EXPLORING THE RELATIONSHIP BETWEEN MATHEMATICS TEACHERS' SUBJECT MATTER KNOWLEDGE AND THEIR TEACHING EFFECTIVENESS

by

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Abstract

The purpose of the study was to explore the relationship between mathematics teachers' subject matter knowledge and their teaching effectiveness. A convenient sample of 19 grade 11 mathematics teachers and 418 students were initially selected for the study and took part in some stages of the study. Of this lot, only 11 teachers and 246 students participated in all the stages of the study. Explanatory Mixed methods research design which entails the use of a co-relational study and a descriptive survey design were employed in the study. Data was collected from the teachers using a self report questionnaire, Teacher Subject Matter Knowledge of Trigonometric Functions Scale (TSMKTFS) and peer evaluation questionnaire, and from students using teacher evaluation questionnaire and Student Trigonometric Functions Performance Scale (STFPS). All the instruments had their validity and reliability accordingly determined. Quantitative data gathered was analysed using descriptive and inferential statistics while qualitative data gathered from teachers' and students' tests were analysed using task performance analysis. It was found that a positive, statistically significant relationship existed between teachers' subject matter knowledge and the composite measure of their teaching effectiveness. The relationships between teachers' subject matter knowledge and students' achievement and also between teachers' subject matter knowledge and students' rating of the teachers' teaching effectiveness were found to be positive and statistically significant. However, the relationships between teachers' subject matter knowledge and teachers' self rating as well as teachers' subject matter knowledge and peers' rating of teachers' teaching effectiveness were not found to be statistically significant though they were positive. Further data analysis showed that there was a difference between the subject matter knowledge of effective and ineffective teachers and also between the students taught by effective teachers and the students taught by the ineffective teachers.

Key terms

Mathematics teacher; subject matter knowledge; teaching effectiveness; student achievement; self rating; student rating, peer rating; trigonometric functions.

Declaration

I declare that EXPLORING THE RELATIONSHIP BETWEEN MATHEMATICS TEACHERS' SUBJECT MATTER KNOWLEDGE AND THEIR TEACHING EFFECTIVENESS is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

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SIGNATURE	**DATE**
(Ugorji Iheanachor Ogbonnaya)	

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List of Abbreviations

DoE	-	Department of Education
DoHE	-	Department of Higher Education
DoBE	-	Department of Basic Education
FET	-	Further Education and Training
GET	-	General Education and Training
ICT	-	Information and communication Technology
MPSPAF	-	Mathematical Production System Performance Analysis Framework
PCK	-	Pedagogical content knowledge
SGB	-	School Governing Board
SMK	-	Subject matter knowledge
SOTL	-	Student opportunity to learn
SPSS	-	Statistical Package for the Social Science
STFPS	-	Student Trigonometric Functions Performance Scale
TSMKTFS	-	Teacher Subject Matter Knowledge of Trigonometric Functions Scale

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Chapter One

Orientation of the Study

1.1 Introduction

There can be little argument against the view that a major desire of many African nations today is to be technologically developed. Such a desire makes sense since the wealth and strength of any nation is no longer dependent only on the abundance of natural resources but more on the technological advancement of that nation. Education and skill have become the most central elements that impact on the development of any nation. Hence, education and skill are core tools for increasing economic competitiveness and promoting social relevance (McGrath & Akoojee, 2007). Today, there is a shift from the orthodoxy of developmental strategy of the previous centuries hinged on available natural resources to a knowledge based economy where education and skill are the major propellers of economic development. Professional skills in the field of engineering, science and technology are now needed more than ever.

This implies that there is a need to have a pool of technologically skilled manpower to fulfil the current millennial economic and technological development goals and this can only be actualised by placing adequate emphasis on the pursuit of science, engineering and technological careers. In South Africa in particular, there tends to be an acute shortage of skilled manpower in the field of science, engineering and technology (Maree, 2010). This could be explained by the consistent students' under-achievement and lack of interest in mathematics (Atagana, Mogari, Kriek, Ochonogor, Ogbonnaya & Makwakwa, 2009; Ogbonnaya, 2010).

To enable our students to compete favourably in the technologically oriented society, they need to be taught the relevant mathematical knowledge and skills. Mathematics as a body of knowledge embodies problem solving, deductive and inductive reasoning, among others things. Proficiency in these areas is essential when pursuing studies in science, engineering and technology. Hence, conventional reasoning would suggest that the starting point to increase the pool of scientists, engineers and technologists will be to ensure that students obtain good passes in mathematics. The poor mathematical performance of students in many African countries (see, for example, Howie & Plomp, 2002; Ogbonnaya, 2007) makes a case for the need to ensure that students' achievement in mathematics in Africa is improved. Among others factors, the problem of students' under achievement in mathematics has been attributed to the poor strategies employed in the teaching and learning of mathematics, poor infrastructure in some schools and lack of adequately trained mathematics teachers (Onwu, 1999; Spreen & Vally, 2006; Stols, Kriek & Ogbonnaya, 2008).

In South Africa, the situation is very critical; there is so much evidence of students' poor performance in mathematics (Howie & Plomp, 2002; Ogbonnaya 2010; Wessels & Nieuwoudt, 2010). A recent survey of topics in mathematics that learners find difficult to learn (Atagana, et. al, 2009) shows that most learners find a number of topics in mathematics difficult to learn. For example, 103 out of the sample of 222 Grades 10-12 students (representing 46% of the sample) reported that they find it difficult to learn Trigonometry. Many students tend to encounter difficulties with trigonometry, particularly the trigonometric functions (Moore, 2009). Trigonometry is a fundamental topic in mathematics that finds several applications in other branches of mathematics as well as in statistics, economics, surveying, architecture and many branches of engineering (Weber, 2005). In fact, "understanding Trigonometric functions is a pre-requisite for understanding topics in Newtonian physics, architecture, surveying, and many branches of engineering" (Weber, 2005:91). Hence, lack of understanding of trigonometric functions implies not fitting into these science, engineering and technology careers.

According to McGrath and Akoojee (2007), the document of the Accelerated and Shared Growth Initiative of South Africa (AsgiSA) lunched in 2005 by the South African government acknowledged the challenge that the shortage of engineering, science and technological skills poses on the developmental initiatives of the country. The challenge could be met according to McGrath and Akoojee (2007) by achieving higher levels of numeracy in the early grades of school and by increasing mathematics and science high school graduation rate, among other things.

This goal of increasing high school graduation rate in mathematics can be achieved in the near future if the students could be helped to overcome the problems they encounter in learning mathematics. To help the students overcome the problems their in learning mathematics, teachers have a great role to play because teachers have the most significant effect on what their students learn (Rice, 2003). Therefore, teachers need to be more effective in their teaching of mathematics. However, the subject matter knowledge (SMK) of a teacher is fundamental to his or her ability to provide effective teaching (Ball, Hill & Bass, 2005; Kreber, 2002). Hence, students' poor performance in mathematics could possibly be linked to teachers' inadequate subject matter knowledge to help the students learn meaningfully.

The starting point to solving the problem of students' underachievement in mathematics would be to embark on a wide range inventory/audit of mathematics teachers' subject matter knowledge and their teaching effectiveness. However, mathematics education over the years has faced the challenge of developing accurate measurement of teachers' subject matter knowledge and teachers' teaching effectiveness (Ogbonnaya & Mogari, 2011). One reason for this is the scarcity of valid and reliable instruments to accurately gauge teachers' subject matter knowledge (Heritage & Vendlinski, 2006) and their teaching effectiveness due to the intangible nature of knowledge and the vagueness of teaching effectiveness. Another possible reason is the difficult task of clearly and precisely extricating the relationship between teachers' subject matter knowledge and their effectiveness in teaching among other variables that influence teaching and learning. Hence, the current study explores the relationship between teachers' subject matter knowledge and their teaching effectiveness using valid and reliable measures of teacher subject matter knowledge and teacher teaching effectiveness.

The specific focus of my study is Grade 11 teachers' subject matter knowledge of trigonometric functions because this topic is reported to be difficult to teach and learn by a considerable number of high school mathematics teachers and students (Atagana et.al, 2009).

1.2 Context

South Africa (also known as the Republic of South Africa) is located on the southern tip of Africa. It shares borders with Botswana, Namibia and Zimbabwe on the north, Mozambique and Swaziland on the east, while to the south and west South Africa borders with the Atlantic and Indian oceans and enclaves the Kingdom of Lesotho. The Republic of South Africa has a land area of 1,219,090 square kilometres and with estimated population of 49 million (SouthAfrica.info, 2011; Statistics South Africa, 2003).

The Republic of South Africa has nine provinces, each with its own executive council, distinctive landscape, population, economy and climate. Pretoria (in Gauteng province) is the capital city of the country. South Africa is a nation of diversity, with a variety of cultures, languages and religious beliefs. The country has eleven official languages but English appears to be the most widely spoken of them. South Africa is ethnically diverse, with the largest Caucasian, Indian, and racially mixed communities in Africa (SouthAfrica.info, 2011; Statistics South Africa, 2003).

Economically, South Africa has an abundant supply of mineral resources. It has the world's 22nd-largest economy by gross domestic product and the world's 32nd-largest labour force. It is the 19th largest world producer of electricity; has the 15th longest network of railway tracks in the world; and has the 17th longest length of roadways (SouthAfrica.info, 2011).

1.2.1 Education in South Africa

The educational system of South Africa is classified into three levels, namely, General Education and Training (GET) (grade 0 to grade 9), Further Education and Training (FET) (grade 10 to grade 12), and Higher Education and Training (Tertiary education) which includes education for undergraduate and postgraduate degrees, certificates and diplomas, up to the level of the doctoral degree (SouthAfrica.info, 2010). The Matric examination is written at the end of Grade 12 and it is a requirement for admission to study at the University.

At the national level, with effect from 2009, there are two departments of education: The Department of Basic Education (DoBE) and the Department of Higher Education (DoHE). The DoBE is responsible for academic control of the first two levels (GET and FET) of education across the country by providing a national framework for school policy, while each of the nine provinces has its own provincial department of education that is in charge of administrative responsibilities at GET and FET levels of education. The DoHE is responsible for academic control of the tertiary level of education across the country by providing a national framework for higher education.

At the school level of the General Education and Training (grade 0 to grade 9) and the Further Education and Training (grade 10 to grade 12) bands, the running of the school is vested in the hands of elected School Governing Board (SGB) members (comprising the principal of the school, elected parents/guardians, and teacher representatives) while the school management team (the principal, the deputy principals and the heads of departments) are responsible for the day to day management of the school under the watchful eyes of the SGB.

The National Department of Basic Education (DoBE) is responsible for the setting and administration of National external examinations in the country. The Matric examination that is written at the end of Grade 12 is one of such examinations. At the provincial level, the provincial departments of education also set and administer some provincial external examination like the Grades 10 - 12 June examination.

Prior to 1994 during the apartheid era, the South African education system was stratified along ethnic and racial lines. There were different education systems for the different races and the school systems were inequitably resourced in terms of both human and material resources (Sayed, 2002; Thomas, 1996). One effect of the fragmented system of education, Sayed noted, was the underdevelopment of mathematics, science and technology education in the Black secondary school system because most of the teachers in the Black secondary schools were trained to teach only humanities and arts subjects.

Since 1994, the South African education system has undergone (and is still undergoing) several structural and curriculum reforms in order to overcome the inadequacies and imbalances created by the segregated and inequitable system of the past and to meet the current development needs of the country (Sayed, 2002; Soudien & Baxen, 1997). For example, in order to increase the provision of education to all citizens, the government introduced the no-fee schools programme where schools in poverty stricken areas classified as no-fee schools do not charge school fees to students but receive all their required funding from the government. Also, meals are provided to the pupils in primary schools in such areas by the government (Kiti, 2008). In order to address the highly fragmented system of education of the past and create an integrated national framework for learning achievements that would “enhance access to, and mobility and quality within, education and training”, the government setup the National Qualifications Framework (NQF) (Republic of South Africa department of Education, 1997, p. 14).

1.2.2 Teacher training/education in South Africa

In the apartheid system of government, teacher education system was racially stratified with separate teacher education colleges for the different races. This created a multiple and separate programmes of teacher education for the different races. This multiple system determined where each teacher was trained and where he/she would eventually teach. The bulk of the training in Black colleges of education was limited to humanities and arts subjects, the effect of which was the underdevelopment of mathematics, science and technology education in the black schools. This is a legacy that the democratic governments since 1994 have sought to remedy (Sayed, 2002). As observed by Adler and Davis (2006), the majority of black teachers trained under apartheid government only had a 3-year College of Education diploma and the quality of their training, especially in mathematics, was poor. So, many of the teachers of mathematics have not had the opportunity to learn further mathematics.

One important step of the first democratic government was the rationalization of teacher education by amalgamating the colleges of education with the universities. Hence, the colleges of education were incorporated into the higher education system thereby moving them from the provincial to the national control (Sayed, 2002). As Sayed observed, the government also established a framework for teacher education which encompasses, among other things, the following:

- The National Education Policy Act (DoE, 1996). This policy empowers the Minister of Education to set guidelines for teacher education and accreditation

- The Norms and Standards for Educators (NSE) (DoE, 2000b) that provides detailed account of what a competent teacher should be.

- The South African Council of Educators (SACE) established in 1996 and charged with the responsibility of teacher registration, discipline, conduct and professional development.

- National Qualifications Framework (NQF) and the South African Qualifications Authority (SAQA) charged with the responsibility of standards and quality assurance of teacher education. The teacher education quality assurance duty of SAQA is shared with the Higher Education Quality Assurance Committee (HEQC) of the Council of Higher Education (CHE).

These and other mechanisms have been used by the democratic governments since 1994 to tackle the challenges and imbalances of teacher education of the past in terms of regulation, governance, curriculum and quality assurance.

1.2.3 Mathematics education in the current South African school system

In the present day South African school system, mathematics is taught as a compulsory subject in GET band (grades 0 -9), after which it becomes an elective subject where the student chooses between mathematics and Mathematical Literacy in grades 10 – 12.

As a result of the expansion of access to mathematics education in the formally disadvantaged Black secondary schools and the change in the curriculum in the new dispensation that brought in new topics into the mathematics curriculum, there was acute shortage of qualified mathematics teachers in those schools. This led to mass recruitment of teachers to teach mathematics even from other subjects. Most of these teachers were not taught some of the topics in the curriculum in their secondary school or during their teacher training education. Hence, one major concern was how well the teachers knew the subject matter to handle the topics effectively.

The universities at their own level have also made concerted efforts since 1994 to increase the pool of qualified mathematics teachers to meet the current need of the country. For example, the Post Graduate Certificate in Education (PGCE) is run by some universities in the country. This programme is normally used as an access route to teaching for non teaching qualified graduates but as noted by Adler and Davis (2006) the programme has failed to attract many mathematics graduates into the teaching profession. Also, the Bachelors of Education degrees (B Ed) run by many higher institutions in the country through which a specialisation in secondary mathematics teaching may be obtained attract manly rejects[1] from the Bachelors of Science programme. Hence, South Africa, like many other nations, still suffers acute shortage of qualified mathematics teachers.

To circumvent the problem posed by the apparent subject matter knowledge inadequacy of some of the mathematics teachers, the national and provincial governments, other government and non-government organisations, the private sector and other stakeholders in mathematics education have been organising subject matter professional development and certification courses for the teachers. Hence, most of the mathematics teachers have obtained their subject matter competency in most of the topics through formal in-service training (for example the Advanced Certificate in Education (ACE) programme) or other professional development programmes (North, Scheiber & Ottaviani, 2010). The ACE programme is a teacher in-service programme designed to enable teachers upgrade their 3-year teaching diploma to a 4-year programme. This was aimed at addressing the inequities of the apartheid era discussed above (Adler & Davies, 2006).

[1] There are lower entry requirements for B Ed programme than for BSc programme; most of the students entering the B Ed programme have not performed well in mathematics in school (Adler & Davies, 2006)

It is against the backdrop highlighted above that mathematics education in South Africa has faced a lot of challenges even after 17 years of democracy and which points to the need for studies like the present one in order to better understand the status of mathematics education especially among the formerly mathematically disadvantaged group.

1.3 Statement of the problem

The problem of this study is to explore the relationship between mathematics teachers' subject matter knowledge and their teaching effectiveness. This will hopefully be done by addressing the following research questions:

1. Is there any relationship between mathematics teachers' subject matter knowledge and their teaching effectiveness?

Teacher effectiveness in the study is used as a composite term comprising students' achievement, peer rating, self rating and students' rating. There may then be a need to determine a possible relationship between teachers' subject matter knowledge and each of the components. Hence, in connection to question one, the study will address the following sub-questions:

i. Is there any relationship between mathematics teachers' subject matter knowledge and their students' achievement?

ii. Is there any relationship between mathematics teachers' subject matter knowledge and their students rating of the teachers' teaching effectiveness?

iii. Is there any relationship between mathematics teachers' subject matter knowledge and their peers' rating of the teachers' teaching effectiveness?

iv. Is there any relationship between mathematics teachers' subject matter knowledge and their self rating of their teaching effectiveness?

2 Is there any difference between the subject matter knowledge of effective teachers and the subject matter knowledge of ineffective teachers?

3 Is there any difference between the subject matter knowledge of the students of effective teachers and the subject matter knowledge of the students of ineffective teachers?

1.4 Significance of the study

Rice (2003) noted the absence of a strong, robust and deep body of research on teacher quality to guide crucial decisions on the teachers to hire, retain and promote. This study helps to fill the perceived research gap in this regard by detailing the indicants of teacher quality through the exploration of the relationship between mathematics teachers' subject matter knowledge and their teaching effectiveness using high quality measures of teachers' subject matter knowledge and their teaching effectiveness. Also, the use of viable approaches to measure teacher effectiveness will provide an effective rating of teachers in order to inform in-service teacher training and teacher professional development programmes

This study also helps to fill the research gap observed by Karp (2010) that it would be desirable to extend the studies of Ball et al. (2005) on developing instrument for measuring teachers' knowledge at elementary school level to middle school and high school. In order to achieve the purpose of this study, an instrument that could be used to measure teachers' subject matter knowledge of trigonometric functions at the high school level was developed.

Ever since the introduction of the National Curriculum Statement (NCS) in 2004, the South African national and provincial departments of education, private organisations, nongovernmental organisations and also the teachers have invested heavily in training and retraining of teachers. Therefore, it would seem reasonable at this point to know if the huge investments on teachers are yielding the much desired dividend of improving their teaching effectiveness.

As observed by Karp (2010), it is important to clarify and formulate a detailed description of the elements of which teachers' subject matter knowledge and skills for teaching a particular content domain consists of and the ways in which the absence of such skills is manifested. This is another need that this study sought to meet.

As stated above students have difficulties learning trigonometric functions but despite the documented students' difficulties as observed by Weber (2005), the educational literature in this area seems sparse. Hence this study will add to the research in the teaching and learning of trigonometric functions and possibly serve as a lead-way to the effective teaching and meaningful learning of trigonometric functions especially in the South African context.

Darling-Hammond (2000) observed that while few people would doubt the importance of subject-specific knowledge for teaching effectiveness, direct evidence that students' achievement is related to teacher subject matter knowledge was surprisingly sparse. There seems to be no recorded assessment of teachers' subject matter knowledge of trigonometric functions to the knowledge of the researcher. Therefore, this study makes a contribution to mathematics education research by carrying out a valid and reliable measure of teachers' subject matter knowledge of this important branch of mathematics. This could be a lead way to another cognitive theory. The study is also significant this time due to its importance in informing the current debate on teacher education and teacher subject matter competence in South Africa.

Over two decades ago Shulman (1986a) observed that "where the teacher cognition programme has clearly fallen short is in elucidation of teacher's cognitive understanding of subject matter content and the relationships between such understanding and the instruction teachers provide for students" (p. 25). This statement is as true today, in the context of South Africa, as it was then in the United States. Studies on the relationship between teachers' subject matter knowledge and their effectiveness have not yielded consistent results over the years which could be attributed to the measures of teachers' subject matter knowledge and their teaching effectiveness used. Hence, the current study makes a further contribution to this issue. It will also, in a way, open up new possibilities for improving mathematics teaching and students' achievement in mathematics.

This study also informs areas of teachers' difficulties/weakness in the teaching of trigonometric functions; this will guide in-service and professional development training for mathematics teachers that will enhance the teachers' teaching effectiveness and consequently students' achievement.

In addition, as posited by Wessels and Nieuwoudt (2010) that in order to plan meaningful professional development for teacher, it is necessary to profile teacher according to their subject matter knowledge of the subject, the study will provide useful model that can be used by national and provincial departments of education, school administrators, school mathematics curriculum developers and other stakeholders for formulating educational policies in terms of teacher recruitment, training and professional development.

To textbook writers, the study will possibly help them to write more quality books that will meet the current needs of mathematics teaching and learning in the country.

1.5 Definition of terms

Effective Teaching

Teaching that helps to achieve the teaching goals (Anderson, 2004) or teaching that produces desired students' academic results (Uchefuna, 2001).

Further Education and Training (FET)

FET refers to grades 10 -12 of the South African school system. It is also referred to as High school level.

General Education and Training (GET)

GET is grades 0 - 9 of the South African school system.

High school

High school refers to the Further Education and Training (FET) band. That is grades 10 – 12.

Matric examination

This is the short form of Matriculation examination. It is a national examination written at the end of grade 12. That is at the exit of FET education. It is used to gain admission to the higher education and training or to secure employment.

Subject Matter Knowledge

This refers to knowledge of the subject content. This goes beyond knowledge facts or concepts in the subject domain but it encompasses knowledge of the substantive and syntactic structure of the subject (Schwab, 1978). Subject matter knowledge is elaborated in section 2.3.2.

Teaching

Teaching is used in this study to mean everything a teacher does to support the learning of his/her students. It means the interactive work of facilitating lessons in classrooms and the entire tasks that arise in the course of the work (Ball, Thames & Phelps, 2008).

Teaching effectiveness

This is a measure of how teaching is able to meet curriculum goals or help students achieve the learning objectives. Teaching effectiveness is discussed in section 2.3.3.

1.6 Outline of Chapters

The study is reported based on the following outline of chapters.

Chapter one - Introduction

This chapter gives the context of the study describing the background of the study, the statement of the problem, the research questions, the significance of the study, a brief definition of terms and structure of the thesis.

Chapter Two - Conceptual framework and literature review

In this chapter, the conceptual framework guiding the study and review of some related literature are presented. The literature is based on the main themes of the study – teacher subject matter knowledge and teacher effectiveness.

Chapter Three – Research methods

This chapter focuses on the methods used in the study including research design, sample selection method, data collection instruments – development of instruments and procedures of data collection, validity and reliability of instruments, pilot study, and ethical issues considered in the study.

Chapter four - Data analyses

This chapter focuses on data analyses methods and procedures. The results of data analyses are presented and the results were used to draw together the findings of the study and answer the research questions.

Chapter five – Discussion

Here, the findings of the study are discussed alongside the implications

Chapter Six - Summary, conclusion and recommendations

This chapter summarises the study and draws a conclusion upon which recommendation were made.

1.7 Conclusion

In this chapter the orientation of the study was established. The study was put into context. The research questions of the study, the significance of the study, were addressed. Also, the definitions of terms as they were used in the study and the structure of the thesis were presented.

1.8 Projection for the next chapter

In the next chapter, the conceptual framework guiding the study and review of some related literature are presented. The literature are based on the main themes of the study – teacher subject matter knowledge and teacher effectiveness.

Chapter Two

Conceptual Framework and Literature Review

2.1 Introduction

Research into the teaching and learning of mathematics has been a major focus of educational studies in the past five decades (Darling-Hammond, 2000; Grouws & Cebulla, 2000). This is accentuated by the growing evidence of the importance of mathematics in the globalised knowledge-based economy and students' poor achievement in mathematics and their lack of interest in pursuing mathematical, scientific and engineering/technological careers (Durant, Evans, & Thomas, 1989; Reynolds & Farrell, 1996).

The importance of mathematics in today's world to a person and to a nation as highlighted in the mathematics National Curriculum Statement (NCS) grades 10 -12 (DoE, 2003: 9) states that "Mathematical competence provides access to rewarding activity and contributes to personal, social, scientific and economic development".

Ogbonnaya (2008) noted that the students' poor achievement in mathematics has become an issue of global concern and for many years the governments of nations, teachers, researchers and other stakeholders in education have tried to understand the variables that are related to students' achievement in mathematics. Given the likelihood that teacher variables of some kind are implicated, a number of research studies have focused on an array of teacher variables presumed to be related to students' achievements in mathematics. For instance, some of the studies focused on teacher qualifications (e.g. Darling-Hammond, 2000; Ogbonnaya, 2007; Rice, 2003), some others on teacher subject majors (e.g. Ogbonnaya & Osiki, 2007; Wilson & Floden, 2003) and some others on teacher classroom instructional practices (e.g. Stols, Kriek & Ogbonnaya, 2008; Grouws & Cebulla, 2000).

The need to improve teachers' effectiveness in teaching mathematics and hence students' achievement in mathematics in South Africa is very critical in the light of the need to increase the pool of scientists, engineers and technologists to meet the government's millennial development goals. However, the teacher factors that can actually make the teachers to be more effective in helping students to achieve better results in mathematics in South Africa have not been identified by any empirical study to the knowledge of the researcher. There is evidence from research literature indicating that students' differential achievements in mathematics are traceable to teacher factors (Darling-Hammond, 2000; Ingvarson et al., 2004; Mogari, Kriek, Stols & Ogbonnaya, 2009; Rice, 2003).
Collias, Pajak, & Rigden (2000) argued that teacher variables are the most important determinant of their effectiveness in teaching. Therefore, teacher variables could provide an explanation for the student's poor achievement in mathematics in South Africa.

One very important teacher variable in teaching is teacher's subject matter knowledge of the subject (Wessels & Nieuwoudt, 2010). Therefore, the present study seeks to explore the relationship between mathematics teachers' subject matter knowledge and their teaching effectiveness in one district in a province in South Africa.

2.2 The hypothetical relationship between teacher subject matter knowledge and teacher teaching effectiveness

The focus of this study is on the relationship between teacher subject matter knowledge and teacher teaching effectiveness. It is conceptualised that a teacher's subject matter knowledge has a remarkable relationship with the teacher's teaching effectiveness. The hypothetical relations between teacher subject matter knowledge and teacher teaching effectiveness among some latent factors that are related to teaching and learning is represented with a schema in Figure 2.1 below.

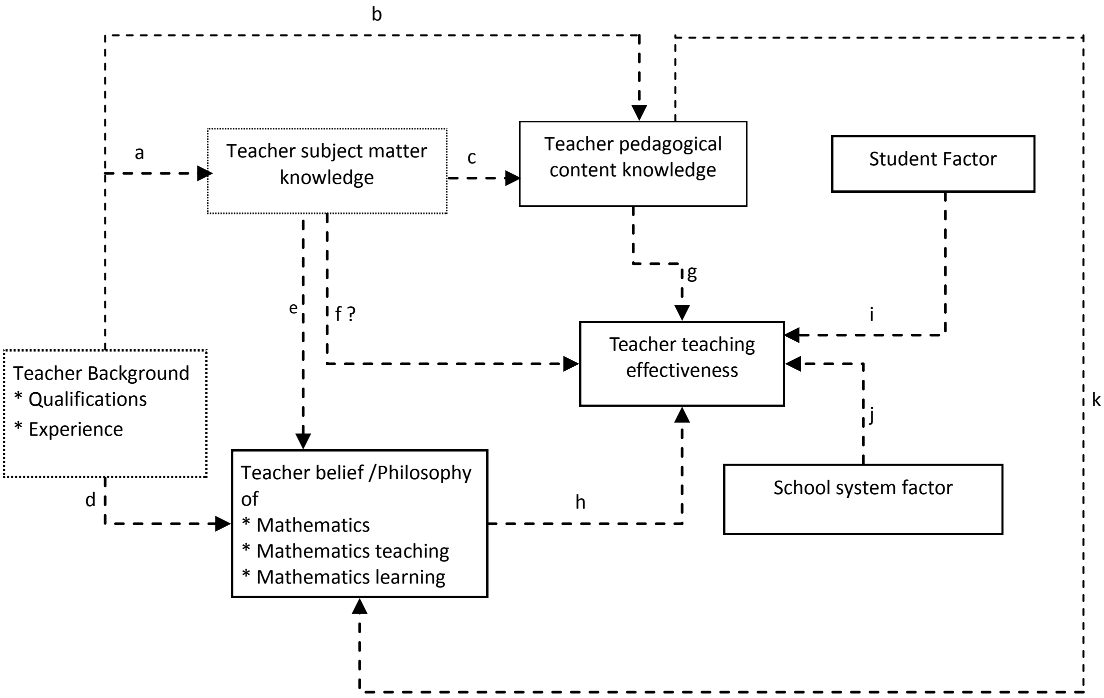

Figure 2.1 Hypothetical diagram of the relation between teacher subject matter knowledge and teacher teaching effectiveness among other factors.

The framework contains seven concepts/variables: Teacher background, teacher subject matter knowledge, teacher pedagogical content knowledge, teacher belief, student factor, school system factor and teacher effectiveness. The respective interlinks between the seven concepts/variables (designated by the letters *a* – *k*) illustrate how one concept/variable relates to the other, the link *f* being the subject for discussion.

It is hypothesised that teacher teaching effectiveness is a function of teacher factors (i.e. teacher subject matter knowledge, teacher pedagogical content knowledge and teacher belief or philosophy), student factor and school system factor. Teacher factors are related to teacher background – teacher qualifications and experience.

2.3 Review of literature on the key variables and the relationship between them

2.3.1 Teacher background

In this study, teacher background is used to encompass teacher qualifications and experiences through which a teacher develops knowledge of the subject matter and some set of beliefs about mathematics and its teaching and learning. See Sections 2.61 and 2.66 for discussions on teacher qualifications and teacher experience respectively.

2.3.2 Subject matter knowledge (SMK)

Even & Tirosh (1995: 2) noted that "the mid 1980's marked a change in conceptions of the teacher's role in promoting learning; which now came to include setting mathematical goals and creating classroom environments to pursue them; helping students understand the subject-matter by representing it in appropriate ways; asking questions, suggesting activities and conducting discussions". Similarly, in South Africa, the Mathematics - National Curriculum Statement stipulates that the teaching of mathematics should help students develop deep conceptual understanding in order to make sense of mathematics and acquire the specific knowledge and skills necessary for the application of mathematics to physical, social and mathematical problem (Department of Education, 2003). These roles demand critical subject-matter knowledge on the part of the teacher.

Teacher subject-matter knowledge has been accorded different conceptualization over the past years. A few decades back, teachers' subject-matter knowledge was defined quantitatively, for example in terms of the academic qualification, the number of courses taken at college or university, teachers test scores on certification test, etc. (Ball, 1991; Even & Tirosh, 1995). In the more recent years, teachers' subject matter knowledge has been defined more qualitatively with emphases on cognitive processes and understanding of facts, concepts and principles, and how the facts, concepts and principles are connected and organized. Conceptualisation of teacher subject matter knowledge in terms of the nature and scope of knowledge of the subject matter have also received more attention (Even & Tirosh, 1995, Shulman, 1986b).

According to Davis (2003), the subject matter of any area of study includes the topics, facts, definitions, concepts, procedures, organizing structures, representations, influences, reasons, truths and connections within and outside the area of study. To Van Billiard (2006), SMK is knowledge that is specific to a given topic or content area. Leinhardt and Smith (1985) with particular reference to mathematics conceptualised mathematical SMK as the knowledge of "concepts, algorithmic operations, the connections among different algorithmic procedures, the subset of the number system being drawn upon, the understanding of classes of student errors, and curricular presentations" (p 247). Drawing from the conceptualisation subject matter knowledge by Leinhardt and Smith (1985), some scholars have conceptualised teachers' subject-matter knowledge as encompassing knowledge and understanding of facts, concepts, and principles and the ways in which they are organized, as well as knowledge of the ways to establish truth in the discipline (Even, 1990; Kennedy, 1990; Shulman, 1986b; Tamir, 1987). This conceptualisation presupposes three aspects of SMK: knowledge of the content of the subject, knowledge of the organization of the content, and knowledge of the methods of inquiry used within the subject. Kennedy (1990) offered an explanation of each of the three aspect of teachers' subject matter knowledge as follows:

Knowledge of the content of the subject includes knowledge of the facts, concepts, principles or laws that have been gathered over years of inquiry into the subject. Content is usually presumed both to increase in volume and to change in character over time. Knowledge of the organization of the content refers to knowledge of the relationships among facts and ideas which students of the discipline have developed. The facts and ideas may not be important in their discrete isolated forms but are made important through the relationships that exit among them. The relationships among the facts and ideas form a body of knowledge such that the significance of any idea or fact is ascertained by its apparent relation to other ideas and facts. Knowledge of the methods of inquiry includes knowledge of assumptions, rules of evidence, or forms of argument that are or can be employed in the discipline.

Shulman (1986b) distinguishes between two kinds of understanding of the subject-matter that teachers need to have. These are ‘knowing that’ and ‘knowing why’. The teacher needs to understand that something is so and also why it is so.

‘Knowing that’ is the most basic level of subject-matter knowledge. It includes declarative knowledge of rules, algorithms, procedures and concepts related to specific mathematical topics in the school curriculum. ‘Knowing that’ is important as a basis for adequate pedagogical content knowledge. ‘Knowing why’ is knowledge which pertains to the underlying meaning and understanding of why things are the way they are. Though "Knowing that" is certainly important it is not enough “Knowing why” enables better pedagogical decisions (Even & Tirosh, 1995).

Shulman (1986b) argues that “teachers must not only be capable of defining for students the accepted truths in a domain. They must also be able to explain why a particular proposition is deemed warranted, why it is worth knowing and how it relates to other propositions" (p. 9).

Drawing from Shulman’s conceptualisation of teacher subject matter knowledge, Ball (1991), argued that subject matter knowledge encompasses an understanding of the intellectual fabric and essence of the subject matter itself.

Similarly, Schwab (1978) argued that subject matter knowledge is more than knowledge of facts and concepts. Schwab conceptualized subject matter knowledge to comprise knowledge of the substantive and the syntactic structures of the subject matter. Knowledge of substantive structures refers to knowledge of facts, concepts and the relationships between the facts and concepts in the subject domain. This knowledge of the substantive structures is what Shulman referred to as ‘knowing that’.

Knowledge of the syntactic structures is knowledge of legitimacy principles for the rules of a subject domain. That is the ‘knowing why’ in the Shulman’s conceptualisation. Teachers’ knowledge of the substantive structures and knowledge of the syntactic structures of the subject domain are both necessary for effective teaching.

This study focused on teachers' understanding of grade 11 trigonometric functions in the National Curriculum Statement (DoE, 2003). The curriculum demands that the students at the end of their learning experience of trigonometric function should be able to

a) Demonstrate the ability to work with various types of trigonometric functions including $y = a\sin(x) + q$, $y = a\cos(x) + q$, $y = a\tan(x) + q$, $y = \sin(kx)$, $y = \cos(kx)$, $y = \tan(kx)$, $y = \sin(x + p)$, $y = \cos(x + p)$, $y = \tan(x + p)$
b) Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae), and
c) Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures about the effect of the parameters k, p, a and q for the functions (DoE, 2003: p 23).

These curriculum assessment guidelines demand knowledge of the facts, concepts and the relationships between the facts and concepts and also knowledge of the legitimacy principles for the rules pertaining to the trigonometric functions. In other words, the curriculum assessment guidelines demands that the teacher of the curriculum should have knowledge of the substantive and the syntactic structures of the subject matter of trigonometric functions to enable him/her facilitate students learning of the curriculum envisioned learning outcomes.

Hence, for the purpose of this study, teachers' SMK is conceptualized to be knowledge of the facts, concepts and the relationships between the facts and concepts of the subject domain (trigonometric functions in this case) and also knowledge of the validity of the rules that apply to trigonometric functions.

2.3.3 Teaching effectiveness

Teaching effectiveness or effective teaching is an elusive term amenable to several meanings and interpretations depending on the context where it is used and who uses it.

The purpose of teaching is to promote learning and the major role of a teacher is to facilitate learning. Teaching entails the application of skills and the carrying out of appropriate activities that enable students to develop and ultimately exhibit the expected learning behaviours. There is evidence that there is a strong relationship between effective teaching and students' academic achievement (see, for example, Rice, 2003). This suggests that teacher quality is an important factor that affects learning (Collias, Pajak & Rigden, 2000) because decisions that teachers take about their teaching can either facilitate or impede students' learning (Wenglinsky, 2002). Goe (2007) observed that although a synthesis of research studies shows that some teachers are more effective in contributing to their students' learning than others, it has been a challenge for any study to systematically explain the significant difference in teachers' skills/characteristics that account for the difference in their teaching effectiveness.

Tsang and Rowland (2005) argued that for a teacher to be effective, he/she must have good mastery of the substantive syntactic structures of the subject which enables him to unpack the subject's content in a way that makes meaning to the students. Such a teacher can be seen to understand the subject and is also able to present it to students in ways that establish a foundation of knowledge that the students can build on. An effective teacher has a strong knowledge base of the subject matter content and also has a repertoire of pedagogical strategies that he/she can draw on in order to bring the lesson home to the students. Put in other words, an effective teacher must have a comprehensive understanding of the subject content and a powerful pedagogical representation of the subject. This is in concert with the primary goal of teacher education that involves the disciplinary education through which subject matter content as well as pedagogical knowledge can be acquired (Adeosun, Oni, Oladipo, Onuoha, & Yakassai, 2009).

Bransford, Brown & Cocking (2004: 188) contended that for teaching to be effective, teachers need "to have a deep understanding of the subject matter and its structure, as well as an equally thorough understanding of the kinds of teaching activities that help students understand the subject matter in order to be capable of asking probing questions''. This implies that mathematics teachers need thorough understanding of the subject matter of mathematics to know how to address the topics in their classes.

Anderson (2004) conceptualised effective teachers as those who through appropriate use of their repertoire of knowledge and skills achieve the teaching goals imposed on them by the authorities or the goals they established for themselves. This implies that an effective teacher facilitates the actualisation of the curriculum goals in his/her class.

Similarly, the South African mathematics revised National Curriculum Statement envisioned, among other things, that the teaching of mathematics can help the students to recognise that mathematics is a creative part of human activity, develop deep conceptual understanding in order to make sense of mathematics, and acquire the specific knowledge and skills necessary for the application of mathematics to physical, social and mathematical problem (Department of Education, 2002: 5). This supposes that effective teaching of mathematics will orchestrate the actualisation of the vision.

These views show that certain teacher characteristics are essential for effective teaching. The characteristics are complementary and interrelated and can be grouped under knowledge of the subject; lesson preparation, organisation and presentation; effective student assessment and communication with the students. These characteristics work together to help a teacher accomplish the curriculum learning goals.

Hence, in this study, effective teaching is conceptualised as the teaching that enables students to achieve their academic learning goals and effective teachers are teachers that possess the appropriate knowledge and skills needed to enable students to achieve the educational learning goals or curriculum standards. Put in other words, effective teachers must possess the knowledge and skills needed to attain the curriculum goals (standards) and must also use the knowledge and skills appropriately in order to accomplish the goals.

2.3.4 Teacher pedagogical content knowledge (PCK)

The notion of PCK was advanced by Shulman (1986b) in which he proposed the consideration of the relationship between teacher content knowledge and teacher pedagogical knowledge. He argued that PCK is an amalgam of content knowledge and pedagogical knowledge that enables the transformation of the subject matter knowledge into pedagogically useful forms. It is the content knowledge that deals with teaching. Shulman conceived PCK to be different from the subject knowledge of the subject expert as well as the universal pedagogical knowledge shared by teachers across disciplines. While Shulman's original conception of PCK has been critiqued and modified by many scholars including Shulman himself, the centrality of subject matter knowledge in PCK have been acknowledged by many researchers (Van Driel, Verloop & De Vos, 1998). At the centre of PCK is the transformation of subject matter knowledge in teaching. To most scholars SMK is an indisputable foundation for the development of PCK (Rohaan, Taconis & Jochems, 2010). Smith (cited in Rohaan et al., 2010) argued that subject matter knowledge is vital for effective teaching because "strong and useful pedagogical content knowledge cannot be built on a shaky content foundation". This suggests that studying the relationship between teacher subject matter knowledge and their effectiveness in teaching (*f* in Figure 2.1) will likely offer great revelation into the knowledge base for effective teaching.

2.3.5 Teacher belief/philosophy

A teacher's belief is his/her personal opinion, viewpoint or construct that can provide an understanding of the teacher's practice (Pajares, 1992; Richardson, 1996). Therefore, a teacher's belief about teaching and learning mathematics is the teacher's viewpoint (philosophy) about the teaching and learning of mathematics. It is an expression of the teacher's values and thoughts about mathematics teaching and learning. In other words, it describes an overall system that guides the teacher's teaching decisions and classroom instructional behaviours. Beliefs are personal principles that an individual constructs from experiences and employs, often unconsciously, to interpret new experiences and information to guide action (Pajares, 1992). Belief systems are dynamic in nature, undergoing change and restructuring as individuals evaluate their beliefs against their experiences (Thompson, 1992).

Mathematics teachers' beliefs play a central role in their teaching (Handal & Herrington, 2003); they are stable sources of teachers' reference in their teaching. They are built up over time, and are related to teachers' theories of mathematics, the nature of mathematics teaching practices, their roles as teachers, and their relationships with their students. Teachers' beliefs about teaching and learning are formulated and reformulated as they go through the stages of teacher development. The beliefs they hold are the basis for their personal knowledge about teaching and learning and have strong influence on their planning, instructional decisions and classroom behaviours (Yang, 2010). Also, belief explains how and why different teachers may have different reasons for selecting a particular content, place different emphasis on the same content and have different styles of teaching (Md. Zain, 2007).

Carter and Norwood (1997) categorized teacher beliefs about teaching and learning mathematics into three: belief about the nature of mathematics, belief about the nature of mathematics teaching, and belief about the process of learning mathematics. These three categories form the basis of mathematical philosophy in the system of beliefs toward teaching and learning mathematics.

A teacher's belief about the nature of mathematics is his or her conception of the nature of mathematics as a whole. Such views form the basis of the philosophy of mathematics, although some teacher's views may not have been elaborated into fully articulated philosophies. The teacher's belief about the nature of mathematics is not necessarily consciously held view; rather it may be implicitly held philosophy (Ernest, 1989). A teacher's belief about the teaching of mathematics is the teacher's conception of the type and variety of teaching roles, actions and classroom activities associated with the teaching of mathematics. A teachers' belief about learning of mathematics is the teacher's mental models or views of learning mathematics. This is closely related to teacher's belief about mathematics teaching. It consists of the teacher's perception of the process of learning mathematics, the behaviours and mental activities the students need to be involved in to learn mathematics, and what constitute correct and ideal learning activities (Ernest, 1989).

Research suggests that teacher's beliefs relate to their classroom practice (Stipek, Givvin, Salmon, & MacGyvers, 2001). The beliefs teachers hold about teaching and learning will determine the teachers' implementation or lack of implementation of curriculum requirements. If teachers' beliefs do not match curriculum demands, it is likely that they will not work in line with the requirements. Teachers' beliefs therefore can make them to be either obstacles to or conveyers of curriculum requirements (Prawat, 1990). Similarly, Cuban (1993) noted that teachers' knowledge, beliefs and attitude explain the core of their instructional practices and hence their teaching effectiveness. Beliefs, Topper (2000) acknowledged, are filters through which teachers make sense of the curriculum and influence the classroom instructional behaviour they display.

A teacher's beliefs are influenced by the teachers' background, SMK and pedagogical content knowledge (PCK). Studies (e.g. Beswick, 2008; De Leon-Carillo, 2007; Raths, 2001) have documented how teachers' beliefs about teaching and learning are influenced by the teachers' trainings, professional developments, and experiences (both as students and as teachers). Beswick (2005) posits that the relationship between teachers' beliefs and their classroom practices though complex and subtle is powerful and crucial determinants of what teachers do in their classrooms.

In summary, teacher's belief has influence on his/her classroom instructional behaviours. Teacher's belief about the nature of mathematics is the basis of the teacher's beliefs about the teaching and learning of mathematics.

2.3.6 School System Factor

School system Factor is used here to refer to the conditions under which the teacher works. This includes the immediate school working condition like the provision of teaching resources, the professional work community and also the educational policies, the curriculum, time allocated for teaching and any other legislation under which the school is operated. Teacher teaching effectiveness can be affected by the conditions within the school (Ingvarson et al., 2004).

2.3.7 Student Factor

Student factor refers to what the students bring to the class. It comprises the nature of the social background of the students - their attitude, motivation, interest and proficiency level, their prior knowledge, goals, beliefs and dispositions they bring into class with them (Ahmad, 2008). These can influence students' classroom interaction and thus influence the teachers' teaching effectiveness.

2.4 Evaluation of teacher subject matter knowledge

The evaluation of subject matter knowledge of mathematics teachers has been a cause for concern in mathematics education. Knowledge is intangible; hence education researchers over the years have used various objective markers that are tangible and practical as proxy measures of teachers' subject matter knowledge. For example, Rice (2003) used teachers' level of qualifications as marker for subject matter knowledge, Darling-Hammound (2000); Greenberg, Rhodes, Ye and Stancavage (2004); Wenglinsky (2002), and Wilson and Floden (2003) used subject majors as measures of teacher knowledge, and some other studies used teachers coursework coverage or grade points average (GPA) at college or university.

The assumption behind the use of the proxy measures is that teachers with higher GPAs, for example, or those that covered more courses at college or university, or teachers that majored in mathematics have more subject matter knowledge of mathematics than their counterparts that had lower GPAs or covered fewer courses at college or university, or did not major in mathematics. However, one major shortcoming with the use of the indirect or proxy measures is that variations in quality of education and training activities could mean that teachers with the same paper credentials possess different levels of subject matter knowledge. The proxy measures do not give a clear indication of the specifics of what the teachers know and do not know about the subject content in which case studies that use them do not closely examine the content, scope or nature of knowledge that would improve teaching (Ball, Lubienski & Mewborn, 2001)

By contrast, the use of cognitive test to measure teachers' subject matter knowledge provides direct measure of the teachers' knowledge in the content domain (Mullens, Murnane & Willett, 1996). It also gives the researcher control over the actual content that is measured (Ahn & Choi, 2004). The use of tests to evaluate teachers' subject matter knowledge is hinged on the philosophy that 'the proof of the pudding is in the eating'. That is to say that the true test of knowing is in the doing. Hence, the true value or quality of a teacher's subject matter content can only be judged when it is tested. Moreover, as argued by Odden, Borman and Fermanich (2004) standard based teacher test scores could be useful in research on teacher effects on student learning as teachers' scores from well designed, practice based teacher evaluation system could be measures of teacher quality that can be used in studying the relationship between teacher quality and student learning.

Much as the use of tests provides a direct measure of teachers' SMK, it may also lead to misleading results if the tests are not aligned with the curriculum the students are expected to be taught by the teachers' or if the cognitive demands (complexity) of the tests questions are not strong enough to differentiate between teachers with deep understanding of the subject matter and teachers with shallow understanding of the subject matter (Ahn & Choi, 2004). For instance, if the test is not aligned with the curriculum the teachers are teaching, it means that the teachers are tested on something different from what they are expected to teach. Hence, the teachers SMK that is relevant to their work and brought to bear in the classroom is not being tested. Also, if the cognitive demand of the test is only at the level of knowledge (recall) and application or performing routine procedures (Bloom, 1956; DoE, 2008), it may be difficult to differentiate between the knowledge of the teachers hence studies that use such a test are often beset with the issues of validity and reliability. However, when teachers' test is of higher cognitive demands (problems that seek conceptual understanding and problem solving/reasoning), it becomes possible to tests teachers understanding of the content above basic skills as their knowledge at the higher levels may be more influential in teaching the content effectively (Ahn & Choi, 2004). Therefore, in using test to measure teachers' SMK it would seem reasonable to align the test content with the curriculum that the teachers are expected to teach and also make the cognitive demands of the test items to include conceptual understanding and problem solving/reasoning questions together with knowledge (recall) and application or performing routine procedures questions in order to differentiate between teachers with varying levels of the subject matter knowledge.

In this study, achievement test was used to evaluate the teachers' subject matter knowledge of the content domain – trigonometric functions.

2.5 Evaluation of effective teaching

The conceptualisation of effective teaching described in section 2.3.3 assumes that effective teachers: are masters of the subject matter, are aware of the students' intended learning goals (curriculum goals) and possess skills which they combine with their knowledge of the subject matter, knowledge of the students' errors/misconceptions and knowledge of curriculum goals to accomplish the students' intended learning goals. The goals guide the effective teachers' planning and delivery of lessons.

Also, effective teachers design appropriate learning units that are linked to the standards and actively pursue these goals. Hence, they set their teaching goals (directly or indirectly) to achieve the curriculum standards. It is also assumed from the definition that teacher effectiveness can be assessed in terms of students' behaviour and learning.

Researchers in education have advocated various measures of teacher teaching effectiveness but there have been some controversies over the capability of each of the measures to effectively give objective, dependable and accurate indication of effective teaching. Students' achievement, students' evaluation of the teachers' teaching, peer evaluation of the teacher, classroom observations, self evaluation, lesson plans, teaching portfolios and students' work-sample reviews are some of the teaching effectiveness evaluation tools (Berk, 2005; Mathers, Oliva & Laine, 2008; Doyle, 2004). The triangulation of multiple-methods and multiple-sources as supported by teacher evaluation experts (Ory & Ryan, 2001) will likely eliminate the potential bias posed by each of the methods and sources and give a more objective measure of teaching effectiveness.

In this study, the triangulation of data from three direct observation sources of teaching effectiveness namely, students' ratings, peers' ratings and teachers' self ratings were combined with indirect source of teaching effectiveness namely students' achievement tests to evaluate teacher teaching effectiveness. It is believed that the triangulation of the sources and method will likely yield a more accurate, valid and reliable measure of teacher teaching effectiveness than when such evaluations rests solely on the use of a method or source (Salsali, 2005).

2.5.1 Students' achievement test.

This entails the use of students' scores in a standardised test to measure the impact of teachers' instruction on the students. Over time, students' academic performance in both internal and external examinations had been used to evaluate teachers' effectiveness (Gallagher, 2004; Milanowski, 2004; Mathers, Oliva, & Laine, 2008). Teachers have significant influence on students' academic achievement because it is teachers that are ultimately responsible for translating educational policy and curriculum intentions into learning opportunities for the students (Afe, 2001). Hence, an effective teacher according to Uchefuna (2001) is one that produces desired students' academic results. Students' examination scores are often used as a measure of educational output because students test scores have been shown to be positively correlated with their high school graduation rate, future employment prospects and adult wages (Currie & Thomas 2001; Hanushek & Raymond 2002). Students' average scores can better reflect the result of a teacher's teaching effectiveness (Lin & Lawrenz, 1999). Therefore, a natural measure of teacher effectiveness might be assumed to be the average achievement test scores of his/her students. Similarly, Le and Buddin (2005) from their synthesis of research pointed out that students test scores can be used as a criterion measure for teacher teaching effectiveness.

In order to use standardised test to evaluate teachers' teaching effectiveness, it is absolutely necessary to ensure that a high level students' opportunity to learn (OTL) is provided by the teachers (Anderson, 2004). In this study, students' opportunity to learn was ascertained and used as inclusion criterion for the teachers and their students.

2.5.2 Student evaluation

Students' evaluation of teachers is one of the most popularly used methods for evaluating teacher teaching effectiveness. The use of students' rating of teachers teaching as a means of evaluating teacher teaching effectiveness is supported by the fact that the most acceptable criterion for measuring effective teaching is the amount of students learning that takes place. According to research, there are consistently high correlations between students' ratings of the amount learned in a subject matter and their overall ratings of the teacher (Theall and Franklin, 2001). According to Greenwald (2002), the reliability and validity of students rating as a measure of teaching performance have been generally supported by researchers. The review of studies by Prebble et al. (2004) on the impact of student assessment of teaching on teaching quality shows that students assessments of teaching are among the most reliable and accessible indicators of teacher teaching effectiveness. Theall and Franklin (2001) also indicated that students are the most qualified source to rate the extent to which teaching is productive, informative, satisfying or worthwhile. Zabaleta (2007) indicated that student ratings of teaching have become a part of the evaluation system in higher education and results from student ratings of teaching effectiveness have been used to make critical judgement in higher education (Beran & Rokosh, 2009). Evidence also shows that high school students are capable of distinguishing effective teachers (Irving, 2004).

2.5.3 Self evaluation

In self evaluation, a teacher carries out self assessment of his/her practices. It is a process that enables teachers to reflect and analyse their own instructions in retrospect. Hence, the teachers review their performances and rate them. A teacher's self evaluation is an important source of evidence of his/her teaching effectiveness and it demonstrates the teacher's knowledge about teaching and perceived effectiveness in the classroom (Cranton, 2001). One main concern about the use of teachers' self rating to evaluate effectiveness is the possible teachers biased estimate of their teaching effectiveness. However, as observed by Berk (2005) a teacher's self rating of his/her teaching effectiveness can provide support for what the teacher does in the classroom and can present a picture of the teacher's teaching that is unobtainable from any other source. Also it seems reasonable that a teacher’s self assessment of his/her teaching should count for something in the teaching effectiveness equations (Berk, 2005).

A teaching effectiveness self evaluation instrument should be structured to seek teacher information in the areas of knowledge of topic/subject, lesson planning/presentation, classroom approach, and teacher-student communication (Seldin, 1999; Berk, 2005). The self rating instrument that was used in the study is similar to the peer rating instrument and it was developed to seek teachers’ information about their knowledge of topic/subject (subject mastery), lesson preparation, organisation and presentation, student assessment and communication with students.

2.5.4 Peer evaluation

Peer evaluation is a process by which the quality of a teacher's work (teacher’s teaching effectiveness) is evaluated by his/her colleagues. It is the process of teachers evaluating teachers. Peers' knowledge of subject matter and their understanding of actual teaching contexts make them uniquely qualified to judge the adequacy or otherwise of classroom instructional process (French-Lazovik, 1981). Also, the organised review of a teacher’s work by his/her colleagues makes the work of the teacher evident. Peer evaluation has been used by a few studies in teacher quality research. For example, Ferguson and Womack (1993) used supervisor evaluation as a measure of teaching effectiveness in their study.

Peer evaluation of teaching requires observation of teaching, review of teaching materials, teacher portfolios and students’ portfolios before completing the peer evaluation instrument.

In this study, teacher peer evaluation was carried out by peers with knowledge of the curriculum, sound instructional background and experience in teaching mathematics at high school level. Two peer evaluators (rater) were used to peer evaluate each teacher’s teaching effectiveness.

2.6 Previous Related Studies on the relationship between Teacher Subject Matter Knowledge and Their Teaching Effectiveness

One goal of mathematics instruction is to enhance students' understanding of both the concepts and the procedures of mathematics. To successfully accomplish this goal, the teachers must be rich in mathematical content and process; they need to have sound knowledge of the concepts, skills, and reasoning processes of mathematics (NCTM[1], 2007). Teachers need to be well versed and deep rooted in the subject to enable them build capacity to pose questions, select appropriate tasks and evaluate students understanding of the subject matter (Grossman, Wilson & Shulman, 1989). Such a deep rooted knowledge teacher will not need to rely heavily on textbooks or present content in a fragmented manner without giving adequate explanations to key concepts of the subject matter or attending to students' difficulties that could arise during teaching (McDiarmid, Ball & Anderson, 1989).

Previous research studies (for example, Sanders & Rivers, 1996; Ma, 1999; Collias, Pajak & Rigden, 2000; Darling-Hammond, 2000) indicated that teacher knowledge matters because

> *To implement standards and curriculum effectively, school systems depend upon the work of skilled teachers who understand the subject matter. How well teachers know mathematics is central to their capacity to use instructional materials wisely, to assess students' progress, and to make sound judgments about presentation, emphasis, and sequencing.* (Ball et al., 2005, p. 14)

Similarly, Rohaan, Taconis and Jochems (2010) emphasised that subject specific teacher knowledge in general is important for effective teaching. Hence, teachers need a thorough understanding of the subject matter to know the topics to address and how to address them in their lessons.

Knowledge is intangible and hence difficult to measure. Therefore, researchers use various approximate and tangible indicators to measure knowledge. Le and Buddin (2005) identified

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that a few studies in teacher education and teacher quality research have used teacher tests (like teacher licensure test) to examine teacher subject matter knowledge while most studies have used subject-specific coursework or subject-specific qualifications or degrees to more broadly examine teacher subject matter knowledge. This review includes the studies that used teacher tests and those that used teacher subject-specific coursework or subject-specific qualifications or degrees, teacher certification, and teacher experience as indicators of teacher subject matter knowledge.

2.6.1 Qualifications

In their study, in which they used teachers' qualifications in terms of having a bachelor's or master's degree in mathematics or not to measure teacher subject matter knowledge and students score on a standardised mathematics test to measure teacher teaching effectiveness, Goldhaber and Brewer (2000) found a positive significant relationship between teacher subject matter knowledge and teacher teaching effectiveness.

Greenwald, Hedges and Laine (1996) in their meta-analysis of studies on the relationship between school resources and student achievement used teacher qualification (measured by having a master's degree or not in the teaching subject) to indicate teacher subject matter knowledge. The study found that there was a significant positive relationship between teacher SMK and students' achievement. Similarly, Goldhaber and Brewer (1997) indicated that teacher advanced degree in the teaching subject was positively related to students' achievement.

Wenglinsky (2000) reported a positive relationship between teacher qualifications and students learning gains. However, he (Wenglinsky, 2000) and Greenberg, Rhodes, Ye and Stancavage (2004) observed that teachers' postgraduate qualifications at Master's or higher level were not significantly related to students' achievement in the subject.

Some other studies that used teacher qualifications to estimate teacher subject matter knowledge found positive correlation between teacher subject matter knowledge and students achievement. For instance, Betts, Zau and Rice (2003) found that teachers' highest degree correlates positively with students' achievement. Rice (2003) found teachers advanced degree in their teaching subjects to be positively related to students' achievement.

A more recent study on the effect of teacher quality on the achievement of students in integrated physics and chemistry (IPC) in the south central region of the United States by Alexander (2008) showed that teacher qualifications is positively related to students achievement in integrated physics and chemistry.

While these studies were carried out outside Africa a similar study in Lesotho, Southern Africa, reported by Ogbonnaya and Osiki (2007) found a statistically significant positive relationship between teacher qualifications and teacher teaching effectiveness at secondary school level.

The studies reported here portend that teacher qualification is related to teacher teaching effectiveness. This is expected to be so where teachers are deployed to teach the subjects in which they obtained the qualifications and also teach at levels where the knowledge the teachers acquired through the qualifications are brought to bear in their teaching. However, in situations, as it is in some countries in Africa (Ogbonnaya, 2007), that teachers are deployed to teach subjects that are outside their qualifications or where they have the paper qualifications but are deficient in the subject matter content, it is likely that one may not find a significant relationship between teacher qualifications and teacher teaching effectiveness. Also, as may be the case reported by Goldhaber and Brewer (1997) that teacher advanced degree in the teaching subject was positively related to students' achievement, it is very likely that the knowledge acquired in advanced degree in a subject may not be significant in teaching at lower levels of education (like secondary and high school levels) to enable the teacher to be more effective. This could be the reason for which some other scholars employed other measures of teacher subject matter knowledge in their studies.

2.6.2 Coursework

A few studies were found that empirically evaluated the relationship between teachers' coursework (subject matter coursework and teaching methodology coursework) and their teaching effectiveness. The studies seem to show positive relationship between teacher coursework and their teaching effectiveness. Ferguson and Womack (1993) used teacher education coursework to measure teacher subject matter knowledge and used peer evaluation (supervisor evaluation) to estimate teaching effectiveness. The study showed that teacher coursework is positively related to teaching effectiveness and explained a greater variation in teaching effectiveness than teacher test score. Similarly, Monk (1994) found that teacher subject matter knowledge of mathematics measured by undergraduate coursework in mathematics was positively related to teacher effectiveness measured by students' improvement. The study further found that teacher additional undergraduate coursework had positive impact on students' achievement in mathematics in advanced courses while it had no effect on students' achievement in remedial courses. This will be expected because at advanced level, higher order level of knowledge may be needed in teaching the students and teachers having more undergraduate coursework will likely employ the knowledge gained from the additional courses in making more impact on the students' learning. However, remedial students are likely to be mainly students that even struggle to grasp basic mathematical concepts. So, in teaching the remedial students, teachers may not need to employ higher order level of knowledge.

Rice (2003) showed that teacher coursework in subject specific area or pedagogy is positively related to students' achievement and that pedagogical coursework seems to contribute to teacher effectiveness at all grade levels, mainly when combined with content knowledge. The study further showed that the effect of teacher content coursework on students' achievements is most pronounced at the high school level. This is possibly so because more subject content is needed to teach at the high school level than at primary or secondary school level.

These findings suggest that the relationship between teacher subject matter knowledge and students' achievement may depend upon the cognitive ability level or grade level of the students. In the physical sciences Monk also found the number of teacher's undergraduate coursework to be positively related to students' achievement gains. Likewise, Wenglinsky (2000) teachers' coursework in mathematics was related to students' achievement

2.6.3 Subject majors

In studying the relationship between teacher SMK and teacher teaching effectiveness, some researchers used teacher subject major as indicator of teacher subject matter knowledge. Their use of this measure could be due to the perceived importance of the relationship between teachers' subject majors and students' achievement that have been acknowledged by some education groups (Thomas & Raechelle, 2000). Some of the studies show a positive connection between teachers' subject majors and teacher teaching effectiveness. Goldhaber and Brewer (1997) found teacher SMK measured by teacher subject major as the most reliable predictor of students' achievement in mathematics and science. Rowan, Chiang and Miller (1997) found that students of teachers that majored in mathematics showed greater achievement gains in mathematics than students of teachers that did not major in mathematics but the effect on students' achievement was rather small.

The greater achievement gains by the students of teachers that majored in mathematics could be because the teachers being specialists in the subject area understood the nitty-gritty of the subject better than the non-mathematics major teachers and as such were in a better position to mediate their students learning that made their students had greater achievement gains.

A review of high school students' performance in mathematics and science by Darling-Hammound (2000) revealed that the most reliable predictor of teaching effectiveness is teachers majoring in the subjects they teach. Similarly, Wenglinsky (2002) and Greenberg, Rhodes, Ye, and Stancavage (2004) said that mathematics teachers having a major in mathematics correlated with higher students' achievement in mathematics. Wilson and Floden (2003) found that students of mathematics teachers with mathematics or mathematics education degrees demonstrate higher academic achievement in mathematics. Nevertheless, they indicated that there could be a limit to which more mathematics knowledge would result to more teaching effectiveness.

A more recent study by Ogbonnaya and Osiki (2007) in Lesotho evaluated the relationship between teachers' subject major and their students' achievement using data from 40 Form C (grade 10) mathematics teachers and their students' achievement scores in a national standardised examination. The study found a positive and statistically significant relationship between the teachers' subject majors and students' achievement. In other words, teachers that majored in mathematics were more likely to help their students achieve better in the subject.

Some studies reported inconsistent relationships between subject major measures of teacher SMK and teacher teaching effectiveness (Ingvarson et al., 2004). Monk (1994) found that when teacher subject matter knowledge was estimated by subject major the study found negative or no relationship between teacher subject matter knowledge and students' achievement. Martin et al. (2000) and Wenglinsky (2000) found that having a major in mathematics was not necessarily associated with teacher effectiveness. These negative or no relationships between teachers' subject majors and students' achievement found in these studies might not necessarily be as a result of the teachers' subject matters knowledge (measured by subject majors) but could be as a result of other factors like school system factor or students factors depicted in the conceptual framework (Figure 2.1). It is also possible that the students' achievement measures used were not based on the curriculum the students were taught by the teachers or that the level of student opportunity to learn the curriculum on which the measures of students achievements were based were less than acceptable (see Anderson, 2004) since the students opportunities to learn the curriculum were not established by these studies.

About a decade ago, Wilson, Floden and Ferrini-Mundy (2001) argued that studies on the relationship between teacher subject matter knowledge and their teaching effectiveness was relatively small and not consistent in their findings. Also, they (Wilson, Floden & Ferrini-Mundy) expressed that the conclusions of the few studies "are provocative because they undermine the certainty often expressed about the strong relationship between college study of a subject matter and teacher quality" (p. 6).

The confounding evidence in these findings makes a case for the need to further investigate the actual relationship that might exist between teacher SMK and their teaching effectiveness taking cognisance of possible methodological caveats especially in terms of measures of teacher SMK that these studies suffered.

2.6.4 Teacher test

In the literature search, some studies were found that used teacher test to evaluate teacher subject matter knowledge. Strauss and Sawyer (1986) in their study used average teachers' scores at district level on a National Teacher Examination to estimate teacher knowledge and average students' test scores to estimate students' achievement in North Carolina in the United States. They found a strong positive correlation between the average teachers' core and the average students' score.

A similar study was conducted by Ferguson (1991) using average teachers' scores at district level on a state teacher licensure test in over 900 districts in the state of Texas in the United States and students average test scores in the districts. Ferguson concluded that there is a positive relationship between teachers average test scores and students average test score. The study further reported that teachers' score accounted for 20-25 percent variation in students' average scores in the districts.

One major limitation of these studies is that teachers' and students' scores were taken at district level. The presence of outliers in the data could lead to overestimation of the effect of teacher subject matter knowledge on students' achievement.

Boyd, Grossman, Lankford, Loeb & Wyckoff (2006) studied the effects of pathways into teaching in New York city using data on students and teachers in grades 3 – 8, the study showed that teachers scores on verbal and literary ability licensure tests correlate positively with students achievement. Also, Ferguson and Womack (1993) in their study found that teacher test score is related to the teaching effectiveness measured by supervisor evaluation (peer evaluation).

2.6.5 Teacher certification

Teacher certification is used in many countries, states, provinces or districts to permit applicants entry into the teaching profession. Certification earns a teacher permit from the authority in charge to teach in schools that require authorization in general, as well as allowing the teacher to teach in particular content areas or subjects across the curriculum. While requirement for teacher certification may vary across countries, states, provinces or districts, most require teachers to have earned a degree or diploma in a content domain. In some cases, the authorities may require candidates to write and pass certification tests/examinations in subject content and or pedagogy before the candidate is certified. Sometimes where the requirements are not completely met, the prospective teacher may be given provisional or temporal certification.

In South Africa, the certification one requires to teach in public school is registration with the South African Council of Educators (SACE). The requirement for SACE registration is a teaching degree/diploma/certificate. Where the prospective teacher holds a non teaching certificate, he may be provisionally registered for one calendar year and will be expected to enrol for a teaching certificate before the end of the one year provisional registration period.

Some studies used teacher certification to estimate teacher knowledge of the subject matter, for example, Cornett (1984a) compared the teaching effectiveness (measured using a teaching functions 4-point rating scale) of 292 fully certified and 191 temporarily certified teachers in North Carolina. The study found no difference in the average score of the two groups of teachers. However, Cornett (1984b) conducted a similar study in Georgia using a locally developed teacher evaluation instrument. The study reported that fully certified teachers on average scored higher than the temporarily certified teachers.

Hawk, Coble & Swanson (1985) compared students' achievements in general mathematics and algebra of properly (fully) and improperly (temporarily) certified teachers. The study found that properly certified teachers were more effective in their teaching in that their students made statistically significant greater gains in both general mathematics and algebra.

The result of a study by Goldhaber and Brewer (2000) showed that grade 12 students of certified mathematics teachers had greater gains in mathematics achievement than students of uncertified mathematics teachers but for science, the study showed that there was no difference in achievement gains between students of certified and uncertified teachers.

Laczko-Kerr and Berliner (2002) in their study of the effect of teacher certification on student achievement, created 109 matched pairs of certified and uncertified teachers within the same school, district or similar districts. The study showed that certified teachers are more effective in their teaching than uncertified teachers. They further reported that students of the uncertified teachers would have had additional 20% growth if they had been taught by the certified teachers.

Alexander and Fuller (2004) explored the relationship between teacher certification and teacher effectiveness measured by students' achievement gains in mathematics from Texas achievement tests scores. The study found that students of certified teachers had greater gains in mathematics examination than students of uncertified teachers. Goldhaber and Anthony (2004) compared the teaching effectiveness of the National Board for Professional Teacher Standards[1] (NBPTS) certified and uncertified elementary school teachers using 3 years comprehensive teacher and student data from North Carolina, USA. Their study showed that teacher teaching effectiveness measured by student achievement gains were higher for students taught by the NBPTS certified teachers than for uncertified teachers.

[1] To be NBPTS certified, each applicant must pass an assessment on content and pedagogical knowledge

Similarly, Boyd, Grossman, Lankford, Loeb and Wyckoff (2006) showed that students of fully certified teachers achieve more than students of teachers that are not fully certified. Boyd, Lankford, Loeb, Rockoff & Wyckoff (2008) also reported a similar finding in poor schools in New York City.

Darling-Hammond (2009) also reported that evidence from recent research studies show that teacher certification is positively related to students' achievement; student achievement gain is most improved when teachers are fully certified.

From the reviews, it was found that some studies suggest that fully certified teachers are more effective in teaching than uncertified or provisionally certified teachers while other studies suggest that teacher certification does not really make any difference in the teachers' teaching effectiveness. The inconsistent findings from the various studies could be as a result of the certification criteria employed in the different contexts on which the studies were based.

2.6.6 Experience

A teacher's teaching experience is used in this study to refer to the number of years the teacher had taught the subject at school. Some studies were found in literature that used teacher teaching experience to estimate teacher teaching effectiveness. Over the years teacher experience has been thought to be associated with teacher teaching effectiveness (Alexander & Fuller, 2005). Greemwald, Hedges, and Laine (1996) in their meta-analysis of data from 60 studies found teachers' years of teaching experience to positively correlate with their teaching effectiveness. One possibility here is that as teachers progress in teaching they are likely to gain more knowledge of the subject and also of students' learning difficulties in the subject and how to help the students overcome the learning difficulties. These make the teacher to likely become more effective over years of teaching.

Hawkins, Stancavage & Dossey (1998) showed that teachers' teaching experience is related to their effectiveness in enhancing their students' achievement but the relationship was not linear; teachers that had fewer than five years of experience were less effective than those that had at least five years teaching experience but benefit of experience levelled off after five years.

Betts, Zau, & Rice (2003) found that teachers' experience significantly correlates with students' achievement in mathematics. Similarly, Rivkin, Hanushek, & Kain (2005) showed that students of experienced teachers achieved better than students of inexperienced teachers.

Similarly, Boyd, Grossman, Lankford, Loeb & Wyckoff (2006) showed from their study in New York with grades 3 – 8 teachers and students that teachers' teaching experience relates to their teaching effectiveness especially in their first few years of teaching,

In his study on teacher quality in Lesotho, Southern Africa, Ogbonnaya (2007) showed teachers' years of teaching experience in mathematics to be significantly related to teacher teaching effectiveness measured by their students' achievement but the benefit of teacher experience in mathematics in Lesotho levels off after about 10 years of teaching. The levelling off of the benefit of experience found in that study could be because the teachers do not continue to develop and acquire more knowledge and skills after 10 years of teaching.

A few studies (e.g. Hanushek, 1997; Martin et al.; 2000 and Wenglinsky; 2002) found that the number of years in teaching was not associated with teaching effectiveness. These contrary findings could be due to the presence of very-well prepared beginning teachers who were highly effective or disenchanted experienced teachers that have lost interest in teaching.

2.6.7 Other reviews

A review of thirty studies on the relationship between teacher subject matter knowledge and teacher teaching effectiveness by Byrne (1983) revealed a combination of both positive and negative small and insignificant relationships. Seventeen of the studies showed a positive but statistically insignificant relationship. A similar finding was recorded by Ashton and Crocker (1987) in their synthesis of studies on the relationship between teacher subject matter knowledge and teacher teaching effectiveness

Anthony and Walshaw (2009) in their analyses of resent studies on effective teaching of mathematics in New Zealand and some international studies found teacher subject matter knowledge to matter and thus argue that "effective teachers develop and use sound knowledge of the subject matter to initiate learning and act responsively towards the mathematical needs of all their students" (p 157).

2.7 Summary

Taken together, despite the few contradictory findings, the studies reviewed seem to provide some support that teacher subject matter knowledge is related to teacher teaching effectiveness and hence students achievement. However, it is important to note, as argued by Wayne and Young (2003), that most of the studies included in the review suffered from methodological flaws like inadequate measures of teacher subject matter knowledge and teaching effectiveness. For instance some of the studies used district wide aggregate teachers' and students' scores on tests to estimate teachers' subject matter knowledge and teaching effectiveness respectively. These studies do not give the individual teachers' or students' scores. Some other studies used teachers coursework covered. These studies do not adequately measure teacher subject matter content in the area as coursework covered does not give indication of the depth of coverage. Also, trainings in different institutions do not guarantee uniformity in coverage and depth.

Teacher teaching effectiveness was measured in most of the studies only by student achievement, the studies did not triangulate this with other measures of teaching effectiveness like students rating of teacher, teacher self rating or peer rating.

It is also important to note that the studies in this area were conducted in other countries; none was conducted in the South African context. Little is known about whether and how teacher subject matter knowledge is related to teacher teaching effectiveness in mathematics in South Africa. Knowing how powerful contextual influence can be on a social science research (see for example, Landry, Amara & Lamari, 2001), one cannot equate the result of studies obtained in other contexts to South Africa and especially against the backdrop of South African mathematics teaching and learning. Hence, it is imperative, much more so at this time in South Africa, to look at the relationship between mathematics teachers' subject matter knowledge and their teaching effectiveness as this could be headway in the improvement of mathematics education in the country.

2.8 Conclusion

The conceptual framework and literature review presented in this chapter were aimed at linking research findings and theory about the relationship between teacher subject matter knowledge and their teaching effectiveness in mathematics with the investigations carried out in this study. Some of the studies and literature indicated that teachers' subject matter knowledge measured by various indicators like qualifications, subject majors, coursework taken by the teachers, certification status, teachers' test scores and length of teaching experience is related to their teaching effectiveness as indicated by their students' achievements or peers' rating of teachers' teaching effectiveness. Some other studies reviewed and literature either found a negative or no relationship between teacher subject matter knowledge and their teaching effectiveness.

The inconsistency in research findings could be as a result of contextual differences or methodological caveats. Moreover, no study was found that linked teacher SMK with their teaching effectiveness measured by students rating of the teacher and/or teachers' self rating or triangulation of the various measures of teacher teaching effectiveness. Also, no study was found that related measures of teacher knowledge and measures of their teaching effectiveness in South Africa.

Hence, the current study was aimed at investigating the relationship between mathematics teachers' subject matter knowledge and their teaching effectiveness in South Africa by triangulating the various indicators of teacher teaching effectiveness.

2.9 Projection for the next chapter

The next chapter focuses on the methodology used in the study including research design, sample selection method, data collection instruments – development of instruments and procedures of data collection, validity and reliability of instruments, and ethical issues considered in the study.

Chapter Three

Research Methodology

3.1 Introduction

This chapter describes the research design, the research sample, data collection instruments, procedure for data collection and data analyses methods. It also includes a discussion of ethical issues considered in the study. The research methodology was based on the objectives of the research outlined in chapter 1. The purpose of the study was to explore the relationship between mathematics teachers' subject matter knowledge and their teaching effectiveness in the South African context.

3.1 Research design

To address the problem of this study set out in section 1.3 and provide answers to the research questions, explanatory Mixed methods research design (Creswell, 2008), which entails the use of a co-relational study and a descriptive survey design were employed in the study. The former was used to determine the relationship between teachers' subject matter knowledge and their teaching effectiveness. Then, qualitative data collected through the descriptive survey helped to elaborate on the quantitative results. Hence, the mixed method approach helped to give a better and deeper understanding of the research problems than using only one of the methods (Creswell, 2008).

3.2 Sampling

The study population is grade 11 mathematics teachers and their students in one education district in a province in South Africa. A convenient sample of 19 grade 11 mathematics teachers and 418 of their (the teachers') students took part in the study. A convenient sampling technique was necessitated by the fact that most of the teachers were not willing to be subjected to a subject matter test. The number of students per teacher ranged from 15 to 54 and all the students of the participating teachers took part in the study. But after data cleansing the total number of students was reduced to 246 (see section 4.1). Grade 11 mathematics teachers and students were chosen for the study because trigonometric functions are taught at grade 11 according to the Mathematics National Curriculum Statement (DoE, 2003) and most grade 11 students in the study by Atagana et. al (2009) reported that they find the topic difficult to learn. The district was chosen for the study because most of the 22 high schools in the chosen district have not been performing well in the Matric examination and also because of the willingness of the management to allow the study to be conducted there. The district and province names are not mentioned here because of ethical reasons.

3.3 Instrumentation

The two main variables of the study are teacher subject matter knowledge and teacher teaching effectiveness. As explained in chapter two there are various measures of the SMK and teacher teaching effectiveness but in this study, teacher SMK was measured using teacher SMK test - Teacher Subject Matter Knowledge of Trigonometric Functions Scale (TSMKTFS). Teacher background information (e.g. qualifications, subject majors and teaching experience) were also collected. Teacher teaching effectiveness was measured using triangulation of students' achievement test, teacher self rating questionnaire, teacher peer rating questionnaire and student rating of teacher questionnaire.

3.3.1 Description of Instruments

3.3.1.1 Teacher Subject Matter Knowledge of Trigonometric Functions Scale (TSMKTFS)

This instrument is a set of test questions developed to measure teachers' subject matter knowledge of Trigonometric functions. It was used to measure teachers' knowledge of content of Trigonometric functions as specified in the grade 11 South African National Curriculum Statement for mathematics. It tested teachers' knowledge of content based on memorized facts or skills, conceptual understanding and problem solving. A detailed explanation of the instrument's development is discussed in section 3.4.1. The instrument is attached in appendix 1.

3.3.1.2 Student Trigonometric Functions Performance Scale (STFPS)

This is the test instrument that was used to measure students' achievement in trigonometric functions. It is based on the Assessment Guidelines for Mathematics as prescribed by the South African National Curriculum Statement (DoE, 2008). It is similar to the Teacher Subject Matter Knowledge of Trigonometric Functions Scale (TSMKTFS) in the sense that the same questions were asked but with a different cover page. The same set of questions was used because it was judged that it would be easier to compare teachers and students performance on a particular question than when different set of questions were used. The instrument is attached in appendix 2.

3.3.1.3 The student evaluation instrument

The student evaluation instrument was used to evaluate the teachers teaching effectiveness from the perspective of the students. It elicited students' information concerning their teacher's teaching in terms of the teachers' display of subject mastery in teaching, lesson organisation/presentation, students' assessment and communication with students. The process of students' evaluation of teachers' teaching effectiveness was discussed in section 2.5.2. The instrument is in appendix 4.

3.3.1.4 Self evaluation instrument

The self evaluation instrument was used to appraise a teachers teaching effectiveness from the perspective of the teacher himself/herself. The instrument is similar to the peer rating instrument and is developed to seek teachers' information about their subject matter knowledge, lesson preparation, lesson organisation and presentation, student assessment and communication with students. The process of teachers' self evaluation of their teaching effectiveness was discussed in section 2.5.3. The teacher instrument is in appendix 5.

3.3.1.5 Peer Evaluation Instrument

The Peer evaluation instrument was used to measure teachers' teaching effectiveness in the area of knowledge of topic/subject (subject mastery), lesson preparation, organisation and presentation, student assessment and communication with students from the perspective of their peers (their fellow mathematics teachers). It is a six-point likert type scale that enabled the teachers' teaching effectiveness to be rated by their peers. The process of peer evaluation of teachers' teaching effectiveness was discussed in section 2.5.4. See appendix 6 for the peer evaluation instrument.

3.3.1.6 Student opportunity to learn instrument

In order to use standardised test to evaluate teachers' teaching effectiveness, Anderson, (2004) observed that it is absolutely necessary to ensure that a high level student opportunity to learn (SOTL) is provided by the teachers. SOTL is an element of curriculum alignment that is used to ensure that what the students are taught is in line with the curriculum (and hence what is being tested). The level of student opportunity to learn was assessed in this study using the student opportunity to learn form (see appendix 3). More detail about the student opportunity to learn instrument is discussed in section 3.3.2.2.

3.3.1.7 Tests Validation instruments

The test validation instruments were used by validators to evaluate the Teacher Subject Matter Knowledge of Trigonometric Functions Scale (TSMKTFS) and the Student Trigonometric Functions Performance Scale (STFPS). The instruments enabled the validators to evaluate the appropriateness of each of the questions in the instruments by indicating the level of relevance of each question in terms of their alignment with grade 11 trigonometric functions content in the National Curriculum Statement. Appendix 16 is the TSMKFS evaluation form while appendix 17 is the STFPS evaluation form.

3.3.2 Development of the instruments

3.3.2.1 Development of TSMKTFS and STFPS

For a valid and reliable inference to be drawn from a study that is based on teacher or student achievement, the study must employ assessments that are aligned with curriculum standards that the teachers are expected to teach or the students learn (Ahn & Choi, 2004; La Marca, 2001). That is, the test must have a high degree of match with the curriculum standards. La Marca (2001) opined that content match and depth match are the two major dimensions of assessing instrument alignment. Content match checks how well test content matches subject area content in the sense that it addresses the broad curriculum standard, the objectives of the curriculum and reflects major emphasis and priorities of the curriculum. Depth match checks how well the test matches the knowledge and skills of the curriculum standard in terms of complexity.

Therefore, to ensure that the tests instruments are aligned with the curriculum standards the following steps were followed in the development and validation of instruments.

Step1:

The first step in the process of developing the tests instrument was to establish a conceptual framework of the instrument by identifying the specific mathematics content (learning outcomes and assessment standards) of grade 11 trigonometric functions from the NCS. This is achieved by analysing the grade 11 NCS trigonometric functions content and depth (using DoE (2007) taxonomical categorisation of content complexity). This helped me to identify the constructs that the instrument would assess.

Trigonometric functions Grade 11

The NCS assessment standards of grade 11 Trigonometric functions (DoE, 2003) stipulate that the teaching of the topic should enable students to do the following:

- Demonstrate the ability to work with various types of functions including:

  $y = \sin(kx)$

  $y = \cos(kx)$

$y = \tan(kx)$;

$y = \sin(x + p)$

$y = \cos(x + p)$

$y = \tan(x + p)$

$y = a \sin(x) + q$

$y = a \cos(x) + q$

$y = a \tan(x) + q$

- Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae).

- Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures about the effect of the parameters k, p, a and q for functions including those listed above.

- Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of functions including those listed above:

  domain and range;

  intercepts with the axes;

  asymptotes;

  shape and symmetry; and

  periodicity and amplitude.

Content complexity of Grade 11 Trigonometric functions

Content complexity is used to differentiate learning expectations and outcomes by considering the amount of prior knowledge, processing of concepts and skills, perceived cognitive demand required, discipline sophistication, number of parts, reasoning, generalization, and application of content structure necessary to meet an expectation or to attain an outcome (Webb, 2010). Content complexity is an important factor when considering tasks students perform, the instructional experiences teachers shape for students, and the relationship among different components of the instructional system. It is frequently used to express a range of levels from simple to high engagement in doing mathematics (Webb, 2010).

The South African mathematics assessment guidelines for Grade 10 -12 (DoE, 2008), used a variation of Bloom's taxonomy to categorise the content complexity of mathematics examination questions at the Further Education and Training (FET) band. The taxonomical categorisation is used to differentiate the level of complexity of questions posed in mathematics examinations. The categories are four, namely knowledge (recall) – level one, performing routine procedures – level two, performing complex procedures – level three, and problem solving – level four. The categories can be taught of as levels of complexity of the questions with the level of complexity increasing from recall questions to problem solving questions.

The first two categories (recall, performing routine procedures) are regarded to demand low-level cognitive skills according to Boston and Smith (2009) classification, while the last two categories (performing complex procedures and problem solving) are regarded to demand high-level cognitive skills (critical and creative thinking skills).

Table 3.1 Analysis of Content complexity of Grade 11 Trigonometric functions using DoE (2008; 13) content complexity levels for mathematics

Curriculum content	Example	Content Complexity level	Annotation
Conversion between numerical, graphical, verbal and symbolic representations	a) Draw the graph of $y = \sin(kx)$, $y = \cos(kx)$, $y = \tan(kx)$	2	This is point by point plot. The student must substitute some values for x to calculate y – forming a table of values. Plot the points on the graph and join the points. The three step procedure is application of algorithm to

			perform known procedures
	b) write the equation of any of the given trigonometric functions graph	2/3	This requires recognising the shape of the trigonometric graphs and calculating/reading of the amplitude, frequency These multiple steps are not quite straight forward but require students to make some decisions as to how to approach the problem or activity. In general it requires some mathematical reasoning
Conversion between numerical, graphical, verbal and symbolic representations	a) Draw the graphs of $y = \sin(x + p)$, $y = \cos(x + p)$, $y = \tan(x + p)$	2	This is point by point plot. The student must substitute some values of x to calculate y – forming a table of values. Plot the points on the graph and join the points. The three step procedure is application of algorithm to perform known procedures
	b) write the equation of any of the given trigonometric functions graph : $y = \sin(x + p)$, $y = \cos(x + p)$,	2/3	This requires 1 recognising the shape of the trigonometric graphs and the horizontal shift of the function. 2 calculating/reading of the amplitude, frequency

	$y = \tan(x + p)$		These multiple steps are not quite straight forward but require students to make some decisions as to how to approach the problem or activity. In general it requires some mathematical reasoning
Investigation of the effects of the parameters *k, p,* and *q* on the functions	Investigate the effect of changing the values of each of k, p and q on $y = sin(kx)$ $y = cos(kx)$ $y = tan(kx)$ $y = sin(x + p)$ $y = cos(x + p)$ $y = tan(x + p)$	4	This investigation requires strategic thinking; it requires reasoning, planning and using evidence to draw conclusions.
Sketch graphs of the functions using the following characteristics: • domain and range; • intercepts with the axes; • asymptotes; •periodicity and amplitude;	Sketch the graphs of the functions $f(x) = cos\ x + 1$ and $g(x) = tan\ 2x$ for $x\ \varepsilon\ [-180^o;180^o]$ on the same set of axes. Show all the intercepts with	3	The student must know the domain and range; Determine how to calculate the • intercepts with the axes; • Know the asymptotes; • Determine how to calculate the Frequency and amplitude; These actions imply more than one step that are could be

	the axes and the co-ordinate of the turning points. Draw the asymptotes using dotted lines	procedural but becomes complex with combination of translation (vertical or horizontal shift)

Step 2

Based on the framework and analysis of the content complexity of Grade 11 trigonometric functions carried out in step 1, a pool of questions to assess the subject matter knowledge content and depth of knowledge of the grade 11 NCS trigonometry functions was generated. The questions were developed to explore teachers' and students' understanding of facts, concepts and the relationships between the facts and concepts of trigonometric functions in line with the conceptualisation of subject matter knowledge of trigonometric functions discussed in section 2.3.2. The pool of questions was sent to 3 high school mathematics teachers with at least 7 years mathematics teaching experience at high school level and also to mathematics subject specialists (learning area facilitators) for evaluation and vetting. They evaluated the appropriateness of the responses that the items elicited in line with the curriculum. They selected a set of questions which by their judgment covered the curriculum and they adjusted the mark allocations to the questions to make up 60 marks.

Step 3

The selected questions made up the pilot instrument. The instrument was further subjected to validation by 5 validators (three high school mathematics teachers and examiners with over 10 years teaching experience and 2 mathematics education researchers lecturing at the university) using the validation forms in appendixes 16 and 17. The questions were retained because based on the validators' ratings each of them had a minimum average assessment rating of 3.5 on a 4-point (1, 2, 3 and 4) validity rating scale.

Step 4

The teacher instrument was piloted with five teachers while the student instrument was piloted with 40 students.

Step 5

The pilot tests were marked by two high school mathematics teachers. They analysed the teachers' and students' responses to the questions and recommended that question 5.1 be written as "Write down the equation(s) of the asymptotes of g(x)" instead of "Write down the equation(s) of the asymptotes" and that the domain of question 5.4 be limited to x ϵ [-180^0; 180^0] instead of x ϵ [-360^0; 360^0]. The changes were effected in the final tests (see appendices 1 and 2).

3.3.2.2 Development of student opportunity to learn instrument

As stated in section 3.3.1.6, in order to use a standardised cognitive test to evaluate teachers' teaching effectiveness, it is necessary to ensure that the teachers provided a high level of student opportunity to learn (SOTL) (Anderson, 2004). Evaluating students' opportunity to learn before administering a test is used to ensure that the students were taught the knowledge/skills being tested. The level of student opportunity to learn was assessed in this study using the opportunity to learn form (see appendix 3). The SOTL form followed the pattern of Anderson (2004) student opportunity to learn instrument. The first column lists all the knowledge/skill being tested while the second column is used to indicate if the knowledge/skill was taught by the teacher. To complete the form the researcher read through the students' notes/portfolios to check if the teacher reviewed or taught the students the knowledge/skills being tested. If the knowledge/skill in the first column was taught or reviewed the researcher ticked 'yes' in the second column otherwise he ticked 'no'. To ensure that what the students were taught was accurately captured, each teacher selected the notes/portfolios of 5 of his/her students that keep good notes for this exercise. Also, using more than one student's note/portfolio helped to ensure that what one student missed out in his/her note/portfolio was captured from another student's note/portfolio.

The student opportunity to learn score for each teacher was computed by adding the total number of 'yes' ticked by the researcher in the second column and converting it to a percentage of the total number of knowledge/skills listed. Since Anderson (2004) recommended that in order to obtain accurate estimate of teacher effectiveness a high level of students' opportunity to learn (at least 70%) is necessary, in this study, only teachers that scored at least 80 % in the student opportunity to learn evaluation were involved.

3.3.2.3 Development of the student evaluation instrument

Marsh & Hocevar (1991) suggested that the following general procedure be followed when developing a student's evaluation of teaching instrument.

- Development of a large pool of items (from literature, existing instruments, interview with students and teachers),
- Pilot the instrument to receive feedback about the items, and
- Consideration of the psychometric qualities of the items while revisions are made.

The development of the instrument began with the search for the characteristics of effective teacher/teaching. Based on a survey of students' and teachers' views of effective teaching and reviews of similar studies, the South African Norms and Standards for Educators documents, the National Curriculum Statement for mathematics and other related literature, a framework of effective teaching that guides the development of the instrument was formulated. The framework specified the domain of interest of the study which according to Berk (1979) is a crucial first step in the development of an evaluation instrument.

By looking through the lens of the framework of effective teaching, a pool of 186 items was developed from the survey of students' and teachers' views of effective teaching and from literature (e.g. the South African Norms and Standards for Educators and the Mathematics National Curriculum Statement for mathematics and other related literature). From the pool items were drawn for each subscale identified in the framework, namely, knowledge of subject content, lesson preparation, lesson organisation, lesson presentation, assessment of students learning, and communication with students.

3.3.2.3.1 Survey of students' and teachers' views of an effective mathematics teacher

The survey of high school teachers' and students' views of the qualities of effective mathematics teacher identified excellent knowledge of the subject matter, ability to communicate the subject clearly, always attend class, helps learners where they don't understand, motivate learners to learn, gives learners opportunity to ask questions and talk in class, pays attention to students learning difficulties, prepares for lesson before coming to class, explains the subject well, provides helpful feedback to students, uses examples that students are familiar with to bring the lesson home, and provides relevant examples as the hallmark of effective mathematics teacher. Items for instrument were developed based on the statements and as much as possible the phrases and wordings used by the students and teachers were retained.

3.3.2.3.2 Combining items from the different sources

The items developed from the survey were combined with items generated from the other sources (for example, similar studies, Norms and Standards for Educators and the Mathematics National Curriculum Statement for mathematics) to generate the 186 items. This comprised items from the six dimensions namely knowledge of the subject, lesson preparation, lesson presentation, motivation to students, communication with students and assessment. Table 3.2 shows some of the items and their sources. 'Te' denotes teacher; 'St' denotes students while 'No' denotes Norms and standards for educators (DoE, 2000b) as the source; 'Lit' denotes literature; 'NCS' denotes the National curriculum statement for Mathematics grades 10 -12 (DoE, 2003) and 'Po' indicates the National education policy act, Number 697 (DoE, 2000a). The Table shows that some of the items had more than one source. For example item 80 – "simplifies the subject matter to learners" was indicated by both teachers and students as a quality of effective teacher in teaching.

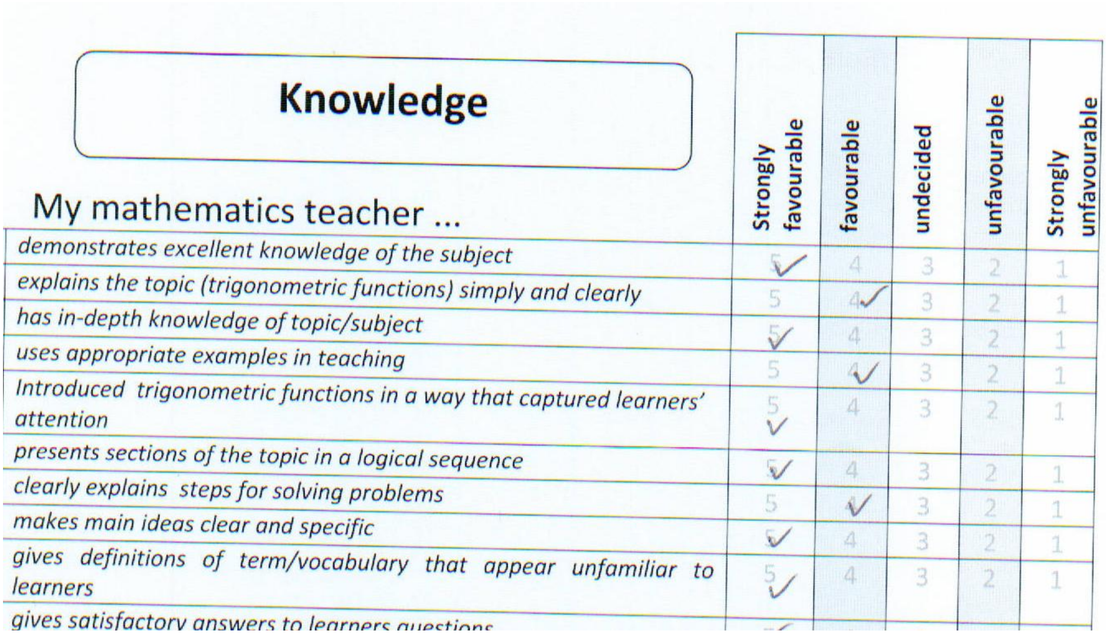
Table 3.2 Some of the items of the evaluation instruments and their sources

		Source					
		Te	St	No	Lit	NCS	Po
75	Listens to learners	✓	✓				
76	Prepares for lesson before coming to class	✓	✓				
77	Gives a lot of work	✓	✓				
78	Explains/communicates well	✓	✓				
79	Well equipped with the content/subject matter	✓					✓
80	Simplifies the subject matter to learners	✓	✓				
81	Well articulates teacher tasks and learner tasks	✓					
82	Depicts innovative qualities in lesson plan	✓					✓
83	Uses easy to follow lesson plan	✓					
84	Always prepared for lesson		✓				
85	Paces lessons so the learners can follow		✓	✓			
86	Sequences lessons in a manner to take cognisance of differing learning abilities of learners		✓				
87	Makes learners pay attention to lesson		✓				

3.3.2.3.3 Vetting of the instrument

The 186 items were vetted by six teachers (three mathematics teachers, two science teachers and an English language teacher) and 10 high school students. They advised that some of the items be removed and also some grammatical changes be made to some of the items. For instance, they suggested that the word 'learners' instead of 'students' should be used since that is the word being used more commonly in the high school system in South Africa. The removal of some the items trimmed down the total number of items to 135. The 135 items were further vetted by a different group of four mathematics and science education researchers who found some of the items to be merely the same in the sense that they elicited the same information. Such items that they found to be duplicates of other items were removed and as a result the items were further brought down to 84 items. The vetting of the instrument was carried out to ensure that the items were explicit and by no means ambiguous so that the items would be interpreted correctly by respondents (Mogari, 2004).

The 84 item instrument was subjected to rating by 7 mathematics teachers and mathematics and science education researchers who judged how favourable each item was with respect to the construct it was purported to measure using a 5 to 1 rating scale where 5 = strongly favourable to the concept; 4 = favourable to the concept; 3 = undecided; 2 = unfavourable to the concept and 1= strongly unfavourable to the concept were used for this purpose. Figure 3.1 below shows how one of the raters rated some of the items with respect to knowledge construct for the students' evaluation instrument.



Knowledge

My mathematics teacher ...

	Strongly favourable	favourable	undecided	unfavourable	Strongly unfavourable
demonstrates excellent knowledge of the subject	5 ✓	4	3	2	1
explains the topic (trigonometric functions) simply and clearly	5	4 ✓	3	2	1
has in-depth knowledge of topic/subject	5 ✓	4	3	2	1
uses appropriate examples in teaching	5	4 ✓	3	2	1
Introduced trigonometric functions in a way that captured learners' attention	5 ✓	4	3	2	1
presents sections of the topic in a logical sequence	5 ✓	4	3	2	1
clearly explains steps for solving problems	5	4 ✓	3	2	1
makes main ideas clear and specific	5 ✓	4	3	2	1
gives definitions of term/vocabulary that appear unfamiliar to learners	5 ✓	4	3	2	1
gives satisfactory answers to learners questions					

Figure 3.1 Example of a rater's rating of some of the item in relation to measuring teachers' knowledge of the subject matter from students' perspectives.

3.3.2.3.4 Selection of items

To select the items for the instrument, I computed the correlation between average rating for each item and the total (summed) score across all item in each subscale (Trochim, 2006). From the correlation coefficients, any item that had a low correlation (less than 0.7) with the total score in the subscale was eliminated. This produced 39 items which was further reduced to 30 items by eliminating the 9 items with the least correlation coefficients. This was done in order to limit the number of items to 30 so that it would not be too long for participants to complete because potential respondents may be less inclined to participate in a survey expected to last for a long time (Galesic & Bosnjak, 2009).

The instrument consisted of the 30 items (see Table 3.3) in a six-point Likert type scale (three points for agreement and three points for disagreement namely: strongly agree, agree, slightly agree, slightly disagree, disagree and strongly disagree). The use of the even number scaling system eliminates the possibility of respondents opting for a mid-point (sitting on the fence or neutral) position (Cohen, Manion, & Morrison, 2007). The six-point rating exhausts the range of possible responses that respondents may wish to give that will enable discrimination between effective teaching/teachers and ineffective teaching/teachers.

Table 3.3 Student evaluation of teaching instrument

	Please indicate the extent of your agreement/disagreement with the following statements about your mathematics teacher, using the following scale. Strongly agree = 6 Agree = 5 slightly Agree = 4 slightly Disagree = 3 Disagree = 2 strongly Disagree = 1 For each question mark X in the appropriate box that corresponds to the extent of your agreement/disagreement. My mathematics teacher ...	Strongly agree	Agree	Slightly Agree	Slightly Disagree	Disagree	Strongly Disagree
1	introduced the topic in a way that captured learners' attention	6	5	4	3	2	1
2	gave definitions of terms/vocabularies that appear unfamiliar to learners	6	5	4	3	2	1
3	gave satisfactory answers to learners questions	6	5	4	3	2	1
4	made lessons relevant and meaningful to learners	6	5	4	3	2	1
5	simplified the subject matter to learners	6	5	4	3	2	1
6	showed sound knowledge of the subject matter	6	5	4	3	2	1
7	showed learners interesting and useful ways of solving problems.	6	5	4	3	2	1
8	started lessons by connecting to previous lessons	6	5	4	3	2	1
9	ended lessons by connecting to future lessons	6	5	4	3	2	1
10	presented sections of the topic in a logical sequence	6	5	4	3	2	1
11	related content to real life examples	6	5	4	3	2	1
12	was always well-prepared for class	6	5	4	3	2	1
13	summarized the main points by the end of lesson	6	5	4	3	2	1
14	was always in class with all necessary materials for teaching topic	6	5	4	3	2	1

	Please indicate the extent of your agreement/disagreement with the following statements about your mathematics teacher, using the following scale. Strongly agree = 6 Agree = 5 slightly Agree = 4 slightly Disagree = 3 Disagree = 2 strongly Disagree = 1 For each question mark X in the appropriate box that corresponds to the extent of your agreement/disagreement. My mathematics teacher ...	Strongly agree	Agree	Slightly Agree	Slightly Disagree	Disagree	Strongly Disagree
15	related ideas to learners' prior knowledge	6	5	4	3	2	1
16	supported lessons with useful class work	6	5	4	3	2	1
17	made use of different teaching techniques	6	5	4	3	2	1
18	motivated learners to pay attention to lesson	6	5	4	3	2	1
19	always attended classes	6	5	4	3	2	1
20	helped learners where they didn't understand	6	5	4	3	2	1
21	encouraged learners to learn	6	5	4	3	2	1
22	was always punctual to class	6	5	4	3	2	1
23	gave individual support to learners when needed	6	5	4	3	2	1
24	adjusted the lessons when learners experienced difficulties in learning.	6	5	4	3	2	1
25	used assessment results to provide extra help to learners	6	5	4	3	2	1
26	explained something in different ways to help learners understand.	6	5	4	3	2	1
27	took extra steps to help all learners learn and achieve success in maths.	6	5	4	3	2	1
28	supported lessons with useful classroom discussions	6	5	4	3	2	1
29	gave feedback to learners about their homework and assignment	6	5	4	3	2	1
30	communicated the topic clearly	6	5	4	3	2	1

3.3.2.3.5 **Factor analysis**

Factor analysis was used to further determine if the items in the instrument measured the theorised constructs and thus strengthen the validity of the instrument (Ogbonnya & Mogari, 2011). Principal components (PC) factor analysis on Statistical Package for the Social Science (SPSS) was used to determine the factor loadings of the items of the instrument. The first step was to carry out a preliminary analysis using the output of the R-matrix. The result revealed that items 29, 19 and 22 of the instrument had respectively 11, 9 and 9 of their one-tailed significant values greater than 0.05. Hence, it was judged better to eliminate the three items to avoid singularity (Field, 2005). The Kaiser-Meyer-Olkin (KMO) measure of sample adequacy was 0.892. The Bartlett's test of sphericity gave a value of .000 (see Appendix 12). The KMO value of 0.892 falls in the range of 'great value' and the highly significant values of Bartlett's test ($p < 0.001$) indicated that factor analysis was appropriate for the data (Field, 2005).

To optimise the factor structure and search for the best explanation of patterns in the data, factor rotation (Varimax with Kaiser Normalization) was applied. Table 3.4 lists the SPSS output of the eigenvalues associated with each factor before extraction, after extraction and after rotation. By Kaiser's criterion (Field, 2005) five factors, that is factors with eigenvalues greater than 1 were extracted.

Table 3.4. Eigenvalues associated with each factor

Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
Component	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	11.887	44.026	44.026	11.887	44.026	44.026	5.268	19.510	19.510
2	2.008	7.438	51.464	2.008	7.438	51.464	4.325	16.017	35.527
3	1.799	6.662	58.126	1.799	6.662	58.126	3.552	13.156	48.683
4	1.250	4.631	62.757	1.250	4.631	62.757	2.756	10.206	58.889
5	1.101	4.077	66.834	1.101	4.077	66.834	2.145	7.945	66.834
6	.951	3.524	70.358						
7	.862	3.193	73.550						
8	.803	2.975	76.525						
9	.662	2.453	78.978						
10	.591	2.187	81.165						
11	.562	2.080	83.246						
12	.545	2.019	85.264						
13	.462	1.711	86.975						
14	.440	1.631	88.606						
15	.392	1.453	90.059						
16	.365	1.350	91.409						
17	.350	1.295	92.704						
18	.296	1.096	93.800						
19	.278	1.028	94.828						
20	.261	.968	95.796						
21	.219	.811	96.607						
22	.199	.738	97.345						
23	.186	.689	98.033						
24	.155	.575	98.608						
25	.152	.562	99.170						
26	.123	.456	99.626						
27	.101	.374	100.000						

Extraction Method: Principal Component Analysis.

The eigenvalues of the factors after rotation are displayed in final part labelled "Rotation Sums of Squared Loading". Rotation helped to optimise the factor structure and equalised the relative importance of the five factors. Before rotation, factor 1 accounted for 44.025%

of the variance while factors 2, 3, 4 and 5 respectively accounted for 7.438%, 6.662%, 4.634% and 4.077%. However, after rotation, factor 1 accounted for 19.510% and factors 2 – 5 accounted for 16.017%, 13.156%, 10.206% and 7.945% respectively.

The extraction of the five factors is also justified by the scree plot of the analysis shown in Figure 3.2. The thunderbolt indicates the point of inflexion. The curve begins to tail off after four factors but drops again after five factors before reaching a stable plateau. From the curve, one could justify retaining three or five factors but I judged it safer to retain five factors according to Kaiser's criterion that five factors should be extracted. Also, that the variables are less than 30 and the average communality (0.6791) is greater than 0.6 (see Appendix 13 for the table of communalities) justifies the extraction of five factors (Field, 2005; Ogbonnaya & Mogari, 2011).

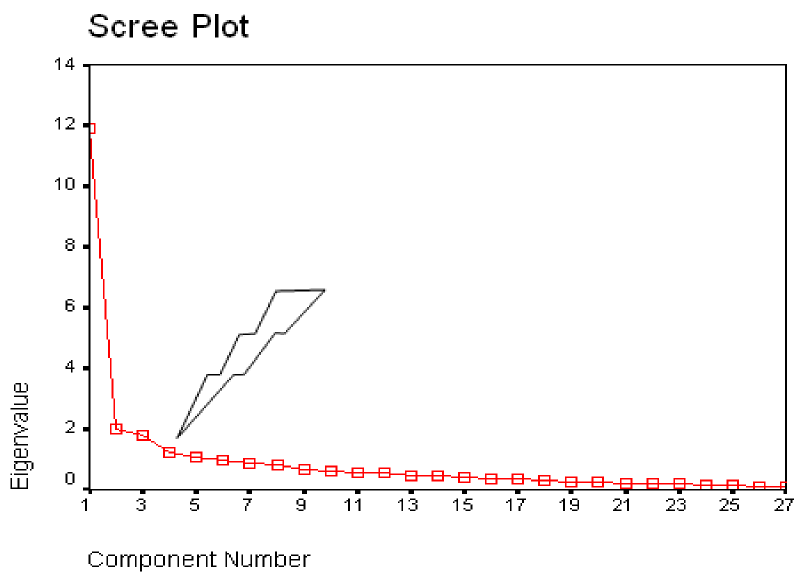


Figure 3.2 Scree plot for the factor extraction

The factor loadings of the 27 items are presented in Table 3.5. Factor loadings less than 0.3 have not been displayed and the values are listed in the order of size of their factor loadings.

Table 3.5 Factor loadings of the 27 items

Rotated Component Matrix[a]

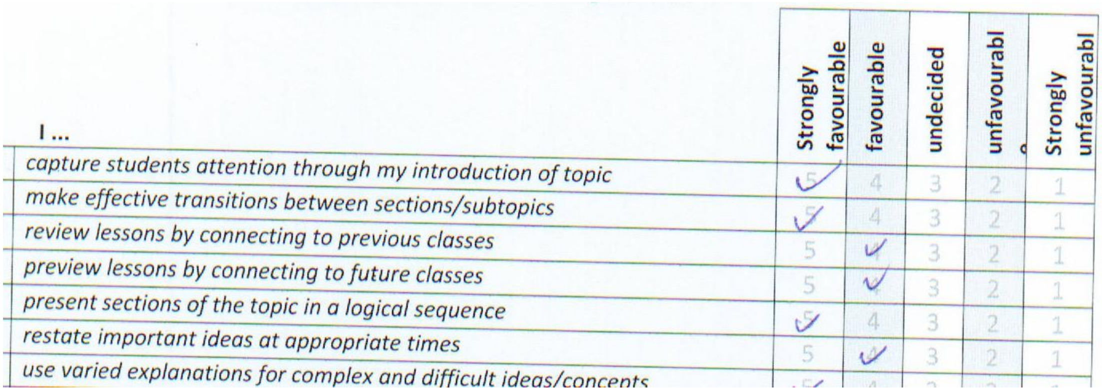
	Component				
	1	2	3	4	5
Q28	.778				
Q23	.766				
Q24	.728				
Q27	.701			.331	
Q17	.677				
Q26	.641			.438	
Q20	.633		.367		
Q13	.503	.399			
Q5		.711			
Q6		.700			
Q4		.634		.395	.389
Q9	.420	.613			
Q7	.399	.591			.413
Q3	.378	.558		.433	
Q12		.553	.549		
Q14			.723		.338
Q16	.341		.655		
Q25	.363		.605	.353	
Q2			.588		
Q10		.487	.561		
Q21	.363	-.326	.506	.423	
Q15	.409	.437	.499		
Q18				.761	
Q1		.371		.623	
Q11		.392		.555	.374
Q30					.750
Q8		.385			.683

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.
a. Rotation converged in 10 iterations.

The result showed that items 13, 17, 20, 23, 24, 26, 27 and 28 loaded to one factor – Factor 1. They were thought to be measuring teachers' lesson presentation. Item 3, 4, 5, 6, 7, 9 and 12 loaded to one factor – Factor 2. They were thought to measure teachers' knowledge of subject content. Items 2, 10, 14, 15, 16, 21 and 25 which loaded to Factor 3 were seen to measure lesson preparation. Items 1, 11 and 18 loaded to Factor 4, they were judged to measure teachers' motivation to students and lastly, items 30 and 8 that loaded to Factor 5 were seen to measure teachers' communication to learners. Hence the final student rating instrument consists of the 27 items (see appendix 4).

3.3.3 Development of the self evaluation instrument

The self evaluation instrument was developed following the same steps used in the development of the student evaluation instrument. From the pool of 186 items obtained from the survey and literature 62 items were drawn based on the vetting of the six teachers. The items were then rated by the seven teachers and researchers that rated the student evaluation instrument in terms of the favourability of each of the items to the constructs they claimed to measure using the same 5 to 1 rating that was used for the student rating instrument. Figure 3.3 below shows part of one rater's rating of some of the items with respect to Lesson presentation construct for the teacher self evaluation instrument.



I ...	Strongly favourable	favourable	undecided	unfavourable	Strongly unfavourable
capture students attention through my introduction of topic	5 ✓	4	3	2	1
make effective transitions between sections/subtopics	5 ✓	4	3	2	1
review lessons by connecting to previous classes	5	✓	3	2	1
preview lessons by connecting to future classes	5	✓	3	2	1
present sections of the topic in a logical sequence	5 ✓	4	3	2	1
restate important ideas at appropriate times	5	✓	3	2	1
use varied explanations for complex and difficult ideas/concepts					

Figure 3.3 A rater's rating of some of the items with respect to Lesson presentation construct for the teacher self evaluation instrument.

3.3.3.1 Selection of items for the final instrument

To select final items for the instrument, the correlation between average rating for each item and total score across all items in each construct (Trochim, 2006) were computed and the items with correlation coefficients lower than 0.7 were eliminated. The process led to a 32 item self evaluation instrument (appendix 5).

3.3.4. Development of the peer evaluation instrument

The peer evaluation instrument is similar to the self evaluation instrument. It was used by other mathematics teachers (colleagues of the participating teacher) to evaluate the participating teacher hence the 32 items in the self evaluation instruments were all retained but the first person pronouns used in the self evaluation instrument were changed to third person pronouns in the peer evaluation instrument. For example, the item "*explained concepts in different ways to help my learners understand*" in the self evaluation was changed to "explained concepts in different ways to help his/her learners understand" in the peer evaluation instrument. The instrument is in appendix 6.

3.4 Validity and reliability of the instruments

Validity and reliability are used to determine the quality of research instruments. The validity of an instrument is the extent to which the instrument measures what it intends to measure. It means that the scores from the instrument make sense, are meaningful and enable the researcher to draw conclusion from the sample to the population (Creswell, 2008). Reliability of an instrument is the instrument's ability to obtain the same response each time the instrument is administered. That is the ability of the instrument to obtain stable and consistent scores (Creswell, 2008).

3.4.1 Validity

Content validity of each of the instruments was carried out. In addition, construct validity of each of the three evaluation instruments were ascertained. Content validity is the extent to which the content of an instrument covers the extent and depth of the topics it is intended to cover (Creswell, 2008). It is a useful concept when evaluating educational tests and research questionnaires (Lewis, 1999). To evaluate content validity of instruments researchers normally consult experts in the field of study to have them identify if the questions are valid (Creswell, 2008). Content validity of each of the instruments was tested by involving experts in the field of mathematics, mathematics education and science education.

3.4.1.1 Validation of the tests instruments (TSMKTFS & STFAS)

The content validity of the *TSMKTFS and STFAS* was carried out by involving expert mathematics educators, examiners and researchers to evaluate and vet the questions in the instruments. Also, validity of the questions was done by a validation process carried out using the tests validation forms in Appendices 15 and 16. The tests instruments were validated by five mathematics educators and researchers. They judged the relevance of each question in terms of alignment in content and complexity with the grade 11 mathematics of the National Curriculum Statement (NCS) grades 10 -12 mathematics (DoE, 2003).

Furthermore, the construct validity of the instruments was established by carrying out content complexity analysis using DoE (2008) assessment guidelines for mathematics content complexity taxonomy of each question in the instruments (see Appendix 17 for the analysis). The result of the analysis in Appendix 17 is in conformity with the content complexity analysis of Grade 11 Trigonometric functions shown in Table 3.1.

3.4.1.2 Validation of student rating instrument

The instrument underwent content and construct validity tests. Content validity of the instrument was established by grounding the instrument on the established framework of effective mathematics teacher/teaching. To further ensure content validity (Creswell, 2008), the instrument was vetted by teachers, students and experts in the field of mathematics, science education and psychometric tests. The experts checked that each item in the instrument relates to what it was meant to measure, that the scale was of appropriate length and that the language was appropriate for high school students for whom English was a second language.

To ascertain that the items actually measured what they are assumed to measure, construct validity was performed on the items. This was done in two phases. In the first phase, the correlation between each item and the total (summed) score across all items in each subscale was computed (Trochim, 2006). The items that correlated highly (0.7 and above) with the summed score in the subscale were selected; dropping out the remaining items. The 30 items that were selected made up the pilot instrument in Table 3.3.

In the second phase of the construct validity, factor analysis was used to further determine if the items in the instrument measured the theorised constructs and thus strengthen the validity of the instrument. The result of the factor analysis was discussed in section 3.4.3.5 under factor analysis. The final instrument was a 27 item instrument. The result showed that the final instrument was valid (see appendix 4 for the final student rating instrument).

3.4.1.3 Validation of self evaluation and peer evaluation instruments

Part of the development process of the self evaluation *and peer evaluation* instruments was the evaluation of the items in the instrument by experts in mathematics and science education. This process was used to ensure the content validity of the instruments. Construct validity was achieved by the process of experts rating of the favourability of each item to the construct it was purported to measure and subsequent calculation of the correlation between the average rating for each item and total score across all items in each construct to select final items with correlation coefficients greater than or equal to 0.7 (see sections 3.4.4 and 3.4.5).

3.4.1.4 Validation of student opportunity to learn trigonometric functions form

The SOTL form was validated by three mathematics educators that verified that the knowledge/concepts that were tested by the tests and listed in column one of the SOTL form conformed to the knowledge/concepts expected to be taught in grade 11 trigonometric functions according to the National Curriculum Statement. This process was carried out using the student opportunity to learn trigonometric functions form validation instrument attached in appendix 18. Each of the items in the instrument were rated to be high in conformity with the grade 11 trigonometric functions assessment standards of the National Curriculum Statement (NCS) by the validators.

3.4.2 Reliability

For a research instrument to be reliable it must demonstrate that if it is administered to similar group of respondents in a similar context it will yield similar result (Cohen, Manion and Morrison, 2007). Principally, there are three types of reliability: stability, equivalence and internal consistency. Reliability in terms of stability and internal consistency of all the instruments that were used in the study were established.

3.4.2.1 Reliability of the two tests instruments (TSMKTFS & STFAS)

Reliability in terms of stability measures consistency over time and over similar samples. This could be examined by test-retest method and correlating the scores of the two tests or using two similar groups and correlating the scores from the two groups. The correlation coefficient must be significant at 95% or higher confidence interval (Cohen, Manion and Morrison, 2007).

The STFAS was pilot tested with 40 grade 11 students from a high performing high school in another district of the province. The students were randomly assigned to two equivalent groups and the average scores of the students in one group on each question was correlated with the average score of the other group on each question. The instrument showed significant correlation coefficient of 0.95 obtained at 99% confidence interval (see appendix 11). Using the Spearman Brown formula $R = 2r/(1+r)$, this gave a reliability of 0.97. The results implied that the test instruments were very reliable.

3.4.2.2 Reliability of student evaluation of effective teaching instrument

The Reliability (internal consistency reliability) of the evaluation instrument was established by calculating coefficient alpha (Cohen, Manion and Morrison, 2007), using data gathered in a pilot study of the instruments from 165 grade 11 students (though only 109 students completed all the 30 items in the instrument) from 9 mathematics classes in 4 high schools in which the students evaluated their teachers' teaching of trigonometric functions. Coefficient alpha (α) value of 0.9473 was obtained from computation using SPSS (see appendix 7 for the SPSS output).

3.4.2.3 Reliability of the self evaluation instrument

The Reliability (internal consistency reliability) of the self evaluation instrument using pilot test data gave coefficient alpha value of 0.92 (see appendix 8). The results show that the instruments was very reliable

3.4.2.4 Reliability of the peer evaluation instrument

Result from pilot study showed that the peer evaluation instrument had coefficient alpha value of 0.95 (see appendix 9). The reliability value confirmed that the instrument was reliable and hence was used for the study.

3.4.2.5 Reliability of student opportunity to learn trigonometric functions form

To calculate the reliability of the students' opportunity to learn trigonometric functions form, the results of two evaluators' (the researcher and a high school mathematics teacher) scores of the opportunity to learn a teacher provided for the students using the instruments were correlated. The result gave a correlation coefficient of 1.0 (see appendix 19 for the SPSS output). This result implies that the instrument is highly reliable in assessing student opportunity to learn grade 11 trigonometric functions that a teacher offers to his/her students.

3.5 Data analysis

3.5.1 Quantitative data analysis

Quantitative data gathered was analysed using descriptive and inferential statistics. Pearson's correlation coefficient (two-tailed) was computed (with SPSS for windows) at 95 % confidence interval between measures of teacher subject matter knowledge and teacher teaching effectiveness. Quantitative data analyses carried out in this study is described in details in section 4.2.

3.5.2 Qualitative Data analysis

The qualitative data gathered from teachers' and students' tests scripts were analysed using analysis of task performance of the teachers' and students' answers to the tests' questions. Detailed explanation of the qualitative data analysis of this study is given in section 4.3.

3.6 Ethical issues

At every level of the study and its reports, ethical practices were ensured by observing the following ethical guidelines.

3.6.1 Informed consent of the participants

The participants (teachers, students and authorities of schools of participating teachers and students) were informed of the purpose and aims of the study before they began to take part in the study. They were also informed about how the result of the study would be used. A consent form that states the purpose and aims of the study and seeking their consent to participate in the study was handed out to the participants. They signed the form before participating in the study and only those that signed the form took part in the study. (See appendix 14 for the consent form). The consent form contained information about the research topic and purpose of the study, an outline of assurance that their participation was voluntary, that their identities and that of the schools would not be revealed in any means in the reporting and that they could withdraw their participation from the study at any time without being prejudiced. Also, permission was sought from the department of education to carry out the study in the district and a letter of authority to conduct the study in the district was obtained from the department of education.

3.6.2 Voluntary participation

Ethics demands that participation in a social science research be voluntary (Babbie, 2001; Creswell, 2008). The participants were told they had the right to take part in the study or refuse to participate. No participant was forced to take part in the study and they were allowed to withdraw at any time in the process. This was stated in the consent letter to the teachers.

3.6.3 Anonymity

It is expedient that the social researcher protects the participants from all possible harm (Gay & Airasian, 2003). In a study like this, harm could come by making it possible for the identities of the participants to be known. Such harm could be overcome by keeping the participants anonymous. This was achieved by not collecting participants' names on the questionnaires and by not writing their names or the names of their schools in the reports. The report was also presented in a manner that the participants or their schools cannot be linked to any information.

3.6.4 Respect for the study site

In order to show respect for the schools that were used for the study, permission from the school authorities, were obtained before commencement of the study. Every effort was made to avoid disrupting class sessions in the course of the study.

3.6.5 Honest reporting of the findings

Data and the findings of the study are reported honestly without changing or altering any data or information. The findings of the study are honestly reported to education research community in conference proceedings and journal articles.

3.7 Summary

The problem of the study was to explore the relationship between mathematics teachers' subject matter knowledge and their teaching effectiveness. To address this problem and provide answer to the research questions, explanatory mixed methods research design was employed. A convenient sample of 19 grade 11 mathematics teachers and their students were selected for the study. A teacher test was developed and used to measure teacher subject matter knowledge while a student test, student evaluation of teacher teaching effectiveness questionnaire, teacher self evaluation of teaching effectiveness and peer evaluation of teacher teaching effectiveness were used to collect data about teachers' teaching effectiveness. The instruments were subjected to various tests for validity and reliability and were found to be reliable and valid for the purpose of the study.

Furthermore, as it is demanded of ethics in social science research, the study adhered to participants' informed consent, voluntary participation, anonymity, respect for the study site and honest reporting of the findings.

3.8 Projection for the next chapter

The next chapter presents the results of the data analyses (quantitative and qualitative) carried out to address the research questions set out in chapter one.

Chapter Four

Results

4.1 Introduction

The purpose of this chapter is to present the results of the investigations carried out and thus provide answers to the research questions the guided this study. The chapter presents the results of the data analyses carried out to address the research questions. The purpose of the study, as articulated in section 1.3 of chapter one, was to explore the possible relationship between mathematics teachers' subject matter knowledge and their teaching effectiveness.

Data was collected from grade 11 mathematics teachers and also from their students and peers. Data was collected from the teachers using a self report questionnaire and a cognitive test. The self report questionnaire was used to collect teachers' demographic information (gender, qualifications, subject specialisation and years of teaching experience) and teachers' self evaluation of their teaching effectiveness. The teachers' test was used to collect information about teachers' subject matter knowledge. Also, a self report questionnaire was used to collect data from the students taught by the teachers' about their evaluation of the teachers' teaching effectiveness while a cognitive test was used to collect data about the students' achievement. Another self report questionnaire was used to collect data from the peers of the teachers about their evaluations of the teaching effectiveness of the participating teachers. In addition, a student opportunity to learn form was used by the researcher to collect information on the opportunity to learn trigonometric functions that each of the teachers provided for his/her students.

A total of 19 teachers and 418 students took part in the study. However, not all the 19 teachers participated in all the stages of the study. For example, while all the 19 teachers completed the teachers' self rating of their teaching effectiveness questionnaire only 11 wrote the teacher subject matter knowledge test. Hence, for the analyses that involved teacher subject matter knowledge, the results of the 11 teachers were used. The number of students for the 11 teachers that wrote the student achievement test was 246.

Both quantitative and qualitative data analyses techniques were used to analyse the data. First, the chapter presents the descriptive statistics of the data collected from the teachers, followed by the correlation analysis and linear regression analysis of teacher subject matter knowledge and their teaching effectiveness. Furthermore, a cognitive task performance analysis of teachers' and students' solutions to the tests questions was carried out. Using these data analyses techniques attempts were made to answer the research questions.

4.2 Quantitative analyses

4.2.1 Descriptive statistical analysis - teachers' demographic information

This section presents the data collected in a table of frequencies. The table was used to present and describe the data collected from the teachers regarding their qualifications, subject specialisation and years of teaching experience.

The teachers' demographic information is shown in Table 4.1. The table shows that the majority of the mathematics teachers that took part in the study were male and accounted for 55% of the participating teachers. It also shows that the majority (approximately 53%) of the teachers had taught for more than 10 years and about 50% had got at least a first degree. Only 12 (approximately 63%) of the teachers had majored in mathematics or mathematics education either at the college level or at the university level.

A comparison of the teachers' participation in the study reveals that the more experienced teachers (those with more 10 years teaching experience) fully participated more in the study than the less experienced teachers. Perhaps, the more experienced teachers were more confident to write the test than the less experienced teachers.

Table 4.1 Teachers' demographic information (N = 19)

	All Teachers	**Partial participation**[1]	**Full participation**
Gender			
Male	11	5	6
Female	8	3	5
Teaching Experience			
0 – 5 years	4	2	2
6 – 10 years	3	1	2
11 –15 years	4	2	2
16 –20 years	2	-	2
Over 20 years	4	1	3
*Void	2	2	-
Qualification			
Teaching certificate	1	1	-
Diploma	2	1	1
Higher Diploma	1	1	-
ACE	3	-	3
Bachelors degree	8	4	4
Masters degree	1	-	1
*Void	3	1	2
Mathematics/Mathematics Education Major			
Yes	12	5	7
No	7	3	4

* No information provided

[1] Some of the 19 teachers did not participate in all the stages of the study (see section 4.1)

4.2.2 Correlation analyses

Correlation analysis was used to ascertain if there is any relationship between mathematics teachers' subject matter knowledge and the measures of the teaching effectiveness of the teachers. Teachers' score on the test (Teacher Subject Matter Knowledge of Trigonometric Functions Scale) was used as teachers SMK while students' achievement was measured using the students score on the student test (Student Trigonometric Functions Performance Scale).

The average test score of the students of each teacher was used as the measure of the teacher's teaching effectiveness in terms of students' achievement. The other measures of teachers' teaching effectiveness were the teachers' self evaluation, the peers' evaluation and students' evaluation of the teachers' teaching effectiveness (appendixes 4, 5 and 6). The instruments for peer evaluation, student evaluation and self evaluation were in a form of Likert type scale, which is normally used as interval scale (see Creswell, 2008; Liu & Mintram, 2005; Raubenheimer, 2004). Hence, Pearson product moment correlation was used instead of Biserial correlation to determine the appropriate coefficients.

Pearson's product-moment correlation coefficient between teacher subject matter knowledge and the measures of teacher teaching effectiveness were calculated using SPSS at 95% and 99% confidence intervals. The correlation result being significant at $p < 0.05$ means that the probability of obtaining the correlation by chance is less than five out of 100 (5%). The correlation result being significant at $p < 0.01$ means that the probability of obtaining the correlation by chance is less than one out of 100 (1%).

4.2.2.1 Correlation between teacher subject matter knowledge and measures of teacher teaching effectiveness.

Table 4.2 shows the correlation matrix of Pearson's product-moment correlation between teacher subject matter knowledge and each measure of the variables defining teacher teaching effectiveness (students' achievement, teachers' self evaluation, students' evaluation and peers' evaluation of the teachers teaching effectiveness).

The table shows that there were positive relationship between mathematics teachers' subject matter knowledge and each measure of the teaching effectiveness of the teachers. However, the correlations between teacher subject matter knowledge and teacher self evaluation together with the correlations between teacher subject matter knowledge and peer evaluation of teachers' teaching effectiveness were not statistically significant but the correlation between teacher subject matter knowledge and student achievement and also between teacher subject matter knowledge and student evaluation of teacher teaching effectiveness were found to be statistically significant.

Table 4.2 Pearson product-moment correlation between teacher subject matter knowledge and teacher teaching effectiveness (N =11)

Variables	Subject matter knowledge	Students achievement	Self evaluation	Student evaluation	Peer evaluation
Subject matter knowledge	1				
Students achievement	.876**	1			
Self evaluation	.439	.581	1		
Student evaluation	.775*	.318	.514	1	
Peer evaluation	.595	.449	.299	-.265	1

** Correlation is significant at $p < 0.01$

* Correlation is significant at $p < 0.05$

4.2.2.2 Correlation between teachers' subject matter knowledge and combined indices of teachers' teaching effectiveness.

The variables defining teachers' teaching effectiveness were combined and correlated with teachers' SMK. The variables of teaching effectiveness were considered to be equally important hence they were given equal weight in the composite variable. Table 4.3 shows the Pearson product-moment correlation coefficient obtained. The table shows that teachers' SMK has a positive significant relationship with combined indices of teachers' teaching effectiveness.

Table 4.3 Pearson product-moment correlation between teachers' subject matter knowledge and combined indices of teachers' teaching effectiveness (N =11)

Variable	r
combined indices of teachers' teaching effectiveness	.804**

**significant at $p < 0.01$

4.2.2.3 Correlation between teachers' subject matter knowledge and students' opportunity to learn.

Teachers' SMK was correlated with students' opportunity to learn measure. Table 4.4 shows the result obtained. The table shows that teachers' SMK has a positive significant relationship with students' opportunity to learn.

Table 4.4 Pearson product-moment correlation between teachers' subject matter knowledge and students' opportunity to learn (N = 11)

Variable	r
Students' opportunity to learn	.572*

*significant at $p < 0.05$

4.2.3 Linear regression analysis

To confirm the results of the correlation analyses obtained above and estimates how the independent variable (teachers' SMK) predicts the dependent variable (students' achievement, students' rating of teachers' teaching effectiveness and students' opportunity to learn), linear regression analysis of teacher subject matter knowledge and each of the indices of teachers' teaching effectiveness and also the students' opportunity to learn measure were carried out using SPSS.

4.2.3.1 Linear regression analysis of teachers' SMK and student achievement

The result of linear regression analysis of teachers' SMK and students' achievement is displayed in Tables 4.5. The Table is the SPSS output of the model summary showing R (the correlation between the observed and predicted values of students' achievement), R squared, adjusted R squared and the standard error. R square is the coefficient of determination; it is the proportion of variation in students' achievement explained by teachers' subject matter knowledge.

Table 4.5 Model summary of linear regression analysis of teachers' SMK and students' achievement (N = 11).

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.876[a]	.768	.742	4.364057585

a. Predictors: (Constant), SMK

The value of R square (.768) implies that 76.8 % of the variation in the students' achievement scores is explained by teachers SMK.

4.2.3.2 Linear regression analysis of teachers' SMK and students' rating

Table 4.6 shows the result of linear regression analysis of teachers' SMK and students' rating of teachers' teaching effectiveness.

Table 4.6 Model summary of linear regression analysis of teachers' SMK and students' rating of teachers' teaching effectiveness (N = 11).

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.775[a]	.601	.544	.38192722

a. Predictors: (Constant), SMK

The Table shows that the value of R squared is 0.601. This implies that 60.1% variation in students' rating of teachers' teaching effectiveness is predicted by teachers' SMK.

4.2.3.3 Linear regression analysis of teachers' SMK and students' opportunity to learn

Table 4.7 shows the result of linear regression analysis of teachers' SMK and students' opportunity to learn. The Table shows that the value of R square is 0.327. This implies that 32.7% variation in students' opportunity to learn provided by a teacher is predicted by the teacher's SMK.

Table 4.7 Model summary of linear regression analysis of teachers' SMK and students' opportunity to learn (N = 11).

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.572[a]	.327	.266	6.21332

a. Predictors: (Constant), SMK

4.2.4 t-Test

To compare the test performances of the teachers who were effective in their teaching and those that were not and also the performances of the students of the two classes of teachers, I employed t-test analytic tool using SPSS. The first step was to categorise the teachers into two groups namely 'Effective' and 'Ineffective' teachers. It was stated in section 2.3.3 that effective teachers are those teachers that enable their students to achieve the educational learning goals or curriculum standards. The National Curriculum Statement (NCS) grades 10-12 (general) subject assessment guidelines for mathematics (DoE, 2007:6) states that a student need to obtain at least 40% in mathematics examination to be graded moderate achievement and 50% to be graded as adequate achievement. Hence, adequate achievement for at least half of the students of each teacher was used as the benchmark for effective teacher. In other words, teachers that produced up to 50 % adequate level student achievement were categorized as 'effective teachers' while teachers that produced less than 50 % adequate level student achievement were categorized as categorised as 'Ineffective teachers.

Based on this categorisation, out of the 11 teachers that took the test[1] and also their students who took the student test, 4 teachers fell under the effective teachers group while 7 fell under the ineffective teachers group.

The scores of the teachers, out of 60 marks, in the test according to their groups are as follows:

Effective teachers	53, 54, 49, and 47
Ineffective teachers	21, 26, 31, 18, 43, 46 and 17

4.2.4.1 t-Test comparison of the effective and ineffective teachers' general performance on the test.

Independent samples t-Test is used here to compare the general performances of the effective and ineffective teachers. Table 4.8 shows the SPSS output of the independent samples t-Test of the two groups of teachers.

Table 4.8 SPSS output of the independent samples t-Test of the effective and Ineffective teachers

Group Statistics

	GROUP	N	Mean	Std. Deviation	Std. Error Mean
SCORE	Effective	4	50.75	3.304	1.652
	Ineffective	7	28.86	11.739	4.437

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means					95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
SCORE	Equal variances assumed	5.554	.043	3.574	9	.006	21.89	6.126	8.036	35.750
	Equal variances not assumed			4.624	7.491	.002	21.89	4.735	10.844	32.941

[1] 8 teachers that initially consented to participate in the study later on declined to write the test but they took part in the other stages of the study.

The result shows that the significance value p (.006) is less than .005 at 95% confidence interval (α= .05) meaning that the *t*-test (with 9 degrees of freedom) was statistically significant. This implies that the average score of the effective teachers (M= 50.75, s = 3.304) differed significantly from the average score of the ineffective teachers (M = 28.86, s = 11.739).

4.2.4.2 t-Test comparison of the two group of teachers' performance on each test question

Having established that the effective and ineffective teachers differ in their general performances in the subject matter knowledge test, the next step was to ascertain the particular items in the test in which these two groups' performance statistically differed significantly. To do this, *t*-test comparing the scores of the two groups was calculated for each item in the test.

Table 4.9 shows the results of the *t*-tests for each of the 20 questions in the test at 95% confidence interval (α= .05). In the table only the two tailed significance (p) values are shown.

Table 4.9 results of *t*-Test comparison of the two groups' performance on each test question

Question	*Significance (p) value (2-tail)*
1.1	.725
1.2	.078
1.3	.038*
1.4	.172
1.5	.017*
2.1	.078
2.2	.078
3.1	.172
3.2	.912
3.3.1	.030*
3.3.2	.596
3.3.3	.197
4.1	.172
4.2	.246
4.3	.078
5	.005*
5.1	.078
5.2	.749
5.3	.725
5.4	.001*

* statistically significant difference

The results show that the significance (p) values of two-tailed t-tests comparisons between the mean scores of effective teachers and the mean scores of ineffective teachers on questions 1.1, 1.2, 1.4, 2.1, 2.2, 3.1, 3.2, 3.3.2, 3.3.3, 4.1, 4.2, 4.3, 5.1, 5.2 and 5.3 had significance values Each of the values is greater than α (.05) meaning that there is no statistically significant difference between the performances of the effective and ineffective teachers on these questions.

Nevertheless, the results in Table 4.9 show that the two-tailed t-tests comparisons between the mean scores of effective teachers and the mean scores ineffective teachers on questions 1.3, 1.5, 3.3.1, 5 and 5.4 had significance values. Each of the values is less than α (.05) which implies that there is statistically significant difference between the performances of the effective and ineffective teachers on questions 1.3, 1.5, 3.3.1, 5 and 5.4.

4.2.4.3 *t*-Test comparison of the students of effective and Ineffective teachers' general performance on the test.

To compare the general performances of the students of the two categories of teachers Independent samples t-Test was used. Table 4.10 shows the SPSS output of the independent samples t-Test of the two groups of students.

Table 4.10 SPSS output of the independent samples t-Test of the students of effective and ineffective teachers

Group Statistics

	GROUP	N	Mean	Std. Deviation	Std. Error Mean
Stud Score	Effective Teachers'. Students	88	25.82	12.120	1.292
	Ineffective Teachers'. Students	158	16.83	8.082	.643

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Stud Score	Equal variances assumed	37.210	.000	6.956	244	.000	8.99	1.292	6.443	11.535
	Equal variances not assumed			6.229	130.977	.000	8.99	1.443	6.134	11.844

The result shows that the significance value p (.000) is less than .005 at 95% confidence interval (α= .05) meaning that the *t*-test was statistically significant. This implies that the average score of the students of the effective teachers statistically differ significantly from the average score of the students of the ineffective teachers.

4.2.4.4 *t*-Test comparison of the two groups of students' performance on each test question.

To ascertain the particular test questions in which the performances of the two categories of student statistically differ significantly, *t*-test comparing the scores of the two groups of students was calculated for each of the test question. Table 4.11 shows the (significance (p) values) results of the *t*-tests for each of the 20 questions in the test at 95% confidence interval (α= .05).

The table shows that the significance (p) values of comparisons on questions 2.1, 4.1, 4.2, 4.3, and 5 had significance values. Each of the values is greater than α (.05) meaning that there is no statistically significant difference between the performances of the students of the effective and the students of the ineffective teachers on these questions.

However, the results in Table 4.11 show that the significance (p) values of comparisons on questions 1.1, 1.2, 1.3, 1.4, 1.5, 2.2, 3.1, 3.2, 3.3.1, 3.3.2, 3.3.3, 5.1, 5.2, 5.3 and 5.4 are all less than α (.05). This implies that there is statistically significant difference between the performances of the two categories of students on the questions.

Table 4.11 Results of *t*-Test comparison of the two groups' performance on each test question

Question	***Significance (p) value (2-tail)***
1.1	.031*
1.2	.000*
1.3	.000*
1.4	.001*
1.5	.024*
2.1	.333
2.2	.034*
3.1	.001*
3.2	.000*
3.3.1	.000*
3.3.2	.002*
3.3.3	.029*
4.1	.526
4.2	.136
4.3	.092
5	.443
5.1	.000*
5.2	.021*
5.3	.004*
5.4	.010*

* statistically significant difference

4.3 Qualitative analyses

In order to complement the quantitative data analyses provided in section 4.2 and hence gain better and deeper understanding of the research problems, qualitative data analysis was employed to further interrogate the data collected. The qualitative analysis entailed task performance analyses of the teachers' and students' solutions to the tests items to highlight hotspots and bottlenecks in their solutions. This section reports on the task performance analyses.

To carry out the analyses, a qualitative data analysis framework – Mathematical Production System Performance Analysis Framework (MPSPAF) was developed through which it was possible to extricate teachers and students performances at various levels in the tasks.

4.3.1 Mathematical Production System Performance Analysis Framework (MPSPAF)

MPSPAF is an application of systems engineering principles and methods and cognitive science research in human cognitive architecture in explaining the process of solving a mathematical problem. In systems engineering, designing solution to a problem basically involves designing an input-process-output system that converts the input to the desired output (Jeffries, Turner, Polson & Atwood, 1981). The input-process-output system is a production system; input is what goes into the system (the raw materials), process is the shaping and forming of the materials or the exchange of information needed to get the result, output is what comes out of the system - the finished product. The output is the product of the process carried out on the input. Similarly, solving a mathematical problem is a production system. It involves an input-process operation to reach the desired output. The input here is the problem (and its conditions/constraints), the process is all that the problem solver invokes (the concepts, formulas, steps, etc) to arrive at the output which is the solution of the problem.

In systems engineering, analysis of a systems performance involves analyses of the input, process and output operations to see if the process yields the desired output given the input

(Rosso, 2005). Drawing from this perspective, it was considered worthwhile to develop an analytical framework that mimicked the input-process-output processes of a production system in mathematics task performance analysis seeing that mathematics problem solving is an input-process-output operation.

The framework was developed to carry out error analyses of teachers' and students' solutions to each test item. It helped to determine the teachers' and students' errors as they processed the input (the problem) to arrive at their solutions. Figure 4.1 shows the input-process-output (production system) model of problem solving as depicted by the framework.

The model shows that the problem solver interacts with the problem to generate the output at the process level. Hence, the problem solver could commit errors in the course of the processing.

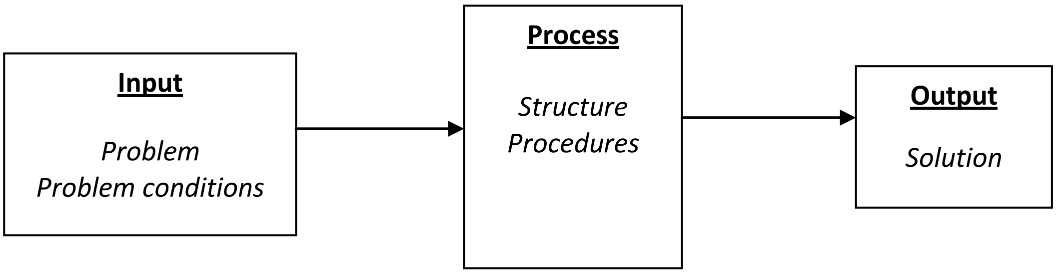


Figure 4.1 Production system problem solving model

When a person encounters a problem he/she draws a structure or representation of the problem (the person's interpretation or understanding of the problem) based on his/her experience and belief about the major variables or factors relevant to the problem (Bruer, 1993). The representation of the problem is the link between the external world and the person's internal processing system (memory) and it shapes the course of the problem solving. A poor initial problem representation can make an easy problem difficult or impossible to solve because there can be no paths from such initial state to the solution (Silver 1987). Based on the structure developed the solver develops an execution procedure (steps) to arrive at the solution.

The idea of representation mentioned above is fundamental to cognitive science. Representations are the symbol structures we construct to encode our experience, process it and store it in our memories (Bruer, 1993). In solving a problem, the solver will select a structure (frame) from the long term memory and copy it to the working memory (Bruer, 1993, Naidoo & Ranjeeth, 2007) and carries out an execution procedure he/she judges is most appropriate to the problem situation to arrive at a solution. In the course of selection of the structure and execution of the procedure steps, errors might get into the problem solving process. Hence the error could be a result of wrong problem solution structure or incorrect execution of the procedures.

Therefore, to identify the errors that students and teachers made in solving the tests question, the framework was designed to capture the two categories of possible errors in mathematical problem solving namely: error in selection of correct solution structures to the problem and errors in the correct execution of procedure steps. The errors are termed structural error (Naidoo & Ranjeeth, 2007) and execution error.

4.3.1.1 Structural error

This type of error is committed when the problem solver failed to draw the appropriate structure or representation of the problem. The person failed to grasp some essential principles to the solution (Naidoo & Ranjeeth, 2007). It is a conceptual error in that the person did not understand the mathematical concept or concepts embodied in the task. That is, he/she did not understand the properties or principals needed in the task. In other words, the person lacks the conceptual knowledge needed to solve the problem.

Example of a structural error from a script in the test

Example 1: question 1.3

Question: if *f(x) = tan 2x*.

1.3 What are the equations of the asymptotes of the function if it is drawn for the domain [–180°, 180°]?

$$\tan 2x = \frac{\sin 2x}{\cos 2x} \quad \text{if } x \in [-180^\circ; 180^\circ)$$

Here the student showed lack of understanding of the concept of asymptotes of tangent function.

Example 2: Question 4. 3

Given the function *f(X) = sin(x-60)*

Determine the amplitude of the function q, if q(x) = $\frac{1}{2}f(x)+\ 2$

Solution (given by a teacher)

$$\text{Amp} = \frac{1}{2} + 2$$
$$= 2\tfrac{1}{2} \text{ and } -2\tfrac{1}{2}$$

The solution show structural error, the teacher showed that he/she did not know that vertical translation of the trigonometric function does not affect the amplitude. Somebody may argue that the teacher's incorrect solution was as a result of his/her inability to recall known fact but this may not be the case as a person's ability to recall facts in a subject domain is influenced by the person's subject matter knowledge in that subject domain (Alexander, Kulikowich & Schulze, 1994).

4.3.1.2 Execution error

This is the type of error committed in the course of putting the procedure steps into action. In this case the structure might be correct but the problem solver failed to carry out the manipulations correctly. This is referred to as executive error (Naidoo & Ranjeeth, 2007). Execution error could be as a result of not executing the procedure in order or as a result of arithmetical (or computational) error. So to differentiate the two types of execution error I classified them as procedural error and arithmetical (computational) error.

4.3.1.3 Procedural error

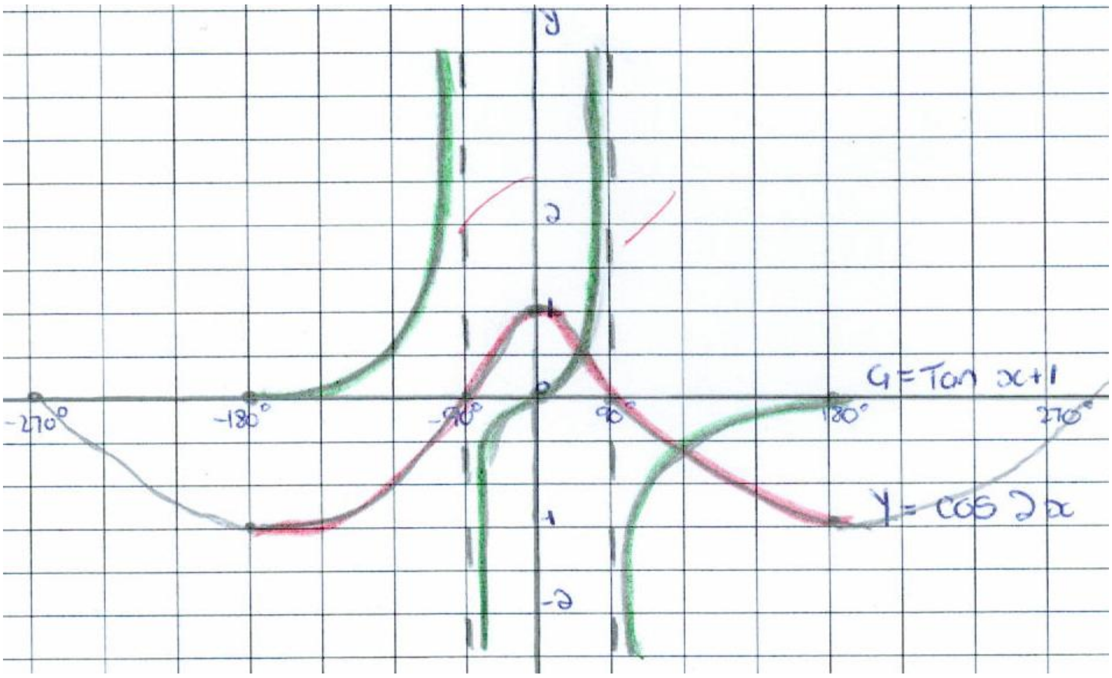
This type of error is committed when one fails to follow the procedural order necessary to achieve a solution. It includes errors as a result of omitting steps or procedures.

Example of a procedural error from the test

Question 5

Sketch the graphs of the functions *f(x) = cos 2x* and *g(x) = tan x +1* for *x* ε *[-180°;180°]* on the same set of axes. Show all the intercepts with the axes and the co-ordinates of the turning points. Draw the asymptotes using dotted lines.

The following Solution was given by a student.



Here the student showed clear conceptual understanding of how to sketch the graph using the properties but did not follow procedure of using uniform scale on the horizontal (x) axis hence had a wrong sketch.

4.3.1.4 Arithmetic/computational error

This is a simpler form of execution error. This type of error is committed when the problem solver employs the correct structure (concept) and procedure in problem solving but somewhere along the line miscalculates or makes a mistake in arithmetic or computational operation like substitution of wrong value to a variable or writing of 1.02 instead of 10.2

Example of arithmetic or computational error from the test

Question 1.4

$f(x) = tan\ 2x$.

If $x = 90^o$, find the values of $f(x)$.

Solution (Given by a teacher)

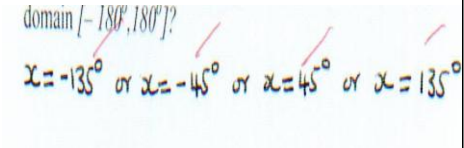
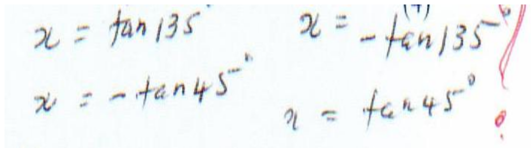
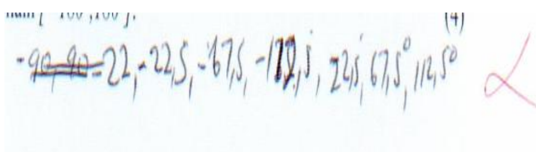
$$f(90) = \tan 2 \times 90^\circ$$
$$f(90) = \tan 180^\circ$$
$$= 1.$$

Here the teacher wrongly computed *tan* 180^o *to be 1.* The teacher knew that *tan 2x* equals *tan* 180^o given that $x = 90^o$. The teacher followed the correct procedure but committed arithmetic error.

4.3.2 Analyses of effective and ineffective teachers performances

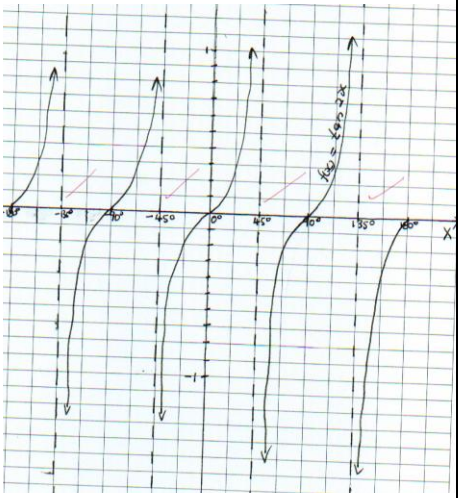
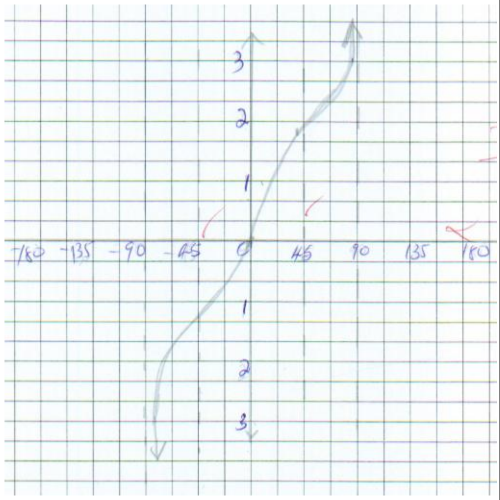
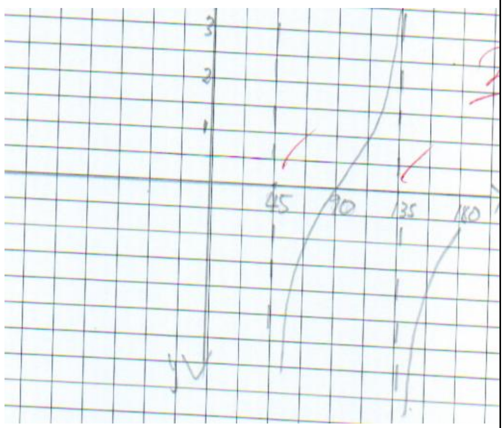
The quantitative analyses carried out earlier showed that there was a statistical significant difference in the general performances of the effective and ineffective teachers in the test and also there were statistical significant differences in their performances in some of the test items. In this section the Mathematical Production System Performance Analysis Framework (MPSPAF) is used to analyse the performance of the teachers on those questions to help address research questions 2 and 3. The analyses include statistics of the number of completely correct, partially correct and completely wrong solutions provided by each category of teachers.

The test items in which there were notable differences in the performances of the effective and ineffective teachers were questions 1.3, 1.5, 3.3.1, 5 and 5.4 (section 4.2.4.2). The analysis is presented in that order in the following section.

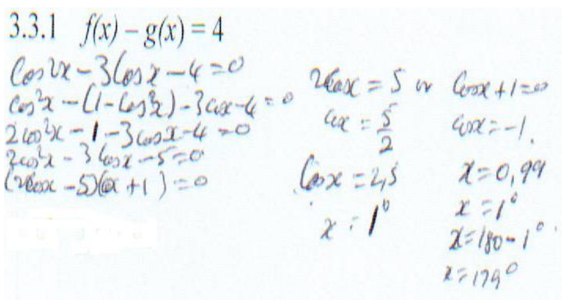
Task 1.3		
	Effective teachers (n = 4)	**Ineffective teachers (n = 7)**
Correct solutions	4	3
partial correct solutions	-	1
completely wrong solutions	-	1
No solutions	-	2
Structural errors	-	-
Procedural errors	-	2
Arithmetic errors	-	-
Vignette	**Vignette A**  	**Vignette B**  **Vignette C** 

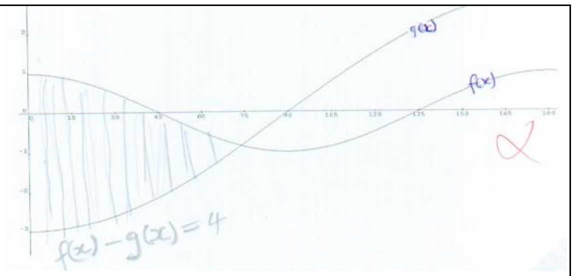
Annotation	The effective teachers seemed to understand the concept of asymptotes, even the effect of the 2 in the function (*$f(x) = tan\ 2x$*).	Vignette B teacher seemed to know the concept but didn't know the procedure of writing equation of asymptote – the teacher wrote $x = tan\ 135^o$ instead of $x = 135^o$. Vignette C teacher also seemed to know the concept but divided 90^o by 4 instead of by 2. Also the teacher made conceptual error in writing equation. If the teacher had written x = -22.5^o one would have said the error was only arithmetic. The other three teachers did not present any solution.

Task 1.5		
	Effective teachers (n = 4)	**Ineffective teachers (n = 7)**
Correct solutions	4	2
partial correct solutions	-	2
completely wrong solutions	-	3
No solutions	-	-
Structural errors	-	3
Procedural errors	-	2
Arithmetic errors	-	-

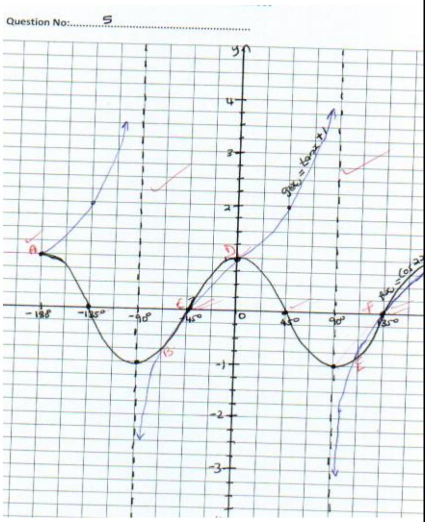
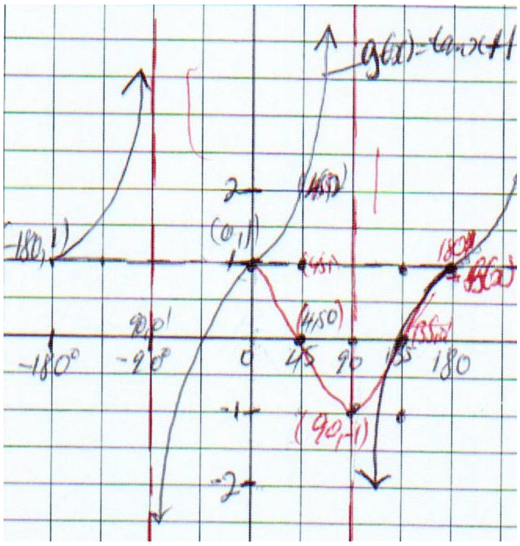
Vignette	**Vignette A** 	**Vignette B**  **Vignette C** 
Annotation	All the effective teachers sketched the graph correctly showing clearly the asymptotes with dotted lines.	Majority of the ineffective teachers seem not to know the properties to use to sketch the graphs – which implies lack of conceptual understanding of the sketching tan graph with the properties. Vignette B teacher drew asymptotes at x = -90^o, -45^o, 45^o and 90^o and also the sketch passes through the asymptote, this shows structural error.

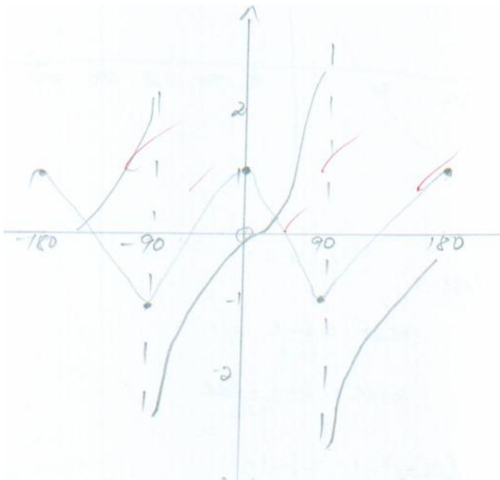
		Vignette C teacher sketched only the right hand side of the function ($45^o \leq x \leq 180^o$) while he/she provided all the equations of the asymptotes in question1.3 this is procedural error. This error was also committed by other two ineffective teachers whose sketches are not shown here.

Task 3.3.1		
	Effective teachers (n = 4)	**Ineffective teachers (n = 7)**
Correct solutions	4	3
partial correct solutions	-	-
completely wrong solutions	-	2
No solutions	-	2
Structural errors	-	2
Procedural errors	-	-
Arithmetic errors	-	-
Vignette		**Vignette A**  **Vignette B**

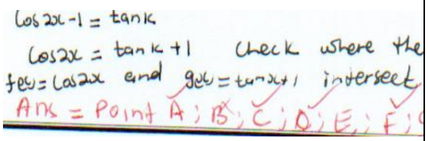
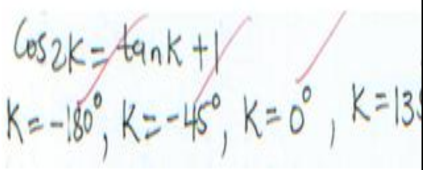
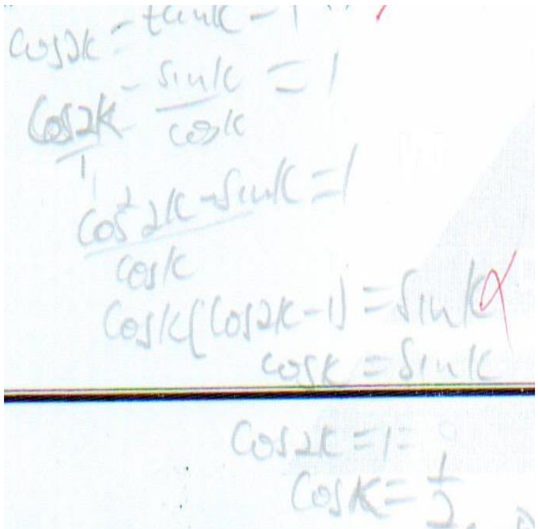
		
Annotation	The effective teachers all correctly read the solution from the graphs showing understanding of multiple representation of trigonometric functions	The two vignettes (A and B) show lack of conceptual understanding of how to solve the problem manifested by most of the ineffective teachers

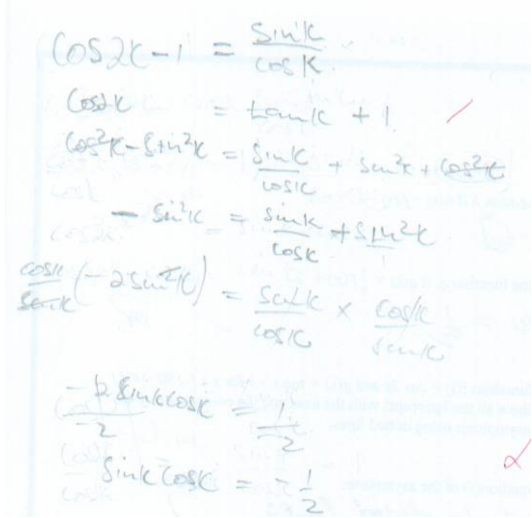
Task 5		
	Effective teachers (n = 4)	**Ineffective teachers (n = 7)**
Correct solutions	4	-
partial correct solutions	-	5
completely wrong solutions	-	-
No solutions	-	2
Structural errors	-	1
Procedural errors	-	4
Arithmetic errors	-	-

<table>
<tr><td>Vignette</td><td>Vignette A

</td><td>Vignette B


Vignette C
</td></tr>
<tr><td>Annotation</td><td>The effective teachers had no problems with sketching trigonometric graphs using properties. This also shows understanding of multiple representations of trigonometric functions by the effective teachers.</td><td>Vignette B teacher could sketch g(x) = tan x +1 but could not complete cos 2x, this is seen as procedural error.

Vignette C teacher sketched cos 2x but had problem with the vertical translation of tan x by 1 unit. This is seen as conceptual error. These errors were also manifested by the most of the other ineffective teachers in solving this problem.</td></tr>
</table>

Task 5.4		
	Effective teachers (n = 4)	**Ineffective teachers (n = 7)**
Correct solutions	2	-
partial correct solutions	2	2
completely wrong solutions	-	1
No solutions	-	4
Structural errors	-	2
Procedural errors	2	1
Arithmetic errors	-	-
Vignette	**Vignette A**  **Vignette B** 	**Vignette C**  **Vignette D**

		 $\cos 2k - 1 = \frac{\sin k}{\cos k}$ $\cos 2k = \tan k + 1$ $\cos^2 k - \sin^2 k = \frac{\sin k}{\cos k} + \sin^2 k + \cos^2 k$ $-\sin^2 k = \frac{\sin k}{\cos k} + \sin^2 k$ $\frac{\cos k}{\sin k}(-2\sin^2 k) = \frac{\sin k}{\cos k} \times \frac{\cos k}{\sin k}$ $\frac{-2\sin k \cos k}{-2} = \frac{1}{-2}$ $\sin k \cos k = -\frac{1}{2}$
Annotation	All the effective teachers simplified and transformed the equation that enable them to link it with the equations of the graphs. The two teachers that did not give complete solutions failed to write all the values of the interception of the two functions. The effective teachers displayed conceptual and procedural understanding of the solution steps as revealed by vignettes A and B.	The ineffective teachers showed lack of understanding of the solution steps as vignettes C and D reveal. They did not connect the given equation to the equations of the graph in order to read off the solutions from the graphs.

4.3.2.1 Summary of errors made by the teachers

In this section, a summary of the errors made by the teachers as were observed from the analyses of teachers' solutions to the question in section 4.3.2 above are provided. Table 4.12 shows the summary of errors made by the teachers according to the two categories.

Table 4. 12 Summary of errors made by the teachers in the 5 tasks

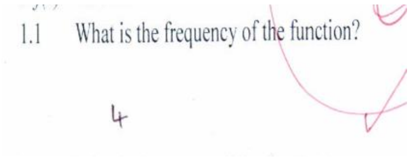
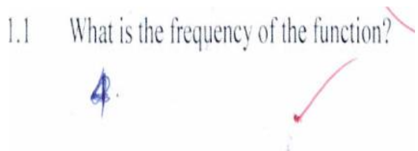
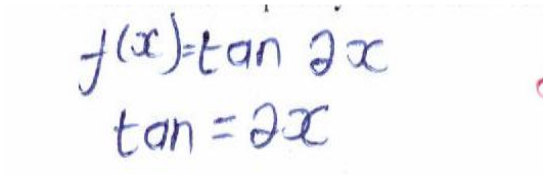
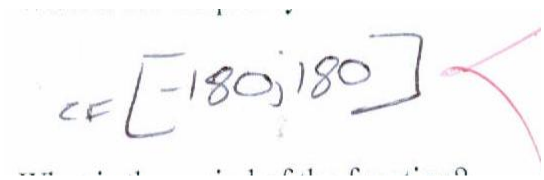
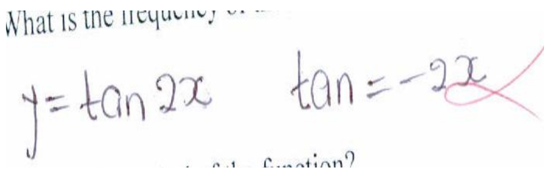
	Effective teachers (n = 4)	Ineffective teachers (n = 7)
Structural errors	0 (0.00%)	8 (1.14%)
Procedural errors	2 (0.50%)	9 (1.29%)
Arithmetic errors	0 (0.00%)	0 (0.00%)

4.3.3 Analyses of students performances

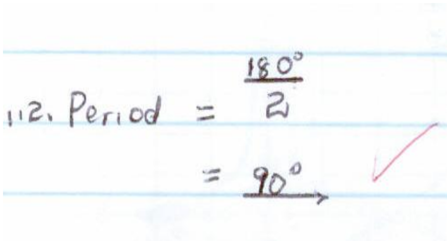
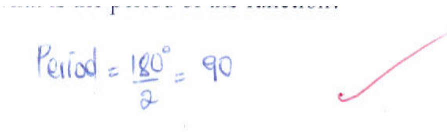
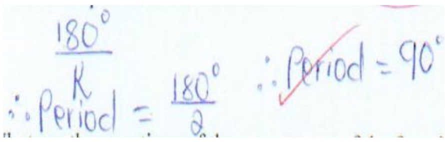
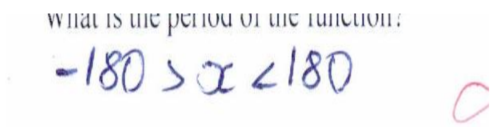
As with the teachers, the quantitative analysis of students' performance showed that there was a statistical significant difference in the general performances of the students of effective teachers and students of ineffective teachers in the test and also there were statistically significant differences in their performances in some of the test items. In this section the Mathematical Production System Performance Analysis Framework (MPSPAF) is used to analyse the performance of the two categories of students on those questions.

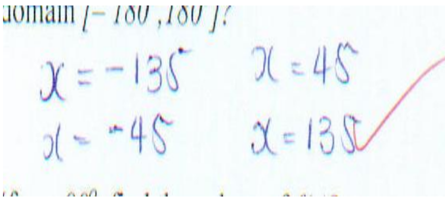
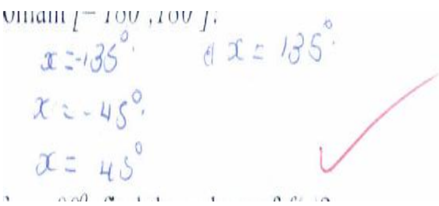
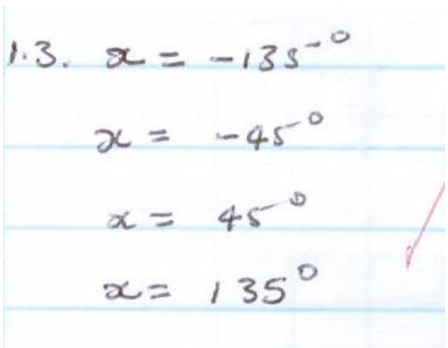
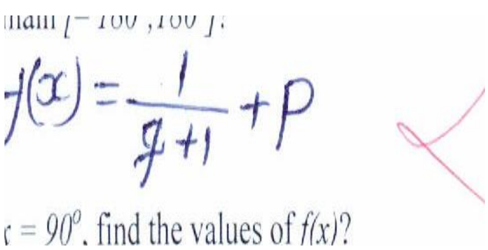
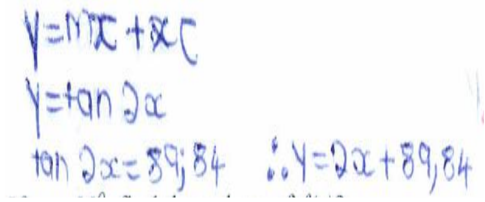
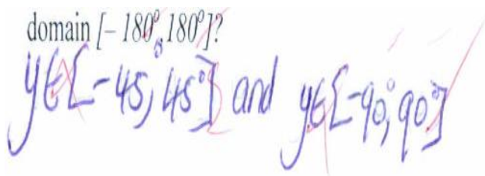
The test items in which there were statistically significant differences in the performances of the students of effective teachers and students of ineffective teachers were questions 1.1, 1.2, 1.3, 1.4, 1.5, 2.2, 3.1, 3.2, 3.3.1, 3.3.2, 3.3.3, 5.1, 5.2, 5.3 and 5.4 (section 4.2.4.4). The analysis is presented in that order.

Task 1.1		
	Effective teachers students (n = 20)	Ineffective teachers students (n = 35)
Correct solutions	2	-
partial correct solutions	-	-
completely wrong solutions	15	28
No solutions	3	7

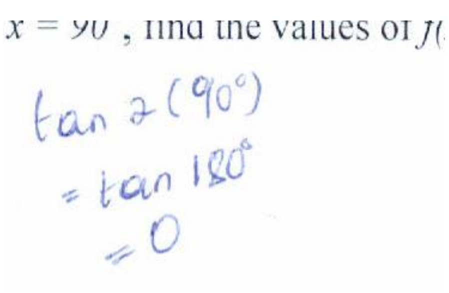
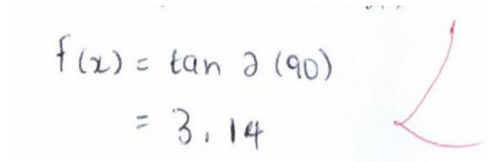
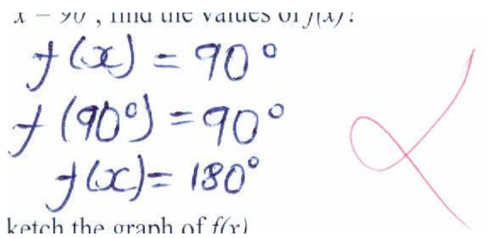
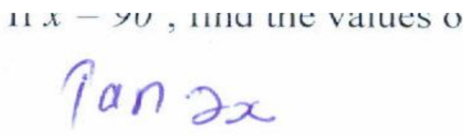
Structural errors	15	28
Procedural errors	3	-
Arithmetic errors	-	-
Vignette	**Vignette A**  **Vignette B**  	**Vignette C**  **Vignette D**  **Vignette E** 
Annotation	While both categories of students seemed to be unfamiliar with the concept, some of the students of effective teachers could relate frequency, 360^{o} and period as explained in some of their test books but the students of the ineffective teachers could not draw up any connection to period and 360^{o}.	

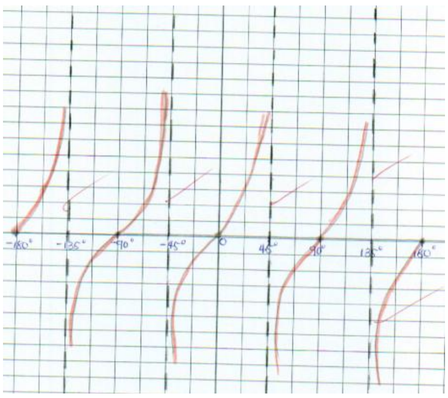
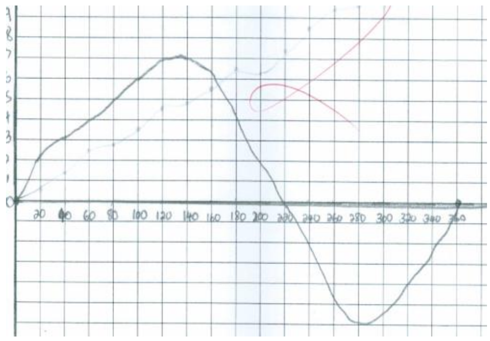
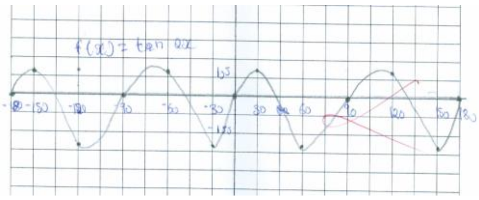
Task 1.2		
	Effective teachers students (n = 20)	Ineffective teachers students (n = 35)

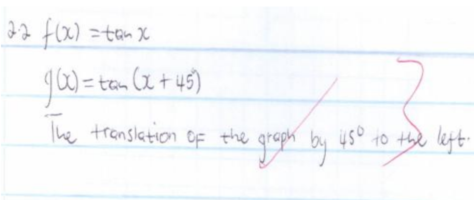
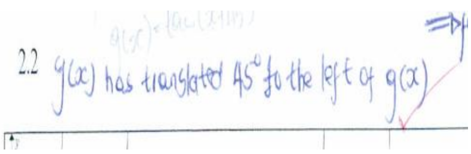
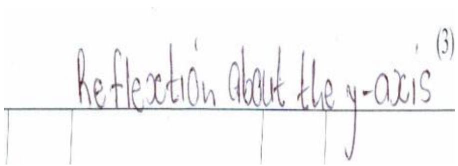
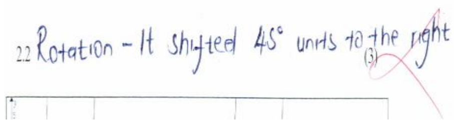
Correct solutions	17	10
partial correct solutions	-	-
completely wrong solutions	3	23
No solutions	-	2
Structural errors	3	23
Procedural errors	1	1
Arithmetic errors	-	-
Vignette	**Vignette A**  **Vignette B**  **Vignette C** 	**Vignette D**  **Vignette E** 
Annotation	This category of student showed understanding of period of the tangent function.	Majority of the students here showed misconception about the period of the tangent function.

Task 1.3		
	Effective teachers students (n = 20)	Ineffective teachers students (n =35)
Correct solutions	17	8
partial correct solutions	-	1
completely wrong solutions	3	23
No solutions	-	3
Structural errors	1	23
Procedural errors	2	3
Arithmetic errors	-	-
Vignette	**Vignette A**  **Vignette B**  **Vignette C** 	**Vignette D**  **Vignette E**  **Vignette F** 

Annotation	These students showed knowledge of asymptotes of tan function.	This category of students displayed general misconception of the asymptotes of tan function.

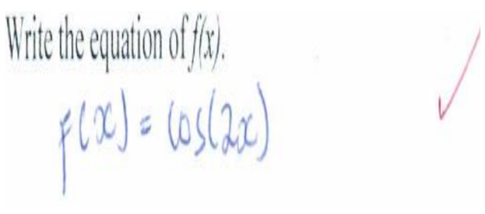
Task 1.4		
	Effective teachers students (n= 20)	Ineffective teachers students (n =35)
Correct solutions	16	13
partial correct solutions	1	1
completely wrong solutions	2	18
No solutions	1	3
Structural errors	2	17
Procedural errors	-	3
Arithmetic errors	-	-
Vignette	**Vignette A** 	**Vignette B**  **Vignette C**  **Vignette D** 
Annotation	Here the student did not only give correct answers but also showed how they arrived at the answers.	While some of the student here got correct answers, majority of the students in this category showed that they did not even understand what the questions really demanded.

Task 1.5		
	Effective teachers students (n = 20)	Ineffective teachers students (n = 35)
Correct solutions	7	4
partial correct solutions	11	10
completely wrong solutions	2	16
No solutions	1	5
Structural errors	3	17
Procedural errors	9	6
Arithmetic errors	-	1
Vignette	**Vignette A** 	**Vignette B**  **Vignette C** 
Annotation	This group of students showed that they knew how to use the properties to sketch the function	There seems to be general lack of understanding on how to use the properties to sketch graph of tan function.

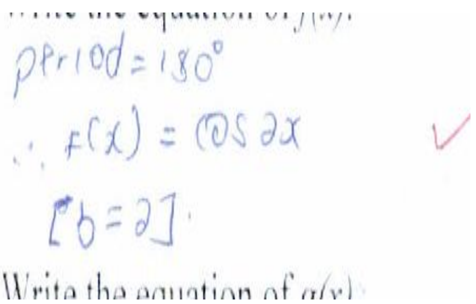
Task 2.2		
	Effective teachers students (n = 20)	Ineffective teachers students (n = 35)
Correct solutions	10	2
partial correct solutions	2	1
completely wrong solutions	8	23
No solutions	-	9
Structural errors	8	23
Procedural errors	2	1
Arithmetic errors	-	-
Vignette	**Vignette A**  **Vignette B**  	**Vignette C**  **Vignette D** 
Annotation	Majority of the students of effective teachers showed understanding of the concept of transformation (shifting) of trigonometric functions graphs.	This group showed lack of conceptual understanding of transformation (shifting) of trigonometric functions graphs.

<table>
<tr><th colspan="3">Task 3.1</th></tr>
<tr><td></td><td>Effective teachers students (n = 20)</td><td>Ineffective teachers students (n = 35)</td></tr>
<tr><td>Correct solutions</td><td>18</td><td>6</td></tr>
<tr><td>partial correct solutions</td><td>-</td><td>8</td></tr>
<tr><td>completely wrong solutions</td><td>2</td><td>22</td></tr>
<tr><td>No solutions</td><td>-</td><td>1</td></tr>
<tr><td>Structural errors</td><td>2</td><td>19</td></tr>
<tr><td>Procedural errors</td><td>-</td><td>3</td></tr>
<tr><td>Arithmetic errors</td><td>-</td><td>-</td></tr>
<tr><td>Vignette</td><td>

Vignette A



Vignette B



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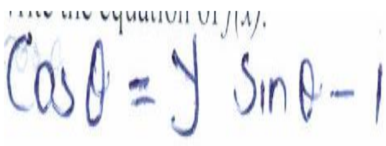
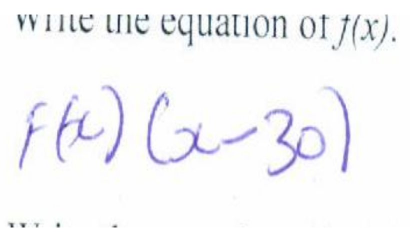
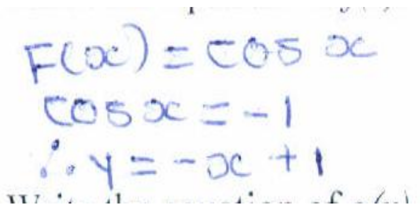
Vignette C



Vignette D



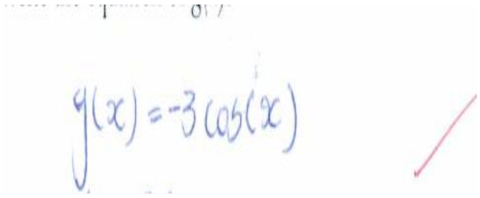
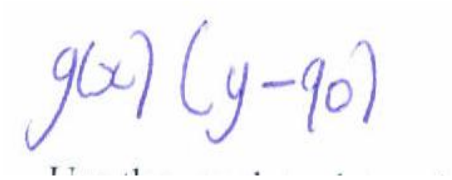
Vignette E



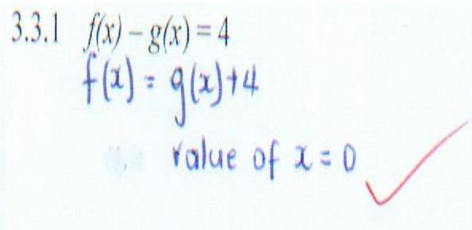
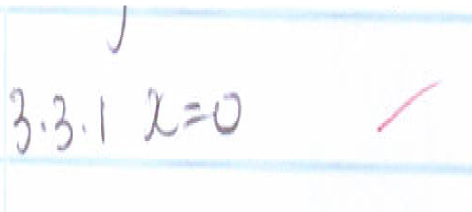
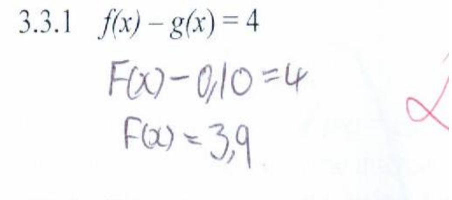
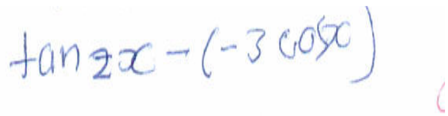
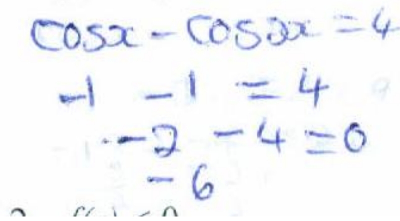
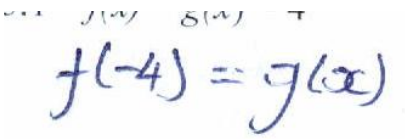
</td></tr>
</table>

		Vignette F Write the equation of $f(x)$. $f(x) = \tan 2x$
Annotation	The students of effective teachers could convert between graphical and symbolic representations in line with the curriculum assessment standards.	This category of students displayed misconception of conversion between graphical and symbolic representations.

Task 3.2		
	Effective teachers students (n = 20)	Ineffective teachers students (n = 35)
Correct solutions	16	8
partial correct solutions	1	5
completely wrong solutions	3	15
No solutions	-	7
Structural errors	3	12
Procedural errors	1	7
Arithmetic errors	-	-
Vignette	**Vignette A** Write the equation of $g(x)$. $g(x) = -3 \cos x$ $a = -3$ $b = 1$ (period $= 360°$).	**Vignette C** $g(x) \cos \theta + 3$

	Vignette B	Vignette D
	 $g(x) = -3\cos(x)$	 $g(x)\ (y-90)$
Annotation	Just as was stated above on Task 3.1, the students of the effective teachers could convert between graphical and symbolic representations in line with the curriculum assessment standards. This shows conceptual understanding of the properties of functions and application of their characteristics to a variety of Problems.	The students of the ineffective teachers could not convert between graphical and symbolic which shows lack of conceptual understanding of the properties of functions and application of their characteristics to a variety of Problems.

Task 3.3.1		
	Effective teachers students (n = 20)	Ineffective teachers students (n = 35)
Correct solutions	9	-
partial correct solutions	1	2
completely wrong solutions	7	26

No solutions	3	7
Structural errors	6	27
Procedural errors	1	1
Arithmetic errors	-	-
Vignette	**Vignette A**  **Vignette B**  	**Vignette C**  **Vignette D**  **Vignette E**  **Vignette F** 

Annotation	Many of the effective teachers' students could correctly read the solution from the graphs showing understanding of properties and multiple representations of trigonometric functions	The students of the ineffective teacher showed lack of conceptual understanding of how to solve the problem manifested by most of the ineffective teachers

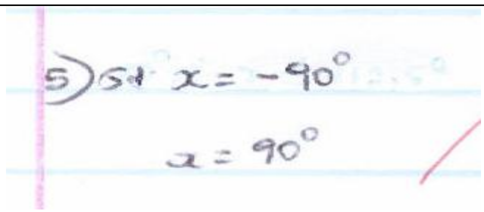
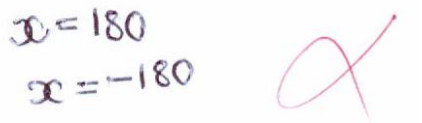
Task 3.3.2		
	Effective teachers students (n = 20)	Ineffective teachers students (n = 35)
Correct solutions	8	2
partial correct solutions	2	7
completely wrong solutions	8	19
No solutions	2	7
Structural errors	4	18
Procedural errors	5	8
Arithmetic errors	-	-
Vignette	**Vignette A** $45^\circ < x < 135^\circ$ **Vignette B**	**Vignette C** $f(x) < 0$ $0 > F(x) \leq 3{,}9$ **Vignette D** KS $x - 30 < 0$ $x = 30$

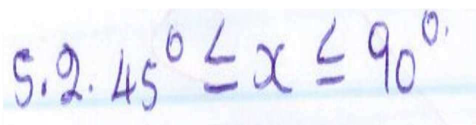
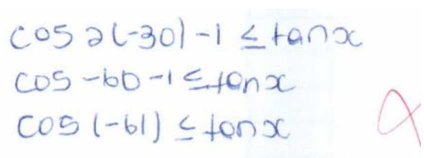
	3.3.2 $f(x) < 0$ $45^{0} < x < 135^{0}$	**Vignette E** 3.3.2 $f(x) < 0$ $f(x) = \tan 2x < 0$ $2x < \tan^{-1}(0)$ $2x < 0$ $\frac{2}{2}x < \frac{0}{2}$
Annotation	Same as in Task 3.1.1 above.	

Task 3.3.3		
	Effective teachers students (n = 20)	Ineffective teachers students (n = 35)
Correct solutions	4	1
partial correct solutions	4	3
completely wrong solutions	10	21
No solutions	2	10
Structural errors	6	20
Procedural errors	8	5
Arithmetic errors	-	-
Vignette	**Vignette A** $0^{0} \leq x \leq 90^{0}$ $135^{0} \leq x \leq 180^{0}$ $\therefore x = 135^{0}$ or $x = \cancel{45^{0}}$ **Vignette B**	**Vignette D** $f(x).g(x) > 0$ $0{,}03 \cdot 0{,}10 > 0$ $0{,}333 > 0$ **Vignette E** $30 - 90 > 0$ $-60 > 0$

	$x \in (135; 180^\circ)$ **Vignette C** 3.3.3 $45^\circ < x < 90^\circ$ and $135^\circ < x$	**Vignette F** $f(x).g(x) > 0$ $f(4) \cdot g(2) > 0$
Annotation	Same as in Task 3.1.1 above.	

Task 5.1		
	Effective teachers students (n = 20)	Ineffective teachers students (n = 35)
Correct solutions	19	8
partial correct solutions	1	2
completely wrong solutions	-	18
No solutions	-	7
Structural errors	-	16
Procedural errors	-	2
Arithmetic errors	1	1
Vignette	**Vignette A** Write down the equation(s) of the asymptotes $x = -90^\circ$ $x = 90^\circ$ **Vignette B**	**Vignette C** $x = 30$ $y = 57{,}2$ **Vignette D**

		
Annotation	Like in Task 1.3, students of the effective teachers showed understanding of the concept of asymptotes of tan function, while the students of the ineffective teachers displayed general misconception of the asymptotes of tan function. This trend was also manifested by their teachers.	

Task 5.2		
	Effective teachers students (n = 20)	Ineffective teachers students (n = 35)
Correct solutions	2	-
partial correct solutions	6	-
completely wrong solutions	11	20
No solutions	1	15
Structural errors	5	20
Procedural errors	12	-
Arithmetic errors	-	3
Vignette	**Vignette A**  **Vignette B**	**Vignette D**  **Vignette E**

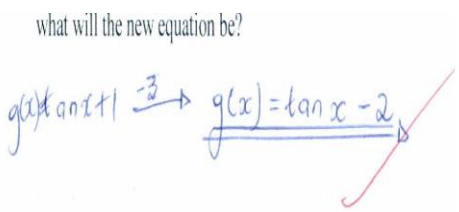
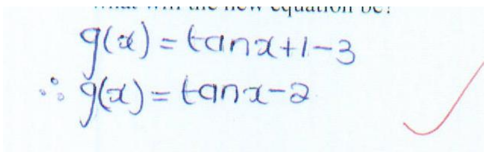
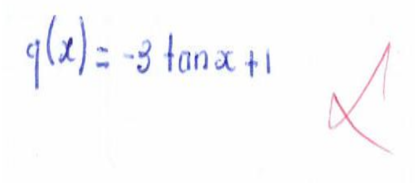
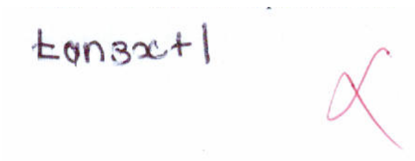
<table>
<tr>
<td></td>
<td>Use the graph to determine the values of x if: cos 2x - 1 ≤ tan x; in the interval [0°;180°]
$\cos 2x - 1 \leq \tan x$
$\cos 2x \leq \tan x + 1$
$0° \leq x < 90°$
$x = 135°$
$\& \ x = 180°$

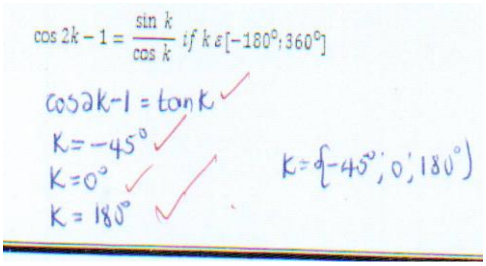
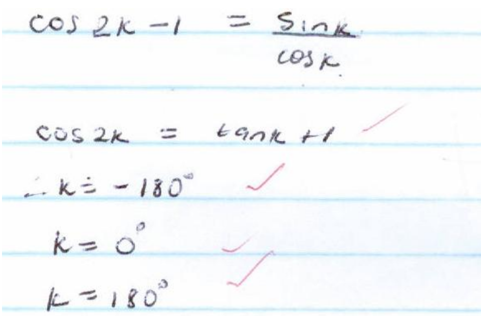
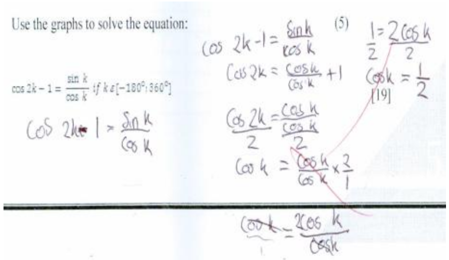
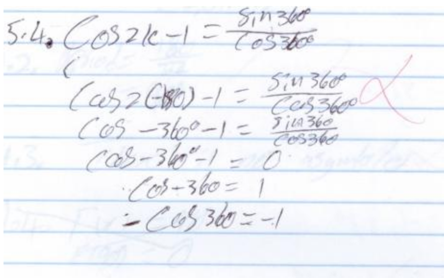
Vignette C

5.4 $45° < x < 90°$</td>
<td>$2x - 1 = 0$
$2x = 1$
$\frac{2x}{2} = \frac{1}{2}$
$x = \frac{1}{2}$

Vignette F

$\cos 2x - 1 \leq \tan x$
$\cos 2x - 1 \leq 0$
$\cos 2x \leq 1$
$2x \leq \cos^{-1}(1)$
$\frac{2x}{2} \leq \frac{0}{2}$
$x \leq 0$</td>
</tr>
<tr>
<td>Annotation</td>
<td colspan="2">This task requires transformation of the inequality to relate it to the function sketched in graphs and then reading off the values from the graph. Some of the students of the effective teachers were able to do this. However, the students of the ineffective teachers could not relate the inequality to the graph which shows lack of the conceptual understanding of the fundamental principle to employ in solving the problem.</td>
</tr>
</table>

Task 5.3		
	Effective teachers students (n = 20)	Ineffective teachers students (n = 35)
Correct solutions	14	7
partial correct solutions	-	1
completely wrong solutions	6	15
No solutions	-	12
Structural errors	6	14
Procedural errors	-	2
Arithmetic errors	-	2
Vignette	**Vignette A**  **Vignette B** 	**Vignette C**  **Vignette D** 
Annotation	Most of the students of the effective teachers showed understanding of the concept of vertical transformation of trigonometric function and how it affects the equation of the function (the symbolic representation). Some of the students of the ineffective teachers also did but majority could not relate the transformation to the equation of the function.	

Task 5.4		
	Effective teachers students (n = 20)	Ineffective teachers students (n = 35)
Correct solutions	3	-
partial correct solutions	6	6
completely wrong solutions	3	13
No solutions	8	16
Structural errors	5	13
Procedural errors	4	5
Arithmetic errors	-	2
Vignette	**Vignette A**  **Vignette B** 	**Vignette C**  **Vignette D** 
Annotation	While this problem seems to have been a challenge to both categories of students, some of the students of the effective teachers were able to simplify the equation and related it to the graphed function but the students of the ineffective teachers generally showed lack of knowledge of how to approach the problem. They (ineffective teachers' students) did not connect the equation to the graph. The two categories of	

	students reflected their teachers as was observed with the teachers' solutions to this task in section 4.3.2.

4.3.3.1 Summary of errors made by the students

In this section, a summary of the errors made by the students as were observed from the analyses of students' solutions to the question in section 4.3.3 above are provided. Table 4.13 shows the summary of errors made by the students according to the two categories.

Table 4. 13 Summary of errors made by the students in the 15 tasks

	Effective teachers' students (n = 20)	**Ineffective teachers' students (n = 35)**
Structural errors	69 (3.45%)	290 (8.29%)
Procedural errors	48 (2.40%)	57 (1.63%)
Arithmetic errors	1 (0.05%)	9 (0.26%)

4.4 Answering of research questions

The results of the data analyses presented above were used to address the research questions posed in this study. The questions are addressed in the following sections.

4.4.1 Research question one

The first research question was: Is there any relationship between mathematics teachers' subject matter knowledge and their teaching effectiveness?

As stated in section 1.3, teachers' teaching effectiveness was used as a composite term measured by students' achievement, peer rating, self rating, and students' rating of teachers' teaching effectiveness. Hence, there was the need to determine the possible relationship between the teachers' subject matter knowledge and each of the measures of the teacher's teaching effectiveness. Therefore, for this question (question one) the following sub-questions were raised:

i. Is there any relationship between mathematics teachers' subject matter knowledge and their students' achievement?

ii. Is there any relationship between mathematics teachers' subject matter knowledge and their students' rating of the teachers' teaching effectiveness?

iii. Is there any relationship between mathematics teachers' subject matter knowledge and their peers' rating of the teachers' teaching effectiveness?

iv. Is there any relationship between mathematics teachers' subject matter knowledge and their self rating of their teaching effectiveness?

The sub questions are addressed respectively.

4.4.1.1 Sub question one

To answer the question (Is there any relationship between mathematics teachers' subject matter knowledge and their students' achievement?), the data were analysed using correlation analysis while statistical inference was taken at 0.01 alpha levels. The result is displayed in Table 4.2. From the table the result ($r = 0.876$; $p < 0.01$) indicated that a statistically significant relationship existed between teachers' subject matter knowledge and students' achievement. Furthermore, the results of linear regression analysis of teacher SMK and students' achievement displayed in Table 4.5 showed that a teacher's subject matter knowledge does explain the variation in students' achievement score. This confirmed the result of the correlation analysis.

4.4.1.2 Sub question two

The second sub question was: Is there any relationship between mathematics teachers' subject matter knowledge and their students' rating of the teachers' teaching effectiveness? The result of correlation analysis between teacher subject matter knowledge and their students rating shown in Table 4.2 ($r = 0.775$; $p < 0.05$) indicated that a statistically significant relationship exited between teachers' subject matter knowledge and students' rating of their teaching effectiveness. This finding was confirmed by the result of linear regression analysis in Table 4.6 showed that a teacher's subject matter knowledge can well predict students' rating of the teacher's teaching effectiveness.

4.4.1.3 Sub question three

The third sub question was: Is there any relationship between mathematics teachers' subject matter knowledge and their peers' rating of the teachers' teaching effectiveness? The result of correlation analysis between teachers' subject matter knowledge and their peers' rating shown in Table 4.2 ($r = 0.595$; $p < 0.05$) indicated that there was a positive relationship between teachers' subject matter knowledge and their peers' rating of the teachers' teaching effectiveness. However, the relationship was not statistically significant at 95% confidence interval.

4.4.1.4 Sub question four

The fourth sub question to the first research question was: Is there any relationship between mathematics teachers' subject matter knowledge and their self rating of their teaching effectiveness? The result of correlation analysis between teacher subject matter knowledge and their peers' rating of the teachers' teaching effectiveness shown in Table 4.2 ($r = 0.439$; $p < 0.05$) indicated that a positive relationship existed between between mathematics teachers' subject matter knowledge and their self rating of their teaching effectiveness but the relationship was not found to be statistically significant at 95 % confidence interval.

4.4.2 Research question two

The second research question was: is there any difference between the subject matter knowledge of effective and ineffective teachers? The results of t-test between the scores of effective teachers and the scores of ineffective teachers in subject matter knowledge test (sections 4.2.4.1 & 4.2.4.2; Tables 4.8 & 4.9) showed that there was a statistically significant difference between the achievements of the effective teachers and the ineffective teachers in the test. Further analyses revealed that the differences existed in the achievement of the effective teachers and ineffective teachers in five questions. Cognitive task performance analyses (section 4.3.2) of the two categories of teachers' solutions to the test questions showed that the effective teachers had more conceptual understanding of the subject matter than the ineffective teachers. Hence, as was shown in the Table 4.12 (Summary of errors made by the teachers in the 5 tasks), the effective teachers committed fewer structural and procedural errors in answering the questions than the ineffective teachers.

4.4.3 Research question three

The third research question was: Is there any difference between the subject matter knowledge of the students of effective and ineffective teachers? In other words, is there any difference between the subject matter knowledge of students taught by effective teachers and students taught by ineffective teachers? Cognitive task performance analyses (section 4.3.3) of the two categories of students' solutions to the fifteen questions showed that, like their teachers, the students of the effective teachers had more conceptual understanding of the subject matter than the students of the ineffective teachers. Hence, as Table 4.13 shows, the effective teachers' students committed fewer errors (structural, procedural and arithmetic errors) in answering the questions than the students of the ineffective teachers.

4.5 Conclusion

In this chapter the analyses of the data meant to provide answers to the research questions were carried out. The results of the analyses showed that positive relationship exists between teacher subject matter knowledge and the measures of teacher teaching effectiveness namely: students' achievement, student rating of teacher teaching effectiveness, peers' rating of teacher teaching effectiveness and teacher self rating of teacher teaching effectiveness. However, only the relationships between teacher subject matter knowledge and students' achievement and between teacher subject matter knowledge and students' rating of teacher teaching effectiveness were found to be statistically significant. It was also found that there was a statistically significant positive relationship between teacher subject matter knowledge and the composite measure (comprising students' achievement, students' rating, peers' rating and teachers' self rating) of teachers' teaching effectiveness.

The analysis further showed that the teacher subject matter knowledge had a positive statistically significant relationship with the student opportunity to learn the subject matter that the teacher provided for the students.

Also, the results showed that there was a statistically significant difference between the subject matter knowledge of effective teachers and ineffective teachers. Qualitative data analysis – cognitive task performance analyses of the teachers and students solutions to the tests questions showed that the effective teachers had more conceptual knowledge of the subject matter and committed fewer errors in the solving the mathematical tasks than the ineffective teachers. Likewise, the students of the effective teachers showed better conceptual understanding of the subject matter and committed fewer errors in the tasks than the students of the ineffective teachers.

4.6 Projection for the next chapter

The next chapter discusses the findings and the implications of the results. It also attempts to relate the findings to the relevant literature.

Chapter Five

Discussion of Findings

5.1 Introduction

This chapter gives a brief account of what was carried out in the study and discusses the findings in relation to the research questions and some literature reviewed. It also presents the implications of the findings.

5.2 Summary of findings

The study set out to explore the relationship between mathematics teachers' subject matter knowledge and their teaching effectiveness. It was found that a positive, statistically significant relationship existed between the participating teachers' subject matter knowledge and the composite measure of their teaching effectiveness. The relationship between teachers' SMK and the students' achievement and also students' rating of the teachers' teaching effectiveness were also found to be positive and statistically significant. However, the relationships between teachers' SMK and teachers' self rating and peers' rating of teacher teaching effectiveness were not found to be statistically significant though they were positive.

Further data analysis conducted showed that there was a difference between the SMK of effective and ineffective teachers in favour of the effective teachers. Also, there was a difference between the SMK of the students taught by effective teachers and the SMK of the students taught by the ineffective teachers in favour of the students taught by the effective teachers. The effective teachers showed better conceptual understanding of the subject matter - trigonometric functions; its properties, the multiple representations, transformations and its translations between different representations. The ineffective teachers on the other hand, generally showed lack of conceptual understanding of the subject matter of trigonometric functions and their associated concepts mentioned above.

5.3 Discussions of the Findings

The findings are discussed here in order of the research questions posed.

5.3.1 Research question one

As stated in sections 4.4.1, the first research question was: Is there any relationship between mathematics teachers' subject matter knowledge and their teaching effectiveness? To this question four sub-questions were raised (sections 1.3 and 4.4.1). Based on the findings of this study the sub-questions are discussed in their order in the following sub-section.

5.3.1.1 Sub-question one

To the first sub-question which was: Is there any relationship between mathematics teachers' subject matter knowledge and their students' achievement?, the result of correlation analysis in Table 4.7 shows that there was a positive statistically significant relationship between teacher subject matter knowledge and their students' achievement. In other words, students whose teachers had more SMK performed better in mathematics than students whose teachers had less SMK. The result of linear regression analysis in Table 4.5 showed that 76.8 % of the variation in the students' achievement in the test used in this study could be attributed to teacher subject matter knowledge.

This finding is in agreement with the findings of Strauss and Sawyer (1986), Ferguson (1991), Wenglinsky (2002), Greenberg, Rhodes, Ye, and Stancavage (2004), Boyd, Grossman, Lankford, Loeb & Wyckoff (2006) and Ogbonnaya and Osiki (2007). This finding is not surprising, since according to Sanders and Rivers (1996) and also Collias, Pajak and Rigden (2000), the influence of teachers is the single-most important factor in determining students' achievement. It is the teacher that interprets and facilitates the curriculum to the students. The teacher is the manager of his/her class; it is the teacher that determines what to teach and what not to teach, when and how to teach it. To be able to do these functions, the teacher relies heavily on his/her subject matter understanding which also influences his/her pedagogical strategies and belief about mathematics teaching and learning (Rohaan, Taconis & Jochems, 2010; Tsang & Rowland, 2005). A teacher with deep subject matter content is able to understand and correct his/her students' learning misconceptions, errors and difficulties. Such a teacher will likely channel his/her instructions to meet varied level of students' readiness.

As observed by Tsang and Rowland (2005), a teacher that has good mastery of the substantive syntactic structures of the subject will be able to unpack the subject's content in a way it makes meaning to the students. On the other hand, a teacher that does not have mastery of the subject matter will not be able to extricate the subject matter content to the understanding of the students.

To enable students to develop deep conceptual understanding of mathematics, and acquire the specific knowledge and skills necessary for the application of mathematics to physical, social and mathematical problem as stipulated in the National Curriculum Statement NCS, (Department of Education, 2002: 5), the mathematics class should expose the students to deep concepts and application of mathematics which is most likely to happen if the teacher is deep rooted in mathematical concepts and applications. This possibly explains why research over the years tends to portend a positive relationship between teacher subject matter knowledge and students achievement in mathematics. This could also throw more light on the current debate about the state of mathematics teaching and learning in the country especially in the light of the recent paradoxical revelation that some teachers could not pass the mathematics examination that was set for students at the level the teachers teach.

Tchoshanov (2011) observed that a teacher with subject matter knowledge that is limited to mathematical procedures only will have less opportunity to influence students' success than a teacher who conceptually understands the subject matter. A teacher cannot help the students beyond the level of his/her own knowledge and ability. It is not possible for a teacher to expose the students to higher order thinking or conceptual knowledge problems when the teacher himself or herself cannot solve higher order thinking problems. Such a teacher will rely heavily on textbooks in his lesson delivery. The teacher may also find it difficult to contextualise his teaching which is very fundamental in helping students develop conceptual understanding of the subject matter.

This also may explain why some teachers tend to avoid teaching some parts of the curriculum or topics. If a teacher lacks the subject matter mastery of a topic or curriculum content, he will likely not go beyond the introductory part of the topic which will only expose the students to the lowest level of content complexity in that topic or subject. Such students will not easily answer questions at the higher levels of content complexity and as such will not easily achieve good results in any standard examination as they may only be able to attend to questions at the recall level. A class taught by a teacher that has limited subject matter knowledge of the subject will likely be characterised by mass failure and under achievement in standardised examinations/test.

5.3.1.2 Sub-question two

The second sub-question to research question one was: Is there any relationship between mathematics teachers' subject matter knowledge and their students' rating of the teachers' teaching effectiveness? The result of correlation analysis in Table 4.7 shows that there was a positive statistically significant relationship between teachers' subject matter knowledge and their students' rating of the teachers' teaching effectiveness. Furthermore, the result of linear regression analysis (Table 4.6) showed that 60.1% variation in students' rating of teachers' teaching effectiveness is explained by teachers' SMK. This means that students whose teachers have more SMK would likely give a more positive rating of their teacher's teaching effectiveness than students whose teachers have less SMK.

While no study was found that explored the relationship between teachers' SMK and students' rating, evidence shows that high school students are capable of distinguishing between effective and ineffective teachers (Irving, 2004). Also students' rating of teachers is related to teachers' teaching effectiveness measured by students' achievement or performance (Theall & Franklin, 2001).

The significant positive relationship found between teacher subject matter knowledge and students' rating of teacher teaching effectiveness suggests that students' rating of a teacher's teaching effectiveness can be used as an indicator of the teacher's subject matter knowledge. This finding can be expected because what a teacher does in class in relation to his/her teaching delivery is a reflection of the subject matter knowledge of the teacher. As argued by Ogbonnaya (2007), a teacher's classroom practices/behaviours are influenced by the knowledge of the subject matter that the teacher possesses. Similarly, Tchoshanov (2011) argued that a teacher's ability to engage students in meaningful classroom discourse is an indicator of the teacher's conceptual understanding of the subject matter. The subject matter knowledge of a teacher is brought to bear in the lesson delivery as the teacher relates the content to the students in such a way that it could be meaningful to the students. The teacher's subject matter knowledge is made manifest in the teacher's presentation, organisation, assessment and communication with the students. These qualities are observable to the students and they (the students) use the qualities to rate the teacher's teaching effectiveness.

5.3.1.3 Sub-question three

The third sub-question to research question one was: Is there any relationship between mathematics teachers' subject matter knowledge and their peers' rating of the teachers' teaching effectiveness? The result of correlation analysis in Tables 4.7 shows that there was a positive relationship between teachers' subject matter knowledge and their peers' rating of the teachers' teaching effectiveness. This agrees with the result of Ferguson and Womack's (1993) study that teachers' subject matter knowledge is positively related to peers' rating of the teachers' teaching effectiveness. However, in the case of this study the relationship was found to be statistically insignificant.

The result shows that while peers may be knowledgeable about the subject matter content and also understand the teaching context, they may not, unlike students, be in position to rate the extent to which teaching is productive, informative, satisfying or worthwhile and hence may not accurately predict teachers' teaching effectiveness the same way that the students who are direct recipients of teachers' teaching could predict. Even though peers' evaluations are not as highly correlated as students', peers can also give an indication of the effectiveness of teachers' teaching from a close range perspective that no other person could give based on their knowledge of subject matter, the school context, the students, the curriculum, the learning outcomes and the subject's assessment guidelines.

5.3.1.4 Sub-question four

Sub-question four to research question one was: Is there any relationship between mathematics teachers' subject matter knowledge and their self rating of their teaching effectiveness? The result of correlation analysis in Tables 4.7 shows that there was a positive but statistically insignificant relationship between teacher subject matter knowledge and their self rating of their teaching effectiveness. This shows that teachers self rating of their teaching effectiveness is positively related to their subject matter knowledge but their self rating of their teaching effectiveness cannot give a predictable indication of their subject matter knowledge. It is likely that teachers gave a superfluous rating of their teaching effectiveness.

This is possible because teachers may feel that giving a superfluous rating of their teaching effectiveness will boost their self ego and make them more socially acceptable. Also, it may be that the teachers out of ignorance had exaggerated views of their teaching effectiveness. Such a feeling of expert teachers will make a teacher rate himself/herself more highly than his/her subject matter expertise. However, the finding shows that teachers' rating of themselves in terms of their effectiveness in facilitating the curriculum and helping students achieve the learning objectives can be a pointer to the teachers' capabilities and inabilities as well as their delivery of the curriculum content. This could be an important source of evidence of the teachers' teaching effectiveness and their knowledge about teaching as argued by Cranton (2001) and Berk (2005). This could be a useful index for formative evaluation of teachers teaching which could inform areas teachers need to be helped in order to improve on their teaching. The formative evaluation can be used to plan in service training and professional development programmes for teachers.

5.3.2 Research question two

The second research question explored if there was any difference between the subject matter knowledge of effective teachers and the subject matter knowledge of ineffective teachers. To answer this question quantitative and qualitative analyses techniques were employed. The results of the quantitative analysis (t-Test) in section 4.2.4.1 (Table 4.8) showed that there was a statistically significant difference between the subject matter knowledge of the effective and ineffective teachers in general based on their achievement across all the questions in the test used in the study. The results showed that, in general, the effective teachers significantly performed better than the ineffective teachers in the subject matter knowledge test. Further analysis of the result in section 4.2.4.2 (Table 4.9) showed that significant differences existed between the subject matter knowledge of the effective and ineffective teachers in some specific questions.

Qualitative analyses (cognitive task performance analyses) of the teachers performances on those specific test questions in which statistically significant differences existed between the subject matter knowledge of the effective and the ineffective teachers showed that the effective teachers displayed more conceptual understanding of the subject matter than the ineffective teachers. The effective teachers committed less structural and procedural errors in all the questions than did the ineffective teachers. In general, the effective teachers showed better conceptual understanding of trigonometric functions; its properties, the multiple representations, transformations and its translations between different representations than the ineffective teachers whose solutions to the problems were characterised by structural and procedural errors; a general lack of understanding of trigonometric functions, the properties of the functions, multiple representations and transformations of trigonometric functions.

The lack of conceptual understanding of the subject matter by the ineffective teachers was made manifest in various ways including consistent use of wrong approaches, formulas and procedures that do not apply to the problem solution. Also, there were cases where solution steps that did not follow logical sequence were used by the ineffective teachers. That gave rise to superfluous answers. In some cases, some of the questions were not attempted by the infective teachers. Not attempting a question may imply the solver's lack of understanding of the question.

Shepard et al., (2005) and Tchoshanov (2011) posited that teachers' effectiveness in teaching depends on teachers' subject matter knowledge because a teacher's conceptual understanding of the subject matter enables the teacher to select instructional and assessment tasks that would help the students achieve the learning goals. This could explain why this study and some other studies (e.g. Ogbonnaya, 2007; Tchoshanov, Lesser & Salazar 2008) over the years tend to convey that more effective teachers have more subject matter knowledge than less effective or ineffective teachers.

5.3.3 Research question three

The third research question explored if there was any difference between the subject matter knowledge of the students of effective teachers and the subject matter knowledge of the students of ineffective teachers. Qualitative analyses technique was employed to answer this question. Cognitive task performance analyses of the students' performances on the 15 questions where statistically significant differences existed between the achievements of the two categories of students showed that the students of the effective teachers displayed better conceptual understanding of the subject matter than the students of the ineffective teachers. The students of effective teachers committed less structural and procedural errors than the students of the ineffective teachers. The solutions of the students of ineffective teachers to the problems were characterised by structural and procedural errors just like their teachers solutions to the problems. This indicates that the students of the ineffective teachers had less conceptual understanding of the subject matter than the students of the effective teachers.

This and many other studies, for example Boyd, Grossman, Lankford, Loeb & Wyckoff (2006); Ogbonnaya and Osiki (2007) and Tchoshanov (2011) have shown that teachers' subject matter knowledge is related to students' achievement. Therefore, students taught by teachers that have more subject matter knowledge are more likely to achieve academic success than students taught by teachers that have less knowledge of the subject matter. It then follows that students of more effective teachers are more likely to also have better knowledge of the subject matter than students taught by the ineffective teachers. This explains why most studies on teacher quality and student learning seem to portend similar results.

Similar to this finding also, Tchoshanov, Lesser and Salazar (2008) used a sample of 22 mathematics teachers and their students and showed that students' performance parallels their teachers' performance on cognitive tests. They observed that the same pattern existed between teachers and students performances in various tasks in the test as was also the case in this study.

This study revealed, as was also observed by Tchoshanov, Lesser and Salazar (2008), that if a teacher's mastery of a concept is low then it can impact on student learning of the concept as well. This is not unexpected as the teacher will either avoid teaching the concept or muddle up the teaching in such a way the students will not grasp the learning outcome expected from the teaching. This possibly explains why the same pattern of errors was observed between the teachers and theirs students in this study.

5.4 Conclusion

The study revealed that there is a significant positive relationship between teacher subject matter knowledge and teacher teaching effectiveness measured by student achievement, student rating of teacher teaching effectiveness, teacher self rating of teacher teaching effectiveness and peers' rating of teacher teaching effectiveness. The relationship between teacher subject matter knowledge and teacher self rating of teacher teaching effectiveness was not statistically significant likewise the relationship between teacher subject matter knowledge and teacher peers' rating of teacher teaching effectiveness. Conversely, the relationship between teacher subject matter knowledge and students' achievement was found to be statistically significant, so also was the relationship between teacher subject matter knowledge and students' rating of teacher teaching effectiveness.

It was also shown that 76.8 % of the variation in the students' achievement in the study could be attributed to teachers' subject matter knowledge while 60.1% variation in students' rating of teachers' teaching effectiveness in the study is explained by teachers' subject matter knowledge.

The findings imply that students' achievement and students' rating of teachers' teaching effectiveness can give an indication of teachers' knowledge of the subject matter. In order words, students' achievement can be seen as a reflection of the teachers' knowledge of the subject matter likewise students' ratings of teachers' teaching effectiveness.

The study also revealed that effective teachers had more knowledge of the subject matter than ineffective teachers; the effective teachers showed better conceptual understanding of trigonometric functions; its properties, the multiple representations, transformations and its translations between different representations. Hence, they (effective teachers) performed better and committed fewer errors in problem solving than the ineffective teachers that generally showed lack of conceptual understanding of the subject matter of trigonometric functions, the properties, multiple representations and transformations of trigonometric functions.

Another revelation from the study is that students are like their teachers; the students of the effective teachers tended to have a better subject matter understanding than the students of the ineffective teachers. The students of effective teachers had more conceptual understanding of the subject matter content and committed fewer errors than the students' of ineffective teachers.

The findings imply that student will achieve better academic success if they are taught by teachers with more knowledge of the subject matter than by teachers with less subject matter knowledge and that the knowledge and errors that students display is a legacy of the knowledge and the errors of the teachers that teach them.

5.5 Projection for the next chapter

The next chapter proffers some recommendation based on the findings and discussion of the findings in this study.

Chapter Six

Summary, Conclusion and Recommendations

6.1 Introduction

This chapter summarises the major findings of the study and draws a conclusion upon which recommendations are made.

6.2 Summary of the study

The purpose of the study was to explore the relationship between teachers' subject matter knowledge and their teaching effectiveness. Data about teacher subject matter knowledge was collected using a cognitive test in trigonometric functions while data about teachers' teaching effectiveness was collected using student test, student rating of teachers' teaching effectiveness questionnaire, peers' rating of teachers' teaching effectiveness questionnaire, and teacher self rating of their teaching effectiveness questionnaire. The data was first analysed quantitatively using correlation analysis (Pearson's moment correlation), linear regression analysis and inferential analysis (t-Test). Subsequently, the data was analysed qualitatively using task performance analysis based on a cognitive task performance analyses framework developed for the purpose of this study.

It was found that a positive, statistically significant relationship existed between teachers' subject matter knowledge and the composite measures of their teaching effectiveness. The relationship between teachers' subject matter knowledge and each of the measures that constituted the composite measure namely students' achievement, students rating of the teachers teaching effectiveness, peers' rating of the teachers' teaching effectiveness and teachers' self rating of their teaching effectiveness were found to be positive. However, only the relationship between teachers' subject matter knowledge and the students' achievement, and the relationship between teachers' subject matter knowledge and the students' rating of the teachers' teaching effectiveness were found to be statistically significant.

The results also showed that there were differences (quantitatively and qualitatively) between the subject matter knowledge of effective and ineffective teachers in favour of the effective teachers. Furthermore, qualitative analyses of the students' performance revealed that there were differences between the subject matter knowledge of students taught by the effective teachers and the students taught by the ineffective teachers in favour of the students taught by the effective teachers; students of the effective teachers showed better conceptual understanding of the subject matter than the students of the ineffective teachers. The students of effective teachers committed less structural and procedural errors than the students of the ineffective teachers.

6.3 Conclusion

The findings of this study have shown that teacher subject matter knowledge is a major player in what the students learn. Students' ability to answer test questions is a reflection of their teachers' ability to answer similar questions. Also, the knowledge and errors that students display are legacies of the knowledge and errors of their teachers. This study has therefore demonstrated the reality of the saying that 'no nation can grow above the knowledge base of its teachers'. The growth or development of any nation depends on the youth, that is the students of that nation and the quality of the students depends on the knowledge the students acquire through the help of their teachers.

It was also found in this study that teachers' knowledge of the subject matter is related to students' rating of the teachers' teaching effectiveness. This implies that a teacher's observable instructional behaviours can be a pointer to what the teacher knows about the subject matter. Nevertheless, a mathematics teacher's instructional behaviour is influenced by the teacher's belief about the nature of mathematics, how mathematics should be taught and how it should be learned. Studies (for example, Entwistle & Entwistle, 2003; Ingvarson et al., 2004; Stols, Kriek & Ogbonnaya, 2008; Wenglinsky, 2002) have shown that students' achievement in mathematics is related to teachers' instructional practices. According to the framework of this study (section 2.2), a teachers' belief system in a particular content area is related to the subject matter knowledge of the teacher in that content area and also related to the teacher’s teaching effectiveness. From all these, there is little wonder then that students’ rating of a teachers’ teaching effectiveness (this was also found in this study to be related to students’ achievement) was found to be related to teachers’ subject matter knowledge.

The study further showed that the effective teachers displayed more conceptual understanding of the subject matter than the ineffective teachers. The lack of conceptual understanding of the subject matter by the ineffective teachers was made manifest in various ways including consistent use of wrong mathematical approaches to solve problems. This consistent use of wrong mathematical approaches to solve problem was also manifested more by the students of the ineffective teachers than the students of effective teachers.

The findings imply that students’ achievement and students’ rating of teachers’ teaching effectiveness can give an indication of teachers’ knowledge of the subject matter and that students will likely achieve better academic success if they are taught by teachers with more knowledge of the subject matter than by teachers with less subject matter knowledge.

6.4 Recommendations

Based on the findings of this study the following recommendations are put forward.

6.4.1 Increase subject matter content in teacher education

More emphases should be given to subject matter content in teacher education training without neglecting teaching methodology. The assumption behind this recommendation is that more knowledge of the subject matter will likely translate to better teaching as the study has demonstrated. The study has shown that teachers need sound knowledge of the subject matter in order to effectively facilitate students learning. It is therefore imperative that the training of teachers, especially in the light of the new curriculum, should give more content courses to the teachers in order for them to deliver effectively. Sound knowledge of the subject matter will enable the teachers to effectively help students construct sound mathematical concepts rather than relying on rote memorisation of mathematical algorithms that hinder conceptual understanding and ultimately lead to poor achievement in mathematics as was evident in the performance of the students of the ineffective teachers in this study.

6.4.2 More subject matter knowledge workshops for serving teachers

Teacher in-service workshops should focus more on subject matter knowledge. As the study has shown that teacher subject matter knowledge is related to students' achievement and student subject matter knowledge, it follows that if teachers have more knowledge of the subject matter the students will also have more knowledge and achieve better as well. It therefore means that if the achievement of students in mathematics is to be improved a starting point is to improve practising teachers' knowledge of the subject. One way of achieving this is to mount concerted subject content workshops for the practising teachers. This is particularly important taking into consideration the subject matter inadequacies of blacks' teacher education during the apartheid era discussed in section 1.2.2. To effectively carry out such workshops for teachers, there may be need to embark on wide range of training needs assessments of the teachers' subject matter knowledge of mathematics. This will make each workshop to be tailor-made and properly channelled to suit specific categories of teachers.

Some studies (e.g. Ball, Lubienski, & Mewborn, 2001) found teacher professional development programmes to be ineffective in helping teachers improve their teaching effectiveness because such professional development programmes are intellectually superficial, disconnected from deep issues of curriculum and learning, fragmented, and non-cumulative. Also, as argued by Little and McLaughlin (1993), professional development programmes that just update teachers' knowledge instead of providing an opportunity for sustained learning about issues to do with curriculum, students or teaching will not help teachers to be more effective in their curriculum delivery.

Importantly, some other studies have shown that teacher in-service trainings (teacher professional development workshops) that are focused on the subject matter that the teachers teach are positively related to students' achievement. Ogbonnaya (2007), for example, found that teachers' professional development is positively related to students' achievement when focused on how students learn particular subject matter, instructional practices that are related to the subject matter and how students understand it, and strengthening teachers' content knowledge of the subject. Similarly, Cohen and Hill (2001) observed that teachers whose professional development focus directly on the curriculum they teach adopt the practices they are taught in such professional development interventions and hence improve their teaching effectiveness. Garet, Porter, Desimone, Birman and Yoon (2001) found that when teachers' professional development is linked directly to their daily experiences and aligned with curriculum they would be more likely to change their instructional practices and gain greater subject matter knowledge and improved teaching effectiveness. Varella (2000) and Franke (2002) found that teacher professional development has positive effects on students' achievement if it is long-termed.

Therefore, the duration of the recommended teacher development workshops should be long enough to enable proper coverage of the content in such a way that the workshops will be beneficial to the teachers. It may not be beneficial that many topics are covered in a one or two days workshop and the teachers leave the workshop not having improved on their content knowledge. As observed by teachers (see Atagana et al., 2009), they (teachers) need regular content training on some topics (including trigonometric functions) in order to improve on their teaching.

6.4.3 Integration of Information and Communication Technology (ICT) into the teaching and learning of mathematics

Some mathematical concepts are difficult to explain with the conventional chalkboard especially when such concepts need three-dimensional visualisation. This makes it very difficult for students to grasp such concepts. Even when such concepts can be explained on the board some of them are so time-consuming that the teachers find it difficult to succinctly help the students to understand especially in large classes. The use of Information and Communication Technology (ICT) can help the teachers explain (and perhaps demonstrate) such concepts in a way that they become easy for the students to grasp.

Research is full of evidence of the benefits of the integration of ICT into teaching and learning especially in mathematics. In South Africa, for instance, Ogbonnaya (2010) showed that the integration of ICT into the teaching and learning of functions helped to improve students' learning of the concepts. One major misconception that students displayed in this study is that of the transformation of trigonometric functions graphs and the sketching of the graphs of the functions using their properties. These concepts can be easily understood if ICT is used to demonstrate the concepts to the students because through the visual presentation of mathematical ideas that ICT offers, students can see the effect of changing the trigonometric functions parameters like period and amplitude, the vertical and the horizontal shifts, etc. Hence, through visual presentation, difficult mathematical ideas can be made more readily understandable (Muijs & Reynolds, 2005).

The integration of ICT into mathematics teaching and learning offers opportunities for cooperative learning among the students (Ogbonnaya, 2010). Cooperation and collaboration among the students enable them to build their own understanding according to Osborne & Hennessy's (2003).

Another benefit that might accrue from the integration of ICT into mathematics education at the secondary school level is that it motivates students to learn and develop a likening for mathematics (Ogbonnaya, 2010). The positive attitude that ICT makes students to develop towards mathematics makes students spend more time learning mathematics which enhances their learning of mathematics and eventually translates to better achievement in mathematics.

6.4.4 Teacher certification and recruitment process to include subject matter tests

Another recommendation that emanates from the findings of this study is that teacher certification/licensing and recruitment in South Africa should be based on teacher success on a teacher licensing and recruitment subject matter knowledge test in addition to college or university certificate. This is important because teachers are prepared in difference institutions and sometimes in different countries based on different curricula and contexts, this makes it difficult to ascertain teachers' subject matter knowledge based on paper qualifications but a subject content test as a prerequisite to certification and recruitment will give a clearer picture of what a teachers knows and does not know thus helping the certification and recruitment agency make a better informed decision. Someone may argue that a degree in education with major or minor in the subject will be enough for certification but Atagana et al. (2009) showed that even teachers with degrees in mathematics and science still find it difficult to teach some of the topics in mathematics and science at Further Education and Training (FET) band. This being the situation, I argue that a better option to ascertain teachers' competence to teach mathematics is by subject matter knowledge test.

6.4.5 Teacher promotion to be based on teacher performance in SMK test and student rating of the teachers teaching effectiveness

This study has demonstrated that students' rating of a teacher's teaching can also give an indication of the subject matter competence of the teacher and also that teacher subject matter knowledge can give an indication of students' achievement. I therefore argue that teacher promotion (which I believe should be based on teachers' teaching effectiveness), should be based on students' ratings of the teachers' teaching effectiveness and also on the teachers' performance on subject matter knowledge test. This will make teachers to be more serious with their core duty which is teaching and by so doing they will be more effective in their teaching.

The promotion of teachers to senior positions in education based on gender and race instead of indices of teaching effectiveness may not encourage teachers to strive for effective teaching.

6.4.6 Teacher class allocation to be based on teacher performance in SMK tests and student rating

Based on my personal discussions with many teachers in the course of this study and also during teacher development activities it was gathered that in some schools teacher allocation to grade levels to teach is not completely based on the teachers' competence to handle the grade levels but based on the choice of the head of department and/or the school principal. For example, one condition to qualify to mark the Matric examination is that a teacher must be a grade 12 teacher and because the markers earn extra money for the marking some heads of departments and principals just allocate their friends to teach grade 12 irrespective of the fact that such teachers may not be competent to teach grade 12. Hence, I posit here that it might be more beneficial to the schools in particular and the nation at large to allocate teachers to grade levels to teach based on their subject matter competence to handle such grades. Through subject matter knowledge test and students rating of teachers' teaching effectiveness, it may be discovered that a teacher will perform better in teaching grade 10 than teaching grade 12, for an example.

6.5 Suggestion for future research

To enhance future research on the relationship between teacher subject matter knowledge and teacher teaching effectiveness, it would seem reasonable to:

Explore the effects of the other factors (students factor and school factor) hypothesised in the conceptual framework to be related to teachers' teaching effectiveness. Such studies may not only consider the relationships between the various factors but can also consider the influence of some of the factors like school factor or student factor on teachers' teaching effectiveness. This is important because factors like students' factor and workplace conditions (for example, the time allocated for mathematics teaching, the number of students in a class, etc.) can exert powerful influence on the motivation of teachers to acquire more subject matter knowledge that will help him/her to be more effective.

Further studies should also explore the relationship between teacher subject matter knowledge and teacher teaching effectiveness using other topics like financial mathematics, geometry, etc. The studies can also replicate this study using other grade levels to see if similar results will be obtained.

There is also the need for an empirical study on the relationship between mathematics teachers' beliefs and their subject matter knowledge and also between mathematics teachers' beliefs and their teaching effectiveness.

6.6 Limitations of the study

One limitation of the present study, in retrospect, is that students' achievement was measured by their success (marks obtained) on the test; this should not be uncritically accepted as there are other goals of education than passing examination. The analysis represents one instrument of evaluation among many; bearing in mind that not everything which is desirable in education is measurable (Jones; Tanner; & Treadaway, 2000). However, achievement on mathematics tests is one significant goal for mathematics teaching and learning and the result should be interpreted in that context.

Also, the study only used one topic (trigonometric functions) and one grade level (grade 11) to explore the relationship between teachers' subject matter knowledge and their teaching effectiveness. So, generalising the findings of this study to other topics in mathematics and other grade levels should be done with caution.

6.7 Epilogue

The present study investigated the relationship between teachers' subject matter knowledge and their teaching effectiveness in mathematics in South Africa has not been carried out before. Thus the findings add to the available body of knowledge in mathematics education especially in the context of South Africa. The study found positive relationships between teacher subject matter knowledge in mathematics and teacher teaching effectiveness measured by students' achievement, students' rating, teacher self rating and peer rating of teacher. Furthermore, the relationships between teacher subject matter knowledge and teacher teaching effectiveness measured by students' achievement and students rating were found to be statistically significant.

Based on these findings recommendations were made that stakeholders in mathematics education in South Africa (as elsewhere) may find useful.

6.8 Final thought

The key to the technological development and economic empowerment of Africa in general and South Africa in particular lies in the hands of the mathematics teachers. With highly knowledgeable and dedicated mathematics teachers, South Africa and Africa will be on the higher pedestal of economic and technological advancement.

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Appendixes

Appendix 1
TEACHER SUBJECT MATTER KNOWLEDGE OF TRIGONOMETRIC FUNCTIONS SCALE (TSMKTFS)

Total: 60 Marks
Time: 2 Hours

Instructions and information

1. This question paper consists of **FIVE** questions. Answer **ALL** questions
2. Number the questions correctly according to the numbering system used in this question paper.
3. An approved calculator (non-programmable and non-graphical) may be used
4. All calculations must be clearly shown.
5. Write neatly and legibly.
6. Use the grid sheets provided for sketching the graphs (one for each question)

This question paper consists of 6 Pages

1. If $f(x) = \tan 2x$.

1.1 What is the frequency of the function? (2)

1.2 What is the period of the function? (2)

1.3 What are the equations of the asymptotes of the function if it is drawn for the domain $[-180^o, 180^o]$? (4)

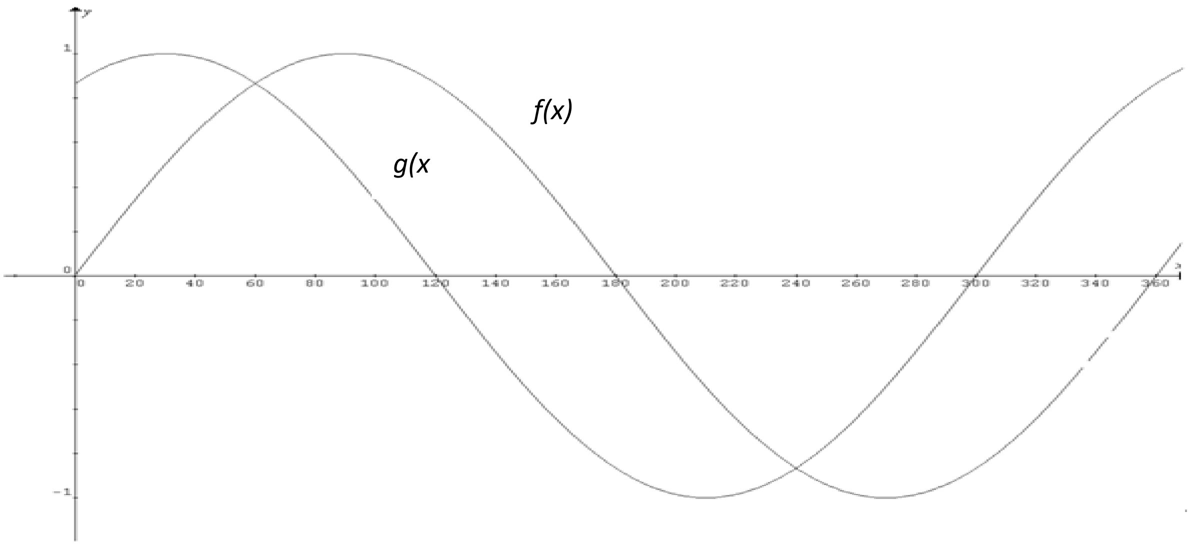
1.4 If $x = 90^o$, find the values of $f(x)$? (1)

1.5 Sketch the graph of $f(x)$ (4)

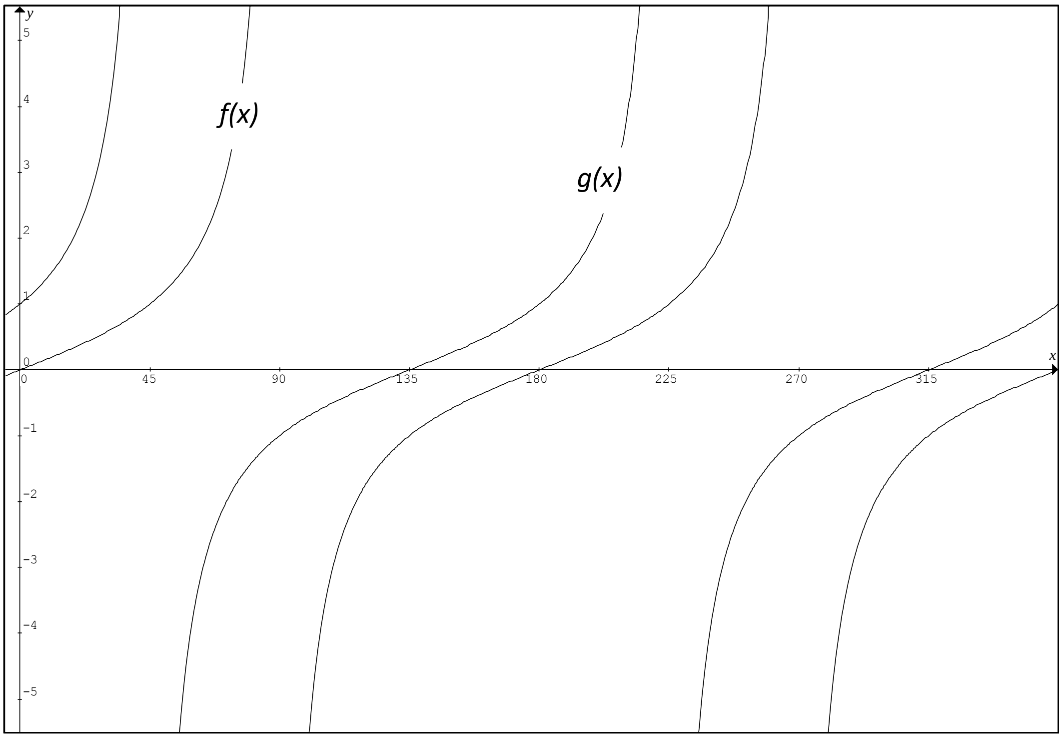
[13]

2. Describe the translation in each case of $f(x)$ to $g(x)$ drawn below for $0^o \leq x \leq 360^o$

2.1 (3)

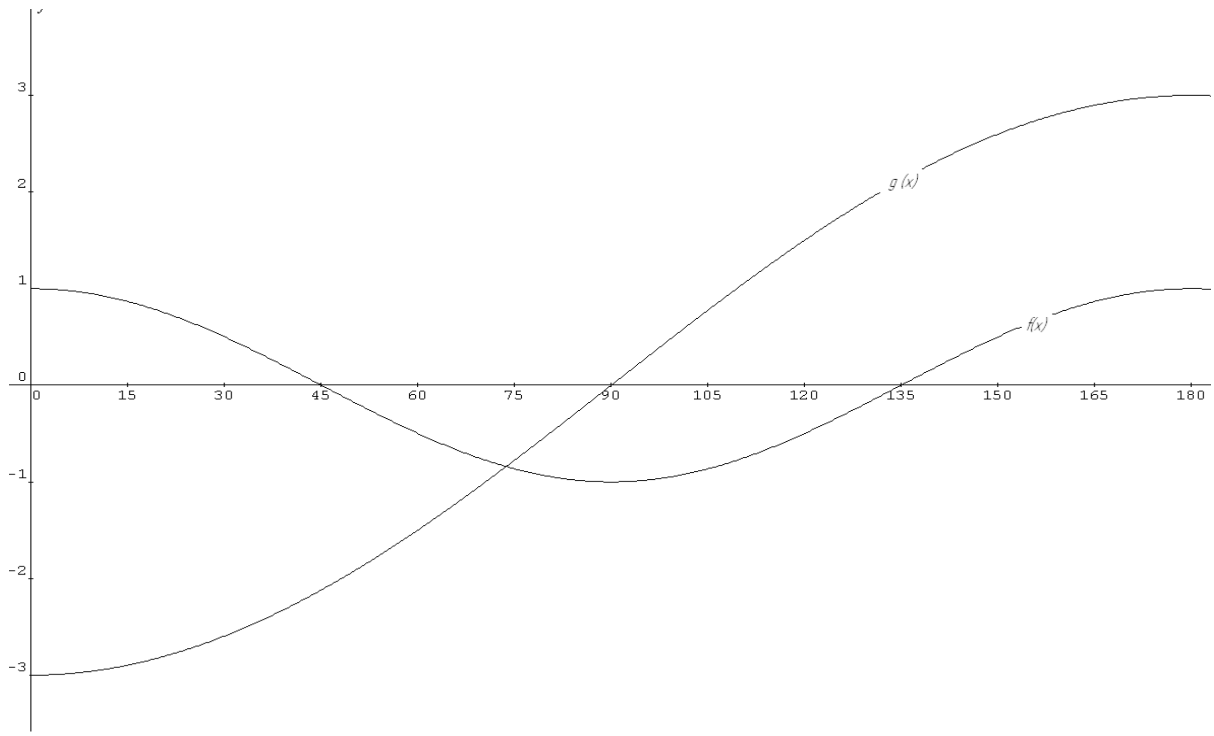


2.2 (3)



[6]

3. The sketch below shows the graphs of *f(x)* and *g(x)*



3.1 Write the equation of *f(x).* (3)

3.2 Write the equation of *g(x).* (3)

3.3 Use the graph to determine the value(s) of *x* for which:

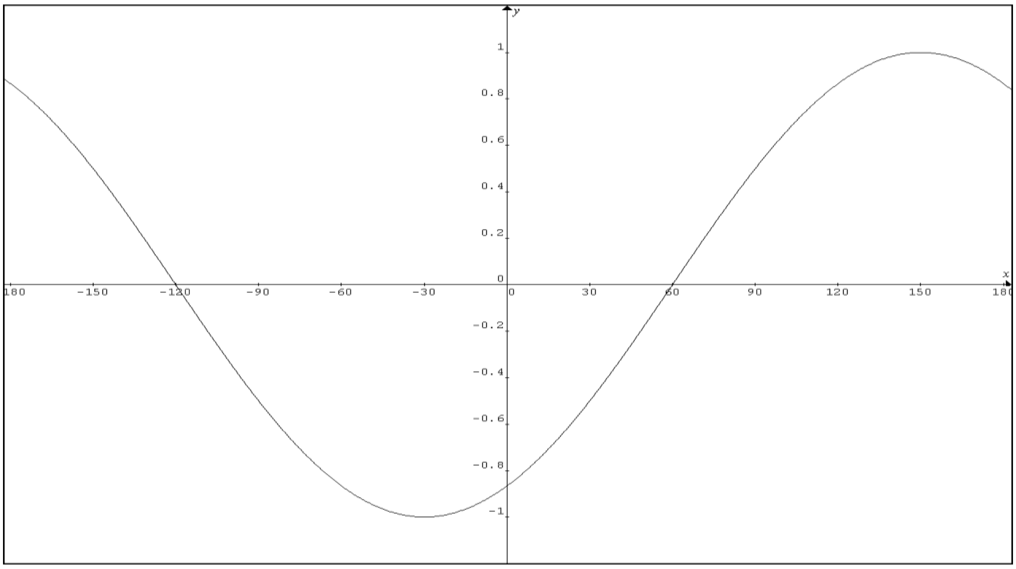
3.3.1 $f(x) - g(x) = 4$ (2)

3.3.2 $f(x) < 0$ (2)

3.3.3 $f(x).g(x) > 0$ (3)

[13]

4. Given the function $f(x) = \sin(x - 60^o)$ for $x \in [-180^\circ; 180^\circ]$



Determine:

4.1 The period of the function *g* if $g(x) = f(2x)$ (3)

4.2 The range of the function *h* if $h(x) = f(x) - 1$ (3)

4.3 The amplitude of the function *q*, if *q(x)* = $\frac{1}{2}f(x) + 2$ (3)

[9]

5. Sketch the graphs of the functions *f(x) = cos 2x* and *g(x) = tan x +1* for *x* ε *[-180°;180°]* on the same set of axes. Show all the intercepts with the axes and the co-ordinates of the turning points. Draw the asymptotes using dotted lines.

(6)

5.1 Write down the equation(s) of the asymptotes of *g(x)* (2)

5.2 Use the graph to determine the values of x if: (3)
cos 2x - 1 ≤ *tan x*; in the interval *[0°;180°]*

5.3 If the graph of *g(x) = tan x +1* is translated downwards by 3 units, what will the new equation be?

(3)

5.4 Use the graphs to solve the equation: (5)

$\cos 2k - 1 = \frac{\sin k}{\cos k}$ if k ε [-180°;180°]

[19]

Appendix 2

STUDENT TRIGONOMETRIC FUNCTIONS PERFORMANCE SCALE (STFPS)

Grade 11

Total: 60 Marks
Time: 2 Hours

Instructions and information

7. This question paper consists of **FIVE** questions. Answer **ALL** questions
8. Number the questions correctly according to the numbering system used in this question paper.
9. An approved calculator (non-programmable and non-graphical) may be used
10. All calculations must be clearly shown.
11. Write neatly and legibly.
12. Use the grid sheets provided for sketching the graphs (one for each question)

This question paper consists of 6 Pages

1. If $f(x) = \tan 2x$.

1.1 What is the frequency of the function? (2)

1.2 What is the period of the function? (2)

1.3 What are the equations of the asymptotes of the function if it is drawn for the domain $[-180^o, 180^o]$? (4)

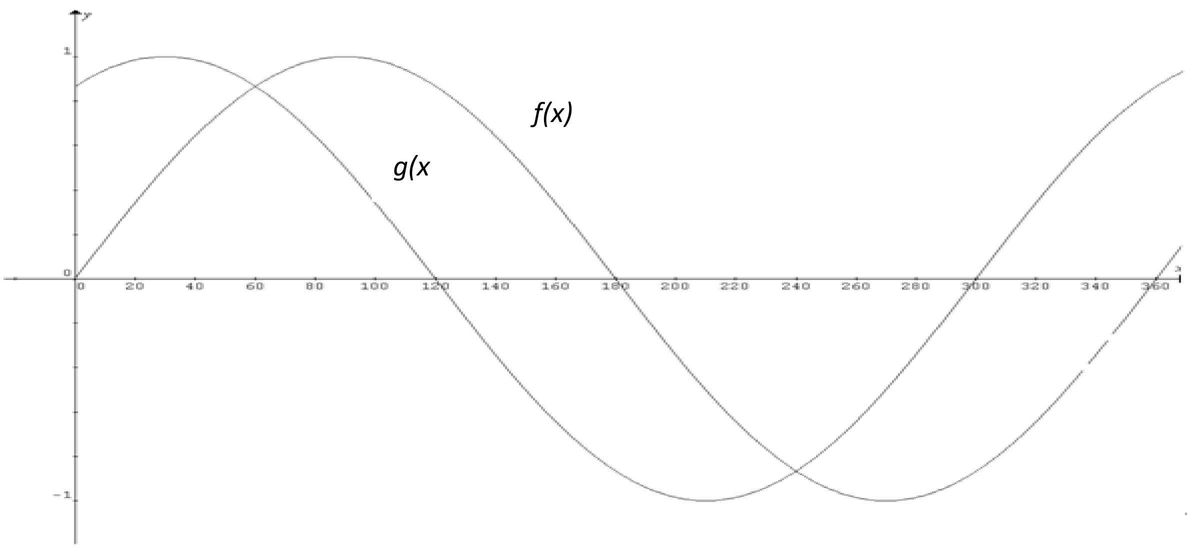
1.4 If $x = 90^o$, find the values of $f(x)$? (1)

1.5 Sketch the graph of $f(x)$ (4)

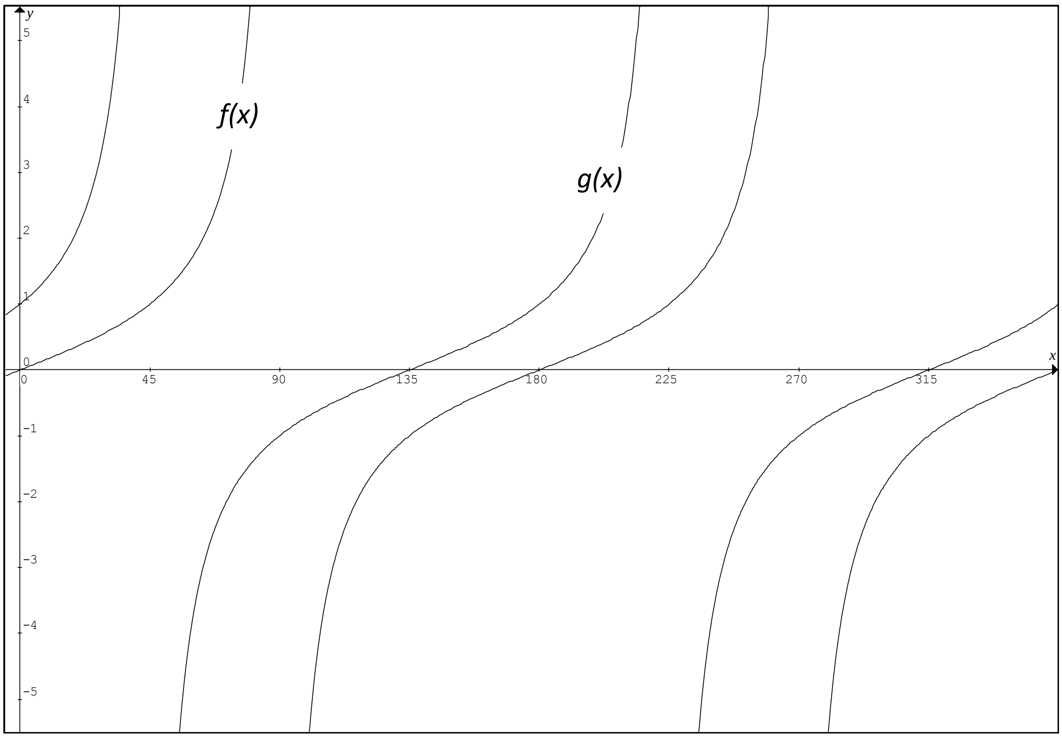
[13]

2. Describe the translation in each case of $f(x)$ to $g(x)$ drawn below for $0^o \leq x \leq 360^o$

2.1 (3)

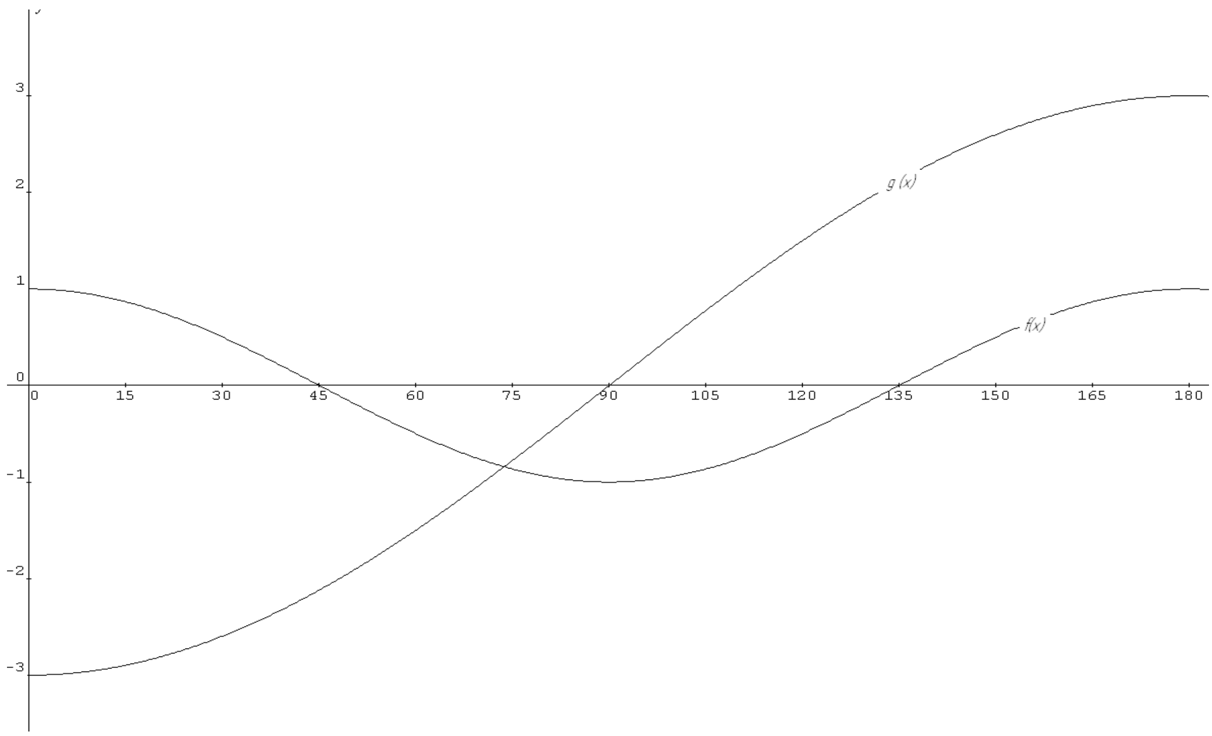


2.2 (3)



[6]

3. The sketch below shows the graphs of *f(x)* and *g(x)*



3.1 Write the equation of *f(x).* (3)

3.2 Write the equation of $g(x)$. (3)

3.3 Use the graph to determine the value(s) of x for which:

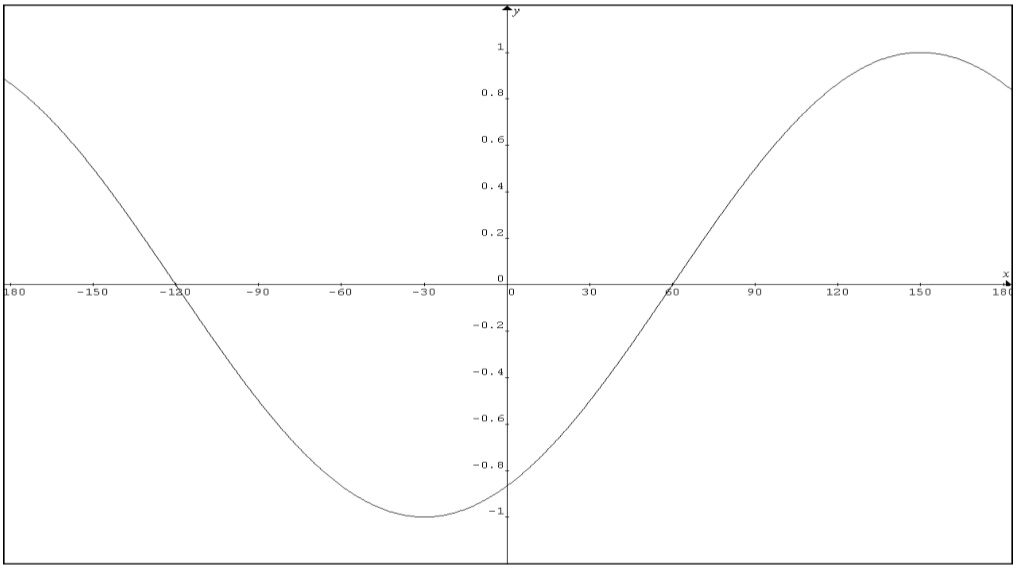
3.3.1 $f(x) - g(x) = 4$ (2)

3.3.2 $f(x) < 0$ (2)

3.3.3 $f(x).g(x) > 0$ (3)

[13]

4. Given the function $f(x) = \sin(x - 60^{o})$ for $x \in [-180^{o}; 180^{o}]$



Determine:

4.1 The period of the function g if $g(x) = f(2x)$ (3)

4.2 The range of the function h if $h(x) = f(x) - 1$ (3)

4.3 The amplitude of the function *q*, if $q(x) = \frac{1}{2}f(x) + 2$ (3)

[9]

5. Sketch the graphs of the functions $f(x) = \cos 2x$ and $g(x) = \tan x + 1$ for $x \in [-180^o;180^o]$ on the same set of axes. Show all the intercepts with the axes and the co-ordinates of the turning points. Draw the asymptotes using dotted lines.

(6)

5.1 Write down the equation(s) of the asymptotes of $g(x)$ (2)

5.2 Use the graph to determine the values of x if: (3)

$\cos 2x - 1 \leq \tan x$; in the interval $[0^o;180^o]$

5.3 If the graph of $g(x) = \tan x + 1$ is translated downwards by 3 units, what will the new equation be?

(3)

5.4 Use the graphs to solve the equation: (5)

$\cos 2k - 1 = \frac{\sin k}{\cos k}$ if $k \in [-180^o;180^o]$ [19]

Appendix 3
Student opportunity to learn form

Student Opportunity to Learn Trigonometric Functions Form

The purpose of this form is to find out what the Grade 11 students were taught by their teacher on trigonometric functions.

The knowledge/skills are listed in column 1. Column 2 is used to indicate if the knowledge/skills were taught or reviewed by the teacher

	Column 1	**Column 2**	
1	Sketch graphs of $y = \sin(kx)$, $y = \cos(kx)$, $y = \tan(kx)$	Yes	No
2	Determine the period of the functions $y = \sin(kx)$, $y = \cos(kx)$, $y = \tan(kx)$	Yes	No
3	Determine the amplitudes of the functions $y = \sin(kx)$, $y = \cos(kx)$, $y = \tan(kx)$	Yes	No
4	Use Characteristics (e.g. domain, intercepts, asymptotes, period, range) to sketch graph	Yes	No
5	Conversion from graphical to symbolic representations of trigonometric functions	Yes	No
6	Sketch graphs of $y = \sin(x + p)$, $y = \cos(x + p)$, $y = \tan(x + p)$	Yes	No
7	Determine the period of the functions $y = \sin(x + p)$, $y = \cos(x + p)$, $y = \tan(x + p)$	Yes	No
8	Determine the amplitudes of the functions $y = \sin(x + p)$, $y = \cos(x + p)$, $y = \tan(x + p)$	Yes	No
9	Describe the translations of the graphs of $y = a\sin(kx)$, $y = a\cos(kx)$, $y = a\tan(kx)$	Yes	No
10	Describe the translations of the graphs of $y = \sin(x + p)$, $y = \cos(x + p)$, $y = \tan(x + p)$	Yes	No

Appendix 4

Student evaluation instrument

Control No.	

Mathematics Teacher Teaching Effectiveness Evaluation
Student Rating Questionnaire

This questionnaire is designed to obtain information about your mathematics teacher's trigonometric functions teaching this academic year. Your response will be anonymous and the information gathered will help us improve the teaching of Mathematics and also help the students to perform better in Mathematics.

Please indicate the extent of your agreement/disagreement with the following statements about your mathematics teacher, using the following scale.

Strongly agree = 6 Agree = 5 slightly Agree = 4 slightly Disagree = 3 Disagree = 2 strongly Disagree = 1

For each question mark ✓ or **X** in the appropriate box that corresponds to the extent of your agreement/disagreement.

	My mathematics teacher ...	Strongly agree	Agree	Slightly Agree	Slightly Disagree	Disagree	Strongly Disagree
1	introduced trigonometric functions in a way that captured learners' attention	6	5	4	3	2	1
2	gave definitions of terms/vocabularies that appear unfamiliar to learners	6	5	4	3	2	1
3	gave satisfactory answers to learners questions	6	5	4	3	2	1
4	made lessons relevant and meaningful to learners	6	5	4	3	2	1
5	simplified the subject matter to learners	6	5	4	3	2	1
6	showed sound knowledge of the subject matter	6	5	4	3	2	1
7	showed learners interesting and useful ways of solving problems.	6	5	4	3	2	1
8	started lessons by connecting to previous lessons	6	5	4	3	2	1
9	ended lessons by connecting to future lessons	6	5	4	3	2	1
10	presented sections of the topic in a logical sequence	6	5	4	3	2	1
11	related content to real life examples	6	5	4	3	2	1
12	was always well-prepared for class	6	5	4	3	2	1
13	summarized the main points by the end of lesson	6	5	4	3	2	1
14	was always in class with all necessary materials for teaching topic	6	5	4	3	2	1
15	related ideas to learners' prior knowledge	6	5	4	3	2	1
16	supported lessons with useful class work	6	5	4	3	2	1

	My mathematics teacher ...	Strongly agree	Agree	Slightly Agree	Slightly Disagree	Disagree	Strongly Disagree
17	made use of different teaching techniques	6	5	4	3	2	1
18	motivated learners to pay attention to lesson	6	5	4	3	2	1
19	helped learners where they didn't understand	6	5	4	3	2	1
20	encouraged learners to learn	6	5	4	3	2	1
21	gave individual support to learners when needed	6	5	4	3	2	1
22	adjusted the lessons when learners experienced difficulties in learning.	6	5	4	3	2	1
23	used assessment results to provide extra help to learners	6	5	4	3	2	1
24	explained something in different ways to help learners understand.	6	5	4	3	2	1
25	took extra steps to help all learners learn and achieve success in maths.	6	5	4	3	2	1
26	supported lessons with useful classroom discussions	6	5	4	3	2	1
27	communicated the topic clearly	6	5	4	3	2	1

Thank you very much. Your participation is greatly appreciated.

Appendix 5

Self evaluation instrument

Control No.	

Mathematics Teacher Teaching Effectiveness Evaluation Self Rating Questionnaire

Dear Grade 11 Mathematics educator,

I am Mr. UI Ogbonnaya, I am a student in the Department of Mathematics, Science and Technology education, University of South Africa. I am interested in determining the link between teachers' subject matter knowledge and their teaching effectiveness.
The enclosed questionnaire is designed to obtain information about your trigonometric functions teaching in Grade11 this academic year. Your response will be anonymous and the information gathered will help us improve the teaching of Mathematics and also help our students to perform better in Mathematics.
I would appreciate your completion of the questionnaire. I realise that your schedule is very busy. However, I hope that the 10-15 minutes it will take you will help us understand how to improve the teaching of Mathematics in South Africa.

Thank you in advance for your participation. If you have any question about the study or any of items in the questionnaire, call me on 012 429 8083 or 0737208026.

Yours truly,

UI Ogbonnaya

Directions

1. This questionnaire asks you to rate your trigonometric functions teaching in Grade11 this academic year based on the extent of your agreement or disagreement with the statements.

2. For each statement mark ✓ or **X** in the appropriate box that corresponds to the extent of your agreement/disagreement.

3. Please give an answer/rating to every question/statement.

Section A: teacher's rating of trigonometric functions teaching

	I …	Strongly agree	Agree	Slightly Agree	Slightly Disagree	Disagree	Strongly Disagree
1	*demonstrated sound knowledge of the curriculum*	6	5	4	3	2	1
2	*demonstrated sound knowledge of students prior conceptions*	6	5	4	3	2	1
3	*clearly explained procedures for solving problems*	6	5	4	3	2	1
4	*explained concepts in different ways to help my learners understand.*	6	5	4	3	2	1
5	*reviewed lessons by connecting to previous classes*	6	5	4	3	2	1
6	*previewed lessons by connecting to future classes*	6	5	4	3	2	1
7	*used a variety of instructional strategies appropriate to the topic*	6	5	4	3	2	1
8	*planned in advance to ensure lessons run smoothly*	6	5	4	3	2	1
9	*ensured that lessons on the topic were well organised*	6	5	4	3	2	1
10	*always related ideas to students' prior knowledge*	6	5	4	3	2	1
11	*created opportunity for students participation in classroom problem solving*	6	5	4	3	2	1
12	*carried the whole class along in my lesson delivery.*	6	5	4	3	2	1
13	*asked questions to monitor students progress*	6	5	4	3	2	1
14	*asked probing questions when student answer was incomplete*	6	5	4	3	2	1
15	*encouraged my students to ask questions*	6	5	4	3	2	1
16	*supported lessons with useful class work*	6	5	4	3	2	1
17	*used different resource materials*	6	5	4	3	2	1
18	*always attended class*	6	5	4	3	2	1
19	*was always punctual to class*	6	5	4	3	2	1
20	*provided inspirational learning environment*	6	5	4	3	2	1
21	*brought lessons home by using examples in the learners' context*	6	5	4	3	2	1
22	*built my teaching on students prior knowledge*	6	5	4	3	2	1
23	*gave individual support to learners when needed*	6	5	4	3	2	1
24	*adjusted lessons when my learners experienced difficulties in learning.*	6	5	4	3	2	1
25	*challenged my students to think through and solve problems, either by themselves or together as a group.*	6	5	4	3	2	1
26	*took extra step to help all my learners learn and achieve success in maths.*	6	5	4	3	2	1
27	*monitored and checked students home work always*	6	5	4	3	2	1
28	*well articulated teacher tasks and learner tasks*	6	5	4	3	2	1
29	*taught in a manner that took cognisance of differing learning abilities of learners*	6	5	4	3	2	1
30	*gave feedback to learners about their homework and assignment*	6	5	4	3	2	1
31	*appropriately integrated assessment in teaching*	6	5	4	3	2	1
32	*used assessment results to provide extra help to my students*	6	5	4	3	2	1

Section B. Demographic Information

1. Gender: [ ] Male [ ] Female

2. How many years have you taught Mathematics at FET band?

[ ] 0 – 5 years, [ ] 6 – 10 years, [ ] 11 –15 years [ ] 16 –20 years, [ ] Over 20 years

3. Qualification(s): ....

4. Subject specialisation: ....

Thank you very much. Your participation is greatly appreciated.

Appendix 6

Peer evaluation instrument

Mathematics teacher teaching effectiveness Evaluation
Peer rating questionnaire

Dear Educator,

I am Mr. UI Ogbonnaya, I am a student in the Department of Mathematics, Science and Technology education, University of South Africa. I am interested in determining the link between teachers' subject matter knowledge and their teaching effectiveness.

The enclosed questionnaire is designed to obtain information about your colleague's trigonometric functions teaching in Grade11 this academic year. Your response will be anonymous and the information gathered will help us improve the teaching of Mathematics and also help our students to perform better in Mathematics.

I would appreciate your completion of the questionnaire. I realise that your schedule is very busy. However, I hope that the 10-15 minutes it will take you will help us understand how to improve the teaching of Mathematics in South Africa.

Thank you in advance for your participation. If you have any question about the study or any of items in the questionnaire, call me on 012 429 8083 or 0737208026.

Yours truly,

UI Ogbonnaya

Directions

4. This questionnaire asks you to rate your colleague's trigonometric functions teaching in Grade11 this academic year based on the extent of your agreement or disagreement with the statements.

5. For each statement mark ✓ or **X** in the appropriate box that corresponds to the extent of your agreement/disagreement.

6. Please give an answer/rating to every question/statement.

Section A: Rating of the educator’s trigonometric functions teaching

	The educator ...	Strongly agree	Agree	Slightly Agree	Slightly Disagree	Disagree	Strongly Disagree
1	demonstrated sound knowledge of the curriculum	6	5	4	3	2	1
2	demonstrated sound knowledge of students prior conceptions	6	5	4	3	2	1
3	clearly explained procedures for solving problems	6	5	4	3	2	1
4	explained concepts in different ways to help his/her learners understand.	6	5	4	3	2	1
5	reviewed lessons by connecting to previous classes	6	5	4	3	2	1
6	previewed lessons by connecting to future classes	6	5	4	3	2	1
7	used a variety of instructional strategies appropriate to the topic	6	5	4	3	2	1
8	planned in advance to ensure lessons run smoothly	6	5	4	3	2	1
9	ensured that lessons on the topic were well organised	6	5	4	3	2	1
10	always related ideas to students’ prior knowledge	6	5	4	3	2	1
11	created opportunity for students participation in classroom problem solving	6	5	4	3	2	1
12	carried the whole class along in his/her lesson delivery.	6	5	4	3	2	1
13	asked questions to monitor students progress	6	5	4	3	2	1
14	asked probing questions when student answer was incomplete	6	5	4	3	2	1
15	encouraged his/her students to ask questions	6	5	4	3	2	1
16	supported lessons with useful class work	6	5	4	3	2	1
17	used different resource materials	6	5	4	3	2	1
18	always attended class	6	5	4	3	2	1
19	was always punctual to class	6	5	4	3	2	1
20	provided inspirational learning environment	6	5	4	3	2	1
21	brought lessons home by using examples in the learners’ context	6	5	4	3	2	1
22	built his/her teaching on students prior knowledge	6	5	4	3	2	1
23	gave individual support to learners when needed	6	5	4	3	2	1
24	adjusted lessons when his/her learners experienced difficulties in learning.	6	5	4	3	2	1
25	challenged his/her students to think through and solve problems, either by themselves or together as a group.	6	5	4	3	2	1
26	took extra step to help all his/her learners learn and achieve success in maths.	6	5	4	3	2	1
27	monitored and checked students home work always	6	5	4	3	2	1
28	well articulated teacher tasks and learner tasks	6	5	4	3	2	1
29	taught in a manner that took cognisance of differing learning abilities of learners	6	5	4	3	2	1
30	gave feedback to learners about their homework and assignment	6	5	4	3	2	1
31	appropriately integrated assessment in teaching	6	5	4	3	2	1
32	used assessment results to provide extra help to my students	6	5	4	3	2	1

Section B. Demographic Information

Please kindly supply the following information about yourself.

1. Gender: [ ] Male [ ] Female

2. How many years have you taught Mathematics?

[ ] 0 – 5 years, [ ] 6 – 10 years, [ ] 11 –15 years [ ] 16 –20 years, [ ] Over 20 years

3. Qualification(s): ....

4. Subject specialisation: ....

Thank you very much. Your participation is greatly appreciated.

Appendix 7

Result of reliability – student rating

```
 ****** Method 1 (space saver) will be used for this analysis ******
_


R E L I A B I L I T Y   A N A L Y S I S   -   S C A L E   (A L P H A)


Reliability Coefficients

N of Cases =    109.0                   N of Items = 30

Alpha =    .9473
```

Appendix 8
Result of reliability – self rating

Reliability

```
 ****** Method 1 (space saver) will be used for this analysis ******
_


  R E L I A B I L I T Y   A N A L Y S I S   -   S C A L E   (A L P H A)


Reliability Coefficients

N of Cases =     13.0                    N of Items = 32

Alpha =    .9158
```

Appendix 9

Result of reliability – Peer rating

Reliability

```
 ****** Method 1 (space saver) will be used for this analysis ******
_


  R E L I A B I L I T Y   A N A L Y S I S   -   S C A L E   (A L P H A)


Reliability Coefficients

N of Cases =     11.0                    N of Items = 32

Alpha =    .9535
```

Appendix 10

Result of reliability – SMK competence rating scale

Reliability

```
 ****** Method 1 (space saver) will be used for this analysis ******
_


  R E L I A B I L I T Y   A N A L Y S I S   -   S C A L E   (A L P H A)


Reliability Coefficients

N of Cases =     11.0                    N of Items = 10

Alpha =    .8947
```

Appendix 11

Correlation of scores the two equivalent group in Pilot test

Correlations

		GRP1	GRP2
GRP1	Pearson Correlation	1	.950**
	Sig. (2-tailed)	.	.000
	N	20	20
GRP2	Pearson Correlation	.950**	1
	Sig. (2-tailed)	.000	.
	N	20	20

**. Correlation is significant at the 0.01 level

Appendix 12
KMO and Barlett's Test

KMO and Bartlett's Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.892
Bartlett's Test of Sphericity	Approx. Chi-Square	1967.608
	df	351
	Sig.	.000

Appendix 13
Table of communalities

Communalities

	Initial	Extraction
Q1	1.000	.702
Q2	1.000	.439
Q3	1.000	.675
Q4	1.000	.741
Q5	1.000	.700
Q6	1.000	.664
Q7	1.000	.729
Q8	1.000	.670
Q9	1.000	.642
Q10	1.000	.622
Q11	1.000	.709
Q12	1.000	.618
Q13	1.000	.550
Q14	1.000	.682
Q15	1.000	.614
Q16	1.000	.595
Q17	1.000	.662
Q18	1.000	.648
Q20	1.000	.645
Q21	1.000	.745
Q23	1.000	.720
Q24	1.000	.645
Q25	1.000	.643
Q26	1.000	.713
Q27	1.000	.757
Q28	1.000	.724
Q30	1.000	.792

Extraction Method: Principal Component Analysis.

Appendix 14
Consent form

3rd August 2010

Institute for Science and Technology Education

University of South Africa (Unisa)

Teacher participation in research study consent form

Title of study: Link between mathematics teachers' subject matter knowledge and their teaching effectiveness

Dear Respondent,

You are invited to participate in an academic research study conducted by Mr. Ogbonnaya Ugorji of the Institute for Science and Technology Education at Unisa.

The purpose of the study is to explore the link between mathematics teachers' subject matter knowledge and their teaching effectiveness. This is with the view to proffering useful solutions to the teaching of mathematics in order to improve students' achievement in the subject.

Please note that:

- Your participation in the study is voluntary. You may choose not to participate and may withdraw your participation at any time without any negative consequences.
- Your information will be treated as confidence and your identity will by no means be revealed in any publication.
- The result of this study will be used for academic purposes only and may be published in the academic journal. I will provide you with a summary of the results of my findings on request.
- Should you have any queries, please do not hesitate to contact me on 012 429 8083 or by email at ogbonui@unisa.ac.za

Please sign this form to indicate that:

- You have read and understood the information above.
- You give your consent to participate in the study on voluntary basis.

____________________________ ____________________________

Respondent's signature Date

Appendix 15

Teacher Subject Matter Knowledge of Trigonometric Functions Scale (TSMKFS)
Instrument Evaluation Form

The attached instrument –Teacher Subject Matter Knowledge of Trigonometric Functions Scale (TSMKFS) was developed to measure Grade 11 mathematics teachers' subject matter knowledge of Trigonometric functions in line with National Curriculum Statement in Mathematics.
I hereby request that you evaluate the questions and indicate the level of relevance of each question to test grade 11 teachers knowledge of trigonometric functions in line with the National Curriculum Statement using the scale:

1 = not relevant; 2 = fairly relevant; 3 = relevant; 4 = highly relevant.

QUESTION	RELEVANCE	Comment on the question (if any)
1.1		
1.2		
1.3		
1.4		
1.5		
2.1		
2.2		
3.1		
3.2		
3.3.1		
3.3.2		
3.3.3		
4.1		
4.2		
4.3		
5		
5.1		
5.2		
5.3		
5.4		

Please, comment the extent to which the instrument covers the entire content of the Grade 11 Trigonometric functions curriculum.

………

……………………

Personal information of Evaluator

Qualification: ……………………………… Status: ....

Signature:... Date: ....

Appendix 16

Student Trigonometric Functions Performance Scale (STFPS)
Instrument Evaluation Form

The attached instrument – student trigonometric functions performance scale (STFPS) was developed to measure Grade 11 mathematics students' subject matter knowledge of Trigonometric functions in line with National Curriculum Statement in Mathematics.
I hereby request that you evaluate the questions and indicate the level of relevance of each question to test grade 11 students knowledge of trigonometric functions in line with the National Curriculum Statement using the scale:

1 = not relevant; 2 = fairly relevant; 3 = relevant; 4 = highly relevant.

QUESTION	RELEVANCE	Comment on the question (if any)
1.1		
1.2		
1.3		
1.4		
1.5		
2.1		
2.2		
3.1		
3.2		
3.3.1		
3.3.2		
3.3.3		
4.1		
4.2		
4.3		
5		
5.1		
5.2		
5.3		
5.4		

Please, comment the extent to which the instrument covers the entire content of the Grade 11 Trigonometric functions curriculum.

………

………..

Personal information of Evaluator

Qualification: ………………………….. Status: ....

Signature:.. Date: ....

Appendix 17

Content complexity of TSMKTFS and STFPS

Question		Content Complexity level	Annotation
1	If *f(x) = tan 2x*.		
1.1	What is the frequency of the function?	2	Involves performance of well-known procedures
1.2	What is the period of the function?	2	
1.3	What are the equations of the asymptotes of the function if it is drawn for the domain $[-180^o, 180^o]$?	2	
1.4	If $x = 90^o$, find the values of *f(x)*?	2	
1.5	Sketch the graph of *f(x)*	3	* Problems are mainly unfamiliar and learners are expected to solve by integrating different learning outcomes. Problems do not have a direct route to the solution
2.1 & 2.2	Describe the translation in each case of *f(x)* to *g(x)* for $0^o \leq x \leq 360^o$	2	
	The sketch below shows the graphs of *f(x)* and *g(x)*	2	
3.1	Write the equation of *f(x).*	2	
3.2	Write the equation of *g(x).*	2	
3.3	Use the graph to determine the value(s) of *x* for which:	2	
3.3.1	$f(x) - g(x) = 4$		Problem involves mathematical reasoning processes
3.3.2	$f(x) < 0$	2	Problem involves mathematical reasoning processes
3.3.3	$f(x).g(x) > 0$	3	
4 4.1	Given the function $f(x) = \sin(x - 60^o)$ for $x \in [-180^o; 180^o]$ Determine: The period of the function *g* if $g(x) = f(2x)$	2	
4.2	The range of the function *h* if $h(x) = f(x) - 1$	2	
4.3	The amplitude of the function *q*, if $q(x) = \frac{1}{2} f(x) + 2$	2	
5	Sketch the graphs of the functions $f(x) = \cos 2x$ and $g(x) = \tan x + 1$ for $x \varepsilon [-180^o; 180^o]$ on	3	* Problems are mainly unfamiliar and learners are

	the same set of axes. Show all the intercepts with the axes and the co-ordinate of the turning points. Draw the asymptotes using dotted lines.		expected to solve by integrating different Learning outcomes. Problems do not have a direct route to the solution
5.1	Write down the equation(s) of the asymptotes	2	
5.2	Use the graph to determine the values of x if: *cos 2x* - 1 ≤ *tan x*; in the interval *[0°;180°]*	2	
5.3	If the graph of *g(x) = tan x +1* is translated downwards by 3 units, what will the new equation be?	2	
5.4	Use the graphs to solve the equation: $\cos 2k - 1 = \frac{\sin k}{\cos k}$ if *k* ε [-180°;180°]	3	This is not a familiar problem. It requires integrating different learning outcomes. Problem mathematical reasoning processes

* see the assessment guidelines for mathematics (DoE, 2007) for an example on Grade 11 sketch of trigonometric functions graph

Appendix 18
Student Opportunity to Learn Trigonometric Functions Form

Instrument Evaluation Form

The attached instrument – Student Opportunity to Learn Trigonometric Functions form was developed to appraise the opportunities teachers provided for students to learn trigonometric functions in grade 11.

I hereby request that you evaluate the knowledge/concept listed in column one in line with its alignment with the assessment standards of Grade 11 trigonometric functions in the National curriculum statement (NCS) and indicate the level of conformity of knowledge/concept with the NCS using the scale:

1 = does not conform to assessment standards
2 = fairly conform to assessment standards
3 = conform to assessment standards
4 = highly conform to assessment standards

	Knowledge/concept	**Level of conformity**
1	Sketch graphs of $y = \sin(kx)$, $y = \cos(kx)$, $y = \tan(kx)$	
2	Determine the period of the functions $y = \sin(kx)$, $y = \cos(kx)$, $y = \tan(kx)$	
3	Determine the amplitudes of the functions $y = \sin(kx)$, $y = \cos(kx)$, $y = \tan(kx)$	
4	Use Characteristics (e.g. domain, intercepts, asymptotes, period, range) to sketch graph	
5	Conversion from graphical to symbolic representations of trigonometric functions	
6	Sketch graphs of $y = \sin(x + p)$, $y = \cos(x + p)$, $y = \tan(x + p)$	
7	Determine the period of the functions $y = \sin(x + p)$, $y = \cos(x + p)$, $y = \tan(x + p)$	
8	Determine the amplitudes of the functions $y = \sin(x + p)$, $y = \cos(x + p)$, $y = \tan(x + p)$	
9	Describe the translations of the graphs of $y = a\sin(kx)$, $y = a\cos(kx)$, $y = a\tan(kx)$	
10	Describe the translations of the graphs of $y = \sin(x + p)$, $y = \cos(x + p)$, $y = \tan(x + p)$	

Please, write any comment you have about the instrument.

………
………………………

Personal information of Evaluator

Qualification: ………………………….. Status: ....

Signature:.. Date: ....

Appendix 19

Correlation of inter-rater reliability of SOTL form

Correlations

Correlations

		Researcher 1	Researcher 2
Researcher 1	Pearson Correlation	1	1.000**
	Sig. (2-tailed)	.	.
	N	10	10
Researcher 2	Pearson Correlation	1.000**	1
	Sig. (2-tailed)	.	.
	N	10	10

**. Correlation is significant at the 0.01 level (2-tailed).