THE PERFORMANCE AND LEARNING DIFFICULTIES OF GRADE 10 LEARNERS IN SOLVING EUCLIDEAN GEOMETRY PROBLEMS IN TSHWANE WEST DISTRICT

by

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DECLARATION

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DEDICATION

This work is dedicated to my lovely husband, Olawale Johnson Olabode, my adorable children, Victoria, Augustus, and Christopher Olabode. I love you all.

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ABSTRACT

There is a growing trend of declining performance in the final year (Grade 12) mathematics examinations in the South African public school system. The study aimed to evaluate Grade 10 learners in the Tshwane West District on their performance and learning difficulties in solving Euclidean Geometry problems. The study utilized a mixed method approach under a pragmatic paradigm to determine the achievement and the challenges experienced by learners when solving Euclidean geometry problems. The Van Hiele levels of geometry thinking and constructivist theory, underpinned the theoretical framework used to determine the actual performance and how they understand geometry concepts. The population was Grade 10 learners, and the sample size was 80 learners, purposively selected from two secondary schools in the Tshwane West district in the Gauteng Province. The Geometric Achievement Test instrument was used to determine firstly, the overall performance of learners in Euclidean geometry, secondly, the level of Grade 10 learners on Van Hiele levels of thinking and lastly to specify the area where learners have the most difficulties when engaging with geometry problems. A semi-structured interview guide and class observation checklist were used to further understand the challenges experienced by grade 10 when they are faced with Euclidean geometry questions.

The findings from this study showed the underperformance of learners in three Euclidean geometry topics: these included parallel lines, congruency, and parallelograms. The findings further indicated that the low performance was a result of a lack of understanding of the computational and spatial thinking that characterises Euclidean geometry. These findings were supported by the quantitative findings of the study.

The pass mark stipulated in Euclidean geometry is 30%; less than 10% of the participants in the GAT obtained 30% and above. The findings of the study showed that less than 5% of the learners obtained a score of 38 on the achievement test, which was the highest score obtained. Of the 80 participants, 1,25% (1 of 80) obtained a score of 2. Euclidean geometry requires learners to use their spatial and logical skills in solving mathematical questions; for this sample, making connections and comprehending the visual and spatial aspects of parallel lines, congruency, and parallelograms were found to be difficult.

The findings also showed that learner lack an understanding of the properties of parallel lines, resulted in difficulties in calculating the magnitudes of unknown angles using these properties. The findings further indicated that Grade 10 learners have difficulty solving geometry requiring knowledge of corresponding, alternating, and co-interior angles. A notable difficulty was that for the learners to apply the conditions of congruency and execute the proof for parallelograms, learners must understand parallel lines and its properties. Teachers must ensure that learners understand the procedures of naming angles correctly. The understanding of properties of parallel lines, congruency, and proof of parallelograms is essential for enhancing learner abilities in Euclidean geometry.

Keywords: Geometrical thinking, learner performance, learning difficulties, parallelograms, congruency, parallel lines, interaction

LIST OF ABBREVIATIONS AND ACRONYMS

Abbreviation/Acronym	Description
ANA	Annual National Assessment
CAPS	Curriculum and Assessment Policy Statement
COVID-19	Corona virus-2019
DBE	Department of Basic Education
FET	Further Education Training
GAT	Geometry Achievement Test
LTSM	Learning and Teaching Support Material
MST	Maths, Science and Technology
NSC	National Senior certificate
PGCE	Post Graduate Diploma in Education
SPSS	Statistical Package for Social Sciences
SSIP	Secondary School Intervention Program
SSCS	Search solve create and share
SACMEQ	Southern and Eastern Africa Consortium from Monitoring Education Quality
TIMSS	Trends in International Mathematics and science studies
2D	Two Dimensional
UNISA	University of South Africa
WEF	World Economic Forum

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CHAPTER ONE

ORIENTATION TO THE STUDY

1.1 INTRODUCTION

This section provides an overview of the learning difficulties of Grade 10 learners in Tshwane West District, in the Gauteng Province, in solving Euclidean geometry problems. It is divided into sub-sections; the motivation for the study, geometry, a branch of mathematics, learning difficulties and poor performance of learners in Euclidean geometry, and poor performance generally in mathematics of South African learners.

1.2 MOTIVATION FOR THE STUDY

There is a growing trend of a declining pass rate in mathematics examinations of the final school grade (Grade 12) in South Africa. Learner performance in mathematics has been declining. Research has shown that learners struggle with mathematics right from elementary education. In the field of mathematics, geometry Euclidean geometry has been highlighted as one of the topics learners find difficult to comprehend. This branch of mathematics presents a myriad of challenges to many secondary school learners. In response to this failure in mathematics by Grade 12, the Secondary School Improvement Programme (SSIP) was introduced by the Gauteng Department of Education (GDE) to improve learner abilities in mathematics to provide improvement in in schools whose pass rate is less than 80% in the final year of schooling.

This concern resonates with the researcher's experience as a mathematics teacher for over two decades. It appears that learners face difficulty in solving Euclidean geometry problems. Present-day learners still struggle to gain the appropriate knowledge of geometry. This often leads to low performance when this aspect of mathematics is raised. Stakeholders such as teachers, parents, and the government are worried about this trend in students’ performance in their final exams. This concern necessitated this study. Combining different research tools such as GAT, interviews, and observations, the researcher examined the performance and learning difficulties of Grade 10 learners in two selected secondary schools in Tshwane West District, Gauteng, Province.

1.3 GEOMETRY

Geometry is an ancient branch of mathematics, the origin of which is multi-fold. Many studies claim that geometry begins from numerous cultures, which include Indian, Babylonian, Egyptian, Chinese, and Greek. Makhubele (2015), describes early geometry as first discovered among the Egyptians when they were re-measuring their land that was flooded and destroyed by the Nile River floods. The re-measuring of land was imperative to Egyptians during this period. The Department of Basic Education (2014), state that the word geometry comes from the Greek word 'geo-metrein'. The word geo means "earth" and mentrein which means "measurement" (Jacobs, 1999). In other words, geometry is the study of science that deals with the measurement of the earth. Makhubele (2015) also defines geometry as a branch of mathematics that deals with the shapes of individual objects, spatial relationships among various objects and the properties of spaces and shapes. Geometry can be divided into two major sections: Analytical geometry and Euclidean geometry. Analytical geometry deals with the study of space and shape using algebra and coordinate systems.

Euclidean geometry is the study of space and shape using a system of logical deduction. Euclidean geometry is based on axioms and theorems. Euclid collated the geometry known at the time into a system of axioms and theorems (Greenberg, 1999). According to Yagamram and Naidoo (2010, as cited in Alex, 2016), Euclidean geometry is connected to every strand in the mathematics curriculum and many real-life situations. Euclidean geometry is a topic in mathematics that is connected to culture, history, art, and design and its relation to life makes it more ‘interesting and stimulating’ (Luneta, 2015). Jacobs (1999) asserts that Euclidean geometry is fascinating and useful because of its wide range of applications to real-life problems (Jacobs, 1999, p. xi). Jacobs argues that many objects around our environments are formed with amazing Euclidean shapes from geometry like the arc of circles in rainbows, hexagons in honeycombs, cubes in salt crystals and spheres in soap bubbles (Jacobs, 1999). Euclidean geometry develops critical thinking (Bhagat & Chang, 2015; Ndlovu & Mji, 2012; Pandiscio, 2015) and logical thinking skills of learners (Pandiscio, 2015; van Putten, Howie & Stols, 2010); promotes the spatial perception of the real world (van Putten et al., 2010); and helps to teach reading and interpretation of mathematical argument (Shongwe, 2022).

1.3.1 Learning difficulties and poor performance in Euclidean geometry

Learning difficulties can precipitate poor performance in learners due to a lack of mathematical knowledge about a topic or concept. Sebsibe (2019:9) refers to learning difficulties as a deficit in learner knowledge about a phenomenon. In the context of this study, learning difficulties are understood as deficiencies a learner has which make learning difficult. This can affect the performance of the learner in mathematics. Euclidean geometry is often perceived as a problematic and difficult topic (Alex, 2016; Luneta, 2015; Sadiki, 2016). Learning difficulties generally result in poor performance, now being recorded in most countries. In the context of South Africa, the topic has become one of those in which learners perform extremely poorly (Makhubele, Nkhoma & Luneta, 2015). Alex (2016) identified teaching practices that are not learner-centred, and which lack basic understanding as the learning difficulties of learners in geometry, as contributing to poor performance. Ndlovu and Mji (2012) categorised learner difficulties experienced in geometry into four categories. The first involves difficulties that show lack of basic geometry knowledge and vocabulary. The second includes learner inability to make logical deductions. The inability of learners to organise in a logical chain from a given conclusion was categorised in the third, and the inability of learners to precisely label and connect arguments was the fourth. This indicates that the level of difficulties learners experienced ranges from fewer difficulties to more difficulties attributed to basic knowledge. Alex (2016) further argues that a lack of basic knowledge also results in differences in the level of learner thinking and the expected level they should have in geometry. This indicates that learners appear to lack basic knowledge in geometry leading to poor performance in this topic.

Some researchers argue that it is difficult to teach and learn Euclidean geometry because of its complexity and abstract nature (Luneta, 2015; Mamiala et al., 2017; Ndlovu & Mji, 2012; Pandiscio, 2015). Van Putten et al. (2010) argue that many teachers are not familiar with the content of Euclidean geometry, therefore, this shows that teachers may experience difficulties in teaching this topic.

Luneta (2013) concurs that teachers lack the skills that are necessary to empower learners to understand the geometry concepts. Some teachers found it difficult to explain the contents of Euclidean geometry to their learners in such a way that they may develop a conceptual understanding of the content learnt (Luneta, 2015). Lack of conceptual understanding in learners leads to errors and misconceptions and even results in learners being unable to answer geometry question in the examination (Luneta, 2015; Makhubele et al., 2014).

A further observation by Luneta (2015), is that errors in geometry occur because of the procedural way geometry is taught to the learners. Luneta (2013) asserts that in the procedure for solving mathematics problems many of the errors made in geometry by learners were conceptual errors, not procedural. Long (2011) defines procedural knowledge as the knowledge that is characterized by habitual repetition which implies creating a building block on the knowledge, but which does not necessarily have a skill or knowledge that connects with other skills or knowledge (Long, 2011). The ideal situation is when procedural knowledge is underpinned by conceptual knowledge.

Several studies have been conducted on learning difficulties in Euclidean geometry, that is errors and misconceptions (Alex, 2016; Bhagat & Chang, 2015; Feza & Webb, 2005; Kesan & Caliskan, 2013; Luneta, 2015; Makhubele et al., 2014; Ngirishi & Bansilal, 2019). As far as the researcher is aware little or no research was conducted in the Tshwane West district relating to the learning difficulties of grade 10 learners in Euclidean geometry.

1.3.2 South African learners' poor performance in Mathematics

Over the years, the poor performance of South African learners in mathematics has emerged as a growing concern (Mabena, Mokogosi & Ramapela, 2021). The poor performance of learners in South Africa is evident in many cross-national and national achievement tests in which most of them did not do well in mathematics. South African learners participated in international studies such as the World Economic Forum (Baller et al., 2016), Trends in International Mathematics and Science Studies (TIMSS, 2019), and Southern and Eastern Africa Consortium from Monitoring Education Quality (SACMEQ, 2013) systemic mathematics assessments. Cross-national studies aimed to provide adequate information for comparing the standards and improving the quality of education in the participating countries. South Africa was ranked number 137 out of 139 in 2015 in WEF and last position out of 143 countries in WEF in 2016.

Similarly, South African Grade 8 learners who participated in TIMSS were rated 37 out of 39 countries (TIMSS, 2019). South Africa was rated the 6th position out of 14 countries that participated in SACMEQ 2013. Spaull (2013) reports that the South African educational system may be among the countries in the world that is the weakest regarding mathematics, particularly at the secondary level, based on the results of the international tests.

Poor performance in mathematics of South African learners similarly occurs in the national assessment tests. An example of the national assessment test in which South African learners do not perform well is the National Senior Certificate (NSC) examination for Grade 12 learners. The National Senior Certificate (NSC) for South African Grade 12 learners provides an exit point for the twelve years in the basic education system but also offers a measure of the health of the educational system in South Africa. The NSC performance trend (2017-2021), revealed, the percentages of Grade 12 learners that achieved 30% and above were51.9% in 2017, 54.8% in 2018, 54.6 in 2019, 53.8 % in 2020 and 57.6% in 2021. Although 7.9% and 3.8% improvements were revealed in 2018 and 2021 respectively, a declining performance trend was experienced in 2019 and 2021 with 3.4% and 3.8% respectively.

The poor performance of South African learners evidenced in the various assessments mentioned above is an indication that there may be obstacles learners experience which led to them performed dismally. According to Mabena, Mokgosi and Ramapela (2021), learners may struggle due to internal issues relating to the learner, and external issues relating to the teacher. In the same vein, Tachie and Chireshe (2013), opine that internal factors are issues that learners encounter on their own. Learner beliefs, a negative attitude toward mathematics, lack of interest in the subject, and language barriers are examples of such phenomena, whereas external factors are factors that are driven by occurrences that are external to the learner. Those variables are created by things over which learners have no influence, such as incompetent teachers, ineffective teaching strategies, teacher attitude and behaviour, or a lack of resources, that prevent learners from understanding mathematics conceptually. These challenges could have influenced the performance in mathematics.

Language issues and poor reading skills, according to Daniyan (2015), may also contribute to low performance in mathematics. Learners who fail to understand the vocabulary used in mathematics teaching and learning may struggle academically. The author further argued that learners who struggle to comprehend the vocabulary employed as an instructional medium lack the capacity to think and learn mathematics. In other words, learners with language issues find it difficult to think about and learn mathematics concepts. Because they do not understand the medium of instruction, this can consequently lead to poor performance in mathematics.

Some studies, Alex (2016), Bhagat and Chang (2015), and Luneta (2015), have claimed that some topics in mathematics, such as algebra, calculus, and geometry, are challenging for learners due to their being abstract in nature, with a practical component that is unfamiliar to learners. Learners perceive topics as difficult because they do not understand the concepts involved.

Another aspect that contributed to poor performance was a loss of interest resulting from aa lack of understanding of Euclidean geometry (Alex, 2016). According to Sinyosi (2015), a lack of knowledge of geometry might lead to learners acquiring a negative attitude toward the subject. This means that when pupils do not grasp Euclidean geometry, they may pay less attention to what they are learning, perhaps resulting in poor performance.

In response to the problem, the government made efforts to minimize the poor performance of learners in mathematics through various school intervention programs like the Dinaledi projects and the secondary school intervention program (SSIP) The Dinaledi project was established in 2011 with a few schools involved, Schools in the Dinaledi project were selected with the aim of increasing the number of learners who can take mathematics and physical sciences as their major subjects from Grades 10 to 12. In addition, the project aimed to improve teacher pedagogical and content knowledge in mathematics and science with the hope of increasing the pass rate of learners in Grades 10 to 12 (Department of Basic Education, 2016). Subsequently, the Dinaledi project was transformed into the Mathematics, Science and Technology (MST) school project to cater for the Grade 8 and 9 learners in mathematics and natural science. The MST project was implemented for increasing the number of learners taking mathematics, science, and technology subjects to improve the pass rate in the subject and to increase teacher capacity.

Another intervention programme is the Secondary School Intervention Program (SSIP) for Grades 10 to 12. SSIP focuses on critical and challenging areas of the curriculum content of the gateway subjects using the best teachers to teach Grades 10 to 12 and specially developed resources. This program has been implemented since 2015 and runs during the weekends and holidays throughout the year. These programmes were also implemented in schools attached to the Gauteng Department of Education.

Despite the intervention strategies implemented by the Gauteng Department of Education to improve the poor performance in underperforming schools, some schools in Gauteng Province, especially in the Tshwane West district still perform poorly in mathematics. This problem was identified through the analysis of Schools' Subject Reports from 2017 to 2021 (DBE, 2014; Department of Basic Education, 2017; Department of Basic Education, 2013). The School Subject Report from the Department of Basic Education provides information on the performance of each school nationwide in each key subject. The School Subject Report revealed that some schools had constantly performed below the achieved level within these periods. In the context of this study, underperforming schools are those that perform below the standard achievement level. The standard achievement level of 30% and above was set by the Department of Basic Education. The results shown in the table 1 below are the pass rates of learners in some of the schools that have performed poorly in mathematics for the past five years in the Grade 12 matric examination.

Table 1: Pass rate of schools underperforming in mathematics from 2017-2021

School	Quintile	2017	2018	2019	2020	2021
A	3	26.0%	42.6%	56.8%	36%	26.7%
B	3	35.0%	55.2%	29.4%	40.9%	28.0%
C	1	29.0%	34.0%	44.2%	22.6%	48.%
D	1	25.0%	42.9%	29.0%	47.8%	32.3%

In the past five years, learners in underperforming schools have had difficulty in answering most of the questions in the Grade 12 matric examination. The poor results in mathematics from these underperforming schools had negative effects on a number of school leavers, for whom there is a minimum university admission requirement. Poor performance in mathematics may become a barrier for learners who may want to further their careers in engineering, science, and technology (Alex, 2016).

The NSC Grade 12 mathematics examination is divided into Paper 1 and Paper 2. From the analysis of Paper 2 in the 2021 NSC examination, Euclidean geometry constitutes the largest part. The Curriculum and Assessment Policy Statement (CAPS), stipulates that Euclidean geometry should constitute 50 marks out of the 150 marks allocated to Paper 2. In other words, Euclidean geometry constitutes 30% to 35% of the total mark of Paper 2. Euclidean geometry is not only an essential part of the South African curriculum but is an essential part of mathematics curricula internationally. Miyazaki, Fujita, and Jones (2017), argue that teaching and learning Euclidean geometry is a key component of any mathematics curriculum.

The elimination of Euclidean geometry as the required Paper 3 in 2008 has resulted in a loss of consistency in the study of space and shapes (Van Putten, Howie & Stols, 2010). Between 2008 and 2012, learners did not have the option to engage with proofs. The researchers also discovered that some teachers did not appear to be teaching Euclidean geometry and that some teachers may have neglected to teach Euclidean geometry and focussed only on teaching other mathematical topics. Van Putten et al. (2010) contended that the teacher's attitude influences the teaching of Euclidean geometry. Since Euclidean geometry was considered an optional paper, teachers could choose whether to teach it. Teachers may select what to do with the Euclidean geometry curriculum and whether to teach it or not Some of the schools were not able to teach their learners the content of Euclidean geometry (Van Putten et al., 2010).

However, Euclidean geometry has been introduced again in the new curriculum, the Curriculum Assessment Policy Statement (CAPS), as a compulsory paper 2 (Van Putten et al., 2010). Teachers who do not have the background knowledge of Euclidean geometry may not be able to teach it effectively. The lack of background knowledge of Euclidean geometry in teachers may constitute one of the difficulties that learners experience.

1.4 RATIONALE FOR THE STUDY

This study is of great importance to the academics, stakeholders, teachers, parents and policymakers in South Africa, and the continent of Africa as a whole. Firstly, this study provides greater insight into the learning difficulties resulting in poor performance in mathematics in general and specifically in Euclidean geometry.

The findings of this study would provide recommendations and strategies to enhance student learning abilities in Euclidean geometry. Employing mixed method approaches to this study enabled the researcher to examine the performance and learning difficulties of Grade 10 learners.

The findings from this study may provide valuable information to mathematics teachers especially teachers who teach in both the Senior Phase and Further Education and Training

(FET) band. When teachers are aware of learner difficulties, they then know where to assist the learners in addressing these difficulties. Hence, learner knowledge of Euclidean geometry may likely improve if learner difficulties in solving Euclidean geometry can be resolved. Tachie and Chireshe (2013) argue that learners with adequate knowledge of geometry may likely pursue sciences, engineering, and technology. Addressing the challenges of learners may improve poor performance in Euclidean geometry, which may increase the number of learners pursuing an engineering career.

1.5 THE PROBLEM STATEMENT

Learning difficulties in Euclidean geometry are a major concern in South Africa, especially in many schools in Gauteng Province. Some of the schools that perform poorly in mathematics are found in the rural part of the Tshwane West District. As previously indicated, four schools had been constantly underperforming over the last five years despite the intervention strategies by the Gauteng Department of Education to improve the learner pass rate in the Tshwane West District. This situation posed a question as to why learners in these schools were performing poorly. Learners in these underperforming schools were having difficulty answering most of the questions in the examination.

Studies have been done regarding learning difficulties in Euclidean geometry (Alex, 2016; Bhagat & Chang, 2015; Musyimi, 2016; Sadiki, 2016). The study of Luneta (2015) aimed at understanding learner misconceptions of Euclidean geometry. This study found that errors and misconceptions have resulted in learners not answering any question related to Euclidean geometry during an examination. Bhagat and Chang (2015) found that the use of GeoGebra can be an effective tool in teaching geometry because it did not require the use of the internet. Musyimi (2016) focuses on Kenyan schools in finding the factors affecting poor performance in Euclidean geometry. Sadiki (2016) found that using Van Hiele levels of geometric thinking was effective in the teaching of Euclidean geometry. As far as the knowledge of the researcher is concerned little or no research that has been done in Tshwane West related to learning difficulties in Euclidean geometry. Hence, there was a need for research in exploring learning difficulties in Grade 10 Euclidean geometry in Tshwane West District. If learners do not have a strong foundation at Grade 10, they continue to have difficulty which might ultimately result in poor performance in the NSC Euclidean geometry examination.

This study explored the in-depth Grade 10 learning difficulties in Euclidean geometry using quantitative and qualitative approaches. The findings obtained from this study gave an overview of the challenges resulting in poor performance. The study further came up with recommendations for addressing these challenges.

1.6 RESEARCH QUESTION

Following the research gaps in the literature identified from the research problem stated in the previous sections, the study tends to provide answers to the following research question.

1.6.1 Main research question

What are the learning difficulties learners experience when solving Grade10 Euclidean geometry problems in Tshwane West District?

1.6.2 Research sub-questions

The following sub-questions have been formulated to answer the main research question:

- How do grade 10 learners perform in solving Euclidean geometry problems?
- What difficulties do Grade 10 learners experience when solving Euclidean geometry problems?
- Why do learners experience difficulties when solving Euclidean geometry problems?
- How do Grade 10 learners engage with Euclidean geometry concepts?

1.7 AIMS AND OBJECTIVES OF THE STUDY

The aim and objectives inform the steps the researcher followed in data collection to seek answers to the research questions formulated in this study.

1.7.1 Aim

The study aim was to explore the learning difficulties experienced by Grade 10 learners in Tshwane West District in solving Euclidean geometry problems.

1.7.2 Objectives

The study objectives are as to:

- Examine the performance of Grade 10 when solving Euclidean geometry problems.
- Examine learners' learning difficulties when solving Euclidean geometry.
- Explore how Grade 10 learners learn Euclidean geometry.
- To suggest possible strategies that can be used to improve the performance and address learners' learning difficulties when solving Euclidean geometry.

1.8 RESEARCH METHODOLOGY AND DESIGN

Research methodology is the process of planning and strategising on how data are collected and analysed to answer the research questions and carry out the purpose of the study (Johnson & Christensen, 2020; McMillan & Schumacher, 2014). Research methodology provides a roadmap as to how the researcher plans to carry out the research. This roadmap explains the research paradigm, research approach, research instrument, research design, population, and sampling, selected to explore the learning difficulties of Grade 10. The roadmap also explains how the data that were collected were analysed to achieve the aims and answer the research questions that will be used to explore the learning difficulties of Grade 10 learners in solving Euclidean geometry questions (see Chapter Four for more detail).

1.8.1 Research paradigm

A research paradigm is a mode of thinking, aligned with a community, to which a researcher fits in terms of assumptions, propositions, thinking and approach to research (Bertram & Christiansen, 2014; Johnson & Christensen, 2020; Okeke & Van Wyk, 2015). Researchers can be in a research community according to their views or perspectives about their research. Bertram & Christiansen (2014) argue that the perspective in which the researcher views the world will influence the way the researcher will research the world.

A researcher can be positioned in a positivist, interpretive or critical paradigm (Bertram & Christiansen, 2014; Okeke & Van Wyk, 2015). The study is positioned within an interpretive paradigm. The paradigm was based on the perspective that there exist important social relationships in a natural setting. The social relationships may not be well understood through the application of the positivist paradigm which is a scientific approach that relies on easily measurable phenomena. The scientific approach aims at establishing laws and principles of universal validity (McMillan & Schumacher, 2010:323). For instance, the scientific approach often neglects to address the reason for the existence of certain behaviours in a context. The interpretive paradigm, therefore, considers the meanings and interpretations of participants in the collection of data. Since this research intended to explore and understand the learning difficulties of the learner in solving Euclidean geometry (Creswell, 2014), the interpretive paradigm provided a suitable methodology as it emphasised studies that were carried out in a natural setting (see Chapter Four for more detail).

1.8.2 Research approach

The study employed a mixed method approach in a sequential explanatory research design by combining quantitative and qualitative approaches in a single study. This research design is the most appropriate for this study considering the nature of the research aims and the questions formulated for the study. For the quantitative aspect of this study, the researcher administered a geometry achievement test to learners in underperforming schools to identify the learning difficulties in solving Euclidean geometry. This provided the researcher with insights into the learning difficulties that Grade 10 learners were experiencing when solving Euclidean geometry problems in Tshwane West District in Gauteng Province.

This performance obtained from the diagnostics test informed about the participants' geometry level of thought as described by the Van Hieles. This informed the qualitative aspect of this study. Semi-structured interviews and lesson observations were applied afterwards by the researcher to follow up on the poor performance and learning difficulties. The researcher went to the classroom to observe how teachers from the underperforming schools taught and interacted with their learners (see Chapter Four for more detail).

1.8.3 Population and Sampling

A population is the total number of people, groups or organisations that could be involved in a study (Bertram & Christiansen, 2014), that is the larger picture of who could be involved in the study. Sampling is the process of determining the people, settings events or behaviours that are needed in the study (Bertram & Christiansen, 2014).

The population for this study was Grade 10 learners and their teachers from Tshwane West District public schools. At the time when this study was conducted, there were 56 public schools in Tshwane West District and the sample was taken purposefully from the underperforming schools (Creswell & Creswell, 2023). The learners came from different socioeconomic backgrounds and the sample was selected using a purposive sampling technique. The purposive sampling was appropriate for this study because the study focused on the schools that were not performing well in Euclidean geometry in the Tshwane West district in Gauteng (see Chapter Four for more detail).

1.8.4 Instrumentation and data collection techniques

This study used the following instruments to collect data in two selected schools: diagnostic tests, semi-structured interviews, and classroom observations. The diagnostic test was designed by the researcher using Grade 9 textbooks and an examination bank. The purpose of the achievement test was to determine learner knowledge and their levels of Van Hiele geometrical thinking.

To ensure trustworthiness, the instrument was given to experts in the field of mathematics education like the professors in the department of mathematics. The researcher used the question bank to change some of the questions so to ensure the reliability of the instrument. The instrument focused on conditions of congruent triangles and the properties of parallel lines which were learnt in Grade 9. The researcher included the parallelogram which was a topic in grade 10.

The instruments were piloted to learners of other underperforming schools to ensure the security of the main study. A pilot study is a mini investigation designed to test instruments and methods to be used in the main study to identify problems that may occur in the main study (Okeke & Van Wyk, 2015). McMillian & Schumacher (2014) argue that conducting a pilot study gives the researcher the opportunity of checking the clarity or ambiguity of sentences, the time for completion, and any problems experienced before the instruments could be used in the main study. The researcher administered the Geometry Achievement Test (GAT) with the help of mathematics teachers in schools that participated in the study.

The semi-structured interview schedule was used which was informed by the diagnostic test results from the participating schools. Semi-structured and open-ended questions were used so that participants were able to explain the learning difficulties that the learners are facing when studying Euclidean geometry. Newby (2010) argues that semi-structured interviews have the advantage of rich data. Another advantage is that the researcher rephrases questions where there may be a need for further clarification on what the researcher intended to find out (McMillan & Schumacher, 2014). Also, the two teachers from selected schools were interviewed to establish their educational qualifications and years of experience. The researcher interviewed the teachers in these schools to ascertain their level of qualification, and to ascertain whether they were qualified to teach grade 10 mathematics.

After the semi-structured interview, the researcher went into the classroom to observe how teachers teach grade 10 geometry and how learners learn this topic, focusing on both the pedagogical and content knowledge of the teacher. McMillan and Schumacher (2014) argued that when a researcher observes the participants over a period, the researcher makes sense of, or interprets, the phenomena in the natural settings regarding learner performance and learning experiences (see Chapter Four for more detail).

1.8.5 Data analysis and interpretation

Data analysis is a process of putting the gathered information in order and giving meaning to it (Barbbie, 2013). The researcher analysed the learner scripts resulting from the diagnostic test by categorising the results using the frequency distribution table. Learner scripts were categorised according to the difficulties learners experience when solving Euclidean geometry problems.

The data on the video and audio tape was transcribed, summarised, and categorised according to the emerging themes that were relevant to the research questions and objectives of the study (Saunders et al., 2013). The researcher began coding soon after the first interviews were conducted as the first set of data served as a basis for data collection and analysis, (Corbin & Strauss, 2008). The researcher identified the themes as related to the research questions and objectives.

The researcher also considered “new” themes which were not anticipated in the collection methods that emerged from the fieldwork. This necessitated the use of open coding which was provided for a detailed examination of the interviews. Generating categories and themes required “a heightened awareness of the data, a focused attention to this data, and openness to the subtle, tacit undercurrents of social life” (Marshall & Rossman, 2011, pp. 158–159). The researcher had prolonged engagements with the fieldwork data as well as the literature review to generate categories which augment those suggested by the questions presented to participants (see Chapter Four for more detail).

1.9 CREDIBILITY AND TRUSTWORTHINESS

According to Creswell (2012), credibility is the process of validating or finding the accuracy of data using data triangulation and member checking (2012, p. 259). However, McMillan & Schumacher (2014) argued that to ensure credibility, research may combine any of the following 10 strategies which include prolonged and persistent fieldwork: multimethod strategies, participant language, verbatim accounts, low-inference descriptors, multiple researchers, mechanically recorded data, member checking, participant review and negative discrepant data. Therefore, the data must be analysed by scrutinising the data to get the theme that represents the experience. Credibility in qualitative research is of the highest significance (Creswell, 2012, p. 259). To ensure the credibility of this study, the researcher and participants agreed with the meaning and description or composition of the experience. The researcher conducted interviews with the participants and ensured that data gathered from the interview correlated with the information collected during observation and data analysis. The researcher informed the teachers and learners about all they needed to know about the study so that the credibility and trustworthiness of the result would not be affected (see Chapter Four for more detail).

1.10 ETHICAL CONSIDERATION

Teachers and learners participated voluntarily and were allowed to terminate their participation at any time during the study (McMillan & Schumacher, 2014, p. 130). The researcher did not force anyone to participate in this study.

Furthermore, the researcher informed the teachers and learners about the procedure and that there was no risk involved in the study. Permission was obtained from the parents of learners that participated in the study. Parents also gave consent by signing the consent form for learners to participate in the study (Okeke & Van Wyk, 2015).

To maintain the confidentiality of the participants, the researcher kept the teacher and learner names anonymous when reporting the result of the study. The researcher ensured that the identities of teachers and learners that participated in the study were protected (Bertram & Christiansen, 2014). Also, the researcher respected the privacy of the teachers and learners by keeping the confidentiality of the data or information obtained from them (see Chapter Four for more detail).

1.11 LIMITATION AND DELIMITATION OF THE STUDY

The study has several limitations that will affect the generalization of the findings of the study. Only two underperforming high schools in the Tshwane West district were sampled. This may not represent the poor performance and learning difficulties of grade 10 learners in the district and the entire Gauteng Provine. Hence, the findings from this study cannot be generalized.

Another limitation was the restrictions during the Covid -19 pandemic. The outcome of this study was influenced by the COVID-19 pandemic during the data collection period, in which researcher access to the learners and classrooms was limited to six lessons for the period of lesson observations. Each week, the researcher observed two one-hour lessons. A period of three weeks will have been enough for the classroom observation.

The study focused only on the difficulties learners are experiencing when learning about parallel lines, conditions of congruency and parallelograms which is only one aspect of Euclidean geometry. These topics according to the CAPS document should only be taught for one week during the first term. The study was conducted over three weeks to collect the necessary data within the stipulated period. In addition, the fact that the study only focused on grade 10 Euclidean geometry meant other aspects of Euclidean geometry were not touched, therefore, the question of credibility may arise because the study is only focused on one aspect of Euclidean geometry. Nevertheless, the findings have relevance to the field of mathematics teachers.

1.12 OUTLINE OF THE THESIS

The thesis is divided into seven chapters.

Chapter One introduces the research concept of the study, as well as the motivation for this study. It further discusses the research problem, research objectives, research questions and research hypotheses. Other sections discussed in this chapter include the scope of the study, limitations of the study, significance of the study, research methodology and ethical clearance.

While Chapter Two presents and discusses in detail the theoretical framework driving this study. Chapter Three comprises a review of the relevant literature based on the objectives of this study covering the subject of geometry, Euclidean geometry, within mathematics education.

Chapter Four addresses the study's research approach, research paradigm, research design, and methodologies employed.

Chapter Five presents the data and the analyses. Chapter Six discusses the findings of the study relating to a diagnostic test, semi-structured interviews, and lessons observations. Chapter Seven concludes the study by summarising the key findings of the study, providing the conclusion, and proposing recommendations for the study as well as suggestions for future studies.

1.13 CHAPTER SUMMARY

This study provides an overview of the learning difficulties and challenges learners encounter in solving Euclidean geometry problems. These challenges have led to poor performance resulting in a lack of appropriate and adequate knowledge of Euclidean geometry. This challenge comes in two forms. One of which is the learner-learner interaction, and the other is the learners-teacher interaction as it relates to the knowledge bank of both the teachers and the learners regarding Euclidean geometry. These concerns have been presented in the different sections in Chapter One. The introduction, the motivation of the study, the statement of the problem, the significance of the study, the preliminary literature review and the theoretical framework and research methodology give a holistic view of the research problem of the study.

CHAPTER TWO

THEORETICAL FRAMEWORK

2.1 INTRODUCTION

The previous chapter provided an overview of the learning experiences faced by grade 10 learners when solving Euclidean geometry problems. The performance and learning difficulties of grade 10 learners are discussed in terms of internal factors and external factors. Chapter two provides insight into the theoretical framework on which this study is anchored. This chapter is further divided into different sections. The first discusses the importance of theories in teaching and learning and the second presents the theories applied to inform this study. In this section, constructivism theory is discussed. The Van Hiele theory, encompassing the Van Hiele levels of geometrical thinking and the Van Hiele Instructional Model are discussed in the third and fourth sections respectively. The rationale for choosing these theories is discussed.

2.2 THE IMPORTANCE OF THEORIES IN TEACHING AND LEARNING

The term theory denotes a set of statements or principles which explain a group of facts and phenomena (Gray & Macblain, 2012). In other words, a set of principles on which the practice or activity has based a supposition or a system of ideas intended to explain a phenomenon. According to Johnson and Christensen (2019), a theory is a generalization or a set of generalizations that is used to describe carefully how a system works in general. This set of principles would undergo various acts of scrutiny, and rigorous testing by passing through the different stages of practice and reflection before it can be regarded as a theory. Theories used in teaching and learning provide an explanation of complex situations in terms of how and why such a system operates in educational contexts (Johnson & Christensen, 2019). Some examples of systems that operate in teaching and learning include how learners develop their cognition, intelligence and memory, and motivation toward learning. Theories explain how and why such situations operate.

Most theories relevant to teaching and learning include in some measure, constructivism, the theory of multiple intelligences, social learning theory and others (Gray & Macblain, 2012).

The theoretical foundation of this study is anchored on two theories namely:

- Constructivism theory
- The Van Hiele theory – Van Hiele levels of geometrical thinking, and the Van Hiele Instructional Model

2.3 CONSTRUCTIVISM THEORY

The constructivism theory is a learning theory first propounded by Jean Piaget, a Swiss scientist in 1964. Other theorists expounded further on the constructs in Piaget's constructivism theory. The central focus of the constructivism theory, and socio-constructivist theory, by the founding fathers such as Jean Piaget, Lev Vygotsky, John Dewey, and Jerome Bruner, is that learners are viewed as active participants in the process of learning (Golder, 2018). Constructivism dovetailed with the cognitive and social development of learners, and the way it affects how they learn Euclidean geometry concepts. This study discusses the perspectives of these constructivists, and socio-constructivists, and juxtaposed their views regarding learner experiences concerning Euclidean geometry.

From Piaget's perspective of constructivism theory, the cognitive skills of children advance as they grow older, and children's cognitive skills are tied to the stages of development (Golder, 2018). This theory explains that every stage of development represents a different type of thinking. Children in stage one cannot think or reason as well as children in stage 2 or even 3. The cognitive skills of children should align with the developmental stage. It follows that transitioning from one stage to another must be in sequential order. Piaget sees the intelligence of children as fixed and universal and as based on the progression from one stage of development to another. This universality implies children of the same stage of development have a similar level of cognitive skill and reasoning. This translates to the way they acquire knowledge and learn. This somewhat fixed development is perceived as a drawback of Piaget's theory.

In 1968, Lev Vygotsky disagreed with the postulates made by Piaget regarding the constructivism theory. He propounds social constructivism, which is a variation of cognitive constructivism. The central focus of Vygotsky's theory of social constructivism is on the collaborative nature of learning. Learners develop knowledge from social interaction with other people in their environments (Orevaoghene, 2020.) The experiences of learners in the Euclidean geometry classroom can be enhanced depending on the level of interaction they have with each other, and their environment.

Regarding the cognitive development of children, the constructivist views overlap. Jean Piaget opines that children's learning experiences are obtained by constantly interacting with their environment. Because children have the cognitive ability to learn on their own in an environment in which they interact, adult assistance is not required for any kind for learning to take place (Mutsvangwa, 2016). Lev Vygotsky, on the other hand, suggests that children's learning experiences can be supported and encouraged by an adult. Jerome Bruner juxtaposes the stages of cognitive development to learning processes. He is of the view that learning is a continuous process, where more knowledgeable individuals can activate the innate abilities of a learner to enhance their learning. John Dewey sees that the learner new experiences as by-products drawn from past experiences which they fit into their own into prior experience.

Overall, these constructivist views were formulated on the learners' abilities toward active participation in the learning process, either encouraged by an adult, or by owning the learning processes in the acquisition of new experiences. Similarly, education critics have harshly criticised the traditional methods of geometry teaching and learning for failing to accept learners as individuals with the ability to develop their own knowledge (Mutsvangwa, 2016). With constructivism, learners take ownership of their learning.

Learners discover new concepts in mathematics by trying to adjust their prior knowledge to allow the understanding of new concepts. For instance, in grade 10, learners need to develop a fresh understanding of new topic which is proof of parallelograms for them to learn the new topic Learners must readjust by revising the knowledge of parallel lines and congruency for them to allow the learning. However, teacher responsibility cannot be avoided but should be considered as an organizer and facilitator of learner knowledge. Gerhard van der Wal (2015) avers that the teacher role in constructivism theory should be to guide and motivate learners.

Sunzuma and Maharaj (2020), aver learners are trying to construct meaning with the process of self-discovery and adaptation when constructing knowledge. Self-discovery is the process of learners acquiring knowledge on their own and adaptation to learning through assimilation and accommodation is central to cognitive constructivism. Accommodation is the process of changing and accepting new knowledge from prior knowledge. So, accommodation is the process of blending the original experience with a new experience. In other words, changing (accommodation) and blending (assimilation) are instrumental to self-knowing.

Regarding social constructivism, Ultanir (2012) argued that to engage learners in learning activities that will meet the needs of learners in a classroom, teachers must deliberately make the classrooms learners-centred and interactive. In such classrooms, powers and responsibilities are shared among the learners. Ultimately, teachers become guardians and motivators in the classroom.

Furthermore, social constructivism focuses on the importance of culture and social interaction (Gray & Macblain, 2012). In the context of the study, culture refers to what occurs in the classroom to which learners belong. In other words, in the same context, learners gained their prior knowledge by interacting with other learners in the same circle as them in the classroom. Ultanir (2012) refers to this interaction as social interaction. The knowledge gained by learners from their culture and social interaction with people in their circle should assist them in their new learning.

The constructivist theory underpins the Van Hiele geometric model. Both constructivism theories are embedded in this geometric model. Clement (2004) argued that the Van Hiele model of geometric thought builds on theories of both Vygotsky's constructivism and the cognitive theory of Piaget.

For this study, the Van Hiele geometrical thinking and constructivist theory were used as the theoretical framework. The rationale for using the Van Hiele model of geometrical thinking is to understand the level of thought of grade 10 learners and the experiences learners face when solving Euclidean geometry problems. Furthermore, this theory helped the researcher to design the Geometry Achievement Test questions. Constructivism theory focused on the perception of the learners, how learners build new understanding of parallelograms based on the understanding of parallel lines and congruency from previous grades. Constructivist theory would guide and frame the semi-structured interview questions and classroom observation schedule to tap into learner understanding and the difficulties they encounter that do not allow them to answer questions on parallelograms. Also, both theories guided the researcher to analyse and interpret the findings of this study. The next session discusses the role of theories in educational research, especially the role of the Van Hiele model in geometrical education research.

2.4 ROLES OF VAN HIELE'S THEORY IN EDUCATIONAL RESEARCH

Theories such as Piaget's cognitive constructivism, Vygotsky's social constructivism, the Van Hiele geometrical theory, and many others play essential roles in educational research and the professional development of teachers. Clement (2004) argued that the Van Hiele geometrical theory has developed and strengthened the area of teaching and learning about proof and conjectures.

The theory also serves as a theoretical framework for research such as assessment, and educational technology, and even with learners with special needs, analyses of textbooks, and curriculum development.

Clement (2004) further argued that the Van Hiele theory is not only applicable to mathematics discipline but also applicable to other disciplines like chemistry. Also, another study was done by Vogelzang, Van Berkel, and Verdonk (2015) to investigate learner understanding of chemical concepts based on the Van Hiele theory. Vogelzang et al. (2015) argued that learners can move from one level of understanding to a higher level. Therefore, the Van Hiele theory can be applied to enhance teaching and learning in other disciplines apart from mathematics, andhas been used, evaluated and modified in various studies (Bansilal & Naidoo, 2012; Jojo, 2015; Khalil et al., 2018; Kilpatrick, 2014; Luneta, 2015; Makhubele et al., 2015; Miyazaki, Fujita, Jones, et al., 2017; Ndlovu & Mji, 2012; Stols et al., 2015; Wijayanti et al., 2018). The theory is helpful in a study when it fulfils the criteria of being tested, criticized, and modified.

Such theories would have been affected by a large body of works by using, evaluating, and revising the theory (Clements, 2004). The Van Hiele theory is an example of such theories.

In the study conducted by Wijayanti et al., (2018), the Van Hiele Levels of thought were modified by creating a level before visualisation which is called the pre-recognition level. The author compared it with another learning model Search, Solve, Create and Share (SSCS). It was discovered that in pre-cognition and visualisation, learners can identify geometric figures both in position and in complicated orientation, but they have difficulties in solving problems in the geometrical questions. The study claims that the SSCS is a model that can be used in solving the geometric problems rather than applying the Van Hiele levels of geometrical thinking. That claim poses a contradiction again to the validity of the use of Van Hiele levels of geometrical thinking.

In summary, the Van Hiele theory is a theory of cognition that can be applied to teaching, if the teacher takes cognisance of the levels of cognitive development of the learners, but it is not a theory of teaching, and does not inform that teacher of what pedagogy to use. The Theory of Instruction is therefore applied. This is because the pedagogy is vital to teaching and learning, specifically in the area of Euclidean geometry.

Most of these studies claim that learners understand proofs and conjectures in the last two stages of Van Hiele's levels of geometrical thinking (Alex & Mammen, 2016; Sadiki, 2016). Similarly, Van Hiele theory has been used as a theoretical framework in a variety of studies on geometry and other related topics including assessment, curriculum developments, educational technology and many more (Alex & Mammen, 2016; George, 2017; Khalil et al., 2018; Luneta, 2015; Sadiki, 2016; Stols et al., 2015). For example, in the study conducted by Khalil et al., (2018), Van Hiele levels of geometric thinking was used as the theoretical framework in which it was discovered that learners developed their understanding of deductive proof in the later phase (three and phase four).

Similarly, Luneta (2015) argues that the application of Van Hiele geometrical models are good descriptors of the performance of learner current and future geometry learning. The Van Hiele levels have been used by teachers to determine the level of geometry in which learners are, to design activities that align with learner level. Concerning this argument from Luneta (2013), this present study adopts the Van Hiele model of geometric thinking as a theoretical framework to explain learning difficulties that grade 10 learners experience while solving Euclidean geometry problems. Luneta (2015) analysed errors student made when solving coordinate geometry problems in the final grade 12 examination in South Africa based on Van Hiele levels of geometry thinking. The analysis was done on the 808 mathematics students Examination conducted in South Africa. The result revealed that grade 12 learners who were supposed to be in levels 3 and 4 of the Van Hiele levels of geometry thinking lack conceptual understanding. They can operate at level 2. The result of the study further revealed that learners lacked conceptual understanding which consequently results in learners not operating at the appropriate level required at this curriculum level.

Secondly, theories in educational research promote a rich understanding of the study. According to Clements (2004), theories may help deepen the research understanding and also encourage creative thinking. The use of theories guides researchers' ways of thinking not to just reproduce what other researchers have said but to think critically and see how such theories affect the study. According to Ngulube, Mathipa and Gumbo (2015), theory gives an inquiry a direction. The theory chosen for this study helped in channelling the researcher's thoughts and provided a focal point to the study.

Apart from the above-mentioned role of theories in educational research, a theory also assists teachers to construct a model that enhances understanding of significant educational problems. According to Clement (2004), this model connected the education and psychological concerns of learners thereby providing teachers with a clear understanding of learner difficulties. As earlier mentioned, the Van Hiele geometric model assisted the researcher to explore deeply the grade 10 learner level of thinking and the experiences they faced when solving geometry problems. Ngulube et al., (2015) argue that educational theories provide a means of interpreting data collected from the study. Thus, the data collected from this study helped the researcher to think about the model that would assist the researcher in analysing the learner experiences that may emerge from this study.

Moreover, theories prevent the fragmentation of understanding by arranging it in a way that learners will make sense of what they are taught (Ngulube et al., 2015). Similarly, according to Alex (2016), the Van Hiele geometrical model is formulated to improve the teaching and learning of geometric concepts. In other words, a well-designed geometric instructional model may enhance learner understanding of geometric concepts. It is of the highest importance that teachers be aware of learner experiences to help them to move from one geometric level to a higher level. Therefore, the Van Hiele geometric model is found to be relevant to this study as it enabled the researcher to explore the learner levels of geometrical thought and their learning experiences in grade 10 when solving Euclidean geometry problems. The next section presents in detail the Van Hiele levels of geometrical thinking.

2.5 VAN HIELE LEVELS OF GEOMETRIC THINKING

For almost fifty years, the Van Hiele levels of geometrical thinking have been widely used in studies related to geometry (Bansilal and Naidoo, 2012; Crompton et al., 2018). According to Rizki et al. (2018), the Van Hiele levels play an important role in the learning of geometry because it influences the learner level of thinking and the instruction given by teachers. The theory originated from the doctorate research of a Dutch couple, Pierre Van Hiele and Dina Van Hiele–Geldof from the University of Utrecht in 1957 (Machisi & Feza, 2021; Vojkuvkova, 2012). The couple were mathematics teachers in a Montessori secondary school in the Netherlands. They were concerned about the learning difficulties when solving geometry problems (Clements, 2004). The theory was the profound solution they came up with to solve their learner difficulties in geometry.

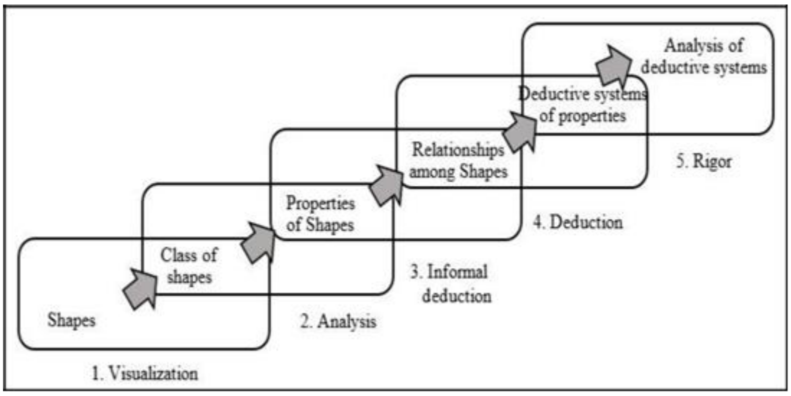
The Van Hiele model of geometric thought assisted learners with learning difficulties in geometry. The theory provided a guide on how geometry could be taught to provide learners with a rich understanding of the topic (Alex & Mammen, 2016). The Van Hiele theory shows how the teacher's instructional model and learners' level of geometry thinking are connected to provide effective teaching and learning experiences. Learner cognitive level develops from one level to the next level in a hierarchal and sequential order. According to Machisi and Feza (2021), learners can only learn geometry successfully if they complete every level in order without skipping one. In other words, the learner must successfully learn and understand the visualisation level before moving to the abstraction level. The cognitive development of learners depends on the ability to assimilate the instruction given, moving from one level to another (Ndlovu & Mji, 2012; Stols et al., 2015). Therefore, understanding geometry instructions is the key for learners to progress from one level to the next level. Learner progression depends on the quality and nature of instruction, not on their age (Ndlovu & Mji, 2012). Moreso, learner progress in geometrical understanding is based on four components of instruction as suggested by Khalil et al., (2018) which include active involvement in well-designed activities, proper objectives of the lesson, the context of the study and active participation in class discussions. Geometric lessons structured around these components will enhance learner understanding in sequential order. Hence, learners move sequentially from one level of understanding to a higher level instead of memorising the content (Khalil et al., 2018a).

The Van Hiele levels of geometry is thought to provide learners with a particular grade to be placed on a particular level. Machisi and Feza (2021) indicated that under normal circumstances learners in the senior secondary are supposed to be in the informal deduction which is level 3 for them to prepare for the formal deduction. However, the situation remains abnormal according to Machisi and Feza (2021), as most learners operate below level 3, as described by the Van Hiele level of geometrical thinking. Most grade 10 learners are operating at the visualisation and analysis levels (Ngirishi & Bansilal, 2019).

Van Hiele levels of geometrical thinking were initially from levels 0 to 4. Subsequently, two decades ago, Spark and the Van Heiles suggested numbering levels from 1 to5, to avoid confusion about a level zero. Clement and Battista (1992) also provided for the pre-recognition level which is also called level 0 (Stols et al., 2015). However, these levels of geometric thinking remain the same but only differ in their numbering. Since the levels describe the same characteristics, the researcher will use the one to five levels of geometric thought. These five levels of geometric thinking are discussed in detail in the section below.

These five levels of geometric thinking are labelled visualisation, analysis, abstraction, formal deduction, and rigour. The discussion on each level of geometric thinking is presented in the section below. Figure 1 illustrates the five levels of geometric thinking.

Figure 1: The Van Hiele theory of geometric thoughts (Van de Walle 2006, p.306)



2.5.1 Level 1- (Visualisation or Recognition)

At this level, learners can determine geometric shapes by their appearances (Clements, 2004). Similarly, learners learn to recognise the geometrical shapes, but they are not able to differentiate between shapes (Khalil et al., 2018; Luneta, 2015; Ndlovu & Mji, 2012a; Stols et al., 2015). According to DBE (2012), the Curriculum and Assessment Policy Statement (CAPS) document, the Foundation Phase (grade 1-3) learners are at this stage.

However, Nojiyeza (2019) considered all learners in primary schools to be at this level of geometric thought. This statement implies that foundation phase learners, and intermediate phase learners are expected to be in level one. Learners at this stage are expected to recognize and name the 2-D shapes without making a distinction between those shapes. Some of these shapes include triangles and quadrilaterals which include squares, rectangles, parallelograms, rhombi, and kites. Learners learn to draw and build these shapes at this phase.

Khalil et al. (2018) aver that shapes are treated as shapes without understanding their characteristics, definitions, or descriptions. Crompton et al. (2018) argue that learners move to the higher level as soon as they can recognise different shapes. For this study, learners who can identify parallel lines, the angle formed by parallel lines and recognise shapes like parallelograms are regarded as having achieved level one. When learners can recognise the triangles, rectangles, squares, parallelograms, rhombus, and kites, then learners can progress to the analysis level.

2.5.2 Level 2- (Analysis or Descriptive Level)

At level 2 learners can recognise and match a particular property to a particular shape but cannot see the relationship between the properties (Khalil et al., 2018; Luneta, 2015; Ndlovu & Mji, 2012; Stols et al., 2015). According to the CAPS document, learners in the intermediate phase are supposed to be at level 2. The intermediate phase includes learners from Grades 4-6. Learner experience moves from recognition of shapes to more detailed descriptions in the intermediate phase. Learners learn the properties of the shapes and match them with the drawings of shapes and state the properties. Learners are introduced to properties of parallel lines, and properties of parallelograms. At this level, they can identify the properties of a parallelogram such as knowing that the opposite sides of a parallelogram are both equal and parallel but cannot link these properties to other shapes. Khalil et al. (2018) argue that learners cannot completely describe an object, although, they can match the properties with the shapes. As learners identify properties of shapes and match the properties with the shape at this stage, they should be ready to move to level 3.

2.5.3 Level 3- (Abstraction or Ordering Level)

As noted above, for learners to be at level 3, they should master level 2 where they are expected to know and understand the properties of 2-D shapes. At level 3 learners should be at the abstraction and relationship-oriented level, that is an initial level in which learners begin the informal proofs and deductive reasoning (Ndlovu & Mji, 2012). However, learners in the senior phase in the introduction of the CAPS document in grades 7-9 revise the clear description and properties of 2-D shapes. According to the CAPS document, grade 7 learners must describe, sort and compare 2-D triangular shapes by focusing on the types of triangles and giving a brief description of quadrilaterals. grade 9 learners are also required to identify and give a clear definition of quadrilaterals they investigate, and prove that two triangles are similar and fulfil the conditions of congruency. Also, they are required to compare parallelograms, rectangles, squares and rhombuses, trapeziums and kites. At this level, learners are expected to combine shapes and properties and deduce a relationship between the properties of each shape. For instance, learners can combine the properties of parallelogram lines to prove that the opposite sides of the parallelogram are equal. According to Stols et al. (2015), learners use logical implication, and class justification inclusion to enhance their reasoning. When learners can fully combine shape properties, and reason deductively, they can progress to the next level.

2.5.4 Level 4 – (Formal deduction)

Level 4 requires formal deduction, where learners prove theorems and understand the role of axioms. (Luneta, 2015). At this level, learners can understand the role of definitions, construct a series of steps, articulate proofs and make and verify conjectures deductively. A good guess or idea about geometry is regarded as a conjecture, that is a set of mathematical statements which appear to be true but has not been formally proven. Learners understand the relationship between proofs, axioms, and conjectures, verified conjectures which resulted in proof (Khalil et al., 2018; Luneta, 2015; Stols et al., 2015).

Learners can reason formally and can rephrase the problem using the correct language. According to the CAPS document, learners in Further Education and Training (FET) at Grades 10-12, are those that are supposed to be at this level. In grade 10, learners can investigate, make conjectures and prove geometry shapes. Through conjecture, learners make a statement that they believe is true or even false but yet to be proven while proof is a rhetorical device for convincing oneself or others that a mathematical statement is true (Krantz, 2011).

At level 4, learners should be able to make conjectures and prove usingformal deduction. Learners must be competent in using the correct language at this stage.

2.5.5 Level 5- (Rigour and Axiomatic or Meta Mathematical level)

As soon as learners can do deductive proofs, they move to level 5. Each level has its specific language which must clearly be understood by learners before they can progress to the next level.

Luneta (2015) referred to the stage as the metamathematical level. At this level, learners employ logical reasoning to make valid arguments. They can analyse and compare the axiomatic system. Axioms, as stated by Krantz (2011), require no proof, and use logical reasoning. If learners can fully understand this level, it shows that learner progression from one level to a higher level may be manifested through the properties the learners possessed.

Vojkavkova (2012) argues that the five properties manifested when learners are progressing from one level to a higher level which include fixed sequence, adjacency, distinction, separation, and attainment. The first property shows learner progression is the fixed sequence. According to Vojkavokova (2012), fixed sequence emphasises that the hierarchy of Van Hiele level of thoughts is important. Learners cannot progress to a higher level without being proficient on the lower level. For example, learners cannot move to level 2 without fully understanding level one. In other words, learners cannot skip the lower level by moving to a higher level. The evidence that learners understand level 3 must be manifested in grade 10 learners of Tshwane West District to show how they are progressing. It is rather unfortunate that the challenges they were learners experiencing in solving Euclidean problems, impeded their progress.

The second property described by Vojkavokova is adjacency. Van Hiele's levels of geometric thinking that were intrinsic in the preceding level become extrinsic in the current level. Whatever concepts learners learn in the previous level become the basis of building the new concept on the current level.

The properties of separation emphasize how two people on different levels cannot understand each other. When this situation arises, learners will not be able to understand the teacher because the teacher did not use the language and symbol that learners understood. It is paramount that teacher should be able to understand learner level and assist them to move to the higher level. Consequently, leading to complete understanding at the next level has five phases which are information, guided orientation, free orientation, explication, and integration. The property of attainment deals with the instructional model for teachers. The Van Hiele instructional model is discussed in the next section.

2.6 THE VAN HIELE INSTRUCTIONAL MODEL

Van Hiele levels of geometric thinking do not work in isolation: they work hand in hand with the instructional model which focuses on the teacher approaches when teaching geometry concepts. Salud, Sobretodo and Hortillosa (2022) argued that the Van Hiele instructional model is crucial to teaching and learning of geometry, because it allows learners to be actively engage in activities that help them to understand geometry. Dina Van Hiele suggests five instructional approaches that teachers can use to assist learners in understanding geometry. According to Sadiki (2016), the five instructional approaches suggested by Dina Van Hiele include interviews, direct orientation, making a clear explanation, free orientation, and integration. The instructional model if properly applied may enable the learners to move from one level to the subsequent level in their geometric thinking. As teachers are aware of learner levels on the Van Hiele model, they may be more prepared to help learners to overcome the learner difficulties they may face when solving Euclidean geometry in grade 10.

2.6.1 Phase 1: Inquiry/ information.

As earlier noted, the Van Hiele levels of thought emanated from social constructivist theory. At this phase, the instructor is expected to test the prior knowledge of learners to find out what they already know about the topic. At this phase, learner level of geometry thinking is determined using teacher – learner interaction by engaging in discussion and asking questions to find out about prior knowledge (Armah et al., 2018; Machisi & Feza, 2021). Inquiry provides the teacher with the background learners have on the topic. Teachers needed to be competent enough to determine whether learners are prepared to learn the concept in geometry. According to Machisi and Feza (2021), it is the responsibility of teachers to ensure learner readiness for the proof of parallelograms by making sure learners revise the topic of congruency and parallelograms. Learners must understand the concept of parallel lines and congruency before proceeding to parallelograms.

2.6.2 Phase 2: Guided orientation

After the teacher understands the learners background on the topic, a new concept is introduced to the learners, and they are guided to explore the new concept. The teacher's role is emphatically stressed as a facilitator who guides the learners (Armah et al., 2018). The teacher uses questions and engagement to discuss and discover about the context discussed. Guided orientation ensures learners understand what is being discussed. Teachers need to engage with learners to facilitate the teaching of parallelograms in grade 10.

2.6.3 Phase 3: Explication

This acquisition of knowledge is assisted by the correct use of language symbols and the teacher's role is to help learners learn the proper use of language and vocabulary. Teachers must ensure that learners use appropriate terminology (Armah et al., 2018). Through discussion, the main idea is for the teacher to raise the learner's understanding level. It was reported in the diagnostic report of the NSC examination that learners have a problem using the correct language when answering the questions on Euclidean Geometry (Department of Basic Education, 2017). Therefore, learners need to understand the correct language and symbols when solving Euclidean Geometry problems.

2.6.4 Phase 4: Free Orientation

In phase 4, learners are expected to think deeply by solving geometry problems using various problem-solving techniques. Since the learners know about the topic under discussion, learners answer open-ended questions independently to help them explore the relationship within the levels of geometry (Armah et al., 2018). Teachers are there to guide the learners so that to whatever learners are thinking is not misconception and ensure proper use of language and symbols.

2.6.5 Phase 5: Integration

This is the concluding phase, also crucial where learners summarise, internalise, and absorb what has been learnt. Learners establish links between objects and relations (Chiphambo & Feza, 2020). The integration shows that a complete summary either by written or oral activities must be done at this stage (Salud, Sobretodo &. Hortillosa, 2022). A careful planning of appropriate lesson activities is needed to ensure they enjoy and have a deep understanding of the Euclidean geometry. In this study, learners would be observed in the classroom to explore how they learn Euclidean geometry in their classrooms through learner- learner and teacher-learner engagement.

2.7 CHAPTER SUMMARY

This chapter discusses the theoretical framework supporting this study. The Van Hiele theory and constructivism theory underpinned this study. The Van Hiele levels of geometrical thinking guided development of the achievement of geometry questions and in the analysis of the grade 10 learners' performance and difficulties in geometry, while the constructivism theory was used to explore learners in the classroom to understand the process of building their understanding of the Euclidean geometry. The rationale of these theories was juxtaposed and the relationship relating from one theory to another regarding the challenges learners face was discussed.

CHAPTER THREE

LITERATURE REVIEW

3.1 INTRODUCTION

The previous chapter presented the theoretical framework which underpins this study. The theoretical framework helped the researcher to locate the study within the research environment. As mentioned in the previous chapter, the Van Hiele model of geometrical thought and constructivism theory was the theoretical framework which underpinned the study; the two theories would assist the researcher in the development of the instruments and in the analysis of the data.

The literature review commences with the meaning of a literature review, and the sources of the literature review. Also, the researcher discusses the purpose of the review in the context of the experiences of learners in solving Euclidean geometry problems. After that, four areas of research related to this study are addressed. The first section covers the performance of learners in mathematics around the world. The second section presents the notion of Euclidean geometry and the performances of learners in this concept area of mathematics. This third section addresses the mathematical proficiency of learners where five strands of proficiency are discussed. In the fourth section, the errors and misconceptions displayed by learners when solving Euclidean geometry are presented. The concluding section presents a gap that exists in the literature as related to the difficulties of learners when solving Euclidean geometry problems.

3.2 LITERATURE IN THE CONTEXT OF THE STUDY

A literature review is a process in which a researcher collects and analyses the past and contemporary literature related to a particular research topic (Creswell, 2012; Gay, Mills, & Airasian, 2011; Newby, 2014). Literature in the research sense includes journal articles, books, conference papers, books and government documents that are relevant to describing the state of the research topic at hand. Newby (2014) further explained that a literature review is a means of connecting a research intention and the results to the research others have done. The review of the literature provides the researcher with the rationale for such a research study.

Consequently, the researcher systematically embarked on the review of related works of literature in teaching and learning Geometry both past and present to justify the need to explore the experiences of learners when solving Euclidean Geometry in the FET. The sources of this literature are classified into primary and secondary sources. These sources are discussed in the two sections below.

3.3 PURPOSE OF LITERATURE REVIEW

Gay et al. (2011) highlighted the four purposes of the literature review. The first purpose is to help the researcher to understand previous studies related to the topic. The review serves as a guide to know what has already been done and not been done in the area of the study. The information that was gathered in this area guided the researcher to avoid engaging in unnecessary duplication of the studies that have already been done.

Secondly, it gives the understanding and insight needed to place the topic within a logical framework. Thus, it provided both support and a benchmark to compare the outcomes from this study with other findings, as noted by Creswell and Creswell (2013). The insight gained from the review of the literature positioned this study within this framework. Hence, a literature review can fill the gap within the research world.

Thirdly, the previous studies provide a rationale for the study and a focus for the present study. Lastly, it enables the researcher to discover the research approaches and designs that were productive to explore the learning experience of learners in solving Euclidian problems.

3.4 EUCLIDEAN GEOMETRY

Euclidean geometry is one of the ten main content areas of FET Phase Mathematics as prescribed by the South African Curriculum statement (DBE, 2011). According to Curriculum Assessment Policy Statement (CAPS), the content areas covered in the FET include number pattern, finance, algebra, calculus, probability, Euclidean geometry, analytical geometry trigonometry and statistics. Department of Basic Education (2011) refer to Euclidean geometry as the arm of geometry that deals with space and shape using logical deduction. Logical deduction is a way of reasoning that can be justified by arriving at a conclusion that is universally accepted (Musyimi, 2016). Also, Jones (2000) states that logical deduction is a way of establishing geometric truth. Euclidean geometry helps learners to improve their reasoning and their ability to think critically and creatively.

Globally, geometry has been tagged to be a problematic arm of mathematics for most learners to learn and teachers to teach (Alex & Mammen, 2016; Bhagat & Chang, 2015; Luneta, 2015; Sadiki, 2016). In a study conducted by Alex & Mammen (2016), the concern was to address the learning difficulties of grade 10 students with geometry in South Africa using the Van Hiele model as a framework. The result of this study revealed that most grade 10 learners could not recognise geometric figures, and learners in general could not link geometry shapes with their properties. As a result of this underlying problem, many grade 10 learners are unprepared for formal proof as described on the Van Hiele levels of geometric thought which is expected to be attained by the grade 10 learners.

In another study, Luneta (2015) sought to identify errors learners made when solving coordinate geometry problems. Of the approximately 108 000 scripts of the 2008 matric examination 1000 scripts were selected, approximately 10%. The study confirmed that learners experienced difficulties in solving geometry problems. According to Lunata (2015), most difficulties learners displayed were revealed through errors learners made in their script. Some learners showed difficulties of not understanding the basic concepts which Luneta called conceptual errors. Other difficulties learners displayed included the inappropriate and misuse of formula (procedural error) and careless errors which are errors learners made unknowingly which might have been corrected if learners had been encouraged to check their work.

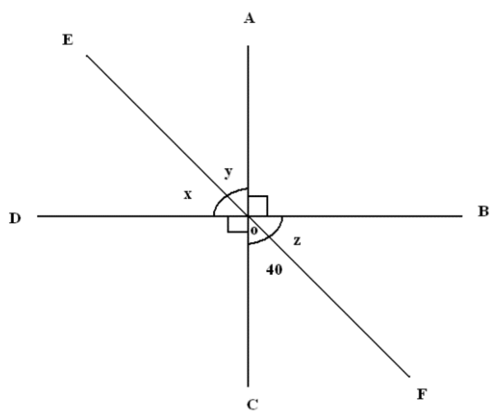
Geometry is difficult for learners not only in South Africa but also other parts of the world. Indian learners also find geometry difficult. Learners have difficulties while trying to understand a particular concept of mathematics like geometry, algebra, and calculus. Geometry especially may be a problematic concept of mathematics as the concepts are abstractions of the real-world (Bhagat & Chang ,2015)

Department of Basic Education (2014) describes the two distinct areas of geometry as analytical geometry and Euclidean geometry. While analytical geometry deals with the study of space and shape using algebra and coordinate systems, Euclidean geometry is the study of space and shape using a system of logical deduction. Euclidean geometry, based on axioms and theorems, was ordered into a logical system by Euclid (Greenberg, 1999).

The study of Euclidean Geometry develops critical and logical thinking skills of learners (Ndlovu and Mji, 2012; Bhagat and Chang, 2015; Pandiscio, 2015; Van Putten et al., 2010; Pandiscio, 2015); promotes and appreciates the spatial perception of the real world (Van Putten et al., 2010); helps to teach reading and interpretation of mathematical argument (Van Putten et al., 2010).

Learners in grade 9 are meant to understand the description of the relationship between angles formed by perpendicular lines, intersecting lines and parallel lines cut by a transversal. grade 9 learners should also know the difference between supplementary and complementary angles. Learners in this grade should know that two angles that add up to 180^o are said to be supplementary and when two angles are added together 90^o the term used is a complementary angle.

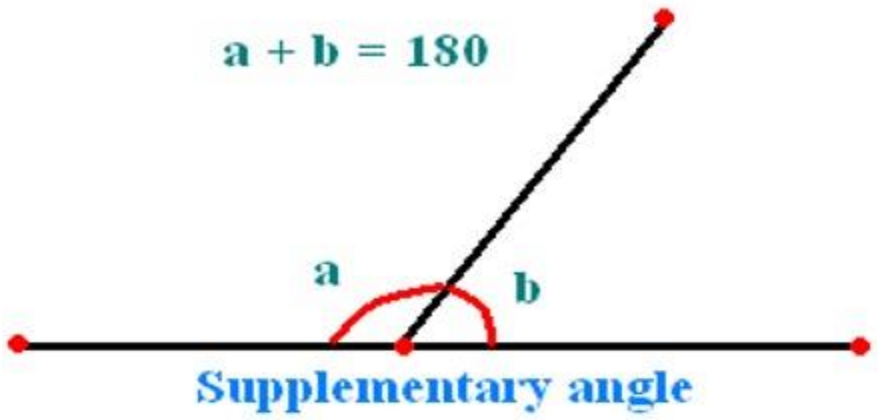
Figure 2: Showing angles that are complementary.



$x + y = 90^o$

AOE and DOE are complementary angles.

Figure 3: Showing supplementary angles.

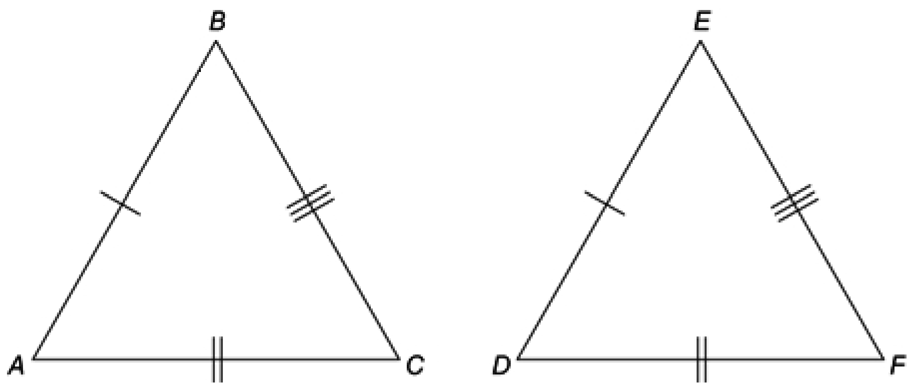


grade 9 learners need to investigate the conditions for congruency of triangles. They should be abreast with the conditions for congruency.

3.4.1 Conditions of congruency:

Two triangles are said to be congruent if the three sides of one triangle are equal in length to the three sides of the other triangle. The condition is known to be the Side, Side, Side (SSS) condition. Figure 4 shows the diagrams in which the corresponding sides of the two triangles are equal.

Figure 4: Congruent triangles



The second condition of congruency is if two sides of an included angle in one of the triangles are equal to the two sides and the included angle in the other. The condition is known to be a Side, Angle, and Side (SAS) condition.

The third condition of congruency is when the two angles and one side of one triangle are equal to two angles and the corresponding side of the other triangle. The fourth condition is when in two right-angled triangles the hypotenuse and a side of one triangle are equal to the hypotenuse and a side of the other triangle.

Learning difficulties are challenges that learners experience when they are learning Euclidean geometry. Learning difficulties if not well dealt with from the root lead to poor performance in mathematics.

3.5 PERFORMANCE OF LEARNERS IN MATHEMATICS

Mathematics is known to be one of the major subjects connected to all sciences and technology-oriented disciplines like medicine, pharmacy, engineering, information technology and many more (Bhagat & Chang, 2015). Therefore, excellent performance in this subject is indispensable for learners to qualify for admission into the sciences and technological professions.

There is a general concern of poor performance in the Trends in International Mathematics and Science Study (TIMSS) assessment, however, Asian countries performed excellently (TIMSS, 2019). For instance, top five countries on the list of high performing countries in mathematics emanated from Asia. These countries are Singapore, Hong Kong, Korea, Chinese Taipei, and Japan. Next to Asian countries is Europe. Northern Ireland topped the list of high performing countries. Other continents that followed Asia and Europe are the United States of America, Australia and then Africa. South Africa is the lowest on the chart representing Africa (TIMSS, 2019). For high performing countries in mathematics, it does not necessarily mean that all learners perform excellently but a lower percentage may be experiencing difficulties in mathematics.

Learners' experiences of poor performance in mathematics are indeed worrisome and concerning. The problem of learners' poor performance is a global concern (Ihendinihu, 2013; Mbugua, Kibet, Muthaa, & Nkonke, 2012; Pournara, Hodgen, & Adler, 2015). Resonating with the learning experiences of learners regarding mathematics indicated by TIMSS (2019) in Africa, Mbugua et al., (2012) shares a similar observation in Kenya regarding poor performance in mathematics Kenya secondary schools. Ihendinihu (2013) and Anaduaka and Okafor (2013) reported similar concern regarding poor performance in mathematics in secondary education in Nigeria.

Previous studies (Mbugua et al., 2012; Tachie and Chireshe, 2013; Umar, Adamu, and Sadiq, 2014) opined that learners may likely contribute to their own poor performance. For instance, Umar et al. (2014) noted that learners who suffer from anxiety, and low self-esteem may have a dislike for mathematics; this may result in poor performance of learners in mathematics. Nevertheless, the shortage of textbooks, classrooms, and a shortage of competent teachers some of the challenges experienced in many of the schools. Adjei (2020), argued that for teachers to be competent he or she must be qualified and equipped to teach Euclidean geometry. Similarly, incompetent teachers find it challenging to assist learners struggling with mathematics (Mbugua et al., 2012; Tachie & Chireshe, 2013) which might result in the poor performance of learners.

Lack of mathematical proficiency in the four strands of mathematics which include conceptual understanding, procedural fluency, strategic competencies, and adaptive reasoning may to contribute to the poor performance of learners (Dlamini, 2017). In lieu of this, the researcher investigated the literature of the countries like Singapore, Hong Kong, Korea, Chinese Taipei, and Japan with high performance in mathematics and evaluated factors that aid high performance.

3.6 PERFORMANCE IN EUCLIDEAN GEOMETRY

Ndlovu and Mji (2012) categorised learning difficulties experienced in geometry into four categories. The first is lack of basic geometrical knowledge and vocabulary. The second is that learners do not have the skill to draw a logical conclusion. The third type of learning difficulty involved learners not having adequate skills to organise information from a given conclusion.

Lastly, learners are not able to link arguments. These categories indicate that the level of difficulties learners experience range from fewer minor challenges to major ones that can be attributed to lack of basic knowledge. Alex (2016) further argued that a lack of basic knowledge also resulted in differences in the level of learner thinking and the expected level they should be in geometry. This indicates that learners appear to lack the necessary knowledge in geometry leading to poor performance in this topic.

Van Putten et al. (2010) argued that many teachers are not familiar with the content of Euclidean geometry. Therefore, this shows that teachers may experience difficulties in teaching this topic.

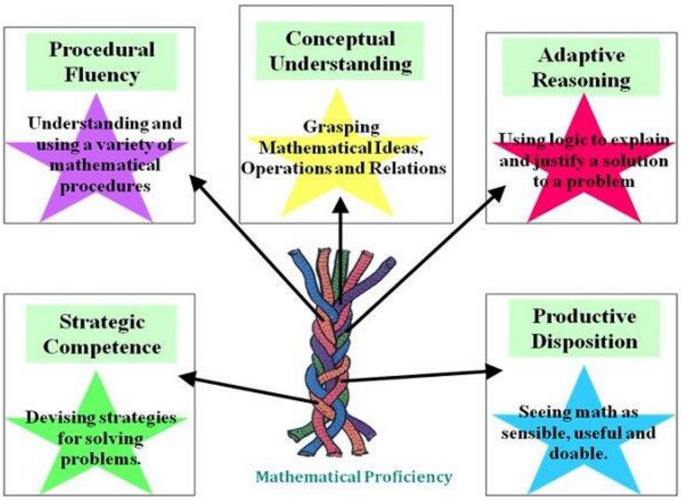
Luneta (2013) concurred that teachers lack the skills that are necessary to empower learners to understand geometry concepts. Some teachers found it challenging to explain the contents of Euclidean geometry to their learners in such a way that the learners will have a conceptual understanding of the content (Luneta, 2015). Lack of conceptual understanding in learners lead to errors and misconceptions and even result in learners being unable to answer geometry question in the examination (Luneta, 2015; Makhbele, Nkhoma, & Luneta, 2014). However, Luneta (2015) asserts that errors in geometry occur because of the procedural way of teaching geometry. This study highlighted errors that learners made when solving grade 10 Euclidean geometry questions. Hurrell (2021), define procedural knowledge as the knowledge of rote learning that is characterised by series of steps to solve mathematical problems or goal. This implies that creating a building block on the knowledge that can be built and does not have a skill or knowledge that connects with other skills or knowledge.

Many of the errors made in geometry by learners were conceptual errors, not procedural errors. Several studies conducted on learning difficulties in Euclidean geometry, focussed on errors and misconceptions (Alex & Mammen, 2016; Bhagat & Chang, 2015; Feza & Webb, 2005; Kesan & Caliskan, 2013; Luneta, 2015; Makhbele et al., 2014; Sandt & Nieuwoudt, 2005). As far as the research on the learning experience of learners is concerned, no such studies have been conducted in the Tshwane district. The proposed study explores the learning difficulties of grade 10 learners regarding proof and conjectures in the Tshwane West District.

3.7 MATHEMATICAL PROFICIENCY

Mathematical proficiency includes five strands that are interwoven. The development of all these strands is required to achieve proficiency. For learners to be mathematical proficient, Kilpatrick, Swafford, and Findell (2001) argued for five interwoven strands, which include conceptual understanding, which is the comprehension of mathematics concepts and operations and relations. Procedural fluency is the skill used in carrying out procedures, flexibility, accurately, efficiently, and appropriately. Strategic competence is the ability to formulate, represent and solve mathematical problems. Adaptive reasoning is the capacity for logical reflection, explanation, and justification. The strand labelled productive disposition is the habitual inclination to see mathematics as sensible, useful, and worthwhile coupled with a belief in diligence and own efficacy. Figure 5 summarised the five strands of mathematical proficiency.

Figure 5:The five strands of mathematical proficiency adapted from National Research Council (2001, p.5).



3.7.1 Conceptual understanding

One factor which affects learners in the study of Euclidean Geometry is lack of conceptual understanding. Conceptual understanding is one of the five interwoven strands of mathematical proficiency. Hiebert and Lefevre (1986), describe conceptual understanding as the web of knowledge that interconnects the existing knowledge to the new information that is just entering the system.

It includes the knowledge of representing mathematics in different ways and how different representations are used for different purposes. Learners can be said to have conceptual understanding if they have the knowledge of geometry concepts. This may lead to the transfer of this knowledge to another situation.

According to Luneta (2015), conceptual understanding is more than manipulation, which consequently leads to the conceptual knowledge of geometrical shapes. This means that a learner can easily identify geometrical shapes and reproduce when asked. Even when learners forget a part of the work, learners can work their way through.

It is the development of necessary skills that can work for different purposes. Makgakga (2011), and Alex and Mammen (2018) clearly stated that learners need to meaningfully and actively engage with mathematics to develop conceptual understanding. Teachers needed to be an expert in knowledge of geometry for learners to develop conceptual understanding.

However, a lack of conceptual understanding results in a loss of interest in geometry consequently leading to the poor performance of learners in mathematics. In Malaysia, Hock et al., (2015) conducted a study which assessed the conceptual understanding of 30 twelve years old primary school learners in Euclidean geometry. The study found that conceptual understanding is needed for learners to learn geometry successfully. A lack of conceptual understanding leads to a loss of interest in geometry.

In South Africa, a similar study was conducted by Alex and Mammen (2018) using a quantitative case study which focused on investigating the understanding of geometry terminology of volunteer pre-service mathematics teachers in a local university. The study revealed that the curriculum affected the performance of learners negatively. Studies have shown that teachers must use multiple representations in other to improve their conceptual understanding of learners during Euclidean geometry lessons (Ngirish & Bailal 2019; Khalid & Embong, 2020). Hence, teachers must ensure that visual, verbal, and real-life representations should complement and supplement each other.

3.7.2 Procedural fluency

Procedural fluency is another key strand that is vital towards building mathematical proficiency. Procedural fluency refers to the understanding of techniques, symbols and steps that are needed to complete a mathematical task successfully. Learners need to have this knowledge because it will facilitate the easy identification of geometrical shapes. The level of procedural fluency of learners has an impact on their learning experiences in Euclidean geometry.

Learners' experiences in Euclidean geometry are tied to procedural fluency and conceptual understanding. That is to say learner needs to understand the principles guiding geometrical concept identification. This knowledge is requisite for other strands of mathematics capacity, such as fluency in procedures. Learners who only memorise and do not have full grasp of shape identification are said to have engaged in rote learning. It takes good understanding about concepts before one can reproduce it after memorisation. Learners needed to be given opportunities to engage with higher-level tasks to promote deep understanding of Euclidean geometry. According to Ally and Christiansen (2013), high level skills include adaptive reasoning, productive disposition, and problem solving which link procedural fluency and conceptual understanding together.

3.7.3 Strategic Competence

The mathematical proficiency of learners goes beyond learners having conceptual and procedural understanding. According to Groves (2012), learners who have the skill of strategic competency are active problem solvers. An active problem solver can formulate, represent, and solve mathematical problems, and interpret and evaluate the solution in the context of the problem.

Strategic competence allows learners to engage with a mathematical problem in a real-life context and make sense of the situation to improve learning. In a study conducted by Ally and Christiansen (2013) to assess the mathematical proficiency of learners in grade 6 using an assessment rubric for teachers, it was revealed that learners hardly had the chance to engage in problem solving activities. When learners engage in problem solving activities, they tend to get a deeper understanding of the Euclidean geometry problem and make sense of their learning**.** Learners must engage in real life problems in Euclidean geometry: this deepens understanding.

3.7.4 Adaptive Reasoning

Kilpatrick, Swafford, and Findell (2001, p. 129) defined adaptive reasoning as the ability of learners to think logically, reflect, explain, and justify relationships among concepts and situations. Adaptive reasoning connects the other three strands of mathematical proficiency, that is it connects conceptual understanding, procedural fluency, and strategic competencies, and it binds the three other strands together. Learners draw on strategic competence to formulate and represent a problem, using heuristic approaches that may provide a solution. Hence, adaptive reasoning takes over when determining the legitimacy of the proposed solution plan. Conceptual understanding provides metaphor and representation that serve as a source of taking into account to determine whether a solution is justifiable and to justify it. Adaptive reasoning, whether a procedure is appropriate. Learners demonstrate that they have the strand of adaptive reasoning by providing sufficient reasoning when encountering a problem. Also, learner needs mathematical claims to others by improving their adaptive reasoning skill which invariably improve conceptual understanding Ally and Christiansen (2013) and Graven (2016) argued that learners are given a lesser opportunity to develop mathematical proficiency by not engaging in adaptive reasoning.

3.7.5 Productive Disposition

Kilpatrick et al. (2001) defined productive disposition as the ability of learners to view mathematics as sensible and worthwhile, with the belief in diligence and one's own efficacy. Learners with a high level of productive disposition value mathematics as an essential subject and believe in a continuous effort to try. They derive much joy in doing mathematics and see every problem as a task achievable. According to Cardinale (2013:46), such learners are "effective learners and doers of mathematics."

3.8 MATHEMATICS TEACHING AND LEARNING IN SOUTH AFRICA

Evidence has shown that the Curriculum Assessment Policies Statement (CAPS) is not achieving the proposed result of improvement in education (DBE, 2018). The CAPS document is good, but it has not had any effect on the performance of learners. Learners still perform below the standard in assessment both internationally and locally. The department of education recently produced a new framework to be used alongside the CAPS documents titled Teaching and Learning of Mathematics framework. The framework aims to help learners to learn mathematics with understanding. The framework incorporates the idea of Kilpatrick's mathematical proficiency and the ideas of many countries in which their learners are performing very well in mathematics. The framework was adapted for South African learners. The framework was developed to help learners to learn mathematics successfully, includes conceptual understanding, procedural knowledge, strategic competence, and reasoning. The implementation of this framework may assist teachers to teach mathematics with greater success.

3.9 ASSESSMENT

Assessment is an ongoing process of identifying gathering and interpreting information about how well a learner performs using various forms (DBE, 2011). Teachers use different modes of assessment for different purposes. Teachers may use a baseline assessment to find information about the prior knowledge of learners. This kind of assessment assists teachers to focus on weak areas of learners. Some assessments help determine the progress of learners, which could be in the form of tests and examinations. Formative and summative assessments are an integral part of learning, not something that is done in haste. Teachers should do their utmost best to see that assessment should be unbiased, transparent, valid, and reliable. For this study, learners were assessed using the Geometry Achievement Test to be able to understand the overall performance of learners in Euclidean geometry (see section 4.6.1).

3.10 ERROR AND MISCONCEPTION

Errors and misconceptions have been identified when solving Euclidean geometry questions (Luneta, 2015; Luneta & Makonye, 2010; Makhbele et al., 2014; Mbusi, 2015). Error and misconceptions are two different words used interchangeably (Gardee & Brodie, 2015). Gardee & Brodie (2015) argue that an error is any solution that deviates from the correct solution. An error occurs because of a misunderstanding between the teacher and learners. Luneta (2015) argued that learners display errors because learners have difficulties in learning geometry. In this case, both teachers and learners operate at different levels of thinking in the Van Hiele theory. An error displayed by learners is an indication of an underlying misconception. Therefore, they (errors and misconceptions) are related but have different meanings and implications.

An error seems to play an essential role in teaching and learning geometry. When learners display errors and misconceptions, it helps teachers to locate the area of learner weakness and the causes of such errors (Makhubele et al., 2014). Luneta (2015) analysed 1000 scripts from the 2008 grade 12 mathematics examination. The purpose of the study was to identify the errors, and misconceptions of learners make when solving coordinate geometry. The conceptual and procedural errors were the typical kinds of errors which learners committed. This study further revealed that learners committed those errors because they did not understand the basic concepts of Euclidean transformation. The cause of these errors was attributed to teachers rarely teaching the concept of geometry in such a way that learners may grasp the concepts.

In another study conducted by Mbusi (2015), the focus of the study was based on misconceptions of pre-service teachers with transformation geometry by using the Van Hiele theory. Mbusi (2015) used an action research approach with 82 learners studying a ' Bachelor of Education in the Foundation Phase. The research instruments for this study included a test and a semi-structured interview. The researcher used content analysis, and a semi-structured interview was used to determine the display of the errors by these 82 pre-service teachers. The study revealed that two types of errors were displayed by the pre-service teachers. Non-systemic and systemic errors were displayed. A systemic error is a misconception caused due to difficulties with the rules learned previously. Lack of necessary skills which consequently leads to faulty procedures. The tendency to consider the geometric figures as material objects resulted in participants using visual perception rather than reasoning based on the properties

(Mbusi,2015). So, in this study, the errors and misconception of learners would be analysed from according to the kinds of error made in the GAT.

Furthermore, Makhbele et al., (2014) conducted research on the kinds of errors learners display when responding to grade 11 Euclidean geometry questions. The Van Hiele theory of geometrical thought was used as a theoretical framework to understand the learner knowledge of geometry (Makhbele et al,.2014). This study took place in a rural Mpumalanga Province in South Africa. A sample of 30 learners was selected randomly from a population of 264. This study revealed that most learners misapplied rules which shows that learners have a weak conceptual understanding. The authors recommended that teachers should use the error analysis approach to determine when learners are continually making errors in the basic calculation (Makhbele et al., 2014).

3.11 INSTRUCTIONAL APPROACH

Many studies related to the teaching of geometry have attributed learner performance in geometry to poor strategies used by teachers (Alex & Mammen, 2016; Giannakopoulos, 2017; Luneta, 2015). Teaching strategies of teachers depend on the level of knowledge of the teacher. Teachers as a facilitator of knowledge are expected to have both knowledge of the subject matters (content knowledge) and professional knowledge (pedagogical content knowledge) (Alex & Mammen, 2016).

Content knowledge contains the content of a discipline which includes the discipline and understanding of the discipline of the specific core concept. Teachers must have the deep knowledge of skills, facts, and concepts and know how geometry concepts are related to other mathematical concepts. However, teachers that lack the content knowledge may not be able assist learners to understand geometry (Sunzuma &Maharaj, 2019).

Pedagogical knowledge covers a universal principle of teaching, learning assessment and classroom management. According to Sunzuma &Maharaj (2019), pedagogical knowledge of how learn geometry, includes that a teacher must be aware of what learners can do, what learners know and how they get to know what they know. Pedagogical content knowledge finally intercepts the content knowledge and pedagogical knowledge, which constitutes the teacher's unique professional expertise and knowledge relevant to making specific content accessible to the student. Content knowledge is a requirement for the pedagogical content knowledge. For this study the researcher used the lesson observation protocol to understand the teaching and learning process in the classroom.

3.12 CHAPTER SUMMARY

This chapter presents scholarly works that explain the empirical underpinnings of learner performance and learning difficulties in Euclidean geometry. This study provided an overview of learning difficulties and challenges faced by learners in solving Euclidean geometry. The central focus of the literature was the performance of learners in mathematics around the world, the notion of Euclidean geometry and the performances of learners in this concept area of mathematics, the mathematical proficiency of learners and the error and misconceptions displayed by learners when solving Euclidean geometry. The next chapter discusses the research methodology and procedures followed in carrying out this study.

CHAPTER FOUR

RESEARCH DESIGN AND METHODOLOGY

4.1 INTRODUCTION

The previous chapter presented a review of the scholarly literature related to the performance and difficulties of grade 10 learners in Euclidean geometry. This chapter addresses the research methodology used in the study. The research methodology details the procedures followed in conducting this research study. Cohen, Manion and Morison (2018) state that a research methodology drives the research design in a realistic way to answer the research questions.

The research paradigm, research approach and the design of the study are presented in this chapter. Thereafter, the researcher describes the population and the sampling technique applied in the study. Furthermore, the researcher also presents instruments used in data collection. The instruments included the Geometry Achievement Test (GAT), a semi-structured interview guide and a classroom observation checklist. The chapter concludes with an explanation of the validity and reliability, credibility, trustworthiness, and dependability inherent in the study, and provides the ethical considerations for the study.

4.2 RESEARCH APPROACH

According to Creswell and Creswell (2018), a research approach is a plan and procedure for conducting research which starts from the point of conceptualizing the study to the point of data collection, analysis, and interpretation. A research approach could be quantitative, qualitative, or constitute mixed methods. This study incorporated both quantitative and qualitative methods as this approach contributed evidence to answer the sub-questions.

A quantitative approach typically presents the collection and summarisation of numerical data. This form of approach is appropriate for testing theories, and examining the relationship that exists among variables with the use of statistical procedures. Hence, the statistical procedures support the generalization or replication of research (Cohen, Manion & Morrison (2018). This approach could only address the hypotheses part and could not address the research questions requiring qualitative descriptions. A qualitative design collects non-numerical data like rich descriptions and pictures. The qualitative approach focuses on exploring and understanding the problem in an inductive way from the individual perspectives by using open ended questions. This study applied a qualitative by using literal descriptions to get an in-depth understanding of the challenges grade 10 learner face when learning Euclidean geometry. This approach could only address the research questions and therefore could not be used alone.

These two approaches were combined and integrated in this study and is referred to as mixed-method approach. A mixed approach incorporates the quantitative and qualitative form of data collection in a single study to answer the research questions (Creswell & Creswell, 2018; Johnson & Christensen, 2016). Quantitative research is based on measurements, while qualitative research aims at eliciting the meanings, feelings, behaviours, and attitudes of a particular population. Creswell (2014) recommends the use of a multi-method approach in research, to achieve better data interpretation and reduce bias. That is, the qualitative approach is valuable for describing people's experiences, which Creswell (2014) believes helps to research life experiences, social processes, organisational structures, and settings. This approach is found to be relevant to the study because it deals with the wider range of the research questions, thus both qualitative and quantitative arms are addressed.

4.3 RESEARCH PARADIGM

The research paradigms as defined by Willis (2007), is a thorough belief system, worldview, or framework that directs study and practise in a particular field (p.8). The research paradigm explains the philosophical assumptions underpinning the choosing of a research approach. As such, the paradigms guide the researcher on the kind of research data that is obtained and the statistical tests that are employed in determining the performance of learners in the mathematical topic, Euclidean geometry. Creswell and Creswell (2018) offer a similar description of a paradigm as a basic belief system or worldview that guides the investigation. Khatri (2020) also described the paradigm as a perspective a researcher uses to consider and evaluate the methodological aspect of the research work based on a particular philosophical grounding. The paradigms describing this study guided the kind of interview questions and the procedures to follow to explore the difficulties that grade10 learners experience. The researcher applied both positivism and interpretivism paradigms in this study. The positivism paradigm enables the researcher to view data collected from the GAT as a single reality through deductive reasoning. The interpretative paradigm was engaged with the data collected through a semi structured interview and the lesson observation by acknowledging a socially constructed reality accessible in multiple ways maintaining objectivity (McMillan and Schumacher, 2014). These perspectives assisted in examining the performance and difficulties of grade 10 learners in Euclidean geometry in the two selected secondary schools in the Tshwane West district in the Gauteng Province.

4.3.1 Positivism Paradigm

Pharm (2018) argued that it could be difficult to use the positivism paradigm when researching social phenomena because of its relationship to human thinking, intention, and attitude which may be challenging to measure or quantify. The current study sought to measure the performance and investigate the challenges of grade 10 learners experience when solving Euclidean geometry problems. It is logical for the researcher to say that the positivist paradigm alone is insufficient to evaluate grade 10 learner achievement because the challenges learners experience needs to be richly understood using semi-structured interviews and the classroom observation. A mixed-method approach was applied in this study. This was achieved by incorporating positivism and interpretivism in this study.

4.3.2 Interpretive paradigm

The central aim of interpretive research is to understand the subjective world of human experience. The interpretive paradigm views knowledge and reality as subjective, socially constructed, and situation specific. Interpretivism is associated with qualitative research which features multiple views of reality (Cohen et al., 2018). According to Creswell and Creswell (2018) and Adom, Yeboah and Ankrah (2016), to understanding the fact about a phenomenon, the researcher makes use of his experience and the view of the research participant. Interpretivism believes that the best way to understand a phenomenon is to view it in its context (Creswell & Creswell, 2023). They see the quantification of data as limited in nature, focusing on a small portion of reality that cannot be utilised without losing the importance of the phenomenon. The central focus of interpretive research is mainly to understand the cause of difficulties learners experience and interpret meaning within the learning that took place in the classroom. Interpretive research aims to comprehend how people experience and make meaning of their worlds. Therefore, it considers human behaviour as too diverse and complex to be adequately captured through the positivist methods of quantitative measurement.

4.3.3 Pragmatic Paradigm

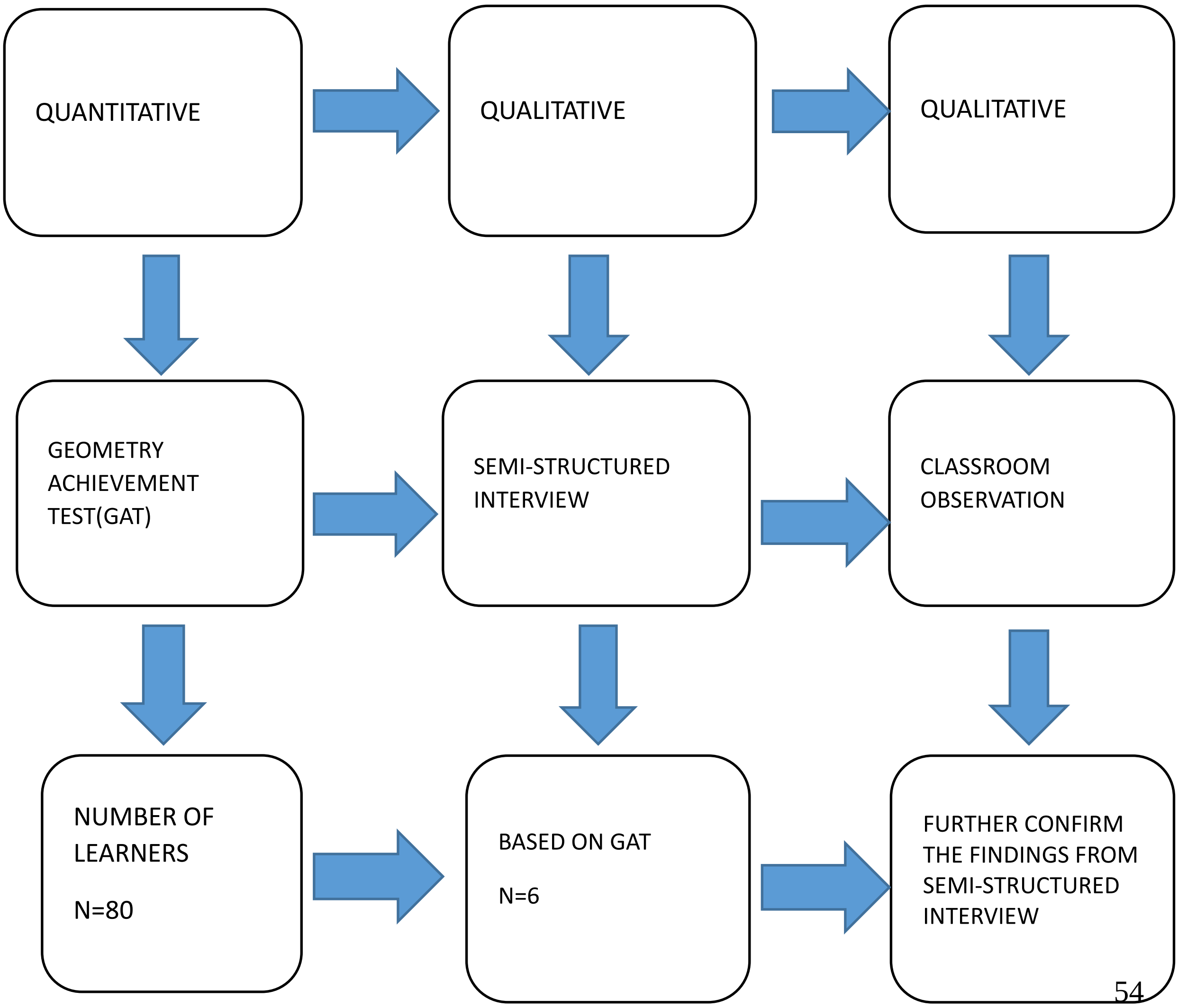
Pragmatism is consequence-oriented research which involves the careful mixing of both qualitative and quantitative research. According to Cohen et al. (2018), the pragmatism paradigm involves the use of “common sense” rather than “anything goes” in choosing what will best work for a study. This paradigm is practically driven and focuses on the research problems and questions. This study was conducted within a pragmatic worldview in which the researcher decided on what works for the study (Cohen et al., 2018). The researcher combined and mixed the quantitative and qualitative paradigms in this single study (Creswell & Creswell, 2023) to answer the main research question on the performances and difficulties of grade 10 learners when proving and solving problems related to parallelograms in particular. Hence, the use of pragmatic paradigms became indispensable in which the researcher employed both inductive and deductive reasoning from the perspectives of quantitative and qualitative points of view. Research suggests that incorporating positivism and interpretivism research paradigms provides rich and comprehensive data to evaluate the performance and learning difficulties of grade 10 learners in Euclidean geometry in the two selected secondary schools in the Tshwane district in the Gauteng Province.

4.4 RESEARCH DESIGN

The research design according to Creswell and Creswell (2018), is the plan in the mode of inquiry that stipulates the procedure in place to answer the research questions. McMillan and Schumacher (2014) similarly state that the purpose of the research design is to describe the general plan. The designs are classified according to quantitative, qualitative, mixed, and analytic methods. The first design uses objectivity and quantities in measuring and describing phenomena. The second which is qualitative uses literal and verbal descriptions to collect data in naturally occurring situations. The third class, mixed methods, combines both quantitative and qualitative methods in a single study. Lastly, analytical research uses document analysis to investigate concepts and events.

This study employed a mixed-method design which is the sequential explanatory design in which quantitative research data is initially collected and analysed followed by the qualitative research. The qualitative builds on the result of the quantitative to give an in-depth understanding that answers research questions. On one hand, the quantitative aspects used in this study is descriptive in nature in the form of a Geometry Achievement Test results to measure the performance of Grade10. Which means the GAT result are purely numerical result. On the other hand, the qualitative is phenomenological in nature which uses the semi-interview to deeper understand from participants' perspectives the learning difficulties learners face when solving geometry problems. A further qualitative aspect is to observe participants to understand how grade 10 learners engage Euclidean geometry in a natural setting which is their classroom. The sequencing of the data collection procedure will be as shown in figure 6.

Figure 6:The sequencing of data collection procedures in the study



Initially, the quantitative study was conducted by administering the achievement test to all the learners who participated in this study. The Geometry Achievement Test was used to measure learner performance, aligned with Van Hiele levels, in solving Euclidean geometry problems and thereafter, the researcher studied learner responses to each question item. The results found in the quantitative phase informed the researcher on the type of data to be collected in the qualitative phase. The qualitative data used to explore deeply on how learners solved Euclidean geometry problems to find reasons/ explanations for non-optimal learners learning experience, the way and method adopted by learners in their learning of Euclidean geometry.

4.5 RESEARCH SITE, POPULATION AND SAMPLING

4.5.1 Research site

The research site is defined as the exact location in which the study takes place (Durdella, 2020). In other words, the place where the researcher conducts research. This study was conducted in two selected schools in the Tshwane West district in Gauteng province.

4.5.2 Population of the study

Mills and Gay (2018) defined population as the total set of elements, whether individuals, objects, or events, which conform with standards to which the result can be generalized. In other words, the research population can be a collection of a set on which the study is focused. McMillan and Schumacher (2014) regard research population as the targeted population. The targeted population for this study was grade 10 learners in Tshwane-West District with 15 secondary schools. Some of the schools that are performing poorly in mathematics are found in the Shoshanguve township where this study took place. The study took place in two selected secondary schools in Shoshanguve which are in the geographical area of Gauteng Province.

4.5.3 Sampling

Sampling, according to Johnson and Christensen (2016), refers to the technique and process of drawing a sample from the population. In other words, a technique of drawing a subset from a larger group, the process in which the researcher selected the participants and the participating schools for the study. The sample should be a group that is representative of the population.

The essence of sampling is to obtain an adequate representation of the targeted population (Christensen et al., 2014). The researcher carefully decides on the types of sampling that will be appropriate for the study.

The participants in this study were identified by analysing the Department of the Basic Education school subject performance report for the five years i.e 2017, 2018, 2019, 2020 and 2021. These reports comprise of various subjects administered at the year-end examination each of the above-mentioned years, of which mathematics was one of them. Schools that participated were selected based on how they constantly performed poorly over the period of five years. The researcher used critical purposive sampling method in selecting the sample for the study. According to McMillian and Schumacher (2014), in purposive sampling the sample selected for the study must represent or inform the investigator about the topic of interest. In this study, the researcher employed a purposive sampling technique in selecting two schools that had been underperforming in mathematics in their grade 12 final examination. These schools were chosen because their learners were among those who performed poorly in mathematics consecutively over the past three years. School A consisted of the grade 10 mathematics class with 110 learners. School B has 90 learners. 40% of the grade10 learners in each of the schools were taken to give equal representation of the participants in both schools. For school A, the sample size was 44 and for school B the sample size was 36, this gives a total of 80 learners as sample size for the study.

4.6 DATA COLLECTION INSTRUMENTS

An instrument in research refers to tools that can be used in data collection to measure, observe, or document information with the purpose of answering the research questions (Creswell, 2014; Gay et al., 2012; Newby, 2014). The data collection instrument connected to the research approach (paradigm) in such a way that each approach has its own data collection instruments. Examples of the quantitative instruments include a survey, standardized tests, and a self-developed test. In qualitative research, measuring tools could comprise of interview questions, and observations.

With regards to data collection instruments in this study, the researcher administered the Geometry Achievement Test (GAT), and followed with semi-structured interview questions in which some open-ended questions were predetermined, while the majority of the interview questions emerged from the results of the Geometry Achievement Test. The final instrument used was the classroom observation checklist which was adapted from the Sepeng Observation protocol of 2010 to suit the present study. However, the GAT and the interview questions were developed by the researcher. The semi-structured interview questions and the classroom observation checklist built on the outcome of the Geometry Achievement Test to carefully answer the research questions.

The Geometry Achievement Test was insufficient to give the researcher a rich understanding of grade 10 learner academic experience when solving Euclidean geometry. Therefore, there was a need for follow up with data collection from the semi-structured interview to ask learners about their learning experience. It was also deemed necessary to observe learners in their classroom to fully understand how learners learn the concepts of parallel line, congruency, and parallelogram (Creswell & Creswell 2023).

4.6.1 Geometry Achievement Test

In basic geometry, learners should acquire skills of reading, conceptual knowledge and procedural skills, communication and thinking skills as reflected in the Van Hiele model. The Geometry Achievement Test in this study is used to measure what learners have already learned in geometry from the previous grade. Consequently, the task revealed specific strengths and weaknesses. The Geometry Achievement Test was administered to the 80 learners to measure their understanding of parallel lines, congruency and the integration of parallel line and congruency in parallelogram. The instrument measured visualisation, analysis, abstraction, formal proof, and rigour. Therefore, the distribution and summary of the question items according to the Van Hiele levels of geometry were found in tables 2 and 3.

The test items consisted mainly of Grades 9 and 10 work extracted from past quality assured departmental examinations; these were administered to 80 learners. It can be assumed therefore, that the instrument, the Geometry Achievement Test, had gone through internal and external moderation to ensure validity and reliability of the instrument. The researcher gave the test instrument to two senior academic lecturers in the Department of Mathematics

Education in the University of South Africa and two mathematics subjects specialists to confirm content validity for the second level check.

4.6.1.1 Development of Geometry Achievement Test (GAT)

The Geometry Achievement Test served primarily three purposes: The first was to ascertain the general performance of the learners in Euclidean geometry. The second was to determine the level of grade 10 learners on the Van Hiele theory of geometric thought. The third was to identify the specific area in Euclidean geometry where learners had experienced the most difficulties. And lastly, to understand the area specific area in the to focus on semi-structured interviews and subsequence classroom observation that will answer research questions.

According to the curriculum and assessment policy statement (CAPS) of the DBE, for learners to progress to the next grade, learners must understand the content of the curriculum in the previous grade (DBE, 2011a). It is on this note that the researcher assumed that the participants would be able to do the grade 9 content. Also, learners should be able to do grade 10 work on the basis that learners are being taught these topics in their present grade.

The first question in the Geometry Achievement Test assessed the learners understanding of parallel lines using a given angle — the second question assessed learners understanding of conditions of congruency. In the concluding question, the researcher wanted to know whether learners can integrate two concepts, and in the first whether the learner can use the principle in understanding the concept of the parallelogram (see Appendix A). Table 2 and table 3 show the distribution and summary of question item in the GAT according to the Van Hiele levels.

Table 2: Distribution of question items according to Van Hiele levels

Question	Visualisation	Analysis	Abstraction	Formal proof	Rigour
1.1.1	✓				
1.1.2(i)		✓			
1.1.3(ii)		✓			
1.2.1		✓			
1.2.2		✓			
2.1.1			✓		
2.1.2			✓		
2.1.3			✓		
2.1.4			✓		
2.2.1	✓				
2.2.2		✓			
2.3.1			✓		
2.3.2			✓		
2.3.3				✓	
2.4.1				✓	
2.4.2				✓	
3.1.1	✓				
3.1.2				✓	
3.2.1				✓	
3.2.2			✓		
3.2.3			✓		

Table 3:The summary of the distribution of questions items according to Van Hiele levels

level	Meaning	What learner can do	Question(s)
1	Visualisation	Learners must be able to recognise shapes	1.1.1; 2.2.1;3.1.1
2	Analysis	Learners must be able to relate properties to the shapes	1.1.2; 1.13;1.2.1; 1.2.2;2.2.2
3	Abstraction	Do informal proof	2.1.1;2.1.2;2.1.3;2.1.4;2.3.1; 2.3.2;3.2.2;3.2.3
4	Formal proof	Relate two properties together and proof	2.3.3;2.4.1;2.4.2;3.1.2;3.2.1
5	Rigour	Learner can analyse axiomatic proof	N/A

4.6.2 Semi-Structured interview

The semi-structured interview, according to Cohen et al. (2018), is a qualitative method of investigation that combines open-ended questions that enable the researcher to seek clarification and elaboration from the responders about the items. This type of interview is considered by Newby (2014) to fit between questionnaires in which there is no freedom to deviate and the evolving interviews with the known or expected result in mind.

Furthermore, the semi-structured interview is flexible, which may likely have certain questions and an interview guide. Cohen et al. (2018), state that in a semi-structured interview, the interviewer can ask follow-up questions which clarify the interviewees' understanding, explore a viewpoint to determine knowledge, or open other explanations and answers to the questions that were not foreseen when the questions are determined. The researcher has asked the follow-up questions to understand the reasons for the learner responses in the Geometry Achievement Test. Although the use of a semi-structured interview could be time-consuming, expensive, and requires an expert in interviewing, its advantages outweigh the limitations. The semi-structured interview guide is provided (See also Appendix C).

4.6.3 Classroom Observation

Classroom observation is a way that the researcher sees and hears what is occurring naturally at the research site to obtain a rich understanding of the phenomenon under study (McMillan & Schumacher, 2014). Classroom observation facilitates a deep understanding of the context and participant behaviour, which allows the collection of more complex data to reflect the importance of the effect of the context. The researcher's role in classroom observation in this study was that of a non-participant observer. The researcher did not participate in the class activities, offer a suggestion, or interact with the learners while observing the situation (Mills & Gay, 2018). The researcher has chosen the non-participatory observation to document how each learner performs closely, whereas with participatory observation, the researcher might engage with activities that distract the researcher from data collection or limit what is being observed. Also, participatory observation can affect the decision of the researcher which consequently affect the validity and reliability of the study.

Three lessons on grade 10 Euclidean geometry were observed twice per week at each of the schools which mean six lessons were observed over the period of three weeks. All 80 learners attended these lessons facilitated by the two teachers in the selected schools. The researcher conducted the observation using a pre-constructed observation schedule for a total of six lessons conducted in each school amounting to twelve observations in total. This arrangement ensured uniformity and commonality in the data collection process and enhanced the validity of data collected. The researcher used a lesson observation schedule adapted from Sepeng (2010). See Appendix D). The Sepeng (2010) observation tool was adapted and contextualized for this study. It is important to note that the researcher did not participate in the lesson but only observed the learning of Euclidean geometry. The lesson observation did not only focus on learner responses during the Euclidean geometry lesson, but also on the nature of the interaction of the teachers and learners, instructional approaches and resources, and teacher content knowledge of the subject.

4.7 PILOT STUDY

Prior to embarking on the main study, the researcher piloted the study to test the research instruments. The main objective to pilot a study is to test the reliability and validity of the instrument as well as to determine if it needed to be modified. Also, a pilot study is essential to establish whether the instrument is relevant, the time allotted is sufficient, the instructions and questions are clear to the participants, and whether it will provide the data necessary to answer the important research questions. Cohen et al. (2018) inform that the purpose of the pilot study (Christensen et al., 2014; Gay et al., 2012) is to modify and adjust the method and instruments prior to the main study. The pilot study uncovers and identifies in advance the potential problems that might come up in the main study. According to Kumar (2011), a pilot study is also a feasibility study. The pilot study was conducted in two selected schools.

First, the researcher piloted the Geometry Achievement Test and discovered that the initial duration allocated for the Geometry Achievement Test was insufficient for the participants. Furthermore, the pilot study of the geometry test also revealed that the test may not provide sufficient data to answer the first sub research question because it did not align with the Van Hiele level one to three of Geometric thinking. The researcher modified the test instrument to align with the Van Hiele levels of geometry thinking to answer the research questions properly. Furthermore, the duration for the test was also extended to 45minutes which allowed the participants to answer the items without time constraints.

After the Geometry Achievement Test was written, the researcher interviewed five purposively selected learners according to how they responded to the achievement test. Three learners were interviewed on the first day after the achievement test and the remaining two learners were interview in the following day. This pilot study gave the researcher a rough estimate time schedule for the interview. Each interview session with each learners took at least 15 to 20 minutes.

4.8 DATA COLLECTION PROCESS

In collecting data, the researcher has divided the data collection process into three phases:

- Administering the Geometry Achievement Test
- Semi-structured interviews
- Classroom observation.

4.8.1 Phase 1: Administering the Geometry Achievement Test

The data collection started with the quantitative aspect which involved 80 grade 10 learners, in two participating schools writing the Geometry Achievement Test. Before administering the test, 80 learners were allocated a code name to ensure anonymity. LSA1 stands for learners 1 in school A and LSB1 learner 1 in school B. A learner with a code name LSA10 described as a learner from the first school in place 10 in the list, who wrote the Geometry Achievement Test. LSA20 and LSB5, for example, can be explained the same way. The seating order enabled the researchers to identify the learners for the second phase of data collection. The test assessed learners on the concepts of parallel lines, congruency, and proofs and conjectures of parallelograms. The test enabled researchers to ascertain the performance and learning difficulties of learners in Euclidean geometry. The duration of the test was 60 minutes which is to be commenced after the school hours so as not to interrupt the school program. The teachers ensured that participants wrote the Geometry Achievement Test in their various classrooms to achieve uncompromised data.

The Geometry Achievement Test served primarily three purposes:

- To ascertain the general performance of the learners in Euclidean geometry.
- To identify a specific area in Euclidean geometry where learners have experienced most difficulties.
- To identify the area where the researcher will focus on during lesson observation and subsequent interviews.

All the 80 learners were informed about the planned Geometry Achievement Test before it was administered. The participating teachers in each school were requested to assist in administering the test. The researcher discussed with both teachers the test and invigilation procedure to be implemented during the test. The researcher delivered the test instrument in both schools on the day the test was written.

4.8.2 Phase 2: Semi Structured Interview

After the Geometry Achievement Test had been marked and analysed the result of the analysis informed the researcher how to sort learner scripts according to the number of questions they answered with Correct (CR), Incorrect (ICR) or Blank responses (BR) using a frequency table. The learners, with poor performance would be identified and then interviewed to determine their challenges. Due to the Covid-19 pandemic, face-to-face interviews were not possible. Therefore, the researcher requested the contact numbers of learners from the principal of each school to conduct a telephonic semi-structured interview. The researcher conducted all the interviews using an interview schedule which is provided in section 4.5.2.3. All interviews were conducted outside of the school hours to avoid interrupting the school activities.

4.8.3 Phase 3: Classroom Observation

The researcher visited each school twice in two weeks to observe the delivery of a grade 10 lesson on Euclidean geometry topic. The rationale for observing each school only twice was so as not to disturb the running of the everyday lesson. This observation took six weeks to conduct. This arrangement ensured uniformity and commonality in the data collection process and thus enhanced the trustworthiness of the data collection. During this visit, the researcher adhered to all Coronavirus covid -19 regulations by having the personal protective equipment like wearing of a mask, to have a bottle of sanitizer to sanitize at every point and check the temperature when entering and leaving the school premises. It is important to mention that due to Coronavirus covid-19 pandemic regulation, social distancing was in place. Social distancing will create change in the school setting particularly the classroom arrangement. The learner's seats were 1.5metres apart which left little or no space for visitors within the classroom she positioned herself away from the learners right at the back of the classroom where the researcher would be able to observe and maintain social distancing and at the same time capture the lesson. In this way, the researcher is protecting herself and the participants from the corona virus. The researcher stayed close to the window to be able to capture every activity that happens in the classroom.

As such, the observation process would be non-participatory concerning the researcher. Furthermore, the lesson observation instrument focused on how learners respond to the teacher when introducing the topic of Euclidean geometry in grade 10 classroom, teaching methods and the strategies employed the lesson; learner involvement, participation, engagement and interaction and strategies employed by learners in proof and conjecture in Euclidean geometry , assessment strategies and the methods used by the teachers, exposition of the teachers content knowledge; recognition of learners previous knowledge and how new knowledge is constructed on the existing knowledge etc.

4.9 VALIDITY AND RELIABILITY

Validity shows the extent to which the scientific explanation of phenomena matches the reality (McMillan & Schumacher, 2014). Similarly, validity is defined as the degree to which an instrument measures what it was intended to measure (Bui, 2009). From a qualitative perspective, validity is regarded as trustworthiness. The instrument needs to be valid so as serve the intended purpose of the instrument. To ensure the validity of the instrument used in this study, Geometry Achievement Test items were drawn from both grade 9 and 10 question banks. The Geometry Achievement Test was reviewed by an expert in the field of Mathematics Education. This process ensured the process met the standards and were valid for the purpose that it was intended.

Interview questions were piloted on a small-scale study to ensure validity. A pilot study was conducted in the school with 40 learners in Tshwane West district. The purpose of this pilot study was to ensure that any area that may give difficulties in the main study is taken care of. The observation checklist was adapted from Sepeng (2010) which has been widely used and found to be suitable for purpose.

Reliability refers to the extent to which an instrument consistently measures what it was intended to measure when it is repeatedly used (Abu et al., 2012; Creswell & Creswell, 2018; Johnson & Christensen, 2016). Strong reliability gives the same result regardless of the number of times that such an instrument is administered. To ensure that the Geometry Achievement Test, interview questions and the observation checklist instruments were reliable, the instruments were pre-tested on grade 10 learners in the pilot study.

4.10 TRUSTWORTHINESS

The study is a mixed method research study: there is, therefore, a need to access the quality of the qualitative part of the study. Trustworthiness is the process of ensuring the integrity and thoroughness of the study (Connelly, 2016; Daniyan, 2015). Connelly (2016) argued that it is important to demonstrate trustworthiness in a study to be acceptable by the readers. For a study to be acceptable means free from mistakes of any kinds (Johnson & Christensen, 2016). Trustworthiness was analogous to validity and reliability in quantitative research. Furthermore, Lincoln and Guba (1985) highlighted the qualities, credibility, dependability, transferability, and confirmability.

4.10.1 Credibility

According to Creswell (2011), credibility is the process of validating or finding the accuracy of data using data triangulation and member checking (p. 259). However, McMillan and Schumacher (2014) argued that to ensure credibility; research may combine any of the following ten strategies which include prolonged and persistent fieldwork, multimethod strategies, participant language, verbatim accounts, low-inference descriptors, multiple researchers, mechanically recorded data, member checking participant review and negative discrepant data. Therefore, data collected in this study was analysed through scrutiny to get the theme that represents the performance of learners when solving Euclidean geometry.

Furthermore, the researcher found the commonality between these themes and compared the those in the literature review. Credibility in qualitative research is of the highest significance (Creswell, 2012: 259). To ensure the credibility of this study, the researcher spent an appropriate amount of time in each of the research sites. Furthermore, the researcher has engaged with the participants by rephrasing many of the questions to ascertain the participant views on their performance.

At the time of the data collection for this study, there was a global pandemic known as Covid 19. Due to covid -19 regulations, the Department of Basic Education issued a guideline as to how learners were to return to school after lockdown. School A used fortnightly rotational programs in which grade 10 learners were in school one week and following week were away from school. While School B used a rotation of three days per week. So, the researcher spent a prolonged period with the participants. Any visit to the school should be well planned. So, all together the researcher spent six weeks in the research site. The researcher spent time with the participants, starting from the administering the test till the end of classroom observation. Also, participants and researcher agreed of the meaning and description of the errors and misconceptions of learners.

The researcher conducted semi-structured interviews with the participants and ensured that data gathered from these interviews correlated with the information collected during the lesson observations in the classrooms. This process is referred to as triangulation. The process of triangulation assisted the researcher to confirm the authenticity of the data collected. Besides, the research informed teachers and learners all they needed to know that affected the credibility and trustworthiness of the result.

4.10.2 Confirmability

Confirmability refers to the strategy in trustworthiness which deals with the extent to which other researchers came to the same conclusion based on the evidence presented (De Klerk & Harmse, 2020). In other words, confirmability explains how well the data and interpretation of findings in a study is genuine and be confirmed by others (Korstjens & Moser, 2018). Confirmability ensures that data were not made up or imagined by the researcher.

The researcher kept a daily activity record in a reflective journal of what had been learnt and done during this study which was shared with the senior lecture in the department of education. This strategy enabled a transparency in the process of the findings and interpretation of data.

4.10.3 Dependability

De Klerk and Harmse (2020) stated that dependability is the stability of the findings over time, that is how consistent the findings would be from time to time. In this study, the researcher used an audit trail; the researcher engaged with a senior colleague who did not have interest in the study. Among other things to ensure dependability the researcher attended online every class which was called ‘tutorial lessons’ throughout the course of this study. The tutorial lessons were scheduled on a weekly basis, during which, the researcher shared the results of this study several times and was corrected by professors, supervisors, and peers who are respected in the field of mathematics. The engagement with research fellows shaped the research study. Hence, the tutorial lesson activities form part of the dependability.

4.10.4 Transformability

Transformability is the degree to which the findings from this can be transferred to other contexts or other settings with other participants (Korstjens & Moser, 2018). The researcher uses the ‘thick description’ to discuss in detail the sample, research site which help readers of the study to have a clear understanding of the context of the study.

Table 4: Strategies of Trustworthiness in qualitative research

Strategies	Definition according to literature	How it applies to my study
Credibility	The truthfulness of the research findings (Korstgen and Moser, 2018)	Detail of the pilot study. Triangulation. Prolong engagement. Member check.
Transferability	Degree to which the result can be transfer to another context	Give detail of the research location under
Dependability	How well the data collection process flow	Detail how my theoretical framework is infusing into my study.
Confirmability	The extent the data collected support the findings and interpretation data	Inviting others to go through the data analysis process

4.11 DATA ANALYSIS

Data analysis and interpretation help in specifying the steps in scrutinising and providing meaning to the quantitative and qualitative data collected (Creswell, 2014). For this study, the researcher employed the use of descriptive statistical procedure to make an informed decision which could characterised the poor performing schools. The researcher followed a sequential mixed method of analysis in which the quantitative strand of the data was analysed first, and this then invariably informs or drives the analysis of the qualitative strand of the data analysis (Onwuegbuzie & Combs, 2011). The result from the analysis of the Geometry Achievement Test informed the researcher about the performance of learners in Euclidean geometry, energized the interviews and informed the classroom observation of learners based on their responses in the Geometry Achievement Test.

The quantitative data was generated from the Geometry Achievement Test. The test was formally divided into three questions as shown in Table 5. The table indicated the first sets of the question which were on parallel lines, the second on congruency and the last part is on proof of parallelograms. The researcher analysed the Geometry Achievement Test using a frequency table. The researcher has used Correct Response (CR), Incomplete (InR) Incorrect Response (IR) and Blank Response (BR) to descriptively analyse the test results.

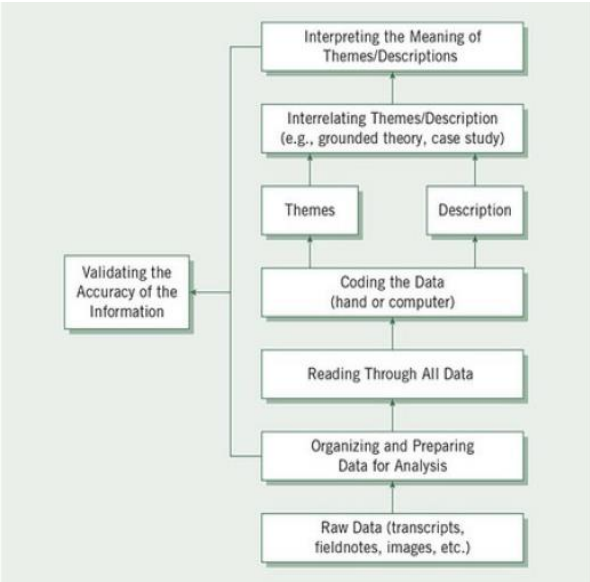
Table 5: showing the question item of GAT according to learners' responses

Question	Sub question	Correct Response (CR)	Incomplete Response (InR)	Incorrect Response (IR)	Blank Response (BR)
1.Parallel lines	1.1				
	1.2				
	1.3				
2. Congruency	2.1.1				
	2.1.2				
	2.1.3				
	2.1.4				
	2.2.1				
	2.2.2				
	2.3.1				
	2.3.2				
	2.3.3				
	2.4.1				
	2.4.2				
3. parallelogram	3.1.1				
	3.1.2				
	3.2.1				
	3.2.2				
	3.2.3				

4.11.1 Process of Qualitative Data Analysis

The researcher followed the process of data analysis to organize and prepare the data for analysis in qualitative analysis as described by Creswell and Creswell (2018). The researcher transcribed the data collected from the semi-structured interview and categorised all data. This was repeatedly read to provide the general overview of data collected through the interview and the lesson observation. The researcher attached notes and underlined the themes that emerged from the data collected and started coding all the data collected. Themes were generated and described. Fig 7 represents the description of themes (Creswell & Creswell, 2018).

Figure 7: Data analysis process in qualitative research (adapted from Creswell & Creswell, 2018, p. 269)



4.12 ETHICAL CONSIDERATION

Ethical consideration is regarded as what is considered right or wrong, good, or bad from the moral perspectives (McMillan & Schumacher, 2014; Newby, 2014). The researcher ensured that ethical practices were ensured in every stage of this study by following all guidelines relating to permission, informed consent, participation, and confidentiality.

4.12.1 Permission

The researcher obtained permission from the Department of Basic Education, Gauteng Province to have access to the research site. Permission was also obtained from the principals and mathematics teachers whose lessons were observed. The ethical clearance certificate was also obtained from the University of South Africa (UNISA) to ensure the research observed all research protocols.

4.12.2 Informed consent

For the learners to participate in this study, informed consent forms were signed by the learners and their parents. Parents were assured that information obtained from this study would be used solely for academic purposes. An overview of what the study is about was provided in the consent form. The purpose of the study, its significance and the rationale of the study was clearly stated on the consent form. It was also indicated on the consent form that the study is not a threat to the lives of their children and their confidentiality is guaranteed (Okeke & Van Wyk, 2015).

4.12.3 Participation

Teachers and learners participated voluntarily and were allowed to terminate their participation at any time during the study (McMillan & Schumacher, 2014). Furthermore, the researcher informed the teachers and learners about the procedure and risk involved in the study. To avoid learners and teachers being harmed, the researcher kept the teacher and learners names anonymous when reporting the results from the study. The researcher also ensured that the identities of teachers and learners that participated in the study were protected (Bertram & Christiansen, 2014).

4.12.4 Confidentiality

Also, the researcher respected the privacy of the teachers and learners by keeping the confidentiality of the data or information obtained from them. Confidentiality refers to, in the context of a research study, an agreement with the researcher about what can be done with the information obtained about a research participant (Johnson & Christensen, 2013). The data gathered from participants were only used for the study, and the data were not in any way used for any other purpose. The participants were assured that their personal information would be treated confidentially and anonymously. Researcher ensures that the identity of participant is unknown to anyone (Creswell & Creswell, 2018). The name of the schools and the participants would not appear at any stage in this study. Instead, this information was coded; they cannot be linked to a specific person or school. Data would be preserved in a computer and stored for 5 years by using data encryption to maintain data security. Keep back-up copies to prevent accident or loss of data. Provide suitable storage for the physical protection of data and prevent unauthorised access, any possible alteration, disclosure, and destruction of the data.

4.13 CHAPTER SUMMARY

In this chapter, the researcher explained in detail the research paradigm, design, and methodology. The research site, population of the study, sample size and the data collection instruments were also described. Furthermore, details on the validity, reliability and ethical issues were explained. The next chapter presents the research findings and data analysis.

CHAPTER FIVE

DATA ANALYSIS AND INTERPRETATION

5.1 INTRODUCTION

In the previous chapter, the research methodology was discussed. Following the research design, the data collection procedures used in investigating poor performance and investigating the learning difficulties encountered by grade 10 learners when studying Euclidean geometry in Tshwane West District in Gauteng Province were discussed. To answer the research questions in this study, the researcher reported the Geometry Achievement Test, semi-structured interview questions, and classroom observations used in the previous chapter. Furthermore, the researcher discussed data analysis procedures.

In this chapter the analysis and interpretation of data from three secondary schools in the Soshanguve township in Tshwane West District is presented. Also, presented is the methodological approach, the analysis of the pilot and the main study. One school participated in the pilot study and the other two schools participated in the main study.

The data analysis is in two phases. In the first phase, the quantitative data collected was analysed to answer the research question.

> How do grade 10 learners perform in solving Euclidean geometry problems?
>
> What difficulties do grade 10 learners experience when solving Euclidean geometry problems?

While in the second phase, the qualitative data was analysed to address the following research questions.

1. Why do learners experience those difficulties when solving Euclidean geometry problems?
2. How do grade 10 engage with Euclidean geometry?

5.2 DEMOGRAPHIC INFORMATION ON PARTICIPANTS

This section presents the demographic information of learners and teachers in this study.

Table 6 shows the total number of learners and the gender of learners from the two schools.

Table 6: Gender composition of grade 10 learners in School A and School B

Schools	Total number	Female	Male
School A	44	22 (59.1%)	18 (40.9%)
School B	36	24 (52.7%)	17(47.2%)

The participants in this study comprised 44 learners with 22 females and 18 males from School A. The total of 36 learners from School B comprised 24 females and 17 males. The participants were purposefully selected (see Chapter 1). The gender composition shows that there were more female learners in each of the schools than males.

Table 7: Demographic information of the teachers

School	Gender	Age	Qualification	Years' experience
School A	Male	30	BEd (Physical Science and Mathematics)	3 years
School A	Male	32	PGCE (Computer Science and Mathematics	6 years
School B	Female	35	BEd Mathematics	5 years
School B	Female	45	Teacher Certificate and BEd Mathematics	10 years

Table 7 shows the demographic information of teachers in School A and School B. The years of experience of teachers range from three years to 10 years of teaching experience. All the teachers had a Bachelor of Education in Mathematics. The age range of the teachers was 30-45 years.

Table 8 shows the overall performance of learners in the Geometry Achievement Test (GAT) for grade 10 learners. The least score obtained from the test was 1 (2%) and the highest score obtained was 19 (38%). It can also be deduced from the table that less than 4 percent of the participants scored above 30%. This indicates the poor performance of learners in Euclidean geometry.

Table 8: The overall performance of learners in grade 10 Geometry Achievement Test

Marks obtained (out of total 50)	Percentage of mark obtained	Frequency (80)
1	2	1
2	4	1
3	6	7
4	8	10
5	10	12
6	12	10
7	14	8
8	16	12
9	18	8
10	20	4
11	22	3
14	24	1
16	32	2
19	38	1

Note the total mark is 50

5.3 METHODOLOGICAL APPROACH

The data analysis adopted in this chapter follows a methodological approach based on constructivism and the Van Hiele five levels of geometry thought which was discussed in Chapter 2. As earlier mentioned in the second chapter, the Van Hiele five levels of geometrical thought were used in the analysis of the Geometry Achievement Test (GAT). Table 9 showed the methodological approach used in this study. Thus, the methodological approach was based on these five levels, namely visualisation, analysis, order, deduction, and rigour, as the main constructs for this study (Armah et al., 2017; Armah & Kissi, 2019; Ngirishi & Bansilal, 2019). As stated by McMillan & Schumacher (2014) the construct of interest is derived from the theoretical framework. The GAT instrument was designed and analysed according to the five levels as described in the framework. Each of the levels has performance indicators or descriptors that are used to measure what learners can do at a certain level (Miyazaki et al., 2017).

For example, a learner at level 1, should be able to answer questions related to the indicators at level 1. The indicator description is that a learner can identify different shapes and give their names. The table below shows the constructs, their definitions and the descriptor or indicator. For the study, the researcher focused on the first four constructs (levels) and their descriptors to be able to answer question one.

Table 9: Methodological approach

Construct/Level	Definitions	Performance indicator/descriptor
Visualisation	The learners can recognise different shapes according to the appearances	-learners identify lines that are parallel -learners can identify a line cutting two parallel lines which is a transversal line.
Descriptive	Learners can recognise and classify shapes with their properties	Learners understand when to use the properties of parallel lines: vertically opposite angles are equal, and alternating angles are equal. corresponding angles are equal, and the sum of co-interior angles are supplementary (= 180^0)
Abstract/ rational	When learners form definitions, distinguish between necessary and sufficient conditions and understand informal proofs	Learners can understand the conditions of congruency. Learners must be able to prove those two triangles that are congruent to each other when they know that: • all the corresponding sides of the two triangles are congruent (SSS) • two corresponding sides and one included angle are congruent (SAS) • two corresponding angles and an included side (AAS) • the hypothenuse and a side of the right-angled triangle. (RHS)
Formal deduction	Learners at this level can establish a theorem within the axiomatic system	Use deductive reasoning to prove a theorem

Rigour	When learners can reason formally about the mathematical system.	When learners can analyse their proofs by use of deductive reasoning

5.4 ANALYSIS OF QUANTITATIVE DATA

The quantitative data was analysed using descriptive statistics. The purpose of descriptive statistics for analysis is to help the researcher to describe, summarize, interpret, and make sense of data collected from the quantitative phase (Cohen et al., 2018; Johnson & Christensen, 2016). In this study the data were analysed and categorised according to correct responses (CR), incorrect responses (IR), incomplete responses (InR) and blank responses (BR) which were adopted from Didi's and Erba's (2015) model to analyse data collected from the GAT. Didi's model was also followed by Makgakga (2017) and Zhou (2020). The CR was used to categorise all responses that are free from errors or mistakes, and which conform with the memorandum. The IR represented all responses that have errors or misconceptions which do not conform with the memorandum or the standard of grade 10 Euclidean geometry. The InR shows any responses that may be partially correct but missing out on some elements of completeness like giving the correct magnitude of the angle but do not give reasons. BR indicated no response was given to the question.

Similarly, each of the questions in the GAT were categorised according to the Van Hiele level of geometry (Ngirishi & Bansilal, 2019). Table 3 on Page 58 shows the summaries of the distribution of questions items according to Van Hiele level of the geometry thought. Three questions were grouped under level 1, five questions were grouped and categorised into level 2 and eight questions fell into level 3. A further five questions were assigned to level 4 and level 5 had no questions, as these questions beyond the scope of the secondary school curriculum. Furthermore, descriptive statistics were used to describe and summarise the data generated from the Geometry Achievement Test (GAT) to describe the overall performance of learners in grade 10 in Euclidean geometry. The researcher calculated the frequency, range which included the minimum, and maximum marks of learners that participated in the GAT, the mean, standard errors and the standard deviation to describe the performance of grade 10 learners (Cohen et al., 2018; McMillan & Schumacher, 2014). The descriptive statistics involved the use of measures of central tendency, measures of dispersion and a histogram of distribution which were generated using the Statistical Package for Social Sciences (SPSS). Statistical software, SPSS, can perform data entry and analysis from simple to complex: the descriptive category is one of them. Since, one of the goals of the study is to describe or summarize, the characteristics of the performance of grade 10 learners on the Euclidean geometry test, the researcher used IBM SPSS statistics version 26, the most recent edition at the time of this study for data analysis. Before analysis of the main study, the researcher presents the pilot study results in section 5.7 below.

5.5 DESCRIPTIVE ANALYSIS OF THE PILOT STUDY

GAT was piloted with a total number of 40 learners in one of the secondary schools in the Tshwane West district which did not participate in the main study. However, the pilot school has the similar characteristics of poor performance as the schools used in the main study. The main purpose of this pilot study was to identify gaps in the GAT that needed to be corrected and improved before the actual investigation could commence (Department of Anaesthesiology and Pain Medicine, 2017; Imtiaz et al., 2020; Lowe, 2019; Malmqvist et al., 2019). However, the researcher briefly discussed the findings in the pilot study to understand the procedure to be followed in the main study (see Table 6). The pilot study revealed that the duration allocated for GAT was insufficient, as the majority of learners were not able to finish within the stipulated time. Hence, the allocated time was increased from 45 minutes to one hour in the main study.

Similarly, the pilot study revealed ambiguity in some of the questions. For instance, question 2.4 was reframed as the question did not clearly state what learners should do. Hence, to remove the ambiguity the question was adjusted from '=' to '≡'. The improvement on the GAT was done with the help of two grade 10 mathematics subject experts from the Tshwane West District secondary schools and a senior lecturer in the Department of Mathematics Education from one of the universities in South Africa. With this adjustment, the researcher believed that the GAT instrument would enable the first research question to be answered. Questions in table 10 are coded as $Q_{1.1.1}$, $Q_{1.1.2}$, $Q_{1.1.3}$ etcetera throughout the analysis of data. Data were analysed using *correct responses (CR), incorrect responses (InR), incomplete responses (IR) and blank* *responses (IR)* in table 10. The CR refers to learners used correct procedures to answers, InR refers to learners used incorrect procedures to solve problems, IR refers to learners either used correct procedures or incorrect procedures to solve problems which were incomplete, and BR refers to learners who did not attempt the questions.

Table 10 Descriptive analysis of the learners' performance in the GAT pilot study

Question	CR		InR		IR		BR	
1.1.1	6	15%	6	15%	22	55%	6	15%
1.1.2	2	5%	3	7.50%	29	72.50%	6	15%
1.1.3	3	7.50%	1	2.50%	30	75%	6	15%
1.2.1	1	2.50%	2	5%	34	85%	3	7.50%
1.2.2	3	7.50%	2	5%	29	72.50%	6	15%
2.1.1	4	10%	17	42.50%	10	25%	9	22.50%
2.1.2	3	7.50%	24	60%	6	15%	7	17.50%
2.1.3	3	7.50%	6	2.50%	14	35%	17	42.50%
2.1.4	3	7.50%	4	10%	15	37.50%	18	45%
2.2.1	24	60%	1	2.50%	5	12.50%	10	25%
2.2.2	3	7.50%	0	0%	24	60%	13	32.50%
2.3.1	1	2.50%	0	0%	30	75%	9	22.50%
2.3.2	14	35%	0	0%	11	27.50%	15	37.50%
2.3.3	1	2.50%	2	5%	22	55%	15	37.50%
2.4.1	1	2.50%	0	0%	17	42.50%	22	55%
2.4.2	1	2.50%	5	12.50%	10	25%	24	60%
3.1.1	1	2.50%	2	5%	13	32.50%	24	60%
3.1.2	1	2.50%	0	0%	11	27.50%	28	70%
3.2.1	1	2.50%	1	2.50%	9	22.50%	29	72.50%
3.2.2	1	2.50%	0	0%	12	30%	27	67.50%
3.2.3	1	2.50%	3	7.50%	10	25%	26	65%

Table 10 depicted the descriptive analysis of the learner performance in the GAT pilot study from Q1 with five sub-items. The percentage of the learners with correct responses (CR) in Q1 varies from 2.5% to 15%. This indicates that few learners can identify and relate the properties of parallel lines that cut across transversal by providing reasons. Q1.1.1 and Q1.12 have the highest and lowest percentage of learners with CR. The percentage of learners with InR is similar to that of CR in which 1.1.1 and 1.1.3 have questions where learners achieved InR. This means that some of the learners can identify parallel lines, however, they have difficulties relating the properties of parallel lines. They cannot give reasons for their answers. It is evident that learners lack conceptual understanding of this concept. This reason concurs with Nahdi and Jatisunda (2020) that learners that lack conceptual understanding cannot make sense of mathematical concepts. Furthermore, 85% of learners gave incorrect responses (IR) in Q1.2.1. The result suggested that many learner errors and misconceptions in solving a problem related to transversal cutting across parallel liness. According to DBE (2011a), learners in grade 10 are supposed to have mastered the concept of parallel lines, already in Grades 8 and 9. Based on the findings above the majority of learners lack conceptual understanding of parallel lines and their properties.

Similarly, Q2 comprises 11 sub-items. The table revealed that the percentages of learners that attained CR in Q2.1-2.4.1 range from 2.5% to 60%. Many of the learners that got CR were found in Q2.2.1 because they were asked to identify two sets of triangles that were congruent to each other. This indicates that few learners could understand the conditions of congruency. Those learners that achieve InR vary between 2.5% to 60%. These findings showed that many learners knew when pair of triangles were congruent to each other but found it difficult to employ the condition of congruency. Learners who gave IR in this question ranged between 15%-60%. Most learners achieved IR in Q2.2.2., which revealed learner errors and misconceptions in applying conditions of congruency to the given figures. However, in Q2.3.1,2.3.3, 2.4.1, and 2.2.4.2 most learners' responses were BR. which are 22.5%, 37.5%,55,5% and 60% respectively.

There were five sub-items in Q3. The learners who attained CR and InR in Q3 are 2.5% and 7.5% respectively. Most of the learners attained between IR and BR 22.5% to 72.5% respectively. This result shows that learners did not understand parallelograms. They lack the procedural and conceptual understanding of parallelograms. The findings of this pilot have confirmed that indeed grade 10 learners grapple with grade 9 parallel lines and congruency of triangles which needed to get a deeper understanding as to what can be the reasons behind this poor performance in GAT.

Table 11: **Descriptive Statistic Analysis of the Pilot**

	N	Range	Minimum	Maximum	Mean		Std Deviation
	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
Pilot study	40	82.33	.00	26.00	10.034	2.13046	14.28357

Table 11 depicted the descriptive statistic of the pilot study. Out of the total number of 40 learners that participated in the GAT, the findings suggested all the learners performed poorly, the minimum and the maximum mark of learners is 0% and 26% respectively. The average mark is 10.034% with a standard deviation of 14.28 respectively. This low standard deviation indicated the performance of learners in this test is clustered around the average mark. Furthermore, the low overall average implied a high number of 38 (95%) participants who did poorly in the test. It was obvious from the data that learners lacked the understanding of parallel lines, congruency, and parallel lines. The lack of understanding of the above concepts was evidenced by learner responses which were displayed by kinds of errors and misconceptions. This above-mentioned concept is evidenced by the kind of errors and misconceptions learners displayed in their responses.

5.6 ANALYSIS OF THE MAIN STUDY

There were 80 learners who were purposefully selected from two schools who participated in the GAT. The GAT consisted of three questions with 22 sub-questions. After the researcher administered the test to the participants, scripts were marked with the aid of the memorandum (Appendix A) to generate the data. The process took the same form used in the pilot study (see section 5.3). The detail of the analysis was discussed in the section below.

5.6.1 Analysis of Q1

Question 1(Q1) assessed learner understanding of parallel lines cut by a transversal line to determine the magnitude of unknown angles and their properties in the given shapes. In these questions, learners are expected to identify the properties of the parallel line when cut by a transversal line and match them with their properties. Learners are to visualize the figure to see the properties of parallel lines cut across a transversal line to form letters like “F” (Corresponding angles), “U” (co-interior angles) and “Z” or “N” (alternate angles). Thus, the Q1 integrated level 1 and 2 of Van Hiele levels of geometric thought.

Table 12 presents the result of learner responses to the GAT on the understanding of parallel lines by using the properties of parallel lines to calculate the magnitude of unknown angles. Learners’ responses were categorized according to Didis and Erbas's (2015) Model also analysed using Van Hiele levels of geometrical thought.

Table 12: Distribution of learners’ responses (with their percentages) to Q1

Correct Response (CR), Incomplete Responses (IR) Incorrect Responses (InR) and Blank Response (BR) according to Van Hiele’s Level of Geometry Thought (VHLGT) N=80 where N represent the number of learners that participated in the main study.

Question	VHLGHT	CR (%)	InR (%)	IR (%)	BR (%)
1.1.1	L1	14(17.5)	53(66.3)	12(15.0)	1(1.3)
1.1.2	L2	8(10.0)	41(51.2)	29(36.3)	2(2.5)
1.1.3	L2	7(8.8)	23(28.7)	46(57.5)	5(5.0)
1.2.1	L2	2.5(2.5)	5(6.3)	63(78.8)	10(12.5)
1.2.2	L2	6(7.5)	5(6.3)	55(68.8)	14(17.5)

Table 12 represents the distribution of learner responses to Q1 with five sub-question items. The table depicted that out of 80 learners that participated in the GAT main study, only about 2(2.5%) of learners provided CR to all the parallel lines questions. The data in table 12 showed that the percentage of CR ranged from 2(2.5%) to 14(17.5%). The highest and the lowest percentage of CR was found in Q1.1.1 and Q1.2.1 respectively. Level 1 questions assessed learners’ ability to recognise shapes while level 2 assessed those that allowed learners to do more than identify the object but also to match the properties with the shape (Ngirishi & Bansilal, 2019; Rizki et al., 2018). Q1.1.1 and Q1.2.1 focused on learners’ identification and analysis of parallel lines, as they were categorised as level 1 (L1) and Level 2(L2) according to the Van Hiele level of geometry thoughts as suggested by Ngirishi & Bansilal (2019) and Rizki et al., (2018). More than three-quarters of the learners that participated in the main study struggled to solve Q1.2.1 which resulted in many learners responding IR. While 10(12%) of learners did not attempt to solve Q1.2.1 at all which resulted in BR. Similarly, the percentage of learners that attained CR in Q1.1.2, 1.2.1 and 1.2.2 were also low.

This finding indicated 67 (83.8%) of the participants in this study were operating in level 1. They could identify parallel lines in GAT. Astuti, Suryadi and Turnudi (2018) argue that any learner who cannot explain the image of a 2D shape with characteristics is operating at level 1. However, only 14 (17.5%) gave correct reasons for how they identified the parallel lines and used properties of parallel lines to solve the unknown magnitudes of the given angles. This suggests that approximately 21% of learners operate at level 2. However, 53 (66.35%) learners were unable to provide correct reasons which resulted in InR. The number of learners who responded to IR was found to be 12 (15%). Learners with IR seem to have a challenge in identifying and matching the appropriate properties of parallel lines and 1(1.3%) BR.

5.6.2 Analysis of Q2

In Q2.1.1- Q2.1.4, learners were given four pairs of 2D shapes and were asked if each pair were congruent to the other and give a reason for their earlier produced answers. Similarly, Q2.2.1-2.2.2,2.3.1 and 2.3.2 follow the same pattern. Also, in Q2.4.1 and Q2.4.2, expected learners to identify the shapes and relate what they see to the properties of the plane figure, allowing them to demonstrate proving. The Q2 assessed the integration of visualisation, analysis, and formal deduction which was level 1,2 and 3 of the Van Hiele level of thought.

Table 13: Distribution of learners' responses (with their percentages) to Q2 with N = 80

Question		CR	InR	IR	BR
2.1.1	L3	11(13.8)	32(40.0)	24(30.0)	13(16.3)
2.1.2	L3	12(15.0)	33(41.3)	20(25.0)	15(18.8)
2.1.3	L3	14(17.5)	25(31.3)	24(30.0)	17(21.3)
2.1.4	L3	12(15.0)	25(31.3)	23(28.7)	20(25.0)
2.2.1	L1	66(82.5)	0(0.0)	7(8.8)	7(8.8)
2.2.2	L2	10(12.5)	5(6.3)	53(66.3)	12(15.0)
2.3.1	L3	2(2.5)	2(2.5)	66(82.5)	8(10.0)
2.3.2	L1	54(67.5)	0(0.0)	18(22.5)	15(37.5)
2.3.3	L4	2(2.5)	7(8.8)	34(42.5)	37(46.3)
2.4.1	L4	2(2.5)	1(1.3)	40(50.0)	37(46.3)
2.4.2	L4	1(1.3)	4(5.0)	34(42.5)	41(51.2)

Table 13 depicted the distribution of learner responses to Q2 with 11 sub-question items. The table revealed that out of 80 learners that participated in the GAT the highest number of CR were found in **Q2.2.1** and the lowest was found in Q2.4.2 which are 82.5% and 1.3% respectively. Also, high numbers of learners were found to attain CR in **Q2.3.2**. This result suggested that most learners were operating at L1. Moreover, the highest number of learners that obtained InR were found in **Q2.1.1.** Most learners that attained InR were able to identify two triangles that are congruent but could not explain the condition for the congruency (Astuti et al., 2018). The majority 66 (82.5%) of learners obtained IR in Q2.3.1 which is at L3 according to the Van Hiele level of geometrical thought. Even in Q2.41 where 40 (50%) of the learners responded IR, quite a few learners (2.5%) responded to CR. The majority 37(46.3%) of participants responded BR to questions 2.3.3 and 2.4.1. This result suggested that most learners had misconceptions, 30% and 16.3% respectively. Most learners that score InR understand when two pairs of shapes are congruent but cannot explain the properties of congruency that made them conclude that the figure is congruent. The conditions for congruency are as follows: the corresponding sides of the two triangles are equal which is (side, side, side (SSS)); two sides of the two triangles and included angles are equal (SAS); two angles and included sides of the two triangles are equal (ASA) and the Right-angle Hypotenuse and Side (RHS) of one triangle are equal in two respective triangles.

Furthermore, Table 13 also revealed that in Q2.2.1-2.4.2, 66 (82.5%) of learners attained CR while 7 (8.8%) learners were categorised IR and BR in Q2.2.1 while in Q2.2.2 the majority of learner responses were categorised IR, 53(66.3%). Similarly, most learners 55 (67.5%) answered Q2.2.2 correctly because learners can identify two that are congruent to each other. However, in Q2.3.1 the case was different because most learners 66 (82.5%) scores were IR. The result revealed that the majority of learners could identify the two triangles. However, their challenges were based on the inability to match the condition of congruency to the pair of 2D shapes that are congruent to each other. This is similar to the difficulties learners faced in Q2.1.1-2.1.4. Learners who could not recognise the properties of shapes struggle with level 2 of Van Hiele level of geometric theory (Ngirishi & Bansilal, 2019). According to the Curriculum Assessment Policy Statement (CAPS), learners in grade 10 must be able to apply the properties of congruence to prove two triangles are congruent (Department of Basic Education, 2011b). If learners understand the congruency properties, they will notice that Side, Side, Side (SSS), or Side, Angle, Side (SAS), are the conditions that allow one to conclude congruency.

Furthermore, not more than 2(2.5%) answered Q2.4.1-2.4.2 correctly. Even, with those learners that tried to answer the questions (InR), the percentage is not more than 5%. Out of 80 learners, 32 (42.5%) of learners responded incorrectly to Q2.4.1 and Q2.4.2. while slightly more than 50% of learners that participated in the GAT responded BR to the same questions.

5.6.3 Analysis of Q3

Questions 3.1 and 3.2 assessed learner understanding of the integration of levels 1 to 3. Learners were asked to identify the shape, prove the congruency of a given plane shape and calculate the magnitude of the unknown angles. Table 14 shows the result of learners' performance of Q3.

Table 14: Learners' responses and percentages to question Q3 where N = 40

Question		CR	INR	IR	BR
3.1.1	L1	29(36.3)	0(0.0)	39(48.8)	12(15.0)
3.1.2	L4	2(2.5)	6(7.5)	34(42.5)	38(47.5)
3.2.1	L4	2(2.5)	1(1.3)	38(47.5)	39(48.8)
3.2.2	L3	4(5.0)	1(1.3)	38(47.5)	37(46.3)
3.2.3	L3	5(6.3)	6(7.5)	27(33.8)	42(52.5)

The percentage of CR ranges from 2.5% to 36.3% (see Table 14). This indicated that few learners were able to identify that the quadrilateral was a parallelogram. This result agrees with the findings of Ngirishi and Bansilal (2019), that some FET learners were still operating at level one. Although the majority of learners 68 (85%) attempted to answer Q3.1.1, 39 (48%) of them were found to give the question IR. Similarly, in Question 3.1.2 and Q3.2.1, the percentage of learners with CR was the same which is 2 (2.5%) out of 80 learners that participated in the test. However, Q3.2.3. seems to have the highest 42(52.5%) numbers of learners responded BR in the test.

5.7 OVERALL PERFORMANCE OF LEARNERS (Descriptive statistics)

The researcher also considered the overall performance of learners in Euclidean geometry by using descriptive statistics. Table 15 below shows the general performance of 80 grade 10 learners in GAT. To get an overview of the general performance of grade 10 learners in Euclidean geometry in two selected schools in the Tshwane West district, the researcher calculates the mean, median, mode standard deviation and skewness of the achievement test. According to McMillan & Schumacher (2014) and Cohen et al.,(2018), the mean is commonly used because it involves all the data by calculating the average score for the data set. Also, the standard deviation explains how far the data set is away from the mean score for the data set. The table below shows the overall result of the main study.

Table 15: Descriptive Statistics of the main study

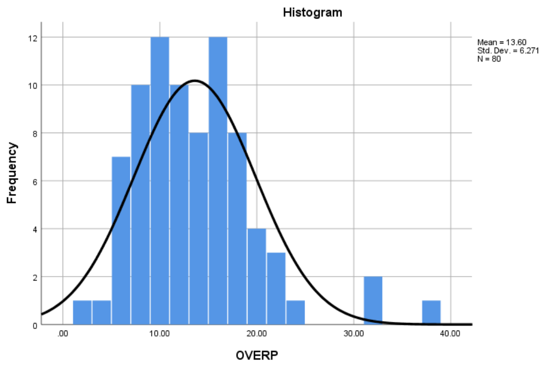
	N	Range	Minimum	Maximum	Mean	Std. Deviation	Skewness	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error
Overall performance	80	36.00	2.00	38.00	13.6000	6.27149	1.258	.269

Table 15 above shows the descriptive statistic of the main study. The mean statistic score for the main study is 13.6 %. The standard deviation of 6.27 %. The lowest mark obtained was 2 (4%) while the maximum was 19 (38%), the mode is 10%, and the median is 12% (see Appendix X).

This overall result revealed by the mean statistic implies that most learners performed poorly on the test. The result suggested learners lack the understanding of parallel lines, congruency, and parallelograms. The low standard deviation statistics compared to the mean indicated the performance of learners in this GAT is clustered around the mean statistic (Zhou, 2019). The range statistics revealed a huge difference between the maximum statistic score and the minimum statistics score. Similarly, the result also revealed that most learners score 10% on the test which indicates that the majority performed poorly in the GAT. This result depicts a positively skewed, 1.258. The graphical representation is shown in Figure 1

Figure 8 presents the graphical representation of the result in table 14. The graph shows that most of the marks of learners fall between 8% and 20% which shows the distribution of the mark is around the mean mark as analysed in table 14. The figure is a positive distribution which indicates that as the curve approaches zero, the right side of the curve becomes longer in which the learner scores are between 2 (4%) and 38 (78%).

Figure 8: The histogram of the result of the main study



This result revealed that most learners performed poorly on the test. Hence result shows that the grade 10 learners in the Tshwane district are not performing well in Euclidean geometry. This result is consistent with the views of Alex & Mammen (2016) and Ngirish & Bansilal (2019) who argue that learners are not performing well in geometry. The result further revealed that most of the learners struggle with levels 1 and 2 whereas learners in this grade were supposed to be in level 3. This seems to be a re-occurring problem from their previous grades. Ngirishi & Bansilal (2019) found out that the majority of learners in Grades 10 and 11 were at level one, as described in the Van Hiele geometry model.

5.8 OVERVIEW OF GRADE 10 LEARNERS PERFORMANCE IN EQULIDEAN GEOEMETRY

A one-sample t-test was used to test for the statistical significance of the improvement in performance of grade 10 learners in Euclidean geometry. A significant difference is shown when the p value is less than 0.05 (Cohen et al., 2018). However, there is no significant difference when the p value is higher than 0.005.

The table 16 indicated the overall analysis of the performance of learners in this study. The analysis revealed a significant difference between the departmental pass mark of 30% and the overall performance of learners in Euclidean geometry. The P-value *of 0.000* which is less than 0.005 at the 95% confidence limit. This suggests that the grade 10 learners in this study performed poorly as compared to the departmental pass mark.

Table 16: The performance of learners in Euclidean geometry

One-Sample Test						
	Test Value = 30					
	T	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
OVERP	-23.389	79	0.000	-16.40000	-17.7957	-15.0043
Q1	-4.255	79	0.000	-7.45875	-10.9480	-3.9695
Q2	-4.349	79	0.000	-5.87075	-8.5576	-3.1839
Q3	-10.731	79	0.000	-19.12475	-22.6720	-15.5775

Similarly, the outcome of Q1 showed that there was a significant difference between the departmental pass requirement mark of 30% and the performance of learners. As the P-value of 0.000 is at the minimum compared to the 0.005 P-value at a 95% confidence level. The result indicated that the learners performed dismally in questions related to parallel lines. However, learners performed better in Q1 compared to the other two topics in Euclidean geometry, as shown in Table 5.16, where t = -4,225.

Moreover, Table 16 shows a significant difference between the pass mark of 30% from the Department of Basic Education and the performance of Q2. The P-value is 0.000, which is less than 0.005 at a 95% confidence level. This result further highlighted poor performance of learners in Q2 where learners could not apply properties of congruency to solve problems, with t = -4,349. This finding concurs with the study of Ngirishi and Bansilal (2019) that found that learners in high schools have challenges to applying deductive reasonings in solving problem related to the properties of congruency.

The analysis of the result in the overall performance showed that there is a significant difference between the department pass mark of 30% and the performance of learners in Q3, at the 95% confidence level, the P-value was 0.000, which was less than 0.005. Learners appeared to have challenges in proving of parallelogram theorems and in their application to solving problems. According to this analysis, most of the learners do not attend this Q3.

5.9 PHASE TWO: ANALYSIS OF QUALITATIVE DATA

The qualitative data were collected from the semi-structured interview and lesson observations. The analysis in this current study responded to research questions 3 and 4 (see sections 1.8.2.3 and 1.8.2.4). Cohen et al., (2018) argues that the role of quantitative analysis in the inquiry was to organize, describe, understand, account for, and explain the data that was collected. The role of qualitative analysis also entails making note of the situation, noting patterns, themes, categories and regularities that emerge from the data (McMillan & Schumacher, 2014). The purpose of the qualitative data analysis in this current study was to provide a follow-up to discover a rich understanding of the causes of learners' difficulties when they are confronted with Euclidean geometry questions.

In qualitative research, the process of data analysis constitutes a way of making sense of the data themselves (Creswell, 2012). The researcher writes down notes, memos thoughts and reflections in the field or during an interview and observation. These steps were taken to be able to answer the research questions by moving from understanding to explanation to institute transparency in the research process.

5.9.1 Semi-structured interviews

As mentioned earlier in Chapter 4, the researcher conducted a semi-structured interview with six learners from the participating schools based on the analysis of difficulties learners displayed in the GAT. The purpose of the semi-structured interview was to acquire an in-depth understanding of the difficulties learners faced in the process of solving Euclidean geometry. Van Hiele's level of geometrical thought which underpin this study were used to analyse the interview data where visualisation, analysis, abstraction, formal deduction, and rigour are used as categories in the methodological approach for this study.

This section discussed the results of the semi-structured interview that were conducted after GAT to further reveal the challenges of learners when engaged with Euclidean geometry. Six learners were interviewed according to the errors and misconceptions they displayed in their GAT. The researcher~~s~~ used pseudonyms for learners 1-6 (L1 – L6) and the questions were asked according to the kind of errors displayed by the learners. The semi-structured interviews were conducted in the English language. However, the participants were allowed to speak freely in the language of their choice to express themselves better. The following questions in table 17 below were asked in relation to errors learners displayed in their written responses.

Table 17: Sample of semi structured interview questions

The learning difficulties with parallel lines:	What are parallel lines?
	How do you identify a parallel line?
	What two angles are the same?
	How to get to know the two angles are the same?
	How do you solve Q1.2.1 and Q1.2.2?
The learning difficulties with congruency:	What do you understand by congruency?
	What are the conditions of congruency?
	What condition of congruency are you using in Q2.1.1-2.1.4?
	How do you answer Q2.2.1 and Q2.3?
	What are the reasons for your answers?
	How do you answer Q3.1
The learning difficulties with parallelogram	What shape is in Q3.1
	How do you use congruency to prove Q3.12?
	How do you proof parallelogram theorem in Q 3.2?
	Explain how you solve 3.2.2 & 3.2.3.-

5.9.1.1 Learning difficulties in parallel lines

Excerpt 1:

The first interview question needed to probe the mistakes learners display while recognising and providing a logical motivation for questions that are related to the properties of parallel lines. The extracts illustrate learners' responses to the semi-structured interview question.

Researcher:	What are parallel lines?
L1:	Hmm, Parallel lines are lines that are… (L1 pause for a while before she continues parallel)
L2:	Parallel lines are the line that are facing the same direction
L3:	Parallel lines are lines that can never meet

L1 responses to the question suggested learner lack the understanding of parallel lines. This was evidenced in the response that the L1 made by trying to rephrase the question back as an answer. '*Hmm, parallel lines are lines that are parallel*. The response from L1 showed that the learner does not have a basic understanding of parallel lines because this made the learner pause for a while before he continues. This indicated that the learners could not find the right word to use. This statement is in line with the result from the GAT where the majority of learners struggled to provide a reason for their response in Q1.1.1. This result also confirmed with Orevaoghene (2020) that learners who do not understand simple terminology like parallel lines will find geometry to be more challenging. According to the CAPS document, learners should be able to define parallel lines from the previous grades. Learners should have understood that two lines are parallel to each other are located on the same plane, are the same distance apart and the two lines do not intercept with each other, L2 and L3 definitions accounted for those learners in the GAT that justified correctly while the two lines are parallel to each other.

Excerpt 2:

Researcher:	What is the relationship between line TC and AB?

Learner L1: they are parallel.

Learner L2: they are parallel.

Learner L3: they are parallel.

Most of the participant responses to this question show correct answers. The majority of learners recognised parallel lines. Most learners see lines TC and AB are parallel to each other which agreed with Cho & Win (2020) was one of the areas learners have least difficulties in this topic, however, the majority of learners are struggling with giving a justification for the answer given in Q1.1.1which are in the following transcript below.

Excerpt 3:

Researcher:	Why do you say Line BC and TC are parallel?
Learner L1	Because they have the same angles Akri, Aba (1&2, 2&1 that is C and D also B and C
Learner L2	because they are facing the same direction, even the arrow is pointed in the same direction.
Learner L3	because two lines are going their separate ways.
Learner L4	the two lines are not close to each other

The motivation provided by L1 showed that the learner was able to connect the parallel lines with the angles formed. L2, L3 and L4 have a challenge in connecting parallel lines and do not give the reason for making their conclusions. This indicated that L2, L3 and L4 lack the ability to think logically. Learners who do not have the capacity for logical thoughts in Euclidean geometry, give a reflection, explanation and lack adaptive reasoning (Khalil et al., 2018; Kilpatrick et al.,2001) The result indicated that most learners lack adaptive reasoning in dealing with parallel lines. This was evidenced in learner L4 when responded *that the two lines are not close to each other*. The learner L4 do not show and understanding of two parallel lines. Also, L4 unable to show distance between two parallel lines. The statement given by these learners indicated Learner L4 lack the understanding of the parallel lines. According to the CAPs learners in grade 10 should have mastered the parallel lines because these are topics which should have been discussed and understood in earlier grades. Learners must have the capacity to reason logically and be able to draw a valid and acceptable conclusion.

Except 4:

Researcher: How do you get angles C1 and C2 in question Q1.1.2 and Q1.1.3 respectively

Learners L4: I did not do it in Q1.1.2 and Q1.1.3 I know our teacher taught us this in grade seven or eight, in 2018 there about but I have forgotten them. I cannot remember them.

Learner L2: Here ma'am, honestly, I don't remember how to find the angles in this part it has been a long that they have taught us. I have to guess the answer here.

Learner 5 I was thinking C1=B1 because they are the same, but I cannot see any other angles that are the same as C2 that's why I wrote C2.

Learner 8: we were never taught these types before that is why I don't know how to do it.

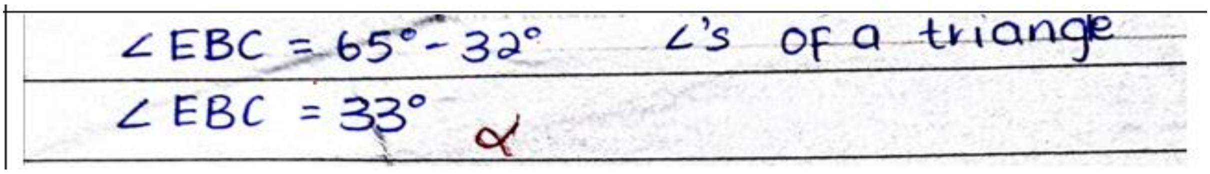
From the excerpts above, L1 cannot remember what has been taught in the previous grades about the angles formed by parallel lines cut across by a transversal so decided to leave the question without attempting it. While L3 do not understand the property of parallel line therefore cannot see any angles that are formed by the parallel lines. This was evident when the learner responded *I didn't do it*. L2 also claimed to have forgotten the concept. This result revealed that most learners in grade 10 still have a weak understanding of the properties of parallel lines. This finding is in line with Ngirishi and Bansilal (2019) that found that the FET learners in Kwazulu-Natal have misconceptions in connection with the properties of parallel lines. From the excerpt above, it is evident that some learners are in grade 10 lack understanding the properties of parallel lines and this might be true of other schools in Tshwane-West district.

Excerpt 5:

Researcher: When I checked your written responses to Q1.2.1(researcher pointed to responses provided by L1 as shown in figure 9) I realised that your answer was incorrect. You seemed to have an idea of what you are doing at one point however, you have the challenge to get the correct answer.

Researcher: Can you explain how you arrive at your answer?

Figure 9: The sample from L1's work on parallel line



Learner L1: I subtracted from 65 degrees 32degree, Ma'am, it gave me 33 degrees, then I was confused about which angle was supposed to be 33 degrees. I can see that the two angles are alternate angles which are 32 degrees, but I made a mistake here not sure which angle should be 33 degrees.

Learner L2: I did not understand what I was doing, I think the total size of the rectangle is 180 degrees, and then I calculated that 32 plus 65 equals 97 after I got 97, I subtracted it from the number of the whole rectangle 180 degrees minus 97degrees. So, my answer was 83 degrees.

The except 5 shows L1 and L2 responses to the interviewer's questions relating to Q1.2.1. The analysis of excerpt 5 suggests that L1 and L2 indicate a misconception in recognising the magnitude of the angles that are formed by parallel lines. The L1 seemed to have a misconception relating two angles that are described as alternate. Although she did not write that the angles alternate with each other. While L2 cannot differentiate between shapes which were evidenced when they responded *that the total size of the rectangle is 180 degrees*. L2 seems not to understand the difference between different plane shapes. This shows that some learners in grade 10 cannot differentiate between plane shapes. L2 called a triangle a rectangle. This indicates that most learners were operating below the visualisation level of Van Hiele's level of geometrical thought. The finding agrees with Atebe & Schafer (2008), Luneta (2015a) and Ngirishi & Bansilal (2019) that found out most learners in grade 10 are operating at the pre-recognition level.

5.9.1.2 Analysis of learning difficulties in congruency

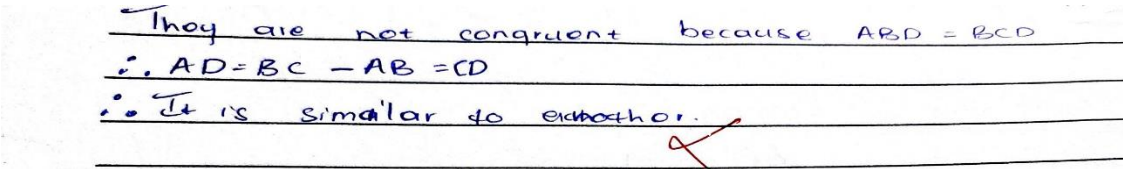
The semi-structured interview questions in this category related to difficulties by learners in the concept of congruency in Euclidean geometry. Learners were scrutinised to follow up on the mistakes learners displayed in the GAT question number two. The following excerpts are from the interrogations with learners.

Excerpt 6:

Researcher:	What is your understanding of congruency? Can you tell me if the figures provided to you in Q2.1.1-2.1.4 are congruent or not?
Learner L04	I remembered that congruent things are the same and have the same size. Ma'am, some of the sizes are not the same.
Researcher	When I check your written responses to these four questions, I noticed you were able to identify correctly one out of the four even in one you do not give a reason for your responses. Do you understand what congruency is?
Learner L6	in those questions, I am not sure of my answers to those questions, and I do not understand the condition of congruency. I did not know the reason I must write. I just guess.

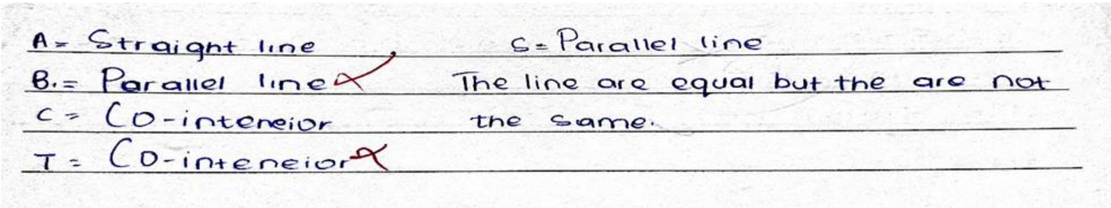
Excerpt 6 shows the analysis of Q2.1.1-2.14. The analysis shows L6 have difficulties in the terms used in congruency of shapes when responding *not understanding* the conditions of congruency. The L6 seemed not to have the basic knowledge of congruency which made the learner result given irrelevant and invalid reasons like that of the samples in Figure 10.and 11. Dhlamini, Chuene, Masha, & Kibirige (2019) argue that learners with a lack of basic understanding of congruency find it difficult to provide complete and relevant logical reasoning to the congruency questions. The findings of the semi-structured interviews explained factors that resulted in learners giving incomplete and irrelevant responses provided in the GAT.

Figure 10: the sample of L6 work on congruency



They are not congruent because ABD = BCD
∴ AD = BC – AB = CD
∴ It is similar to eachother.

Figure 11 : the sample of L6 on congruency



A = Straight line
B. = Parallel line
C = Co-interior
T = Co-interior
S = Parallel line
The line are equal but the are not the same.

Excerpt 7:

Researcher	I see that you answered Q2.2.1 correctly but you struggle to get Q2.2.2 Could please explain why your answer?
Learner L4	I know that the two triangles ΔPQR and ΔFED are congruent, not clear what reasons I must give
Learner L5	The sizes and shapes are the same

5.9.1.3 Analysis of Q2.2 and Q2.3

From this excerpt for Q2.2. and Q2. L4 and L5 can recognise when two shapes are congruent to each other but L4 has no idea of the condition of congruency. Therefore, this learner does not provide any reason for question 2.2 while learner L5 cannot figure out the condition to be used. This excerpt indicated learners have misconceptions about the conditions of congruency. Most learners found it difficult to give logical reasons to use the terms found in congruency with the SSS, SAS, ASA and RHS.

Except 8

Researcher	You are asked to prove that line BC is equal to line EF. Can you explain your answers?
Learner L1	The lines BC and EF are parallel to each other

Learner L3	I see that the C1 =F1 because they are alternate.

The except shows learners' L1 and L3 responses to Q2.4. L1 believed line BC and EF parallel to each other which is incorrect and irrelevant to the given proof while learner L3 could see that C1 and F1 alternate with each other. The reasons given by L3 may be correct however, it is irrelevant to the proof of Line BF and CE. Here, L3 expected to see that line FC was added to both lines BC and EF instead of giving an inappropriate reason and out of context. Dhlamini et al. (2019) argued that learners provide incorrect and irrelevant reasoning and lack spatial orientation. Learners with spatial orientation can be able to transfer their prior knowledge to the new context. Which most of the participants struggle to do in this study.

5.9.1.4 Analysis of the difficulties in parallelogram

This subsection analysed the difficulties of according to the responses given by L1, L3 and L5 in relation to parallelogram during the semi-structured interviews. The analysis is found in Excerpt 9 and 10 below.

Excerpt 9

Researcher	I can see that you got the answer in 3.1 wrong. Can you explain to me how you arrive at your answer?
Learner L1	I see a rectangle
LearnerL3	I see a diagonal
Learner L4	it looks like two triangles
Learner L5	Congruent

Excerpt 9 shows the learners' responses to the interview question related to the parallelogram. Most learners in grade 10 understands the properties of a parallelogram and most all the interviewees were able to respond correctly, however, excerpt 9 shows that most learners still have difficulties with visualising parallelograms despite how the figure was rotated. This is evidenced in most learners’ responses. Learner L1 believed the figure to the figure is a rectangle. This response indicates that learner L1 struggled with class inclusion. Alex &

Mammen (2016), argued that class inclusion is a relational property between two shapes. For example, there is a class inclusion between a rectangle and a parallelogram. A rectangle is a parallelogram, however; a parallelogram cannot be a rectangle. The L1 response shows that most learners have challenges with class inclusion. This finding is consistent with Cho & Win's (2020) and Ngirishi & Bansilal's (2019) findings. Cho & Win (2020) found out that the majority of grade 8 learners in Myanmar have a misconception about class inclusion. Similarly, Ngirishi & Bansilal (2019) found in their study that a high number of learners in the FET grades in KwaZulu -Natal have misconceptions concerning class inclusion. Learners' responses L3, L4 and L5 show that some learners have a poor understanding of the properties of a parallelogram. Their responses indicated a lack of conceptual understanding.

Excerpt 10

Researcher	I see that in your responses to the GAT, you do not respond to Q3.1.2. Why don't you answer Q 3.1.2?
Learner L1	I did not write anything because I do not understand what to do
Researcher	Can you explain to me what you did in Q3.2?
Learner L6	I added the 102 degrees and 36 degrees together, so I got 138 degrees, I subtracted 138 degrees from 360 degrees, and I got 242 for my angle x. For my value of y, I only added the value together

Excerpt 10 shows learners L1 and L6 responses to the interview question related to a calculation involving parallelograms. The L1 responses show a lack of understanding which was evidenced when the learner L1 responded *I do not understand*. Here learners are supposed to understand that the opposite angles of the parallelogram are equal which give a reason for the angle x to be 36 degrees instead of adding them together. L6 shows a misconception, in that learners added the alternating angles instead of equating them. Furthermore, to get the value of angle y learners are expected to do it this way: $x = 36°$. Figure 5.3a and 5.3b present the correct answer to the question 3.1.2 which assessed learners on properties of parallelogram to find the unknown angles in the questions.

Figure 12: The correct answer to question 3.1.2

$\hat{X} + X\widehat{W}U = 180°$ [$\angle'$ s in a $\Delta = 180°$]

$\hat{X} = \hat{V} = 36°$

$X\hat{U}W = y$

$y = 180° - 36° - 102° = 42°$

OR

Figure 13: The correct answer to question 3.1.2

$\therefore \hat{X} + \widehat{W} + \hat{V} + \hat{U} = 360$ [$Sum\ of\ \angle s\ in\ Quad$]

$36° + 36° + \hat{U} + \widehat{W} = 360°$

$\hat{U} = \widehat{W}$

$72° + 2\hat{U} = 360°$

$U = 298°/2$

$X = 149° - 102° = 42°$

The analysis of this interview question showed that learners made mistakes with basic arithmetic which indicated a lack of conceptual understanding of parallelograms. Learners are unable to transfer the knowledge of the properties of parallelograms into the calculations which indicated the justification for most of the learners leaving their script blank in the GAT. Luneta (2015) argues that conceptual understanding is a critical issue.

5.10 LESSON OBSERVATION

The researcher observed a total of six lessons from a school in the Tshwane West district. In School A each of the lessons was forty minutes. The lessons observed on Euclidean geometry included parallel lines, congruency of triangles and parallelograms. The lesson observations were observed from April 12th -23rd 2021. Learners' productive, evocative, evaluative, and reflective skills were taken note of during the lesson observation which was adapted from the original observation schedule by the Sepeng (2010). Also, strategies used by the teachers were monitored during the period to know whether the method of teaching also posed a challenge to the learning of Euclidean geometry in grade 10. The lesson observations were carried out using the lesson observation schedule from Sepeng (2010). The lesson observation schedule was adapted to suit this study. The researcher observed the learners in their classroom by using the four components of the Sepeng (2010) which includes productive skills, evocative skills, evaluative skills, and reflective skills. The four skills were discussed in sections 5.10.1-5.10.4. The researcher took a photograph of the learners' written work. The researcher sat at the right corner of the classrooms the researcher captured the learners' activities during the observation of the lessons.

The purpose of this lesson observation was to enhance and complement the data collected from the GAT to explore the challenges learners faced in learning Euclidean geometry in their natural space, and, to triangulate the data collected from the semi-structured interview, and the test. to give a rich understanding of the learning difficulties experienced by grade 10 learners in the Tshwane West district. The researcher used italics to represent direct quotations from the participants (Cohen et al., 2018).

5.10.1 The productive skills of learners in Euclidean geometry lesson

In productive skills, the finding revealed that most of the learners were able to read and write read and write during Euclidean geometry lessons that were observed. However, most of the learners have difficulties answering some of the questions given to them by their teachers correctly and some learners could not answer some questions at all The productive skills empower learners to demonstrate their understanding of Euclidean geometry through reading and writing form (Zhou, 2019). Daniyan (2015) argues that learners who find it difficult to read and write during mathematics lessons: such learners will find it difficult to understand the concept being taught. Again, some learners struggled with geometry terms. Learners find it difficult in differentiating between some terms however teacher assisted learners to differentiate between terms. For instance, learners were unable to differentiate between parallel and perpendicular lines in the first lesson when the teacher was introducing the lesson. Also, most learners are not able to transfer the concept being taught to be able to complete the given task on their own. For example, in a particular lesson observed learners were given homework to state whether a pair of triangles are congruent or not and they were justifying their answers with reasons. The figure below shows the diagram the working learner was asked to do as part of the homework.

Figure 14: Sample of learner work

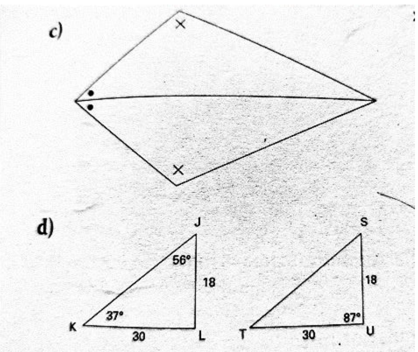


Fig 14 shows the diagram of a sample of the learner's homework. In this sample, learners were asked in questions c and d respectively to state whether the pair of triangles that were given are congruent to each other. Also, learners were asked to give reasons for the answers and justify why two triangles are congruent. Learners were expected to know that in the two questions that the triangle given were congruent. In question (c) learners were expected to give the reason that two angles in each of the triangles are equal and aside, because common to each triangle was equal. The reasoning expected to be given for (c) was angle, angel, and side (AAS). Question (d) also shows that the two triangles are congruent and justified on the condition that all three sides of both triangles are congruent to each other.

Some of the learners could not transfer the knowledge of congruency to answer the question. Below is a sample of the learner's work in figure 5.5.

Figure 15: The sample of a learner responses

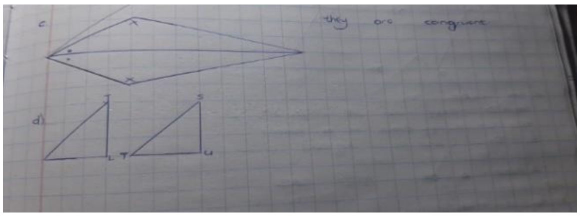


Figure 15 shows the learner response according to the question in Figure 14 above. The learner wrote that the answer to question (c) was congruent and did not justify the reason for why they were congruent. The learner cannot see the angles and the side that was congruent to each other in question c and the sides were congruent. Also, the second response of the learner in the sample showed that the learner did not complete the labelling and could therefore not complete the question. This shows that the learner lacks the understanding of congruency and cannot transfer the knowledge of what had been taught in class to the homework. Luneta (2015) found out that learners struggled to answers questions in examination is as result of lack the basic understanding of geometry.

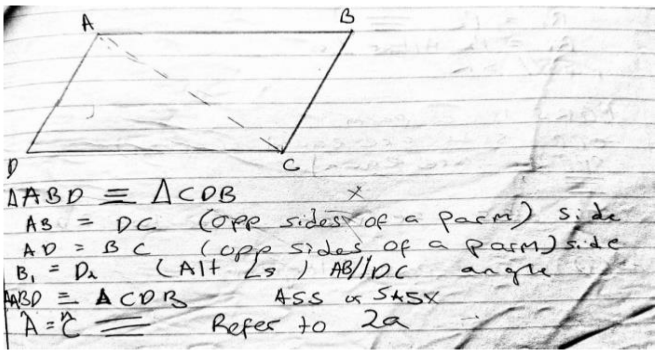
5.10.2 The evocative skills of learners in Euclidean geometry

The findings revealed that most learners during observation were found to be passive. In almost six lessons observed, most learners were quiet and listen to what the teacher presented to them. Only a few learners were raising their hands to ask or answer, and the teacher focused on those few learners. Zhou (2019) argues that evocative skill is a skill that allows learners to ask a question to seek clarity. Asking questions promotes the critical, creative and problem-solving skills of learners (Chin & Osborne, 2008). The researcher~~s~~ discovered that learners do not ask the teacher any questions for seeking clarity or enhancing understanding of the geometry concept being taught. Learners were not bold to challenge the teachers by asking a question to have a clear understanding of the Euclidean geometry concept. In most of the lessons, teachers do most of the talking. Most learners struggle with the new information given to them.

5.10.3 Evaluating skills of learners in Euclidean geometry

In evaluating skills, the finding reveals that some learners could assess their peers' work on the concept however most learners were incapable of assessing other learners' work. Evaluating skills refer to high-order thinking skills that determine how to make a judgement about the work done (Alhassora et al., 2017). According to Makgakga (2011, p20), evaluating skills "eradicate errors and misconception".

Figure 16: Sample of learner's work showing errors and misconceptions

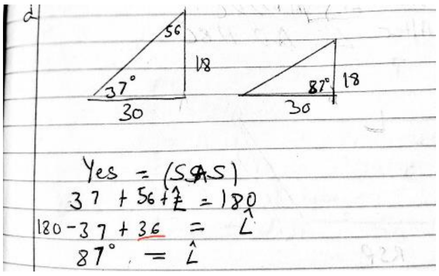


Learners with evaluating skills could see errors and misconceptions made. During the six classroom observations, the teacher asked each learner to exchange their books when discussing the correction so that learners can be able to identify errors while they mark their partners work. After marking, the teacher asked learners about some of the errors learners made during the corrections which they discussed. The researcher observed the process of evaluation three times during the classroom visit. On some of the days when the teacher was not teaching during those periods learners were assessing their partners work. In one of the classes, the teachers asked about some of the mistakes they observed. Only one learner was able to point out errors that other learners made. The others were busy with the correction without understanding what the teachers were doing. Figure 5.6 was a sample of learners' work showing some of the errors one of the learners made that were identified by the other learner. The learner is supposed to prove that ∆ABD is congruent to ∆ CDB, that the magnitude of angles A and C are equal and to prove that angles ABC and ADC are equal. Figure 6 shows a sample of learner work where the learner was expected to draw the diagonal from B to D, but the learners draw a diagonal line from A to C which did not represent the given statement. This learner appears not have understood what must be done to answer the parallelogram question. Learner could not identify the error made. This finding is consistent with Alhassora et al. (2017) who found that Malaysian learners with lack of evaluating skills have difficulties in solving geometry questions.

5.10.4. Reflective skill of learners in Euclidean geometry

Learners are always overwhelmed with errors and misconceptions throughout the lessons observed. The findings from this lesson observation revealed that learners couldn't solve the homework given to them and ended up coming to the classroom without solving their homework problem, sometimes they couldn't give logical reasoning for their answers. Figure 17 presents a sample of learner's work who cannot reflect on a given answer.

Figure 17: sample of learner's work showing misconceptions



This learner does not give a reason for the answer given and made an error when the learner was supposed to write 56^{o} then wrote 36^{o}, such learners cannot reflect and think back on the error made, a reflective skill allows learners to make a conscious decision towards becoming critical thinkers and active learners. Learners must make hypothesized and informed decisions in Euclidean geometry through logical reasoning (Yuni et al., 2021). In Figures 5.5, 5.6 and 5.7, the errors that learners displayed during the lesson observation included not giving logical reasoning and making both procedural and conceptual errors. The findings from this study are consistent with the study Poon and Leung (2016) that found that learners who cannot reflect through logical reason have difficulties performing well in mathematics.

Based on the lesson observation it was discovered that the teaching strategies used by teachers were the conventional teaching method of talking and writing method, while learners sit and listen to the teacher. The approach did not affect the learners' understanding. Similarly, time was a challenge for the teacher were unable to cover most of the concepts for learners to fully understand lessons. The teacher has insufficient time to check learners' work due to the covid -19 protocol adjustment. Most learners' books were piled up by the teachers who did not have sufficient time to check learners' work.

In terms of resources, teachers that were observed used textbooks. Although teachers have access to a computer and smart board devices. In all the lessons observed, teachers do not use any information technology in any of the lessons.

5.11 CHAPTER SUMMARY

This chapter presents the analyses and interpretation of data from three secondary schools in the Tshwane West district. One school participated in the pilot study and the other two schools participated in the main study. This chapter presented the data collected from qualitative and quantitative instruments. The data instrument for the quantitative approach was the Geometry Achievement Test. Data collected from this instrument was used to answer research question one which was to identify what difficulties grade 10 learners experience when solving Euclidean geometry problems. A semi-structured interview guide and classroom observation checklist were instruments used for the qualitative approach used to provide rich and comprehensive data for answering research questions two and three, which were to ascertain why learners experience those difficulties when solving Euclidean geometry problems and how grade 10 learners learn Euclidean geometry.

CHAPTER SIX

DISCUSSION OF FINDINGS

6.1 INTRODUCTION

The previous chapter presented the analysis of the Geometry Achievement Test (GAT) and the data collected from the semi-structured interviews and lesson observations. The researcher analysed the semi-structured interview data, and the following themes emerged from the data collection: difficulties with parallel lines, congruency, and parallelograms. Also, the class observations were analysed to provide a better understanding of how grade 10 learners in selected secondary schools learn Euclidean geometry in their classrooms.

This chapter discusses the results of the study that investigated the performance and the learning difficulties that grade 10 learners experience when solving Euclidean geometry problems, as revealed from the quantitative and qualitative data. The purpose of the study was to explore the areas in grade 10 Euclidean geometry in which learners experienced difficulties (Creswell & Creswell, 2018; Mills & Gay, 2018). Thus, the quantitative analysis was done primarily to the determine the performance and challenges that learners experience when solving Euclidean geometry problems.

Thereafter, the qualitative analysis was done to respond to the research sub-questions, which will be answered later in this chapter. The research sub-questions were as follows:

- Why do learners experience difficulties when solving Euclidean geometry problems?
- How do grade 10 learners engage with Euclidean geometry concepts?

The discussion section in this chapter indicates how the qualitative findings complement the quantitative results (Creswell & Creswell, 2018). Similarly, the discussion was done in line with the theoretical framework and the literature review. Similarly, this chapter also summarizes the discussion of performance and learning difficulties of learners in two selected secondary schools in the Tshwane West district.

6.2 QUALITATIVE AND QUANTITATIVE DATA

The researcher addressed the findings from the quantitative and qualitative data collected via the GAT, semi-structured interviews, and lesson observation in this section. As stated previously in chapter 5, one of the goals of administering the GAT was to ascertain the overall performance of learners on problems requiring the concept of parallel lines, congruency, and parallelograms (section 5.6 – 5.8). These sections interpreted the GAT results on the performance of learners using the Erba and Didis (2015) model, descriptive statistics, and statistical analysis. Furthermore, the findings from the semi-structured interview and lesson observation were also interpreted to explain the GAT findings (Creswell & Creswell, 2018). The purpose of the semi-structured interview was to explore the learning problems encountered when tackling grade 10 Euclidean geometry problems. Furthermore, the lesson observation assisted in understanding geometry learning in grade 10, given the challenges that learners faced in their natural space. The researcher presented the qualitative findings to explain the quantitative findings in this section by using the themes that emerged from the data analysis.

6.2.1. Overall performance in Euclidean geometry

The findings from the GAT revealed the low performance of learners in the cluster of topics parallel lines, congruency, and parallelograms. The findings revealed a response to the first research question that grade 10 learners are not performing well in Euclidean geometry. As earlier noted, Euclidean geometry in the context of this study, means the concept of parallel lines, congruency, and parallelograms.

The Erba and Didis (2015) model indicated learner percentages of correct, incomplete, incorrect, and blank responses to provide an overview of the performance of learners in parallel lines, congruency, and parallelograms. As mentioned previously, the CR was used to categorize all responses that were devoid of errors or mistakes, indicating a complete grasp of the topic that was consistent with the memorandum. The IR reflected all responses that contained errors or misconceptions, indicating that learners lacked one or more types of comprehension and did not comply with the memorandum or the standard of grade 10 Euclidean geometry. The InR displayed any responses that were partially right but lacked some components of completeness, such as providing the proper magnitude of angle but failing to provide explanations. BR indicated that there was no response to the question. On the other hand, the result indicated the descriptive and inferential analysis of the findings of learners in the GAT, which reflect the summary of the performance of learners when solving problems related to parallel lines, congruent triangles, and their proofs, as well as those related to parallelograms. The statistical analysis software used in this study is IBM SSPS V26. In this software, the researcher used the one-sample T-test to determine whether there was a statistically significant difference between the performance of learners in GAT and the departmental standard value of 30% (Cohen et al., 2018; Mills & Gay, 2018); 30% is the grade 10 pass. In in the test at a 95% confidence level, the p-value is greater than 0.05 this means there is no statistical difference between the learner performance and standard value. Otherwise, the two values are significant if the p-value is less than 0.05.

The findings from this study showed the underperformance of learners in the three broad topics of Euclidean geometry in grade 10 which include parallel lines, congruency, and parallelograms. The overall p-value of learners is *0.000*, which is less than 0.05. This implies that there is a significant difference which show poor performance of learners. In other words, this finding supported the claim that grade 10 learners struggle with Euclidean geometry, as also stated by Luneta, (2015) and Ngirishi and Bansilal, (2019). The finding indicated that most learners struggled to understand parallel lines, congruency, and the parallelogram. This finding is also in line with the international studies that confirm that learners have difficulties in understanding geometry concepts generally (Zulnaidi & Zamri 2017), The overall mean score of the GAT was 13.6% and a standard deviation of 6.27% (see section 5.7.0). Most learners in this study scored below 10%. This finding is consistent with that of other researchers, for example, Luneta (2015) and Ngirishi and Bansilal (2019), who also found that learners do not perform well in geometry because they lack an understanding of the foundational geometry concepts. This result revealed that learners in this study seemed to lack an understanding of the properties of parallel lines when cut across by a transversal line, the conditions of congruency, and the proof of a parallelogram (Cho & Win, 2020; Luneta, 2015a; Ngirishi & Bansilal, 2019). The researcher discusses the details of the findings in each of the question items.

6.2.2 Performance and difficulties in parallel lines

The result of the GAT on parallel lines indicated low performance as compared to the departmental value of 30%. The *p-value = 0.0000*, which was less than 0.05 at a 95% confidence level. This shows that there was a significant difference in support that learners performing poorly because the p-value is less than 0.05. The GAT revealed that the *M = 22.5413* and the *S = 15.679* in parallel lines, which also indicated the poor performance of learners with questions on understanding the properties of parallel lines (Luneta, 2015; Orevaoghene, 2020; Zulnaidi & Zamri, 2017). In this question, learners were expected to identify parallel lines, provide the names of angles formed by the properties of parallel lines cut across by transversal lines, and calculate the sizes of angles formed by the properties of parallel lines. Although most of the learners (83.3% when combining the CR and InR) were able to identify the parallel lines (see table 12, question 1.1.1) few learners were able to identify parallel lines. However, some learners are not able to identify two or more equal angles that are corresponding, alternate, or co-interior (table 12, question 1.1.2-1.1.3). These findings concur with Mapedzamombe (2020) that learners were confused with the properties of parallel lines viz alternating, corresponding and vertically opposite angles. For example, even when learners were asked to calculate the magnitude of any given angle, most learners had incorrect responses, and some learners had blank responses, which indicated poor performance of learners in parallel lines despite it being a topic dealt with in the lower grades (Luneta, 2015). Also, some of the learners were not able to express in clear terms what they experienced because of limited vocabulary and communication skills. The poor performance of parallel lines indicated a lack of understanding of the properties of parallel lines (Astuti et al., 2018).

During the semi-structured interview, L1 said, *"Hmmmmn, parallel lines are lines that are parallel,"* which was similar to what was found during the classroom observation. This result suggests that learners lack a basic understanding of the terminology used in geometry, as was also found by other researchers (Alex & Mammen, 2018; Orevaoghene, 2020). Alex and Mammen (2018) and Orevaoghene (2020), found there are a significant relationship between learner performance and the knowledge of terminology in geometry. Learners in the study do not have a basic understanding of the terminology of parallel lines and its properties which may appear to be one of the reasons for poor performance in GAT. Because the participants in this study do not understand the basic terminologies in geometry like parallel lines, perpendicular lines, corresponding angles, alternating angles, and supplementary angles, they may struggle to see the relationship between two parallel lines.

Similar results were found in one of the lessons, where learners were not able to distinguish between parallel and perpendicular lines because they don't understand those terms, as was also found by other researchers (Alex & Mammen, 2018; Luneta, 2015). They were unable to show their understanding of the properties of parallel lines. Conceptual understanding begins when learners can have the knowledge of terms and are able to utilize those terms (Alex & Mammen, 2018). However, learners failed to see the angle formed by the alternate, or corresponding angles, or see that the co-interior angles are supplementary. Although the teacher emphasized during one of the lessons observed that the corresponding angle is equal and forms a letter F, the sum of co-interior angles are supplementary angles that form a letter U and alternate angles form letter N, learners failed to see the angles that are formed using the properties of parallel lines. This result indicated that learners lack the conceptual understanding of the properties of parallel lines.

According to reports of those learners with blank responses, the questions were too difficult for them. So, they failed to understand what must be done, as was also observed by Moila (2017). When learners were engaged during the interview to find out the reasons for the blank response, they replied that they had not been exposed to questions that were challenging. This was revealed when L6 said, *"We never understood this concept before, which was why I don't know how to answer it*." This indicates lack of understanding of parallel lines may result in learners giving BR. This result is in line with Luneta (2015) that when learners lack conceptual understanding, learners cannot respond to a given question.

Similarly, when learners were asked to calculate the value of unknown angles in given triangles constructed in a parallelogram, learners struggle to use the properties of parallel lines to determine the unknown angles. However, most learners in this study could not calculate the magnitude of angles using the properties of parallel lines. This shows that because learners could not understand the properties of parallel lines, they cannot calculate the magnitude of the unknown using alternate angles and co-interior angles. When interviewed learners said *they do not understand because of the way the teacher was teaching them, he is too fast*. This finding indicated that the teaching approach used by the teacher was not learner centred. This was also supported by the lesson observation in which the teacher observed she/he was always under pressure to finish the curriculum, and therefore the teacher did not pay attention to whether the learners understood the concept being taught (Sunzuma & Maharaj, 2019). During lesson observation, the teacher was unable to check learner work in two of the six lesson observation opportunities.

As noted earlier, the parallel lines questions are at Van Hiele levels 1 and 2. Learners in grade 10 were supposed to have been at Van Hiele level 3 of geometrical thinking, it was notable that most learners seemed to struggle with the properties of parallel lines; this mismatch of levels contributed to their poor performance in geometry. They do not understand corresponding angles, alternate angles, and co-interior angles as properties of parallel lines and cannot make sense of them. It would therefore be difficult for learners to understand geometry at a higher level and make sense of it. The finding is consistent with other researcher studies (Luneta, 2015; Ngirishi & Bansilal, 2019), which found that most learners in FET were operating below level 3, the level required for FET level geometry. This means that learners who participated in the GAT seem to lack an understanding of the properties of parallel lines, which would contribute to their poor performance in that concept. This finding is supported by other research studies (Luneta, 2015; Malatjie & Machaba, 2019).

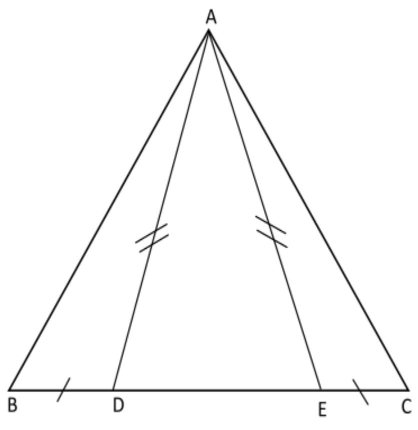
6.2.3 Performance and difficulties in congruency

Learners were expected to identify a pair of congruent shapes and give a reason why those shapes are congruent when answering the GAT questions related to congruency. Also, learners were expected to prove that two pairs of 2D shapes were congruent with each other. The performance of learners in this question indicated a poor understanding of congruency. The p-value is 0.000, which was less than 0.05 at 95% confidence level. This finding suggests that grade 10 learners in this study perform poorly as compared to the departmental pass mark of 30%. The result indicates poor performance due to the p-value less than 0.05. The poor performance in congruency was caused by some mistakes learners displayed when answering questions related to congruency. The mean score and the standard deviation in congruency are *M = 24.13* and *SD = 12.07* respectively. Some of the mistakes include the learner inability to use the conditions, like the one provided by L6 (see section 5.9.1.2). L6 reported that the two angles and one corresponding side of these triangles are the same. *Learners responded that the two triangles are not congruent, but they are similar*. The quantitative results indicated a lack of understanding of congruency to prove that a pair of triangles were congruent to each other, as also found in another study (Casanova et al., 2021). Learners are supposed to know when two triangles are congruent because their corresponding sides are equal, meaning that the two triangles are the same shape and sides. However, what the learners did contradicts what was expected of them. Most learners provided responses that are irrelevant to the concept of congruency. Similar results were found in the interview, where most learners stated that they did not understand the conditions of congruency. Some learners could not see that the two triangles were congruent. Most learners were unable to see three sides of one triangle equal to the corresponding sides of the other triangles to conclude that the triangles were congruent. These findings are supported by Casanova et al., (2021) that learners cannot apply the conditions of congruency to solve parallelogram problems. Learners should be able to identify the shapes in the diagram before they can relate what they see to the properties of the plane figure, which then allows them to demonstrate proof. For example, learners were given an isosceles triangle but could not identify the shape due to a lack of understanding of the properties of shapes. Learners should have seen that two adjacent sides are equal, and the base angles are also equal. Learners must apply these properties to congruency to prove that these two triangles are congruent to each other. The result of this study indicated that learners lack an understanding of the conditions of congruency. The findings of this study agreed with those of Casanova et al. (2021) and Sadiki (2016), who found that the poor performance of learners in congruency was due to a lack of understanding of congruent conditions.

Question 2.2.1 tested learners' ability to identify three triangles given that two are congruent triangles, and at Van Hiele level 1, 82.5% of learners correctly answered 2.2.1 questions. However, if they respond correctly, they still have the challenge of correctly naming the congruent triangle according to their corresponding angles. Ndlovu &Mji (2012) also found the same result. They got the answer correct due to the fact they chose the right figure, but considering the labelling of the figure, most of the participants seemed to be incorrect. Learners in grade 10 are supposed to be able to name congruent triangles according to the corresponding angles.

Question 2.3.1 assessed learner understanding of the "common side." When two triangles are congruent, they may share the same line, as shown in Fig. 18. Learners were given that line BD equals line EC and that line AD equals line AE. Learners see that BE consists of both lines BD and DE, while CD consists of lines DE and EC. Therefore, DE is common to both lines BE and CD. Instead of learners responding that DE is common to both lines, most learners gave incorrect answers (48%) and blank responses (52%). They cannot identify common lines shared by two congruent angles. Similarly, in Questions 2.3.3 and 2.4.3, learners were given two triangles that had the same vertices and sides, with two sides being equal. Learners should use the conditions of congruency to prove that the corresponding three sides of a triangle in one triangle are congruent to the other triangle that shares the same line, which is Side, Side, Side (SSS). Also, learners are supposed to use the common vertex with the two sides of one triangle to prove congruency to another triangle. Most learners (42.5%) responded incorrectly, and 51.2 percent of their responses were blank.

Figure 18: triangles with common sides



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This finding confirms that most learners perform poorly because they do not understand the condition of congruency which made them to provide incorrect and blank responses. These has been shown by the learners who struggled to prove that the two triangles are congruent when sharing common vertex. Most of the learners responses were responding to the question which appeared that they were just guessing (Casanova et al., 2021; Dhlamini et al., 2019). According to the DBE (2011), learners in grade 10 should have mastered the conditions of congruency in grade 9 to be prepared for the proof of a parallelogram in grade 10. The results show that learners did not master this condition of congruency well in previous grades. These findings show that the challenges faced by learners could be as result of lack of prior knowledge. Dhlamini et al., (2019) suggest that teachers must be trained to address the geometric thinking of learners because a lack of prior knowledge results in learners making irrelevant and blank responses.

6.2.4 Performance and difficulties in parallelograms

These questions tested the learner understanding of the properties of the parallelogram to prove that the two opposite sides and angles of the parallelogram are equal. The performance of learners in this question was poor. The p-value is 0.000, which is less than 0.05 at 95% confidence. However, most participants in this study seemed to have difficulties proving that two parallel sides of a parallelogram are equal. Even when they are given two sets of triangles, that are congruent to each other with a common line. Learners write that the two opposite angles are equal, but rather than using the properties of a parallel line and congruency discussed in the previous section to prove that they choose to justify their answers. Learners have difficulties in writing. This finding showed that learners lack the basic knowledge of proof in Euclidean geometry. Learners who do not understand the properties of parallel lines and congruency, cannot prove that the opposite sides of a parallelogram are parallel to each other, which leads to their low performance in these questions. Many learners seem to have difficulties with the proof of a parallelogram because they are unable to demonstrate that the opposite sides and angles of a parallelogram are also equal. Most learners who can demonstrate that the opposite angles of a parallelogram can be calculated do so because they understand applying properties of parallel lines and congruency. Learners must understand different shapes and their properties, as well as learn geometry in a hands-on and exciting manner, to perform well in geometry (Makhubele, 2012).

6.3 QUALITATIVE FINDINGS

In this section, the researcher discusses the findings obtained from the qualitative data obtained through the semi-structured interviews with six learners and the lesson observations. As noted in Chapter 5, the purpose of administering the semi-structured interview was to explore the learning difficulties of learners when solving grade 10 geometry problems. Moreover, the lesson observation helped in understanding how geometry is being taught in grade 10 along with the challenges of learning Euclidean geometry that emerge when learners are in their natural space. This section discusses the findings of the qualitative data obtained from the semi-structured interviews and the lesson observations. The semi-structured interviews were used to explore the difficulties learners faced and their explanations for their low performance, which was identified in the quantitative analysis. Data from the semi-structured interview produced the following themes: learner misconceptions in geometry, and a lack of conceptual understanding. Moreover, the lesson observation was used to get a deep understanding of the process learning in their natural space. The themes that emerge from the qualitative analysis include conception and misconception in Euclidean geometry; lack of conceptual understanding in geometry; learner interactions; attitudes toward Euclidean geometry; teaching approach; and teaching and learning resources.

6.3.1 Learner misconceptions of geometry

The findings of the semi-structured interviews indicated that most learners could identify parallel lines; identify when pairs of objects were congruent to each other and be able to identify parallelograms. However, the participants in the study struggled to reflect on and interpret the diagram, which made it difficult for learners to explain the answers they see when they are probed by the researcher. This means that most learners cannot visualize geometric images mentally. Visualization is more than just identifying shapes and figures; it requires learners to mentally imagine geometry. According to Ngirishi and Bansilal (2019), "visualization" is the ability to interpret and reflect on mental images even when diagrams are not provided. This means that some participants in this study still grapple with how to think deeply and give a proper explanation regarding the shapes that were asked to be compared to see if they were congruent. This result indicated that most learners in grade 10 appeared to be operating at the level 1 of Van Hiele's geometrical thinking. This finding concurs with those Alex & Mammen (2012). Ngirishi and Bansilal (2019) that found most learners in senior high school were not operating at the level proposed by Van Hiele level of geometric thought. Alex and Mammen's (2012) findings indicated that most learners were operating below level 1. Ngirishi and Bansilal (2019), indicated most FET learners are operating between levels 1 and 2. This finding also indicated that some grade 10 learners struggle with Levels 1 and 2. This is a concern because the CAPS document clearly indicated that learners in grade 10 should have learnt parallel lines, and the conditions of congruency, from their earlier grades for them to learn the concept of a parallelogram in grade 10.

However, the findings of this study contradict what learners are expected to know in grade 10. The learners in this study cannot calculate the magnitude of unknown angles using the relationship of alternate and corresponding angles, when two lines are parallel. They cannot prove that the three sides of one triangle are congruent to the three sides of another triangle (SSS). Also, the participants in this study fail to see that when two angles and included side of one triangle are equal to two angles and included angles of another triangle then the two triangles are congruent (ASA) This underlines the challenges faced by these participants and points to the reason why the majority of learners could not answer questions related to the parallelograms: they do not understand those topics taught in the previous grades. Therefore, it is the errors and misconceptions of learners that underpin, and explain, their poor performance in Euclidean geometry.

6.3.2 Lack of conceptual understanding

The findings of the interview indicated that most learners lack conceptual understanding as described by Hiebert and Lefevre (2013). Most of the learners that participated in the interview were asked to justify the answers they provided in their GAT. They found it difficult to explain how they arrived at their answer, which shows a lack of understanding of the geometry concept. Most of the mistakes that learners make show that they are unable to provide an explanation for the answers they have provided. When learners were asked the reason for identifying geometry shapes, they struggled to justify the reason. Some learners were confused when they were asked to explain their answers about the symbol used for congruency of triangles. They cannot understand the differences between the two symbols, congruency, and similarity. Instead of learners writing the congruency symbol '≡ 'they write the similarity symbol '///'. This result indicated that learners are confused with the congruency symbol. According to Luneta (2015), learners must comprehend and be proficient in the geometry because of the importance of geometry in everyday life. Lack of conceptual understanding has a negative impact on the performance of learners. For instance, learners were unable to identify parallel lines when they were asked about the relationship between lines AB and TC. The findings from this study confirmed the findings of Luneta (2015) that learners must comprehend and be proficient in geometry because of its importance in everyday life.

6.3.3 Instructional approach

The result of the lesson observations indicated that the teaching approach used by the teacher seems to be a talk and chalk approach, which is a conventional teaching method (Alex & Mammen, 2018; Chimuka, 2017; Chiphambo & Feza, 2020). In many of the lessons observed, most of the learners were calm and attentive to what the teacher presented to them, while others were busy writing what was on the board. This finding shows that most of the learners were passive during the lesson. Only a few learners raised their hands to ask or answer questions, and the teacher concentrated on those few learners. According to Chimuka (2017), the instructional approach method influences geometry learning. The instructional approach used by the teacher in teaching Euclidean geometry tends not to reach all the learners because the teacher was focusing on a few learners who raised their hands during those lessons. Tswanwani et al., (2014) found that learners with low performance get less attention from their teachers. This approach was teacher-centred, where the teacher talked, and learners did not have the boldness to interact with the teacher. Consequently, learners were passive during teaching and learning, where they had little or nothing to say. They seemed to be confused, which was shown on their faces that they were confused. The findings suggest that the teacher-centred approach used by the teacher in teaching Euclidean geometry might have made it difficult for effective learning to take place. This kind of approach does not support learning geometry but seems to have a negative impact on the performance of learners because they are not actively involved during teaching and learning. The findings in this study supported Adjei (2020), whose study indicated that when teachers use the talk and chalk method, teachers tend to dominate the teaching and learning process, which may have a negative effect on the performance of learners in geometry thinking. However, studies suggested that teachers should use an instructional approach that engages learners with hands-on activities that can promote a deeper understanding of Euclidean geometry to improve the performance of learners (Adjei, 2020; J. K. Alex, 2016; Armah & Kissi, 2019; Chimuka, 2017; Chiphambo & Feza, 2020). Teachers should use a teaching approach in their classroom that meets the needs of diverse learners for learners to have a deeper understanding of Euclidean geometry in grade 10.

6.3.4 Learner-learner interaction

The result of the lesson observation shows that the engagement of learners with other learners was limited, and these were supported in Khalil et al's (2018) study. Learners in this study seemed to have little or no opportunity to interact with other learners because of the seating arrangement in the classroom. During the six classroom sessions, the seeting arrangement was the same, in which each learner mainly sat alone, far apart from each other. Learners work according to the teacher instructions (Khalid & Embong, 2019; Khalil et al., 2018). This made it difficult for learners to interact with each other (Khalid & Embong, 2019). Cooperative learning has been shown to increase learner interaction, mastery of geometry, geometry retention, and learner confidence. (Arslan, 2020; Chan et al., 2021). Teachers should encourage cooperative learning in their classroom to improve the teaching and learning of geometry. The findings of Chan et al. (2021) were that when learners sit in a group, it promotes collaboration between learners.

6.3.5 Teaching and learning resources

The result showed that the teacher seemed not to utilize the teaching resources provided for the teaching of geometry by the Department of Basic Education to enhance the teaching and learning of Euclidean geometry in the province. Instead, the teacher mainly relied on the textbooks when preparing and presenting the lessons. Even though the teacher has access to several textbooks that can be used as references (Rahaju et al., 2019; Rahayu & Jupri, 2021) The teacher only used a textbook throughout the six lessons of observation. Most learners do not have textbooks. Because of the COVID-19 protocol, learners were not allowed to share their textbooks. However, learners were provided with copies of the work that needed to be done. Lack of a textbook may have an impact on the performance of learners, as noted by Zhou, (2019). Teachers need to use different textbooks in the preparation to has a better all-around understanding of the topic. Also, to aid the teaching of their lessons and to help learners who need to seek clarification in geometry class. The classroom was provided with a smartboard, which teachers were supposed to utilize for teaching geometry using the Euclidean geometry resources. This finding indicates that some schools are equipped with smart boards, but some teachers do not know how to use them. This shows that teachers need training on how to use the computer for teaching geometry. Though a smart board was provided, if the teachers do not understand how to use it, learners cannot benefit from these resources. This finding suggests that teaching and learning resources must be utilized competently for learners to have learning that will benefit them.

6.3.6 Learner attitude towards Euclidean geometry

The result of the classroom observation indicated that some learners do not do their homework given to them, as also discussed by Makgakga (2016) and Zhou (2019). Some learners do not bother to do their work because they do not understand. They are frustrated and seem not to enjoy doing mathematics. This finding suggested that learners seemed to have a negative attitude towards their learning of Euclidean geometry in grade 10. Learners do not see Euclidean geometry as something worthwhile that requires persistence, so it becomes difficult for learners to perform well. This result supported Sanyasi's (2015) claim that the attitude of learners toward mathematics influences their performance. which indicated that learners with a positive attitude will enjoy doing geometry, which may have a positive effect on their performance in mathematics.

6.4 CHAPTER SUMMARY

This chapter discussed the results and the analysis of the study that investigated the performance and the learning difficulties that grade 10 learners experienced when solving Euclidean geometry. The quantitative and qualitative data revealed the areas in grade 10 Euclidean geometry in which learners experienced difficulties. Learners still struggle with the properties of parallel lines and condition which learners supposed to have mastered in the previous grades which lead to the poor performance in proofs of parallelograms

CHAPTER SEVEN

SUMMARY, CONCLUSION AND RECOMMENDATIONS

7.1 INTRODUCTION

The previous chapter presented a discussion of the findings of the Geometry Achievement Test (GAT). This was done in relation to the data collected from the semi-structured interview and the lesson observations. Three main themes emanating from the collected data are enumerated here. These include performance and learning difficulties of grade 10 learners in parallel lines, congruency, and parallelograms. This chapter summarizes the major findings of the study and this centres on the themes earlier mentioned here and reported in chapters five and six. The chapter gives a summary of the study, details the major findings of the study, as well as providing a conclusion for each of the themes identified. That is, the performance and learning difficulty of grade 10 learners. The chapter concludes with a recommendations section, limitations of the study and suggestions for future studies.

7.2 SUMMARY OF THE CHAPTERS

This thesis is divided into seven chapters. Chapter 1 introduced the context of the study by identifying the research gaps in the literature as it concerns the performance and learning difficulty of grade 10 learners. This was presented in different sub-sections. The statement of the problem, the motivation of the study, the significance of the study, the assumptions of the study, the aim and objectives of the study, the hypotheses, and the delimitation of the study were outlined.

Chapter 2 detailed the two theoretical frameworks guiding this study. The Van Hiele theory of geometrical thinking and Vygotsky's social constructivism are the theoretical frameworks on which this study is anchored . The Van Hiele theory reinforces geometrical thinking in relation to visualisation, analysis, abstraction, informal deduction, rigour and axiomatics. Vygotsky's social constructivism explains that learners are active participants in every process of their learning, where their involvement is highly crucial and the role of teachers cannot be underestimated in social constructivism.

Chapter 3 presented a literature review of the related literature on the notion of geometry, the performance of learners in mathematics, performance in Euclidean geometry, and mathematics teaching and learning in South Africa.

Chapter 4 discussed the research methods applied in the study. These focused on the sample design, study population, sample size and sampling techniques, data collection instruments, data collection procedures, analysis of the data and ethical considerations.

Chapter 5 presented the results of the analysis of the collected data. The descriptive and inferential statistics were presented and discussed, and the results were presented in tables and figures. Chapter 6 presented the discussion of key findings and Chapter 7 concluded the study and presented the conclusions and recommendations in line with the objectives of the study.

7.3 KEY FINDINGS

The major findings of this study are presented here in relation to the research questions asked in Chapter 1 of this study and it is presented here:

- To what extent does the grade 10 perform when solving Euclidean geometry problems?
- What difficulties do grade 10 learners experience when solving Euclidean geometry problems?
- Why do learners experience difficulties when solving Euclidean geometry problems?
- How do grade 10 learners engage with Euclidean geometry?

7.3.1 Major findings from research question one

The first research question aimed at establishing the difficulties grade 10 learners experience in solving Euclidean geometry problems. The findings of the study showed the low performance of grade 10 learners in the two selected secondary schools in the Tshwane West district in the Gauteng Province were a result of a lack of conceptual understanding that characterizes Euclidean geometry. This is why grade 10 learners have trouble in Euclidean geometry. This was further buttressed in the quantitative findings of the study.

Since 30% is the benchmark for a pass mark in Euclidean geometry stipulated in these schools, less than 40% of the participants in the GAT obtained 30% and above. The findings of the study showed that less than 10% of the learners obtained a score of 38% on the achievement test, which was the highest score obtained. Only 1% of the participants obtained a score of 4%. This translates to a mean score of 13.6 and a standard deviation was 6.27. Because Euclidean geometry requires learners to use their spatial and logical skills in solving mathematical questions, making connections, and comprehending the visual and spatial aspects of parallel lines, congruency and parallelograms have resulted in perceptual difficulties for grade 10 learners.

Since learners have a lack of understanding of the properties of parallel lines, this results in difficulties in calculating the magnitudes of unknown angles using the properties of parallel lines. Explaining this further, the quantitative findings showed that grade 10 learners still have problems solving geometry where the characteristics of corresponding, alternating, and co-interior angles, when parallel lines are cut by a transversal, are required. It is further challenging for grade 10 learners to apply the condition of congruency and do the proof of parallelograms. This finding is also in line with other studies conducted (Ngirishi & Bansilal, 2019; Orevaoghene, 2020; Škrbec & Čadež, 2015) where it was found that learners have difficulties in understanding geometry concepts.

7.3.2 Major findings from research question two

The result of the qualitative aspect of the study shows that learners experience difficulties in understanding grade 10 Euclidean geometry concepts because they lack understanding of the geometry terminology. Learner's lack of prior knowledge and conceptual understanding of geometry is evident when learners provide irrelevant and blank responses. The qualitative result also indicates that the conventional instructional approach of teacher, the learner-learner's lack of interaction, the lack of teaching and learning resources, and the learner attitude are some of the reasons for the difficulties of learners in Euclidean geometry.

Furthermore, the finding suggests that learners without an understanding of the properties of parallel lines and congruency struggle to understand the proof of a parallelogram. Before understanding the concept of a parallelogram in grade 10, learners must first understand the concepts of parallel lines and congruency. Also, learners' scores seem higher in the congruency section than in parallelogram section of the GAT. Hence, this indicates that they did not have a grounded conceptual understanding in previous grades, perhaps only some rote procedures and so could not retain the information. Most of the mistakes learners made show that teachers need to be prepared and address mistakes from the earlier grades so that learners have a clear understanding of the parallel line and its properties, and the condition of congruency, so that when they reach grade 10, they will be able to understand the geometry of the parallelogram. Also, the finding revealed that the teacher's instructional approach, the interaction between learners and other learners, the teaching resources, and the attitude of learners toward learning Euclidean geometry in grade 10 can have a negative effect on the performance of learners. The finding also indicated that teachers must be conversant with the geometric thinking of learners and their progression to help learners move from lower geometry to a higher one. Teachers must engage in in-service training, workshops, and seminars that enhance their content knowledge and pedagogical knowledge to keep abreast of the latest developments in the teaching of Euclidean geometry in grade 10. It is suggested that teachers move away from conventional teaching methods and focus more on the learner understanding.

7.3.3 Major findings from research question three

The researcher observed that the teacher used a traditional instructional approach in teaching geometry. The teaching approach does not encourage learners to be active during the lesson which may have negative impact on the performance of learners in Euclidean geometry. Qualitative results also revealed that there was little or no collaboration between learners during the lesson observed. Learners do not use information technology to teach geometry.

7.4 RECOMMENDATIONS

Based on the outcome of this study giving an insight into the learning difficulties and performance grade 10 learners faced, the following recommendations are made:

- Teachers must ensure that learners understand the procedures for the naming of the angles correctly.

- The understanding of properties of parallel lines, congruency, and proof of parallelograms are essential to the understanding of grade 10 Euclidean geometry.
- Teachers must engage with activities and meet the needs of diverse learners in their classrooms.
- The study suggests that teachers can use digital technologies such as GeoGebra and Sketchpad to teach Euclidean geometry.
- The classroom should be a learning space for learners where they engage with one another to improve their geometric thinking.
- Learner-to- learner interactions and as wells as learners-to- teachers interactions should be encouraged. This may enhance learners learning abilities.

7.5 RESEARCHER VOICE AND THE THEORETICAL FRAMEWORK

This study was underpinned by the Van Hiele model of geometric thinking. The outcomes of this study provide insight into the performance and difficulties of grade 10 Euclidean geometry. According to the Van Hiele levels of geometrical thinking, learners in grade 10 are supposed to be at Level 3. However, the analysis of the quantitative and qualitative data in this study indicated that most learners struggled between levels 1 and 2 of Van Hiele level geometry.

It is clearly indicated that geometry learning is hierarchical; learners cannot move to a higher level of geometry if they do not master and make sense of the level below. The result of this study suggests that learners need to understand and make sense of parallel lines and congruency before learners can make sense of proof of parallelogram. A teacher should be conversant with the geometrical thinking of learners and their progression to help learners move from a lower level to a higher level of geometric thinking.

7.6 LIMITATIONS OF THE STUDY

This study is a mixed-methods research project that focuses on the grade10 learners selected from two underperforming secondary schools in the Tshwane West District. Hence, the findings from this study cannot be generalized.

Another limitation was the study took place during the COVID -19 pandemic. The outcome from this study was influenced by the COVID-19 pandemic during the data collection period, in which researcher's access to the learners and classrooms was limited to six lessons for lesson observations. Each week, the researcher observed two one-hour lessons. A period of three weeks will have been enough for the classroom observation.

7.7 CONCLUSION

This study explored the challenges of the grade 10 learners in Euclidean geometry that result in poor learner performance in the two selected secondary schools in the Tshwane West district in Gauteng province. A sequential explanatory mixed method design was used with a quantitative method to examine the performance of grade 10 learners in Euclidean geometry. Thereafter, the qualitative research method was used to explore learner performance in the learning of geometry in a grade 10 classroom in selected secondary schools. A Geometry Achievement Test (GAT), semi-structured interviews and a classroom observation check list were the instruments used to gather the information that emerged from this study. The analysis of GAT revealed that most learners have difficulties justifying their answers, naming the angles correctly, and understanding the properties of parallel lines, congruency, and proof of parallelograms, which may be due to a lack of conceptual and procedural understanding of Euclidean geometry. The factors that affect learning Euclidean geometry in selected grade 10 classrooms as observed in the classrooms included the teachers' conventional teaching approach, a lack of learner-to learner interaction, an underutilization of teaching and learning resources by the teachers, and the attitude of learners toward the topic of geometry. The outcome of this study offered an understanding of the challenges that grade 10 learners experience when solving Euclidean geometry, especially in the participating schools. The study was conducted in response to the poor performance of learners in the district. This research in this area of study may contribute to the performance in Euclidean geometry, which may improve the performance of learners in mathematics.

For future research Euclidean geometry in grade 10 is crucial to providing a strong foundation as learners go through the FET phase. As learners prepare for the Senior Certificate

Examination, it is essential that mathematics teachers have knowledge of the difficulties of learners so that they can prepare them for the graded task of exiting the FET Phase.

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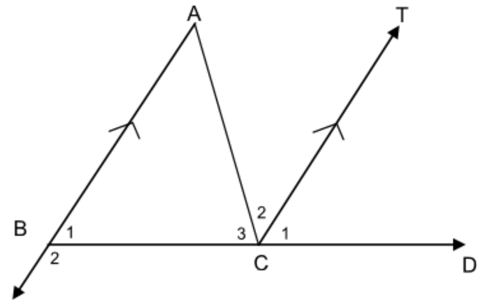
APPENDIX A: GEOMETRY ASSESSMENT TEST

Instructions:
1. Answer all questions.
2. Write Neatly and Legibly
3. Leave open lines between your answers.
4. Draw a line after each question

Question 1: Parallel Lines.

1.

1.1



1.1.1 In the figure above

What is the relationship between AB and TC?

(2)

__________________________ Reason _______________________

1.1.2 If $C_1 = 65^o$ and $C_2 = 43^o$.

(i) Write down the angle that is equal to C_1 and give reason for your answer?

(2)

______________________ Reason ________________________

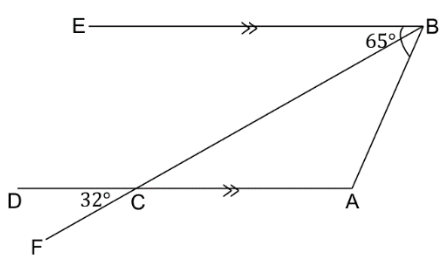
(2)

(ii) Write down the angle that is equal to C_2 and give reason for your answer?

______________________ Reason _________________________

(2)

1.2 In the diagram below $\angle$ ABE = 65° and $\angle$ DCF = 32°



1.2.1 Calculate the size of $\angle$ EBC. Give a reason for your answer. (2)

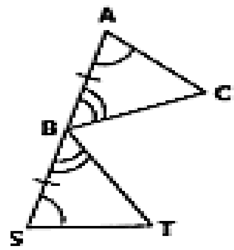
1.2.2 Calculate the size of ∠ CAB. Give a reason for your answer. (2)

Question 2: Congruency

2.

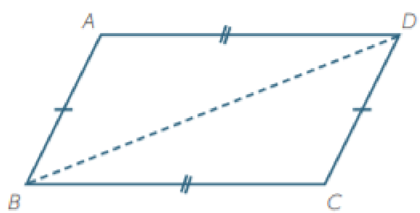
2.1 Determine whether the following figures are congruent or not using the given information. If the figures are congruent, name them in correct order and write down the reason for the congruency.

2.1.1



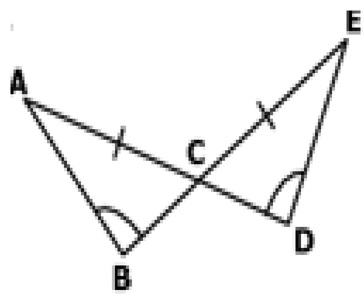
(2)

2.1.2



(2)

2.1.3



(2)

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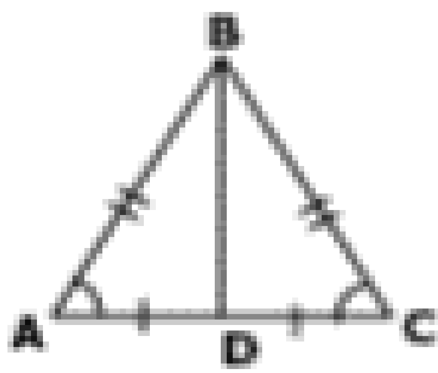
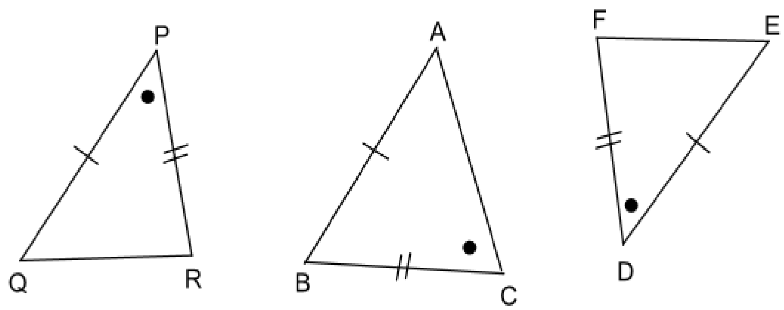
2.1.4



(2)

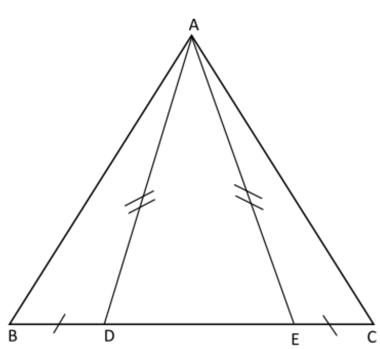
2.2



2.2.1 Which triangle is congruent to ΔPQR? (1)

2.2.2 Explain your answer in 2.2.1 (1)

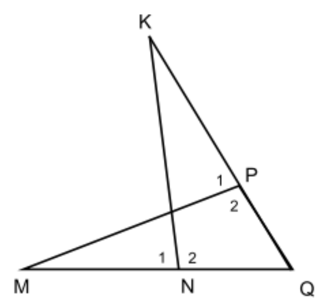
2.3 In ΔABC, D and E are points on BC such that BD = EC and AD = AE.



2.3.1 Why is BE = CD (1)

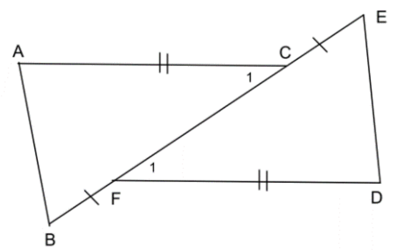
2.3.2 Which triangle is congruent to ΔABE (1)

2.3.3 In the figure below ΔKNQ and ΔMPQ have a common Vertex Q and P is a point on KQ and N is a point on MQ. KQ = MQ and PQ = QN.



Prove with reasons that $\Delta KNQ \equiv \Delta MPQ$ (4)

2.4 In the diagram below AC=DF, AB=DE and BF = CE



2.4.1 Prove that BC =EF (2)

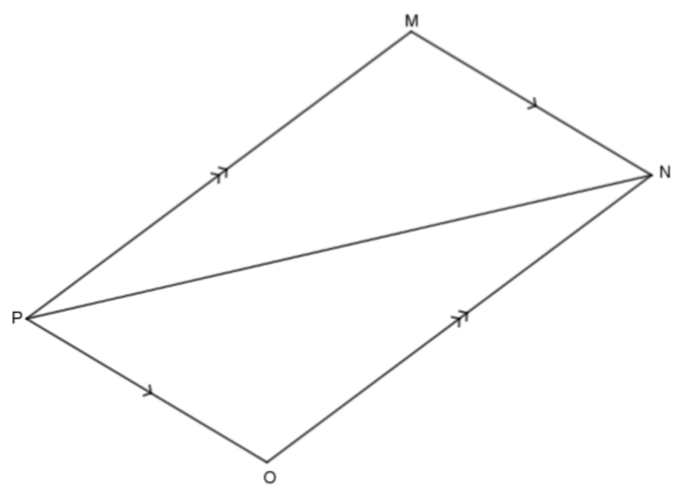
Statement	Reason

2.4.2 Prove that $\Delta ABC \equiv \Delta DEF$ (4)

Statement	Reason

Question 3: Quadrilaterals

3.1 MNOP is a quadrilateral with Diagonal PN, MN∥PO and MP∥NO



3.1.1 The quadrilateral above is

____ (1)

3.1.2 Use congruency to prove that PM = NO.

Hints: ΔPNM and ΔPNO to be congruent (5)

____

____

____

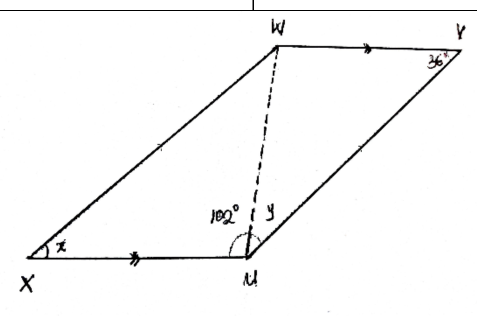
____

____

3.2

3.2.1 Quadrilateral XWVU with sides XW ∥ UV and XU∥WV is given. Also given is $\hat{X} = y$ and $\hat{V} = 36^\circ$; $X\hat{U}W = 102^\circ$ and $W\hat{U}V = x$. Prove that XWVU is a parallelogram by using the congruency of the two triangles GIVE THOSE TRIANGLES. (6)

Statement	Reason



3.2.2 Determine the value of y. (3)

3.2.3 Determine the Value of *x*. (3)

Thank you for your time.

Total: 50marks

APPENDIX B: MEMORANDUM FOR THE GEOMETRY ASSESSMENT TEST

Memorandum

Question 1				
Question	solution		Marks Allocated	Total
1.1	Statement	Reason		
	$\hat{B} = \hat{C} = 65°$ $\hat{C}_2 = \hat{A} = 43°$ $\hat{B}_2 = 180° - 65°$ $= 115°$	Corr. ∠S and AB// TC✓**A** Alt ∠S and AB// TCPA ∠S On a Str. Line or adj. Suppl. ∠SPA	Correct statement: ½ Mark Reason: ½ Mark Correct Statement: ½ Mark Reason: ½ Mark Correct Statement: ½ Mark Reason: ½ Mark	

			(3)
1.2 1.2.1	$\hat{ACB} = \hat{DCF} = 32°$ (Vertically opp ∠'s) $\hat{EBC} = \hat{ACB} = 32°$(Alt ∠'s EB// DA)	Statement and reason: 1 Mark each	(2)
1.2.2	$\hat{CAB} + \hat{ACB} = 180°$(co-int. ∠'s: EB//DA)✓**S/R** $\hat{CAB} = 180° - 65°$**PM** $\hat{CAB} = 115°$**PA** OR $\hat{CAB} + \hat{ACB} + \hat{ABC} = 180°$ [∠'s of a **PS/R**] $\hat{CAB} = 180° - (32° + 33°)$ [$\angle ABC = 65° - 32°$]✓**M** $\hat{CAB} = 180° - 65° = 115°$	Statement and reason: 1 Mark Substitution: 1Mark Answer: 1 Mark	(3)

	$C\hat{A}B = 115°$**PA**		
1.3 1.3.1	In Δ PMN $\hat{Q}_1 = 55^o$ (Corresponding ∠'s ; MN// QR)✓**S/R** $55^o + 2x - 5^o + 40^o$ (3 ∠'s of ΔPQO =180°)*P***R** $5x + 90° = 180°$***P*R** $\therefore \hat{X} = 18°$✓**CA**	1 Mark Statement: 1 Mark Reason: 1 Mark Answer: 1 Mark	(4)
1.3.2	$M\hat{O}N = 2x - 5°$ $= 2(18°) - 5$ ✓ **M**	Substitution/Method: 1 Mark	

	$= 31°P$ **CA**		Statement and Reason: 1Mark Answer: 1Mark	(3)
2.1	ΔABC ≡ ΔSBT, by ASA✓**A** ΔABD ≡ ΔCDB, by SSS✓**A** ΔABC ≡ ΔEDC, by AAS✓**A** ΔABD ≡ ΔCBD, by ASA✓**A**		Statement and Reason: 1 Mark each	(4)
	Statement	Reason		
2.2 2.2.1	$\Delta DEF \equiv \Delta PQR$✓**A**	S ∠ S✓ **A**	Statement: 1 Mark Reason: 1 Mark	(2)
2.2.2 2.3.3	BD +DE = CE+DE $\Delta ACD\ and\ \Delta ABE$ In $\Delta KNQ\ and\ \Delta MPQ$	Answer✓**A** (S ∠S) ✓**A** Common✓**A**	Answer 1mark Statement and Reason: 1 Mark	(2)

	1. Q̂=Q̂	Given✓**A**	Statement and Reason: 1 Mark.	
	2. NQ = PQ	Given✓**A**	Statement and Reason: 1 Mark.	
	3. KQ = M.	(S ∠S) ✓**A**	Statement and Reason: 1 Mark.	
	4. $\Delta KNQ \equiv \Delta MPQ$		Statement and Reason: 1 Mark.	(4)
2.4.1	BF = CE	Given	Statement and Reason: 1Mark	
	BF +FC = CE+FC $\therefore BC = EF$	Adding FC on both sides	Statement and Reason: 1 Mark	(2)
2.4.2	In Δ ABC and ΔDEF			
	AC = DF	Given✓**S/R**	Statement and Reason: 1 Mark.	
	AB= DE	Given✓**S/R**	Statement and Reason: 1 Mark.	
	BC = EF	Proved✓**S/R**	Statement and Reason: 1 Mark.	

	In Δ ABC = ΔDEF	SSS✓**S/R**	Statement and Reason: 1 Mark.	(4)
2..1	In Δ PMN and ΔNOP 1. $M\hat{P}N = P\hat{N}O$ [alternate ∠'s MP//NO] ✓ **S/R** 2. $M\hat{N}P = \angle N\hat{P}O$ [alternate ∠'s MN//PO] ✓**S/R** 3. PN= PN [common] ✓**S/R** 4. $\Delta PMN \equiv \Delta NOP[\ll S]$✓**S/R** 5. $PM = NO$✓**S/R**		Statement and Reason: 1 Mark. Statement and Reason: 1 Mark. Statement and Reason: 1 Mark. Statement and Reason: 1 Mark. Statement and Reason: 1 Mark	(5)

<table>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>3.2.1</td><td><table>
<tr><td>Statement</td><td>Reason</td></tr>
<tr><td>$X\hat{U}V = U\widehat{W}V$</td><td>Alt $\angle s; XU \parallel WV$)✓S/R</td></tr>
<tr><td>$V\hat{U}W = X\hat{W}V$</td><td>Alt $\angle s; XW \parallel UV$)✓S/R</td></tr>
</table></td><td>Statement and Reason: 1 Mark.

Statement and Reason: 1 Mark.

Statement and Reason: 1 Mark.</td><td></td></tr>
</table>

	In ΔXWU and ΔVUW side WU=WU	Common side)✓**S/R**	Statement and Reason: 1 Mark.	
	$\therefore \Delta XWU \equiv \Delta VUW$	Congruent (AAS)✓**S/R**	Statement and Reason: 1 Mark.	
	XW = UV and XU=WV	Congruent triangle (AAS)✓**S/R**	Statement and Reason: 1 Mark.	
	X̂=V̂	Congruent Triangle (AAS) ✓**S/R**		
	$\therefore XWVU\ is\ a\ parallelogram$	The opposite sides of Quad are =**)** ✓**S/R**	Statement and Reason: 1 Mark.	(7)
3.2.2 3.2.3	XWVU is a parallelogram, $\therefore \hat{X} = \hat{V}$ [opp. $\angle s$of a parallelogram are =] ✓**S/R** Ŵ = Û [opp. $\angle s\ of\ a\ Parallelogram\ are$] ✓**S/R**		Statement and Reason: 1 Mark. Statement and Reason: 1 Mark. Statement and Reason: 1 Mark.	(3)

	$\therefore y = 36°$✓**S/R** $\hat{X} + X\widehat{W}U = 180°$ [$\angle'$ s in $a\ \Delta = 180°$] ✓**S/R** $\hat{X}=\hat{V}= 36°$ ✓ $X\hat{U}W = x$ X = 180° -36° - 102° = 42°✓**S/R** OR $\therefore \hat{X} + \widehat{W} + \hat{V} + \hat{U} = 360$ [$Sum\ of\ \angle s\ in\ Quad$] ✓**S/R** $36°+36°+ \hat{U} +\widehat{W} =360°$	Statement and Reason: 1 Mark. Statement and Reason: 1 Mark. Statement and Reason: 1 Mark. Statement and Reason: 1 Mark. Statement and Reason: 1 Mark.	(3)

	$\hat{U} = \widehat{W}$ $72° + 2\hat{U} = 360°$ ✓**CA** U=298°/2 X =149°-102° = 42°✓**CA**	Statement and Reason: 1 Mark. Answer :1Mark Answer:1 Mark **Total Marks**	(3) 55

APPENDIX C: SAMPLE OF LEARNER'S WORK

Code No: SB026

03/50

Instructions: 1. Answer all questions.

2. Write Neatly and Legibly

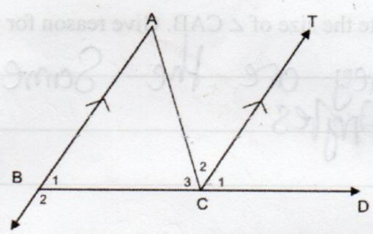
3. Leave open lines between your answers.

4. Draw a line after each question

Question 1: Parallel Lines.

1.

1.1



1.1.1 In the figure above

What is the relationship between AB and TC? (2)

They are parraller lines Reason because They Joined each other

1.1.2 If $C_1 = 65^o$ and $C_2 = 43^o$.

(i) Write down the angle that is equal to C_1 and give reason for your answer? (2)

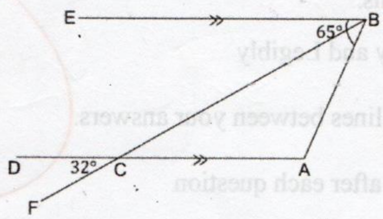
43° Reason they are Close to each other

(ii) Write down the angle that is equal to C_2 and give reason for your answer?

C1 Reason they are parraller (2)

3

1.2 In the diagram below ∠ ABE = 65° and ∠ DCF = 32°



1.2.1 Calculate the size of ∠ EBC. Give reason for your answer. (2)

180° because the Angle are not there Same as parraller lines

1.2.2 Calculate the size of ∠ CAB. Give reason for your answer. (2)

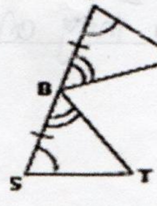
CA = 30° They are the Same Corresponding
AB = 30° Angles

Question 2: Congruency

2.

2.1 Determine whether the following figures are congruent or not using the given information. If the figures are congruent, name them in correct order and write down the reason for the congruency

2.1.1

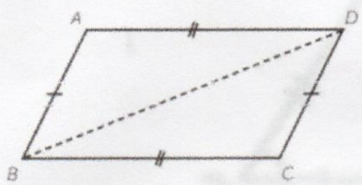


(2)

4

ST = they are Side Side Side
AC = They are S S S
Ab = SSS

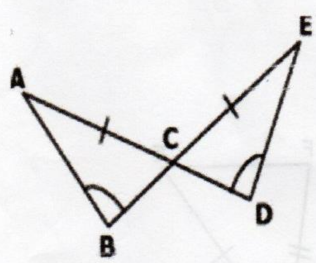
2.1.2



(2)

Abd = are equal to Dcb
So this is Squire rectangle

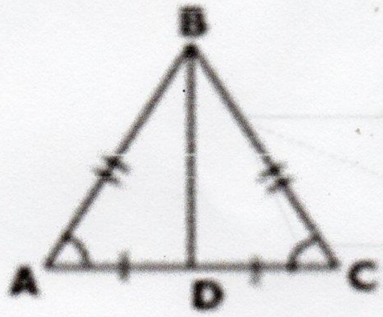
2.1.3



(2)

They are Congruencay Which Means
When you turn it Around They will
Still be the ~~DA~~ Same or Joined eizy
way

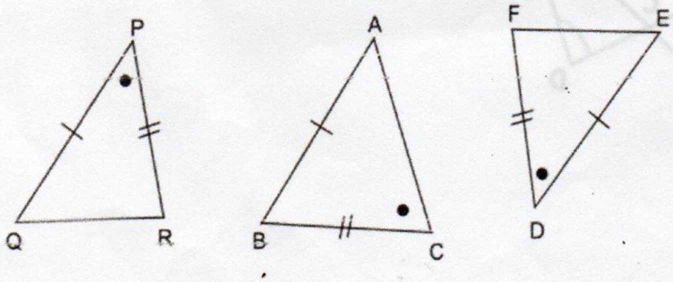
2.1.4



(2)

A B D are equal
B D C egaal as Abd

2.2



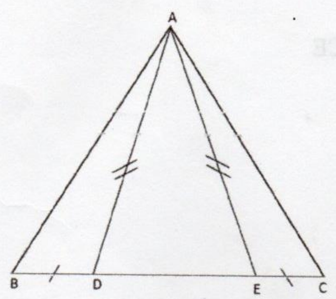
2.2.1 Which triangle is congruent to ΔPQR? (1)

feD 1

2.2.2 Explain your answer in 2.2.1 (1)

because they can rotate and give us Same Answer.

2.3 In ΔABC, D and E are points on BC such that BD = EC and AD = AE.



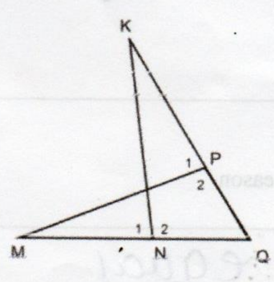
2.3.1 Why is BE = CD (1)

they are equal

2.3.2 Which triangle is congruent to ΔABE (1)

ADe 1

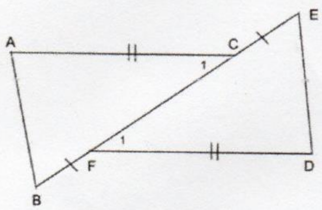
2.3.3 In the figure below ΔKNQ and ΔMPQ have a common Vertex Q and P is a point on KQ and N is a point on MQ. KQ = MQ and PQ = QN.



Prove with reasons that $\Delta KNQ \equiv \Delta MPQ$ (4)

Knq = They are not equal to Mpq
Mpq = is equal or bigger than Knq

2.4 In the diagram below AC=DF, AB=DE and BF = CE



2.4.1 Prove that BC =EF (2)

Statement	Reason
bC	equal
ef	equal
AC =DF, Ab=DE and bF=CE	They are All equal

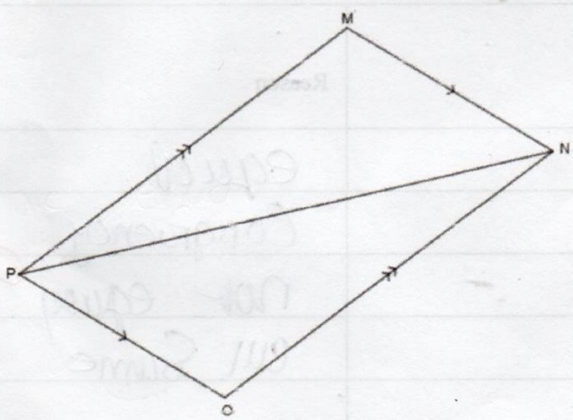
2.4.2 Prove that $\Delta ABC \equiv \Delta DEF$ (4)

Statement	Reason
Abc	equal

Def	equal
, f C	not equal

Question 3: Quadrilaterals

3.1 MNOP is a quadrilateral with Diagonal PN, MN||PO and MP||NO

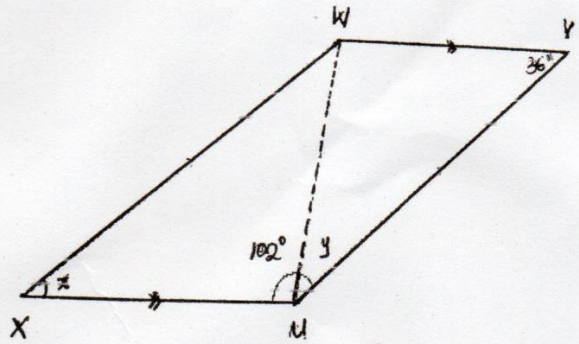


quadrilatoral Square

3.1.1 The quadrilateral above is quadrilateral, rectangle, Square (1)

3.1.2 Use congruency to prove that PM =NO.

Hints : ΔPNM and ΔPNO to be congruent. (5)



3.2.2 Determine the value of y. (3)

3.2.3 Determine the Value of x. (3)

Thank you for your time

Total: 50marks

APPENDIX D: SEMI-STRUCTURED INTERVIEW

1. **Researcher:** How do you understand parallel lines?

Learner:____

2. **Researcher:** How do you arrive at your answer in 1.1?

Learner:____

3. **Researcher:** How do you arrive at your answer in 1.2?

Learner:____

4. **Researcher:** How do you arrive at your answer in 1.3?

Learner____

5. **Researcher:** What do you understand by congruent triangles?

Learner____

6. **Researcher:** How do you know that two triangles are congruent?

Learner:____

____

____

7. **Researcher:** What do you understand by parallelogram?

Learner:____

____

____

8. **Researcher:** I can see that you did not answer question 3.1. Why don't you answer it?

Learner:____

____

____

9. **Researcher:** Why don't you answer question 3.2?

Learner:____

____

____

APPENDIX E: OBSERVATION SCHEDULE

DEMOGRAPHIC DETAILS

1. Name of School: ____

2. Physical Address of School: ______________________________________

____

3. Postal Address of School: ______________________________________

____

4. Tel: ______________________ Fax: ______________________

5. Name of Principal: __________________________

Male		Female	

6. Name of Teacher: __________________________

Male		Female	

7. Grade Observed: __________ 8. Number of Learners: ________

OBSERVING CLASSROOM PRACTICE

1. How does teaching and learning of Mathematics occur? (Please list e.g., *whole class*)

   (i) ____________________ (ii) ____________________

   (ii) ____________________ (iv) ____________________

2. How is the classroom arranged? (*Furniture*)

   ____

3. What methodology / approach is being used? ____

   ____

4. Which resources are used?

   ____

5. How does the teacher deal with correct or incorrect responses?

   ____

   ____

OBSERVING TEACHING AND LEARNING IN THE CLASSROOM

The PEER system underlies the lessons in a classroom situation. It might not be possible to incorporate all of them in a particular lesson, but each lesson will contain some aspects of this system. Please tick (✓) your rating.

A	**PRODUCTIVE SKILLS**	Excellent	Good	Average	Needs	Not
1.	Learners are able to do reading on the concept being taught.					
2.	Learners write notes on the concept taught.					
3.	Learners are able to solve problems given as exercises.					
4.	Learners are able to relate and apply the concept in real life problems.					
5.	Learners are able to use their knowledge of and experience in the concept in formulating their own responses.					
6.	Learners are able to accomplish work given on the concept independently					
7.	Learners are able to define and describe learned terms encountered when dealing with the concept.					
8.	Learners are able to follow the steps in solving exercises based on the concept.					
9.	Learners competently use technology (calculators) in areas where it is required in the concept.					
10.	Learners are able to deal with problems in real and abstract context using the concept.					

11.	Learners’ ways of making decisions in problem solving is enhanced.					
B	**EVOCATIVE SKILLS**	Excellent	Good	Average	Needs More Attention	Not Applicable to the
1.	Learners ask questions for clarification.					
2.	Learners ask questions to consolidate their understanding of the concept					
3.	Learners are puzzled by certain areas of the concept and hence very inquisitive.					
4.	Learners are able to interpret new information on the concept.					
5.	Learners ask critical questions to ensure that methods used are appropriate.					
6.	Learners use their referencing skills to acquire better understanding of the concept.					
C	**EVALUATIVE SKILLS**					
1.	Learners are able to do self-assessment tasks in the concept learned.					
2.	Learners are capable of evaluating their own work on the concept.					
3.	Learners are able to evaluate procedures followed in problem solving in the concept.					

4.	Learners are able to identify errors committed when dealing with the concept.					
5.	Learners are able to discuss pros and cons in using specific methods to solve problems.					
6.	Learners are able to identify incorrect ways of solving problems.					
7.	Learners have alternative ways to solve problems based on the concept.					
D	**REFLECTIVE SKILLS**	Excellent	Good	Average	Needs More Attention	Not Applicable to the
1.	Learners are constantly engulfed in the world of "exploration in errors."					
2.	Learners reflect on errors committed in solving problems and work towards eliminating those errors.					
3.	Learners are able to respond to questions testing their comprehension of the learned concept.					
4.	Learners are able to select and use appropriate methods in solving problems.					
5.	Learners are able of hypothesizing in problem solving.					
6.	Learners can reflect on the decision they made in solving a particular problem.					

EXAMPLES OF ERRORS CORRECTED

Please provide examples of errors corrected when dealing in the topic being evaluated.

APPENDIX F: GDE RESEARCH APPROVAL LETTER

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	05 November 2020
Validity of Research Approval:	08 February 2021– 30 September 2021 2019/355A
Name of Researcher:	Olabode AA
Address of Researcher:	154 Justice Mahomed Street
	Sunny Side
	Pretoria
Telephone Number:	084 975 8280
Email address:	54020883@mylife.unisa.ac.za
Research Topic:	Learning difficulties of Grade 10 Euclidean Geometry in Tshwane West District, Gauteng Province.
Type of qualification	Masters
Number and type of schools:	2 Secondary Schools
District/s/HO	Tshwane West

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

09/11/2020.

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

1. *Letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

APPENDIX G: ETHICS APPROVAL FORM

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2020/06/10

Ref: **2020/06/10/54020883/21/AM**
Name: Mrs AA Olabode
Student No.: 54020883

Dear Mrs AA Olabode

Decision: Ethics Approval from 2020/06/10 to 2023/06/10

Researcher(s): Name: Mrs AA Olabode
E-mail address: 54020883@mylife.unisa.ac.za
Telephone: +27849758280

Supervisor(s): Name: Dr T Makgakga
E-mail address: makgasw@unisa.ac.za
Telephone: +27123376004

Title of research:

Performance and learning difficulties of Grade 10 in solving Euclidean geometry in Tshwane West District, Gauteng Province.

Qualification: MEd Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2020/06/10 to 2023/06/10.

*The **medium risk** application was reviewed by the Ethics Review Committee on 2020/06/10 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.
2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

APPENDIX H: LETTER TO SCHOOL PRINCIPAL

Mrs Adedayo Olabode
803 Dalbergia,
154 Justice Mahomed Street
Sunnyside,
Pretoria.
0002.

Cell: 0849758280. Work: 0124401228 Email address: 54020883@mylife.unisa.ac.za
Or dayowaleola@gmail.com

Dear Principal,

Re: Permission to do research in your school

My name is **Adedayo Olabode**. I am currently doing a master's degree in mathematics education under the supervision of **Dr Tšhegofatšo Makgakga**. The purpose of my study is to explore the learning difficulties of grade 10 learners in solving Euclidean Geometry. As part of the research, I need to collect data from schools. I therefore ask for your permission to allow me to use your school as a site for this research to be carried out, and permission to work with grade 10 mathematics teachers and learners.

The collection of data will involve the administration of a diagnostics test to grade 10 learners, interviews with these learners and teachers, and observing both teachers and learners during instruction. The results from this study will inform both policy and practice. Classroom observations will be conducted during school time. However, Diagnostics test, interviews with teachers and learners will only be conducted after contact time, that is, between 14H00 and 15H00. You will also be provided with the transcript of these interviews. The names of the school, learners and teachers will not be exposed; the school and participants will be referred to by a pseudonym.

After reading this letter you have a right to agree or not to agree. The participation of your school in this project is voluntarily and should you wish to withdraw at any stage of the research you are free to do so. Should you wish to get more information, my telephone number is: 0849758280.

Hoping to hear from you soon.

APPENDIX I: LETTER TO LEARNERS

Mrs Adedayo Olabode

803 Dalbergia,

154 Justice Mahomed Street

Sunnyside,

Pretoria.

0002.

Cell: 0849758280. Work: 0124401228 Email address: 54020883@mylife.unisa.ac.za

Or dayowaleola@gmail.com

Dear ……………...

Re: Request for your participation in research

My name is **Adedayo Olabode**. I am currently doing a master's degree in mathematics education under the supervision of **Dr Tšhegofatšo Makgakga**. The aim of my study is to explore the learning difficulties of grade 10 learners in solving Euclidean Geometry. I plan to work with grade 10 teachers and learners. I therefore ask for your permission to participate in this research. In one of the lessons, learners in your classroom will observe to understand the difficulties of grade 10 when learning Euclidean Geometry.

After reading this letter you have a right to agree or not to agree. Your participation in this project is voluntarily and should you wish to withdraw at any stage of the research you are free to do so. Should you wish to get more information, my telephone number is: 0849758280.

Hoping to hear from you soon.

Regards,

Adedayo Olabode

APPENDIX J: LETTER OF PERMISSION FROM THE PARENTS

Mrs Adedayo Olabode
803 Dalbergia,
154 Justice Mahomed Street
Sunnyside,
Pretoria.
0002.
Cell: 0849758280. Work: 0124401228 Email address: 54020883@mylife.unisa.ac.za
Or dayowaleola@gmail.com

Dear parent,

Re: Request for your child to participate in research

My name is **Adedayo Olabode**. I am currently doing a master's degree in mathematics education under the supervision of **Dr Tšhegofatšo Makgakga**. The aim of my study is to investigate the causes of poor performance in mathematics. If you allow your child to participate in this research, he/ she will participate in a series of activities related to this research. I will be administering a diagnostic test and conducting interviews and observing your child to identify areas in which learners may be having difficulties. Results from these tests will simply be used to identify areas learners may be having difficulties with and will not count against your child's grade.

The benefits of this research study consist of identifying areas in which teachers and learners need to focus to improve their Euclidean Geometry skill and prepare learners better for their Senior Certificate Examination. Participation is completely voluntary. Your child's name and program results will not be released without your permission. I am only interested in seeing how to provide your child with the best education.

If you have any questions, please feel free to contact me. My telephone number is: 0849758280.

Hoping to hear from you soon.

Mrs. Olabode Adedayo.

APPENDIX L: CONSENT FORM FOR THE PARENTS

---- Please sign and return the bottom portion of this consent form as soon as you have read the letter above.

I, the parent/ legal guardian of ...., acknowledge that the researcher has explained to me the need for this research, explained what is involved and offered to answer any questions. I freely and voluntarily consent to my child's participation in this research. I understand all information gathered during the research will be completely confidential.

Name of learner: ....

Signature of parent/ legal guardian: ....

Date: ....

APPENDIX M: CONSENT FORMS FOR THE PRINCIPAL AND PARTICPATING TEACHERS

Consent forms: To the principal and to all the participating teachers

I …………………………………………… (please print your name in full) the principal/ a grade 10 mathematics teacher agree to be a participant in the research conducted by Adedayo Olabode in which he will be investigating the learning difficulties of grade 10 in Euclidean Geometry.

- I give consent to the following: My school participates in the research.
  Yes □ or No □ (use a cross to indicate your selection)

- To give lessons in my class(es) for context-based problem-solving activities.
  Yes □ or No □ (use a cross to indicate your selection)

- To administer an achievement test in my class(es).
  Yes □ or No □ (use a cross to indicate your selection)

- To be interviewed.
  es □ or No □ (use a cross to indicate your selection)

To be observed during lessons.

Yes □ or No □ (use a cross to indicate your selection)

Signed: …………………………………

Date: …………………………………

APPENDIX N: SPSS RESULT FOR THE GEOMETRY ACHIEVEMENT TEST

```
DESCRIPTIVES VARIABLES=OVERP Q1 Q2 Q3
  /STATISTICS=MEAN STDDEV RANGE MIN MAX SKEWNESS.
```

Descriptives

Descriptive Statistics

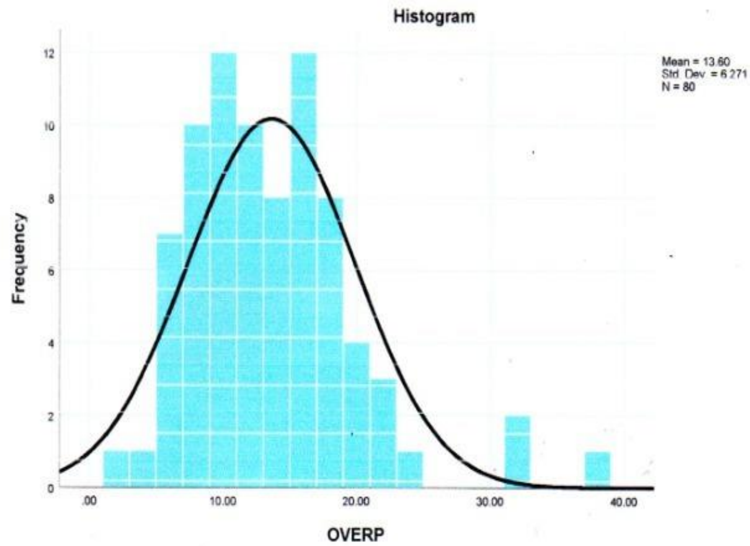
	N	Range	Minimum	Maximum	Mean	Std. Deviation	Skewness
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic
OVERP	80	36.00	2.00	38.00	13.6000	6.27149	1.258
Q1	80	70.00	.00	70.00	22.5412	15.67906	.710
Q2	80	54.17	.00	54.17	24.1292	12.07375	.268
Q3	80	66.67	.00	66.67	10.8753	15.93989	1.068
Valid N (listwise)	80						

Descriptive Statistics

	Skewness
	Std. Error
OVERP	.269
Q1	.269
Q2	.269
Q3	.269
Valid N (listwise)	

```
FREQUENCIES VARIABLES=OVERP
  /STATISTICS=STDDEV MEAN MEDIAN MODE
  /HISTOGRAM NORMAL
  /ORDER=ANALYSIS.
```

Frequencies




```
DATASET ACTIVATE DataSet5.
FREQUENCIES VARIABLES=Q1.1.1 Q1.1.2 Q1.1.3 Q1.2.1 Q1.2.2 Q2.1.1 Q2.1.2 Q2.1.4
Q3.2.1 Q3.2.3 Q3.2.2
    Q3.1.2 Q3.1.1 Q2.4.2 Q2.4.1 Q2.3.3 Q2.3.2 Q2.3.1 Q2.2.2 Q2.2.1 Q2.1.3
  /ORDER=ANALYSIS.
```

Frequencies

```
[DataSet5] C:\Users\ADEDAYO OLABODE\Documents\SBL&SCLPERFORMANCE19122021.sav
```

Statistics

		Q1.1.1	Q1.1.2	Q1.1.3	Q1.2.1	Q1.2.2	Q2.1.1	Q2.1.2
N	Valid	80	80	80	80	80	80	80
	Missing	0	0	0	0	0	0	0

Statistics

		Q2.1.4	Q3.2.1	Q3.2.3	Q3.2.2	Q3.1.2	Q3.1.1	Q2.4.2
N	Valid	80	80	80	80	80	80	80
	Missing	0	0	0	0	0	0	0

Statistics

		Q2.4.1	Q2.3.3	Q2.3.2	Q2.3.1	Q2.2.2	Q2.2.1	Q2.1.3
N	Valid	80	80	80	80	80	80	80
	Missing	0	0	0	0	0	0	0

Frequency Table

Q1.1.1

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	14	17.5	17.5	17.5
	InR	53	66.3	66.3	83.8
	IR	12	15.0	15.0	98.8
	BR	1	1.3	1.3	100.0
	Total	80	100.0	100.0	

Q1.1.2

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	8	10.0	10.0	10.0
	InR	41	51.2	51.2	61.3
	IR	29	36.3	36.3	97.5
	BR	2	2.5	2.5	100.0
	Total	80	100.0	100.0	

Q1.1.3

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	7	8.8	8.8	8.8
	InR	23	28.7	28.7	37.5
	IR	46	57.5	57.5	95.0
	BR	4	5.0	5.0	100.0
	Total	80	100.0	100.0	

Q1.2.1

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	2	2.5	2.5	2.5
	InR	5	6.3	6.3	8.8
	IR	63	78.8	78.8	87.5
	BR	10	12.5	12.5	100.0
	Total	80	100.0	100.0	

Q1.2.2

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	6	7.5	7.5	7.5
	InR	5	6.3	6.3	13.8
	IR	55	68.8	68.8	82.5
	BR	14	17.5	17.5	100.0
	Total	80	100.0	100.0	

Q2.1.1

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	11	13.8	13.8	13.8
	InR	32	40.0	40.0	53.8
	IR	24	30.0	30.0	83.8
	BR	13	16.3	16.3	100.0
	Total	80	100.0	100.0	

Q2.1.2

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	12	15.0	15.0	15.0
	InR	33	41.3	41.3	56.3
	IR	20	25.0	25.0	81.3
	BR	15	18.8	18.8	100.0
	Total	80	100.0	100.0	

Q2.3.1

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	2	2.5	2.5	2.5
	InR	2	2.5	2.5	5.0
	IR	66	82.5	82.5	87.5
	BR	10	12.5	12.5	100.0
	Total	80	100.0	100.0	

Q2.3.2

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	54	67.5	67.5	67.5
	IR	18	22.5	22.5	90.0
	BR	8	10.0	10.0	100.0
	Total	80	100.0	100.0	

Q2.3.3

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	2	2.5	2.5	2.5
	InR	7	8.8	8.8	11.3
	IR	34	42.5	42.5	53.8
	BR	37	46.3	46.3	100.0
	Total	80	100.0	100.0	

Q2.4.1

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	2	2.5	2.5	2.5
	InR	1	1.3	1.3	3.8
	IR	40	50.0	50.0	53.8
	BR	37	46.3	46.3	100.0
	Total	80	100.0	100.0	

Q2.4.2

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	1	1.3	1.3	1.3
	InR	4	5.0	5.0	6.3
	IR	34	42.5	42.5	48.8
	BR	41	51.2	51.2	100.0
	Total	80	100.0	100.0	

Q3.1.1

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	29	36.3	36.3	36.3
	IR	39	48.8	48.8	85.0
	BR	12	15.0	15.0	100.0
	Total	80	100.0	100.0	

Q3.1.2

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	2	2.5	2.5	2.5
	InR	6	7.5	7.5	10.0
	IR	34	42.5	42.5	52.5
	BR	38	47.5	47.5	100.0
	Total	80	100.0	100.0	

Q3.2.1

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	2	2.5	2.5	2.5
	InR	1	1.3	1.3	3.8
	IR	38	47.5	47.5	51.2
	BR	39	48.8	48.8	100.0
	Total	80	100.0	100.0	

Q3.2.2

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	4	5.0	5.0	5.0
	InR	1	1.3	1.3	6.3
	IR	38	47.5	47.5	53.8
	BR	37	46.3	46.3	100.0
	Total	80	100.0	100.0	

Q3.2.3

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	CR	5	6.3	6.3	6.3
	InR	6	7.5	7.5	13.8
	IR	27	33.8	33.8	47.5
	BR	42	52.5	52.5	100.0
	Total	80	100.0	100.0	

APPENDIX O: TURNITIN DIGITAL RECEIPT

turnitin

Digital Receipt

This receipt acknowledges that Turnitin received your paper. Below you will find the receipt information regarding your submission.

The first page of your submissions is displayed below.

Submission author:	Adedayo Olabode
Assignment title:	Complete dissertation/thesis FINAL
Submission title:	The performance and the learning difficulties of grade10 in ...
File name:	NERS_IN_SOLVING_EUCLIDEAN_GEOMETRY_IN_TSHWANE_WE...
File size:	1.53M
Page count:	152
Word count:	45,235
Character count:	251,939
Submission date:	28-Mar-2023 03:43AM (UTC+0200)
Submission ID:	2048601189

APPENDIX P: TURNITIN REPORT

The performance and the learning difficulties of grade10 in solving Euclidean geometry in Tshwane West

ORIGINALITY REPORT

30%	28%	12%	11%
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Title: THE PERFORMANCE AND LEARNING DIFFICULTIES OF GRADE 10 LEARNERS IN SOLVING EUCLIDEAN GEOMETRY PROBLEMS IN TSHWANE WEST DISTRICT

by

Prof Caroline Long

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MC Long, Professor, Senior Research Associate, University of Johannesburg, 2020-

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