THE ROLE OF THE PROBLEM – BASED APPROACH IN THE PERFORMANCE
OF GRADE 9 LEARNERS IN SOLVING WORD PROBLEMS.

by

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"I declare that 'The role of the problem-based approach in the performance of Grade 9 learners in solving word problems' is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references."

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Signature

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Summary: In this study, the role of the problem-based approach on the performance of Grade 9 learners in solving word problems is investigated. Traditional approaches have produced learners whose performance in mathematics is not satisfactory and who are not sufficiently equipped with critical and problem skills that are necessary in this dynamic world. Problem-based approach is among the current reform efforts recommended for teaching and learning mathematics. For this approach to be successful, learners need vital tools such as problem solving strategies, which many learners in this country lack. The emphasis in this study was therefore on exposing learners to a variety of problem solving strategies through the problem-based approach. Problems solved throughout the investigation were non-routine, word problems. The results show that awareness of these strategies improves learners' problem solving performance and attitudes towards mathematics. Based on this investigation, recommendations are made concerning effective implementation of this approach to the teaching and learning of mathematics.

Key words: Problem-based approach, problem solving strategies, performance, Grade 9 learners, problem solving, word problems.

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CHAPTER ONE
ORIENTATION

1.1   Introduction and background

Poor performance of learners in mathematics is a great concern to everybody who has an interest in education. Workshops and in-service training have been provided to assist teachers to produce better results but these attempts had a seemingly low impact (Graven 2002:66). Learners are still struggling and performing unsatisfactorily in mathematics, others decided to drop mathematics altogether. The new curriculum, Outcomes Based Education (OBE) has been introduced in the FET and requires all learners to take either Mathematical Literacy or Mathematics, hence no learner can avoid taking mathematics. This implies that much more effort will be needed in order to change learners' attitudes towards mathematics. According to Department of Education (2003:9), everybody requires mathematical skills in their everyday lives. These include skills in financial issues such as hire purchase, mortgage bonds, investments, understanding house plans, sewing patterns, use of medicine and cooking. It furthermore points out that the work place requires the use of fundamental numerical and spatial skills in order to meet these demands efficiently. The Department continues by saying that, in order to be participating citizens in a developing democracy, it is essential for people to have acquired a critical stance with regard to mathematical arguments presented in the media and other platforms.

It is critical that mathematics teaching and learning should develop learners' logical and analytical thinking skills. It should produce confident problem solvers who will be able to live meaningfully in this challenging and rapidly growing technological society. The Department of Education (2002:4) adds that mathematics teaching and learning should develop in learners: A love for mathematics as well as the necessary confidence and competence to deal with any mathematical situation without being hindered by fear of mathematics.

Unfortunately our learners do not have the skills mentioned above and a poor performance is not only seen in mathematics classrooms and examinations but also through research studies.

The Third International Mathematics and Science Study (TIMSS) conducted in 1997 was the largest study ever undertaken on Mathematics and Science which involved over forty-five countries. In South Africa, the Human Science Research Council (HSRC) conducted this study with the intention to inform stakeholders in education about proficiency of learners at key points. The results of the study revealed that:

- ❖ Scores of South Africans are very low as compared to other countries.
- ❖ In this country, learners' problem solving techniques are generally poor (Ramnarain 1999:5).

Today's world needs workers who are equipped with problem-solving skills and abilities, technologically literate and with mathematical thinking abilities (NCTM 1989). Tynjala (1999:356) adds that employers need from their employees not only a good command of relevant knowledge but diversified social, communication, and cooperation skills, ability to work in different contexts with different experts from other field, but also the ability to come up with solutions for challenging and complex problems. Traditional instruction does not adequately equip our learners with skills and abilities mentioned above. This instruction is mainly teacher-centred, knowledge and skills are acquired first and then applied later to solve problems (Wessels & Kwari, 2003:75). Wessels & Kwari further pointed out that observations revealed that knowledge acquired in the classroom does not transfer well to the profession one chooses in life. One cannot predict all problems one would meet in one's lifetime to prepare methods of solving them while in school.

The results of TIMSS study above imply relatively obvious that the teaching of mathematics in South Africa does not generally incorporate problem solving to develop the mathematical skills indicated earlier. In addition, the traditional teacher-centered approaches predominant in classrooms marginalize the role of mathematical problem-solving in that "the processes and the activities of problem-solving take on a

secondary role with the answer as the main goal" (Ramnarain 1999:59). I believe the fact that these approaches are still dominant can be attributed to the way in which teachers view mathematics.

In the next section, I will discuss the traditional view of mathematics and contradict it with a more contemporary view in which mathematics is seen as a human construction, thus fallible.

1.2   Views about mathematics

A person's beliefs about mathematics influence his or her views about what mathematics is and how mathematics teaching and learning should take place. According to the traditional views, mathematics is "regarded as a meaningless formal game played with marks on paper following rules" (Ernest 1991:10). It is also perceived as a 'fixed' static body of knowledge (Romberg & Kaput 1999:4). The teacher has to demonstrate how a manipulation has to be carried out, emphasizing rules and algorithms that need to be mastered; and students are expected to memorize facts and to practice procedures until they have mastered them (Romberg & Kaput 1999:4). Then they have to recall these during examinations. Students are "graded not on understanding of the mathematical concepts and reasoning but on their ability to produce the right symbol string- answers using strategies they have no need to articulate" (Kaput 1999:133). This approach to mathematics instruction left many people with a perception that mathematics is mysterious and conceptually inaccessible except to a few geniuses. Learners who are products of this form of instruction perform poorly in general problem solving as discussed previously. These factors together with the results of research that has been conducted in mathematics contributed to a shift in views on the nature of mathematics and mathematics learning and teaching.

Contrary to the traditional view, mathematics is currently viewed as a human activity. This view is concerned with what mathematicians do and have done with all the imperfections inherent in any human activity. Mathematics is also regarded as a

dialogue between two or more people tackling mathematical problems, as such it is a set of socially distributed practices. As a human activity "mathematics cannot be isolated from its history, its sociological implications and its applications" (Ernest 1991:35). This view of mathematics as a human construction of knowledge, fallible and changing, requires a change of mindset of teachers who still adhere to the traditional view of mathematics.

This study is based on the latter view of mathematics and a brief discussion of how learning and teaching are considered in this view, follows.

1.3 The teaching and learning of mathematics

Mathematics teaching and learning should aim at promoting the development of all forms and levels of high quality mathematical thinking. The aim should be to assist learners to become autonomous thinkers with a strong, but realistic faith in their own potential and ability in mathematics (Wessels & Kwari 2003:1). Teaching and learning should also help learners to obtain mathematical competence in such a way that it can contribute to and be used as a tool in problem solving. These could be achieved when the teaching and learning of mathematics embrace the constructivist view. The theory of constructivism will be discussed in detail in chapter two.

1.3.1 Learning

According to the theory of constructivism, learning takes place when learners construct their own knowledge. (Murray, Olivier & Human 1998:270) point out that attempts should be made to establish individual and social procedures to monitor and improve these constructions. They go on to point out that learning occurs when learners grapple with problems for which they have no routine methods of solution. Teachers should provide learners with meaningful experiences so that learners make quality constructions.

1.3.2 Teaching

Teaching is taken as the facilitation of learners' construction of knowledge and not the transmission of knowledge from the teacher to the learners. The teacher removes

himself/herself from showing or telling how to do every tiny procedure and explaining every concept (Van de Walle 1998:53-4). Teaching should be aware of learners' real-live experiences, guide them to formulate problems, and develop strategies to find solutions in a range of contexts. By exploring problems, learners learn to verify and interpret results and they also learn to generalize solutions. As a result they become better problem-solvers who are confident in addressing real-world problem situations. Teaching and learning will be given more attention in the next chapter. It is however vital to indicate that the teaching of mathematics should be portrayed as a human activity, fallible and in which learners actively take part. According to Cobuild dictionary, fallible means someone or something is not perfect and is likely to make mistakes. According to quasi-empiricism, (Ernest 1991:35), "Mathematicians are fallible and their products, including concepts and proofs, can never be considered final or perfect". Mathematics is open to revision as new challenges or meanings emerge (Ernest 1991:36). One of the approaches that the researcher believes portrays mathematics as a human activity is the "problem - based" approach in which learners actively solve problems.

1.4   Problem – based approach (PBA)

Before one goes deeper into this approach, it is important to define briefly the concepts "problem" and "problem solving".

1.4.1   A problem

According to Oxford Reference Dictionary (1986:662), a problem is "a doubtful or difficult matter requiring solution". It is a question that exercises the mind.

Schoenfeld (1989:87-89) describes a mathematical problem as "a task (a) in which the student is interested and engaged and for which he wishes to obtain a resolution and (b) for which the student does not have a readily accessible mathematical means by which to achieve a solution". This definition presumes that one has to be engaged in solving a problem; a task is not a problem until you have made it your own. Dolan and Williamson (1983:ix) point out that a problem involves more than simply deriving

an answer, it also means selecting an appropriate strategy. In this study word problems mean mathematical problems (as described by Schoenfeld) set in contexts.

1.4.2   Problem-solving

To solve a problem means to find a way where no specific way is known off-hand, to find a way out of a difficulty, to find a way around an obstacle, to attain a desired end that is not immediately attainable by appropriate means (Ernest 1991:284). Therefore, in a problem-based approach, learning takes place while learners are engaged in solving a mathematical problem.

1.4.3   What is problem – based approach

A problem-based approach to mathematics teaching and learning is based on the approach in which the teacher "expects students to solve problems or make sense of mathematical situations for which no well defined procedures exist" (Erickson 1999:516). According to Davis (1992:237), in this approach, "instead of starting with "mathematical" ideas and then "applying" them, we would start with *problems* or *tasks* and as a result of working on these problems the learners will be left with a residue of "mathematics". Davis further indicates that mathematics is what is left over after one has worked on the problems**.** Hiebert, J; Carpenter,TP; Fennema,E; Fuson,K; Human,PG; Murray,H; Olivier,AI and Wearne,D (1996:17) add by saying that not all learners will have the same residue since their prior learning and the kind of operations they used will affect the knowledge they will have acquired at the end of their activities. This residue includes strategies that learners will have developed in the process of solving problems. Different residues will be dealt with in 2.4.2.

The following factors are important in problem-based approach:
- ❖ Learners are expected to problematise mathematics. They should wonder how things are and should take the initiative to inquire, to search for solutions and to resolve incongruities (Hiebert et al 1996:14).
- ❖ Instructional flexibility should prevail in classrooms. The teacher should assess how learners interact with the problem and make changes where necessary. Through

proper questioning, the teacher should also help learners draw out their thinking (Erickson 1999:519).

❖ Social interaction and reflection are vital. Kilpatrick (1985:10) points out the importance of working in groups by indicating that such groups may also be instruments for developing problem-solving skills and strategies. When learners interact with each other, opportunities are created for learners to talk about their thinking and this talk encourages reflection. Constructivists view reflection as a major source of knowledge on all levels of mathematics (Murray, H; Olivier, AI & Human, PG 1998 : 271).

❖ A positive classroom atmosphere or environment should be created. For example, the teacher himself/herself should be enthusiastic about problem solving, encourage students to bring problems from their personal experiences, recognize and reinforce willingness and perseverance, praise students for getting correct solutions, and during problem-solving, emphasize the selection and use of problem-solving strategies (Charles, R; Lester, F & O' Daffer, P 1997: viii).

❖ Tasks selected should be "Motivating situations that consider students' interests and experiences, local contexts, puzzles, and applications". They should be of "Reasonable difficulty levels that challenge, yet not discourage" (Erickson 1999: 517).

❖ Learners should construct their own methods of solving problems individually or collectively using their prior knowledge (Hiebert et al 1996).

1.4.4   Strategies commonly found in literature

According to Van de Walle (1998:40), strategies for solving problems are "identifiable methods of approaching a task that are completely independent of the specific topic or subject matter". The Ohio Department of Education (1980a :7) is of the opinion that "strategies represent procedures that one can use to make a problem clearer, simpler, smaller, or more manageable".

Possibly the best-known strategies for solving problems are based on Polya's four-step framework.

These are:

❖ Understand the problem.

❖ Devise a plan or decide on an approach for tackling the problem.

❖ Carry out the plan.

❖ Look back at the problem, the answer, and what you have done to get there.

Once the problem is understood, the next step is the strategy selection process. This is where a learner matches a useful tool or approach with a problem at hand.

Lenchner (1983:10) points out that for any given problem, strategies may be used singly or in combination. Different problem solvers use different strategies in solving the same problem. It is therefore important that a learner should have a variety of these strategies.

1.5   Problem Statement

Experience as a Mathematics teacher has shown that learners have difficulties understanding mathematics and their difficulties become severe when they have to solve contextual problems. Learners seem to lack the necessary strategies for solving such problems (Ramnarain, 1999:7). In most mathematics classrooms learning is based on rote memorization and routine drill. Emphasis is seldom placed on learning mathematics by actively solving non-routine problems and developing different strategies and skills for problem solving. This study is intended to assist learners (through problem- based approach) develop different strategies for solving contextual problems and to improve their performance in problem solving.

This study addresses the following research questions:

1   What does the "problem- based approach" entail?

2   What approaches do learners in traditional mathematics classrooms use to solve problems?

3   What effect does direct instruction in problem solving strategies have on problem solving performance of learners?

4   What are the learners' beliefs and attitudes towards word problems in particular and mathematics in general before and after intervention?

5   What recommendations (based on the findings) can be made regarding implementation of problem-based approach?

1.6   Aim and objectives

The Aim was to investigate the role of problem-based approach on the performance of grade 9 learners in solving word problems.

This aim was achieved through the following objectives:

- ❖ To find out what the problem-based approach to teaching and learning entails.
- ❖ To determine problem solving approaches of learners in solving contextual problems before intervention.
- ❖ To provide explicit instruction in problem solving strategies and to determine the effect of this on learners' problem solving performance.
- ❖ To determine learners' beliefs and attitude towards word problems in particular and mathematics in general.
- ❖ To make recommendations regarding the implementation of problem- based approach.

1.7   Motivation for research

Through classroom experience as a mathematics teacher, I came to realize that most learners are struggling in mathematics and some of them have a negative attitude towards the subject as such they don't care whether they pass or fail the subject. I have also realized that learners find solving word problems very difficult. It is very worrying to realize that even learners who are said to be 'good' at mathematics find it very difficult to answer word problems. This situation causes much concern because the approach that is advocated in Outcomes Based Education (OBE) requires learners to work on contextual problems. These contextual problems are expressed in words, rather than in symbols. For example in the grade 9 end of year examinations, questions in the Continuous Tasks for Assessment (CTA) are contextual problems. My observation as an educator and discussions with some educators revealed that learners have difficulty answering this type of questions. Ntshohi (2005:84) indicated that learners are usually drilled on procedures and algorithms for solving ready formulated equations and get limited exposure to contextual problems. The same difficulties are also experienced by grade 12 learners on the few word problems they encounter in class and during external examinations. As an educator and marker in grade 12 examinations I realized that learners' performance in "word problems" is unsatisfactory and some learners don't even bother to answer such questions.

Mathematics teaching has to prepare adaptive learners who are able to apply what they learned in school in different and challenging situations that they may encounter in their lives and at the work place (Hanley 1994, internet source, retrieved in 2002). Outcomes Based Education (OBE) in South Africa aims at producing such learners. It proposes learner-centred approaches where learners construct their own knowledge by solving problems, working in groups and sharing ideas with each other. I believe the problem-based approach is one such approach that could be beneficial to learners. Through this approach, mathematics becomes meaningful and I hope with time, learners will develop skills and strategies for solving problems within mathematics, other disciplines and in everyday life. In most OBE classrooms however, teacher-centred methods are still predominant. One therefore becomes interested in exploring the problem-based approach with the intention of gathering information that will shed more light on the advantages and or challenges of implementing this approach. In the problem-based approach (PBA), however, the teacher does not suggest procedures or strategies for solving problems; learners should come up with appropriate strategies. Learners from previously disadvantaged schools lack knowledge of these strategies. It is for this reason that learners should be taught or made aware of various strategies with the hope that they will not only improve their problem solving ability, but also their achievement in mathematics. This study endeavours to encourage curriculum developers, INSET providers and educators to revisit their views about mathematics teaching and learning.

1.8   Research methodology

1.8.1   Research design and methodology

This study may be termed descriptive research. According to Schumacher & Macmillan (1997:281), descriptive research deals with the way things are presently or were in the past. It does not involve manipulation of variables. The study is descriptive in the sense that the literature study on the traditional and problem – based approaches to teaching and learning mathematics will be presented. The role of

problem-based approach in promoting mathematical understanding will be clearly discussed.

The research is also quasi experimental in that the groups of learners used were intact, already established classes of learners (Schumacher and Macmillan 1997:335). There was no random assignment of subjects. Two grade 9 classes of learners with mixed abilities at a secondary school were used for the study. One class was used as a control group and the other class as the experimental group. The experimental group received explicit instruction in problem solving strategies through a problem - based approach for six weeks and the control group received normal traditional instruction.

The grade 9 learners were selected for this study because their end of year tasks (CTA) consist of contextual problems and during this investigation, problems solved were contextual.

Learners from this secondary school were chosen because the school was easily accessible to the researcher.

This study may also be called evaluation research. According to Schumacher & Macmillan (1997:22), evaluation research determines the merit and worth of a particular practice and can stimulate further research. In this investigation, the problem – based approach was implemented with the experimental group and its worth evaluated. The researcher provided instruction to the experimental group. Cobb and Steffe (in Clouthier and Shandola 1993:327) cite the following reasons for researchers acting as teachers:

- ❖ Researchers "cannot rely solely on theoretical analysis to understand children's mathematical realities".
- ❖ There is no substitute for the intimate interaction involved when teachers explore learner's construction of mathematical knowledge.

1.8.2   Collection of data

Data were collected through class tasks, tests, questionnaires, classroom observations, informal interviews and journals.

The learners from both groups first wrote a pre-test (see appendix A). This test consisted of word problems. Learners also filled out a questionnaire on their beliefs

and attitudes towards solving word problems in particular and mathematics in general. The experimental group was then given explicit instruction in different problem solving strategies through a problem- based approach by the researcher. Instructions were provided for about 30-45 minutes per day, in about 30 lessons (which is approximately 20 hours). During this period the learners in the experimental group worked on short tasks that took about 15 minutes as well as extended tasks that could be completed in between 1 to 3 lessons. They worked individually and in groups. The control group received the traditional instructions by their teacher.

At the end of this period the learners from both groups wrote a post-test (see appendix B) and the performances of the two groups in the tests were compared.

The tasks and tests given to learners were mathematical problems in which learners were expected to show how they arrived at their answers.

The tasks and tests given aimed at determining whether:

- ❖ Learners can interpret the information given in the problem correctly.
- ❖ They can select the correct strategy for solving the problem.
- ❖ They can use the selected strategy correctly.
- ❖ Learners can eventually get the correct solution.

Learners from the experimental group were encouraged to record in their journals, their feelings about the lessons, the difficulties, their successes, their suggestions and everything they considered important. The researcher also recorded the classroom observations on a daily basis so as to know where or what improvements to make before the next lesson. Informal interviews were conducted regularly during lessons to gain more understanding of learners' thinking.

The same questionnaire (appendix C) was given to both groups after the pre- and post tests to determine their overall beliefs about mathematics as well as their feelings and attitudes towards solving word problems in particular and mathematics in general.

1.8.3   In-depth review of the literature

An in-depth review of the literature was carried out. According to De Vos (1998:128-129), the literature review is very important in research:

- ❖ It provides a substantially better insight into dimensions and complexity of the problem.
- ❖ In reading about a specific topic, the researcher may shape the research question or hypothesis through the identification of alternative conceptions of the problem or variables that had not previously occurred to him.
- ❖ The literature review provides the frame-work of the research and identifies the area of knowledge that the study is intended to expand.
- ❖ It equips the researcher with a complete and thorough justification for the subsequent steps, as well as with a sense of the importance of the undertaking.
- ❖ It makes one aware of the research that has already been done on the problem at hand.

1.9   Chapter division

Chapter 1 is an introduction and background of the topic. In this chapter concerns regarding poor problem-solving performance of learners who receive instruction through the traditional approaches to teaching and learning have been highlighted. It became apparent that there is a need to change from these traditional to the contemporary approaches. One such contemporary approach, which was introduced briefly, is the problem-based approach, on which this study is based. Finally, the problem was formulated and followed by the aim of the study and the accompanying objectives.

Chapter 2 is the review of literature in which more information concerning the problem at hand is presented.

Chapter 3 is based on methodology and research design. This chapter deals with how the research was conducted, how sampling was done, which instruments were used to collect data and how those instruments were used to provide as much information as possible to the research questions at hand. The null hypotheses were also stated in this chapter.

Data was presented in **chapter 4**, followed by analysis and discussions of findings.

Chapter 5 is a summary of the findings and conclusions including recommendations as well as limitations of this study.

1.10 Conclusion

This chapter has provided introduction and background to the investigation, followed by the statement of the problem. The research questions relating to the problem were stated. The aim of the study and the accompanying objectives addressing the research questions were formulated. The research design and methodology of how the research will be conducted were also presented. Lastly the division of chapters for this study was outlined.

The following chapter is a literature study intended to gain more understanding of the problem and to address the first research question.

CHAPTER TWO
LITERATURE STUDY

2.1 Introduction

Solving of problems has been the source of mathematical knowledge since the olden times. In trying to solve problems encountered in their daily lives, people invented their own ways of solving problems. They developed new concepts, discovered new ideas and shared their findings with others. Although problem solving has been here for a very long time, people have different views on the teaching of problem solving. These different views leave learners with different kinds of mathematical "residue". According to Davis (1992:11), the 'residue' is what learners take away from the classroom. Wessels & Kwari (2003:71) identify two main approaches to problem solving. These are:

- ❖ Teaching mathematics for problem solving or learning to solve problems (traditional approach) and
- ❖ Teaching mathematics through problem solving (contemporary approach). In this second approach, problem solving is regarded as the vehicle for learning mathematics.

In the discussions to follow, the traditional approach will be discussed briefly and the main part will concentrate on the contemporary approach, which is referred to as problem – based approach, in this study. The problem-based approach supports teaching of mathematics as a meaningful activity to develop learners' understanding. Different mathematical understandings that are promoted through problem- based approach will be discussed briefly. Polya's framework for problem solving together with different phases in this model will be looked into. This framework will be followed by discussions of different strategies used in problem solving. Strategies, as it will be seen in the discussions, are vital tools in the problem solving process. The

suggestion is that they should be explicitly taught to learners. Assessment forms an important part of every instruction therefore, the last part of this study looks briefly at assessment in problem-based approach.

The literature study that has been conducted on the problem-based approach addresses the first research question for this study, which is: What does problem- based approach entail?

2.2   Approaches to the teaching of problem solving

2.2.1   The traditional approach

This approach is the most common approach found in schools. The teacher first teaches the concepts, facts and procedures and expects learners to solve problems by using or applying the knowledge and procedures just taught in class. Learners are not given opportunity to use their own methods and tend to be too dependent on their teachers. Learning usually takes place through memorization, osmosis and imitation (Kilpartrick 1985:8).

2.2.1.1   Memorization

In this teaching strategy, an algorithm is developed for a class of problems and learners are 'programmed' to follow the algorithm to obtain a solution. Since the algorithm can be used in certain problems, students have difficulty in recognizing when the algorithm is appropriate. Sometimes heuristic suggestions are treated as procedures to be followed and attempt to organize them into algorithms. Kilpatrick (1985:9) says "In these approaches students are taught to recite to themselves a list of steps in problem solving… Such approaches are difficult for students to manage and students find it difficult to classify problems by type".

2.2.1.2   Osmosis

Students are given lots of problems to solve assuming that they will pick up appropriate strategies during practice. In this approach, mathematical content is

regarded as the sum total of topics covered. This means emphasis is on quantity rather than quality of solutions. This strategy thus ignores the effects of learners' negative attitudes and beliefs about themselves as problem solvers.

2.2.1.3   Imitation

In imitation, learners analyze the difference between their solutions and those of a model student, an analysis that then becomes the basis for remedial instruction.

These ways of teaching and learning have not produced learners who can use their knowledge in solving problems in different walks of life. Carpenter & Lehrer (1999:19) point out that it is not at all easy to anticipate all the skills that learners will need in future or the problems they will encounter over their lifetime. It is therefore important that we prepare them to learn new skills, acquire knowledge and adapt their knowledge to solve new problems.

2.3   Problem-based approach

The problem-based approach is believed to equip learners with knowledge and skills that will be beneficial throughout their lifetime. This approach is informed by constructivism.

2.3.1   Constructivism

The constructivists believe that knowledge cannot be transmitted from the teacher to the learner. They view learning as the active construction of knowledge by learners themselves. This does not mean that the learners have to progress on their own or that there is nothing a teacher can do to assist learners in their learning. The teacher has to provide learners with opportunities that stimulate thought and mental activity, which are likely to help learners achieve in mathematics (Orton 1994:38).

Constructivism does not rule out nor prescribe any teaching method, but attention should be placed to uniqueness of learners, their abilities, and their needs. Orton (1994:48) further indicates that constructivism is certainly being interpreted as

advocating discovery and inquiry- based learning, incorporating opportunities for discussion, negotiating and exchanging of ideas.

Constructing mathematical knowledge involves more than acquiring new concepts. It also involves re-constructing prior knowledge. Silver, Kilpatrick & Schlesinger (1990:7) cite an example in which learners who had a misconception on the use of an equal sign were assisted. To these learners an equal sign was considered as 'doing something' signal, as a result although learners understood the equation like "$3x + 5=17$" because there is a single term on the right—the result of doing something, they however; had difficulty solving an equation like '$3x +5= x +17$". It seemed unfinished because the right side still has an addition to be performed. After a sequence of appropriate activities, learners re-organized their understanding so that an equal sign was seen as a relational symbol rather than a 'do something signal'.

When learning is seen as knowledge construction and reorganization, teachers can consider the special ways each student learns, and they can begin to view instruction not as piling of little bricks of knowledge but as an effort to help students make sense out of their world. Learners are seen as active minds making meaning out of their experiences by constantly reconstructing and reorganizing their knowledge (Silver et al 1990:7). During the learner's participation in the construction of knowledge, the learner gradually derives a lot of concepts which form large units of interrelated ideas. Olivier (1989:11) calls these large units of interrelated ideas in the learner's mind schema, which he believed are stored in memory and can be retrieved and utilized. The interaction between a child's schemas and new ideas involves what Piaget originally cited as assimilation and accommodation.

Assimilation: If a new familiar idea is encountered, it can be incorporated directly into an existing schema that is very much like the new idea. A new idea is interpreted or recognized in terms of an existing (concept) schema. The new idea contributes to our schemas by expanding existing concepts and by forming new distinctions through differentiation (Olivier 1989:11).

Accommodation: According to Steffe & Wiegel (1996:491), accommodation is regarded as a modification of a conceptual structure in response to a perturbation. A perturbation may be interpreted as a synonym for a cognitive conflict.

Olivier (1989):11) points out that during accommodation, a new idea may be quite different from existing schemas, a schema may be relevant but not adequate to assimilate the new idea. It is then necessary to reconstruct and reorganize our schema. Such reconstruction leaves previous knowledge intact as part or subset or special case of the modified schema. Learners construct and reconstruct knowledge in order to make meaning of the material and hence to understand better.

2.3.2   Learning and teaching in problem-based approach

Problem-based approach was defined in chapter one. By letting learners 'grapple' with problems for which no well - defined procedures exist, it is believed learners' own understandings of mathematical concepts and skills in problem solving will be developed. Teaching does not take place through the transmission of knowledge from the teacher to the learners; instead learners actively take part in their construction of knowledge. Learners are encouraged to problematize the subject. According to Hiebert et al (1996:12), problematizing the subject most likely leads to understanding.

2.3.2.1   Prior-learning in instruction

In problem-based approach, knowledge that learners bring to the classroom is regarded as the most important factor influencing learning (van Rooyen & van de Merve 1996). Learning can be meaningful only when the information presented to learners links up with their previous experience in a particular field. Bell (1993:11) indicates that it is important then that teachers begin lessons with tasks that allow learners to use and to show their current knowledge. The teacher should then build on this knowledge and help learners develop their own methods, rather than expect learners to put aside their current knowledge and "possibly unsuccessfully, to pick up a new method" (Bell 1993:11). The advantage of creating connections between new knowledge and existing knowledge is that well connected knowledge is remembered better (Hiebert & Carpenter 1992:75). However, Murray et al (1998:278) argue that

even though learners' previous knowledge is vital, it may sometimes be limiting thus giving rise to misconceptions through limited exposure to a concept or through experiences of a limited kind. It is for this reason that assessing learners' knowledge continuously is crucial. Each lesson or topic should start with some questions designed to expose any misunderstandings or misconceptions that learners might have in the topic to be treated. It is necessary to point out that, errors and misconceptions are as important as correct responses. Errors are a result of learners' efforts to construct their knowledge. As far as Olivier is concerned "misconceptions form part of a pupils conceptual structure that will influence new learning, mostly in a negative way, because misconceptions generate errors" (1989:18). After assessing learners, a teacher can use the information obtained to guide his/her instruction and reflect about its effectiveness (Silver et al 1990:21).

2.3.2.2 Reflection

In solving problems, learners learn by reflection. Teaching is based on the fact that learners 'learn by doing' and thinking about what they do, that is, reflection. Reflection "involves the conscious examination of one's own actions and thoughts" (Carpenter & Lehrer 1999:22). Learners become aware of what they know and what they do not know and find ways of addressing their lack of knowledge where necessary. The major value of problem solving occurs when learners step back and reflect on the way they actually used to solve the problem and on whether the strategies could be improved so as to obtain optimum results.

The problem- based approach develops learners' meta-cognition. Meta-cognition refers to conscious monitoring and regulation of one's own thought processes. Learners become aware of their knowledge and understanding or lack of both. They are able to make decisions about whether to continue with the strategy they have chosen, or whether to switch to another strategy or whether to change their point of view (Van de Walle 2004:54). It is important to point out that meta-cognition can be learned and there is evidence that learners who have learned to monitor and regulate

their own problem solving behaviours do show improvement in problem solving (Van de Walle 2004:55).

2.3.2.3   Cooperation

Cooperation is one important strategy through which problem solving can be taught to learners. Learners work in groups in which they bring out their ideas into the open, where they can be refined and defended. Learners can hear their poorly formulated expressions phrased more precisely by their peers. They can also assist each other in grappling with ideas they themselves have just begun to understand (Silver et al 1990: 23). During social interaction with others, learners get an opportunity to clarify concepts and procedures in ways difficult to do alone (Kilpaltrick 1985:10). The main aim of problem- based approach is to promote learners' understanding of mathematics and its application in their lives.

2.4   Mathematical understanding

Mathematical understanding can be viewed from two perspectives, the functional and the structural understanding.

2.4.1   Functional perspective

From the functional perspective, understanding is defined "in terms of the ways in which students contribute to and share in the collective activity of the here and now". This view focuses on the activities that take place in the classroom (Hiebert et al 1996:15). These activities include:

- ❖ The role of the teacher.
- ❖ The role of the learners.
- ❖ The classroom environment and
- ❖ The selection and sequencing of tasks.

2.4.1.1   The role of the teacher in problem- based approach

In order to ensure that learners' construction of knowledge becomes meaningful, the teacher has to play a very active and challenging role. The teacher has to develop a 'mathematical discourse community' of learners that do not only problematize mathematics but also share in searching for solutions. She/he has to select and present tasks that engage learners in reflective inquiry that promotes meaningful learning (Clarke 1997:280, Hiebert et al 1996:16).

Motivation and perseverance are very important factors in successful problem solving. As such the teacher should ensure that the character of the problem solving tasks does not change after students begin working. He/she should also keep the cognitive demands of high level tasks from declining (Erickson 1999:519). In order for learners not to become de-motivated, the teacher should therefore share relevant information with learners as long as it does not prevent learners from problematizing the subject. The learners will make a very slow progress if they have to rediscover everything on their own. Cobb, Wood, Yackel, Nicholas, Wheatley, Trigatti & Perlwitz (1991:12) point out that since conflict is an important aspect of learning, the teacher should also highlight conflicts between alternative interpretations or solutions. The teacher should help learners evaluate one another's suggestions and critically reflect on them by anticipating objections and consequences.

Succeeding in the problem- based approach is not an easy task for a teacher, most importantly because learners are challenged to develop their own strategies for solving problems. Teachers must help these learners to develop their informal strategies into more formal ones which can be applied in other situations (Wubbels, Korthagen & Broekman 1997:2). Tynjala (1999) indicates that a teacher must have sound pedagogical content knowledge. That is, subject content knowledge and the knowledge of how learners learn mathematics. Brophy (1991:352) adds by saying that where the teachers' knowledge is explicit, better connected and more integrated, they tend to teach the subject more dynamically, represent it in more varied ways, and respond fully to the comments and questions of learners. Where pedagogical content knowledge is limited, teachers rely on traditional methods of teaching. A teacher who

knows how learners learn, is able to provide learners with different opportunities to construct knowledge. He or she can easily adapt his/her teaching styles and strategies so as to meet the different learning styles of learners. Mahanye (1996) adds that the teacher must also be confident enough to display initiative and drive, and must continuously reflect on and review his or her classroom practice.

To sum up, one may say a teacher should not be a transmitter of knowledge but a facilitator of knowledge. He/she must be a co-learner with his students in which classroom experiences are "a crucial source of pedagogical problems whose resolution involves the reorganization of their knowledge and beliefs about learning and teaching" (Cobb et al 1991:8).

2.4.1.2   Role of learners

Hiebert et al (1996:16) say that during problem solving, learners share the responsibility for developing a community of learners in which they participate. Thus all learners discuss alternative strategies and or different ways of viewing important mathematical ideas. Learners explain to each other as to why their conjectures and conclusions make sense and why a particular procedure or strategy they have used is valid for the given problem (Carpenter & Lehrer 1999:26). Learners recognize that learning means learning from others, taking advantage of others' ideas and the results of their investigations. They learn to listen to ideas of others and even though they may not agree with them, they become aware that they have to support their own positions with evidence and to reconcile the differences between positions. In this way mathematics is not merely a collection of ways to get answers but a language of thought (Carpenter & Lehrer 1999:26). Students view themselves as a community of learners with all members having different but important roles to play in assisting each other (Forman 1996: 121). According to Ramnarain (1999:83), peer interaction is very important as it can help build up the self-confidence of learners. He further points out that learners feel less anxiety and tension when they express their ideas to peers. Learners strive to become self-directed individuals who routinely engage in constructing, symbolizing, applying, and generalizing mathematical ideas. These,

according to the NCTM (1989:125), are essential for developing the capabilities for learners' life long learning.

From the above discussions, it becomes evident that communication plays a vital role in mathematics instruction. It is thus necessary to briefly look at concerns regarding learners whose home language differs from the language of instruction.

Second language learners in classrooms that support problem-based approach

There are concerns that learners whose main language is not the language of instruction are at a disadvantage in classrooms where communication plays a vital role (Murray et al 1998:282). In South Africa, for example, majority of learners receive instruction in English, which is not their first language. One would expect these learners' participation in class discussions to be limited as they sometimes do not understand the teacher during teaching and worst of all they don't understand the language used in the tasks or problems. This means that sometimes getting incorrect solutions may not have been caused by lack of knowledge but mainly by lack of understanding of the language used. Limited proficiency in the language of instruction is also responsible for learners' comprehension difficulties. Experience has shown that this difficulty in comprehension also contributes to the poor performance of learners. It is not surprising that many learners do not only dislike word/contextual problems but perform badly in these problems. Setati (2002:13) argues that code-switching can be used to assist learners who have a problem with language. Code-switching occurs when an individual alternates between two or more languages. Although Setati points out that there are people who are against code-switching, she believes that it has many advantages. Learners' main language can be used as a code for encouragement, to focus or regain learners' attention, to clarify, enhance or to reinforce lesson material. In South Africa, research has shown that code switching has been used successfully to enable learner-learner and learner-teacher interactions. Using learners' main language opens up opportunities for exploratory talk, and thus for meaning making (Setati 2002:15).

However Adler (in Setati 2002:15) identifies code switching as one of the dilemmas in teaching and learning mathematics. Where the language of instruction is different

from the learners' main language, the teacher has difficulty in deciding whether or not to code switch, especially in public. The teacher also has difficulty in deciding whether to encourage learners to code-switch during group and class discussions. The problem is caused by the fact that learners have to access the language of instruction, as critical assessment will be in this language. Adler concludes by saying "teachers in multilingual classrooms face a dilemma of whether to switch or not to switch languages in their day to day teaching". If they stick to English, learners often don't understand. Yet if they "resort" to Setswana (i.e., they switch between English and Setswana) they must be "careful", as learners will be denied access to English and being able to "improve". It seems therefore that "the dilemma of code-switching cannot necessarily be resolved but can be managed" (Setati 2002:15).

Learners can also be advised to use journals or "learning logs" to record their feelings, frustrations, uncertainties, triumphs or they can use them to record their progress with language or mathematics in general. For second language learners, these logs "can help them to formulate their ideas, and expression of those ideas, before displaying them in class, or at risk on a test" (Silver et al 1990:22). Murray et al (1998:283) point out that the issue of language remains serious and should be researched and debated further.

2.4.1.3   Classroom environment

For meaningful learning to take place, classroom environment should be conducive to learning (Boaler 1993:346, Hiebert et al 1996:16). The environment must be safe for learners to express their ideas without fear of ridicule or embarrassment. In mathematics classrooms, learners do not only learn mathematics but also ways of behaving. "They learn what value their peers and their teacher place on mathematical ability, on verbal facility, on competition, on cooperation, on hard work, and on getting by". They learn whether to try, or to appear to try, or to ignore the teacher. "Classrooms are communities where people agree to behave in certain ways, and where they carry on an extended dialogue even when only a few are talking" (Silver et al 1990:8). Learners need to learn mathematics in classrooms that are microcosms

of mathematical culture. These are classrooms in which the values of mathematics as sense making are reflected in every day activities (Schoenfeld 1989:88).

2.4.1.4   Selection and sequencing of problems

There are different types of problems usually used in mathematics classrooms. Orton and Frobisher (1996) categorized mathematics problems into three main categories. These are routine, environmental and process problems. The three categories are defined as follows:

❖ Routine problems use knowledge and techniques already acquired by a student in a narrow and synthetic context. Routine problems include:

Drill exercises, for example, *324 × 32*

Simple translation problems: Involve translating the words into a simple and single mathematical expression. For example*:*

> *Palesa has R50. She spends a quarter of her money on chips. How much money does she have left?*

Complex translation problems: They involve more than one step, at least two steps. For example:

> *Tennis balls come in packs of four. A carton holds 24 packs. Mr. Samuels ordered 1200 balls. How many cartons did Mr. Samuels order?*

❖ Environmental problems are also referred to as real-life problems. These are set in contexts that represent the real or practical world, or as close as possible to the applied problem. For example,

> *How much paper of all kinds does your school use in one month?*

❖ Process problems are "set in a mathematics context in contrast to real problems. They concentrate on mathematics itself (purely mathematical) and on the mathematical thinking processes for arriving at a solution". The procedure or method of getting the solution is considered the most critical step in the solution (Charles et al 1997:vi, Wessels & Kwari 2003:73). For example:

There are 18 animals in Thwala's farm yard. Some are chickens and some are pigs. If you count their legs you get 58. How many of the animals are chickens and how many are pigs?

These different kinds of problems serve different purposes in the mathematics curriculum. **Drill exercises** provide learners with practice in using an algorithm. They help learners maintain mastery of basic computational skills. **Simple and complex translation problems** provide learners with experience in translating real-world situations into mathematical expressions**. Process problems** exemplify the "processes inherent in thinking through and solving a problem". They develop general problem solving strategies (Lester & Charles 1982:10). **Environmental problems** give learners opportunities to use their mathematical knowledge and skills to solve problems. Through these problems learners see the usefulness of mathematics in their everyday life.

In this study process problems are mainly used for the above reasons and also because they can be used to introduce mathematical concepts. The given example on process problems can be used to introduce linear simultaneous equations. With process problems, a single problem can be solved using many different strategies. The process problem cited above can be solved by using a table, guess and check, equations etc.

In selecting tasks, the teacher should draw on the knowledge of the subject and learners' own thinking (Hiebert et al 1996:16). Knowledge of the subject encourages learners to be exposed to key ideas. Knowledge of students thinking allows the teacher to select tasks linked with students experience and for which learners can see the relevance of the ideas and skills already possessed. The Department of Education (2003:42) adds that "it is through engaging learners in situations of a mathematical nature experienced in their lives that the teacher will bring home to learners the usefulness and importance of mathematical ways of thought in solving problems in such situations. Since problems used are non-routine, Erickson (1999:516) indicates that problems chosen should be interesting and challenging. They should have

multiple solution strategies, multiple representations and multiple solutions. However he warns that although problems chosen should be challenging, they should be of reasonable difficulty level so as not to discourage learners.

Although real world problems should be used in classrooms, Hiebert et al (1996:18) advises that the real world context should not be the only context used since this would be a very narrow approach. Students should solve problems in many different contexts including those contextualized entirely within mathematics. It is however important to note that even though context enhances facilitation of learning, it must be remembered that learners interact with the context in different and individual ways. It is therefore necessary that learners should bring their own 'context' to a task (Boaler1993:346). Extended problems should be amongst the problems done in class because as Bereiter (1992:354) says "they can be more challenging and realistic and that the longer discussion allows for analysis of underlying principles and alternative problem solving strategies".

The selection of tasks is as important as sequencing of tasks. In sequencing tasks, the most important consideration is the learners' prior knowledge. Murray et al (1998:277) believe that students first work on activities involving new concepts that they engage in informally using whatever skills they have available. The teacher then gradually introduces more generally accepted terminology and more rigorous reasoning processes as the students become able to give meaning to these. Murray et al also warn that in sequencing tasks, studying special or easy cases first does not make the development of concepts and skills easier, it merely hampers understanding, and this is due to what they call "limiting constructions".

Ernest (1991:239) points out that although some people believe that learning is hierarchical (that is, teaching should start with concepts or skills which are prerequisites to the learning of subsequent skills or knowledge),
  ❖ the uniqueness of learning hierarchies is not confirmed, and

❖ the hierarchy of concepts is also rejected because of the complex interconnections between concepts, the acquisition of concepts can be a life-long affair.

This means that there is no one hierarchy that best describes the sequence or structure of every learner's knowledge acquisition. Cangelosi (1996:13 - 26) however, indicates that integration of topics is very important. When topics are isolated, for example, fractions from percents and ratio, geometry from algebra, one gets a feeling that there are many concepts to be learned, whereas only a few concepts are to be introduced and content revolves around those concepts. Learners also find it hard to see the connection or relationship between these concepts.

One may sum up by pointing that tasks and tools selected should aim at providing a "window on students thinking, not just so that the teacher can provide more appropriate instruction for specific students, but also so the teacher can construct better models for understanding students' thinking in general" (Carpenter & Lehrer 1999:31).

2.4.2   The structural perspective:

According to Hiebert et al (1996:17), in this view, understanding means "representing and organizing knowledge internally in ways that highlights relationships between pieces of information. The structural perspective focuses on what learners take from the classroom. What learners take from the classroom has been referred to as the 'residue' in section 2.1. This residue is influenced by the knowledge the learner brings to a problem situation and by the nature of the problem solved. The following kinds of residue have been highlighted**:**

2.4.2.1   Insights into structure

Learners who problematise mathematics develop a deeper understanding of the subject matter.

2.4.2.2   Strategies for solving problems

Hiebert et al (1996:17) identify two kinds of strategies produced during problem solving.

❖ The first kind includes particular procedures used for solving particular problems and the general approaches or ways of thinking needed to construct the procedures. The procedures that are left behind will to a large extent depend on the problems that were solved. Hiebert et al further point out that these procedures are the skills that are ordinarily taught at school. (Elaboration of different strategies will be found in 2**.**6)

❖ The second kind is the meta-strategic residue. By problematizing situations, students learn how to construct their own strategies and can adapt them later to solve new problems. In constructing new methods, learners integrate conceptual knowledge with procedural skills. This is important because learners can "rely on their conceptual understandings to drive their procedural advances". This has not been the case in traditional instruction where learners possess understandings that they do not use to inform their procedures. The result is memorization and execution of procedures they do not understand (1996:17).

2.4.2.3   Disposition towards mathematics

Disposition refers to "the attitudes and beliefs that students possess about mathematics" (Van de Walle 2004:55). The way mathematics problems are treated in a classroom influence learners' attitudes (for example likes, dislikes, and preferences) and beliefs towards the subject either in a positive or negative way. These beliefs and attitudes in turn influence learners' orientation toward future activities. Problematizing mathematics makes learners aware that mathematics is a human activity in which they too can participate. As a result learners' attitudes and beliefs are influenced positively. It is worth noting that learners who believe they can solve problems and enjoy doing so tend to become good problem solvers. Likewise, learners with positive attitude towards mathematics and problem solving feel the pleasure and satisfaction in solving challenging problems. They are willing to persevere, attempt a problem several times

and even search out for new problems. The same cannot be said about learners with negative attitude and beliefs (Van de Walle 1998:51).

In their research, Yusof and Tall (internet source:8) hypothesized that students become more successful if they are consciously aware of more successful thinking strategies. This awareness and success at solving non-routine problems not by rote, but in a supportive problem-solving environment develops a positive attitude towards mathematics. Their research findings on "Changing attitude to University Mathematics through problem-solving" confirmed their hypothesis.

2.5   A framework for the process of problem solving

According to the Collins Cobuild dictionary (1995:673), a framework is "a particular set of rules, ideas, or beliefs which you use in order to deal with problems or to decide what to do". A framework for problem solving is used to guide learners and problem solvers in mathematics. Most formulations of a problem solving framework attribute some relationships to Polya's problem solving stages (Wilson, Fernandez & Hadaway 1993:58). Polya identified four stages in problem solving, but other frameworks have more than four stages/phases. For example Sternberg's framework (Dikgomo 2004) has seven stages.

2.5.1   Polya's model for problem-solving

As has been stated in the first chapter, the model for problem solving used in this study is mainly based on Polya's four phases of problem solving. Lester (1985:61-62) however, believes that this model does not consider the meta-cognitive actions, which are guiding forces in problem-solving. He also pointed out that failure at efforts to improve problem-solving may be due to large part on over-emphasis of development of heuristics skills at the expense of meta-cognitive skills to regulate activities. This study therefore, acknowledges and will consider the importance of these skills.

Polya's four phases are:
   ❖   Understand the problem

- ❖ Devise a plan or decide on an approach for attacking the problem
- ❖ Carry out the plan
- ❖ Look back at the problem, the answer, and what you have done to get there.

To Polya, teaching learners how to think was of primary importance. However Wilson et al (1993:60) warn that in some cases teaching learners how to think may be transformed to teaching learners 'what to think' or 'what to do'. This may be caused by an emphasis on procedural knowledge about problem solving as seen in most textbooks. According to Wilson (1993:61), the linear models of problem solving as shown in figure 1 below are very inconsistent with genuine problem solving as they do not promote Polya's goal of teaching learners to think.

```
┌─────────────────┐          ┌─────────────────┐
│  ┌───────────┐  │          │  ┌───────────┐  │
│  │ Read      │  │          │  │ Read      │  │
│  └─────┬─────┘  │          │  └─────┬─────┘  │
│  ┌─────▼─────┐  │   ┌────┐ │  ┌─────▼─────┐  │
│  │ Decide    │  │   │ or │ │  │ Know      │  │
│  └─────┬─────┘  │   └────┘ │  └─────┬─────┘  │
│  ┌─────▼─────┐  │          │  ┌─────▼─────┐  │
│  │ Solve     │  │          │  │ Plan      │  │
│  └─────┬─────┘  │          │  └─────┬─────┘  │
│  ┌─────▼─────┐  │          │  ┌─────▼─────┐  │
│  │ Examine   │  │          │  │ Solve     │  │
│  └───────────┘  │          │  └─────┬─────┘  │
│                 │          │  ┌─────▼─────┐  │
│                 │          │  │ Check     │  │
│                 │          │  └───────────┘  │
└─────────────────┘          └─────────────────┘
```
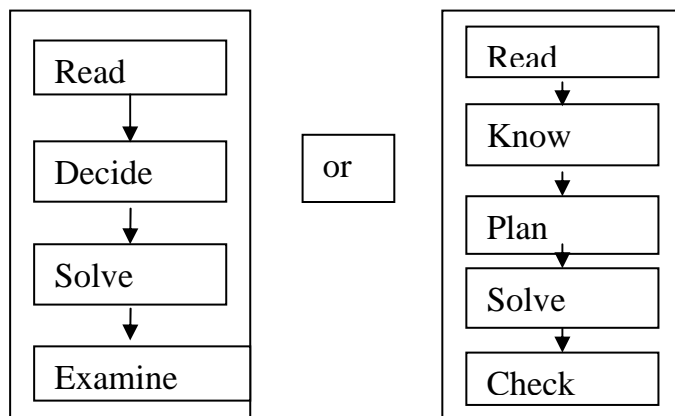
Figure 1   Linear model of problem solving

"By their linear nature, all of these traditional models have the following defects:
- ❖ they depict problem solving as a linear process
- ❖ they present problem solving as a series of steps
- ❖ they imply that solving mathematics problems is a procedure to be memorized, practiced, and habituated
- ❖ they lead to an emphasis on answer getting" (Wilson et al 1993: 61).

Wilson et al further point out that genuine or real problem solving is dynamic and cyclic in nature. A learner who has just understood the problem may attempt to devise a plan but realizes that there is a need to understand the problem better. Sometimes

after the plan has been found, the learner may be unable to proceed. This may force the learner to make a new plan, to develop new understandings, or to pose a new related problem to work on. Wilson et al illustrate the dynamic and cyclic nature of problem solving in the figure below:
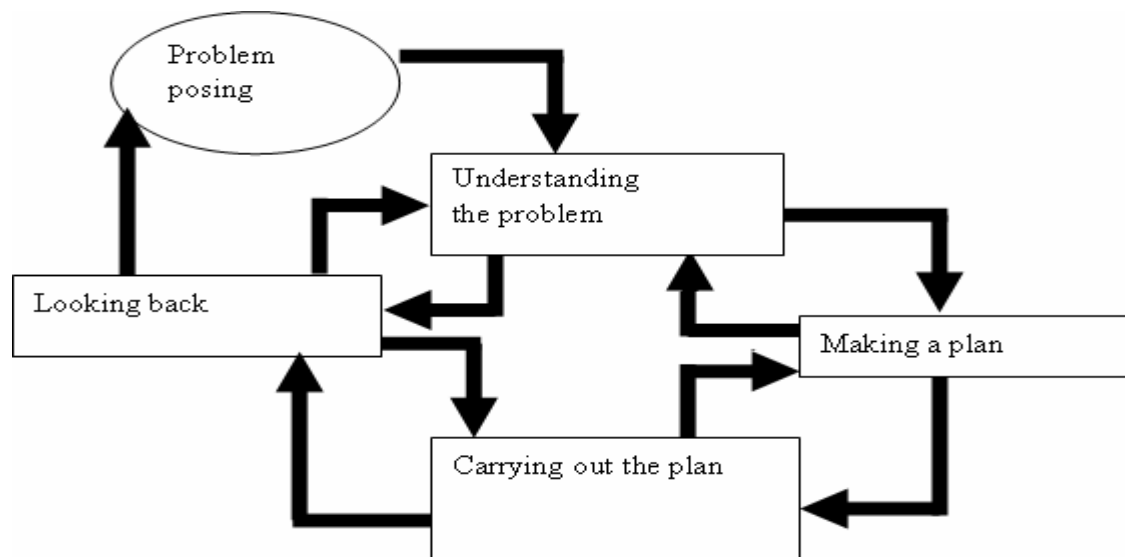
Figure 2   Cyclic model of problem solving

The different stages in problem solving as formulated by Polya and used in this study are going to be discussed individually. It is important to point out that these stages are interrelated, success of the whole problem solving depends on careful consideration of activities at every stage.

2.5.1.1   Understanding the problem

The learners are encouraged to think deeply about the problem at hand and to find out what is known and what needs to be done. They decide on what information is important and what is unimportant. Where necessary the problem may be reformulated in learners' own words (Van de Walle 1998:40). Lenchner (1983:9) believes that at this stage it is important that learners are allowed to ask questions as long as their questions concern the problem. The teacher should decline to answer questions related to the process of solving the problem. Usually when learners first

start working on problem solving they ask questions which clearly indicate their dependence on their teacher. Yusof and Tall (internet source:3) report that when learners become stuck on the problem, their questions included: What shall we do now? Is this the right way to follow? They also show enormous resistance which goes away after some time. When learners do not have questions, it is advisable that the teacher asks questions that prompt their questions.

Through questioning the teacher can also pinpoint learners' difficulty in understanding the problem, especially in cases where language may be a barrier. In South Africa, for example, where learners learn in second language, it is very important that learners are assisted to understand the question properly. Words or phrases that learners do not understand should be discussed. Lack of understanding due to language problems does not only lead to incorrect solutions but also to lack of interest in solving the problem. In some cases learners may think that they understood the problem from the onset, only to find out later that ambiguity surfaces after they have started solving. For example, if a problem refers to a person's work for one week, the learners may not realize until they have started solving the problem that the number of work days in a week is unclear. It is important to discuss this difficulty and clarify it before learners continue (Lenchner 1983:10).

Trying to understand the problem is calming as it gets the problem solver to do something productive without having to decide what to do. Good problem solvers know the advantage of spending time on this phase whereas poor problem solvers rush to the next stage (Van de Walle1998:40).

2.5.1.2   Devising the plan

Once the learners have understood the problem, they then have to decide on a plan of action in solving the problem. It is time to think or reflect on ideas that may be brought to the problem. These may be mathematical concepts and procedures or they may be general processes or strategies (Van de Walle 1998:41). At this stage, one asks questions such as: What do I know? What do I need to do to solve the problem? How can I obtain more information or data to seek the solution? However, it is important to

remember that information that may be brought to the problem or that is needed to solve a problem is unique to each individual. The teacher may find it useful to question learners about their work so as to diagnose their strengths and weaknesses related to the problem. Hints may be provided where necessary

2.5.1.3   Carrying out the plan

Through a plan one gets a general outline of what has to be done. It is very important however, to remember that conceiving a plan is more and very useful if the learner has not received the plan from outside or accepted it from the authority of the teacher. The teacher should encourage learners to solve problems on their own. The usefulness in conceiving a plan on their own (or with little help) is that the plan cannot be forgotten easily. At this stage learners should be reminded that if the chosen plan does not work, they should try an alternative plan suggested in the previous phase (Lester & Charles 1982:37). If their plan seems to work, they should continuously check each step. The learner himself/herself should be convinced of the correctness of each step (Polya 1956:12). The mistake usually done by learners is to rush into this stage and ignore other stages. Although this step is important, success of the whole problem solving process also depends on serious consideration of the first two stages.

2.5.1.4   Looking back

This phase allows the learner to reconsider and re-examine the path that led to the solution and the solution/result itself. In this way they do not only consolidate their knowledge but also develop their ability to solve problems. One should check the result and the arguments used. Lenchner (1983:24) believes that the reasonableness of the answer can be achieved when learners write their answers in complete sentences. Writing answers in complete sentences would result in learners reviewing the statement of the problem or the question being asked and in the detection of a possible error.

During the looking back phase, learners can be advised to derive the result differently whenever possible. Polya argues that the most important duty of a mathematics teacher is to make learners aware that mathematics problems have connections with

each other and with other problems outside mathematics. Lester (1985:47) points out that it is not enough to train learners to check their work and to ensure that the solution satisfies the conditions of the problem. The right direction might be for teachers to focus more attention on solution attempts and less on correct answers. Post problem sessions in which learners share their attempts and discuss their reasons for the choices they made might be one way to bring about this change. According to Polya (1956:14) looking back at the solution is believed to be very interesting for students who have tried their best and have 'the consciousness of having done well'.

2.6 Analysis of different strategies used in problem solving

To teach problem solving successfully, teachers must do more than just assigning problems to learners, they must teach strategies (Dolan & Williamson 1983:ix). The teacher should therefore use appropriate examples and provide learners with meaningful experiences aimed at understanding and learning of strategies. Learners need to be taught a variety of strategies so as to have a wide choice during problem solving. Most textbooks however, channel students to the translation method only: read the problem, write the equation, check the result. Even though this method is powerful and versatile, teaching only one strategy gives learners the impression that all problems can be solved using this strategy. For learners who are unable to write an equation and are without alternate strategies, would not find a solution (Dolan & Williamson 1983:x). Having a variety of strategies to use during problem solving may also influence a learner positively. For example, a learner who attempts to solve every problem through trial and error feels frustrated and loses pleasure in doing the task after a succession of errors. "If this learner had more strategies at her or his disposal, the affective response might had been different" (Boekaerts et al 1995:243).

In solving problems, once the problem is understood, one then selects appropriate strategy or strategies. Here the problem solver matches a useful tool or approach with the task/problem at hand. One can however, only choose an appropriate strategy if he/ she has a variety of strategies from which to choose. Van de Walle (1998:51) points out that strategies that can be used to approach a variety of problems develop with

experience over a long period of time as problems are solved and one reflects on how the problems were solved.

In his research, Ramnarain (1999) investigated how direct instruction in problem solving strategies affected the performance of learners in mathematics. It was found that the strategies-based teaching improved the problem-solving performance of learners as opposed to traditional teacher-centred approaches. The strategies-based teaching has been found to be most effective in engaging learners in the processes of mathematical thinking. By selecting and applying strategies, learners are believed to use both cognitive and meta-cognitive processes. In conclusion on his research on grade 8 learners, Ramnarain (1999:143) concluded that "the implementation of a strategies-based teaching and learning leads to an improvement of attitude of learners towards problem-solving in particular and mathematics in general".

Sigurdson; Olson & Mason et al (1994:375) investigated three teaching approaches:
- ❖ Problem-process approach (where the focus is on meaning and the processes that are being used).
- ❖ Algorithmic-practice (Use of rules or algorithms without meaning attached).
- ❖ Meaning approach, where only meaning is the focus.

Their findings indicate that there was a noticeable improvement in the achievement of low-achievers who were exposed to the problem-process approach.

According to Hembree (1992:264), a meta-analysis of a number of studies focusing on training of sub-skills has shown that such training resulted in increased problem solving performance in mathematics. These sub-skills include drawing of diagrams or pictures, translating from verbal statements to equations, including skills in guess and test strategy.

Different strategies that can be taught to learners

There are many strategies that can be used during problem solving. For example, Malouff (internet source, retrieved in 2004) identified fifty types of problem-solving

strategies. In these discussions the following strategies, as identified by Ohio Education Department, are emphasized as they seem more appropriate for junior secondary learners. It is very important however, to point out that although there is strong support for explicitly teaching problem solving strategies, "one acknowledges that knowing these strategies does not guarantee success at problem solving" (Ramnarain 1999:70).

These strategies are:

2.6.1   Guess and check

One effective strategy in solving certain problems is to make a reasonable guess of the answer. As a strategy the key element is 'and check'. In checking, the problem solver makes an educated guess and then checks it against the conditions of the problem. If the guess fits the conditions, the guess may have solved the problem. Otherwise the guess may have to be improved. This process is repeated until an acceptable answer has been determined.

Guess and check strategy can be used in solving problems similar to the one below.

Happy Holiday hotel is famous for its cheerful bed bugs. In every single bed there are three bed bugs and in every double bed there are 5 bed bugs. If there are 90 bed bugs altogether, how many of each bed are there?

The learner can choose different numbers of single beds and double beds, and check whether the total of bed bugs is 90. If the total is not 90, the learner chooses other numbers until the correct answer, 90 bedbugs is obtained. For example, if the number of single beds is 8, and that of double beds is 10, the total of bedbugs is

$$8 \times 3 = 24$$

$$10 \times 5 = 50$$

The total is 74 and not 90. Trying 10 single beds gives $10 \times 3 = 30$, $90 - 30 = 60 \Rightarrow$ $60 \div 5 = 12$. Ten single beds and 12 double beds will have a total of 90 bed bugs.

2.6.2   Look for a pattern

According to Van de Walle (1998:54), mathematics is the science of pattern and order. As such, pattern searching can be found in many activities. Dolan & Williamson (1983:35) adds that patterns play an integral role in the discovery and application of mathematical concepts, learners should be taught:

❖ to analyze patterns and make generalizations based on their observations

❖ to check the generalization against known information, and finally

❖ to construct a formal proof to verify the generalization.

Although the learners at this level (grade 9 learners) are not mature enough to deal with construction of a formal proof, they are however, mature enough to be introduced to the first two goals.

The following example can be solved using the above strategy.

Suppose Mvula is offered a job and the employer said she would pay his salary as follows: R1 the first day, R2 the second day, R4 the third day, R8 the fourth day and so on to the end of the month. How much will Mvula be paid for working on the tenth day?

By putting the numbers in a sequence, a pattern appears. The learner can now complete the sequence up to the tenth day.

1, 2, 4, 8, 16, 32, …

2.6.3   Construct a table

This strategy is usually used with a *look for a pattern strategy*. Data organized in a tabular form helps one to easily discover a pattern and any information that is missing. Charts and tables are a major form of communication within mathematics (Van de Walle 1998:54). The information in the above problem can be organized in a table in the following way

Day	1	2	3	4	5	6	10
Salary	1	2	4	8	16	32	512

2.6.4 Account for all possibilities

Account for all possibilities is a strategy that is commonly used with *look for a pattern and construct a table.* This strategy does not mean that one has to examine all possibilities rather that one accounts for them in a systematic way.

For example:
> *Last week a carpenter made some three- legged stools and some four-legged chairs. The total number of stools and chairs was 30, and the carpenter used 103 legs in all. How many chairs did the carpenter make? (Draw a table of all possibilities…).*

Number of three legged stools	10	11	12	13	14	16	17
Number of four legged stools	20	19	18	17	16	14	13
Total of legs	110	109	108	107	106	104	103

From the table, one can see that 17 three-legged stools and 13 four legged stools will give a total of 103 legs in all

2.6.5 Act it out

Visualizing what is involved in the problem may sometimes be the main obstacle posed by a problem. To overcome such difficulty of picturing how the actions occur and how they are related, one might find it helpful to use people or objects exactly as described in the problem. In cases where it is not possible to use people or objects, items may be used that represent the people or objects. Acting out the problem may itself lead to the answer or to another strategy that may help in obtaining the answer (Lenchner 1983:35).

The Ohio Department of education (1980a:14) believes that "Going through the motions' seems to give the problem a 'concreteness' that makes it much easier to discover relationships among elements which lead to solution". This strategy is said to be more suitable for young children.

For example:

Suppose one buys a rare stamp for R15, sell it for R20, buy it back for R25, and finally sell it for R30. How much money was made or lost in the buying and selling of this stamp?

Lenchner goes on to say that many people erroneously conclude that a profit of R15 was made. However when this situation is acted out with a friend using slips of paper to represent the money, the correct answer is R10.

2.6.6   Make a model

It is not always possible or convenient to act out the conditions of a problem. It may however still be important to visualize the problem. In such cases, it is sometimes possible to make a model of the problem. In this case, a model may refer to a simple model, an object, a diagram or a pictorial representation. After drawing a diagram, a series of logical steps eventually lead to the solution of a problem. Dolan & Williamson (1983:57-58) identify two essential steps in applying this strategy.

❖ Decide on a model that is appropriate for the problem. Since there is usually more than one appropriate model, selection of one of these models depends on the ability to perceive what is important in the problem and to a large extent on previously acquired knowledge. Learners should therefore be provided with experiences with a wide variety of types of models that are useful in problem solving.

❖ Use the model selected to aid in solving the problem. In some instances, the model itself is the solution, whereas in others it may only be one of possible solutions and its alteration may lead to other solutions.

In Singapore, models play an important part in problem solving in primary schools. Through the use of models, learners develop a deeper understanding and comprehension of the problem's structure and its known and unknown quantities. Most importantly, models enable pre-algebra students to gain experience with the fundamentals of algebraic thinking in a concrete and representational manner (Ferruci; Yeap & Carter 2001:26 - 29)

Look at the following problem where this strategy is applied.

Mr. Seretse had R80. He spent $\frac{3}{5}$ of his money for a mathematics study guide and $\frac{1}{10}$ for a calculator. How much did he spend for both items?

A student might begin by drawing a rectangle that represents R80 and then divide this rectangle into 5 equal parts to represent fifths of R80.

Next the original rectangle is divided again into 10 equal parts to represent tenths of R80
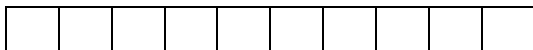
Lastly three of the larger areas can be shaded to represent $\frac{3}{5}$ of the money spent on the study guide. Similarly, one of the smaller areas can be shaded to show $\frac{1}{10}$ of the money spent on a calculator.

The problem is completed as follows:

10 units represent R80

1 unit represents R8

7 units represent R8 × 7 = R56

Therefore Mr Seretse spent R56 for both items.

2.6.7 Work backward

This strategy begins with the goal rather than what is given. Sometimes a problem may contain a series of actions that are better understood and clarified by working back from the end to a desired point in the action sequence (Ramnarain 1999:80). The

main task here is to find the starting point. This strategy can be used in solving the following problem:

> *Thandile was broke when he received his weekly allowance on Monday. On Tuesday he spends R25.50 of the allowance. On Wednesday, his brother pays him the R20.70 he owes him. How much is Thandile's allowance if he now has R45.00?*

To solve this problem, one has to work backwards, starting with the money Thandile has now.

R45.00 – R20.70 (the money given by his brother) = R24.30. Now add the money spend on Tuesday, i.e. R25.50. The total becomes R49.80, which is Thandile's weekly allowance.

2.6.8   Solving a simpler problem

In solving a problem that appears difficult or complicated, one may find it helpful to start by solving one or more similar problems that have simpler conditions. It may happen that the solution of the simpler problems may lead to the solution of the more difficult problem (Lenchner 1983:28). The general idea is to modify or to simplify the quantities in a problem so that the resulting problem/task is easier to understand and analyze.

For example:

> *Before their netball game, 7 girls shook hands once with each other. How many hand- shakes took place?*

It becomes much easier when one starts simplifying the problem by solving it for 2 girls, then increasing the number to 3 girls, 4 girls up to 7 girls.

This becomes: 2 girls   1 handshake

3 girls   3 handshakes

4 girls   6 handshakes

5 girls   10 handshakes

6 girls   15 handshakes

7 girls   21 handshakes

One could see that a pattern emerges, and it becomes easy to find the required number of handshakes

2.6.9   Write an equation or open sentence

Equations, like charts and tables are major tools of communication in mathematics. In using this strategy, a problematic situation is translated into an equation. This is done because an equation is easier to work with; it can easily suggest a familiar pattern or it may be a useful way to communicate an idea (Van de Walle 1998:56). An equation is a powerful tool used in algebra. Algebra involves using a mathematical short hand to represent different quantities and the relationships among them. Letters of the alphabet are usually used as variables to represent the unknown quantities in the problem, and the conditions in the problem are represented by an equation or inequality. By solving the equation or inequality, one would have found the way to the solution of the problem at hand (Lenchner 1983:39).

For example:

> *Two apples weigh the same as a banana and a cherry. A banana weighs the same as nine cherries. How many cherries weigh the same as one apple?*

In this problem there are three unknown quantities, so three variables are used. Suppose a, b, and c are chosen to represent the weight of one apple, one banana and one cherry respectively.

This information can be used using these two equations

> *Two apples weigh the same as a banana and a cherry. The equation becomes:* $2a = b + c$*. A banana weighs the same as nine cherries, gives* $b = 9c$*, solving these two equations yields:* $a = 5c$*.*

The last equation gives us an expression for the weight of one apple in terms of the weight of cherries. Five cherries weigh the same as one apple (Lenchner 1983:40).

2.6.10   Deduction/Logical Reasoning

Logic is the cornerstone upon which mathematics is built as such this is one of the powerful techniques frequently used in mathematics. This strategy may take the form of a process of elimination although learners at junior high school level may not be ready for complete development of formal logic. According to Piaget, they are entering formal operational stage. They should however be introduced to logic in a manner consistent with the level of understanding of learners in this age group. In using this strategy, students will eliminate possible solutions from a given set to determine a correct answer.

For example:

> *Use the given clues to find the required number amongst this list.*
> *9135, 5731, 8361, 7591, 5241, 3715, 5263, 3175, 2537, 1693, 5313,*
> *3174, 7621*
> *These are the clues:*
> a.   *The sum of the first and last digit is equal to the sum of the two*
>   *middle digits.*
> b.   *The four digits in the numbers are all different.*
> c.   *The hundreds digit is smaller than the thousands digit and the*
>   *tens digit.*
> d.   *All of the digits are odd*

By following the clues logically, the required number is 3175

2.6.11   Change your point of view

One may be blocked in finding a solution to a particular problem. This may be caused by one's decision that there is only one way to approach the solution, or one may have made an incorrect assumption about the given information. In this case it is always important to re read the problem and change ones point of view (Lenchner 1983:43).

2.6.12   Looking back strategies

These strategies are referred to as looking back strategies since they are applied mainly after the solution to a problem has been obtained. According to Ohio Department of Education (1980b:20), these include:

Generalize- This strategy is used to extend conclusions obtained during problem solving to more general and far reaching situations.

Check the solution- Sometimes an original problem may become symbolic in the process of finding a solution, one may lose sight of the original problem. This strategy forces learners to check the answer against the requirements of the problem.

For example, learners may be given this problem:

> *When five consecutive numbers are added together, their sum is 155.*
> *Find the numbers*.

A learner may let one of the numbers be n, the next numbers will be $n + 1$, $n + 2$, $n + 3$, $n + 4$. After finding the appropriate equation and solving it, n is found to be 29. A learner who does not *check his/her solution* may say an answer to the above problem is 29.

Checking the answer against the problem will indicate that one has to give the five numbers. The correct answer will be 29, 30, 31, 32 and 33

Find another way to solve it-- this strategy makes learners aware that there are many ways to solve a problem. In trying to find another way to solve a problem, one may develop different and sometimes better ways to confront and deal with the problem.

Study the solution process- -studying the solution process puts the problem into perspective. "It makes the activity a more complete learning experience than answer-getting. Concepts and relationships involved in the problem emerge and fit more logically into a student's personal mental structure" (Ohio Department of Education 1980b:27).

Learners can be encouraged to use this strategy by:
  ❖ Asking them to describe verbally to the class how the problem was solved.
  ❖ Having one learner teaching the solution of a problem to another so that the other learner can solve a similar problem.

❖ Requiring learners to record the solution process with reasons for every step.

❖ Having learners respond to how the answer would be changed or the solution procedures altered by varying certain elements in the problem.

❖ Giving the same problem to learners but without numbers. Asking them how they would find answers if they had the necessary numerical information (Ohio Department of Education 1980b:28).

As has been said earlier these strategies can be used singly or in combination depending on the problem being solved. Again one would also emphasize that teaching learners these strategies does not guarantee their success at problem solving. Problem solving is a very complex process whose success does not only depend on knowledge of strategies, but also on meta-cognitive skills, affective factors and knowledge base in the domain of mathematics. Lester (1994:666) adds that problem solving ability develops slowly over a long time. Thus learners must solve many problems in order to improve their problem solving ability.

2.7   Assessment in problem-based approach

Assessment is a very important part of any instruction. In a problem-based approach learners are not assessed only at the end of a topic or during examinations on how well they can recall the memorized facts and procedures. Instead assessment is an ongoing activity that is integral to instruction. Learners should be assessed on their understanding of concepts and procedures, on their use of different problem solving strategies, and on their reasoning and communication skills. Assessment should also focus on learners' disposition towards mathematics. Information on perseverance and willingness to attempt problems should also be assessed regularly (Van de Walle 2004:61-62).

Assessment should provide both teacher and learners information about their growth towards mathematical power and problem solving ability, not only on mastery of procedural skills. It is important then that multiple means of assessment be used. These include both formal and informal means such as classroom observations, interviews, tests, projects, homework, examinations etc. Learners should get

opportunity to assess themselves, thus giving them more responsibility towards their work. This means self-assessment and peer assessment should be encouraged. Teachers should give guidance so that these assessments are meaningful.

2.8   Conclusion

In this chapter, a literature study was done so as to gain more understanding on problem- based approach and problem solving strategies. To understand problem-based approach better, one felt that it is important to start by looking briefly at the traditional approach to problem-solving. From the literature study it became clear that in the traditional approach, learners do not learn mathematics in a meaningful way. They do not understand mathematics and they develop negative attitudes towards mathematics.

In the problem- based approach, opportunities are created in which learners begin to see mathematics as a human activity. Their prior and informal knowledge is utilized fully. Considering learners' prior knowledge does not only promote understanding but helps learners begin to see the usefulness of knowledge that they already have. According to the literature, it is important that learners do not depend entirely on the teachers but should use their own strategies when solving problems. However, due to the kind of instruction that our learners have been exposed to, our learners may not be aware of these strategies. They are not even aware that during instruction their own strategies are acceptable. As such instruction should focus on problem solving and teachers should provide learners with explicit instruction in problem solving strategies. Research by Hembree (1992), Sigurdson et al (1994), Ramnarain (1999), Yusof and Tall (internet source) has shown that instruction in problem solving strategies improves learners' problem solving performance and their attitudes towards mathematics.

In teaching problem solving, Polya's model was found to be very useful. However, teachers should be careful not to encourage learners to recite the phases in this model instead; the importance of each phase should be emphasized. This can be achieved

when the teacher models the problem solving process, as s/he solves non-routine problems on the board. The learners also become aware that the phases are not linear but cyclic.

To make learners better problem solvers, instruction should also focus on their knowledge base, affective issues as well as their meta-cognitive skills. Lester (1985:45) points out that even an ideal combination of strategies/approaches may not bring about success if attention is not given to the 'guiding forces' of problem solving (meta-cognitive aspects). Shaughnessy (1985:403) on the other hand thinks that, people's confidence in themselves as problem solvers and their beliefs and feelings about mathematics can exert a strong influence on their ability to solve problems.

One would like to close this part by saying that although problem- based approach is very demanding and challenging both for the teacher and the learners, the end-results are however, very rewarding and ever lasting.

CHAPTER THREE
RESEARCH METHODOLOGY

3.1   Introduction

In the previous chapter, a literature study on problem-based approach and problem solving strategies was presented to address the first research question and for the clear formulation of the problem.

To address the second, third and fourth research questions, an investigation into the effect of providing direct instruction on problem solving strategies was carried out. This investigation was conducted in an environment that supports problem-based approach. The role of the teacher in the classroom was therefore that of a facilitator. The teacher provided learners with meaningful experiences from which they can construct their knowledge.

It is important to point out that in this investigation the researcher acted as both a teacher and a researcher. This was done for the following reasons:

- ❖ Although many teachers are aware of problem-solving, few teachers understand the difference between a traditional approach and problem-based approach.
- ❖ For those teachers who understand what problem-based approach entails, majority are neither sure of how to implement this approach in their classrooms nor are they interested even to try it (due to their own valid reasons).

To ensure that the learners are exposed to a problem-based environment, the researcher decided to conduct the intervention herself and determined the value of this approach.

This study may therefore be called evaluation research. According to Schumacher & Macmillan (1997:22-23), evaluation research determines the merit and worth of a particular practice and can stimulate further research.

In this investigation, a problem-based approach was implemented with the experimental group and its worth evaluated. The researcher provided instruction to the experimental group.

In this study both quantitative and qualitative methods of gathering information were used, for example, questionnaires and classroom observations. The researcher felt that one method would not provide enough information. On the other hand by combining the two research methods, the researcher collected sufficient information to draw reasonable conclusions.

3.2   Demarcation of the study

The target population in this inquiry was the grade 9 learners. The grade 9 learners were selected because this study deals with solving of contextual problems and grade 9 end of year tasks, CTAs (Continuous Tasks for Assessment) consist of this type of problems.

Learners from a school in Ficksburg, in the Free State were used for the study. This is a township school with about 1200 learners, of which about 300 are in grade 9. Although the medium of instruction at this school is English, the home language of most learners is South Sotho. The issue of language is pointed out here because it may influence the results of this research. However, the researcher tried to minimize the influence of language by code-switching where necessary.

The school was chosen for its accessibility to the researcher who was an educator at the school. Permission to carry out this investigation was granted by the principal of the school. It was agreed that study time periods after school hours would be the suitable time for the investigation. The mathematics teacher of these learners was also made aware of the investigation.

3.3   Sampling

Groups of learners used were intact, already established classes of learners (Schumacher and Macmillan 1997:335) and there was no random assignment of

subjects. Two classes of mixed ability grade 9 learners at a secondary school were used. One class was used as a control group while the other class as the experimental group. Each group consisted of 45 learners. This figure was determined by the average number of learners in each grade 9 class at the school. The researcher wanted to experience the real challenges facing teachers who would like to implement (or are implementing) this approach in such big classrooms. Reducing the number of learners would not portray a true picture.

3.4   Data gathering instruments

Information was obtained through various instruments such as questionnaires, tests, classroom observations, journals and informal interviews.

3.4.1   Questionnaires

The same questionnaire was given to both the control and experimental groups before and after intervention. It consisted of seventeen questions that were divided into four sections.

(See appendix C).

<u>Section A</u>: Consisted of two fill in questions about gender and age of learners.

<u>Sections B and C</u>: Consisted of multiple- choice questions about learners' attitudes to mathematics in general and problem solving (word problems) in particular.

Learners chose responses from the following five options:

(a) False   (b) Partially false   (c) Don't know   (d) Partially true   (e) True

<u>Section D</u>: This section consisted of an open-ended question. Learners were given one mathematical problem stated in both numerical form and in words. They were required to say, with reasons, which of the two problems they enjoyed solving. This question was posed to determine learners' attitudes towards word problems.

3.4.2   Pre- and Post-tests

The pre-test was given to both groups before the intervention. It consisted of eight contextual problems. The questions were taken from different sources, such as,

articles, books and Olympiads. Some of the questions were taken directly from the sources, whilst others were modified. The problems used were chosen for the following reasons:

- ❖ They were non-routine
- ❖ They can be solved through a variety of strategies
- ❖ They were challenging yet of reasonable difficulty (mathematics required in solving them was within their syllabus).

The pre-test was given to learners on the first week of February 2005. It was written during study time at school. The researcher marked it manually using the rubric shown in Table 1.

The post-test was given to both the experimental and control groups after the intervention. The questions in the post-test were very similar to those in the pre-test. This test was administered during one study period on the third week of March 2005. Just like the pre-test, it was also marked manually with the same rubric that was used for the pre-test.

The rubric was used to assess the learners' achievement. The rubric enabled the researcher to quantify the problem solving processes of the learners. In using the rubric, the emphasis was placed not only on the correct answer, but also on the understanding of the problem and the solution processes that learners used to solve the problems. The rubric catered for the different strategies that learners used. By using this rubric every effort that the learner has put in trying to solve the problem is recognized. For the above reasons, it was considered appropriate for assessing problem solving.

Number of points	Observed characteristics of the student's solution
0	• Blank paper • Numbers from problem recopied-no understanding of problem evidenced • Incorrect answer and no work shown
1	• Inappropriate strategy shown-Problem not shown • Approach unsuccessful-different approach not tried • Attempt failed to reach a sub goal
2	• Inappropriate strategy-but showed some understanding of the problem • Appropriate strategy used-did not find the solution or reach a sub goal but did not finish the problem • Correct answer and no work shown

3	• Appropriate strategy but
	  o -ignored a condition in the problem
	  o -incorrect answer for no apparent reason
	  o -thinking process unclear

4	• Appropriate strategy or strategies • Work reflects understanding of the problem • Incorrect answer due to a copying or computational error
5	• Appropriate strategy or strategies • Work reflects understanding of the problem • Correct answer

Table 1   **Rubric for marking problem solving tests**

In both tests learners were expected to write answers in the spaces provided in their question papers. Learners were strongly requested not to write answers only but to show all the working details that led to their answers. They were also advised to use the sheets of paper provided where the spaces were not enough. Those sheets were then stapled to their answers.

3.4.3   Classroom observations

The learners were observed daily as they solved the problems. The observations were recorded as comments on how the learners were progressing during problem solving and these comments / observations were then reviewed after every lesson. The observations assisted the researcher to identify areas that needed improvements before the next lesson. They also indicated learners' progress on a daily basis.

Classrooms observations concentrated on the following areas:

- ❖ What learners do first when confronted with a problem.
- ❖ Whether appropriate strategies are used.
- ❖ What learners do when the chosen strategy fails.
- ❖ Whether learners in a group are free to express their opinions during discussions.
- ❖ Whether learners were able to explain their solutions to each other in the group and the whole class.
- ❖ What learners do after they have solved the problem.

In order to gain as much information as possible during observations, the researcher had to move around and interview (informally) learners as they solved problems. From these informal interviews the researcher wanted to detect the thinking processes of learners during problem solving. Learners' responses during the interview were also recorded.

3.4.4   Learners' journals

Learners were advised to use the back of their exercise books as journals. They were requested to record their feelings about the lessons, difficulties encountered as well as their progress as often as they can. They were also free to note down questions they would like to ask in the next lesson. Although the researcher could have used some of the information from these journals for her research, these were mainly for learners' benefit**.**

3.5   Validity and reliability of data gathering instruments

In order to obtain valid and reliable data, measuring instruments should be checked for validity and reliability.

3.5.1   Validity

A measuring instrument is said to be valid if it measures what it is supposed to measure. According to Mulder (1989:216), validity can vary depending on the

purpose of a test, therefore various forms of validity exist. In this study content validity was looked into as it was found to be most appropriate.

Content Validity: "By content validity, it is meant how well the test succeeds in covering the field with which the test is concerned" (Mulder 1989:217). Mulder goes on to point out that this validity depends on the opinions of informed persons.

In this inquiry, the tests and the questionnaire were given to two colleagues to determine their content validity. These measuring instruments were also sent to a lecturer at Unisa to give his opinions about them.

3.5.2   Reliability

According to De Vos (2002:168), reliability refers "in general to the extent to which independent administration of the same instrument (or highly similar instruments) consistently yields the same (or similar) results under comparable conditions". De Vos further points out that reliability is primarily concerned not with what one measures but with how well it is being measured. To ensure reliability of the two tests, a rubric was used in marking learners' work.

The pilot study conducted was also found very useful in establishing the validity and reliability of both tests and the questionnaire.

3.5.3   The pilot study

A pilot study "is a small scale survey, i.e. a trial run" (Steffens and Botha 2002:164). During this study a small sample that is representative of the target population was chosen and the same questions and statistical analysis as for the main investigation were used.

De Vos (2002:210) identifies the following aspects of a pilot study:

- ❖ Study of the literature
- ❖ The experience of the experts
- ❖ Preliminary exploratory studies
- ❖ Intensive study of strategic units

These will be discussed briefly.

Study of the literature

This study is intended to update the researcher with existing knowledge related to her inquiry. Through the study of the literature the prospective researcher can formulate her problem clearly. In this inquiry literature was obtained from different sources such as books, publications, journals, internet sources, etc.

The experience of experts

Persons trained in a specialized area or those who have been active for many years in a specific area are valuable resources during research. Ciliers (1973) in (De Vos 2002:213) indicates that information gained from these human resources can help one delineate the problem more sharply and to gain valuable information regarding the technical and practical aspects of the investigation.

Experts were consulted during this study. These included experienced mathematics teachers. Some of these teachers have already completed their masters' degrees in mathematics education and some are currently furthering their studies in this field. Information from these experts was obtained mainly through informal interviews.

Preliminary exploratory studies

Through these studies a researcher obtains an overview of the actual situation where the investigation will be executed. Through preliminary exploratory studies the researcher can be alerted to unforeseen problems that may emerge during the main investigation. These studies are important with a view to practical planning of the inquiry such as transport, finance, time factors and accessibility of the respondents (De Vos 2002:213). This aspect of the pilot study was also addressed. After discussions with the school principal, the researcher was granted permission to carry out the investigation during study period. As such accessibility of the participants was not a problem.

<u>Intensive study of strategic units</u>

The intensive study involves exposing a few cases that are similar to the main investigation, to exactly the same procedures as planned for the main investigation, in order to modify the measuring instruments (Yegidis & Weinbach, 1996) in (De Vos 2002:214). This is also referred to as field-testing the instruments prior to the main investigation.

For the pilot study the two tests and the questionnaire were given to five grade 9 learners. The tests were marked by the researcher. Informal interviews with these learners provided the researcher with information regarding clarity of questions and whether or not the time allocated to these instruments was enough. The pilot study assisted the researcher to make changes to the instruments where necessary.

3.6   Intervention programme

The experimental group was given explicit instruction in problem solving strategies through a problem-based approach. Lessons were conducted four times a week from the beginning of February 2005 until the second week of March 2005. Twenty four lessons were conducted in all.

The teacher (also the researcher) started by making learners aware of what problem solving is and introduced them to the phases in problem solving (Polya's model was used as a framework). Learners were also made aware of the importance of problem solving strategies by indicating that in solving problems, they do not necessarily have to follow a method or strategy used by their teacher. If they know different strategies they can use any appropriate strategy that they understand better and they will still get the same answer as that obtained by the teacher. This was done to capture their interest and to make them see these sessions as useful and not a waste of time. Without learners being interested in these sessions, not much will be achieved.

Problem solving strategies were introduced to learners with problems exemplifying different strategies. First the problem was given and solved by the whole class. Then learners worked individually and in groups in solving a collection of problems

requiring the use of a particular strategy. For example, to introduce 'working backward' strategy, a problem that could be solved by that strategy would be given and the whole class came up with ideas to solve it. After it had been solved, learners were informed that the problem solving strategy used is called 'working backward'. However if a different strategy was used to solve the given problem, that strategy was discussed and learners requested to find another way of solving the problem and guided into using the targeted strategy. Learners were then given more problems to solve individually and in groups. They were also advised to solve one problem in different ways where possible. This procedure was followed until all the problem solving strategies as indicated in chapter two were introduced. Learners were then given more problems in which they had to choose the most appropriate strategy for solving a particular problem. Learners compared and discussed their strategies and answers. Problems used during lessons were mainly contextual problems.

While the teacher illustrated an example for learners, the problem solving process was modelled; thinking aloud and moving back and forth through the different phases of the process.

The following were emphasized during intervention:

❖ Learners were always advised to make sure that they understood the problem before solving it. Sometimes the whole class worked together in trying to understand the problem. When a common understanding was reached, learners moved on to the next phase of problem solving.

❖ Both individual and group-work were encouraged: Learners worked individually and then moved to their respective groups where their strategies were discussed. The whole group would then agree on appropriate strategy/strategies for solving a particular problem. Learners were encouraged to "think aloud" as they solve problems.

❖ That the problem solving process is not linear. Even after the problem was solved, learners may still feel it is necessary to go back to the problem to find out whether they have indeed understood what the problem required.

❖ After the problem was solved learners checked whether their answers made sense. Reflection on the solution process was emphasized.

❖ A learner from each group (with the help of other group members) had to explain their solution process to the whole class, answer questions from other learners in the class concerning the strategy used or the way the problem was solved. The emphasis was therefore more on the solution process than on the answer.

❖ The teacher facilitated the learning process. Guiding learners and providing support where necessary but most importantly asked the 'why' questions to probe learners to explain their solutions.

To minimize the influence that might be brought about by language, learners were also allowed to code-switch where necessary.

3.7   Statistical methods used

The arithmetic mean, standard deviation, range, maximum and minimum scores are explained briefly and an indication of how they are interpreted in this study is made.

The arithmetic mean (which will be referred to as 'mean') is the most important and frequently used measure of central tendency and the score of each learner is used in computing it (Schumacher & Macmillan 1997:11).

A high mean score in a test indicates high performance or achievement in a test. On the other hand, the lower the mean score is, the lower the performance of learners in a test.

In this analysis therefore, a high mean score will indicate that learners' problem solving performance in the given test is high and vice- versa.

Standard deviation is a numerical index that tells how the scores are distributed from the mean. The larger this numerical index is, the wider the scores are distributed around the mean and vice-versa (Schumacher & Macmillan 1997:18).

Range is the difference between the highest and lowest score in a distribution (Schumacher & Macmillan 1997:17). It indicates to what extent scores are distributed

in a test. In this study the lowest and the highest scores will be presented and the range calculated.

Maximum score is the highest score in a distribution / test
Minimum score is the lowest score in a distribution / test

T- test

A t-test was used as it is considered an important tool in determining the significance of differences in calculated means. It helps one to decide whether to keep or reject hypotheses.

The null hypotheses will be tested so as to determine whether or not they should be retained or rejected. They will be tested by making use of the calculated means of both groups in different tests.

To determine whether the mean scores in the given tests are significantly different, a t-test will be used. First a t-test for independent data will be used to compare the means of both groups in the pre-test. This test will also be used again to determine whether or not there is a significant difference in the mean scores of the experimental and the control groups in the post-test. A t- test for dependent data will be used to compare the means of the experimental group in both tests.

The results of the t-test will be interpreted in the following way:

If the calculated t-test value is greater than the critical t-value (Table value) on the 0,05 level but not on the 0,01 level, the null hypothesis is rejected on the 5% level of significance. It can therefore be stated with 95% confidence that there is a statistically significant difference in the mean scores being compared.

If a calculated t-value is bigger than the critical t-value at (particular degrees of freedom) on the 0,01 level, the null hypothesis is rejected on the 1% level of significance, meaning there is a 99% confidence that a statistically significant difference exists between the means that are being compared.

On the other hand, if the calculated t- value is smaller than the critical value on the 0,05 level, the null hypothesis may be retained. Retaining the null hypothesis means that there is no statistically significant difference in the mean scores of both groups.

3.8   Analysis and interpretation of results

Results of both the pre- and post-tests were presented in tables, bar-charts and line graphs. Tables, bar–charts and line graphs were chosen because they represented information in a simple way. Analysis and interpretation of the findings was substantiated by the information from the literature study.

An analysis of learners' work in the pre-test was done to determine learners' approaches in solving problems in traditional classrooms hence answering the second research question. Analysis of learners' work during class observations and in tests was made to determine whether or not learners were able to select the correct problem solving strategy or strategies in solving the given problems. Analysis was also done to determine whether or not learners used the chosen strategy correctly. The mean scores of the pre-test of both groups were compared by means of a t-test to find out whether there was a significant difference in these scores or not. This was done to determine whether or not the abilities of the two groups could influence the findings of the research. A two - tailed test for two sets of independent data was chosen because there was no reason to believe that there will be a significant difference in the performance of both groups. If the calculated t-test value is smaller than the table value at the chosen degrees of freedom, the null hypothesis will not be rejected. If the calculated value is larger, the null hypothesis will be rejected. The same test was also conducted for the mean scores of both groups in the post-test. The results of the t-test revealed whether or not explicit instruction in problem solving strategies improved the problem solving performance of the learners.

Comparison of the mean scores of both groups in the pre-test was done to ensure that abilities of the learners in the two groups did not differ significantly. The following null hypothesis was put forward:

There is no significant difference in the performance of the control group and experimental group in solving word problems in the pre-test.

A t-test was computed to determine whether there is a significant difference in the performance of the experimental group before and after intervention. This was done to determine the effectiveness of instruction in problem solving strategies with the experimental group and to answer research question three. The following null hypothesis was thus put forward:

There is no significant difference in the problem solving performance of the experimental group before and after intervention.

The results of the questionnaire were divided into themes, presented in tables and analyzed. Frequencies of learners' responses before and after intervention (intervention applies to experimental group only) in both groups were compared. This was followed by the comparison of the responses of the experimental group before and after intervention.

The results of the questionnaire were used to answer research question four which seeks to determine learners' beliefs and attitudes towards mathematics in general and problem solving in particular.

3.9 Synthesis

The methodology that was used for this study was explained above. The reasons for the researcher taking part in intervention and why researchers act as teachers were put forth. It was apparent that both quantitative and qualitative data gathering instruments were used so as to obtain as much information as necessary. It was also pointed out how data will be presented and analyzed in the next chapter.

CHAPTER FOUR
PRESENTATON, ANALYSIS AND
INTERPRETATION OF FINDINGS

4.1   Introduction

In the previous chapter methodology carried out in this study was explained. In this chapter findings from the investigation carried out will now be presented in order to address research questions two, three and four. The results of the pre- and post-tests (see appendix A and B) will be analyzed by means of relevant statistical tools such as the mean, the standard deviation, range including minimum and maximum values obtained in the tests. These statistical values will not only assist one in comparing the performance of both groups but will help one in deciding whether or not the intervention improved learners' performance in problem solving. A t- test will be used to determine whether the mean scores obtained are statistically different thus confirming or rejecting the null hypothesis developed. Through the analysis of learners' responses in the questionnaire, The researcher will be able to deduce how the intervention has influenced learners' perceptions about mathematics and their attitude towards word problems in particular and mathematics in general.

This chapter will be concluded with a summary of the main findings and conclusions drawn during analysis and interpretation of results.

The findings from the tests will now be presented.

4.2   The pre-test

The pre-test consisted of eight non-routine, word problems (See appendix A). Maximum score of each question was five thus giving a total score of 40. The marking rubric for scoring the questions is shown in Table 1.

Summary of results is given below:

	Experimental group	Control group
Total score	40	40
X	7,13	6,93
SD	4,45	4,19
Maximum score	17	18
Minimum score	1	1
Range	16	17
N	45	45

Table 2   Results of Statistical analysis of pre-test scores

Legend: X = arithmetic mean

SD = standard deviation

N = number of learners who wrote the test

The pre-test will be analyzed further by looking at the mean scores, frequency of scores, the mean score per question and learners' responses in each question.

4.2.1   The mean scores

The mean scores for the pre-test are presented in the following table.

Experimental group	7,13
Control group	6,93

Table 3   Mean scores of both groups in the pre-test.

The mean score for the experimental group was slightly higher than that of the control group as a result; one may think that the performance of the experimental group was better than that of the other group. It was therefore important to use statistical techniques to determine whether or not there was a significant difference in the mean scores of these groups. A t-test was used.

The t-test for two independent sets of data was calculated. This test was chosen because data from two different groups was being compared. The calculated t-value

was 0,2172. This value was smaller than the critical value for a two-tailed test at both 5% and 1% level (2,5396) at 88 degrees of freedom. Therefore, the null hypothesis was not rejected. This means there was no statistically significant difference in the mean scores of both groups in the pre-test. It can be concluded that there was no significant difference in the performance of these two groups in the pre-test. (See appendix D for calculations of t- test).

4.2.2   Frequency of scores

Scores of both groups in the pre-test are presented in Figure 3.
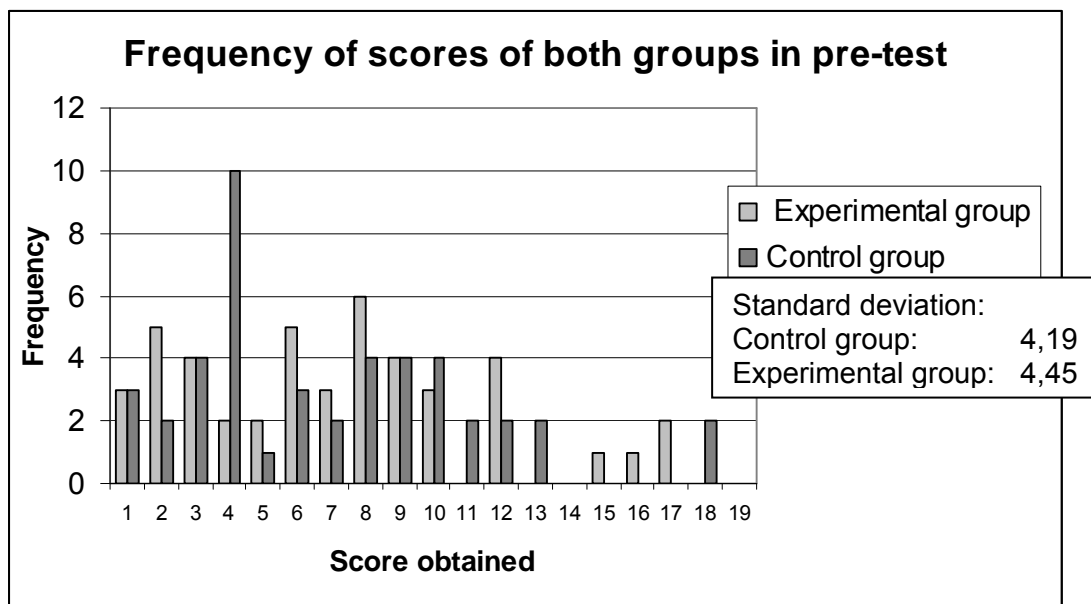
Figure 3   Bar-chart showing frequency of scores of both groups in the pre-test

From the above two representations, it can be seen that the frequencies of the different scores differed very slightly (with not more than 3) except for score 4. Ten learners in the control group obtained 4 marks compared with 2 in the experimental group. Generally the performances of both groups were similar.

4.2.3   The mean score per question.

(Maximum score: 5)

ITEM	1	2	3	4	5	6	7	8
Control	1,84	1,09	0,82	0,22	0,60	1,16	0,62	0,78
Experimental	2,09	1,18	0,56	0,36	0,33	1,40	0,76	0,76

Table 4   Mean score for each question in the pre-test

The above means show that the performance of both groups in the pre-test was almost the same and this can also be deduced when one looks at the following line graph (Figure 4).
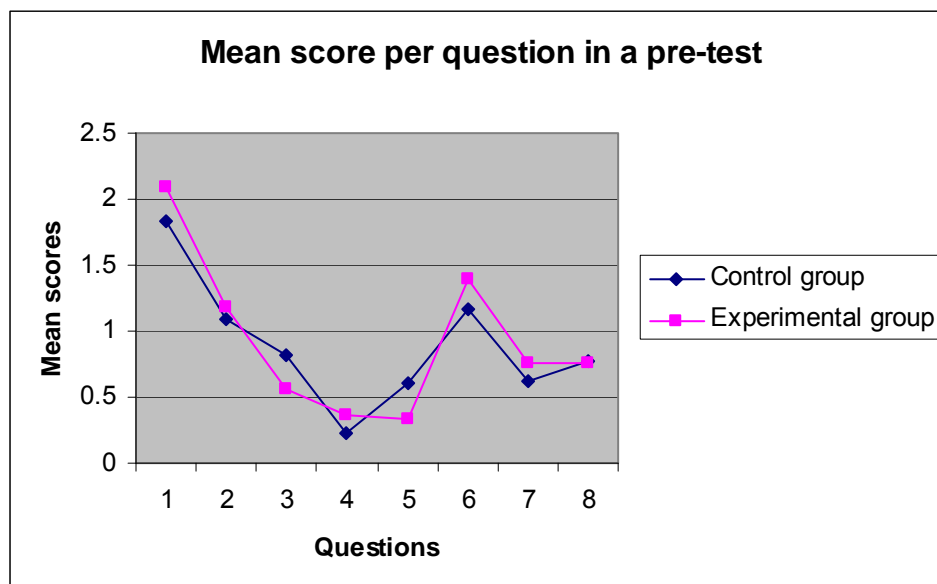
Figure 4   Line graphs showing mean scores per question in the pre-test.

The two graphs closely overlapped indicating that the problem solving ability of learners in both groups did not differ much. This was also confirmed by a t-test.

In general the two groups performed unsatisfactorily in the pre-test. A majority of learners did not score any marks for questions 3,4,5,7 and 8 hence the mean scores were less than 1. For the other three questions also learners scored low marks as evident in Figure 3. Examples of responses that are included in the discussions to follow reveal that learners had limited knowledge, skills and strategies essential for problem solving.

4.2.4   Analysis of responses from both groups

For each of the questions 1, 2 and 6, learners' mean scores in both groups were greater than 1. As a result these questions were grouped together in the following discussions. The mean scores ranged between 1,09 and 2,09 out of a possible score of 5 (See Table 5). On the other hand for questions 3, 4, 5, 7, and 8, learners' mean scores are less than 1 (almost 0) as such these will also be discussed together.

Question 1

The total score for this question was 5 and the mean scores were 1.84 and 2.09 for control and experimental groups respectively.

Most learners recognized the pattern; however, a majority of these learners were not able to answer the question correctly. Several learners' responses were as follows:

$$1p + 3p + 5p + 7p + 9p + 11p + 13p + 15p + 17p + 19$$
$$= 100$$
$$\therefore \text{Bus will picking up a } \underline{100} \text{ passengers.}$$

Other learners did not complete the pattern to the 10th stop as indicated in the example below:

$$\textcircled{U} = 1, 3; 5; 7, 9, 11$$

Eighteen learners in the experimental group and 12 learners in the control group obtained four marks and higher.

Question 2

The total score for this question was 5 and the mean scores were 1.09 and 1.18 for control and experimental groups respectively.

Most learners wrote down the answer without showing how it was found. The researcher believed that these learners might have found the answer by trial and error. On the other hand it is possible that learners did not think that the solution process was important but only the answer as it is usually emphasized in traditional classrooms. This confirmed what was pointed out by Wilson et al (1993:61), that traditional models of problem solving lead to an emphasis on getting the answer.

Question 6

The mean scores for this question were 1.16 and 1.40 for control and experimental groups respectively. The maximum score for this question was 5.

During discussions with learners it became evident that this was one of the questions that was clearly understood by most learners since they were involved in sports competitions in one way or the other. It is important then as indicated by the Department of Education (2003:42) that tasks selected should be related to learners'

experiences so that they become meaningful and learners build on these experiences. Learners were however, not able to solve the problem, but at least most attempted this

Several learners divided 8 by 2 and said 4 games will be played.

It became obvious that learners lacked adequate skills or strategies to solve this problem. The best way to find the solution to question 6 was to draw a diagram showing different teams. However an interesting solution by one of the learners is as follows:

Questions 3, 4, 5, 7 and 8
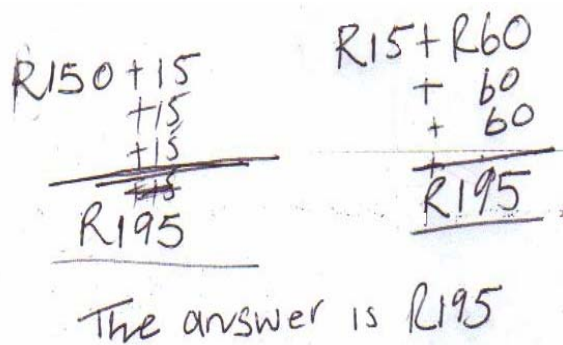
As indicated earlier, the mean score for each of these questions was smaller than 1 out of a possible score of 5.

In questions **three, four and five** learners either did not write anything or copied some of the numbers appearing in the questions without any attempt to solve them. This reaction may be caused by the fact that question three involved fractions, question four involved percent and question five involved ratio. These concepts are

related, though they are not portrayed as such in most classrooms. This seems with the argument that isolating topics gives an impression that there are many concepts to be learned and learners do not easily see the relationship among topics thus they may take longer to understand (Cangelosi 1996). A learner who does not understand one of these concepts may also have difficulty working with the others. Therefore the researcher felt that learners were not competent in working with these concepts.

In question **seven**, it was evident that although learners understood this question, they did not know what to do with the given information. An observation here was that some learners gave their answers in rands instead of days indicating that they never looked back at the problem to see whether their answer made sense or not**.** The following responses from learners' work reveal the above observation.

 One learner wrote:
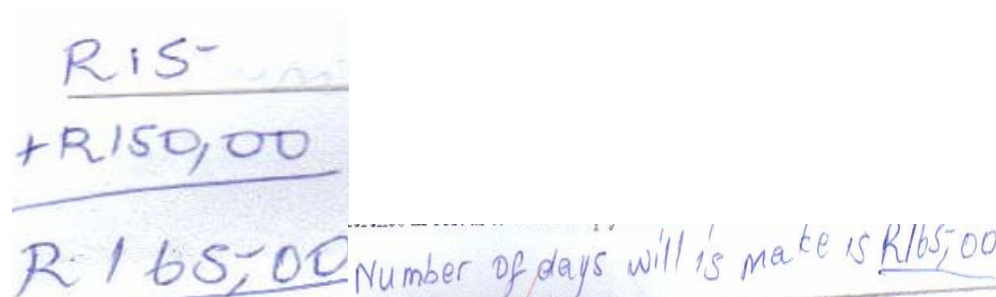
Another learner wrote:
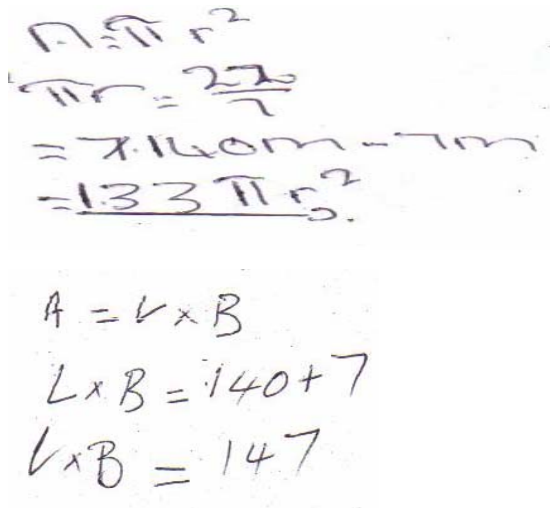
During problem solving one may lose sight of the original problem and looking back strategies force one to check the answer against the requirements of the problem (Ohio Department of Education 1980b:20). Reflection is one of the major aspects of problem solving and in this case learners did not reflect on their solutions. This is one of the issues that needs attention in teaching by problem-based approach.

In question **eight,** learners wrote the formulae for the area of the rectangle and/or formulae for the area of a circle but did not know what to do with these formulae. Below are examples of what some learners wrote:

$$\pi \cdot \frac{2}{3} \pi r^2$$
$$\pi r = \frac{22}{7}$$
$$= 7.140m - 7m$$
$$= \underline{133 \pi r^2}$$

$$A = L \times B$$
$$L \times B = 140 + 7$$
$$L \times B = 147$$

In most mathematics classrooms, the mistake made by educators is to do an example on the board using the formulae after which learners are given problems to practice using the formulae. If there is no example done for these learners, then it is difficult for them to use these formulae in solving problems even though they know the formulae. Hiebert et al (1996:17) believe these learners possess knowledge that they do not use to inform their procedures, thus resulting in memorization and execution of procedures without understanding.

The common observation made while marking and during informal interviews with learners was that they believed the answer was more important than the solution method leading to the answer. It could be that this practice was based on traditional instruction which contributes to this belief. Kaput (1999:133) points out that in these classrooms, learners are graded not on understanding of the mathematical concepts and reasoning, but on their ability to produce the right symbol string- answers. It was not surprising therefore that in some instances learners wrote only the answer and in others, where they were uncertain they did not write anything. Lester points (1985:47) out that the right direction might be for teachers to focus more attention on solution attempts and less on correct answers.

4.3   The post-test

(See appendix B)

The post-test was administered immediately after the intervention with the experimental group, which was aproximately six weeks after the pre-test.

Questions in this test were similar to those in the pre-test. Questions in both tests were matched. e.g. question 1 of pre-test was similar to question 1 of post-test. The same marking rubric that was used for the pre-test was also used for the post-test.

Table 5 shows statistical analysis of results for the pre-and post-tests for the two groups. This analysis assisted in making comparisons on the general performance of these two groups in the tests.

	Experimental group		Control group	
	Pre-test	Post-test	Pre-test	Post-test
Total score	40	40	40	40
X	7,13	12,57	6,93	8,44
SD	4.45	4.80	4.19	3.31
Maximum score	17	27	18	17
Minimum score	1	3	1	2
Range	16	24	17	15
N	45	45	45	45

Table 5   Summary of results of both groups in the two tests

4.3.1   The mean scores

It can be seen from Table 5 that in the pre-test, the mean score of the experimental group (7,13) was slightly higher than that of the control group (6,93) therefore, to compensate for this difference, the mean scores in the post-test were adjusted. This was done to minimize the possibility that selection based on mathematical ability will be a threat to this study (Schumacher & Macmillan 1997:376).

The means were adjusted by 0,1 because the mean of the experimental group in the pre-test was 0,1 higher than the mean of both groups in a pre-test.

The mean score of the control group in the post-test was increased by 0,1 and that of the experimental group was lowered by 0,1.

The adjusted means are shown below:

Groups	Mean before it was adjusted	Mean after it was adjusted
Control	8,44	8,54
Experimental	12,57	12,47

Table 6   Adjusted means of post-test

4.3.2   Frequencies of scores

Figure 5 below shows that the lowest scores were 2 and 3 for the control and experimental groups respectively. The highest score for the control group was 17 while that of the experimental group was 27.

The scores of both groups in the post-test are represented in the following bar chart
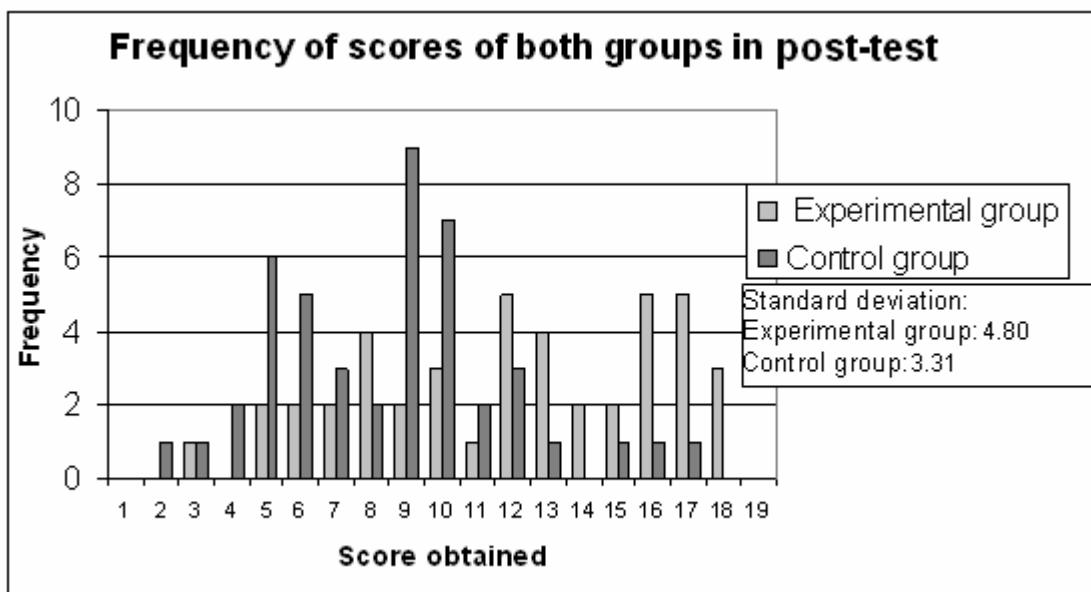
Figure 5   Bar-chart showing frequency of scores in a post-test.

These results revealed that most of the learners in the control group obtained ten marks and lower, whereas in the experimental group numbers of learners obtaining different marks were evenly distributed.

4.3.3   The mean score per question

The mean scores for each question in the post-test are shown in the following table:

Question	1	2	3	4	5	6	7	8
Experimental group	2.96	1.6	1.80	1.09	0.7	1.38	2.98	0.51
Control group	1.87	1.04	1.16	0.87	0.78	0.6	1.04	0.93

Table 7   Mean score per question in a post-test

Table 7 shows that the experimental group performed better than the control group in six out of eight questions. The group that received instruction in problem solving strategies performed better than the group that did not. These findings support Ramnarain's (1999:143) conclusion that explicit instruction in problem solving strategies improves learners' performance in mathematics.

4.3.4   Strategies revealed in learners' work

From learners' work in the post-test, it was apparent that learners in the experimental group were now aware of different strategies because they tried to use those strategies in the test. Their work revealed that they used tables, drew diagrams, used trial and error, arranged numbers systematically and looked for patterns.

Here are few examples of some strategies that learners used to solve No. 5:

(a)

(b)
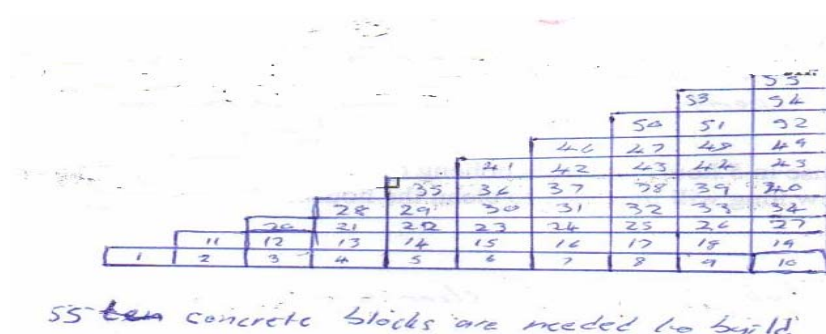
(c)

steps	4	5	6	7	8	9	10
Con. blocks	10	15	21	28	36	45	55

55 concrete blocks are needed to build 10 steps

However, it is interesting to point out that learners avoided algebraic methods to solve problems. It is also important to point out that very few learners left out the working details of the problem. One would believe that learners were now aware of the importance of writing out the whole solution process and not only the answer. I believe the explanations that were required during class discussions contributed to this behaviour. According to Lester (1985:47) post problem sessions in which learners share and discuss reasons for their choices during problem solving reveal the importance of writing out the solution process.

The control groups' use of strategies was still very limited.

In questions five and eight, the control group performed slightly better than the experimental group, however, one cannot say much because for both groups, the mean score is less than 1 in these questions, which is still a very unsatisfactory performance. One concludes by pointing out that although the mean scores per question had increased in most of the questions for the experimental group, the overall performance was generally still low. According to Kantowski (Lester 1985:43), students' ability in problem solving increases gradually over time and numerous skills and procedures involved develop at different rates. The results of the post-test also revealed what was said earlier that improved performance in problem solving does not only depend on the knowledge of problem solving strategies but also on many other factors (Lester 1985:44).

Learners' solutions indicated that in some instances, their content knowledge was limited and in others it was clear that learners were unsure of which strategies to use.

4.3.5   Comparison of mean scores

A t-test for independent data was calculated to compare the means and to test the null hypothesis:

> *There is no significant difference in the mean scores of both control*
> *and experimental groups in the post-test.*

The calculated value was 4,4706 which is larger than the critical value for two-tailed test at both 5 % and 1 % level of confidence. Since the calculated value was larger than the critical value at both of these levels, null hypothesis was rejected (see 4.2). Therefore the following conclusion was made:

> *There is a significant difference in the mean scores of the control and*
> *experimental groups.*

It can be deduced from the above conclusion that the experimental group performed better than the control group in the post-test.

4.4   The experimental group

A comparison of results obtained by the experimental group in both tests was made to determine the effect of explicit instruction in problem solving strategies on learners' achievement in solving non-routine problems.

4.4.1   Comparison of pre- and post-tests

Figure 6 shows a bar chart of frequencies of scores from the pre- and post-tests. The total score for all questions was 40 in each test.
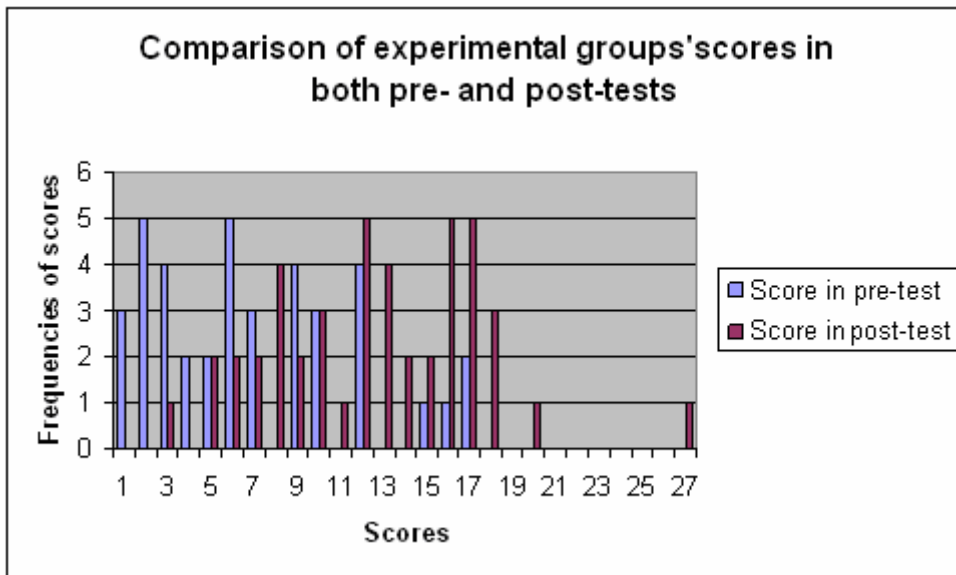
Figure 6   Bar-chart showing frequencies of scores in both tests

The figure reveals that the frequency of learners obtaining eleven marks and more, increased in the post-test as compared to the pre-test. This result meant that learners performed better in the post-test than in the pre-test. The mean scores per question for pre- and post-tests were compared using the line graphs (see figure 7).
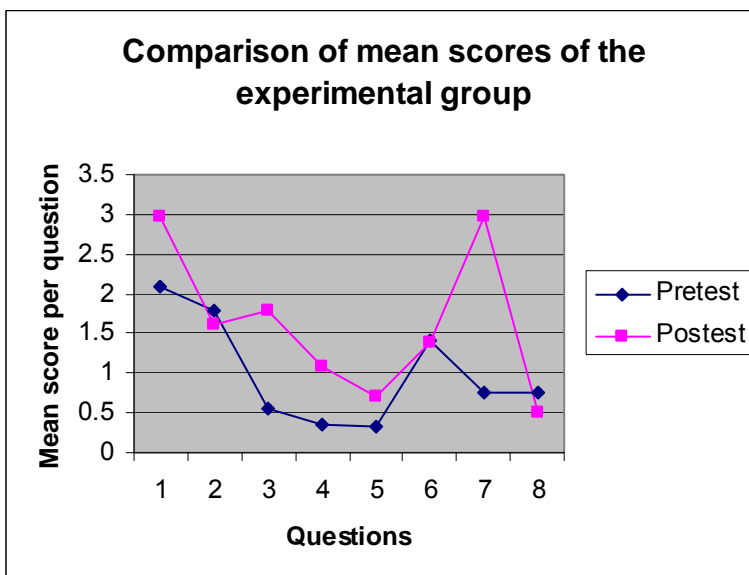
Figure 7   Line graphs showing mean scores of experimental group in both tests.

The line graph for the experimental group mostly lies above that of the control group indicating better performance for the experimental group than the control group.

It was therefore important to conduct a t-test to determine whether or not the mean scores obtained in these two tests were significantly different. The result assisted in addressing the following null hypothesis:

There is no significant difference in the mean scores of the experimental group in the pre- and post-tests.

A *t*-test for dependent data was calculated. The calculated value at 44 degrees of freedom was 7,0597. This calculated value is bigger than the critical values at both 0,05 and 0,01 levels and as such the null hypothesis was rejected .

Rejecting null hypothesis meant: *that there was a statistically significant difference in the mean scores of the pre- and post-tests*

These results indicated that there was a statistically significant difference in the performance of the experimental group in both tests. The experimental group performed significantly better in the post-test as compared to the pre-test.

It was concluded that instruction in problem solving strategies improved learners' performance in problem solving.

SYNTHESIS

From all the above discussions the following points became apparent.

❖ In most mathematics classrooms the type of problems which learners usually solve are routine problems which do not develop learners' problem solving skills. In these classrooms too, learners are expected to practice on exercises similar to that worked out by teachers as examples following the exact method used. As a result, learners lack knowledge of different tools or strategies necessary to solve problems, as such they perform badly in cases that require them to solve non-routine problems.

❖ In classrooms where learners are exposed to both routine and non routine problems and where they are equipped with different skills and strategies learners are likely to perform better in mathematics. In this investigation learners who were instructed on different strategies performed slightly

better than those who were not. Their selection and application of different strategies has also improved slightly. However, it needs to be remembered that success at problem solving takes time and depends on many factors. It was not surprising then that the increase in performance is slight.

4.5   The questionnaire

The learners' questionnaire (see appendix C) consisted of 17 items and four sections.

Section A: Consisted of items 1 and 2 about learners' gender and age respectively.

The results of the first two items are shown below:

AGE	FEMALE		MALE	
	Experimental	Control	Experimental	Control
13	1	1		1
14	10	8	8	9
15	9	10	11	10
16	4	4	2	2
Total	24	23	21	22

Table 8   Frequency distribution of age and gender by group of learners.

Table 10 shows that there was not much difference in the two groups as far as gender and age of learners were concerned. The influence of age and gender on the results is thus minimized. The age of learners in both groups ranges from 13 to 16 years. There were 24 females and 23 males in the experimental and control groups respectively. In the experimental group there were 21 males and in the control group there were 22 males.

4.5.1   Learners' beliefs and perceptions

Section B of the Questionnaire (Appendix C)

Items 3, 4, 5, 6, 7, 8, 12 and 15 addressed learners' beliefs and perceptions about mathematics teaching and learning. Items 12 and 15 were included under this theme as it became evident during analysis of data that they fit better under beliefs and perceptions than in attitudes.

4.5.1.1 Summary of learners' responses.

From the summary below it became apparent that learners' responses in the two groups in the pre-test did not differ much except for items five, seven and eight. Since responses of the two groups differed very slightly in five out of eight items, it was concluded that learners' beliefs about mathematics in the two groups were almost the same before intervention.

Table 9 shows distribution of responses of both groups in the pre- and post-tests in percentages. (Responses of the experimental group are written in **bold**).

Pre stands for responses before intervention.

Post stands for responses after intervention with the experimental group.

Items	3		4		5		6		7		8		12		15	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
False	22.2	22.2	2.2	8.9	24.4	42.2	37.8	37.8	11.1	4.4	51.1	35.6	4.4	8.9	2.2	4.4
	17.8	**26.7**	**2.2**	**0**	**15.6**	**6.7**	**51.2**	**40**	**6.6**	**4.4**	**13.3**	**35.6**	**4.4**	**62.2**	**2.2**	**71.1**
Partially False	6.7	8.9	15.6	0	17.8	6.6	20	13.3	17.8	6.6	4.4	6.6	2.2	0	0	11.1
	4.4	**2.2**	**0**	**0**	**6.7**	**8.9**	**15.6**	**6.7**	**2.2**	**0**	**0**	**0**	**0**	**6.7**	**2.2**	**4.4**
Don't Know	8.9	2.2	4.4	8.9	11.1	6.6	4.4	8.9	4.4	13.3	11.1	8.9	2.2	2.2	2.2	2.2
	8.9	**2.2**	**13.3**	**0**	**6.7**	**0**	**2.2**	**8.9**	**0**	**4.4**	**4.4**	**22.2**	**2.2**	**2.2**	**0**	**2.2**
Partially False	28.9	13.3	17.7	13.3	17.8	26.7	24.4	28.9	13.3	11.1	8.9	15.6	17.8	8.9	4.4	2.2
	15.6	**31.1**	**8.9**	**8.9**	**13.3**	**20**	**22.2**	**28.9**	**4.4**	**4.4**	**2.2**	**11.1**	**13.3**	**8.9**	**2.2**	**6.7**
True	33.3	53.3	60	68.9	28.9	17.8	13.3	11.1	53.3	77.8	22.2	33.3	73.3	80	91.1	80
	53.3	**37.8**	**75.6**	**91.1**	**57.8**	**64.4**	**8.9**	**15.6**	**86.9**	**86.7**	**80**	**31.1**	**80**	**20**	**93.3**	**15.6**

Table 9   Distribution of responses on learners' beliefs about Mathematics

The analysis of each item of the questionnaire before and after intervention will be discussed below. I believe that to make analysis simpler and clearer, either the first two choices (false and partially false) or the last two choices (partially true and true) can be used. Using either of those provides one with more or less the same information. The third choice (Don't know) does not give much information. I chose to use the responses of the last two choices.

Comparisons were based on the responses of the experimental group because it was the group that received intervention.

Item 3: One learns mathematics best by memorizing facts and procedures.

Learners' responses to item 3 are shown in the table below:

	Pre	Post
Partially True (A)	15.6	31.1
True (B)	53.3	37.8
Sum (A+B)	68.9	68.9

Table 10   Responses to item 3 in percentages

For this item the majority of learners believed that mathematics is learned best by memorizing facts and procedures, 68.9 % in the pre- and post-tests. According to Kilpatrick (1985:8), in traditional classrooms the teacher first works out a problem on the board after which learners are then expected to memorize the procedures in order to apply them later. This way of teaching mathematics leaves the learners with the belief that the best way to learn mathematics is to memorize facts and procedures and recall them whenever necessary. It is interesting to point out that even after intervention; a majority of learners in the experimental group still maintained their previous belief. This shows that it takes a long time to change a person's belief. As a result for problem-based approach to bring about the desired change in learners' beliefs, it is important that it is applied from early grades.

Item 4: Mathematics is about solving problems.

	Pre	Post
Partially True (A)	8.9	8.9
True (B)	75.6	91.1
Sum (A+B)	84.5	100

Table 11   Responses to item 4 in percentages

In the pre-test 84.5% of learners stated that mathematics is about solving problems. This percentage increased to 100% after intervention. High percentage of learners who gave a positive response in the pre-test may be due to learners believing that by

recalling and applying procedures they were involved in problem solving. After intervention all learners in the experimental group agreed with item 4.

From responses in item 3 and 4, it became apparent that learners believed that mathematics is learnt best by applying facts and procedures even though they still maintained that mathematics was about solving problems. Similar beliefs about mathematics were also reported by Dossey and colleagues (Wilson et al 1993:46). They found that students in grades 3, 7 and 11 in the US believed that mathematics is useful, but mainly involves memorizing and following rules.

Item 5:   Mathematics is about inventing new ideas

	Pre	Post
Partially True (A)	13.3	20
True (B)	57.8	64.4
Sum (A+B)	71.1	84.4

Table 12   Responses to item 5 in percentages

In item 5 there was an increase from 71.1% (before intervention) to 84.4% (after intervention) in learners who believed that mathematics is about inventing new ideas. When one recalls how teaching and learning takes place in most traditional classrooms this huge positive response before intervention is surprising. It is likely that learners did not understand what it was meant by inventing ideas and they believed that when they were recalling facts they were inventing new ideas. It is important though, to point out that this percentage increased after intervention indicating that problem-based approach made a positive contribution to learners' beliefs.

Item 6: I usually understand a new idea in mathematics quickly.

	Pre	Post
Partially True (A)	22.2	28.8
True (B)	8.9	15.6
Sum (A+B)	31.1	44.4

Table 13   Responses to item 6 in percentages

Responses in the pre-test showed that a small number (31.1%) of learners understood a new idea quickly. In most mathematics classrooms, teachers encourage memorization at the expense of understanding as a result, learners struggle to understand since their prior knowledge is not linked to new knowledge when they memorize. According to Bell (1993:11), learning becomes meaningful and learners understand better when their current knowledge is taken into account during instruction. After intervention the percentage of learners who understood a new idea quickly, rose slightly to 44.4%. During intervention learners were encouraged to bring their prior knowledge into what is being taught so that they see the usefulness of knowledge already possessed.

Item 7: I have to work very hard to understand mathematics.

	Pre	Post
Partially True (A)	4.4	4.4
True (B)	86.9	86.7
Sum (A+B)	91.3	91.1

Table 14   Responses to item 7 in percentages

Responses in the pre-test revealed that 91.3% of learners have to work very hard to understand mathematics. As has been pointed out in item 6 the way mathematics is taught makes it difficult for learners to connect new and previous knowledge as a result they find it difficult to understand. After the intervention the percentage of learners who agreed with the above item did not change. In this study word problems were used and learners had to work very hard to understand what the problem requires and to solve it. The responses to item 7 are thus not surprising.

Item 8: The mathematics learned in school has little or nothing to do with the real world.

	Pre	Post
Partially True (A)	2.2	11.1
True (B)	80	31.1
Sum (A+B)	82.2	42.2

Table 15   Responses to item 8 in percentages

Before intervention 82.2% of learners responded that mathematics learned in school has little or nothing to do with the real world. In lessons teachers usually use drill exercises and problems devoid of context (that is problems written in symbols). As such learners do not see the relevancy of what they are being taught in their lives. The above figure decreased dramatically after intervention, with only 42.2% still maintaining this belief. These results are in line with what the Hiebert et al (1996:16) mentioned earlier that tasks selected should be linked with students experience so that they can see the relevance of the ideas and skills they already possessed. In this research, problems used were based on learners' every day experiences and learners were encouraged to use any method they like in solving them. It is very important therefore to expose learners to different problems in mathematics because according to Lester & Charles (1982), these problems serve different and important purposes in mathematics.

Item 12: **The teacher must always show me which method to use to solve a given word problem.**

	Pre	Post
Partially True (A)	13.3	8.9
True (B)	80	20
Sum (A+B)	93.3	28.9

Table 16   Responses to item 12 in percentages

In the pre-test 91.1% of learners indicated that the teacher must always show them the method to use in solving the given problem. This percentage decreased to 28.9 after intervention. The above results support what was discussed earlier by Kilpatrick (1985) about how teaching and learning takes place through the traditional approach. Here learners rely on the teacher and are not given opportunity to use their own methods and to discuss and defend their different solution strategies. During intervention learners were free to use any strategies or methods they like to solve problems. As such they became less dependent on their teacher. Ntsohi (2005:54) pointed out that allowing learners to use their own methods gives them courage, boosts their ego and sustains their motivation to learn.

Item 15: I feel the most important thing in mathematics is getting the correct answer.

	Pre	Post
Partially True (A)	13.3	8.9
True (B)	80	20
Sum (A+B)	93.3	28.9

Table 17   Responses to item 15 in percentages

In the pre-test 95.5 % of learners think the answer is the only important thing in mathematics. The emphasis that teachers put on the correct answer may have contributed to this belief. Kaput (1999:133) pointed out that usually learners are graded not on understanding but on their ability to produce the right answer. Learners are not aware that mathematics involves much more than that.

During intervention learners became aware that the solution strategies were as important as correct answers. They were encouraged to explain and discuss their solutions. Lester (1985:47) advised that the right direction in teaching mathematics should be for teachers to focus more attention on solution attempts and less on correct answers. Post problem sessions in which learners share their attempts and discuss their reasons for the choices they made might be one way to bring about this change.

SYNTHESIS

❖ The results of the pre- and post- tests above show that learners believe that even though mathematics is about inventing ideas and solving problems, it (mathematics) is learnt best by memorizing facts and procedures. This shows that even after intervention, learners still believe that the best way to learn mathematics is to memorize rules and procedures. This shows that it is not very easy to change a person's belief especially the belief one held for a long time.

❖ It also became evident that learners took long to understand a new idea in mathematics and they thus have to work very hard to understand mathematics.

❖ Most learners believed that mathematics learned in school has little or nothing to do with the real world, that the teacher must always show them the appropriate method to use to solve a problem and that getting the answer is the most important thing in mathematics. However, the results of the post-test revealed that learners have changed their beliefs. They indicated that mathematics learned in schools was important in their lives, they can use their own methods in solving problems and that mathematics involves more than getting the correct answer. Learners have started to appreciate the importance of the solution process.

4.5.2   Learners' attitudes towards word problems and mathematics

As pointed out earlier, the second theme will be on the attitudes of learners towards mathematics and word problems. Attitude plays a big role in problem solving performance of learners.

The following items were addressing the above theme:

9   I feel confident in my ability to solve mathematics word problems

10   I feel anxious when asked to solve mathematics word problems

11   Solving mathematics word problems is a great pleasure for me

13   I am willing to try a different approach when my attempt fails

14   When confronted with a word problem, I want to give up right away

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16   When I have finished working on the problem, I look back to see whether my
        answer makes sense.

From the table below it became obvious that responses of learners from both groups
before intervention did not differ much. As such I concluded that learners' attitude
towards mathematics was almost the same in the two groups. Item 16 disclosed a big
difference in the responses of these two groups. In the control group 71.1% of learners
stated that they looked back to see whether their answers made sense after solving the
problem while 46.6% of the experimental group stated that they looked back. It is
important to point out that these responses of the control group were not reflected on
the way they performed in the pre-test. The responses in the test revealed that learners
gave answers with incorrect units to some of the questions. For example, in question
6, some learners gave answers in rands instead of days. Examples of learners'
responses to question 6 are given in 4.2.4.

**Summary of the two groups' responses before and after intervention are
displayed in the table below.** (Experimental group's responses are in **bold**).

Items	9		10		11		13		14		16	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
False	15.6	26.7	20	26.7	13.3	13.3	26.7	31.1	51.1	44.4	22.2	15.6
	15.6	**0**	**20**	**17.8**	**8.9**	**8.9**	**15.6**	**8.9**	**33.3**	**57.8**	**46.7**	**15.6**
Partially False	15.6	13.3	11.1	15.6	8.9	8.9	6.7	4.4	4.4	17.8	6.7	6.7
	26.7	**2.2**	**13.3**	**6.7**	**6.7**	**4.4**	**6.7**	**0**	**13.3**	**15.6**	**2.2**	**2.2**
Don't Know	20	6.7	8.9	0	11.1	4.4	4.4	6.7	2.2	8.9	0	0
	13.3	**4.4**	**11.1**	**6.7**	**8.9**	**11.1**	**8.9**	**4.4**	**4.4**	**11.1**	**4.4**	**2.2**
Partially True (A)	13.3	22.2	28.9	33.3	11.1	17.8	6.7	17.8	6.7	17.8	22.2	31.1
	28.9	**28.9**	**28.9**	**24.4**	**28.9**	**13.3**	**35.6**	**17.8**	**20**	**2.2**	**13.3**	**24.4**
True (B)	35.5	31.1	31.1	24.4	55.6	55.6	55.6	40	35.6	11.1	48.9	46.7
	15.6	**64.4**	**26.7**	**44.4**	**46.7**	**62.2**	**33.4**	**68.9**	**28.9**	**13.3**	**33.3**	**55.6**
Sum (A+B)	48.8	53.3	60	57.7	66.7	73.4	62.3	57.8	42.3	28.9	71.1	77.8
	44.5	**93.3**	**55.6**	**68.8**	**75.6**	**75.5**	**69**	**86.7**	**48.9**	**15.5**	**46.6**	**80**

Table 18   Summary of responses on attitudes to Mathematics

Different responses of experimental group before and after intervention were analyzed. As in the previous theme analysis was based on the sum of the last two choices.

Items 9 and 11 were paired as they both aimed at revealing learners' positive attitudes.

Statement 9: I feel confident in my ability to solve mathematics word problems.

Statement 11: Solving mathematics word problems is a great pleasure for me.

 Responses for these statements are shown below.

Items	9		11	
	Pre	Post	Pre	Post
False	15.6	0	8.9	8.9
Partially false	26.7	2.2	6.7	4.4
Don't know	13.3	4.4	8.9	11.1
Partially true (A)	28.9	28.9	28.9	13.3
True (B)	15.6	64.4	46.7	62.2
Sum (A+B)	44.5	93.3	75.6	75.5

Table 19   Responses to item 9 and 11 in percentages

Looking at the responses of experimental group in both tests one realizes that the percentage of learners who felt that they were confident to solve word problems (item 9) increased from 44.5 % in pre-test to 93.3 % in the post-test. Learners who relied on their teacher to first work out an example for them lack the confidence in solving problems on their own. It was mentioned earlier that a learner who lacks problem solving strategies looses interest in solving a problem after attempts with one method have failed. On the other hand a learner who knows different strategies feels confident in solving challenging problems by trying out these strategies. Exposing learners to different strategies and allowing them to use their own methods as well as giving learners many word problems to solve does not only increase their skills in solving such problems, but also increases their confidence in solving them. Peer interaction also contributes to learner's self- confidence. It is important that teachers encourage learners to work in pairs or in groups during problem solving so as to promote peer

interaction. For item 11, there was a small decrease (75.6 to 75.5) in the percentage of learners who believed solving mathematics word problems was a great pleasure for them. In this study (as has been stated earlier) all problems given to learners were non-routine word problems and these required learners to work very hard. As such there was no increase in learners' pleasure in solving word problems after intervention.

Items 10 and 14 were also paired as they revealed negative attitudes.

Item10: I feel anxious when asked to solve mathematics word problems.
Item14: When confronted with a word problem, I want to give up right away

Item	10		14	
	Pre-test	Post-test	Pre-test	Post-test
False	20	17.8	33.3	57.8
Partially false	13.3	6.7	13.3	15.6
Don't know	11.1	6.7	4.4	11.1
Partially true	28.9	24.4	20	2.2
True	26.7	44.4	28.4	13.3
Sum (A+B)	55.5	68.8	48.4	15.5

Table 20   Responses to items 10 and 14 in percentages

The percentage of learners who felt anxious when asked to solve word problems (item 10) increased from 55.5 % to 68.8 %. For item 14 there was a noticeable decrease in the percentage of learners who would give up right away when confronted with a word problem. In the pre-test 48.4 % responded positively compared to 15.5% in the post-test.

Experience has shown that a common mistake made by teachers is to avoid giving learners word problems for the fear that learners do not understand this type of problems. Responses in post-test showed that although majority of learners were still anxious when asked to solve problems they were however; brave enough to work on the problem instead of giving up. In this study all problems that learners solved were expressed in words. Initially most learners were reluctant to solve these problems indicating that they did not know how to solve them. After a few lessons, when they were aware of different strategies they enjoyed trying them out in solving word

problems and comparing their answers. It is very important that learners are encouraged to attempt a problem even if they are unsure of the correct strategy to use because in the process they will be discovering and learning more about the problem. Classroom environment should thus allow learners to make mistakes without fear that they will be embarrassed.

Items 13 and 16 were also treated together as they aimed at revealing positive attitudes (perseverance and reflection).

Item 13: I am willing to try a different approach when my attempt fails
Item 16: When I have finished working on the problem, I look back to see whether my answer makes sense.

Item	13		16	
	Pre	Post	Pre	Post
False	15.6	8.9	46.7	15.6
Partially false	6.7	0	2.2	2.2
Don't know	8.9	4.4	4.4	2.2
Partially true(A)	35.6	17.8	13.3	24.4
True (B)	33.4	68.9	33.3	55.6
Sum (A+B)	69	86.7	46.6	80

Table 21   Responses to items 13 and 16 in percentages

The percentage of learners who stated that they were willing to try a different approach when the first attempt failed rose from 69% in the pre-test to 86,7% in the post-test. This showed that most learners were now willing to try a different approach when their attempts failed. The increase in percentage was also found in statement 16 where 46% of learners indicated before intervention that they looked back to see whether their answers made sense and 80% gave the same response after intervention. In solving non-routine problems, it is not always the first attempt that works, but one has to try different approaches until the appropriate one is found. In classrooms where learners are not given this opportunity of struggling with a problem and trying different approaches, learners lose interest quickly after the first attempt. One possible reason may be learners' overdependence on their teacher. During intervention learners

became aware of the importance of perseverance and use of different strategies which contributed positively to problem solving. These findings support assertions that learners with positive attitudes towards mathematics persevere in their attempts to solve problems (Van de Walle 1998:51). The results also showed that after intervention learners began to understand the importance of the 'looking back' stage in problem solving. According to Lester (1985:47), looking back at a problem does not only help a learner to determine whether their answers make sense but also helps a learner to see whether all conditions in a problem have been met. It also gives learners the opportunity to study the solution process.

Looking at the responses of the control group, it is evident that in cases where there were changes in their responses, these changes were small as compared to those of the experimental group.

4.5.3   Synthesis

From the responses one may summarize the above by saying that the teaching approach used by the teacher has an impact on learners' attitude to problem solving and mathematics.

In classrooms that support traditional teaching approach learners have mixed feelings and attitudes about problem solving. Although learners enjoy mathematics they are however, not confident to solve word problems and feel anxious when asked to solve them. They give up right away when confronted with problems, they do not try a different approach and do not look back to see whether the answer makes sense or not. In problem- based learning, where teaching is learner- centred, learners' attitudes are different. They are more confident and persevere in their attempts to solve problems and usually look back to see whether their answers are sensible and they study the solution process.

The results in this section indicated that after intervention, learners' attitudes towards mathematics and word problems have improved. This result is in line with the argument that one of the residues that learners take away from the classroom is their

disposition towards mathematics. This residue is influenced by the kind of problems solved and the manner in which they were dealt with in class.

4.5.4   Learners' preference for word problems

In section D (see Appendix C), learners' preference to the way in which problems are set is investigated. Item 17 read thus:

17   Given these two mathematics problems, which one will you like to answer? Give reasons for your choice.

a   Find the value of x in: $\frac{1}{4} x + 5 = 25$

b   Mpho spends a quarter of her money on chips and R5 on soft drinks. Together she had spent R25. How much did she have initially?

The purpose of this item was also to determine the attitudes of learners towards word problems. This item required learners to give their reasons why they like or dislike the two problems.

(Responses are given in percentages)

N = 45 for each group.

	Experimental group		Control group	
	PRE-TEST	POST-TEST	PRE-TEST	POST-TEST
a	46.7	37.8	62.2	64
b	53.3	62.2	37.8	35.6

Table 22   Responses to section D in percentages

In Table 22, learners who preferred word problems chose 'b' and those who did not chose 'a'.

The results of the pre-test indicated that the percentage of learners in the experimental group who preferred word problems was 53.3 compared to 37.8 in the control group. It is not easy to say what may have caused this difference. I would think it is likely that since learners in the experimental group were aware that they are taking part in a research study they responded the way they thought would please the researcher. Schumacher & Macmillan (1997:190) refer to this behaviour as the Hawthorne effect. Before this investigation was conducted permission was obtained from parents to use

their children in this study and learners became aware that they were to be participants.

These learners' past experiences with word problems might also have contributed to the way they responded. These two groups might have had different experiences previously. According to Ntsohi (2005:55) learners' attitudes and feelings towards the subject may be the result of their experiences and exposure to the way the subject was taught. For example, Ramnarain (1999:118) discovered that learners' attitudes to the subject are closely related to the personality and teaching approach of the teacher.

It is also likely that learners in the experimental group understood the language of learning and teaching (LOLT) better than those in the control group because one of the common reasons for learners in the control group choosing 'a' was that they do not understand 'b' which is given in words. As has been pointed out earlier some of the difficulties that learners encounter in learning mathematics are related to language. Besides the above discussions, it can be seen from the results of the post-test that there was some slight increase in the number of learners in the experimental group who liked to solve word problems, 53.3% to 62.2% in the post-test. This meant that although learners were exposed to many word problems, many of them still don't like solving them. This was not surprising when one looked at learners' reasons for their choices.

Below are some of the reasons for learners' choices.

Those learners who chose "a" gave the following reasons:
- ❖ They like 'a' because they know a lot about solving it.
- ❖ It is easy / simple. I don't think very hard.
- ❖ I understand the method of 'a'.
- ❖ You don't read too much, I don't enjoy word problems.
- ❖ I like working with 'x', 'x' is mathematics.
- ❖ The teacher teaches x or shows me the method to use.
- ❖ Sometimes I don't understand.
- ❖ With 'b' you don't know if you multiply or plus.

These reasons supported the arguments about the way mathematics is taught, about the importance of language in teaching and learning and the kinds of problems solved. Learners who are exposed to different problems become aware that the same problem can be expressed in words and in symbols. However if learners always work with problems like 'a' and the teacher does an example first, learners may not see the similarity when the same problem is written in words and they tend to rely too much on the teacher. Learners end up having negative attitudes to word problems because these problems require them to read, think and decide which operations or method to use. Most learners also said that they didn't like 'b' because they did not understand the question. Similar reasons for disliking word problem were also reported by Ramnarain (1999:121). A learner who is not competent with language of learning and teaching (LOLT) usually avoids word problems, not because such a learner dislikes them but because most of the time a learner does not understand what the problem requires.

Some of those who chose 'b' said:
- ❖ Because I understand what to do.
- ❖ 'X' is difficult.
- ❖ Because we sometimes do them in class.
- ❖ It is not hard to think and I can do it.
- ❖ Word problems make sense.
- ❖ I work with money at home.

Learners who don't have much difficulty with LOLT usually understand word problems better though experience has shown that despite their understanding most learners struggle to solve them. Other learners who chose 'b' felt that 'a' does not make sense because they don't know the importance of finding 'x', but when the problem is in words, the problem makes sense to them. Teachers usually rush to use variables without ensuring that learners understand what variables are and why they are used. This explains why learners tend to use these variables without any

understanding of what they are doing. Other learners prefer to solve 'b 'because they work with money at home thus emphasizing that bringing learners' everyday context in the classroom is very important.

Some learners did not write reasons for their choices and others tried to solve the problem they had chosen. This may be caused by limited knowledge of language used, that is, they may not have known how to express themselves in English or they may not have understood what they had to do. Of all the learners from both groups, only one learner indicated that the two problems were the same and that he liked them both.

Some of the learners' responses were contradictory. For example, some learners who indicated in section B that solving mathematics problems is a great pleasure also pointed out in section C that they chose 'a' because they don't like solving word problems. It is likely that these learners were confused or that they did not give these items much thought.

4.6 Lesson Observations

❖ At the initial stage of intervention, most learners requested that they should be shown the method to use first. They strongly pointed out that their teacher always worked out a similar problem for them on the board before giving them work to do. This observation was similar to that reported by Yusof and Tall (Internet source). Learners were however encouraged to read through the problem several times and try any of the strategies done earlier or any method they think may help them. They eventually became independent and started to apply these strategies in solving problems.

❖ It became apparent that learners liked using their own methods. They enjoyed mathematics more when they saw their different methods producing the same results. This observation is contrary to what happens in classrooms where learners have to practice on exercises following methods done earlier by the teacher. Allowing learners to use their own methods encourages them to try these methods in solving problems and boosts their confidence and motivation.

When the first strategy they used failed, learners were willing to try a different approach.

❖ Initially, learners were shy to explain their solutions to the whole class. In most mathematics classes learners are rarely given opportunity to discuss and defend their ideas (Ramnarain 1999:124). With time, however most overcame this shyness and enjoyed discussions and explaining their solutions to group members and the whole class. They made sure that they looked back at the problem and the solution process before they presented their solution to the class.

❖ Lesson observations also revealed that learners liked working in groups. This is probably because in OBE classrooms, group work is very common. The other possible reason may be that learners enjoyed assisting each other as was stated by Silver (1990:23) in the literature study.

❖ Learners were however struggling with language. Although they were able to read what was written, they had difficulty with comprehending the problem. Initially they attempted to solve the problem without understanding clearly the information presented and what the question required. More time was now spent on comprehension of the problem. With time, learners understood the importance of understanding the problem before it could be solved. Code-switching was done where necessary but it was expected of learners to present their solutions in English, which was the medium of instruction.

❖ Generally the progress was slow.

It is important to point out that encouraging learners to explain their solutions is very beneficial to their understanding of mathematics. Learners' explanations help the teacher to diagnose misconceptions. According to Reed (1999:47), diagnosing misconceptions is made challenging by the fact that immature strategies can often produce correct answers. Sometimes learners with correct answers gave incorrect explanations whereas those who obtained incorrect answers had a partial understanding of the problem. The above findings were also observed during the intervention.

4.7   Learners' journals

Learners did not use their journals much even though they were encouraged to complete them on a daily basis. Most of the entries made were about their feelings towards mathematics before and during intervention. They pointed out that they used to think that mathematics was very difficult, that they were not aware that they can use any method they like to solve the problems. It is quite obvious that these feelings have changed after intervention.

4.8   Conclusion

In this chapter, results of the investigation were presented, analyzed and interpreted. These results will be summarized as follows:

❖ Learners who were explicitly taught problem solving strategies began to apply these strategies in solving non- routine word problems. This is quite opposite from most traditional classrooms where learners expect their teacher to show them the method to use in solving problems. If these learners get stuck, they cannot proceed on their own, since they are not aware of other strategies to try and they are too dependant on their teacher.

❖ Learners became more systematic and organized in their approach to problem solving. They ensured that they understood the problem before attempting to solve it and looked back to see whether their answers made sense or not.

❖ Learners who were exposed to problem solving strategies performed significantly better than those who were not in the post-test. They showed improvement in their selection and application of different strategies in solving problems, however they progressed slowly due to limited content knowledge and difficulty with language.

The way mathematics is taught influences learners' beliefs and attitudes towards mathematics. In this investigation, the experimental group was exposed to the problem-based approach and was given instruction in problem solving strategies. In lessons they were free to solve problems using whichever strategy they like. They

worked individually and in groups and were probed to explain their solutions among other group members and to the whole class and they became aware that:

- ❖ They do not have to wait for their teacher to demonstrate the method to use
- ❖ They can use any appropriate method / strategy they like
- ❖ Their opinions are valued in classrooms
- ❖ The correct answer is not all that is important in mathematics but the solution process is equally important. Learners also realized the value of explaining and reflecting on their methods.

Experience has shown that in most mathematics classrooms, work given to learners consists mostly of problems devoid of context, hence they are meaningless and very far from learners' everyday experiences. Learners therefore believe that mathematics learned in school has nothing to do with their everyday life. If problems used are from learners' experiences, as was the case during intervention, learners' beliefs are different. It is very important then that learners are exposed to different kinds of mathematical problems in an environment that encourages them to try their best without fear of being embarrassed and hence boosting their confidence.

Researchers such as Dossey and colleagues (Wilson 1993) reported that even though learners believe that mathematics is about solving problems and inventing new ideas, they also believe that mathematics is learnt by memorizing and involves rules and procedures. The same belief has been revealed in this investigation. However, as I indicated earlier, for this believe to change, problem-based approach should start at an early age and continue throughout learners' schooling.

The general finding in this chapter is that teaching learners problem solving strategies has positive impact on their mathematical performance including their attitude to the subject. It becomes apparent then that learners were left with different residue.
These are:

- ❖ Better understanding of mathematics
- ❖ Different strategies for solving problems, and
- ❖ Positive attitudes and beliefs about mathematics.

CHAPTER FIVE
SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1   Introduction

This chapter presents the summary of findings from the theoretical background and empirical investigation. The fifth research question will be answered by making recommendations based on the findings. Weaknesses or limitations of this study followed by suggestions for further research will also be spelt out.

5.2   Findings

5.2.1   Theoretical background

Chapter two was on theoretical background for this study. The main aims were to get a deeper understanding of the topic investigated and to address the first research question in which the researcher explored problem -based approach in detail. The two approaches to problem solving were discussed, the traditional approach and the contemporary approach (Problem-based approach). The traditional approach is still the most dominant approach in most mathematics classrooms even though reforms in education recommend contemporary approaches (Cangelosi 1996:31). In looking at the problem-based approach, much emphasis was put on different problem solving strategies which are vital tools during problem solving.

5.2.2   The empirical investigation

Chapter three was based on the methodology for this study. It described how the investigation was carried out to address the second, third and fourth research questions. Two groups of learners, the control and experimental groups wrote a pre-test on non-routine mathematics word problems. Learners' responses and approaches in solving word problems in the pre-test were analyzed to address the second research

question. To address research question three, the experimental group was then given explicit instruction in problem solving strategies for six weeks. The control group received instruction in the normal traditional mathematics classroom. At the end of the six weeks, the two groups wrote a post-test.

In order to deal with the fourth research question, a questionnaire on learners' beliefs and attitudes towards mathematics in general, and word problems in particular was given to both groups with pre- and post-tests.

Null hypotheses were presented and tested in chapter four.

Information was collected through tests, questionnaires, observations and journals.

Data from the investigation were presented, analyzed and then interpreted. To determine learners' performance in the two tests, learners' scores were analyzed statistically by calculating the mean, standard deviation, maximum and minimum scores, and the range. Learners' responses in solving individual items in the pre- test were analyzed in order to gain insight into their problem solving approaches. The findings addressed the second research question. T-tests were used to test hypotheses stated in chapter three and to determine whether or not instruction in problem solving strategies improved performance. Results of responses were then used to determine the change in learners' performance, beliefs about mathematics and attitudes towards word problems and mathematics. The results revealed the following:

- ❖ In traditional classrooms learners have limited problem solving strategies; as such they rely too much on their teachers when solving problems. Too much emphasis is placed on the correct answer at the expense of the solution method leading to the correct answer.

- ❖ Learners who were explicitly taught the problem solving strategies had a wide choice of strategies to choose from during problem solving. They did not depend on their teachers to workout an example for them before giving them problems to solve. These learners were more organized and systematic in the way they solved problems. Their selection and application of appropriate strategies improved. They became aware of the importance of understanding the problem before it could be solved, they also looked back to determine

whether their answers made sense and their performance in problem solving improved.

❖ Discussions are important in the learning of mathematics. Learners enjoy discussing their solutions with each other and comparing their solutions as such they become aware that in mathematics the solution process is very important. Through discussions learners' misconceptions can be identified and corrected in a positive environment.

❖ Learners who learn mathematics through a problem-based approach become more confident to solve word problems and persist in their attempts when the first one fails. Their beliefs about mathematics and attitudes towards mathematics change positively.

❖ Success at problem solving takes time and does not only depend on the knowledge of problem solving strategies.

5.3   Recommendations

Outcomes Based education (OBE) aims at producing independent and life-long learners with critical thinking skills. This means teaching of mathematics should strive to equip learners with knowledge and skills that will be useful not only at school but in their lives presently and in future. In order to achieve these and to address the fifth research objective, the following recommendations are made:

❖ Learners should be given a variety of non-routine problems.
❖ Teachers should use different teaching strategies.
❖ Problem solving strategies should be explicitly taught to learners.
❖ Teacher development is crucial.

Discussions of the above recommendations are made below.

Learners should be given a variety of non routine problems to solve:

Development of mathematical knowledge was a result of people solving problems in their lifes, therefore learners should also be given problems to solve so that in the process their mathematical knowledge grows. The emphasis should be on non-routine problems that will require learners to use their prior knowledge and link it to new knowledge. To encourage critical thinking, these problems must be those requiring

more than rote application of a previously learned formula. From the discussions in the previous chapter it was clear that learners do not like word problems. This may be caused by the fact that learners are seldom given these kinds of problems to solve in class because they are considered demanding and learners usually perform badly in word problems. The researcher then recommends that learners be exposed to word problems and problem solving from very early grades of their schooling. They should learn from these early grades the importance of Polya's problem solving stages and where possible a link should be made between the word problem and the corresponding algebraic statement or equation. In this way learners will be aware that one problem can be written either in words or in symbols.

Teaching and problem solving strategies

The educator should move away from traditional ways of teaching mathematics or at least use different teaching strategies so as to accommodate all learners. These strategies should provide learners with opportunities to actively construct their mathematical knowledge. Learners should be active, discover ideas for themselves, make conjectures, experiment, formulate and test hypothesis and make conclusions. Learners should be allowed to use their informal strategies and be guided into developing these strategies to more formal ones. According to Ntsohi (2005:86), mathematical expertise is characterized by "the ability to do mathematics, ability to solve mathematical problems, ability to communicate mathematically and ability to reason mathematically". Teaching approaches used in the mathematics classroom should thus allow learners to interact socially during construction of knowledge.

The educator should therefore facilitate, motivate and encourage learners to do their best but most importantly he / she has to create a positive learning environment.

The literature study and the findings in this study reveal that explicit instruction in problem solving strategies is very important. Educators should start exposing learners to different problem solving strategies because their use engages the learners in processes inherent to mathematical thinking.

Teacher development

There are many changes taking place in education, as such it is very beneficial to prepare and develop teachers to cope with these changes. Most teachers of mathematics have formalist views of mathematics and these views become apparent in the way they teach and these are passed on to learners. Current changes in mathematics curriculum require teachers to shift their views, knowledge and practices and grant learners opportunities to play an active role in their learning. It is important that teachers are offered training in these new approaches to the teaching and learning of mathematics so that they " 'fit' more closely with the roles, philosophies and values underpinning the new mathematics curriculum" (Graven 2002:65).

It is not easy for teachers to change their beliefs about how mathematics should be taught even when they realize that the way they have been doing things is not successful. They therefore need more than the 'normal' in-service training of 2-5 days but a lot of support in their classrooms. It is common that teachers attend training and when they get to their classrooms implementation of what was discussed during training becomes very confusing (Graven 2002:66) to the extent that they revert back to their comfort zones. I would therefore strongly recommend classroom-based support and regular cluster meetings where teachers share their successes and frustrations.

I also believe that where possible learning facilitators should not only rely on documents (learner and teacher portfolios) when checking /monitoring implementation of curriculum, they should visit teachers regularly and find out what support they need and decide on intervention strategies. Those teachers (model teachers) who have better understanding of implementing the new curriculum can also be requested to assist others.

5.4   Synthesis

For mathematics teaching and learning to be successful, teachers should equip learners with problem solving skills. Recommendations made show that these skills can only be developed when learners solve a variety of non-routine mathematical problems, some which are word problems.

Since competency at problem solving takes time, learners should start problem solving at early grades. It is the teachers' responsibility to expose learners to a wide choice of problem solving strategies as these are vital tools when problems are solved. Learners are in a position to apply these strategies correctly when the teacher becomes a facilitator of knowledge and create a positive learning environment for all learners. It is however not possible for a teacher to equip learners with these skills if the teacher's knowledge is inadequate hence the importance of teacher development can not be underestimated. In-service training and regular classroom based support coupled with cluster meetings have been recommended as possible solutions where a teachers' knowledge is limited.

5.5   Limitations and weakness of study

In this study there are several limitations / weaknesses identified.

The first one is the time constraint. As it has been pointed out in the earlier discussions, expertise at problem solving takes time, therefore if longer time had been given to this investigation then better results would be obtained. I believe that at least six months to one year would be appropriate.

Secondly, I believe that learners were sometimes confused by two teachers with different teaching approaches, their teacher with his traditional approach and the researcher with contemporary approach with emphasis on the solution process. It would have been better if these learners were taught mathematics by the researcher only during the investigation period. It was also not easy to give learners as much homework as one would like because they had work from their teacher as well.

Thirdly, since the researcher was also the teacher, she/he might have unconsciously influenced the results of this investigation. For example, learners might have been taught in a way the researcher wanted the results to go.

Lastly, in this study, a very small sample was used, as such the results cannot be generalized to the larger population. The researcher believes that a more extensive study involving a larger group and other social groupings is crucial for the results to be generalized.

5.6   Suggestions for further research

Problem-based approach is not common in most mathematics classrooms even though the curriculum and research emphasize its importance in mathematics. As indicated in Chapter two, this approach is characterized by active participation of learners, exchange of ideas as well as social interaction amongst learners. The researcher's observation is that learners are organized in groups in most OBE classrooms and research can be done to determine the extent of which these groups are utilized to enhance learner participation and social interaction. I would also suggest that research be done on the extent of support offered by the department for effective implementation of new mathematics curriculum. Implementing new curriculum is very challenging for educators and it is necessary that they get regular support from the Department of Education. The findings will assist in planning for intervention strategies so that educators feel confident to deliver the curriculum effectively.

5.7   Conclusion

In this chapter summary of findings from the previous chapters and recommendations were made.

The main aim of the study has been realized and it became clear that:

In order for mathematics to be meaningful to learners, it should be taught through the problem- based approach. This approach develops learners' problem solving and critical thinking skills which will be helpful in solving problems at school and from their everyday lives. The researcher strongly thinks that for successful implementation

of this approach, teachers need much support from their fellow teachers, parents and the Department of Education.

Learners lack problem solving skills, as such explicit instruction in problem solving strategies is critical. From the findings it can be concluded that instruction in these strategies through a problem-based approach improves learners' problem solving performance. It also affects positively learners' attitudes towards word problems in particular and mathematics in general. Emphasis is made on problem-based approach because instruction through other approaches may yield different results.

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Appendix A

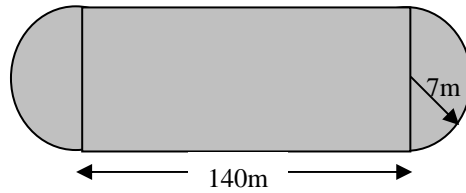**PRETEST**   Time: $1\frac{1}{2}$ hrs

Answer **all** questions

Show all your working in the spaces provided

You may use calculators where necessary.

1   An empty bus is picking up passengers at the following rate. One passenger jumped on at the first stop, three jumped on at the second stop, five jumped on at the third stop and seven jumped on at the fourth stop and so on. How many passengers got on at the 10th stop?

2   Palesa needs money to buy an ice cream. She has 20 coins in her pocket. Some are 5c and some are 10c coins. She needs more than R1,50. What is the least number of 10c coins that she must have?

3   Letsitsa started out for the gym. He passed the post-office at 2:40pm and he was one fourth (1/4) of the way. He passed the bank at 2:47pm and he was one half (1/2) of the way. If he continued walking at the same speed, at what time will he get to the gym?

4   The area of a rectangle is 20 x 10 squared cm. If each of its sides is increased by 10%, find the percentage by which the area increases.

5   If 12 technology students in grade 9D make 36 toys in 4 days, how many toys would 6 students make in 6 days? (Assume learners work at the same rate).

6   In a single elimination soccer tournament, teams are eliminated when they lose. If 8 teams are involved, how many games have to be played in order to determine a tournament winner.

7   Video Town rental shop charges a basic fee of R150.00, as well as R15 per day to rent a television. Crazy Video television rental shop charges a basic fee of R15 but has daily rate of R60 per day to rent a television. For what number of days will it make no difference in cost as to which shop you rent?

8   Mr Ngwenya has a rectangular plot as shown in the diagram below. The shaded area shows the part of land that he wants to use for growing his vegetables. Calculate the area of the land that will be used to grow his vegetables?

7m

140m

Appendix B
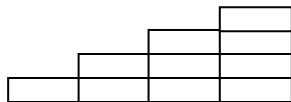
POST TEST Time: $1\frac{1}{2}$ hrs

Answer **all** questions

Show all your working details in the spaces provided

You may use calculators where necessary

1   Ten concrete blocks are needed to build 4 steps as in the sketch. How many concrete blocks are needed to build 10 steps?

2   There are 18 animals in Naledi's farmyard. Some are chickens and some are cows. Naledi counted 50 legs in all. How many of the animals are chickens and how many are cows?

3   Two girls have R50 to spend. They spent half (1/2) of their money on burgers and a quarter (1/4) of their change on soft drinks. How much did they spend altogether?

4   The area of a rectangle is 20 x 10 cm squared. If each of its sides is increased by 10%, find the percentage by which the area increases.

5   Maneo can clean a house in 5 hours and Moleboheng can clean the house in 4 hours. If they work together, how long will they take to clean the house?

6   There are 4 basketball teams in a tournament. The teams are lettered A through D. Each team plays each of the other teams twice. How many games are played all together?

7   Thabang has R100 pocket money and Mpho has R40. They are both offered temporary jobs at different companies. Thabang gets R10 a day and Mpho is paid R25 a day. If they do not spend their pocket money or their daily wages, after how many days will they have the same amount of money?

8   The inner boundary of a race track is formed by two opposite sides of a 400m square joined by two semi-circles as shown in the diagram. Rankhasa wants to plant evergreen grass in the shaded area. He first has to know the area of the shaded part. Could you please help him calculate the area of the shaded part?
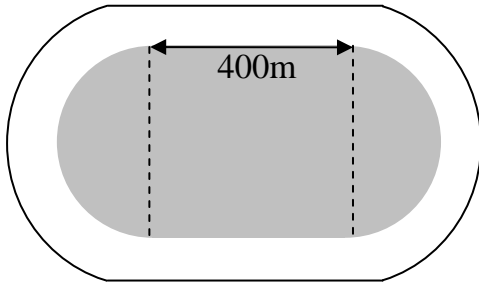
Appendix C

QUESTIONNAIRE

Instructions:

Answer all questions

Do not write your name

All answers will be treated confidentially

Section A (write your answers in the spaces provided)

1   What is your sex? ---------------

2   How old are you? ---------------

Answer section B and section C by making a circle around the letter that you have chosen.

Section B

3   One learns mathematics best by memorizing facts and procedures

  (a) False   (b) Partially false   (c) Don't know   (d) Partially true   (e) True

4   Mathematics is about solving problems

  (a) False   (b) Partially false   (c) Don't know   (d) Partially true   (e) True

5   Mathematics is about inventing new ideas

  (a) False   (b) Partially false   (c) Don't know   (d) Partially true   (e) True

6   I usually understand a new idea in mathematics quickly

  (a) False   (b) Partially false   (c) Don't know   (d) Partially true   (e) True

7   I have to work very hard to understand mathematics

  (a) False   (b) Partially false   (c) Don't know   (d) Partially true   (e) True

8   The mathematics learned in school has little or nothing to do with the real world

  (a) False   (b) Partially false   (c) Don't know   (d) Partially true   (e) True

Section C

9   I feel confident in my ability to solve mathematics word problems

  (a) False   (b) Partially false   (c) Don't know   (d) Partially true   (e) True

10   I feel anxious when asked to solve mathematics word problems

  (a) False   (b) Partially false   (c) Don't know   (d) Partially true   (e) True

11   Solving mathematics word problems is a great pleasure for me

   (a) False   (b) Partially false   (c) Don't know   (d) Partially true   (e) True

12   The teacher must always show me which method to use to solve a given word problem

   (a) False   (b) Partially false   (c) Don't know   (d) Partially true   (e) True

12   I am willing to try a different approach when my attempt fails

   (a) False   (b) Partially false   (c) Don't know   (d) Partially true   (e) True

13   When confronted with a word problem, I want to give up right away

   (a) False   (b) Partially false   (c) Don't know   (d) Partially true   (e) True

14   I feel the most important thing in mathematics is getting the correct answer

   (a) False   (b) Partially false   (c) Don't know   (d) Partially true   (e) True

15   When I have finished working on the problem, I look back to see whether my answer makes sense.

   (a) False   (b) Partially false   (c) Don't know   (d) Partially true   (e) True

Section D

16   Given these two mathematics problems, which one will you like to answer? Give reasons for your choice.

a   Find the value of x in: $\frac{1}{4} x + 5 = 25$

b   Mpho spends a quarter of her money on chips and R5 on soft drinks. Together she had spent R25. How much did she have initially?

Appendix D

Calculation of the t-value to compare means of the control and experimental groups in the pre-test.

Mean of control group   (X) = 6.93, Standard deviation = 4.185472711~ 4,19
Mean of experimental group (Y) = 7.13, Standard deviation = 4.446653491~4,45
The above values were calculated from scores of learners represented in barchart, fig.3.

Formulae used are taken from Mulder 1989:149. (Independent data).
The same formula was also used to compare means of these two groups in the post-test.

$$t = \frac{|\overline{X} - \overline{Y}|}{S_{\overline{X} - \overline{Y}}}$$

Where **t** is the t- value, **X** is the mean of the control group, **Y** is the mean of experimental group and $S_{\overline{X} - \overline{Y}}$ is the standard error of the difference between means.

$$S_{\overline{X} - \overline{Y}} = \hat{S} \sqrt{\frac{1}{Nx} + \frac{1}{Ny}}$$

Where $\hat{S}$ Is estimated std. deviation of population,

Nx Is number of learners in the control group and Ny Is number of learners in experimental group

$$\hat{S} = \sqrt{\frac{NxSx^2 + NySy^2}{Nx + Ny - 2}}$$

Sx is standard deviation of control group and **Sy** is standard deviation of the experimental group

$$\hat{S} = \sqrt{\frac{(45)(4,185472711)^2 + (45)(4,446653491)^2}{45 + 45 - 2}}$$

$$\hat{S} = \sqrt{\frac{788,3181817 + 889,7727271}{88}} = 4,366831216$$

$$S_{\overline{X} - Y} = 4,366831216 \sqrt{\frac{1}{45} + \frac{1}{45}} = 0,920608853$$

$$t = \frac{|6,93 - 7,13|}{0,920608853} = 0,2172$$

Calculation of t-value to compare means of the experimental group in the pre-test and post-test. Formulae for related data is as in Mulder 1989 :144.

$$t = \frac{\sqrt{N-1} \; \Sigma D}{\sqrt{N \Sigma D^2 - (\Sigma D)^2}}$$

N is the number of learners in the experimental group
D is the difference in each learners scores in the pre- and post test. N=45, $\Sigma D = 251$

$$t = \frac{\sqrt{44} \; 251}{\sqrt{45 \times 2636 - (251)^2}}$$

$$t = \frac{1664945645}{\sqrt{118620 - 63001}}$$

$$t = \frac{1664945645}{235,86805} = 7,059736$$