurring events.

In addition, I felt that video recording would afford me the opportunity to see the events as they occurred long after leaving the site. Such gave me an opportunity to review the activities and

interactions in the mathematics classroom that were useful in the analysis of data. Furthermore, video recording provided permanent records and it gave me an opportunity to view the lessons several times during data analysis. Hence re-analysis of the lessons would be possible which I thought was essential for checking my interpretation, explanation and description of the opportunities created for the participation of immigrant learners when learning linear programming in Grade eleven. In this way, I endeavored to ensure reliability and trustworthiness of the results. Also using video recordings helped me to capture most of the interactions and conversations that went on between the teachers and immigrant learner(s) which I might have missed during observation as I focused more on particular events.

Classroom observations were conducted for one week (at most five lessons at each school) to capture a sequence of learning. Each lesson was forty five minutes long or ninety minutes when it was a double period. These observations were carried out during linear programming lessons which lasted for forty five minutes or ninety minutes when it was a double period. These were enough for the teachers to cover linear programming content. The assumption here is that linear programming content and the activities designed or selected by teachers reflect what they understood or considered to be important. As Cornbleth (1990) has argued, what teachers do in classrooms communicates messages about their understanding of curriculum and meaning of knowledge.

Prior to my data collection, I checked all the consent forms to ensure that I was allowed to video record immigrant learners. All learners were willing to participate. When I arrived at the schools, the teachers informed me that they had made sure that the seating arrangement accommodated researchers. Each learner had his own desk. The desks were arranged so as to have columns and spaces in between so that the teachers moved freely in between. Due to the large number of learners in the mathematics classroom, the desks were paired and the other columns were against the opposite walls of the classroom. The teachers' desks were in front but very close to the first row of learners.

I had planned to observe linear programming lessons continuously for five days so that I can capture the learning sequence. But it was not possible because of school activities that took place

in the schools. School B was involved in two activities, sports day and cultural day during data collection week. Furthermore, there was a staff meeting which coincided with the mathematics lesson. The mathematics teacher at School B made me aware of these disruptions except when there was going to be a staff meeting. At School C the teacher was involved in an accident so I had to suspend data collection. At School A the learners were writing monthly tests organized by the department of basic education. The nature of things as they are, prompted me to be mindful of connections among the presuppositions I brought to the inquiry, my experience of the situation as it presented itself. This meant that I had to be flexible and accommodate these disruptions in my research plan. This was done by extending the data collection process by a week at School A and B. At School C, I collected data two months later which was when the teacher had reported back to work.

On disruption, Valero and Vithal (1998) give an account of two research studies in Columbia and South Africa, both of which experienced serious disruptions to the researchers' plans. With these stories, they note that dominant research paradigms are situated in Western cultures with assumptions of stability, predictability and consistency, situations that are less common in the rest of the world. They, therefore, argue for closer attention to the context of the particular situation in which research occurs. Though Valero and Vithal recognize the higher likelihood of disruption in relatively unstable environments, they point out the general instability throughout our rapidly changing world. For any context, especially one in which a researcher gives up many of the traditional controls, they highlight the importance of disruptions, the need to have a less linear and deterministic approach to method. The next section presents an account of interviews, the second method of data collection to collaborate observations.

3.3.4 Interviews

Opie (2004) argues that the kinds of interviews available to researchers lie in a continuum ranging from structured to unstructured interviews. Cohen *et al.* (2000: 270) elaborate this point further by saying that interviews differ mainly in "degree of structure, which, itself reflects the purpose of the interview". In this study a semi-structured interview was employed due to its flexibility as it would enable the researcher to probe for more insight in an idea. Opie (2004) points out that a semi-structured interview allows for depth of feelings to be ascertained by

providing opportunities to probe and expand the interviewee's response. He further points out that the main advantage of a semi-structured interview is that it allows the researcher to clarify questions or change the wording, to deviate from the prearranged questions depending on the interviewee's response. Moreover, a semi-structured interview provides the overall shape of the interview and prevents aimless rambling (Opie, 2004) as compared to unstructured interview.

Cohen *et al.* (2000: 268) argue that interviews are conducted for numerous purposes, such as "to evaluate or assess a person in some respect ... to gather data as in surveys or experimental situations". In this study, the purpose of interviews was multifaceted. First, interviews were conducted in order to find out the home languages and languages of learning and teaching of immigrant learners at their country of origin. Furthermore, what materials were prepared for those from French, Portuguese, Spanish background? Secondly, clinical interviews were also aimed at ascertaining how immigrant learners take up opportunities created in the mathematics classroom when solving a linear programming task. Lastly, the information gathered through reflective interviews with the teachers was to validate my interpretation of the data.

3.3.4.1 Teacher pre-observation interview

The pre-observation interview was conducted a day before lesson observations at each site. The pre-observation interview provided me with information about the activities, handouts or resources that the teachers were going to use. In particular what they wanted to achieve by engaging their immigrant learners in these activities and how they were to use them. During the course of the classroom observations, the focus was on how the teacher created opportunities for the use the languages brought by immigrant's learners. This was informed by literature which argues that the main difficulty for learners who are learning in an additional language is acquiring English, the language of instruction and learning linear programming. While this happens in different dimensions with many other learners in different context, it is exacerbated by the fact that some immigrant learners are not fluent in the official languages in South Africa.

3.3.4.2 Reflective interview

Reflective interviews were conducted after having analysed the classroom observation data. These interviews were semi-structured and therefore, were less formal and the conversation in

the interview was guided by the responses to the questions posed by the researcher. The semi-structured interview further gave me the opportunity to probe further if necessary (Cohen *et al.*, 2000; Opie, 2004). To avoid a situation where the researcher would end up imposing views of the cultural situation, the researcher talked less to allow the respondents to talk more. All the interviews were video recorded with consent from the participants and later transcribed. This also helped to record nonverbal communication, which also facilitated data analysis (Yin, 2009). In addition to recording the interviews, I took notes while interviewing the teachers. This was so because the use of a video recorder does not eliminate the need for taking some points (McMillan & Schumacher, 2006), as it helps reformulate questions.

Video recordings of the lessons were used during the interview process and focused the teacher's attention on specific incidents during the lessons observed. In addition, the interview shed more light on what I had observed and improved my analysis of classroom observations as I was able to explicitly communicate my assumptions and interpretations with the teachers. I was aware that my interpretations of what I observed might change as I re-analyze the data or as I interviewed the teachers as they might agree or disagree with some of my assumptions. The reflective interview served as a form of triangulation in light of my findings as well as of current literature.

However, I was aware that interviews present challenges in terms of interviewing and listening skills as potential information might have been missed due to poor questioning or listening skills. It was necessary that I maintain control, probe gently, be able to manage personal space and be non-judgmental (Opie, 2004).

3.3.5 Clinical interview

A clinical interview was administered on three Grade eleven immigrant learners, one in each of the three different schools. The three immigrant learners are John at School A and he arrived in South Africa in the year 2010, Bheki at School B and Allen at School C started schooling in South Africa. The profiles of each of these learners are discussed in Chapter 8. During the clinical interview, these learners were required to solve a linear programming task and explain what they were doing, whilst at the same time being questioned about their reasons for embarking on a set of procedures. At School A, I observed that John, from the DRC, had

difficulty in expressing his thinking in English. So he requested to solve the task before I asked him questions. I noted down issues that needed to be probed further during the interview while he was solving the task. This was done in order to give him space to express his thoughts productively. The following is an excerpt from one of the interviews to show what the learner said:

Thulie: I have a task that I would like you to solve. I will ask you questions as you solve it

John: Okay, our teacher told us...

Thulie: Alright, I would like you to solve the task...eh but it is English.

John: It is okay, our teacher give us tasks written in English in class.

Thulie: Read the task aloud and maybe if you have a question you can ask and I will explain

John: Okay, but let me try and solve it on my own and perhaps you ask the questions after I have finished solving

John requested that he solve the task quietly before I interrupt him. This was because he was not able to work on the task and communicate his thinking simultaneously. In order to address the issues of allowing John to solve the task and talk separately, the clinical interview with John comprised two parts: doing the task and then talking about the process of solving the tasks. The second part of the clinical interview involved asking John questions about his method or actions he employed when solving the task. The environment was relaxed and learners were free to express themselves in whatever manner and language they felt like using. This happened after the teaching of linear programming by their teacher.

3.4 Advantages and disadvantages of the methods of data collection

Videoed lesson observations in this research study focused on classroom instruction and learning activities. According to Cohen & Manion (1980: 103) the main advantage of observation is that the researcher “can discern ongoing behavior as it occurs and is able to make appropriate notes on its salient points”. On the other hand, videoing has technical problems of focusing and quality of sound. To minimize these videoing problems I asked a knowledgeable person to show me how to operate the video. The actual recordings were done after a few pilot recordings. Different viewing positions were used during observations to avoid strictly focusing on the immigrant learners. Each immigrant learner participated either in response to a teachers’ question when asked to read from the handout or when asked by the teacher to write a solution on the chalkboard.

Since the study used observations, the presence of the researcher might have influenced the results particularly if the researcher is new in the environment and stays for a short period as the participants could have been on guard regarding what they say and do (Opie, 2004). Opie further contends that “people consciously or unconsciously, may change the way they behave when being observed” (Opie, 2004: 122).

Observations and interviews are time consuming to transcribe due to too much data if recorded. However, recording of observations and interviews enable both verbal and non-verbal behavior to be noted which cannot be achieved with a questionnaire.

3.5 Piloting instruments

Having developed the instruments, the next step was to check whether they would work as anticipated. When an interview is one of the methods of data collection, Opie (2004) states that one should carry out a pilot study to weed out any ambiguities in the questions and to check the length of the interview. I could not carry out a full study to pilot the instruments as this implied finding another school with similar conditions. Piloting the instruments helped me in ensuring that all the equipment was functioning properly. I was able to identify potential questions for the interview and to improve my interviewing skills. In addition, it helped me to see whether the interview questions were related to the specific research questions and the research problem. There were no modifications that were a result of the piloting. All data collected was transcribed and analysed hence the next section discusses the process of analyzing the data.

3.6 Data analysis

Data analysis in qualitative research incorporates a number of different processes. Scholars in this area argue that the first stage involves close engagement with raw data or exploring systematically what the data is saying (Denzin & Lincoln, 2006; Yin, 2009). These scholars argue that data analysis involves bringing order to the data that has been collected, summarizing and looking for patterns and themes. The second stage involves developing interpretations. According to Yin (2009) developing interpretations means making sense of the results and attaching meaning and significance to the patterns and themes that the researcher identified during analysis.

Denzin and Lincoln (2006) take the view that, although sense making occurs during data analysis, at some point making sense of data needs to be done against some specific theoretical frames. My research is framed by Gee's discursive perspective (1999, 2005a, 2011) which regards language as a social tool for communication, thinking and also as a building tool. The analytic framework design was based on the theoretical construct of Discourses (with a capital D) in the contexts of linear programming in the mathematics classrooms. Discourse analysis therefore became a useful device as "it is the study of language in use in the world" (Gee, 2011: i). It is a way of understanding social interactions in the mathematics classroom. In the next section I discuss the first step of data analysis, which involves transcription of the video recorded data of classroom observations and interviews.

3.6.1 Transcription

The first step of data analysis involved transcribing all interviews and classroom observations video recordings to convert them into text so that I should have written texts. I transcribed all the interviews and classroom observations with the assistance of colleagues who are fluent in French, Xitsonga and Setswana since some of the utterances were in these languages. This task was time consuming and very difficult. In the transcription, we aimed for consistency while acknowledging the methodical process that transcription involves and the challenges inherent in attempting to produce accurate re-presentation of recorded conversations (Lapadat, 2000; Tilley & Powick, 2002). During the transcription process, I transcribed everything in English or isiZulu, writing the isiZulu words in isiZulu and translating them to English; English words in English. So did the French colleague, writing French words in French and then translating them to English. The colleague fluent in Xitsonga wrote words in Xitsonga and translated them to English. A colleague fluent in Setswana wrote words in Setswana and translated them to English.

At the end of the transcription, I listened to video recordings while looking at the written text to make sure that what was said was represented. The most common error was when the speakers were switching languages; there were omissions which we corrected together with my colleagues. Reading the transcripts and listening to video recordings more than once reduced these omissions and non-verbal cues were also added where necessary.

Another challenge I faced when transcribing is that when listening to audio recordings, many words were not audible enough. This was mainly because of the background noise, a speaker's pronunciation, or poor positioning of the recording device could obscure individual words, series of words or even complete utterances. Therefore, I have not included any utterance in my thesis that represents obscured speech. At times, in my transcription, I knew a word could be only one or another. My point is that we make judgment calls all the time when we listen to speech especially recorded speech data. Indeed, no two individuals pronounce a word exactly the same way. Therefore, it is important to briefly say something about the re-presentation of data used and why these were chosen to give the reader better access to the transcripts in the analysis.

The transcription was originally numbered according to utterances. When selecting excerpts, I have kept the original numbering so that it may be possible to relate back to the original transcription. The utterances have been chunked into stanzas (Gee, 2005a: 127). Each utterance has been broken down into lines containing one piece of new information (Gee, 2005a: 125). This makes it easy to refer to a whole stanza, a line or a more specific utterance.

In the transcription I use Gee's criteria for identifying each line as being an idea unit through which people give information and a tone unit in which people stress intonation. I have adopted Gee's use of underlined text to denote the part or parts of a line that carry the major stress and capital letters will denote emphasis by the speaker. To denote a hearable pause, short pause or a hesitation I will use "...". Inaudible text and non-linguistic messages such as gestures, laughs, sighs have been described in parenthesis where possible. Utterances in other languages like French, isiZulu are put inside square brackets and in italics. Next to each utterance is the translation in English.

Gee (2011: xi) mentions that representing any dialogue in a transcript is a challenge and states that "any speech data can be transcribed in more or less detailed ways such that we get a continuum of possible transcripts ranging from detailed transcripts to much less detailed ones". He further cautions that if the data include more details in the transcripts it would end up "being trees that obscure the forest, it is the forest we are after". This suggests that discourse analysis is based on the details of speech (gaze, gesture and action) that are arguably deemed relevant in the

context where the speech was used and that are relevant to the arguments the analysis is attempting to make (Gee, 2005a). A discourse analysis is not based on all the physical features present, not even all those that might be meaningful.

Setati (2003) whose study focused on multilingual classrooms argues that qualitative data is always a re-presentation of what actually happened. Re-presentation of multilingual data “is inevitably a process of selection and is informed by theory, research question, tools of analysis and the purposes of re-presenting the data” (Setati, 2003: 294). This re-presentation can alter the findings and conclusions of the research. For the purposes of this study I have made an attempt to include sufficient linguistic and non-verbal detail in an attempt to make available the transcript closer to the actual discussion, whilst making sure that it is readable in the new representation of the transcript.

3.6.2 Discourse analysis

The above argument highlights the fact that analysis is understood as a combination of close engagement with data, interpretation of data and theorizing. Hence the next stage of data analysis involved interaction of data, analysis and theory in an attempt to identify features in the text such as discourses. A discourse is a theme that relate to identities in relation to the social activity enacted (Gee, 2005a). The analysis provided an insight into how teachers created opportunities for the participation of immigrant learners during the teaching of linear programming and how immigrant learners take up these opportunities in the mathematics classroom. Researchers argue that during teaching, classroom discourse may be viewed as ranging from procedural discourses to conceptual discourses. Setati (2002a) states that procedural discourse is used to describe discourses that focus on the procedural steps to be taken to solve the tasks.

In the context of my study procedural discourses related to what the immigrant learners did algorithmically to solve a linear programming task without indicating or necessarily requiring any understanding of the mathematical processes involved. In solving linear programming tasks this may include practices like the use of inequalities, use of tables, drawing a straight line, shading the region represented by inequalities. In my experience as a high school teacher, tables are used as a means of making inequalities more accessible to the learners in order to generate

correct graphical representations. It was necessary for me to ascertain the extent to which immigrant learners used the procedure to acquire the inequalities and formulate understanding that would generate a relationship resulting in a graphical representation.

From my experience, linear programming tasks require that immigrant learners work through multiple tasks and constraints including mathematical, contextual and linguistic challenges. The linguistic challenge included unfamiliarity with everyday words and phrases like ‘at least’ and ‘at most’ when representing them using mathematics symbols. This confusion resulted in immigrant learners misrepresenting these phrases leading to different linear inequalities. As a result immigrant learners might not be able to show the expected solution to the tasks.

3.6.3 Framework of the discourse analysis

In order to begin to clarify and organize the large amounts of data collected, I established a coding system. Initially, transcripts from the classroom observations, and interviews were classified in light of a variety of themes related to opportunities to learn linear programming. These were organized chronologically, with the lesson considered to be the unit of analysis. Within the lessons, examples of dialogue from classroom observation transcripts that had a clear beginning and end were designated as episodes. Three themes and the relationship among them soon emerged as central and these became the focus of data analysis. The themes that emerged were as follows:

- Evidence of opportunity to learn linear programming
- Teacher’s actions when teaching linear programming
- Learners’ actions when learning linear programming

Establishing the themes as the focus of data analysis showed that some of the opportunities were linked to specific teacher’s action.

3.6.4 Classroom Discourses

The analysis was concerned with analyzing how and what Discourses emerged during teaching and learning linear programming to immigrant learners. Gee’s (2005a, 2011) method for

discourse analysis and a selection of his analytic tools was used. The selected tools of enquiry are *social languages*, *situated meanings* and *Discourse models*, which he refers to as “figured worlds” (Gee, 2011: 170).

Social languages: This is the style or variety of language (mixture of languages) that helps to enact a social activity and is associated with a particular social identity (Gee, 2011: 156). They are embedded in certain identities that are enacted when one uses language in different context or when interacting with the teacher or another immigrant learner. Gee (2005a) argues that different people have different access to the activities enacted and he suggests the following questions as a guideline for exploring *social languages* in any given piece of text which were very useful:

Guiding questions:

1. What *social languages* are involved?
2. What socially situated identities and activities do the *social languages* enact?
3. What Discourses models are involved?

The *social languages* used by teachers during the teaching are evidences of the opportunities to learn linear programming. The analysis focused on how teachers linguistically supported immigrant learners in order to understand the *social languages* presented during teaching. Teachers provided linguistic support by drawing on languages brought by immigrant learners in the mathematics classrooms. This was seen as a reflection of the opportunities that were established and the teacher’s reactions about them during teaching of linear programming.

Gee (2005a) argues that when we use language we build a reality by building seven things which he refers to as the ‘seven building tasks’. A *social language* is the language aspect of a discourse; that contains the clues or cues that guide the seven building tasks. Therefore, he suggests the following questions to ask given a piece of language:

Table 3. 2: Building tasks and question asked

Building task	Question to be asked
1. Make things significant in certain ways (to give them meaning):	how does the teacher or immigrant learner make use of language to make this text significant?
2. Get recognized as engaging in a certain sort of activity:	What social activity is this piece of language being used to enact?
3. To get recognized as taking on a certain identity or role (to build an identity):	What identity is this piece of language being used to enact?
4. To signal what sort of relationship we have, we want to have or are trying to have with our listener's:	What sort of relationship is this piece of language seeking to enact with the immigrant learner or teacher in the linear programming lesson?
5. To convey a perspective on the nature of the distribution of social goods:	What perspective on social goods is this piece of language communicating?
6. To render certain things as connected or relevant to other things:	How does this language make one thing relevant or irrelevant?
7. To make certain sign systems and certain forms of knowledge and beliefs relevant or privileged (inequalities, graphs):	How does this piece of language privilege sign system?

Situated meanings: Gee (2005a: 64) stresses that “*situated meanings* are assembled out of diverse features, “on the spot” as we speak, listen and act”. Hence in context we assemble the features that will constitute the pattern or *situated meaning* that a word will have. It is apparent that different context requires one to call upon different patterns to be assembled. Therefore a *social language* triggers a *situated meaning* of a word or phrase in the context of linear programming.

Guiding questions

1. What *situated meanings* do the word or phrase mean in the context of linear programming?

2. What *situated meaning* for the word or phrase is it reasonable to assign to immigrant learners?
3. What *situated meaning* is it reasonable from the point view of the Discourse in which these words were used or other Discourses, to assume are potentially attributable to these words by immigrant learners?

Discourse models: Situated meanings reveal Discourse models. Gee (2005a: 61) states that “*Discourse models* are theories (story lines, images, explanatory framework) that people hold often unconsciously and make sense of the world and their experiences in it”. Theories are simplifications of reality which focuses on helping us understand complicated realities resulting in focusing on important things and leaving out some of the details.

Guiding questions:

1. What *Discourse models* are relevant in this piece of language? What must I assume the immigrant learners or teacher, feel, value and believe consciously or not in order to talk (write), act and / or interact in this way?
2. Are there differences between the *Discourse models* that are affecting espoused beliefs and those that are affecting actual actions and practices?
3. How is the relevant *Discourse model* helping to reproduce, transform or create social, cultural, institutional, and/ or political relationships? What Discourses are these Discourse models helping to reproduce transform or create?

In order to identify each of the building tasks the following questions were used as a guide:

1. Which key words/ phrases/ sentences are indicative of the building tasks? Each building task is likely to be indicated by a certain word or phrase or even a paragraph. It is important that these are easily recognizable in any text that is analysed.
2. What are the situated meanings of these texts? Providing definitions of the key indicators for a building task is a step towards a justification for matching certain words/ phrases to certain building tasks.

3. In what language (s) are these words given? The languages could be French, Xitsonga or English or code switching. Within these is ordinary language or mathematical language used in a sentence or paragraph?

The organization of the analysis is such that each building task is discussed individually using Gee's tools of enquiry mentioned above guided by the questions which will be listed below. At each level of the analysis a brief discussion is undertaken and an overall discussion for each episode is done before moving on to the next episode.

Discussion on building tasks:

Gee (2005a: 110-113) states that a discourse analysis involves asking questions about the seven building tasks and he provides twenty six questions one can ask when analysing text and further argues that the extent of the convergence of the answers to these questions may be used as one measure of the validity of the analysis. I decided to list the questions below:

Building significance

1. What are the situated meanings of some of the words and phrases that seem important in the situation?
2. What situated meanings and values seem to be attached to places, times, bodies, people, objects, artifacts and institutions relevant in this situation?
3. What situated meanings and values are attached to other oral and written texts quoted or alluded to in the situation ("intertextuality")?
4. What discourse models seem to be at play in connecting and integrating these situated meanings to each other?
5. What institutions and/or Discourses is being (re-)produced in this situation and how are they being stabilized or transformed in the act?

Building activities

6. What is the larger or main activity (or set of activities) going on in the situation?
7. What sub-activities compose this activity (or these activities)?
8. What actions compose these sub-activities and activities?

Building identities

9. What identities (roles, positions), with their concomitant personal, social and cultural knowledge and beliefs (cognition), feelings (affect), and values seem to be relevant to, taken for granted in, or under construction in the situation?
10. How are these identities stabilised or transformed in the situation?
11. In terms of identities, activities and relationships, what Discourses are relevant (and irrelevant) in the situation? How are they made relevant (and irrelevant) and in what ways?

Building relationships

12. What sorts of social relationships seem to be relevant to, taken for granted in, or under construction in the situation?
13. How are these social relationships stabilised or transformed in the situation?
14. How are other oral or written texts quoted or alluded to so as to set up certain relationships to other texts, people or Discourse?
15. In terms of identities, activities and relationships, what Discourses are relevant (and irrelevant) in the situation? How are they made relevant (and irrelevant) and in what ways?

Building politics (distribution of social goods)

16. What social goods (e.g. status, power, race and class or more narrowly defined social networks and identities) are relevant (and irrelevant) in this situation? How are they made relevant (and irrelevant), and in what ways?
17. How are these social goods connected to the Discourse models and Discourses operative in the situation?

Building connections

18. What sorts of connections - looking backward and/or forward - are made within and across utterances and large stretches of the interaction?

19. What sort of connections are made to previous or future interactions, to other people, ideas, texts, things, institutions and Discourses outside the current situation (this has to do with “intertextuality”)
20. How is “intertextuality” (quoting or alluding to other texts) used to create connections among the current situation and other ones or among different Discourses?
21. How do connections of the sort in 18, 1 and 20 help (together with situated meanings and Discourse models) to constitute “coherence”- and what sort of “coherence” in the situation?

Building significance for sign system and knowledge

22. What sign systems are relevant (or irrelevant) in the situation (e.g. speech, writing, images and gestures)? How are they made relevant (and irrelevant) and in what ways?
23. What systems of knowledge and ways of knowing are relevant (or irrelevant) in the situation? How are they made relevant (and irrelevant) and in what ways?
24. What languages in the sense of “national” languages like English, Setswana, isiZulu are relevant (or irrelevant) in the situation?
25. What social languages are relevant (or irrelevant) in the situation? How are they made relevant (or irrelevant) in the situation? How are they made relevant (and irrelevant) and in what ways?
26. How is quoting or alluding to other oral or written texts (“intertextuality”) used to engage with issues covered in question 22-25?

Gee does not only give us tools, he further provides guidance on how one can go about doing discourse analysis. I picked some guidelines that I felt would be appropriate for my analysis, and I analyzed most of the utterances with a mathematical background borrowing linguistic tools. The borrowed linguistic tools will be used just like linguists who use mathematical diagrams like frequency tables and charts in their analysis. Therefore, I did not go into deeper linguistic cues that a linguist could use for the same data.

The next stage involved analysis of the immigrant learner’s solution to the task during clinical interview.

3.6.5 Immigrant learners' activity from a linear programming perspective

The immigrant learners' activity involved mainly the solving of a linear programming task presented in English. Morgan (2006) states that during the solution processes, the immigrant learners are making use of whatever semiotic processes, symbols and language with which they are familiar in order to engage with the task. I am only able to examine how immigrant learners talk about the activity in which they were involved and how their talk is demonstrating their knowledge of linear programming. The activity tool is examined from the solution presented by the learners and the question asked after they had solved the task. Thus research question three is addressed by what the immigrant learners did and my observation while they were solving the task.

Gee (2005a) points out that all the building tasks are interlinked and it is impossible to examine one without the others coming into play. Yet in reality one cannot do an analysis that examines all of the building tasks simultaneously. Thus the analysis is at three levels; first I look at what the immigrant learners make significant, how the immigrant learners build identities and relationships. This means how the immigrant learners talk about themselves in relation to others and the world. The second part of the analysis I looked at how they build sign system and knowledge and how they build connections while solving a linear programming task. The final phase of the analysis looks at social goods, that is the social benefits or disadvantages that the immigrant learners perceive. This will allow me to look at the opportunities created in the teaching of linear programming.

3.6.5.1 Questions to guide the analytic process

What does the immigrant learner highlight as significant?

See this through what the immigrant learner bring to the fore in their response to the task and questions asked by the researcher, activities and gestures, why they make certain connections (or not) through what they emphasise.

What sign system and knowledge are used to talk about the activity?

See this through the use of mathematical Discourse, use of symbols or unknowns, use of or reference to methods or procedures.

What connections (or disconnections) are evident from looking at the immigrant learners' discourse?

See this through the way in which the immigrant learners make connections with the mathematics domain as a whole (or not), or the way in which it sets up associations with specific parts of the mathematics domain (or other domain).

What activities is the immigrant learner involved in?

See this through what immigrant learners did when attempting the linear programming task which will be examined through the activity.

Identity: How do immigrant learners view themselves in relation to the linear programming tasks?

See this through the way in which immigrant learner's talk about their involvement in doing the problem, for example being in control or not coping

What relationships emerge with respect to teacher and other learners involved in the learning of linear programming?

See this through the way in which immigrant learners put forward in terms of who is in control, especially through the way in which they talk about (or do not talk about) their involvement and others in the activity of solving linear programming.

Politics: what social goods are perceived by the immigrant learners as being accessible?

See this through the way in which the immigrant learners see themselves positioned with respect to others in terms of the benefits or disadvantages that such a positioning might imply and in terms of how the identity put forward enables or inhibits them from accessing those social goods.

3.6.5.2 Analysis of immigrant learner's activity

The written responses produced by the immigrant learners were examined at three levels. These three levels are then used to gauge whether immigrant learners are supported when learning linear programming. This will make me conclude whether immigrant learners have access to linear programming discourse in each of the levels. The levels are described below.

Constraint inequalities

From the written responses that the immigrant learners have produced in attempting linear programming task I determined the extent to which they demonstrated an understanding of the social languages that showed constraint inequalities from the given task as well as the situated meanings of some key words in the task. The text produced by the immigrant learners was examined for certain features used to suppose that immigrant learners are recognized in the linear programming discourse. This suggests that I looked for evidence in the text for relevant constraints inequalities. The evidence in the text was any of the four instances which are a hierarchy of understanding. The instances are my judgment and the evidence lead to assigning the immigrant learners' understanding to one of the following instances (which are in order from little to extensive understanding) discussed below:

Table 3. 3: Degree of relevant constraint inequalities

Instances	Degree of relevant constraint inequalities reflected in immigrant learner's solution to the given task
Instance 1	<ul style="list-style-type: none"> - No understanding of constraint inequalities to be extracted from the task. - Inequalities simply written down and cannot be connected to aspects of the task being solved. - No evidence that ideas reflected by the text are connected to the task being solved.
Instance 2	<ul style="list-style-type: none"> - Some evidence of extracting the inequalities from the task. - Some misrepresentation or misinterpretation of the inequality sign had occurred. (E.g “does not exceed 4000” represented by a greater than inequality sign [meaning $4000 < x$] instead of a less than inequality sign [$x < 4000$])
Instance 3	<ul style="list-style-type: none"> - definite recognition of the constraint inequalities represented - Some incomplete representation of inequalities/not all expected inequalities shown in the text.
Instance 4	<ul style="list-style-type: none"> - Evidence of the correct interpretation of all aspects of constraint inequalities. - Complete and clear representation of all the inequalities to be extracted from the given task.

Drawing/ or representing inequalities graphically and identifying a feasible region

From the text it was possible to determine how immigrant learners went about representing inequalities on a Cartesian plane. There was a procedure evident in the written responses that were used to find the points to plot in order to draw the line. The procedure was a table of x and y values, or the dual intercept method or even guess work. The instances were a hierarchy of understanding (which are in order from little to extensive understanding) discussed below:

Table 3. 4: Degree of relevant graphical representation of inequalities

Instances	Degree of relevant drawing/ or graphically representation of inequalities and identifying feasible region
Instance 1	<ul style="list-style-type: none">- no evidence of a procedure or a linear function- appear to be random use of numbers such that it is not possible to represent the inequality graphically (representation of immigrant learners inequality)
Instance 2	<ul style="list-style-type: none">- some evidence of method present in establishing relationship but method not clear
Instance 3	<ul style="list-style-type: none">- Appropriate method, dual intercept or table- Presentation not systematic- Implementation not well ordered i.e less methodological
Instance 4	<ul style="list-style-type: none">- evidence of clear and correct method used to represent inequalities graphically- organized, well structured work to guide the representation of inequalities on a Cartesian plane

Mathematically correct/optimising

The text produced by the immigrant learner reflected the extent to which the mathematics presented was correct and applicable to the linear programming task. It was important that the text produced by the immigrant learner showed evidence of the level of understanding of an objective function when optimizing. The instances were a hierarchy of understanding (which are in order from little to extensive understanding) discussed below:

Table 3.5: Degree of relevant mathematically correct/ optimising

Instances	Degree of relevant mathematically correct/ optimizing
Instance 1	<ul style="list-style-type: none">- no evidence of correct and appropriate mathematics applicable to the task- no mathematical working shown- incorrect mathematics.
Instance 2	- some evidence of correct formulae applicable to the task but is insufficient to enable progress.
Instance 3	-some meaningful and appropriate mathematical relationships. For example, in order to calculate maximum income using the objective function.
Instance 4	-Correct mathematics and establishment of an equation that leads to a correct solution to the task.

The overall analytic framework that made me recognize immigrant learners as part of the linear programming discourse looked as follows: an examination of the text to identify constraint inequalities, graphical representation and calculation of maximum income. Analysis of the immigrant learners' work was followed by analysis of the immigrant learner talk about solving the task to ascertain the seven building tasks mentioned in the previous section, enabled me to see patterns pointing to certain Discourse models in order to answer my research question.

In summary, the overall analytic framework looked at the examination of the immigrant learners' activities followed by an analysis of the seven building tasks, which enabled me to see patterns that indicated evidence of certain Discourse models. This enabled me to address Research Question 3. The next discussion focuses on issues pertaining rigor in qualitative research.

3.7 Rigor in qualitative/case study research

The issues of validity of the explanations made were ensured by either providing evidence from the transcripts or by using the literature. Opie (2004) calls it evidence based process. He argues that claims that are made should be backed-up by refereed sources from journals and academic books. Another tool that I used is argument (Opie, 2004) based on either the data itself or

statements from the literature. In using the literature, I tried as much as possible to use data from the original sources.

Opie (2004) views reliability as the property of the whole data gathering process. While reliability and validity are often used in quantitative research as measures of quality in a piece of research, LeCompete & Preissle (1993) argue that qualitative research appropriates selectively the tools of quantitative research. Consequently, issues of reliability, validity and generalizability turn to be unworkable in qualitative research. In qualitative research, credibility is used instead of validity, confirmability as opposed to reliability and transferability instead of generalizability. As indicated earlier the study uses triangulation by procedures (Opie, 2004) as a means of ensuring credibility of the research. The next section discusses issues pertaining to transferability.

3.7.1 Transferability

This study is qualitative in nature. It sought to provide a thick description of three cases. Therefore, the issue of generalizing the findings is impossible. Lincoln & Guba (1985) point to the fact that in qualitative research generalizability is interpreted to mean comparability and transferability of findings to other individuals working within similar conditions. Lincoln & Guba (1983: 36) argue that “some degree of transfer is possible if enough ‘thick description’ is available about both the sending and receiving contexts to make a reasoned judgement possible”. They suggest that it is therefore important that a researcher provides a clear and detailed description of the phenomenon so that others can decide the extent to which the findings of the research may apply to their situations.

Opie (2004) argues that the value of qualitative research lies in the adequacy, appropriateness and extent to which the description made would suite a teacher working in similar conditions to relate his or her decision making to those described. I am therefore aware that the outcomes of this research may not be generalisable even to the overall practice of the cases themselves. But I am confident that they could possibly highlight the opportunities created by teachers as they make an effort to teach linear programming to immigrant learners in Grade eleven.

3.7.2 Confirmability

Cohen *et al.* (2000) draw on the work of LeCompte & Preissle (1993) to argue that the ‘canons of reliability in quantitative research are impractical in qualitative research’. According to Lincoln & Guba (2004), in qualitative research reliability is construed as dependability as it involves member checks or confirmability. Aligning myself with Lincoln and Guba, I conducted follow up interviews with the three teachers after all the data had been collected and analysed. These interviews were to confirm or refute my assumptions and interpretation of the teachers’ opportunities for the participation of immigrant learners. For instance, I noticed that the teacher at the urban environment switched between two additional languages English and French. This led me to claim that his use of languages is to support immigrant learners from Francophone background (for example learners from the DRC). The teacher illuminated this point further during the interview when he said (*your attention is drawn to the bold text*)

*Leo: For me I try to use **French** so that **they understand** but it is a **problem for those immigrant learners from Portuguese speaking countries, but with French I try...***
(Reflective interview November 2011)

3.7.3 Credibility

Credibility means the results of a qualitative study are believable and trustworthy from the perspective of the participants or subjects in the research itself (Shenton, 2004). This is mainly because, qualitative research attempts to describe or explain the event, group or phenomenon of interest from the perspective of participants and they are best situated to judge the credibility of the findings in a qualitative study.

Lincoln & Guba (1985) explain that to ensure credibility, researchers need to employ multiple research methods, such as interviews and participant observations to study the same phenomena. Researchers further have to develop an understanding of the organization being studied before data collection begins. This is to ensure that credible findings and interpretations will be produced. These researchers further argue that researchers should provide external check on the inquiry process especially the human sources from which the raw data come from.

In this study, peer consultations were conducted with colleagues throughout the study. Issues such as the sampling, methodology, the theoretical framework and the framing of the study were discussed in order to establish credibility. Furthermore, to ensure credibility data was collected from various sources. Shenton (2004) argue that such an approach helps to test the credibility of the findings. In this study, mathematics teachers from different backgrounds were involved. In addition to that, the schools were selected from different sites in South Africa. In this thesis I have also present a detailed description of how data was collected and analyzed (Merriam, 1998).

3.8 Ethical considerations

There were ethical issues surrounding the integrity of the schools as well as “informed consent, guarantees of confidentiality, beneficence and non-maleficence” (Cohen, Manion & Morrison, 2000: 279) of the immigrant learners that participated in the study. Ely (1991: 223) contends that “the very naivety of many research participants makes it the more imperative that we are careful to protect them.” As with establishing rigour, I believe that ethical issues in any research study are determined and that it is upon the researcher to ensure that all undertakings are ethically considered, conducted, interpreted and reported. Cohen *et al*, (2000: 315), states that “...researchers inhabit the world that they are researching and their influence may not be neutral”. This suggests that the issue of consent cannot be overlooked. I now turn to a discussion of how I ensured that my research was standard, moral, fair and worthy.

Cohen *et al*. (2000) argue that carrying out research demands that the researcher obtains consent and cooperation of the participants and the institutions that provide research facilities. In this study four institutions are involved; University of South Africa (UNISA) under whose name the study was conducted, two secondary schools in Gauteng province and one in the Limpopo province. Grade eleven mathematics teachers and their Grade eleven immigrant learners from these schools were involved in the study.

Cohen *et al* (2000) further point out that the issue of access and acceptance is very crucial in pursuing a study. As the study was conducted under the name of UNISA, I applied for ethical clearance from the UNISA research ethics committee and was granted ([see appendix A](#)). As the study was conducted in two public schools and one private school, I sought permission to

conduct the study from the Department of Basic Education ([see Appendix B](#)). I chose to undertake this study at three research sites; urban, township and rural environment. Details of how I got to know the schools are presented in Chapter 4, which is about gaining access to the research sites. This was to make sure that the different classroom settings in South Africa are represented. The representation was important because it captured immigrant learners' experiences from different contexts. Also, I wanted classrooms that enroll immigrant learners.

Permission to collect data was sought from the principals of the three schools. Three letters were prepared for the participants of the study ([appendix C, D, E](#)). One letter stating the research topic, the second letter was inviting the participants to take part in the study and the third letter was the consent form. According to Kent (1996: 19), aspects of informed consent that need to be borne in mind are:

- a) Give information that is relevant to the subjects' decisions about whether to participate.
- b) Make sure that the subjects understand that information
- c) Ensure that participation is voluntary
- d) Obtain parental consent when dealing with children

With the above guidelines in mind, the letters were read and explained to the participants. Immigrant learners were asked to take the letters home and request permission from their parents or guardians to participate in the study. All learners returned the letters signed by them and their parents or guardians. All teachers identified for the study agreed to participate.

The letters indicated the research topic, purpose of the study and explained the need for videoing of lessons and the interviews. Letters also addressed issues around confidentiality of results, anonymity of school and participants, privacy, freedom of participation, respect and fairness (Cohen *et al.*, 2000). In this regard, I decided to use pseudonyms for the teachers and learners involved in this study. The names of the schools are not mentioned, I only refer to them as School A, School B and School C.

I also provided the principals with copies of letters and consent forms that were sent to the parents of the learners and the teacher regarding the same issues. The consent form issued to immigrant learners and their parents, the following was given:

- An information letter about the project which included a brief outline of the research topic and aims ([see Appendix E](#))
- An overview of the activities (clinical interview) in which the immigrant learners were to be involved ([see Appendix F](#))
- Participation in the study was purely voluntary
- An assurance that the research and the findings would in no way be any reflection of individual immigrant learners, their families or their school.
- An explanation of how the research findings are likely to be used (Adler & Lerman, 2003)
- A set of tick boxes in which consent could be indicated for transcripts and recordings to be used for educational publications and by other researchers (Silverman, 2001) ([see Appendix C](#))

To protect participants, their confidentiality, anonymity, non-identifiability and non-traceability as well as that of the schools was ensured during reporting (Cohen *et al.*, 2000: 292). All names used in the transcripts are pseudonyms.

This study used interviews and observations as data gathering instruments. Hence one needed to address ethical issues regarding each instrument as well. Ethical issues concerning observation were dealt with in the preceding paragraphs. Cohen *et al.* (2000) warns that interviews can be biased due to the characteristics of both the researcher and respondent. Kvale in Cohen *et al.* (2000) identified informed consent, confidentiality and the consequences of interviews as problematic areas that need to be addressed in interviews. Oppehein cited in Opie (2004, 112) argues that bias in interviews can be minimized if “the interviewer... maintains control of the interview, to probe gently but incisively and present a measure of authority and assurance of confidentiality”. Tuckman (1972) cited in Cohen *et al.* (2000: 279) exemplifies the problems of interviews by saying that:

“... the interviewer should brief the respondent as to the nature or purpose of the interview (being as candid as possible without biasing responses) and attempt to make the respondent feel at ease. He should explain the manner in which he will be recording responses, and if he plans to record he should get the respondent’s assent. At all times the interviewer must remember that he is a data collection instrument and try not to let his own biases, opinions, or curiosity affect his behavior ...

I indicated to the teachers that the purpose of the interview was to confirm or refute my assumptions and interpretations of what I observed in the lessons and to assist me in understanding the opportunities created and presented to immigrant learners while learning linear programming. I further asked them to allow me to video record the interviews as this would enable me to re-listen to the interviews during data analysis. They had no objection to this and I video recorded all interviews. Tuckman cited in Cohen *et al.* (2000) argues that interviews could be both threatening and stressful to respondents as they often do not know in advance the contents of the interview and the interview environment is usually unfamiliar. In response to this I conducted interviews in rooms allocated by the principals with the hope that they would feel more comfortable. I tried as much as I could to be non-judgmental while being sensitive. At School A the interview was conducted in the teachers’ office, School B in the school library and School C in the Home Economics’ room.

3.9 Conclusion

I began this chapter by indicating that the study uses case study as its research methodology. I went further to show that interviews and classroom observations for five days were employed in collecting the data. I also indicated that the study uses qualitative methods of data collection and analysis. I highlighted the limitations of both the methodology and instruments used in the data collection. Lastly, I indicated that the collected data was analyzed using a discourse perspective. With respect to issues concerning rigor in research, I pointed out that tools of quantitative research (reliability, validity and generalizability of findings) cannot be applied directly on qualitative research. To this end, I concerned myself with credibility instead of validity, transferability instead of generalizability and conformability instead of reliability.

The methods were suited for this study as there was a need to interact with the school environment, the teachers and learners in their environment. On the other hand, it should be

expected that the method is somewhat personal in that no researcher observes, interviews or studies documents exactly like another (Stake, 1978). Threats to reliability were reduced by the use of a combination of verbatim accounts and video recorded data (Hitchcock & Hughes, 1995; Yin, 2009; Schumacher & McMillan, 1993).

In the next Chapter I discuss the process of gaining access to the research sites.

CHAPTER 4

GAINING ACCESS AND LOCATION OF RESEARCH SITES

4.1 Introduction

This chapter gives an account of the process of gaining access to research sites and participants. I start by discussing the literature which argues that gaining access is a process which depends on the relationship established by the researcher. This suggests that in order to successfully conduct a study, researchers must first gain the cooperation and trust of participants by establishing a good working relationship and rapport with them. The researcher must of necessity earn the trust and acceptance of the participants. I then discuss issues surrounding immigrants in South Africa in order to describe the background for some of the challenges I faced to gain entry and collect data. This is followed by a discussion of the schools that is the research sites including the environments in which they operate.

The process of acquiring formal and academic access is important in educational research as to ensure the collection of valid and reliable data that is needed to answer research questions (Opie, 2004; Hatch, 2002). This is important for ethical research. Formal access involved obtaining written permission from responsible authorities and administrators such as provincial departments of basic education and school principals, and participating immigrant learners' parents or guardians to allow the researcher to enter into schools and collect data. Academic access is equally important. It involves epistemic access to vital data. This was assured when participating immigrant learners freely and voluntarily provide data imperative to the study.

4.2 Gaining access

Gaining access involved negotiating approaches to participants who acted as gatekeepers (Hammersley & Atkinson, 2007) to the process of data collection. Although the issue of gaining access to participants was "complicated and nuanced" (Harrington, 2003), it was about power and identity relationships of the researcher and participants. Harrington thus argues that it is crucial that researchers establish roles and relationships that participants find acceptable.

Glesne and Peshkin (1999: 33) define access as a process. According to them,

...it refers to acquisition of consent to go where you want, observe what you want, talk to whomever you want, obtain and read whatever documents you require, and do all of this for whatever period of time you need to satisfy your research purposes .

Hammersley and Atkinson (2007), Wanat (2008), and Beynon (1983) argue that the process depends on successful negotiations on the relationship between the researcher and the researched or gatekeepers and/or participants. They give examples of relationships they developed with their research participants. Wanat (2008: 200) reports that he gained access to different educational sites by developing “empathetic relationships” with gatekeepers. Similarly, Beynon (1983: 40) states that in order to gain access into the schools he found common interests which he refers to as bridges and reports “weaving” his way into a school. The experiences of the scholars mentioned above suggest that a relationship of trust between the researcher and the researched needs to be developed. When the relationship of trust is developed it is understood to be the key that grants access to a research site and epistemic access (Hornbeger & Johnson, 2008). In addition, Hornberger (1992: 160) states that it is not sufficient to enter the research site basically by being located in it; to a certain extent, “I enter it by establishing social relationships with its members”. This is because being allowed into a research site does not always guarantee participants’ cooperation.

The process of access in multilingual schools which enrol immigrant learners is similar to the process of access to any other school in South Africa. In all cases, a researcher has to deal with a highly institutionalised setting (Creese, 2011). Nevertheless, the process of access in multilingual schools with immigrant learners who (some of them) have witnessed xenophobic attacks was a challenge because they did not want to be identified as foreigners. The investigation of immigrant learners was conducted at a time of increasing movement of people across borders due to economic and political difficulties in Southern Africa. According to Martin-Jones (2011), the movement of people across borders result in linguistic practices moving across time and space, changing as people go, taking with them old affiliations, at times shedding these affiliations and building up new identities which might be in terms of language. In the process of movement and change linguistic practices come to constitute a terrain for competition, a point of negotiation and a place where some practices are valued more highly than others.

Hammersley and Atkinson (2007: 52) argue that periods of change and transition may be understood as challenging by some participants to research projects. The participants may wish to turn observers away from them thus causing “conflict of interest” as they do not feel safe under the glare of close observation and scrutiny that comes with studies such as this one. Yet such research might provide important research opportunities for the researcher. Such reluctance from desired participants may, however, distract and slow down the process of access.

4.2.1 Access to research sites

Gaining access to schools was at four levels. First I had to negotiate access with the responsible provincial department of education. Secondly it involved access into the schools through the principals; thirdly, access into the mathematics classroom through teachers and fourthly access to immigrant learners through their parents and guardians.

In the section below, I outline the process of gaining access to the research sites. However, I will first discuss issues surrounding immigrants in South Africa. In so doing I paint a background to some of the challenges I faced as I negotiated access and during data collection.

4.3 Immigrants in the rainbow nation

The issues surrounding immigrants in South Africa reflect the strong negative views by some South African nationals against immigrants who intend to stay in the country, some permanently. The literature presented discusses the negative attitude reflected in the public discourse and thus explains the tension and violence directed towards immigrants. However, migration to South Africa is not new. In the past, male labour migrants from neighbouring countries like Lesotho, Malawi, Mozambique, Swaziland or Zimbabwe were employed to work in the mines and on the farms. While working in South Africa, they were accommodated in migrant hostels (miners) with the understanding that they would return to their country of origin once their contracts of employment expired. Furthermore, thousands of refugees from Mozambique arrived in South Africa in the 1980’s fleeing civil war in that country. But the flow of immigrants in search for a better life in South Africa has increased since 1994 after the end of apartheid when the country opened up to the world.

Post apartheid South Africa is a country that experienced dark historic times that were full of racial discrimination and inequity. In the apartheid era, the word “*kaffir*” was used derogatively to address dark-skinned people (Perbedy & Crush, 1998), in other words; Africans. Giving names to disliked groups of people did not stop with the end of apartheid. Since the collapse of apartheid, the word “*makwerekwere*” has come into use with reference to specifically dark-skinned foreigners from other African countries (Ongori & Agolla, 2007). Matsinhe (2011: 295) states that, “the phantom of *makwerekwere* has been constructed and deployed in and through public discourse to render Africans from outside the borders orderable as the nation’s bogeyman.” The public discourse created a strongly negative view about immigrants to some South African citizens which is expressed in various ways. The negative view tends to be concentrated in the poorer areas such as the townships and rural areas. In these areas, immigrants are perceived as unwelcome competitors who grab employment and business opportunities away from South African citizens, particularly when they reminisce that there were denied these during the times of apartheid. Due to these reasons, xenophobia resulted in waves of violence against African immigrants in the form of brutal attacks on their property, as well as displacement and killing of immigrants (Matsinhe, 2011).

Researchers in this area explain the notion of xenophobia in South Africa based on the accusation of foreigners for crime, unemployment and spread of HIV/AIDS as a demonstration of frustration by poor and unemployed citizens (Morris, 1998; Tshitereke, 1999; Harris, 2002; Neocosmos, 2008). According to Tshitereke (1999), the frustration of the poor and unemployed is due to the problem and slow progress of redressing the inequalities of the past. Tshitereke further suggests that the issue of hostility towards foreign nationals emanates from prolonged seclusion and isolation of South Africans during apartheid, particularly black South Africans, which prohibited contact not simply with the world beyond Africa but with Africa and South Africa itself.

The collapse of apartheid ended South Africa’s seclusion and isolation, and opened the borders which led to newly-found exposure to the outside world. This meant that South African citizens were all of a sudden exposed to foreigners, many of whom were unable to speak any one of the South African indigenous languages (Morris, 1998). Indeed, some of the migrants from other

African countries are quite different from South African; visible in terms of physical features, their different customs and behaviour, and their clothing styles. Furthermore, the culture of African foreigners differs from the South African culture and makes them easily identifiable.

Harris (2002) suggests an explanation that situates xenophobia within the post-apartheid period, wherein broad social institutions such as the media produced negative representations of Africa and African foreign nationals as “illegals”, “illegal aliens”, “illegal immigrants”, criminals and drug traffickers, most notably by the South African police service and the department of home affairs. But a relevant question to ask in this context is: why were white foreigners and Asians in South Africa not targeted? For example, some of the Asian foreigners speak Mandarin, speak English with difficulty, and cannot speak any of the indigenous South African languages.

Neocosmos (2008) proposes an explanation that locates anti-African xenophobia within South African self-government. He argues that it is because of the anti-rural and pro-urban character which during the apartheid era ruralised and devalued black lives, on the one hand, whilst urbanising and valuing white lives, on the other. The post-apartheid state simply shifted this rural/urban binary opposition to Africa/South Africa, such that Africa is perceived as rural and backward and South Africa as urban and modern.

The literature discussed above highlights the tension that resulted in violence directed at black foreigners including teachers in South Africa and also witnessed by some children. It was therefore not surprising to note that some of the immigrant learners did not want to be identified as foreigners. In the paragraphs that follow I outline in detail the process of gaining access to the three schools identified for this study. I start by discussing the process of locating the schools and their neighbourhoods.

4.4 Locating the schools

During my short stay in South Africa (2006-2008), I learned that the nearest provincial department of basic education to the schools grants permission to conduct research provided that a research proposal is submitted to the department for ethical clearance. However, locating the schools which enrol immigrant learners was a challenge for me. This is because, being an

immigrant myself, I am not familiar with most of the schools that enrol immigrant learners in South Africa. Nonetheless, I was involved in assisting orphan and vulnerable children from different schools at a site around Mabopane in Pretoria. Some of these children who came to the site were migrants from different countries in Africa. For example, they migrated from Zimbabwe, Mozambique, Democratic Republic of Congo (DRC) and Malawi.

These children came to the site on Saturdays for counselling sessions, extra lessons in mathematics and science. Unfortunately, I did not have formal contacts with their schools. Therefore, in order to be directed to their schools and to be introduced to the school staff, I approached the immigrant learners whom I was assisting with their school mathematics. I explained to the learners that I wanted to visit their school in order to conduct a study that explores how their mathematics teachers create learning opportunities when teaching linear programming. Hammersley and Atkinson (2007) clearly state that in the process of negotiating access it is necessary to tell the participants the purpose of the research. I noticed that they did not want to be identified as foreign nationals, at least not in their schools. This might have been due to the fact that they had witnessed xenophobic attacks. So I had to find alternative research sites.

4.4.1 Alternative research sites

Following the unsuccessful negotiation with the immigrant learners at the site in Mabopane, I decided to adopt an alternative research site. I approached colleagues I met in 2006 while I was studying at the University of the Witwatersrand in Johannesburg. I approached one colleague who is a principal at one of the schools in Kagiso, west of Johannesburg. I met her through a language research group at Wits. She played a major role in assisting me with the location of the schools. We started by identifying areas with a high population of immigrants. The areas happened to be around central Johannesburg neighbourhoods, west of Johannesburg and a village in the Limpopo province which I will refer to as Sitani in this thesis. We then searched for schools which we believed had a high percentage of immigrant learners. I will refer to the schools as School A in Central Johannesburg neighbourhood, School B west of Johannesburg and School C in Sitani, a village in the Limpopo province.

I then applied for permission to conduct research in the Gauteng and Limpopo Department of Education which was granted. I then presented myself to the research sites.

4.4.2 Presenting myself at the empirical fields/ research sites

During the process of negotiating access to the schools, I tried to build positive relationships that would eventually allow me to access mathematics classrooms with immigrant learners. My aim was to foreground certain social categories that were identifiable, relevant and shared by gatekeepers (Harrington, 2003). I thus highlighted three aspects of my identity to the principals, teachers and immigrant learners: I was an immigrant postgraduate student from Swaziland studying at the University of South Africa (UNISA); I have taught secondary mathematics in Swaziland, and thirdly, that I was a researcher. I gave evidence of my status as a researcher and provided official proof of my student status at UNISA.

Gee (2005a) contends that we all have multiple social identities. These identities constitute a valuable resource for building bonds of similarity and attraction (Harrington, 2003: 610). I introduced myself as a teacher and migrant from Swaziland because I wanted to draw similarity and attraction between my concerns of conducting a study with mathematics teachers teaching immigrant learners. When I introduced myself as a researcher I wanted to reassure my participants about the academic value of my investigation.

I also had to decide in what terms I was going to present my research interest with immigrant learners. Based on the research literature aforementioned, I was aware of potential negative attitudes held by South African citizens towards immigrants. I was thus concerned that disclosing my research focus would minimize the possibility of access to the classrooms. However, when requesting that they participate in the study, I stated clearly the focus of my research project in the hope that teachers who held negative views regarding immigrant learners would naturally choose not to participate in the study. I was able to secure entry to classrooms with immigrant learners at the schools and accessed classroom interactions where more than three languages were used by the teachers as well as the immigrant learners.

The table below illustrates the diversity of the teachers in the different schools where data were collected. In section 4.6 I will present information about the contexts in the three settings. In the table below, two teachers are migrants from the DRC and Zimbabwe and one is a South African citizen. The home language of the teacher from the DRC is Lingala while the LoLT in the DRC is French, but now he is teaching mathematics through the medium of English. The home language of the teacher from Zimbabwe is Shona. The teacher from South Africa grew up in the Gauteng province which is known for its multicultural and multilingual nature. Therefore, it was not surprising that he understands the eight out of the eleven official languages in South Africa.

Table 4. 1: Secondary school teachers and classroom related information

Name of school	Location	Teacher (pseudonyms)	Nationality	Languages spoken by the teacher	Languages the teacher understands
School A (private)	Urban	Mr Leo	DRC	English, French and Lingala	isiZulu
School B (public)	Township	Mr Jele	South Africa	English, isiZulu, Setswana, Afrikaans, Sesotho	Siswati, Ndebele, isiXhosa
School C (public)	Rural	Mrs Mpho	Zimbabwe	Shona, English	Ndebele

The three teachers involved in my study are qualified to teach mathematics at secondary level. They all have Bachelor's degree in education majoring in mathematics. At the time of data collection, Leo had taught mathematics for more than ten years in South Africa. Mrs Mpho had taught more than ten years in Zimbabwe and it was her second year teaching mathematics in South Africa. Mr Jele had taught mathematics for eight years. The table above shows some of the languages the teachers speak and understand. It is important to note that the data collection took place without any intervention, and so teachers were doing things their normal way. That is, teachers did things the way they would have done without an outsider in the schools, or in their classrooms. The data consisted of field notes, interviews with teachers, clinical interview with immigrant learners.

In the next section I present the reception I received from the principals of the schools and their mathematics teachers.

4.4.3 Principals of schools and mathematics teacher's interpretation of researcher

My identity as an immigrant from Swaziland, a researcher and a colleague was never challenged at the three schools. According to Harrington (2003), the researcher's claims must be recognised and approved by gatekeepers in order to be granted access. As Harrington points out, participants and, in my case, principals have "power" because they are not "passive recipients of a researcher's impression management strategies, but are active in accepting, rejecting, or modifying the researcher's identity claims" (2003: 617). However, gaining access to the schools and classrooms did not guarantee access to immigrant learners. Hammersley and Atkinson (2007) affirm that, access is not only a matter of gaining entry into a community or institution, but also a matter of being in a situation where data collection is successful. My focus was on immigrant learners in the mathematics classroom, so I had to negotiate access to them with them as well. I discuss this negotiation below.

4.4.4 Access negotiations with immigrant learners

The mathematics teachers introduced me to immigrant learners in their mathematics classrooms. At School A the mathematics teacher introduced me to immigrant learners from the DRC, Malawi and Zimbabwe. I decided to work with immigrant learners from the DRC. The reason was because they were now exposed to an environment where the language of learning and teaching was English instead of French which was the language used in their home country. Furthermore, the mathematics teacher is from the DRC and he shares two languages with the immigrant learners. The languages are a home language which is Lingala and French the LoLT in the DRC. Studies have shown that mathematics teachers sometimes use the learners' home languages as a resource during the teaching and learning of mathematics (Setati, 2008; Nkambule, Setati & Duma, 2010; Webb & Webb, 2008; Moschkovich, 2007). Since the teacher shares the same home language (Lingala) and French I wished to see how he would create opportunities for the immigrant learners from his country of origin.

At Schools B and C the mathematics teachers introduced me to immigrant learners from Mozambique. However the immigrant learners from both schools did not want to be associated with Mozambique or be seen as foreigners. These learners seemed to view the study as posing a threat to them when seen as foreign nationals. According to Lee (1993: 4) sensitive research is that “which potentially poses a substantial threat to those who are involved or have been involved in it”. Research is also considered sensitive when those studied view the research as somehow frightening (Van Meter, 2000) in terms of their identity. The immigrant learners’ related sensitivity to the study due to unpleasant incidents (Lee, 1993) discussed earlier, and fears of scrutiny and exposure (Payne, Dingwall, and Carter 1980) of their identity. These learners had witnessed the xenophobic attacks and they identified themselves as South Africans as shown in the excerpts below, in which Bheki and Allen (not their real names) raises the issue of South African identity documents:

Bheki: Why do you want to interview us? Is it not going to interfere with my identity document? I was born in South Africa and my mother is South African. (Informal interview, April, 2010)

Allen: I was born in South Africa but my mother is from Mozambique and my father from Angola. Is this not going to change my identity document? (Informal interview, October, 2011).

The concern raised by Bheki and Allen above shows that they viewed the study as sensitive and were not comfortable that their teacher had introduced them as immigrant learners. So I had to persuade them that this was not a problem and that even if they were South African citizens, I would still wish to work with them. They accepted formally by indicating in a consent form that they would participate in the study. Briefly, negotiations with principals and teachers gave me access and entry to mathematics classrooms with immigrant learners, but did not provide me with access to immigrant learners.

4.4.5 Presentation of the research topic

I presented my research topic in broad terms, saying that I wanted to observe Grade 11 linear programming lessons. I mentioned to the teachers that my aim was to investigate how teachers create opportunities for the participation of immigrant learners when teaching linear

programming. In other words, I wanted to find out what they do in a mathematics classroom with immigrant learners that they would not do if there were no immigrants. Furthermore, I wished to explore how immigrant learners solve a linear programming task presented in English. I gave the principals and mathematics teachers an information letter which in clear language explained the purpose of my research project.

Even though defining a research topic in broad terms is an approach that is contested and appears in previous accounts of access (Hammersley and Atkinson, 2007; Beynon, 1983; Klaas, 2006; Taylor and Bogdan, 1998), a number of scholars report for instance that informed consent from the start of the research is “neither possible nor desirable” (Hammersley and Atkinson, 2007: 57). During access negotiations, they used “neutral topics” (Beynon, 1983: 39), “toned down” the language of their research proposal (Klaas, 2006), or were “truthful, but vague and imprecise” (Taylor and Bogdan, 1998: 33). These scholars argue that for the researcher to be able to disclose the specificity of a research topic, both the researcher and researched would need to share a set of theoretical and political orientations which is unlikely to be the case and thus an unrealistic criterion to meet.

In all the three classrooms, I presented myself in the same way as with the principal, that is, as being an immigrant from Swaziland, a teacher and a researcher. During the two week observations, classroom interactions were conducted in English as well as in learners’ home languages at School B. At School A the teacher switched between English and French languages which are both additional languages to immigrant learners. At School C the teacher used English only. In the next section I present the information about the schools.

4.5 The schools

The three schools are located in central Johannesburg, West of Johannesburg and East of Limpopo (see figure 3.1 page 56). Most people migrate into these areas in search of jobs in the mining companies. The location of the schools represented schools from urban, township to rural areas. Therefore, a mix of village, semi-urban, and town schools were used to get a representative sample of the places with migrants in the country.

4.5.1 School A

4.5.1.1 Location

Central Johannesburg neighbourhood is a community of migrants from all over South Africa as well as the rest of Africa. It has a high density of African migrants (Staats, 2001). It was originally marked as an area for the rich; one where the air was purer due to its position over the smoky mining town of Johannesburg. It turned out to be a multiclass area, attracting migrants from abroad who came to South Africa seeking a new life. During the apartheid era, it was one of the integrated areas in South Africa (<http://www.wikipedia.org>). Nowadays, some of the languages spoken by the people who live in this community include Portuguese, French, English, Gujarati, Mandarin, Chichewa, Arabic, Amharic, Swahili, Tigrinya, Igbo, Kikongo, Lingala (Sunday Times, 20/10/2010) just to mention a few. This suggests that migrant learners may be simultaneously engaged in the multidimensional process of preserving certain cultural practices in terms of language, at the same time adopting alternative language practices from their host country. Such an observation emphasizes the complex, diverse and hybrid nature of dispersion of people and language. It can be noted that language is used to confront the present and the future by developing new practices in order to establish a new sense of place, belonging and role across generations.

School A is located in Central Johannesburg. It is named after one of the greatest scholars in Africa. According to Muyumba (2004), the scholar (Diop) was a mathematician and nuclear physicist. He is one of the founders of African studies (Africology). Africology is defined as the scientific study of Africa and African people and their cultures throughout the world. The African scholar worked mainly on the history of Africa and Egypt. He was determined to challenge the history and culture of Africans by classical western studies as well as by the explorers and colonisers through systemic study analysis. For example, he argued that Egypt is a classical African civilisation (Kilele, 2001). Some of his arguments are based on “evidence of physical anthropology which reveal a definite African of Egypt” (Diop, 1989: 9 cited in Muyumba, 2004). Furthermore, Diop presented linguistic evidence that proves that Egyptians identified themselves as “*kmtyw or Kemetiu*” meaning black people. He points to the linguistic affinity of ancient Egyptians with the Wolof language spoken in Senegal and argues that lexical

items are shared with ancient Egyptian or the language of the Nile. Surely this school has something special for the African people.

School A is a private school situated in an environment where the population is mainly immigrants from all over Africa. It was established in 1994 and the buildings look quite old. It has two floors, the ground floor is occupied by Grade eight and nine classrooms, the first floor is occupied by Grade ten and eleven classrooms and the second floor is occupied by Grade twelve classrooms. The administration block is on the first floor. According to the director of the school, School A was established to cater for all learners from all over Africa as well as to educate humanity. At the time of data collection, there were twenty three teachers in this school, a secretary and a security officer. Six teachers out of the twenty three are originally from the DRC and can speak English and French fluently. Three teachers are from Ghana and can also speak French, four from Malawi, five from Zimbabwe and five are South African citizens. The director of the school is a Congolese as well as the security officer who monitors the coming in and going out of the learners during school hours.

French, the second highest spoken additional language in Africa (La Francophone dam le Monde, 2007), is offered as a subject at this school because the majority of immigrant learners are from French speaking countries. Immigrant learners with a Francophone background learn it as a second language and learn English as a first language. This suggests that immigrant learners from the DRC or Francophone background are exposed to an environment that supports the use of French. The environment in which School A is located does accommodate immigrant learners. In other words being an immigrant is regarded as being ‘normal’ in this school and thus they are not shy to identify themselves as such.

4.5.1.2 Flow of activities

School A has a student population of about eight hundred learners. Punctuality is enforced, as can be seen by the security that locks the gate by 8 o’clock. According to the director of the school, it has children who are immigrants from many parts of Africa. For example, children from former Portuguese, French and Spanish colonies so speak those languages. In its enrolment, 85% are South African citizens and 15% are learners from all over Africa. The language of

teaching and learning is English, but they offer French lessons as a second language to learners with a French background. The Grade eleven mathematics classroom selected for the study had five immigrant learners -three from the DRC, one from Malawi and one from Zimbabwe. Each period lasted for forty minutes or eighty minutes when there is a double period. On Wednesdays, the time for lessons is shortened by five minutes to allow for sporting activities in the afternoon.

4.5.1.3 Teaching materials

Given the fact that the school has immigrant learners from all over Africa, the perspectives of the teacher concerning teaching materials for learners from French, Spanish and Portuguese background were sought during the pre-observation interview. The teacher mentioned that there is no material prepared for these learners:

Thulie *Are there any materials prepared in French for those from French speaking countries or Portuguese for those coming from Portuguese speaking countries? Any material support provided for these learners?*

Leo *Everything is in English, they have to try and learn English. We are teaching in English but for me mathematics sometimes I switch to French for those who understands little English (November, 2011)*

During classroom observations the teacher had enough handouts for all learners. The handouts were all in English, with linear programming tasks for learners to solve. The teacher mentioned that there are facilities like a photocopying machine.

4.5.2 School B

4.5.2.1 Location

The second school is located in the industrial hub of Western Gauteng Province. There is a lot of migration into the area due to the presence of mining industries there. The mining industry has formed an integral part in the development of the town. Minerals such as gold, manganese, iron asbestos and lime are extracted. Multilingualism is spread as there are many languages spoken by the people such as Xitsonga, Setswana, Southern Sotho, Northern Sotho, isiXhosa, and Tshivenda.

School B is located in a township environment in the west of Johannesburg, which I will refer to as Ekuthuleni, in the Western part of the Gauteng Province. It was established in 2005. The buildings are well designed; the layout of classrooms and the administration blocks are well planned. The township where learners reside is poverty-stricken, as seen from the corrugated shacks and Reconstruction and Development Programme (RDP) houses surrounding the school. Learners at this school come from mainly the surrounding areas. Some of the learners are immigrants from countries like Malawi, Mozambique and Zimbabwe just to mention a few. The mathematics classroom that I observed had two immigrant learners from Mozambique.

School B is one of the few townships schools in Gauteng recognized for great academic performance. Since its inception, School B has shown great achievements both in academic and in extra curriculum activities. Its achievement is evident in its Grade twelve results which have been outstanding and according to the Principal its success depends on a number of strategies. One of the strategies includes teachers attending workshops which helped them understand the outcome-based education (OBE) curriculum. Furthermore, learners are supported in the form of study classes and extra lessons during the school terms and holidays. In 2008 the Grade twelve pass rate was 100% and this was the school's first examination. In 2009 the pass rate was 96% and in 2010 the pass rate was 99%. In extra curriculum activities like music competition, the school has also won a number of trophies which are displayed in the Principal's office.

As far as cleanliness goes, the inside of the school looks well kempt due to regular clean-up by learners as well as grounds men. Furthermore, the classrooms are in a good state from inside and outside.

4.5.2.2 Flow of School Activities

School B has a student population of more than nine hundred. Punctuality in the morning is enforced, as can be seen by the security at the gate. School activities on Monday and Friday start with assembly where various activities such as singing religious songs by learners, prayer and announcements by the principal or deputy principal of the school. Lessons commence soon after assembly. Each period lasts forty five minutes or ninety minutes when it is a double period. On Wednesdays, the time for lessons is shortened to allow for sporting activities in the afternoon.

4.5.2.3 Teaching materials

The perspectives of the teacher concerning the school resources were sought through informal talks and during the pre-observation interview. The teacher was happy with the general resources, considering that facilities like photocopying were in good working condition. For instance he commented thus: *“Here I think we have enough facilities”...* (Interviewed 24 February, 2011). The head of mathematics department also reiterated this, saying *“it’s ok. It only depends on the teachers, whether they really want to photocopy (handouts) for the students”*. During classroom observations the teacher had handouts with linear programming tasks for learners to solve. He rarely used a textbook. These handouts were always in surplus. In the next section I discuss School C situated at Sitani, a village East of Limpopo province.

4.5.3 School C

4.5.3.1 Location

Sitani is located in the East of Limpopo province which is found in the Northern part of South Africa. Tourism and wildlife play a dominant role in this area. This is due to the Palabora Mining Company (PMC), which has developed over the years into a tourist centre. PMC's open cast mine is Africa's widest man-made hole at almost 2,000 meters wide. Furthermore, there is Kruger National Park which is one of the largest game reserves in Africa.

Sitani is one of the poorest areas in South Africa and despite the altered political circumstances the discrepancy between poor and wealthy people has not changed. There are approximately 50000 inhabitants of Sitani. They are exclusively dependent on working in the surrounding mines and farms. The mines will probably close down within the next twenty years, leaving many of these people unemployed. It was observed that many people live under poor circumstances and cannot escape the vicious cycle of poverty and unemployment. However, there are some development projects, like those aimed at promoting tourism. This is to shift the community focus away from a mining area to an area that thrives on tourism (<http://www.pafound.co.za>).

The buildings at School C are well kempt; the layout of classrooms and the administration blocks are well planned. Learners at this school come from the surrounding areas and some from distant

places. Some of the learners are immigrants from Mozambique. Learners do not want to be identified as foreign as seen in the extract below:

Teacher: Some of the learners are from Mozambique because... (interrupted)

Learners: No we are not maam, we are South Africans...

(classroom observation October, 2011)

The mathematics classroom that I observed had two immigrant learners from Mozambique. The mathematics classroom seemed overcrowded with fifty two learners. The inside of the school looked clean. Furthermore, the classrooms are in a good state from inside and all learners had a desk and a table.

4.5.3.2 Flow of School Activities

School C has a student population of more than eight hundred. Some of them travel long distances to school and as a result they are late for the first lessons. During the week of data collection, I observed that some of the learners were standing outside the school gate because the security locks the gate at half past seven in the morning. According to the mathematics teacher, the first lesson always delays because of learners coming late.

School activities on Monday and Friday start with assembly where various activities take place such as singing religious songs by learners, prayer and announcements by the acting principal or deputy principal. Lessons commence soon after assembly. Each period lasted for forty five minutes or ninety minutes when it is a double period. On Wednesdays, like in the other schools, the time for lessons is shortened by five minutes to allow for sporting activities in the afternoon.

4.5.3.3 Teaching materials

Facilities like photocopying were available, however most learners did not have mathematics textbooks. The teacher was happy with the general resources that she can use to support the learners. For instance she commented thus: *“I make copies for learners because most of them do not have their textbooks, and sometimes I use material I get from other mathematics teachers from the surrounding schools”... (Interviewed October, 2011)*. During classroom observations

the teacher had handouts with linear programming tasks learners had to solve. These handouts were in surplus.

The section that follows shows the context in terms of languages learners indicated that they speak, understand and can read and write. The languages were sought through a learner questionnaire issued before classroom observation. The learner questionnaire allows for the exploration of the role of the languages spoken at home, distinguishing between learners mainly speaking a language that is different from the language of assessment, other official languages and information on the specific country where the students or their parents were born. Therefore, the analysis of a learner questionnaire at each site shows the languages represented in the mathematics classroom. However, self reported data isn't always reliable. I have decided to present these languages from the three sites at the same time. The aim is not to compare them but to present the information in one section. I will refer to the mathematics classrooms as Classroom A at School A, Classroom B at School B and Classroom C at School C.

4.6 Languages represented in Classrooms A, B and C

This section presents the home languages, languages learners speak at home, languages learners speak, languages learners understand, and languages learners read and write at school and mathematics classrooms. The languages learners read and write are presented at the same time in Table 4.6. I start by presenting the learners' home languages.

4.6.1 Learners' home languages in Classrooms A, B and C

Linguists argue that all humans are successful, barring any developmental problems or specific disability, in mastering a home language regardless of their social, cultural, economic and political divisions (Halliday, 1994; Gee, 1994). I argue that they already come equipped with a resource and it makes sense not to leave it out of their learning repertoire. In this regard, the table below presents the learners' home languages that were represented in Classrooms A, B and C.

Table 4. 2: Learners home languages in Classrooms A, B and C

Home Languages	Classroom A		Classroom B		Classroom C	
	number	%	Number	%	Number	%
English	3	12	0	0	0	0
Afrikaans	1	4	0	0	0	0
IsiZulu	8	31	14	30	1	2
Sesotho	0	0	4	9	2	4
Setswana	3	12	13	28	0	0
Sepedi	2	8	4	9	2	4
isiXhosa	2	8	9	19	0	0
Xitsonga	1	4	2	4	45	87
Tshivenda	1	4	0	0	0	0
Isiswati	0	0	1	2	2	4
IsiNdebele	0	0	0	0	0	0
Lingala	3	12	0	0	0	0
Chichewa	1	4	0	0	0	0
Shona	1	4	0	0	0	0
French	0	0	0	0	0	0
Portuguese	0	0	0	0	0	0
Total	26		47		52	

The above table shows that there are eleven home languages represented in Classroom A. Eight out of the eleven official languages in South Africa are represented in this classroom. Three of the learners' home languages are not indigenous to South Africa and are home languages of the immigrant learners. These home languages are Lingala, Chichewa and Shona, which are home languages to immigrant learners who migrated from the Democratic Republic of Congo (DRC), Malawi and Zimbabwe respectively.

In Classroom B there are seven home languages, of the learners, out of the eleven official languages of South Africa. The immigrant learners from Mozambique had Xitsonga as their home language which is also one of South Africa's official languages. Furthermore, the data shows that the majority of the learners in this mathematics classroom have isiZulu followed closely by Setswana as their home languages. In Classroom C there are five home languages, of the learners, out of the eleven official languages of South Africa. The majority of the learners in this mathematics classroom (forty five out of fifty two - 87%) had Xitsonga as their home

language. The immigrant learners are from Mozambique and all had Xitsonga as their home language.

When looking at the home languages of the learners in these mathematics classrooms, isiZulu, Sepedi and Xitsonga are common to all three classrooms. While isiNdebele, is not home language to any of the learners in the classrooms observed. English and Afrikaans are home languages of learners in Classroom A. A point worth noting is that learners in Classroom A indicated that they were immigrants while learners at the other schools, that is classroom B and C, did not indicate that they are immigrants. One of the reasons might be that they are second generation immigrants. That is to say, they were born in South Africa with foreign born parents and so don't see themselves as immigrants.

The next Table 4.3 presents the languages these learners speak at home. It is interesting to note that they speak additional languages other than their home languages thus encouraging them to the culture of speaking more than one language, which is common in South Africa. This raises the issue of South Africa being a plural multilingual society. In such a situation children use a range of languages at home and have access to linguistic resources that they draw on as necessary for their meaning-making needs.

4.6.2 Languages learners speak at home in Classrooms A, B, and C

For many learners, the speaking of English is a second language activity. Indeed, in some areas English is spoken primarily at school with most communication outside the school being undertaken in a home language like in the rural environment. Adler (2001) argues that in urban and township settings there are opportunities for the learners to acquire English informally outside the classroom though the degree varies. And, of course, this is not likely to be the English of the mathematics classroom and possibly not much of the more general academic language used in classrooms, so there is also the possibility that any superficial 'mastery' of English might lead teachers to assume the learner is more able to operate with mathematical English than they actually are. Table 4.3 presents the languages learners speak at home. It was interesting to find out during the analysis that English is one of the languages some learners speak at home as shown in the table. However, there was no qualitative information about what

‘speak English at home’ means, in terms both of quantity, what domains of English, level of formality, who they spoke it with, etc.

Table 4. 3: Languages learners speak at home in Classrooms A, B and C

Languages learners speak at home	Classroom A		Classroom B		Classroom C	
	Number	%	Number	%	Number	%
English	18	69	7	15	12	23
Afrikaans	3	12	0	0	1	2
IsiZulu	10	38	22	47	2	4
Sesotho	0	0	10	21	5	10
Setswana	4	15	18	38	0	0
Sepedi	2	8	5	11	4	8
isiXhosa	2	8	7	15	1	2
Xitsonga	1	4	4	9	51	98
Tshivenda	1	4	1	2	1	2
Isiswati	0	0	3	6	2	4
IsiNdebele	0	0	3	6	0	0
Lingala	3	12	0	0	0	0
Chichewa	1	4	0	0	0	0
Shona	1	4	0	0	0	0
French	3	12	0	0	0	0
Portuguese	0	0	2	4	3	6

The number of learners who speak the languages at home is more than those who had indicated that these languages were their home languages. This suggests that learners were not expected to give a single response but they would indicate all the languages they speak at home. This showed that learners shared fluency in the speaking of the languages. That is to say even though their home language was Xitsonga; they spoke Portuguese at home, as the Mozambicans were so heavily colonised that Portuguese was spoken at home by uneducated people. Alternatively, even though their home language was Lingala, they were able to speak other languages like French. It was therefore not surprising to hear learners speaking other languages even though it was not their home language during lesson observations.

In Classroom A, English is spoken by 69% of the learners at home. The immigrant learners from the DRC indicated that they speak French, Lingala and English. Immigrant learners from Malawi

indicated that they speak English and Chichewa, while those from Zimbabwe speak Shona and English at their homes. This is because Zimbabwe and Malawi were once British colonies and English was the colonial master's language.

In Classroom B, two of the learners who had isiXhosa as their home language indicated that they did not speak it at home but instead spoke English and another African language which is isiZulu. This is not surprising since an analysis by Setati (2008) indicated that parents, teachers and learners positioned themselves with regard to the political role of English and other politically favorable languages such as isiZulu. IsiZulu is the language most frequently spoken by 47% of the learners at home in Classroom B. Immigrant learners indicated that they spoke Portuguese, isiZulu and Xitsonga at home. In Classroom C, 98% of the learners speak Xitsonga and one, an immigrant learner speaks Portuguese at home. Twelve of the learners indicated that they speak English at home. Some of the learners code-switch at home and speak a mix of two or more languages. I can claim that the disparity between home languages and languages spoken at home sometimes is due to marriages between people from different societies like the case of Bheki and allens parents.

The next data will look at languages understood by the learners in the mathematics classroom.

4.6.3 Languages learners understand in Classroom A, B and C

The degree of understanding a language varies. Some learners may understand a language but cannot speak it. Others' might have minimal understanding especially when they have learned English informally outside school. Therefore, the table presents languages that learners indicated that they understand. However, the degree of their understanding of the languages was not established.

Table 4. 4: Languages learners understand in Classroom A, B and C

Languages learners understand	Classroom A		Classroom B		Classroom C	
	Number	%	Number	%	Number	%
English	26	100	47	100	52	100
Afrikaans	2	8	2	4	6	12
IsiZulu	17	65	36	77	27	52
Sesotho	8	31	22	47	29	56
Setswana	7	27	36	77	6	12
Sepedi	6	23	18	38	13	25
IsiXhosa	12	46	12	26	3	6
Xitsonga	2	8	6	13	52	100
Tshivenda	1	4	1	2	7	13
Isiswati	14	54	15	32	13	25
Isindebele	13	50	9	19	2	4
Lingala	3	12	0	0	0	0
Chichewa	1	4	0	0	0	0
Shona	1	4	0	0	0	0
French	3	12	0	0	0	0
Portuguese	0	0	2	4	3	6

Learners in the three Classrooms indicated that they all understand English, which covers a wide range of understanding. In Classroom A, more than 50% of the learners understand isiZulu, isiSwati, isiXhosa and isiNdebele. The immigrant learners indicated that they understood their home languages and English, while the three immigrant learners from DRC understood French as well.

In Classroom B, the most frequently understood African languages are Setswana and isiZulu, which are understood by 77% of the learners including the two immigrant learners. The other learners (33%) indicated that they understand isiXhosa, isiSwati or isiNdebele which means they understand isiZulu even though they did not indicate on the learner questionnaire. Two of the learners understand Afrikaans; these are some of the learners who indicated that English is one of the languages they speak at home.

During the clinical interview, one of the immigrant learners indicated that she understood six out of the eleven official languages of South Africa as well as Portuguese. While the second one understood ten out of the eleven official languages and some little Portuguese. The two immigrant learners in Classroom B have adapted into the culture of the township where it is common to find individuals that understand many languages especially in Gauteng Province.

The next data presents languages spoken by learners.

4.6.4 Languages learners speak in Classroom A, B and C

The languages learners speak are sometimes the ones they are exposed to in their communities where they live. It might not be the languages they speak at home as the table below shows.

Table 4. 5: Languages learners spoke in Classroom A, B and C

Languages learners speak	Classroom A		Classroom B		Classroom C	
	Number	%	Number	%	Number	%
English	19	73	45	96	46	88
Afrikaans	2	8	2	4	0	25
IsiZulu	14	54	31	66	13	21
Sesotho	3	12	16	34	11	2
Setswana	6	23	34	72	1	13
Sepedi	3	12	12	26	7	2
IsiXhosa	3	12	8	17	1	94
Xitsonga	1	4	3	6	49	4
Tshivenda	1	4	1	4	2	13
Isiswati	1	4	5	11	7	0
IsiNdebele	2	8	1	2	0	0
Lingala	1	4	0	0	0	0
Chichewa	1	4	0	0	0	0
Shona	1	4	0	0	0	0
French	3	12	0	0	0	0
Portuguese	0	0	2	4	3	6

During the analysis of the learner questionnaire, I observed that some learners indicated that they could not speak English which indicates a range of proficiency. In Classroom A, there were

seven learners, Classroom B two, and six in Classroom C. The most frequently spoken African language in Classroom A was isiZulu. Two immigrant learners from DRC indicated that they could not speak Lingala, their home language.

In Classroom B, Setswana and isiZulu were the most frequently spoken languages. The immigrant learners indicated that they could also speak isiZulu. This was obvious during a clinical interview with one of them as his utterances were mostly in isiZulu. In Classroom C, Xitsonga was the most frequently spoken African language. The three immigrant learners indicated that they could speak Portuguese.

The next section presents languages learners could read and write

4.6.5 Languages learners read and write in Classrooms A, B and C

Languages learners read and write are presented in one table as mentioned earlier. My assumption is that people learn a given way of reading and writing by participating in the distinctive social and cultural practices of different social and cultural groups, in this case the school. According to Gee (2003) when these groups engage participants to read and write in certain ways they are also taught ways of how to interact, act, talk, know, believe and value in certain ways. But it is likely that they have engaged in a variety of literacy practices in different contexts. For example, learning to read and write for religious contexts can be very different from learning to read and write for a mathematics class and they also imply different ways of thinking and being. This suggest that the amalgam of individual literacies will be individual in each case, even if we can determine major features, such as schooled literacy. However, even here, though the literacy of school mathematics is very different from the literacy of, say, school history. He asserts that “people do not read and write generally, they read and write specific sorts of texts in specific ways and these ways are determined by the values of different social and cultural groups” (Gee, 2003: 5).

Table 4.6 presents the languages learners can read and write in Classrooms A, B and C.

Table 4. 6: Languages learners read and write in Classroom A, B and C

Languages learners read and write	Classroom A		Classroom B		Classroom C	
	Number	%	Number	%	Number	%
English	22	85	47	100	51	98
Afrikaans	6	23	0	0	6	12
IsiZulu	8	31	24	51	8	0
Sesotho	3	12	4	9	1	2
Setswana	3	12	24	51	0	0
Sepedi	3	12	4	9	5	10
IsiXhosa	2	8	8	17	1	2
Xitsonga	1	4	1	2	51	98
Tshivenda	1	4	0	9	0	0
Isiswati	1	4	4	2	3	6
isiNdebele	1	4	1	0	0	0
Lingala	0	0	0	0	0	0
Chichewa	1	4	0	0	0	0
Shona	1	4	0	0	0	0
French	3	12	0	0	0	0
Portuguese	0	0	0	0	3	6

Four learners in Classroom A indicated that they could not read and write English, yet it is the LoLT. The immigrant learners from the DRC indicated that they could read and write English with difficulty but cannot read and write Lingala, their home language. In Classroom B, all learners indicated that they can read and write English even though two of them had indicated that they could not speak it. One immigrant learner indicated that she could not read and write Xitsonga her home language but could read and write isiZulu. The other immigrant learner indicated that he could read and write Xitsonga and isiZulu. During classroom discussions in Classroom B (page 149), most utterances were in isiZulu and English which these learners indicated that they understood, could speak, read and write.

In Classroom C, 98% of the learners indicated that they could read and write English and Xitsonga except for one learner. The three immigrant learners could read and write Portuguese. Allen could read and write Portuguese, isiZulu and Xitsonga.

4.7 Conclusion

The chapter discussed the process of negotiating access to the research sites. The discussion highlighted the challenges encountered while fulfilling the research agenda. The chapter has shown that the relationship between the researcher and the teachers is always about partnerships between the researcher and participants. The discussion has highlighted the importance of agreement when conducting a study. In this way, research becomes a collaborative project with the research participants. This meant that working with the teachers included negotiations to ensure that the researcher and teachers became equal partners. In other words, they both belonged to and participated equally in the research process.

The chapter also discussed issues surrounding immigrant learners in South Africa. These issues portray immigrants as criminals and create negative views which result in tension and violence. As a result some immigrant learners especially in the township and rural environment did not want to be identified as being foreigners. The chapter further highlighted the location of the schools where data was collected. In discussing the locations of the schools, an attempt was made to include some of the activities that might attract some of the people to migrate to these areas in search for job opportunities.

The chapter further showed an analysis of the learner questionnaire which quantifies the home languages of the immigrant learners and of local learners. The home languages of immigrant learners in Classroom A are Lingala, Chichewa and Shona which are not indigenous to South Africa. Xitsonga is the home language of immigrant learners at School B and School C. The analysis of the questionnaire further shows languages learners indicated that they speak, languages they speak at home, languages they understand and languages they can read and write. The data is interesting because immigrant learners from the DRC indicated that they could not speak Lingala their home language but they understood it.

The next three Chapters present an analysis of data collected during lesson observation in Classrooms A, B and C. The aim is to explore how teachers responded to language diversity as they created learning opportunities for the participation of immigrant learners in the mathematics classroom. That is to say what did teachers do when immigrant learners were present in the

mathematics classroom that they would not do when absent. In Chapter 5, I start by discussing the analytic framework employed when searching for evidence of opportunities for the participation of immigrant learners during teaching and learning of linear programming. The analytic framework is used in all the chapters that present an analysis of the lesson observation.

CHAPTER 5

TEACHING IN TWO ADDITIONAL LANGUAGES: FOCUS ON CLASSROOM A

5.1 Introduction

In the following chapters, Chapters 5, 6 and 7, I provide an analysis of the specific practices that teachers used to support immigrant learners that they would ordinarily not use if immigrant learners were not present during the teaching and learning of linear programming. The aim was not to map out the full range of practices, but rather to identify those practices which would be appropriate to answer my research questions. Other selections would have produced other readings but in a study such as this one, it is important to be selective. Therefore, the chapters focus on data collected in an urban, township and rural settings during lesson observation in response to the following research questions:

1. How do teachers create opportunities for the participation of immigrant learners when teaching linear programming?
2. How are languages within the immediate environment of immigrant learners used during the teaching and learning of linear programming?

I have selected ‘stanzas’ which I discuss as episodes to provide as evidence of opportunities created by teachers for the learning of linear programming in the three mathematics classroom settings. Selecting ‘stanzas’ does not mean that sections that were not selected in the process of analysing the data were not considered, nor that, they are less important. The ‘stanzas’ selected were judged as most appropriate and relevant in responding to the research questions.

For the purposes of analysing, the transcript was divided according to linear programming content discussed on a particular day and time leading to the format of the transcript and the interpretation of the content of the transcripts (Setati, 2003). Therefore, the division of the ‘stanzas’ occurs when there is a shift in perspective. Such a change allowed me to notice shifts in social interaction between teacher and immigrant learners or local learners and gave me an insight into how the teachers supported immigrant learners when teaching linear programming.

I used Moschkovich's (2002b) situated socio-cultural view when investigating teaching and learning practices by mathematics teachers when creating opportunities for the participation of immigrant learners. To do this I examined how teachers supported immigrant learners during construction of inequalities to find constraints, graphical representation of constraints inequalities and when optimising a linear function. I have chosen to focus on these discourses because these are the key processes necessary for participating in linear programming discourse. Furthermore, the use of other resources which will allow me to understand how they use language during the teaching and learning of linear programming will also be discussed.

The teaching at each site is examined using the seven building tasks (Gee, 2005a, 2011) discussed in the analytic framework developed in Chapter 3 (section 3.6.4). Following each of these analyses is a discussion to identify developments that point to possible discourse models (figured world) that were at play at that time. These discussions, together with the concluding remarks at the end of the analysis of each site are intended to identify the discourse models that were influencing teachers as they taught linear programming in classrooms with immigrant learners. Furthermore, the teacher's language practices during the teaching are discussed. The aim is to find out how teachers responded to diversity and used languages in the immediate environment of immigrant learners as a support during the teaching of linear programming.

The next section presents an analysis of the lesson observation in Classroom A.

5.2 Teaching in Classroom A – urban environment

This section presents a discourse analysis of data collected during lesson observations in Classroom A. The start of the teaching sequence was on a Monday and lessons were observed consecutively for four days. I start by presenting an overview of the lessons that were observed. This is followed by a presentation of the languages used during teaching and learning of linear programming. The aim is to show that on some occasions, the teacher linguistically supported immigrant learners during the teaching and learning of linear programming. Lastly, I present an analysis of the lesson observation according to episodes to show as evidence of the opportunities that were created by teachers for the participation of immigrant learners in the mathematics classroom.

5.3 Overview of the linear programming lessons

Lesson 1

As I entered the mathematics classroom together with the teacher, he observed that there were papers on the floor. So he started by asking learners to pick up the papers that were on the floor. He even mentioned that they must keep their school clean because one day Jesus will come and find their school dirty and they will be left behind. All the learners laughed at what the teacher was saying. There were twenty six learners, ten boys and sixteen girls, in Classroom A. The teacher mentioned that they had already started solving linear programming problems so now they will be given a feasible region where in one example they will have to find the constraint inequalities. In the second example learners will find coordinates of the feasible region when given the constraint inequalities.

The teacher told learners that in order to find constraints, learners have to find equations of lines bounding the feasible region. The teacher introduced two methods of finding the equation of a straight line; the dual intercept method and the gradient intercept method. He referred to these methods as a Congolese method (dual intercept method) and a South African method (gradient intercept method). He started by showing learners how to find the equation of a line using the gradient intercept formula ($y = mx + c$). The teacher mentioned that when using the formula, they must first find the gradient of the line which is denoted by m in the formula and the y -intercept denoted by c . The teacher then asked learners to define gradient without the use of the variables x and y . He asked one learner how he would define gradient if his mother were to ask for the definition of gradient. He wanted learners to think of gradient as rise on the vertical axis over run on the horizontal axis. He did not want the definition which is common in mathematical lessons which is change in y divided by change in x . His argument was that the mother will not understand because she does not know change in y over change in x .

The teacher then showed learners how to use the dual intercept formula to find equations of a straight line in order to write down the constraint inequalities defining a feasible region. He referred to the method as a Congolese method $\frac{x}{a} + \frac{y}{b} = 1$. He asked learners to identify the intercept from the given graph in the task they were solving at that time.

In this lesson, the strategy that the teacher used was showing the learners how to use the above formulae to find equations of the lines bounding the feasible region. As he was writing on the chalkboard, he would ask the learners questions and they responded in a chorus form. After showing the learners both methods, he asked them which method was easy. They all preferred the dual intercept method. The teacher then told learners to use both methods when finding inequalities. Learners were working in pairs and discussing the procedures. The teacher was moving among learners' desks checking their work and he observed that they were solving using one method, the dual intercept formula. As a result, he insisted that they use the gradient intercept formula. The learners' response was that they like Congo because he had referred to the method as the Congolese method. This suggests that the teacher seemed to build cohesion in a context that has polarised learners. The teacher then asked learners to solve another problem ([Appendix M](#)) while he was checking their work. Most learners did not finish and they were supposed to finish the work at home.

Lesson two

The teacher started by going over the homework while learners were checking their work to see if it was correct. Then he read the task below aloud for learners to write down in their exercise books. As he was reading he would introduce French. He mentioned that he does not know how to pronounce the word properly in English but he knows how the word is pronounced in French.

Example 2: A school wants to take learners on an outing to an *aquarium*. At least fifty-five learners must be transported. There are two types of minibuses available. Type A can carry fourteen passengers and type B can carry ten passengers. There are at most three types A buses and at most four type B buses. A maximum of five drivers are available. Let x be the number of type A buses and y be the number of type B buses.

In the sketch, the inequalities are shown.

- a) Write down the inequalities that satisfy the given constraints.
- b) Write down all the feasible solutions to the problem.
- c) If all five drivers must drive and the type B bus uses less petrol than type A bus, how many buses of each type should be used to transport learners in the most economical way?
- d) If it costs R30 per learner to travel on type B and R40 per learner for type A, how much must the school pay the bus company to hire the buses for the outing?

Learners were writing down the task, which indicates that he had not made copies. Then the teacher asked learners to solve the task. He further gave learners linear programming tasks where they were given a feasible region and they were asked to find the constraints ([Appendix M](#)). The learners did not finish solving the tasks so the teacher told them to finish solving the tasks at home. One of the examples was taken from a 2009 senior examination paper for Grade 12. This might be because he wanted to familiarise learners with the type of questions asked for assessment and also to see if they have mastered linear programming content and ready for assessment.

Lesson three

The teacher started by going over the steps involved when solving a linear programming task. He mentioned that in linear programming, they will be given a problem where they are expected to write down the constraints inequalities which are the limitations. He explained the meaning of one limitation in the task he had given learners *availability* in the context of linear programming. According to the teacher the implication of *availability* is less than or equal to and he related the meaning to the limitation of time. He also explained the meaning of *maximum* as something that cannot go beyond the given value. On this day the teacher switched from English to French when explaining the meaning of the words like *availability*, *at least* and *maximum* in relation to the tasks he was discussing with the learners. He then gave learners a task to solve and most of the learners did not finish so they had to do the work as homework.

Learners solved the tasks either individually or in twos or threes. The teacher was checking their work and asking them questions related to what they were doing. He would make comments related to what each learner had done. He emphasised that when constructing inequalities they must make sure that they make sense. For example, learners should not add two quantities that were not possible to combine. He gave an example of adding mangoes and bananas to equal mango-banana which does not make sense. At one point he noted that one learner had written an inequality incorrectly and referred to the way he had written the inequality as *ingozi*. This is an isiZulu word referring to danger. The word was used as a catch-phrase, to grab the learners' attention. Furthermore, it is possible that people who don't speak isiZulu might be familiar with it from road signs. The teacher was interacting with the learners in isiZulu, one of the widely

spoken languages in South Africa. He went on to write the inequality on the board for all learners to see the manner the learner had written the inequality so that they may not repeat the same mistake. The teacher then mentioned that they would be writing a linear programming test. Learners requested that it be an open book test to which he agreed and told them that learning linear programming is about understanding. So if they want to open their books it is okay.

Lesson four

During this lesson the teacher was revising a linear programming test to help them prepare for their monthly test. The teacher started by going over linear programming content and made learners aware how some of them lost marks while attempting the tasks. Learners were correcting their mistakes as the teacher was explaining. Then he showed learners how to solve one problem.

In the next section I present the number of utterances in different languages used by the teacher when teaching linear programming in Classroom A.

5.4 Teacher's language practices in Classroom A

During teaching and learning of linear programming in Classroom A, the teacher used either English only, switched from English to French or used French only during his teaching. On some occasions, the teacher switched from English to isiZulu. Table 5.1 presents the language practices and the frequency (in terms of the teacher's utterances) during teaching and learning of linear programming in Classroom A. I decided to indicate the number of utterances when the teacher switched languages.

Table 5. 1: Teacher’s language practice in Classroom A

Teachers language practices	Text	Frequency (number of teachers utterances)	
English only	Now from here you can represent all the constraints in inequality and indicate the visible region.	Lesson 1	
		Lesson 2	
		Lesson 3	
		Lesson 4	
		Total	
Code switching (English, French and isiZulu)	[<i>Si vous voulez que y-intercepte x soit égale à zéro. Si vous souhaitez que la valeur de x-intercepte vous mettriez y égale à zéro</i>] So if you want y-intercept you make x equal to zero. If you want the value of x -intercept you make y equal zero	Lesson 1	2
		Lesson 2	2
		Lesson 3	-
		Lesson 4	6
		Total	10
French only	[<i>alors ... sens ... hein plus grand que deux Ralto</i>] at least two units of Ralto	Lesson 1	2
		Lesson 2	4
		Lesson 3	3
		Lesson 4	5
		Total	14

The above table shows that the teacher switched from English to French on some occasions. This was to make sure that immigrant learners from a Francophone background were not excluded. He also switched from English to isiZulu on some occasions. This suggests that the teacher owns resources of language that he used during the teaching and learning of linear programming. However, there were no instances of using Lingala during the lessons.

In the next section I present the evidence of opportunities that enabled immigrant learners to participate when linguistically supported by their mathematics teacher during the teaching and learning of linear programming. I start by showing evidence that while the teacher was reading from the handout with the task written in English, he would introduce French so that immigrant learners shared the same meaning of the word as the other learners.

5.5 Analysis of the teaching in Classroom A

Episode 1: Switching from English to French

In the following episode the teacher read aloud the task to the learners. Reading to the learners provided the opportunity to introduce immigrant learners to new genres. All learners, as listeners, were engaged while developing background knowledge and increasing their comprehension skills. While reading, the teacher emphasized that he did not know how to pronounce some English words like *aquarium*, so he incorporated French as shown in the stanza below.

59. Teacher A school wants to take learners on an outing, a school wants to take learners on an outing to an *aquar...aquarium*, eh! How do you pronounce it? I don't know in English *aquarium* or *aquarium*? Because in French we pronounce it as *aquarium* (teacher writes *aquarium* on the chalk board and pronounce *aquarium* in French) in an outing to an *aquarium* full stop. At least fifty five learners must be transported...There are two types of minibuses available...Type A...can carry fourteen passengers ... and type B can carry ten passengers ...There are at most three type A buses ...at most three type B buses ... A maximum of five drivers are available, let x be the number of type A buses full stop, let x be the number of type A buses ... y be the number of type B buses ...

Analysis of Episode 1

Activity

The teacher was reading a linear programming task written in English and immigrant learners as well as local learners were writing down in their exercise books. The teacher introduced French while reading and wrote the word on the chalk board so that all learners including immigrant learners see the spelling of the word which they might not be able to catch while he was reading.

Significance

In this excerpt, the significance of reading and writing is clear in this episode as the teacher made the proper pronunciation of English words significant. This suggested that English is the most highly valued language and proper pronunciation would enable learning of the language and improvement in comprehension skills to aid all learners engage in the task. The teacher was also signaling that not everyone is equally competent in all languages and so lending support to and expressing solidarity with the immigrant learners. Furthermore, the spelling of *aquarium* is the same in both French and English so he might have been trying to impact a strategy of spotting

cognates across languages. On the other hand he might have been simply genuinely stumbling over the word *aquarium* and wrote it on the board to reassure himself and the learners.

Identity

The teacher positions himself as someone who can understand the English word *aquarium* but cannot pronounce it properly in English. In spite of that, he could pronounce it in French. His use of ‘I’ refers to himself and his use of ‘we’ refers to himself and those learners who speak French. So he then positions himself alongside the Francophone learners in the class.

Relationship

The text shows a ‘we’, ‘I’ and a ‘you’, the use of ‘we’ suggested that the teacher is building a relationship with immigrant learners from a francophone background. When he used the first person pronoun ‘I’ to refer to himself, he is acknowledging the multilingual nature of the classroom and positing an expertise with the English speakers. This may be a strategy to encourage them to take on this expert role with their immigrant peers.

Connection

The teacher is building or sustaining connections to immigrant learners who speak French. He is also building and sustaining disconnections with people who speak English.

Politics

In this situation, the proper pronunciation of the English word *aquarium* is relevant. The teacher makes it relevant by positioning himself as not being able to pronounce the word properly in English. The proper pronunciation is connected to success in learning mathematics through the medium of English. The teacher might also be aware that improper pronunciation might confuse learners therefore; he has to present his position in relation to French and English.

Sign system and knowledge

Knowledge of English pronunciation is necessary so that the listeners catch the word and relate it to its meaning.

Discussion

The analysis shows an opportunity for the immigrant learners with a Francophone background as the teacher acknowledges the multilingual nature of the mathematics classroom. This shows that there is an awareness of language differences in this mathematics classroom. In view of that, to engage in the differences between the immigrant learners' languages, the teacher incorporates this reality by introducing French pronunciation since the spelling of the word *aquarium* is the same in French and English. Choosing French pronunciation in an environment where the language of learning and teaching is English created an environment in which French is welcomed and intended for those learners who have been learning through the medium of French. Then it seemed appropriate that the teacher decided to include French, particularly so that immigrant learners who have been learning mathematics through the medium of French are directly involved. Certainly, if there were no learners who had French as their language of learning and teaching before migrating to South Africa, he would not have switched to French. The switch related to the need for immigrant learners to fit in the new environment and fully integrate into their socio-cultural milieu. I argue that this is one way of providing immigrant learners with learning opportunities when reading from a handout. How did he draw on French during his teaching process?

The next episode was selected because it shows how the teacher used French, the language that immigrant learners' understand, as a resource to support them during the teaching and learning of linear programming. The teacher translated some of the phrases when negotiating the symbolic meaning of *availability*, *maximum* and *at least* in French.

Episode 2: Negotiating the meaning of key words in linear programming

The text below shows how the teacher guided immigrant learners to develop the mathematical meanings of key words and phrases in linear programming through the opportunity provided by the tasks presented in English. These meanings will then be represented using inequality signs. Mathematical symbolic representations enable all learners to gain the ability of extracting mathematical processes and effective decision making. In this excerpt, the teacher is supporting immigrant learners' activity by shaping the development of word or phrase meaning by drawing on discursive resources. According to Gee & Clinton (2000), competency in mathematics

demonstrates control over the specialized discourse. The excerpt below shows how the teacher guided all learners including immigrant learners when negotiating the symbolic meaning of the words *availability*, *at least* and *maximum* in order to derive constraints.

79. Teacher There are words that will indicate that this is a constraint...like *availability*, okay? Less than or equal to, greater or equal to. Here they talk about *availability*, what does *availability* mean?
80. Learners Less than or equal to.
81. Teacher *Availability* means I cannot go beyond this number, okay?
82. Learners Yes
83. Teacher As I cannot go beyond this... (Teacher points to 20)...less or equal to and here they say *availability*. Aaah *availability*, ...that this is a constraint when they say *availability*...*Availability* implies less or equal to [*et cela signifie moins de*], therefore will have the constraint here... paint shop can handle the maximum of eight units per month ... [*qui va donner le maximum*] Maximum, it's a constraint ... It means I can't go beyond that one... it means as I am producing x -units of Ralto and y -unit of Quatro, if I take x plus y (teacher writing $x+y$) those are the total unit that I will produce. It means when I will take them into the paint, the paint say I cannot take more than eight per month to produce, do you see that?
84. Learners Yes (chorus)
85. Teacher And they say at least two unit of Ralto must be produced each month, [*ya au moins deux*]_at least two units of Ralto. [*alors ... sens ... hein plus grand que deux Ralto*] This is the first constraint ...they say at least two units, at least two units means what?
86. Learners Greater than (chorus)
87. Teacher Greater than or equal, at least two units of Ralto. ... It means x must be at least two (writing $x \geq 2$ on the chalk board), I cannot produce less than two units for Ralto [*Je ne peux produire moins pour que deux Ralto*]. ...the number of unit of Ralto must be at least third of the number of Quatro, Ja! Which constraint is that? Many people fail that question ... you do not get it? ...They say, look at here, furthermore they say number of unit of Ralto which is x , the number of unit of Ralto which is x must be at least...right? A third of a number of Quatro, a third of y ...

Analysis of Episode 2

Activity

The text indicated that the teacher negotiated the mathematical meaning and symbolic representation of the words *availability*, *at least* and *maximum*. We also see that in the text the teacher code switched English and French, which indicated that he was assisting immigrant learners with a francophone background. It is interesting to note that in utterance 85 he reformulated the English phrase in two different ways in French. The activity suggested an opportunity for immigrant learners to be exposed to meanings in two different languages during teaching and learning of linear programming in mathematics classroom in South Africa.

Significance

The text portrays the meaning of key words like *available*, *maximum*, *at least*, significant when deriving constraints. The introduction of French supported immigrant learners so that they could understand and relate the meaning to the mathematical meaning.

Identity

The text portrayed the teacher as someone who possessed languages as resources which he could use to assist immigrant learners with a francophone background understand the content. Therefore, he provided explanations in French as well as in English.

Relationship

The teacher developed a relationship with his French speaking learners and interprets meanings of key words in French. This assisted immigrant learners to interpret the linear programming task based on their previous experiences. However, it can be argued that the use of '*plus grand que*' might mislead immigrant learners into saying 'greater than' and leaving out the 'equal to' part. This shows the complexity in translation during the teaching and learning process.

Connection

The text indicates that the teacher is connecting the everyday meaning of *available* to the inequality less than or equal to (meaning $x \leq k$). This is evident in utterance 83. This suggests that he is aware that particular words can take a complete differently meaning from those assumed in everyday contexts (Walkerdine, 1988) and can hinder access to the objective understood for this lesson.

Politics

The text showed that the teacher made relevant the mathematical meaning of *available*, *maximum*, *at least* which might hinder access to the meaning sought in the task. The teacher related the situated meaning of these phrases in the context of the task in English and French to enable immigrant learners to understand. We see that in utterance 83 which shows that the teacher reinforced the mathematical meaning as immigrant learners learning in a third language

might not have a clue. Therefore, he provided the mathematical meaning of *availability* in linear programming to ground it in their world.

Sign system and knowledge

The text portrayed the knowledge of mathematical meaning of key words in the task such as *available* as less than or equal to (meaning $x \leq k$), knowledge of *maximum* as less than or equal (meaning $x \leq k$) and knowledge of *at least* as greater than or equal (meaning $x \geq k$) to which learners could use to derive constraints.

Discussion

The analysis showed that the teacher switched between English and French in order to enable immigrant learners to understand the situated meaning of *available*, *at least* and *maximum* which are key words in linear programming. Such an opportunity might lead them to successful constructing of constraint inequalities for the given linear programming task. The analysis further shows that there was a smoother transition from French to English which suggested that immigrant learners were not excluded or seen as having language problems as the teacher was prepared to work with them. He was aware of their francophone background and also able to build on their previous experiences. I would argue that immigrant learners were fortunate to be connecting concepts in both additional languages. The teacher's resources of language became tools with which he could support immigrant learners from a francophone background and the related social goods during the teaching and learning of linear programming. This provided an insight into the role of languages and how immigrant learners might develop understanding under the guidance of a teacher who is able to use languages they learnt from their country of origin.

In the next episode, the teacher further switches from English to French when explaining how to use the dual intercept method when representing a linear function on a graph.

Episode 3: Graphing linear functions

To draw a linear function you need to find three points. The teacher was explaining to learners how to use the dual intercept method when finding the coordinates of the two points. He introduced French as he was explaining to the learners as shown in the following excerpt.

95. Teacher You must always label your axis, labeling the axis is got marks. You don't miss the mark...
96. Learners Yes (chorus)
97. Teacher The line y is equal to mx (meaning $y = mx$) see that? If I wrote y is equal to mx ($y = mx$), generally the equation of a line is equal to mx plus c (meaning $y = mx + c$), right?
98. Learners Yes (chorus)
99. Teacher ...normally C represents...the y intercept...always C represent the y - intercept. It means always when C is not there...it means C equals to what?
100. Learners Zero (chorus)
101. Teacher Zero. It means already your line must pass by the origin zero...zero (teacher writing $(0, 0)$ on the chalk board), every time you see that you have the equation line without a C , your line passes by the origin zero...zero (meaning $(0, 0)$). [*Si vous voulez que y -intercepte x soit égale à zéro. Si vous souhaitez que la valeur de x -intercepte vous mettriez y égale à zéro*] So if you want y -intercept you make x equal to zero. If you want the value of x -intercept you make y equal zero. You know what y wants, then you're gonna look for another point how? Give yourself an x value, make it two or three, then you find the corresponding for the y . And then you're gonna join the point to the origin, and that's how you have your line. Jah, work together. This is called number of units of Ralto (writing on the board x -axis). And number of units of Quatro (writing on the y -axis), This is y and this is x , okay? [*adressons d'abord la question d'un 'scale'... hachurons le dessus et le bas*] Let us first address the issue of a scale... we shade below we shade above... Here, I think our scale if I was to look at a number, our scale I can take two...two (writing $(2; 2)$ on the chalkboard). Our scale can be two...four...six...eight...and ten. On the y two, four, six, eight, ten. Here, I think the first question when y is equal to zero for the first line, I think it's easy. Two x plus three third, three third is what? It's ten over three, right?

Analysis of Episode 3

Activity

The teacher gave instructions and guidance on how to draw a graph in order to represent a linear function. He used French and English when giving these instructions, we see that in utterance 101 line 11, where he did not simply translate but also provided different information in French to what he does in English. He is presumably making a judgment that the francophone learners may not be aware of this convention but that the other learners do.

Significance

He made significant labeling of the axis as well as knowing that the line represented by $y = mx$ passed through the origin. He reminded learners about the importance of labeling the axes and the text was significant when he told learners about marks which definitely they would not like to miss in an examination.

Identity

The teacher used the pronouns like *you*, *your line*, *you see*, *you have*, which suggested that they are accountable as learners and how linear programming is used to evaluate them. He positioned himself as a competent mathematics teacher who possessed the knowledge and someone who knew what will earn those marks in an examination.

Relationship

The relationship developed in this text was that of a teacher giving instructions to learners on what they should do and show when drawing straight line graphs.

Connection

The text made a connection of the line which passed through the origin, $y = mx$, with the general equation of a straight line, $y = mx + c$. We see that in utterance 97.

Politics

The teacher provided privilege of the knowledge of the proper graphical representation of linear functions where the intercept is zero which is in the form of $y = mx$ and not reflecting the constant term denoted by c .

Sign system and knowledge

Knowledge of the equation of a linear function passing through the origin and relating it to the general equation of a straight line is shown by the text. The teacher created an opportunity for immigrant learners by introducing French when teaching about the scale on the axes (utterance 101) and the convention used to define a region.

Discussion

The teacher created an opportunity to enable immigrant learners to understand how to draw straight line graphs using the dual intercept method. He introduced French on some occasions to make sure that immigrant learners understand how to calculate the coordinates of the intercepts. The analysis also showed that the teacher emphasized the importance of the scale on both axes and mentions its significance in an examination.

In the next episode, the teacher introduces learners to the steps to follow when solving a linear programming task. While he was talking, immigrants and the other learners were writing down the steps the teacher was presenting so that they may refer to when given a linear programming task.

Episode 4: Finding constraints given a feasible region

The teacher presented the approach to be followed when solving a linear programming task in a form of steps when solving a linear programming task. He used social language, language aspect of linear programming as shown in the following excerpt:

8. Teacher ...in linear programming...we give you a problem...you are going to...derive constraints...after the constraints you are going graph them ... you are going to draw the graph ...after graphing them you find a feasible region...After finding the feasible region...name the points of the feasible region...Then calculate the solution of the problem ...Then you write the objective function... From there you determine which point gives you the maximum or the minimum value ... If you are given a problem, I told you when, when it's profit you maximise, when it's costs you minimise okay! Okay!
9. Learners Yes (chorus)

Analysis of Episode 4

Activity

The teacher was presenting the steps that all learners in this linear programming lesson had to follow when solving a linear programming task. While presenting the steps, he was using the social languages in linear programming in order to improve fluency in the language which might result in immigrant learners' confidence in their ability.

Significance

The text portrays the appropriate presentation of a solution to a linear programming task significant and he provides learners with the steps involved when solving such a task. The text further suggests that he is encouraging immigrant learners to approach a given linear programming task in the manner he has presented. We see that in his use of prepositions like "...After finding the feasible region...name the points of the feasible region..." which indicates that, for example, name the points in the feasible region after representing the constraints inequalities graphically. If these steps are applied appropriately, immigrant learners will be able to find the correct solution.

Identity

The text shows that the teacher spoke as though he represented a body of knowledge. In utterance 8 he said "... in linear programming...we give you a problem ...".The use of *we* suggested that he is part of the mathematics community that makes rules about how linear programming tasks are dealt with. He has changed the meaning of 'we' here from its use in episode 1. The action evoked might be an expert guide giving step by step instructions to inexperienced followers.

Relationship

In the text we see an introduction of a 'we' and an 'I' representing experts in linear programming and 'you' representing listeners which are learners taking advice from an expert, the teacher. Therefore, the relationship implied by the text is that of the teacher and learners who are to follow the steps presented to them when solving linear programming tasks. We see that in the teacher's repeated use of the phrase "...you are going to..."

Connection

The text showed a connection of the steps all learners had to follow when solving a linear programming task. We see that in his use of "...after constrains ... after graphing... after finding the feasible...then calculate... then you... from there" which suggested an order in which these steps had to be done, thus encouraging all learners to follow a procedure.

Politics

The text indicated the necessary steps to be followed when solving linear programming tasks. The steps are made relevant when the teacher used linear programming social language and spoke from the position of a member of the mathematics community. So if learners followed the steps accordingly they will be successful in solving the linear programming task.

Sign system and knowledge

The teacher improved fluency in linear programming social languages as well as furthered conceptual understanding of the necessary steps learners had to follow when solving linear programming tasks. He carefully developed the social language in linear programming as shown in the teacher's utterance.

Discussion

The analysis of this episode showed that the teacher provided learners with step-by-step instructions when solving linear programming tasks. The teacher presented the knowledge to all learners in such a way that they were not given an opportunity to think of other methods like using the search line when maximizing and minimizing. He also used the social language of linear programming and according to Khisty and Chval (2002) it was through the take-up of conventional language that mathematical ideas were seeded. Consequently, the meanings that all learners including immigrant learners will construct ultimately descend from those captured through the kind of language the teacher used. I can argue that the responsibility for distinguishing between terms and phrases as well as sensitizing immigrant learners to their fine distinction weighs heavily with the teacher. In my view the teacher might greatly influence the linear programming meanings made by learners.

In the next episode, the teacher shares his experience as a teacher who has taught in the DRC as well as in South Africa by introducing two methods in terms of the two nationalities. The two methods of finding an equation of a straight line are the gradient intercept method ($y = mx + c$) and the dual intercept method ($\frac{x}{a} + \frac{y}{b} = 1$). In my view, there seems to be an interesting tension between teaching the students the discipline specific language of linear programming whilst also

maintaining social relations with them. He does this more to express solidarity with the learners' bicultural situation and to present them with an explanation as to why there are these two methods.

Episode 5: Constructing inequalities

26. Teacher I will teach you two techniques, one technique from South Africa and one technique from Congo... (learners laughing) Jah! This one we do not do it here, but I will give you. Then you will choose which one is easier for you. You know that the standard form of an equation of a line is y equal to mx plus c (writing $y = mx + c$ on the chalkboard) which is the general form. Then you change it to the standard form which is ax plus by equal to c (writing $ax + by = c$ on the chalkboard). The first thing is to determine the gradient and the y intercept. What is the y intercept?
27. learners Where the line meets the y -axis
43. Teacher Second method, Congolese method! Congolese method Jah! Look at here Jah! Another method Jah! Okay another method
44. Learners Yes Congolese method (learners laughing)
45. Teacher Now look at here you use the formula which says x over a plus y over b is equal to one,
(writing $\frac{x}{a} + \frac{y}{b} = 1$ on the chalkboard) where a is... a is where a is the x -intercept, b is the y intercept okay? EASY formula Okay now you look at here (teacher points to positions of intercepts on the graph) What is the x -intercept? YES

Analysis of Episode 5

Activity

The teacher gave instructions on how to use the two well known formulae of finding equations of straight lines in order to find constraint inequalities which he referred to as a South African and a Congolese method (see utterance 26).

Significance

In this text, the teacher made the two well known methods significant and he further referred to the methods in terms of the two nationalities; his country of origin as well as some immigrant learners and the country where he taught local and immigrant learners from his country. This suggested that the teacher is creating an environment that privileges one of the nationalities.

Identity

The teacher assumed the role of being a teacher in South African classrooms and he taught his immigrant learners and local learners' techniques he taught in Congo and South Africa. He positioned himself as a teacher who shared knowledge, skills and approaches used in Congo and South Africa. This is evident in utterance 26. Furthermore, the use of the verb 'give' suggest a 'gift' which puts him in the position of one able to bestow gifts and also them as people who should then be grateful for having been given it. It also presents knowledge as something which can be 'handed over', that is transmitted, as opposed to knowledge that is co-constructed.

Relationship

The teacher positioned himself as someone who possesses knowledge and the learners are positioned as recipients. The relationship portrayed by the text is that of a teacher who has experience in teaching mathematics in two countries. Immigrant learners are learning linear programming in South Africa but he doesn't actually push them towards one or other method. He is keeping their options open and legitimizing (potentially) prior knowledge that they may already have of linear programming.

Connection

The teacher connected well known methods of finding equations of straight lines to two nationalities instead of referring to dual intercept method and gradient intercept method. The aim is to make significant by relating mathematical concepts to personal biography. Such may encourage all learners to remember the two techniques since Congo is their country of origin and now they are learning linear programming in South Africa.

Politics

The situation represented in the text indicated that Congo and South Africa are made relevant when finding an equation of a straight line when the teacher referred to the two techniques, dual intercept and gradient intercept method. Congo is also privileged, this is depicted when the teacher said "...this one we do not do it here but I will give you..."

Sign system and knowledge

The text reflected the knowledge of gradient intercept and dual intercept method which learners had to know in order to find equations of straight lines. The teacher provided the privilege of the knowledge and sign systems by claiming that they were techniques from the two countries. This is evident in utterance 26 and suggested that learners had to understand both methods in order to apply when solving linear programming tasks.

Discussion

The analysis of the text showed that the teacher used a mnemonic strategy which is a way of avoiding the complex language of linear programming. Referring to the two methods in terms of the two nationalities instead of dual intercept and gradient intercept method might assist immigrant learners since they were still in the process of learning English. Yet they knew how to apply the formulae. Immigrant learners and the other learners in this mathematics classroom may use these ideas as an opportunity to remember these two approaches. However, the complex language is the content in a very real sense so one would ask, is it good practice to avoid it that way?

In the next Episode, the teacher noted that one learner had written an inequality incorrectly. He interacted with the learners in isiZulu.

Episode 6: Incorrect representation of inequality

In linear programming, inequalities are used to specify the set of numbers a variable can be assigned. Sometimes inequalities can be combined to show a range such that it is the minimum value, the variable and the maximum value like $12 \leq x \leq 20$. In such a representation the inequality sign faces the same direction as the variable in the middle. However, in the stanza below, the teacher noted that one learner had written one inequality incorrectly and he interacted with the learners in isiZulu, one of the most widely spoken languages in South Africa (Statistics South Africa, 2001: 8), when warning learners on how to write inequalities as indicated below:

Teacher's reaction on a learner's inequality circled below

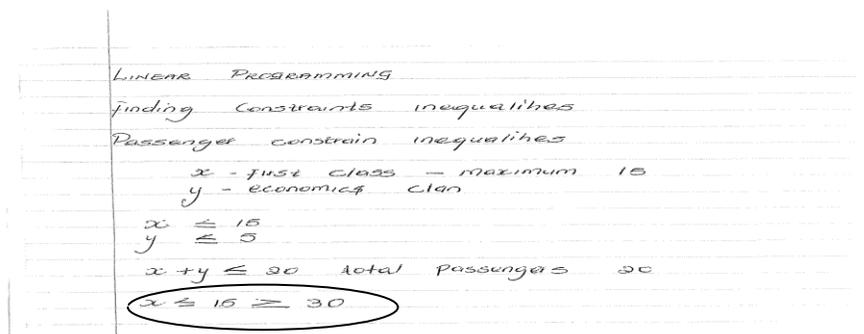


Figure 5. 1: Learner's incorrect inequality

66. Learner Sir, ...(teacher went to check the learners work)
67. Teacher Jah! (looking at the learners inequalities) What! What! What! Come on look at here (teacher moving to the chalkboard) we learn from our mistake (teacher writing inequality on the chalkboard $x \leq 15 \geq 30$) guys what is this? [Ingozi! Ingozi!] danger! Danger! How can YOU say $x \leq 15 \geq 30$ ingozi! [danger] You put them side by side facing each other (teacher using gestures) [ingozi] oh! danger no no, [ingozi!] danger what! what! what! no! no! no [ingozi!] okay guys now carry on (learners continued to write down their inequalities while the teacher checked their work and commenting to what they had written)
68. Learner Sir, here are my inequalities
69. Teacher What? okay look at here, x is the number of people, you are saying x is less than number of kilos, does it make sense? Wrong!
70. Learner Hmm it doesn't make sense (learner realising that the constraint does not make sense)
71. Teacher Wrong, it does not make sense, guys I told you make sure that your constraints make sense

Analysis of Episode 6

Activity

The teacher used isiZulu word to draw the learners' attention and used mainly English to warn learners not to represent inequalities incorrectly. In this case you cannot write inequalities in the manner they had written because there is no meaning conveyed.

Significance

He draws the learners' attention by mentioning that "we learn from our mistakes" and went on to write on the board so that learners could pay attention to what he referred to. He takes the opportunity to teach all learners from the mistake of an individual learner.

Identity

He positions himself as someone who knew how inequalities were supposed to be written and learners had to pay attention so that they do not make the same mistake. The use of 'you' is referring to the learner and asked by someone who knows. What is also interesting is his use of the 'we' to include them as well, so he is positioning himself as an insider and the learners as emerging insiders in this community of practice.

Relationship

The word 'you', referred to the immigrant learner and 'we', to the teacher and all learners. Therefore, the text showed his relationship with the learners as that of a teacher and learners taking advice. He is the lone authority and all the learners are aligned together. This shows the difference from the positioning shown in episode 1 where he aligned with the Congolese learners.

Connection

The text indicated that the teacher connected incorrect representation to dangerous situations which would not lead them to success.

Politics

The teacher made relevant knowledge of correct representation of inequalities to all learners not just to the learner who had made the mistake. We noticed that when he asked learners to pay attention and said "...look at here we learn from our mistake". He takes one individual's mistake and then claims it for everyone, including himself. So here he is illustrating how the community of practice develops in general terms and making mistakes a valid part of the learning process which is a very democratic stance.

Sign system and knowledge

The text indicated that knowledge of proper representation of inequality sign is very important, suggesting a representation that is not valued in mathematics.

Discussion

In this analysis, what was attention-grabbing was the language the teacher used to draw the learners attention to what he was saying. The teacher switched from English to isiZulu. He further proceeded to write the incorrect inequality on the chalkboard so that all the learners in this linear programming lesson could see what he was talking about and reflect upon it. That is to say he did not only interact with the learner who had written the inequality incorrectly, but instead involved all the learners in the classroom i.e. immigrants as well as local learners. By shaping this representation with other learners he allows them to extend their current modes of operating thus creating opportunities that enable them to think mathematically. The teacher drew upon ways of talking familiar to the majority of learners by using languages in their immediate environment. He positions himself as the figure of authority as shown by the relationship he sustained with the learners. The discourse model that the teacher was operating with is that we learn from each other's mistakes. Therefore, it was necessary to make all learners aware so that they do not repeat the same mistake when writing inequalities. It further shows the importance of making errors in order to become a member of this discourse community.

Table 5.2 in the next page presents the discourse models in Classroom A as well as the languages used during teaching and learning of linear programming in the episodes chosen for analysis.

5.6 Teacher's Discourses in Classroom A

Table 5. 2: Teacher's Discourse model in Classroom A

Episodes	Linear programming discourse		Language use			Discourse model Figured world- Success model
	Procedural	Conceptual	English	French	isiZulu	
1		X	X	X		Read so that listeners catch and understand the words
2		X	X	X		Show examples to learners including examination tips
3	X		X	X		Give strategies to learners for drawing straight line graphs
4	X		X			Give precise linear programming information
5	X		X			Devise a way to assist learners remember formulae
6		X	X		X	All learners may learn from each other's mistake

The above table shows that the teacher switched from English to French or from English to isiZulu on some occasions. He interacted with the learners in isiZulu one out of five (20%) episodes and in French three out of five (60%) episodes from the sample I have presented in this chapter. The use of French is to foster conceptual understanding of the content that was under discussion on that particular day and time. For example, the teacher included the meaning of key words like 'at least' in French. The interaction in French encouraged immigrant learners to draw on their prior knowledge (this is shown in Chapter 8 section 8.6.1). However, switching from English to French was not always conceptual as in the discussion of 'aquarium' illustrated in episode 1. He uses code-switching for more than just conceptual development. Knowing the

meaning of key words contributes significantly to finding the correct solution in a given linear programming task. Furthermore, the teacher used several expressions in English which also benefited the other learners who are second language learners.

English is used mainly for procedural discourse and it is noted that it was used two out of five Episodes (40%). In Episode 3, the teacher focused on a procedure of drawing straight lines. He used both English and French, which are additional languages of the immigrant learners.

5.7 Conclusion

The analysis of the teaching of linear programming in Classroom A shows that the teacher switched from English to French and he aligned himself with immigrant learners from the DRC. The immigrant learners were exposed to two additional languages. However, on some occasions he switched from English to isiZulu and it positioned him as the figure of authority that represented the mathematics discipline and knew how inequalities should be represented. The switch from English to French related to conceptual discourse. This implied that the teacher wanted immigrant learners from French speaking countries to understand the linear programming content and processes that were under discussion at that particular time. Linear programming content served as a bridge for crossing languages especially when explaining key words.

The socio-cultural practice related to language use in these linear programming lessons indicated a match between home and school language practices. This approach gave immigrant learners the opportunity for considering their ideas because the teacher created an environment that mirrors their own in order to promote success. This was because the teacher from Congo (DRC) has possession of effective resources to encourage immigrant learners' participation in the mathematics classroom. When the teacher used French he might have developed mathematical thinking by building from immigrant learners' cultural resources. The manner in which he used these languages showed that he linguistically supported the immigrant learners to cater for language diversity in the mathematics classroom. In this classroom, local learners were exposed to an environment where French was used as a resource while learning linear programming.

Furthermore, the use of isiZulu in the ‘ingozi’ example functions as a discourse marker that the learners should pay attention to what’s coming next.

The teacher’s approach in Classroom A is interesting in that French played a major role in his teaching in the presence of immigrant learners from Francophone countries. He switched between English and French which are both additional languages to the learners and have a developed mathematics register. The *aquarium* example discussed in episode 1 may be just random, a word the teacher genuinely struggled with, precisely because it is a cognate in both English and French, or it may be that it is a strategic intervention to demonstrate solidarity as an additional language learner or to open up space for them to express their difficulties in crossing languages. One important question to ask, is whether the use of these two languages disadvantaged local learners while advantaging immigrant learners? This kind of code switching has not previously been reported in the literature about the teaching of mathematics in multilingual classrooms in South Africa.

In the next chapter, I present an analysis of data collected in Classroom B situated in a township environment. The immigrant learners are from Mozambique. One arrived at the age of five while the other is a second generation, born in South Africa with at least one foreign parent. Their home language is Xitsonga and they speak Portuguese at home. I begin by providing an overview of the five lessons that were observed.

CHAPTER 6

TEACHING IN MANY LANGUAGES: FOCUS ON CLASSROOM B

6.1 Introduction

This chapter presents an analysis of data that was collected through video recording in Classroom B. Video recording of lessons was done over a period of five days. As mentioned in chapter 3 (page 59), it was not possible to observe the lessons on five consecutive days. The first day was followed by a sports day and day two followed by a culture day then a weekend. Day three was on a Monday followed by a staff meeting on the next day. Lessons four and five are the only ones that were observed on consecutive days. The reflective interview was conducted three months after the videoing of the lessons. The time gap between the video-recording of the lesson and the teacher interview gave me an opportunity to transcribe all the five lessons and to study the lesson transcripts carefully in preparation for the reflective interview. I watched the video several times identifying critical incidents across all five lessons.

The lesson transcripts were analysed based on the linear programming content the teacher was teaching on that particular day. The aim was to find out whether the teacher made any specific attempt to accommodate immigrant learners. The analysis is divided into two sections. The first part shows the languages the teacher used in the teaching of linear programming. That is to say whether the teacher used English only, home language only or switched between languages. The purpose is to show that the languages that the teacher draws on are the languages the majority of the learners speak, understand, read and write. The second part attempts to identify discourse models that shed light on how teachers present learning opportunities to immigrant learners.

I start by presenting an overview of the lessons that were observed in Classroom B.

6.2 Overview of the linear programming lessons

There were forty seven learners in the mathematics classroom - twenty three boys and twenty four girls, and each had a desk and a chair. There were spaces in between the desk for the teacher to move around when checking the learners' work. The section that follows provides an

overview of what happened in each of the five linear programming lessons observed. During the pre-observation interview, the teacher explained to me that:

“I use handouts most of the time in my lessons. I rarely use a textbook but do advise learners to consult their textbooks for clarification of concepts or as a reference”.

(Pre-observation interview March, 2011).

This was evident during lesson observation and these handouts were always in surplus. The only time he referred learners to a textbook was on day five.

Lesson 1

On this day, the lesson was in the afternoon and it was a double period. This meant that the total time for this lesson was ninety minutes. The teacher began the lesson by informing learners that they would be doing linear programming and I was introduced as a researcher who will observe the lessons about linear programming as he had explained to them earlier. The teacher began by distributing two handouts to the learners. On one handout was an explanation of some of the terminology used in linear programming. In the second handout the teacher had written down the steps learners have to follow when grappling with a linear programming task.

The strategy that the teacher used was to ask one learner to read from the handout to the whole class. He would then explain the key concepts by writing important points on the chalkboard. He emphasized the procedural steps when solving a linear programming task to the whole class. On this day learners were made aware that in linear programming they would be given a statement about a real life problem. As a result the solution to a linear programming task will always be found in the first quadrant in a Cartesian plane. He reminded learners to use the dual intercept method when drawing straight lines in order to represent inequalities. The teacher emphasized to the learners that a solid line was drawn to represent x is greater or equal to k ($x \geq k$) and y is less than or equal to k ($y \leq k$). While a broken line will represent a strict inequality like x is less than k ($x < k$) or x is greater than k ($x > k$).

The introduction was followed by the teacher distributing the third handout with three questions. Learners were expected to make inequalities from the given statements. The teacher asked the learners to solve the problems individually for a few minutes. This was then followed by a

teacher led discussion. During the discussion the teacher and learners switched between English, isiZulu and Setswana. The two immigrant learners were also participating, at one point the teacher asked Lizzy, one of the immigrant learners, to read from one of the handouts while Bheki (another immigrant learner) answered a few questions posed by the teacher. After constructing inequalities the teacher asked learners to finish the task at home by drawing graphs to represent the feasible region. This day was followed by a sports day so lesson two was observed a day after.

Lesson 2

Lesson 2 was in the morning. The teacher started by asking one learner to read the problem they had to finish at home for the whole class. He then asked for a volunteer to work out the problem on the chalkboard. While the learner wrote the solution on the chalkboard, the teacher noted that twenty nine learners did not do their homework. He asked the twenty nine learners to see him in his office at break time. He mentioned that he wanted to teach them a lesson so that they start taking things seriously. He then asked the volunteer to stop writing the solution on the chalkboard.

The teacher distributed one handout with a linear programming task about a patient in a hospital bed who needed certain amounts of proteins, vitamins and iron which they had to solve. He asked one learner to read the first problem two times. After the learner had read the problem he asked one learner to briefly tell the whole class her understanding of the problem. The learner explained her understanding mainly in Setswana with occasional switching between English, Setswana and isiZulu. The teacher asked another learner whose home language is isiZulu to share his understanding of the task to the whole class. These learners communicated their understanding in a language they were comfortable with which was their home languages.

The strategy used during lesson two was mainly a teacher led whole class discussion where the teacher guided learners on how to extract inequalities from a given statement. Once they had the inequalities they were asked to draw the feasible region while the teacher checked their work. The teacher then drew the feasible region on the chalkboard for all the learners to see. While he drew, he was engaging the learners by asking them questions such as “what is the y intercept”?

This activity was followed by the teacher distributing the second handout. The teacher asked one learner to read the statement to the whole class. He mentioned that the information was represented in a form of a table. The teacher asked learners to work individually for a few minutes and make constraints. This was followed by a teacher led discussion. Again learners were to draw the feasible region as homework.

Lesson 3

Lesson 3 was on a Monday. The teacher started by thanking learners who went to church. He also mentioned that others went to watch soccer and some were disappointed by the results of the match. It is worth mentioning that on that weekend the two popular teams with a large following in South Africa, Orlando Pirates and Kaizer Chiefs were playing. He reminded learners of the work they were supposed to do as homework and asked them if they had done it. Learners indicated confidently that they had done their homework. They responded confidently because they had been punished for not finishing their work for lesson two. The teacher asked one learner to read the task to the whole class. He then led a whole class discussion soliciting solutions from the learners. At the same time learners were marking their work. The teacher then asked one learner to sketch the graph showing the feasible region on the chalkboard.

This was followed by the teacher explaining to learners how to find a maximum profit using two methods. He expected learners to listen very carefully and put pens down, meaning they should stop writing and pay attention to what he was saying. The first method involved finding the coordinates of the vertices of the feasible region and substituting the values of x and y in the profit function in order to determine which point gives the greatest value. The second method involves using a search line by making y the subject of the formula in the profit function and use the gradient of the profit function to move over the feasible region using a ruler. He emphasized that learners pay attention to the scale on the graph and adjust the ratio by using their knowledge of equivalent fractions. For example if the gradient was one over two and the scale was in multiple of hundreds then that will mean the gradient will be one hundred over two hundred. The teacher then asked learners to find the maximum profit using a search line with reference to their graphs. He advised them to use the gradient ratio according to their scale on the two axes.

Lesson 4

Lesson four lasted for forty five minutes and activities consisted of a task where learners were given the feasible region and they had to form a table in order to construct inequalities. The teacher started by distributing handouts and then asked one learner to read the task on one hand out to the whole class. The task was about a company which manufactured minibuses. The production of these minibuses involved body work and engine work. The factory had a maximum number of hours to produce these minibuses. When learners were finding constraints, the teacher advised them to draw a table before writing down the inequalities. He guided learners in a form of teacher led discussion in which he occasionally discouraged chorus responses.

A second handout with another task was distributed where learners had to find inequalities represented by the feasible region. He drew a sketch diagram on the board and reminded learners on how to find inequalities when given a feasible region. He mentioned that one method involves using the gradient intercept method. He asked one learner to read the problem to the whole class after which learners were given an opportunity to write down the constraints. He was checking the learners' work as they were working individually. This was then followed by a teacher led discussion. The teacher on this day was evaluating some of the learners' responses. One of the constraints that surprised learners was $500 \leq y \leq 1000$. It was at this time that the teacher requested eight learners whose home language is Xitsonga to see him after the lesson so that I can choose who to work with.

While they solved the questions the teacher noted that they had problems in finding the coordinates of the point of intersection of two graphs. He then reminded them how to solve equations simultaneously using the elimination or substitution method. Again one learner read another problem on the handout where they were given a feasible region. Learners were able to find the constraint that represented the feasible region. Learners were given homework which they were to discuss the following day.

Lesson 5

Lesson five was ninety minutes long. It started with one learner writing solutions of the homework on the chalkboard with the help of other learners while the teacher checked the other learners' work. He then reminded learners how to use a search line to find the maximum point.

The teacher then referred learners to their textbooks where they checked the linear programming content. He then asked them to solve some of the problems on areas where they found constraints. This was followed by an activity where the teacher asked learners questions which they answered individually. Questions such as "what is linear programming? Define a feasible region; Give us a scenario from which we can construct inequalities etc". Learners were very active mainly because a learner who responded correctly was given a pen as a motivation.

The learners' responses to the teacher's questions indicated that they understood the concept. They gave interesting scenarios. However at some point the teacher noted that there were concepts which they had not mastered which he would explain to the learners all over again. From my observation the teacher was hard working and dedicated to his work.

In the next section I present an analysis of the activities during the lessons. These activities show that the teacher and learners were actively involved. Further I present a frequency table showing the languages used by the teacher during the lesson. The analysis showed that the teacher at various times used English only, home language and switched between languages. The aim is to show the language resources the teacher used in his teaching of linear programming.

In the following section I present an analysis of the languages used by the teacher.

6.3 Teachers' language practices in Classroom B

Generally the teacher's use of languages varied between: English only, switching between isiZulu and English; and isiZulu only in all the five lessons observed as presented in the table below.

Table 6. 1: Teacher’s language practices in Classroom B

Teachers language practices	Text (frequency)	Frequency (number of teacher’s utterances)	
English only	T ... greater or equals to (writing $x + y \geq 200$) greater or equals to because they said at least two hundred okay second one second one yes. The second one yes	Lesson 1	61
		Lesson 2	30
		Lesson 3	23
		Lesson 4	25
		Lesson 5	35
		Total	174
Code switching (English, Setswana and isiZulu)	T ...there are other things [<i>kumele</i>] that we must take into consideration number one [<i>ukuthi ihhola</i>] is that the hall cannot accomodate iSwaneville [<i>yonke</i>] all people those are the first constraints...	Lesson 1	57
		Lesson 2	31
		Lesson 3	41
		Lesson 4	37
		Lesson 5	67
		Total	233
Home language only	T ...[<i>kushoda ini?</i>] what is missing T ...[<i>ane!</i>] by the way Jah! [<i>Silungisa lapho angithi...</i>] we are correcting this, is it so... T ...[<i>nani la niyabona ukuthi abantu abaletha imali abafana angithi?</i>] you can all see that we need boys here	Lesson 1	2
		Lesson 2	4
		Lesson 3	3
		Lesson 4	1
		Lesson 5	4
		Total	14

Table 6.1 shows that the most frequent language practice of the teacher was when switching between isiZulu, Setswana and English which accounted for 233 of the occurrences. This is followed by using English only 174 times. The use of isiZulu only was minimal, which was fourteen occurrences.

The majority of the learners in this mathematics classroom indicated that they could speak English, Setswana and isiZulu which suggested that they could use one or the other depending on their preference, which is mainly related to their social context. The languages spoken by the immigrant learners were the languages most of the local learners in the community spoke. In this setting, it was observed that the teacher valued the discourse and language diversity that the learners brought to the classroom. Gee (2005a) argues that identity encompasses teachers’ values and ways of being. At this school, the teacher had a strong sense of his own identity which was also obvious in his way of using languages when teaching linear programming, that of being a

multilingual teacher teaching multilingual learners in a township school. There was no pressure to use English. The manner in which the languages were used in Classroom B was the same as the manner in which language is used in their immediate environment. Even though English is the language of learning and teaching, learners were free to present their ideas in whichever language they felt comfortable with.

In the next section I present an analysis of the data collected during the teaching and learning of linear programming. The first episode presents a linear programming task and how the teacher assisted learners in extracting constraint inequalities.

6.4 Analysis of the teaching in Classroom B

In this episode the teacher had issued a handout with the task presented below. He asked one learner, which happened to be one of the immigrant learners (Lizzy), to read the task aloud to the class.

A party is to be arranged. Let x be the number of girls and y the number of boys that will attend this party. Make inequalities (constraints), if you are given the following information.

There must be at least 200 people at the party. However, the hall can only fit 600 people. We need at least 150 girls and at most 400 boys and the ratio of boys to girls must not be more than 2:1. The organisers must make at least R4 000 on entrance fees. Each boy pays R20 to attend and each girl pays R15.

The task resembled linear programming tasks, which are characterized by the use of a specific *social language* (Gee, 2005a) consisting of phrases like: ‘at least 200 people’, ‘can only fit 600 people’, ‘we need at least 150 girls’, ‘at most 400 boys’, ‘their ratio must not be more than 2: 1’. These particular phrases conveyed specific meaning to the reader which could be represented symbolically via inequalities. Gee (2005) argues that *social languages* trigger *situated meanings* in relation to the activity the social group is enacting and make visible and recognizable their social identity. This suggested that learning linear programming involved understanding the statement and produced meaning in relation to the given task recognizable to those affiliated with the domain.

The teacher asked learners to make sense of the problem before constructing inequalities, in my view he wanted learners to think of a number of issues before writing down the inequalities. While they were thinking he mentioned instances which learners were familiar with like the maximum number of learners that could be accommodated in a classroom. He also mentioned that the community where learners live could not be accommodated in a hall. In my view the teacher contextualized the task in order to motivate the learners and make concepts and operations more meaningful. This further illustrated the usefulness of linear programming and acted as a source of opportunities for applying mathematical reasoning and understanding.

According to Gee (1999) the meanings of some words were tied to learners' experiences, feelings and actions. These experiences were not stored as images but as perceptions of the world. One can therefore argue that there might be no meaning experienced unless one can associate the word or phrase with a person's experiences. However, the context of the learning environment in which the task was presented influenced the way learners might have interacted with the task. In most cases learners judge their teachers by their actions and assign those values to mathematics and learning of mathematics that teachers demonstrated rather than the ones they espouse. The teacher's utterances when guiding the learners is presented in episode 7 below:

Episode 7

81. Teacher Let us try and make sense from the statement you cannot charge the same price for all [*i-price eyi-one angithi abanye bancono kunabanye*] you understand why? This one is a simple scenario ne! Simple scenario someone [*u-inthenda ukuthi abene party but one kuna*] There are other things [*kumele*] take into consideration number one [*ukuthi ihhola*] cannot accommodate iSwaneville [*yonke*] those are the first constraints [*ukuthi*] okay now lets go now you have to organize a party ...a hall must have... [*ihhola kumele ibe nama*] seats that will accommodate people you have invited can you just say no I will invite iSwaneville [*yonke but wazi kahle ukuthi ihhola is like iklassi Jah! iklassi*] cannot accommodate more than sixty learners okay right anyway [*kumele sicale ngama*] constraints. Let's try to draw a table (teacher draws a table on the board) number one we said it's girl and x represents the number of girls and y the number of boys what are the two critical things here? we said there are two critical things go through the statement (name of learner)
82. Learner (reading from handout) A party is to be arranged. Let x be the number of girls and y the number of boys that will attend this party. Make inequalities (constraints), if you are given the following information. There must be at least 200 people at the party. However, the hall can... can... can only fit 600 people. We need at least 150 girls and at least at most 400 boys and the ratio of boys to girls must not be more than 2:1.

Analysis of Episode 7

Activity

The teacher's repetitive use of "you cannot..." suggested that he gave advice which might influence the learners way of thinking on what to consider, what the maximum number of people to be invited is. So the immigrant learners as well as local learners listened carefully until the teacher asked them a question "what is the constraint" before responding to the teacher's advice when making sense of the statement. Learners identified the nature of the activity, and in the process made an effort to figure out and brainstorm which are valued activities in mathematics.

Significance

What is significant in the above text is the teacher's repetitive use of the modal verb "you cannot charge..., you cannot accommodate iSwaneville, ...you cannot accommodate more than sixty learners" which will give direction to the learners' sense making and might influence their identities in relation to the task.

Identity

In the first part of the incident, the teacher used the pronoun "let us try and make sense..." which meant that the teacher and learners were in the same position. But then the teacher's repeatedly use of *you cannot* show learners' social involvement as listeners. Addressing all learners as listeners suggested that they might respond according to his views. He further said "those are the constraints". The teacher positioned himself as someone who knew what can and cannot be included in the solution of the task.

Relationships

Initially, the teacher uses *us* which positioned him and learners as making sense of the task, but then as he continued he then uses *you* suggesting that he was now positioning himself as a teacher and them as learners. He then said "right anyway kumele sicale ngama [*we must first find*] constraints. Let us try to draw a table". At this point, his use of "us" suggested that the teacher was including the learners and assumed that they would think and act as he has advised them.

Connections

The teacher placed the idea within a suitable context, by giving an example of their community and their classroom. All learners in this classroom were familiar with these two situations and might have an understanding of the task situation and appreciate the value, capability and features to apply their understanding when solving the task.

Politics

The mathematical meaning of ‘at least 200 people’ which is a minimum’ and ‘can only fit which is a maximum’ in relation to the task is made relevant in this situation by drawing on a context familiar to the learners. This meaning is connected to successful construction of the correct inequality for the given task.

Sign system and knowledge

Knowledge of the mathematical meaning of ‘can only fit 600 people’ and ‘at least’ in relation to the context is privileged in this situation and the teacher switched between English and isiZulu, the languages most of the learners indicated that they understand.

Discussion

In the above analysis the teacher contextualized the task in order to develop all learners’ understanding of linear programming concepts. Furthermore, the teacher’s utterances showed that he explicitly suggested that, as a mathematics teacher, he knew what could and could not be included in the solution of the task. This indicated that all learners should listen to what the teacher said and note down the relevant points. This was the learning opportunity for all learners in this linear programming lesson.

His actions might influence the learners’ actions towards the task and I argue that he was providing support by linking the task to their experiences. He wanted learners to recognize the social meaning and then associate the text with linear programming. In the process, the learners’ responsibility is to adapt to the mathematics Discourse thus giving priority to agreement rather than comprehension by accepting an established meaning and values of the text in an

unquestioning way. The Discourse model that is emerging is that learners have to adapt to the mathematics Discourse.

In the next episode, I present an analysis to show the opportunity created for the situated meaning of at least one hundred and fifty girls and at most four hundred boys in the context of the task.

Episode 8: The situated meaning of $0 \leq y \leq 400$

The social language that the teacher negotiated with the learners was *y less than or equal to four hundred* in the context of the given task. Generally from a mathematical point of view, the meaning of the phrase “*y is less than or equal to four hundred*” includes all numbers less than four hundred including negative numbers. However in the context of linear programming negative numbers are not included in the solution of the task. Therefore the teacher introduced the symbolic meaning of *at most four hundred* in the given task as $0 \leq y \leq 400$. The teacher’s utterances in relation to the symbolic meaning of *at most four hundred* trigger its situated meaning among learners. The teacher designs his language such that learners realised that negative numbers are not included because *y* represents the number of boys. In short it has to be zero or any number less than four hundred.

149. Teacher The second one sithe kumele ibe x plus y greater or equal to two hundred and x plus y is less or equal to six hundred from the two sentences [*angithi*] the first one [*beyithi*] at least two hundred people to attend the party ne! And then they have x plus y is less than or equal to six hundred at least one hundred and fifty girls that will be x is greater or equal to one hundred and fifty next one y is less than or equal to four hundred now what will happen if I include zero like $0 \leq y \leq 400$ (teacher writing on the chalkboard) ... now if you are saying y is less than four hundred [*omunye umuntu athi*] another person says no! wait [*mani*] let us include zero lets say now the number $u-y$ represent the number of boys lets say okay right lets say the number of boys there must be greater than zero but less than four hundred. Whats the difference between the two? (Teacher pointing at $0 \leq y \leq 400$ and $y \leq 400$) what is the difference between the two? Bear in mind that [*la sikhuluma*] nge y which represents the number of boys [*angithi*] is it?

150. Learners Yes

151. Teacher [*Omunye ethi*] okay since now they said the number of boys must be less than four hundred [*athi*] said the first one will be this one (teacher pointing at y is less than 400) but the other candidate or other learner *athi* no! Even if [*bathi*] they say y is less than four hundred but we need to include zero. Whats the importance of including u -zero in this case? Yes

152. Learner It is because you do not know how many boys will attend so zero will help you to find out [*kuthi bayingakhi bafana*] maybe [*abazo athenda*] between kuzero no four hundred
153. Teacher zero to four hundred another trial yes
154. Learner even if [*kuza umfana oyi-one*] one boy attends it is still okay
155. Teacher it is still correct yes okay if i-y [*yala ngu*] four hundred [*kuphela*] only what ... what but ... what are you saying even if [*kuyi-one uzo attenda ahlangane nalamantombazane lana*] mix with the girls...
156. Learner *isitabane* [he is gay] (learners laughing)

Analysis of Episode 8

Activity

The teacher guided learners and the learner's activity meant that their response should be in line with the teacher's advice. This is shown by the learner's response thus, "even if *kuza umfana oyi one* [even if one boy attend] it is still okay". Another response which showed guidance by the teachers utterance was the use of the word "...*isitabane*" which suggested an insight to the mathematical meaning of *at most four hundred*.

Significance:

The teacher highlighted the importance of including zero in this situation by the use of the expression "bear in mind that *la sikhuluma nge y* [here we are talking about y] which is the number of boys" suggesting that learners should think carefully. This expression emphasized the significance of zero in the inequality and he wanted learners to relate the meaning to the specific context. When he used "...*la sikhuluma*...[here we are talking about y], he included learners in his discussion who took it up as shown in utterance 154 suggesting the significance of zero as suggested by the teacher.

Identities

The teacher stresses the point by saying "if I include zero..." The use of *I* in this utterance referred to himself, then he changed to "*omunye umuntu athi* [someone says] no! Let us include zero" ... The use of *us* referred to the teacher and an unknown person who knows that zero has to be included. The teacher then changed and asked learners "...what is the difference between the two"? These utterances were directed to the learners from a perspective of someone who knows that there is a difference between $0 \leq y \leq 400$ and $y \leq 400$). He was not negotiating with the learners but asking them for the difference. The approach is influenced by his identity of being a

mathematics teacher and he created the contrast in such a way that learners see the difference when he makes reference to the number of *boys* which could only be represented by positive numbers. He positioned himself as someone who knew that the minimum should be zero in this context.

Relationship

The relationship is that of a teacher and learners as further evidenced in utterance 153 which is in response to one of the learner's responses (utterance 152) suggesting that he was seeking an additional input from the learners perhaps a response which closely matched his view. His relationship to the learners was that of someone who had important knowledge to share. The learners' response showed understanding of the symbolic meaning by saying, "even if *kuza umfana oyi*-one [even if one boy attend] it is okay", was probably drawn on the advice of someone who knew even though it was expressed as though it were his own suggestion. The teacher was someone who was to give advice and suggestions while learners had to listen and respond in accordance with the teacher's suggestion.

Connections

When the teacher said "we have to include this zero", this rendered the presence of boys and their number relevant to the situation.

Politics

The number of boys which has to be within the given range is made relevant by the introduction of an unknown person and asking learners for their opinion.

Sign system and knowledge

The learner's use of the adverb "even if" in line 4 is with respect to the given condition that is presence of one boy was welcome. This suggested a common understanding with the teacher who then re-voiced the learner's utterance and utterance 155 triggered one learner to draw on an understanding which draws jokes (utterance 156).

Discussion

The analysis suggested that when the teacher negotiated the symbolic meaning of $0 \leq y \leq 400$ the learners' responses were based on the teacher's advice. Learners were not asked to explain the meanings according to their views. This was suggested in the relationship constructed by the teacher as his identity reflected someone who knew what should be included in the situation. The learners' understanding of the situated meaning of the symbolic meaning is also reflected by the use of jokes like (*isitabane*), which refers to an individual who has the physical features of one sex and the psychological characteristics of the other (Molamu, 2004). Furthermore, the word *isitabane* is used in the popular South African Soap opera, *Generations* South African Broadcasting Corporation (SABC 1). This suggested that during the teaching and learning of linear programming, using languages consisted of sets of practices which include ways of speaking and writing, describing, arguing, explaining and joking. The teacher associated the activities they were doing with language practices common in the immediate environments of these learners during the lessons. This suggested that the teacher is sensitive to the challenges that learners face as they worked to master linear programming.

The language behaviors in the linear programming lesson were typical language behaviors in the township and the teacher created opportunities for such which led to an understanding of the situated meaning of $0 \leq y \leq 400$. It became more evident that the same learners use these languages as a resource in different contexts that is at home and at school. This suggests that learners are not adopting new ways of using languages in order to accomplish new tasks and to interact in new ways but more to the nature of valuing, believing, thinking, in short Discourses (Gee, 2005a) of the community. I argue that this is one way the teacher provided learners with opportunities to use language(s) in their immediate environment during the teaching of linear programming.

The teacher's approach took multilingual behaviour as a resource by allowing multilingual behaviour in the classroom, which lead to an understanding of the concept under discussion at that time. However, separating the different contexts for their language practices and the use of language within the immediate environment as a resource might have not provided immigrant learners with an opportunity to understand the symbolic meaning of $0 \leq y \leq 400$. The Discourse

model that emerged was that the teacher had an important knowledge and skill to share (Down-to-earth model) during the lesson.

In the next episode, the teacher noted that immigrant learners as well as local learners did not finish the task they were supposed to finish at home. There were twenty nine learners out of forty seven who had not done their homework.

Episode 9

In the following extract taken from the transcript of Lesson two, the teacher observed that some learners had not finished the activity from lesson one. In most cases learners were expected to finish their activities at home.

266. Teacher Right lets have one learner [*ozosinikeza*] to give us amaconstraints any volunteer (learner moving towards the board to write down the constraints. While she was writing the teacher noted that some learners had not done their home work) wow wow wow Asenze kube simply bangakhi abangabhalanga? (learners stood up) one, two three four five six seven eight nine ten eleven twelve thirteen fourteen fifteen sixteen seventeen eighteen nineteen twenty twenty one twenty two twenty three twenty four twenty five twenty six twenty seven twenty eight twenty nine, [*bayi*] they are twenty nine okay sit down. (to the learner who was writing the solution on the board) [*hhayi yiyekete*] stop, lets check number four please make sure that ngeshort break niyeza, short break, short break quarter past ten and I will make sure that no one comes to class [*angaka bhali*] short break [*asenze*] let us continue with linear programming ...that will be the best punishment from there you will start taking things seriously lets talk about number four number four ...because of the sports day yesterday [*yiko nibese nidecida ukungangabhali*] ...number four lets do linear programming

Analysis of Episode 9

Activity

The activity that the teacher enacted was that of a teacher who wanted learners to take responsibility for their learning by doing their homework. Learners on the other hand responded accordingly by owning up. They were not pressurized or the teacher did not even check their work individually.

Significance

The significance of the text is that learners should do their homework. It further showed that to discourage learners from the malpractice, the teacher has to discipline them.

Identity

The teacher's use of "please make sure" depicts a person in authority. His use of "I will" suggested that he wanted learners to do their best when as far as linear programming was concerned in order to succeed.

Relationship

The text portrayed a teacher speaking to his learners about the right thing to do in a mathematics class irrespective of other school activities on the previous day.

Connections

The teacher made a connection that if learners did not do their homework that meant they did not see the value of what was intended to be achieved. This is evident in the statement: "from there you will start taking things seriously".

Politics

Doing school work is relevant and connected to success and if homework is not done learners will not succeed in mastering the skills. It is made relevant by instilling discipline to those who fail to do their homework; we see that in the following statement: "that will be the best punishment..."

Sign system and knowledge

In the text, the knowledge that was presented was that of a teacher giving warning to learners to desist from failing to do their homework, which was a sign of not taking (things) their lessons seriously, which is evident in the statement "...from there.....seriously". This form of language is commonly used when learners are given warnings.

The discourse model that is emerging is that finishing the worksheet is a necessary condition for demonstrating understanding of linear programming. The learners' experience was evident on day three, as was shown by the following extract below:

414. Teacher everyone [*ubhalile*] finished writing
415. Learners Yes
416. Teacher [*ngibuza*] I ask for the last time everyone [*ubhalile*] finished writing?
417. Learners Yes
418. Teacher [*Singachekha*] can we check?
419. Learners Chekha (very confidently) check
420. Teacher [*hhayi niright*] you are alright, number four (name of learner) go through the statement everyone has a handout (name of same learner to read question 4) double period ne! yes (name of same learner to read) everyone has a handout (teacher making sure that they all had a handout) name of learner number four

The above extract shows that learners on this day had finished their activity which is expected in the school Discourse. Gee (2005) argues that the values instilled are the ones that guide the social practices of the learners when they grew up.

Discussion

It is worth noting that the above interaction suggested the teaching and learning of identities where learners are encouraged to play decisive roles about their learning of linear programming as evidenced by their taking greater responsibility. These were the qualities which teachers and society aimed to aspire even though not mathematical. They were essential for enhancement of the social basics. Immigrant learners also were not excluded hence this suggested that they were provided with opportunities to learn.

The above analysis showed that learning linear programming was influenced by participation as well as by completing worksheets in school or at home. The belief was that mathematics was not a spectator sport. One ought to learn by 'doing', not just by 'watching' others solve the tasks or discuss their solutions. This discourse was the tool that guided the learning of mathematics. Since the learners engaged in the practices of the mathematics community they began to understand and rely less on the teacher for support. The teacher believed that learners who had not done their home work should be punished so they would not repeat the offence. It is one of

the theories that parents as well as teachers believed in order to make sure that the child or learner in this case completed the tasks at school or at home. Also, immigrant learners were treated the same as local learners hence this enabled them to make sure that they did their work at home. The discourse model that emerged was that mathematics was all about doing your work at school or at home; a sense of responsibility.

Episode 10: Language use is not restricted to using appropriate grammar

The episode below illustrate that the teacher was aware of the different ways that language was used in mathematics lessons as well as how it functioned in linear programming lessons. This gave the teacher an important tool for making linear programming accessible to learners since they are learning it in a second or third language as shown in this episode.

The incident discussed below is in relation to the task presented below.

Question 5 (Appendix C)

5. In a certain week a radio manufacturer makes two types of portable radios, M (mains) and B (battery). Let x be the number of type M and y be the number of type B. In the sketch the shaded area represents the feasible region.

5.1 Write down the constraints to the linear programming problem given:

5.1.1 At most 60 of type M and 100 of type B can be manufactured in a week

The teacher had asked learners to write down the constraints, so he was discussing their responses in a whole class teacher led discussion. In this episode, learners have been expressing inequality sign as “less than”, “greater than”. In the extract below, one immigrant learner (Bheki) decides to use the phrase “smaller than” instead of “less than”.

(Learners response Bheki)

- | | |
|--------------|--|
| 620. Bhek | x is less than or equals to sixty |
| 621. Teacher | x is less than or equals to sixty because bathe at most angithi second one |
| 622. Bheki | y is smaller or ...(learners laughing) |
| 623. Teacher | there is nothing wrong with that y is smaller or equals to... |
| 624. Bheki | y is smaller or equals to one hundred |

625. Teacher y is smaller or equals to one hundred the next one two point one point two (teacher reading from handout) at least eighty radios in total must be produced in a week to cover costs yes

Analysis of Episode 10

Activity

The teacher supported Bheki's participation (see utterance 624) (an immigrant learner) and he further re-voiced his response to the task. Bheki on the other hand shared his idea to the whole class while other learners laughed because of his use of 'smaller than' instead of 'less than'. The teacher took the opportunity to highlight this for the students thus demonstrating the value of their contribution in the learning of linear programming.

Significance

The teachers' utterance of "there is nothing wrong with that" shows the significance of Bheki's contribution and he further re-voiced his response to the task (utterance 623).

Identity

Bheki ignored the learners who laughed at how he verbalised the inequality and continued the way he understood and thought, about the task (624). The teacher as the knowledgeable other supported Bheki (utterance 623). This implied that 'smaller than or equal to' may be used to explain 'less than or equal to' in linear programming even though they have expressed it as 'less than' during the lessons.

Relationship

The text shows a feature of commonality between the teacher and Bheki's idea. This suggested that the teacher put emphasis on the importance of giving someone a chance to share his ideas in order to advance the learners understanding of the concept which would end up advancing the classes understanding as well. This was seen when other learners stopped laughing and tended to listen to Bheki's contribution while focusing on the linear programming concept.

Connection

When learners laughed at the way Bheki expressed his understanding suggested a disconnection to how they had verbalised the inequality. But Bheki connected ‘smaller than’ to ‘less than’ and seemed confident especially because the teacher supported his idea.

Politics

Relevant information in this text is the mathematical meaning of ‘at most’ which can be verbalised as ‘smaller than’ or ‘less than’. The teacher made it relevant by revoicing Bheki’s utterance in (utterance 623).

Sign system and knowledge

Bheki seemed to have recognised that ‘at most one hundred’ means ‘less than one hundred’ even though he used a different phrase ‘that is smaller than one hundred’. The teacher supported his idea and also positioned it as productive for their learning. The knowledge system was that the less than inequality sign may be verbalised as ‘less than’ or ‘smaller than’.

Discussion

For the teacher to cater for a diverse group of learners and the changing demographics it requires an understanding that learners should have opportunities to reason and construct their understanding. This encourages participation because learners bring their experiences to the learning not focusing on what they are lacking because this will result in a discontinuity model (Moschkovich, 1996). Moschkovich (2002b) showed that there is a need for teachers to pay attention to the mathematics and not on how it is expressed by using appropriate grammar. This would result in learners learning the language to express mathematics as well as linear programming concepts simultaneously.

In the above extract, the learner offered his thought and the teacher paid attention to the learner’s engagement. The teacher repeated the offered phrase, and in my view encouraged Bheki to respond to the task the way he wanted. The teacher is not hooked on a particular phrase, like ‘less than’ by overlooking ‘smaller than’ which is an equally valid possibility but the more conventional usage is ‘less than’. This indicated that he wanted learners to focus on the

phenomena being discussed as formal English language is not sufficient when learning the language of linear programming. All learners needed to understand the ways social language was used in linear programming.

This further suggested that if the learner was ready to learn linear programming content, fluency in the language of learning and teaching should not be an obstacle. The analysis suggests that the teacher was also aware of the variation in language ability among learners' background knowledge. That encouraged all learners to participate especially if they understood the language demands inherent in the linear programming tasks. In such an environment learners can participate successfully. Learners therefore used the resources as means for learning linear programming as they received appropriate support from the teacher. The teacher broke the authoritative approach and considered the immigrant learner's experience as a resource. The discourse model that emerged was that attention to mathematics facilitates learning.

However, I argue that the teacher could have used this as a useful teaching opportunity about the conventional nature of sign systems. The mathematics is primary, but that does not make the language a trivial feature (Moschkovich, 2006). As discussed earlier, in some senses the language is the mathematics. Explicit stressing of mathematics language is also important.

In the next episode the teacher presented an opportunity to teach the method of using a search line when finding the maximum point in the feasible region.

Episode 11: Search line method when optimising

In this episode, the teacher introduced the method of optimizing a linear function by finding a point in the feasible region that gave a maximum profit using a search line. When given an objective function ($P = mx + ky$), to draw a search line, the teacher encouraged learners to make y the subject of the formula in order to write the equation of the search line in the gradient intercept form ($y = mx + c$). When given an objective function, P is expressed in terms of x and y which means it is the subject of the formula. So in this incident the teacher makes learners aware that they have to make y the subject of the formula in order to use the gradient when finding the point that gives the maximum profit.

509. Teacher This is your profit line ne! (teacher pointing at the profit function) Its P... [*hhayi asiyekele labo p*] let us stop using p its two hundred x plus two hundred y ($20000x + 10000y$) ngithe the first step make y the subject of the formula angithi. If you make y the subject of the formula it simply means you have to transpose this angithi now it will be ten thousand y equal to p minus twenty thousand x ($10\ 000y = P - 20\ 000x$) ne! the next step divide by what? ...by ten thousand [*angithi*] right now divide nge ten thousand this is what we get (writing $y = -2x + p/10\ 000$ on the chalkboard) ... this one (teacher circling $p/10000$) we are not interested [*ngayo uyangithola*]. The only part esi-interested [*kuyo*] in is the part ye gradient ...bear in mind from Grade nine you know that straight line is given by y equal to mx plus c . ($y=mx+c$). Therefore the coefficient of x is known as a gradient now gradient in this case its negative two over one (writing $-2/1$) siyavuma
510. Learners Yes
511. Teacher Now we have to apply amaequivalent fractions, [*niyakhumbula ama*] you remember equivalent fraction from Grade one [*kuthi*] one over two is the same as four over eight [*niyakhumbula*] you remember? (teacher interrupted by a learner with a document to be signed) Right [*asiye ne! sithi*] half is the same as two over four equals to four over eight and so forth. [*ukusho ukuthi ngamanye amagama*] in other words half is the same as one thousand over two thousand ne! now you take this gradient then now you will be controlled by your scale [*lapha*] here. [*i-scale sakho sicounte ngani ngama*] your scale is in one hundred or two hundred ... right
512. Learners [*Ngama*] it is one hundred

Analysis of Episode 11

Activity

The text illustrated that the teacher taught learners how to use the gradient of a line when using a search line method to find the maximum point in a feasible region in order to calculate the maximum profit when given a profit function. We see that in utterance 509. The use of the second person pronoun “you” in this context suggested that the steps to be followed are directed to the learners by someone who knows what has to be done in relation to the given task.

Significant

The episode shows the significance of the coefficient of x (gradient) in the equation of the profit function when expressed in a standard form of a straight line. This coefficient has to be related to the scale on the axes.

Identity

The teacher used specific language like in utterance 509 which positioned him as someone who knows what learners have to do when solving the task. He further directed the learners’ attention

to ‘making y the subject of the formula’ and ignored P thus emphasizing the use of x and y , we see that in “It is P ... [*hhayi asiyekele labo P*] let us ignore P ”

Relationship

When the teacher uttered the following “we are not interested” suggested that the teacher is including learners in the activity of using a search line because it seems that the use of “we” in this context is referring to the teacher and as well as immigrant learners.

Connections

The teacher made a connection between the gradient in a standard form of an equation of a straight line and the equation of the search line.

Politics

The teacher emphasized the relevance of the gradient when drawing a search line using parallel lines. The gradient can be related to the scale that the learners had chosen using knowledge of equivalent fractions and he said “you will be controlled by your scale”.

Sign system and knowledge

The text showed the teacher’s use of mathematical processes like drawing parallel lines which has equal gradient.

Discussion

The teacher provided learners with support by introducing the idea of gradient when introducing the search line to find the maximum profit. In the process the search line concept is developed linking the profit function to the gradient function in terms of x and y , which learners are familiar with. It was clear from the above utterance that the teacher presented instructions to learners on how to find the maximum using the search line method, but at the same time he wanted learners to practise the skills he had taught them. The discourse model that is emerging is that mathematics is about x and y , learners ought to be careful and be efficient in mathematical behaviour and examination (assessment wise).

In the next episode, the teacher encouraged learners to practise what he was teaching them.

Episode 12

In order for learners to understand messages conveyed to them, the teacher engaged with mathematical language and immigrant learners did not only listen but also participated by using their graphs. This suggested that the quality and nature of linear programming input plays a major role during the process of teaching and learning. A large quantity of input in linear programming would not foster understanding of the content if all learners could not apply the skills learnt. Therefore, pushing learners beyond their current knowledge and understanding, this is learning.

513. Teacher use your scale *la* the way you have calibrated. If you calibrated [*ngama*] in fifty it simply means you have to change your gradient [*ukuthi ihambe*] to be in fifty or according to your scale let us take one example [*wena*] your scale is fifty, one hundred one fifty two hundred and so on... now it simply means you have to change this two [*uyibhale*] write in terms of fifty's and one hundred ...so [*ungayibhala*] you write as two hundred over one hundred [*niyangithola*] you understand?. it simply means vertically you have to start from two hundred horizontally you start one hundred... then after [*ungatshinthisa*] you change the scale you move over the feasible region using a ruler. I repeat you move over a feasible region using a ruler the last point that will touch your ruler in the feasible region is the point that will give you a maximum profit. I repeat everyone [*uzoyenza*], everyone will use i-scale [*sakhe*] whether amaten, hundred... if your gradient [*wena*] your calibration [*yakho asithathe lama*] hundred, its hundred two hundred.... and so forth even vertically its one hundred, two hundred and so forth now there comes your gradient your gradient is two, [*ngithi kuwe*] plot two [*uzomplottha kuphi*] two because half of one hundred is fifty according to your scale, half of fifty is twenty five, half of twenty five is twelve point five... two will be close to zero and you will be tempted to say two is zero... now to avoid that use equivalent fractions to say okay if it is zero (teacher drawing x-axis on the board) gradient negative two [*ngitheni kuwe*] change this (teacher circling 2) [*uyibhale*] write it according to your scale, multiply by ten to get negative twenty over ten... but now [*kunama*] there is hundred which means we multiply by one hundred... my gradient will be negative two hundred over one hundred (meaning -200/100) it simply means my starting point is... I have to go two hundred units down and one hundred to the right... my two hundred on the x-axis (marking a point on the x-axis) one hundred on the y-axis (marking a point on the y-axis) ...[*ngitheni*] take a ruler now move it on the feasible region make sure [*kuthi yini ofuna ukuyichekha la, sithe*] the last point that will touch on the feasible region is the minimum like in that case okay! The first point that will give you a minimum [*yile yile*] (pointing to A)

Analysis of Episode 12

Activity

The text shows evidence of a teacher encouraging learners to draw a search line according to the scale on the axis of the graphs drawn. This implies that learners have to refer to their graphs when attempting this approach.

Significance

The teacher made the calibration significant because he emphasized that each learner used his or her scale when drawing the search line in order to find the maximum point. This allows learners to take full control of their work.

Identity

He positioned himself as a teacher, with learners as listeners. The learners had to follow his advice of how to use the scale shown on their graphs. This approach will position them as active role players in relation to the linear programming activity.

Relationship

The text portrayed the teacher and learners as depicted in "...you have to change your gradient... you can write..." which suggested that they do as they are told by someone who knows how it is done, a 'you' and 'I' relationship.

Connection

The teacher made a connection between the scales shown on the learners' graph that represents a feasible region to equivalent fractions in relation to the gradient of the search lines representing an objective function.

Politics

The relevant 'social good' in this situation is the importance of relating the gradient of the search line to their graphs that they had drawn. This is made relevant when he instructs learners to use their scale on the axes to find equivalent fractions of the gradient the way they have calibrated. Their individual work will then be connected to the gradient of the search line.

Sign system and knowledge

The text shows evidence of knowledge of the gradient of a straight line and how to draw parallel lines so that all learners may be able to optimize a linear programming task. This shows that the teacher is giving learners an opportunity to practice the skills he has taught them for meaningful purposes (Self reliance), since learners have to learn how to use diagrams as part of the construction of mathematical knowledge.

Discussion

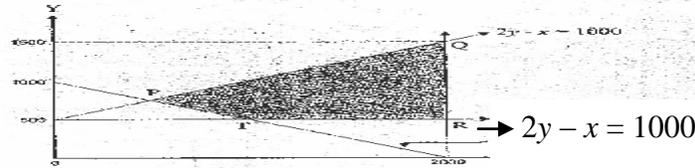
The incident showed a teacher who interacted with his learners in a confident way in order to help them use their graphical representations of the tasks thus building confidence in what they are doing when optimizing a linear function. The analysis showed that the teacher was aware that immigrant learners would not always use the same scale when representing their inequalities geometrically; therefore he had to mention that their scale was equally correct. What is important was that they used it appropriately. This led to an opportunity to understand how to use their scale when optimizing an objective function. The task made learners aware of their own mathematical capability.

Episode 13

In this incident, learners were given a task showing equation of the lines bounding the feasible region. The first question required learners to write down the inequalities. There were two methods of writing inequalities when given the equation of the lines as well as the graph showing the feasible region. One method involved making y the subject of the formula in the given equation. Another method was to calculate the gradient and substitute in the gradient intercept method. It is worth noting that learners solved this example for the first time hence the question asked by the learner in this incident.

5.2

An entrepreneur manufactures two types of furniture pieces: chairs and tables. The costs are R250 per chair and R200 per table. He sells each chair for R300 and each table for R400. He makes x chairs and y tables each month, so that the points (x, y) lie only in the shaded (feasible) region below.



- 2.1 Write down the inequalities which describe the feasible region.
- 2.2 Determine the coordinates of P and T.
- 2.3 Determine the minimum total cost.
- 2.4 Determine the maximum profit.

_____ → $x + 2y = 2000$

- 2.1 Write down the inequalities which describe the feasible region.
- 2.2 Determine the coordinates of P and T.
- 2.3 Determine the minimum total cost.
- 2.4 Determine the maximum profit.

Star Schools (2009: 203)

Figure 5. 2: Linear programming Task

The first question required learners to write down the inequalities that describe the feasible region. Gugu asked the teacher how one can find inequalities when given a feasible region. While she was talking, the majority of the learners said “*amaconstraints* (those are constraints)” to which the teacher requested learners not to respond in that manner as shown in the following extract.

638. Gugu [kulehandout osinikeze yona siwathola njani amainequalities] how do we find inequalities?
639. Learners [amaconstraints] those are constraints
640. Teacher [hhayi ningamlwisi] do not fight here
641. Gugu [ngithathe lama] I took the equation [ngathola lo] I got...
642. Teacher [mamela u-aware ukuthi lento abasinikeze yona ane!] The only thing *la basebenzise* the word inequalities [ngoba bakunikile la but angikukhombise ukuthi sithola kanjani] now [bheka la, bheka la] look here look there is one graph [iphasa ku P uyayibona] that graph passes the y-axis where? [ikhathe kuphi?]
643. Learners [ku] at one thousand
644. Teacher one thousand ne! kux-axis [ikhathe kuphi?] where does it cut the x-axis?
645. Learners [Kutwo] at two thousand

Analysis of Episode 13

Activity

The teacher made sure that all learners were respected and took this opportunity to explain to the class how to write inequalities when given a feasible region and equation of the boundary lines.

Significant

The significance of the text is the learners' question, utterance 638, to which other learners responded in an inappropriate manner according to the teacher (utterance 640). The teacher did not ignore Gugu's question, further he made sure that the other learners listen to what she said. The teacher further became conscious of the learning opportunity the question could create and thus decided to illustrate to the learners (utterance 642). He took the learner's question as a starting point when he explained to the class.

Identities

The teacher positioned Gugu as a capable learner in relation to the other learners in this interaction and advised them to listen to her instead of responding in an inappropriate manner. Further in utterance 642, the teacher showed his personal involvement.

Relationship

The teacher identified that the learner did not understand because of the way the equations of the boundary line were written. The teacher took this as an opportunity to explain to the whole class how to find inequalities when given equations of the boundary lines.

Connections

The text illustrated that the teacher made a connection between obtaining the inequality by first finding the intercepts of each line and relating to the given equations.

Politics

The teacher made relevant the method of changing the subject of the formula to make y the subject in order to find the inequality when given an equation. He emphasised the importance of finding intercepts from the graph and relate to the given equation.

Sign system and knowledge

The knowledge that the text indicated was the approach of finding the inequalities when given a feasible region and equations of the boundary lines. The teacher emphasised that learners change to a general format of gradient intercept method ($y = mx + c$).

Discussion

The episode above showed a teacher who cares about the learning environment. A safe and welcoming environment with minimum fear about learner's questions in linear programming was essential. Learning was enhanced when the teaching occurred in a safe environment rather than in a threatening one. When the environment is threatening, learners may feel inhibited during the teaching and learning process because of unfamiliarity with the other learners in the mathematics classroom. The episode showed that the teacher was vigilant in creating a safe and welcoming environment. When a learner fears being embarrassed, it prevents him or her from making optimal use of the chance availed by the teacher during teaching. This could also lead to them withdrawing from social interaction yet it is critical to learning linear programming content. Therefore, the teacher in Classroom B was conscious of providing all learners a safe and less inhibiting environment.

The episode showed a teacher who interacted with his learners in a confident way in order to help them understand the process of finding inequalities. Furthermore, the analysis showed how a learner's question influenced the teacher-learner interaction. The question guided the classroom activity and this is one opportunity that supported the participation of immigrant learners in this context. The discourse model that emerged was that learning occurred in a safe environment.

The next section presents the teachers discourses in Classroom B.

6.5 Teacher's Discourses in Classroom B

Table 6. 2: Teacher's Discourses in Classroom B

Episodes	Linear programming discourse		Language use			Discourse model (Success model)
	procedural	conceptual	English	Setswana	isiZulu	
7		X	X		X	Learners have to adapt to linear programming discourse
8		X	X		X	Teacher has important knowledge to share
9						Mathematics is learnt by doing –sense of responsibility
10	X	X	X			Attention to mathematics facilitate linear programming discourse
11		X	X		X	Mathematics is about x and y
12	X	X	X		X	
13		X	X		X	Learning occurs in a safe environment

The table shows that the teacher used isiZulu and English to promote conceptual understanding of linear programming content in five out of seven episodes in the selected data for the episodes. This implies that during the teaching and learning of linear programming, the language behaviours are typical language behaviours in the township and the teacher created opportunities for their use. It became more evident that the same learners use these languages as a resource in different contexts at home and at school. However, separating the different contexts for their language practices and the use of language within the immediate environment as a resource might have not provided learners with a learning opportunity.

6.6 Conclusion

The above discussion shows that when learners studied linear programming content, they learnt the mathematical meaning of words, phrases as well as the inequalities of the phrases like ‘at most’ five learners (meaning less or equal to five learners). In the process of learning the content and its mathematical meaning, geometric representation, the languages used by the teacher and learners during the lesson was the kind of languages used in the immediate environment of immigrant learners in this mathematics classroom. Furthermore, in linear programming there is the use of technical terms developed by mathematicians, for example ‘feasible region’, ‘search line’ etc. Immigrant learners needed to communicate mathematically (Pimm, 1991) using these technical terms. The teacher’s Discourse needed to be that of mathematicians while at the same time using everyday languages in order to support immigrant learners. In such an environment, learners are provided with opportunities to ask questions and share their ideas in whole class discussion. It gave them a high quality mathematics experience in a caring environment. I argue that such experiences provide immigrant learners with an opportunity to hear and share enough of the language to use as a resource thus enabling them to succeed.

In the next chapter, I present an analysis of the teaching in Classroom C.

CHAPTER 7

TEACHING IN ENGLISH ONLY: FOCUS ON CLASSROOM C

7.1 Introduction

In this chapter, I present an analysis of data that was collected through video recording and teacher interview in Classroom C. Video recording of lessons was done over four consecutive days. On the last day of the teaching sequence, the lesson was interrupted because the Grade 11 learners' were writing a Geography common test. Furthermore, teachers were to attend a memorial service for a colleague from a neighbouring school. This meant that the lesson lasted for forty five minutes instead of the scheduled ninety minutes.

The classroom observation was analysed based on the linear programming processes under discussion on that particular day. The teacher's approach was mainly to instruct learners on how to solve tasks and learners were given an opportunity to solve the linear programming tasks individually or as a group while the teacher checked and responded to the learners' questions where necessary. As indicated in chapter 4 section 4.6.1, most learners in this classroom have Xitsonga as their home language and they indicated that they were South African citizens. Only three learners were immigrants from Mozambique. One of the immigrant learners indicated that he did not understand English. The purpose of the analysis was to explore the classroom practices as to whether or not they accommodated immigrant learners. In the section that follows, I begin the analysis by providing an overview of those four lessons.

7.2 Overview of the linear programming lessons

The classroom was packed as there were fifty two learners - twenty eight boys and twenty four girls, in this classroom and each learner had a desk and a chair. Even though the class was full, there was space in between the desks for the teacher to move around when checking the learners' work. The section that follows provides an overview of what happened in each of the four linear programming lessons observed.

Lesson one

On this day most learners were not attending the mathematics lesson because they had to attend a disciplinary session as most of them had not attended a study. On this day the teacher told learners that they would be doing linear programming instead of trigonometry which they had done the week before. She started by introducing the researchers who were interested in finding out how they handled linear programming and also mentioned to the learners that she would be teaching them. The teacher started by introducing learners to the definition of linear programming and she explained that they must have background knowledge of linear inequalities. So she started by reminding learners how to represent a linear inequality on a Cartesian plane. She asked learners to draw an example of a linear function. She mentioned that the important thing was to indicate where the line meets the vertical and horizontal axes. She further emphasised that they did not have to form a table of x and y values but to simply find the intercept. While she explained some learners simply copied into their exercise books. She reminded them that they should first understand so they know what they were doing. Learners were given an example of lines as follows; $x = -6$; $y = 4$ and $y = mx + c$ which they were to show on a Cartesian plane.

The teacher then introduced learners to inequalities and emphasised that the symbol used was either less than ($x < k$) or greater than ($x > k$) which was represented by a broken line on the Cartesian plane. Then when it was greater than or equal to (meaning $y \geq mx + c$) or less than or equal to ($y \leq mx + c$) it was represented by a bold line. She went on to tell learners that in order to find the region represented by the inequality they had to choose a point on one side of the line to see if it satisfied the given inequality. The teacher then wrote inequalities on the board so that learners could attempt them as class work. She checked their work and assisted those who had difficulty. Learners were to finish the work at home.

Lesson two

The teacher started the lesson by going over homework and she wrote the solution on the chalkboard for learners to check their work. Then she introduced learners to the concept of 'a feasible region', which was indicated by shading. She mentioned that there are two types of regions; one where the region was bounded and the other, not bounded. The unbounded region

had an infinite number of solutions while the bounded did not. She then wrote questions on the chalkboard so that learners could represent the inequalities graphically to indicate the feasible region by shading. She notified learners that in order to find a feasible region they should shade the unwanted region with a light pencil so that once they had identified the region they would erase and shade the feasible region only. Or they may use a coloured pencil. Learners were able to draw inequalities and identified a feasible region. She noticed that the learners who had missed the first lesson had problems because they could not draw a straight line graph. So she started showing learners how to draw a straight line and choose a point above or below the line to find the region satisfying the inequality in order to show the feasible region. The solution of one of her example had negative values and she emphasised that if there are no negative values in the solution they have to draw the first quadrant only. Another point that she noted was that learners named coordinates as if it was a ratio, like four is to seven. She made learners aware that these were coordinates and not representing a ratio therefore they should use a semicolon when writing them down. Learners also confused equations of x -axis and y -axis so she chose a few points on the x -axis and asked learners to state the coordinates of the points. They observed that the value of the x -coordinates is zero and that is why its equation is x is equal to zero. The same applies to the y -axis - they chose points on the line and noticed that the second coordinate which is the y -coordinate is always equal to zero therefore its equation is y is equal to zero.

Lesson three

On day three the teacher introduced learners to making inequalities from a given statement but before introducing the learners she corrected learners work for the previous day. This was how to show a feasible region on a Cartesian plane. Learners were again reminded on how to represent a feasible region by choosing a point above or below the line. She noted that one learner was confusing the coordinates with each value, that is taking each value at a time and substituting in the inequality for both x and y . And she explained to the learner and emphasised that “we are taking a point which has an x -coordinate and a y -coordinate not a digit”. Then the class moved on to the day’s work which was about finding constraint inequalities when given a statement. The teacher distributed a hand-out with the explanation of the meaning of statements like ‘not less than’, ‘does not exceed’, ‘greater than’ etc ([Appendix P](#)). All the linear programming phrases were explained to the learners and their corresponding mathematical meaning they

represent like ‘not less than’ means ‘greater than’ ($x > k$). Another thing that learners were made aware of was the importance of writing the equation of the x -axis and the y -axis in order to show that the region is in the first quadrant. She emphasised that learners should always show these two inequalities, but they should not expect to be given marks for writing them down. She introduced learners to variables and how they are written as two variables (like in $2x + 3y < 4$) or as one variable (like in $x > 5$). She emphasised that sometimes they may be written in such a way that they are comparing each other like in $y = 2x$, which means y is twice x .

Lesson four

On this day learners were introduced to optimization using a search line. The teacher gave learners a linear programming task which they had to solve and indicate a feasible region. Then she showed them how to draw a search line according to their scale and move a ruler in the region in order to find the last point to touch the ruler. The point will give the value of the maximum. While they solved the second example the geography teacher indicated that they are suppose to write a geography common test together with the other Grade eleven learners. On the following day learners were to write a test on linear programming.

In the next section I present a table to show the language practices of the teacher during the teaching of linear programming at School C.

7.3 Teacher’s language practices in classroom C

During the teaching of linear programming, the teacher used English only. On some occasions she reminded learners of the importance of communicating in English because the language of assessment is English. During the interview, the teacher had mentioned that the school needs a library and should introduce a culture of reading. However, the learners communicated in their home languages when working in groups. The language background of the learners showed that the majority had Xitsonga as their home language. The teacher did not share fluency of this language with the learners.

The table shows his language practices when teaching linear programming in the rural environment.

Table 7. 1: Teacher’s language practices in Classroom C

Teachers language practices	Text (frequency)	Frequency (number of utterances)	
English only	... These are the key words and you are supposed to know them. ‘Greater than’ or ‘equal to’, ‘at least’. If I say I want at least three boys it means I want three or more. Minimum. I want a minimum of four boys means I want four or more. May not be less than, Not less than. If it is not less than it is supposed to be equal or more. So these are the key words we use in linear programming...	Day 1	ALL UTTERANCES WERE IN ENGLISH
		Day 2	
		Day 3	
		Day 4	
Code switching (English, learners home language)		Day 1	0
		Day 2	0
		Day 3	0
		Day 4	0
		Total	0
Home language only		Day 1	0
		Day 2	0
		Day 3	0
		Day 4	0
		Total	0

Table 7.1 shows that the teacher’s utterances were in English only in all the four lessons observed. She indicated that she cannot speak or understand Xitsonga, the home language of the majority of the learners in the mathematics classroom including immigrant learners. Another reason was that the linear programming tasks were in English and therefore, all the linear programming discourse was in English.

In the next section I present episodes to show how the teacher created opportunities for immigrant learners to participate during the teaching and learning of linear programming content.

7.4 Analysis of the teaching in Classroom C

The teacher needed to define linear programming to the learners. The definition emphasised the knowledge of inequalities. Her explanation consisted of the use of the mathematics register like Cartesian plane, inequalities, maximum, minimum, straight line as shown in episode 14.

Episode 14: Definition of linear programming

11. Teacher So let us first write the definition so that when you are doing the topic you know exactly what you are doing. So we are saying, linear programming is a mathematical method that uses linear inequalities to solve problems and these problems involve finding the maximums and the minimums. And now we have to concentrate on this linear inequalities (*teacher underlines linear inequalities*). For you to be able to solve linear programming problems you are supposed to have a background of linear inequalities on how to show inequalities on a Cartesian plane. ...you should know how to draw a straight line on a Cartesian plane. And that one is not for me to show you. You should know how to draw a straight line on a Cartesian plane...

Analysis of episode 14

Activity

The texts indicate that the definition of linear programming triggered the knowledge of linear inequalities and learners were supposed to know how to draw straight line graphs. So she then decided to focus on linear inequalities and their representation “...we have to concentrate on this linear inequalities”

Significance

The text portrayed the definition of linear programming as significant as stated in this utterance “...definition so that when you are doing the topic you know exactly what you are doing...” The word exactly implied that in the definition all the processes are mentioned.

Identity

The text suggested that the teacher was a member of a community of mathematician who knew what linear programming was and she wanted them to write down the definition. This definition will enable them to know the processes involved in linear programming and they will identify themselves with linear programming content.

Relationship

The text indicated that the teacher started by showing a relationship with the learners when she used ‘us’ (line 1 in utterance 11), then she referred to learners by using ‘you’ (line 1 utterance 11). The use of ‘you’ is to refer to learners who must know the definition. When writing the definition she used the pronoun “we” (line 2), which refers to the mathematics community.

Connection

The teacher connected knowledge of representing straight line graphs on a Cartesian plane to representing inequalities graphically. The processes will be used when solving linear programming tasks. We see that in utterance 11, line 2.

Politics

The text makes the definition of linear programming relevant in the situation. In the definition the processes involved and the mathematics register is made explicit to the learners.

Sign system and and knowledge

In the text the knowledge that was presented was the definition of linear programming so that learners knew it and were also challenged to think when writing. However, this definition was presented in a second language only, which some learners might not understand.

Discussion

The teacher felt that learners needed to write down the definition of linear programming so that they knew what they were doing. Writing down the definition of linear programming would give learners the opportunity to read it more than once. When the learners were writing the definition down, they also engaged with the given information.

Prior knowledge had to be elicited especially, how to draw straight line graphs on a Cartesian plane. Learners were not excluded from the discussion because as they drew the graphs of linear functions they were involved in the learning process. This shows the mediating power of the multimodal affordances of the mathematics register, ‘doing’ as ‘thinking’ and ‘learning’.

In the next episode the teacher emphasized the correct use of mathematical language and she was teaching learners correct use of the mathematics language.

Episode 15: Mathematical language

15. Teacher I think you know... what is a Cartesian plane? Jah! I am not talking Greek here ...
(Silence)
17. Teacher ...now a Cartesian plane you draw the y -axis and the x -axis ...now when you are drawing a straight line ... on a Cartesian plane you only need two points, which points? ...you don't need to make a table of values. ...which points do we use to draw a line?
18. Learners The x -value and the y -value
19. Teacher The x -values, we don't say the x -value. Can you use the correct term?
20. Learners The x -intercept and the y -intercept
21. Teacher Yeah, that's good. We use the y -intercept and the x -intercept, is it?
23. Teacher So there is no need for you to make a table for values using a lot of values. Only this two allow you to draw a straight line... y -intercept we are going to find it where? y -intercept we are taking which value? For the y -intercept. We want to find out where our graph, our line cuts the y -axis isn't it?

Analysis of Episode 15

Activity

The activity shown is how to draw a straight line on a Cartesian plane and to use the correct mathematics language when expressing these ideas (see utterance 19).

Significance

The significance in the text was the proper use of mathematical language when referring to x -intercept and y -intercept as shown in the teacher's utterance 21.

Identity

The text indicated a teacher who was a member of a mathematical community, we see that in the use of plural form of the pronoun 'we' to refer to a community of mathematicians. These mathematicians provide influence for the standards to be introduced by teachers during the teaching and learning processes (utterance 19) like the proper use of the mathematical register.

Relationship

The text reflects a ‘we’ and a ‘you’ which suggests a teacher teaching learners how to communicate mathematically. According Pimm (1987: 203), learners needed to “learn how to express themselves like mathematicians”. This suggests that learners need to learn to use the mathematics register in order to express their ideas in an appropriate manner.

Connection

The text indicated a connection between Cartesian plane, y -intercept, x -intercept and a straight line graph in utterance 21. It further made a disconnection with a table of values when drawing a straight line graph as depicted in utterance 23. The importance of the difference is to enable learners choose the best method.

Politics

The text provided the privilege of the knowledge of the dual intercept method when drawing a straight line graph. The teacher makes it relevant by emphasising the correct use of the intercept term. (utterance 19).

Sign system and knowledge

The situation shows knowledge of dual intercept method when drawing straight line graphs as well as the proper use of mathematical language when expressing mathematical ideas.

Discussion

The teacher shaped learners’ mathematical responses when communicating their thinking about how to draw a straight line graph using the best method. Sfard and Kieran (2001: 70) emphasized that “the art of communicating has to be taught”, it requires pushing the talk in mathematically enriching ways. The teacher made a conceptual connection (Kazemi & Franken, 2004) in assisting all learners’ thoughts on the proper use of mathematical language.

Episode 16: Inequalities on a graph

The teacher teaches learners how to represent inequalities graphically using either a solid line or a broken line as shown in the excerpt below:

55. Teacher Showing inequalities on a Cartesian plane...I think you know the general term ... What is the general equation for a linear function?
56. Learners y is equal to mx plus c (meaning $y = mx + c$)
57. Teacher ...now when we are looking at inequalities we are using the symbols...
58. Learners Yes (chorus)
59. Teacher We are not using the symbol equal to (teacher writing $y = mx + c$)...we use... less than or equal to (\leq), then less than ($<$)...we use greater than or equal to (\geq), then greater than ($>$). So these are the four symbols we are going to use for inequalities.
60. Learners Yes (chorus)
61. Teacher Now if you have y is greater than mx plus c ($y > mx + c$), when you are drawing the line you use a dotted line...But when it is y is greater than or equal mx plus c , we use bold lines... (teacher drawing a bold line and a dotted line...silence)
62. Teacher And this dotted line is showing that all the points on the line are not included...(teacher pointing at the dotted line she had drawn)
67. Teacher But if I say x is greater or equal to four ($x \geq 4$)... I am now including ...that is when you are using the bold line ...show that the element on the line are included ... For ($<$) less than and greater than ($>$) we use a broken lines ... we have to draw our Cartesian plane; don't forget to label this Cartesian plane... So that one who will be marking will know which one is your x -axis and which one is your y -axis...
83. Teacher ...and then I will show you how to shade, to shade the unwanted region

Analysis of Episode 16

Activity

The teacher assisted learners to develop skills in order to represent inequalities graphically by drawing a broken line to represent less than (meaning $y < mx + c$) or greater than (meaning $y > mx + c$) or bold line to represent less or equal to (meaning $y \leq mx + c$) or greater or equal to (meaning $y \geq mx + c$) and then shading the unwanted region. In utterances 61, 62, 67, learners are given steps to follow when representing inequalities on a Cartesian plane.

Significance

The teacher made the text significant by stressing the difference in representation between the bold line and broken line instead of just saying you draw a line. Her use of "...I will show you how to shade the unwanted region... you choose any point..." also grabbed the learners' attention to what she said and to do as told.

Identity

The text positioned the teacher as someone who was in control. This was evident when she used the first personal pronoun for instance in utterance 67 and her use of 'we' in utterance 59. This suggests that she represented a body of knowledgeable of people who knew how to represent inequalities geometrically.

Relationship

The teacher's utterances suggest that the learners were taking advice from a knowledgeable person, the teacher.

Connection

The text connected algebraic representation of inequalities to the geometric representation suggesting that learners should have knowledge of these concepts in order to represent a region correctly.

Politics

The social goods represented in the situation are the introduction of a person marking examination scripts and the power that the person has. Therefore the knowledge of proper representation of the Cartesian plane is relevant.

Sign system and knowledge

The text shows the knowledge of the general equation for a linear function, inequalities, drawing Cartesian plane, and labelling the axes as important especially in an examination.

Discussion

The teacher made all learners aware of the steps to follow when representing inequalities geometrically. All learners including the immigrant learners had access to the necessary skills and knew what was expected when attempting a linear programming task and representing it geometrically. She positioned herself as a figure of authority in this linear programming lesson. Learners took instructions from a knowledgeable other. This was one opportunity for encouraging the participation of learners when learning linear programming discourse.

The next episode shows the teacher when guiding learners on how to extract inequalities from a given linear programming task.

Episode 17: Extracting inequalities

In this episode the teacher made all the learners including the immigrant learners aware of the important words to follow when constructing inequalities from a given linear programming task as shown below:

247. Teacher ... we are finding the constraints, making inequalities from given word problems... when you are reading a question, ... we have got some key words which we are suppose to note...and those key words are going to tell you which sign are you going to use. Are you going to use greater than, are you going to use less than, are you going to use greater than or equal to. So for less than ... for this symbol less than ($<$) ...we use the word less than, smaller than if you see a word smaller than it means you are going to use this symbol ...
249. Teacher ... May not reach we use less than. For greater than we have got key words more than, must exceed ... ($>$). Then less than or equal to ... we use ... at most... it means will be the maximum
251. Teacher ... Also the word maximum, may not exceed, not more than. These are the key words and you are supposed to know them. Greater than or equal to, at least if I say I want at least three boys. It means I want three or more, Minimum I want a minimum of four boys it means I want four or more. May not be less than, Not less than. If it is not less than it is suppose to be equal or more. So these are the key word we use in linear programming...
253. Teacher So when you are writing the constrains, so it means you have to follow these words, and we have got this two constrains x is greater or equal to zero and y is greater or equal to zero, they are suppose to be there always....

Analysis of Episode 17

Activity

The activity in this episode indicates that the teacher is explaining the mathematical meaning of key words that learners have to know when constructing inequalities from a given statement in linear programming.

Significance

The teacher made her utterance significant when she told learners to pay attention to some words in a given statement for example when he said "...when you are reading a question, ... we have got some key words which we are suppose to note...and those key words are going to tell you

which sign are you going to use...”. This suggested that knowing the mathematical meaning of key words in linear programming will enable the learner to construct the correct inequalities. She stressed the importance of knowing the meaning of key words by saying “...when you are writing the constraints, so it means you have to follow these words...”

Identity

The identity assumed by the teacher was that of a knowledgeable other who wanted her learners to know how constraints were derived from given statements. Their knowledge of key words will make them succeed in constructing correct inequalities.

Relationship

The text indicated a teacher teaching and directing learners what to look for in a given statement to write down the correct mathematical meaning as depicted in: “for this symbol less than ($<$) ...we use the word less than, smaller than if you see a word smaller than it means you are going to use this symbol ...”

Connection

The teacher made a connection between key words and their mathematical meaning in linear programming in order to construct correct inequalities in a given statement.

Politics

The politics is about mathematical meaning of key words which must be correctly represented as inequalities. The mathematical meaning of key words is made relevant when she mentions that in utterance 249, 251 and learners are encouraged to follow these words (see utterance 253).

Sign system and knowledge

The text indicated knowledge of correct inequality symbol to assign when given key words in a linear programming task.

Discussion

The analysis suggests that the teacher made significant the knowledge of key words in linear programming which she explained to her learners. Her view of knowledge might be that proper explanation might lead to learners understanding the meaning of the concepts in relation to the linear programming tasks. What is relevant is that all learners' recognized certain patterns in their experiences of the world which constituted one of the many situated meanings of words they had come across. It seem that the teacher spotted an opportunity of entry that she can use for moving all learners towards more highly developed and mathematically grounded understanding of key words in linear programming. This suggests an opportunity for success that would allow all learners including immigrants to engage with linear programming productively and make use of appropriate language to support their understanding. They would be able to use notations appropriately when given a linear programming task.

The table below summarises the teacher's discourses in Classroom C.

7.5 Teacher's Discourses in Classroom C

Table 7. 2: Teacher's Discourses at play in Classroom C

Episodes	Linear programming discourse		Language use	Discourse model
	procedural	conceptual	English	
14		X	X	Learners have to adapt to linear programming discourse
15	X		X	Teacher has linear programming knowledge to share
16		X	X	Teacher has linear programming knowledge to share
17		X	X	Explanation necessary when teaching linear programming content

The above table illustrates that during the teaching and learning of linear programming in Classroom C, the teacher spoke in English only. Her utterances are procedural as well as conceptual. The classroom practices position the teacher as a figure of authority in the classroom in terms of possessing linear programming knowledge. All learners are listening and taking advice from the teacher.

7.6 Conclusion

The analysis of the teaching of linear programming in Classroom C indicates that the teacher interacted with all learners using English only. The opportunities that supported learners when learning linear programming included the writing down of the definition of linear programming so that learners could refer to it at any time. She further presented steps that all learners could follow when representing inequalities geometrically and emphasised that when drawing a linear function they have to find two points which are the x and y -intercepts. The teacher further emphasised the mathematical meaning of key words that all learners have to follow when constructing inequalities. The focus of the analysis has been on how the teacher created learning opportunities for the participation of immigrant learners during the teaching and learning of linear programming.

As a follow-up, to the issues raised in chapters 5, 6 and 7, I conducted clinical interviews with three immigrant learners, one at each site. The analysis of the clinical interview is presented in the next chapter.

CHAPTER 8

IMMIGRANT LEARNERS SOLVING A LINEAR PROGRAMMING TASK

8.1 Introduction

In this chapter, the analysis of the clinical interview is examined and scrutinized to show how immigrant learners interact with a linear programming task. The chapter focuses on an analysis of data collected while conducting a clinical interview with three immigrant learners; John from the urban environment (School A), Bheki (School B) from the township environment and Allen from the rural environment (School C). The profiles of these learners will be discussed just before an analysis of each of their clinical interview data. The clinical interview was conducted at the end of lesson observation at each site.

The chapter illustrates how immigrant learners solve a linear programming task. The aim is to explore how immigrant learners take up the learning opportunities created by teachers during the teaching and learning of linear programming. Therefore, the question will be explored by the following question:

What do immigrant learners do when solving a linear programming task?

In attempting to answer this question, three sub-questions were posed:

- What practices do immigrant learners engage in to participate effectively in linear programming discourse?
- What underlying discourse models are at play in the text that immigrant learners produce with regard to linear programming?
- How do these discourse models inform us about the immigrant learner's opportunities in linear programming?

What emerged in the analysis of the lesson observations were the opportunities created by teachers through practices which positioned them as possessing linear programming knowledge. Furthermore, the language practices showed that teachers in Classroom A and B used code switching to support learners when learning linear programming. In Classroom A the teacher switched from English to French. In Classroom B the teacher switched from English to isiZulu or Setswana. These teachers valued and built on immigrant learner's linguistic skills while

modelling the use of English during teaching and learning. While in Classroom C the teacher used English only. The language practices of the teachers might or might not have an influence on resources immigrant learner's draw on when solving linear programming tasks. Therefore, as a follow up to the opportunities created by teachers when teaching linear programming, this chapter presents practices and resources that immigrant learners draw on when interacting with a given linear programming task presented in English. These practices were elicited in a clinical interview that was conducted at the end of the lesson observation. Immigrant learners at this stage were supposed to have mastered the linear programming discourse.

In this chapter I used the situated socio-cultural perspective (Moschkovich, 2002b) to explore the way the immigrant learners interacted with a linear programming task and an explanation of their work. Consequently, the immigrant learners' explanations, their actions and the use of other resources allowed me to understand how they used language to be recognised as mathematics learners with or without expertise in linear programming discourse. The focus in this Chapter, namely the linear programming activities included practices such as explaining a linear programming process, generalizing when optimising, translating ordinary language to mathematical notation (mathematical language) to obtain and write down a set of constraints that are linear equations or linear inequalities, graphical representation of constraints and other characteristics of linear programming discourse.

Moschkovich (1996, 1999, 2002b) describes how language is understood to act as a go between speech and the learning that is taking place in learners who are learning mathematics in more than one language. In her article, Moschkovich (1996) refers to a discontinuity model and situated model. She argues that the discontinuity model has gradually been seen as a deficit model regarding real meaning because the learners' experiences and use of their preferred languages when expressing their thinking were seen to be 'obstacles' to learning. The situated model on the other hand, takes into consideration the learner's experiences, preferred languages to express their thoughts as resources when learning mathematics.

In this regard Moschkovich (2002b) contends that we shift the focus to a model that views learners enacting a Discourse (Gee, 2005a; 1999) with its associated discourse models that guide

the way in which we understand the world within the context of that Discourse. She emphasizes that Discourses vary socially, culturally, historically across individuals, communities, purposes and time (Moschkovich, 2003). In the case of the study reported in this thesis, the classroom settings gave different contexts for analyzing linear programming discourses which might influence immigrant learners' perspectives of their classroom communities. The settings were multilingual classrooms with immigrant learners in an urban, township and rural environment respectively.

In what follows I discuss the framework of the discourse analysis, then how I identified the linear programming content analysed. The linear programming content analysed is about three processes namely; construction of constraint inequality, drawing a graph to represent a feasible region and optimising an objective function. The discussion of the linear programming content will be followed by the framework of the analysis of the immigrant learner's activities which was according to the three content categories. This is followed by the analysis of the learner's task activities which is preceded by a profile of each immigrant learner, finally a discussion of the findings.

8.2 Framework of the discourse analysis

Learning linear programming includes understanding the language in which the text is presented as well as the language of the mathematics in the text. In response to the questions posed, the learner has to use an appropriate method and language to present the text. As discussed earlier, Gee (2005a) argues that texts are constructed in specific social languages which are used to present information and construct questions in each genre, for example, in linear programming.

So when immigrant learners wrote down the solution and gave an explanation of the solving process, they designed their oral or written language using appropriate linear programming discourse to have patterns that reflect linear programming context. In this case social languages gave an insight into how they enact and shape Discourses in their classrooms, and their approaches were seen as reflection of the opportunities that were established or not during teaching, and their feelings about them.

In order to examine and understand how immigrant learner's engaged with linear programming discourse, I used Gee's (2005a) seven building tasks discussed earlier. To recap, building tasks help in explaining explain how the immigrant learner used language "in tandem with actions, interactions, non-linguistic symbol systems, mathematical objects, tools, technologies and distinctive ways of thinking, valuing, feeling, and believing" (Gee, 2005a: 10) to build their world.

8.2.1 Questions that guided the analytic process based on the seven building tasks

I made use of Gee's (2005) list of twenty six questions about the seven building tasks and selected the following that are relevant in analyzing immigrant learners' utterances:

- i. What does the immigrant learner highlight as *significant*?
- ii. What social *activity* is the immigrant learner enacting?
- iii. What *identity* is the immigrant learner assuming in relation to linear programming?
- iv. What *relationship* is the immigrant learner signalling to have with others?
- v. What implications does the language used by the immigrant learner have for the distribution of *social goods*?
- vi. How does the immigrant learner *connect* linear programming to other mathematics Discourse?
- vii. How are certain *sign system and knowledge* relevant or privileged by the immigrant learner?

Discussion on:

1. What is being made significant?

Identify this through what the immigrant learner brings to the forefront in their response to the task and questions asked by the researcher, activity and gestures, why they make certain connections (or not) and through what they emphasise.

2. What social *activity* is the immigrant learner enacting?

Identify this through what immigrant learners did when attempting the linear programming task which will be examined through their activity.

3. What *identity* is the immigrant learner assuming in relation to linear programming?

Identify this through the way in which immigrant learners talk about their involvement in solving the task, for example being in control or not coping

4. What *relationship* is the immigrant learner signalling to have with others?

Identify this through the way in which immigrant learners put forward in terms of who is in control, especially through the way in which they talk about (or do not talk about) their involvement and others in the activity of solving linear programming.

5. What implications does the language used by the immigrant learner have for the distribution of social goods (given *status*)?

Identify this through the way in which the immigrant learners see themselves positioned with respect to others in terms of the benefits or disadvantages that such a positioning might imply and in terms of how the identity put forward enables or inhibits them from acquiring those social goods.

6. How does the immigrant learner *connect or disconnect* linear programming to other mathematics discourse?

Identify this through the way in which the immigrant learner make connections with the mathematics domain as a whole (or not), or the way in which it sets up associations with specific parts of the mathematics domain (or other domain)

7. How are certain *sign system and knowledge* relevant or privileged by the immigrant learner?

Identify this through the use of mathematical discourse, use of symbols or unknowns, use of or reference to methods or procedures.

Linear programming was chosen as content to do this research because it is a well defined discourse. Therefore, learning linear programming may be viewed as an initiation to this well defined discourse. If learning linear programming is an initiation to a certain type of discourse, such learning involves becoming proficient in specifically mathematical ways of expressing your understanding. Learning linear programming must therefore lead to special uses of mathematical symbols which are regulated by meta-rules (Sfard, 2001).

The next section presents the content analysed from the data collected during the clinical interview of immigrant learners. The immigrant learners were asked to solve a linear programming task presented in English and explained their thinking in the process. I start by presenting categories into which the utterances were organised in order to identify linear programming content to be analysed.

8.3 Identifying linear programming content to be analysed

The content of any situation serves to give meaning within which a situation can be interpreted. According to Gee (2005a) a situation has no meaning if there is no content because the situation happens in a particular context. By examining utterances, it was possible to explain the content by having text organised into categories. The categories into which the immigrant learner's utterances were organised are according to the linear programming content indicated below. The indicators are put into words that the text reflected together with examples from the immigrant learner's utterances to clarify the text.

Linear programming involves a number of aspects, like extracting constraints inequalities from given statements, representing the inequalities graphically and optimising an objective function, which immigrant learners should be familiar with. These aspects are meant to portray a real life problem situation where immigrant learners are expected to apply mathematical algorithms to solve the linear programming tasks. This suggests that immigrant learners should have mastered certain linear programming processes which they would apply in understanding and solving the task. The processes that are the focus are discussed below:

8.3.1 Constraints inequalities

Linear programming enables learners to develop the ability to model the problem situations. Fundamental to such modelling is the ability to focus on the relevant factors in a problem situation; to identify and make assumptions about relationships among those factors; to develop a model using algebraic or other concepts that represent the situation; and manipulate the model to develop plausible lines of action to resolve the original problem. The relationships in linear programming can include a set of constraints that can be represented algebraically or geometrically using linear equations or linear inequalities.

The immigrant learners had to transfer ordinary language to mathematical language or to symbolic notation.

For example given the following linear programming task; A manufacturer of kitchen units make two types of units, Ralto and Quatro, in a workshop which is available for only twenty days each month. Suppose he makes x units of Ralto and y units of Quatro each month. It takes two days to put one unit of Ralto together and three and a third days to put together one unit of Quatro. The paint shop can handle a maximum of eight units per month. At least two units of Ralto must be produced each month. Furthermore the number of units of Ralto must be at least a third of the number of quarto. Write down the constrain inequalities (*Task given to learners in Classroom A*).

In the given example learners have to focus on factors like the constraint of time which is the number of days the workshop is available per month (inequality will be $2x + 3\frac{1}{3}y \leq 20$), number of units the paint shop can handle per month ($x + y \leq 8$). Number of units produced per month, that is at least two units of Ralto produced per month ($x \geq 2$) and the number of units of Ralto must be at least a third of the number of Quatro ($x \geq \frac{1}{3}y$).

8.3.2 Drawing a graph to represent a feasible region

Graphical representation involves drawing the boundary line using the dual intercept method or the gradient intercept method. In both methods, two or three points are identified and joined by a straight line. In an inequality, the solution lies in a region, referred to as a feasible region, in which the constraint is satisfied. To find the feasible region, we choose a convenient point that is either above or below the line and substitute its coordinates into the constraint to determine if it's satisfied. If it is, then all points on the same side of the line as this point will also satisfy the constraint. Inequalities are usually represented by shading the region to indicate a feasible region.

The procedure that should be used when representing the feasible region is learned through participation in mathematical discourse. If learners know how to use any representation, it may be because the symbols bring to mind a collection of past

experiences and relate the learner to a variety of decisions that seem appropriate in the context. The function of a graphical representation is to represent the task containing the constraints. It brings order into the learner's already existing mental picture and knowledge in order to connect them together (Dossey, McCrone, Giordane & Weir 2002). From my experience, graphical representation is strongly influenced by a teacher's demands and learners' habits.

Example: with reference to the example above, the four inequalities that will be represented graphically are: i) $2x + 3\frac{1}{3}y \leq 20$; ii) $x + y \leq 8$; iii) $x \geq 2$ and iv) $x \geq \frac{1}{3}y$ as shown below:

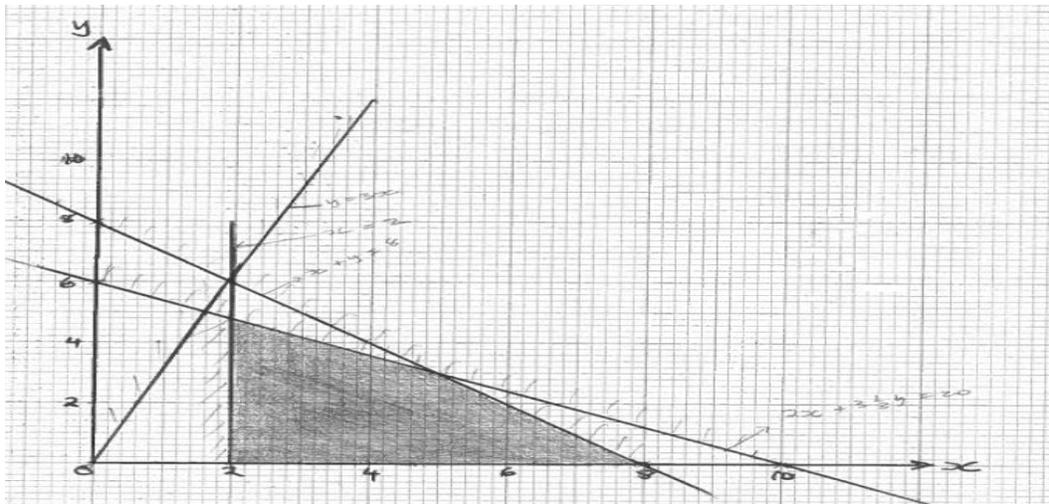


Figure 8. 1: Linear programming task; model solution

8.3.3 Optimising an objective function

A mathematical optimisation model consists of an objective function and a set of constraints expressed in the form of a system of equations or inequalities. The objective function describes the behaviour of the measure of effectiveness or what is best in the given situation. If the objective function is to describe the behaviour of the measure of effectiveness, it must capture the relationship between that measure and those variables that cause it to change.

When optimising, learners should find the value of the objective function at the extreme point by using either a search line method or the coordinates of the vertices of the feasible region. The search line method requires learners to draw a set of lines parallel to the line representing the objective function in order to find an extreme point in the feasible region. The coordinates of the extreme point are then substituted in the objective function to calculate the maximum or minimum value.

The second method involves finding the coordinates of the vertices of the feasible region. These coordinates are then substituted in the objective function to find the point that gives a maximum value when given a profit function or minimum value when given a cost function.

For example: Units of Ralto yield a profit of R700 per unit, while units of Quatro yield a profit of R900 per unit. Write down the equation of the profit function P in terms of x and y .

The objective function will be $P = 700x + 900y$

The section that follows discusses the tools of analysis of the immigrant learner's text.

8.4 Framework of the analysis of the immigrant learners' activities

The immigrant learners' written responses were examined in the three content categories described above. These three content categories are then used to determine whether immigrant learners were supported when learning linear programming. This assisted me in concluding whether immigrant learners have access to linear programming discourse in each of the content categories.

8.4.1 Analysis of constraint inequalities:

From the written responses that the immigrant learners have produced in attempting the linear programming task, I determined the extent to which they demonstrated an understanding of the social languages that showed constraints inequalities from the given task as well as the situated meanings of some key words in the task. The written responses were examined for certain features which can be used to identify the extent to which an understanding is evident. Therefore,

I searched for evidence in the text for relevant constraints inequalities. The possible evidence in the text might be any of the four instances which are a hierarchy of understanding. The instances are my judgement and the evidence will lead to assigning the immigrant learners understanding to one of the following instances (which are in order from little to extensive understanding). Four instances could have been possible, namely:

Table 8. 1: Degree of correct identification of constraints inequalities

Instances	Degrees of relevant constraints inequalities reflected in immigrant learner’s solution to the given task
Instance 1	<ul style="list-style-type: none"> - no understanding of constraints inequalities to be extracted from the task. - inequalities simply written down, cannot be connected to aspects of the task - No evidence that ideas reflected by the text are connected to the task.
Instance 2	<ul style="list-style-type: none"> - some evidence of extracting the inequalities from the task. - some misrepresentation or misinterpretation of the inequality sign had occurred, (E.g “does not exceed 4000” represented by a greater than inequality sign [meaning $4000 < x$] instead of a less than inequality sign [$x < 4000$])
Instance 3	<ul style="list-style-type: none"> - definite recognition of the constraints inequalities represented - not all expected inequalities shown in the text.
Instance 4	<ul style="list-style-type: none"> - evidence of the correct interpretation of all aspects of constraints inequalities. - complete and clear representation of all the inequalities to be extracted from the given task.

8.4.2 Analysis of graphical representation of inequalities: feasible region

From written responses it was possible to determine how immigrant learners went about representing inequalities on a Cartesian plane. For example, there might be a procedure evident in the written response that is used to find the points to plot in order to draw the line. The procedure may be a table of x and y values, the dual intercept method or even guess work. The instances are a hierarchy of understanding (from which are in order from little to extensive understanding) discussed below.

Table 8. 2: Degree of correct execution of graphs showing inequalities

Instances	Degrees of relevant drawing/ or graphical representation of inequalities and identifying feasible region
Instance 1	<ul style="list-style-type: none">- no evidence of a procedure or a linear function- appear to be random use of numbers such that it is not possible to represent the inequality graphically (representation of immigrant learners inequality)
Instance 2	<ul style="list-style-type: none">- some evidence of method present in establishing relationship but method not clear
Instance 3	<ul style="list-style-type: none">- Appropriate method, dual intercept or table- Presentation not systematic- Implementation not well ordered i.e less methodological
Instance 4	<ul style="list-style-type: none">- evidence of clear and correct method used to represent inequalities graphically- organized, well structured work to guide the representation of inequalities on a Cartesian plane

8.4.3 Analysis of mathematically correct/optimising

The immigrant learners' written response should reflect the extent to which the mathematics presented is correct and applicable to the linear programming task. It is important that the text shows evidence of the level of understanding of an objective function when optimising. Again, the instances are a hierarchy of understanding (which are in order from little to extensive understanding) discussed in Table 8.3.

Table 8. 3: Degree of relevant mathematically optimising

Instances	Degrees of relevant mathematically correct/ optimizing
Instances 1	<ul style="list-style-type: none">- no evidence of correct and appropriate mathematics applicable to the task- no mathematical working shown- incorrect mathematics.
Instances 2	- some evidence of correct formulae applicable to the task but is insufficient to enable progress.
Instances 3	-some meaningful and appropriate mathematical relationships. For example, in order to calculate maximum income using the objective function.
Instances 4	-Correct mathematics and establishment of an equation that lead to correct solution to the task.

Summary

The overall analytic framework that enabled me to recognize immigrant learners as part of the linear programming discourse is the following: firstly an examination of the written response looking for constraints inequalities relating to the information given in the task, secondly a graphical representation of the constraints inequalities and thirdly optimising using the correct equation of the objective function. The analysis of the immigrant learners' written work was then followed by analysis of the seven building tasks which facilitated the identification of patterns pointing to certain Discourse models which answered my research questions.

In the next section I present a brief background to John who is an immigrant in Classroom A. John is an interesting case especially because he arrived in South Africa towards the end of the year 2010 and enrolled in Grade 11 at School A.

8.5 John's background

John is sixteen years old. He was born in the Democratic Republic of Congo (DRC), was brought up there and did his schooling up to Grade 11 through the medium of French. He had sufficient competence in French, speaks it at home, with friends in the neighbourhood and sometimes at school. Furthermore, John speaks English with his friends who do not speak French. He moved

to South Africa with his parents who settled in Johannesburg towards the end of the year 2010. The family joined members of the community from the DRC who had settled in Johannesburg earlier. There are families from DRC some of which have connections with his family such that they visit each other, do mathematics together and attend social functions together. He joined School A in October 2010, which is the fourth term of the school year in South Africa. John was admitted to Grade ten.

Some teachers at School A and people from the surroundings speak French which benefits John because there are opportunities for speaking French in this environment. The security guard who mans the school gate is, like John, Congolese and speaks French. So is the school's director, and some of the teachers. According to John, his family members speak Lingala and French at home. But most people in the neighbourhood of School A speak English (Sunday Times, 20/10/2010) because most of them are from different countries around the world. Some of the people speak English because they had learnt English in South Africa as a useful national and global language. Other people speak English because they come from places where English is spoken and taught.

In building his new life in Johannesburg, John had been able to draw on the range of experiences that he had back in the DRC. For example, he was then learning through the medium of English in South Africa. He claimed, "I picked up English from my friends on the street back in the DRC" (*Interview November 2011*). The move to Johannesburg also means that he has begun to construct a new identity at school and in the neighbourhoods. John is exposed to multiple languages such as isiZulu, siNdebele, siSwati, Afrikaans, Setswana. He is also exposed to different cultural practices and different views of the world in this environment. According to Martin-Jones (2000: 23) "these become resources on which individuals and groups may draw on as they take on different identities in different domains of their lives".

Certainly, John was then learning mathematics through the medium of English at secondary school. The English he learned from friends back in the DRC was now a resource in South Africa. Prior to migrating, he had developed his mathematical concepts in French during primary and some secondary school years. John was now studying English as a first additional language because Lingala, his home language, is not studied in South Africa. He is fortunate because he

was also learning French, the language of learning and teaching in the DRC, as a second additional language.

In the next section an analysis is done of John's work and then summarised. I start with analysis of Johns' written activities which is then followed by an analysis of his explanation in the interviews.

8.6 Analysis of the learners' tasks

For reference, I introduced each learner's task together with its solution before the analysis of his written work. The analysis will be followed by a discussion to identify possible discourse models that are at play. The discussion and the concluding remarks at the end of the analysis identify the influences that the discourse models had on the immigrant learners as they solve the linear programming tasks and explained how they have solved it. I start with an analysis of John's clinical interview.

8.6.1 John

John had to work through multiple questions which included mathematical, contextual and linguistic challenges. The language challenges included some unfamiliarity with everyday words and phrases. For example, they were expected to understand phrases such as "short video", "printed binder" as well as other words that are germane to the linear programming task. Furthermore, phrases such as "at least four thousand" and "not expected to exceed four thousand" might seem familiar to someone fluent in English, but may be misinterpreted or not understood by someone who has mostly learnt English outside a formal learning environment, leading to very wrong translation to mathematical inequalities. In terms of linear programming content, immigrant learners need to demonstrate dexterity with constraints, graphing and optimising.

8.6.1.1 John's task

A local health board is producing a guide for healthy living. The guide should provide advice on health education, healthy lifestyles and the like. The board intends to produce the guide in two formats: one will be in the form of a short video; the other as a printed binder. The board is currently trying to decide how many of each type to produce for sale. It has estimated that it is

likely to sell no more than 10 000 copies of both items together. At least 2 000 copies of the binder and at least 2 000 copies of the video could be sold, although sales of the video are not expected to exceed 4 000 copies. Let x be the number of videos sold, and y the number of printed binders sold.

5.1 Write down the constraints inequalities that can be deduced from the given information.

5.2 Use graph paper to represent these inequalities graphically and indicate the feasible region clearly.

5.3 The board is seeking to maximise the income, I , earned from the sales of the two products. Each video will sell for R50 and each binder for R30. Write down the objective function for the income.

5.4 Determine graphically, by using a search line, the number of videos and binders that ought to be sold to maximise the income.

The next section shows the solution to the above task for reference.

8.6.1.2 Model solution

5.1 x represent numbers of videos and y represent numbers of binders

No more than 10000 copies, of both items together, so the inequality is $x + y \leq 10000$

At least 2000 copies of binders, so symbolically it is $y \geq 2000$

At least 2000 copies of video, although sales are not expected to exceed 4000, so inequality will be $2000 \leq x \leq 4000$.

5.2 The graphical representation of the constraints inequalities is presented below. The feasible region is shaded and the search line is represented by a broken line.

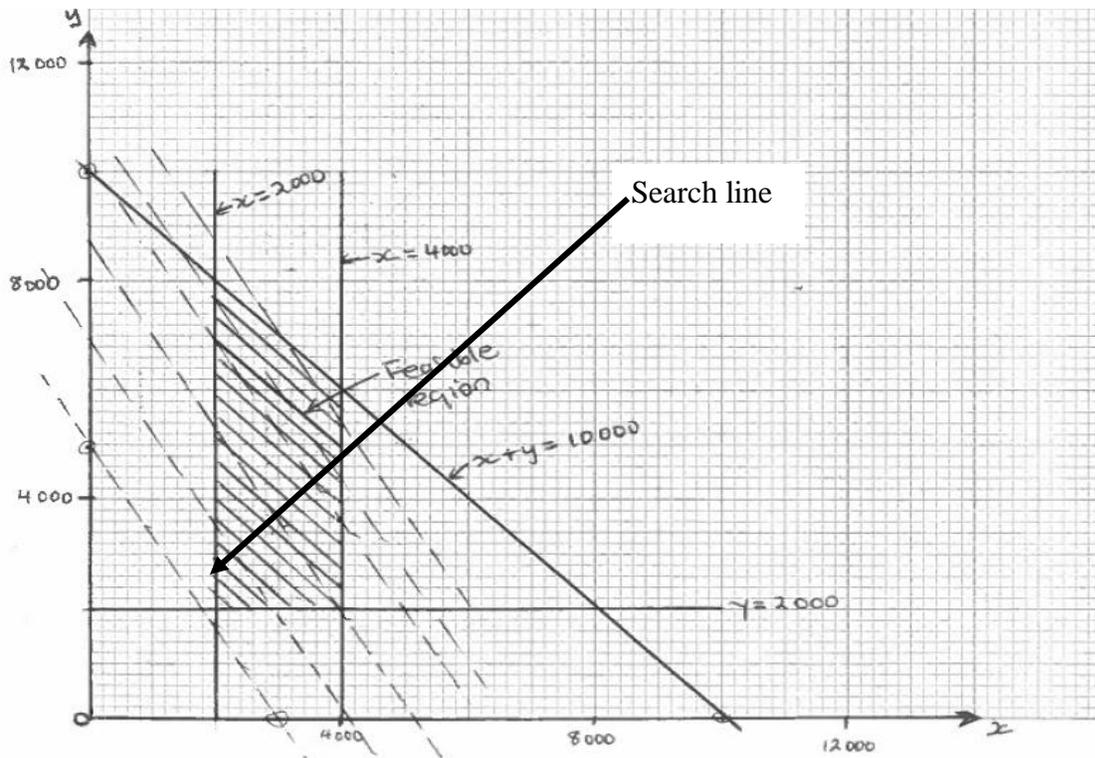


Figure 8. 2: Linear programming task solved by John; model solution

5.3 Income (I) = $50x + 30y$ is the objective function. The value of I can be found in one of two ways.

1. Taking each vertex of the feasible region in turn and calculating I for the $(x; y)$ values for these points, you find that $I(4000; 6000)$ gives the maximum value of I .
2. Draw the search line for the objective function $I = 50x + 30y$, x and y represent the number of video's and binders sold, and I represent the income to be maximized. To maximize the income, slide the ruler parallel to the search line for the objective function and find the coordinates of the extreme point, which is $(4000; 6000)$ of the feasible region (the last point touched by the ruler) and substitute it into the objective function

The maximum income will be $50(4000) + 30(6000) = 380\ 000$

In order to draw the search line, you make y the subject in the objective function (I);

Objective function: $I = 50x + 30y$

Search line $y = I - \frac{50}{30}x$

(Graham & Graham, 2010)

Below I present an analysis of John's activity which will be followed by an analysis of his talk about solving the task.

8.6.1.3 Analysis of John's written work

John requested time to solve the task before I asked him questions about his approach. Thus I was not able to access his thinking or information processing while he was actually working on the task. This might be because his thinking might be obscured by having to explain what he was doing. He might also have been intimidated by a one-on-one situation with someone unfamiliar. What was needed was to allow John time to solve the linear programming task and I needed to make sure that the environment was less intimidating. In addition John needed the opportunity to develop his higher order thinking about the task so that he could communicate his ideas more easily. The analysis will therefore discuss John's solution to the linear programming task. This was followed by an analysis of John's verbal explanation of his solution processes.

The focus of the analysis has been on John's processes and the explanation he gave with regard to the questions posed by the researcher on how he solved the linear programming task. In this analysis the immigrant learner's written work is categorised using the four instances of learners' activity mentioned earlier (Table 8.1, 8.2 and 8.3).

1. Constraints inequalities

The text indicates clearly all the inequalities to be extracted from the given task. There is evidence in John's written work that he recognized that the first inequality is obtained by adding x and y to get $x + y \leq 10\,000$. The text also reflects the minimum and possible maximum number of videos that must be sold, which we see in the second inequality. John's text suggests that he is able to extract inequalities from the given task. This indicates that he was able to identify situated meanings of key words needed to represent the given information algebraically in the form of inequalities. One interesting inequality is where he denoted an understanding of the

condition of selling videos. The sale of the videos was to be more than 2 000 but less than 4000. He started by writing the inequalities separately and then denoted with a curly bracket that the two can be represented as one inequality as shown below:

Inequality 1 ① $x + y \leq 10\,000$ $x \rightarrow$ no. of video
 $y \rightarrow$ number of binders

Inequality 2 ② $x \leq 4\,000$ } $2\,000 \leq x \leq 4\,000$
 $x > 2\,000$ }

Inequality 3 $y \geq 2\,000$

Figure 8. 3: John's solution

John’s activity is categorised as **instance 4**: The text shown in Figure 8.3 indicates evidence of correct interpretation of all aspects of the given linear programming task.

The next part presents the graphical representation of the inequalities shown in Figure 8.3

2. Drawing/ or representing inequalities graphically and identifying a feasible region

John represented the inequalities, Figure 8.4, graphically as shown below.

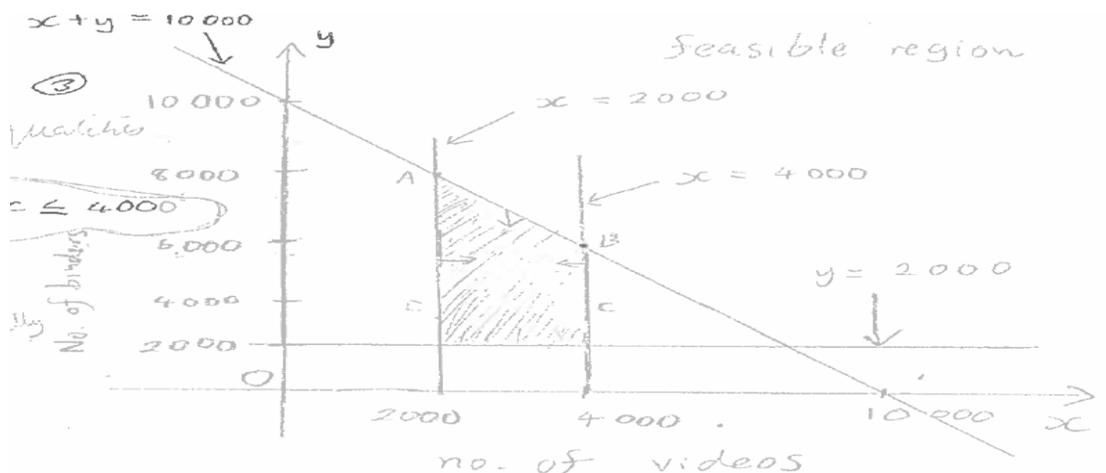


Figure 8. 4: John's feasible region

The graph shows a correct representation of the inequalities identified by John. Each boundary line is labelled to guide the reader. The region representing the inequalities is indicated by arrows. That is to say, for $x + y \leq 10\,000$ John drew an arrow pointing below the line as shown in the diagram. All the inequalities that John extracted from the given task are correctly represented on the Cartesian plane. This leads John to the correct solution indicated by shading which is the feasible region.

John's graphical representation of the inequalities is categorised as **instance 4**. The text shows evidence of clear and correct method used to represent inequalities graphically. The text is organized, well structured and guide the representation of inequalities on a Cartesian plane.

The next section shows the linear programming practices that John called upon when optimising the objective function

3. Mathematically correct/optimising

So far, John's written work throughout implies that he was able to set up inequalities from the linear programming task. Correct mathematics was shown in the establishment of an objective function. The inequalities are represented graphically showing the feasible region. The text below shows the objective function and his method of finding the maximum number of videos and binders to be sold in order to maximize the income.

profit function

$$I = 50x + 30y$$

Vertices from feasible region =

(2 000; 2 000) ^D; (2 000; 8 000) ^A; (4 000; 2 000) ^C; (4 000; 6 000) ^B

3) Profit/maximisation

$$I_D = 50(2\,000) + 30(2\,000)$$

$$= 100\,000 + 60\,000$$

$$= 160\,000$$

$$I_A = 50(2\,000) + 30(8\,000)$$

$$= 100\,000 + 240\,000$$

$$= 340\,000$$

$$I_C = 50(4\,000) + 30(2\,000)$$

$$= 200\,000 + 60\,000$$

$$= 260\,000$$

$$I_B = 50(4\,000) + 30(6\,000)$$

$$= 200\,000 + 180\,000$$

$$= 380\,000$$

Max ^{income} profit @ (4 000; 6 000)

Figure 8. 5: John's optimising solution

Figure 8.5 displays that John pulled out coordinates of the vertices of the feasible region and labelled each point A, B, C and D to guide the reader in his calculation. And at the end he indicated that maximum income is at the point (4000; 6000). John's working is categorised as **instance 4**. His work shows evidence of correct inequalities extracted from the given task, correct graphical representation of inequalities and calculation of maximum income. However it must be noted that the question required John to use a search line when optimising. Even though his approach is correct, he did not use the prescribed method. This might be due to the language of the task or he did not know how to use a search line.

Discussion

From the analysis of John's written responses, it became clear that the mathematics shown in the text was correct and related to the linear programming task. First of all John extracted correct inequalities from the given task. The constraints inequalities are then represented graphically to show the feasible region. The graph shows that John is successful in identifying the feasible region without any problem. The written response indicated an equation representing the profit function. However, in the last line of the solution John changed to maximum income and cancelled profit as indicated in the last line. This suggests that John is aware that the problem is not looking for maximum profit but for maximum income as indicated in the question paper.

Furthermore, John's text implied that there is a reliance on some procedure which seemed to enable him to succeed when tackling the problem. The way he has presented his solution is as if he was recalling some procedure he has done before. It is obvious that John is not excluded from the discourse of linear programming and this is because of the strategy he used.

The next section presents an analysis of John's explanation in the interview.

8.6.1.4 Analysis of John's explanation during the interview

In the incident below John explains how he has solved the task. The discussion is based on my observations as he was solving the task and some of the things he did during the process of solving the task, as well as his talk. From my experience, learners have to understand the task in order to extract the inequalities; however, John indicated that he did not understand what the task was about because he does not understand English. My focus is on how the teacher created opportunities for accessing linear programming discourse if the language of the task is the problem.

In this incident, John explains that the teacher gives French explanations during his teaching in order to help them understand linear programming presented in English. But in the absence of the teacher he uses a dictionary to find meaning of some words.

Episode 1: Teacher providing support

19. John *[Okay, parfois notre professeur explique Français parce que il sait que nous sommes du Congo et il peut parler Français, donc ... il ... nous explique et nous comprenons]* (relaxed) sometimes our teacher explains in French because he knows we are from Congo and he can speak French, so ... he ... explains to us and we understand...
20. Thulie Okay, do you ask him to explain?
21. John *[Oui, des fois nous lui demandons, mais nous comprenons en Français puis nous exécutons la tâche en Anglais...]* yeah sometimes we ask him, but we understand in French then solve the task in English...
22. Thulie Let us say you are doing your homework, what do you do in that case?
23. John *[Des fois, nous utilisons notre dictionnaire pour vérifier le sens de certains mots en Français et alors nous comprenons]* Sometimes we use our dictionary to find the meaning of some words in French and then we understand
27. Thulie Did you do linear programming in Congo?

28. John *[Non, mais nous avons fait des inégalités linéaires... qui consiste à résoudre les inégalités et les dessiner le graphiques...]* No, but we had done linear inequalities (with confidence)...that is solving inequalities and drawing graphs ...

Analysis of episode 1

Significance

John stressed the idea that he needed an explanation in French for some words which the teacher provided during the teaching process otherwise he used his dictionary. This shows that John encountered both English and French during teaching and learning linear programming. The significant gain of language duality contributed to the development of manoeuvring between English and French. In this way, John's English language was built upon social interactions with his mathematics teacher. He also placed significance to the knowledge of inequalities he gained in the DRC which enabled him to succeed in learning linear programming (utterance 28).

Identity

John positioned himself as a learner who could communicate linear programming through French. During the teaching he asked the teacher to explain some of the linear programming content which he did not understand when presented in English. We see that in utterance 19, "sometimes our teacher explains in French because he knows we are from Congo and he can speak French, so ... he ... explains to us and we understand..." He further stated that "sometimes we ask him..." However, in the absence of the teacher, John said that he used a dictionary (utterance 23) to find the meaning of some words in order to understand the content presented in English. During the interview John missed both his teacher and the dictionary which act as a support during mathematics lessons.

Relationship

John positioned himself as a learner who is supported by a teacher from the same linguistic background and is bonded by the teacher's switching from English to French. Furthermore, John uses a dictionary to find meaning of words when alone doing his homework, "...sometimes we use our dictionary to find meaning of some words in French and then we understand" (utterance 23). John's actions shows confidence and desire for establishing social relationship with the linear programming discourse. His identity is therefore linked to the desire for relationship and

recognition in the discourse. The identity is thus realized through access to material resources such as the use of dictionaries. It can be argued that John's recognition in the Discourse is directly related to what he could do using the material resources.

Connection

John made a connection of solving linear programming tasks to linear inequalities, the mathematical practices of drawing linear inequalities and solving of linear inequalities (utterance 28).

Politics

John's response to the researchers' question of whether they had learnt linear programming in the DRC was: "we had done linear inequalities..." and to show familiarity with inequalities he went on thus "...that is solving inequalities and drawing graphs..." The response by John showed that he saw himself as being advantaged by the content he learnt in the DRC. He went on to state his knowledge about inequalities that is solving inequalities and representing them graphically. This shows that he acknowledges what he learnt in the DRC and he is now applying when solving linear programming task.

Sign system and knowledge

John has recognised that to solve linear programming you need knowledge of linear inequalities and he appeared to be familiar with the graphing of inequalities. We see that in utterance 28 "we had done linear inequalities..."

Discussion

In this episode John positioned himself as a learner who needed support from the teacher. He constructed himself as a learner who could communicate linear programming through French. The opportunity for John to develop meaningful linear programming content was guided by the teacher when providing French explanations. It is evident that French plays a major role in his learning of linear programming. Therefore, the learning environment created by the teacher is responsible for creating and controlling the opportunities under which John, can succeed in this mathematics classroom.

John stressed his knowledge of inequalities and that he only needed support when finding meaning of words in order to link or make a connection to his previous knowledge of inequalities. Therefore, the concepts he learnt through the medium of French do not need to be re-taught in English in order for John to succeed (Cummins, 2000). The context is such that the teacher is aware that there are learners who do not understand English but understand French. As a result the teacher incorporated French explanations while teaching (As presented in Chapter 5) but also these learners take up these opportunities by sometimes requesting the teacher to provide explanations where necessary. John share a culture and languages with the mathematics teacher; since they are from the same country and might share the same values. The shared cultural knowledge in terms of language, (shared by John and his teacher) probably contributed towards him feeling confident when learning linear programming. John has the resources necessary to support him, especially the language.

In the next episode John explains what he had to look for in order to extract the correct inequalities. It is worth mentioning that before he started solving the task he circled key words as shown below:

A local health board is producing a guide for healthy living. The guide should provide advice on health education, healthy lifestyles and the like. The board intends to produce the guide in two formats: one will be in the form of a short video; the other as a printed binder.

The board is currently trying to decide how many of each type to produce for sale. It has estimated that it is likely to sell no more than 10 000 copies, of both items together. At least 4000 copies of the video and at least 2 000 copies of the binder could be sold, although sales of the binder are not expected to exceed 4 000 copies. Let x be the number of videos sold, and y the number of printed binders sold.

5.1 Write down the constraint inequalities that can be deduced from the given information.

5.2 Use graph paper to represent these inequalities graphically and indicate the feasible region clearly.

5.3 The board is seeking to maximise the income, I , earned from the sales of the two products. Each video will sell for R50 and each binder for R30. Write down the objective function for the income.

5.4 Determine graphically, by using a search line, the number of videos and binders that ought to be sold to maximise the income.

5.5 What maximum income will be generated by the two guides?

① - in equality

② - graph

③ - maximum

Furthermore, he had numbered three things he was going to do; he has numbered firstly inequality, secondly graph and then maximum.

Episode 2: Extracting inequalities

57. Thulie In linear programming you have to understand the problem, you said you did not understand the first part of the problem, what do you do if you do not understand what the task is all about?
58. John [*... Pour cette question, j'ai regardé les mots-clés ... il ya des mots clés que vous devez connaître leur signification ... le professeur nous a dit que ces mots nous donner le signe de l'inégalité ... hein, comme inférieur ou supérieur. Parfois, inférieur ou égal à ... et supérieur ou égal à ... hein ... jah*] For this question, I looked at the key words...there are key words that you must know their meaning...the teacher told us that those words give us the sign of the inequality...eh like less than or greater than. Sometimes less than or equal to...and greater than or equal to...eh jah...
59. Thulie Key words that you must know... like which words?
60. John [*Comme ... au moins hein ... tout au plus ... maximum ... minimum ...jah*] Like ...at least eh...at most ...maximum ...minimum jah...
61. Thulie Okay, what are the key words in this task?
62. John [*ici il ya au moins deux mille ... et cela signifie plus de deux mille ... l'inégalité sera ... hein supérieure ou égale à deux mille...*]_eh... here there is at least two thousand ...and it means more than two thousand ...the inequality will be ... eh greater than or equals to two thousand
63. Thulie Fantastic ...okay... eh... anything else?
64. John [*... il ya celui-ci pointant sur ne dépasse pas ... eh ne dépasse pas quatre mille ... et cela signifie moins de quatre mille ... et c'est cette inégalité.*] (looking at the words he underlined before solving the task eh) ... there is this one (*pointing at does not exceed*) ...eh... does not exceed four thousand ...and it means less than four thousand...and it is this inequality.
65. Thulie Okay continue...
66. John [*hein ... alors ... avec celui-ci ... hein il ya au moins deux mille.... aussi ... ne dépasse pas quatre mille ... ok ... je pense...*] (silence) (eh ...so ...with this one ...eh there is at least two thousand andalso ...does not exceed four thousand...okay I think.... (silence)

Analysis of Episode 2

Significance

John attached significance to the situated meaning of key words in linear programming suggesting that to him knowledge of the mathematical meaning of key words played a significant role when extracting inequalities in a given linear programming task. We see that in utterance 58 “I looked at the key words...there are key words that you must know their meaning”. Furthermore, he started by circling those key words before even writing their symbolic meaning.

Identity

John used the first person pronoun “I” to position himself with respect to the given task. It is depicted that in “I looked for key words” (utterance 58). This implies that he sees himself as being involved in the activity of solving the linear programming task by extracting the appropriate inequalities and following the teachers advice “the teacher told us ...” (utterance 58). At the same time John spoke from a position of authority when he said “there are key words that you must know” (utterance 58). Even though it is most probable that he is drawing from the teachers talk, John indicated it when he said the teacher told us.

Relationship

John commented that “the teacher told us that those words give us the sign of the inequality...” which positions him as a learner under some knowledgeable other in this situation. However, John appropriated the teacher’s expertise by following his instruction. John had confidence that he was doing the right thing to solve the task. He was drawing on the advice of the teacher when he said “here there is at least two thousand ...and it means more than two thousand ...the inequality will be ... eh greater than or equals to two thousand” illustrate that John positioned himself as involved in the process of solving linear programming. John as a learner seemed to have experienced key words and their corresponding mathematical meaning which led him to the correct inequalities. He had a good degree of access to the Discourse surrounding linear programming and this enabled him to end up with the correct symbolic meaning leading to a correct solution to the task.

Connections

John’s response in utterance 58 implies that he understood the link between the situated meaning of key words like “at least”, “does not exceed” and their symbolic meaning in linear programming. We see that in utterance 62 and 64. John made a connection between these words and their symbolic meaning to end up with inequalities to be represented in the task.

Politics

John acknowledged that key words guide you when extracting inequalities given a linear programming task and he appeared to view knowledge of key words in linear programming the

only way of getting correct inequalities within the linear programming discourse. This seemed to suggest that his knowledge of the symbolic meaning of key words seemed to advantage him in linear programming.

Sign system and knowledge

John interpreted the phrases appropriately for each inequality showing an understanding of the situated meanings of the key words in this linear programming task he was solving. We see that in utterance 62 and 64 he says "...does not exceed four thousand ...and it means less than four thousand...and it is this inequality". Further John translated greater than to French as he was explaining.

Discussion

There is an opinion that knowing the symbolic meaning of key words is the preferred approach to finding correct inequalities from a given linear programming task. In this episode, John foregrounded key words which suggest that he understood the link between the situated meaning of key words to linear programming and their symbolic meaning. He seemed to have been instructed to behave in a particular manner and thus his observation that within linear programming you do not have a choice of approaches. This preferred approach comes from someone who knows, the mathematics teacher, in the Discourse of school mathematics. John highlighted aspects of the linear programming task, make-up and the authority of the teacher. Such behaviour enabled him to engage appropriately with the task.

The analysis above indicates that John's teacher assisted him in developing linear programming practices matching those of mathematicians, which are appropriate for this environment (Barwell, 2007; Herbel-Eisenmann, 2010). This suggested an opportunity for John to experience continuity of meanings of key words in linear programming which had a positive influence on his participation in linear programming. The teacher considered the information presented to be in French, so that linear programming content could be meaningful. He developed immigrant learners' knowledge of social languages by using French in that context. The teachers' appreciation of some of the challenges immigrant learners face when learning English, the language of learning and teaching (LoLT) in South Africa, as well as mathematical language

might improve some of the meaning of symbols that John faced. Such an appreciation aids John in the construction of his identity in relation to the linear programming discourse. Furthermore, it might also facilitate John's commitment to the learning of linear programming.

In the next episode, John had one of his inequalities represented as $2000 \leq x \leq 4000$. In his explanation he indicated that he is using the knowledge of inequalities he learnt in the DRC.

Episode 3: Experience brought

68. John [Oui ces sont mes inégalités ... eh oui] yes these are my inequalities ...eh yes
69. Thulie Okay, I am fascinated by this inequality, you have this inequality (pointing at $2000 \leq x \leq 4000$) what does it represent?
70. John [Il représente ne dépasse pas quatre milles, ne dépasse pas signifie moins que (pointant à moins de 4000) et ... hein ... il ya au moins deux mille dans la question ... alors ... sens ... hein plus grand que ... x supérieur deux mille ... qui dans cette partie est (pointant à $2000 \leq x$) pour obtenir cette inégalité...] It represent does not exceed four thousand, does not exceed means less than] (pointing at less than 4000) and ...eh... there is at least two thousand in the question... so ... meaning ... eh greater than...x greater two thousand...which is this part (pointing at $2000 \leq x$) so getting this inequality...
71. Thulie Alright, continue
72. John [Oui ... oui je sais qu'il ya une limite ... ce qui signifie pas plus de quatre mille et pas moins de deux mille (en utilisant des gestes) ... hein ... alors une inégalité comme ça ... est généralement représentée comme ceci ... Nous l'avons fait au Congo, donc je sais qu'il doit être écrit comme ceci.] Yes a ...yes I know that there is a limit... which means not more four thousand and not less than two thousand (using gestures) ...eh so ... so an inequality like this ... is usually represented like this... We did this in Congo, so I know it has to be written like this

Analysis of Episode 3

Significance

John has made a correct representation of inequalities, an important aspect in linear programming. This could be inferred from his use of words like "...so an inequality like this ... is usually represented like this..." This suggests that John valued formal presentation of inequalities when solving linear programming task. It could have been that he viewed this type of presentation as the proper way of writing such inequalities. He had confidence in his abilities to set inequalities.

Identity

John was confident, in utterance 68 he says, "...yes these are my inequalities" and furthermore, the use of the first person pronoun "I" in utterance 72 thus, "I know that there is a limit... which means not more than four thousand and not less than two thousand... so an inequality like this ... is usually represented like this... We did this in Congo, so I know it has to be written like this". These comments gave an impression that John was talking like someone who knows that the inequalities represent the information in the task.

Relationship

John's response that "...we did this in Congo..." confirms that he did not do this on his own in Congo, hence the use of "we". When he put this forward he positioned himself as a learner under some knowledgeable other (Lave and Wenger, 1991) in this situation, and therefore it has to be this way.

Connection

The use of the phrase "so an inequality like this ... is usually represented like this... we did this in Congo, so I know it has to be written like this" connects with the mathematical practices he learnt in DRC. The use of the pronoun 'we' refers to himself and other learners in the DRC. The use of the adverb "usually" positions John as someone who is familiar with this kind of Discourse in mathematics.

Politics

John observed that the appropriate presentation of inequalities is important in linear programming. That it is a good representation of inequalities that has a limit of minimum and maximum values. He projected the experience he brought in such a way that it is associated with a certain status.

Sign system and knowledge

John's language-in-use here is that of someone who was familiar with the discourse of inequalities. We see that in utterance 72, "so an inequality like this ... is usually represented like

this...” his use of the adverb ‘usually’ suggested someone who is familiar with the practice of mathematicians.

Discussion

John drew attention to the representation of the inequality in this text which suggested that he holds a strong link between inequalities and linear programming. The inequality written by John reflects that he understood the information given in the task and it seemed to enable him to engage meaningfully about how to present such inequalities. Even though he indicated that he did not understand what the task was about because of English, he is able to come out with a mathematical representation of the task. This preferred method of representing inequalities comes from somewhere in the Discourse of school mathematics.

In the next episode, John explained the approach when optimising in order to maximise the cost function.

Episode 4: Steps followed/ optimizing: mathematics method

To optimise a linear programming function, learners have to first find the objective function and then the point that will give a maximum value of the objective function. To find the point that will give a maximum value, learners use a search line or substitute coordinates of the vertices of the feasible region in the objective function. John had this to say:

101. John *[Oui ... il sera de trouver le point qui va donner le maximum ... la question ... Je dois maximiser ... donc j'écris la formule ... qui est la fonction de profit ... ici]* Yes... it will be finding the point that will give maximum...the question ...I have to maximize... so I write the formula ... which is the profit function... here (pointing at what he had numbered 4, the objective function)
102. Thulie Is that the objective function?
103. John *[Oui, le profit fonctionne de fois. Mais ils disent ... hein ... la question est la fonction objectif ...]* Yes, the profit functions sometimes. But they say ... eh... the question is objective function...
104. Thulie So what is the objective function here?
109. John *[Euh ... c'est ... cinquante x plus trente y]* Eh... it's... fifty x plus thirty y (writing $I = 50x + 30y$)
110. Thulie Okay, continue
113. John *[Ce point maximisera le profit]* This point will maximize the profit (pointing at 4 000; 6 000)

114. Thulie So what is the maximum value? I saw that you were calculating again
115. John [*La valeur maximale est R380 000 ... Je voulais juste être sûr de mes valeurs...*] The maximum value is R380 000...I just wanted to be sure with my values...]

Analysis of Episode 4

Significance

The text showed that John highlighted the equation of the objective function. What was significant with him was to write the equation for the objective function before maximising. "...I have to maximize... so I write the formula..." he therefore engaged with the task using the equation he had written and he does not use the search line as expected. John put forward that in order to optimize, you have to write the equation of the objective function. He seemed to be in control of the necessary steps to be followed which involves mathematical language.

Identity

The text indicated that there is some confidence in John when it comes to solving this part of the problem. He said "...I have to maximize... so I write the formula... This point will maximize the profit... I just wanted to be sure with my values..." The use of such words gave the impression that he is confident of the work he has presented and just has to double check using the calculator. The identity that he assumed is that of someone who is in control of what he was doing. The identity is indicated by the use of the first pronoun "I" when referring to what he did when solving the task. He was speaking like an expert in Linear Programming discourse.

Relationship

John said "the profit functions sometimes. But they say ... eh... the question is objective". Through this statement John brought into the picture somebody within the linear programming discourse (even though this is back grounded) that somehow asked these questions as if the meanings of the profit function and objective function are interchangeable.

Connection

There is a strong connection between the equation for the objective function and maximum income calculated using the vertices of the feasible region. Once John came out with the equation, he substituted the coordinates of the vertices of the feasible region and identified the

point that gave a maximum value. He ignored the part that required him to use a search line. The text therefore suggested that John could not make a connection between a search line and the objective function but used another preferred method which is inappropriate since the question required him to use a search line method.

Politics

John acknowledged that the equation for objective function can be used to find the maximum income and he appeared to view the equation to be the preferred method for finding maximum value in linear programming discourse. We see that in “I have to maximize... so I write the formula...” (Utterance 101).

Sign system and knowledge

The equation of the objective function appeared to be the first thing that John is supposed to find. He was able to talk about the relationship between the points that will give maximum, the process of maximising fluently. For example, he said “...it will be finding the point that will give maximum ...I have to maximize... so I write the formula ... which is the profit function...” (Utterance 101) John also indicated in his working which point would maximise the income. In his working, he referred to maximum profit and then changed to maximum income apparently with reference to the objective function. He also mentioned that sometimes the question asks for profit function, we see that in utterance 103, which suggests that John is making a connection to profit function but at the same time he realised that the question was asking for maximum income. However, John did not use the prescribed method of the search line. Perhaps he did not know how to do it which suggests lack of procedural knowledge.

Discussion

John’s recognition of the need to write down an equation for the objective function suggested that he had some access to the linear programming discourse, even though he did not use a search line method to calculate the maximum income. Furthermore, the manner in which he substituted the values of x and y in the objective function implied that he was trying to apply some learnt strategies without further reasoning. This is an indication that his access to linear

programming discourse was not at the peripheral level. In other words, John was able to recognize that this task has a particular method for its solution.

Moving into the next episode, we see that John positioned himself as an immigrant learner who understood mathematical concepts better through the medium of French, a language he has been exposed to through primary and some secondary years of his schooling.

Episode 5: Language choice

A number of researchers have shown that learners may express themselves and understand mathematics better in a language that they are fluent in rather than in a language that they are still learning (Kazima, 2006; Moschkovich 2002b). So I wanted to know the language John used when learning mathematics as shown below:

85. Thulie Now I want to know which language do you use in mathematics?
86. John [*Je fais mon travail en Français pour ne pas faire des erreurs ... J'utilise toujours le Français ... en Anglais je ne sais pas certains mots ... donc le Français je connais ... j'ai fait les mathématiques en Français...*] I do my work in French so that I do not make mistake... I always use French ... English I do not know some of the words... so French I know...have done mathematics in French...
87. Thulie When do you use English?
88. John [*Je n'utilise pas beaucoup l'Anglais en mathématiques ... parfois j'échange... mais pour les mathématiques, j'utilise le Français...*] I do not use much English in mathematics...sometimes I swap ... but for mathematics I use French...
89. Thulie Do you sometimes find yourself thinking in English and sometimes in French?
90. John [*Non ... parce que ... En mathématiques je pense en Français ... sauf lors de la lecture du manuel ... car ils sont en Anglais?*] No... because ... in mathematics I think in French... except when reading from the textbook... since they are English?]

Analysis of Episode 5

Significance

John said, "...I do my work in French so that I do not make mistakes... I always use French... have done mathematics in French..." (Utterance 86) and his emphasis is on the word 'do not make mistakes'. By including the word 'always', makes his obvious use of French in mathematics significant. John further states that, "...in mathematics I think in French..." and this serves further to make French significant when learning linear programming.

Identity

John positioned himself as not being able to do mathematics in English and uses French for thinking and emphasises that "...I do my work in French so that I do not make mistakes..." He further mentioned that he used English when reading textbook thus "...except when reading from the textbook... since they are English..." (Utterance 90). This suggests that his preferred language when learning linear programming is French. John seems to be confident when using French.

Relationship

John said "...except when reading from the textbook... since they are English..." which suggests an 'I' and 'them' observation of the players involved in preparing linear programming content in mathematics textbook. It seems that his use of 'they' was more generally to refer to the textbooks recommended at his school. Perhaps if he was given a choice he would read material in French.

Connection

John was making a connection of the mathematics he learnt through the medium of French and the linear programming he is learning through the medium of English. He said, "... I do my work in French so that I do not make mistakes... I always use French ... English I do not know some of the words... so French I know...have done mathematics in French..." which suggested that he was confident that he could succeed as long as his solution to the linear programming task was presented correctly in the mathematical language.

Politics

John privileged French when doing linear programming. "...I do my work in French so that I do not make mistakes..." There was a sense that if he tried to use English he might make mistakes and be disadvantaged in some way. However, he recognised that he had to read English textbooks at school.

Sign system and knowledge

John raised the issue of the relationship between meaning of some words in English and mathematical concepts which is complicated. Especially mathematical meaning of some English

words as opposed to everyday usage “... I always use French ... English I do not know some of the words... so French I know... have done mathematics in French...” According to Schleppegrell (2007) academic language may differ from one discipline to another. Therefore research argues that academic uses of language as well as the meaning of individual words need to be explicitly taught for immigrant learners to fulfil the discourse requirements privileged in academic settings. Teaching academic language, English, would enable John to understand the material encountered in linear programming. This is because the language of academic texts has distinctive features and meanings that may present a contrast to the language used in informal spoken interaction. But John was aware of the different meanings of words he then decided to use French in order to be sure of the mathematical meaning as the teacher knows French. Linking it to French terminology could perhaps be helpful, that is French / English terminology.

Discussion

John constructed himself as a learner who could communicate linear programming concepts in French as opposed to English. The episode showed that the experience of migrating from the DRC to South Africa led to an opening of opportunities for John to learn and use English at school. John was responding to the new circumstances which involved, drawing on and refashioning practices associated with his cultural values while constructing his new identity within the private and public domain of his life. The language choices that John was making in this new environment included efforts to take up new languages at the heart of his response to these circumstances. John was still relying heavily on his French to facilitate and negotiate new relationships and construct new identities (Martin-Jones, 2011). These identities were contextualized, that is, were located in experience and opportunities that arose during the teaching and learning of linear programming.

8.6.1.5 Any opportunity for John to learn linear programming?

The analysis of John’s activity and explanation in the interview has shown that access to linear programming meant that his practices involved more than engaging with the text, it involves extracting inequalities from the given task, representing these inequalities graphically, knowing and using graph paper, calculators, ruler, and pencil in order to be recognizable in the learning of linear programming. Sometimes John did not understand the information written in English for

the given text, but was able to write text recognisably within the linear programming discourse. He was able to do this because he had learnt how to communicate through signs and symbols (semiotic domain), that text associated with a set of practices recruit one or more possibilities, such as oral or written language, equations, symbols or graphs to communicate distinctive type of meanings distinctive to that domain (Gee, 2003). For example, John capitalised on the meaning of the following phrases in the linear programming task he was solving: ‘no more’, ‘at least’, ‘does not exceed’, which are of cardinal importance to finding constraints inequalities. If you do not know the mathematical meaning of these words then you are not likely to come out with the correct inequalities.

It is obvious from the analysis of his activity that John could transfer knowledge and skills acquired in one language to his new language of learning and teaching (Cummins, 2000). Even though he did not understand the language in which the task was presented, John believed he could solve the task. John negotiated the English by straight memorisation of the meaning or implication of some key words in linear programming and followed the teacher’s steps presented during the lesson. When faced with several English sentences, John was able to find correct constraints inequalities by focusing on the mathematical meaning of key words and ignoring most of the other words. He could translate key words into mathematical symbols. He further provided a solution based on his understanding of the key words he recognized in the question. The solution indicated that John understood the steps involved when solving linear programming tasks. In conclusion, the analysis shows that despite having limitations to his understanding of English John has had an opportunity to learn linear programming at this school thanks to teacher support and his own use of learned strategies.

In the next section I present an analysis of Bheki. I start by presenting his profile. This will be followed by an analysis of his activity and then his talk about the activity.

8.6.2 Bheki’s background information

Bheki is eighteen years old. He was born in South Africa to a Mozambican father and a South African mother. He lives with both parents and other members of his family. Bheki speaks Xitsonga, isiZulu and sometimes Portuguese at home with his father. He understands eight of the

eleven official languages of South Africa which are English, Afrikaans, isiZulu, Sesotho, Setswana, Sepedi, isiXhosa, Xitsonga, isiNdebele and isiSwati. Bheki can speak English, isiZulu, Setswana, Xitsonga and Portuguese. He can write and read English, isiZulu and Xitsonga. It is worth noting that his language practices are typical of South African learners who grew up in the Gauteng province, which is known for its multicultural and multilingual nature. Bheki is studying Xitsonga as a home language and English as a second language.

This section presents an analysis of the clinical interview conducted with Bheki. I asked him questions while solving the task. But as mentioned earlier I will present an analysis of his activity before analysing his talk. The analysis of Bheki's activity is according to the categories presented in Tables 8.1, 8.2 and 8.3. I start by presenting the task and its solution for reference.

8.6.2.1 Task 2

A local health board is producing a guide for healthy living. The guide should provide advice on health education, healthy lifestyles and the like. The board intends to produce the guide in two formats: one will be in the form of a short video; the other as a printed binder.

The board is currently trying to decide how many of each type to produce for sale. It has estimated that it is likely to sell no more than 10 000 copies, of both items together. At least 4 000 copies of the video and at least 2 000 copies of the binder could be sold, although sales of the binder are not expected to exceed 4 000 copies. Let x be the number of videos sold, and y the number of printed binders sold.

5.1 Write down the constraints inequalities that can be deduced from the given information.

5.2 Use graph paper to represent these inequalities graphically and indicate the feasible region clearly.

5.3 The board is seeking to maximise the income, I , earned from the sales of the two products. Each video will sell for R50 and each binder for R30. Write down the objective function for the income.

5.4 Determine graphically, by using a search line, the number of videos and binders that ought to be sold to maximise the income.

8.6.2.2 Model solution: Task 2

5.1 x number of videos and y is the number of binders

No more than 10000 copies, of both items together, so $x + y \leq 10000$

At least 4000 copies of video, so $x \geq 4000$

At least 2000 copies of binder, although sales are not expected to exceed 4000, so

$2000 \leq y \leq 4000$

5.2

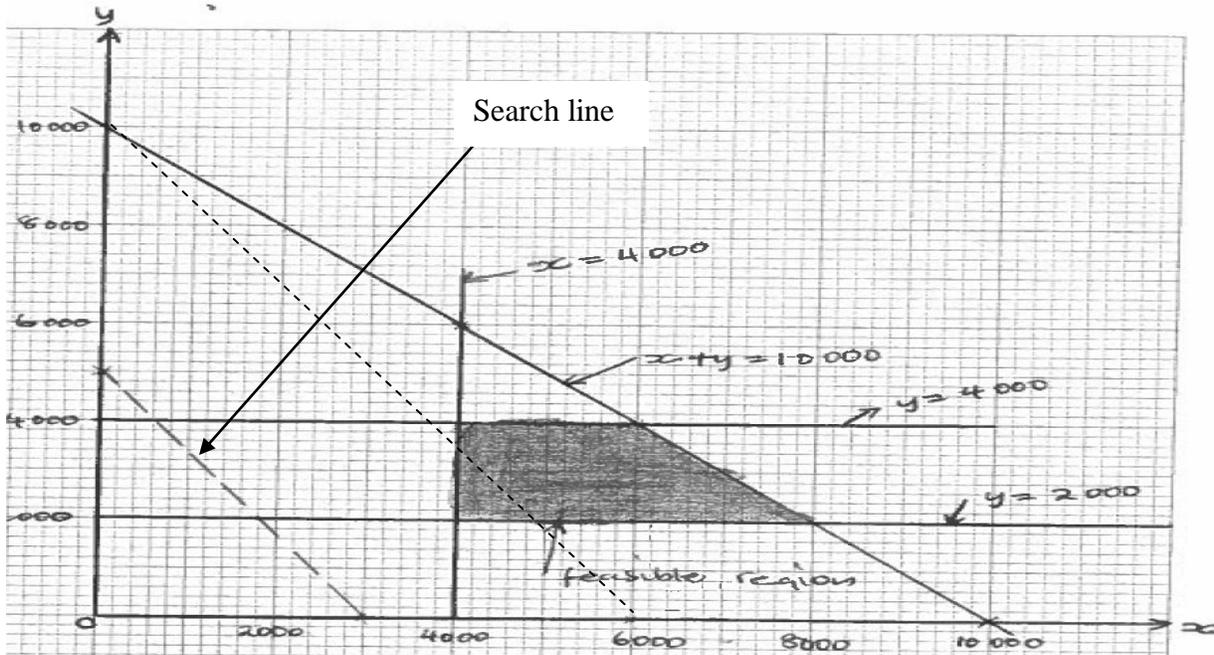


Figure 8. 6: Model feasible region for linear programming Task 2

5.3 Income (I) = $50x + 30y$

I can be found in one of two ways.

1. Taking each vertex of the feasible region in turn and calculating I for the $(x; y)$ values for these points, you find that $I (8000; 2000)$ gives the maximum value of I .
2. Draw the search line for the objective function. Then draw the objective function $I = 50x + 30y$, the line representing any other profit will be parallel to the search line and find the extreme point (the last point where the ruler touches the feasible region), which is $(8000; 2000)$ and substitute it in the objective function

The maximum income will be $50(8000) + 30(2000) = 460\,000$

To draw the search line, make y the subject of the formula; $y = (I - 50x)/30$

(Graham & Graham, 2010)

8.6.2.3 Analysis of Bheki's written work

The focus of this text is on the inequalities that Bheki constructed from the given task. These inequalities show that Bheki understood the social languages and situated meanings of some key words in the task to produce the responses shown in figure 8.7 below.

Constraints

$x = \text{Videos}$
 $y = \text{printed binder}$

Inequality 1 $x \geq 4000$
Inequality 2 $y \geq 2000$
Inequality 3 $4000x + 2000y \leq 10000$
Inequality 4 $y \leq 4000$

Figure 8. 7: Bheki's constraints inequalities

The above text indicates an understanding of the variables representing videos, x and binders, y . In the initial text of inequality 3, Bheki appeared to have multiplied x by the minimum number of videos and y by the minimum number of binders. However he cancelled the 4 000 and the 2 000 and the text showed that he realised that it was a misrepresentation and from his inequality it is evident that he interpreted the information in the task correctly. Bheki has four inequalities which indicate an understanding of all the inequalities to be extracted from the task. This suggests that Bheki interpreted all the aspects of the task correctly. His work is categorised as instance 4.

Instance 4: Evidence of correct interpretation of all aspects of the tasks

The work that follows shows the graphical representation of the inequalities shown in figure 8.7 to indicate the feasible region.

Graphical representation of constraints

The constraints obtained by Bheki were then represented graphically to show the feasible region indicated below:

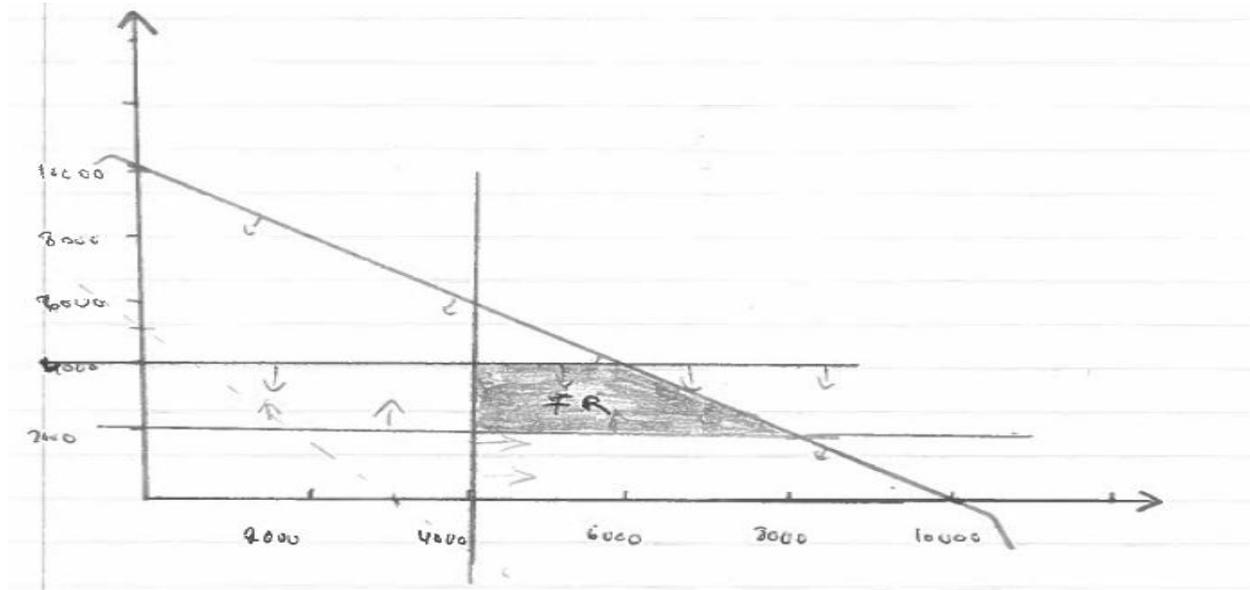


Figure 8. 8: Bheki's feasible region

The graphical representation of the inequalities shown in Figure 8.7 represents the feasible region. For each inequality the region is indicated by arrows. The shaded region represents the region where the inequalities are satisfied and labelled FR. FR stands for feasible region. The text shows all inequalities identified and are represented correctly even though Bheki did not label the boundary lines. There is no method shown on the graph, maybe because these are simply straight line graphs either x or y lines.

Instance 4: The text shows evidence of correct representation of inequalities on a Cartesian plane

The next section shows Bheki maximising the objective function using the search line method.

Objective function/ search line

To obtain the coordinates of the point that will give a maximum, a search line is drawn. In order to draw a search line given the objective function as $P = 50x + 30y$ you make y the subject and find the gradient of the line. Then one has to change the gradient according to the scale using equivalent fractions. Bheki sketched the first line as shown below.

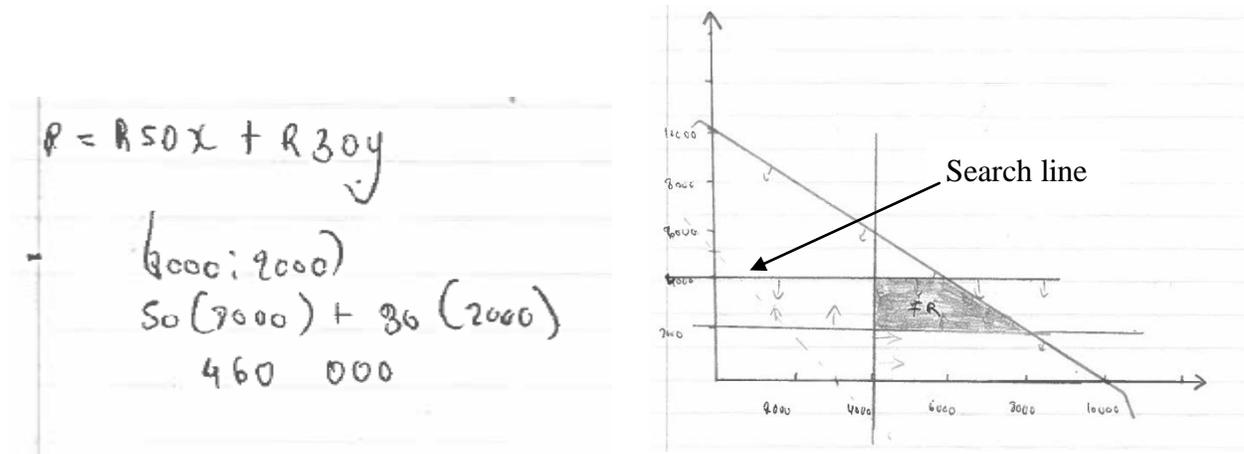


Figure 8. 9: Bheki's search line method

The objective function is presented in the first line. This is followed by one of the coordinates of the vertices of the feasible region which he obtained by using a search line indicated by a broken line. The coordinates of the point are then substituted in the equation of the objective function (second and third line). This demonstrates the correct establishment of an equation of the objective function which led Bheki to the correct solution. His work is categorised as instance 4.

Instance 4: The text is appropriate and fully correct which lead Bheki to a correct solution

Discussion of Bheki's written work

The analysis of Bheki's work illustrates correct inequalities extracted from the given task. These inequalities are then correctly presented graphically to show the feasible region. Bheki's written responses demonstrate that he mastered the skills necessary to enable him engage with the linear programming task successfully. His attempt shows an excellent understanding of the linear programming discourse and the methods used are clearly evident in the text.

The next section will present an analysis of Bheki's explanation while solving the task.

The context is about a manufacturer who produces a health guide in two formats, a video and pamphlets in the form of binders. The episode shown below focuses on how Bheki extracted the inequalities. The interaction went as follows:

8.6.2.4 Analysis of Bheki’s explanation during the interview

Episode 6: Extracting inequalities

In this episode Bheki was finding the inequalities represented in the task. So he started by looking for key words like “no more than” and assigned the correct inequality sign as shown below:

62. Bheki x is the number of videos and y is the number of pamphlets and then at least forty thousand copies which means x is greater or equal to forty thousand and y is greater or equals to twenty thousand [*lapha bathi*] here they say estimate [*ukuthi izothenga*] that they will sell no more than ten thousand copies which means forty thousand x plus twenty thousand y is less than or equal ten thousand
63. Thulie How do you know that the inequality is less than?
66. Bheki [*angithi bathi no more than ukusho ukuthi akufunekanga ukuthi amavideos nama banners andlule ku ten thousand copies*] they say no more than in the question, so the number should not exceed ten thousand copies.
68. Bheki y is less or equal to forty thousand which means [*amaprinta akufunekanga andlule ku forty thousand ama constraints lawa*] the printed material should not be more than forty thousand copies
74. Bheki both means the two items the video and the binders that’s why I say forty thousand x and twenty thousand y is less or equal to ten thousand
76. Bheki Although...
80. Bheki It means it lies between the two of them

Analysis of Episode 6

Significance

Bheki recognised the significant role that correct inequalities play when solving linear programming tasks. We see that in the use of the modal verb ‘should’ which indicated that he makes ‘no more’ significant “...they say no more than in the question, so the number should not exceed ten thousand copies...” shows that Bheki was indicating that ‘no more’ is represented by a less than inequality sign.

Identity

From Bheki's utterances, we see that he was in control, he knows the symbolic meaning of some words in linear programming and it enabled him to write down the correct inequalities. "...both means the two items the video and the binders that's why I say forty thousand x and twenty thousand y is less or equal to ten thousand..." his use of the words *that's why* suggested that he was justifying his actions and was confident with his inequality.

Relationship

Bheki said, "...they say no more than in the question, so the number should not exceed ten thousand copies..." through the use of this statement, Bheki brought into the picture some people within the Discourse of linear programming that assign the meaning of these phrase 'no more'. It seemed Bheki was developing self confidence in that he seemed able to make a decision about the authenticity of the given statements.

Connections

Bheki's response suggested that there is a link between some key words in linear programming and their symbolic representation using inequalities. This is implied in his statement when he said "...they say no more than in the question" and he connects it with "...so the number should not exceed ten thousand copies..." He interpreted the question correctly by connecting with the correct inequality sign in this context.

Politics

Bheki acknowledged that knowing the symbolic meaning of some words in linear programming would enable him to write down the correct inequalities representing the information given in the task. We see that when he says, "...they say no more than in the question, so the number should not exceed ten thousand copies..." while he talks about 'they', referring to those who prepared the task, Bheki privileges knowledge of these words.

Sign system and knowledge

Bheki seemed to have recognised that he should approach the task by searching for key words or phrases that would lead to inequalities. We see that when he says, "...they say no more than in

the question, so the number should not exceed ten thousand copies...” This suggested that the key words are ‘no more’ and he assigned the correct inequality sign. This suggests an opportunity to find the inequality sign.

Discussion

Bheki has recognised that knowledge of the mathematical meaning of key words in the task was necessary. He correctly assigned inequalities in the given linear programming task which shows his knowledge of the phrases in the given linear programming task. This seemed to enable him to engage with the task in a meaningful way. Bheki’s recognition of key words suggested that he had access to the Discourse of linear programming he was learning in a second language.

In the next section I present his approach when representing the inequalities obtained in figure 8.7 graphically.

Episode7: Graphical representation

- 85. Thulie ... draw the graph?
- 86. Bheki ...the graph, the feasible region
- 92. Bheki ...I can draw it in my exercise book (learner drawing axes)... BEFORE I sketch [...*sengiyachekha kutsi the scale sitawufitha yini?*] I check if the chosen scale will fit
- 94. Bheki ... Which means x plus y is greater or less than ten thousand...
- 96. Bheki ... which means it is going to be one thousand noma (or) one thousand five hundred, noma (or) two thousand
- 98. Bheki ...my x intercept is going to be ten thousand
- 102. Bheki ...then less than four thousand... this one is... x is less than four thousand I am going to ... x is four thousand then y is forty thousand then it is going to be less than..
- 104. Bheki [...*nansi ifeasible region*] here is my feasible region

Analysis of Episode 7

Significance

The text shows that Bheki drew attention to the importance of showing a correct scale on the graph before representing the inequalities graphically. We see that in his use of the word ‘before’ with emphasis in utterance 92 when he says “... BEFORE I sketch ... I check if the chosen scale will fit...”

Identity

In handling the task appropriately, Bheki was enacting the identity of a learner who was in not constrained in the linear programming discourse, and we see that when he uses the 'I' repeatedly as follows; "...I can draw it ... Before I sketch ... I check if the chosen scale will fit..."

Relationship

Bheki does not indicate any relationship with other people. He seemed to be regulating his solution processes when he said "...I can draw it in my exercise book ... Before I sketch ...I check if the chosen scale will fit". His use of my exercise book suggested that he had confidence and could even use a squared paper instead of a graph paper.

Connections

Bheki was making a connection to drawing inequalities with correct scale on the x and y axes so that it was relevant to the given information. Bheki says "... Before I sketch... I check if the chosen scale will fit..." he was showing the connection between the importances of a proper scale in order to draw a meaningful graph that can be easily understood.

Politics

The text shows the relevance of choosing a correct scale before representing the functions geometrically. Bheki made it clear that "... Before I sketch ... I check if the chosen scale will fit..." and then he identified intercepts on the axes. This suggested that in order to represent functions graphically, it was important to think of a scale which was according to the values of the intercepts.

Sign system and knowledge

He seemed confident with the knowledge of graphs as could be seen in his use of "... I check if the chosen scale will fit..." His use of the verb check means that he wanted to examine the scale in order to establish whether the given conditions would fit.

Discussion

The analysis suggests that Bheki was able to draw the graph by first making sure that the scale was correct. This is the first step that leads to a correct graph especially when given constraints inequalities. The connection that Bheki made between the equations of the lines and the scale suggested an ability to represent the inequalities graphically. He seemed to be aware that in order to draw a line you first have to decide on your scale. In my experience as a mathematics teacher it is necessary to connect scale to the equation that can be represented graphically. His graph on Figure 8.8 does show a consistent scale. Bheki had confidence in what he was doing.

In the next episode, Bheki is using the search line method to find the maximum value of the objective function.

Episode 8: Optimising using a search line

118. Bheki search line ...*[lana i-scale asilingani kufuna kuthi sizoyibeka lana kufuna kuthi sitimeze 100 izonginika five thousand then I five thousand is between four thousand and six thousand then kufuna ukuthi sizoyibeka lana. Then lena si times nge 100 nayo isinika three thousand which is between two thousand and four thousand nayo sizoyibeka lana then kufuna kuthi sidrawe i-broken line broken line to maximise lapha bathi to maximise the income]* To draw a search line we must make sure that the scale is the same, so we multiply by one hundred to get five thousand which is between four thousand and six thousand. Then this one we multiply by one hundred to get three thousand which is between two thousand and four thousand then we plot here...then we have to draw a broken line to maximise here they say maximise income...
120. Bheki I am going to use the search line
122. Bheki *[Lapha]* here... I am going to move the ruler *[lana ku]* in the feasible region then the last point to touch the ruler is my point *[elizonginika]* which is going to give me maximum profit

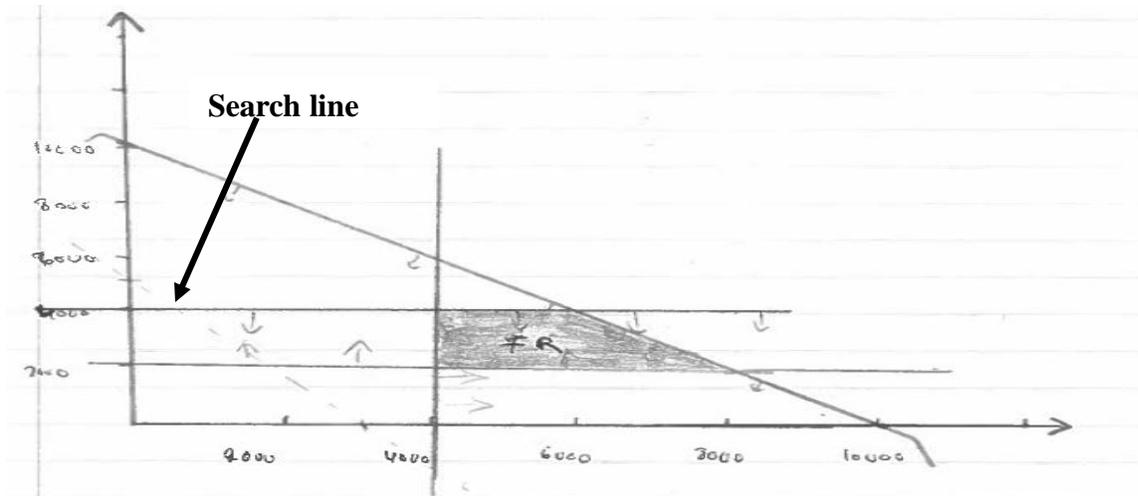


Figure 8. 10: Bheki's search line solution

Analysis of Episode 8

Significance

Bheki attached significance to the scale in order to use the search line on this graph. "...to draw a search line we must make sure that the scale is the same, so we multiply by one hundred to get five thousand which is between four thousand and six thousand. Then this one we multiply by one hundred to get three thousand which is between two thousand and four thousand then we plot here..." (Utterance 118).

Identity

Bheki was able to follow the steps by using the search line, first he was aware that in order to draw the search line the coefficients of x and y in the objective function should be multiplied by a multiple of 100 suggesting that he was in control, he was not at a disadvantage, he appeared to be confident as he engaged with the task. We see this in the way he states what he is doing "...I am going to use the search line ...I am going to move the ruler..." (Utterance 118).

Relationship

Bheki made reference to 'we' when he says "...we must make sure...we multiply by... this one we multiply by..." When he introduced 'we' he referred to himself and perhaps other learners, and with the community of mathematicians which suggested that this is how it is done by

everyone. The explanation suggested that the procedure is done in class with the teacher perhaps or other learners. We see that in his repeated use of the “we” to support his position.

Connections

He made a connection of a scale and the coefficient of x in the objective function. This connection suggests that a correct scale would lead to a meaningful representation of a search line. We see that in his emphasis of multiplying the coefficient of x and y in the objective function. This is done in order to draw the search line according to the scale on the axis.

Politics

Bheki was aware of his ability to engage with the task, and seemed to be confident with his approach to the task. He is privileging the use of points that are easily identifiable on the axes in order to plot and draw a search line.

Sign system and knowledge

Bheki gave an impression that he had an understanding of how to draw and use a search line. We see that not only in the skills he displayed when engaging with the task, but also when he explained that, “... I am going to use the search line...I am going to move the ruler...” which meant that the ruler would be moving parallel to the search line. The movement is denoted by a dotted line. This was another confidence in knowledge of drawing of parallel line using an appropriate tool at that time.

Discussion

Bheki did not ignore the context of the question as he referred to the scale, multiplying to make sure that it was appropriately represented on the graph. Furthermore, he used his knowledge of how to use a search line to find the last vertex touched by the ruler. This led him to the correct solution. Bheki was using his knowledge of linear programming processes. From his explanation, the discussion suggested that Bheki understood the task sufficiently as a result he was able to arrive at the correct solution.

In the next episode, Bheki turned his focus from a narrative of what he did when solving the task for the clinical interview to a misunderstanding he had in another linear programming task he had written in a linear programming test before the clinical interview. Apparently his solution was not correct because he did not construct all the inequalities that were represented in the text.

Episode 9: Bheki's challenges when solving linear programming tasks

134. Bheki But sometimes [*kunekubhala kuthi*] they write half straight, [*mine angizange ngibone kutsi ihalf yi constraints*] I did not recognise half as a constraints
135. Thulie ...so in other words are you saying it was tricky?
136. Bheki no [*bebabhale half not i-fraction so angikhonanga kuyisolva*] they had written half in words not symbolically, I was not able to solve it... otherwise I would have solved it
137. Thulie Oh! Now [*ngiyabona bebabhale half ngemagama*] I see it was not symbolically which means you are used to symbols like one over two?
138. Bheki [*Yebo ngoba its mathematics ngangayiboni kutsi*] yes because it is mathematics I did not realise it is one of the constraints

Analysis of Episode 9

Significance

Bheki said, "...I did not recognise half... I would have solved it ... I did not realise..." (Meaning half written in words instead of a symbol one over two), and his emphasis is on the word "I would have solved it". By including the word 'would' he made his inability to solve the task significant, and it emphasised that the manner in which the half was written played a role in his failure to extract the correct inequality.

Identity

His identity in relation to the task shows that he was disadvantaged by not knowing that half written in words is one of the inequalities. This suggested that he was unable to write the correct inequality (line 136) in relation to the task. The lack of understanding distanced him from the task we see that in utterance "I did not recognise half as a constraint ..."

Relationship

Bheki's positioning was such that he appeared to view the task as being compiled by some authority or body of people in the mathematical community in such a way that it was difficult for

him to understand as a result it confused him. "...they had written half in words not symbolically, I was not able to solve it... otherwise I would have solved it..." which suggested a sense of those who gave such tasks.

Connections

There seemed to be a connection that Bheki is making with this type of question with respect to changing words to symbols. "...I did not recognise half... I would have solved it ... I did not realise..." implies some kind of responsibility as a result of earlier specification in the linear programming tasks. Bheki could not make connections of half in words and the mathematical symbol at that time he was suppose to.

Politics

The task appeared to be difficult when expressed in ordinary language because they require conversion to mathematical meaning. We see that when Bheki repeatedly made statements like "...I did not recognise half... I would have solved it ... I did not realise..." which implied that he was expecting symbols in linear programming because it is some accepted practice in mathematics. The meaning of half may have passed him by because it was written in ordinary English.

Sign system and knowledge

Bheki had identified his mistake and it appeared that he had recognised that half in words has the same mathematical meaning as the symbol $\frac{1}{2}$. He seemed to be disadvantaged by the use of words instead of mathematical symbols. "...I did not recognise that half is one of the constraints... they had written half in words not symbolically, I was not able to solve it... otherwise I would have solved it... because it is mathematics I did not realise..."

Discussion

Writing words instead of mathematical symbols in linear programming might not lead to the correct solution. In the episode above, because of the positioning by some authority, Bheki was unable to write down the constraints inequality. This created confusion and highlighted the skill

he was lacking, namely converting words to another representation like symbols and it limited him in the broader political sense (Gee, 2005a).

Most of Bheki's explanations were in isiZulu which suggested that he might process linear programming in this language. It is worth mentioning that Bheki's home language is Xitsonga. During classroom observation, most utterances were in isiZulu (Table 4. 8 in Chapter 4) and his teacher's home language is isiZulu.

8.6.2.5 Any opportunities for Bheki to learn linear programming?

The episodes discussed above suggest that Bheki used an approach learned in class during the teaching and learning of linear programming. He was aided by making connections in the text to key words and their mathematical meaning. Bheki did not ignore the real life context as he mentioned that people like to watch videos than reading pamphlets. The nature of the context is familiar to Bheki and he was not excluded in the discourse. He was able to move between ordinary English and mathematical English which was not a source of difficulty. Bheki focused on keywords that helped him switch to mathematical meaning and aided his understanding of linear programming. For example in the episode, no more than was used to provide the inequality appropriate to the given expression. Allowing Bheki to engage with the activity enhanced the development of the content under discussion. The development of linear programming depended on the extent to which he was regarded as an active participant in the learning process and not a listener.

In the second and third episode he was focusing on the correct scale to represent the information graphically. These episodes showed that he had mastered the necessary skills and is aware that correct and readable graph will lead to proper values of maximum and minimum point when optimising.

The next section presents an analysis of Allen's clinical interview. I first present his profile. This is followed by an analysis of his written work and the analysis of his explanation as he was solving task 2. Task 2 is the same task solved by Bheki. (Solution showed on pages 228 and 229).

8.6.3 Allen's profile

Allen is seventeen years old. He was born in South Africa to an Angolan father and a Mozambican mother. He lives with his mother and other family members not far from the school. His father migrated from Angola while his mother migrated with her family from Mozambique. According to Allen, the family speaks mainly Portuguese at home. Her mother speaks Xitsonga and Portuguese. Siswati and isiZulu are the other languages spoken in the community where Allen's family live.

Allen is in an environment that does not support the speaking of English. So he has less support for English, the language of learning and teaching. This environment has been referred to as a foreign language learning environment (Setati, Adler, Reed and Bapoo, 2002). These researchers argue that in such an environment, the language of learning and teaching is only heard and spoken at the school.

Some teachers and learners at School C speak Setswana, Shona, Tshivenda and Xitsonga. The school guard who mans the gate speaks isiZulu. He is exposed mainly to Xitsonga because many people in the surrounding speak it. When he grew up he was exposed to a range of languages and is now able to draw on languages like Siswati, isiZulu, Xitsonga and English. He rarely speaks Portuguese with his friends.

During the interview Allen mentioned that he speaks English only at school or during classroom sessions that are taught in English. He also mentioned that he does not speak English either outside the classroom or at home. Allen also understands isiXhosa and isiNdebele. He started his schooling in South Africa. Allen is studying Xitsonga as a home language and English as second language. However, he was reluctant to share some of his background information. His argument was that it might result in changing his South African identity document, because maybe the information is not reflected.

8.6.3.1 Analysis of Allen's written work

In linear programming learners have to make sense of the problem written in English. They are expected to extract inequalities, solve the linear programming task in English, this expectation

poses a very challenging task particularly because children from non-fluent families typically encounter English only in school and may not grow proficient in English (Adler, 2001; Setati *et al*, 2002). The challenge is reflected in Allen's inequalities below.

Constraints inequalities

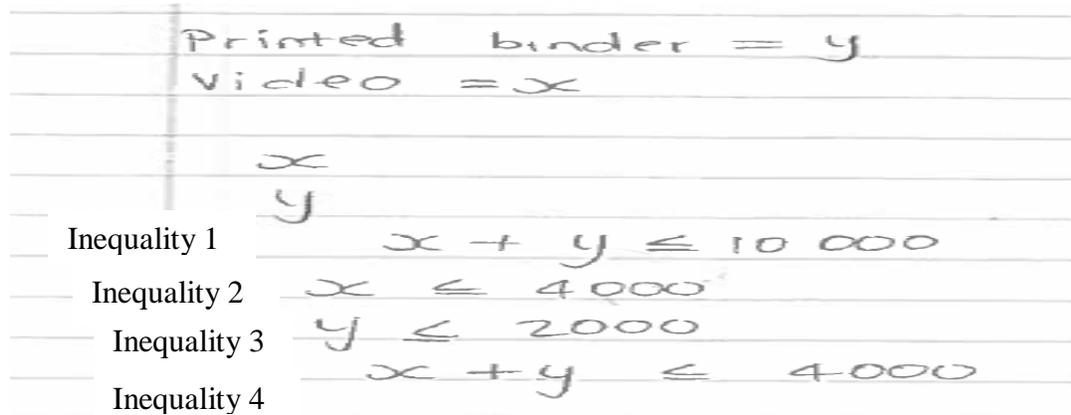


Figure 8. 11: Allen's inequalities

The above inequalities indicate that Allen wrote down the correct letters for the binder and video. The first inequality is correct suggesting that he was able to interpret part of the information in the task correctly. However, the second inequality shows a misrepresentation of the inequality sign to be representing at least four thousand videos. The same applies to inequality 3. He had less than two thousand instead of greater than two thousand. This suggests that Allen interprets 'at least' as 'less than or equal to' instead of 'greater than or equal to'. Inequality four is not connected to the binder not having to exceed 4000 however it is not clear why he added the ' $x + y$ '.

Instance 2: Evidence of misrepresentation of the inequality sign.

Representing inequalities graphically

5.1	$x + y \leq 10\,000$	10 000 ; 0
	$x \leq 4\,000$	0 ; 10 000
	$y \leq 2\,000$	
	$x + y \leq 4\,000$	4 000 ; 0
		0 ; 4 000

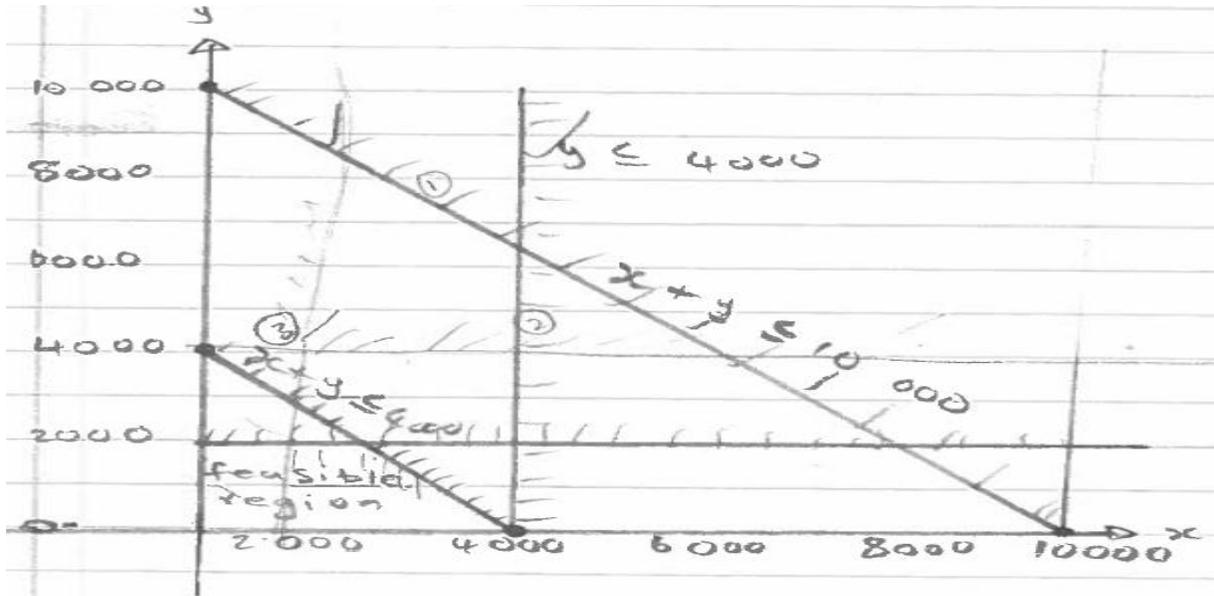


Figure 8. 12: Allen's feasible region

The text indicates a correct representation of the inequalities obtained from the first part. Allen used the dual intercept method to find the points to plot in order to draw the lines $x + y = 10\,000$ and $x + y = 4\,000$. The text shows correct regions representing the inequalities. Allen indicated the feasible region according to his inequalities. However because some of the inequalities constructed in the first part are not correct his solution is also not correct. But the focus is not on correct or wrong solutions but on how he engages with linear programming.

Instance 4: evidence of correct representation of inequalities on a Cartesian plane

Mathematically correct optimising

$$P = 50x + 30y$$
$$P = 50(3000) + 30(2000)$$
$$= 150\ 000 + 60\ 000$$
$$\therefore P = 210\ 000$$

Figure 8. 13: Allen's optimising solution

The text indicates a correct equation of the objective function. The maximum point is in relation to his feasible region. There is substitution of x and y values in the objective function to find the value but it is not clear how he obtained the extreme point. The maximum point is in relation to his feasible region. However the solution is not correct because of the inequalities which he interpreted incorrectly.

Instance 4: Evidence of correct mathematics

Discussion

Allen's attempt at the task showed that he was not able to interpret 'at least' given in the task correctly. This resulted in incorrect inequalities because of misrepresentation of the inequality sign. However his approach was appropriate and does show practices necessary in linear programming. This is not surprising, most multilingual learners struggle with the interpretation of phrases like 'at least' and 'at most' (Adler, 2001).

Numerous literature has shown that language is one of the stumbling blocks for mathematics understanding, especially for learners who are learning in a second language (Setati, 2002). Research points to the fact that learners understand and draw on their experiences when the learners' home language is used as a support to a second language such as English (Nkambule, Setati & Duma, 2010).

The next section looks at how he explained the processes involved when solving the task. I asked him questions while solving the task. He started solving the task by constructing the inequalities. I must mention that my focus was not on whether the solution is correct or wrong but on how immigrant learners interact with a linear programming task.

8.6.3.2 Analysis of Allen's explanation during the interview

From my experience, learners learning in a second language appear to find linear programming difficult. These difficulties are frequently pointed to their inability to construct correct inequalities due to lack of English language proficiency. The incident below shows that one of the factors that led to incorrect inequalities when solving the linear programming task was assigning an incorrect inequality signs to the phrase 'at least'.

Episode 10: Extracting inequalities

40. Allen At least four thousand copies per video. Okay, at least four thousand. At least two thousand of the copies can be sold. At least is less than or equal to...(silence)
41. Thulie Write down the inequality sign to represent at least four thousand, is that okay
42. Allen Two thousand is for y, it's separate. For four thousand less or equal to x and ...two thousand less or equal to y. $x + y$ is less or equal to four thousand ($x + y \leq 4\ 000$). Less or equal to?
43. Thulie You've got the constraints?
44. Allen Yes, (silence looking at his inequalities) Four thousand.
 [O *mise di statement*] you mixed the statement] $x + y$ is less or equal to four thousand ($x + y \leq 4000$) Its not four thousand, it's ten thousand
 It's more likely more than ten thousand per piece for both items.
 [Ya , *ten thousand ne ele ya x +y*] Ten thousand is for $x + y$
 I cannot explain now...my inequalities are confusing...I will use the graph paper to represent this inequality graphically and indicate the feasible region (scratching his head...long pause...)
45. Thulie Okay...you want to represent your constraints graphically?
46. Allen Yes, but I am not sure [*ngatsi*] it is not making sense to me now...but...anyway [*amaconstraints ami lawa*]...These are my constraints...yes... counting...one two three four... jah! All of them?

Analysis of Episode 10

Significance

Allen attached significance to the meaning of ‘at least a certain number’ as less than or equal to and he appeared to be consistent with the meaning which resulted to incorrect inequalities. However, his expression of, “I cannot explain now...my inequalities are confusing...I will use the graph paper to represent this inequality graphically...” (utterance 44) emphasised the difficulty that he experienced with the task. Later on in his explanation he repeats (see utterance 46) which emphasises the idea that he cannot explain the meaning of his inequalities.

Identity

Allen seemed to be talking from a position in which he lacked confidence. He started his explanation with a statement that suggested that the meaning of ‘at least’ in the task has confused him (utterance 40). In his discussion of the constraints inequalities and the solution of the problem reduced to an impossible situation, and Allen expresses his confusion by stating, “...I cannot explain now...my inequalities are confusing...” (utterance 44).

Relationships

Allen made references to “...you mixed the statement...” (see utterance 44, line 2), the use of the second person pronoun ‘you’ refers to the person who is doing the problem. He uses *you* in the utterance, whereas he uses “I” when he says, “...I cannot explain now...my inequalities are confusing...” (utterance 44, line 6). In the first instance Allen seem to see that there is a problem with his inequalities, so he changes the subject of his utterance to you, implying some sort of blame that is attached to this fictitious person.

Connections

Allen made a connection between his inequalities and their graphical representation which might have directly helped him in understanding the relationship between number of videos and binders to be sold. It is depicted in utterance 45.

Politics

The text shows that Allen could not relate the mathematical meaning of ‘at least’ to the inequality ‘greater than or equal to’ and he seemed to attribute this lack of understanding to something within the problem.

Sign system and knowledge

Allen gives the impression that his understanding of the phrase ‘at least’ is less than or equal to. However, his interpretation of the relationships given for the number of binders and videos to be sold in the problem should be between four thousand and ten thousand yet his inequality means that it should be less than four thousand. It appears as though Allen realized that his interpretation is not appropriate but suggested that he will represent the feasible region geometrically. From his explanation this appears to be a guess, because his reasoning behind the total number to be sold is not clear in his explanation in utterance 44. Allen then decided to move on to representing the inequalities graphically perhaps a geometric representation will be easy for him to explain.

Discussion

Allen misinterpreted the meaning of ‘at least’ in the context of the linear programming task. This resulted in wrong inequalities which he was not able to explain what they were representing. The analysis has shown that Allen appeared to be unable to access the necessary interpretation techniques that would allow him to make more meaningful inequalities towards a correct solution. This situation deprived him of a rich opportunity to master the meaning of ‘at least’. This resulted in Allen showing a lack of confidence when he explained the inequalities he had obtained. He ended up blaming certain actions to an unknown person as he explained his inequalities. By doing this he could be seen to only take responsibility for certain parts of the solution, whilst other actions (the more confusing) are attributed to others, and are possibly seen as being out of his control.

As argued earlier, learners can only arrive at the correct solution of the linear programming task if the inequalities are appropriate to the task. Misinterpretation will lead to an inappropriate inequality sign on the information found within the problem as shown in Allen’s work. He

needed support to interpret 'at least' in another language like his home language Xitsonga. Khisty & Chval (2002) argued that second language learners, like Allen, learn when they hear and see words in multiple contexts, especially words with multiple meanings in each context. The limited exposure of Allen to English impedes his construction of inequalities as he attached significance to the meaning of 'at least' which was assigned an incorrect inequality.

In the next section, Allen explains his understanding of the task situation which suggests that his challenge was mainly assigning the wrong inequality sign to the key word (i.e. at least).

Episode 11: Allen understands the task

47. Thulie ...What is the problem about nje...if you can explain...?
48. Allen A local health is producing a guide for healthy living...(silence)
50. Thulie ...you say a local health is producing a guide for health living. What is a guide by the way?
51. Allen Is on how to guide people on how to live.
52. Thulie ...it is about advising people to lead a healthy life. [*So ba khicita*] they make two...
53. Allen A short video and printed binders
54. Thulie [*Ngikuphi labafuna kukukhicita kube kunengi*] which one do they want to produce a lot?]
55. Allen Sorry
56. Thulie ...there is two thousand and there is four thousand?
57. Allen Yes.
58. Thulie Which one is four thousand?
59. Allen Is the video.
60. Thulie ...Do you have any suggestions? Why are they deciding to produce four thousand videos, are they not expensive?
61. Allen They are expensive but they can be real, like so many people like to see things on TV.
68. Thulie So if [*bakuhlanganisa koyi-two...*] if they add the two it shouldn't exceed a certain number. What is that number?
69. Allen Ten thousand.
70. Thulie ...Why are they having a limit instead of a minimum?
73. Allen They can produce more than what is needed by the people.

Analysis of Episode 11

Significance

Allen attached significance to the videos produced by the health guide we see that when he says “...they are expensive but they can be real, like so many people like to see things on TV...” This suggested that people would buy the videos even though they are expensive.

Identity

Allen seemed to understand the reason behind the production of the health guide, it is evident when he said, “...Is on how to guide people on how to live...” and further says “...they can produce more than what is needed by the people...” which showed that he understood the context of the task.

Relationship

There was the introduction of people, which suggested that Allen distanced himself from the people involved in the problem. We see that when he says “...is on how to guide people on how to live... they can produce more than what is needed by the people...”

Connection

From the above text, it could be said that Allen was connecting understanding the information better when watching the videos and disconnecting understanding when reading from the binders even though videos are expensive. His argument is that “...they are expensive but they can be real, like so many people like to see things on TV...”

Politics

The text indicates the relevance of producing videos that will be meaningful to the people. This leads to the reality in the information conveyed in the message. We see that when he said “...they are expensive but they can be real, like so many people like to see things on TV...”

Sign system and knowledge

The text suggested that Allen was aware that a minimum number of videos should be produced. We see that when he said “...they can produce more than what is needed by the people...”

Discussion

The analysis suggested that Allen understood the information in the task and he attached significance to the number of videos to be produced. He is making a connection between understanding digital information nowadays. His knowledge of the sign system showed that he was also aware that a minimum number should be produced. This suggested an opportunity when guided because his understanding was elicited to ensure comprehension of the language of the problem. Mercer & Sam (2006) argues that the guiding role of more knowledgeable member of the community in the development of children's knowledge and understanding can be important for their induction into the discourses.

In the extract that follows, Allen explained his graphical representation of the inequalities. In this particular situation, at the beginning of the episode Allen made an error in utterance 97. It took some guidance from the interviewer (Thulie) to resolve the issues embedded in the finding of intercepts and plotting them on the axis to draw the line representing $x + y = 10000$ as shown in the episode below. This episode indicate that facilitating Allen explanation of his approaches required patience on the part of the interviewer as there were many long pauses as he considered what to do and how to verbalise his actions.

Episode 12: Graphical representation

In order for Allen to represent the inequalities graphically, the first step was to draw the graph of a straight line which meant introducing the equal sign in place of the inequality sign. Which means $x + y = 10000$; $x + y = 4000$; $x = 4000$ and $y = 2000$. In the excerpt that follows, Allen used the dual intercept method. In his first step he does not introduce the equal sign in place of the inequality sign so he gets $x > 10000$, then he drew $x = 10000$. I then explained to him that he has to find the coordinates in order to draw a line as shown in the exchange below:

95. Allen Will let Y equal to zero(Y=0)
96. Thulie Yes, so what is x?
97. Allen x is less than or equal to ten thousand.(writing $x < 10000$)
Aaa (disagreeing) When your writing. When your drawing a line, you make this one an equal line (pointing at $x < 10000$).
98. Thulie Yes, this one an equal sign (pointing at $x + y \leq 10000$)
So make this one an equal sign. x plus y equals to 10000 and then when x is zero y is ten thousand it means it's the point zero and ten thousand(0;10000) not a line. Is it?

99. Allen Ya!
100. Thulie So it's zero, you said which one is zero?
101. Allen We said this one is zero(pointing at intercept on graph)
108. Allen It's zero ten thousand (0; 10000).
109. Thulie And then you join the two. That is your line.
110. Allen Aha!(glad to see the results)
111. Thulie Is it solid; is it solid or dotted first of all?
112. Allen It's solid.
113. Thulie It's solid, tha's okay. Now lets show the region represented by the inequality.
114. Allen Okay, pick a point. Which side are we going to shade, this one or that one? Let's choose zero and zero(0;0) . And we substitute the, zero plus zero is less or equal to Y. Zero plus zero is less or equal 10 000. So which is it? Is it false, you take the false side? Zero plus zero is equal or less than ten thousand. This statement is true, we shade the. We sahde the top part.
120. Allen The tricky ones are the ones where they are two.
We are going to shade this one. And this one we are going to say when y is equal to zero, x is four thousand, and four thousand is to zero(4000;0) is this point. And when x is equal to zero, it's zero is to four thousand (0; 4000) point zero is to four thousand is this one. And we shade below? We shade above.

Analysis of Episode 12

Significance

Allen was focusing on finding the coordinates of the intercepts of the line to represent the inequality $x + y \leq 10000$. We see that when he said "...will let x equal to zero and find y , and the let y equal to zero to find x ..." it showed that he attached significance to the dual intercept method. Unfortunately he introduced an inequality sign instead of an equation. Even though he attached significance to the dual intercept method

Identity

The text shows Allen's position as a learner who lacked the necessary skills to draw a sloping line, but after assisting him he spoke like someone in authority, a teacher. This identity is indicated in utterance 114 and 120. His repeated use of "...we are going to shade..." is like he was giving instructions to his students.

Relationship

There is the repeated use of 'we' in utterance 114 and 120 suggesting a teacher and learners. He was speaking like a teacher, instructing the person interviewing him, asking her questions. He answered most of the questions to which he seemed to know the answers.

Connections

At first the text shows a disconnection between an equation of a line and an inequality sign (utterance 97). However after engaging with the person interviewing him (utterance 98 to 110) he quickly recalled the necessary steps to follow as depicted in (utterance 114 and 120).

Politics

The text shows that Allen is privileging the dual intercept method. We see that when he said "...will let x equal to zero and find y , and the let y equal to zero to find x ". The loud voice signals some kind of authority and he wanted to grab the researcher's attention to his preferred method.

Sign system and knowledge

Allen gave an impression that he was able to explain the dual intercept method that could be used in order to draw the graphs of the inequalities "...I have to find the points where the line meets the two lines ...I put x equal to zero in this inequality ... then you get four thousand so the point is zero is to four thousand which is this one...then when y is zero... x is 4000 again it is this point...less than is below so I shade above the line..." However his knowledge was lacking something.

Discussion

The analysis of the text indicated that at the beginning Allen was not able to find the coordinates of the y -intercepts. He needed clarification and guidance when it appeared that there was an error or misunderstanding in his presentation of the work. The text showed that Allen disconnected coordinates of the intercepts and the equation of the line he wanted to graph graphically. His understanding was not going to lead him to the correct solution, but after interacting with the researcher, Allen was able to remember the mathematical practices involved in linear programming, like when showing the feasible region. Supporting Allen enabled him to move towards a deeper understanding about the method of finding the x and y -intercepts.

In the next episode, Allen shows how to optimise an objective function.

Episode 13: Optimising

The solution procedure recommended by the curriculum is highly prescriptive. When optimising, learners find the objective function, use the search line method or substitute the coordinates of the vertices of the feasible region. When finding the objective function, learners have to select the correct arithmetic operation and transform the given information into an objective function as illustrated in the episode below.

77. Allen ...In question five point three the binder is seeking for a maximising profitable of selling video for fifty rand and binder for thirty rand.
78. Thulie Oh, okay. [*A se balane*] lets write, now [*ngicabanga go the sithole iprofit function*] I think we must find a profit function. Let's try that one before we draw the profit function.
79. Allen P is equal to fifty x...
80. Thulie Why x?
81. Allen This fifty rand they are for the videos and our videos we said are x.
82. Thulie x is the number of videos, okay.
83. Allen And thirty y.
84. Thulie Because ibinders i-thirty rands, okay.
122. Allen Let's maximize the profit.
Let's maximize the profit.
When the profit we take on top and write furthest.
123. Thulie On top, what do you mean by on top?
124. Allen On our feasible region we mean the point that is on top and that is the right furthest.
125. Thulie Okay
126. Allen That will give us the maximum. Two thousand! And then our x is going to be...(silence)...our scale.
127. Thulie ...divide it into two.

Analysis of Episode 13

Significance

Allen attached significance to the point on top and furthest which he obtained from the feasible region. We see that when he says "...the point that is on top and that is the right furthest..." (utterance 122, 124). However it is not clear how he ended up choosing the point.

Identity

His explanation of the procedure suggested that something was missing. We see that in the repeated use of 'we', 'our' which shows that he is trying by all means to include the interviewer in his solution process. We see that in "... we mean the point...that will give us..." (utterance

122, 124,, 126). His position in relation to the task is dependent on guidance from the interviewer.

Relationship

Allen says "...our feasible region...our x...let us..." which suggested the players involved when maximising. In this context it seems the people involved are the interviewer and himself finding the maximum together with Allen. He has developed a relationship with the interviewer who has assisted him when drawing a correct representation of the inequalities.

Connection

Allen was making a connection between an equation of an objective function and the furthest point in the feasible region which was used to find maximum value of the sales. However, it was not clear how he decided on the extreme point when he says "...And so our x is going to be three thousand, and then we substitute in our profit equation..."

Politics

The furthest point to the right in the feasible region is relevant but does not indicate the method used to find this point which gives the maximum value. His use of the conjunction "so" indicate the reason for his action and is confident about the value R210 000. (utterance 128).

Sign system and knowledge

Allen was able to give an explanation of his approach he used when calculating the value of the maximum using the objective function. He goes further to explain what he in utterance 128. The impression was that he understood that to maximise you substitute a point in the objective function he was able to represent as an equation. However, there is a gap in his knowledge of how he obtained the extreme point suggesting something missing.

Discussion

Allen in this excerpt highlighted the equation of the objective function and the furthest point in the feasible region. This suggests that he made a connection between maximising and the furthest point to be substituted in the profit function to find the maximum value. However it is

not clear how he decided on this furthest point. He was able to engage with the task in a meaningful conversation about how to maximize the objective function. His conversation depicted that there is something lacking, which is how to arrive at the furthest point. Allen was able to retrieve information from the text and made use of the given information to express the objective function. This led him to arriving at a solution which he is able to talk about confidently even though his solution is not correct.

8.6.3.3 Any opportunity for Allen to learn linear programming?

The analysis showed that Allen's written work does not lead him to the correct feasible region. This was a result of his consistent misinterpretation of the inequalities when representing 'does not exceed four thousand', 'at least two thousand' and 'no more than' and assigning a wrong inequality sign. Misinterpretation led to the implementation of inappropriate inequality sign on the information found within the problem. One of the reasons for misinterpretation of the phrase 'at least' might be that he is simultaneously acquiring English, a language of instruction and learning linear programming in English. He has to cope with the double task of learning a new language and to understand the key words and syntax of mathematics in the new language (Durkin & Shine, 1991). This showed that something was lacking.

However, his working did demonstrate access to linear programming discourse processes. We see evidence that Allen was able to represent graphically his constraints inequalities appropriately. It can be argued from his work that Allen was able to employ some sort of method or procedure. He was able to engage with the task and to bring it to completion. In addition to this, we see from Allen's discourse that he was able to engage in discussions about the task fairly meaningfully because of such factors as having an understanding of what an inequality is, even though this understanding is not correct. What also emerged from the discussion is that Allen had some sort of understanding of the perceived value of linear programming tasks, even though he seems not to be able to verbalise exactly why this is seen to be. Especially when giving reasons on the production of more videos. Furthermore, it is evident from the ability to give reasons for certain content of the task situation (such as why produce more videos than binders, why a minimum number of videos) in a given constraints that access to the Discourse surrounding linear programming is not a challenge for him. The apparent lack of resources for accessing the

meaning of the inequalities may appear to place Allen at a disadvantage because this led to wrong solution to the task even though he knew the procedures to follow. In conclusion, the analysis shows that Allen had a lack of continuity when learning linear programming despite his use of learned strategies. The consequence of this communication gap will clearly result in poor performance on the part of Allen. Therefore, to support Allen in linear programming there is a need to build on and link with the languages he brings from his community, like Xitsonga.

8.7 Conclusion

This chapter presented an analysis of data collected while conducting a clinical interview with John, Bheki and Allen. The focus of the analysis has been on how immigrant learners interact with a linear programming task which was presented in English. The analysis shows that the experiences of these learners do not differ from other second language learners'; they are learning linear programming in an additional language. However, John's experience is interesting. The experience of John was interesting in that French played an important role in his learning. Furthermore, during teaching of linear programming, the teacher switched between English and French, and not English and Lingala his home language. John's preference of French was also evident during the clinical interview. John preferred to give explanation in French. This made me conclude that John learnt linear programming in two additional languages, French and English which both have a developed mathematics register. This kind of code switching is also interesting because it has not been reported in South Africa.

CHAPTER 9

SO WHAT AND WHERE TO FROM HERE?

9.1 Introduction

At the commencement of this study the problem I identified was the influx of immigrants in South Africa. Some of these immigrants hailed from former French, Spanish, Portuguese and English colonies. These migrants travel with school going children who join mainstream classrooms in South Africa. The home languages of these migrants are indigenous to their home countries and not all of them are spoken in South Africa. Examples are Lingala, Chichewa from the DRC and Malawi respectively. The other problem identified was the hostility which was reported against immigrants in South Africa. I wanted to understand how teachers created opportunities for the participation of immigrant learners in a country that has been reported to be hostile towards them. My aim was to establish what discourses are at work in the teaching and learning of linear programming in a Grade 11 mathematics classroom with immigrant learners. I was of the view that immigrant learners may have to deal with minimum participation which seemed to be connected to past xenophobic attacks witnessed by school going children.

Research on mathematics education in multilingual classrooms in South Africa has not explored practices that immigrant learners call upon when learning mathematics. The South African context was marked as if there are no immigrants in the mathematics classroom. I was motivated to explore the classrooms with immigrant learners in South Africa, a context that has not been previously explored in mathematics education. I thus explored what mathematics teachers do differently in classrooms with immigrant learners as opposed to classrooms with none. To try and understand the discourses, I asked questions related to the language in use, suggesting that the opportunities created by teachers will be determined by language and how teachers made use of these resources available to immigrant learners.

The work was situated in three different settings and the opportunities created are developed in each specific context. These opportunities are useful and established by the relationships which are made possible by the different resources which are available within the contexts.

Now, as I come to the end of this journey and write the final chapter of the thesis, I look back at where it all started. I recognise that teaching in multilingual classrooms with immigrant learners is complex and deeply situated within individual settings and social practices. Any particular approach to teaching linear programming in classrooms with immigrant learners opens certain spaces at the very time that closes others, and these spaces can be both productive and constraining. For instance, because some immigrant learners lack fluency in the LoLT, some do not share a home language with the teacher or with the other learners. These problems are reflected in mathematics education internationally.

In this conclusion, I will provide an overview of the journey travelled to produce this thesis, provide a summary of the findings and discuss the contribution this study has made. A methodological reflection is also presented and then the chapter concludes by discussing some of the limitations of the research.

9.2 An overview of the journey

At the beginning of the thesis I showed that migration to South Africa is not new, however, in the past blacks were not recognised as immigrants. The new South Africa was designed to change this legacy and take South Africa into a new democratic future. A future in which there would be a more equitable distribution of educational opportunities and access to skills that would ensure that black immigrants have a future in South Africa. South Africa is signatory to United Nations policies which support immigrants in their destination country. These migrants are then immersed in the South African culture and have the same rights as the South African citizens. As a consequence, school going children are found in multilingual classrooms where the LoLT is English or Afrikaans. With the increase in diversity within and between societies, the teaching of mathematics becomes complex. The complexity is due to the fact that some teachers are not fluent in the immigrant learners' home languages and can therefore not use them when interacting with their teachers.

In Chapter 2 I presented research that point to the importance of language when teaching and learning mathematics. I argued that within this context of multilingualism, the language in which mathematics is taught is crucial, particularly because studies have shown that learners use home

languages when solving mathematical tasks. Other researchers argue that the cultural practice of the immigrant learners must be taken into consideration because they play an important role in their learning. Therefore, while policy might be prescriptive when it comes to the language that must be used during teaching and learning, the language in education policy is relatively open; there are spaces to use the learners' home languages during teaching and learning. And research shows that teachers do call upon resources of language. The question was how about immigrant learners who some of them are not fluent in these languages. How do teachers open up opportunities for their participation? While these spaces were theoretical open, whether or not they were recognised (or not) would only be seen by considering individual cases in the field of practice.

Before considering the case study I needed to understand the environments where immigrants are located. I decided to focus in an urban, township and rural environments. In particular I argued that in the urban environment it is normal to be an immigrant and identify oneself as such. While in the township and rural environments immigrant learners identify themselves as South African citizens. And it is reflected in the languages used when communicating.

In Chapter 3 the focus therefore moved to a consideration of the schools that enrol immigrant learners at the three different settings, in order to produce an account of the specific teaching approaches that create opportunities for participation of immigrant learners. I showed that I purposely selected three different schools, School A in the urban, School B in the township and School C in the rural environments. In the urban environment immigrant learners migrated from the DRC where the LoLT is French and their home language is Lingala. In the township environment immigrant learners are born from parents who migrated from Mozambique. In the rural environment immigrant learners are born from parents who migrated from Angola and Mozambique and speak Portuguese at home. The home language of the immigrant learners in the township and rural environments is Xitsonga.

In Chapter 4 the process of gaining access to classrooms that enrol immigrants is outlined and I argue that it was a process that depended on negotiation with the gate keepers who are the Principals of the schools. I outlined how the relationship of trust was established between the

researcher and the participants because gaining access did not guarantee data collection. In the same chapter the issues surrounding immigrants in South Africa are also presented. These issues portray immigrants as not welcome by some South African citizens. For many years black African immigrants had lived in peace and harmony with their South African neighbours. Some of the Africans from different parts of Africa are married to South Africans. Therefore for those who had some children, their children are South African by birth. Yet many black South Africans did not accept them because one of their parents is a foreigner.

In this chapter, the classrooms where the data was collected are introduced and discussed in terms of the language background of the learners. The context shows that some of the home languages of the immigrant learners are not indigenous to South Africa. And some of these learners speak languages like Portuguese, French, Lingala or Chichewa at home.

The focus of the study now moved to understand how teachers created learning opportunities for the participation of immigrant learners in the three different settings identified. Using theoretical resources derived mainly from Gee's (2005a) discourse analysis tools, I was able to give an in-depth analysis and interpretation of how teachers used language not as a communication tool but to build reality. The analysis provided an insight into how immigrant learners are supported by code switching. This was the focus of Chapters 5, 6 and 7. In the urban environment, the analysis is presented in Chapter 5, the teacher switched between English and French which are additional languages of the immigrant learners. In the township environment, the analysis is presented in Chapter 6 and the teacher switched between English, isiZulu and Setswana. In the rural environment, the analysis is presented in Chapter 7 and the teacher used English only.

Next I considered clinical interviews with three immigrant learners, one from each site to understand how they respond to linear programming tasks presented in English. The analysis of the clinical interviews of the three immigrant learners, John, Bheki and Allen is presented in Chapter 8. John migrated from the DRC where the LoLT is French. He enrolled in School A situated in the urban environment. His home language is Lingala. Bheki was born in South Africa and his parents migrated from Mozambique. His home language is Xitsonga and he speaks Portuguese at home. Allen was also born in South Africa. His parents are from

Mozambique and Angola. His home language is Xitsonga and they speak mainly Portuguese at home.

The overall analysis revealed that teachers in the urban and township settings are fluent in the immigrant learners' languages and therefore supported them by code switching. While the language that the teacher shares with the learners in Classroom C is English so her interactions were mainly in English. The urban and township settings were operating within a social setting that was part of the social fabric operating with forms of unity connected into a globalised and networked world. As a result at the school level, selection of key words in linear programming were translated to support immigrant learners grounded on their prior knowledge that lead to the construction of correct inequalities in a language that they understand. The choices were supported by an interpretation of official discourses based on the linear programming content under discussions. Within this context, learning linear programming was exemplified as a practical accomplishment which supported learning identities established on the mathematical language in linear programming.

In the rural setting the social setting was closed together within a context operating with more traditional authority relationship. However, it was built within a caring environment and recognised the need to build self confidence in its learners. These learners were able to develop identities through the spaces created within the context to learn and practice linear programming. In the following section of this chapter an overview of the findings is presented and each question is discussed in relation to the conceptual framework that guided the study.

9.3 Summary of findings -teachers creating learning opportunities

In the previous section I gave an overview of the journey of the study. In this section I highlight the findings. The study investigated how teachers in these multilingual mathematics classrooms supported immigrant learners and the learning opportunities they created for their participation. It addressed the following questions:

1. How do teachers create opportunities for the participation of immigrant learners when teaching linear programming?

2. How are languages within the immediate environment of immigrant learners used during the teaching and learning of linear programming?

The above questions are answered from a socio-cultural perspective and discuss the underlying discourse models, and then the discourse models that emerged as teachers taught the way that they did in order to create opportunities for the participation of immigrant learners.

9.3.1 Underlying discourse models of the teachers

What came out clearly at the end of the case study was that teachers in Classroom A and B share fluency of the languages immigrant learners brought into the mathematics classrooms and linguistically supported them. Such created opportunities for the use of the languages that the immigrant learners brought into the classrooms as a resource. While the teacher in Classroom C did not share fluency of the learners home languages and therefore provided English explanations only during the teaching and learning of linear programming. The evidence from the discourses indicates that: firstly, the teachers emphasized the importance of the meaning of key words in linear programming; secondly, the teachers emphasized certain tools with which learners can engage the tasks in some mathematically meaningful manner when representing the inequalities graphically; and thirdly, learners follow certain steps when solving a linear programming task.

Through the analysis of data collected during lesson observation and interviews with teachers and learners the following discourse models emerged:

- Attention to mathematics facilitates the learning of linear programming,
- Teachers have important knowledge to share with their learners.
- Mathematics has its own language and it is about manipulation of symbols.
- Read so that listeners catch and understand the words
- Show examples to learners including examination tips
- Give strategies to learners for drawing straight line graphs
- Give precise linear programming information
- Devise a way to assist learners remember formulae
- All learners may learn from each other's mistake

- Learners have to adapt to linear programming discourse
- Mathematics is learnt by doing –sense of responsibility
- Mathematics is about x and y
- Learning occurs in a safe environment
- Explanation necessary when teaching linear programming content

The discourse models that emerged are developed from a success model (Gee, 2005a) which may influence many patterns of the teacher’s actual behaviours. According to Gee (2005a: 82) we are all dominated by many practiced discourse models. If we happen to be in an environment where the daily behaviours correspond to the success model, “the daily observations and social practices reinforce explicit ideological learning in regard to the discourse model for success”. The teachers who come to see the participation of immigrant learners in terms of the success model, created opportunities so that they become confident and establish themselves through the success of others, and the expectations of the school (Gee, 2005: 83). When teachers lack fluency of the learners’ languages, with regard to the success model they come to use the LoLT only which is English. However, in order to achieve success, it is necessary to interpret phrases correctly in linear programming in a language that learners understand like French, isiZulu or Xitsonga.

9.3.2 Learners’ take up of the opportunities created by mathematics teachers

The discussion above highlights some of the discourse models that teachers enacted in order to create opportunities for the participation of immigrant learners when teaching linear programming. In this section I discuss findings which show how immigrant learners responded to a linear programming task. Firstly the findings reveal that immigrant learners responded to linear programming discourse according to key words. Secondly immigrant learners’ engagement or not with the task showed that they have tools which enabled them or not, to engage with the task. The immigrant learners’ written work indicated a procedure and some meaning attached. As a result they reacted to linear programming tasks based upon their understanding of where and how the specific linear programming task fits into the discourse, how they might handle the representation of the task, mathematically and graphically.

9.3.3 The discourse of the immigrant learners

The discussion addresses the question: *what practices do immigrant learners engage in to participate effectively in linear programming discourse?* I discuss these discourses with specific reference to, firstly, what enabled immigrant learners to correctly solve the linear programming task, and secondly, what led to wrong solution of the linear programming task presented in English.

9.3.3.1 The discourses of immigrant learners who found the correct solution

The immigrant learners', John and Bheki, who demonstrated some sort of confidence on knowledge of key words in linear programming and following a procedure, were more enabled when solving linear programming tasks. Connected to this is the recognition of what is required mathematically in linear programming (like setting up correct linear inequalities or use of a formula like the objective function and search line). The use of the formula for objective function is related to the immigrant learner's ability to recognise the task situation and correctly set up the supporting expressions. With respect to the learners' talk about their approach to solving linear programming task, John made links between linear programming and solving inequalities. He seemed to attach significance in proper presentation of inequalities. Bheki also attached significance in appropriate scale on the axes before representing inequalities graphically. Furthermore, when using the search line to optimize, Bheki first made sure that he multiplies the coefficient of the x and y variables in the objective function in order to position the search line on the graph appropriately. Bheki and John were able to situate themselves as active role players in the solving of the given linear programming tasks. They were not just passive recipients of a task with no possible action.

9.3.3.2 The discourses of Allen who was unable to find the correct solution

In doing the task Allen used a procedure but was unable to find the correct solution because he did not interpret the mathematical language appropriately to obtain and write down correct inequalities. Even though carrying out of the method did enable him to interact with the task to some level, he was not able to find the correct solution of the task (because, for example, the inequality sign was incorrectly assigned or related to 'at least'). Nonetheless, his working did demonstrate skills necessary when solving a linear programming task, especially when

representing the inequalities graphically. Allen did recognise the feasible region which was according to his inequalities extracted from the task. His working demonstrated a basic understanding of linear programming discourse but lacked appropriate interpretation of the mathematical meaning of key words in the task. As a result wrong inequalities led to an inappropriate solution.

In terms of Allen's explanation, he made reference to key words, but was not able to assign correct inequalities to some of them like 'at least'. His talk lacked confidence within the context of linear programming. At the political level Allen expressed feelings of authority away from himself especially when he was not able to explain the meaning of his inequalities in relation to the task.

9.3.4 Underlying discourse models of immigrant learners

From the discourses of the immigrant learners, *what underlying discourse models are at play in the text that immigrant learners produce with regard to linear programming?*

Given the situation mentioned above, immigrant learners either arrived at the correct solution or did not arrive at the correct solution of the linear programming task. The immigrant learners who were able to arrive at the correct solution are John and Bheki each (in their own way) did and said things that showed that there are opportunities that facilitated the handling of the linear programming task. This refers to the extent to which they come to be able to interact with the task. The evidence from the discourses indicates that: firstly, these immigrant learners locate the tasks within the discourse of linear programming according to key words given in the task; secondly, they have certain tools with which they are able to engage the task in some mathematically meaningful manner when representing the inequalities graphically; and thirdly, they follow the steps presented by the teachers. These learners interacted with the linear programming task based upon their understanding and then decided how they carry out the necessary steps in the social context of school mathematics in order to be successful. As a result the immigrant learners displayed a success model.

The evidence from Allen's working and talk suggests that he generally does have adequate procedures that allowed him to interact with the task. But, had not mastered the mathematical

meaning of key words in linear programming and thus was not successful in solving the task correctly.

9.3.5 The success model and the immigrant learners' opportunities to participate

The immigrant learners depended on the opportunities created by their teacher's during the teaching and learning of linear programming. These learners mastered the mathematical meaning of key words in linear programming and the necessary steps to follow when solving a task. Their participation in the solving of a linear programming task during the clinical interview helps in explaining whether they had opportunities during teaching and learning in the classrooms.

The immigrant learners in this study came from three different school environments. From a socio-situated perspective, the environment in which immigrant learners' learnt linear programming has a significant influence on how they solved the task successfully. For example in the urban environment, there are opportunities to choose a language that they understand as the teacher switched between English and French during the teaching and learning of linear programming. In the township environment the teacher switched between English, isiZulu and Setswana. In these environments, the teachers are proficient in the languages that immigrant learners brought into the mathematics classroom. Furthermore these teachers valued these languages brought into the linear programming lessons. As a result they were able to use them as a support during the teaching and learning of linear programming. This approach created the opportunity for a choice of language and thus enabled immigrant learners to consider their ideas in a language they are comfortable with. The environment created by the teachers promoted success.

However, if the teacher is not proficient in the languages brought by the learners to the linear programming lesson that teacher cannot draw on them like in Classroom C. Therefore there is not much the learners can do when interacting with the teacher. Then English is predominantly used during teaching and learning of linear programming, yet these learners encounter English only at school. At home English is not spoken but instead Portuguese and Xitsonga. This approach obstructs the most determined efforts to succeed.

9.4 The contribution of this study

This study provides how immigrant learners are supported in multilingual classrooms in South Africa at a particular time in its history. Every one of us is immersed in stories. Where we come from—our histories—and who we are in this hierarchical, racialism, gendered, and class-based world—our locations—matter in what we say and do. Who we are shapes what we see.

Immigrant learners that the study focused on are black African migrants who share race with the majority learners in South Africa. In the past, race defined power in South Africa. Today language plays an equal role. The immigrants are immersed in an environment where the majority are blacks and learn in a second language. In this context, nine out of the eleven official languages are not given the same status as English or Afrikaans. So as migrants cross borders, they bring diverse languages which make the situation even more complex. The influx of immigrants makes not only English the language of power, but also a thing of accumulation of languages that are all often forced into silence that cannot be heard nor understood by fellow citizens. However, in reality immigrant learners switch to their choice of language when solving a linear programming task.

Through all the disruption, the core of identity and language remains something for immigrants to rely on. The immigrant learners speak Portuguese, French, Chichewa, Lingala, Xitsonga at home and help keep up their languages which they can use as a support in their learning of linear programming. The immigrant learners are confident to draw on their choice of language to understand linear programming, their approach and what the tasks mean despite the context, we begin to understand the learning not as similar to what has been reported in the context of language diversity. Furthermore, in a context where the teacher shares proficiency and culture with the immigrant learners, they use their languages as a support through code switching.

However, the situation becomes even more complex in the urban environment because switching was between English and French. Switching to French during teaching and learning in a South African context creates inequalities between other immigrant and local learners who are not fluent in the language. The inequalities are created because other immigrant learners and local learners do not get the opportunity of understanding a second explanation in French. As a result

French becomes a social good in a South African context as some immigrant learners have access to meaning or explanation in two additional languages.

9.5 Recommendations to mathematics education

This section presents the recommendations that the study makes to mathematics education. The study has shown that the discourse models that emerged during the analysis of the teaching and learning of linear programming are the basis of discourses produced by immigrant learners in South Africa. The success model made the teachers in Classroom A and B to draw on the languages brought into the mathematics classroom. The teachers focused on mathematical meaning of key words in linear programming and switched between English and the languages learners were comfortable with and thus exposing them to meaning in many languages. What emerged in this study is that the language switching can only be possible if the teacher is proficient in these languages.

In this regard, the study supports the argument that we cannot expect to get uniform practices across the settings through policy regulation, particularly in a context such as South Africa. In this context, some of the official languages are not given the same status as English or Afrikaans to support the learning process. The teaching approaches work to support access to linear programming knowledge differently in the three settings. In these settings, the teachers are doing what they genuinely believe is the best they can do within their context—they are working very hard to produce the kind of immigrant learner that will have a future in South Africa. However, their interpretations of what is required are determined largely by their own resources they have available.

The study supports the view that quality mathematics teaching cannot be legislated. It suggests that if we want to develop some kind of uniformity in quality across the settings it will not be done through regulations connected to written documents. It will be through a project that will involve the resources of the teachers themselves and the development of a discursive understanding of mathematics education and of the responsiveness of the immigrant learners within the specific post apartheid South African education context. Mathematics teachers and others who have an interest in producing quality teaching material need to come together, to

build relationships, to share resources and to use these as a basis for their own practice to open equal opportunities to all.

9.5 Methodological reflections

This has been a journey for me which I enjoyed particularly because I had the support from my supervisor. The research drew on Gee's (2005a) discourse analysis. I briefly reflect on this approach. Gee's approach to discourse analysis resulted in description using the seven building tasks for understanding how the students build meaning. This, while being productive in terms of enabling thick and rich descriptions to be produced and therefore ensuring theoretical validity, as well as analytic descriptions, contributed to the production of the thesis. The study showed that the methodology and the analytic tools can be adapted and refined so as to appropriately address the research question. What has also emerged from this study, though, is that this methodological and analytical style is no simple matter because the constructs are not easily communicated, making it at times difficult to identify, and put the necessary language in place in order to report on the phenomena.

9.6 Limitations of the study

The case study while providing rich and thick descriptions cannot be generalised to other contexts – all that it can do is provide accounts of the opportunities created by teachers when teaching linear programming to immigrant learners through this communication. However, having said this, while these are specific cases, the theoretical work done in producing the accounts and the tools developed for interrogating the empirical field are productive for further work in the field of mathematics education. The research has also illuminated some key challenges for teaching and learning of immigrant learners in multilingual classrooms in South Africa. In addition, since the cases represent urban, township and rural environments this illuminates possibilities and opens up questions for further study more generally.

9.7 Conclusion

This chapter provided an overview of the journey that produced the thesis. The research provided some insights into the complexity of teaching mathematics in classrooms with immigrant learners in South Africa. It has confirmed that context really does play a key role and that

proficiency in the learners languages are of crucial importance. Viewing teachers and immigrant learner's discursive practices from the situated-sociocultural perspective highlighted the factors that contribute to the success of immigrant learners in school. The success of these learners included correct interpretation of mathematical meaning of key words. As a result immigrant learners' engagement with the task showed that they have tools with which they are able to engage the task in a procedure and some meaning attached to it. They responded to linear programming task based upon their understanding of where and how linear programming task fit into the mathematics Discourse. The lack of the proficiency between the teachers and the learners languages result in lack of access to linear programming knowledge.

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APPENDIX A: ETHICS CLEARANCE UNISA



Ms Thulisile Nkambule (35118164)
College of Science, Engineering and Technology
Unisa
Pretoria

19 August 2010

TO WHOM IT MAY CONCERN

Permission to conduct PhD Research Project

Ref: 007/NT/2010

The request for ethical approval for your PhD research project entitled: "Immigrant Learners Learning Linear Programming in Multilingual Classrooms in South Africa" refers.

The College of Science, Engineering and Technology's (CSET) Research and Ethics Committee (CREC) has considered the relevant parts of the studies relating to the abovementioned research project and research methodology and is pleased to inform you that ethical clearance is granted for your study as set out in your proposal and application for ethical clearance.

Therefore involved parties may also consider ethics approval as granted. However, the permission granted must not be misconstrued as constituting an instruction from the CSET Executive or the CSET CREC that sampled interviewees (if applicable) are compelled to take part in the research project. All interviewees retain their individual right to decide whether to participate or not.

We trust that the research will be undertaken in a manner that is respectful of the rights and integrity of those who volunteer to participate, as stipulated in the UNISA Research Ethics policy. The policy can be found at the following URL:

http://cm.unisa.ac.za/contents/departments/res_policies/docs/ResearchEthicsPolicy_apprvCounc_21Sept07.pdf

Yours sincerely



Prof L Labuschagne
Chair: CSET CREC



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APPENDIX B: GDE RESEARCH APPROVAL LETTER



UMnyango WezeMfundo
Department of Education

Lefapha la Thuto
Departement van Onderwys

Enquiries: Diane Buntting (011) 843 6503

Date:	15 September 2010
Name of Researcher:	Thulisile Nkambule
Address of Researcher:	227 Visagie Street
	Pretoria
	0134
Reference No:	D2011/28
Telephone Number:	012 429 4917 / 079 025 3455
Fax Number:	086 659 6806
Email address:	nkambt@unisa.ac.za
Research Topic:	Immigrant learners Learning Linera Programming in multilingual classrooms in South Africa
Number and type of schools:	THREE SECONDARY SCHOOLS
District/s/HO	Johannesburg North; Johannesburg West and Tshwane North

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

Permission has been granted to proceed with the above study subject to the conditions listed below being met, and may be withdrawn should any of these conditions be flouted:

- 1. The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
- 2. The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
- 3. A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*

Office of the Chief Director: Information and Knowledge Management
Room 501, 111 Commissioner Street, Johannesburg, 2000 P.O.Box 7710, Johannesburg, 2000
Tel: (011) 355-0809 Fax: (011) 355-0734

4. A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.
5. The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.
6. Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.
7. Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year.
8. Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.
9. It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.
10. The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.
11. The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.
12. On completion of the study the researcher must supply the Director: Knowledge Management & Research with one Hard Cover bound and one Ring bound copy of the final, approved research report. The researcher would also provide the said manager with an electronic copy of the research abstract/summary and/or annotation.
13. The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.
14. Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards



Shadrack Phele MIRMSA
 [Member of the Institute of Risk Management South Africa]
 CHIEF EDUCATION SPECIALIST: RESEARCH COORDINATION

15 September 2010

The contents of this letter has been read and understood by the researcher.	
Signature of Researcher:	<i>Mkawbute</i>
Date:	<i>16 September 2010</i>

APPENDIX C: INFORMATION LETTER TO THE PRINCIPAL

IMMIGRANT LEARNERS LEARNING LINEAR PROGRAMMING RESEARCH PROJECT

Dear principal

My name is Thulisile Nkambule. At present I am studying towards a Doctor of Philosophy (PhD) in Mathematics Education at the University of South Africa. As part of my studies I am doing a study on how immigrant learners learn linear programming in a multilingual classroom in South Africa.

I am requesting permission to work with one of your staff members and his/ her Grade 11 learners in your school in this study. If you allow them to participate, they would be asked to allow me to observe linear programming lessons in their classroom for five days in three months at agreed upon times. I will also have your teacher to participate in a reflective interview focusing on his/ her observed lessons. Furthermore, he will be requested to select two immigrant learners for a clinical interview.

I am also requesting for permission to video-record the lessons and the interviews so that I can ensure that I make an accurate record of what the teacher says and do. When the recordings have been transcribed, the teacher will be provided with a copy of the transcript, so that he can verify that the information is correct.

I intend to protect the anonymity and the confidentiality of the teacher and learners' responses. Their names and contact details will be kept in a separate file from any data that they supply. This will only be able to be linked to their data by me. In any publication emerging from this research, they will be referred to by pseudonyms. If for any reason they would like their real names to be used in the publications, they will need to make written request to me.

A brief summary of the findings will be available to the teacher once the research has been completed. The findings might also be presented at academic conferences and published in national and international academic journals.

Please be advised that the participation of your school in this research project is voluntary. Should you wish to withdraw at any stage, or withdraw any unprocessed data you have supplied, you are free to do so. Your decision to participate or not, or to withdraw, will be completely independent of your dealings with the University of South Africa.

If you are happy for your school to participate, please indicate that you have read and understood this information letter by signing the accompanying consent form and returning it to me.

Should you require any further information do not hesitate to contact me.

Ms Thulisile Nkambule (cell number: 0790253455)

Consent form for the principal

IMMIGRANT LEARNERS LEARNING LINEAR PROGRAMMING RESEARCH PROJECT

Researcher: Ms Thulisile Nkambule

Supervisor: Prof. M. Setati Phakeng

I agree that the school can participate in the project named above. The details of lesson observations and interviews have been explained to me. An information letter has been given to me to keep.

I give consent to the following: (Tick to indicate your selection)

Video recording of the lessons:

Yes No

The possible future use of the videotext for teaching purposes:

Yes No

The participants being interviewed:

Yes No

Tape recording of the participants' interview:

Yes No

Principal signature

Date

Witness

Date

Researcher

Date

APPENDIX D: INFORMATION LETTER TO THE TEACHER

IMMIGRANT LEARNERS LEARNING LINEAR PROGRAMMING RESEARCH PROJECT

Dear Teacher

My name is Thulisile Nkambule. At present I am studying towards a Doctor of Philosophy (PhD) in Mathematics Education. As part of my studies I am doing a study on immigrants learners learning linear programming in multilingual classrooms.

Your Principal has given me permission to send you this letter to invite you to participate in this research project. Once you have read the letter you can decide whether you want to participate or not. Should you agree to participate, I will ask you to allow me to observe your teaching in one of your Grade 11 mathematics classrooms for five consecutive days over a period of three months at agreed upon times. I will also ask you to participate in a reflective interview focusing on your observed lessons. With your permission, the lessons will be video-recorded and interviews tape-recorded so that I can ensure that I make an accurate record of what you say and do. When the tape has been transcribed, you will be provided with a copy of the transcript, so that you can verify that the information is correct.

I will protect your identity and your responses will be kept confidential. Your name and contact details will be kept in a separate file from any data that you supply. In any publication emerging from this research, you will be referred to by a pseudonym. I will remove any references to personal information that might allow someone to guess your identity. Should you wish that your real name be used in the publications, you will need to make written request to me.

Once the research has been completed, a brief summary of the findings will be available to you. It is also possible that findings will be presented at academic conferences and published in national and international academic journals.

Please know that your participation in this research project is voluntary. Should you wish to withdraw at any stage, or withdraw any unprocessed data you have supplied, you are free to do so.

If you would like to participate, please indicate that you have read and understood this information by signing the accompanying consent form and returning it to me.

Should you require any further information do not hesitate to contact me.

Ms Thulisile Nkambule

0790253455

Consent form for the teacher

IMMIGRANT LEARNERS LEARNING LINEAR PROGRAMMING RESEARCH PROJECT

Researcher: Ms Thulisile Nkambule

Supervisor: Prof M. Setati Phakeng

I----- agree to participate in the project named above. The details of lesson observations and interviews have been explained to me. An information letter has been given to me to keep.

I give consent to the following: (Tick to indicate your selection)

Video recording of the lessons:

Yes No

The possible future use of the videotext for teaching purposes:

Yes No

.....
Signature of participant

.....
Date

.....
Signature of witness

.....
Date

.....
Signature of researcher

.....
Date

Teacher consent form for video recording

IMMIGRANT LEARNERS LEARNING LINEAR PROGRAMMING RESEARCH PROJECT

Iam aware of the data collection processes in the research project as stated in the letter.

I give consent to the following:

Being interviewed at some point during the study (Tick to indicate your selection)

Yes or no

The tape recording of my interview with the researcher

Yes or no

.....
Signature of participant

.....
date

.....
Signature of witness

.....
date

.....
Signature of researcher

.....
date

APPENDIX E: INFORMATION LETTER TO THE PARENT/GUARDIAN

IMMIGRANT LEARNERS LEARNING LINEAR PROGRAMMING RESEARCH PROJECT

Dear Parent or guardian

My name is Thulisile Nkambule. At present I am studying towards a Doctor of Philosophy (PhD) in Mathematics Education at the University of South Africa. As part of my studies I am doing a study on immigrant learners learning linear programming in multilingual classrooms.

Your child's mathematics teacher and principal have given me permission to send you this letter to invite your child to participate in this research project.

Children whose parents agree that they participate in this study will be video recorded for five consecutive days over a period of three months during mathematics lessons. Children whose parents do not agree that their children be video recorded will be kept away from the focus of the video recorder. They will not be deprived from the lesson.

I intend to protect your child's anonymity and confidentiality. Their real names will not be used in the final report. I will remove any reference to personal information that might allow someone to guess their identity. The results of the research may be reported at conferences, in journals and to research funders. In case I need to use the information in the video recording for conferences or for teaching purposes, the children's faces will be hidden from public viewing.

Please note that your child is not forced to participate. Should you require any further information please contact me.

Please complete the consent form if you agree that your child be part of this research project.

Ms Thulisile Nkambule
0790253455

APPENDIX F: INFORMATION LETTER TO THE IMMIGRANT LEARNER

IMMIGRANT LEARNERS LEARNING LINEAR PROGRAMMING RESEARCH PROJECT

Dear learner

My name is Thulisile Nkambule. At present I am studying towards a Doctor of Philosophy (PhD) in Mathematics Education at the University of South Africa. As part of my studies I am doing a research on immigrant learners learning linear programming in multilingual classrooms. Your mathematics teacher and Principal have given me permission to send you this letter to invite you to participate in this research project.

If you agree to participate in this study you will be requested to be present during the time when I will be observing lessons in your class. Only linear programming lessons will be observed.

I intend to protect your anonymity and confidentiality. Your real name will not be used in the final report. I will remove any reference to personal information that might allow someone to guess your identity and of your teacher. In case I need to use the information in the video recording for conference or for teaching purposes, your faces will be hidden from public viewing.

Remember that you are not forced to participate. Should you require any further information contact me.

If you agree to be part of this research project please complete the consent form.

Ms Thulisile Nkambule
0790253455

Consent form for the learner

IMMIGRANT LEARNERS LEARNING LINEAR PROGRAMMING RESEARCH PROJECT

Researcher: Ms Thulisile Nkambule

Supervisor: Prof M. Setati Phakeng

I, agree to participate in the project named above. The details of lesson observations and interviews have been explained to me. An information letter has been given to me to keep.

I give consent to the following: (Tick to indicate your selection)

Video recording of the lessons:

Yes No

The possible future use of the videotext for teaching purposes:

Yes No

.....
Signature of learner

.....
Date

.....
Signature of witness

.....
Date

.....
Signature of researcher

.....
Date

Learner consent form for video recording

IMMIGRANT LEARNERS LEARNING LINEAR PROGRAMMING RESEARCH PROJECT

Ia mathematics learner at , am aware of all the data collection processes in the project as stated in the information letter.

I give consent to the following: (Tick to indicate your selection)

Being interviewed during the study

Yes no

Being tape recorded during the interviews

Yes no

.....
Signature of learner

.....
Date

.....
Signature of witness

.....
Date

.....
Signature of researcher

.....
Date

APPENDIX G: LESSON OBSERVATION IN CLASSROOM A

LESSON ONE

1. Teacher *(When we entered the Grade 11B, there were papers on the floor and the teacher uttered the following to the learners)* Grade 11B picks up the papers! Okay! Pick up the papers and put them in the dust bin before we start the work of the day. Right! Grade 11B, arrange your desk propely. You guys you don't want to keep your school clean. Put it up there. One day you will be surprised, Jesus will come and find your school dirty. You are going to remain you see! We do not know whether, whether may be the Jesus talks. You don't know [*students laugh*] May be Jesus looks like men or what? We do not know you see! Okay thank you sit down. Eh! We have a visitor, maam Thulie, right. Jah! She is from UNISA. She is studying at Unisa doing mathematics education right. She is interested to see how we are handling linear programming okay. We have already started right. She will assist us, eh! She is well received, she is received.
2. Thulie Thank you
3. Teacher Maam I do not know where you will sit, there at the back or front, sit where you want. There or here where you want, right.
4. Thulie I will sit at the back
5. Teacher Okay
6. Thulie Ninjani?
7. Learners Siyaphila
8. Teacher Okay guys let's see... [*Teacher writes linear programming on the board*]okay! Eh! This is linear programming eh, okay! Lets see, eh okay ...okay well so far! What we have done in linear programming is, we give you a problem, we give you a problem, you are going to from there you find derive constraints, after the constrains you are going graph them okay! you are going to draw the graph okay, after graphing them you find a feasible region. After finding the feasible region name the points of the feasible region. Then calculate the solution of the problem Okay. Then you write the objective function, from the objective function given the problem. From there you determine which point gives you the maximum or the minimum value okay. If you are given a problem, I told you when, when its profit you maximise, when its costs you minimise okay! Okay!
9. Learners Yes
10. Teacher Today we are we are going to do like backwards.I will ... I will give you the constraints no sorry the graph. From the graph you are going to determine the equation of the constraints ... okay, the constraints ...example one Jah...[*teacher drawing graph showing feasible region on the board, learners copying the diagram*] one, two, three, four, six... one, two, six, eight...make the lines straight, make the lines parallel to the y-axis and x-axis.
11. Teacher get ready don't waste time, Okay hurry up! hurry up!...okay ready get ready i want to read the problem okay right [*teacher reading problem to the learners*]
- [*Teacher writing on the board*] The graph above, the graph above was, was used in the solution of a problem by means of linear programming, linear programming okay. Now carry on, (*Teacher reading problem*) The shaded quadrilateral, the shaded quadrilateral, the shaded quadrilateral, ABCD, ABCD. The shaded quadrilateral ABCD indicates the feasible area full stop, indicates the feasible area full stop. Indicates the feasible area full stop. First question [*teacher writes a) on the board*]
- a) Write down the set of inequalities, write down the set of inequalities, which defines the feasible area, which defines the feasible area, write down the set of inequalities which defines the feasible area full stop which defines the feasible area full stop. [*Teacher writes b) on the board*]

26. Teacher Now those are the easy one to find. These are the inequalities which are easy to find. Now look at the two lines, how do we find the other inequalities... Those are difficult! How do you find the inequalities? I will teach you two techniques, one technique from South Africa and one technique from Congo. (*learners laughing*) Ja! This one we do not do it here, but I will give you. Then you will choose which one is easier for you. Ja! from South Africa look at here, look at here, number three okay? You know that the standardform of an equation of a line. Then you determine the gradient and the the y-intercept. The first thing is to determine the gradient and the y intercept. For instance line c, What is the y intercept?
27. Learner Eight
28. Teacher The line cuts the y-axis at ...eight! Ja! Therefore your c is eight! Okay! You understand?
29. Learners Yes
30. Teacher Ja! Then you find the gradient, look! [*name of learner*] what is gradient? What's the gradient? Whats the gradient Your mother ask what's the gradient? Tell your mother what is gradient? Swimming pool gradient? If your mother ask what are you going to tell her?
31. Learner change in y over change in x...[*learner interrupted by teacher*]
32. Teacher Your mother! Ah! [*emphasis*] your mother don't know understand change in y over change in x, what are you going to tell her?
33. Learner ...rise over run...
34. Teacher Gradient is rise from the horizontal, gradient is the rise from the horizontal. Now remember I told you gradient is rise over run, do you still remember that? Rise over run. The rise is on the y-axis and the run is on the x-axis. Now, will the gradient be positive or negative? How, which position gives the negative gradient which position gives the positive gradient? Ja! which position? Remember I told you when you rise like that (*teacher pointing upwards direction*) the rise will give positive gradient and when you descend like that (*teacher pointing downward*) the gradient is negative. You know that? Ja! Now which means ja! now when the rise is up it is positive gradient and when it is downwards it is negative gradient. Now if you look at the line, will the gradient be positive or negative?
35. Learner s negative gradient
36. Teacher Look at here, my my look at the rise from four to eight it gives me my rise and my run is four (*teacher pointing at the graph*) which means my gradient is then we know its negative eight over four which is negative two
37. Learner s negative two
38. Teacher so gradient is negative eight over four which is equal to negative two. Therefore the equation of the line is y is equal to negative two x plus.....plus what? Right!
39. Learners plus eight
40. Teacher Okay now I can say y is equal negative two x plus eight. Which becomes y plus two x equal to eight. Then you we want to define the feasible region. How is the feasible region? Is it above the line or below?
41. Learner It is below the line [*learners using gestures*]
42. Teacher Okay therefore the constraints the inequality will be y plus two x is equal to 8, my graph is y is equal to two x plus eight ($y = 2x + 8$). because the feasible region is above it will be greater than or equals to eight ($y \geq 2x + 8$)...
- 20:20
43. Teacher Congolese method! Congolese method Ja! Look at here Ja! Another method Ja! Okay another method
44. Learner Yes Congolese method
45. Teacher Now look at here you use the formula which says x over a plus y over b is equal to one, where a is, a is where a is the x-intercept, b is the y intercept okay? Easy formula Okay now you look at here [*teacher pointing at positions of intercepts on the graph*] What is the x-intercept? Yes

46. Learners the x-intercept is four and the y intercept is 8.
47. Teacher Which one is representing a?
48. Learners Four
49. Teacher From there the equation becomes x over four plus y over eight equal to one. From there you find the LCM of four and eight. The LCM of four and eight is what? Yes
50. Learners It is eight
51. Teacher From there you multiply each term by eight. X over four times eight plus y over eight times eight equal to one times eight. You get two x plus y equal to eight which is what you have here
52. Learners Aha!
53. Teacher Ja! Now which one is easier for you?
54. Learner Congolese method is easy
55. Teacher The congolese method is easier, hahaha hahaha (*Teacher and learners laughing*) now [*Teacher writes $2x + y \geq 8$*]
56. Learner Yes congolese method is easier
57. Teacher Now you guys solve for the fourth one, find inequalities for the fourth constraints...use both methods, guys use both methods. The South African method and the Congolese method
Pause... *Learners working in pairs while the teacher was moving around checking what they were doing,*
58. Teacher who can explain what is a feasible region Ha! Guys no. You did not see that. Remember after you find standard formula you put it in a general formula ay plus bx is equal to c . After standard formula convert it to general formula.
59. Teacher [*Reading second example for learners to try*] A school wants to take learners on an outing, outing. A school wants to take learners on an outing, a school wants to take learners on an outing to an aquar..., aquarium, eh! how do you pronounce it I don't know in English aquarium or aquarium? Because in French we pronounce it as aquarium [*teacher writes aquarium on the board and pronounce aquarium in French*] in an outing to an aquarium full stop. At least fifty five learners must be transported, at least fifty five learners must be transported, at least fifty five learners must be transported okay, now full stop, There are two types of minibuses available. There are two types of minibuses available full stop Type A, type A can carry fourteen passengers one four, type A can carry fourteen passengers one four, and type B can carry ten passengers, type B can carry ten passengers, type B can carry ten passengers full stop. Okay, There are at most three type A buses, there are at most three type B buses full stop. A maximum of five drivers are available, let x be the number of type A buses full stop, let x be the number of type A buses full stop. And y be the number of type B buses, full stop. And y be the number of type b buses full stop. Now they give you points here and here [*teacher drawing graph showing a feasible region and label the axes thus*] one, two, three, four, five now type A, type B on the axis. now one, two, three, four and five.
60. Learners We cannot see your graph sir, where is it going?
61. Learner Where are you eyes?
62. Teacher Okay, the feasible region is here over here [*teacher pointing at boundary of feasible region*]
63. Learners We cannot see your diagram sir?

64. Teacher It goes from somewhere to four, right and from five to five [*teacher pointing at intercepts on the graph*] This one comes from five to five, at three we have a vertical line. Jah! Hurry up! Hurry up. Okay [*teacher continue to read the problem*] In the sketch coma, in the sketch coma, the inequalities are shown, the inequalities are shown full stop first question [*teacher writes a*] on the board] first question a) Write down the inequalities that satisfy the given constraints full stop. Write down the inequalities that satisfy the given constraints full stop. Write down the inequalities that satisfy the given constraints full stop. [*teacher writes b*] on the board. b) Write down all the feasible solution to the problem. Write down all the feasible solution to the problem solution to the problem Okay to the problem. c) If all five drivers must drive, if all five drivers must drive, okay, and the type b bus, and the type B bus, uses less petrol than the type A bus coma okay. How many buses of each type, how many buses of each type how many buses of each type, should be used should be used to transport learners in the most economical way question mark. How many buses of each type should be used to transport the learners in the most economical way question mark. d) okay, If it costs R30 per learners to travel on type B, if it costs R30 per learner to travel on type B, If it costs R30 per learner to travel on type B and R40 per learner for type A, and R40 per learner for type A and R40 per learner for type A, how much must be sorry the school pay, how much must the school pay, the bus company, the bus company to hire the buses, to hire the buses for the outing question mark Jah, okay yes. Jah now solve the problem. let me find coordinates of the feasible region, let us use, how do we find coordinates of the feasible region? I will remind you. You guys keep on forgetting. You ignore also... From one what do you find? you find y is equal eight minus two x. I take y equal to 8 minus two x, I call it three. Now three into where?
65. Learners three into two
66. Teacher three into two. I get what here? I get two times brackets eight minus two x plus x equal to twelve. ($y = 2(8 - 2x) + x = 12$). Now I get sixteen minus four x plus x equal to twelve ($16 - 4x + x = 12$). Right now sixteen taken that side, here it will be minus three x equal to minus eh! minus four ($-3x = -4$). okay right!
67. Learner Yes!
66. Learner Sir, ...(went on to check learners work)
67. Teacher Jah! (looking at the learners inequalities) What! What! What! Come on look at here (moving to the chalkboard) we learn from our mistake (writing inequality on the chalkboard $x \leq 15 \geq 30$) guys what is this? *Ingozi! Ingozi!* [danger! Danger!] How can you say $x \leq 15 \geq 30$ *ingozi!* [danger] You put them side by side facing each other (using gestures) *ingozi* oh! [danger] no no, *ingozi!* [danger] what! what! what! no! no! no *ingozi!* okay guys now carry on (learners continued to write down their inequalities while the teacher checked their work and commenting to what they had written)
68. Learner Sir, here are my inequalities
69. Teacher What? okay look at here, x is the number of people, you are saying x is less than number of kilos, does it make sense? Wrong!
70. Learner Hmm it doesn't make sense
71. Teacher Wrong, it does not make sense, guys I told you make sure that your constraints make sense

68. Teacher Now x is equal to four over three. Now four over three into three into three it gives me what? It gives me y is equal to four no! Sorry. Eh! It gives me y is equal to eight minus two times four over three ($y=8-2(4/3)$), equal eight minus eight over three which gives me twenty four minus eight over three. jah sixteen over three, you know how to solve that. Coordinates of A are four over three and sixteen over three. Ja! Now which one maximise nine x plus three y now?
Pause
69. Teacher okay, you can find which one maximise nine x plus three y okay! Let us move to the second example. Second example ja!
- Revising a test and the first question was on financial mathematics
70. Teacher Jah, what have you found?
71. Learner 16(sixteen)
72. Teacher 16(sixteen)
73. Learners And 20(twenty)
74. Teacher 16(sixteen) and 20(twenty). And between here and here is six (pointing on the board) You see minus $6x+480 = X+60$, $-6X-X = 60 -480$, $-5X = 420$, X is equal to what?
75. Learners Negative seven
76. Teacher Aa! This is $-7x$, sorry. Then you say this is 60, right? Then once you find 60 you place one of your question, you find Y
77. Teacher Okay, now we do 1.2. 1.2 is a full linear programming problem, now here you are going to construct everything by yourself. 1.2 Are you ready? 1.2 now is the full linear problem where they give you a problem where you have to set all the constrains which are the limitations and the aah positive function. I told you, you see? You going to read, let us read the problem. A manufacturer of kitchen unit makes two type units Quanto, suppose he make a X unit of Quanto and Y unit of Quanto. It takes two days to put one unit Y unit, X : number of units of Ralto per month because he is producing per month. And Y : number of units of Quatro per month, okay? Here they define, you must know your x and you must know your Y . Now they say it takes two hours to put one unit of ralto together and three and a third days to put one... to put together one unit of Quatro, okay? It reads, you see here one unit of Ralto two hours
78. Learners Two days
79. Teacher Two days, sorry. Two days. One unit of quatro three, they say what? Three aah... Three and a third. You put a big three, days. To put together a unit of Quatro. Now this is done in a workshop which is available for only twenty days each month, okay? You see I have told you their name, their words that we indicate this is a constrain like availability, okay? Less than or equal to, greater or equal to. Here they told about availability, what does availability mean?
80. Learners Less than or equal to.
81. Teacher Availability means I cannot go beyond this number, okay?
82. Learners Yes
83. Teacher As I cannot go beyond this less or equal to and here they say availability. Aaah availability, they say what? How many? Twenty days, twenty days (writing on the board) you know already that the is a constrain when they say availability. Availability implies less or equal to, **[et cela signifie moins de]**, therefore will have the constrain here. If what you unit of ralto is two days, now all the X unit of Ralto will... I want to produce will take me two times X days plus am also producing Quatro, what do you need? One unit three and a third. On the Y you take three and a third Y . all this represent the time we spend in the workshop and this time the available time is twenty days($2X+3$) you see the first constrain. The constrain of time. Okay, the constrain of time. Then we continue, the "paint shop" can handle the maximum of eight units per month. Ya, you see? Those are names: **[qui va donner le maximum]** Maximum, it's a constrain. They say a paint shop can handle a maximum of eight unit per month (reading the question).You see already a constrain, this is another constrain.

Here paint, paint (writing it on the board) maximum of eight units per month. As am which constrain is the? You see, the paint unit can only take eight units per month, maximum of eight units per month. Maximum, meaning? It means I can't go beyond that one, it means as I am producing X-units of Ralto and Y-unit of Quatro, if I take X+Y those are the total unit that I will produce. It means when I will take them into the paint, the paint say I cannot take more than eight per month to produce, do you see that?

84. Learners

Yes

85. Teacher

[*alors ... sens ... hein plus grand que deux Ralto*] And they say at least two unit of Ralto must be produce each month, at least two units of Ralto. [*ya au moins deux*]. This is the first constrain, second constrain their here (pointing on the board).And they say at least two units, at least two units means what?

86. Learners

Greater than

87. Teacher

Greater than or equal, at least two units of Ralto. Ralto their X, right? It means X must be at least two($X > 2$), I cannot produce less than two units for Ralto(spoke French). And it says feather more, the number of unit of Ralto must be at least third of the number of Quatro, ya. Which constrain is that? Many people fail that question, ya many people fail, you not get it? Huh? What's the constrain the? They say, look at here, feather more they say number of unit of Ralto which is X, the number of unit of Ralto which is X must be at least, must be at least, right? A third of a number of Quatro, a third of $Y(X >)$.This is the constrain, that's all.

88. Learners

Aah!

89. Teacher

You don't get this one? The number of unit of Ralto X must be at least a third of a number of Quatro. Always you must add this constrain, X and Y belong to \mathbb{N}_0 which means a positive number. Ya you see, I don't know how much I think it was six marks.

90. Learners

Five

91. Teacher

Five marks. Now from here you can represent all the constrain in inequality and indicate the visible region. You represent those constrain, their how many? Their four, four constrain: one, two, three, four, and five. Ya, now you see where you do now the X-intercept and the Y-intercept here. Ya, represent now those constrains inequality, you know when here that this one is twelve.

92. Teacher

This is one I can give only to a girl, ya! Now who got the highest? I guess it's you? (pointing a student)You don't know yourself? You tell them, the one who got the highest. How much did you get in the test?

93. Learners

Thirty four out of fifty.

94. Teacher

Thirty four out of fifty. You got what? Okay, she is the one.

95. Teacher

You must always label your axis, labelling the axis is got marks. You don't miss the mark; linear programming is in the exam. The exam is already at the department and linear programming is the.

96. Teacher

You forgot ten. No, when Y is equal to zero. If you put zero, zero is theWhen X equals to zero then Y is equal to zero. When Y is equal to zero, X is equal to zero. Zero, zero one point. When Y is equal to three for instance Y equals to three. You put three here and X equals to one, you are join zero zero (0; 0) in that form. Always you must know, look, look at here. The line $Y = MX$, see that? If I wrote $Y = MX$, generally the equation line is $MX + C$, right?

97. Learners

Ya.

98. Teacher

Come in. C, look at here. C, normally C represents what? This is C. The Y intercept, always C represents the Y intercept. It means always when C is not the, it means C equals to what?

99. Learners

Zero

100. Teacher

Zero. It means already your line must pass by the origin zero zero (0, 0), every time you see that you have the equation line without a C, your line passes by the origin zero zero (0, 0). [*Si vous voulez que y-intercepte x soit égale à zéro. Si vous souhaitez que la valeur de x-intercepte vous mettriez y égale à zéro*] So if you want y-intercept you make x equal to zero. If you want the value of x-intercept you make y equal zero]. You know what Y wants, then you're gonna look for another point how? Give yourself an X value, make it two or three, then you find the corresponding for the Y. And then you're gonna join the point to the origin, and that's how you have your line. Ya, work

together. This is called number of units of Ralto (writing on the board X-axis) .And number of units of Quatro (writing on the Y-axis), This is Y and this is X, okay? [*adressesons d'abord la question d'un 'scale'... hachurons le dessus et le bas*] Let us first address the issue of a scale... we shade below we shade above Here] I think our scale if I was to look at a number, our scale I can take two two (2; 2). our scale can be two, four, six, eight, ten. on the Y two, four, six, eight, ten. Here, I think the first question when Y is equal to zero for the first line, I think it's easy. Two X plus three third, three third is what? It's ten over three, right?

101. Learners Ya
102. Teacher Ten over three Y, when X equals to zero Y equals to what? When I first want to represent the equation, I first represent the line by introducing equal to twenty. First you ignore the inequality sign and you represent a equal sign. When X equals to zero, X, Y. When X is zero, Y is equal to what?
103. Learners Six
104. Teacher Y equals to six and Y is zero X equals to?
105. Learners Ten
106. Teacher Ten and then you come here, when X zero Y is six(plotting the points on the graph) and for Y is zero X is ten then you join, this is the line two X three quarter Y equals to twenty($2x + \frac{3}{4}Y = 20$), okay? And for this line where is the solution, is it above the line or below the line?
107. Learners Below.
108. Teacher Below the line, you just put an arrow the to just show that it's below. The second one is easy, right?
109. Learners Yes sir.
110. Teacher This is eight eight (8; 8), eight eight (8; 8). When X is zero Y is eight and Y is zero X is eight, and then you join the line. This is the line $x + Y = 8$. Then our solution is still below because it's less than. Now X greater than two($x > 2$), where to from the? X greater than two is easy. You found it..... You see t is line? When X is greater or equal to one over three Y, now if you want to make Y the subject of the formula it will be now what? Remember your suppose to multiply both sides by three.
111. Learners Ya.
112. Teacher It will be three X greater or equal to Y, okay? It means your inequality equation will be Y is less or equal to three X, you see? It's the same thing, if you see the formula transforming Y is less or equal to three X. Let us first represent this one; X is greater or equal to two. First I have to represent the line X is equal to two.
113. Learners It's equal to two(simultaneously with the teacher)
114. Teacher X equals to two is here, this is the line X equals to two($x = 2$). Because they say X must be greater($x >$), my solution is at the right, okay? Now I have to represent this line which is equivalent to Y equals to three X. First I have to represent the line $Y = 3X$ I told you this line because there is no C first you know it will pass by here (*pointing on the graph*) and then I have to look for another point; any point, give yourself a value. When X, like here when X equals to two for instance Y will be equal to six; X two Y six(plotting the point on the graph) you see my line will go up the. This is the line Y less or equal to three X. Ya and they say Y is less or equal to three X, which side. Eeh, let's see. If I take a point here (plotting a point) look at here, if I take this point inside here. Let me take a point this side and that side, check which one will be my area. If I take this point here, this point is X equal to two Y equals to four, if I replace X by two here my Y will be six. Ya, my Y will be six. Ya it means, is this fall in the region? Am looking for where ya, am sorry. Y must be less than here, if I replace X by two I will find Y equals to six but six is six. No it's four less than six ($4 < 6$), ya you see because if I take this point Y is four($y = 4$) X is two($x = 2$), right? Now am looking for all the points such that when I replace the Y here, the Y value must less than three times the X value, okay? But if I see this point, this point which is here Y is four and X is two. Now, if I represent here point four and two (4; 2) this the Y value and this is the X value. I want, I want my Y value to be less than three times the X value this is what I want. But if I look at here, four less than ya. Four is less than six ($4 < 6$), true or false?

115. Learners True.
116. Teacher True. It means this point is a solution, so my solution is this side not that side. It's this side of the line. Now if you look at the, what's the feasible region? The feasible region is where all the, all the it means where they meet and they meet here. This side if I see, it must come here. It touch here, sorry. It touch the, it touch here it go the and the. Ya, this is my feasible region (high lighting the area) which is got the point A, B, C, D. which side?
117. Learners Where your shading
118. Teacher I wanted to choose any point below the line and any point above it, to check which one. I think X equal to two, I think when X equal to two and Y equals to four is going to be below the line. You understand? You don't understand, no. It means you choose any points this side and that side. You see, this is the point. The point is above, right? And this point is below the line, okay? Take, let me choose any point which is below the line, you say when X is equal to two is equal to four. You see this point here is below the line, if I want to take any point above the line I can take one and you see, if I take one and four(1;4) Y will be above the line, then you check the coordinates. And if they don't correspond or satisfy the inequality it means it is the region where. You understand? Ya you guys wait, ya I take one point two point three (1.2.3) unit of Ralto in the profit of, they say they want the equation of the profit function. Ya, that one easy right?
119. Learners Yes.
120. Teacher Ya, that one easy
121. Teacher Remember your equation must make sense when your working, you must be able to read. They must make sense, don't compare maybe you say money then this side you say people, then the other side is money then you put them together. No sense, right?
122. Learners Yes!
123. Learner1 Okay, first identify what X is.(this two learners discussing amongst themselves)
124. Learner 2 The number of buses
125. Learner1 X is the number of what?
126. Learner2 X is the number of the first class, and Y... Write down.
127. Learner 1 You should write down
128. Learner3 Sir(asking for assistance from the teacher)
129. Teacher You don't understand? You see that one, let me see. They say the gradient of are... You see x squared, what is X?
130. Learners The number of passangers.
131. Teacher X is the number of person. You see people, you see people greater than luggage. You see the number of people less than the luggage. But what are you doing? You see your comparing the number of people to luggage, you put them together. What, what?
132. Learners (They are laughing out loud)
133. Teacher I say I repeat. Your equation must make sense. This one is? Numbers of people on first class(talking to a leaner)
134. Learner4 Sir, what is this?(asking for assistance from the teacher)
135. Teacher Look at that boy, I only say(french words) the constrains are equal.
- CLASS DISCUSSING
136. Teacher What, what? Look at here, hey hey our Mr guy what is this? X is less or equal to fifteen greater or equal to thirty($x + y \leq 30$)? How could you say X is less or equal to fifteen, side to side. You put them right, Y is less than five greater than twenty. Okay, now it makes sense. Carry on!

137. Teacher (Explaining to a learner) Okay, look at here. X is the number of people, the number of people less than the number of kolos. Does it make sense?
138. Learner Mm...
139. Teacher No, it just a problem.
140. Learner2 It doesn't make sense.
141. Teacher You compare kilo, people people(Now talking to another learner). Now why is Y less than five? Where did you get the fifty five?
142. Learner Y is less than
143. Teacher Shh..Silence, let us see this well, we learn from our mistakes. Ah this here, look at this here they say the aircraft caters for a total of twenty passangers, see the sentence. Now, they say air craft class A caters for class A(writing X on the board) caisters for a maximum of fifteen. And therefore this maximum, look at here becaus this one caters for a maximum of twenty. You say the maximum in the now favors five $Y \leq 5$ (talking of another learner's work). Now what's your problem here?(asking the learner). Who can tell us, What's your problem? He say, look at here(going to the board). The aircraft caters for a maximum of twenty, this is maths. This is mathematics, twenty but the first class caters for a maximum off fifteen($X \leq 15$). Therefore, therefore the other one must get a maximum of five.
144. Teacher Look at here, I don't. Your all(french being said) where you have the all, what I have here here it can be twenty, this can be total. I must put them the total, I must put them the total. You see from here if I add them side by side I will have twenty, okay? This one you will use for yourself, this one; what, what? The maximum of five here which is normal, which is normal because here when you get the maximum of fifteen it i obviius because the total will be twenty and that one the maximum will be five but mind you I don't know if it will make sense here how so, but when they say a maximum you must cont out all the class A because am catering class A and class B and put them together but here they gave me specifically. I think this constrain you can put it ($Y \leq 5$) but it might be redundant, it might be redundant you see? From this one, this one was here if you add them. This is a consequence, this an automatic consequence of this(Indicating on the board). Automatically having this and this, I must have this ($X + Y \leq 20$). It is obvious if you use this constrain, if you use this constrain it will be redundant, it will be redundsnt but at least you raised the question you raised something but it will be redundant.
145. Teacher Now represent graphically, we have three constrains. Represent them graphically. Now represent them graphically, you know your constrains.
146. Learner4 Is this equation right?
147. Teacher He is right, you know wha because this is the number of KG for per all the passangers. It means that all the X passangers will have to thirty times X Kg ($30 X \text{kg}$). This is one at twenty for one passanger, twenty times Y will be total for all passanger of Y in the economic class. If I add them it will be the total Kg that all the passanger it must carry in the aircraft and this aircraft can not cater for more than six hundred, you understand. It makes sense! Guys, I see you understand.
148. Learner2 Thirty X plus twenty Y is less than or equal to six hundred ($30X + 20Y \leq 600$) and then from the what do we do?
149. Tearcher Ya, your problem. You see when you draw you first, this is your equation. It will be five and fifteen(drawing a graph of the equation on the board) when X equals to Y, X is equal to fifteen. This one it will be here, it will be here(drawing lines to help finding the feasible region). And all my feasible region will be here. It will be down here, that is why it is redundant. Youu see this one? Because it is just the consequence of two here, it will be redundant. You can put it but it does not play it is not really a constrain.

150. Teacher (Talking to the learner) Yes this is what? Is this on correct! Where did you get it from? Guys, don't write things that you don't know. Don't write thing you don't know. I remember the comedian from Cameroon, they say what do you do eh for a living? Listen, what I see I do and what I don't see I don't do. What I see I do and what I don't see I don't do, ya! You, your writing things you don't know. You see already one constarin is already redundant, you see that one constrain is not part of the feasible region, it is redundant.
151. Teacher (Looking in the book of a learner) What your feasible region, we should. This constrain is $X+Y$, $X+Y$. The number of Kg is redandunt, why? You see when you look at the problem the lugage, the lugage constrain does not play any role in the feasible region. Being the or not, if I know the number of passangers I don't know what is redundant here because it is a problem or it is a two reflection of the aircraft. In the aircraft they need a person to know the maximum number of it but is problem the number of passanger is enough. You take the constrain. I don't know, maybe is just for the problem. I don't think it's reality because in the air craft, you must know the number of people and the lugage you are taking. Okay, you can not exceed the capacity of the aircraft. Okay, good. Now use your income to determine how many passangers, find the solution. Find the maximum.
152. Teacher (learner explaining their solution) No don't add, it mango plus banana. It equal to what?
153. Learner Equals to banana mango.
154. Teacher Mango plus banana equals to what? Mango banana
155. Learner (Laughing)
156. Teacher No you can't add because those are X and other are Y.

APPENDIX H: CLINICAL INTERVIEW JOHN

1. Thulie: Good day John
2. John: Bonjour
3. Thulie: How are you?
4. John: Je suis bien madame
5. Thulie: Wow, that is all I understand in French, but you may use any language as we work together, French or English.
6. John: Ok! Mais je vais essayer d'utiliser l'anglais, j'apprends l'anglais comme ma langue maternelle ... donc je vais essayer (okay but I will try and use English, I am learning English as my first language...so I will try)
7. Thulie: What about French?
8. John: J'apprends le français comme seconde langue au lieu d' autres langues de chez nous (I learn French as a second language instead of the other home languages)
9. Thulie: Did you learn English at school in Congo?
10. John: Au Congo, nous apprennent le Français à l'école et on parle le Lingala à la maison avec notre mère. Mais nous parlons Français avec notre père. L'Anglais, nous apprenons de nos amis (In Congo we learn French at school and we speak Lingala at home with our mother. But we speak French with our father. English we learn from our friends)
11. Thulie: When did you arrive in South Africa?
12. John: Vers la fin de l'année dernière je me suis inscrit *Octobre 2010*... (Towards the end of last year I enrolled October 2010...)
13. Thulie: I have a task that I would like you to solve. I will ask you questions as you solve
14. John: D'accord, le professeur nous a dit... (Okay, the teacher told us...)
15. Thulie: Can you tell me the information in your own words, what the task is about?
16. John: Je ne peux pas parce que ... parce que je ne comprends pas l'anglais ... (*hochant la tête*) (I cannot because ... because I do not understand English... (*Nodding his head*))
17. Thulie: Can you tell me in your own words what is written here?
18. John: no, no
19. Thulie: How do you cope then during the lesson?
20. John: Okay (*relaxe*), parfois notre professeur explique Français (parce que) il sait que nous sommes du Congo et il peut parler Français, donc ... il ... nous explique et nous comprenons (Okay (*relaxed*) sometimes our teacher explains in French *parce que* (because) he knows we are from Congo and he can speak French, so ... he ... explains to us and we understand...)
21. Thulie: Okay, do you ask him to explain?
22. John: Oui, des fois nous lui demandons, mais nous comprenons en Français puis nous exécutons la tâche en Anglais... (yeah sometimes we ask him, but we understand in French then solve task in English...)
23. Thulie: Let us say you are doing your homework what do you do in that case?
24. John: Des fois, nous utilisons notre dictionnaire pour vérifier le sens de certains mots en Français et alors nous comprenons (Sometimes we use our dictionary to find meaning of some words in French and then we understand)
25. Thulie: You mean you find meaning of the words in French
26. John: Oui, nous avons le dictionnaires de l'Anglais au Français ou du Français a l'Anglais .Nous cherchons donc le mot Anglais dans le dictionnaire et alors nous savons sens en Français. Il devient facile (Yes, we have English to French dictionaries or French to English dictionary. So we look for the English word in the dictionary then we know meaning in French it becomes easy)
27. Thulie: Did you do linear programming in Congo?
28. John: Non, mais nous avons fait des inégalités linéaires (*avec confiance*) ... qui consiste a résoudre les inégalités et les dessiner le graphiques... (No, but we had done linear inequalities (*with confidence*)...that is solving inequalities and drawing graphs ...)
29. Thulie: Alright, I would like you to solve the task...eh but it is English.
30. John: C'est bon, le professeur nous donne des tâches écrites en Anglais en classe aussi. (It is okay, the teacher give us tasks written in English in class also)
31. Thulie: Read the task aloud and may be if you have a question you ask, I will explain
32. John: (*reading aloud*) A local health ...board is ...producing a guide ... guide for healthy living. The guide should provide ... advice on health ... health education ... healthy lifestyles ... and the like. The board

- intends ... to produce ... the guide in two formats ... one will be ... will be in the form of a short video ... the other as a print printed binder...
33. Thulie: Do you understand the information you have just read, what is it telling us?
34. John: Non je ne comprends pas mais je sais qu'il ya deux choses (No I don't understand but I know there are two things (*John pointing at the word two*)...)
35. Thulie: Okay continue reading...
36. John: (*reading task*) the board is ... is currently trying to decide ... to decide how many of each type to produce for sale for sale...it ... it has estimated that it is likely ... likely to sell no more than ten thousand (10 000) copies ... copies of both ...both items together. At least four thousand (4 000) copies of the video and at least two thousand (2 000) copies of the binder could be sold ... al ... although sales of the binder are not expected to exceed four thousand (4 000) copies.
37. Thulie: Please solve the task, I will ask questions as you solve
38. John: D'accord, mais laissez-moi essayer de le résoudre par moi-même et peut-être vous poser les questions après que j'ai terminé la résolution (Okay, but let me try and solve it on my own and perhaps you ask the questions after I have finished solving)
39. Thulie: I have no problem with that (*John working at the task, first started by underlining the following phrases; two formats, one will be, the other, how many of each, no more than 10 000, both items, at least four thousand, at least two thousand, not to exceed, left out expected then after a few minutes he showed me the solution*)
40. John: C'est ici ma solution (This is my solution)
41. Thulie: Ah! This is fantastic! John, before you solved the tasks I observed that you started by underlining these words (*pointing at the words he underlined*), is there any reason for doing that?
42. John: Ces sont les contraintes, je reçois des inégalités ... non oui les Signes tels que l'inégalité de moins que pour ne pas dépasser ... moins pour pas plus de ... plus grande que pour au moins ... et j'ai les tous donc j'ajoute... (These are the constraints, I get inequalities...no yes the inequality signs like less than for not to exceed...less than for no more than...greater than for at least... and I have both so I add...)
43. Thulie: Good, you say for both you add, meaning you add what?
44. John: (*silence*) ... j'ajoute x et y ... oui x et y... ()...I add x and y ...yes x and y...)
45. Thulie: Okay, let us say you add the two formats of the health guide...
46. John: Je ne sais pas mais je sais qu'il ya deux choses, la vidéo et quoi? (I don't know but I know that there are two things, the video and what?)
47. Thulie: The other thing is a binder
48. John: oui ... j'ai donc ajouter les deux ... et ils sont représentés par X et Y ... les questions dit pas plus de dix mille pour les tous donc il doit être inférieur à ...(yes...so I add the two...and they are represented by x and y... the questions says no more than ten thousand for both so it has to be less than...)
49. Thulie: ...Now when underlining you were leaving out some words like in this one, not expected to exceed four thousand. The word expected is not underlined. Any reasons for that?
50. John: cherche des mots-clés ... hein mots pour m'aider avec les inégalités ... hein je veux dire que je veux dire donc que je veux les inégalités pour que je me concentrer sur ces mots ... les mots-clés... (I look for key words...eh words to help me with inequalities...eh I mean I want inequalities so I focus on those words...the key words...)
51. Thulie: Yeah? So what does that mean?
52. John: Je veux dire je n'ai pas beaucoup d'attention à quelques mots ... parce que je sais si je prends les mots clés tels que de ne pas dépasser ... hein, je sais que mon inégalité est moins que ... tant attendu ... hein ... (*rires*) ... comme ici mon inégalité est x est inférieur à quatre mille ... jah ... (I mean I do not pay much attention to some words...because I know if I pick the key words like not to exceed...eh I know that my inequality is less than...so expected ... eh...(*laughs*)...like here my inequality is x is less than four thousand...jah...)
53. Thulie: Can you explain to me as to how you solved the task?
54. John: En mathématiques ... hhm ... Je viens de suivre les étapes que l'enseignant utilisait en classe... (In mathematics ... hhm ...I just follow the steps the teacher was using in class...)
55. Thulie: If you follow the steps... you follow the steps...how do you know that you have greater than or less than like here (*pointing at one inequality*)?
56. John: La question est similaire à celle que notre professeur nous a demandé de résoudre en classe (the question is similar to the one our teacher asked us to solve in class)
57. Thulie: In linear programming you have to understand the problem, you said you did not understand the first part of the problem, what do you do if you do not understand what the problem is all about?

58. John: ... Pour cette question, j'ai regardé les mots-clés ... il ya des mots clés que vous devez connaître leur signification ... le professeur nous a dit que ces mots nous donner le signe de l'inégalité ... hein, comme inférieur ou supérieur. Parfois, inférieur ou égal à ... et supérieur ou égal à ... hein ... jah (For this question, I looked at the key words...there are key words that you must know their meaning...the teacher told us that those words give us the sign of the inequality...eh like less than or greater than. Sometimes less than or equal to...and greater than or equal to...eh jah...)
59. Thulie: key words that you must know... like what?
60. John: Comme ... au moins hein ... tout au plus ... maximum ... minimum ...jah (Like ...at least eh...at most ...maximum ...minimum jah...)
61. Thulie: Okay, what are the key words in this task?
62. John: ici il ya au moins deux mille ... et cela signifie plus de deux mille ... l'inégalité sera ... hein supérieure ou égale à deux mille..._(eh... here there is at least two thousand ...and it means more than two thousand ...the inequality will be ... eh greater than or equals to two thousand)
63. Thulie: fantastic ...okay eh... anything else?
64. John: ... il ya celui-ci (*pointant sur ne dépasse pas*) ... eh ne dépasse pas quatre mille ... et cela signifie moins de quatre mille ... et c'est cette inégalité. ((*looking at the words he underlined before solving the task eh*) ... there is this one (*pointing at does not exceed*) ...eh does not exceed four thousand ...and it means less than four thousand...and it is this inequality.)
65. Thulie: Okay continue...
66. John: hein ... alors ... avec celui-ci ... hein il ya au moins deux mille... aussi ... ne dépasse pas quatre mille ... ok ... je pense... (*silence*) (eh ...so ...with this one ...eh there is at least two thousand andalso ...does not exceed four thousand...okay I think.... (*silence*))
67. Thulie: This is fascinating,
68. John: Oui ces sont mes inégalités ... eh oui (yes these are my inequalities ...eh yes)
69. Thulie: Okay, I now understand what you mean by key words. I am fascinated by this inequality, you have this inequality (*pointing at $2000 \leq x \leq 4000$*) what does it represent?
70. John: Il représente ne dépasse pas quatre milles, ne dépasse pas signifie moins que (*pointant à moins de 4000*) et ... hein ... il ya au moins deux mille dans la question ... alors ... sens ... hein plus grand que ... x supérieur deux mille ... qui dans cette partie est (*pointant à $2000 \leq x$*) pour obtenir cette inégalité... (It represent does not exceed four thousand, does not exceed means less than (*pointing at less than 4000*) and ...eh... there is at least two thousand in the question... so ... meaning ... eh greater than...x greater two thousand...which is this part (*pointing at $2000 \leq x$*) so getting this inequality...)
71. Thulie: Alright, continue
72. John: Oui ... oui je sais qu'il ya une limite ... ce qui signifie pas plus de quatre mille et pas moins de deux mille (*en utilisant des gestes*) ... hein ... alors une inégalité comme ça ... est généralement représentée comme ceci ... Nous l'avons fait au Congo, donc je sais qu'il doit être écrit comme ceci. (yes an ...yes I know that there is a limit... which means not more four thousand and not less than two thousand (*using gestures*) ...eh so ... so an inequality like this ... is usually represented like this... We did this in Congo, so I know it has to be written like this)
73. Thulie: okay, give me an example of a number satisfying the inequality...that is any number you can think of...can it be one thousand five hundred?
74. John: ..._Pas mille cinq-cents (*hochant la tête*) ... oui pas mille cinq-cents... ... (le nombre est plus de deux mille ... alors peut-être disons trois mille ... oui trois mille est un exemple ...) (...not one thousand five hundred (*nodding his head*)...yes not one thousand five hundred... ...(the number is more than two thousand...so may be let's say three thousand ...yes three thousand is one example...)
75. Thulie: Okay, good what is our next inequality?
76. John: Bon ca c'est pour x, (Okay this is for x)
77. Thulie: Yes which is representing the binders... now we have to look at the number for the videos
78. John: (*silence*) oui ... c'est y... oui Y qui représente la deuxième chose ... hein ((*silence*) yes... that is y... yes y which represents the second thing...eh)
79. Thulie: what are the key words now?
80. John: ok laissez-moi voir ... il ya au moins quatre mille exemplaires pour les vidéos ... qui signifie supérieur à quatre mille ... oui c'est plus grand que l'inégalité (okay let me see...there are at least four thousand copies for the videos... (*pointing at the words at least four thousand on the handout*) which means greater than four thousand...(writing down inequality) yes greater than is the inequality (*noticing that it is the same inequality that he had written*))

81. Thulie: Good, let us move on to this one with ten thousand, I can see you added x and y, any reason for adding the two?
82. John: IL est dit ... c'est dit... des copies des toutes les questions en même temps ... donc j'ai ajouté car tous signifient ajouter... (It says... it says... (*Looking at the question paper*) ...copies of both items together...therefore I added because both means add...)
83. Thulie: I can see, in other words it is easy to write inequalities by just looking at the key words and knowing which inequality it represents. But then there are operation sign which are denoted by words like together
84. John oui ... alors ... hein ... Je me concentre sur les mots clés ... Je sais que je vais trouver la solution même si je ne comprends pas ou réside le problème ... (yes... so ...eh... I just focus on the key words ...I know I will get the solution even when I do not understand what the problem is all about...)
85. Thulie: Now I want to know do you do your work in English. Do you think in French?
86. John: Je fais mon travail en Français pour ne pas faire des erreurs ... J'utilise toujours le Français ... en Anglais je ne sais pas certains mots ... donc le Français je connais ... j'ai fait les mathématiques en Français... (I do my work in French so that I do not make mistake... I always use French ... English I do not know some of the words... so French I know...have done mathematics in French...)
87. Thulie: When would you use English?
88. John: Je n'utilise pas beaucoup l'Anglais en mathématiques ... parfois j'échange... mais pour les mathématiques, j'utilise le Français... (I do not use much English in mathematics...sometimes I swap ... but for mathematics I use French...)
89. Thulie: Do you sometimes find yourself thinking in English and sometimes in French?
90. John: Non ... parce que ... En mathématiques je pense en Français ... sauf lors de la lecture du manuel ... car ils sont en Anglais? (No... because ... in mathematics I think in French... except when reading from the textbook... since they are English?)
91. Thulie: When do you use English?
92. John: Je viens de lire en Anglais parce que le matériel est en anglais ... eh puis ma compréhension est meilleure en Français ... alors je peux m'exprimer en Français ... alors quand je pense à un problème hein, je répond en Français ... si heureusement en mathématiques que nous écrivons des solutions sans expliquer Jah ... (I just read in English because the material is English...eh then my understanding is better in French...then I can express in French...so when thinking eh about a problem I respond in French...so fortunately in mathematics we write solutions without explaining jah...)
93. Thulie: So it is when you are asked to explain that you swap into English....
94. John: Oui ... (yes ...)
- Thulie: What about when you do not understand?
95. John: Oui j'utiliser le Français ...Le Français Je comprends plus facilement ... et mon professeur comprend ... il vaut donc mieux ... Il nous a expliqué... (yeah I use French ... French I understand it is easier... and my teacher understands... so it is better...he explains to us...)
96. Thulie: It is easier in French. So what sort of times would it be easier in English?
97. John: Hein, je ne suis pas sûr ... Je ne suis pas sûr ... car l'Anglais est plus difficile ... je continue à apprendre ... mais le Français je connais (...eh I am not sure ... I am not sure...because the English is harder... am still learning... but French I know)
98. Thulie: Indicate the feasible region by shading because I can see you have arrows...
99. John: Okay (Okay (*shading the feasible region the region where the arrows meet*))
100. Thulie: What will be your next step, because you said you follow steps?
101. John: Oui ... il sera de trouver le point qui va donner le maximum ... la question ... Je dois maximiser ... donc j'écrie la formule ... qui est la fonction de profit ... ici (Yes... it will be finding the point that will give maximum...the question ...I have to maximize... so I write the formula ... which is the profit function... here (*pointing at what he had numbered 4, the objective function*))
102. Thulie: Is that the objective function?
103. John: Oui, le profit fonctionne de fois. Mais ils disent ... hein ... la question est la fonction objectif ... (Yes, the profit functions sometimes. But they say ... eh... the question is objective function...)
104. Thulie: So what is the objective function here?
105. John: Laissez-moi voir ... c'est ... hein... (Let me see... it is ... eh...)
106. Thulie: Look at the question ...we are told that each video cost how much? And each binder cost how much?

107. John: Oui, nous avons donc le liant pour R30 chacun, c'est donc trente y) (*écrit 30y*) ... et la vidéo à R50 chacune , donc c'est cinquante x (*écrire 50x*) ... (Yes, so we have binder R30 each, so it is thirty y) (*writing down 30y*) ... and video R50 each so it is fifty x (*writing down 50x*)...
108. Thulie: Okay, so what is the objective function?
109. John: Euh ... c'est ... cinquante x plus trente Y ($I = 50x + 30y$) (Eh... it's... fifty x plus thirty y (*writing I = 50x + 30y*))
110. Thulie: Okay, continue
111. John: (*looking at the feasible region, then he mark points inside the region, then he writes down the coordinates of the vertices of the feasible region...next he substituted each coordinate in the profit function*) (ok, j'ai calculé la valeur de la fonction de profit utilisant chaque point ... ((*swaping to English*)...point at vertices of the feasible region...eh then it is point B...I marked the points
112. Thulie: Okay continue (*John double check the values of the function at each point using his calculator*)
113. John: Ce point maximisera le profit (*pointant à 4 000; 6 000*) This point will maximize the profit (*pointing at 4 000; 6 000*)
114. Thulie: So what is the maximum value? I saw that you were calculating again
115. John: La valeur maximale est R380 000 ... Je voulais juste être sûr de mes valeurs... (The maximum value is R380 000...I just wanted to be sure with my values...)
116. Thulie: Great! (*still fascinated by the work that John did, I had to ask him once more about the language he uses when learning mathematics*) Just tell me John, when thinking ... or let me say when solving mathematical tasks, which language do you use? Is it English or French?
117. John... Je fais tout en Français, mais alors je dois l'écrire en Anglais. Mais je comprends mieux en Français ... certains des mots sont les mêmes ... (...I do everything in French, but then I have to write it in English. But I understand better in French...some of the words are the same...)
118. Thulie: What do you mean the same?
119. John: L'orthographe de certains mots, comme solution ...c'est la prononciation qui diffère sinon ils sont les mêmes... (The spelling of some of the words, like solution... it is the prononciation that differs otherwise they are the same...)
120. Thulie: Okay
121. John: ...I do everything in French, but then I have to write it in English. But I understand better in French...some of the words are the same...)
122. Thulie: Thank you very much

APPENDIX I: LESSON OBSERVATION IN CLASSROOM B

lesson 1

1. Teacher Good afternoon class. May I introduce Mrs Nkambule from UNISA. She is here for linear programming as i have told before. She is studying mathematics education at UNISA. She is here for Linear programming. As you know we have started with inequalities to represent a region okay. Today we are going to start with smething else we call linear programming (teacher distributing handouts)
2. Learners Yes
3. Teacher Everyone has a handout? Right I had indicated that ukuthi when we do linear programming you will always be given a statement, from the statement kusho ukuthi uchuze x and y uze nama-equations, inequalities then singasho ukuthi number one is the steps, number two is the algebraic expression ne! now (*name of learner*) read what is on the first handout on steps
4. Learners (*Learner reading*) Decide on what you are going to call x and y What must be made maximum or minimum at the end must be equal to x and y. Two remember x is greater or equal to zero y is greater or equal to zero. Represent the constraints algebraically. represent the information on a graph and determine feasible region
5. Teacher minimum sorry (*some students arrived late*) one student made an announcement concerning their sports day which was to be held the following day
6. Learner (*Learner making an announcement*) Good afternoon class, maniphuma la you are asked to assembly outside when the bell rings in order to be briefed about what is goint to happen tommorrow, nazi ukuthi kwenzakalani tomorrow right
7. Teacher right number one, decide what you are going to call x and y. What must be made maximum or minimum at the end must be equal to x and y right
8. Learners some learners said Yes, others said sure, others said siyabonga in response to the announcements
9. Teacher number two, remember x ibe greater or equal to zero and x ibe greater or equal to zero right?
10. Learners yes
11. Teacher number three represent the constraints algebraically ne!
12. Learners yes
13. Teacher Number four represent the information on a graph and determine what?
14. Learner feasible region
15. Teacher right! Use dual intercept method ne! And now chekha la for greater or equal and less than or equal use a solid line. For greater than or less than you use a dotted line. Do you remember that? You use what?
16. Learners yes a dotted line
17. Teacher remember that x and y are members of real numbers colour the area (x,y Real). X and y are members of integers, use dots remember to name your axes properly. Find the gradient of the profit function which can be determine graphically or algebraically. Calculate minimum or maximum profit depending on what you have been asked. if it is cost you minimize and if it is profit you maximise. niyangithola? and something like that...
18. Learners yes
19. Teacher Lets go to the algebraic representation, right! If you have a statement that says y is not more than or less than 10, when you represent that .. i-answer kumele ibe what? Kumele ibe y is less than or equal to ten ($y \leq 10$), right?
20. Learner yes
21. Teacher Lets check the second one, y is at least two hundred i-answer kumele ibe more or equal to two hundred. Kufana nokuthi sithi kuwe i-passing mark is at least fifty percent. It simply means that your mark should be more than fifty percent. Uyangithola?

22. Learners Yes
23. Teacher x is not less than ten. If something is not less than ten, it is definitely is sure greater than...
24. Learners greater than ten
25. Teacher greater than ten, uyangithola?
26. Learners Yes
27. Teacher next one, x and y are together greater than at least one hundred ($x + y \geq 100$)
28. Learners at least one hundred
29. Teacher Once you see the word together it simply means summation angithi?
30. Learners Yes
31. Teacher You simply add the two ne!
32. Learners yes
33. Teacher now uthe x plus y batheni?
34. Learners at least
35. Teacher At least usho greater than...
36. Learners one hundred
37. Teacher greater than one hundred
38. Learners yes
39. Teacher now chekha the ratio. The ratio of x to y is at least three is to four. Now the ratio x is to y is at least what? Three to what../
40. Learners three to four
41. Teacher (teacher writes 3:4 on the board) three is to four i-ratio, now you know what, i-ratio, now if you have something like (teacher writes $\frac{1}{2}$ on the chalk board) this is just the same as one is to two ne (teacher writes 1:2 on the chalk board) so x is at least two times y would mean x is greater or equal to two y ($x \geq 2y$).
42. Learners Yes
43. Teacher niyangithola?
44. Learners Yes
45. Teacher y is at most three times x usho ukuthi y is less or equal three times x ($y \leq 3x$). Niyangithola?
46. Learners Yes
47. Teacher Like i-ratio yethu la! For every educator! Kumele kube na thirty two learners uyangithola?
48. Learners yes
49. Teacher If you have sixty four learners it simply means nina kumele nibe na two educators niyangithola? Kusho ukuthi leratio le ayibalanci. Especially if sowumdalanyana njengathi kumele leratio i one is to thirty (*learners laughing*) right x is at least two times y, two times is double angithi?
50. Learners yes
51. Teacher Now bathi x is greater and equal to two y and y is at most three times meaning y is less than three x angithi now (teacher writing at least and at most on the board) ne! Make sure kuthi you understand these words! (Teacher emphasizing)not necessarily ukuthi everytime you have to say eh! y is less than what ever particular number or y is greater than.. but instead of using abogreater than or less than we use also what? we use abo at least and nabo at most niyangithola?
52. Learners yes
53. Teacher right any question? Okay (teacher distributing handouts) everyone uyitholile i-handout? Everyone? Bring the surplus! right from your handout ne! An important thing in a statement will be what? Important

54. Learners terminology
55. Teacher terminology right! Number one is implicit constraints you are given the definition ukuthi if you say something is an implicit statement it simply means that ama-inequalities that arise what how?
56. Learners naturally
57. Teacher ne! now next when do we do linear programming? When we are given a real life situation and we will see now from the first example, we will be given a real life situation ne! So when we say something arises naturally you cannot say we are having negative three girls at the end of the three days (learners laughing) uyangithola! or negative six point five boys yini into ewrong lapho? number one...
58. Learners Is the sign
59. Teacher Is the sign number two?
60. Learners points
61. Teacher points (learners laughing) just imagine sowuthi aha wena lento ye three point five... That's a serious insult now uyazi mamela yazi we said last time, mamela we said last time every time for one to get a feasible region you have to represent your constraints kufirst quadrant why? kufirst quadrant because x is greater than zero ku first quadrant and y is greater than zero ku first quadrant ne! it simply means before you can write whatever okuzoba yi constraints your first constraint kuzoba yi x is greater or equals to zero and y is greater or equals to zero. Its either you have positive girls or no girls uyangithola? Angithi? there is nothing wrong if uzothi kiclass linale twenty eight boys there is nothing wrong in that like kithatha le-issue yabo boys school or yabo girls school. there is nothing wrong in that Ja! Then number two the constraints this is just i-terminology you use for linear programming ane but chekha ukuthi bathini ngeconstraints bathi the inequalities that you make from the given information
62. Learners given information
63. Teacher given information ne! If sithi the number of boys laskolweni kumele kube at least twenty what will be the constraints? The number of boys ne! in this class ne! Is at least twenty what will be the constraints?
64. Learners greater than and equal to twenty
65. Teacher yes (pointing at one learner)
66. Learner greater or equal to twenty
67. Teacher greater or equal to twenty angithi? Number one sithi u-identifaya ukuthi which one utayibits x or which one utayibitsa y but since in this case sidila na only boys utayibitsa x or y ne?
68. Learners yes
69. Teacher angithi but basically amaconstraints or inequalities uwathola from the given statement or given information uyangithola? Any questions siright
70. Learner yes
71. Teacher chekha number three
72. Learners feasible region
73. Teacher feasible region angithi siyenzile from last weekend ukuthi ifeasible region is the area on the graph that satisfies the above constraints
74. Learners that satisfies
75. Teacher ane! Ja! Silungisa lapho angithi... That angithi
76. Learners yes
77. Teacher right alright an objective function also called what?
78. Learners a search line
79. Teacher is a straight line graph you draw to help you solve the problem that one we will look at later. Let us start with number one. (Name of learner) asked to read the problem

80. Learner *(reading from handout)* A party is to be arranged. Let x be the number of girls and y the number of boys that will attend this party. Make inequalities (constraints), if you are given the following information. There must be at least 200 people at the party. However, the hall can only fit 600 people. We need at least 150 girls and at most 400 boys and the ratio of boys to girls must not be more than 2:1. The organisers must make at least R4 000 on entrance fees. Each boy pays R20 to attend and each girl pays R15.
81. Teacher Let us try and make sense from the statement you cannot charge i-prize eyi-one angithi abanye bancono kunabanye you understand why? This one is a simple scenario ne! Simple scenario someone u-inthenda ukuthi abene party but one kuna.. There are other things kumele take into consideration number one ukuthi ihhola cannot accomodate iSwaneville yonke those are the first constraints ukuthi kay now lets go now ukuthi now you have to organize a party ihhola kumele ibe nama seats that will accomodate abantu oba invithile can you just say no I will invite iSwanevielle yonke but wazi kahle ukuthi ihhola is like iklassi Ja! and iklassi cannot accomodate more than sixty learners okay right anyway kumele sicale ngama constraints. Lets try to draw a table (*teacher draws a table on the board*) number one we said its girl and x represents the number of girls and y the number of boys what are the two critical things la? we said there are two critical things go through the statement (*name of learner*)
82. Learner *(reading from handout)* A party is to be arranged. Let x be the number of girls and y the number of boys that will attend this party. Make inequalities (constraints), if you are given the following information. There must be at least 200 people at the party. However, the hall can only fit 600 people. We need at least 150 girls and at most 400 boys and the ratio of boys to girls must not be more than 2:1. The organisers must make at least R4 000 on entrance fees. Each boy pays R20 to attend and each girl pays R15.
83. Teacher Okay I will give you one (*teacher writes attendance in the table*) the first one is attendance what will be the second one?
84. Learners entrance fees
85. Teacher entrance fees (teacher writes fees in the table) okay seninamo amaconstraints lets introduce another column or another row yama totals. Whats the total ye-attendance?
86. Learners six hundred
87. Teacher six hundred (teacher writes 600 in the table)
88. Learners two hundred, others one fifty (teacher stops writing) five fifty
89. Teacher what is the total attendance? Kumele babe bangakhi abantu aba-athendayo
90. Learners two hundred
91. Teacher Ah must have a restriction
92. Teacher The first statement ithi there must be at least two hundred people at the party ne!
93. Learners yes
94. Teacher which means when we add the number of girls aand the number of boys kumele sibe at least two hundred
95. Learners two hundred
96. Teacher Thats the first thing angithi
97. Learners yes
Okay lets say la two hundred ... Anela sithe x le sayibitsa y okay
98. Learners yes
99. Teacher Which simply means this will be x plus y two hundred (teacher writing $x + y \geq 200$) what will be the sign la (pointing at the space between $x + y$ and 200)?
100. Learners greater or equals to
101. Teacher greater or equals to (writing $x+y \geq 200$) greater or equals to because they said at least two hundred okay second one second one yes. The second one yes
102. Learner x is

103. Teacher x is...
104. Learner x is greater or equals to fifteen
105. Teacher x is greater or equals to fifteen no!
106. Learner i think Eh!!
107. Teacher les or equal to four thousand
108. Learners the at least four thousand, that means that x plus y is greater or equals to four thousand
109. Teacher okay asibuyela la (pointing at table) now you are saying when you add one x plus y, bear in mind ukuthi that the people babhadal different amount angithi
110. Learners different amount yes
111. Teacher but now what you are saying okay whatever is the number of girls plus the number of boys must equal to two hundred
112. Learner yes
113. Teacher What happened to that fifteen rand and twenty rand, what happened to fifteen rand and twenty rand? Angithi basically what you are saying la mamela the fee abazongena ngayo i-fifteen rand and nabazongena nge twenty rand.
114. Learners yes
115. Teacher From whatever category ye fifteen rand and category yabazongena nge twenty rand
116. Learners twenty rand
117. Teacher Whatever category yabazongena nge fifteen rand or nge twenty rand we expect ukuthi we must have what? At least four thousand rand
118. Learners four thousand rand
119. Teacher four thousand okay what should be the sign yala?
120. Learners greater than or equal to fifteen rand plus twenty rand is greater or equals to four thousand rand ($R15 + R20 \geq R4000$)
121. Teacher greater than or equals to four thousand but mathematically it does not make sense to say that fifteen rand plus twenty rand is greater or equals to four thousand it simply means we must include something.
122. Learners Oh!
123. Teacher yes (pointing at learner)
124. Learners fifteen x plus twenty y greater or equal to four thousand (teacher writing 15 and 20 in the table)
125. Teacher then fifteen x plus twenty y greater or equal to four thousand $15x + 20y \geq R4000$, another constraints yes another constraints angithi you do not know ukuthi how many girls will attend that party
126. Learners yes
127. Teacher and in this case nani la niyabona ukuthi abantu abaletha imali abafana angithi?
128. Learners laughing
129. Teacher angithi, we are not sure ukuthi how many girls will attend the party and how many boys will attend the party angithi it might happen ukuthi kuba yihalf company, i-half company makuyi couple angithi?
130. Learner yes
131. Teacher what will be another constraint another constrain right let us go through the statement. (*Teacher reading from handout*) There must be at least two hundred people at the party that one we have considered it
132. Learners yes

133. Teacher However the hall can only fit six hundred people angithi, lets have that one lets compare the two statements ane! The first one, there must be at least two hundred people at the party ane at the same time we are saying the hall can only fit six hundred people whats the maximum whats the minimum we said there is a maximum and a minimum
134. Learners yes
135. Teacher because at the same time you are saying the hall or whatever bathini kumele babe greater than two hundred but at the same time bangabi babe not more than six hundred, niyangithola kuthi ngithini now the second constrain should be what?
136. Learners $x + y$ is less or equal to six hundred another constrain.
137. Teacher $x + y$ is less or equal to six hundred another constrain? We need at least one hundred and fifty girls as organisers mina no no ntokazi, singaba happy uma kuna na at least one hundred and fifty girls in the party. Its a constrain on its own its a constrain on its own what is the constrain what will be the constrain yalapho? if you are saying we need at least one hundred and fifty girls and at most four hundred boys what would be the constraints?
138. Learner x is greater or equals to one hundred and fifty and y is less than or equal to four hundred
139. Teacher okay lets say x is greater or equal to one hundred and fifty enye
140. Learners y is less than four hundred
- 141 Teacher y is less than four hundred another constraints, ratio
142. Learners ki-ratio
143. Teacher What will be the constraints
144. Learners y over x is less or equal to two over one
145. Teacher (teacher writes y over x is less or equal to two over one) i-ratio yala
146. Learners yes
147. Teacher alright, any question so far right now lets write down all the constraints angithi now if you make y the subject of the formula, this will be $y \leq 2x$ from there (teacher pointing at ratio) angithi, you times y by one and x by two angithi should be $y \leq 2x$ this one ya-ratio right
148. Learners yes
149. Teacher The second one sithe kumele ibe $x + y$ greater or equal to two hundred ($x + y$ and $x + y$ is less or equal to six hundred from the two sentences angithi the first one beyithi at least two hundred people to attend the party ne! And then bane $x + y$ is less than or equal to six hundred at least one hundred and fifty girls that will be x is greater or equal to one hundred and fifty next one y is less than or equal to four hundred now what will happen if I include zero like $0 \leq y \leq 400$ angithi now if you are saying y is less than four hundred omunye umuntu athi no! mani let us include zero lets say now the number angithi $u - y$ represent the number of boys lets say okay right lets say the number of boys there must be greater than zero but less than four hundred. Whats the difference between the two? (teacher pointing at $0 \leq y \leq 400$ and $y \leq 400$) what is the difference between the two? bear in mind that la sikhuluma nge y which represent the number of boys angithi.
- 150 Learners yes
151. Teacher Omunye ethi okay since now they said the number of boys must be less than four hundred athi the first one will be this one (teacher pointing at y is less than 400) but the other candidate or other learner athi no! Even if bathi y is less than four hundred but we need to include zero. whats the importance of including u -zero in this case? yes
152. Learner It is because you do not know how many boys will attend so zero will help you to find out kuthi bayingakhi bafan maybe abazo attenda between kuzero no four hundred
153. Teacher zero to four hundred another trial yes
154. Learner even if kuza umfan oyi one it is still okay
155. Teacher it is still correct yes okay if $i - y$ yala ngu four hundred kuphel what what but what are you saying even if kuyi-one uzo attenda ahlngane nalamantombazane lana

156. Learner isitabane (learners laughing)
157. Teacher right we have to include this zero ne because now it simply means the number of boys meaning in this party abafana bayafuneka la, the number of boys is greater or equals to zero but at the same time we have to limit them angithi
158. Learners yes
159. Teacher bangaphaphi babebanengi there must be less than what?
160. Learners four hundred
161. Teacher Right! For in entrance fee we said its what?
162. Teacher + Ls its fifteen x plus twenty y must be greater or equals to four thousand the third one right what we can do la we can simplify this okay which means we divide by what?
163. Learners divide by five
164. Teacher divide by five, if we divide by five this will be what? It will be three x plus four y is greater or equal to eight hundred ne! Ja! Now bear in mind that once seka so its like the inequalities we did previously, we have to sketch the graphs ja! uyangithola
165. Learners yes
166. Teacher It means just sketch the graphs *-(as learners were about to sketch the graphs the teacher asked them to sketch later)* let us check the next question right okay, let us go through the second question *(name of learner to read the question on handout)*
167. Learners (Learner reading) The company... (as he was reading the teacher noticed that learners were drawing the graph of the first question)
168. Teacher Ah! Mamela, we making a chance to write the constraints okay right
169. Learner (reading) The company, a company has two types of airplanes, a seven o seven (707) and a seven four seven (747). let the number of seven o seven s be x ..
170. Teacher seven o sevens
171. Learner let the number of seven o seven s (707) be x and the number of seven four sevens (747) be y and make inequalities if given the following information. A seven o seven (707) carries two hundred and fifty (250) passengers and a seven four seven (747) carries three hundred (300) passengers. the company is required to transport at least three thousand passengers per day. the company only has sixteen pilots available each day.
172. Teacher okay sharp go through the statement this time ufaka i-confidence
173. Learner The company, a company has two types of airplanes, a seven o seven (707) and a seven four seven (747). let the number of seven o seven s be x let the number of seven o sevens (707) be x and the number of seven four sevens (747) be y and make inequalities if given the following information. A seven o seven (707) carries two hundred and fifty (250) passengers and a seven four seven (747) carries three hundred (300) passengers. the company is required to transport at least three thousand (3 000) passengers per day. the company only has sixteen pilots available each day.
174. Teacher lets take two minutes and write down the constraints. Use the handout to write your inequalities. From the statement write down the constraints
175. Learner
176. Teacher They told you that the company has 707 and 747 ne! And you name 707 x ne! And 747 being y meaning everything ye 707 ufaka i-sign ye x ne! Anything ye 747 ufaka y ne! Right you inequality
177. Learner x plus y is greater or equals to three thousand
178. Teacher x plus y is greater or equals to three thousand
179. Learners yes

180. Teacher okay right if uthi wena x plus y is greater or equals to three thousand bear in mind that sithe x is seven o seven, the passengers are seven o seven plus seven four seven benza three thousand. What happened to two fifty and three hundred. There is two fifty and three hundred there is two fifty somewhere yes!
181. Learner x is greater or equals to two fifty and y is greater or equals to three hundred
182. Teacher why uthi x is greater or equals to two fifty because there is nothing like at least la angithi right
183. Learners x is equal to two fifty and y is equal to 300
184. Teacher ibuyaphi le!
185. Learner two fifty x plus three hundred y is greater or equals to three thousand
186. Teacher two fifty x plus three hundred y is greater or equals to three thousand thats a correct one two fifty x plus three hundred y is greater or equals to three thousand the second one yes!
187. Learner x is greater or equals to two fifty
188. Teacher x is greater than two fifty, ibuyaphi leyo! Asichekheni le niyabona asichekheni i-statement, bathe i-company must use at least four seven four seven and at least two seven o seven i-constrain will be ...
189. Learner x is greater or equals to two and y is greater or equals to four
190. Teacher x is greater or equals to two and the other one will be y is greater or equals to four right the second one is correct, another constraint another constraints ... Hhayi come on the last one. There is a statement lapha ngama number of pilots angithi, kanti when you are given something you decide to ignore it no! there is something about that yes the company only has sixteen pilots available each day what will be the constraints for that?
191. Learner x plus two y
192. Teacher x plus...
193. Learner x plus y is greater or equals to sixteen
194. Teacher Ah! Ah! Funda lestatement sithini, the company only has sixteen pilots available each day
195. Learner two x plus four y is greater or equals to sixteen
196. Teacher Why two x plus four y is greater or equals to sixteen?
197. Learner Because each day the company uses at most two seven o seven and four seven four seven
198. Teacher okay! What happens if you say now the constraints will be x plus y is less or equals sixteen you see it doesn't matter whether bangena ku seven o seven noma ku seven four seven but the point is the company has only sixteen pilots per day ne! May be the constraints is x plus y is less than or equals to sixteen, x plus y is less or equals to sixteen we cannot say x plus y is greater or equals to sixteen because bona bana sixteen pilots or less the min- the maximum is sixteen pilots they cannot use more than sixteen pilots uyangithola right chekha amaconstraints the first one
199. Learner two fifty x plus three y is greater or equals to three thousand
200. Teacher the second one
201. Learner x is greater or equals to two and y is greater or equals to four
202. Teacher the third one
203. Learner x plus y is less or equals sixteen
204. Teacher right let us check the third question this time ibe-correct ne! Sechaba sechaba question three
205. Learner reading from handout the the dairy delivers...
206. Teacher at least (name of learner) reads the word correctly, one learner last year read diary wrongly
207. Learner The dairy delivers milk and orange juice. A house wife can order a maximum of twenty eight bottles per week. She does not want more twelve bottles of orange juice. She must have at least two bottles of milk. If the milk costs eight rand per bottle and the orange juice twelve rands per bottle what is the maximum that she spends per week.

208. Teacher uh its interesting its interesting go through the statement again
209. Learner The dairy delivers milk and orange juice. A house wife can order a maximum of twenty eight bottles per week. She does not want more twelve bottles of orange juice. She must have at least two bottles of milk. If the milk costs eight rand per bottle and the orange juice twelve rands per bottle what is the maximum that she spends per week.
210. Teacher okay sharp two minutes write down the constraints just tell our constraints constraints (Learners writing constraints in their exercise books, teacher going around then wrote the following problem on the board) Mr X wants to buy at least two skirts at R90 each and at least one dress at R 140
211. Teacher listen if I give you a test out of one hundred ane! What will be the maximum ? I repeat you are given a test out of one hundred ne! Now wena u-brilliant, what will be the maximum mark you can get? Mark
212. Learners one hundred
213. Teacher one hundred ane! Meaning you cannot get more than one hundred angisho
- 214 Learners Yes
215. Teacher What will be the constraints yalapho? Your mark will less or equal to one hundred or your mark will be greater or equal to one hundred?
216. Learners less
217. Teacher It will be less or equal to one hundred ane! Right your mark will be at most one hundred or your mark will be at least one hundred?
218. Learners *others said* at least, *others* at most *then teacher going around checking what learners have written*
219. Teacher Now sithi a house wife can order a maximum of twenty eight bottles per week. What will be the constraints yalapho? Will it be greater than twenty eight or will it be less than twenty eight?
220. Learners less than twenty eight
221. Teacher more likely less than twenty eight alright the constraints, lets check the constraints yes
222. Learners eh y is les
223. Teacher Okay lets start by defining sibitsa x and which one is y
224. Learner okay the milk will be x and the juice will be y
225. Teacher the number of milk bottles will be represented by x and the number of orange juice bottles will be represented by y yes
226. Learner x plus y is less or equal to twenty eight
227. Teacher x plus y is less than or equal to twenty eight is that okay?
228. Learners y is less than or equal to twelve
- 229 Teacher y is less than or equal to twelve
- 230 Learner and then x is greater than two
231. Teacher greater or equals to two right let us find the cost function, the cost will be R8 ane!
232. Learners Yes
233. Teacher The cost will be (teacher writes on the board $C(x) = R8 + R12$) the R8 will be for x and the 12 will be for y (teacher writes $C(x) = R8x + R12y$) ane! Sharp. Then you will use this to draw a search line you still remember that ne!
234. Learners yes
let us remind ourselves of the constraints one x plus y is less or equal to twenty eight, two y is less or equal to twelve, three x is greater or equal to two. For your homework you have to sketch the feasible region.

235. Teacher Lets go through this statement and write down the constraints. Mr X wants to buy at least two skirts at ninety rands each and at least one dress at one hundred and forty each and has only six hundred rands to spend what will be the constraints yala? Ah ah ready...(*teacher reading statement again*) Mr X wants to buy at least two skirts at ninety rands each and at least one dress at one hundred and forty each and has only six hundred rands to spend what will be the constraints? according to the statement you have number of skirts and number of dress right the limit is six hundred meaning that whatever that you want to spend imele ibe not more than what?
236. Learners six hundred
237. Teacher ready right
238. Learners yes
239. Teacher yes wena
240. Learner I said one eighty x plus one forty y is less or equal to six hundred ($180x + 140y \leq 600$)
241. Teacher one eighty! Uthe one eighty skirts? Uthe two times ninety?
242. Learner yes
243. Teacher Oh bathe uzothenga two skirts, no! Bathe at least two skirts meaning he can buy three, four as long as angisho whatever i-expenditure yakhe ingeke ibe more than six hundred
244. Learners yes
245. Teacher yes (*name of learner*)
246. Learner x plus y is less or equal to six hundred
247. Teacher oh x plus y is less or equal to six hundred
248. Learner yes
249. Teacher what will be x and what will be y?
250. Learners skirts is x and y amadress
251. Teacher oh ngamanye amagama amaskirts akhe namadress akhe kumele abe less than six hundred rands ushonjalo u-right? Yes
252. Learners ninety x plus one hundred and forty y is less than or equal to six hundred
253. Teacher Good! Ninety x plus one hundred and forty y is less than or equal to six hundred another constraints yes
254. Learner x is greater or equal to two
255. Teacher x is greater or equal to two simple as that if he wants to buy at least two skirts x is greater or equal to two and y?
256. Learner y is greater or equal to one
- Right, now check the third handout now let us be serious number one on the third handout (*name of learner*) read question one for us
257. Learner A ma-nu--facturer of liquid det det...
258. Teacher be serious (*teacher asked another learner to read question on handout*)

259. Learner A manufacturer 1. A manufacturer of liquid detergent uses two basic chemical ingredients, Mix A and Mix B. The detergent is packaged and sold to two separate markets: Household market (H) and Commercial market (C). The products are sold in five litre bottles. For the household detergent each five litre bottle requires four litres of Mix A and one litre of Mix B, whereas the corresponding composition of the commercial detergent is two litres of Mix A and three litres of Mix B as shown in the table.

Chemical composition of Detergents per five litre bottle:

Ingredients	Household Detergent (H)	Commercial Detergent (C)
Mix A	4 litres	2 litres
Mix B	1 litre	3 litres

On a weekly basis the company has supplies of no more than 20 000 litres of Mix A and 15 000 litres of Mix B. Furthermore the company can buy no more than 4 000 containers a week for H and 4 500 containers a week for C. Let x be the number of five litre bottles for H and y be the number of five litre bottles for C.

1.1 Write down the inequalities representing the availability of Mix A and Mix B and the availability of the containers for H and C

1.2 Represent the inequalities on graph paper using the scale 2cm = 2 000 litres on both axes.

1.3 Shade the feasible region on the graph

LESSON 2

260. Teacher You were suppose to answer 1.1 to 1.3

261. Learners yes

262. Teacher write (name of learner) lets go through the statement sa number one wow wow there are three questions you were suppose to sketch, let us start with the question ya-number one from this handout

263 Learner A manufacturer 1. A manufacturer of liquid detergent uses two basic chemical ingredients, Mix A and Mix B. The detergent is packaged and sold to two separate markets: Household market (H) and Commercial market (C). The products are sold in five litre bottles. For the household detergent each five litre bottle requires four litres of Mix A and one litre of Mix B, whereas the corresponding composition of the commercial detergent is two litres of Mix A and three litres of Mix B as shown in the table.

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264. Teacher right question 1.1 (*one learner stood up to talk to the teacher*)

265. Learner 1.1 Write down the inequalities representing the availability of Mix A and Mix B and the availability of the containers for H and C

266. Teacher Right lets have one learner ozosinikeza amaconstraints any volunteer (learner moving towards the board to write down the constraints. While she was writing the teacher noted that some learners did not do their home work) wow wow wow Asenze kube simply bangakhi abangabhalanga? (learners stood up) one, two three four five six seven eight nine ten eleven twelve thirteen fourteen fifteen sixteen seventeen eighteen nineteen twenty twenty one twenty two twenty three twenty four twenty five twenty six twenty seven twenty eight twenty nine, bayi twenty nine okay sit down. (*to the learner who was writing the solution on the board*) hhayi yiyekele lets check number four please make sure that ngeshort break niyeza, short break, short break quarter past tenand I will make sure that no one comes to class angaka bhali short break asenze i-linear programming ...that will be the best punishment from there you will start taking things seriously lets talk about number four number four ...because of the sports day yesterday yiko nibese nidecida ukungangabhali ...number four lets do linear programming
267. Learner If H contributes two rand forty cents to the profit per five litre bottle....(learner interrupted)
268. Teacher number four...
Checking learners handout okay number two, number two...right number three number three a patient in a hospital needs ...number three ne!
269. Learner A patient in a hospital needs at least 18 grams of protein, 6 milligrams of vitamin C and 5 milligrams of iron per meal, which consists of two types of food, A and B. Type A contains 9 grams of protein, 2 milligrams of vitamin C and no iron per mass unit. Type B contains 3 grams of protein, 2 milligrams of vitamin C and 5 milligrams of iron per mass unit. The energy value of A is 800 kilojoules and that of B is 400 kilojoules per mass unit. A patient is not allowed to have more than 4 mass units of A and 5 mass units of B. There are X mass units of A and Y mass units of B on the patients plate.
270. Teacher read once more
271. Learner A patient in a hospital needs at least 18 grams of protein, 6 milligrams of vitamin C and 5 milligrams of iron per meal, which consists of two types of food, A and B. Type A contains 9 grams of protein, 2 milligrams of vitamin C and no iron per mass unit. Type B contains 3 grams of protein, 2 milligrams of vitamin C and 5 milligrams of iron per mass unit. The energy value of A is 800 kilojoules and that of B is 400 kilojoules per mass unit. A patient is not allowed to have more than 4 mass units of A and 5 mass units of B. There are X mass units of A and Y mass units of B on the patients plate.
272. Teacher Right your understanding anyone *name of learner* your understanding anyone who will share with us your understanding before we take out the constraints ngifuna in-undersatnding.. Your understanding
273. Learner kuna patient kasbedlela, kubatla..(Tswana utterance)
274. Teacher Food A
275. Learner Type A i-containa
276. Teacher Ayina iron..
277. Learner The energy value ka A is 800 kilojoules and kani ya b ngifour hundred kilojoules i-patient ayikavumeleki ibe more than four mass unit of A
278. Teacher Ja kasetswana (name of learner) kuthiwani mhlawumbe uyazi angazi ngesiZulu amaproteins kuthiwa yini? Yes sisi
279. Learner (*learner noted that there was a printing error*) sir I want to ask if it is there are X mass unit of A because it says there are X mass units of and
280. Teacher Kushoda ini?
281. Learners A

282. Teacher kushoda A yini leyo.. Okay Ja yes your understanding? This is according to (name of learner) ja your understanding?
283. Learner yes ngiyiunderstandile
284. Teacher your understanding
285. Learner ngiyiunderstandile
286. Teacher your understanding
287. Learner (learners laughing) oh my understanding
288. Teacher Now in terms of equations ke ne ibeni-understande ngakhona ke you think ningakhona ukukhipha amaconstraints
289. Learners yes
290. Teacher ningakhona ukukhipha amaconstraints now what will be your constraints because now some where some how it seems is it possible is it correct to say letho amaconstrain amaprotein amaprotein abe yione
291. Learner proteins
292. Teacher proteins kube yi x amavitamins kube yi y and ama-iron kube yi other letter which will be x and which will be y? Now you anticipate how many equations from the statement angithi because angithi bathe whatever kunama proteins kunama vitamins and the iron angithi
293. Learner yes
294. Teacher and the type B ina the same thing angithi. So that your understanding from your understanding who can give us amaconstrain who can give us amaconstrain say ala yilawa this should be the constarin yala or one of the constrain will be yes your understanding Ja yala kule-plate lakhe kune mass unit ya A ne mass unit ya B and soshilo ukuthi akumelanga athole more five i think of mass A something like that and more than five of mass B or A ne! *name of learner* your constrain constrain now lets start drawing a table see if ngeke senze kube simply we can just say type A and type B (*teacher drawing a table on the board*) right singathi la protein then la sithi amavitamins then la kube iron. right for those labenza i-natural science bayazi ukuthi ini ene protein ibenevitamin C okay. nincede thina abangezi ukuthi yini eneprotein ne vitamin C yini ene iron
295. Learner Fish, liver
- 296 Teacher fish livers okay vitamin c
297. Learners citrus fruits
- 298 Teacher Eh
299. Learners citrus fruits
300. Teacher protein
301. Learners peanuts
302. Teacher okay right lets take our constraints lets chekha amaconstraints for type A and type B for type A batheni? Lets go through the statement again
303. Learner Apatient in a hospital needs 18 grams of protein
304. Teacher 18 grams of protein ne!
305. Learner six milligrams of vitamin C and five milligrams of iron per meal
306. Teacher whoa whoa whoa okay for protein bawukanyi
307. Learners 18 milligrams
308. Teacher 18 ane! (*teacher writes 18 on the side of the table*) mamela that 18 is not for type or for type B but bathe at least 18 once bathi at least 18 is greater than or less than
309. Learner is greater than
310. Teacher okay and what else? Ah ah yes

311. Learner Type A contains nine grams of protein two milligrams of vitamin C and no iron per mass unit
312. Teacher ukuphi okay start afresh (*name of learner*)
313. Learner A patient in a hospital needs 18 grams of protein and six milligrams of vitamin C
314. Teacher and six milligrams of vitamin C
315. Learners vitamin C
316. Teacher (teacher writes 6 next to the table) for iron batheni?
317. Learner five milligrams
318. Teacher five milligrams right (teacher writes five by the side of the table) right inequalities kawuvela angithi meaning (teacher writes greater or equals to sign) oaky
319. Learner which consists of two types of food, A and B.
320. Teacher ja
321. Learner Type A contains nine grams of protein
322. Teacher whoa whoa whoa bathe type A contains
323. Learner nine grams
324. Teacher nine grams ne! Okay
325. Learner yes, two milligrams of vitamin C
326. Teacher two milligrams, type A
327. Learners yes
328. Teacher yes (teacher writes 2 on the board)
329. Learner no iron
330. Teacher no iron (teacher writes 0) yes
331. Learner type B contains three grams of protein two milligrams of vitamins and five milligrams of iron per mass unit
332. Teacher teacher writes 3, 2 and 5 on the board ne
333. Learner The energy value of A is eight hundred kilojoules and the of B is four hundred kilojoules per mass unit.
334. Teacher right there now lets represent this now sibhale algebraically, which one will be x and which one will be y?
335. Learners Type A will be x and type B will be y
336. Teacher (*teacher writes x and y on the table*) okay lets write down ama-inequalities what will be the first inequality
337. Learner x plus y will be greater or equals to eighteen
338. Teacher x plus y eh!
339. Learner yes
340. Teacher x plus is greater or equals to 18, x plus y is greater or equals to eightee from which row from which row kuna row one row two and row three from which row
341. Learner from row one
342. Teacher from row one ubona x plus y is greater or equals to eighteen yes!
343. Learner nine x plus three y is greater or equals to eighteen
344. Teacher nine x plus three y is greater or equals to eighteen (*teacher writes $9x + 3y \geq 18$*)
345. Learners Yes
346. Teacher yama proteins angithi
347. Learner yes
348. Teacher second one yes

349. Learner two x plus three y is greater or equals to six
350. Teacher two x plus three y is greater or equals to six third one
351. Learner five y is
352. Teacher five y is greater or equals to five right lets go back to the statement sibone ukuthi is this not a mistake (teacher circling greater or equals to five) is it correct to write greater or equals to five? Ku-right masithi greater or equals to five or kumele sithi less or equals to five which one is correct we will get it from the statement yes (name of learner)
353. Learner
354. Teacher I was saying is it correct to have this sign? Kumele sithi greater or equals to five noma less or equals to five
355. Learner less or equals to five
356. Teacher yini esigayidile ukuthi la kumele sithi less or equals to five noma sithi greater or equals to five. Basically that is my question. Angithi mamela angithi lasifake uzero, why sifake uzero? because bathe itype A ayinama iron
357. Teacher+Ls ayinama iron
358. Teacher Angithi i-type A ayina Iron, angithi then i-type B ngiyo ene-iron angithi
359. Learners yes
360. Teacher We were told that for this patient kumele i-iron yakhe ibe is either greater than five or less than five or equals to five. Which one is correct?
361. Learners
362. Teacher Bathe at least five or at most five or not more than five batheni?
363. Learners at least five (others at most five) not more than five
364. Teacher Okay okay okay, woza Nhlanhla omncane
365. Learner it is greater or equals to five
366. Teacher i-right igreater or equals to five i-reason yokuthi ibe right yini?
367. Learner naku lapho kucala khona a patient in a hospital bed needs at least eight grams of protein and six milligrams of vitamin C and five milligrams of mass unit
368. Teacher so it means i-correct angithi
369. Learners hmm
370. Teacher now that means five y is greater or equals to five ($5y \geq 5$) lets represent that graphically to identify the feasible region lets do that two minutes two minutes... Lets do this together before you can start let us simplify these inequalities. Its chkha the first one (teacher simplifying first inequality $9x + 3y \geq 18$) you can divide both side by three and if you divide by three to get three x plus y greater or equals to six ($3x + y \geq 6$) angithi the second one
371. Teacher+Ls x plus y is greater or equals to three ($x + y \geq 3$) the last one will be y is greater or equals to one ($y \geq 1$) now sketch identify your feasible region two minutes(learners drawing graphs to identify feasible region while teacher checking their skethces and making comments to guide them) mamela wena ngeke ucale ngeku schetha you have to find the intercepts first then bese uyaskethca unless...just sketch the graph three x plus y is greater or equals to six find the x-intercept and y-intercept we said ku linear programming we only focus ku first quadrant angithi, there is no need for drawing that cross i know siyayithanda icross but there is no need for that cross (name of learner) just sketch the graph its time consuming why ubhale i-table because bengikubhalele. Leave a space lento uzoyibhala later...there is one question engifuna siyibuke kancane the statement esithi a patient is not allowed to have more than four mass units of A and five mass unit of B
372. Learners yes

373. Teacher what will be the constraints yalapho?
374. Learner x is less or equals to four
375. Teacher x is less or equals to four
376. Learner y is less or equals to five
377. Teacher writing on the board) x is less or equals to four ($x \leq 4$) and y is less or equals to five ($y \leq 5$) Ja! So ready ... Cedile ase sibone (teacher drawing two axes on the board while learners finishing their graphs) ... Teacher drawing lines) x is equal to four ushada to the left y is less than or equals to five nay i-y is equal to five you shade below y is greater or equals to one nayi i-one yakho you shade above now you left with x plus y is greater or equals to three when you let y is equal to zero it simply means your x is equal to three your x-intercept is three (teacher marking the position on the graph) at the same time when you let x equal to zero, y will be equal to three ne! then your y-intercept is three ne!
378. Learners yes
379. Teacher then you have to sketch the graph (teacher joining the two points) then you shade above ne!
380. Teacher + Ls above
381. Teacher siza kule angithi (pointing at $3x + y \geq 6$) when you let y is equal to zero you have to divide by three both sides angithi
382. Learners yes
383. Teacher It simply means x will be equals to two eh! If x is zero y will be six your y intercept is equal to six (teacher drawing graph) then you shade above this will be your feasible region. I think so far we can get our constraints but if uchkha i-number two in you handout ne! number two kunumber two already you are given the graph ku this one you have to go through the statement ku number two already you are given the graph uyabona meaning we will do that later for now you are given a statement from the statement you draw a graph any questions? (teacher attending to one staff member who was by the door)
384. Learner discussing constraints
385. Teacher (Teacher distributing handout) surplus surplus surplus let us start with number four let us go through the statement (name of learner to read)
386. Learner (reading from handout) 4. A certain motorcycle manufacturer produces two basic models, the 'Super X' and the 'Super Y'. 'These motorcycles are sold to dealers at a profit of R20 000 per 'Super X' and R10 000 per 'Super Y'. A 'Super X' requires 150 hours for assembly, 50 hours for painting and finishing and 10 hours for checking and testing. The 'Super Y' requires 60 hours for assembly, 40 hours for painting and finishing and 20 hours for checking and testing. The total numbers of hours available per month is 30 000 in the assembly department, 13 000 in the painting and finishing department and 5 000 in the checking and testing department. This can be summarised by the following table:

Department	Hours for Super X	Hours for Super Y	Max hours available per month
Assembly	150	60	30 000
Painting and finishing	50	40	13 000
Checking and testing	10	20	5 000

Let x be the number of 'Super X' and y be the number of 'Super Y' models manufactured per month.

- 4.1 Write down the set of constraints inequalities
- 4.2 Use graph paper to represent the constraints inequalities
- 4.3 Shade the feasible region on the graph paper.
- 4.4 Write down the profit generated in terms of x and y.
- 4.5 How many motorcycles in order to maximise the monthly profit?

4.6 What is the maximum monthly profit?

387. Teacher right from the statement we are given a table now now this table liya-indicatha kuthi eh! For Super X and Super Y kunamadepartment ayi three ne! Kune department ye Assembly ne! Kune department ye painting and trainingne! Kune department ye checking and testing ne! and now the last column yile important (teacher pointing at last column on handout) this column ne! yama maximum nama minimum ne! but now babhale maximum hours available ne! because for i-assembly kunelimit yakhona ne! for painting and finishing kunemaximum hours and for checking and testing nakhona kune maximum hours ne! lets write down the constraints bathe let x be the number of Super X ne! and y be the number of Super y for the manufacture of the models per month angithi lets write down the constraints on the same sheet there is a space just write down the constraints (teacher checking students work) write down the constraints...constraints...name of learner constraints
388. Learner ayi bengisengakayitholi
389. Teacher Ubhalile
390. Learner angikakayibhali
391. Teacher usenzani? Name of learner constraints
392. Learner one fifty x plus sixty y
393. Teacher one fifty x plus sixty y
394. Learner less or equals to thirty thousand
395. Teacher less or equals to sixty thousand ja! Second one yes
396. Learner fifty x plus forty y is less or equal to thirteen thousand
397. Teacher fifty x plus forty y is less or equal to thirteen thousand right next one
398. Learner ten x plus twenty y is less or equals to five thousand
399. Teacher ten x plus twenty y is less or equals to five thousand from that ya checking and testing from that you simplify the constraints ne! i-Physics inini
400. Learner last period
401. Teacher Oh!
- LESSON 3**
402. Teacher Right eh sanibonani
403. Learner s Yebo!
404. Teacher Ninjani
405. Learners sikhona wena unjani
406. Teacher Lets hope niyile esontweni
407. Learners a few siyile others asikayi
408. Teacher Right abayile bathole amablessings, abanye baye estadium babuya ba dissappointed
409. Learners yes
410. Teacher abanye ekuseni bavukile beze eskolweni bazofunda
411. Learners yes
412. Teacher The homework was 4.1 to 4.3 right
413. Learners yes
414. Teacher everyone ubhalile
415. Learners yes
416. Teacher ngibuza for the last time everyone ubhalile
417. Learners yes

418. Teacher singachekha
419. Learners Chekha (very confidently) check
420. Teacher hhayi niright number four (name of learner) go through the statement everyone has a handout (name of same learner to read question 4) double period ne! yes (name of same learner to read) everyone has a handout (teacher making sure that they all had a handout) name of learner number four
421. Learner (reading from handout) 4. A certain motorcycle manufacturer produces two basic models, the 'Super X' and the 'Super Y'. 'These motorcycles are sold to dealers at a profit of R20 000 per 'Super X' and R10 000 per 'Super Y'. A 'Super X' requires 150 hours for assembly, 50 hours for painting and finishing and 10 hours for checking and testing. The 'Super Y' requires 60 hours for assembly, 40 hours for painting and finishing and 20 hours for checking and testing. The total numbers of hours available per month is 30 000 in the assembly department, 13 000 in the painting and finishing department and 5 000 in the checking and testing department. This can be summarised by the following table:
422. Teacher okay asikhiphe amaconstraints yes first one
423. Learner one fifty x plus sixty y is less or equals to thirty thousand ($150x + 60y \leq 30000$)
424. Teacher one fifty x plus sixty y is less or equals to thirty thousand ($150x + 60y \leq 30000$) second one yes
425. Learner fifty x plus forty y is less or equals to thirteen thousand ($50x + 40y \leq 13 000$)
426. Teacher fifty x plus forty y is less or equals to thirteen thousand third one ($50x + 40y \leq 13 000$)
427. Learner ten x plus twenty y is less or equals to five thousand ($10x + 20y \leq 5000$)
428. Teacher that's four point (4.1) one now four point two (4.2) ne!
429. Learners yes
430. Teacher Now if you have your exercise books you can use them instead of a graph paper ne!
431. Learner yes
432. Teacher Now to represent the constraints anyone just sketch it on the board (one learner stood up to draw the sketch on the board) while you sketch I will check the other graphs (teacher checking learners exercise books)....(once the learner had finished sketching the teacher checked the sketch on the board) okay one i-right i-feasible region but kunento eyi-one engasiright. i-calibration but everything is correct except that the way e-calibratwe ngakhona hhayi asicorrect but right four point one, four point two, four point three done what the question for four point four?
433. Learner (chorus learners reading from handout) write down
434. Teacher not i-choir (name of learner to read question)
435. Learner write down the profit generated in terms of x and y
436. Teacher reading the question again) write down the profit generated in terms of x and y okay now chekha from the statement bathe a certain motocycle manufacturer produces two basic models the Super X and the Super Y. now second statement sitheni These motocycles are sold to dealers at a what?
437. Teacher + Ls at a profit of twenty thousand rands
438. Teacher Now you have to introduce i-formula , equation ye profit equals twenty thousand what x angithi
439. Learners yes
440. Teacher plus ten thousand what?
441. Teacher +Ls ten thousand y
442. Teacher so now we have P equals to twenty thousand x plus ten thousand y right asiye four point five niyangithola?
443. Learner reading)how many motocycles are produced in order to maximise the monthly profit

444. Teacher Maximum profit angithi now you have to find maximum profit? There are two methods or more than two methods of finding a maximum profit pens down asimameleni ane! Now eh! Angithi you know ukuthi le-line lirepresenta i-y (marking the axis) this one represents x (marking axis)
445. Learners yes
446. Teacher Now Super X and Super Y you know you have to find the maximum profit. Now lets write down amacoordinates from the the corners or vertex of the feasible region lets say this will be A, B, C and and that D (teacher labeling the vertices of the feasible region on the sketch diagram) Now what are the coordinates of A?
447. Learners zero is to zero (0;0)
448. Teacher teacher writing (0;0) now it simply means if you substitute each point in the equation of profit the one that is going to give you a bigger value is the one that is going to give you a maximum profit commonsense
449. Learners Hmm
450. Teacher niyangithola kuthi ngithini now if you use coordinates zero is to zero (0;0) it simply means it simply means bowungena motro X and mtor Y, Since now i-x i-representa Super X and y i-representa Super y now if wena uthi the coordinates (0;0) is the one that is going to give us maximum profit it simply means the profit is zero ngoba if you substitute in P equals to twenty thousand times zero (20 000 x 0) is zero plus ten thousand time zero (10 000 x 0) is zero it simply means it is time consuming to substitute that point for profit. sasitheni last time i-linear programming i-dila nge maximum profit ne! no one wants a minimum profit unless otherwise lets check the second coordinate which one?
451. Learners zero is to two hundred (0; 250)
452. Teacher okay, is it correct kuthi zero is to two fifty (0; 250)?
453. Learner zero is to one fifty(0; 150)
454. Teacher no no I am saying are the coordinates of this point correct (0; 250) from the sketch? What are the coordinates of C sithe I C nayi la (teacher pointing at point C on the sketch diagram) what are the coordinates of C now now now to save time point C is the point of intersection of two graphs meaning ngulakukhatha khona amagraph ayi-two niyangithola ukuthi ngithini
455. Learners Hmm
456. Teacher now ngipheni lamagraphs lawo lawomalayini ayitwo lapho akhatha khona
457. Learners chorus
458. Teacher No! asikavuleli i-choir yes (pointing at one learner)
459. Learner five x plus four y isless or equal to one twenty ($5x + 4y \leq 120$)
460. Teacher five x plus four y isless or equal to one twenty ($5x + 4y \leq 120$) and which one?
461. Learner x plus two y (interrupted)
462. Teacher x plus two y niyavuma?
463. Learners yes
464. Teacher x plus two y is less or equal to five hundred ($x + 2y \leq 500$) angithi now the point of intersection of the two graphs ne! now to solve for i-coordinates it simply means you have to solve for x and y simultaneously. Basically you can write these as ama-equation phela niyangithola ukuthi ngithini five x plus four y is the same as equal to thirteen thousand ($5x + 4y = 13\ 000$) the second one is x plus two y equals to five hundred ($x + 2y = 5000$) then use what ever mehod ofuna ukuyisebenzisa whether elimination or substitution method angithi?
465. Learner yes
466. Teacher But the simplest method la yi-elimination angithi lets multiply equationtwo by five if you multiply equation two by five one equation one will be five x plus four y equals to thirteen lets multiply equation two by five this will be

467. Teacher + Ls five x plus ten y equals to twenty five hundred ($5x + 10y = 2500$) (*teacher heard one learner saying ten thousand*)
468. Teacher Hhawo five hundred times five, five hundred times five is ten thousand you must be joking but ngiya-understanda there is one thing engi sure ngayo its either u favour i-chiefs noma i-pirates asiyekele angithi. This will be equation one and equation two now to use elimination sekusimply kekhona angithi you have to eliminate one variable now lets say equation two minus equation one this will be five x minus five x
467. Learners zero
468. Teacher ten y minus four y
469. Learners six y
470. Teacher six y equal to twenty five minus thirteen
471. Learner twelve
472. Teacher twelve angithi bathi twelve hundred, twelve hundred angithi (1200) now you can divide by six
- 473 Learners two hundred
474. Teacher now if y is two hundred what will be the value of x lets save time sisubstitute la now its x plus two times two hundred equals to five hundred angithi
475. Learner yes
476. Teacher to save time two times two hundred is four hundred when you transpose it will be five hundred minus four hundred it will be what?
477. Learners one hundred
478. Teacher one hundred it simply means therefore coordinates C will be what?
479. Learner one hundred is to two hundred (100; 200)
480. Teacher +Ls one hundred is to two hundred ne! right another coordinate is , coordinate D angithi asiyeeni
481. Teacher what will be the coordinates of D?
482. Learners two hundred is to zero (200;0)
483. Teacher two hundred is to zero (200;0) yes (teacher pointing at one learner who had a question)
484. Learner andiyiunderstandi five x plus four y is equal to thirteen thousand but at the same time you have (interrupted)
485. Teacher ane malume you have fifty x plus forty y equals to thirteen thousand divide by ten through out what is fifty divide by ten
- 486 Learner five
487. Teacher forty divide by ten four and thirteen thousand divide by ten thirteen hundred so
488. Learner What is the statement here? Why divide by ten?
489. Teacher because ngiyona common factor uyangithola angithi uyasimplifaya (name of learner) you see me after this thank you
490. Learners laughing
491. Teacher Right you have coordinates A, coordinates B, coordinates C and coordinate D angithi
492. Learners yes
493. Teacher angithi sithe you use the coordinates of the vertices of the feasible region angithi now we said the coordinates of A is out because we said it is zero zero, what you have to do you just substitute all these coordinates ezokupha i-number e big obvious ngiyo ezokunikeza i-maximum. lets save time coordinates B our profit is p is equal to twenty thousand x plus ten thousand y lets use coordinates B, coordinates B will be p is equal to twenty thousand times zero plus ten thousand times two fifty ($P = 20\ 000 (0) + 10\ 000 (250)$) which is two million five hundred thousand
494. Learner two point five million

495. Teacher right lets take coordinate C coordinate C just say twenty thousand times one hundred plus ten thousand times two hundred ($20\ 000 \times 100 + 10\ 000 \times 200$)
496. Learner four million
497. Teacher coordinates D use coordinate D
498. Learners four million
499. Teacher coordinate D is four million serious
500. Learners yes
501. Teacher that means that coordinate is incorrect but for now okay sharp right asibuyeleni ku question yethu ku four point five how many...
502. Learners motorcycles
503. Teacher How many motorcycles which one will give us maximum profit because mamela mamela mamela okay sharp mamela lets say the answer is coordinates A that will simply mean zero motorcycle X and two hundred motorcycle Y coordinate A usinika i-maximum. Lets check coordinate B we said coordinates B usinikeza what?
504. Learners two and half million
- 505 Teacher two and half million ne! and we said coordinates C and coordinates D what?
506. Learners four million, four million
507. Teacher ayikaze yenzeka le it simply means one of the coordinates is not correct ane! And I suspect coordinate C is not correct that is the first method of finding maximum profit lets check another method of finding maximum profit I said there are two, lets use the second one, this one is time consuming because now sometimes you may find kutsi during the examination your feasible region ayina four coordinates or vertices or corners or whatever inamore than four and the question will be three marks, how many minutes for three marks kumele kube one and half minutes ubhalele three marks. but the way esislove ngakhona its time consuming now what you have to do when using the other method you just take equation of the profit you make y the subject of the formula and you start by using i-gradient. Bear in mind that i-gradient its all about the movement in the y-axis over the movement in the x-axis and basically sithi i-gradient is change in y over change in x angithi. meaning if your gradient is negative five over four it simply means if uhamba verticly it simply means you must go five units down. lets apply it nihlakaniphile mara.
508. Learners hmm (listening)
509. Teacher This is your profit line ne! (teacher pointing at the profit function) Its P... hhayi asiyekele labo p its two hundred x plus two hundred y ($20000x + 10000y$) ngithe the first step make y the subject of the formula angithi. If you make y the subject of the formula it simply means you have to transpose this angithi now it will be ten thousand y equal to p minus twenty thousand x ($10\ 000y = P - 20\ 000x$) ne! the next step divide by what? by ten thousand angithi right now divide nge ten thousand this is what we get ($y = -2x + p/10\ 000$) ane this one (teacher circling $p/10000$) we are not interested ngayo uyangithola. The only part esi-interested kuyo is the part ye gradient bear in mind from Grade nine you know that straight line is given by y equal to mx plus c. ($y=mx+c$). Therefore the coefficient of x is known as a gradient now gradient in this case its negative two over one ($-2/1$) siyavuma
510. Learners yes
511. Teacher Now we have to apply amaequivalent fractions, niyakhumbula ama equivalent fraction from grade one kuthi one over two is the same as four over eight niyakhumbula (teacher interrupted by a learner with a document to be signed) Tigt asiyene! sithi half is the same as two over four equals to four over eight and so forth. ukusho ukuthi ngamanye amagama i-half is the same as one thousand over two thousand ne! now you take this gradient then now you will be controlled by your scale lapha. i-scale sakho sicounte ngani ngama one hundred or two hundred labasketch right

512. Learners ngama one hundred
513. Teacher use your scale la the way o-calibrathe ngakhona. If u-calibrathe ngama fifty it simply means you have to change your gradient ukuthi ihambe ngamafifty or according to your scale let us take one example wena your scale uhambe fifty, one hundred one fifty two hundred and so on now it simply means you have to change this two uyibhale in terms of fifty's nama one hundred so what will be ungayibhala as two hundred over one hundred niyangithola. it simply means vertically you have to start from two hundred horizontally you start kuphi one hundred niyangithola, then after ungatshintha iscale you move over the feasible region using a ruler. I repeat you move over a feasible region using a ruler the last point that will touch your ruler in the feasible region is the point that will give you a maximum profit. I repeat everyone uzoyenza, everyone will use i-scale sakhe whether amatens, hundred ngithe if your gradient wena i-calibration yakho asithathe lama hundred, its hundred two hundred and so forth even vertically its one hundred, two hundred and so forth now there comes your gradient your gradient is two, ngithi kuwe plotha two uzomplotha kuphi two because half of one hundred is fifty according to your scale, half of fifty is twenty five, half of twenty five is twelve point five i-two izoba close to zero and you will be tempted to sayu-two u-zero. now to avoid that use equivalent fractions to say okay if it is zero (teacher drawing x-axis on the board) gradient negative two ngitheni kuwe change this uyibhale according to your scale, multiply by ten to get negative twenty ovr ten. but now kunam hundred which means we multiply by one hundred. my gradient will be negative two hundred over one hundred (-200/100) it simply means my starting point is I have to go two hundred units down and one hundred to the right. my two hundred on the x-axis (marking a point on the x-axis) one hundred on the y-axis (marking a point on the y-axis) ngitheni take a ruler now move it on the feasible region make sure kuthi yini ofuna ukuyichekha la, sithe the last point that will touch on the feasible region is the minimum like in that case ane! the first point that will give you a minimum yile yile (pointing at A) but la I am not sure whether the point elizonginikeza) i-maximum is this or that point. Ane ngina two points already sengilithachile leli already it is out kufuna uchekhe this point and that point ukuthi which one elizokunikeza i-maximum. You have seen that what you have to do go back to your question do that the point elikunikeza i-maximum profit sketch i-graph yakho ye-profit using a dotted line (teacher showing dotted line)
514. Learners yes
515. Teacher That will simply mean i-point elikunikeza i-profit is this and you have to sketch dotted line eh i-line lakho la! Niyangithola ukuthi ngithini? i-dotted line meaning ukuthi once there is a dotted line eliskefchiwe that is the point from the equation ye profit elikunikeza what?
516. Teacher +Ls elikunikeza i-maximum siyatholana? If i-question ithi calculate the number of motorcycles that will give you a maximum profit wena from that point you have to find what? The values of x and y.
517. Learner If the correct answer is C that will mean ingakhi? Its one hundred is to two hundred (100; 200) that will simply mean point C isinikeza what?
518. Learner Maximum profit
519. Teacher Therefore we need one hundred Super X and two hundred Super Y so that we can find i-maximum profit any question any question any question. Mane sengini-confusile. Yes(teacher pointing at one learner who wanted to ask aquestion)
520. Learner I-maximummasiyithola siyibhala ngama coordinates?
521. Teacher It will depend kuthi i-question ithini. If i-question ithe calculate the maximum profit what will be your final answer?
522. Teacher +Ls four million
523. Teacher four million because the other one isinikeza two and a half million and the other one isinikeza four million so now iyiphi eningi? Ngikupha two and half milliona and four million which one are you going to take?
524. Learner I will take four million

525. Teacher take four million if i-question ithi calculate the maximum profit but if ithi how many how many motor cycles that will be what?
526. Learner Maximise the profit
527. Teacher Now the motorcycles that will maximise will be the C kumele kube one hundred is to two hundred (100; 200) ane? But chekha under your calibration, i-answer akumelanga kuthi kube one hundred is to two hundred but nina for convenient sake any question
528. Learners Eh sir ku-exam situation lets say ...(inaudible)
529. Teacher Right andithi niyakhumbula kuthi ngitheni masicala ngithe this question uzoyithola ku-exam ne? ina two marks or three marks bathi calculate the number of motorcycles or whatever in order to maximise the profit now that is why there are two methods. The first method yile calculate ngamapoints. sonkhe sibonile kuthi le-method is time consuming maningakaboni ke-sharp but mina ngibonile kuthi its time consuming niyangithola right there is an alternative method. yini its this one esiyisebenzisile for the fact kuthi bathi write down the equation of the profit wena nawuyibhala le uyibhala p is equal to that that uthole a-mark for that, the next question but calculate the maximum profit. Number one you indicate the profit line ku-region, number two indicate amasubstitution. Substitute between twenty thousand times ten and ten thousand times that that uyabona in most cases if instruction ithi calculate the value of whatever show all your calculation. Yazi benzanjani its because amanye amaquestion afuna wenzenjani? lets take i-question yama surds
530. Learners yes sir
531. Teacher square root twelve plus square root fifty divide by square root seventy two ($\sqrt{12} + \sqrt{50}/\sqrt{72}$) ungaphancha ku-Calculator and get i-answer but they want you to show your working because come on you cannot get seven marks for punching, you have to show your working.

LESSON 4

532. Teacher right I am not sure whether go through the statement
533. Learner learner reading from handout A BizBuz motor assembly factory employs you as a production planner at the factory. Your job will be to advise the management on how many of each model should be produced per week in order to maximise the profit on the local production. The factory is producing two types of minibuses: Quadrant and Shosholoza. Two of the production processes that the minibuses must go through are bodywork and engine work. The factory cannot operate for less than three sixty hours on engine work for the minibuses. The factory has a maximum capacity of 480 hours for bodywork for the minibuses. half an hour of engine work and half an hour of bodywork is required to produce one quadrant. one over three hour of engine work and one over five hour of bodywork is required to produce one shosholoza. The ratio of shosholoza minibuses to quadrant minibuses produced per week must be at least three is to two. A minimum of 200 quadrant minibuses must be produced per week. let the number of quadrant mini buses manufactured in a week be x. let the number of shosholoza minibuses manufactured in a week be y. Two of the constraints are x is greater or equals to two hundred and three x plus two y is greater or equals to two thousand one hundred and sixty
534. Teacher Right asikhipheni amaconstraints yes (pointing at one learner)
535. Learner x plus y is greater or equals to three sixty
536. Teacher x plus y is greater or equals to three sixty
537. Learner one over two x plus one over three x is less or equals to three sixty
- one over two x plus one over three x is less or equals to three sixty now mamela the simplest method is to draw a table (teacher drawing a table on the board with five columns and three rows) because now we say quadrant is x and shosholoza kube y now from the first chekha from the first statement bathe half hours for engine work bathe half hour for body work kusho ukuthi i-quadrant ifuna ibe ne half engine work and one third body work (teacher writing the fractions half and one third in the table) at the same time one third angithi for engine work and one fifth for bodywork angithi.

538. Learners yes
539. Teacher now the last column we reserve it for maximum now i-company cannot afford ku-operatha for less than three hundred and sixty hours angithi okay asiye ni uthe...
540. Teacher+Ls half x plus a third y is less or equal to three hundred and sixty (teacher writing on the board)
541. Teacher ibuyaphi lena lesina three sixty?
542. Learner silence
543. Teacher why uthe greater than or equals to three hundred and sixty
545. Learner cannot operate less than three hundred and sixty hours
546. Teacher right right la
547. Learner half x plus one over five y is less or equals to four hundred and eighty
548. Teacher half x plus one over five y is less or equals to four hundred and eighty next one
549. Learner x is greater or equals to two hundred
550. Teacher x is greater or equals to two hundred minimum of two hundred
551. Learner y over x is greater or equals to three over two
552. Teacher meaning i-ratio y over x is equal to two y is greater or equals to three x so far lebeningekho sure seniyabona kancane kancane
553. Learner three x plus two y ...
554. Teacher from kuphi
555. Learnersilence
556. Teacher ...Where?
557. Learners yes
558. Teacher now angithi the next question ithi you have to represent the constraints graphically you have done that indicate the feasible region you have done that eh! If the profit on one quadrant is twelve thousand rand and that one from shosholoza is four thousand rand P is equal to twelve thousand x plus four thousand y but we can add another question ethi ke calculate the number of whatever to give the maximum profit angithi la the only thing bebayifuna? is the equation of the profit yes right (teacher distributing another handout)...surplus surplus now except for today we will look at questions where we will be given a sketch graph wena from the sketch graph kufuna ukhiphe amaconstraints now kancane nje sithe if ... lets say for argument sake you have something like this right as your feasible region sithi kuwesiphe amaconstarints (teacher drawing a graph representing a feasible region) kunamaconstrain ayi two la the first one
559. Learner x is greater or equal to two
560. Teacher x is greater or equals to two
561. Learner y is less than or equals to three
562. Teacher y is less than or equals to three third one
563. Learner four x plus ...
564. Teacher y is equal to...
565. Learner l intercept is six
566. Teacher y intercept is six you start with something like y is equal to m x plus c now y intercept is six which means the only value is c is six now y is equal to m x plus six
567. Learner yes
568. Teacher now what is the value of m (teacher underlying m in y equals to mx plus six)
569. Learner negative six over four
570. Teacher negative six over four negative simply means that y is negative six over four x plus six. Now you can simplify this to negative three over two x plus six but now chekha i-shading is it below or

- above?
571. Learners below
572. Teacher below, now what is i-sign yala (teacher pointing at $y \geq -3x/2 + 6$)
573. Learner less than or equals to
574. Teacher less than or equals to chekah i-activity for today five point two ne! five point two five point two go through the statement
575. Learner (reading from handout) an entrepreneur manufactures two types of furniture pieces: chairs and tables. The costs are two hundred and fifty rand per chair and two hundred rand per table. He sells each chair for three hundred rand and each table for four hundred rand. he makes x chairs and y tables each month, so that the points (x:y) lie only in the shaded (feasible) region below
576. Teacher nizwile angithi...was she fast
577. Learners yes
577. Teacher two point one write down the inequalities which describe the feasible region. Ne! you have a space lapho write down the constraints two minutes
578. Learner writing constraints in the spaces provided
579. Teacher constraints through first one
x is greater or equals to two hundred x is greater or equals to two hundred
580. Learners other learners said x is less than two hundred
581. Teacher x is less than two hundred Ja! Yiyona the second one
582. Learner y is greater or equals to five hundred
583. Teacher y is greater or equals to five hundred oh! Wrong
584. Learners i-wrong?
585. Teacher i-right? Bhokani kahle okay yes
586. Learner y is greater than five hundred but less than one thousand five hundred
587. Teacher yes
588. Learner Ha! (unbelievable)
589. Teacher angithi kune dotted line lapha it simply means you consider everything e-less than one thousand and five hundred but greater than five hundred angithi but now you have to chekha i-inequality ku one thousand five hundred akukameli ibe less than or equals to
590. Learners it is less than
591. Teacher It is less because if it is less or equal to it was going to be a solid line since lelidotted line it simply means less right right okay the last one last one
592. Learner its y...
593. Learner y equals negative three over four x...
594. Teacher ibuyaphi i-negative three over four yes Nhlanhla yes
595. Learner y is negative three over four x y is greater than or equals to negative one over two x plus two thousand
596. Teacher negative one over two x plus two thousand straight two point two determinethe coordinates of P and T coordinates of P and Q lets just discuss it singakhulumi ngeplural form angithi ungathi asiyazi unless usitjela something indirect yes
597. Learner simultaneous equations..
598. Teacher simultaneous equations angithi because i-coordinate ya P na T is a point of yi-intersection laku intersecta khona two graphs angithi that means usolva simultaneously two minutes (learners solving problem individually) yes Boyzana nawe hhayi kabi singathuki hhayi kabi asiyeni asiyeni the coordinates of P and T coordinnate of T there is one value that is known what is it?

599. Learner it is y equals to five hundred

600. Teacher which means you just substitute five hundred to the equation of that line elikhathayo then you get the value of x asiyeni asiyeni hh! (teacher checking individual work) coordinates yes

601. Learner I am not done

602. Teacher yess oh sorry coordinates of T (name of learner)

603. Learner coordinates of T are five hundred five hundred

604. Teacher coordinates of P coordinates of P chekha ukuthi yenza i-sense we are saying i-coordinate ya T is y five hundred (teacher writing on the board) angithi coordinates a T now somewhere its coordinates of P if you use your commonsense before calculating uyaestimatha kuthi the value of x kumele ibe less tha five hundred

605. Learners yes

606. Teacher angithi now once the value of x is five hundred la nala five hundred something is wrong ne! so you can tell me that five hundred is wrong

607. Learner twohundred and fifty

608. Teacher now is guess work ucalulatihile noma you divided by two yes or no

609. Learner (teacher checkin thework of one learner)

610. Teacher five hundred five hundred

611. Learner one thousand five hundred

612. Teacher let us solve it kanye kanye we said the value of y is five hundred angithi to find the value of x ku T you have to substitute kule equation angithi

612. Learner yes

613. Teacher Nw the equation yala angithi ngu x plus y is equal to two hundred if the value of y is five hundred this should be five hundred times two angithi meaning x plus one thousand is equal to two thousand angithi so now la ngi one thousand meaning T is one thousand coma five hundred value of P

614. Learner five hundred coma seven hundred and fifty

615. Teacher five hundred coma seven hundred and fifty okay right thanks chekha the handout engininike yona last week, now chekha number two number two yes le le le (teacher pointing question in his handout) number two niyayibona i-number two okay okay number two

616. Learner (learner reading problem from the handout) In a certain week a radio manufacturer makes two types of portable radios, M (mains0 and B (battery). Let x be the number of type M and y be the number of type B. In the sketch, the shaded area represents the feasible region

617. Teacher okay now asichekhe question two point one (teacher reading question) write down the constraints to the linear programming problem given: two point one point one at most sixty of type M and one hundred of type B can be manufactured in a week. What is the constraints yalapho now if uchekha from the graph una two lines lapho, one line is horizontal and one line is vertical it simply means what will be the constraints

618. Learner x is greater or equals to sixty

619. Teacher x is greater or equals to sixty (name of learner) ha! Do you have a feasible region after sixty

620. Learner x is less than or equals to sixty

621. Teacher x is less than or equals to sixty because bathe at most angithi second one

622. Learner y is smaller or ...(learners laughing)

623. Teacher there is nothing wrong with that y is smaller or equals to...

624. Learner y is smaller or equals to one hundred

625. Teacher y is smaller or equals to one hundred the next one two point one point two (teacher reading from handout) at least eighty radios in total must be produced in a week to cover costs yes

626. Learner sixty x plus one hundred y...

627. Teacher sixty x plus one hundred y...

628. Learner x plus y is greater or equals to eighty
629. Teacher x plus y is greater or equals to three sixty
630. Learners yes
631. Teacher twopoint one point three it takes two over three hours to assemble a type M and one over two hour to assemble a type B. The factory works a maximum of sixty hours per week. What will be the constraints ah! Ah! Hhayi ichoir
- 632 Learner two over three x plus one over two y is less than or equals to sixty
633. Teacher two over three x plus one over two y is less than or equals to sixty
- 634 Learner yes
635. Teacher if the profit on type M is forty rands and on typ rands, write down the equation in terms of x and y which will represent the profit P ah ah there is something wrong lapho
- 636 Learner forty x plus eighty is equal to p
637. Teacher forty x plus eighty is equal to p any question? Suggestion recommendations..
638. Learner kulehandout osinikeze yona siwathola njani amainequalities
639. Learners amaconstraints
640. Teacher hhayi ningamlwisi
- 641 Learner ngithathe lama equation ngathola lo...
mamela u-aware ukuthi lento abasinikeze yona ane! The only thing la basebenzise the word inequalities ngoba bakunikile la but angikukhombise ukuthi sithola kanjani now bheka la, bheka la there is one graph a phasa ku P uyayibona that graph ku y-axis ikhathe kuphi?
642. Learners ku one thousand
643. Teacher one thousand ne! kux-axis ikhathe kuphi?
644. Learners kutwo thousand
645. Teacher two thousand angithi ane now angithi ngithe i-gradient is the movement ku x, it simply means kuvertical its one thousand units down and two thousand units to the right now it simply means i-gradient is equal to minus one thousand over two thousand ne! then cancela ane it simplify to negative one over two and in a straight line y is equal to mx plus c your m is minus one over two, then y is minus one over two x plus c; c is the y-intercept angithi and c is one thousand (teacher writes $y = -1/2x + 1000$) angithi. into abayenzile laphayana they just multiply by two to get two y equal to negative x plus two thousand then transpose x mayiya ngale kuba two y plus x equal to two thousand of which is the same thing abakunikeze yona uyangithola? okay right amacoordinates kaP no Q iquestion ithi detrmine the coordinates of P and Q wena what you have to check wena uchekha i-position ya P ukuthi ikuphi. You still remember masenza i-parabola ne! if iquestion ithi determine the coordinates of A and B uchekha iposition ya A and x-axis if A aku x-axis uya chekha kuthi igragh yakho its where layikhatha khona if yes it simply means A is the x-intercept if ikhatha ku y it simply means y i-yintercept ne! if the graph ngilayithena khona it simply means it is the turning point ne!
646. Learners yes

647. Teacher But now if it is any other point uyachekha ukuthi is it the point of intersection na? if sithi point of intersection kulaph amagraph ayitwo lapho akhatha khonane! And kupoint of intersection there is one thing kuthi the two graphs will share the same value of x and the same value of y ne! asicale ke sisabuyela emuva sathi okay the coordinates of T always sicale ngento esimply ne! makubekwe ukudla la kucala bayakunika amastarters hhayi kuthi kusuka nje wena sowucala ngale ngasemuva unless otherwise right right the simplest la ane! is to find the coordinates of T. why sithi the simplest angithi sithe la the point of intersection it simply means you have to equate two graphs u-solve simultaneously uthi equation one enye equation two then you have to use either elimination noma substitution method angithi but in this case there is no need ukuthi wenze amaelimination what you have to do, the value of y is known it sis line imply means you need the equation of this line and the equation of this line (teacher pointing at lines on the graph) is given nayi x plus two y is equal to two thousand what you have todo yu just substitute the value of y so that you find the value of x
648. Learners yes
649. Teacher Now the value of y is 500, then you have x plus two times five hundred equals to two thousand (teacher writes $x + 2(500) = 2000$) then you have x plus one thousand equals to two thousand. Subtract one thousand on both sides of the equation x is equal to two thousand minus one thousand ($x = 2000 - 1000 = 1000$) now if you write it as a coordinate it will be T (1000; 500) uyangithola asiye ku P ke now, the value of y is unknown ne unless masinga yenza by inspection but inspection iyalimaza ngalesinye sikhathi because awubi sure but anticipate proof that the value of y is seven hundred and fifty what you have to do you have to use this equation (pointing at at the line with equation $2y - x = 1000$) and the same equation x plus two y equal to two thousand ($x + 2y = 2000$) and you are given the equation of this line. two y minus x is equal to one thousand and lenye two y plus x is equal to two thousand solve simultaneously angithi its equation one and equation two singathi equation one minus equation two for a change asithi eqn one plus equation two this will be two y plus two y equals to four y then x minus x equals to zero ne! then four y equals to what three thousand then divide by four ke y is equal to seven hundred and fifty uyangithola mama next handout question three Nelson avoid kukhuluma ngamaplural form unless otherwise usitjela something question three next handout
650. Learner i have a question
651. Teacher okay
652. Learner what about two x if una one thousand over to thousand i-wrong yini why do you include i-negative sign my gradient is positive half
653. Teacher manisumulwisa i-question yakhe ithi what will happen if your gradient is two. If you have one thousand over two thousand iwrong mayinjalo. Why incde i-negative sign or le-answer is negative one thousand over two thousand ne! Ubani ongasizisa lapho kaspeedy. yena iquestion yakhe ithi what will happen to your gradient if wena your gradient, you have just said since i-gradient is your movement of vertical over i-movement over horizontal movement and your graph i-khatha ku one thousand positive one thousand angithi and ikhatha ku positive two thousand angithi and now if yena athi i-gradient is positive one thousand over two thousand is it right na?
654. Learner No!
655. Teacher Why sifake i-negative sign? Yes
656. Learner if siya downwards sithi negative
657. Teacher if siya downwards sithi negative but now wena kefake one thousand over two thousand you just simplify this but if it happens that ikuexam wena uphetse i one thousand over two thousand nomakara but kera understanda kutsi maybe kuneelement ye reasoning because if akakhoni kusimplify uyangithola right example three ne you have triple three point five best way

LESSON 5

one learner writing solution of homework on the board while the teacher was checking their work individually: she had $y \leq 100$; $x \leq 60$; $x + y \geq 80$ and $P = 40x + 80y$

658. Teacher Eh! Sorry for number one ane! Do you have one constraints ethi sixty x plus one hundred is less or equal to one hundred and twenty ($60x + 100y \leq 120$)
659. Learners no!
660. Teacher Angithi mamela mamela for three point one point one (3.1.1) mama mani kancane (referring to the learner who was writing the solution on the board and she went to sit down) angithi bathe at most sixty of type M and 100 of type P akire
661. Learners yes
662. Teacher Eh now your constraints should be what? X is less than or equal to sixty and y is less than or equal to one hundred
663. Learners yes
664. Teacher three point one point two (3.1.2) at least eighty radios angithi nakuthi radios you combine both type M and typep so x plus y is greater or equals to eighty ($x + y \geq 80$)
665. Learners chorus x plus y is greater or equals to eighty ($x + y \geq 80$)
666. Teacher It takes an hour to assembly eh! Type M and an hour to assembly type P. Eh the factory wants a maximum of sixty hours per week what will be the constraints yalapho?
667. Learners x plus y is less or equal to sixty ($x + y \leq 60$)
668. Teacher x plus y is less or equal to sixty ($x + y \leq 60$) yi three point bani leyo
669. Learners three point one point three (3.1.3)
670. Teacher teacher continued to check learners work while the learner who was writing the solution on the board stood up and continued to write another inequality as follows $y \leq \frac{3}{4}x + 120$ and she was not sure so the teacher requested the class to help her) Msizeni akunyakazwe lets check amaconstraints ku three point one angithi sengu three point one sicorrect angithi this one is correct (teacher pointing at $y \leq 100$ and $x \leq 60$ correct
671. Learners yes, thre point one point three (3.1.3) x plus y is less or equal to sixty ($x + y \leq 60$)
672. Teacher x plus y is less or equal to sixty ($x + y \leq 60$) angithi i-maximum is sixty?
673. Learners yes
674. Teacher then three point two (3.2) (learner and teacher checking question other learners laughing) hhayi ngiya-understanda
675. Learners continue to write the solution on the board) $P=40x + 80y$
Iphelele na? is that the profit? Angithi $P = 40x + 80y$ is the equation angithi now next question bathe draw the search line angithi that represents the profit on the graphthe search line what you have to do ne! you just draw or can redraw this angithi nenzenjalo?
676. Learnerssilence
677. Learners yes
- 678 Teacher from five vertices ane! Yiphi ezokunikeza i-maximum profit nguleyo pair oyifunayo angithi because bathi pair you have to find your answer as coordinates the x is whatever say twenty it simply means twenty of type M and thirty of type P but that coordinate mawuyisubstituttha kumele ikunikeze what? i-maximum profit (teacher drawing a sketch diagram then pointing at the shaded part) then if this is your feasible region lets say this is A, B, C, D and E which coordinates ezokunikeza i=-maximum profit?
679. Learners Coordinate E, some said D

680. Teacher Which coordinate?
681. Learners a few E and some D others ED
682. Teacher No! No! coordinates should be E or D or C not ED (teacher pointing at one learner) yes
683. Learner Coordinate D
684. Teacher sure! Ungayejele (learners laughing)
685. Learner yes coordinate D
686. Teacher Okay name of learner
687. Learner coordinate E
688. Teacher E uyi prova kanjani?
689. Learner Sir angithi ngithole i-gradient which is which is eighty is to forty then ngathi negative eighty over forty
690. Teacher negative eighty over forty u-right
691. Learners no! its negative forty over eighty
692. Teacher ane your profit is forty x plus eighty y ($40x + 80y$) sithe mayikaso you have to make y the subject of the formula. If you make y the subject of the formula (name of learner) this equals to y equal to negative forty x plus P ($80y = -40x + P$) angithi
693. Learners yes
694. Teacher Then when you divide by eighty both sides nayi i-gradient yakho (teacher circling $-40/80$ in the equation $y = -40/80x + p/80$) simplify your gradient and it will simplify to $(-1/2)$. i-half then (name of learner) wenze so?
695. Learner no, it was the other way round ($-80/40$) I divided eighty by forty
696. Teacher wenze so (teacher showing sign of opposite) sithini thina abanye u-right? La this is eighty and this is forty (teacher pointing at the two intercepts $(0;80)$ and $(40; 0)$) ubeka i-ruler yakho la then you move it over this region bear in mind kuthi sitheni. sithe when you move your ruler over the feasible region, the last point to touch your ruler will be a maximum profit (teacher pointing one learner to move ruler over the feasible region)
697. Learner learner moving ruler and showing other learners that the point is E
698. Teacher okay okay
699. Learners ki-E, pointing at point E
700. Teacher Asenze nje angithi now there is an argument between D and E okay no problem find the coordinates of D and E then substitute ku P, the one ekunikeza i-maximum number will be your maximum profit. Coordinates of D what will be the coordinates of D, now you know the value of x.
701. Learner its sixty
702. Teacher writing sixty, lets find the value of y at this point (pointing at D)
703. Learners two point five (2.5)
704. Teacher two point five (2.5) guess work? How hhayi khuluma utheni forty, sixty wase uthi lena fifty (marking on the sketch diagram) whats the equation of this line?
705. Learners y is equal to mx plus c ($y=mx +c$)
706. Teacher a straight line you call y is mx plus c ($y=mx +c$) what will be your m and what will be your c? yes then the value of c is one twenty (120) m is?
707. Learners half ($1/2$)
708. Teacher half ($1/2$) wenze njani kuthola half?
709. Learners mumbling
710. Teacher singalwi, sibhalile mara what will be the coordinates of, ngibuza for the last time angithi nisolvile i-equation ithi find the pair

711. Learners yes
712. Teacher the question ithe determine the pair that will give you the maximumprofit? Twenty two hundred (20; 200) okay? Lana sixty fifty (60;50) nithole so? Ihamba kanjani? Twenty so?
713. Learners iyakhona ku one hundred
714. Teacher iyakhona ku one hundred yes
715. Learner mumbling
716. Teacher kuphi three point point one (3.1.1) batheni? Hhayi the last question asina-problem ngayo angithi this is y less than and this one is x less than sixty my question is now did you find the value of x at E?
717. Learners no!
718. Teacher Okay lets test ke, ane la bathe p is equal to forty x plus eighty y ($p=40x + 80y$) we substitute, $P = 40(60) + 80(50)$ just say forty times sixty plus eighty times fifty
719. Learners six thousand four hundred (R6400)
- 720 Teacher six thousand four hundred (R6400), the second one? $40(20) + 80(100)$
721. Learner eight thousand eight hundred rands
722. Teacher which coordinate ekunika I maximum profit?
723. Learners E
724. Teacher sure! You are correct E is the coordinate ekunika i-maximum profit. (teacher showing them using a search line) let us assume thatthis is our ruler then move it over the feasible region then E is the maximum
725. Learners yes
726. Teacher any question? Turn to page forty onen ne! page 41 page 41 now number four okay number four if you start on page twenty six, mawuyizwa i-difficult i-linear programming meaning you can also use your text book ane! Angithi angiyisebenzisi mina i-textbook. page 36 check 37 now chekha le-question elandelayo u-question 8 what which.. now if uchekha ku page two thirty six, ku there is a love letter lapha yama constraints bakutjela kuthi what is a constraints ane! its a limiting condition, niyayi bona angithi ku page two thirty seven that the boundary line of a is solid when y is less than or equals to mx plus c ($y \leq mx + c$) or y is greater or equals to mx plus c ($y \geq mx + c$) lento ebesikhuluma ngayo right now chekha lapho text of a graph solution is part of a solution only when all boundary line is a solid line chekha elandelayo linear programming the variables... linear programming is the constraints... linear programming the feasible region... next page linear programming to minimise...linear programming to maximise...linear programming to optimise ...and linear programming search line...linear programming objective function.. just go through the statements lets try question seven, page two thirty nine okay asiye ni ngespeed ane question six it says i-answer from question one page what sir?
727. Learner page what sir?
728. Teacher page two thirty nine what about question six and question five. Six is obvious sithatha from question one from question one bathe which graphical representation in question three b lets start nga question three A right question three bathi refer to question two
729. Learners question two bathi refer to question one
730. Teacher lets start with question one lets start withquestion two ha! Ha! Eh! Eh! Number two
731. Learner taking their exercie books and working
732. Teacher Angithi le siyenza kuclasswork ane! Butif you have some problem I think nibhadele for carrer exhibition trip by tomorrow
733. Learners no
734. Teacher nina animamali
735. Learner sinayo
736. Teacher Amaparents, yibo abangacomplaina

737. Learners next week sir
738. Teacher i-carrer exhibition inext week, i-next week
739. Learners yes
740. Teacher asiyeni, kumele nibhadale benzanjani
741. Learners laba...
Eyi comeon if by Friday i-number iseseyincane you think kumele i-organizwe kanjani i-transport? Hhayi hambani niyozama lets do number eleven just number two ane number one nithe amagraph lets do number two niyabona sicale ngezinzima sagcina ngezilula. Fundakahle okay sharp (name of learner) go through the statement sa number two. ngifuna kunikhombisa something kuthi sithathe ngo top down. sicale ngezinzima ngezi difficulty sigcina ngezisimply yes go through the statement
742. Learner The owner of a sports shop wants to buy sports wear. Can x and y be a set of real numbers? Explain this constraints
- 743 Teacher any volunteer? Yes
- 744 Learner I said yes because...
745. Teacher uthe yes? Angithi mamela uthe yes okay yes (name of another learner)
746. Learner yes
747. Teacher can x and y be a set of any real number? Asenze i-constrain yalento oyishoyo
748. Learner sir I say yes...
749. Teacher yes nawe set of real numbers yini? Is negative fifteen a real number?
750. Learners no!
751. Teacher ha! Ha! Negative fifteen what are set of real numbers? Yes(pointing at one learner)
752. Learner positive square root numbers but not negative square root numbers
753. Teacher yazi nina nidla amahhabhula nihlulwa ngamabhanana (*learners laughing*) negative fifteen is negative fifteen a real number? Yes or no?
754. Learner it's a real number
755. Teacher it's a real number angithi ngibuza i-question can x and y be any real number
756. Learner no
- 757 Teacher lento kumele ibesimply because we said one of the constraints its x greater or equals to zero and y is greater or equals to zero. Shokuthi ngeke sibe nama negative numbers or negative whatever amasports wear or whatever. Singaba namasports wear a-negative?
758. Learners no!
759. Teacher yes! Nivumelana kuthi anikaboni?
760. Learners laughing
761. Teacher ngiya-understanda right lets check number three how will the constraints influence the graphical representation? Ngiyazi asinabo x or two x (name of learner) yes How will these constraints influence the graph?
762. Learner akire..
763. Teacher how will these constraints influence the graph? Your answer
764. Learner There will be no negative numbers
765. Teacher u-right? i-question i-open akuna exact answer eshaya khona but the way abeke ngakhonai-right asiye wena uthini? If the constraints athi y is negative what will be your solution what will be your solution what will be your conclusion calculate the number yale statement find the answer is negative fifteen what will be your conclusion? you calculate and you find that whatever your final answer is negative what will be your conclusion?or kuzokwenzakalani lapho?

766. Learners there is no solution

767. Teacher now lets say during the exam you solve and find that your answer is negative yinie-wrong lapho? Number one its either your calculation is wrong which means you have to go back and check your calculation.

768. Learner yes

769. Teacher now you go through ubheke and find that everything is correct but you still end up with a negative number

770. Learner mara..(learner interrupted)

771. Teacher Ane siku linear programming we use the first quadrant which means x and y is positive now mamela you are solving for y you find that the value of y is negative what will be your conclusion you will scratch yes (pointing at one learner)

772. Learner I will say that is my gradient

773. Teacher Oh sowuyatjintja nyalo you find that the value of y is negative now its gradient sithi kuwe find the coordinates of E, coordinates of E you know that the value of y is one hundred angithi you find the value of x and it becomes negative from the graph you know kuthi kumele ibe positive but wena when you substitute you find that the value is negative twenty instead of positive twenty what will be your conclusion ne! when do we write no solution?

774. Learner when there is error

775. Teacher It is highly impossible kutsi ku-real life situation ingaba negative question three question three bathe refer ku question two ane first write an equality eh! For each constraints and write standard form. The answer yes

776. Learner x plus y is less or equals to four

777. Teacher Right number b what will be the constraints yalapho

778. Teacher +Ls eight hundred x plus four hundred y is less or equal to fifty thousand

779. Teacher sesi-right which graphical representation below represent the constraints in three point one sorry three point one point one (3.1.1) question a or b or c? bear in mind kuthi leconstraint for a is x plus y is less or equal to forty ($x + y \leq 40$)

780. Learner b

781. Teacher right, number six ke refer to question five number four uthi draw the three representation one for the inequality on the same axis. Hha! Nizoyenza leyo number six. Lets see lapha kune-present yenu from maam ene sithe ngumaam bani? Ngitjele igama kuthi sithe ngumaam bani from which institution? maningangitjeli sihambanayo lepresent siyonika amaGrade eight

782. Learner hha!

783. Teacher yes

784. Learner Maam Nkambule from UNISA

785. Teacher Maam Nkambule from UNISA. Right close your books close your books ngibuza i-question ungitjeli answer mawuright uthola i-pen mawu wrong sihamba nawo

786. Learners hha!

787. Teacher What is a feasible region yes

788. Learner i-feasible region are set of points that are set of ponits formed by inequalities (learner using gestures)

789. Teacher ngubani othi yena angayibeka kahle? Simnike

790. Learners Mnike

791. Teacher another question what is a feasible region

792. Learner ngulapho amalayini ahlangana khona nakahlangana idirection ... (learner interrupted)

793. Learner laughing..

794. Teacher yes continue!
795. Learner a set of points that satisfy all the lines on the graphs
796. Teacher satisfy on the graph... how do you find a maximum profit?
797. Learner you use your gradient
798. Teacher how do you use your gradient to find maximum profit?
799. Learner usebenzisa i-broken line kulegraph then ubeke i-ruler on the feasible region the last point to touch your ruler will be the one to give you a maximum profit yakho
800. Teacher when you use your gradient yini into e-critical okumele uyibheke. Ane you say you move your ruler over feasible region you are saying use gradient, use ruler you move over feasible region ane there is one thing something kulegraph okumele uyibheke, uyichekhe kuthi is it possible to use that gradient before you ..ungakhona
801. Learner sign
802. Teacher sign do we all agree that i-gradient is always negative
803. Learners no
804. Teacher Was it possible la ukuthi i-gradient ibe positive (teacher pointing at the example they were solving on the board)
805. Learners no
806. Teacher Mamela mamela hha! Angithi la it was grade 9 or grade 8 labengibafundisa now if a gradient is negative it simply means the line goes down and positive the lines go up. Uthi yena one of the things i-sign ye-gradient right and what else?
807. Learner uchekha amacoordinates ukuthi
808. Teacher amacoordinates ukuthi enzenjani? No lets take this as a feasible region. Number one you will observe that your gradient is negative forty over eighty and what else okumele uyichekhe futhi? Abanye bazothola more than one pen
809. Learners Hha!
810. Teacher Ngizowanikeza amagrade eight
811. Learner I think you check ama-intercept, then ubeka i-ruler yakho
812. Teacher Okay mamela lets say along the axes you count ngama hundred and your gradient is half uvele u-plothe half yes?
813. Learner no you change to the scale on the graph
814. Teacher uyitshintja kanjani? Lets say your gradient is half.
815. Learner you multiply by 100
816. Teacher okay all those who got pens stand up... (learners stood up) okay sit down if it happens that the number of those bamile is more than abahlezi I will give them a second pen
817. Learners no!
818. Teacher okay let me give you another chance what is linear programming?
819. Learner best solution on the graph
820. Teacher what is a constrain? (one learner who answered before) sowuyitholile give others a chance
821. Learner a limiting condition in linear programming
822. Teacher Give us any statement esingasirepresenta ngama constraints
823. Learner obaba wami ufuna amantombazana amabili nomama wami ufuna amantombazana amane both mabaphelele kufuna babe yisix, amantombazana u-x and abafana u-y

824. Teacher ubani ofuna abafana ngoba from your statement ubaba ufuna amantombazana kanti nomama ufuna amantombazan (learners laughing)
825. Learners two x plus four x izidingo zabazali...
826. Teacher Give us a scenario where we will construct inequalities
827. Learner ubaba uKhaphela ufuna ukuhlanyela amasunflower awu sixty

APPENDIX J: BHEKI'S CLINICAL INTERVIEW

1. Thuli Good afternoon Bheki
2. Bheki Good afternoon
3. Thuli How are you today?
4. Bheki I am fine, ngiyaphila
5. Thuli I have visited your school once more today, you remember I promised to come and ask you to solve one problem on linear programming. And I would like to ask you a few questions while solving or while working on this problem. You may respond using any language. So may you please solve this question, it is numbered question five.
6. Bheki question five
7. Thuli Yes question five
8. Bheki Bheki reading problem softly...
9. Thuli you may read aloud, I like it when you read aloud so that we are together
10. Bheki (reading problem aloud and as he was reading he underlined some words) a local health board is producing a guide for healthy living. The guide should provide advice on health education, healthy lifestyles and the like. The board intends to produce the guide in two formats: one will be in the form of a short video; the other as a printed binder. The board is currently trying to decide how many of each type to produce for sale. It has estimated that it is likely to sell no more than ten thousand copies of both items together. At least four thousand copies of the video and at least two thousand copies of the binder could be sold, although sales of the binder are not expected to exceed four thousand copies. Let x be the number of videos sold and y the number of printed binders sold.
11. Thuli okay, okay just In summary what can you say this question is all about what what what
12. Bheki A person may be in a clinic is giving a health guide for people so is making may be of a video of for people so is making a video he wants that the video must not exceed four thousand forty thousand that a local health is producing
13. Thuli Oh this is all about making videos for the community oh!
14. Bheki that this health body is not producing anything but ama videos for the community
15. Thuli oh! This health body is producing something for the community
16. Bheki yes
17. Thuli Manje this health body ingeke i=produce anything
18. Bheki ayi-produce anything besides ama copies namavideos
19. Thuli okay seiyachubeka seyisinika ini? You can use any language, if you prefer Zulu you can use it
20. Bheki I can use any language?
21. Thuli Seyiyachubeka seyisinika ini?
22. Bheki $i-x$ kufuna kube yinumber yama videos and $i-y$ i-number yama
23. Thuli Oh andisinika ne certain numbers
24. Bheki selling this thing noma?
25. Thuli oh mawucabanga why why bathi must not exceed four thousand? Bentelani? Is it selling this thing noma...kuyatsengisa yini/
26. Bheki kuyathengisa may be abanamasupliers lanag..
27. Thuli Oh! Inshort baminimaza ini/
28. Bheki Baminimaza amacosts

29. Thuli Oh! That is why bangafuni kuexceeda four thousand which means noma batsengisa, they must make sure that batsengisa a certain number. Batsini ngale certain number?
30. Bheki a certain number ukusho ukuthi...
31. Thuli kufuna bayitsengise?
32. Bheki kufuneka bathengise twenty thousand yamabanner ne forty thousand yama videos
33. Thuli Oh mawubuka nguyiphi lenamanombolo lamanengi which onelabafuna kuyitsengisa kakhulu?
34. Bheki Ngamavideos
35. Thuli Why?
36. Bheki because amavideos abantu bayawa-understanda than printed banners.
37. Thuli oh (both laughing) than..
38. Bheki than amapafulethi
39. Thuli Oh chubeka
40. Bheki Lana bathi x kubeyinumber yama videos and y kube yinumber yamabhayindasi
41. Thuli Oh uyayiunderstanda leproblem lokusho kutsi i-linear programming in short what is it all about?
42. Bheki i-linear programming is about solving real life problems a certain number must exceed a certain number must be exceeded
43. Thuli lokusho kutsi kuna what they call amarestrictions
44. Bheki yes ama-restriction let me give an example of amaexercise books abantwana, akufunekanga ukuthi ibe less than the number of students ingamane ibe extra
45. Thuli okay ngiyabona kunanankha emagam nankha, local ubonile kutsi it is for the community which is good then is giving a guide for health living. It is not free though but it is for selling that is why akhuluma nga four thousand copies. Nakatsi at least four thousand
46. Bheki at least kufuneka ixceede forty thousand
47. Thuli For example unganginika nje any example unganginika nje any number more than four thousand
48. Bheki five thousand is it?
49. Thuli which means kufuna five thousand
50. Bheki four thousand one hundred
51. Thuli okay okay makatsi at least two thousand usho kutsini
52. Bheki at least kufuna indlule ku two thousand may be two thousand one hundred two thousand five hundred
53. Thuli then bayasho yini bayasitjela utsi masekutsengisa koyi-two?
54. Bheki akufunekanga ukuthi kundlule ku forty thousand
55. Thuli oh not to exceed four thousand oh chubeka
56. Bheki ngenze amaconstrain?
57. Thuli oh yes yini kepha amaconstrain? That is good i-linear programming ine terminology as if you knew kuthi i-first question is about constraints ungewabhal emaconstrain
58. Bheki Ngingawabhala
59. Thuli okay ngibhalele ke
60. Bheki x is equal to...
61. Thuli Oh that is good as you solve it you may talk aloud llike it mawukhuluma when you talk aloud

62. Bheki x is the number of videos and y is the number of pamphlets and then x is at least forty thousand and y is greater or equal to twenty thousand. $x + y$ is less than or equal to ten thousand.
63. Thuli $x + y < 10000$
64. Bheki $x + y < 10000$
65. Thuli Oh
66. Bheki $x < 10000$ and $y < 10000$
67. Thuli Okay
68. Bheki $y < 40000$ and $x < 40000$
69. Thuli where are we the first one
70. Bheki $x < 10000$ and $y < 10000$
71. Thuli it is likely to sell no more than ten thousand copies
72. Bheki of both items
73. Thuli ehe both means the two
74. Bheki both means the two items the video and the binders that's why I say forty thousand x and twenty thousand y is less or equal to ten thousand
75. Thuli Oh! At least four thousand copies x , at least two thousand copies y
76. Bheki although...
77. Thuli although sales of the binder are not expected to exceed four thousand copies
78. Bheki four thousand ...
79. Thuli $2000 < y < 4000$ and $x > 2000$ or $x = 2000$
80. Bheki It means it lies between the two of them
81. Thuli yes that is good!
82. Bheki so now we have the constraints
83. Thuli Good, so now we have the constraints, then question two is to use the graph paper to represent these inequalities graphically. Do you have graph paper can you draw the constraints on the graph paper in order to find the feasible region?
84. Bheki i-graph paper?
85. Thuli Yes can you draw the graph?
86. Bheki the graph, the feasible region
87. Thuli yes, eh! Yini ifeasible region?
88. Bheki feasible feasible is the is the...
89. Thuli is the region ngoba kuyasho nje
90. Bheki i-region where all constraints are satisfied
91. Thuli oh! I region where all these constraints are satisfied okay asichubeke let us draw the region we will need it to solve
92. Bheki I can draw it in my exercise book (learner drawing axes) Before I sketch sengiyachekha kutsi the scale sitawufitha yini?

93. Thuli Oh the scale...asesiyibuke leyo kutsi itsini...itsi the board is currently trying to decide how many of each so they are not sure which means before bacale ba produce they must decide..
94. Bheki which means x plus y is greater or less than ten thousand..
95. Thuli yes x plus y is less than ten thousand sishiye li four thousand na two thousand ngoba nangu so they don't want to produce more than ten thousand that is good
96. Bheki which means it is going to be one thousand noma one thousand five hundred, noma two thousand
97. Thuli Ja! Two thousand is fine hmm
98. Bheki my x intercept is going to be ten thousand
99. Thuli okay good
100. Bheki learner drawing) less than or equal to ten thousand I am going to shade downwards..
101. Thuli okay good
102. Bheki then less than four thousand..
this one is
 x is less than four thousand I am going to ... x is four thousand then y is forty thousand then it is going to be less than..
103. Thuli yaphuma kahle lena le between two thousand and four thousand which one is our feasible region asiyishede kahle
104. Bheki nansi ifeasible region
105. Thuli asiyishede kahle
106. Bheki learner shading...then here they are saying the board is seeking to maximise the income, I , from the sale of the two products ,each video will sell for R50 and each binder for R30. write down the objective function for the income
107. Thuli Ths board is seeking to maximise kusho kutsini loko what is to maximise?
108. Bheki kusho ukuthi bafike kucertain number that is going to give them the greatest value
109. Thuli That is going to give them a greatest value
110. Bheki greatest value yeprofit that they are looking for
111. Thuli greatest value yeprofit and as you can see each video is going to sell for R50 and each binder R30. as you can see i-video is expensive but banftu bayayitsandza
112. Bheki yebo bantu bayayitsandza
113. Thuli okay bantfu bayayitsandza okay write down the objective function what is the objective function?
114. Bheki objective function kusho sort of the equation for the profit
115. Thuli equation ye profit and they call it I
116. Bheki then determine graphically, by using a search line, the number of videos and binders that ought to be sold to maximise the income
117. Thuli search line yini isecond search line
118. Bheki isecond search line i-search manje lana i-scale asilingani kufuna kuthi sizoyibeka lana kufuna kuthi sitimeze 100 izonginika five thousand then I five thousand is between four thousand and six thousand then kufuna ukuthi sizoyibeka lana. Then lena si times nge 100 nayo isinika three thousand which is between two thousand and four thousand nayo sizoyibeka lana then kufuna kuthi sidrawe i-broken line broken line to maximise lapha bathi to maximise the income
119. Thuli oh! Lapha batsiteni find maximum income by using a search line asithole the point how are you going to find the point?
120. Bheki I am going to use the search line

121. Thuli Oh use the search how
122. Bheki lapha I am going to move the ruler lana ku feasible region then the last point to touch the ruler is my point elizonginika imaximum profit
123. Thuli uyi kuyasetjentwaa la open the windows kub
124. Bheki this is the last point to touch my ruler
125. Thuli what are the coordinates
126. Bheki the coordinates are eight thousand and twenty thousand because it is x and y if manje ngifuna uku prove straight ukuthi ngiyo ngenzafifty thousand into thirty thousand la ku objective function x equal to eight thousand and y twenty thousand then ngithola and it gives me four hundred and fifty thousand
127. Thuli this is the maximum income that will be generated
128. Bheki i-maximum income ayifanani ne maximum profit?
129. Thuli ayifanani ngoba asikakhiphi emacosts but lapha we are not told how much it is kwakha lama videos kanye nalama binders so lapha yimaximum income.
130. Bheki oh yes so this is the maximum income
131. Thuli cha niyiunderstandile i-linear programming
132. Bheki ngiyiunderstandile kepha ekucaleni besingaboni kahle
133. Thuli Good because i-linear programming you have to understand the problem and the extract the inequalities from the given statement
134. Bheki But sometimes kunekubhala kuthi half straight, mine angizange ngibone kutsi ihalf yi constrain
135. Thuli oh bebabhale emafraction so in other words are you saying it was tricky?
136. Bheki no bebabhale half not i-fraction so angikhonanga kuyisolva otherwise I would have solved it
137. Thuli Oh! Now ngiyabona bebabhale half ngemagama which means you are used to symbols like one over two?
138. Bheki Yebo ngoba its mathematics ngangayiboni kutsi it is one of the constraints
139. Thuli so now sowuyabona kutsi ungabhala in words and also use the symbols you are used to like one over two which is the same as half in words.
140. Bheki Yebo
141. Thuli ngiyabonga kakhule Bheki I wish you all the best in your studies. Seyikhalile i-bell what is the next subject?
142. Bheki it is English, bye.

APPENDIX K: LESSON OBSERVATION IN CLASSROOM C

1. Teacher. Today we have our visitors from the university of South Africa isn't it?
2. Learners Yes mam
3. Teacher. And they have come to observe our lesson and I will be teaching you isn't, I think it will be good if they introduce themselves because it might be difficult for me to pronounce their names.
4. Thulie Good morning class
5. Learners Good morning maam
6. Thulie How are you?
7. Learners Fine thanks and you?
8. Thulie I am also good. Am happy to be here, I am Mrs Thulisile Nkambule. Yes, Nkambule I am from Swaziland but I am studying at unisa. She is Rebecca Maleka from (using gestures showing that she is from around). She is doing mining engineering at Unisa.
9. Teacher. Its okay thank you, Okay last week we were busy with trigonometry isn't it?
10. Learners Yes maam
11. Teacher. Last week we were doing trigonometry isn't? Let us switch off the trigonometry for a little bit, I am going to introduce a new topic. I am going to introduce a new topic isn't. Okay this week we will be doing linear programming, that is what we will be doing for this week, linear programming. Let us first find out what is linear programming, what is it all about isn't. So let us first write the definition so that when you are doing the topic you know exactly what you are doing. So we are saying, linear programming is a mathematical method that uses linear inequalities to solve problems and these problems involve finding the maximums and the minimums. That is what you call linear programming, linear programming it is a mathematical method that uses linear inequalities to solve problems involving maximums and minimums. And now we have to concentrate on this linear inequalities (*teacher underlines linear inequalities*). For you to be able to solve linear programming problems you are supposed to have a background of linear inequalities on how to show inequalities on a cartesian plane. How to show inequalities on a cartesian plane, So we are supposed to...eh if I had known in time I was going to tell you to bring the graph books because it is easy to work on a graph book than on your line books, so tomorrow make sure you bring your graph book at least we draw these inequalities on a cartesian plane where we can spare the paper. So for you also to draw to show inequalities on a cartesian plane you should know how to draw a straight line on a cartesian plane. And that one is not for me to show you. You should know how to draw a straight line on a cartesian plane. For example when we are given y is equal to two x plus six ($Y=2x+6$) this one is a linear function, isn't it?
12. Learners Yes
13. Teacher +Ls And it is a linear function and we should be able to draw this linear function on a cartesian plane, isn't it?
14. Learners Yes
15. Teacher I think you know what is a cartesian plane? ah.. I am not talking Greek here you know what a cartesian plane isn't?
16. Learners Yes

17. Teacher You know what a cartesian plane is, now a cartesian plane you draw the y-axis and the x-axis. And now when you are drawing a straight line , when you are drawing a straight line on a cartesian plane you only need two points, which points? for you to draw a straight line on a cartesian plane, you need two points, only two points are enough for you to draw on a cartesian plane eh... eh...a straight line on a cartesian plane, you don't need to make a table of values. You only need two points, which points do we use to draw a line?
18. Learners The x-value and the y-value
19. Teacher The x-values, we don't say the x-value. Can you use the correct term?
20. Learners The X-intercept and the y-intercept
21. Teacher Yeah, that's good. We use the y-intercept and the x-intercept, is it?
22. Learners Yes
23. Teacher So there is no need for you to make a table for values using a lot of values. Only this two allow you to draw a straight line. So the y-intercept, y-intercept we are going to find it where? y-intercept we we are taking which value? For the y-intercept. We want to find out where our graph, our line cuts the y-axis isn't it?
24. Learners Yes
25. Teacher It cuts the graph where? Eh!
26. Learners when x is equal to zero
27. Teacher Good, when X is equal to zero. So we are taking when x is equal to zero then we substitute in this equation isn't it? (teacher pointing at the line y is equal to two x plus six).
28. Learners Yes
29. Teacher To find the value of y when y is equal to zero, so can you find out and tell me the intercept? Yes(teacher pointing at a learner)
30. Learners Y is equal to six
31. Teacher So when you put x is equal to zero ($x=0$) we substitute into two x , x with zero and we are saying our intercept is zero six (0,6). So you plot zero six there(showing a point on the y-axis) that is your y-intercept. Then our second point we are using x-intercept. We are using the x-intercept because before you draw the inequality... you show the inequality on a cartesian plane, you are suppose to be able to draw a straight line. is it? You are suppose to know how to draw a straight line.
31. Learners (chorus) Straight line
32. Teacher Then we must move on to the inequalities, okay. The x-intercepts, this is on the previous knowledge we have already done it, we are just recapturing the previous knowledge. Okay, when x-intercepts is found where? x-intercept is found where? Don't just copy what is on the board, you have to copy at the same time thinking, isn't it?
33. Learners Yes
34. Teacher And remember I said to you, don't copy what you do not know. First understand the concept then you copy, because I can see that some of you are just copying. Don't just copy what you don't know. You have to understand first and then you copy, is it?
35. Learners Yes maam
36. Teacher Ehe...(pointing at a learner)
37. Learners x-intercept is when y is equal to zero
38. Teacher Good, x-intercept is when y is equal to zero.
39. Learners y is equal to zero (unison with teacher)
40. Teacher When y is equal to zero, so when y is equal to zero in our linear function we substitute now. Where we have y we are substituting with zero, is it?

41. Learners Yes mam
42. Teacher So let us substitute and see what we get zero is equal to two x plus six ($2x+6$), then you solve for x. We solve for x, if you solve for x what do you get as the value of x? Can you solve it? Solve it.
43. Learners Yes mam
44. Teachersolve it.....x is negative three ($x = -3$) so we are saying x is minus three. Our x-intercept is minus three and zero $(-3,0)$. It is minus three and zero. So you plot minus three zero on your graph. This is zero six $(0,6)$ and minus three zero $(-3,0)$ it may be there (pointing at a point on the cartesian plane) is it?
45. Learners Yes mam
- 46 Teacher Then you plot it, then after plotting you join the points using a ruler. Is it?
47. Learners Yes mam
48. Teacher You draw a straight line. You draw your straight line now and this line is y is equal to two x plus six ($y = 2x+6$) (*teacher labelling the line*)
49. Teacher We have a situation where we have an equations like x is equal to four ($x=4$), x is equal to four. We need to draw that line x is equal to four. Any one to come and draw the line on the cartesian plane fast and show this line on a cartesian plane. Fast, we don't have time. We don't have the other period. We have to find out if you can draw this lines on a cartesian plane.
50. Learner (*one learner stood up to draw the line*)
51. Teacher Don't forget to label, to show which one is the x-axis and which one is the y-axis (*teacher reminding the learner*). Okay, that's good. So we have got x is equal to four is this line, a vertical line. So you have to know this line because we are going to use then when showing inequalities on a cartesian plane. Y is equal to minus six ($y = -6$) y is equal to minus six. I want any one to come and show y is equal to minus six on the cartesian plane, y is equal to minus six on a cartesian plane. Y is equal to minus six. Yes (*teacher pointing at a learner*). Try to be fast we have a few minutes. Don't rub, just use the same cartesian plane. Alright It's fine. so that is how we show straight line on a cartesian plane. Anyone problem with that ?
52. Learner No
53. Teacher So let us begin with inequalities. Can I rub this?
54. Learner Yes
55. Teacher Showing inequalities on a cartesian plane. Showing inequalities on a cartesian plane. I think you know the general term, the general eh....term for a linear function which is? What is the general equation for a linear function, the general equation for a linear function.
56. Learner Y is equal to mx plus c ($y = mx + c$)
57. Teacher Good, y is equal to mx plus c. That is the general function of a linear function, now when we are looking at inequalities we are using the symbols, for inequalities means they are not equal isn't it?
58. Learner Yes
59. Teacher We are not using the symbol equal to. So we are using the symbols greater than or less than, we use...Sorry less than or equal to, then less than, we use greater than or equal to, then greater than. So these are the four symbols we are going to use for inequalities.
60. Learner Yes
61. Teacher Now if you have y is greater than mx plus c ($y > mx + c$), when you are drawing the line you use a dotted line. We use a dotted line. When showing this on a cartesian plane (*teacher pointing at y equal to m x plus c*). But when it is y is greater than or equal mx plus c, we use a bold lines. A dotted line is this one (*teacher pointing at the dotted line*) A bold line is this one (*pointing at the solid line*) isn't it?
62. Learner Yes

63. Teacher And this dotted line is showing that all the points on the line are not included. Let me give you an example. If we have what? x is greater than four ($x > 4$) when you are told to list the elements of that set. Can you give me the numbers which are greater than four? Can you list the elements greater than four? Eh!
64. Learner 5,6,7 ... (in unison)
65. Teacher 5,6,7 up to infinity list all of them. Isn't? You see that four is not included?
66. Learner Yes
67. Teacher But If i say x is greater or equal to four ($x \geq 4$), x is greater or equals to four ($x \geq 4$). I am now including four, so when we are listing the set four, five,six (4,5,6) that is when you are using the bold line, the bold line is to show that the element on the line are included. is it? And a broken line it means elements on the line are not included, are we together? Okay we have when y is less or equal to mx plus c ($y \leq mx + C$) again we use bold line. y is less or equal to mx plus c ($y \leq mx + C$) we use a bold line. For ($<$) less than and greater than ($>$) we use a broken lines. For greater than or equal to or less than or equal to we use a bold lines, is it? For example, let us do the showing of inequalities on a cartesian plane. Okay, we have to draw our cartesian plane, don't forget to to label this cartesian plane. Label the x -axis and the y -axis. So that one who will be marking will know which one is your x -axis and which one is your y -axis. Okay, we will start with the simple one. we want to show this inequality on a cartesian plane x is less than five($x < 5$). Let us say x is less than three. x is less than three, x is less than three. Now if it is x is less than three the line we are going to draw it is x is equal to three. Is it dotted or bold? Its dotted is it?
68. Learner Yes
- 69 Teacher It's dotted because three is not included. So it is dotted, so there (pointing at the x -axis) you write your three. Already you know how to draw this line, that is why I say you are suppose to know how to draw these lines, you have x is equal to three, is it?
70. Learner Yes
71. Teacher. And x is equal to three it is this line(*drawing the dotted line*) and we said it is dotted. We want now to show the inequality, we want to show the inequality. X is less than three, now normally we shade the unwanted region and the region which is left clear is the one which has got our solutions, is it?
72. Learner Yes
73. Teacher We have x is less than three ($x < 3$), x is less than three ($x < 3$) is it on the right side of the line or the left side of the line?
74. learner The left
75. Teacher. The left side is it. So it means we have to shade this unwanted region.You shade, you don't want it, is it?
76. Learner Yes
77. Teacher So this region is showing all the elements which are less than three. So this one which is not shaded is our region. With the solutions, is it?
78. Learner Yes
79. Teacher Y is greater than or equal to minus four. If you have understood, can any one come and show this inequality on a cartesian plane?
80. Teacher Don't rub, just use the same, just use it so that you don't waste time. Okay, thank you so much. Thank you. Okay, we are saying y is greater than or equal to minus four, is it?
81. Learner Yes
82. Teacher First of all, find out if your line is broken or bold and here we are including minus four, so it is bold, is it? Then you write the equation y is equal to minus four, that is the line you are going to draw. y is equal to minus four. Then we are saying greater than or equals to, looking at our line is above or below our line?

83. Learner Below
84. Teacher Greater than minus four, elements which are greater than minus four, greater than or equal to minus four they are minus four, minus three, minus two, minus one up to infinity (-4,-3,-2,-1...upto infinity..) Going up, isn't it? So this is the wanted region. So he has shaded the unwanted region. Which is correct, which is below the line. Any one who is confused? I started with simple ones? Where we have y is greater than or equal to minus four. That line is simple to draw. Is it? and also x is less than three, you know how to draw x is equal to three. Now suppose you are given this one, two x plus five y is greater than or equal to ten, greater or equal to ten ($2x + 5y \geq 10$). First of all I want some one to come and draw the line and then I will show you how to shade, to shade the unwanted region. Any one to come and draw the line? for this one we are using two x plus five y is equal to ten ($2x+5y=10$), isn't it so? We are using two x plus five y is equal to ten ($2x+5y=10$) that is an equation we are going to draw the line, that is an equation we are going to draw. I just want you to draw the line, is it? is it bold or dotted?
85. Learner Bold
86. Teacher It's bold, I just want you to draw the line and then I will show you how to shade it (*learner drawing the line*). Thank you,thank you so much, I think it is correct. Now you have the line, we said for you to draw the line you need two points, the x -intercept when y is zero an y -intercept when x is equal to zero. Then you get your points and you draw the line.Now we want to show the inequality, can I rub this so that it can be clear, are we clear? And we said it is a bold line, isn't it?
87. Learner Yes
88. Teacher It is a bold line, now how to shade the unwanted region, remember we said we shade the unwanted region, is it? Our inequality is which one? Its two x plus five y greater or equal to ten ($2x+5y \geq 10$) that is our inequality is it?. We want to find out which region are we going to shade and we shade the unwanted region, we shade the unwanted region. So what you need to do is, you can see that we have below the line and above the line. you choose any point either this side of the line or that side of the line. Can you choose which points are we going to take? Which point are we going to take, which point are we going to take? It either above the line or below the line. Which points can we....you choose any point. Any point here or any point there. Give me the point . Give me the point. Give me any point which is above the line, a point which is written in ordered pairs. It should be coordinates.
89. Learner Four seven (4,7)
90. Teacher Okay, four seven (4,7) is the point, is it? So we have four seven (4,7) we are taking four,seven(4,7) as our point which is above the line. So what we are going to do is, we substitute that point in our inequality, isn't it? Let us do our substitution, what is the value of x ?
91. Learner Four
92. Teacher Then we put the value of x over the and what is the value of y ?
93. Learner Seven
94. Teacher Seven, put it there and then here is eight plus twelve($8+12 \geq 10$)Eight plus twelve, so we are saying twenty is greater than ten($20 > 10$),is it true? So we are saying twenty is greater than ten. Is it correct?
95. Learners Five times seven is thirty five($5 \times 7 = 35$) (*learner sporting teachers mistake*)
96. Teacher Eight plus thirty five greater or equal to thirty five($8+35 \geq 10$) which makes it forty three ($43 \geq 10$), thank you very much. Some have already copied.
97. Learners (*Laughing*)
98. Teacher That time I told you earlier, do not copy what you don't know.
99. Learners Yes

100. Teacher If you concentrate you will see the mistake but if you do not, you will just copy with wrong answers which is not good. Don't copy before you digest, so we have fourty three which is greater than ten($43 > 10$) isn't? Is it true? True or false?
101. Learners TRUE
102. Teacher It is true. So we are saying the coordinates which we have taken to this side (teacher pointing at the side of (4; 7) it is satisfying our inequalities, so it means that side is the wanted region because any point on that side of it is satisfying, our inequality is the wanted region. If it is not satisfying it means it is unwanted region, isn't it? So it means we are not going to shade this side because we have taken a point on this side and it is satisfying our inequality. So the unwanted region is this, so that is how we shade the unwanted region. You draw the line, you chose any point above or below the line and if it satisfies then that is the region you want. If it doesn't satisfy, then you automatically shade it. Classwork.
- 103 Learners We are not carrying our class work books.
104. Teacher Write somewhere but you are suppose to carry your maths book everyday, everyday that is why I said you just take your book when you see my face. So if you don't see my face for two weeks. Jah But you need to read. Yeah, during study. I told you some of you they don't take their maths book until they see my face, if you don't see my face for a week it means the book will be in the bag or under the pillow for that week. Okay your classwork (*teacher writing problem on the board*). Don't forget that when you are showing inequality the line is either dotted or bold. Don't make a mistake to draw a bold line where you need to draw a dotted line. If you are confused you raise your hand up so that I can attend to you. Try to be fast. I said if you are confused raise your hand so that I can come and help you. You can pair them into two or three (*referring to the inequalities*). No problem. So that you don't waste the papers and don't use free hand because you have the ruler.
105. Teacher If you understood it this tme it means your are going to get a distiction, when you are done you will have to show me before you move on to the next question. When you are done with first question you call me, if you are confused you have to ask remember I said don't write what you don't know. when you are confused you have to ask so that I can show you. don't write what you do not know
106. Teacher Michelle (checking learners work) you are suppose to draw a dotted line remember isn't it, check with your inequality its not greater than or equal to. Please don't forget to use the dotted line, some of you are forgetting to use the dotted line. Please avoid taking points that are nearer to the line, is it? Because you may take a point which is on the line because it is just a sketch you don't have graph paper so that you know that the points are not on the line. Just take points that are far away from the line, either above or below. They should be far away, isn't it? so that you know exactly which side is the point.
- 107 Learners Yes maam
108. Learners (Other learners) Alright
109. Teacher So that you know which side is the point. Okay, tomorrow we will continue with inequalities and our visitors will be here up till we finish the topic of linear inequalities. So tomorrow if you've a graph book at home please make sure you bring it, your graph book so that at least we use a square paper because when it comes to other concepts on feasible region when we need to find the maximum, as we have said that linear inequality deals with solving inequalities finding maximum and minimum. so thank you very much.
- 110 Learners Thank you maam
- LESSON 2**
111. Teacher Okay, good morning.
- 112 Learners Morning maam
113. Teacher How are you?

114. Learners Fine thanks
115. Teacher Yesterday I gave you work to do, did you finish the work? Okay let us continue with our topic, today will be dealing with the feasible region. Now feasible regions we are talking about a region where we find all the solutions of given inequalities. Feasible region we are talking of a region where we find all the solutions of a given inequality. (*writing on the board*) now we are saying it is the region where we find all the solutions of the given inequalities. Now the solution can be found as an infinitely large area on the region. Here we are having our x-axis and our y-axis and we have our first inequality (*being a bold line*) we have our second inequality (*dotted line*) and we have our third inequality like that (*bold line*). So the area we are saying it is infinity, because we are looking at all this area. The one which is shaded is where we find the solutions, the one which is shaded is where we find the solutions of our inequality. We've got our first inequality, second inequality and third inequality. And we have got to solve them simultaneously, isn't it?
116. Learners Yes
117. Teacher On the same graph, so the space... the region which is shaded is what we call the feasible region because all the solutions satisfy our inequalities, all the solutions satisfy all the four inequalities. This one, this one, this one and this one (*pointing at the lines on the cartesian plain*). Our region is infinity because there is no boundary, it keeps on going. As long as this line keeps on going and this line keeps on going we don't have a boundary (*pointing at lines*). So we have it as a infinity large area, so we are saying this is our feasible region. Feasible region is normally shown by shading it, you shade the feasible region. when you shade the unwanted region using a pencil then you take a colour and you shade the feasible region is it, after shading it you erase the pencil so that you are left with the feasible region. You can find your feasible region at an enclosed area, you can find your feasible region at an enclosed area. This one we said the area is infinity is it? It keeps on going, the solutions are infinity you can't finish them. For this one, you can't finish them. But when we have our enclosed area, we have our x-axis and the y-axis. (*Some learners arrived late making noise*) Those who are coming in don't make noise. okay, we have this one which is our first inequality (*represented by a dotted line*) and we have this one as our second inequality (*represented by a bold line*) and we can also have this one (*bold line*) as the other inequality. So we are saying area enclosed by the three inequalities; the first inequality, the second inequality and the third inequality is this one (*shading the area*). So we have got our feasible region there, that is the area enclosed by the three inequalities which is the feasible region. So it means in that region we are going to find all our solutions for our three inequalities, if you take any point from our feasible region and you substitute in any one of the inequality it will satisfy the inequality, it will be true. Any point which you take inside and on this line and which is on this line will not satisfy it, remember we said about a dotted line we said that all the points are not included but when the line is bold it means that all the points are included.
118. Teacher You can take a point on the line or in the feasible region but you cannot take a point (*pointing at the dotted line*) because it doesn't satisfy the other inequality. Any point on the dotted line doesn't satisfy the inequality. So I want to give you some questions, now that you know how to draw the straight line and you know how to shade the unwanted region, I will just give you the questions and I want you to try and find the feasible region in the given inequalities. You have to find the feasible region for the given inequality. Are we together?
119. Learners Yes mam
120. Teacher Find the feasible region for the given inequalities. You now know how to draw the straight line and to shade the unwanted region, we have learned that yesterday. So we taking it as class work. Three problems I have, it's like (*writing the following on the board*) find the feasible regions of the following. Now when you are drawing the feasible region, you are going to draw the inequalities on the same cartesian plane isn't it?
- 121 Learners Yes mam

122. Teacher We have one, two, three, four for part A. This four inequalities are suppose to be drawn on the same cartesian plane, you don't draw them separately because what we want is to find the area which is enclosed by the four inequalities, is it?
123. Learners Yes maam
124. Teacher So do the first one. You now know how to draw the lines, isnt it? If it greater than you use a dotted line and if it greater than or equal to you use a bold line. Use your classwork excecise book. If you dont know what to do you ask, we shade the unwanted region, shade the unwanted region as we were doing yesterday. You draw your cartesian plane, draw your first inequality and find out which region are you to shade. You shade it; shade it very, very light; very ,very light because you are going to erase it. And maybe this one is the other inequality, you shade it. Then if you have got another line like this, if you are shading this side then you shade it. you now see the region is it, this part which is not shaded is our feasible region. So you take a colour, a different colour to a pencil. You can use a pencil then you shade it like that, and then after that you take a rubber and eraser and you remove this (*showing the shaded sides of the lines which is unwanted*) that is why I said it should be light so that you remove and our feasible region will be left. When we are shading we shade the unwanted region so that we know which region is left unshaded. That region which is left unshaded we know that it is our feasible region. So do question one then I will see, do it fast. And we say as a reminder for you to draw a straight line you need two points, where your line cuts the x-axis and where your line cuts the y-axis and don't forget that your line is dotted or bold. Check the sign before you draw your line. I said if you don't know what to do, ask. (*learners working individually or as group while the teacher was attending to learners questions. She noticed that learners had similar problems mainly because half the class was absent*)
125. Teacher I am noticing that the are people who where not here yesterday.
126. Learners Yes
127. Teacher Let us do question one together, let us do question one together cause otherwise I will be troubling you. Okay, we have got the first inequalities. You can see that if we substitute.. If we find the x-intercept here, the y-intercept then x is equal to zero here our y will be negative, isn't it?
128. Learners Yes
129. Teacher So we include the negatives but if the are no negatives just take the positives quadrant but if there are the negatives you have to include the negative quardrant. So we draw our y-axis and our x-axis. So let us calculate the first one, the first one is three x plus four y is greater than twelve ($3x + 4y > 12$). Remember we said when we are looking for the y -intercept and the x-intercept we take it as an equation. We have three x plus four y equals to twelve ($3x+ 4y=12$), we said for us to draw a straight line we need two points where our graph...where our line is going to cut the y-axis and where our line is going to cut the y-axis. And cuts the y-axis when x is zero and when y-axis x-axis when y is zero. So we are saying when x is zero, when x is equal to zero we are suppose to put zero here (*pointing at the equation on three x (3x) isn't it?*) Then we have three multiplied by zero plus four y (4y) is equal to twelve and this one will dissappear (*meaning the 3 multiplied by zero*), isnt it?
130. Learners Yes
131. Teacher We are left with four y (4y) is equal to twelve ($4y = 12$). Then we divide by four is it?
132. Learners Yes

133. Teacher If we divide by four we get y is equal to three ($y = 3$) so our graph is going to cut our y -axis at three, so you plot here. So let say three is here for y (*drawing on the cartesian plane*) and then we move on to finding the value of x that is when y is equal to zero, Now when y is zero we are putting zero the (*pointing at $4y$*) so it is $3x$, if we put zero here it disappears (meaning the $4y$ after substituting y with 0) $3x = 12$. Then our x is equal to four ($x=4$) then we have x is equal to four there (*writing the x coordinate on the x -axis*) then you write your four there. then you take a ruler and you draw a line, sorry our line is not suppose to be bold. our line is not suppose to be bold, our line is suppose to be dotted because we have greater than here and if we have greater than or equal to then it is bold and if it is greater than only the it is dotted. we have a dotted line here, like that isn't it? That is our dotted line. Now we want to find out which part to shade, we want to find out which part to shade. We take this point zero zero ($0,0$) let us take this point zero zero and it is on this side, is it?
134. Learners Yes
135. Teacher If we take zero zero, we have three x plus four y is greater than twelve ($3x+4y > 12$) if you can take zero zero ($0,0$) or you can take minus one and minus three ($-1,-3$), minus one and minus three ($-1,-3$) their also that... this side. If we take zero we are going to get zero is greater than twelve ($0 > 12$). Because we are going to put zero times three we get zero and zero times four we get zero. Zero plus zero is zero ($0+0 = 0$) is it?
136. Learners Yes
137. Teacher zero is greater than 12? Is it false or true?
138. Learners FALSE
139. Teacher False. So If it is false we shade that side and if it is true we shade the other side. We always shade the side which is false, that is why I said shade using a lighter pencil is it? Then we move on to another inequality which is two x minus three y ($2x - 3y$).
140. Learners two x minus three y ($2x - 3y$).
141. Teacher. Two x minus three y less than sixteen ($2x - 3y < 16$) like that and the again we find the intercepts, when x is zero what is our y ? When x is zero what is our y ?
142. Learners Minus five, one over three
143. Teacher Minus five and one third, let us say it is there (*drawing on the y -axis*) then x ...when y is zero, what is x ?
144. Learners Eight
145. Teacher Eight, then we draw a line.
146. Learners Maam it is dotted line!
147. Teacher It is dotted, thank you. It is dotted because we have less than we don't have the equal sign, is it?
148. Learners Yes maam
149. Teacher Then again we are going to find the region we are going to shade, okay, let us take seven and two ($7,2$). We take seven and two ($7,2$) then our x here is seven which is fourteen our y is minus six less than sixteen ($14 - 6 < 16$) fourteen minus six what do we get?
150. Learners Eight
151. Teacher Eight is less than sixteen, is it true or false?
152. Learners True
153. Teacher True, remember we said if it is true that is the region we want. Is it?
154. Learners Yes maam
155. Teacher So we shade the other one, so we have taken our point above the line. So we are shading this part. Is it okay with those who where not here yesterday?
156. Learners Yes maam

157. Teacher So we take our third inequality, our third inequality is simple. We have x is greater than or equal to zero, x is greater than or equal to zero. Our line x is greater than or equal to zero is this line (*pointing the y-axis*) the equation of this line is x is equal to zero, so greater is this side (*pointing the positive side*) so less is this one, so we are shading this one because it is greater we are shading the less side because the greater one is the one we want. Then for y is greater than or equal to zero, y is equal to zero is this line so greater is above isn't it? and less is below so we shade, our inequality is greater so we shade this one. So what type of a solution is that? its an infinity, isn't it? it an infinity. So you take your pencil and you shade this side, that is where we have our feasible region. That is our feasible region.
158. Learners Feasible region (simultaneously with the teacher)
159. Teacher And then after that, remove this pencil. So tat this part which is shaded will be the remaining one. So that is it, do question one and two.
160. Teacher. All intercepts are suppose to be labelled(talking to a learner)
161. Teacher Please make sure if you have a problem you ask.
162. Learners Yes maam
163. Teacher Don't write what you don't know.
164. Teacher Okay can you please look at the board, we have got the x -axis and the y -axis as usual isn't it?
- 165 Learners Yes mam
166. Teacher And we have got two equations, $x=0$ and $y=0$. Most of you are confused which lines are these, $x=0$ is which line?
167. Learners The y ...
168. Teacher The y -axis because if you... All the points here x is zero
169. Learners Okay
170. Teacher. Isnt it?
- 171 Learners Yes maam
172. Teacher If you are writing for example if I have one, two, three, one , two, three this will be negatives isn't it? The coordinate for this point, what is the coordinate for this point?
174. Learners One is to zero (1, 0), zero is to one (0,1)
175. Teacher. Not zero is to one, zero one(0,1) and this one?
176. Learners Zero is to two.
177. Teacher You see you think it is a ratio this one, it is not zero is to one. It's a semi collon zero one(0;1) and then here?
178. Learners zero three (0; 3)
179. Teacher You can see that always x is equal to zero, that is why we call it x is equal to zero. So x is equal to zero is the y -axis and $y =0$ is the x -axis. Here y is always zero on this line, that is the reason why we call the equation of this line y is equal to zero, and the equation of this line $x =0$. So if their talking of the equation x is greater or equal to zero we are taking the equation x is equal to zero which is the y -axis. Do you understand?
180. Learners Yes maam
181. Teacher So continue.
182. Teacher (*Addressing a learner*) You see that above and below it won't work, if you want to use that one make y the subject of the formular. Minus $3y$ is greater or equal to $4x$ minus 12. Y is less or equal to, whenever you divide with a minus the sign changes then we have four x divided by three plus four then you look at the sign which is less or equal to which is now below the line. if you want to use that method,no problem but you will have to make y the subject of the formula.

183. Teacher Okay, let settle down it is almost time up. You are almost late, tomorrow our visitors will still be here as we continue with our topic on linear inequalities

LESSON THREE

184. Teacher okay, okay, we have y is less than x and we have $2x$ plus $3y$ is greater than minus 6. We are doing part B. Okay, okay we have $2x$ plus $3y$ is greater than minus six and we have $4x$ minus $3y$ is greater or equal to minus 12 and then what else? $2x$ plus.... we have it. $2x$ plus $3y$ is less or equal to twelve and then we have x is less or equal to zero. So this one is not there,

185. Learners Yes maam

186. Teacher So we have got the four inequalities, our maximum is we want for the y values its eh... When y is zero x is six. So we have one, two, three, four, five, six. Any one want to come and show us line number one? $2x$ plus $3y$ is greater than minus 6, come quickly we don't have time we need to have another concept.

187. Teacher Use equal to (*Talking to the learner who is writing on the board*)

188. Teacher So okay, we have y is equal to minus 2 and x is minus 3, Use a dotted. okay the shading part? Why do you shade that part? Which point have you used so that you can shade that part?

189. Learners Silent

190. Teacher substitute and see, you are using greater than. Is zero greater than minus 6? You rub that side and you shade that side.

191. Learners Yes maam

192. Teacher Number two? Equation number two, try to be fast. Any one.

193. Learners Number two!

194. Teacher And where are going to shade? He is saying he is using seven two (7; 2). Okay, is it true or false?

195. Learners FALSE

196. Teacher Go and write false, which side have you take your point? Is it above or below the line? (*Talking to the learner who is writing on the board*)

197. Teacher Okay, okay, (7,2) is this point so if it is false you will shade it. Shade that side, next one.

198. Learners Two is below and seven is up

199. Teacher Seven is here (*pointing at it*)

200. Learners What about two?

201. Teacher Is above, hah..we are taking a point, we are taking a point which is (7,2) we are taking a point A which is (7,2) not digit two and digit seven. Ahaa...we are taking a point don't confuse yourself. We are taking a point A which is (7,2) and where is point A? Where is point A?

202. Learners Above

203. Teacher It's above isn't it? So we shade above because it is false, we don't take a digit seven and two you take a point, you choose a point if you want this one B which is five four (5; 4) so you are taking five four (5,4) and B is above and if you take zero zero(0,0) it's below, if you take one one (1,1) it's below. Not one and one (*pointing on both axes*) Don't confuse yourself, isn't it? Next one.

204. Teacher Four, because I told you don't spoil your mind with a calculators. Your suppose to divide minus twelve with minus three is positive four. Don't spoil your mind with a calculator always you want to punch a calculator. One plus one you want to punch a calculator, two plus two you want to punch a calculator. we use an equal sign when we are solving, don't use... its negative. okay, we don't have a board ruler you can use free hand sketching. okay,side to shade. which one are you to take?(3,2) okay, it minus, its minus

205. Learners *Laughing*

206. Teacher You see your calculator now, this time you don't have a calculator

207. Learner Twelve

208. Learner That's true

209. Teacher Okay, which side are you to shade?

210. Learners This side

211. Teacher Jah. Six is greater than minus twelve which is true, isn't it?

212. Learners Yes maam

213. Teacher And we have taken it to this side, so we are going to shade the other side. Last inequality, that one is simple, is for every one. Which line is that? which line is that?

214. Learners x is less than zero ($x < 0$).

215. Teacher Okay, Hah? Less is that side which we are shading it is the left side, x is less than zero it is the left side.

216. Learners x is less than zero.

217. Teacher Left side isn't it? That is what we want, we shade the right side. The one which you don't want

218. Learners Oh, the right side.

219. Teacher Okay, that's okay. Now we want our feasible region.

220. Learners Feasible region (*chorus with the teacher*)

221. Teacher Feasible region, yes come and shade now the feasible region. (*Name of learner*) sit down, I have nominated that one. Make it fast we don't have time, we have a lot to cover today. Eh.. Don't erase, just shade first and then erase. Shade it first. That is our feasible region!

222. Learners Yes maam

223. Teacher If you have got it correct just mark yourself a good. Can I rub here?

224. Learners Yes maam

225. Teacher Part c, number one. y is less than x , any one to come fast. That line is simple to draw, it is the easiest $y < x$. Which equation are you to draw? Which line are you to draw? Hah..? What is the equation of that line?

226. Learners Broken line

227. Teacher. Broken line but what is the equation? We want to transference it from inequality to equation from $y < x$. What will be the equation?

228. Learners Y is equal to x ($y = x$),

229. Teacher Y is equal to x ($y = x$) that's good. we are going to draw the line $y = x$. Don't waste time, when y is equal to x it is easy, it means whenever you have x is equal to one and y is one, when x is two and y is two and x is zero y is zero. You don't know the line $y = x$? You are in grade 11 and you don't know the line $y = x$? don't waste time, any one? You don't know it? The intercepts passes through the origin. now choose a point. Where is the point you have chosen?

230. Learner Zero four (0,4) it is this side (pointing on the y -axis),

231. Teacher We shade it because it is false. Next equation, next equation. Six and three, fast plot it. Shade now.

232. Learner Am going to use five four (5,4)

233. Teacher. 5 and ? Okay which point is that?

234. Learner Pointing at his chosen point.

235. Teacher Put it, so that people won't be confused. It's four and five. So he is taking four and five (4,5)

236. Learner Thirteen is greater than six it true, so it is the side we want we don't shade it.

237. Teacher Next one, try to be fast. Less noise and concentrate

238. Learners It's dotted!

239. Teacher It's a dotted line, yeah do it. Okay the last one, the last one. Try to be fast. The last one, any one want to try that one? Okay that line passes through the origin again, isn't it?
240. Learners Yes
241. Teacher When ever you don't have a y intercept it means your line passes through the origin. The intercept is zero zero, you can try another value. For example take x is equal to one ($x = 1$) so that at least we have zero zero and another point, take x is equal to one. It is not an intercept , your just taking x is equal to one because it is just passing through the origin. And the value of y , y will be 1. Now plot that point one and two third.
242. Learner where?
243. Teacher Two thirds is zero comma six, I said don't spoil your minds with a calculator. It's zero comma six, six. It recurring. Plot one and zero comma six here and then you draw your line passing through all that, yes you have to check if it is dotted or bold and yes it is passing through the origin. Okay, then choose which side you shade, your taking which one? (-1,2) okay. Put y two then less or equal to. Is it true or false?
- 244 Learners FALSE
245. Teacher Two is less than minus two thirds? It's false, so we are shading that side. So any one to come and show us our feasible region? The one which is not shaded. Going down, its infinity.
246. Teacher It's okay, that is our feasible region isn't it? Do we have any questions on shading and finding the feasible region? Because that thing is very important on finding the feasible region, that is linear programming part. Because after getting your constrains your suppose to show them on a cartisian plain and show the feasible rgon, so if you don't know how to find the feasible region your in trouble. I think it is enough, practice makes things perfect. If you keep on practicing and practicing, I think you will be fine. So let us move on to today's business.
247. Teacher Today's business we are finding the constraints, making inequalities from given word problems, making inequalities from given word problems. Just take one and pass, because some of this questions are too long for me to write so am going to use that, just take one and pass. When you are writting down constraints when you are reading a question, we have got key words, we have got some key words which we are suppose to note, we have got some key words. If you look at your work sheet we have got some key words to note and those key words are going to tell you which sign are you going to use. Are you going to use greater than, are you going to use less than, are you going to use greater than or equal to. So for less than if you look at you work sheet, for less than for this symbol less than ($<$) the word which are used there. We use the word less than, smaller than if you see a word smaller than it means you are going to use this symbol isn't it?
248. Learners Yes maam
249. Teacher And may not reach, may not reach we use less than. For greater than we have got key words more than, greater than word, must exceed that is for greater than ($>$). Then less than or equal to, less than or equal to we use less than or equal to; at most, at most it means will be the maximum isn't it?
- 250 Learners Yes
251. Teacher And also the word maximum, may not exceed, not more than. These are the key words and you are suppose to know them. Greater than or equal to, at least if I say I want at least three boys. It means I want three or more, Minimum I want a minimum of four boys it means I want four or more. May not be less than, Not less than. If it is not less than it is suppose to be equal or more. So these are the key word we use in linear programming, they are the key words we use in linear programming.
252. Learners Linear programming

253. Teacher So when you are writing the constraints, so it means you have to follow these words, and we have got this two constraints x is greater or equal to zero and y is greater or equal to zero, they are suppose to be there always. Is it? Normally they don't give marks for this ones, if they say write constraints for the given statement. this ones their just for you to write but you are not given any marks for this ones, don't write this two and you think already you have two marks yet the is still four more constrains to be written. So let us look at the example which is given on the work sheet, we are going to study it and see how we are to write the constraints. And this constrains a written in different ways, they can be written with one variable Example you where having inequalities wrtten in one variable like x is greater than or equal to five. The is only one variable, i think you know what a variable is? X is the variable and we have got X and Y it is our variable. They can be written in one variable, they can be written in two variables (two x plus three y is less than four ($2x+3y < 4$). It is another way of writing the constraints it depend on the statement isn't it?
254. Learners Yes
255. Teacher And also they can be written when they are comparing each other for example you can have $y = 2x$, we are saying for us to get to y we are multiplying x by two, isn't it?
256. Learners Yes
257. Teacher So it means y is twice x . So we are comparing two, its either we are comparing or we are writing it in one variable or two variables. Okay let us read the example, a television manufacturing company and normally when you are writing constraints make sure you read and you understand the question, read and understand the question is it?
258. Learners Yes
259. Teacher A television manufacturing company makes two types of televions, they makes two types of ?
260. Learners Television
261. Teacher Ordinary screen tv's and flat screen tv's, those are the two. No more than five normal screen Tv's and but no less three flat screen must be produced daily. So they are not suppose to produce not more more than five normal screen tv and also not less than three,, from those two statements we can have two inequalities. There are eleven workmen at the manufacturing work shop, their eleven work man isn't it? And i takes one man to assenble the ordinary tv and two men to assemble the flat screen tv in a day. The number of ordinary tv produce daily must be at least half the number of flat screens produced daily. you see now these words are being used? now you know that when they say at least which sign am are to use, where there is not less which sign i am to use. That is why i said thia words are very important, now you will name...normaly when you are being give an examination they will name the variables for you, isn't it? They are saying let the number of ordinary tv let it be x , ordinary let it be x .Then the flat tv let it be y , write down the constraints in that statement.We have our first constraints we are going to get from the statement where er have not more than five ordinary tv are to be made daily, not more than five ordinary tv's are to be made daily. Not more than, so they should be what? If they are not more than they should be less or equal to, so our variable for ordinary tv is x . So we are saying our variable will be x is less or equal to five bacause they are not suppose to be more. .Not more than, not more it is less or equal to. And also for the flat tv they are not suppose to be less than, not less than three, not less than three. Not less than three, so they are suppose to be greater or equal to three, they suppose to manufacture three or more flat tv's per day. Because the condition is that they are not suppose to be less than three, you see?

262. Learners Yes
263. Teacher Okay, they are eleven work men, they are eleven work men and it takes one single work man, one man to do an ordinary tv. And two men to assemble a flat screen and they are only eleven. The are those who will assemble a normal tv and those who will assemble a flat screen and their must not be more than eleven because we only eleven workers is it?
264. Learner Yes
265. Teacher So we are saying for ordinary we need only one person and for flat we are saying we need two x plus two y ($2x+2y$) must not exceed eleven which is less o equal eleven. We should no have more eleven workers where are you going to get the other one company has eleven workers. The number of ordinary tv produced daily must be at least half the number of flat screen. the number of ordinary tv's produced must be half the number of flat screens produced. So the number of ordinary tv must be half of flat one. How do we write that one? The number of ordinary tv's must be half, the number of ordinary tv produced must be at least half the flat . So at least is greater or equal to and we have x is greater or equal to half of you can say is greater or equal to two x, isn't it? cause it should be half. at least is greater or equal to. you know, you can now see that this words are important, so if you don't know these words you can't write any inequalities. Then after that we can ask you to find the feasible region, that is the question. You are to make the inequalities, from the inequalities then find the feasible region.And for the feasible region we don't have any problem, the practice today is to make the inequalities isn't it?
- 266 Learners Yes maam
267. Teacher Just make the inequalities. So have a small excersise, just try it . I know the problem is English.I told you your suppose to communicate in English, always. But you say we are Xitsongas we are not English people, what what. Some of you were saying how can you speak in English we are not white. But now are you asked in Xitsonga?
- 268 Learners No
269. Teacher Okay, I have given you the sheet and the class work. Do that and post it in some excersise, we are just finding some inequality, some constraints is it? Try it. Read and understand the question. Read and understand the questions
270. Teacher okay, you have got your own plan and own paper. Heh? You just write with you hand not with your mouth
271. Teacher Distributing questionnaire to find languages learners understand, speak, read and write. Asking researchers to explain to the learners
272. Thulie If your do not understand you better say and let us explain, ehh.. Number one wether your a female or male, boy or girl. And then how old are you, five, six years old or fourtee. Are you a south african citizen? Cross yes or no. If you are not, what is your citizen ship.Am swazi. okay , number five when did you arrive in south africa, 2005, 2000 , 2008. Now tick your home language, please write write xhosa, I forgor xhosa I wrote sesotho twice.So delete one sesotho and write xhosa in all of them, where the is sesotho you write xhosa if you are xhosa. Its one home langauge, the language you speak at home if you speak afrikaans you tick, if you speak zulu at home you tick.Okay, if you speak more than one langauge at home, you tick as many as you want. Languages you understand, just tick all the languages you understand if you understand all the official languages plus portugise you write it down the and if you understand Xitsonga, you write Xitsonga.
273. Learner I speak Xitsonga
274. Thulie Yes, Xitsonga you indicate, South African Citizen, you write no I am from Mozambique and so on
275. Teacher You are from Mozambique..(interrupted by learners)
276. Learners No we are not from Mozambique, we are South Africans...
277. Teacher Tomorrow our visitors are still here, tomorrow we need to finsh on this topic so that we go back to tri cause next week we are wruting a common test.

LESSON FOUR

278. Teacher It's up to you to pass or to fail.
279. Learners Yes maam
280. Teacher Don't force any one to write, I am not going to force any one to write because myself I do not repeat it is you who is going to repeat, isn't it? So if you don't write the work I gave you it's your own problem.
281. Teacher Okay, number four. Here we are saying let the number of litters used be x and let the number of concertrated be y .
282. Learners There, twelve x plus twenty three x ($12x + 23x$).
283. Teacher Huh?
284. Learners $12x + 23y$ (*correcting the mistake made by the teacher on the board*)
285. Teacher $23y$ thank you.. For number four these are the inequalities and for qusion five . Okay, but I can' t manage to write the question for you what i'll do is that I will write the question on the board for you. The question for today is...
286. Teacher Okay your suppose to write a test today for maam Isnt it? So I have asked for just a few minutes so you will manage.
287. Teacher Please can you make the inequalities, make the inequalities because we don't have time. Make the inequality then you will copy the question later. Make the inequality and then draw, take one copy and. Just make the inequality. We are taking the number of planes to bwe x and the number of boats to be y . The question is: "write down the constrains of the above, including also any implicit constrains, include the implicit constrains. The implicit constrains are x is less than zero, x is greater than zero and y is greater than zero and it is eight marks. This question was written by the grade twelve on tuesday on common test this question, so make the inequalities here from the statement given(pointing the question) make the inequality and it is eight marks for that. Second question, sketch the contraits on the grid provided, given an answer sheet and clearly indicate the feasible region, seven marks for that. write down the objective function for maximizing the profit in the form of P is equal to ($P =$),so you are writing the constrains from this three statements here,this statement it is for profit. So let us do it together, what is the first inequality. We are say this once are suppose be the is it?
288. Learners Yes
289. Teacher Because we are saying they are suppose to manufacture something, so from this statement the company is to manufacture from forty to one fifty planes, so the inequality will be like what? Any one? forty to one fifty, so what will be the inequality. They have to manufacture from fort to one fifty. X is greater than forty but less than one fifty because they have to manufacture from forty to one fifty. so we write it like this forty is less or equal to, x is less or eqaul to x . The inequality for B , y is greater than 10 but less than one twenty. All togeher not more than 200, $x + y$ is less or equal to two hundred, then now the profit on a..... for a plane is sixty rand and for boat is one hundred and so the total profit we are going to get we know we are going x planes.Each one having a profit of sixty, so the total profit for planes should be what?
290. Learners Sixty
291. Teacher So we have sixty ($60x$) and for is Y is $100y$ and you get the total of P . Write down the objective function for maximizing profit, this one is the objective function for maximizing profit. Objective function for maximazing the profit. Sometimes tey say to get the minimum profit and when is . And you have two marks for writting this one,and linear programing question are easy to answer. So we want to show the inequality on a cartision plain and show te feasible region, is it?
292. Learners Yes mam

293. Teacher You have to show the feasible region, feasible region. Remember our maximum here is two hundred. So make sure you choose a suitable scale ,make sure you choose a suitable scale that will have two hundred. What can we have?
- 294 Learners Twenty
295. Teacher. Twenty,
296. Learners Fifty
297. Teacher Fifty, is it? Is it too much? What about twenty five?
- 298 Learners Twenty five.
299. Teacher Use twenty five, you see that our box is two centimeters. So we are saying two centimetres (2cm) divided by twenty five (25mm), two centimeters to represent twenty five millimeters (25mm), we have a positive quadrant isn't it? We don't have negatives. So it is zero, twenty five, fifty, seventy five, one hundred up to two hundred (0, 25,50,75,100 up to 200). Do it fast, use a pencil. And we are using two centimetres to represent twenty five units (2cm : 25). We are using two centimetres to represent twenty five units (2cm: 25). We don't have negatives. Try to be fast, this one we are taking them separately, look at this one. We don't combine this. We take them separately, we take x is greater than or equal to forty then we take x is less than or equal to one fifty. You take x is greater than or equal to forty, then you take x is less than one forty on its own. These ones are two inequalities. And this one will be y is greater than or equal to ten and y is less or equal to one twenty. You write them separately. when you say you take x to be zero, it means x will disappear here. Just put your x and y will be two hundred, stop wasting time by working, some of these things are simple. Then if y is zero it will disappear, you put your hand like that and x will be two hundred, simple. You, you want to substitute zero plus..... okay for x here for both x and y because this one y is going to be two hundred and x will be two hundred isn't it, so on both axis I don't know if that one will suit.
- 300 Teacher Don't copy the question, that will waste time. Because I want to teach you how to find your maximum profit. I want to teach you how to find the maximum profit because here they have asked you a certain question, use a search line on your graph to determine how many each toy table should be sold to generate maximum profit. That is the question being asked, otherwise we have done part of it part four and part five we haven't done it. Don't use a pen cause you will make a mistake and it will cause a mess, use a pencil. Use a pencil so that if you will make a mistake

APPENDIX L: ALLEN'S CLINICAL INTERVIEW

1. Thulie I can see you have these constrain, tell me what do they represent?
2. Allen eh ...they gave us...x for video and y for binders, x plus y less than ten thousand means the board is likely to sell no more than ten thousand items of both items together so...it is the sum of the video and the binders to be sold...*(silence)* they say the board has estimated to sell no mre than ten thousand...i added the two which are representing videos and binder...
3. Thulie so that is your first inequality
4. Allen yes this is my first inequality
5. Thulie tell me about your next inequality, what does it represent?
6. Allen the second inequality is x less than four thousand mean at least four thousand of the video could be sold. At least means less than or equal to four thousand...
7. Thulie What does at least mean according to your understanding in this question
8. Allen At least mean less than or equal to...a number... Say lapha basinike at least four thousand so it means less than four thousand
9. Thulie Let me say I ask for at least R2 from you, how much can you give me?
10. Allen I can give you R1, because you asked for at least R2...
11. Thulie So that is why you have this inequality
12. Allen yes because they say... videos is represented by x and y represent binders...so I know that x is less than four thousand....that is why I have this inequality (pointing at x less than four thousand)
13. Thulie the third inequality represents which restriction?
14. Allen ...*(silence learner reading handout)*...they say at least two thousand copies of binder could be sold...so at least means less than or equal to therefore i have y is less than two thousand...which is my inequality...
15. Thulie Now you have x plus y is less than or equal to four thousand...*(Allen interrupting)*
16. Allen yes...yes... It is sales of the binder and videos...not expected to exceed four thousand...jah and here we have videos at least four thousand...*(silence)*...jah that is how I have written my inequalities...
17. Thulie let us look at your inequalities again...you have this one which is x plus y is less than ten thousand and again x plus y less than four thousand, can you tell me in your own words your understanding of the given task...
18. Allen ...*(reading the task, looking at his solution)*...it means the board is selling binders and videos but the sale should not be more than ten thousand and also not more than four thousand so it is between four thousand and ten thousand...*(silence)*...so I looked for the numbers which must be sold...and I know at least is less than that number...like four thousand here... and ten thousand in the first one...
19. Thulie okay, but you have y is less than two thousand...
20. Allen yes ...jah...they want the binder ... They want to sell less than two thousand of the binders...
21. Thulie my aim is not to mark your work but to find out how you have solved the task so let us move on to the next part where you have drawn the graph to represent your inequalities...I can see you have these points; *pointing at* (4000; 0) and (0; 4000) ...

22. Allen yes, i put x equal to zero in this inequality (pointing at $x + y \leq 4000$) then you get four thousand so the point is zero is to four thousand which is this one...then when y is zero...x is 4000 again it is this point (*pointing at 4000 on the axis*)...less than is below so I shade above the line...the other lines are x equal to four thousand which is this line ...less than means... I shade this side (*using gestures pointing to the right of the line*) and y is two thousand which is this line...less than mean below the line so I shade above the line...
23. Thulie okay so for x plus y less than ten thousand ...
24. Allen Oh yes ...less than ten thousand is below...so I shade above the line... And this is my feasible region...
25. Thulie okay...now let us look at your objective function...
26. Allen yes...it is fifty x plus thirty y here...my point is three thousand and two thousand...
27. Thulie How did you find the point...i noticed that you were calculating...
28. Allen ngithethe this points here, here, this one and this one...eh (*silence*)...then I got two hundred and ten thousand ...so it is ...(*silence*)
29. Thulie Ngiyalefuna. No it's fine you can start discussing the, e gahle...
30. Allen (Reading a statement) this is linear programming, so it's the short video. Let the short video be x.
31. Thulie Nope. Tsonga, whatever is fine. Ningate le bone mina (don't mind me), mina ntabota I will ask you. If I speak siSwati will you understand?
32. Allen We understand.
33. Thulie Then ya, any language is fine.
34. Allen And printed file be y.
So x plus y is less or equal to then thousand.($x+y < 10000$)
35. Thulie Why do you say x plus y is equal to ten thousand?
36. Allen Less or equal to ten thousand. Because the statement says to sell not more than ten thousand copies.
37. Thulie Ten thousand.
38. Allen Yes.
39. Thulie Thank you.
40. Allen At least four thousand copies per video. Okay, at least four thousand. At two thousand of the copies can be sold. At least is less than or equal to...(silence)
41. Thulie Write down the inequality sign to represent at least four thousand, is that okay
42. Allen Two thousand is for y, it's separate. For four thousand less or equal to x and ...two thousand less or equal to y. $x + y$ is less or equal to four thousand($x + y \leq 4000$). Less or equal to?
43. Thulie You've got the constraints?
44. Allen Yes, [*silence looking at his inequalities*] Four thousand.
O mise di statement(you mixed the statement) $x + y$ is less or equal to four thousand($x + y \leq 4000$)
It's not four thousand, it's ten thousand.
It's more likely more than ten thousand per piece for both items.
Ya , ten thousand ne ele ya $x + y$ (ten thousand is for $x + y$)
I cannot explain now...my inequalities are confusing...I will use the graph paper to represent this inequality graphically and indicate the feasible region (scratching his head...long pause...)
45. Thulie Okay...you want to represent your constraints graphically?

46. Allen Yes, but I am not sure ngatsi it is not making sense to me now...but...anyway amaconstraints ami lawa...yes...[counting...one two three four...] jah! All of them?
47. Thulie Lante nbone (let me see), printed by now. Okay, okay. What is the problem about nje? If ne ngancasela (if you could just explain). Ikhuluma ngane (what does it talk about)?
48. Allen A local health is producing a guide for healthy living...(silence)
49. Learner Another learner came in to ask for something (inaudible)
50. Thulie Sorry about that...you say a local health is producing a guide for health living. What is a guide by the way?
51. Allen Is on how to guide people on how to live.
52. Thulie I don't want the guide. (*Laughs along with the students*). Yes to give them advice. Yes, it is about advising people to lead a healthy life. So ba khicita two things (they make two things), what is khicita? Eh, is producing.
53. Allen A short video and printed bundles
54. Thulie Guphe la ba funa go khicita ngube ngunenge (which one wants a lot to be produced?)
55. Allen Sorry?
56. Thulie Nguphe, which one will be, produced the most? Is it the printed or the video? A ngithe the is two thousand and the is four thousand?
57. Allen Yes.
58. Thulie Which one is the four thousand?
59. Allen Is the video.
60. Thulie Why? Do you have any suggestions? Why are the deciding to produce four thousand videos, are they not expensive?
61. Allen They are expensive but they can be real, like so many people like to see things on TV.
62. Thulie Oh, that's very good. Because people nowadays are technologically.
63. Allen Yes.
64. Thulie So la maphepha?
65. Allen They won't work.
66. Thulie They will find them on the streets.
67. Allen Yes.
68. Thulie So if baguhlanganise bo itwo, that's $X + Y$ it shouldn't exceed a certain number. That number is?
69. Allen Ten thousand.
70. Thulie Yes, ten thousand. Why are they having a limit instead of a minimum?
71. Allen That number should not exceed this.
72. Thulie Why ba ngathe open (why aren't they saying open), gubenje teminimum se funa ku producer iten thousand (we want to produce a minimum of ten thousand). Instead of saying maximum.
73. Allen They can produce more than what is needed by the people.
74. Thulie Oh, yes that is very good. They can end up with.... Yet this is business. Is it business or? They also want money
75. Allen This going to give them benefits.
76. Thulie Nangala go ma videos and printed they have maximums. They don't want minimum. How much is a binder, ba se tselile (have they told us)?
77. Allen No they didn't. In question five point three the binder is seeking for a maximising profitable of selling video for fifty rand and binder for thirty rand.
78. Thulie Oh, okay. A se balane (lets write), now ngicabanga go the sithole iprofit function (I think we

- must find a profit function). Let's try that one before we draw the profit function.
79. Allen P is equal to fifty X
80. Thulie Why X?
81. Allen This fifty rand they are for the videos and our videos we said are X.
82. Thulie X number, okay.
83. Allen And thirty Y.
84. Thulie Because ibinders i-thirty rands, okay.
85. Allen five comma two (5.2) Determine the profit line by using this.
86. Thulie Sigakhona go ita (can we do it)? Is it possible to draw a le (this one)? Let's just sketch.
87. Allen This one they just want us to draw the graph.
88. Thulie For the first one, an ngithi we will use the graph to find the maximum?
89. Allen Yes.
90. Thulie Will it be possible for us to sketch? It is not the accurate one, it is just a sketch. Just a sketch and from the sketch we can estimate.
91. Allen The constrains is two thousands. $X + Y = 0$. $X < 10\ 000$
92. Thulie Come again, which one is that one?
93. Allen The first one.
94. Thulie Let's start with the co ordinates.
95. Allen Will let Y equal to zero($Y=0$)
96. Thulie Yes, so what is X?
97. Allen X is less than or equal to ten thousand.($X > 10\ 000$)
Aaa (disagreeing) When your writing. When your drawing a line, you make this one an equal line.
98. Thulie Yes, this one an equal sign
So make this one an equal sign. $x + y = 10\ 000$ and then when x is zero y is ten thousand it means it's the point zero and ten thousand(0;10000) not a line. Is it?
99. Allen Ya!
100. Thulie So it's zero, you said which one is zero?
101. Allen We said this one is zero(*pointing at intercept on graph*)
102. Thulie Y, okay. It's ten thousand zero (10000;0), write the co ordinates ten thousand zero(10000;0)
103. Allen Ten thosand zero(10000;0) (*saying it simultaneously with Mrs Nkambule*)
104. Thulie Ten thosand zero (10000;0) (*saying it simultaneously with the learners*), yes and then plotte that point. It becomes a point not a line
Ten thousand and zero.
105. Allen Yes that's the point. So we erase the line, that's that's the point. No, no problem sisaibona later (*meaning will see it later*). Then Fine, make x zero, y zero now. Which one? x zero now.
Yes x.
Yes.
106. Allen Then it's zero equals to ten thousand.
107. Thulie Again, plotte the point.
108. Allen It's zero ten thousand (0; 10000).
109. Thulie And then you join the two. That is your line.
110. Allen Oh! (glade to see the results to drawing their line)
111. Thulie Is it solid; is it solid or dotted first of all?
112. Allen It's solid.
113. Thulie It's solid, tha's okay. Now lets show the region represented by the inequality.

114. Allen Okay, pick a point. Which side are we going to shade, this one or that one? Let's choose zero and zero(0;0) . And we substitute the, zero plus zero is less or equal to Y. Zero plus zero is less or equal 10 000. So which is it? Is it false, you take the false side? Zero plus zero is equal or less than ten thousand. This statement is true, we shade the. We shade the top part.
115. Thulie Yes, that is true. You are correct but always remember that less is below the line. Always less than is below the line. Okay!
116. Allen Greater than is above the line.
- It is like what we did, you said if the inequality sign says the point is below. We shade the one that we don't want.
117. Thulie Ya, the one we don't want. Am saying that is correct, but always remember that when you have less than that region is below.
118. Allen So this one is going to be a line?
- Yes, now that one is a line. (*Laughing*). It's vertical instead of being slope.
119. Thulie So x is less than we are going to shade this side?
120. Allen The tricky ones are the ones where they are two.
- We are going to shade this one. And this one we are going to say when y is equal to zero, x is four thousand, and four thousand is to zero(4000;0) is this point. And when x is equal to zero, it's zero is to four thousand (0; 4000) point zero is to four thousand is this one. And we shade below? We shade above.
121. Thulie Okay, okay now let's maximize it.
122. Allen Let's maximize the profit.
- Let's maximize the profit.
- When the profit we take on top and write fetherst.
123. Thulie On top, what do you mean by on top?
124. Allen On our feasible region we mean the point that is on top and that is the right fetherst.
125. Thulie Okay
126. Allen That will give us the maximum. Two thousand! And then our x is going to be, our scale.
127. Thulie Let's rectify, divide it into two.
128. Allen Ah, this two thousand?
- You divide it by two here because lana (here) is two thousand and then here it will be three thousand.
- And so our x is going to be three thousand, and then we substitute in our profit equation. Fifty by three thousand plus thirty by three thousand.
- Just take out a calculator.
- Seven hundred and fifty thousand plus...
- Just multiply the first one.
- one hundred and fifty thousand ma o se hlanganesa(when you add them together), bala(write) the balance mo.
129. Thulie This one is now fluent in Xitsonga, ufike nini (when did u arrive here)
130. Allen Maam, I was born here.
131. Thulie Ya, you were born here and now you have adapted to the culture of speaking many languages. The people here speak many langauges.
132. Allen One hundred and fifty thousand, fifty thousand, sixty thousand to one hundred and fifty thousand. Oh, two hundred and ten thousand. So this is our maximum profit,
133. Thulie Is it maximum profit or maximum scale?
134. Allen It's maximum income.
135. Thulie Yes it's maximum income because you do not know the cost price. I think that is fine, thanks a lot. Seniya go phe? Ne ya go enye iclass(so you will be going to another class?)
136. Allen Yes mmaam
137. Thulie Thank you very much.

APPENDIX M: HANDOUT IN CLASSROOM A

Question 1

A small aircraft company decided to offer flights to an exclusive game reserve close to the Kruger National Park. Tourists can travel first class or economic class.

Depending on which class is travelled, the costs per flight per person are:

Return fare first class: R3 200

Return fare economic class: R2 100

- The luggage restrictions per person are:
- 30kg for a first class passenger
- 20 kg for an economic class passenger

A maximum of 600kg of luggage can be transported.

- The aircraft seats a total of 20 passengers
- The aircraft caters for a maximum of 15 first class passengers.

Let the number of first class passengers be x and the number of economic class passengers be y

1.1 Write down the inequalities that describe the constraints (6)

1.2 Write down the income equation in the form $I = \dots$ (2)

1.3 Draw the graph of the inequalities and shade the feasible region(4)

1.4 Use your income equation to determine how many passengers of each class should be transported to ensure a maximum income. Label the point chosen with a M. Show all your working detail. (3)

1.5 Calculate the maximum income (2)

(17)

APPENDIX N: JOHN'S CLINICAL INTERVIEW TASK

A local health board is producing a guide for healthy living. The guide should provide advice on health education, healthy lifestyles and the like. The board intends to produce the guide in two formats: one will be in the form of a short video; the other as a printed binder.

The board is currently trying to decide how many of each type to produce for sale. It has estimated that it is likely to sell no more than 10 000 copies, of both items together. At least 4000 copies of the video and at least 2 000 copies of the binder could be sold, although sales of the binder are not expected to exceed 4 000 copies. Let x be the number of videos sold, and y the number of printed binders sold.

5.1 Write down the constraint inequalities that can be deduced from the given information.

5.2 Use graph paper to represent these inequalities graphically and indicate the feasible region clearly.

5.3 The board is seeking to maximise the income, I , earned from the sales of the two products. Each video will sell for R50 and each binder for R30. Write down the objective function for the income.

5.4 Determine graphically, by using a search line, the number of videos and binders that ought to be sold to maximise the income.

5.5 What maximum income will be generated by the two guides?

0-10000
graph
graph

APPENDIX O: HANDOUTS IN CLASSROOM B

TEACHER'S TIMETABLE (School B)

Lesson observed G (11) for five days (8:00-15:00) 8 periods of 45 minutes each.

Day 1 (Thursday)		G(11)					G (12)	
Day 2 (Monday)				G(12)	G(12)		G(11)	G(11)
Day 3 (Tuesday)		G(12)	G(12)				G(11)	
Day 4 (Thursday)			G(11)		G(12)			
Day 5 (Monday)	G(12)						G(11)	G(11)

Friday- culture day

Wednesday – sports day

Friday – staff meeting

Steps

1. Decide what you are going to call x and y . What must be made maximum or minimum at the end must equal to x and y .
2. Remember $x \geq 0, y \geq 0$
3. Represent the constraints algebraically
4. Represent information on a graph and determine feasible region
 - Use dual intercept method
 - \leq and \geq use solid line
 - $<$ and $>$ use Dotted line
 - $x, y \in \text{real}$ Colour the area
 - $x, y \in \text{integers}$ use dots
 - name axes properly
 - Find gradient of profit function (Determine graphically or algebraically)
 - Calculate minimum or maximum profit

Algebraic representation

y is not more than or less than 10

$$\Rightarrow y \leq 10$$

y is at least 200

$$\Rightarrow y \geq 200$$

x is not less than 10 $\Rightarrow x \geq 10$

x and y together are at least 100

$$\Rightarrow x + y \geq 100$$

Ratio of x to y is at least 3 to 4

$$\Rightarrow \frac{x}{y} \geq \frac{3}{4}$$

x is at least two times y

$$\Rightarrow x \geq 2y$$

y is at most 3 times x

$$\Rightarrow y \leq 3x$$

Important terminology

1. **Implicit constraints:** Inequalities that arise naturally
2. **Constraints:** Inequalities that you make from the given information
3. **The feasible region:** The area on the graph that satisfies the above constraints
4. **The objective function (often called a search line):** The straight line graph you draw to help you solve the problem

Making inequalities (constraints)

Question 1

A party is to be arranged. Let x be the number of girls and y the number of boys that will attend this party. Make inequalities (constraints), if you are given the following information.

There must be at least 200 people at the party. However, the hall can only fit 600 people. We need at least 150 girls and at most 400 boys and the ratio of boys to girls must not be more than 2:1. The organisers must make at least R4 000 on entrance fees. Each boy pays R20 to attend and each girl pays R15.

Question 2

A company has two types of airplanes, a 707 and a 747. Let the number of 707's be x and the number of 747's be y and make inequalities if given the following information. A 707 carries 250 passengers and a 747 carries 300 passengers. The company is required to transport at least 3 000 passengers per day. The company must use at least four 747's and at least two 707's each day. The company has 16 pilots available each day.

Question 3

The dairy delivers milk and orange juice. A housewife can order a maximum of 28 bottles per week. She does not want more than 12 bottles of orange juice. She must have at least 2 bottles of milk. If the milk costs R8, 00 per bottle and the orange juice R12, 00 per bottle, what is the maximum that she spends per week?

Question 4.

1. A manufacturer of liquid detergent uses two basic chemical ingredients, Mix A and Mix B. The detergent is packaged and sold to two separate markets: Household market (H) and Commercial market (C). The products are sold in five litre bottles. For the household detergent each five litre bottle requires four litres of Mix A and one litre of Mix B, whereas the corresponding composition of the commercial detergent is two litres of Mix A and three litres of Mix B as shown in the table.

Chemical composition of Detergents per five litre bottle:

Ingredients	Household Detergent (H)	Commercial Detergent (C)
Mix A	4 litres	2 litres
Mix B	1 litre	3 litres

On a weekly basis the company has supplies of no more than 20 000 litres of Mix A and 15 000 litres of Mix B. Furthermore the company can buy no more than 4 000 containers a week for H and 4 500 containers a week for C. Let x be the number of five litre bottles for H and y be the number of five litre bottles for C.

- 1.1 Write down the inequalities representing the availability of Mix A and Mix B and the availability of the containers for H and C
- 1.2 Represent the inequalities on graph paper using the scale 2cm = 2 000 litres on both axes.
- 1.3 Shade the feasible region on the graph
- 1.4 If H contributes R2.40 to the profit per five litre bottle produce and C contributes R2.00 per five litre bottle, write down the equation for the profit (P), and determine the gradient of the profit line.
- 1.5 Draw the profit line as a dotted line on the graph.
- 1.6 Determine how many five litre bottles of each type of detergent must be produced on a weekly basis to maximize profit. (You may read off the answer from the graph)
- 1.7 Determine the maximum profit per week.

Question 5**(Sketch)**

2. In a certain week a radio manufacturer makes two types of portable radios, M (mains) and B (battery). Let x be the number of type M and y be the number of type B. In the sketch the shaded area represents the feasible region.

2.1 Write down the constraints to the linear programming problem given:

2.1.1 At most 60 of type M and 100 of type B can be manufactured in a week.

2.1.2 At least 80 radios in total must be produced in a week to cover costs.

2.1.3 It takes $\frac{2}{3}$ hour to assemble a type M and $\frac{1}{2}$ hour to assemble a type B. The factory works a maximum of 60 hours per week.

2.2 If the profit on a type M is R40 and on type B is R80, write down the equation in terms of x and y which will represent the profit (P).

2.3 Draw the search line that represents the profit function.

2.4 Use the graph to determine the pair $(x; y)$ in the feasible region where the profit is a maximum.

2.5 What is the maximum weekly profit?

2.6 The manager is informed that the workers union plans a strike for the following week, which will result in only 50 hours being worked. How many radios of each type should now be manufactured for a maximum profit and what will the maximum profit now be for the week?

Question 6

3. A patient in a hospital needs at least 18 grams of protein, 6 milligrams of vitamin C and 5 milligrams of iron per meal, which consists of two types of food, A and B. Type A contains 9 grams of protein, 2 milligrams of vitamin C and no iron per mass unit. Type B contains 3 grams of protein, 2 milligrams of vitamin C and 5 milligrams of iron per mass unit. The energy value of A is 800 kilojoules and that of B is 400 kilojoules per mass unit. A patient is not allowed to have more than 4 mass units of A and 5 mass units of B. There are X mass units of A and Y mass units of B on the patients plate.

3.1 Write down in terms of x and y :

3.1.1 The mathematical constraints which must be satisfied

3.1.2 The kilojoule intake per meal

3.2 Represent the constraints graphically on graph paper. Use the scale 1 unit = 20 mm on both axes. Shade the feasible region

3.3 Deduce from the graphs, the values of x and y which will give the minimum kilojoule intake per meal for the patient.

Question 7

4. A certain motorcycle manufacturer produces two basic models, the ‘Super X’ and the ‘Super Y’. ‘These motorcycles are sold to dealers at a profit of R20 000 per ‘Super X’ and R10 000 per ‘Super Y’. A ‘Super X’ requires 150 hours for assembly, 50 hours for painting and finishing and 10 hours for checking and testing. The ‘Super Y’ requires 60 hours for assembly, 40 hours for painting and finishing and 20 hours for checking and testing. The total numbers of hours available per month is 30 000 in the assembly department, 13 000 in the painting and finishing department and 5 000 in the checking and testing department. This can be summarised by the following table:

Department	Hours for Super X	Hours for Super Y	Max hours available per month
Assembly	150	60	30 000
Painting and finishing	50	40	13 000
Checking and testing	10	20	5 000

Let x be the number of ‘Super X’ and y be the number of ‘Super Y’ models manufactured per month.

- 4.1 Write down the set of constraints inequalities
- 4.2 Use graph paper to represent the constraints inequalities
- 4.3 Shade the feasible region on the graph paper.
- 4.4 Write down the profit generated in terms of x and y .
- 4.5 How many motorcycles in order to maximise the monthly profit?
- 4.6 What is the maximum monthly profit?

Question 8

A local health board is producing a guide for healthy living. The guide should provide advice on health education, healthy lifestyles and the like. The board intends to produce the guide in two formats: one will be in the form of a short video; the other as a printed binder.

The board is currently trying to decide how many of each type to produce for sale. It has estimated that it is likely to sell no more than 10 000 copies, of both items together. At least 4 000 copies of the video and at least 2 000 copies of the binder could be sold, although sales of the binder are not expected to exceed 4 000 copies. Let x be the number of videos sold, and y the number of printed binders sold.

- 5.1 Write down the constraint inequalities that can be deduced from the given information.
- 5.2 Use graph paper to represent these inequalities graphically and indicate the feasible region clearly.
- 5.3 The board is seeking to maximise the income, I , earned from the sales of the two products. Each video will sell for R50 and each binder for R30. Write down the objective function for the income.

5.4 Determine graphically, by using a search line, the number of videos and binders that ought to be sold to maximise the income.

5.5 What maximum income will be generated by the two guides?

Question 9

12. BizBuz motor assembly factory employs you as a production planner at the factory. Your job will be to advise the management on how many of each model should be produced per week in order to maximise the profit on the local production. The factory is producing two types of minibuses: Quadrant and Shosholoza.

Two of the production processes that the minibuses must go through are bodywork and engine work.

- The factory cannot operate for less than three sixty hours on engine work for the minibuses.
- The factory has a maximum capacity of 480 hours for bodywork for the minibuses.
- half an hour of engine work and half an hour of bodywork is required to produce one quadrant.
- one over three hour of engine work and one over five hour of bodywork is required to produce one shosholoza.
- The ratio of shosholoza minibuses to quadrant minibuses produced per week must be at least three is to two.
- A minimum of 200 quadrant minibuses must be produced per week.

Let the number of quadrant mini buses manufactured in a week be x .

Let the number of shosholoza minibuses manufactured in a week be y .

Two of the constraints are:

$$x \geq 200$$

$$3x + 2y \geq 2160$$

12.1 Write down the remaining constraints in terms of x and y to represent the above mentioned information (4)

12.2 Use the attached graph paper (diagram sheet) to represent the constraints graphically. (3)

12.3 Clearly indicate the feasible region by shading it. (1)

12.4 If the profit on one Quadrant minibus is R12 000 and the profit on one Shosholoza minibus is R4 000, write down an equation that will represent the profit on the minibuses. (2)

Question 10

A clothing company manufactures white shirts and grey trousers for schools.

- A minimum of 200 shirts must be manufactured daily.
- In total, not more than 600 pieces of clothing can be manufactured daily.
- It takes 50 machine minutes to manufacture a shirt and 100 machine minutes to manufacture a pair of trousers.
- There are at most 45 000 machine minutes available per day.

APPENDIX P: HANDOUT IN CLASSROOM C

NOTES
LINEAR PROGRAMMING 02

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After completing this module, you should be able to identify and write down constraints.

CONSTRAINTS...

- Limiting conditions or restrictions that are translated into inequalities. Because linear programming involves real quantities $x \geq 0$ and $y \geq 0$ are always constraints (1st quadrant)

$<$	<ul style="list-style-type: none"> Less than Smaller than May not reach Less than or equal to At most Maximum May not exceed Not more than 	<ul style="list-style-type: none"> More than Greater than Must exceed Greater than or equal to At least Minimum May not be less than Not less than
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Key words: write on the board

LISTING THE CONSTRAINTS...

- Read the question very carefully.
- Before you list a constraint, re-read the relevant parts again.
- Look carefully at the key words to decide which inequality sign to use.
- Types of constraints:
 - One variable: e.g. $x \geq 2$ number, or $x \leq 5$ number
e.g. $y \geq 2$ number, or $y \leq 5$ number
 - Two variables (with a given total):
e.g. $ax + by \geq \text{total}$, or $ax + by \leq \text{total}$
 - Two variables (compared to one another):
e.g. $y \geq ax$, or $y \leq ax$

Example:

A television manufacturing company makes two types of televisions, ordinary-screen TVs and flat-screen TVs. No more than 5 ordinary-screen TVs and but no less than 3 flat-screen TVs must be produced daily. There are 11 workmen at the manufacturing workshop and it takes 1 man to assemble the ordinary-screen TV and 2 men to assemble the flat-screen TV per day. The number of ordinary-screen TVs produced daily must be at least half the number of flat-screen TVs produced daily.

Let the number of ordinary-screen TVs produced daily = x
Let the number of flat-screen TVs produced daily = y

1. One variable constraint:
"No more than 5 ordinary-screen TVs" $\therefore x \leq 5$
"No less than 3 flat-screen TVs" $\therefore y \geq 3$

2. Two variables constraint (with a given total):
"There are 11 workmen at the manufacturing workshop and it takes 1 man to assemble the ordinary-screen TV and 2 men to assemble the flat-screen TV per day" Note: There are only 11 workmen available this means the total number of men assembling TVs can be at most 11:
 $\therefore x + 2y \leq 11$

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3. Two variables constraint (compared to one another):
"The number of ordinary-screen TVs produced daily must be at least half the number of flat-screen TVs produced daily."
"Number of ordinary-screen...at least...number of flat-screen." $x \geq \frac{1}{2}y$
(rewrite with y as subject)

REINFORCEMENT EXERCISE:

For questions 1, 2 and 3 assume that the problem involves a recording company that records classical CDs and DVDs.

1. List these "one variable" constraints:
 (a) The company must record a minimum of 20 CDs per month.
 (b) The company may not record more than 50 DVDs per month.
 (c) The number of CDs recorded per month must exceed 35.

Let the number of classical CDs recorded each month be x
Let the number of classical DVDs recorded each month be y

2. List these "two variables and a given total" constraints:
 (a) It takes 12 man hours for the sound engineers to record a CD and 23 man hours for the sound engineers to record a DVD and the total number of man hours available per month is 25 000.
 (b) The total number of recordings made per month must be at least 250.

3. List these "two variables compared to one another" constraints:
 (Write each constraint with y at the beginning of the constraint)
 (a) The company must record at least half as many DVDs as CDs.
 (b) The number of CDs recorded each month must be no less than the number of DVDs.
 (c) The company must record at most 4 times as many CDs as DVDs.

4. Read this and then write out all the constraints:
 The school canteen makes a cool drink by mixing water and concentrate. Her cool drink budget allows her to use at most 30 litres of concentrate per day. She has twenty litre-bottles in which to serve the cool-drink in (she washes out the containers on a daily basis). In order for the drink to taste good she must use at least 5 times as much water as concentrate.

5. Read this and then write out all the constraints:
 A surf clothing manufacturer makes 2 styles of wetsuits: short wetsuits and long wetsuits. Since the short wetsuits sell more quickly, the manufacturer must not make more than 12 long wetsuits per week. The time needed for cutting and sewing each style of wetsuit is shown in the table:

	Short wetsuits	Long wetsuits	Hours available
Cutting of fabric	1 hr	2 hrs	32 hrs
Sewing of fabric	3 hrs	4 hrs	84 hrs

6. Which of the previous 2 questions had:
 (a) constraint(s) with one variable only, constraint(s) with 2 variables and a given total? and constraint(s) with 2 variables compared to one another?
 (b) no constraints with 2 variables compared to one another?

IM (8/7)

APPENDIX Q: LEARNER QUESTIONNAIRE

1. What is your gender? (X) Male----- Female -----
2. What is your Age? -----
3. Are you a South African citizen? (X) Yes No
4. If no when did you arrive in South Africa, (year)? What is your citizenship?
5. Tick your home language

Languages	
English	
Afrikaans	
IsiZulu	
isiXhosa	
Setswana	
Sepedi	
Sesotho	
Xitsonga	
Tshivenda	
IsiNdebele	
isiSwati	
Other	

6. Tick the languages you speak at home

Languages	
English	
Afrikaans	
IsiZulu	
isiXhosa	
Setswana	
Sepedi	
Sesotho	
Xitsonga	
Tshivenda	
IsiNdebele	
isiSwati	
Other	

Other languages you speak at home:

7. Tick the languages you can understand, speak, write and read

Languages you understand

Languages	
English	
Afrikaans	
IsiZulu	
isiXhosa	
Setswana	
Sepedi	
Sesotho	
Xitsonga	
Tshivenda	
IsiNdebele	
isiSwati	
Other	

Languages you speak

Languages	
English	
Afrikaans	
IsiZulu	
isiXhosa	
Setswana	
Sepedi	
Sesotho	
Xitsonga	
Tshivenda	
IsiNdebele	
isiSwati	
Other	

Languages you read and write

Languages	
English	
Afrikaans	
IsiZulu	
isiXhosa	
Setswana	
Sepedi	
Sesotho	
Xitsonga	
Tshivenda	
IsiNdebele	
isiSwati	
Other	