A STUDY OF COINTEGRATION MODELS WITH APPLICATIONS

by

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Abstract

This study estimates cointegration models by applying the Engle-Granger (1989) two-step estimation procedure, the Phillip-Ouliaris (1990) residual-based test and Johansen’s multivariate technique. The cointegration techniques are tested on the Raotbl3 data set, the World Economic Indicators data set and the UKpppuip data set using statistical software R. In the Raotbl3 data set, we test for cointegration between the consumption expenditure, and income and wealth variables. In the world economic indicators data set, we test for cointegration in three of Australia’s key economic indicators, whereas in the UKpppuip data set we test for the existence of long-run economic relationships in the United Kingdom’s purchasing power parity. The study finds the three techniques not to be consistent, that is, they do not lead to the same results. However, it recommends the use of Johansen’s method because it is able to detect more than one cointegrating relationship if present.

Keywords: cointegration; stationarity; nonstationarity; Augmented Dickey-Fuller test; error-correction model; unit root; Engle-Granger method; Phillip-Ouliaris methods; variance ration test; Johansen’s method.
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DECLARATION BY STUDENT

I declare that the submitted work has been completed by me the undersigned and that I have not used any other than permitted reference sources or materials nor engaged in any plagiarism. All references and other sources used by me have been appropriately acknowledged in the work. I further declare that the work has not been submitted for the purpose of academic examination, either in its original or similar form, anywhere else. Declared on the (date):

Signed:

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Chapter 1

Introduction

Empirical research in economics is based on time series. Therefore, it is standard to view time series as the realisation of a stochastic process. Model builders can use statistical inference in constructing and testing the equations that characterise relationships between economic variables. The two central properties of many economic time series are nonstationarity and time-volatility (Wei, 2006). These two properties have led to many applications in both economics and statistics.

Nonstationarity is a property common to many applied time series. This means that a variable has no clear tendency to return to a constant value or linear trend. It is generally correct to assume that economic processes have been generated by a nonstationary process and follow stochastic trends. One major objective of empirical research in economics is to test hypotheses and estimate relationships derived from economic theory, among other such aggregated variables (Pfaff, 2006).

The classical statistical methods used in building and testing large simultaneous equation models, such as Ordinary Least Squares (OLS), were based on the assumption that the variables involved are stationary. The problem is that the statistical inference associated with stationary processes is no longer valid if time series are a realisation of nonstationary processes. If time series are nonstationary it is not possible to use OLS to estimate their long-run linear relationships because it would lead to spurious regression. Spurious regression is a situation in which there appears to be a statistically significant relationship between variables but the variables are unrelated. A few decades ago the difficulty of nonstationarity was not well understood by model builders. However, this is no longer the case because the technique of cointegration has been introduced according to which models containing nonstationary stochastic variables can be constructed in such a way that the results are both statistically and economically meaningful.
Cointegration is an econometric concept which mimics the existence of a long-run equilibrium among economic time series. If two or more series are themselves nonstationary, but a linear combination of them is stationary, then they are said to be cointegrated (Wei, 2006). We should be concerned about cointegration because it is a possible solution to nonstationarity found in many economic time series, and if time series are nonstationary the assumptions upon which OLS estimation rest are violated, rendering its application inappropriate.

Previously, the usual procedure for testing hypotheses concerning the relationship between nonstationary variables was to run OLS regressions on data which had initially been differenced. Data are differenced in order to reduce nonstationary series to stationarity. Although this method is correct in large samples, it may give rise to misleading inferences or spurious regressions in small samples. Moreover, estimation of a single equation framework with integrated or nonstationary variables tends to create the following problems: non-standard distribution of the coefficient estimates generated by the process not being stationary, explanatory variables generated by the process that display autocorrelation, the existence of more than one cointegrated vector and tendency to weak exogeneity (Banerjee et al., 1993).

The remedy for problematic regressions with integrated variables is to test for cointegration and to estimate a vector error-correction model to distinguish between short-run and long-run responses, since cointegration provides more powerful tools when the data sets are of limited length. The technique of cointegration and the error-correction model have both been used before in modelling a number of studies, for example, in modelling Danish gasoline demand (Bentzen et al., 1995), the road transport energy demand for Australia (Samimi, 1995), demand for coal in India (Kulshreshtha and Parikh, 1999), coal demand in China (Chan and Lee, 1997) and the United Kingdom’s final user energy demand (Fouguet et al., 1997). In these studies, the multivariate Johansen cointegrating framework was used to ascertain the cointegrating rank.

The main interest in this study is to estimate cointegrating models and explain their applications to different sets of data using the three main methods of testing for cointegration and related relationships. The results of this study can be used to assess the impact of a temporary or permanent shock on economic variables in an economy. These methods are

- the Engle-Granger method (Engle-Granger, 1987)
- the Phillips-Ouliaris residual-based tests, namely a variance ratio and a multivariate trace
the Johansen’s procedure which builds cointegrated variables directly on maximum likelihood estimation instead of relying on OLS procedures (Johansen and Juselius, 1988).

In this study, the above techniques for testing for cointegration are explored using statistical software $R$. These methods are tested on three data sets:

- the Raotbl3 data set, extracted from the $R$ package urca
- the World Economic Indicators case study on Australia’s economic indicator extracted from the United Nations World Economic Indicators
- the United Kingdom purchasing power parity (UKpppuip) data set extracted from the urca $R$ package.

These data sets were selected because of the availability of data from the urca $R$ package and/or on the Internet. In the Raotbl3 data set, we test for cointegration between the consumption expenditure, income and wealth series; in the world economic indicators data set, we test for cointegration in three of Australia’s key economic indicators; whereas in the UKpppuip data set we test for the existence of long-run economic relationships in the United Kingdom purchasing power parity.

Not all variables in the analysed data sets showed the existence of long-run relationships, and in the World Economic Indicators data set series was cointegrated on second order differencing. These data sets are attached in Appendices A, B and C, respectively.
Chapter 2

A theoretical overview

2.1 Stationarity and nonstationarity

Time series data consist of observations, which are considered as a realisation of random variables that can be described by some stochastic processes. The concept of “stationarity” is related to the properties of this stochastic processes. In this study, the concept of “weak stationarity” is adopted; meaning that the data are assumed to be stationary if the means, variances and covariances of the series are independent of time, rather than the entire distribution.

Nonstationarity in a time series occurs when there is no constant mean $\mu$, no constant variance $\sigma^2$, or both of these properties. It can originate from various sources but the most important one is the unit root.

2.2 Unit root

Any sequence that contains one or more characteristic roots that are equal to one is called a unit root process. The simplest model that may contain a unit root is the AR(1) model.

Consider the autoregressive process of order one, AR(1), below:

$$Y_t = \phi Y_{t-1} + \epsilon_t$$  \hspace{1cm} (2.2.1)

where $\epsilon_t$ denotes a serially uncorrected white noise error term with a mean of zero and a constant variance.
If $\phi = 1$, equation 2.2.1 becomes a random walk without drift model, that is, a nonstationary process. When this happens, we face what is known as the unit root problem. This means that we are faced with a situation of nonstationarity in the series. If, however, $\phi < 1$, then the series $Y_t$ is stationary. The stationarity of the series is important because correlation could persist in nonstationary time series even if the sample is very large and may result in what is called spurious (or nonsense) regression (Yule, 1989). The unit root problem can be solved, or stationarity can be achieved, by differencing the data set (Wei, 2006).

2.3 The augmented Dickey-Fuller (ADF) test

In section 2.2, it was stated that, if $\phi = 1$, equation 2.2.1 becomes a random walk model without drift, which is known as a nonstationary process. The basic idea behind the ADF unit root test for nonstationarity is to simply regress $Y_t$ on its (one period) lagged value $Y_{t-1}$ and find out if the estimated $\phi$ is statistically equal to 1 or not. Equation 2.2.1 can be manipulated by subtracting $Y_{t-1}$ from both sides to obtain

$$Y_t - Y_{t-1} = (\phi - 1)Y_{t-1} + \epsilon_t$$

(2.3.1)

which can be written as

$$\Delta Y_t = \delta Y_{t-1} + \epsilon_t$$

(2.3.2)

where $\delta = (\phi - 1)$, and $\Delta$ is the first difference operator.

In practice, instead of estimating equation 2.2.1, we shall estimate equation 2.3.2 and test for the null hypothesis of $\delta = 0$ against the alternative of $\delta \neq 0$. If $\delta = 0$, then $\phi = 1$, meaning that we have a unit root problem and the series under consideration is nonstationary. It should be noted that under the null hypothesis $\delta = 0$, the $t$-value of the estimated coefficient of $Y_{t-1}$ does not follow the $t$-distribution even in large samples (Erdogdu, 2007). This means that the $t$-value does not have an asymptotic normal distribution. The decision to reject or not to reject the null hypothesis of $\delta = 0$ is based on the Dickey-Fuller (DF) critical values of the $\tau$ (tau) statistic. The DF test is based on an assumption that the errors of term $\epsilon_t$ are uncorrelated.

However, in practice, the errors of the term in the DF test usually show evidence of serial correlation. To solve this problem, Dickey and Fuller have developed a test known as the Augmented Dickey-Fuller (ADF) test. In the ADF test, the lags of the first difference are included in the
regression equation in order to make the error term $\epsilon_t$ white noise and, therefore, the regression equation is presented in the following form:

$$\Delta Y_t = \delta Y_{t-1} + \alpha_i \sum_{i=1}^{m} \Delta Y_{t-i} + \epsilon_t. \quad (2.3.3)$$

To be more specific, the intercept may be included, as well as a time trend $t$, after which the model becomes

$$\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \alpha_i \sum_{i=1}^{m} \Delta Y_{t-i} + \epsilon_t. \quad (2.3.4)$$

The testing procedure for the ADF unit root test is applied to the following model

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \rho \sum_{j=1}^{\rho} \delta_j \Delta y_{t-j} + \epsilon_{it} \quad (2.3.5)$$

where $\alpha$ is a constant, $\beta$ the coefficient on a time trend series, $\gamma$ the coefficient of $y_{t-1}$, $\rho$ is the lag order of the autoregressive process, $\Delta y_t = y_t - y_{t-1}$ are first differences of $y_t$, $y_{t-1}$ are lagged values of order one of $y_t$, $\Delta y_{t-j}$ are changes in lagged values, and $\epsilon_{it}$ is the white noise.

The ADF test can be tested on at least three possible models:

(i) A pure random walk without a drift. This is defined by using the constraint $\alpha = 0$, $\beta = 0$ and $\gamma = 0$ in equation 2.3.5. This leads to the equation

$$\Delta y_t = \Delta y_{t-1} + \epsilon_t. \quad (2.3.6)$$

Equation 2.3.6 is a nonstationary series because its variance grows with time (Pfaff, 2006).

(ii) A random walk with a drift. This is obtained by imposing the constraint $\beta = 0$ and $\gamma = 0$ in equation 2.3.5, which yields to the equation

$$\Delta y_t = \alpha + \Delta y_{t-1} + \epsilon_t. \quad (2.3.7)$$

(iii) A deterministic trend with a drift. For $\beta \neq 0$, equation 2.3.5 becomes the following deterministic trend with a drift model

$$\Delta y_t = \alpha + \beta t + \Delta y_{t-1} + \epsilon_t. \quad (2.3.8)$$

The sign of the drift parameter ($\alpha$) causes the series to wander upward if positive and downward if negative, whereas the size of the absolute value affects the steepness of the series (Pfaff, 2006).
The parameter of interest in the ADF model is $\gamma$. For $\gamma = 0$, the $y_t$ sequence contains the unit root and hence is integrated of order $d = 1$.

The choice of the number of lags ($p$) in this study is based on the significant lag of the autocorrelation function ($ACF$) and the partial autocorrelation function ($PACF$) plots. We take the value of $p$ to be the number of lags at which the $ACF$ cuts off or the number of lags of the $PACF$ that are significant.

The test procedure for unit roots is similar to statistical tests for hypothesis, that is:

(i) Set the null and alternative hypothesis as

$$H_0 : \gamma = 0 \quad (2.3.9)$$

$$H_1 : \gamma < 0 \quad (2.3.10)$$

(ii) Determine the test statistic using

$$F_\tau = \frac{\hat{\gamma}}{SE(\hat{\gamma})} \quad (2.3.11)$$

where $SE(\hat{\gamma})$ is the standard error of $\gamma$.

(iii) Compare the calculated test statistic in 2.3.11 with the critical value from Dickey-Fuller table to reject or not to reject the null hypothesis.

(iv) The ADF test is a lower-tailed test, so if $F_\tau$ is less than the critical value, then the null hypothesis of unit root is rejected and the conclusion is that the variable of the series does not contain a unit root and is nonstationary.

The DF and ADF tests are similar since they have the same asymptotic distribution. Although there are numerous unit root tests, such as the Phillips-Perron test and the Schmidt-Phillips test, the most notable and commonly used is the ADF test, which will be used in this study.

### 2.4 Cointegration tests

On the basis of the theory that integrated variables of order one, $I(1)$, may have a cointegration relationship, it is crucial to test for the existence of such a relationship. If a group of variables are individually integrated of the same order and there is at least one linear combination of these
variables that is stationary, then the variables are said to be cointegrated. The cointegrated variables will never move far apart, and will be attracted to their long-run relationship. Testing for cointegration implies testing for the existence of such a long-run relationship between economic variables. This study considers a number of cointegration tests, namely the Engle-Granger method commonly known as the two-step estimation procedure, the Phillips-Ouliaris methods and the Johansen’s procedure.

2.4.1 Engle-Granger method

As we have stated, the regression of nonstationary series on other series may produce spurious regression. If each variable of the time series data is subjected to unit root analysis and it is found that all the variables are integrated of order one, \( I(1) \), then they contain a unit root. There is a possibility that the regression can still be meaningful (i.e. not spurious) provided that the variables cointegrate. In order to find out whether the variables cointegrate, the least squares regression equation is estimated and the residuals (the error term) of the regression equation are subjected to unit root analysis. If the residuals are stationary, that is \( I(0) \), it means that the variables under study cointegrate and have a long-term or equilibrium relationship. The Engle-Granger method is based on the idea described in this paragraph.

In the two-step estimation procedure, Engle-Granger considered the problem of testing the null hypothesis of no cointegration between a set of variables by estimating the coefficient of a statistic relationship between economic variables using the OLS and applying well-known unit root tests to the residuals to test for stationarity. Rejecting the null hypothesis of a unit root is evidence in favour of cointegration.

In the literature, there are a number of studies that apply Engle-Granger’s two-step estimation procedure. A summary of some of them follows:

The study by Lee (1993) applied the two-step estimation procedure similar to that used by Engle and Granger to examine cointegration relationships between total consumption and disposable income on Japanese data from January 1961 to April 1987. This study investigated whether seasonality in income cointegrates with that in consumption and identifies reasons in support of the empirical cointegration relationship. The results indicated that income and consumption series are integrated of order one at both the long-run (yearly) and the short-run (seasonal) frequency. Results further indicated that both income and consumption series are nonstationary and that
the seasonal pattern has a significant variation over the period, although the seasonal pattern for consumption was more regular.

There is vast literature that explores whether spot and future prices for oil are linked in a long-run relationship. One particular study was undertaken by Maslyuk and Smyth (2009) to examine whether crude oil spot and future prices of the same and different grades cointegrate. The null hypothesis of no cointegration was tested against the alternative of cointegration in the presence of a regime shift on series monthly data from the United States Western Telematic Inc. and United Kingdom Brent using the two-step estimation procedure. The results revealed that there is a cointegration relationship between spot and future prices of crude oil of the same grade, as well as spot and future prices of different grades. Results further indicated that spot and future prices are governed by the same set of fundamentals, such as the exchange rate of the US dollar, macroeconomic variables, and the demand and supply conditions, which are similar and interrelated for crude oil in North American and European markets.

The relationship between female labour force participation (FLFP) and total fertility rate (TFR) has received a lot of attention in the literature on demography and economics. A study of twenty-eight countries from the Organisation for Economic Cooperation and Development (OECD) using panel unit roots, panel cointegration and a panel Granger causality framework revealed a long-run relationship between FLFP and TFR. This study found that there is either uni-directional long-run Granger causality running from FLFP to the TFR or bi-directional Granger causality between the two variables depending on how female labour participation rate is measured and the time period. It was concluded that there is an inverse relationship between FLFP and TFR (Maslyuk and Smyth, 2010).

2.4.2 Phillips-Ouliaris methods

Phillips-Ouliaris introduced two residual-based tests namely:

- the variance ratio test
- the multivariate trace statistics.

These residual-based tests are used in the same way as the unit root tests, but the data are the residuals from the cointegrating regression. These tests seek to test a null hypothesis of no cointegration against the alternative of the presence of cointegration using scalar unit root tests applied
to the residuals. Phillips-Ouliaris methods are based on residuals (differences between the observed and expected values) of the first order autoregression, AR(1), equation. The multivariate trace statistics has the advantage over the variance ratio test in that it is invariant to normalisation, that is, whichever variable is taken to be the dependent variable, the test will yield the same results (Pfaff, 2006).

In the literature, there are no studies directly linked to the application of the Phillips-Ouliaris cointegration test only. However, there are a few studies in which cointegration has been tested using other techniques including the Phillips-Ouliaris methods. These studies will now be discussed.

The study by Cancer (1998) developed the asymptotic theory for residual-based tests and quasi-likelihood ratio tests for cointegration under the assumption of infinite variance errors. He extended the results of the Phillips-Ouliaris methods, which were derived under the assumption of square-integrable errors. He also investigated whether the Phillips-Ouliaris methods are robust to infinite variance errors. The results showed that, regardless of the index of stability $\alpha$, the residual-based tests are more robust to infinite variance errors than the likelihood ratio-based tests.

Theoretical models of pricing-to-market suggest that the profile of economic exposure may be asymmetric between periods when the real exchange rate appreciates and is symmetric between periods when it depreciates. This hypothesis was tested using time-series data on export prices for eight commodities exported from the United Kingdom to the United States during the period 1981 to 1988 (Kanas, 1997). During this period, there was a long-term real depreciation of the pound against the dollar from January 1985 up to April 1988, followed by a long-term real appreciation from February 1985 to April 1988. Kanas (1997) tested for cointegration between the export price of each commodity and the real exchange rate applying the Phillips-Ouliaris methods. The results were generally in favour of this hypothesis.

The study by Choi (1994) of spurious regression and residual-based tests for cointegration when regressors are cointegrated, analysed the asymptotic null distribution of residual-based cointegration tests such as the Phillips’ $\hat{Z}_\alpha$ and the ADF tests when regressors are cointegrated in comparison to the Phillips-Ouliaris methods.

The results showed that the null distributions of residual-based cointegration tests differed from those derived from the use of the Phillips-Ouliaris methods. The practical implication of these
results is that we need to test not only for the presence of a unit root for individual series, but also for the presence of cointegrating vectors for the regressors prior to performing residual-based tests for cointegration.

2.4.3 Johansen’s procedure

Johansen’s procedure builds cointegrated variables directly on maximum likelihood estimation instead of relying on OLS estimation. This procedure relies heavily on the relationship between the rank of a matrix and its characteristic roots. Johansen derived the maximum likelihood estimation using sequential tests for determining the number of cointegrating vectors. His method can be seen as a secondary generation approach in the sense that it builds directly on maximum likelihood instead of partly relying on least squares. In fact, Johansen’s procedure is nothing more than a multivariate generalisation of the Dickey-Fuller test. Consequently, he proposes two different likelihood ratio tests namely

- the trace test
- the maximum eigenvalue test.

This procedure is a vector cointegration test method. It has the advantage over the Engle-Granger and the Phillips-Ouliaris methods in that it can estimate more than one cointegration relationship, if the data set contains two or more time series. In the literature, studies on Johansen’s procedure include the following.

The ongoing debate among energy economists about the relationship between energy use and output growth has led to the emergence of many views. One investigation on the causal interaction between energy use and output growth for Canada was undertaken by Khalifa and Sakka (2004). They used time series properties to develop a vector error-correction (VECM) model to test for multivariate cointegration and Granger-causality. The empirical finds from this analysis indicated that output growth, capital, labour and energy use share two common stochastic trends. In particular, output growth and energy use were found to be moving together towards a stable long-run equilibrium relationship, that is, consistent with causality running in both directions.

Masih et al. (1996) used Johansen’s cointegration analysis to study the relationship between energy use and gross domestic product (GDP) in a group of six Asian countries, including India, Pakistan, Malaysia, Singapore, Philippines and Indonesia. The results indicated that there were cointegration relationships in energy use and GDP among countries like India, Pakistan and Indonesia.
However, no cointegration was found in the case of Malaysia, Singapore and the Philippines. The flow of causality was found to be running from energy to GDP in India and GDP to energy in Pakistan and Indonesia.

Yang (2000) considered the causal relationship between energy use and GDP in Taiwan. Using different measures of energy consumption, he found a bi-directional causality between energy and the GDP. This result contradicts that of Cheng and Lai (1997), who found that there is a uni-directional causal relationship from GDP.

Belloumi (2009) applied the Johansen’s cointegration procedure to examine the causal relationship between per capita energy consumption (PCEC) and per capita gross domestic product (PCGDP) for Tunisia during the 1971-2004 period. In order to test for Granger-causality in the presence of cointegration among variables, a vector error-correction model (VECM) was used instead of a vector autoregressive (VAR) model. His results indicate that the PCGDP and PCEC for Tunisia are related by one cointegrating vector and there is a long-run bi-directional causal relationship between the two series and a short-run uni-directional causality from energy to GDP. The source of causation in the long-run was found to be the error correction terms in both directions. Hence, an important policy implication resulting from this analysis is that energy can be considered as a limiting factor to GDP growth in Tunisia. It was argued that Tunisia’s economy is energy dependent and is relatively vulnerable to energy shocks.

Finally, Asufu (2000) tested the cointegration relationship between energy use and income in four Asian countries using cointegration, Johansen’s procedure and error-correction analysis. He found that cointegration runs from energy to income in India and Indonesia, and that there is a bi-directional causality in Thailand and the Philippines.
Chapter 3

Testing for cointegration

In this chapter we discuss the various methods of testing for cointegration which include

- the two-step estimation procedure developed by Engle and Granger in 1987
- the Phillips-Ouliaris residual-based tests
- the Johansen procedure which builds cointegrated variables directly on maximum likelihood estimation instead of relying on OLS estimates.

3.1 The Engle-Granger method

Engle and Granger developed this crucial technique in 1987. This technique entails cointegrated variables which are discussed at length including a proof of Granger’s representation theorem, which connects the moving average, the autoregressive, and the error correcting representation for cointegrated systems.

According to both statisticians, the steps for determining whether two integrated variables cointegrate of the same order are the following:

- pre-test each variable to determine its order of integration, and
- estimate the error-correction model.

If the integrated variables are found to be integrated to the same order, then it must be tested whether these variables are cointegrated (Johansen, 1988).
3.1.1 Pre-test each variable

Pre-testing each variable aims at determining the order of integration of each variable. By definition, cointegration necessitates that two variables be integrated of the same order. This is done using the Augmented Dickey-Fuller (ADF) unit root test to infer the number of unit roots (if any) in each of the variables under investigation. The testing procedure for the ADF unit root test is applied to the following model

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{j=1}^{\rho} \delta_j \Delta y_{t-j} + \epsilon_t$$

(3.1.1)

where $\alpha$ is a constant, $\beta$ the coefficient on a time trend series, $\gamma$ the coefficient of $y_{t-1}$, $\rho$ is the lag order of the autoregressive process, $\Delta y_t = y_t - y_{t-1}$ are first differences of $y_t$, $y_{t-1}$ are lagged values of order one of $y_t$, $\Delta y_{t-j}$ are changes in lagged values, and $\epsilon_t$ is the white noise.

Once the hypothesis of the unit root test is rejected, we estimate the long-run equilibrium relationship in the form of an OLS regression line

$$y_t = \beta_0 + \beta_1 x_t + \epsilon_t$$

(3.1.2)

where $\beta_0$ is the $y$-intercept, $\beta_1$ is the slope, and $\epsilon_t$ is the error term.

The parameter estimates in equation 3.1.2 are estimated from

$$\hat{\beta}_1 = \frac{\sum (x_t - \bar{x}_t)(y_t - \bar{y}_t)}{\sum (x_t - \bar{x}_t)^2}$$

(3.1.3)

where $\bar{x}_t$ and $\bar{y}_t$ is the mean of $x_t$ and $y_t$ respectively.

The value of $\beta_0$ is estimated from

$$\hat{\beta}_0 = \bar{y}_t - \hat{\beta}_1 \bar{x}.$$  

(3.1.4)

The estimated regression line is then given in the form

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x.$$  

(3.1.5)

If the variables cointegrate, an OLS regression equation 3.1.5 yields a “super-consistent” estimator (Enders, 2004). This means that there is a strong linear relationship between the variables under study. The strong linear relationship can be tested in either of the following ways.

(a) The value of $\hat{\beta}_1$ falls between 0.5 and 1.

(b) The plot of $y_t$ against $x_t$ shows coordinates appearing in an increasing or decreasing direction.
In order to determine if variables cointegrate, we test for unit roots on the residual sequence in the equation 3.1.2 using the ADF test. The residual sequence, denoted by $\epsilon_t$, is a series of estimated values of the deviation from the long-run relationship. They are estimated from

$$\epsilon_t = y_t - \hat{y}_t$$  \hspace{1cm} (3.1.6)

where $\hat{y}_t$ are values from the predicted equation 3.1.5. Testing for unit roots on residuals aims at determining whether these deviations are stationary or not. If they are stationary, then the series cointegrate. If the residuals are not stationary, there is no cointegration. The ADF test is performed on the following model

$$\Delta \hat{\epsilon}_t = a_1 \hat{\epsilon}_{t-1} + \epsilon_t$$  \hspace{1cm} (3.1.7)

where $\Delta \hat{\epsilon}_t$ are the estimated first differenced residuals, $\hat{\epsilon}_{t-1}$ are the estimated lagged residuals, $a_1$ is the parameter of interest representing the slope of the line and $\epsilon_t$ are errors obtained in fitting both differenced residuals.

Since the $\epsilon_t$ sequences are residuals from a regression equation, there is no need to include the intercept term in equation 3.1.7. To test the hypothesis on $a_1$ to determine whether the residuals are stationary, we follow the following steps.

(i) Set both the null and alternative hypothesis as

$$H_0 : a_1 = 0$$ \hspace{1cm} (3.1.8)

$$H_1 : a_1 < 0$$ \hspace{1cm} (3.1.9)

(ii) Determine the test statistic using

$$F_{\hat{\epsilon}_t} = \frac{\hat{a}_1}{SE(\hat{a}_1)}$$ \hspace{1cm} (3.1.10)

where the value of $SE(\hat{a}_1)$ is the standard error of $\hat{a}_1$, the estimate of $a_1$.

(iii) Compare the calculated test statistic in 3.1.10 with the critical value from the Dickey-Fuller table to reject or not to reject the null hypothesis.

(iv) If $F_{\hat{\epsilon}_t}$ is greater than the critical value, we do not reject the null hypothesis, $H_0$.

The rejection of $H_0$ implies that residuals are stationary. This further implies that the variables under study are cointegrated.

The next step is to estimate the error correction model (ECM) which will be done in the next section.
3.1.2 Estimate the error correction model

An error correction model is defined as a dynamic model in which the movement of a variable in any period is related to the previous period’s gap from the long-run equilibrium. Although it may be possible to estimate the long-run or cointegrating relationship, \( y_t = \beta x_t + \epsilon_t \) economic systems are rarely in equilibrium, as they are affected by institutional and/or structural changes that might be temporary or permanent. For example, extra income in the form of a birthday bonus may raise someone’s expenditure pattern in one or two months and then his/her expenditure gradually goes back to equilibrium. Since equilibrium is rarely observed, the short-run evolution of variables (short-run dynamic adjustment) is important. A simple dynamic model of a short-run adjustment model is given by

\[
y_t = \alpha_0 + \gamma_0 x_t + \gamma_1 x_{t-1} + \alpha_1 y_{t-1} + \epsilon_t
\]  

(3.1.11)

where \( y_t \) is the dependent variable, \( x_t \) is the independent variable, \( y_{t-1} \) and \( x_{t-1} \) are lagged values of \( y_t \) and \( x_t \) respectively, \( \alpha_0, \alpha_1, \gamma_0, \gamma_1 \) are parameters, and \( \epsilon_t \) is the error term assumed to be \( \epsilon_t \sim iN(0, \delta^2) \).

The problems associated with the use of the short-run model are the following:

- **Multicollinearity:** This is a situation in which two or more independent variables in a multiple regression model are highly correlated.
- **Spurious correlation:** This is a situation in which two variables have no causal connection, yet it may be inferred that they do as a result of a certain third unseen factor.

These problems are solved by estimating the first differences of equation 3.1.11 to obtain

\[
\Delta y_t = \alpha_0 + \gamma_0 \Delta x_t + \gamma_1 \Delta x_{t-1} + \alpha_1 \Delta y_{t-1} + \epsilon_t.
\]  

(3.1.12)

This, however, introduces problems of

- loss of information about the long-run equilibrium
- the economic theory is differenced away.

The possible solution is to adopt the error-correction mechanism (ECM) formulation of the dynamic structure. We set up the ECM as follows:

\[
y_t = \alpha_0 + \gamma_0 x_t + \gamma_1 x_{t-1} + \alpha_1 y_{t-1} + \epsilon_t.
\]  

(3.1.13)

Subtracting the term \( y_t \) on both sides leads to

\[
\Delta y_t = \alpha_0 + \gamma_0 \Delta x_t + \gamma_1 \Delta x_{t-1} - (1 - \alpha_1) y_{t-1} + \epsilon_t.
\]  

(3.1.14)
Subtracting the term \( \gamma_0 x_{t-1} \) on both sides, equation 3.1.14 becomes

\[
\Delta y_t - \gamma_0 x_{t-1} = \alpha_0 + \gamma_0 x_t - \gamma_0 x_{t-1} + \gamma_1 x_{t-1} - (1 - \alpha_1) y_{t-1} + \varepsilon_t, \tag{3.1.15}
\]

\[
\Delta y_t = \alpha_0 + \gamma_0 \Delta x_t + (\gamma_0 + \gamma_1) x_{t-1} - (1 - \alpha_1) y_{t-1} + \varepsilon_t. \tag{3.1.16}
\]

Reparameterisation reduces equation 3.1.16 to

\[
\Delta y_t = \gamma_0 \Delta x_t - (1 - \alpha_1) \left[ y_{t-1} - \alpha_0 \left( \frac{\gamma_0 + \gamma_1}{1 - \alpha_1} \right) x_{t-1} \right] + \varepsilon_t. \tag{3.1.17}
\]

Taking \( \beta_0 = \frac{\alpha_0}{1 - \alpha_1} \) and \( \beta_1 = \frac{(\gamma_0 + \gamma_1)}{(1 - \alpha_1)} \) the equation 3.1.17 becomes

\[
\Delta y_t = \gamma_0 \Delta x_t - (1 - \alpha_1) \left[ y_{t-1} - \beta_0 - \beta_1 x_{t-1} \right] + \varepsilon_t \tag{3.1.18}
\]

which is the ECM with \(-(1 - \alpha_1)\) as the speed of adjustment, and \( \varepsilon_{t-1} = y_{t-1} - \beta_0 - \beta_1 x_{t-1} \) as the error-correction mechanism which measures the distance of the system away from equilibrium.

The coefficient of \( \varepsilon_{t-1} \) should be negative in sign in order for the system to converge to equilibrium. The size of the coefficient \(- (1 - \alpha_1)\) is an indication of the speed of adjustment towards equilibrium in that

- small values of \(- (1 - \alpha_1)\), tending to \(-1\), indicate that economic agents remove a large percentage of disequilibrium in each period
- larger values, tending to \(0\), indicate that adjustment is slow
- extremely small values, less than \(-2\), indicate an overshooting of economic equilibrium
- positive values would imply that the system diverges from the long-run equilibrium path.

The ECM satisfies the assumptions of classical normal linear regression model (CNLRM). These assumptions include:

- A linear regression model.
- Residuals are normally distributed.
- There is no serial correlation among residuals.
- The number of observations must not exceed the number of parameters to be estimated.
- There is no perfect multicollinearity.

This means that diagnostic tests have to be conducted on the error-correction mechanism in order to determine whether any of these assumptions have not been violated. These tests include...
• a normality test
• a heteroscedasticity test
• a serial correlation test
• a misspecification test.

3.1.2.1 Normality test

We use the Jarque-Bera test to determine whether the ECM is normally distributed. This test measures the difference in kurtosis and skewness of a variable compared to those of the normal distribution (Jarque and Bera, 1980).

In the Jarque-Bera test, we set the null and alternative hypothesis as follows:

\( H_0: \) The variable is normally distributed.
\( H_1: \) The variable is not normally distributed.

The test statistic is

\[
JB = \frac{N - k}{6} \left[ S^2 + \frac{(K - 3)^2}{4} \right]
\]

(3.1.19)

where \( N \) is the number of observations, \( k \) is the number of estimated parameters, \( S \) is the skewness of a variable, and \( K \) is the kurtosis of a variable.

We reject the null hypothesis if the \( p \)-value \( \leq \) level of significance, or if the \( JB > \chi^2(2) \).

3.1.2.2 Heteroscedasticity test

Heteroscedasticity results from a sequence of random variables having different variances. It implies that during regression analysis there is non-consistent variance. Heteroscedasticity is tested using the Langrange multiplier, also known as Engle’s Arch LM test (Engle, 1982). The test procedure is as follows:

\( H_0: \) There is no heteroscedasticity.
\( H_1: \) There is heteroscedasticity.

The test statistic is

\[
LM_E = nR^2
\]

(3.1.20)

where \( n \) is the number of observations, and \( R^2 \) is the coefficient of determination of the augmented residual regression.

We reject the null hypothesis if the \( p \)-value \( \leq \) level of significance and conclude that there is heteroscedasticity.
3.1.2.3 Serial correlation test

Serial correlation is cross-correlation of a signal (white noise) with itself. It may be caused by

- nonstationarity of dependent and explanatory variable
- data manipulation (averaging, interpolation and extrapolation)
- incorrect functional form.

Ljung and Box (1978) suggested the use of Ljung-Box test to test the assumption that the residuals contain no autocorrelation up to any order $k$. The test procedure is as follows:

$H_0$: There is no autocorrelation up to order $k$.

$H_1$: Autocorrelation exists up to order $k$.

The test statistic is

$$Q_{LB} = T(T + 2) \sum_{j=1}^{k} \frac{r_j^2}{T - j} \quad (3.1.21)$$

where $T$ is the number of observations, $k$ is the highest order of autocorrelation for which to test, and $r_j^2$ is the $j^{th}$ autocorrelation.

We reject the null hypothesis if the $p$-value $\leq$ level of significance and conclude that autocorrelation exists up to order $k$.

The major drawback of this test is deciding which lag order ($k$) to use. Ljung and Box (1978) suggested the maximum number of lags to use should be $T^{\frac{4}{5}}$ where $T$ is the number of observations.

3.1.2.4 Misspecification test

Misspecification is as the result of

- incorrect functional form
- inclusion of irrelevant variables
- exclusion of relevant variables.

The consequences of a misspecified regression are

- the residuals are not normally distributed
- there is serial correlation
- regression is inconsistent with actual working of the economy
- the parameter estimates are not robust to samples used.

To test for misspecification, Ramsey (1969) suggested the use of Ramsey’s reset test. This is a likelihood test that compares the likelihood function of original regression to an augmented regression.

### 3.1.3 Limitations of the Engle-Granger method

Although the Engle and Granger procedure is easily implemented, it has several defects:

- The estimation of the long-run equilibrium regression requires that the researcher place one variable on the right-hand side as the dependent variable and use the other variable on the left-hand side as the independent variable. For example, in the case of two variables, it is possible to run the Engle-Granger method for cointegration by using the residuals from either of the following two “equilibrium” regression equation.

$$y_t = \beta_{10} + \beta_{11} x_t + \epsilon_{1t} \quad (3.1.22)$$

or

$$x_t = \beta_{20} + \beta_{21} y_t + \epsilon_{2t} \quad (3.1.23)$$

As the sample size grows infinitely large, the theory indicates that the test for a unit root in the $\epsilon_{1t}$ sequence becomes equivalent to the test for a unit root in the $\epsilon_{2t}$ sequence. Unfortunately, the properties of large samples on which this result is derived may not be applicable to the sample sizes usually available to economists. In many cases available sample sizes are smaller than the required sample size on which the theory is based.

- The two-step estimation procedure is based on the principle that, irrespective of which variable is chosen for normalisation, the same results will be attained if variables are interchanged. In practice, it is possible to find that one regression indicates that the variables are cointegrated, whereas reversing the order indicates no cointegration. For example, in investigating the relationship between income and expenditure, if income is placed on the left-hand side as the dependent variable, it is possible to conclude that income and expenditure cointegrate, but the reverse is not necessarily true. This is a very undesirable feature of the procedure, because the test for cointegration should be invariant to the choice of variable selected for normalisation.

- Engle-Granger’s two-step estimation procedure relies on a two-step estimator. Recall that the first step in the two-step estimation method of pre-setting each variable to determine
the order of integration generates residual series $\hat{\epsilon}_t$ used in the second step. This is used to estimate the regression equation of the form $\Delta \hat{\epsilon}_t = a_1 \hat{\epsilon}_{t-1} + \epsilon_t$ as in equation 3.1.7. Thus, the coefficient $a_1$ is obtained by estimating a regression equation using residuals from another regression. This implies that any error introduced in the first step is carried over into the second step, making the results unreliable.

Fortunately, several methods have been developed that could avoid these defects. These include the Phillip-Ouliaris methods, Johansen’s procedure, and the Stock and Watson maximum likelihood estimators. These tests allow the researcher to test for restricted versions of cointegrating vector(s) and the speed of adjustment parameters. They rely heavily on the relationship between the rank of a matrix and its characteristic roots.

### 3.2 Phillip-Ouliaris methods

In section 3.1, it has been shown that the second step of the Engle-Granger’s method is an ADF test applied to the residuals of the long-run equation. Phillips and Ouliaris introduced two residual-based tests, namely the variance ratio test and the multivariate trace statistic (Phillips-Ouliaris, 1988). The latter of these tests has the advantage that it is invariant to normalisation, that is, for whichever variable is taken to be the dependent variable, the test will yield the same results. These tests are used in the same way as unit root tests but the data are residuals from cointegrating regressions. They are implemented on matrices or multivariate series and are both based on residuals of the first order vector autoregression equation

$$\mathbf{z}_t = \mathbf{\hat{H}} \mathbf{z}_{t-1} + \hat{\xi}_t$$

(3.2.1)

where $\mathbf{z}_t$ is partitioned as $\mathbf{z}_t = (\mathbf{y}_t, \mathbf{x}_t')$ with a dimension of $\mathbf{x}_t$ equal to $(m = n + 1)$, $\mathbf{\hat{H}}$ is a regression coefficient, and $n$ is equal to the number of variables under study.

#### 3.2.1 The variance ratio test

The variance ratio statistic $\hat{P}_u$ is defined as

$$\hat{P}_u = \frac{T \hat{\omega}_{1,2}}{T^{-1} \sum_{t=1}^{T} \hat{u}_t^2}$$

(3.2.2)

where $\hat{u}_t$ are the residuals of the long-run regression equation

$$\mathbf{y}_t = \mathbf{\hat{\beta}} \mathbf{x}_t + \hat{u}_t.$$  

(3.2.3)
The conditional variance $\hat{\omega}_{11.2}$ is derived from the conditional matrix $\hat{\Omega}$ of $\hat{\xi}$, that is the residuals of equation 3.2.1, and is defined as

$$\hat{\omega}_{11.2} = \hat{\omega}_{11} - \hat{\omega}_{21} \hat{\Omega}_{22}^{-1} \hat{\omega}_{21}. \quad (3.2.4)$$

The covariance matrix $\hat{\Omega}$ has been partitioned as

$$\hat{\Omega} = \begin{bmatrix} \hat{\omega}_{11} & \hat{\omega}_{21} \\ \hat{\omega}_{21} & \hat{\Omega}_{22} \end{bmatrix} \quad (3.2.5)$$

and is estimated as

$$\hat{\Omega} = T^{-1} \sum_{t=1}^{T} \hat{\xi}_t \hat{\xi}_t' + T^{-1} \sum_{s=1}^{L} \omega_{sl} \sum_{t=1}^{T} (\hat{\xi}_t \hat{\xi}_{t-s} + \hat{\xi}_{t-s} \hat{\xi}_t') \quad (3.2.6)$$

with weighting function $\omega_{sl} = (1 - s) / (l + 1)$.

A variance ratio test is a residual-based test that seeks to test a null hypothesis of no cointegration using scalar unit roots applied to equation 3.2.3. The null hypothesis is formulated in terms of the conditional variance parameter $\omega_{11.2}$ as follows:

$$H_0 : \omega_{11.2} \neq 0 \quad (3.2.7)$$

against

$$H_1 : \omega_{11.2} = 0. \quad (3.2.8)$$

Therefore, the variance ratio test measures the size of the residual variance from the cointegrating regression of $y_t$ on $x_t$ versus $T^{-1} \sum_{t=1}^{T} \hat{u}^2_t$ against that of a direct estimate of the population conditional variance of $y_t$, given $x_t$ versus $\hat{\omega}_{11.2}$. If cointegration exists between variables, the variance ratio should stabilise asymptotically, whereas if a spurious (nonsense) relationship is present, it would be reflected in the cointegrating regression and the variance ratio should diverge.

### 3.2.2 The multivariate trace statistic

The multivariate trace statistic, denoted as $\hat{P}_z$ is defined as

$$\hat{P}_z = Tr(\hat{\Omega} M_z^{-1}) \quad (3.2.9)$$

with $T$ is the number of observations, $M_z^{-1} = t^{-1} \sum_{t=1}^{T} z_t z'_t$, and $\hat{\Omega}$ is estimated in equation 3.2.6.

The advantage of the Engle-Granger two-step procedure is its ease of implementation. However, its results are dependent on how the long-run equation is specified. In some cases it might be easier
to identify which variable enters on the left as the dependent variable. For example, in accessing the relationship between income and expenditure, it is easy to say that expenditure depends on income. Unfortunately, this is only true in some cases. It is therefore advisable to employ the cointegrating test of Phillips-Ouliaris, which yields the same results irrespective of the variable which enters as the dependent variable, that is, they are invariant to normalisation.

One deficiency of the two methods (two-step procedure and Phillips-Ouliaris) is that one can only estimate a single cointegrating relationship. However, if one deals with more than two time series, it is possible that more than one cointegrating relationship will exist, which calls for the use of vector cointegration techniques like Johansen’s procedure.

3.2.3 Limitation of Phillips-Ouliaris methods

This method can only estimate single cointegrating relationships. If one deals with more than two time series, it is possible that more than one cointegrating relationship exists. The remedy to this limitation is to use Johansen’s procedures.

3.3 Johansen’s procedure

Johansen’s method takes as a starting point the vector autoregression (VAR) of order \( p \) given by

\[
X_t = \Pi_1 X_{t-1} + \Pi_2 X_{t-2} + \ldots + \Pi_p X_{t-p} + u_t
\]  

(3.3.1)

where \( X_t \) is an \( n \times 1 \) vector of variables that are integrated of order one, that is, \( I(1) \), \( u_t \) is an \( n \times 1 \) vector of innovations while \( \Pi_1 \) through \( \Pi_p \) are \( m \times m \) coefficient matrices. Reparameterising the equation 3.3.1, that is, subtracting \( X_{t-1} \) on both sides, leads to

\[
\Delta X_t = \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \ldots + \Gamma_{p-1} \Delta X_{t-p+1} - \Pi X_{t-p} + u_t
\]  

(3.3.2)

where \( \Gamma_1 = \Pi_1 - I, \Gamma_2 = \Pi_2 - \Gamma_1, \Gamma_3 = \Pi_3 - \Gamma_2 \) and \( \Pi = I - \Pi_1 - \Pi_2 - \ldots - \Pi_p \). The matrix \( \Pi \) determines the extent to which the system is cointegrated and is called the impact matrix.

Returning to the general reparameterised equation 3.3.2, if we consider the first equation of the system as:

\[
\Delta X_{1t} = \gamma'_{11} \Delta X_{t-1} + \gamma'_{12} \Delta X_{t-2} + \ldots + \gamma'_{1p-1} \Delta X_{t-p+1} - \Pi'_{1} X_{t-p} + u_{1t}
\]  

(3.3.3)

where \( \gamma'_{ij} \) is the first row of \( \Gamma_j \), \( j = 1, 2, \ldots, p - 1 \), and \( \Pi'_{1} \) is the first row of \( \Pi \). Here \( \Delta X_{1t} \) is stationary, that is, \( I(0) \), \( j = 1, 2, \ldots, p - 1 \) are all \( I(0) \), \( u_t \) is assumed to be \( I(0) \) and so
for a meaningful equation, $\Pi'X_{t-p}$ must be stationary, $I(0)$.

If none of the components of $X_t$ are cointegrated, they must be zero. On the other hand, if they are cointegrated, all the rows of $\Pi$ must be cointegrated but not necessarily distinct. This is because the number of distinct cointegrating vectors depends on the row rank of $\Pi$ (Harris, 1995).

The matrix $\Pi$ is of order $m \times m$. If it has rank $m$, that is, $m$ number of linearly independent rows or columns, then it forms a basis for $m$-dimensional vector space. This implies that all $m \times 1$ vectors can be generated as linear combinations of its row. Any of these linear combinations of the rows would lead to stationarity, meaning that $X_{t-p}$ has stationary components if the rank of $\Pi$ is $r < m$.

We may write $\Pi = \beta \alpha'$ for suitable $m \times r$ matrices, $\beta$ and $\alpha$. Here

$$\alpha' = \begin{bmatrix} \alpha_1' \\ \alpha_2' \\ \vdots \\ \cdots \\ \alpha_r' \end{bmatrix}$$

(3.3.4)

$$\beta = \begin{bmatrix} \beta_1 & \beta_2 & \cdots & \beta_r \end{bmatrix}$$

(3.3.5)

Then $\Pi X_{t-p} = \beta \alpha' X_{t-p}$ and all linear combinations of $\alpha' X_{t-p}$ are stationary. It should be noted that we have to perform the ADF test to access the order of integration of each variable before applying Johansen’s procedure. Johansen’s procedure estimates the VAR subject to $\Pi = \beta \alpha'$ for various values of $r$ number of cointegrating vectors, using the maximum likelihood estimator assuming $u_t \sim iidN(0, \Sigma)$. His estimate can thus be rewritten as

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \cdots + \Gamma_{p-1} \Delta X_{t-p+1} - \beta \alpha' X_{t-p} + u_t.$$  

(3.3.6)

The question is, how do we detect the number of cointegrating vectors?

Johansen proposed two likelihood ratio tests namely:

- The trace test
- The maximum eigenvalue.
3.3.1 The trace test

The trace test tests the null hypothesis of \( r \) cointegrating vectors against the alternative hypothesis of \( n \) cointegrating vectors. The test statistic is given by

\[
j_{\text{trace}} = -T \sum_{i=r+1}^{n} \ln(1 - \hat{\lambda}_i). \quad (3.3.7)
\]

3.3.2 The maximum eigenvalue

The maximum eigenvalue test, on the other hand, tests the null hypothesis of \( r \) cointegrating vectors against the alternative hypothesis of \( (r + 1) \) cointegrating vectors. Its test statistic is given by

\[
j_{\text{max}} = -T(1 - \hat{\lambda}_{r+1}) \quad (3.3.8)
\]

where \( T \) is the sample size, and \( \hat{\lambda}_i \) is the \( i^{th} \) largest canonical correlation.

3.3.3 Limitation of Johansen’s procedure

This method assumes that the cointegrating vector remains constant during the period of study. In reality, it is possible that the long-run relationships between the underlying variables change. The reason for this might be technological progress, economic crisis, changes in people’s preferences and behaviour accordingly, policy or regime alteration and institutional development. This is especially the case if the sample period is long.

To take this into account, Gregory and Hasen (1996) have introduced tests for cointegration with one and two unknown structural break(s). However, such tests do not form part of this study.
4.1 Introduction

In this chapter we applied the Engle-Granger method, the Phillip-Ouliaris methods and Johansen’s procedure to test for the existence of cointegrating relationships in the Raotbl3 data set using statistical software $R$. This data set contains the time series used by Holden and Perman (1994). In this study the Raotbl3 data set is included as Appendix A. More details about the data are provided in the data appendix of a book by Bhaskara (1994).

Three time series and three dummy variables are given in this data set. The three dummies are as a result of structural changes such as strikes, wars and natural disasters that did not make it possible to capture data in specific time periods. Data frames are quarterly data (times series objects) from the United Kingdom starting in 1966:4 (1966, April) until 1991:2 (1991, February) for the following six variables all transformed to natural logarithms to reduce variances:

i. (lc) Real consumption expenditure.

ii. (li) Real income.

iii. (lw) Real wealth.

iv. dd682 Dummy variable for 68:2.

v. dd792 Dummy variable for 79:2.
vi. $dd_{883}$ Dummy variable for 88:3.

The three time series (lc, li and lw) above are defined from

(i) CAAB-seasonally adjusted real consumers’ expenditure in millions of pounds, 1985 prices
(ii) AIIX-seasonally adjusted nominal consumers’ expenditure in millions of pounds, current prices
(iii) AIIW-seasonally adjusted nominal personal disposable income in millions of pounds, current prices

The three time series can be found in Table 5 of *Economic Trends Annual Supplement* (1992).

The wealth series is defined as seasonally adjusted gross personal financial wealth in millions of pounds, current prices, and is compiled from several editions of *Financial Statistics* in order to obtain a consistent series.

Constant price series for income and wealth for the year 1985 were obtained from the current price series using the implicit consumers’ expenditure deflator (AIIX/CAAB). The variables lc, li and lw are the natural logarithm of the constant price consumption, income and wealth series. The dummy variables are defined as follow:

(i) $DD_{682} = 1$ in 1968:2, -1 in 1968:3 and 0 otherwise,
(ii) $DD_{792} = 1$ in 1979:2, -1 in 1979:3 and 0 otherwise,
(iii) $DD_{883} = 0$ up to and including 1988:2 and 1 thereafter.

### 4.2 Descriptive summary

The following commands import and display the data set described above in a statistical package *R*. The data set is stored under the file name “Raotbl3”.

```r
> library(urca)
> data(Raotbl3)
> attach(Raotbl3)
> Raotbl3
```

A descriptive summary of the Raotbl3 data set in natural logarithm is provided.
> summary(Raotbl3)


Figure 4.1: The consumption, income and wealth time series plot.

From figure 4.1, the plots of the consumption and income series do not vary about a fixed level, indicating nonstationarity in the mean but not in the variance, while the plot of the wealth series varies about a fixed level with constant variance, indicating nonstationarity in both the mean and the variance. This can be confirmed after testing for unit roots.
Table 4.1 below provides a descriptive summary of the data set.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum</th>
<th>Median</th>
<th>Mean</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>lc</td>
<td>10.48</td>
<td>10.77</td>
<td>10.79</td>
<td>11.14</td>
</tr>
<tr>
<td>li</td>
<td>10.58</td>
<td>10.90</td>
<td>10.89</td>
<td>11.24</td>
</tr>
<tr>
<td>lw</td>
<td>12.72</td>
<td>13.06</td>
<td>13.14</td>
<td>13.77</td>
</tr>
</tbody>
</table>

In all three variables, the means and the medians are not far away from each other. This probably indicates that the series in the data set are slightly symmetric. However, this does not indicate normality. A test for normality is performed.

### 4.3 Test for normality

The null and the alternative hypothesis of this set are the following:

- $H_0$: The series in the data set are normally distributed.
- $H_1$: The series in the data set are not normally distributed.

Using the Shapiro test, we test for normality of the variables to see if the series in the data set are well modelled by a normal distribution using the following R command:

```r
> shapiro.test(lc)
Shapiro-Wilk normality test
data: lc
W = 0.9458, p-value = 0.0004748
> shapiro.test(li)
Shapiro-Wilk normality test
data: li
W = 0.9618, p-value = 0.005767
> shapiro.test(lw)
Shapiro-Wilk normality test
data: lw
W = 0.9043, p-value = 2.517e-06
```

Table 4.2 provides the summary results of the normality test using Shapiro-Wilk test.

From the above R command, the p-values are very small which means the null hypothesis of normality can be rejected at the 1% level of significance for all series. This implies that the time series
Table 4.2: Normality test on the *Raotbl3* data set

<table>
<thead>
<tr>
<th>Variable</th>
<th>Shapiro-Wilk</th>
<th>p-value</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>lc</em></td>
<td>0.9458</td>
<td>$4.475 \times 10^{-4}$</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td><em>li</em></td>
<td>0.9618</td>
<td>$5.767 \times 10^{-3}$</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td><em>lw</em></td>
<td>0.9043</td>
<td>$2.517 \times 10^{-6}$</td>
<td>Reject the null hypothesis</td>
</tr>
</tbody>
</table>

of all the variables are not normally distributed.

In the next command, we test for the presence of unit roots in each of the series (consumption, income and wealth) using the ADF unit root test. Testing for unit root implies testing for nonstationarity in the series.

### 4.4 Augmented Dickey-Fuller (ADF) test for unit roots

We pre-test each variable to determine the order of integration using the ADF unit root test. This is because cointegration necessitates that variables be integrated of the same order. By determining the order of integration, we use the function `ur.df` in *R* on each variable. For each variable tested for unit root, we set both the null and alternative hypotheses as

\[
H_0 : \delta = 0 \quad \text{(4.4.1)}
\]

\[
H_1 : \delta < 0 \quad \text{(4.4.2)}
\]

After inspection of the behaviour of the ACF plot for the *lc* series, it was found that its residuals become white noise after lag five. This means that the ACF plot for the *lc* series cuts off at lag five. Since this series shows an increasing trend in figure 4.1, we test the ADF test on the trend model and lag five as follows:

```r
> lc.ct=ur.df(lc,lags=5,type='trend')
> summary(lc.ct)
```

Coefficients:

| Estimate  | Std. Error | t value | Pr(>|t|) |
|-----------|------------|---------|----------|
| (Intercept) | 0.8782632 | 0.3723647 | 2.359 | 0.0206 * |
| z.lag.1   | -0.0837027 | 0.0355904 | -2.352 | 0.0210 * |
| tt        | 0.0005533 | 0.0002246 | 2.463 | 0.0158 * |
| z.diff.lag1 | -0.0505981 | 0.1040693 | -0.486 | 0.6281 |
\[ z.\text{diff.lag2} \quad 0.1501023 \quad 0.1027518 \quad 1.461 \quad 0.1478 \]
\[ z.\text{diff.lag3} \quad 0.2411832 \quad 0.1036023 \quad 2.328 \quad 0.0223 \ast \]
\[ z.\text{diff.lag4} \quad -0.0810911 \quad 0.1078641 \quad -0.752 \quad 0.4543 \]
\[ z.\text{diff.lag5} \quad 0.2129158 \quad 0.1067675 \quad 1.994 \quad 0.0493 \ast \]

---

Multiple R-squared: 0.1993, Adjusted R-squared: 0.1333

Value of test-statistic is: -2.3518 3.6798 3.057

We are fitting the ADF model

\[ Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \alpha_i \sum_{i=1}^{m} \Delta Y_{t-i} + \epsilon_t. \]  \hfill (4.4.3)

In terms of the coefficient estimate, the above model becomes

\[ lc_t = 0.8782632 + 0.0005533 t - 0.0837027 lc_{t-1} - 0.0505981 \Delta lc_{t-1} + \ldots + 0.2129158 \Delta lc_{t-5} \] \hfill (4.4.4)

where \( lc_t \) is the consumption function series, \( tt \) is the trend, \( lc_{t-1} \) are the lagged values of \( lc \) and \( \Delta lc_{t-1} \) are the first difference lagged values of the consumption function. The parameter of interest in equation 4.4.4 is the estimated coefficient of \( lc_{t-1} \) which is \(-0.0837027\). The calculated test statistic \(-2.3518\) is derived from

\[ F_r = \frac{\delta}{SE(\delta)} = \frac{-0.0837027}{0.0355904} = -2.3518. \] \hfill (4.4.5)

As extracted from \( R \), the critical values for the unit root test are given in table 4.3.

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>6.50</td>
<td>4.88</td>
<td>4.16</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>8.73</td>
<td>6.49</td>
<td>5.47</td>
</tr>
</tbody>
</table>

Since the calculated test statistic \(-2.3518\), falls in the non-rejection region, that is, to the right of the \( \tau \) (tau) critical values, we cannot reject the null hypothesis for presence of unit roots at the 10% level of significance. This means that the consumption series contains a unit root.
Figure 4.2 shows a plot of the ADF unit root test for the consumption series. From this plot the residuals appear to vary about a fixed level. Time series that exhibit this phenomenon are said to be nonstationary in the mean but not in the variance. This confirms that the consumption series contains a unit root.

The ADF test for unit roots in real income ($li$) and real wealth ($lw$) respectively can be obtained from the following commands:

> lic=ts(li)
> lcc=ur.df(lic, lags=5, type='trend')
> summary(lcc)

Coefficients:

|            | Estimate | Std. Error | t value | Pr(>|t|) |
|-------------|----------|------------|---------|----------|
| (Intercept) | 1.5208232| 0.6263415  | 2.428   | 0.0173 * |
| z.lag.1     | -0.1434285| 0.0592676  | -2.420  | 0.0177 * |
| tt          | 0.0009068 | 0.0003671  | 2.470   | 0.0155 * |
| z.diff.lag1 | -0.0697724| 0.1097974  | -0.635  | 0.5268   |
| z.diff.lag2 | 0.1145049 | 0.1118766  | 1.023   | 0.3090   |
| z.diff.lag3 | -0.0248040| 0.1098202  | -0.226  | 0.8219   |
z.diff.lag4  0.2221217  0.1090673  2.037  0.0448 *
z.diff.lag5  0.0864713  0.1089450  0.794  0.4296

-----

Multiple R-squared: 0.1359,  Adjusted R-squared: 0.06477
Value of test-statistic is: -2.42 4.4181 3.0514

> kl=ts(lw)
> kl2=ur.df(kl, lags=5, type='trend')
> summary(kl2)

Coefficients:

|                | Estimate | Std. Error | t value | Pr(>|t|) |
|----------------|----------|------------|---------|----------|
| (Intercept)    | 0.3774736| 0.2591340  | 1.457   | 0.1489   |
| z.lag.1        | -0.0302509| 0.0203009  | -1.490  | 0.1399   |
| tt             | 0.0004702 | 0.0002252  | 2.088   | 0.0398 * |
| z.diff.lag1    | 0.1644410 | 0.1067375  | 1.541   | 0.1271   |
| z.diff.lag2    | -0.0001526| 0.1089781  | -0.001  | 0.9989   |
| z.diff.lag3    | 0.0465937 | 0.1099422  | 0.424   | 0.6728   |
| z.diff.lag4    | -0.0509364| 0.1163995  | -0.438  | 0.6628   |
| z.diff.lag5    | 0.2179245 | 0.1138548  | 1.914   | 0.0590   |

Multiple R-squared: 0.1247,  Adjusted R-squared: 0.0526
Value of test-statistic is: -1.4901 1.7591 2.207

Table 4.4: Summary of the ADF test for unit root in the variables (with a trend and intercept)

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF test statistic</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption series (lc)</td>
<td>-2.3518</td>
<td>Fail to reject the null hypothesis</td>
</tr>
<tr>
<td>Real income (li)</td>
<td>-2.4200</td>
<td>Fail to reject the null hypothesis</td>
</tr>
<tr>
<td>Real wealth (lw)</td>
<td>-1.4901</td>
<td>Fail to reject the null hypothesis</td>
</tr>
</tbody>
</table>

The ADF critical value for unit root at the 10% level of significance is −3.15. From table 4.4, it can be deduced that the three variables are nonstationary, each contain at least one unit root, that is, they are integrated of order one, \( I(1) \). We now test whether the integration is of the same order.
4.5 Augmented Dickey-Fuller (ADF) test for the order of integration

Here we test whether the series is possible $I(2)$, that is, whether they contain a second order of integration. This test is achieved by supplying the differenced series in the `ur.df` R function to test for presence of unit roots.

Figure 4.3: A plot of 1st differences of consumption, income and wealth series

From figure 4.3 it can be observed that the first difference series for the Raotbl3 data set are stationary both in the mean and the variance. This is because they fluctuate about the zero mean and exhibit constant variance. Since the ACF and the PACF plots cut off at lag five, we test the ADF test for the possible second order of integration using the no trend model and five lags as follows:

```r
> lc2.ct=ur.df(lc2,type="none", lags=5)
> summary(lc2.ct)
```

Coefficients:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| z.lag.1  | -0.28585   | 0.14698 | -1.945   | 0.05507 |
| z.diff.lag1 | -0.69758 | 0.15368 | -4.539   | 1.82e-05 *** |
Figure 4.4 exhibits a pattern of series stationary in the mean and in the variance. To verify this assertion we conduct a formal stationarity test (ADF test) on the residuals of the consumption series.

For the differenced real income and differenced real wealth respectively, the following commands are used:

> li2=diff(li)
> li2.ct=ur.df(li2,type="none", lags=5)
> summary(li2.ct)

Coefficients:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| z.lag.1  | -0.4853    | 0.2083  | -2.329   | 0.02219 * |

Residual standard error: 0.01374 on 90 degrees of freedom
Multiple R-squared: 0.579, Adjusted R-squared: 0.5602
F-statistic: 30.94 on 4 and 90 DF, p-value: 3.366e-16
Value of test-statistic is: -1.945
\begin{verbatim}
> lw2=diff(lw)
> lw2.ct=ur.df(lw2,type="none", lags=5)
> summary(lw2.ct)

Coefficients:

                  Estimate Std. Error t value  Pr(>|t|)
z.lag.1         -0.51732   0.19326  -2.677  0.0089 **
z.diff.lag1    -0.30022   0.19100  -1.572  0.1197
z.diff.lag2    -0.28261   0.17803  -1.587  0.1161
z.diff.lag3    -0.23015   0.16523  -1.393  0.1672
z.diff.lag4    -0.26658   0.14166  -1.882  0.0632 .
z.diff.lag5    -0.05817   0.11911  -0.488  0.6265

Residual standard error: 0.04313 on 90 degrees of freedom
Multiple R-squared: 0.3959,  Adjusted R-squared: 0.3691
Value of test-statistic is: -2.6769

The critical values for the ADF test for the order of integration are shown in the table 4.5 below.

<table>
<thead>
<tr>
<th>τ1</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.6</td>
<td>-1.95</td>
<td>-1.61</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.5: Critical values for the ADF order of integration test

Table 4.6 summarises the results of the ADF test for the order of integration.
\end{verbatim}
Table 4.6: Summary for ADF test for unit roots in variables (in 1\textsuperscript{st} difference form)

<table>
<thead>
<tr>
<th>Variable</th>
<th>test statistic</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differenced Consumption series (∆lc)</td>
<td>−1.9450</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>Differenced Real income (∆li)</td>
<td>−2.3292</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>Differenced Real wealth (∆lw)</td>
<td>−2.6769</td>
<td>Reject the null hypothesis</td>
</tr>
</tbody>
</table>

The ADF critical value at the 10\% level of significance is −1.61. Based on the ADF test, the first difference variables are stationary, which implies that variables $lc$, $li$ and $lw$ are integrated of order one, $I(1)$. Now we can test for the existence of a long-run relationship between the variables, that is, cointegration.

4.6 Tests for cointegration

There are various techniques to test for cointegration. The following techniques, the Engle-Granger method, the Phillips-Ouliaris methods and Johansen’s procedure, were applied to the \textit{Raotbl3} data set.

4.6.1 Engle-Granger method

In \textit{R}, the Engle-Granger two-step procedure is implemented in \textit{R} using the following commands.

```r
> lc=ts(lc,start=c(1966,4), end=c(1991,2), frequency=4)
> li=ts(li,start=c(1966,4), end=c(1991,2), frequency=4)
> lw=ts(lw,start=c(1966,4), end=c(1991,2), frequency=4)
> ukcons=window(cbind(lc,li,lw),start=c(1962,2),end=c(1991,2))
> lc.eq=summary(lm(lc~li+lw, data =ukcons))
> li.eq=summary(lm(li~lc+lw, data =ukcons))
> lw.eq=summary(lm(lw~li+lc, data =ukcons))
```

To implement the Engle-Granger method on \textit{Raotbl3} data set, we begin by regressing the consumption, income and wealth series on each other and then assess the model fit. If variables cointegrate, the resulting OLS regression yields a “super-consistent” estimator of the cointegrating parameters.
By this we mean that there is a very strong relationship between the estimated parameters. Taking the consumption series as the dependent variable and the other (income and wealth) as the independent variables, we yield the following regression equation

\[ lc = -0.178458 + 0.910971li + 0.079761lw. \]  

(4.6.1)

Call:

\texttt{lm(formula = lc \sim li + lw, data = ukcons)}

Coefficients:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| (Intercept) | -0.178458 | 0.103491 | -1.724 0.088 |
| li | 0.910971 | 0.013457 | 67.697 <2e-16 *** |
| lw | 0.079761 | 0.007764 | 10.274 <2e-16 *** |

Multiple R-squared: 0.9918, Adjusted R-squared: 0.9916

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

The \( p \)-values of the independent variables are very small (\( 2 \times 10^{-16} \)); this means that these regression coefficients are statistically significant at the 0.001 level of significance. The R-squared value is 0.9918, meaning that 99.18% of the variations in the consumption series are explained by changes in income and wealth series which reflects the true real-life situation. When income and wealth series are taken to be the dependent variables respectively, the following regression equations are obtained:

\[ li = 0.302998 + 1.075667lc - 0.077579lw, \]  

(4.6.2)

\[ lw = 1.0681 - 5.4625li + 6.6315lc. \]  

(4.6.3)

It is interesting to see that there is a negative relationship between income and wealth, meaning that having income (high or low) does not necessarily mean that you are wealthy. To determine if the variables actually cointegrate, we test whether the residuals from the regression relationship(s) are stationary. To extract and store the residuals of the regression equations, we use the following commands:

\begin{verbatim}
> error.lc=ts(resid(lc.eq),start=c(1967,2),end=c(1991,2),frequency=4)
> error.li=ts(resid(li.eq),start=c(1967,2),end=c(1991,2),frequency=4)
> error.lw=ts(resid(lw.eq),start=c(1967,2),end=c(1991,2),frequency=4)
\end{verbatim}

The residuals of these three long-run relationships are stored as objects \textit{error.lc}, \textit{error.li} and \textit{error.lw} in R respectively. When we plot the residuals \textit{error.lc} in figure 4.5 we observe that they are stationary both in mean and variance.
From the plot it is observed that all the lags are insignificant (within the confidence bound), we therefore test the none trend with zero lags ADF test on residuals as follows:

```r
> ci.lc=ur.df(error.lc,lags=0,type='none')
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
z.lag.1  -0.62472   0.09605  -6.504  3.63e-09
---
Multiple R-squared: 0.3427, Adjusted R-squared: 0.3286
Value of test-statistic is: -6.504
> ci.li=ur.df(error.li,lags=0,type='none')
> summary(ci.li)
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
z.lag.1  -0.62593   0.09644  -6.49  3.87e-09 ****
---
Multiple R-squared: 0.3395, Adjusted R-squared: 0.3253
Value of test-statistic is: -6.49
> ci.lw=ur.df(error.lw,lags=0,type='none')
> summary(ci.lw)
Coefficients:
```
Estimate Std. Error t value Pr(>|t|)
z.lag.1 -0.21469 0.07911 -2.714 0.00793 **
z.diff.lag -0.32469 0.09841 -3.299 0.00137 **

Multiple R-squared: 0.2472, Adjusted R-squared: 0.231

Value of test-statistic is: -2.7137

In performing the ADF test on residuals we are actually fitting the following model

$$\Delta \hat{\epsilon}_t = a_1 \hat{\epsilon}_{t-1} + \epsilon_t$$

(4.6.4)

where $\epsilon_t$ is the residual from the long-run regression relationship, and $a_1$ is the estimated regression coefficient of the lagged residuals from the long-run relationship, which is the parameter of interest.

The null and alternative hypotheses are set as

$$H_0 : a_1 = 0$$

(4.6.5)

$$H_0 : a_1 < 0$$

(4.6.6)

If we cannot reject the null hypothesis in equation 4.6.5, we conclude that the residuals contain a unit root and, hence the variables do not cointegrate. Instead, rejection of the null hypothesis in equation 4.6.5 above implies that residuals are stationary, hence the variables cointegrate.

As extracted from Hamilton (1994) the critical values for the test are given in table 4.7.

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.31</td>
<td>-3.77</td>
<td>-3.45</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.7: Critical values of ADF test for residuals

Table 4.8 summarises the ADF test results on residuals of the regression equation.

<table>
<thead>
<tr>
<th>ADF test statistic</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>consumption -6.50</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>income -6.49</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>wealth -2.71</td>
<td>Fail to reject the null hypothesis</td>
</tr>
</tbody>
</table>

The ADF critical value at the 10% level of significance is -3.45. Given that the consumption, income and wealth series are integrated of order one, that is, $I(1)$, and that the residuals are
stationary as shown in the summary in table 4.8, we can conclude that the three variables do not cointegrate.

In the next step, the error correction model (ECM) for the cointegrated series, that is, consumption, wealth and income functions are specified. In other words we fit the ECM model below:

$$\Delta lc_t = \beta_0 + \beta_1 \Delta li_t + \beta_2 \Delta lw_t + \alpha^* \varepsilon_{t-1} + u_t$$  \hspace{1cm} (4.6.7)

where $\alpha^* = -(1 - \alpha_1)$, $\varepsilon_{t-1} = [lc_{t-1} - \beta_0 - \beta_1 li_{t-1} - \beta_2 lw_{t-1}]$ as in equation 3.1.18. Using the necessary R command the error correction model is set out as follows:

```r
> lc.d=diff(lc)
> li.d=diff(li)
> lw.d=diff(lw)
> leq2=lag(error.lc)
> ecm=summary(lm(lc.d~li.d+lw.d+leq2))
```

Call:
`lm(formula = lc.d ~ li.d + lw.d + leq2)`

Coefficients:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| (Intercept) | 0.004774   | 0.001417 | 3.369    | 0.00110 ** |
| li.d     | 0.235293   | 0.073202 | 3.214    | 0.00179 ** |
| lw.d     | 0.025621   | 0.031233 | 0.820    | 0.41411    |
| leq2     | -0.097765  | 0.081055 | 1.206    | 0.0018**   |

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Residual standard error: 0.01299 on 94 degrees of freedom
Multiple R-squared: 0.1242, Adjusted R-squared: 0.09625
F-statistic: 4.444 on 3 and 94 DF, p-value: 0.005767

The estimated ECM is

$$\Delta \hat{lc}_t = 0.0048 + 0.2353 \Delta \hat{li} + 0.0256 \Delta \hat{lw} - 0.0977 \hat{\varepsilon}_{t-1}. \hspace{1cm} (4.6.8)$$

From the above equation we see that $\alpha^* = -0.0977$ enters with a correct sign (negative) but is large, that is, tends to 0 indicating that the speed of adjustment to equilibrium is slow. We can conclude that, ceteris paribus (keeping other factors constant), consumption and income series converge to a long-run cointegrating equilibrium.
4.6.1.1 Diagnostic error tests

In R, diagnostic tests are performed using the following commands:

```r
> jaque.bera.test(lag(error.lc), lag=3)
> arch(lag(error.lc),lag.single=3)
> box.test(lag(error.lc), lag=1,type="Ljung-Box")
> reset(lag(error.lc),type="regressors")
```

Table 4.9 summarises the results of the diagnostic test on residuals from the Raotbl3 data set.

<table>
<thead>
<tr>
<th>Test</th>
<th>Test statistic</th>
<th>p-value</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jarque-Bera</td>
<td>4.29</td>
<td>0.1169</td>
<td>Normally distributed</td>
</tr>
<tr>
<td>ARCH-LM</td>
<td>1.03</td>
<td>0.3101</td>
<td>No heteroscedasticity</td>
</tr>
<tr>
<td>Ljung-Box</td>
<td>26.64</td>
<td>0.0420</td>
<td>Serial correlation</td>
</tr>
<tr>
<td>Ramsey Reset</td>
<td>2.49</td>
<td>0.288</td>
<td>No misspecification</td>
</tr>
</tbody>
</table>

The p-values in table 4.9 are compared with the 0.10 level of significance.

4.6.2 Phillips-Ouliaris methods

The Phillips-Ouliaris methods are implemented by using two residual-based tests, namely

- the variance ratio test
- the multivariate ratio test.

4.6.2.1 The variance ratio test

In R, the variance ratio test is implemented in the function `ca.po` as follows:

```r
> lc=ts(lc,start=c(1966,4),end=c(1991,2),frequency=4)
> li=ts(li,start=c(1966,4),end=c(1991,2),frequency=4)
> lw=ts(lw,start=c(1966,4),end=c(1991,2),frequency=4)
> ukcons=window(cbind(lc,li,lw),start=c(1967,2),end=c(1991,2))
> pu.test=summary(ca.po(ukcons,demean='const',type='Pu'))
```

Test of type Pu detrending of series with constant only

Coefficients:

```r
Estimate Std. Error t value Pr(>|t|)
```

43
Table 4.10 summarises the results of the variance ratio test from the Raotbl3 data set.

<table>
<thead>
<tr>
<th>Level of significance</th>
<th>10%</th>
<th>5%</th>
<th>1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical value</td>
<td>33.67</td>
<td>40.52</td>
<td>53.87</td>
</tr>
<tr>
<td>Decision</td>
<td>Reject $H_0$</td>
<td>Reject $H_0$</td>
<td>Reject $H_0$</td>
</tr>
</tbody>
</table>

The test statistic for the variance ratio test ($\hat{P}_u$) is 58.91. From table 4.10, we observe that this calculated test statistic is larger than the critical value extracted from $R$ at the 1% level of significance. We therefore reject the null hypothesis of no cointegration against the alternative of the presence of cointegrating variables. This leads to the same conclusion reached while using the Engle-Granger two-step estimation procedure, that is, there is not a long-run relationship (cointegration) between consumption, income and wealth series in the Raotbl3 data set.

### 4.6.2.2 The multivariate trace statistic

In $R$, the multivariate statistic is implemented in the `ca.po` function as follows:

```r
> lc=ts(lc,start=c(1966,4),end=c(1991,2),frequency=4)
> li=ts(li,start=c(1966,4),end=c(1991,2),frequency=4)
> lw=ts(lw,start=c(1966,4),end=c(1991,2),frequency=4)
> ukcons=window(cbind(lc,li,lw),start=c(1967,2),end=c(1991,2))
> Pu.test=summary(ca.po(ukcons,demean='const',type='Pu'))
> Pu.test
```

Test of type Pu detrending of series with constant only

Coefficients:

| Estimate Std. Error t value Pr(>|t|) |
|-------------------------------------|
| (Intercept) -0.178458 0.103491   -1.724 0.088 |
| z[, -1]li 0.910971 0.013457 67.697 <2e-16 *** |
### Output of the multivariate trace statistic is

```r
> pz.test
Response lc:
Call:
lm(formula = lc ~ zr)
Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.022293  0.087754  -0.254 0.80003
zrlc      0.853380   0.085127 10.025  < 2e-16 ***
zrli      0.118152   0.078442  1.506  0.13544
zrlw      0.024679   0.009372  2.633  0.00992 **

Residual standard error: 0.01351 on 92 degrees of freedom
Multiple R-squared: 0.9945,   Adjusted R-squared: 0.9943
F-statistic: 5566 on 3 and 92 DF,  p-value: < 2.2e-16
```

```r
Response li:
Call:
lm(formula = li ~ zr)
Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.13002    0.10555  1.232  0.2211
zrlc      0.49383    0.10239  4.823 5.58e-06 ***
zrli      0.52867    0.09435  5.603 2.18e-07 ***
zrlw     -0.02429    0.01127 -2.155  0.0338 *

Residual standard error: 0.01625 on 92 degrees of freedom
Multiple R-squared: 0.9945,   Adjusted R-squared: 0.9943
F-statistic: 3697 on 3 and 92 DF,  p-value: < 2.2e-16
```

Response lw
Call:
\texttt{lm(formula = lw ~ zr)}

Coefficients:
\begin{verbatim}
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.31857   0.27819  -1.145  0.255 
zrlc        0.13079   0.26986   0.485  0.629 
zrli       -0.07984   0.24867  -0.321  0.749 
zrlw       0.98357    0.02971  33.106  <2e-16 ***
---
Value of test-statistic is: 88.0345
Critical values of Pz are:
\begin{verbatim}
   10pct  5pct  1pct
critical values  80.2034  89.7619 109.4525
\end{verbatim}

Table 4.11 summarises the results of the long-run relationship of the consumption, income and
wealth series using the Phillips-Ouliaris test.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Level of significance & 10\% & 5\% & 1\% \\
\hline
Critical value of $\hat{P}_a$ & 33.69 & 40.52 & 53.87 \\
Critical value of $\hat{P}_z$ & 80.20 & 89.76 & 109.45 \\
\hline
\end{tabular}
\caption{Phillips-Ouliaris:Cointegration test}
\end{table}

The calculated test statistics of the variance ratio test ($\hat{P}_a$) and the multivariate trace ($\hat{P}_z$) are
58.91 and 88.03 respectively. Since both tests are upper-tailed tests, the null hypothesis is rejected
if the test statistic is greater than the critical value. This implies than the null hypothesis is
rejected at 1\% significance level with the variance ratio test, but can only be rejected at the 10\%
significance level with the multivariate trace statistic.

4.6.3 Johansen’s method

Johansen proposes two likelihood ratio tests namely:

- The trace test
- The maximum eigenvalue.
4.6.3.1 The trace test

The trace test tests the null hypothesis of $r$ cointegrating vectors against the alternative hypothesis of $n$ cointegrating vectors. If $r = 0$, it means that there is no relationship among the variables that is stationary. In R, the trace test is implemented in `ca.jo` function as follows:

```r
> summary(ca.jo(data.frame(lc, li, lw), type="trace", ecdet="const"))
```

Test type: trace statistic, without linear trend and constant in cointegration

Eigenvalues (lambda):

```
[1] 3.037824e-01 1.028445e-01 2.280181e-02 4.874698e-16
```

Values of test statistic and critical values of test:

<table>
<thead>
<tr>
<th></th>
<th>test 10pct</th>
<th>5pct</th>
<th>1pct</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r \leq 2$</td>
<td>2.24</td>
<td>7.52</td>
<td>9.24 12.97</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>12.76</td>
<td>17.85</td>
<td>19.96 24.60</td>
</tr>
<tr>
<td>$r = 0$</td>
<td>47.89</td>
<td>32.00</td>
<td>34.91 41.07</td>
</tr>
</tbody>
</table>

Eigenvectors, normalised to first column:

(These are the cointegration relations)

```
lc.l2 1.00000000 1.0000000 1.0000000 1.0000000
li.l2 -0.95747202 -0.8679656 -1.8045735 -0.9901210
lw.l2 -0.04852978 -0.1045337 0.2832673 -0.5369134
constant 0.29129705 0.0234724 5.0886246 7.0576721
```

Weights $W$:

(This is the loading matrix)

```
lc.d 0.2422490 -0.18249990 -0.0002770341 -7.625075e-15
li.d 0.4914478 0.05256545 0.0046028620 -2.131011e-14
lw.d 0.3021879 0.10564804 -0.0483327582 -8.724908e-15
```

4.6.3.2 The maximum eigenvalue

The maximum eigenvalue test, on the other hand, tests the null hypothesis of $r$ cointegrating vectors against the alternative hypothesis of $(r + 1)$ cointegrating vectors. In R it is implemented in the `ca.jo` function as follows:

```r
> summary(ca.jo(data.frame(lc, li, lw), type="eigen", ecdet="const"))
```

Test type: maximal eigenvalue statistic (lambda max), without linear trend
and constant in cointegration

Eigenvalues (lambda):

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>3.037824e-01</td>
<td>1.028445e-01</td>
<td>2.280181e-02</td>
</tr>
</tbody>
</table>

Values of teststatistic and critical values of test:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>test</td>
<td>10pct</td>
<td>5pct</td>
</tr>
<tr>
<td>r &lt;= 2</td>
<td>2.24</td>
<td>7.52</td>
</tr>
<tr>
<td>r &lt;= 1</td>
<td>10.53</td>
<td>13.75</td>
</tr>
<tr>
<td>r = 0</td>
<td>35.12</td>
<td>19.77</td>
</tr>
</tbody>
</table>

Eigenvectors, normalised to first column:

(These are the cointegration relations)

<table>
<thead>
<tr>
<th>lc.l2</th>
<th>li.l2</th>
<th>lw.l2</th>
<th>constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>lc.l2</td>
<td>1.00000000</td>
<td>1.00000000</td>
<td>1.00000000</td>
</tr>
<tr>
<td>li.l2</td>
<td>-0.95747202</td>
<td>-0.86796562</td>
<td>-1.8045735</td>
</tr>
<tr>
<td>lw.l2</td>
<td>-0.04852978</td>
<td>-0.10453373</td>
<td>0.2832673</td>
</tr>
<tr>
<td>constant</td>
<td>0.29129705</td>
<td>0.02347248</td>
<td>5.0886246</td>
</tr>
</tbody>
</table>

Weights W:

(This is the loading matrix)

<table>
<thead>
<tr>
<th>lc.d</th>
<th>li.d</th>
<th>lw.d</th>
<th>constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>lc.d</td>
<td>0.2422490</td>
<td>-0.18249990</td>
<td>-0.0002770341</td>
</tr>
<tr>
<td>li.d</td>
<td>0.4914478</td>
<td>0.05256545</td>
<td>0.0046028620</td>
</tr>
<tr>
<td>lw.d</td>
<td>0.3021879</td>
<td>0.10564804</td>
<td>-0.0483327582</td>
</tr>
</tbody>
</table>

Table 4.12 summarises results of Johansen’s cointegration methodology on the Raotbl3 data set.

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>Alternative</th>
<th>test statistic</th>
<th>10%</th>
<th>5%</th>
<th>1%</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_{trace}$ test</td>
<td>$r \leq 2$</td>
<td>$r &gt; 2$</td>
<td>2.24</td>
<td>7.52</td>
<td>9.24</td>
<td>12.97</td>
</tr>
<tr>
<td></td>
<td>$r \leq 1$</td>
<td>$r &gt; 1$</td>
<td>12.76</td>
<td>17.85</td>
<td>19.96</td>
<td>24.60</td>
</tr>
<tr>
<td></td>
<td>$r = 0$</td>
<td>$r &gt; 0$</td>
<td>47.89</td>
<td>32.00</td>
<td>34.91</td>
<td>41.07</td>
</tr>
<tr>
<td>$J_{max}$ test</td>
<td>$r = 2$</td>
<td>$r = 3$</td>
<td>2.24</td>
<td>7.52</td>
<td>9.24</td>
<td>12.97</td>
</tr>
<tr>
<td></td>
<td>$r = 1$</td>
<td>$r = 2$</td>
<td>10.53</td>
<td>13.75</td>
<td>15.67</td>
<td>20.20</td>
</tr>
<tr>
<td></td>
<td>$r = 0$</td>
<td>$r = 1$</td>
<td>35.12</td>
<td>19.77</td>
<td>22.00</td>
<td>26.81</td>
</tr>
</tbody>
</table>

From table 4.12, it can be observed that
(i) the null hypothesis of no cointegration \( r = 0 \) against the alternative of presence of one or more cointegrating vector is rejected at the 10\% level of significance in both techniques (trace test and maximum eigenvalue). This implies that cointegration exists between the consumption, income and wealth series of the Raotbl3 data set.

(ii) the null hypothesis \( r \leq 1 \) and \( r \leq 2 \) against the alternative of the existence of two or three cointegrating vectors is not rejected by both tests. This means that there is no more than one cointegration relationship in the Raotbl3 data set.

4.7 Concluding remarks

Since the three methods used to test for cointegration are not consistent, that is, they do not yield the same results, it can be concluded that there is no cointegration between the consumption, income and wealth time series in the Raotbl3 data set. The results of this analysis can be used to assess the impact of a temporary shock such as a birthday bonus, or a permanent shock such as annual salary raise on an economy.
Chapter 5

Testing for Cointegration in the World Economic Indicators

5.1 Introduction

World economic indicators are specific indices and measures that not only indicate the overall wealth of the global economy, but also provide some insight into its future. Some economic indicators use statistics to illustrate the ups and downs of particular trends in economic activities. The most commonly used world economic indicators are the rates of inflation, the unemployment rates, the real gross domestic product (GDP) growth rates, GDP per capita, GDP purchasing power parity, amounts of foreign direct investments, populations living below the poverty line, and current account balances. In this data set we assess the existence of long-run equilibrium (cointegration) in the following Australia’s economic indicators:

- Total Producer Price Index Manufacturing (PPI): This is an index that shows the cost of resources needed to produce manufactured goods during the previous month. The PPI measures change in effective prices received by domestic producers of the manufacturing sector for that part of their output which is sold on the domestic market.

- Domestic Producer Price Index of Finished goods (DPPI): This measures how much money manufacturers and wholesalers pay for finished goods and materials.

- Consumer Price Index (CPI): This is a measure which estimates the average price of the consumer goods and services purchased by households. The CPI measures change in a constant market basket of goods and services from one period to the next within the same
If cointegration exists in these economic indicators, it implies that they are procyclic, that is, they move in the direction of the economic movement (or cycle) of a country. This means that the movement of economic indicators is directly proportional to the trend of Australia’s economic performance. If cointegration does not exist, it means that these economic indicators are countercyclic, that is, they are inversely related to economic performance (Pauly, 2003). Consequently, the PPI, the DPPI and the CPI series in this data set are transformed to natural logarithms creating new variables, which are defined as:

- \( ppi \) equivalent to the natural logarithm of \( PPI \)
- \( dppi \) equivalent to the natural logarithm of \( DPPI \)
- \( cpi \) equivalent to the natural logarithm of \( CPI \).

This transformation is a remedy to the violation of the assumptions of normality in the dataset such as constant variance and independence of the error term.

More details about the data set can be extracted from United Nations World Economic Indicators (http://quanis1.easydata.co.za/TableViewer/tableView.aspx). In this study the world economic indicators data set is included as Appendix B.

### 5.2 Descriptive summary

A descriptive summary of the data set in natural logarithm is provided.

<table>
<thead>
<tr>
<th></th>
<th>ppi</th>
<th>dppi</th>
<th>cpi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min.</td>
<td>4.387</td>
<td>4.368</td>
<td>4.403</td>
</tr>
<tr>
<td>1st Qu.</td>
<td>4.501</td>
<td>4.458</td>
<td>4.507</td>
</tr>
<tr>
<td>Median</td>
<td>4.546</td>
<td>4.560</td>
<td>4.578</td>
</tr>
<tr>
<td>Mean</td>
<td>4.585</td>
<td>4.569</td>
<td>4.579</td>
</tr>
<tr>
<td>3rd Qu.</td>
<td>4.699</td>
<td>4.669</td>
<td>4.651</td>
</tr>
<tr>
<td>Max.</td>
<td>4.808</td>
<td>4.758</td>
<td>4.734</td>
</tr>
</tbody>
</table>

Figure 5.1 is a plot of the total \( ppi \) for manufacturing, the \( dppi \) of finished goods and the \( cpi \) time series.
The plot of the \textit{ppi}, \textit{dppi} and \textit{cpi} series indicates nonstationarity in the mean and in the variance with an increasing trend. This implies that transformation and/or differencing is required to reduce the time series to stationarity.

Table 5.1 below provides a descriptive summary of the data set.

<table>
<thead>
<tr>
<th></th>
<th>Minimum</th>
<th>Median</th>
<th>Mean</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textit{ppi}</td>
<td>4.387</td>
<td>4.546</td>
<td>4.585</td>
<td>4.808</td>
</tr>
<tr>
<td>\textit{dppi}</td>
<td>4.368</td>
<td>4.560</td>
<td>4.569</td>
<td>4.758</td>
</tr>
<tr>
<td>\textit{cpi}</td>
<td>4.403</td>
<td>4.578</td>
<td>4.579</td>
<td>4.734</td>
</tr>
</tbody>
</table>

From table 5.1, it may be observed that the minimum value, the median, the mean and the maximum value are close to each other. This indicates that this data set is symmetric but does not necessarily show that it is normally distributed.
5.3 Test for normality

We perform a test for normality as follows: Using the Shapiro-Wilk test, we test for normality of variables to see if the series in the data set are well modelled by a normal distribution using the following R command:

```r
> shapiro.test(ppi)
Shapiro-Wilk normality test
data: ppi
W = 0.8458, p-value = 0.006748

> shapiro.test(dppi)
Shapiro-Wilk normality test
data: dppi
W = 0.9618, p-value = 0.004767

> shapiro.test(cpi)
Shapiro-Wilk normality test
data: cpi
W = 0.9044, p-value = 2.517e-03
```

Table 5.2 provides the summary results of the normality test using the Shapiro-Wilk test.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Shapiro-Wilk</th>
<th>p-value</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>ppi</td>
<td>0.8458</td>
<td>6.748 \times 10^{-3}</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>dppi</td>
<td>0.9618</td>
<td>4.767 \times 10^{-3}</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>cpi</td>
<td>0.9044</td>
<td>2.517 \times 10^{-3}</td>
<td>Reject the null hypothesis</td>
</tr>
</tbody>
</table>

From the above R command, the p-values are very small (less than 0.01) which means the null hypothesis of normality can be rejected at the 1% level of significance for all series. This implies that the time series of the variables are not normally distributed. However, this was expected, since all variables in this data set were transformed by taking a natural logarithm.

5.4 Augmented Dickey-Fuller (ADF) test for unit roots

In this section we perform the ADF test for unit roots to determine if the ppi, the dppi and the cpi series are nonstationary. After inspection of the behaviour of the ACF plot for the ppi series,
it was found that its residuals become white noise after lag five. This means that the ACF plot for the ppi series cuts off at lag five. Since this series shows an increasing trend in figure 5.1, we test the ADF test on the trend model and lag five as follows:

```r
> la=ts(ppi)
> df=ur.df(ppi,lags=5,type='trend')
> summary(df)
```

```
# Augmented Dickey-Fuller Test Unit Root Test#

Coefficients:

| Estimate  | Std. Error  | t value | Pr(>|t|) |
|-----------|-------------|---------|----------|
| (Intercept) | 0.929926    | 0.465990 | 1.996    | 0.0540   |
| la.lag.1   | -0.210870   | 0.106813 | -1.974   | 0.0565   |
| tt         | 0.001775    | 0.001031 | 1.723    | 0.0940   |
| la.diff.lag1 | 0.440848    | 0.173157 | 2.546    | 0.0156 *|
| la.diff.lag2 | 0.004497    | 0.186146 | 0.024    | 0.9809   |
| la.diff.lag3 | 0.138793    | 0.182940 | 0.759    | 0.4533   |
| la.diff.lag4 | 0.024497    | 0.176146 | 0.139    | 0.6809   |
| la.diff.lag5 | 0.338793    | 0.192940 | 1.756    | 0.5433   |

Multiple R-squared: 0.216, Adjusted R-squared: 0.1007
F-statistic: 1.874 on 5 and 34 DF, p-value: 0.1249
Value of test-statistic is: -1.9742
Value of the test statistic is: -2.1568
Value of the test statistic is: -3.1018
Critical values for test statistics:

1pct  5pct  10pct
tau3 -4.15 -3.50 -3.18
phi2  7.02  5.13  4.31
phi3  9.31  6.73  5.61

Table 5.3 provides summary results of the ADF unit root test.
Table 5.3: Summary of the ADF unit root test

<table>
<thead>
<tr>
<th>Variable</th>
<th>Test statistic</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>ppi</td>
<td>−1.9742</td>
<td>Fail to reject null hypothesis</td>
</tr>
<tr>
<td>dppi</td>
<td>−2.1568</td>
<td>Fail to reject null hypothesis</td>
</tr>
<tr>
<td>cpi</td>
<td>−3.1018</td>
<td>Fail to reject null hypothesis</td>
</tr>
</tbody>
</table>

The critical value at the 10% level of significance of tau3 is −3.18. The results of the ADF unit root test indicate that all variables in this data set are nonstationary. This is because we fail to reject the hypothesis of the presence of unit roots.

From figure 5.2, it can be observed that the residuals of the ppi series are nonstationary both in the mean and the variance. This suggests differencing and/or transformation to reduce nonstationarity to stationarity. In this way we are determining the order of integration.

Recall that cointegration requires that the series in the data set should be integrated (nonstationary) of the same order and their linear combination must be stationary. Now that we have shown in table 5.3 that all the variables are integrated, in the next section we determine the order of integration.
5.5 Augmented Dickey-Fuller (ADF) test for order of integration

Here we test whether the series is possible \( I(2) \), that is, whether it contains a second order of integration. This test is achieved by supplying the differenced series in the `ur.df R` function to test for the presence of unit roots. In R, the order of integration is determined by using the ADF unit root test on a differenced series. We difference the series to transform a nonstationary series to stationarity. Recall that the original series portrayed an increasing trend in figure 5.1 and the ACF is significant at lag five. Therefore, the ADF test for the possible second order of integration is tested on trend model with lag five as follows:

```r
> la.ct=diff(ppi)
> la.ct1=ur.df(la.ct,lags=5,type='trend')
> summary(la.ct1)
```

# Augmented Dickey-Fuller Test Unit Root Test#

Coefficients:

| Estimate       | Std. Error | t value | Pr(>|t|) |
|----------------|------------|---------|----------|
| (Intercept)    | 0.0088469  | 0.0076142 | 1.162    | 0.2536   |
| la.ct.lag.1    | -0.7113754 | 0.3132107 | -2.271   | 0.0298   *|
| tt             | -0.0001833 | 0.0002734 | -0.670   | 0.5072   |
| la.ct.diff.lag1| 0.0707510  | 0.2680152 | 0.264    | 0.7934   |
| la.ct.diff.lag2| -0.0457867 | 0.2363041 | -0.194   | 0.8476   |
| la.ct.diff.lag3| -0.0499768 | 0.2039751 | -0.245   | 0.8080   |
| la.ct.diff.lag4| -0.03972   | 0.13448   | -0.295   | 0.7690   |
| la.ct.diff.lag5| -0.06029   | 0.13181   | -0.457   | 0.6495   |

Value of test-statistic is: -2.3812

Value of test-statistic is: -2.2717

Value of test-statistic is: -3.8251

Critical values for test statistics:

<table>
<thead>
<tr>
<th>1pct</th>
<th>5pct</th>
<th>10pct</th>
</tr>
</thead>
<tbody>
<tr>
<td>tau3</td>
<td>-4.15</td>
<td>-3.60</td>
</tr>
<tr>
<td>phi2</td>
<td>7.02</td>
<td>5.13</td>
</tr>
<tr>
<td>phi3</td>
<td>9.31</td>
<td>6.73</td>
</tr>
</tbody>
</table>
Figure 5.3 is a plot of the first difference residuals of the \textit{ppi} series. From this plot it can be observed that the mean of the residuals is constant and the \textit{PACF} is significant at lag five, that is, the \textit{PACF} cuts off at lag five. This confirms that the trend model with lag five is appropriate to this series in testing for the order of integration.

Figure 5.3: A plot of the 1\textit{st} difference residuals of the \textit{ppi} series

![Residuals](image)

![Autocorrelations of Residuals](image)

![Partial Autocorrelations of Residuals](image)

Table 5.4 provides summary results of the first order differenced ADF unit root test.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Test statistic</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differenced \textit{ppi}</td>
<td>$-2.3812$</td>
<td>Fail to reject $H_0$</td>
</tr>
<tr>
<td>Differenced \textit{dppi}</td>
<td>$-2.2717$</td>
<td>Fail to reject $H_0$</td>
</tr>
<tr>
<td>Differenced \textit{cpi}</td>
<td>$-3.8251$</td>
<td>Reject $H_0$</td>
</tr>
</tbody>
</table>

Using the critical value at the 5\% level of significance, table 5.4 indicates that the null hypothesis of the presence of the first order integration for the \textit{cpi} is rejected. This implies that only the \textit{cpi} is integrated of order one, that is, \textit{I}(1;1). To obtain the same order of integration for all the variables in the data set, we re-difference the series and perform the unit root test again as follows:
```r
> la.ct2=diff(la.ct)
> la.ct3=ur.df(la.ct2,lags=5,type='trend')
> summary(la.ct3)
```

Coefficients:

```
Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.0010109 0.0077187 0.131 0.896621
la.ct2.lag.1-2.2249989 0.5516184 -4.034 0.000319 ***
tt -0.0001356 0.0003125 -0.434 0.667217
la.ct2.diff.lag1 0.7918639 0.4474334 1.770 0.086291 .
la.ct2.diff.lag2 0.3490917 0.3247018 1.075 0.290363
la.ct2.diff.lag3 0.0099522 0.1869345 0.053 0.957873
la.ct2.diff.lag4 -0.0810911 0.1078641 -0.752 0.4543
la.ct2.diff.lag5 0.2129158 0.1067675 1.994 0.0493 *
```

Value of test-statistic is: -4.0336
Value of test-statistic is: -4.2299
Value of test-statistic is: -4.7550

Critical values for test statistics:

```
1pct 5pct 10pct
tau3 -4.15 -3.50 -3.18
phi2 7.02 5.13 4.31
phi3 9.31 6.73 5.61
```

Table 5.5 provides the summary results of the ADF unit root test for the second order of integration.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Test statistic</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second difference ppi</td>
<td>-4.0336</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>Second difference dppi</td>
<td>-4.2299</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>Second difference cpi</td>
<td>-4.7550</td>
<td>Reject the null hypothesis</td>
</tr>
</tbody>
</table>

Using the critical value at the 5% level of significance, table 5.5 indicates that the null hypothesis of the second order difference unit root test is rejected in all series of this data set. This means that all the variables are integrated of order two, that is, $I(2; 2)$. In the following subsections, tests for the presence of long-run relationships (cointegration) are provided by testing for the stationarity
of the residuals of the series.

5.6 Tests for cointegration

5.6.1 Engle-Granger method

This two-step estimation procedure starts with fitting the linear regression models of the series, taking each variable, one at a time, as the dependent variable and the rest as independent variables. Residuals from the regression models are extracted and stored in statistical software $R$. The ADF test for unit root is then tested on the estimated residuals. If the residuals are stationary, the variables under investigation have long-run relationships. Hence, cointegration exists among variables.

Taking $ppi$ as the dependent variable, the regression model is fitted as follows:

```r
> data2=cbind(ppi,dppi,cpi)
> ppi.eq=summary(lm(ppi~dppi+cpi),data=data2)
> ppi.eq
```

The resulting model is

\[ ppi = 0.5497 + 1.1963dppi - 0.3124cpi. \] (5.6.1)

Equation 5.6.1 shows that there is an inverse relationship between the $ppi$ and the $cpi$. This implies that as one variable increases the other one decreases. It can also be deduced that the model fits the data set well, $R^2 = 0.9572$. This means that 95.72% of the variations in the $ppi$ are explained by changes in the $dppi$ and the $cpi$. 

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When the \( dp_{pi} \) and the \( cpi \) are taken to be the dependent variables, the following regression models are fitted:

\[
dp_{pi} = -0.86152 + 0.35597pp_{i} + 0.82953cpi, \tag{5.6.2}
\]

\[
cpi = 1.02681 - 0.09816pp_{i} + 0.87593dp_{pi}. \tag{5.6.3}
\]

The estimated residuals of equations 5.6.1, 5.6.2 and 5.6.3 are extracted and stored as:

\[
> \text{error.ppi}=(\text{resid(ppi.eq)})
\]
\[
> \text{error.dppi}=(\text{resid(dp_{i}.eq)})
\]
\[
> \text{error.cpi}=(\text{resid(cpi.eq)})
\]

The ADF test for unit root on the estimated residuals is performed as:

\[
> \text{summary(}\text{ur.df(error.ppi, lags=1, type='none'))}
\]

The value of the test statistic is: -2.3734

The value of the test statistic is: -2.4588

The value of the test statistic is: -2.0856

As extracted from Hamilton (1994) the critical values for the test are given in table 5.6.

![Table 5.6: Critical values of ADF test for residuals](image)

Table 5.7 provides summary results for the ADF unit root test for the nonstationarity of residuals in the estimated regression model.
Table 5.7: Engle-Granger cointegration test

<table>
<thead>
<tr>
<th>Variable</th>
<th>Test statistic</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>error.ppi</td>
<td>-2.3734</td>
<td>Fail to reject the null hypothesis</td>
</tr>
<tr>
<td>error.dppi</td>
<td>-2.4588</td>
<td>Fail to reject the null hypothesis</td>
</tr>
<tr>
<td>error.cpi</td>
<td>-2.0856</td>
<td>Fail to reject the null hypothesis</td>
</tr>
</tbody>
</table>

The ADF critical value at the 10% level of significance is -3.45. From table 5.7 the null hypothesis of the presence of unit roots in the residuals of the regression model is not rejected at the 10% level of significance. This implies that the residuals of the estimated regression models are nonstationary.

Figure 5.4: A plot of the ADF unit root test on the residuals of the ppi series

![Residuals plot](image)

Autocorrelations of Residuals

![Partial Autocorrelations of Residuals](image)

Figure 5.4 shows that the residuals of the ppi series are nonstationary. This confirms that the variables in the data set do not cointegrate.

Based on the two-step estimation procedure, it can be concluded that Australia’s PPI for manu-
facturing, the \textit{DPPI} for finished goods and its \textit{CPI} do not cointegrate.

In the next step, the error correction model (ECM) is fitted as follows:

\[ \Delta ppi_t = \beta_0 + \beta_1 \Delta dppi_t + \beta_2 \Delta cpi + \alpha^* \varepsilon_{t-1} + u_t \]  \hspace{1cm} (5.6.4)

where \( \alpha^* = - (1 - \alpha_1) \), \( \varepsilon_{t-1} = [ppi_{t-1} - \beta_0 - \beta_1 dppi_{t-1} - \beta_2 cpi_{t-1}] \) as in equation 3.1.18. Using the necessary \textit{R} command the error correction model is set out as follows:

\begin{verbatim}
> ppi.d=diff(ppi)
> dppi.d=diff(dppi)
> cpi.d=diff(cpi)
> leq1=lag(error.ppi)
> ecm=summary(lm(ppi.d~dppi.d+cpi.d+leq1))
\end{verbatim}

\begin{verbatim}
Call:
lm(formula = ppi.d ~ dppi.d + cpi.d + leq1)
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.005678   0.001417  4.369  0.00510
  dppi.d    0.978583   0.073202 13.546  0.00379 **
     cpi.d    0.679321   0.031233  21.656  0.00379 **
     leq1    -0.790765   0.081055  9.775  0.0018**
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 0.7899 on 44 degrees of freedom
Multiple R-squared:  0.7642,   Adjusted R-squared:  0.70625
F-statistic: 5.444 on 3 and 44 DF,  p-value: 0.005767
\end{verbatim}

The estimated ECM is

\[ \Delta \hat{ppi}_t = 0.0057 + 0.9785 \Delta \hat{dppi} + 0.6793 \Delta \hat{cpi} - 0.7908 \hat{\varepsilon}_{t-1}. \]  \hspace{1cm} (5.6.5)

From the above equation, \( \alpha^* = -0.7908 \) enters with a small correct sign (negative), that is, tends to \(-1\) indicating that the speed of adjustment to equilibrium is high. We can conclude that, ceteris paribus (keeping other factors constant), Australia’s economic indicators do not cointegrate and the ECM pushes the economy back to equilibrium at a high rate.

5.6.1.1 Diagnostic error tests

In \textit{R}, diagnostic tests are performed using the following commands:
> jaque.bera.test(lag(error.ppi), lag=3)
> arch(lag(error.ppi), lag.single=3)
> box.test(lag(error.ppi), lag=1, type="Ljung-Box")
> reset(lag(error.ppi), type="regressors")

Table 5.8 summarises the results of the diagnostic test on residuals from the Raotbl3 data set.

<table>
<thead>
<tr>
<th>Test</th>
<th>Test statistic</th>
<th>p-value</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jarque-Bera</td>
<td>3.26</td>
<td>0.2116</td>
<td>Normally distributed</td>
</tr>
<tr>
<td>ARCH-LM</td>
<td>2.07</td>
<td>0.2101</td>
<td>No heteroscedasticity</td>
</tr>
<tr>
<td>Ljung-Box</td>
<td>13.64</td>
<td>0.0020</td>
<td>Serial correlation</td>
</tr>
<tr>
<td>Ramsey Reset</td>
<td>4.45</td>
<td>0.0328</td>
<td>Misspecification</td>
</tr>
</tbody>
</table>

The p-values in table 5.8 are compared with the 0.10 significance level.

5.6.2 Phillips-Ouliaris methods

The Phillips-Ouliaris methods are implemented by using two residual-based tests.

5.6.2.1 The variance ratio test

In R, the variance ratio test is implemented in the function ca.po as follows:

> Pu.test=summary(ca.po(data2, demean='const', type='Pu'))
> Pu.test

########################################
# Phillips and Ouliaris Unit Root Test #
########################################

Test of type Pu detrending of series with constant only

Coefficients:

Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.5497 0.3105 1.770 0.0841 .
z[, -1]dppi 1.1963 0.2169 5.515 2.12e-06 ***
z[, -1]cpi -0.3124 0.2743 -1.139 0.2614

Value of test-statistic is: 5.5393

Critical values of Pu are:
critical values 33.6955 40.5252 53.8731

Table 5.9 summarises the results of the variance ratio test from the economic indicators data set.

<table>
<thead>
<tr>
<th>Level of significance</th>
<th>10%</th>
<th>5%</th>
<th>1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical value</td>
<td>33.67</td>
<td>40.52</td>
<td>53.87</td>
</tr>
<tr>
<td>Decision</td>
<td>Fail to reject $H_0$</td>
<td>Fail to reject $H_0$</td>
<td>Fail to reject $H_0$</td>
</tr>
</tbody>
</table>

The test statistic for the variance ratio test ($\hat{P}_n$) is 5.54. From the above analysis, we observe that this calculated test statistic is smaller than the critical values as extracted from $R$ at the 10% level of significance. Therefore, at all significance levels the null hypothesis of no cointegration is not rejected.

5.6.2.2 The multivariate trace statistic

In $R$, the multivariate trace statistic is implemented in the function `ca.po` as follows:

```r
> Pz.test=summary(ca.po(data2,demean='const',type='Pz'))
> Pz.test
```

Response `ppi`:

Coefficients:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| (Intercept) | 0.4780 | 0.2235 | 2.139 | 0.0388 * |
| zrppi | 0.7959 | 0.1122 | 7.095 | 1.59e-08 *** |
| zrdppi | 0.4239 | 0.1989 | 2.131 | 0.0395 * |
| zrcpi | -0.3213 | 0.1921 | -1.673 | 0.1024 |

Response `dppi`:

Coefficients:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| (Intercept) | 0.09041 | 0.08092 | 1.117 | 0.271 |
| zrppi | -0.06707 | 0.04062 | -1.651 | 0.107 |
| zrdppi | 1.08963 | 0.07201 | 15.131 | <2e-16 *** |
| zrcpi | -0.04005 | 0.06954 | -0.576 | 0.568 |

Response `cpi`:

Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.174910 0.075116 2.329 0.0252 *
zrppi -0.006555 0.037702 -0.174 0.8629
zrdpbi 0.104021 0.066848 1.556 0.1278
zrcbi 0.866235 0.064553 13.419 3.38e-16 ***

Value of test-statistic is: 15.1071
Critical values of Pz are:

<table>
<thead>
<tr>
<th>Level</th>
<th>10pct</th>
<th>5pct</th>
<th>1pct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical</td>
<td>80.2034</td>
<td>89.7619</td>
<td>109.4525</td>
</tr>
<tr>
<td>Decision</td>
<td>Fail to reject $H_0$</td>
<td>Fail to reject $H_0$</td>
<td>Fail to reject $H_0$</td>
</tr>
</tbody>
</table>

Table 5.10 summarises the results of the multivariate trace statistic test from the economic indicators data set.

<table>
<thead>
<tr>
<th>Level of significance</th>
<th>10%</th>
<th>5%</th>
<th>1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical value</td>
<td>80.2034</td>
<td>89.7619</td>
<td>109.4525</td>
</tr>
<tr>
<td>Decision</td>
<td>Fail to reject $H_0$</td>
<td>Fail to reject $H_0$</td>
<td>Fail to reject $H_0$</td>
</tr>
</tbody>
</table>

The test statistic for the multivariate test statistic ($\hat{P}_z$) is 15.11. From the above analysis, we observe that this calculated test statistic is smaller than the critical values extracted from R at the 10% level of significance. Therefore, at all levels of significance the null hypothesis of presence of no cointegration is not rejected.

### 5.6.3 Johansen’s method

Johansen’s method is implemented on two likelihood ratio tests, namely:

- The trace test
- The maximum eigenvalue.

#### 5.6.3.1 The trace test

In R, the trace statistic is implemented as follows:

```R
> summary(ca.jo(data2,type='trace',ecdet='const'))
```

```
# Johansen-Procedure #
```

65
Trace statistic, without linear trend and constant in cointegration

Eigenvalues (lambda):

\[
\begin{array}{c}
[1] 5.875376e-01 \\
2.607680e-01 \\
1.616397e-01 \\
1.135093e-15
\end{array}
\]

Values of test statistic and critical values of test:

\[
\begin{array}{cccc}
test & 10\text{pct} & 5\text{pct} & 1\text{pct} \\
r \leq 2 & 7.40 & 7.52 & 9.24 & 12.97 \\
r \leq 1 & 20.09 & 17.85 & 19.96 & 24.60 \\
r = 0 & 57.29 & 32.00 & 34.91 & 41.07
\end{array}
\]

Eigenvectors, normalised to first column:

These are the cointegration relations:

\[
\begin{array}{cccc}
\text{ppi.l2} & \text{dppi.l2} & \text{cpi.l2} & \text{constant} \\
\text{ppi.l2} & 1.000000 & 1.000000 & 1.000000 & 1.000000 \\
\text{dppi.l2} & -0.484825 & -1.893377 & 0.682246 & -4.221935 \\
\text{cpi.l2} & -0.316992 & 1.282587 & -2.207354 & 2.920107 \\
\text{constant} & -1.838918 & -1.808761 & 2.408345 & 1.317851
\end{array}
\]

Weights W:

This is the loading matrix:

\[
\begin{array}{cccc}
\text{ppi.l2} & \text{dppi.l2} & \text{cpi.l2} & \text{constant} \\
\text{ppi.d} & -0.007918 & -0.312298 & -0.025432 & -2.390012e-13 \\
\text{dppi.d} & -0.010171 & -0.063305 & -0.033532 & -2.505351e-15 \\
\text{cpi.d} & -0.011029 & -0.050507 & 0.035569 & -9.709437e-14
\end{array}
\]

5.6.3.2 The maximum eigenvalue

The multivariate trace statistic is implemented as follows:

\[
> \text{summary(ca.jo(data2,type='eigen',ecdet='const'))}
\]

# Johansen-Procedure #

Maximal eigenvalue statistic (lambda max), without linear trend and constant in cointegration

Eigenvalues (lambda):

\[
\begin{array}{c}
[1] 5.875376e-01 \\
2.607680e-01 \\
1.616397e-01 \\
1.135093e-15
\end{array}
\]

Values of test statistic and critical values of test:

\[
\begin{array}{cccc}
test & 10\text{pct} & 5\text{pct} & 1\text{pct} \\
r \leq 2 & 7.40 & 7.52 & 9.24 & 12.97
\end{array}
\]

66
Table 5.11 provides summary results of Johansen’s cointegration method.

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>Alternative</th>
<th>Test statistic</th>
<th>10%</th>
<th>5%</th>
<th>1%</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r \leq 2 )</td>
<td>( r &gt; 2 )</td>
<td>7.40</td>
<td>7.52</td>
<td>9.24</td>
<td>12.97</td>
<td>Do not reject ( H_0 )</td>
</tr>
<tr>
<td>( r \leq 1 )</td>
<td>( r &gt; 1 )</td>
<td>20.09</td>
<td>17.85</td>
<td>19.96</td>
<td>24.60</td>
<td>Reject ( H_0 )</td>
</tr>
<tr>
<td>( r = 0 )</td>
<td>( r &gt; 0 )</td>
<td>57.29</td>
<td>32.00</td>
<td>34.91</td>
<td>41.07</td>
<td>Reject ( H_0 )</td>
</tr>
</tbody>
</table>

From table 5.11 it can be observed that

- cointegration exists among Australia’s economic indicators, since the null hypothesis of no cointegration \( (r = 0) \) is rejected

- there is at most one cointegration vector relationship between these variables, since the null hypothesis of \( r \leq 2 \) is not rejected.
5.7 Concluding remarks

From the results of the analysis, it is observed that the Engle-Granger method and the Phillips-Ouliaris methods indicate no cointegration yet the Johansen’s method indicates presence of cointegration in Australia’s economic indicators. Since the three methods are not consistent, that is, they do not yield the same results, it can be concluded that the $PPI$ (Manufacturing), the $PPI$ (Finished goods) and the $CPI$ do not cointegrate. This means that these economic indicators are countercyclic, that is, they move in opposite directions of Australia’s economy.
Chapter 6

Testing Cointegration Results for the \( UK_{pppuip} \) Data Set

6.1 Introduction

In this data set we test for cointegration in the purchasing power parity (\( PPP \)) and the uncovered interest rate parity (\( UIP \)) for the United Kingdom. The \( PPP \) is the economic concept that continuously adjusts exchange rates between countries in order to denote the purchasing power of each country; that is, \( PPP \) refers to the use of the long-term equilibrium exchange rate of two countries to equalise purchasing power.

Purchasing power is the number of goods/services that can be purchased with a unit of currency. A parity condition occurs when the difference in the interest rate between two countries is equal to the expected change in exchange rate between the countries’ currencies. It can be expressed as

\[
(i_1 - i_2) = E(e)
\]

where, \( i_1 \) represents the interest rate in country 1, \( i_2 \) represents the interest rate in country 2, and \( E(e) \) represents the expected rate of change in the exchange rate.

In the \( UK_{pppuip} \) data set, we use quarterly data spanning a range from 1972:Q1 to 1987:Q2 where Q1 and Q2 are the first and second quarter respectively. In this study the \( UK_{pppuip} \) data set is included as Appendix C.

The variables are

- \( p_1 \) : UK wholesale price index.
\( p_2 \): Trade weighted foreign wholesale price.

\( i_1 \): Three month Treasury Bill rate.

\( i_2 \): Three month Eurodollar rate.

\( e_{12} \): UK effective exchange rate.

\( d_{oilp0} \): World oil price at period \( t \).

\( d_{oilp1} \): World oil price at period \( t-1 \).

Figure 6.1 is a plot of the \( UKpppuip \) data set.

Figure 6.1: A plot of \( p_1, p_2, e_{12}, i_1, i_2, d_{oilp0} \) and \( d_{oilp1} \) series

Figure 6.1 indicates nonstationarity in the mean with an increasing trend for the \( p_1, p_2 \) and the \( e_{12} \) series. For the \( i_1 \) and the \( i_2 \) series the plot indicates nonstationarity in both the mean and the variance. Moreover, the series of the \( d_{oilp0} \) and the \( d_{oilp1} \) appear to be stationary. These results can only be confirmed after performing a formal unit root test for nonstationarity.
6.2 Descriptive summary

A descriptive summary of the \textit{UKpppuip} data set in natural logarithm is provided.

\begin{verbatim}
> library(urca)
>attach(UKpppuip)
>
> summary(UKpppuip)

p1  p2  e12  i1
Min. :3.400 Min. :3.847 Min. :-4.900 Min. :0.04679
1st Qu.:3.967 1st Qu.:4.288 1st Qu.: -4.641 1st Qu.:0.08771
Median :4.486 Median :4.530 Median : -4.514 Median :0.10368
Mean :4.361 Mean :4.494 Mean : -4.538 Mean :0.10182
3rd Qu.:4.801 3rd Qu.:4.777 3rd Qu.: -4.429 3rd Qu.:0.11560
Max. :4.995 Max. :4.841 Max. : -4.263 Max. :0.15418
i2  doilp0  doilp1
Min. :0.04860 Min. : -0.52843 Min. : -0.52843
1st Qu.:0.06471 1st Qu.: 0.00000 1st Qu.: 0.00000
Median :0.08687 Median : 0.00000 Median : 0.00000
Mean :0.09105 Mean : 0.03581 Mean : 0.03476
3rd Qu.:0.10953 3rd Qu.: 0.05625 3rd Qu.: 0.05384
Max. :0.16924 Max. : 0.96554 Max. : 0.96554
\end{verbatim}

Table 6.1 provides a descriptive summary of the \textit{UKpppuip} data set.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum</th>
<th>Median</th>
<th>Mean</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>3.400</td>
<td>4.486</td>
<td>4.361</td>
<td>4.995</td>
</tr>
<tr>
<td>$p_2$</td>
<td>3.847</td>
<td>4.530</td>
<td>4.494</td>
<td>4.841</td>
</tr>
<tr>
<td>$e_{12}$</td>
<td>-4.900</td>
<td>-4.514</td>
<td>-4.538</td>
<td>-4.263</td>
</tr>
<tr>
<td>$l_1$</td>
<td>0.048</td>
<td>0.104</td>
<td>0.102</td>
<td>0.154</td>
</tr>
<tr>
<td>$i_2$</td>
<td>0.049</td>
<td>0.087</td>
<td>0.091</td>
<td>0.169</td>
</tr>
<tr>
<td>doilp_0</td>
<td>-0.528</td>
<td>0.000</td>
<td>0.036</td>
<td>0.966</td>
</tr>
<tr>
<td>doilp_1</td>
<td>-0.528</td>
<td>0.000</td>
<td>0.035</td>
<td>0.966</td>
</tr>
</tbody>
</table>
6.3 Test for normality

The null and the alternative hypothesis of this set are set as follows:

\( H_0 \): The series in the data set are normally distributed
\( H_1 \): The series in the data set are not normally distributed

Using the Shapiro-Wilk test, we test for the normality of the variables to see if the series in the data set are well modelled by a normal distribution using the following R command:

```r
> shapiro.test(p1)
Shapiro-Wilk normality test
data: normality
W = 0.8997, p-value = 9.973e-05

> shapiro.test(p2)
Shapiro-Wilk normality test
data: p2
W = 0.9025, p-value = 0.0001275

> shapiro.test(e12)
Shapiro-Wilk normality test
data: e12
W = 0.9637, p-value = 0.06355

> shapiro.test(i1)
Shapiro-Wilk normality test
data: i1
W = 0.9823, p-value = 0.5102

> shapiro.test(i2)
Shapiro-Wilk normality test
data: i2
W = 0.9309, p-value = 0.001783

> shapiro.test(doilp0)
Shapiro-Wilk normality test
data: doilp0
W = 0.6285, p-value = 2.931e-11
```

72
data: doilp1
\[ W = 0.6237, \text{ p-value } = 2.439 \times 10^{-11} \]

Table 6.2 provides the summary results of the normality test using the Shapiro-Wilk test.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Shapiro-Wilk</th>
<th>( p )-value</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1 )</td>
<td>0.8997</td>
<td>( 9.973 \times 10^{-5} )</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>( p_2 )</td>
<td>0.9025</td>
<td>( 1.275 \times 10^{-4} )</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>( e_{12} )</td>
<td>0.9637</td>
<td>( 6.355 \times 10^{-1} )</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>( i_1 )</td>
<td>0.9823</td>
<td>( 5.102 \times 10^{-1} )</td>
<td>Do not reject the null hypothesis</td>
</tr>
<tr>
<td>( i_2 )</td>
<td>0.9309</td>
<td>( 1.783 \times 10^{-3} )</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>( doilp_0 )</td>
<td>0.6285</td>
<td>( 2.931 \times 10^{-11} )</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>( doilp_1 )</td>
<td>0.6237</td>
<td>( 2.439 \times 10^{-11} )</td>
<td>Reject the null hypothesis</td>
</tr>
</tbody>
</table>

Since the \( p \)-values are very small (less than 0.01), the null hypothesis of normality can be rejected at the 1% level of significance for all variables except the \( i_1 \). This implies that most of these time series variables are not all normally distributed.

### 6.4 Augmented Dickey-Fuller (ADF) test for unit roots

The aim of the unit root test is to test for nonstationarity of the variables in the data set. A nonstationarity test is performed by testing for the existence of unit roots in each variable of the data set. We set the null and the alternative hypotheses as

\[
H_0 : \delta = 0 \tag{6.4.1}
\]

\[
H_1 : \delta < 0. \tag{6.4.2}
\]

If the null hypothesis of \( \delta = 0 \) is not rejected, it means that the variable contains a unit root, that is, it is nonstationary.

After inspection of the behaviour of the ACF plot for the \( p_1 \) series, it was found that its residuals become white noise after lag six. This means that the ACF plot for the \( p_1 \) series cuts off at lag six. Since this series shows an increasing trend in figure 6.1, we test the ADF test on the trend model and lag six as follows:
> pl.ct=ur.df(pl,lags=6,type="trend")
> summary(pl.ct)
Test regression trend
Call:
  lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.09688 0.05032  1.925  0.06039 .
z.lag.1      -0.01464 0.01459 -1.004  0.32070
  tt           -0.00027 0.00044 -0.603  0.54932
z.diff.lag1   0.48694 0.14258  3.415  0.00134 **
z.diff.lag2   -0.09225 0.15186 -0.608  0.54650
z.diff.lag3   -0.01516 0.15255 -0.099  0.92125
z.diff.lag4   0.01502 0.15246  0.099  0.92196
z.diff.lag5   -0.07088 0.15255 -0.469  0.64107
z.diff.lag6   -0.16149 0.12488 -1.293  0.20241
Multiple R-squared: 0.7543, Adjusted R-squared: 0.7115
Value of test-statistic is: -1.0038 8.5796 10.7345

The estimated model is

\[ p_{1t} = 0.09869 - 0.000265tt - 0.014645p_{1t-1} - 0.48694\Delta p_{1t-2} - \ldots - 0.16149\Delta p_{1t-6}. \quad (6.4.3) \]

The calculated test statistic is derived from

\[ F_{\tau} = \frac{\hat{\delta}}{SE(\hat{\delta})} = \frac{-0.0146446}{0.0145885} = -1.0038. \quad (6.4.4) \]

As extracted from R, the critical values for the ADF unit root test are given in table 6.3.

<table>
<thead>
<tr>
<th></th>
<th>1%</th>
<th>5%t</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau)</td>
<td>-4.04</td>
<td>-3.45</td>
<td>-3.15</td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td>6.50</td>
<td>4.88</td>
<td>4.16</td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>8.73</td>
<td>6.49</td>
<td>5.47</td>
</tr>
</tbody>
</table>

Since the calculated test statistic, -1.0038, falls within the non-rejection region, that is, to the right of the \(\tau\)(tau) critical values, we cannot reject the null hypothesis for the presence of unit
roots at the 10% level of significance. This means that the variable, UK wholesale price index, \( p_1 \), in the data set contains a unit root.

Figure 6.2 is a plot of the residuals of the foreign wholesale price \( (p_1) \). From the figure, we observe that the series is nonstationary in the mean but not in the variance.

The ADF test output for unit roots for all other variables in this data set is obtained using the following \( R \) commands:

\begin{verbatim}
> p2.ct=ur.df(p2,lags=6,type='trend')
> summary(p2.ct)
> plot(p1.ct)
> e12.ct=ur.df(e12,lags=6,type='trend')
> summary(e12.ct)
> i1.ct=ur.df(i1,lags=6,type='none')
> summary(i1.ct)
> i2.ct=ur.df(i2,lags=6,type='none')
\end{verbatim}
> summary(i2.ct)
> doilp0.ct=ur.df(doilp0,lags=6,type='none')
> summary(doilp0.ct)
> doilp1.ct=ur.df(doilp1,lags=6,type='none')
> summary(doilp1.ct)

Table 6.4 summarises the results of the ADF unit root test for the *UKppp* data set.

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF test statistic</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale price index (<em>p</em>₁)</td>
<td>-1.0038</td>
<td>Fail to reject the null hypothesis</td>
</tr>
<tr>
<td>Weighted foreign wholesale price (<em>p</em>₂)</td>
<td>-1.4112</td>
<td>Fail to reject the null hypothesis</td>
</tr>
<tr>
<td>Effective exchange rate (<em>e</em>₁₂)</td>
<td>-2.0664</td>
<td>Fail to reject the null hypothesis</td>
</tr>
<tr>
<td>Treasury bill rate (<em>i</em>₁)</td>
<td>-0.4923</td>
<td>Fail to reject the null hypothesis</td>
</tr>
<tr>
<td>Eurodollar rate (<em>i</em>₂)</td>
<td>-0.9492</td>
<td>Fail to reject the null hypothesis</td>
</tr>
<tr>
<td>World oil price at period <em>t</em> (<em>doilp₀</em>)</td>
<td>-4.1297</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>World oil price at period <em>t₁</em> (<em>doilp₁</em>)</td>
<td>-4.1042</td>
<td>Reject the null hypothesis</td>
</tr>
</tbody>
</table>

The ADF critical value at the 10% level of significance is −3.15. Table 6.4 shows that the null hypothesis of presence of unit roots for variables *doilp₀* and *doilp₁* is rejected. This means that all other variables are nonstationary except for the world oil price.

It is possible for the world oil price to be stationary since its price is fixed by international organisations outside a country, for example the Organisation of Petroleum Exporting Countries (OPEC). Such prices are relatively stable because they are not affected by structural fluctuations in a country’s economic activities. For example, if employees strike in a country like South Africa, the price of a barrel of oil on the international market is not affected, although the exchange rates in South Africa may be affected.

Since the first five variables contain a unit root, it is possible that they are integrated. In the next section, the ADF test for the order of integration is performed.
6.5 Augmented Dickey-Fuller (ADF) test for order of integration

In this test, each variable in the data set is differenced to reduce the nonstationary series to stationarity, and the ADF test is performed on the differenced series for a possible second order of integration, $I(2)$, using the following R commands:

```r
> p1t=diff(p1)
> p1t.ct=ur.df(p1t)
> plot(p1t.ct)
```

Figure 6.3: A plot of the first difference $p_1, p_2, e_{12}, i_1, i_2, doilp_0$ and $doilp_1$ series

From the plot, it can be observed that the first difference series are nonstationary in the mean but not in the variance. This means that the no trend model is appropriate in testing for the unit root. Inspection of the ACF plots showed that the ACF’s of the series in this data set are significant at
lag six. We therefore test for the possible second order of integration using the ADF test for unit root on the no trend model with six lags are follows:

```r
> p1t=diff(p1)
> p1t.ct=ur.df(p1t, lags=6,type='none')
> summary(p1t.ct)
> p2t=diff(p2)
> p2t.ct=ur.df(p2t, lags=6,type='none')
> summary(p2t.ct)
> e12t=diff(e12)
> e12t.ct=ur.df(e12t, lags=6,type='none')
> summary(e12t.ct)
> i1t=diff(i1)
> i1t.ct=ur.df(i1t, lags=6,type='none')
> summary(i1t.ct)
> i2t=diff(i2)
> i2t.ct=ur.df(i2t, lags=6,type='none')
> summary(i2t.ct)
```

For the UK wholesale price index, the output is given by:

```r
> summary(p1t.ct)
```

Coefficients:

| Estimate  | Std. Error  | t value | Pr(>|t|)  |
|-----------|-------------|---------|-----------|
| z.lag.1   | -0.04719    | -0.928  | 0.3582    |
| z.diff.lag1| -0.05744    | -0.408  | 0.6853    |
| z.diff.lag2| -0.11241    | -0.833  | 0.4089    |
| z.diff.lag3| -0.10667    | -0.789  | 0.4343    |
| z.diff.lag4| -0.03972    | -0.295  | 0.7690    |
| z.diff.lag5| -0.06029    | -0.457  | 0.6495    |
| z.diff.lag6| -0.26352    | -2.000  | 0.0513    |

Multiple R-squared: 0.1351, Adjusted R-squared: 0.006268
Value of test-statistic is: -0.928

Critical values for test statistics:

1pct  5pct  10pct
Figure 6.4 is a plot of the ADF test on the first difference $p_1$ series.

Figure 6.4: A plot of the ADF unit root test for the first difference $p_1$ series

Table 6.5 summarises the results of the ADF test order of integration.

Table 6.5: Summary for ADF test for the order of integration

<table>
<thead>
<tr>
<th>Variable</th>
<th>Test statistic</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differenced Wholesale price index ($\Delta p_1$)</td>
<td>$-0.928$</td>
<td>Fail to reject the null hypothesis</td>
</tr>
<tr>
<td>Differenced Weighted foreign wholesale price ($\Delta p_2$)</td>
<td>$-1.5796$</td>
<td>Fail to reject the null hypothesis</td>
</tr>
<tr>
<td>Differenced Effective exchange rate ($\Delta e_{12}$)</td>
<td>$-2.1458$</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>Differenced Treasury bill rate $\Delta i_1$</td>
<td>$-3.6266$</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>Differenced Eurodollar rate $\Delta i_2$</td>
<td>$-2.7766$</td>
<td>Reject the null hypothesis</td>
</tr>
</tbody>
</table>

The ADF critical value at the 10% level of significance is $-1.61$. Only three variables in table 6.5 are integrated of order one, $I(1, 1)$, the presence of a long-run equilibrium relationship (cointegration) is tested for in the following sections.
6.6 Tests for cointegration

6.6.1 Engle-Granger method

In the two-step estimation procedure, we fit a regression line of nonstationary variables taking each variable, one at a time, to be the dependent variable and the rest as independent variables. Thereafter, we test for the stationarity of the residuals from each estimated regression line. If the residuals are found to be stationary, then the variables cointegrate. In R this is done as follows:

```r
> data1=cbind(p1,p2,e12,i1,i2)
> p1.eq=summary(lm(p1~p2+e12+i1+i2),data=data1)
> p1.eq
Call:
  lm(formula = p1 ~ p2 + e12 + i1 + i2)
Coefficients:
  Estimate Std. Error t value Pr(>|t|)
(Intercept)  -2.3008   0.8004 -2.875  0.00568 **
p2             1.6131   0.0631  25.566 < 2e-16 ***
e12            0.1197   0.1228  0.975  0.33383
i1            -0.7080   0.4609  1.536  0.13004
i2             0.3116   0.4421  0.705  0.48372
Signif. codes:  0 ***  0.001 **  0.01 *  0.05 .  0.1  1
Residual standard error: 0.0655 on 57 degrees of freedom
Multiple R-squared: 0.9847, Adjusted R-squared: 0.9836
F-statistic: 918.2 on 4 and 57 DF, p-value: < 2.2e-16
```

Considering \( p_1 \) as the dependent variable, the estimated regression line is

\[
\hat{p}_1 = -2.30081 + 1.61307p_2 + 0.11967e_{12} - 0.70801i_1 + 0.31163i_2. \tag{6.6.1}
\]

From the output we observe that only \( p_2 \) is statistically significant (\( p\text{-value} = 2 \times 10^{-16} \)) at the 1% significance level. Since the \( R^2 = 0.98 \) is higher and the correlation coefficient between \( p_2 \) and
\( e_{12} \) is also high (-0.88), we suspect serial correlation or misspecification or both in the residuals of \( p_1 \), but we have to perform diagnostic tests to confirm this.

Taking the other remaining variables as dependent variables, we fit the following regression lines:

\[
\hat{p}_2 = 2.27064 + 0.57021p_1 + 0.06968e_{12} + 0.37518i_1 + 0.15738i_2. \quad (6.6.2)
\]

\[
\hat{e}_{12} = -6.0199 + 0.1370p_1 + 0.2256p_2 + 0.76101i_1 - 2.2714i_2. \quad (6.6.3)
\]

\[
\hat{i}_1 = 0.15960 - 0.05615p_1 + 0.08417p_2 + 0.05272e_{12} + 0.52827i_2. \quad (6.6.4)
\]

\[
\hat{i}_2 = -1.06968 + 0.02773p_1 + 0.03962p_2 - 0.17659e_{12} + 0.59285i_1. \quad (6.6.5)
\]

The residuals of the estimated regression lines above are extracted from the following commands:

\[
> \text{error.p1=(resid(p1.eq))}
\]
\[
> \text{error.p2=(resid(p2.eq))}
\]
\[
> \text{error.e12=(resid(e12.eq))}
\]
\[
> \text{error.i1=(resid(i1.eq))}
\]
\[
> \text{error.i2=(resid(i2.eq))}
\]

Nonstationarity in the estimated residuals of the regression lines is tested for using the ADF test as follows:

\[
> \text{p1.lc=ur.df(error.p1,type='none')}
\]
\[
> \text{summary(p1.lc)}
\]
\[
> \text{plot(p1.lc)}
\]

Figure 6.5: A plot of ADF test on residuals of wholesale price.
Figure 6.5 shows that the residuals of long-run relationships are stationary in mean but not in variance. This suggests that taking $p_1$ as the normalisation variables there is no evidence of cointegration in the $UKppwip$ data set. The nonstationarity of the estimated residuals of the regression lines for other variables is tested using the the ADF test as follows:

```r
> p2.lc=ur.df(error.p2,type='none')
> p2.lc
> e12.lc=ur.df(error.e12,type='none')
> e12.lc
> i1.lc=ur.df(error.i1,type='none')
> i1.lc
> i2.lc=ur.df(error.i2,type='none')
> i2.lc
```

# Augmented Dickey-Fuller Test Unit Root / Cointegration Test #

The value of the test statistic is: -2.4662
The value of the test statistic is: -2.1394
The value of the test statistic is: -3.5934
The value of the test statistic is: -4.1602
The value of the test statistic is: -4.0841

As extracted from Hamilton (1994) the critical values for the test are given in table 6.6.

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-4.31</td>
<td>-3.77</td>
<td>-3.45</td>
</tr>
</tbody>
</table>

Table 6.7 summarises the results of the ADF test for the stationarity of residuals.
Table 6.7: Engle-Granger cointegration test

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF test statistic</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale price index ($p_1$)</td>
<td>−2.4662</td>
<td>Fail to reject the null hypothesis</td>
</tr>
<tr>
<td>Weighted foreign wholesale price ($p_2$)</td>
<td>−2.1394</td>
<td>Fail to reject the null hypothesis</td>
</tr>
<tr>
<td>Effective exchange rate ($e_{12}$)</td>
<td>−3.5934</td>
<td>Fail to reject the null hypothesis</td>
</tr>
<tr>
<td>Treasury bill rate ($i_1$)</td>
<td>−4.1602</td>
<td>Fail to reject the null hypothesis</td>
</tr>
<tr>
<td>Eurodollar rate ($i_2$)</td>
<td>−4.0841</td>
<td>Fail to reject the null hypothesis</td>
</tr>
</tbody>
</table>

The ADF critical value at the 1% level of significance is −4.31. Based on Engle-Granger’s two-step procedure, we cannot reject the null hypothesis of no cointegration in all five variables.

In the next step, we estimate the following the error-correction model:

\[
\Delta p_1 = \beta_0 + \beta_1 \Delta p_2 + \beta_2 \Delta e_{12} + \beta_3 \Delta i_1 + \beta_4 \Delta i_2 + \alpha^* \varepsilon_{t-1} + \varepsilon_t \tag{6.6.6}
\]

where \(\alpha^* = -(1 - \alpha_1)\) as the speed of adjustment to equilibrium, and \(\varepsilon_{t-1}\) is the error-correction mechanism. In R this is done as follows:

```R
> p1.l=lag(error.p1)
> p1.d=diff(p1)
> p2.d=diff(p2)
> e12.d=diff(e12)
> i1.d=diff(i1)
> i2.d=diff(i2)
> ecm1=summary(lm(p1.d~p2.d+e12.d+i1.d+i2.d+p1.l))
```

Call:

\(\text{lm(formula = p1.d \sim p2.d + e12.d + i1.d + i2.d + p1.l)}\)

Coefficients:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| (Intercept) | 0.004774 | 0.001417 | 3.369 | 0.00110 ** |
| p2.d | 0.235293 | 0.073202 | 3.214 | 0.00179 ** |
| e12.d | 0.027621 | 0.031233 | 0.820 | 0.41411 |
| i1.d | 0.02623 | 0.01324 | 0.987 | 0.43211 |
| i2.d | 0.07624 | 0.013726 | 0.107 | 0.43211 |
| p1.l | 0.07865 | 0.081055 | 1.206 | 0.0812 |

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 0.01299 on 94 degrees of freedom
Multiple R-squared: 0.1242, Adjusted R-squared: 0.09625
F-statistic: 4.444 on 3 and 94 DF, p-value: 0.005767

The estimated ECM is

\[ \Delta \hat{p}_t = 0.0048 + 0.2353 \Delta \hat{p}_{t-1} + 0.0276 \Delta \hat{e}_{12} + 0.0262 \Delta \hat{i}_1 + 0.0762 \Delta \hat{i}_2 + 0.0787 \hat{\varepsilon}_{t-1}. \] (6.6.7)

From the above equation we observe that \( \alpha^* = 0.0787 \) enters with the wrong sign (positive); this is an indication that the system diverges from equilibrium with time.

### 6.6.1 Diagnostic error tests

In R, diagnostic tests are performed using the following commands:

```R
> jaque.bera.test(lag(error.p1), lag=3)
> arch(lag(error.p1), lag.single=3)
> box.test(lag(error.p1), lag=1, type="Ljung-Box")
> reset(lag(error.p1), type="regressors")
```

Table 6.8 summarises the results of the diagnostic test on residuals.

<table>
<thead>
<tr>
<th>Test</th>
<th>Test statistic</th>
<th>p-value</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jarque-Bera</td>
<td>18.267</td>
<td>0.3169</td>
<td>Normally distributed</td>
</tr>
<tr>
<td>ARCH-LM</td>
<td>52.724</td>
<td>0.000</td>
<td>Heteroscedasticity</td>
</tr>
<tr>
<td>Ljung-Box</td>
<td>140.19</td>
<td>0.9640</td>
<td>No serial correlation</td>
</tr>
<tr>
<td>Ramsey Reset</td>
<td>34.7334</td>
<td>0.8754</td>
<td>No misspecification</td>
</tr>
</tbody>
</table>

The p-values in table 6.8 are compared with the 5% level of significance.

### 6.6.2 Phillips-Ouliaris methods

The Phillips-Ouliaris methods are implemented by using two residual-based tests.

#### 6.6.2.1 The variance ratio test

In R, the variance ratio test is implemented in the function `ca.po` as follows:

```R
> pu.test=summary(ca.po(data1, demean='const', type='Pu'))
```
> pu.test

########################################################
# Phillips and Ouliaris Unit Root Test #
########################################################
Test of type Pu
detrending of series with constant only
Call:
  lm(formula = z[, 1] ~ z[, -1])
Coefficients:

          Estimate Std. Error t value Pr(>|t|)
(Intercept) -2.30081  0.80040  -2.875  0.00568 **
z[, -1]p2  1.61307  0.06309   25.566 < 2e-16 ***
z[, -1]e12  0.11967  0.12278   0.975  0.33383
z[, -1]i1  -0.70801  0.46090  -1.536  0.13004
z[, -1]i2  0.31163  0.44207   0.705  0.48372
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 0.06547 on 57 degrees of freedom
Multiple R-squared: 0.9847,  Adjusted R-squared: 0.9836
F-statistic: 918.2 on 4 and 57 DF,  p-value: < 2.2e-16
Value of test-statistic is: 1.2458
Critical values of Pu are:
  10pct  5pct  1pct
critical values 45.3308 53.2502 71.5214

Table 6.9 summarises the results of the variance ratio test from the UKpppuip data set.

| Table 6.9: Results from the variance ratio test |
|-------------------------------|---|---|---|
| Level of significance | 10% | 5% | 1% |
| Critical value        | 45.3308 | 53.2502 | 71.5214 |
| Decision              | Fail to reject $H_0$ | Fail to reject $H_0$ | Fail to reject $H_0$ |

The test statistic for the variance ratio test ($\hat{P}_u$) is 1.2458. From table 6.9, we observe that this calculated test statistic is smaller than the critical values extracted from R at the 10% level of significance. Therefore, the null hypothesis of no cointegration is not rejected at all levels of significance.
6.6.2.2 The multivariate trace statistic

In R, the multivariate trace statistic is implemented in the function `ca.po` as follows:

```r
> pz.test=summary(ca.po(data1,demean='const',type='Pz'))
> pz.test
Value of test-statistic is: 55.1917
Critical values of Pz are:

<table>
<thead>
<tr>
<th></th>
<th>10pct</th>
<th>5pct</th>
<th>1pct</th>
</tr>
</thead>
<tbody>
<tr>
<td>critical values</td>
<td>168.8572</td>
<td>182.0749</td>
<td>209.8054</td>
</tr>
</tbody>
</table>
```

Table 6.10 summarises the results of the multivariate trace statistic from the UKppui data set.

<table>
<thead>
<tr>
<th>Level of significance</th>
<th>10%</th>
<th>5%</th>
<th>1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical value</td>
<td>168.8572</td>
<td>182.0749</td>
<td>209.8054</td>
</tr>
<tr>
<td>Decision</td>
<td>Fail to reject $H_0$</td>
<td>Fail to reject $H_0$</td>
<td>Fail to reject $H_0$</td>
</tr>
</tbody>
</table>

The test statistic for the multivariate test statistic ($\hat{P}_z$) is 55.1917. From table 6.10, we observe that this calculated test statistic is smaller than the critical values extracted from R at the 10% level of significance. Therefore, the null hypothesis of no cointegration is not rejected at all levels of significance.

6.6.3 Johansen’s method

Johansen’s method is implemented on two likelihood ratio tests namely:

- The trace test
- The maximum eigenvalue.

6.6.3.1 The trace test

```r
> summary(ca.jo(data1,type="trace",ecdet="const"))
```

# Johansen-Procedure #

Test type: trace statistic, without linear trend and constant in cointegration

Eigenvalues (lambda):
Values of test statistic and critical values of test:

\[
\begin{array}{c|cccc}
\text{r} & \text{test} & 10\text{pct} & 5\text{pct} & 1\text{pct} \\
\hline
\text{r} \leq 4 & 5.09 & 7.52 & 9.24 & 12.97 \\
\text{r} \leq 3 & 16.03 & 17.85 & 19.96 & 24.60 \\
\text{r} \leq 2 & 36.86 & 32.00 & 34.91 & 41.07 \\
\text{r} \leq 1 & 60.93 & 49.65 & 53.12 & 60.16 \\
\text{r} = 0 & 105.15 & 71.86 & 76.07 & 84.45 \\
\end{array}
\]

Eigenvectors, normalised to first column:

(These are the cointegration relations)

\[
\begin{array}{cccccccc}
\text{p1.l2} & \text{p2.l2} & \text{e12.l2} & \text{i1.l2} & \text{i2.l2} & \text{constant} \\
\hline
\text{p1.l2} & 1 & 1 & 1 & 1 & 1 & 1 \\
\text{p2.l2} & -0.7346918 & -0.9207675 & -1.361829 & -1.9915177 & -1.4252911 & -1.0358045 \\
\text{e12.l2} & -0.9704284 & -1.7829624 & -2.109588 & 0.6873137 & -0.7242021 & -0.2570326 \\
\text{i1.l2} & -2.8848474 & -7.3432095 & 23.583651 & -0.7875323 & 2.5025109 & -0.4401157 \\
\text{i2.l2} & -2.8286769 & -0.4397012 & -21.948283 & 1.7337995 & 0.8246376 & -0.4631123 \\
\text{constant} & -5.1797144 & -7.5542800 & -7.790391 & 7.5913467 & -1.6406824 & -0.6387833 \\
\end{array}
\]

Weights W:

(This is the loading matrix)

\[
\begin{array}{cccccccc}
\text{p1.d} & \text{p2.d} & \text{e12.d} & \text{i1.d} & \text{i2.d} & \text{constant} \\
\hline
\text{p1.d} & -0.063116180 & -0.008089024 & 0.0028462619 & 0.0005926796 & -0.011717664 & 1.330204e-15 \\
\text{p2.d} & -0.087829681 & 0.016994213 & 0.0013870250 & 0.0329530362 & 0.007204453 & -5.855598e-15 \\
\text{e12.d} & -0.047349896 & 0.089044841 & -0.000323234 & -0.101051588 & -0.029084086 & 2.014059e-14 \\
\text{i1.d} & -0.007442938 & 0.025918893 & -0.0056791100 & 0.0131966312 & -0.020372446 & -2.054232e-15 \\
\text{i2.d} & 0.039061585 & 0.009907056 & 0.0132306063 & 0.0454082868 & -0.010762483 & -9.438451e-15 \\
\end{array}
\]

6.6.3.2 The maximum eigenvalue

> summary(ca.jo(data1,type="eigen",ecdet="const"))

```
# Johansen-Procedure #
```

Eigenvectors (lambda):

\[
\begin{array}{cccccccc}
\text{r} & \text{test} & 10\text{pct} & 5\text{pct} & 1\text{pct} \\
\hline
\text{r} \leq 4 & 5.09 & 7.52 & 9.24 & 12.97 \\
\text{r} \leq 3 & 16.03 & 17.85 & 19.96 & 24.60 \\
\text{r} \leq 2 & 36.86 & 32.00 & 34.91 & 41.07 \\
\text{r} \leq 1 & 60.93 & 49.65 & 53.12 & 60.16 \\
\text{r} = 0 & 105.15 & 71.86 & 76.07 & 84.45 \\
\end{array}
\]

```
[1] 5.214764e-01 3.304515e-01 2.932623e-01 1.667568e-01 8.128293e-02 -1.392413e-16
```

Eigenvectors, normalised to first column:

(These are the cointegration relations)

```
p1.12  p2.12  e12.12  i1.12  i2.12  constant
p1.12  1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
p2.12 -0.7346918 -0.9207675 -1.361829 -1.9915177 -1.4252911 -1.0358045
e12.12 -0.9704284 -1.7829624 -2.109588  0.6873137 -0.7242021 -0.2570326
i1.12 -2.8848474 -7.3432095 23.583651 -0.7875323  2.5025109 -0.4401157
i2.12 -2.8286769 -0.4397012 -21.948283  1.7337995  0.8246376 -0.4631123
constant -5.1797144 -7.5542800 -7.790391  7.5913467 -1.6406824 -0.6387833
```

Weights W:

(This is the loading matrix)

```
p1.12  p2.12  e12.12  i1.12  i2.12  constant
p1.d  -0.063116180 -0.008089024  0.0028462619  0.0005926796 -0.011717664  1.330204e-15
p2.d  -0.087829681  0.016994213  0.0013870250  0.0329530362  0.007204453 -5.855598e-15
e12.d -0.047349896  0.089044841 -0.000323234 -0.101051588 -0.029084086  2.014059e-14
i1.d  -0.007442938  0.025918893 -0.0056791100  0.0131966312 -0.020372446 -2.054232e-15
i2.d  0.039061585  0.009907056  0.0132306063  0.0454082868 -0.010762483 -9.438451e-15
```

```
6.6.3.2 The maximum eigenvalue
```

```R
> summary(ca.jo(data1,type="eigen",ecdet="const"))
```

```
# Johansen-Procedure #
```

```
Eigenvectors (lambda):
```

```
[1] 5.214764e-01 3.304515e-01 2.932623e-01 1.667568e-01 8.128293e-02 -1.392413e-16
```

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Values of test statistic and critical values of test:

<table>
<thead>
<tr>
<th>r &lt;= 4</th>
<th>10pct</th>
<th>5pct</th>
<th>1pct</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.09</td>
<td>7.52</td>
<td>9.24</td>
</tr>
<tr>
<td>r &lt;= 3</td>
<td>10.95</td>
<td>13.75</td>
<td>15.67</td>
</tr>
<tr>
<td>r &lt;= 2</td>
<td>20.83</td>
<td>19.77</td>
<td>20.00</td>
</tr>
<tr>
<td>r &lt;= 1</td>
<td>26.07</td>
<td>25.56</td>
<td>24.14</td>
</tr>
<tr>
<td>r = 0</td>
<td>44.22</td>
<td>31.66</td>
<td>34.40</td>
</tr>
</tbody>
</table>

Eigenvectors, normalised to first column:

(These are the cointegration relations)

<table>
<thead>
<tr>
<th>p1.12</th>
<th>p2.12</th>
<th>e12.12</th>
<th>i1.12</th>
<th>i2.12</th>
<th>constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>p1.12</td>
<td>1.000000</td>
<td>1.000000</td>
<td>1.000000</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>p2.12</td>
<td>-0.7346918</td>
<td>-0.9207675</td>
<td>-1.361829</td>
<td>-1.9915177</td>
<td>-1.4252911</td>
</tr>
<tr>
<td>e12.12</td>
<td>-0.9704284</td>
<td>-1.7829624</td>
<td>-2.109588</td>
<td>0.6873137</td>
<td>-0.7242021</td>
</tr>
<tr>
<td>i1.12</td>
<td>-2.8848474</td>
<td>-7.3432095</td>
<td>23.583651</td>
<td>-0.7875323</td>
<td>2.5025109</td>
</tr>
<tr>
<td>i2.12</td>
<td>-2.8286769</td>
<td>-0.4397012</td>
<td>-21.948283</td>
<td>1.7337995</td>
<td>0.8246376</td>
</tr>
<tr>
<td>constant</td>
<td>-5.1797144</td>
<td>-7.5542800</td>
<td>-7.790391</td>
<td>7.5913467</td>
<td>-1.6406824</td>
</tr>
</tbody>
</table>

Weights W:

(This is the loading matrix)

<table>
<thead>
<tr>
<th>p1.12</th>
<th>p2.12</th>
<th>e12.12</th>
<th>i1.12</th>
<th>i2.12</th>
<th>constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>p1.d</td>
<td>-0.063116180</td>
<td>-0.008089024</td>
<td>0.0028462619</td>
<td>0.0005926796</td>
<td>-0.011717664</td>
</tr>
<tr>
<td>p2.d</td>
<td>-0.087829681</td>
<td>0.016994213</td>
<td>0.0013870250</td>
<td>0.0329533062</td>
<td>0.007204453</td>
</tr>
<tr>
<td>e12.d</td>
<td>-0.047349896</td>
<td>0.089044841</td>
<td>-0.0003203234</td>
<td>-0.1010515588</td>
<td>-0.029084086</td>
</tr>
<tr>
<td>i1.d</td>
<td>-0.007442938</td>
<td>0.025918893</td>
<td>-0.0056791100</td>
<td>0.0131966312</td>
<td>-0.020372446</td>
</tr>
<tr>
<td>i2.d</td>
<td>0.039061585</td>
<td>0.009907056</td>
<td>0.0132306063</td>
<td>0.0454082868</td>
<td>-0.010762483</td>
</tr>
</tbody>
</table>

Tables 6.11 summarises results of Johansen’s cointegration method on the *UKpppauip* data set.
Table 6.11: Johansen’s trace tests and maximum eigenvalues results

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>Alternative</th>
<th>Test statistic</th>
<th>10%</th>
<th>5%</th>
<th>1%</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_{\text{trace}} ) test</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r \leq 4 )</td>
<td>( r &gt; 4 )</td>
<td>5.09</td>
<td>7.52</td>
<td>9.24</td>
<td>12.97</td>
<td>Fail to reject ( H_0 )</td>
</tr>
<tr>
<td>( r \leq 3 )</td>
<td>( r &gt; 3 )</td>
<td>16.03</td>
<td>17.85</td>
<td>19.96</td>
<td>24.60</td>
<td>Fail to reject ( H_0 )</td>
</tr>
<tr>
<td>( r \leq 2 )</td>
<td>( r &gt; 2 )</td>
<td>36.86</td>
<td>32.00</td>
<td>34.91</td>
<td>41.07</td>
<td>Reject ( H_0 )</td>
</tr>
<tr>
<td>( r \leq 1 )</td>
<td>( r &gt; 1 )</td>
<td>60.93</td>
<td>49.65</td>
<td>53.12</td>
<td>60.16</td>
<td>Reject ( H_0 )</td>
</tr>
<tr>
<td>( r = 0 )</td>
<td>( r &gt; 0 )</td>
<td>105.15</td>
<td>71.86</td>
<td>76.07</td>
<td>84.45</td>
<td>Reject ( H_0 )</td>
</tr>
<tr>
<td>( J_{\text{max}} ) test</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r = 4 )</td>
<td>( r = 5 )</td>
<td>5.09</td>
<td>7.52</td>
<td>9.24</td>
<td>12.97</td>
<td>Fail to reject ( H_0 )</td>
</tr>
<tr>
<td>( r = 3 )</td>
<td>( r = 4 )</td>
<td>10.95</td>
<td>13.75</td>
<td>15.67</td>
<td>20.20</td>
<td>Fail to reject ( H_0 )</td>
</tr>
<tr>
<td>( r = 2 )</td>
<td>( r = 3 )</td>
<td>20.83</td>
<td>19.77</td>
<td>20.00</td>
<td>20.81</td>
<td>Reject ( H_0 )</td>
</tr>
<tr>
<td>( r = 1 )</td>
<td>( r = 2 )</td>
<td>26.07</td>
<td>25.56</td>
<td>24.14</td>
<td>23.24</td>
<td>Reject ( H_0 )</td>
</tr>
<tr>
<td>( r = 0 )</td>
<td>( r = 1 )</td>
<td>44.22</td>
<td>31.66</td>
<td>34.40</td>
<td>39.79</td>
<td>Reject ( H_0 )</td>
</tr>
</tbody>
</table>

From table 6.11, it can be observed that

(i) the null hypothesis of no cointegration (\( r = 0 \)) against the alternative of the presence of one or more cointegrating vector is rejected at the 10% level of significance in both techniques (trace test and maximum eigenvalue). This implies that cointegration exists among all variables of the data set.

(ii) the null hypotheses of \( (r \leq 1) \) and \( (r \leq 2) \) are rejected at the 10% level of significance. This implies that there are at most two cointegration vectors in the \( UKpppuip \) data set.

(iii) the results of Johansen’s method further imply that multicointegrating relationships can be tested for in the \( UKpppuip \) data set. However, testing for multicointegration is not part of this study.

6.7 Concluding remarks

In the \( UKpppuip \) data set, seven variables are tested for cointegration, of which five showed the existence of a long-run relationship. Although the Engle-Granger and the Phillips-Ouliaris methods indicated that there is no cointegration amongst the five variables, Johansen’s method indicated presence of cointegration and further showed the existence of at most two cointegration vectors.
Chapter 7

Conclusions and recommendations

In conclusion, the Engle-Granger method requires testing whether variables are integrated of the same order. This is done using the ADF unit root test. If they are integrated and of the same order, we examine the long-run equilibrium relationship by fitting the OLS estimator in order to assess whether there is a linear relationship. Using the error correction model, we test whether the residuals of the long-run relationship are stationary. If all conditions are satisfied, then the two or more variables under investigation cointegrate.

The advantage of the Engle-Granger method over the other techniques is its ease of implementation. However, its results are dependent on how the long-run equilibrium equation is specified. In some cases it might not be easy to identify which variable enters on the left as the dependent variable. It is therefore advisable to employ the cointegration tests of the Phillips-Ouliaris methods, which yield the same results irrespective of the variable which enters as the dependent variable, that is, they are invariant to normalisation.

According to the Phillips-Ouliaris methods, two residual-based tests, namely the variance ratio test and the multivariate trace statistic, are employed to test for cointegration. These tests measure the size of the residual variance from the cointegrating regression of the variables under study. If the residual variance is greater than the conditional variance, then the variables cointegrate. However, one deficiency of the Phillips-Ouliaris methods is that one can only estimate a single cointegrating relationship. Nevertheless, if one deals with more than two time series, it is possible that more than one cointegrating relationship exists which calls for the use of vector cointegration techniques like Johansen’s procedure.
Johansen’s method employs the trace test and maximum eigenvalues to test for the existence of one or more cointegrating vectors in the data set. This procedure is used mostly on multivariate data sets where we suspect the existence of more than one cointegrating relationship. However, it can also be used to verify the results of other cointegration techniques. This method assumes that the cointegrating vector is constant during the period of study. In reality, it is possible that the long-run relationships between the underlying variables change because of changes in technological progress and/or economic crises. In order to remedy this limitation, Gregory and Hasen (1996) have introduced tests for cointegration with one and two unknown structural break(s).

This study finds all the cointegration techniques tested not to be consistent. That is, all three methods do not lead to the same results. In the data analysis performed in chapter 4, 5, and 6, the Engle-Granger method and the Phillips-Ouliaris methods indicated no cointegration whereas the Johansen’s method indicated presence of cointegration. We recommend the use of Johansen’s method because it is able to detect more than one cointegrating relationship if present.
### Chapter 8

**Appendix**

#### 8.1 Appendix A

**The Raotbl3 data**

```r
> Raotbl3

<table>
<thead>
<tr>
<th>lc</th>
<th>li</th>
<th>lw</th>
<th>dd682</th>
<th>dd792</th>
<th>dd883</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966.4</td>
<td>10.4831</td>
<td>10.5821</td>
<td>12.9481</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>1967.1</td>
<td>10.4893</td>
<td>10.5800</td>
<td>12.9895</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1967.2</td>
<td>10.5022</td>
<td>10.5990</td>
<td>13.0115</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1967.3</td>
<td>10.5240</td>
<td>10.6262</td>
<td>13.0411</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1967.4</td>
<td>10.5329</td>
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### 8.2 Appendix B

**Australia’s Economic Indicators**

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United Nations World Economic Indicators [http://quanis1.easydata.co.za/TableViewer/tableView.aspx](http://quanis1.easydata.co.za/TableViewer/tableView.aspx).

