Given a plane, prove that it is a plane.

Draw a line in the plane. If the line is not parallel to the plane, extend it to meet the plane at a point. Then draw a line parallel to this line in the plane. This line is parallel to the given line and the plane is a plane.
Draw a base AB 1" long with centre A and radii 1"

draw a base AB 1" long with centre A and radii 1".

with arc of radius 8" describe arc anz, cutting

cut with arc of radius 8" describe arc anz, cutting

join BC, D, AD

join BC, D, AD

produce it to C making AC equal to AB

produce it to C, making AC equal to AB

then ABCD is the parallelogram required.

other ABCD is the parallelogram required.

proof: In quadrilateral ABCD, AD is parallel to BC common

<ABC = <ADB

<ADC = <BDC

<ABC = <ADB

AD is parallel to BC

AD is parallel to BC
To measure an angle.

Place the middle point of the straight line of the protractor at the point where the two lines meet and make the angle. Let the straight line of the protractor lie along one of the lines, then the figure at which the other line crosses the plate gives a measure of the angle.

If we make a triangle on paper of any shape or size, and cut it out and then tear off the three angles and add them together we get a straight line. This proves that the three angles of the triangle together make two right angles.
Corollaries to Prop. II

Cor. I  All the angles of a rectangle are rt. angles.

Given a rectangle ABCD having \( \angle ABC \) a rt. \( \angle \).

To prove the other \( \angle \)s are rt. \( \angle \).

Proof. Because \( AB \) and \( CD \) are \( || \) and \( BC \) meets them.

\[ \angle ABC + \angle BCD = 2 \times \text{rt.} \angle \text{s}; \text{ but } \angle ABC \text{ is a rt. } \angle \text{ given} \]

\[ \therefore \angle BCD \text{ is a rt. } \angle \]

again, because \( AD \) and \( BC \) are \( || \) and \( AB \) meets them.

\[ \angle BAD + \angle ABC = 2 \times \text{rt.} \angle \text{s} \]

\[ \therefore \angle BAD \text{ is a rt. } \angle \]

and \( \angle ADC = \angle ABC \) (opps. \( \angle \)s of \( \text{par} \))

\[ \therefore \angle ADC \text{ is a rt. } \angle \text{ Q.E.D.} \]

Cor. I  All the sides of a square are equal.

Given a square \( ABCD \) having \( AB = BC \).

To prove all its sides are equal.

Proof. \( AB = CD \) (opposite \( \angle \)s of \( \text{par} \) and \( AD = BC \) (opps. \( \angle \)s of \( \text{par} \))

but \( AB = BC \) (given).

\[ \therefore AB = BC = CD = DA \text{ Q.E.D.} \]
1. Draw a \( \triangle \) whose sides are 2", 3" and 4" long. Measure its angles and find their sum.

\[
\begin{align*}
\angle ABC &= 30^\circ \\
\angle ACB &= 47^\circ \\
\angle BAC &= 104^\circ \\
\text{Sum} &= 181^\circ
\end{align*}
\]
Proposition 2. Theorem.

Opposite sides and angles of a parallelogram are equal, and the diagonals bisect the parallelogram and each other.

\[ \text{Given a parallelogram } ABCD \]

\[ \text{To prove that } AB = CD, AD = BC, \angle ABC = \angle ADC, \angle BAD = \angle BCD; \]

\[ \text{that } AC, BD \text{ bisect the parallelogram and each other.} \]

\[ \text{Cons. Join } AC \]

\[ \text{Proof in } \Delta ABC, ACD \text{ we have} \]

\[ \angle BAC = \text{all } \angle ACD \text{ (since } AB \text{ is } || \text{ to } CD \]

\[ \angle BCA = \text{all } \angle DAC \text{ (since } BC \text{ is } || \text{ to } AD \]

\[ \text{and } AC \text{ common,} \]

\[ \therefore AB = CD, BC = AD, \angle ABC = \angle ADC \text{ and} \]

\[ \angle BAD = \angle BCD \text{ (since their parts are equal).} \]

\[ \text{Also because } \Delta ABD = \Delta ADC, \]

\[ \therefore AC \text{ bisects the parallelogram.} \]

\[ \text{Similarly } BD \text{ bisects the parallelogram.} \]

\[ \text{Now join } BD \text{ and let it cut } AC \text{ in } O \]

\[ \text{Then in } \Delta AOB, COD \text{ we have } AB = CD \text{ (proved)} \]

\[ \angle BAO = \angle OCD \text{ (all } \angle s) \]

\[ \text{and } \angle AOB = \angle COD \text{ vertically opposite} \]

\[ \therefore A \text{ are equal in all respects.} \]

\[ \therefore AO = OC + BO = OD. \quad \text{Q.E.D.} \]
He 2.

\[ \angle ABC = 53^\circ \]
\[ \angle BCA = 37^\circ \]
\[ \angle CAB = 91^\circ \]
\[ \text{Sum} = 181^\circ \]

He 3.

\[ \angle ABO = 53^\circ \]
\[ \angle AOB = 36^\circ \]
\[ \angle OAB = 91^\circ \]
\[ \text{Sum} = 180^\circ \]
Proposition 1  
Theorem

If two sides of a quadrilateral are equal and parallel the figure is a parallelogram.

Given a quadrilateral ABCD having AB and CD equal and parallel.

To prove AD is \( \parallel \) to BC.

Cons: join AC

Proof: In \( \triangle ABC, ADC \) we have

\( AB = CD \) (given) AC common

and included \( \angle \)s \( BAC, ACD \) equal (all \( \angle \)s)

\( \therefore \) \( \angle ACB = \angle DAC \)

and these are all \( \angle \)s

\( \therefore \) AD is \( \parallel \) to BC. Q.E.D.
Problem 4.

\[ L_{PQR} = 38^\circ \]
\[ L_{QRP} = 53^\circ \]
\[ L_{QPR} = 90^\circ \]

\[ \text{Sum} = 181^\circ \]

Problem 5.

\[ L_{ABE} = 37^\circ \]
\[ L_{AEB} = 90^\circ \]
\[ L_{EAB} = 53^\circ \]

\[ \text{Sum} = 180^\circ \]
This Book deals mainly with areas.

Definitions:

- A parallelogram is a four sided figure which has opposite sides parallel.
- A rectangle is a parallelogram which has one of its angles a right angle.
- A square is a rectangle which has two adjacent sides equal.
- A trapezium is a four sided figure which has two of its sides parallel.
- A rhombus is a four sided figure which has all its sides equal, but its angles are not right angles.
\[ \angle DEF = 66^\circ \]
\[ \angle DFE = 90^\circ \]
\[ \angle FDE = 23^\circ \]
\[ 179^\circ \]

\[ \triangle ABD \]
\[ AB = 13\, \text{cm} \]
\[ BD = 5\, \text{cm} \]
\[ AD = 12\, \text{cm} \]
\[ \angle DAB = 23^\circ \]
Given a base $AB$ of a set line $CD$

To draw an isosceles $\triangle ABC$ having one of its sides equal to $a$ 

Take from the vertex $A$ the line $FA$ equal to $a$

Join $AB$ et $F$ 

At $F$ a radial $FA$ equal to $5$$\text{ cm}$, and let $AG$ meeting

Then $AGB$ is the required.
If we want to lay out one line at right angles to another in making a tennis court or football field, we may use this 3:4:5 rule. Thus we may measure 3 yds along one line, and 4 yds from one end of it, then if the distance between the other two ends is 5 yds the first two lines are at right angles.

Exercise 6.

1.
Ex.E. 2.

They make a Hexagon.

No 3.

All angles = 90° Square.

No 4. The 3.4.5 Rule.

AC is a tangent to the ⊙

Dy BE are tangent to the circle.
AC, AB, AD are radii of a circle. BC, CA, BE, CD have the same radius but the other lines are equal.

Any radius will do.
No. 9

equilateral triangle

Reunion 9/4/20

\[ \angle ABC = 58^\circ \]
\[ \angle ACB = 90^\circ \]
\[ \angle BAC = 45^\circ \]

\[ \text{Sum} \ 183^\circ \]
It is impossible to draw a \( \triangle \) whose sides are 3, 4, 7 cm or ins.

You Cannot.

A circle drawn on the hypotenuse (long side) of a right-angled \( \triangle \) as diameter passes through all 3 vertices of a \( \triangle \).

3.

A tangent
10. Draw an isosceles triangle.

11. \( \angle BAC = 40^\circ \)
\( \angle BCA = 70^\circ \)
\( \angle ABC = 70^\circ \)

12. Scale drawings with dimensions.

Definition:
When a straight line meets another straight line so as to make the adjacent angles equal, each is a right angle, and one line is said to be perpendicular to the other.

If a straight line meets another straight line the adjacent angles so formed are together equal to two right angles.

Given a st. line AB meeting another st. line CD at B.

To prove the adjacent angles ABC, ABD are together equal to 2 rt. Ls

Proof: Case I. Where \( \angle ABC = \angle ABD \) each of these angles is \( \pi \) rt. L, and their sum is \( 2 \) rt. L.

Case II. Where \( \angle ABC \) is not equal to \( \angle ABD \).
Proposition 11  Theorem.

If a straight line meet two other straight lines so as to make a pair of adjacent angles together equal to two rt. ls then the two straight lines are in one and the same straight line.

Given: A rt. line $AB$ meeting two other rt. lines $BC, BD$ at $B$. So as to make $\angle ABC$ and $\angle ABD$ together equal to two rt. ls.

To prove $BC + BD$ are in the same rt. line.

Proof. Suppose $BD$ is not in line with $BC$, let $BE$ be drawn in line with $BC$. Then the rt. line $AB$ meets the rt. line $EC$ at $B$.

Given: $\angle ABE$ and $\angle ABC$ together equal to $2$ rt. ls but $\angle ABD$ and $\angle ABC$ together equal $2$ rt. ls (given).

$\therefore \angle ABE + \angle ABC = \angle ABD + \angle ABC$

$\therefore \angle ABE = \angle ABD$

$\therefore BD$ is in line with $BE$, and $BE$ is in a rt. line with $BC$.

$\therefore BD$ and $BE$ are in one and the same rt. line.

Q.E.D.

Definition: When two right angles together make two right angles they are said to be Supplementary.

When two angles together make a right angle they are said to be Complementary.
Proposition 3. Theorem

When two straight lines intersect the vertically opposite angles are equal.

\[ \begin{align*}
\text{Given:} & \quad \text{two straight lines } AB, CD \text{ intersecting at } O \\
\text{To prove:} & \quad \angle AOC = \angle DOB \quad \text{and} \quad \angle AOD = \angle COB
\end{align*} \]

\text{Proof:}

Because \( AO \) meets the straight line \( CD \) at \( O \)

\[ \angle AOD \text{ is the supplement of } \angle AOC \]

again because \( DO \) meets the straight line \( AB \) at \( O \)

\[ \angle AOD \text{ is the supplement of } \angle DOB \]

\[ \therefore \angle AOC \text{ and } \angle DOB \text{ are supplements of the same angle and are therefore equal.} \]

\text{In the same way it may be shown that}

\[ \angle AOD = \angle COB. \]

\( \text{Q.E.D.} \)
Proposition 41. Theorem
13th June 1924

If one side of a triangle be produced, the exterior angle so formed is greater than either of the interior angles.

Given a $\triangle ABC$ with one side $BC$ produced to $D$.

To prove that $\angle ACD$ is greater than $\angle ABC$ or $\angle BAC$.

Proof. Take the lines $AB$ and $BC$ that form $\angle ABC$, and keeping the angle between them the same, slide $AC$ along $BD$ until $B$ reaches $C$.

Then $ECF$ is the new position of $\triangle ABC$.

Now $AB$, in moving to its new position $EC$, has passed right over $AC$ leaving $AC$ outside $ECF$.

$\therefore \angle ACD$ is greater than $\angle ECF$ or $\angle ACD$ is greater than $\angle ABC$.

In the same way by sliding $BA + AC$ along $CG$ in the direction of $G$ we can show that $\angle BCG$ is greater than $\angle BAC$.

But $\angle BCG = \angle ACD$ (Vertically opposite).

$\therefore \angle ACD$ is greater than $\angle BAC$.

Q.E.D.

PARALLELS.

Definition. Parallel straight lines are lines which are in the same plane and will not meet however far they are produced in either direction.

Problem

Through a given point to draw a straight line parallel to a given straight line.
Given a straight line AB and a point P

To draw through P a line || to AB

Cons 1) Place a set square with one edge in the line AB
2) Lay a ruler against one of the other sides of the set square
3) Slide the set square along the ruler until it comes to the P
4) Draw a line through P along the same edge of the set square as lay in AB.
Proposition 5. Theorem

If a straight line cuts two other straight lines so as to make
(1) the alternate angles equal or (ii) a pair of corresponding angles equal,
or (iii) two interior angles on the same side of the cutting line together
equal to two right angles, then the two straight lines are parallel.

Given a straight line EF cutting two other straight lines AB and CD
at G and H, as so as to make either:

(i) \( \angle AGH \) equal to the alt. \( \angle GHD \)
or (ii) \( \angle EGB \) " corresponding \( \angle GHD \)

(iii) \( \angle BGH, GHD \) together equal to two rt. \( \angle \).

To prove \( AB \parallel CD \)

Proof. (i) If \( AB \) and \( CD \) are not \( \parallel \) they will meet
when produced either towards \( B \) and \( D \) or towards
\( A \) and \( C \).
If possible let them meet not at a point \( M \) in
the direction of \( B \) and \( D \).
The \( \triangle GMH \) is a \( \triangle \) and we have the two \( \angle \) \( \angle AGH \)
equal to an interior opp \( \angle GHD \), which in a \( \triangle \)
is impossible.

\( AB \) and \( CD \) cannot meet when produced
\( AB \parallel CD \)

(ii) \( \angle EGB = \angle GHD \)
but \( \angle AGH = \angle EGB \) (vertically opposite)

\( \therefore \) \( \angle AGH = \angle GHD \) and these are \( \angle \)
\( \therefore \) \( AB \parallel CD \)

(iii) when \( \angle BGN + \angle GHD = 2 \mathbf{r} \)
\( \angle BGN + \angle AGH = 2 \mathbf{r} \) (adjacent \( \angle \))

\( \therefore \) \( \angle BGN + \angle AGH = \angle BGN + \angle GHD \)
\( \therefore \) \( \angle AGH = \angle GHD \)
but these are alternate \( \angle \)
\( \therefore \) \( AB \) and \( CD \) are parallel.

Q.E.D.

If a st line cut two parallel st lines it makes

(i) the alternate angles equal
(ii) the corresponding angles equal, and
(iii) the two interior angles on the same side of it together
     equal to two right angles.

Given two || st lines AB, CD with a st line EF cutting
them at G and H.

To prove (i) that \( \angle AGH = \text{alt} \angle GHD \)
(ii) the \( \angle EGB = \text{corresponding} \angle GH \)
     and (iii) \( \angle BGH + \angle GH = 2 \times \angle GH \)

Proof (i) If \( \angle AGH \) is not equal to \( \angle GHD \) one must be the greater.
If possible, let \( \angle KGH = \angle GHD \)
These are alternate \( \angle \). \( \therefore \) KG is || to CD

but AB is || to CD (given)

\( \therefore \) we have 2 intersecting st lines AB and KG both || to third CD
which by Playfair's Axiom is impossible

\( \therefore \) \( \angle AGH \) cannot be unequal to \( \angle GHD \)

\( \therefore \) \( \angle AGH = \angle GHD. \)
(ii) \( \angle EGB = \angle LAGH \) (Vertically opposite).

\[ \text{but } \angle LAGH = \angle LGHD \text{ (proven)} \]
\[ \therefore \angle EGB = \angle LGHD. \]

(iii) \( \angle BGH + \angle LAGH = 2 \times \angle 5 \) (adjacent \( \angle 5 \))

\[ \text{but } \angle LAGH = \angle LGHD \text{ (proven)} \]
\[ \therefore \angle BGH + \angle LGHD = 2 \times \angle 5 \]

\( \therefore \text{D.E.D.} \)

**Proposition 7.** Theorem. Aug 15th 1924.

Straight lines which are parallel to the same line are parallel to one another.

\[ \begin{array}{c|c}
A & B \\
\hline
x & y \\
\hline
c & d
\end{array} \]

Given two st lines \( AB \) and \( CD \) both \( \parallel \) to a third line \( XY \).

To prove that \( AB \parallel CD \)

Proof: If \( AB \) and \( CD \) are not \( \parallel \) they will meet when produced and then we shall have 2 intersecting \( \angle \)s both \( \parallel \) to a \( 3^{rd} \), which by Playfair's Additon is impossible.

\[ \therefore AB \parallel CD \]

They are \( \parallel \).

\( \therefore \text{D.E.D.} \)

Aug 25th 1924

Corollary I. Any two angles of a triangle are together less than two right angles.

Corollary II: Only one perpendicular can be drawn to a straight line from a given point without it. For if possible, let \( AB \) and \( AE \) each be perpendicular to the st line \( BC \).

Then in \( \triangle ADE \) we have

the two \( \angle s \) \( ADE, AED \) together equal to two \( \parallel \angle s \)

and this is impossible.
Proposition 8. Theorem

If one side of a triangle be produced, the exterior angle so formed is equal to the sum of the two opposite angles, and the three interior angles of the triangle are together equal to two right angles.

Given a triangle ABC with one side BC produced to D.

To prove (i) the exterior \( \angle LACD \) is equal to the sum of the 2 interior opposite angles \( \angle ABC, \angle BAC \), and (ii) \( \angle LABC + \angle LBCA + \angle LACB = 2 \times \angle LACD \).

Proof: Through C draw \( CE \parallel AB \).

Because \( AB \parallel CE \) and \( AC \) meets \( \angle ABC \).

\[ \angle LABC \quad \text{corresponding to} \quad \angle LEC \]

\[ \angle LACD = \angle LABC + \angle LACB \]

To each of these equals add \( \angle LABC \).

Then \( \angle LACD + \angle LACB = \angle LABC + \angle LACB + \angle LACB \)

But \( \angle LACD + \angle LACB = (2 \times \angle LACD \quad \text{adjacent to} \quad \angle LACB) \).

\[ \therefore \angle LABC + \angle LBCA + \angle LACB = 2 \angle LACD \]

Q.E.D.

Definition \( \angle 25, \text{1924} \).

An axiom is a self-evident truth.

If equals are added to equals the sums are equal.

Playfair's Axiom states that two intersecting straight lines cannot both be parallel to a third.

Reductions from proposition 8.

It is evident that a triangle can have only one \( \parallel \) or one obtuse angle. At least two angles of a triangle must be acute.
Corollary III

all the interior angles of any rectilinear figure, together with four right angles are equal to twice as many right angles as the figure has sides.

\[ \frac{1}{2} \cdot 6 \cdot 6 = 18 \]

\[ \frac{1}{2} \cdot 4 \cdot 4 = 8 \]

\[ 2n = 4 \cdot 10 \]

\[ 10 = 4 \cdot 6 + 4 \]

a total of 6 at 6

Sth 3rd 1924. adding 6 at 6 to this we get 10 at 6

ie twice as many at 6 as the figure has sides.

Exercise. Draw 6 circles of 2'' diam.

In one place an equilateral 6 in the 2nd a square in the third a regular pentagon (5 sides) in the 4th a regular hexagon (6 sides) in the 5th a regular octagon; in the 6th a regular decagon (10 sides).

\[ 2 \times \text{the no of sides} = 10 \]