A Low Variance Timing Recovery in Turbo Receivers

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***Abstract*—This paper presents a fast converging timing recovery algorithm for digital receivers in low signal to noise ratio. This solves the problem of traditional slow converging phase-locked loop-based timing recovery in low noise environments. In this proposal, the receiver exploits the soft decisions computed at each turbo decoding iteration to provide reliable estimates of soft timing signal which in turn improves the decoding time. The proposed alternative tends convergence in about 10 iterations and derived sequential minimization technique approaches a theoretical Cramer-Rao bound with unbiased estimates. The proposed scheme is adaptive to any modulation and is insensitive to carrier offsets apparent in receivers. We have shown through simulation that the proposed method outperforms conventional timing extraction methods with respect to jitter performance. We therefore note that this timing synchronization method is suited to cellular communication receivers, where delay variation is intolerable.**

***Index Terms*—iterative turbo principle, matched filter output, turbo codes, soft information exchanges.**

I. INTRODUCTION

In 1993, Berrou *et al.* developed the revolutionary iterative "turbo" receiver for decoding two dimensional product-like concatenated codes [1]. The impressive performance of turbo codes triggered the application of this powerful coding technique to digital communications at low SNR environments [2, 3]. Since then the iterative turbo principle has been extended to several receiver functions such as joint signal detection and decoding, equalization and decoding and recently iterative signal synchronization [4]-[6]. In most classical timing phase estimations, timing recovery and decoding process have been separated with little penalty; timing recovery uses an instantaneous decision device to provide tentative decisions that are adequately reliable to estimate the timing phase error [7]. In such situations, however, the timing recovery process assumes that the neighboring symbols are independent at high SNR and the theoretical framework is normally based on least mean square (LMS) and traditional phase-locked loops (PLL) [8].
Such a framework is susceptible to local minima and often presents additional block processing complexities which fail in low SNR. Since turbo receivers operate in low SNR, the future of wireless communication systems' cell planning schemes and battery conservation of portable receivers will have to rely on joint timing recovery and decoding algorithm. Ref. [9, 10] has shown that classical soft-input/soft-output (SISO) iterative detection/decoding algorithm embed timing parameter estimation in the decoding process. For instance in [10], combined iterative decoding, equalization and timing error estimation is performed with modified forward and backward recursions in the SISO decoders using a per-survivor processing algorithm. Such methods are reliable but increase the receiver's design complexity with vast memory requirement. Inorder to reduce the complexity involved in designing the decoder structure, soft information provided at each iteration by a conventional turbo decoder can be used to derive reliable information on timing error estimation. This is the essence of turbo principle synchronization technique [6]. Though recent research attention is focused towards this turbo synchronization method [11, 12], it is clear that more attention should be directed towards achieving fast converging timing recovery process. In mobile receivers, particularly cellular applications, the issue of fast convergence and jitter variation in steady state synchronization is very critical. The objective of this paper is therefore to develop a mathematical framework for such turbo synchronization with a focus on faster convergence and jitter reduction. The objective is achieved through Sequential Unconstrained Extremization Techniques (SUET) from turbo functions and polyphase filtering bank (PFB) in place of interpolators hence viable with incommensurable sampling rate applications.
This paper is organized as follows. In section II, turbo system model is presented. In section III, improved soft timing framework is proposed. Simulations are performed in sections IV and V. Conclusions are drawn in section VI.

II. SYSTEM MODEL

The baseband-equivalent of such a turbo-coded communication encoder and decoder structure is depicted in Figure 1. It consists of two recursive systematic convolutional encoders (RSC) which are separated by a pseudo-random L-bits interleaver (INT), puncturing the output of the encoders increases the transmitted code rate from 1/3 to 1/2. On the other hand the receiver consists of modified turbo decoder with two separate soft-In/Soft-Out maximum likelihood a posteriori (MAP) decoders which are connected with interleaver (INT) and deinterleaver (DEINT). The most widely used MAP algorithm is the recursive Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm [4]-[6], which permits easy calculation of the log likelihood ratio (LLR) on block by block basis. BCJR algorithm,

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however, requires priori knowledge of the channel parameters. Fortunately, the feedback loop provides prior soft bit information from one decoder to the other hence the name ‘turbo code’. It works iteratively, i.e. the decoding improves with consecutive iterations upto certain threshold limit when performance degrades. Implementing joint timing recovery and decoding helps converge the number of the iterations faster.

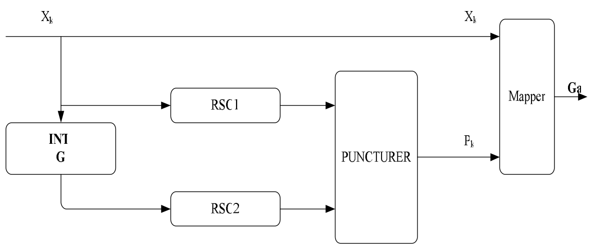

Figure 1a. Structure of a turbo encoder

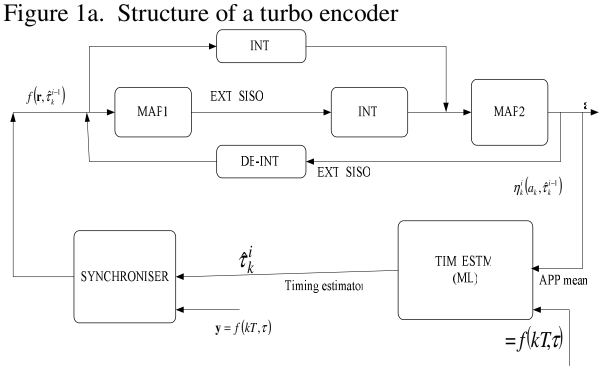

Figure 1b. Structure of modified turbo decoder.

III. THEORETICAL CONSIDERATIONS

The received complex sampled signal is

$$\mathbf{r} = \mathbf{HGa} + \mathbf{n} \tag{1}$$

Where $\mathbf{H}$ is a K-size convolution matrix created from delayed samples of pulse shape $\mathrm{h}(t)$ by set of $\boldsymbol{\tau} = [\tau_1, ..., \tau_K]^T$. $\mathbf{G}$ is an interleaving matrix (obtained by permuting the columns of an identity matrix), $\mathbf{a} = [a_1, ..., a_K]^T$ takes values from a finite alphabet set χ and are the transmitted complex symbols with consecutive symbol duration, T. Vector $\mathbf{n}$ is independently identically distributed (i.i.d.) Gaussian noise with variance No/2.

In order to have sufficient statistics in the decoding-timing recovery module, (1) must be sampled within a set of $T_S \le T/(1+\alpha)$, intervals, where α is the roll-off of the transmitting square root raised cosine filter waveform.

The received sampled vector is passed through discrete polyphase bank [14] of oversampled matched filters with assumed outputs taking the form

$$\mathbf{y} = f(kT + \tau_k) \tag{2}$$

Or explicitly $y\left(\frac{nT_s}{M} + \frac{n_k}{M}\right)$ where n is the number samples per symbol, M is the M-filter stages in a polyphase filter bank and k is the kth complex symbol whose timing phase is yet to be determined. Computing the output of (2) at instants of the argument yields the solution to the problem of time recovery.

The solution to the problem of timing recovery is two fold; estimating timing information and determing the steady state location of the timing instants. Fortunately, several optimization solution to (2) exist, with time-varying timing phase among complex symbols one would naturally present the problem as min-max solution using modified variable *metric* methods (MVMM) [13].

A. *Estimating timing information*

In general case the imperfection of the wireless channel, leads to multi-dimensional random walk jittering timing phase, $\boldsymbol{\tau} = \{\tau_1 ..., \tau_K\}$, within a transmitted frame; however, for the purpose of simplicity in analysis only, we assume vectorial phase in a frame as depicted in [5-6]. The result will be then generalized for relevant emphasis in this work’s timing phase parameter estimation.

Consider an estimation of $\hat{\boldsymbol{\tau}}$ of $\boldsymbol{\tau}$ subject to a trial value $\tilde{\boldsymbol{\tau}}$. This estimate may be seen as the solution of the maximization problem

$$\hat{\boldsymbol{\tau}} = \arg\max_{\tilde{\boldsymbol{\tau}}} \Lambda(\tilde{\boldsymbol{\tau}}) \tag{3}$$

Where

$$\Lambda(\tilde{\boldsymbol{\tau}}) = \ln p(\boldsymbol{r} \mid \tilde{\boldsymbol{\tau}}) \tag{4}$$

$$p(\boldsymbol{r} \mid \tilde{\boldsymbol{\tau}}) = \int_{\boldsymbol{a}} p(\boldsymbol{a})(\boldsymbol{r} \mid \boldsymbol{a}, \tilde{\boldsymbol{\tau}}) d\boldsymbol{a} \tag{5}$$

Where $p(\mathbf{a})$ is the a prior probability mass function, and logarithmic function of second factor of the integrand in (5) is defined as

$$\ln p(\boldsymbol{r} \mid \boldsymbol{a}, \tilde{\boldsymbol{\tau}}) = \Re\left\{\sum_{k=0}^{K-1} a_k^* y(kT + \tilde{\tau}_k)\right\} \tag{6}$$

Here, in-phase baseband channel model is considered without loss of generality. In most theory, complex signal model is desirable. In order to solve for (3), we take the derivative of (4) with respect to $\tilde{\boldsymbol{\tau}}$ and we equate to zero, that is,

$$\begin{aligned} &\frac{\partial}{\partial \tilde{\boldsymbol{\tau}}} \ln p(\boldsymbol{r} \mid \tilde{\boldsymbol{\tau}}) \\ &= \int_{\boldsymbol{a}} \frac{p(\boldsymbol{a}) p(\boldsymbol{r} \mid \boldsymbol{a}, \tilde{\boldsymbol{\tau}})}{p(\boldsymbol{r} \mid \tilde{\boldsymbol{\tau}})} \frac{\partial}{\partial \tilde{\boldsymbol{\tau}}} \ln p(\boldsymbol{r} \mid \boldsymbol{a}, \tilde{\boldsymbol{\tau}}) d\boldsymbol{a} \\ &= \boldsymbol{0} \end{aligned} \tag{7}$$

If we use Baye’s rule in the first factor of the integrand in (7), we have a posteriori conditional probability density function (PDF) of the transmitted vector $\mathbf{a}$

$$\frac{p(\boldsymbol{a}) p(\boldsymbol{r} \mid \boldsymbol{a}, \tilde{\boldsymbol{\tau}})}{p(\boldsymbol{r} \mid \tilde{\boldsymbol{\tau}})} = p(\boldsymbol{a} \mid \boldsymbol{r}, \tilde{\boldsymbol{\tau}}) \tag{8}$$

The Maximum likelihood estimation problem in (3) now becomes

$$\begin{aligned} &\frac{\partial}{\partial \tilde{\boldsymbol{\tau}}} \ln p(\boldsymbol{r} \mid \tilde{\boldsymbol{\tau}}) \\ &= \int_{\boldsymbol{a}} p(\boldsymbol{a} \mid \boldsymbol{r}, \tilde{\boldsymbol{\tau}}) \frac{\partial}{\partial \tilde{\boldsymbol{\tau}}} \left\{ \Re\left\{\sum_{k=0}^{K-1} a_k^* y(kT + \tilde{\tau}_k)\right\}\right\} \\ &= \mathrm{E}_{\boldsymbol{a}} \left\{ \frac{\partial}{\partial \tilde{\boldsymbol{\tau}}} \ln p(\boldsymbol{r} \mid \boldsymbol{a}, \tilde{\boldsymbol{\tau}}) \mid \boldsymbol{r}, \tilde{\boldsymbol{\tau}} \right\} \\ &= \boldsymbol{0} \end{aligned} \tag{9}$$

Since $\tilde{\tau}$ appears in both factors of the expectation problem in (9), the solution of (9) follows iterative turbo decoder principle whose outputs are soft information described by a posteriori mean [6]

$$\eta_k^i\left(\boldsymbol{r},\hat{\tau}_k^{(i-1)}\right) \cong \int_{\boldsymbol{a}} a_k\, p\left(\boldsymbol{a} \mid \boldsymbol{r},\hat{\tau}_k^{(i-1)}\right) d\boldsymbol{a}$$
$$= \sum_{\alpha_\vartheta \in \chi} \alpha_\vartheta\, p\left(a_k = \alpha_\vartheta \mid \boldsymbol{r},\hat{\tau}_k^{(i-1)}\right) \qquad (10)$$

The second factor in (10) is the marginal a posteriori probability (APPs) computed from extrinsic SISO exchanges off LLRs from the second MAP decoder to the first MAP decoder as depicted in figure 1b.

$$\Lambda_{21}\left(a_k \mid \boldsymbol{r},\hat{\tau}_k^i\right) = \log\left(\frac{p\left(a_k = +1 \mid \boldsymbol{r},\hat{\tau}_k^i\right)}{p\left(a_k = -1 \mid \boldsymbol{r},\hat{\tau}_k^i\right)}\right) \qquad (11)$$

Since fast converging estimator is required for our problem, Modified Newton-Raphson (MNR) in [13] is applied to (9) to give a finite discrete approximate solution, that is

$$\hat{\boldsymbol{\tau}}^{(i)} = \hat{\boldsymbol{\tau}}^{(i-1)} - \left(\frac{\partial^2 \tilde{\Lambda}}{\partial \tilde{\boldsymbol{\tau}}^2}\right)^{-1}_{\tilde{\boldsymbol{\tau}} = \hat{\boldsymbol{\tau}}^{(i-1)}} \left(\frac{\partial \tilde{\Lambda}}{\partial \tilde{\boldsymbol{\tau}}}\right)_{\tilde{\boldsymbol{\tau}} = \hat{\boldsymbol{\tau}}^{(i-1)}}$$
$$\hat{\boldsymbol{\tau}}^{(i)} = \hat{\boldsymbol{\tau}}^{(i-1)} - .\boldsymbol{\Omega}^{-1}\boldsymbol{\Psi} \qquad (12)$$

Where
$\tilde{\Lambda} = f\left(\eta_k, \mathbf{y}\right)$, as explicitly depicted in (2) and (10).

Where second term in (12) is governed by choice of the directional unit vector $\boldsymbol{u}$. $\boldsymbol{\Omega}$ is the *Hessian matrix* which must be a nonpositive definite in extremization problem for solution to exist from its inverse. MNR mitigates a problem of $\boldsymbol{\Omega}$ singularity, by choosing the $\boldsymbol{u}$ such that

$$\begin{cases} \boldsymbol{u}^T \boldsymbol{\Omega u} < \boldsymbol{0} \\ \boldsymbol{u}^T \boldsymbol{\Psi} \le \boldsymbol{0} \end{cases} \text{if Eigen}(\boldsymbol{\Omega}) < 0 \qquad (13)$$
$$\boldsymbol{\Omega u} = -\boldsymbol{\Psi} \qquad \text{if Eigen}(\boldsymbol{\Omega}) \ge 0$$

B. Updating timing phase estimates

The estimated timing offset finally interpolates the outputs of discrete polyphase bank of matched filters constructed with M set of weights that are selected from iterative decoder memory of pointer index. Synchronization attained with increase in turbo iterations, generating reliable estimates that maximize the LLRs, hence the selection of the maximum eye opening sample to improve the decoder APP function computation. The process repeats as long as convergence is not attained.

The variance consideration of the estimator may be given by

$$\sigma_{\varepsilon}^2 = \mathrm{E}_{\mathbf{a}}\left\{\hat{\varepsilon}_{\max} - \mathrm{E}_{\mathbf{a}}\left(\hat{\varepsilon}_{\max}\right)\right\}^2$$
$$= \mathrm{E}_{\mathbf{a}}\left[\hat{\varepsilon}_{\max}^2\right] - \left[\mathrm{E}_{\mathbf{a}}\left(\hat{\varepsilon}_{\max}\right)\right]^2 \qquad (14)$$

Where

$\hat{\varepsilon}_{\max} = \tau - \hat{\tau}_{\max}^i$ is the error in estimating the timing phase offset of the proposed algorithm. This error maximizes the LLF presented in (3).

For minimum variance steady-state considerations, expression in (14) is partially differentiated with respect to estimate errors $\hat{\varepsilon}$ and the resulting set to zero. Rigorous derivations are beyond the scope of this paper.

IV. COMMUNICATION LINK SET-UP

To verify the performance of our turbo aided timing recovery scheme, we simulated a baseband communication system transmitting differential quadrature phase shift keying (DQPSK) symbols in MATLAB. We considered convolutional turbo code (031,027) with net rate ½ after puncturing process. The interleaver length set to blocks' sizes of 512 and square root raised cosine signaling pulse with roll-off of 0.25 was used. 1000 blocks were transmitted over additive white Gaussian noise (AWGN), terrestrial wireless medium. The received signal was passed through anti-aliasing filter and then sampled at a rate higher than the baud rate. Discrete matched filter was embedded at the input of the decoder as polyphase bank of synchronizer.

V. PERFORMANCE RESULTS

Figure 2 shows the estimator variance approaching Cramer-Rao bound and indicating unbiased estimator in low SNR up-to 0.35 as iteration increases. In Figure 3 the number of iterations was set to 10, it is clear that for known normalized variances, the performance of the synchronizer improves with decrease in variance. In addition, known timing indicates good performance. This indicates that timing synchronizer uses soft information to get good estimates that converge to true channel offsets and on the other hand, the turbo iterative decoder uses this soft timing signal to converge faster. In Figure 4 it is seen that increasing the iterations improves performance, however whenever the number of iterations exceeds 10, the performance degrades as seen in Figure 2. This means that system converges at only 10 iterations or explicitly in about 10 symbols' period. To explain our hypothesis of low jitter solution further, our proposed alternative was compared with the well-known work of Mueller and Muller in [7]. In Figure 5, we notice that at start-up time, the algorithm in [7] demonstrates some jittering effects and converges to steady state after 10 symbols or loop cycles. On the other hand the proposed algorithm starts up with minimal jittering effects and converges towards steady state region within a preset 10 iterative cycles. For simulation iterations beyond this value, the estimator is biased.

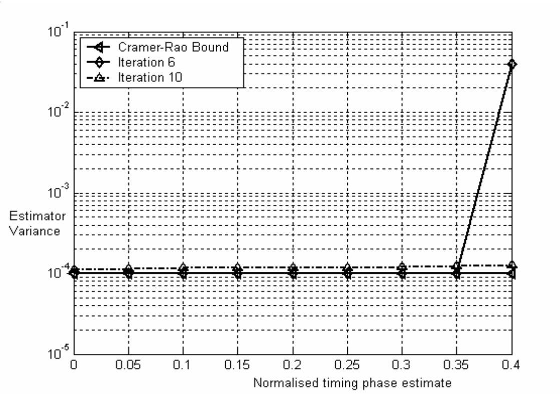

Fig. 2. Estimator variance performance

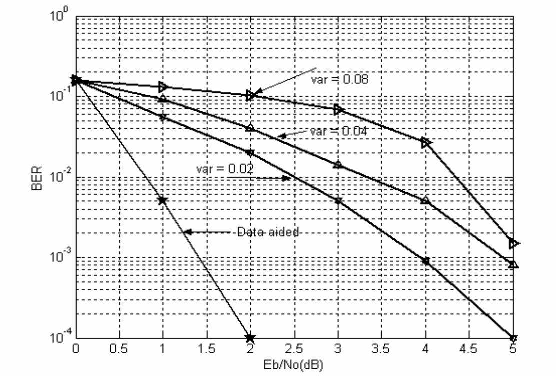

Fig. 3 BER Vs Eb/No at different normalized variances

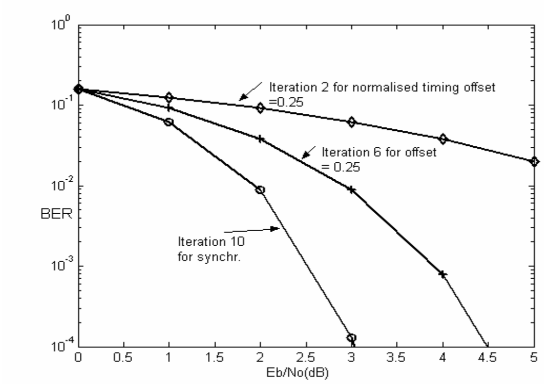

Fig. 4 BER Vs Eb/No at normalized timing offset 0.25

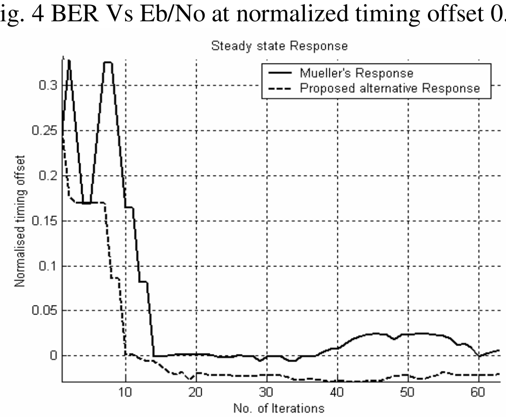

Fig. 5 Jittering criterion at acquisition and steady state

VI. CONCLUSION

Joint synchronization and decoding of turbo codes is likely to give good results in low SNR environments than ever before. In such applications timing recovery is extremely difficult with traditional methods. Deriving good timing estimator function is crucial in both decoding process and steady-state sampling phase. A low variance timing offset estimator with promising results is reported in Figure 3. The variance is also bounded even when the number of iterations is increased beyond 10. The overall performance index in Figure 4 shows that the bit error rate decreases with signal to noise ratio to a maximum of 10 iterations. The performance may start to degrade after 10 iterations due to biasness response of the system in further iterations. The most interesting results are depicted in Figure 5. In comparison of our contribution with the work proposed by Mueller and Muller, our alternative algorithm indicates a lower jitter at start-up time of synchronization. This is a good indication of the solution viability in cellular network applications where high jitter performance is critical. The proposed solution also reaches steady-state earlier than the Mueller and Muller's work hence a better solution for timing recovery problem in mobile receivers.

However, the combined SUET-PFB and turbo symbol synchronization is complex and pilot-aided schemes available in GSM/GPRS/EDGE standards outperform it.

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