Cramer-Rao Bound on Timing Recovery for GSM Receivers

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***Abstract*—The problem of timing recovery in cellular receiver systems is critical. This is because synchronization functions must follow rapid parameter changes inherent in mobile systems. The ultimate goal is therefore to achieve low bit error rate on the recovered information. Traditional timing recovery methods have over-relied on phase-locked loops for timing information adjustment. However, associated schemes do not exploit code properties. This leads to difficulties in synchronizing digital receivers with transmitters' end separated by lossy channels. In this paper we present a class of fast converging timing recovery algorithm for GSM receivers in low signal to noise ratio. In the proposed method, the receiver exploits the soft decisions computed at each turbo decoding iteration to provide reliable estimates of soft timing signal which in turn improves the decoding time. The proposed method converges in about 10 iterations and the derived sequential minimization technique approaches a theoretical Cramer-Rao bound with unbiased estimates. Though an 8-PSK GSM baseband communication system is simulated, it was found that this model worked well with most modulation schemes. The proposed scheme is also insensitive to carrier offsets recovery. Simulation showed that the proposed method outperforms conventional timing extraction methods with respect to jitter performance.**

***Index Terms*—iterative turbo principle, matched filter output, turbo codes, soft information exchanges.**

I. INTRODUCTION

Cramer-Rao bound (CRB) is a lower bound on the error variance of any unbiased estimate, and as such provides a benchmark for practical estimators [23, 24]. The CRB is known to be asymptotically achievable for a large enough number of observations, under mild regularity conditions [15]. In many cases, the statistics of the observation depend not only on the vector parameter to be estimated, but also on a nuisance vector parameter we do not want to estimate [25]. In linearly modulated signals the CRB synchronizers have been applied [25]. However, timing recovery problem in multilevel GSM modulation scheme with unknown data symbols as nuisance parameters is still a challenging task. This is due to computational complexities involved in low SNR scenarios and mobility of the receivers. Fortunately, Berrou *et al.* developed the revolutionary iterative "turbo" receiver for decoding two dimensional product-like concatenated codes [1]. The impressive performance of turbo codes has triggered the application of this powerful coding technique to digital communications at low SNR environments [2, 3]. Several receiver functions such as combined signal detection, equalization, symbol synchronization, respectively, are now possible with turbo decoding [4]-[6].

In most classical timing phase estimations, timing recovery and decoding process have been separated with little penalty; timing recovery uses an instantaneous decision device to provide tentative decisions that are adequately reliable to estimate the timing phase error [7]. In such situations, however, the timing recovery process assumes that the neighboring symbols are mutually independent at high SNR and the theoretical framework is normally based on least mean square (LMS) and traditional phase-locked loops (PLL) [8].

Such a framework is susceptible to local minima and often presents additional block processing complexities which fail in low SNR. Since turbo receivers operate in low SNR, the future of wireless communication systems' cell planning schemes and battery conservation of portable receivers will have to rely on combined timing recovery and decoding algorithm. Refs. [9, 10] have shown that classical soft-input/soft-output (SISO) iterative detection/decoding algorithm embed timing parameter estimation in the decoding process. For instance in [10], combined iterative decoding, equalization and timing error estimation is performed with modified forward and backward recursions in the SISO decoders using a per-survivor processing algorithm [21]. Such methods are reliable but increase the receiver's design complexity with vast memory requirement. Inorder to reduce the complexity involved in designing the decoder structure, soft information provided at each iteration by a conventional turbo decoder can be used to derive reliable information on timing error estimation. This is the essence of turbo principle synchronization technique [6] [22]. Though recent research attention is focused towards this turbo synchronization method [11, 12], less effort has

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been directed towards achieving fast converging timing recovery process.

In the recent past, GSM transmission systems have over-relied on traditional forward error correction codes in [20], to either save bandwidth or reduce power requirements. However, not much of coding gain is feasible, moreover, symbol synchronization depends on pilot information conveyed a long with message information. Though such data-aided synchronizers are faster, they are bandwidth inefficient. In cellular receivers, bandwidth conservation, convergence and jitter variation in steady state synchronization is very critical. Thus, to recover majority of transmitted symbols and frames with minimum error, time and channel impairments become the fundamental question. The objective of this paper is therefore to develop a mathematical framework for such turbo-aided synchronization with a focus on faster convergence and jitter reduction. This goal is achieved through simulations of combined maximum likelihood estimation, modified Newton-raphson minimization model and matched filtering. Performance bound of the estimator has been reviewed in this sequel with regards to Cramer-Rao bound.
This paper is organized as follows. Section I, provided a broad overview of the problem area, related work and results achieved by other researchers. In section II, turbo system model is presented. In section III, improved soft timing framework is proposed. Simulations tests and results shown in sections IV and V. Conclusions are drawn in section VI.

II. SYSTEM MODEL

The baseband-equivalent of such a turbo-coded communication encoder and decoder structure is depicted in Figure 1. It consists of two recursive systematic convolutional encoders (RSC) which are separated by a pseudo-random L-bits interleaver (INT), puncturing the output of the encoders increases the transmitted code rate from 1/3 to 1/2. On the other hand the receiver consists of modified turbo decoder with two separate soft-In/Soft-Out maximum likelihood a posteriori (MAP) decoders which are connected with interleaver (INT) and deinterleaver (DEINT). The most widely used MAP algorithm is the recursive Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm [4]-[6], which permits easy calculation of the log likelihood ratio (LLR) on block by block basis. BCJR algorithm, however, requires priori knowledge of the channel parameters. Fortunately, the feedback loop provides prior soft bit information from one decoder to the other hence the name 'turbo code'. It works iteratively, i.e. the decoding improves with consecutive iterations upto certain threshold limit when performance degrades. Implementing combined timing recovery and decoding helps in convergence.

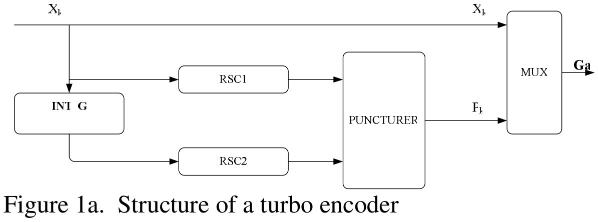

Figure 1a. Structure of a turbo encoder

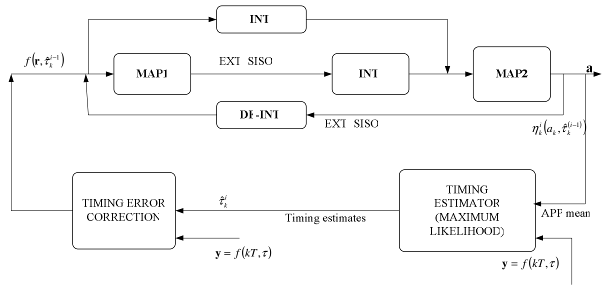

Figure 1b. Structure of modified turbo decoder

III. THEORETICAL CONSIDERATIONS

The received signal is

$$\mathbf{r} = \mathbf{HGa} + \mathbf{n}\ , \qquad (1)$$

where after sampling the received vector becomes $\mathbf{r} = [r_{-M} \ldots r_0 \ldots r_{N-1+M-1}]^T$. Here, we have collected $N+2M$ samples from the received pulses, N is the number of transmitted symbols, $2M+1$ is the samples of the observation interval (so-called pulse truncation) and where we let $M \to \infty$. for best noise performance.

$$\mathbf{H} = \begin{pmatrix} h(\tau) & & \\ . & & \\ . & . & \\ . & & \\ h(N_h T + \tau) & & h(\tau) \\ & & . \\ & . & . \\ & & . \\ & & h(N_h T + \tau) \end{pmatrix}, \qquad (2)$$

is a convolution matrix created from delayed samples of pulse shape $h(t)$ by τ. $\mathbf{G}$ is an interleaving matrix (obtained by permuting the columns of an identity matrix of size L-bits), $\mathbf{a} = (a_1, \ldots, a_N)$ are the transmitted symbols with consecutive symbol duration, T and $\mathbf{n}$ is independently and identically distributed (i.i.d.) Gaussian noise with variance No/2.
In order to have sufficient statistics in the decoding-timing recovery module, (1) must be sampled at $T_S \le T/(1+\alpha)$, intervals, where α is the roll-off of the transmitting square root raised consine waveform.
The received sampled vector is passed through discrete matched filters whose outputs take the form

$$\mathbf{y} = f(kT+\tau) = \sum_{k=1}^{k=N} a_k x(s - kT - \tau) + w(s) \qquad (3)$$

Computing the output of (3) at correct instants of the argument, $\{kT+\tau\}$ yields the solution to the problem of time recovery.
The solution to the problem of timing recovery is two fold; estimating timing phase signal τ and determining the steady state location of the timing instants through suitable update mechanisms. Traditionally this has been achieved using a combination of timing error detector and phase locked-loops implemented without decoding algorithm in mind [7][8] and [14] or with complexities introduced to the decoder [10]-[12].

In the next section we show how decoder functions can be improved with timing recovery with little modifications. We further introduce the concepts of a low variance design of the timing recovery in digital mobile receivers.

A. Estimating timing information

The problem addressed in this section is the estimation of $\hat{\tau}$ of τ subject to a trial value $\tilde{\tau}$. This estimate may be seen as the solution of the maximization problem

$$\hat{\tau} = \underset{\tilde{\tau}}{\operatorname{argmax}}\, \Lambda(\tilde{\tau}). \tag{4}$$

Here,

$$\Lambda(\tilde{\tau}) = \ln p(\mathbf{r} \mid \tilde{\tau}) \tag{5}$$

and

$$p(\mathbf{r} \mid \tilde{\tau}) = \int_{\mathbf{a}} p(\mathbf{a})(\mathbf{r} \mid \mathbf{a}, \tilde{\tau})d\mathbf{a} \tag{6}$$

Where, $p(\mathbf{a})$ is a prior probability mass function. The logarithmic function of second factor of the integrand in (6) is defined as

$$\ln p(\mathbf{r} \mid \mathbf{a}, \tilde{\tau}) = \Re\left\{\sum_{k=0}^{N-1} a_k^* y(kT + \tilde{\tau})\right\} \quad , \tag{7}$$

where $y(kT + \tilde{\tau})$ corresponds to the matched filter output evaluated at $kT + \tilde{\tau}$.

In order to solve for (4), we take the derivative of (5) with respect to $\tilde{\tau}$ and we equate to zero, that is,

$$\begin{aligned}&\frac{\partial}{\partial \tilde{\tau}} \ln p(\mathbf{r} \mid \tilde{\tau}) \\ &= \int_{\mathbf{a}} \frac{p(\mathbf{a})p(\mathbf{r} \mid \mathbf{a}, \tilde{\tau})}{p(\mathbf{r} \mid \tilde{\tau})} \frac{\partial}{\partial \tilde{\tau}} \ln p(\mathbf{r} \mid \mathbf{a}, \tilde{\tau}) d\mathbf{a} \; . \\ &= \mathbf{0}\end{aligned} \tag{8}$$

We notice that the evaluation of (8) requires the knowledge of the priori probabilities, $p(\mathbf{a})$ of the transmitted symbols at the receiver. However, in this problem, we assume that such information can only be derived from posteriori information. If we invoke Baye's rule in the first factor of the integrand in (8), we have a posteriori conditional probability density function (PDF) of the transmitted vector $\mathbf{a}$. We can then represent it as

$$\frac{p(\mathbf{a})p(\mathbf{r} \mid \mathbf{a}, \tilde{\tau})}{p(\mathbf{r} \mid \tilde{\tau})} = p(\mathbf{a} \mid \mathbf{r}, \tilde{\tau}) \tag{9}$$

Since from (1) vector, $\mathbf{r}$ is a function of vector, $\mathbf{n}$ but $\mathbf{n}$ is not a function of τ, we can ignore $\mathbf{n}$ in the following definition without loss of generality. We had indicated that interleaving matrix is an identity matrix, hence (9) becomes

$$p(\mathbf{r} \mid \mathbf{a}, \tilde{\tau}) = f_{\mathbf{n}}(\mathbf{r} - \mathbf{H}(\tau)\mathbf{a}) = \exp\left(-\frac{\|\mathbf{r} - \mathbf{H}(\tau)\mathbf{a}\|^2}{N_0}\right). \tag{10}$$

The Maximum likelihood estimation problem in (4) now becomes an expectation problem given as follows

$$\begin{aligned}&\frac{\partial}{\partial \tilde{\tau}} \ln p(\mathbf{r} \mid \tilde{\tau}) \\ &= \int_{\mathbf{a}} p(\mathbf{a} \mid \mathbf{r}, \tilde{\tau}) \frac{\partial}{\partial \tilde{\tau}} \left\{\Re\left\{\sum_{k=0}^{N-1} a_k^* y(kT + \tilde{\tau})\right\}\right\} . \\ &= \mathrm{E}_{\mathbf{a}} \left\{\frac{\partial}{\partial \tilde{\tau}} \ln p(\mathbf{r} \mid \mathbf{a}, \tilde{\tau}) \mid \mathbf{r}, \tilde{\tau}\right\} \\ &= \mathbf{0}\end{aligned} \tag{11}$$

Since $\tilde{\tau}$ appears in both factors of the expectation problem in (10), the solution of (10) is non-trivial. According to [13], an iterative approach that generates a set of values for $\hat{\boldsymbol{\tau}} = \{\hat{\tau}^1 \ldots \hat{\tau}^i \ldots \hat{\tau}^n\}$ is a possible solution. Theoretically it is possible to prove that within the limit as $n \to \infty$ the sequence of timing estimate converges to a desired solution [15]. However, the proof is analytically complex. Fortunately, an iterative receiver employing turbo codes will help in achieving convergence faster [6]. The turbo principle in [1] provides soft information described by posteriori means as illustrated in figure 1b and given by

$$\begin{aligned}\eta_k^i\left(\mathbf{r}, \hat{\tau}^{(i-1)}\right) &\cong \int_{\mathbf{a}} a_k\, p\left(\mathbf{a} \mid \mathbf{r}, \hat{\tau}^{(i-1)}\right) d\mathbf{a} \\ &= \sum_{\alpha_m \in \mathrm{A}} \alpha_m p\left(a_k = \alpha_m \mid \mathbf{r}, \hat{\tau}^{(i-1)}\right)\end{aligned} \quad . \tag{12}$$

The second factor in (12) is the marginal a posteriori probability (APPs) computed from extrinsic SISO exchanges off log likelihood ratios (LLR) from the second maximum a posteriori (MAP) decoder to the first MAP decoder. Detailed derivation of (12) can be obtained from [2]. The log likelihood ratio as depicted in figure 1b is thus given by

$$\begin{aligned}&\Lambda_{21}\left(a_k \mid \mathbf{r}, \hat{\tau}^i\right) \\ &= \log\left(\frac{p\left(a_k = +1 \mid \mathbf{r}, \hat{\tau}^i\right)}{p\left(a_k = -1 \mid \mathbf{r}, \hat{\tau}^i\right)}\right).\end{aligned} \tag{13}$$

Since fast converging estimator is required for our problem, Modified Newton-Raphson in [13] is applied to (10) to give a numerical solution, that is

$$\hat{\tau}^{(i)} = \hat{\tau}^{(i-1)} - \left(\frac{\partial \tilde{\Lambda}}{\partial \tilde{\tau}}\right)_{\tilde{\tau} = \hat{\tau}^{(i-1)}} \left(\frac{\partial^2 \tilde{\Lambda}}{\partial \tilde{\tau}^2}\right)^{-1}_{\tilde{\tau} = \hat{\tau}^{(i-1)}}, \tag{14}$$

where $\tilde{\Lambda} = f(\eta_k, \mathbf{y})$, as explicitly depicted in (3) and (12) and i and $(i-1)$ denotes the current and the previous turbo iterations.

B. Updating timing phase estimates

We begin the iteration by assuming that the $(i-1)$th timing offset estimate is zero. The estimated timing offset finally updates the early and late samples of the discrete matched filter output and optimal synchronization is attained when the early and late samples become equal [17]. The new timing estimate will be based on the discrete matched filter output $y(s)|_{s=kT+\hat{\tau}^{i-1}}$ as shown in Figure 2 and the mean of posterior probabilities $\eta_k^{(i-1)}$ from the previous iteration as shown in Figure 1. This can be seen in the following expression

$$\left(\frac{\partial\tilde{\Lambda}(\tilde{\tau})}{\partial\tilde{\tau}}\right)_{\tilde{\tau}=\tau^{(i-1)}} \approx \frac{\tilde{\Lambda}\left(\hat{\tau}^{(i-1)}+\Delta\hat{\tau}\right)-\tilde{\Lambda}\left(\hat{\tau}^{(i-1)}-\Delta\hat{\tau}\right)}{2\Delta\hat{\tau}} \quad . \tag{15}$$

Thus, at low SNR (15) is well approximated by

$$=1/\Delta\hat{\tau}\sum_k \Re\left\{\eta_k^{*(i-1)}\begin{pmatrix} y\left(kT+\hat{\tau}^{(i-1)}+\Delta\hat{\tau}\right) \\ -y\left(kT+\hat{\tau}^{(i-1)}-\Delta\hat{\tau}\right)\end{pmatrix}\right\}, \tag{16}$$

and it easy to show that the second derivative in (14) is

$$=1/\Delta\hat{\tau}^2\sum_k \Re\left\{\eta_k^{*(i-1)}\begin{pmatrix} y\left(kT+\hat{\tau}^{(i-1)}+\Delta\hat{\tau}\right) \\ +y\left(kT+\hat{\tau}^{(i-1)}-\Delta\hat{\tau}\right) \\ -2y\left(kT+\hat{\tau}^{(i-1)}\right)\end{pmatrix}\right\}, \tag{17}$$

where $\Delta\hat{\tau}$ is an adjustable advance/delay parameter that satisfies $0<\Delta\hat{\tau}<T/2$.

C. Lower Bound on timing error variance

Our goal is to arrive at a lower bound on timing estimation error variance in face of a time-varying timing offset. We model the timing offset as a random walk, according to

$$\begin{aligned}\tau_{k+1} &= \tau_k+\omega_{k+1}=\tau_{-1}+\sum_{j=0}^{k+1}\omega_j \\ &= \tau_k+(k+1)\Delta T\end{aligned} \tag{18}$$

where $\omega_k \subset \mathrm{N}(0,\sigma_\omega^2)$ are i. i. d. of kth symbol and σ_ω^2 determines the severity of the timing jitter. The random walk is chosen because of its simplicity and because of its ability to model a wide range of mobile channels. We assume a perfect acquisition by setting $\tau_{-1}=0$. In [14], Cramer-Rao bound (CRB) on the timing estimation error variance for generic channel is presented. Timing offset is assumed constant over the duration of the packet length without loss of generality. Hence, CRB gives a lower bound on the estimation error variance of unbiased estimators of deterministic parameters $\boldsymbol{\tau}=[\Delta T,\tau]^T$ In [15], the CRB is demonstrated by

$$\mathrm{E}_{\mathbf{r}}\left[(\hat{\tau}_i-\tau_i)^2\right]\geq CRB_i(\boldsymbol{\tau}) \tag{19}$$

where $CRB_i(\boldsymbol{\tau})$ is the ith diagonal element of the inverse of the *Fisher information matrix* $\mathbf{J}(\boldsymbol{\tau})$. The (i,j)th element of $\mathbf{J}(\boldsymbol{\tau})$ is given by

$$\mathbf{J}(\boldsymbol{\tau})=\mathrm{E}_{\mathbf{r}}\left[-\frac{\partial^2}{\partial\tau_i\partial\tau_j}\ln\left(p(\mathbf{r}\mid\boldsymbol{\tau})\right)\right] \tag{20}$$

The probability density $p(\mathbf{r}\mid\boldsymbol{\tau})$ of $\mathbf{r}$, corresponding to a given value of $\boldsymbol{\tau}$, is called the likelihood function of $\boldsymbol{\tau}$. The expectation $\mathrm{E}_{\mathbf{r}}[.]$ is with respect to $p(\mathbf{r}\mid\boldsymbol{\tau})$. Equivalently (15) can be re-written as

$$\mathbf{J}(\boldsymbol{\tau})=\mathrm{E}\left\{\left[\frac{\partial}{\partial\boldsymbol{\tau}}\ln p(\mathbf{r}\mid\boldsymbol{\tau})\right]\left[\frac{\partial}{\partial\tau}\ln p(\mathbf{r}\mid\boldsymbol{\tau})\right]^T\right\}. \tag{21}$$

From a detailed proof in [16], we obtain Cramer-Rao bound as

$$\frac{\mathrm{E}\left[(\tau-\hat{\tau})^2\right]}{T^2}\geq\frac{2\sigma^2(2N-1)}{\left(\frac{2\pi^2}{3}-1\right)N(N+1)}.$$

$$\frac{\mathrm{E}\left[\left(\Delta T-\hat{\Delta}T\right)^2\right]}{T^2}\geq\frac{2\sigma^2}{\left(\frac{2\pi^2}{3}-1\right)(N-1)N(N+1)} \tag{22}$$

where σ is standard deviation of noise and other parameters retain their definitions as we have given earlier.

IV. SIMULATION TESTS

To verify the performance of our turbo aided timing recovery scheme, we simulated a baseband communication system transmitting 8-constellation alphabet for phase shift keying (8-PSK) symbols in MATLAB. We considered convolutional turbo code (031,027) with punctured net rate ½. The interleaver length set to blocks' sizes of 460 bits and square root raised cosine signaling pulse with roll-off of 0.25 was used. 1000 blocks were transmitted over Rician distributed flat fading channel and additive white Gaussian noise (AWGN). The received signal was passed through anti-aliasing filter and then sampled at a rate higher than the baud rate. Discrete matched filter was embedded at the input of the decoder as early-late gate synchronizer. Refer to Figure 1b.

V. RESULTS

Figure 3 shows the estimator variance approaching Cramer-Rao bound and indicating unbiased estimator in low SNR up-to 0.35 as iteration increases.

In Figure 4, the number of iterations was set to 10. It is clear that for known normalized variances, the performance of the synchronizer improves with decrease in variance. In addition, known timing indicates good performance. This indicates that timing synchronizer uses soft information to get good estimates that converge to true channel offsets and on the other hand, the turbo iterative decoder uses this soft timing signal to converge faster. In Figure 5, it is seen that increasing the iterations improves performance, however whenever the number of iterations exceeds 10, the performance degrades as seen in Figure 3. This means that system converges at only 10 iterations or explicitly in about 10 symbols' period.

Figure 6 gives a comparision between the early-gate synchronizer, the proposed method and data-aided GSM frame error rate performance. Though data-aided gives the best performance, both power and bandwidth requirements make it a prohibitive option.

Typical GSM radio channel model performance is depicted in Figure 7.

For simplicity in radio channel modeling, a Rician multipath flat fading is considered with a Rician K-factor denoted by $K = \beta^2 / 2\sigma_0^2$. Here, β and σ_0^2 are the amplitude of the specular path component (dominant LOS) and variances of Gaussian random channel samples with zero-means.

For smaller, Rician factor K, the performance of synchronizer is worse than for large K. This attributes to the fact that, a Rician distribution is best modeled as a Rayleigh distribution when K is small. The resulting Rayleigh distribution provides poor soft decoder outputs and consequently poor soft timing signal for the next decoder iterations.

To explain our hypothesis of low jitter, the proposed method was compared with the well-known work of Mueller and Muller in [7]. In Figure 8, we notice that at start-up time, the algorithm in [7] demonstrates some jittering effects and converges to steady state after 10 symbols. On the other hand the proposed algorithm starts up with minimal jittering effects and converges to steady state within 10 iterations.

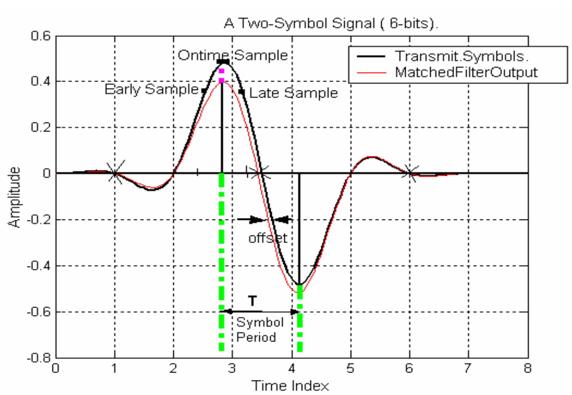

Fig. 2 demonstrates timing recovery problem

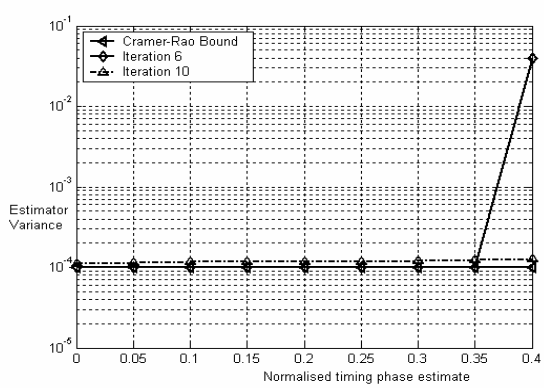

Fig. 3. Estimator variance performance

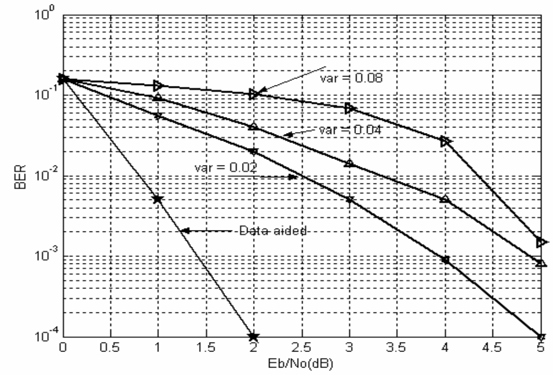

Fig. 4 BER Vs Eb/No at different normalized variances

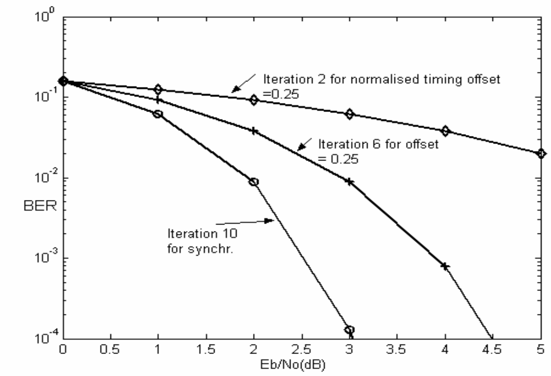

Fig. 5 BER Vs Eb/No at normalized timing offset 0.25

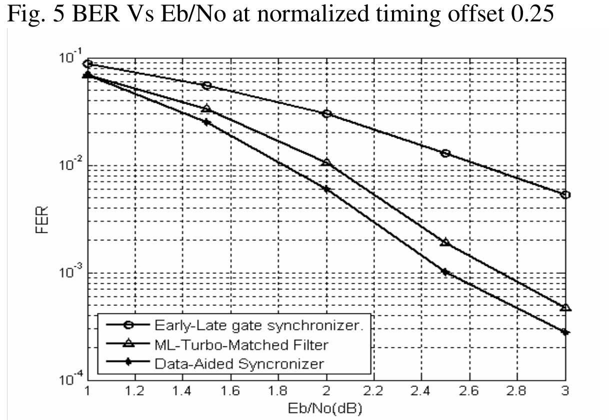

Fig. 6 compares performance of FER Vs Eb/No at normalized timing offset 0.25 and Iteration set to 10.

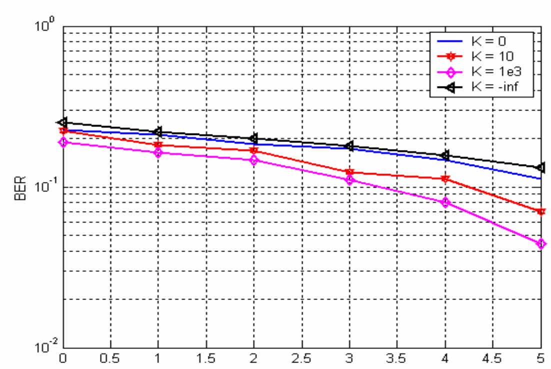

Fig. 7 Multipath fading iterative timing recovery performance for different Rician factor-K in dB at 8-PSK modulation GSM system.

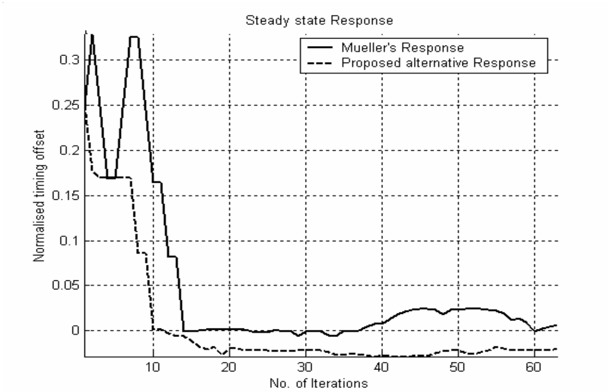

Fig. 8 Convergence criterion from normalized timing offset of 0.25.

VI. CONCLUSION

Combined synchronization and decoding of turbo codes is likely to give good results in low SNR environments. In such applications timing recovery is extremely difficult with traditional methods. Deriving good timing estimator function is crucial in both decoding process and steady-state sampling phase. In Figure 4, a low variance timing offset estimator with promising results is reported. The variance is also bounded even when the number of iterations is increased beyond 10. The overall performance index in Figure 5 and 6 shows that the bit error rate decreases with signal to noise ratio to a maximum of 10 iterations. The estimator performance may start to degrade after 10 iterations due to biased response of the system. The timing phase is therefore, tracked and locked during this period and optimum matched filter outputs are now generated to the APP decoder. Deep fading degrades iterative synchronizer performance as shown in figure 7. The most interesting results are depicted in Figure 8. In comparison of our contribution with the work proposed by Mueller and Muller, our proposed method indicates a lower jitter at start-up time of synchronization. This is a good indication of the solution viability in cellular network (e.g. GSM) applications where high jitter performance is critical. The proposed solution also reaches steady-state earlier than the Mueller and Muller's work hence a better solution for timing recovery problem in mobile receivers. Moreover, the proposed scheme is both bandwidth and power conservation efficient for use in GSM standards.

However, the proposed scheme is still complex and a scheme of training pilot sequences employed in GSM standards provides lower bit error-rate.

VII. REFERENCES

[1] C. Berrou, A. Glavieux and P. Thitimajshima, "Near Shannon limit error-correcting coding and decoding: turbo codes," in *Proc. ICC 1993*, 1993, pp. 1064-1070.

[2] C. Berrou and A. Glavieux, "Near optimum error correcting coding and decoding: turbo codes," *IEEE Trans. Commun*., vol. 44, no. 10, pp. 1261-1271, 1996

[3] J. Hagenauer, "The turbo principle: tutorial introduction and state of art," in *Proc. Int. Symposium on Turbo Codes and Related Topics*, pp. 1-11, Brest, France, September 1997.

[4] M. Sellathurai and S. Haykin, "Turbo-Blast for wireless communications: theory and experiments," *IEEE Trans. Sig. Proc.*., vol. 50, no. 10, pp. 2538-2546, 2002

[5] C. Herzet, V. Ramon, L. Vandendorpe, "Iterative soft-decision directed timing estimation for turbo receivers," *IEEE 9th Symposium on Commun., and Vehicular Technology*, pp. 97-101, 2003

[6] N. Noels, V. Lottici, A. Dejonghe, H. Steendam, M. Moeneclaey, M. Luise, L. Vandendorpe, "A theoretical framework for soft-information-based synchronization in iterative(turbo) receivers," *EURASIP J. Wireless Commun., and Network.,* vol. 2, pp. 177-129, 2005.

[7] K. H. Mueller and M. Muller, "Timing recovery in digital synchronous data receivers," *IEEE Trans. Commun.,* vol. Com-24, pp. 516-532, 1976.

[8] J. G. Proakis and M. Salehi, *Communication systems engineering*. New Jersey: Prentice Hall, 2002.

[9] J. R. Barry, A. Kavcic, S. W. McLaughlin, A. Nayak, and W. Zeng, "Iterative timing recovery," *IEEE Signal Processing Mag*., vol. 2, pp. 89-102, 2004.

[10] P. Kovintavewat, J. R. Barry, M. F. Erden, and E. M. Kurtas, "Robustness of per-survivor iterative timing recovery in perpendicular recording channels," IEEE *Trans. Magnetics*, vol. 10, pp. 807-808, 2005.

[11] B. Mielczarek, *Turbo codes and channel estimation in wireless systems*, PhD Thesis, Signals and Systems, Chalmers University of Technology, Goteborg, 2002.

[12] J. M. Walsh, C. R. Johnson, and P. A. Regalia, "Joint synchronization and decoding exploiting the turbo principle," in *Proc. Conf. Inform. Sciences and Systems*, 2004

[13] D. Tabak and B. C. Kuo, *Optimal control by mathematical programming*. New Jersey: Prentice Hall, 1971.

[14] H. Meyr, M. Moeneclaey, and S. Fechtel, Digital *communication receivers: Synchronization, channel estimation, and signal processing*. New York: John Wiley and sons, 1998.

[15] H. L. Van Trees, Detection*, Estimation and Modulation Theory*. New York: John Wiley and sons, 1968.

[16] A. Nayak, J. Barry and S. McLaughlin, "Joint Timing Recovery and Turbo Equalization for Coded Partial Response Channels," *IEEE Trans. Magnetic*s, vol. 38, no.5, pp. 2295-2297, Sept 2002.

[17] F. J. Harris and M. Rice, "Multirate digital filters for symbol timing synchronization in software defined radios," *IEEE journal on selected areas in commun.*, vol. 19, no. 12, pp. 2346-2357, 2001.

[18] F. J. Harris, Multirate *signal processing for communication systems*. New Jersey: Prentice Hall, 2004.

[19] C. L. Philips and H. T. Nagle, *Digital control system analysis and design*, 3rd ed. New Jersey: Prentice Hall, 1998.

[20] A. Shanmugam and A. R. Abdul Rajak, "Optimal design of forward error correcting codes for wireless communication", www.academicjournal.com, vol. 14, 2005.

[21] A. Polydoros and K. M. Chugg, "Per Survivor processing (PSP)", www.phys.uoa.gr/el-lab/polydoros/psp/psp.html, [Accessed on: 18/11/2005]

[22] N. Noels, C. Herzet, A. Dejonghe*, et al., "*Turbo synchronization: an EM interpretation," In Proc. IEEE international Conference on Communications (ICC '03), Anchorage, Alaska, USA, May 2003.

[23] H. Cramer, *Mathematical Methods of Statistics*. Princeton, NJ: Princeton Univ. Press, 1946.

[24] C. R. Rao, "Information and accuracy attainable in the estimation of statistical parameters," *Bull. Calcutta Math. Soc.,* vol. 37, pp. 81-91, 1945.

[25] N. Noels, H. Wymeerisch, H. Steendam and M. Moeneclay, "The true Cramer-Rao bound for timing recovery from a bandlimited linearly modulated waveform with unknown carrier phase and frequency," *IEEE Transactions on Communications journal*, vol. 52, no. 3, pp. 473-483, March 2004.

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