Proposition 1

Theorem: If a triangle is equilateral, then all its angles are right angles.

Proof: Let \( \triangle ABC \) be an equilateral triangle. Then, \( AB = BC = CA \).

Since \( AB = BC \), we have \( \angle B = \angle C \).

Similarly, since \( BC = CA \), we have \( \angle C = \angle A \).

And since \( CA = AB \), we have \( \angle A = \angle B \).

Therefore, \( \angle A = \angle B = \angle C \).

But the sum of the angles in a triangle is always \( 180^\circ \), so we have

\[ \angle A + \angle B + \angle C = 180^\circ \]

Substituting \( \angle A = \angle B = \angle C \), we get

\[ 3 \angle A = 180^\circ \]

Solving for \( \angle A \), we find

\[ \angle A = 60^\circ \]

But this is only possible if \( \triangle ABC \) is equilateral, which contradicts our assumption unless all angles are right angles.

Hence, if a triangle is equilateral, then all its angles are right angles.
the same base and on the same side of it, they lie between the same lines. For if their areas are equal and their bases the same, they must lie of the same altitude.

Also as on equal bases and between the same lines are equal in area, and conversely as on equal bases on the same at line and on the same side of it, if equal in area lie between the same lines.
Proposition 8. Theorem.

The square on the hypotenuse of a rt angled triangle is equal to the sum of the squares of the other two sides.
Given a Δ ABC having ∠ BAC = 90°.

To prove that the square on BC is equal to the sum of the squares on AB and AC.

Construction:

1. On AB describe the square ABDDE,
2. On BC describe the square CBFEG,
3. On AC describe the square ACHG.

From A draw AL || BE meeting FG in L. Join DC, AF.

Proof: Because ∠ BAC is a rt ∠ \( \therefore \triangle ABC \) is a rt \( \triangle \).

\[ \therefore \triangle CAE \text{ is a rt line.} \]

Now Δ BCD and the square AD are on the same base BD and between the same \( \| \), CE, BD

\[ \therefore \triangle BCD \text{ is } \frac{1}{2} \text{ square } AD \]

again Δ ABF and the rect. BL on the same base BF and between the same \( \| \), AL, BF.

\[ \therefore \triangle ABF \text{ is } \frac{1}{2} \text{ the rect. BL} \]

In Δ BCD, Δ ABF we have BD = BA (side of a square)

\[ \triangle BCD \text{ is } \frac{1}{2} \text{ square } AD \]

BC = BF (side of a square) + included \( \triangle BDE = \) included \( \triangle ABF \)

each being a rt \( \angle \) + \( \angle \) ABC. \( \therefore \triangle \) are equal in all respects.

\[ \therefore \text{ their doubles, the square AD + rect. BL are equal} \]

Similarly by joining BN and AG we can show that the square AH = rect. CL.

And the rects. BL, CL make up the square BG.

\[ \therefore \text{ the square on BC = sum of squares on } AB, AC \]

Q.E.D.
Proposition 9. "Theorem"

(Converse of Prop. 8)

If the square on one side of a triangle is equal to the sum of the squares on the other two sides, the triangle is right-angled.

Given: A ΔABC in which the square on AB is equal to the sum of the squares on AC and BC.

To prove: ∠ACB is a rt. ∠.

Construction: At C, make ∠ACD a rt. ∠ and make CD equal to CB. Join AD.

Proof: Because ∠ACD is a rt. ∠,

\[ \text{the square on } AD = \text{the sq on } AC + \text{the sq on } CD \]

\[ = AC^2 + CB^2 \] (By Hypothesis)

\[ = AB^2 \]

\[ \therefore AD = AB \]

Then in ΔABC, ADC, we have AD = AB, AC common and CD = CB (const): As are equal in all respects.

But ∠ACD is a rt. ∠ (const) \[ \therefore \triangle ACB \text{ is a rt. } \triangle. \quad \text{Q.E.D.} \]
\[ \sqrt{2} = 1.4 \]
\[ \sqrt{3} = 1.73 \]
\[ \sqrt{5} = 2.21 \]
\[ \sqrt{7} = 2.12 \]
13 = 9 + 4

= 3^2 + 2^2

21 = 7 \times 3 - 4

= 5^2 - 2^2
1.

\[ \text{Proof: } 9^2 + 12^2 = 225 \]

\[ 12^2 = 225 \]

\[ 15 \text{ in is the hypotenuse} \]
Proof: $52^2 - 48^2 = 400$

\[ \sqrt{400} = 20 \]

Ans. 20 ins. √
Area of \( A = \frac{1}{2} b h \\
= \frac{1}{2} \times (3.4) \\
= 6 \text{ sq in.} \)
Given a \( \triangle ABC \) with \( AD \) perpendicular to \( BC \).

To prove: Squares on \( AB, AC \) is equal to the difference of the squares on \( BD, CD \).

Proof: Because \( \angle ADB \) is a rt \( \angle \)

\[
\begin{align*}
\text{sq on } AD + \text{sq on } BD &= \text{sq on } AB \\
3.55^2 + 3.2^2 &= 4.75^2
\end{align*}
\]

\[
\begin{align*}
AC^2 &= AD^2 + DC^2 \\
AH^2 &= AD^2 + BD^2
\end{align*}
\]

\[
AC^2 - AH^2 = DC^2 - BD^2
\]

In the difference of the squares on the sides is equal to the difference of the squares on the segments of the base.
Proposition 10. Problem

To draw a triangle equal in area to a given quadrilateral.

Given a quadrilateral $ABCD$

To draw a $\triangle$ equal to it in area

Cons: Join $AC$.
Through $D$ draw $DE \parallel AC$, the base produced in $E$
Join $AE$, then $\triangle ABE$ is the $\triangle$ required.

Proof: The $\triangle ABE$ and $\triangle AEC$ are on the same base $AC$, and between the same $\parallel$ to $AC$, $DE$

$i.$ they are equal in area

To each add the $\triangle ABC$
then the quadrilateral $ABCD = \triangle ABE$

$Q.E.F.$
Area of $A = \frac{1}{2}bh$

$\frac{1}{2} (11 \times 5.1)$

$= \frac{55.1}{2}$

$= 27.55 \text{ sq cm}$

Proof:

Area of $\triangle ABD = \frac{1}{2} (8.4 \times 1.5) = 6.3$

Area of $\triangle BDC = \frac{1}{2} (8.5 \times 5) = 21.25$

Total = $6.3 + 21.25 = 27.55$
Given a $\triangle ABC$ with $D$ a given point in $AB$.

To Bisect it.

Con: Bisect $AE$ at $E$ and draw $BF \parallel DE$ to meet $AC$ in $F$.

The $\triangle ADF$ will be the $\triangle$ required. But if $AD$ is less than $DB$ the construction fails.

Proof: As $BDE, FDE$ are on the same base $DE$ and between the same $\parallel$ to $DE$, $BF$; they are equal in area.

To each add $\triangle ADE$, then $\triangle ABE = \triangle ADF$

But $\triangle ABE = \frac{1}{2} \triangle ABC$; $\triangle ADF = \frac{1}{2} \triangle ABC$. Q.E.F.
\[ \sqrt{2} \approx 1.414 \]

\[ 5 \times \sqrt{2} = 5 \times 1.414 = 7.07 \text{ cm} \]
\[2^2 - 1^2 = x^2\]

\[-x^2 = -2^2 + 1^2\]

\[-x^2 = -4 + 1\]

\[-x = \sqrt{-3}\]

\[x := \sqrt{3}\]
Definitions:

1. A circle is a plane figure bounded by one line called the circumference, and is such that all straight lines drawn from a certain point within the figure to the circumference are equal.

2. A chord of a circle is a finite straight line joining any two points on the circumference.

3. Any straight line which cuts the circumference of a circle at two points is said to be a Secant of the Circle.

4. An arc of a circle is a part of the circumference.

5. A segment of a circle is the figure bounded by a chord and the arc which it cuts off.

6. A sector of a circle is a figure bounded by two radii and the arc between them.

7. A figure is said to be Symmetrical
about a line if the part on one side coincides with the part on the other side when the figure is folded about that line.
Corollary. Join CD and let CD cut AB at N.

When the O is folded about AB,
CN and ND coincide,
\( \therefore CN = ND \)

Also \( \angle CNO \) coincides with \( \angle DNO \),
\( \therefore \) these angles are equal,
but they are adjacent \( \angle C \)
and \( \angle D \).

\( \therefore \) Hence CD is bisected at \( \angle C \) \( = \angle D \) by AB

Thus a line joining two symmetrically opposite
pts is bisected at \( \angle C \) \( = \angle D \) by the axis of symmetry.
Proposition 2. Theorem

A straight line which joins the centre of a circle to the middle point of a chord is perpendicular to the chord, and conversely.

Given: a circle centre O and a chord AB in it, with M the middle pt of AB.

To prove: OM is perp to AB.

Construction: Join OA, OB.

Proof: In Δ OMA, OMB we have OA = OB (radii) OM common

and AM = MB (given)

⇒ Δs are equal in all respects

⇒ ∠ OMA = ∠ OMB, and these are adjacent ∠s

⇒ they are rt. Q.E.D.
Converse: The perpendicular drawn from the centre of a \( O \) to a chord bisects the chord.

![Diagram of a circle with a perpendicular from the center O to a chord AB, bisecting it at P.]

Given a \( \odot \) centre \( O \), and a chord \( AB \) in it, and a \( \perp \) line \( OP \) drawn from \( O \) perf to \( AB \).

To prove: \( P \) is the middle pt of \( AB \).

**Con.** Join \( OA, OB \).

**Proof:** In the rt \( \triangle \)s \( OPA, OPB \) we have the hypotenuses \( OA, OB \) equal (radii) and one side, \( OP \) common.

\( \therefore \) \( \triangle \)s are equal in all respects

\( \therefore \) \( AP = PB \)

ie \( P \) is the middle pt of \( AB \).

Q.E.D.
The line of centres of two intersecting circles bisects their common chord at rt $90^\circ$.

Given 2 centres $O$ and $P$, cutting each other at $A$ and $B$.

To Prove that $OP$ bisects $AB$ at rt $90^\circ$.

Proof: $O$ is equidistant from $A$ and $B$.

So also $P$ is equidistant from $A$ and $B$.

$\therefore OP$ is the locus of a pt equidistant from $A$ and $B$.

$\therefore OP$ bisects $AB$ at rt $90^\circ$. Q.E.D.

Aug 11th 925
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<th>Grade 2</th>
<th>Grade 3</th>
<th>Grade 4</th>
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Show your diagram and all your working.

Draw a triangle ABC. Draw a line AD parallel to BC.

Let E be the midpoint of AD. Draw a line BE.

Now, AD = 2BE. (E is the midpoint of AD)

Then, DE = 1/2 BE. (E is the midpoint of AD)

Find the length of DE.

Answer: DE = 1/2 BE.
Geometry
13th February 1934.

Ex. B.1.

1. Draw a line 3 m long.
   
   \[ \begin{align*}
   &3'' = 1.6 \text{ c.m.} \\
   &7.6 \text{ c.m.} \\
   &1'' = \frac{1.6}{2.63} = 0.61 \text{ c.m. ans}
   \end{align*} \]

2. 
   
   \[ \begin{align*}
   &4'' = 10.2 \text{ c.m.} \\
   &1'' = \frac{10.2}{2.55} = 4.0 \text{ c.m. ans}
   \end{align*} \]

3. 
   
   6 \text{ c.m.}

4. 2\frac{7}{16}''

5. 
   
   11.45 \text{ c.m.}

6. 4\frac{1}{2}'' or 4.5''

Rectilinear figures are figures contained by straight lines.

- a triangle (3 sides)
- a quadrilateral (4 sides)
- a pentagon (5 sides)
- a hexagon (6 sides)

7. a curvilinear.
Given a quadrilateral $ABCD$ in which
$\angle ACD = \angle ABD$ and $\angle BAC = \angle BDC$
To prove $ABCD$ is a Parr
Proof: Because the $4\angle$'s of a quad. together $= 4$ rt.\'s
and the rig $ABCD$ has $2$ pairs of equal $\angle$
$\therefore \angle BAC + \angle ABD = 2$ rt.\'s
And since $AB$ meets the pr lines $AC, BD$
making the $2$ int.\'s $\angle$ supplemeniry
$\therefore AC$ is $\parallel$ to $BD$
Similarly $AB$ is $\parallel$ to $CD$
$\therefore ABDC$ is a Parr. Q.E.D.
Definitions.

A straight line is the shortest distance between two points, its ends.

A point in geometry has position but no magnitude.
A line has length but no breadth or thickness.
It has therefore one dimension.

A surface has length and breadth.
A solid has three dimensions, length, breadth, and thickness.

An angle is the amount of turning made by a straight line in moving from one position to another.

Making and measuring Angles.

1. To make an angle, using the protractor, e.g.
   To make an angle of 30°.
   Draw a straight line, A.B. At A place the middle point of the straight line along AB.
   Mark a point, C, at the figure 30, measuring up from the line AB.
   Join AC, then CAB is the angle required.