<table>
<thead>
<tr>
<th>Regular Figure</th>
<th>No of equal sides</th>
<th>Angle at centre</th>
<th>Angle at ( \frac{360}{n} )</th>
<th>1 Interior Angle</th>
<th>2n-4 Exterior Angle</th>
<th>Sum of Exterior Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>120°</td>
<td>120°</td>
<td>60°</td>
<td>2</td>
<td>120°</td>
</tr>
<tr>
<td>Square</td>
<td>4</td>
<td>90°</td>
<td>90°</td>
<td>90°</td>
<td>4</td>
<td>90°</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>72°</td>
<td>108°</td>
<td>6</td>
<td>72°</td>
<td>360°</td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>60°</td>
<td>120°</td>
<td>60°</td>
<td>8</td>
<td>60°</td>
</tr>
<tr>
<td>Octagon</td>
<td>8</td>
<td>45°</td>
<td>135°</td>
<td>45°</td>
<td>12</td>
<td>45°</td>
</tr>
<tr>
<td>Decagon</td>
<td>10</td>
<td>36°</td>
<td>144°</td>
<td>36°</td>
<td>16</td>
<td>36°</td>
</tr>
</tbody>
</table>

Corollary 4: If the sides of a polygon are produced in order, the sum of the exterior \( L^\circ \)'s formed is equal to \( 4n + L^\circ \).

Consider the 7-sided figure \( ABCDEFG \) with its sides produced in order at every vertex, the sum of the exterior \( L^\circ \) and the interior angle is two \( 180^\circ \):

- The sum of all the \( L^\circ \)'s is \( 14 \times 180^\circ \), but by the previous corollary, the sum of the interior angles of a 7-sided figure is \( 2n-4 \times 180^\circ \) where \( n = 7 \), so in this figure it is \( 10 \times 180^\circ \):

- The sum of the exterior \( L^\circ \)'s is \( 14 \times 180^\circ - 10 \times 180^\circ = 4 \times 180^\circ \)

Alternate proof: \( 7 \times 2 \times 180^\circ = 14 \times 180^\circ - L^\circ \)

\( L^\circ \) at centre = \( 4 \times 180^\circ \) : interior \( L^\circ \) of figure = \( 10 \times 180^\circ \).
Proposition 16: Theorem

If two to have two sides of the one equal to two sides of the other, each to each, and one side equal to one side, namely, either the sides adjacent to the equal sides in each, or sides opposite to equal sides; then the as are equal in all respects.

Given \( \triangle ABC, \triangle DEF \) in which

\[ BC = EF, \quad \angle ABC = \angle DFE \quad \text{and} \quad \angle ACB = \angle DEF. \]

To prove \( \Delta s \) are equal, in all respects.

Proof: Since the \( \angle s \) of a \( \Delta \) are together equal to \( 2 \pi \) in either case the \( 3^{\text{rd}} \) angles are equal. Apply \( \triangle ABC \) to \( \triangle DEF \) so that \( B \) falls on \( E \) and \( BC \) is on \( EF \), then because \( BC = EF \) \( C \) will fall on \( F \) also because \( \angle ABC = \angle DFE \) \( A \) will fall along \( DE \) and because \( \angle ACB = \angle DFE \)

\[ \therefore BC \quad \text{will be along} \quad DF. \]

The \( A \) the pt of intersection of \( AB + AC \) will coincide with \( D \) the pt of intersection of \( DE + DF \)

\[ \therefore \Delta s \quad \text{coincide} + \quad \text{are equal in all respects.} \]

Q.E.D.
Proposition 17. Theorem

If two pr. $\triangle ABC$ and $\triangle DEF$ have their hypotenuses equal and one side of the one equal to one side of the other, the $\triangle s$ are equal in all respects.

Given $\triangle ABC$, $\triangle DEF$, $\triangle DEF$ at $\triangle ABC$ having their hypotenuses $AB$, $DE$ equal and $AC$ equal to $DF$.

To prove that the $\triangle s$ are equal in all respects.

Proof: apply $\triangle ABC$ to $\triangle DEF$, so that $A$ falls on $D$ and $AC$ lies along $DF$. Then because $AC = DF$, $C$ will fall on $F$. Let $B$ fall on $G$ on the side of $DF$ opposite $E$.

Now $\triangle DEF$ and $\triangle DFG$ are $\cong$.

$\therefore \triangle EFG$ is a $pr.$ line and $\triangle EFG$ is a $\triangle$ but $DE = DG$.

$\therefore \angle DEF = \angle DGF$.

Then since $\triangle DEF$, $\triangle GDF$, we have $\angle DEF = \angle DFG$ (pr. $\cong$).

$\therefore \angle DEF = \angle DGF$ (pr. $\cong$) + $DF$ common.

$\therefore \triangle DEF$ and $\triangle DGF$ are equal in all respects.

$\therefore \triangle ABC$, $\triangle DEF$ are equal in all respects.

Q.E.D.

The angles at the base of an isosceles triangle are equal.

Given a \( \triangle ABC \) having \( AB \) equal to \( AC \).

To prove \( \angle ABC = \angle BAC \).

Let \( AF \) cut \( BC \) in \( E \).

**Proof.**

**Cons.** Draw \( AF \) bisecting \( \angle BAC \).

Let \( AF \) cut \( BC \) in \( E \).

**Proof.**

Fold \( \triangle ABC \) about \( AF \) then because \( \angle BAE = \angle CAE \), \( AB \) will lie along \( AC \), and because \( AB = AC \), \( B \) will lie along \( AC \) also.

\( \therefore \angle ABC \) coincides with \( \angle ACB \) and is equal to it. \( \therefore \angle ABC = \angle ACB. \)

Suppose \( AB \) be produced to \( G \) and \( AC \) to \( H \), then \( \angle CBG = \angle BCH \) for these are supplements of equal angles \( \angle ABC, ACB \).
Proposition 10. Theorem.

If two sides of a triangle are equal, the sides opposite to them are equal.

Given a $ABC$ having $LABC$ equal to $LACB$.

To prove $AB = AC$.

Proof: Suppose $AB$ is not equal to $AC$, one must be greater. If possible, let $AD = AC$, where $D$ is a point in $AB$ or $AB$ produced.

Join $AD$. Then $AD = AC$ because $AC = AD$.

$	herefore LACD = LADC$, and $ABC$ is greater than $LABC$.

The interior opposite $L$.

$LADC$ is greater than $LACB$.

$LACD$ is greater than $LACB$, the part greater than the whole, which is impossible.

$AB$ cannot be unequal to $AC$.

$AB = AC$ Q.E.D.

---

Proposition 11. Theorem.

If one side of a $ABC$ is greater than another, the opposite the greater side is greater than the opposite the less.

Given a $ABC$ having $AC$ greater than $AB$.

To prove $LABC$ is greater than $LACB$.

Consider a circle with center $A$ and radius $AB$. Describe an arc cutting $AC$ in $D$. Join $BD$.

Proof. In $ABD$, because $AB = AD$.

$LABD = LADB$.

But the exterior $LADB$ is greater than interior opposite $LACB$.

Again the whole $LABC$ is greater than its part $LABD$.

Still more, than, is $LABC$ greater than $LACB$.

Q.E.D.
The text appears to be a geometric proof, possibly involving triangles and angles. The handwriting is quite clear, and the layout suggests a step-by-step explanation of a mathematical problem or theorem.

The text seems to be discussing the relationship between angles and sides in a triangle, possibly proving a theorem such as the Angle-Angle-Side (AAS) congruence criterion.

The diagram includes a triangle with vertices labeled A, B, and C, and an angle marked at vertex A.

The proof likely involves the use of theorems such as the Side-Angle-Side (SAS) or the Angle-Side-Angle (ASA) congruence criteria, depending on the context of the problem.

The text is written in a clear, legible manner, with each step of the proof being carefully explained. The use of symbols and notation is consistent with standard geometric practices.
If any two \(\triangle\) have 2 sides of one equal to 2 sides of the other respectively, and the other included \(\triangle\) equal, the \(\triangle\) are equal in all respects.

Prop. 15. Theorem.
If two \(\triangle\) have the 3 sides of one equal to the 3 sides of the other respectively & the included to equal, the \(\triangle\) are equal in all respects.

Given \(\triangle ABC\), \(\triangle EDF\), in which
\(AB = DE\), \(BC = EF\), \(AC = DF\)

To prove \(\triangle ABC\) are equal in all respects.

Proof apply \(\triangle ABC\) to \(\triangle EDF\), so that \(B\) falls on \(E\) and \(BC\) on \(EF\), then, because
\(BC = EF\), \(C\) will fall on \(F\). Let \(A\) fall at \(G\) on the side of \(EF\) opposite \(D\) then \(EGF\) is the new position of \(\triangle ABC\). Join \(DG\). In \(\triangle EDG\) because \(ED = EG\)
\(\angle LEDG = \angle EQD\)
also in \(\triangle BDG\), because \(FD = FG\)
\(\angle LFDG = \angle LGD\)
the whole \(\angle LEDF = \angle LEGF\). Then in \(\triangle EDF, EGF\) we have \(ED = EQ\), \(FD = FG\), and included \(\angle LEDF = \angle LEGF\)

\(\therefore\) \(\triangle ABC, DEF\) are equal in all respects.

But \(\triangle EGF\) is to \(\triangle ABC\) in a new position.
\(\therefore\) \(\triangle ABC, DEF\) are equal in all respects. Q.E.D.
Proposition 19. Theorem.

The locus of a pt equidistant from two fixed pts is a pt line bisecting at it meets the line joining the two fixed points.

Given: two pts A and B.

To prove: that the locus of a pt equidistant from A and B is the perpendicular bisector of AB.

Construction: Draw CD bisecting AB at rt L at the pt M.

Take any pt P in CD.

join PA, PB.

Proof: In \( \triangle PMA, \triangle PMB \) we have

\[
AM = BM \text{(Cons.)} \quad PM \text{ common, } \angle \text{ included in } \triangle PMA, \triangle PMB \text{ equal}
\]

Seeing \( \rt L \) (Cons).

\[
: \text{As are equal in all respects } \therefore \]

\[
PA = PB
\]

\( \therefore P \) is equidistant from A and B.

In the same way it may be shown that any other pt in CD is equidistant from A and B also that any pt outside CD is not equidistant from A and B.

\( \therefore \) CD which bisects AB and rt L is the locus of a pt equidistant from A and B.

Q.E.D.
Definition: If a point may move but is subject to certain restrictions, the path which it traces out is called locus.

Example: A position point moves but keeps at a constant distance $R$ from a fixed point $P$. The path it traces out is a circle, in other words the locus of the moving point is a circle whose center is the fixed pt $P$ and whose radius is the constant distance $R$. When we speak of a locus being a Circle, it is the Circumference of the circle that is meant.

Example 2: The locus of a point which moves keeping at a constant distance from a pt line is a pt line || to the given pt line.

Exercise: Draw a 4.5 cm, line. Draw the locus of a pt which moves, keeping 1 cm from the side.
Note: Fig. 1 and Fig. 2 refer to Fig. 1 in the text.

Example: Given a point on a given circle, find the tangent at that point.

Diagram: A circle with a point P on its circumference. A tangent line TP is drawn from P to touch the circle at P. The center of the circle is marked as O. The radius OP is perpendicular to TP at P.

Steps:
1. Draw a circle with center O.
2. Choose a point P on the circumference of the circle.
3. Draw a line segment OP.
4. Draw a line PT from P that is perpendicular to OP.
5. PT is the tangent to the circle at P.

Theorem: The tangent at a point on a circle is perpendicular to the radius at the point of tangency.

Proof: Let TP be the tangent at P to the circle with center O. Draw OP. Since TP touches the circle at P, OP is perpendicular to TP by the definition of a tangent. Therefore, OP is perpendicular to TP, as required.
Proposition 21

Problem

To describe a \( \Delta \) having its sides equal to three given straight lines, any two of which are together greater than the third.

Let \( A, B, C \) be the 3 given straight lines, any two being together greater than the third.

Take any straight line \( DE \) and with centre \( D \) and radius equal to \( B \) describe a \( \odot \) cutting \( DE \) at \( E \) with Centre \( D \) and radius equal to \( B \) describe a \( \odot \) \( KF \) with centre \( E \) and radius equal to \( C \) describe a \( \odot \) \( HF \) cutting \( KF \) at \( F \), join \( DF, EF \) then \( DEF \) is the req \( \Delta \)

\( DE = A, \ DF = B \), and \( EF = C \). Q.E.F
Proposition 22. Problem
To bisect a given angle

Given an angle \( \angle ABC \).
It is required to bisect it.

Construct with centre \( B \) and any convenient radius describe an arc cutting \( AB \) in \( D \) and \( BC \) in \( E \) with centres \( D \) and \( E \) and the same radius describe arcs meeting in \( F \).

Join \( BF \).
Then \( BF \) bisects the angle \( \angle ABC \). Join \( DF, EF \).

Proof: In \( \triangle BDF, BFE \) we have
\[ BD = BE \text{ (radii), } BF \text{ common,} \]
and \( DF = EF \text{ (radii of equal } \odot) \)
\[ \therefore \triangle BDF = \triangle EBF \]
\[ \therefore \angle DBF = \angle EBF \]

Q.E.F.
Proposition 23 Problem.
To bisect a given finite straight line.

Given a finite straight line AB.
It is required to bisect it.

Construction: With centre A and any convenient radius draw arc
With centre B and any radius draw arcs cutting
the first arc at C and D.
Join CD and let CD cut AB in M.
Then AB is bisected at M as required.

Join AC, AD, BC, BD.

Proof. In the As ACO, BCD we have
AC = BC, AD = BD and CD common
i.e. sides equal in all respects
i.e. ∠ACO = ∠BCD.

Then in the As ACM, BCM we have
AC = BC (radii of equal circles) CM common
and the included sides ACM, BCM equal
i.e. sides equal in all respects
i.e. AM = MB
i.e. AB is bisected at M.

Q.E.F.
Exercise 1. \( AB = 3.7'' \)

Exercise 2. \( AB = 6.9 \text{ cm} \)
Proposition 24. Problem

To draw a perpendicular to a given straight line from a given point in it.

Given a straight line AB with a Pt P in it.

It is required to draw through P an rt. line perf to AB.

Com with cent in P and any convenient radius describe a C cutting AB in C and D.

With Centres C and D and any convenient radius describe arcs intersecting in Q.

Join PQ.

Then PQ is the reqd line.

Join QC, QD.

Proof: As CPQ, D PQ we have

PC = PD  (radius of same C)
QC = QD  (radius of equal C's) and PQ common

As are equal in all respects

L GPC = L GPD, and these are adjacent angles

PQ is perf to AB. G.E.F.

Proposition 25. Problem

To draw a perpendicular to a given rt. line from a given point outside it.
Draw a right angle at the given sides.

Draw a right angle at the given sides.

Draw a right angle at the given sides.

Draw a right angle at the given sides.

Draw a right angle at the given sides.
The length of CD = 4.8
Given a straight line AB and a point P outside it.

It is required to draw from P a straight line perpendicular to AB.

Construct with centre P and any radius draw a circle cutting AB at C and D.

With centres C and D and the same radii describe arcs intersecting at Q.

Join PQ.

Then PQ is the line required.

Join PC, PD, QC, QD, and let PQ cut AB in M.

Proof: In the AS CPQ, DPQ we have

CP = DP (radii of the same circle)
CQ = DQ (radii of equal circles) and PQ common

i.e. are equal in all respects

i.e. \( \angle CPQ = \angle DPQ \)

Then in AS CPM, DPM we have

CP = DP, PM common

\( \angle \) included \( \angle SCPM, DPM \) equal (proved)

i.e. AS are equal in all respects

i.e. \( \angle PMC = \angle PMD \)

And these are adjacent \( \angle s \)

i.e. PM is perpendicular to AB.

Q.E.F.
Proposition 26. Problem.

At a given pt in a pt line to make an angle equal to a given angle.

Given an angle $ABC$, a pt line $DE$ and a pt $F$. It is required to draw at $F$ an angle equal to the $ABC$.

Cons. With centre $B$ and any convenient radius describe an arc cutting $AB$ in $G$ and $BC$ in $H$ with centre $F$ and the same radius draw an arc cutting $DE$ in $K$. Join $GH$.


Then $LLFK$ is the angle required.

Proof. In $\triangle GBH$, $L F K$ we have

$BG = FL$ (cons)
$BH = FK$, and $GH = LGK$.

Therefore equal in all respects.

$\therefore L LF K = L ABC$. 
$\therefore L LIFE = L ABC \quad Q.E.F.$
Exercises

1. On a given line AB 3.5" long describe an equilateral triangle.

2. On AB 1.7" long as base describe an isosceles triangle having each of its sides double AB.

3. On a given base 2.6" long describe an isosceles A having each of its sides 1.9" long.
Exercise: Draw a triangle $ABC$ and draw the perpendicular bisectors of the three sides. What do you notice about these three new lines?

We notice that the perpendiculars meet at a certain point $P$ in the $\triangle ABC$.

4. Describe a $\odot$ of given radius which shall pass through two given points. When is this problem incapable of solution? (Write out this in full.)

5. Find a point equidistant from two given pts $A$ and $B$.

6. With ruler and compasses make a right angle. State your construction and give a proof.
This problem is incapable of solution when the radius given is shorter than half the distance between the given points.

Given a radius \( r \) with points \( A \) and \( B \), it is required to draw a circle which shall pass through \( A \) and \( B \).

Construction:

1. With centres at \( A \) and \( B \), draw two arcs of the given radius.
2. Describe arcs intersecting in \( P \).
3. With \( PB \) and the same radius, describe a circle cutting through \( A \) and \( B \).

This is the circle required.

Q.E.F.
- the Pt required.
Given a line AB.

It is required to draw through B a perpendicular to make the angle at C equal to L.

Construction:

1. From B produce a line towards E.
2. With centre B and any radius describe an arc cutting AE in D and O.
3. With centres O and D and any radius describe arcs.
with compasses and ruler make a rt L.

Given a pt line AB
It is required to draw a pt line from B at its
L° to AB.

Cons: with centre B and any convenient radius
draw a circle cutting AB in C.
with centre C and the same radius describe an arc
cutting the circle in D.
with centre D and the same radius describe another
arc cutting the circle in E.
with centres D and E and any convenient radius
describe arcs intersecting in F
join BF. Then ABF is the L required.

Proof join DE, DF, EF, EB, BD.

Proof
Proposition 27. Theorem

Through a given pt. to draw a pt. line parallel to a given pt. line.

Given a pt line AB and a pt C outside it.
It is required to draw through C a pt. line || to AB.

Con. Place a pt. square DEF with one edge DE in the line AB.
Lay a ruler PR against the edge DF.
Slide the pt. square along the ruler until its edge DE passes through the pt. C.
When D'E'F' is the new position of DEF, the pt. square, draw the line D'C' along its edge.
Then D'C' is the line required.

Proof: Because the pt. line PR on the ruler meets the pt. line DE and D'E' making the \( \angle E'DF \) equal to the \( \angle EDF \) (each being the same \( \angle \) of the pt. square),
\[ \text{D'E'} \parallel \text{AB}. \quad \text{Q.E.F.} \]
**Proposition 237. Problem**

To draw a line parallel to a given st line

[2nd method]

Given a st line AB and a pt P outside it.

It is required to draw through P a st line \( \parallel \) to AB

Cons. Take any pt Q in AB. Join PQ.

At P make \( \angle QPR \) equal to \( \angle PBQ \)

Produce RP to S

Then RS is \( \parallel \) to AB

Proof. Because PQ meets the 2st lines AB and RS

making the alternate \( \angle BQP, QPR \) equal (Cons)

\( \therefore \) RS is \( \parallel \) to AB.

Q.E.F

**Exercise**

Draw a realistic 4.

Through its vertices draw st lines \( \parallel \) to the opposite sides

so as to form a new 4.
On Loci (Exercises).

1. The locus of the centre of the rolling circle $O$ is a circle whose centre is the centre of the fixed circle, and whose radius is the sum of the two radii.
The locus of the
centre of a rolling \( \odot \)
is the \( \odot \) of the \( \odot \) whose
Centre is the centre of the fixed \( \odot \) and whose radius
from is the centre of the given \( \odot \) is the centre of the rolling
\( \odot \).

The locus of the middle pt of a pt line
is a pt line drawn \( \parallel \) to the given pt line
of unlimited length.