STATISTICAL MODELLING OF PRIVATE EQUITY INVESTMENTS IN SELECTED EMERGING MARKETS IN AFRICA: EVIDENCE FROM GARCH AND VAR MODELS

by

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DECLARATION

“I declare that **“STATISTICAL MODELLING OF PRIVATE EQUITY INVESTMENTS IN SELECTED EMERGING MARKETS IN AFRICA: EVIDENCE FROM GARCH AND VAR MODELS** is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.”

“I further declare that I have not previously submitted this work, or part of it, for examination at UNISA for another qualification or at any other higher education institution.”

I further declare that I submitted the thesis to originality checking software and that it falls within the accepted requirements for originality.

SIGNATURE

(CHRICENCIA MAKANYARA MURAPE)

Date

DEDICATION

I dedicate this thesis to my late parents, Isaac and Otilia Murape; my late grandparents, Keresiya, James, Mary and Peter; and my children Valentine and Tadiswanashe

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ABSTRACT

The study examines statistical properties and volatility dynamics of an emerging investment asset class, Listed Private Equity (LPE) investments, in selected markets in Africa. There has been widespread acknowledgement that traditional valuation techniques have failed to explain cross-sectional stock returns in emerging and developing markets. From the content review put forward in this study, it is evident that country-specific risk is a priced factor in investments in Africa. The study hypothesised the notion and predicated that asset returns as a product of economic rationality is followed by its statistical properties in the return distribution and volatility dynamics. The study used monthly data for LPE's and selected country-specific data to examine the statistical properties, volatility dynamics and their relationship to country risk factors for countries that were found active in this asset class – South Africa, Ghana, Egypt and Botswana for the period 2010-2020. The study utilised the GARCH (1,1), EGARCH, TGARCH, GARCH-In-Mean, FIGARCH, FIEGARCH, DCC MGARCH, and VAR models to fit the data. These findings provide information that is used for portfolio compilation, asset pricing, and a better understanding of the structural dynamics of LPE returns, thereby laying the groundwork for the development of new valuation tools for this asset class.

Three main findings were: firstly, LPEs for countries under study exhibited stylised effects in the form of volatility clustering, asymmetric effects, and leptokurtic distributions, but there is no evidence of leverage effects or structural breaks. The study found evidence that LPE's are positively skewed with excess kurtosis. The returns for South African LPE's are closer to normal distribution, whilst other countries exhibited non-normal distributions, typical of financial data. Egypt displayed traces of volatility associated with jump diffusions and Ghana volatility series indicated that the investment carries a risk premium commensurate with the inherent risk. The data for Botswana was stationary and was found unsuitable for further tests using GARCH modelling. Secondly, there is no evidence of data asymmetries present in the long-term volatility dynamics of the data under study. Despite the diverse economic systems in which these investments operate, their responses to positive or negative shocks are the same. This indicates that private equity investments in these African markets have homogenous dispositions in the long term; therefore, though these investments are defensive assets, geographical selection cannot be used to diversify returns. Thirdly, the study provided evidence that country-specific factors have a weak influence on the LPE's of the data under study, refuting the hypothesis that country risk elements are a priced factor in these markets.

The study makes several contributions: it is the first to examine the statistical properties and volatility dynamics of LPEs in selected African markets. Second, the thesis extends previous findings on statistical modelling by examining an unexplored market in the body of knowledge, the LPE's in African markets, which has different dispositions to traditional asset classes. Lastly, the study adds to the LPE literature by providing valuable insights regarding LPEs in an African context, as an emerging asset class and a financing tool for economic development.

KEYWORDS:

LPE investments, statistical modelling, volatility dynamics, Africa, country-specific factors, GARCH models, VAR models, spillover effects, valuations, emerging markets

ISIFINQO

Ucwaningo luhlole izakhiwo zezibalo kanye nokuguquguquka okuguquguqukayo kwesigaba sokutshalwa kwezimali sempahla esafufusa, okungukuthi, uhlu lokulinganayo kwangasese (ULK) okwaziwa nge-(LPE) ezimakethe ezikhethiwe e-Afrika. Kube nokuvunywa okusabalele ukuthi amasu okulinganisa endabuko ahlulekile ukuchaza imbuyiselo yesitoko ehlukene ezimakethe ezisafufusa nezisathuthuka. Kusukela ekubuyekezweni kokuqukethwe okubekwa phambili kulolu cwaningo, kusobala ukuthi ubungozi obuqondene nezwe elithile buyinto enenani lokutshalwa kwezimali e-Afrika. Ocwaningweni kwacatshangelwa futhi kwabikezelwa ukuthi imbuyiselo yempahla njengemikhiqizo yokuhluzeka kwezomnotho ilandelwa izici zayo zezibalo ekusatshalalisweni kwembuyiselo, kanye nokuguquguquka kokushintshashintsha. Idatha yanyanga zonke yokutshalwa kwezimali kwe-ULK kanye nedatha ekhethiwe eqondene nezwe elithile yasetshenziswa ukuze kuhlolwe izakhiwo zezibalo kanye nokuguquguquka kanye nobudlelwano bazo nezinto eziyingozi zezwe emazweni atholakale esebenza kulesi sigaba sempahla, okungukuthi, iNingizimu Afrika, iGhana, iGibithe kanye neBotswana isikhathi sonyka wezi-2010–2020. Amamodeli e-GARCH (1,1), EGARCH, TGARCH, GARCH-In-Mean, FIGARCH, FIEGARCH, DCC MGARCH kanye ne-VAR asetshenziswe ukuze kulingane idatha. Okutholwe ocwaningweni kuhlinzeka ngolwazi olungasetshenziselwa ukuhlanganiswa kwephothifoliyo kanye nentengo yempahla. Okutholakele kungase futhi kube usizo ekutholeni ukuqonda okungcono kokuguquguquka kwesakhiwo sembuyiselo ye-LPE, ngaleyo ndlela kubekwe isisekelo sokuthuthukiswa kwamathuluzi amasha okulinganisa alesi sigaba sempahla.

Okuthathu okutholakele okuyinhloko kube kanje: Okokuqala, ukutshalwa kwezimali kwe-ULK emazweni angaphansi kocwaningo kubonise imiphumela eyenziwe ngesitayela nangendlela yokuhlanganisa okuguquguqukayo, imiphumela ye-asimetrikhi nokusatshalaliswa kwelipthokhithi, kodwa abukho ubufakazi bemithelela ezuzisayo noma ukuhlukana kwesakhiwo. Lolu cwaningo luveze ubufakazi bokuthi ukutshalwa kwezimali kwe-ULK bekugudluzwe kahle nekhithosisi eyeqile. Imbuyiselo yokutshalwa kwezimali kwe-ULK yaseNingizimu Afrika ibisondele ekusabalaliseni okuvamile kuyilapho amanye amazwe ebonise ukusatshalaliswa okungajwayelekile, okuyisimo sedatha yezezimali. IGibithe ibonise iminonjana yokuntengantenga okuhlobene nokusabalala okweqayo, futhi uchungechunge lwe-Ghana oluguquguqukayo lubonise ukuthi ukutshalwa kwezimali kwe-ULK kunengozi engqala ehambisana nobungozi obukhona. Idatha yaseBotswana ibimile futhi itholwe ingakulungele ukuhlolwa okwengeziwe kusetshenziswa imodeli ye-GARCH. Okwesibili, abukho ubufakazi be-asimetrikhi yedatha kukuntengantenga -ashukumisayo kwesikhathi eside yedatha engaphansi kocwaningo. Ngaphandle kwezinhlelo zezomnotho ezehlukene lapho lezi zimali zitshalwa khona, izimpendulo zazo ekushaqisweni okuhle noma okungekuhle zitholwe zifana. Lokhu kukhombisa ukuthi ukutshalwa kwezimali kwamasheya azimele kulezi zimakethe zase-Afrika kunesimo esifanayo esikhathini eside. Ngakho-ke, nakuba lokhu kutshalwa

kwezimali kuyimpahla evikelayo, ukukhetha kwendawo akukwazi ukusetshenziselwa ukuhlukanisa imbuyiselo. Okwesithathu, ucwaningo lunikeze ubufakazi bokuthi izici eziqondene nezwe elithile zinethonya elibuthakathaka ekutshalweni kwezimali kwe-ULK ngokuphathelene nedatha esacwaningwayo, okuphikisana nenkoleloze yokuthi izici zezwe ezisengozini ziyisici senani kulezi zimakethe.

Ucwaningo lunikeza iminikelo eminingana. Okokuqala, ucwaningo kwaba ngelokuqala ukuhlola izakhiwo zezibalo kanye nokuguquguquka kokushintshashintsha kokutshalwa kwezimali kwe-ULK ezimakethe ezikhethiwe zase-Afrika. Okwesibili, ucwaningo lwengeza kulokho okutholwe ocwaningweni lwangaphambilini lemodeli yokubala ngoba lubandakanya ukuhlolwa kwesigaba sempahla esingahloliwe emkhakheni okhona wolwazi, okungukuthi, i-ULK ezimakethe zase-Afrika. Indlela yalesi sigaba sempahla ihlukile kulezo zezigaba zempahla evamile. Okokugcina, ucwaningo lwengeza emibhalweni ye-ULK ngokuhlinzeka ngemininingwane ebalulekile mayelana ne-ULK esimweni sase-Afrika njengesigaba sempahla esafufusa kanye nethuluzi lokuxhasa ngezimali lokuthuthukiswa komnotho.

AMAGAMA ABALULEKILE:

Ukutshalwa kwezimali kwe-ULK, imodeli yokubala, ukuguquguquka kokushintshashintsha, i-Afrika, izici eziqondene nezwe elithile, amamodeli e-GARCH, amamodeli e-VAR, imiphumela yokuchitheka, izilinganiso, izimakethe ezikhulayo

KGUTSUFATSO

Phuputso e hlahlobile dintho tse ikgethang tse amanang le dintlha tse hlaloswang ka dipalo le boemo ba diphetoho tse sa tsitsang le tse sa lebellwang tsa tlwaelo ya mmaraka ya ho phahama le ho theoha ka potlako e kgolo, ya matsete a moruo o holang, a tshwanang ka dintho tse itseng le ho sebetsa ka ho tshwana mmarakeng, a bitswang mofuta wa ho tsetela kgwebong ya poraevete e sa hwebeng mmarakeng wa diabo (LPE) mebarakeng e kgethilweng Aforika. Ho bile le kananelo ya batho ba bangata ya hore mekgwa ya boholoholo ya ho fumana boleng ba setoko e hlolehile ho hlalosa dipoello tse fapaneng tsa ditoko tse fapaneng ka nako e itseng mebarakeng ya moruo wa dinaha tse holang. Ho tswa tekolong botjha ya dikahare tseo ho buuwang ka tsona phuputsong ena, ho a bonahala hore kotsi ya naha e itseng e tswalwa ke ditheko tsa matsete Aforika. Phuputsong, ho bile le mohopolo le kakanyo ya hore dipoello tsa diasete e le dihlahiswa tse molemo haholo mothong di latelwa ke dintho tsa tsona tse ikgethang tse amanang le dintlha tse hlaloswang ka dipalo mmoho le boemo ba diphetoho tse sa tsitsang le tse sa lebellwang tsa tlwaelo ya mmaraka ya ho phahama le ho theoha ka potlako e kgolo. Datha ya kgwedi le kgwedi ya matsete a LPE le datha e kgethetsweng naha e itseng di sebedisitswe ho hlahloba dintho tse ikgethang tse amanang le dintlha tse hlaloswang ka dipalo le boemo ba diphetoho tse sa tsitsang le tse sa lebellwang tsa tlwaelo ya mmaraka ya ho phahama le ho theoha ka potlako e kgolo le kamano ya tsona le mabaka a amanang le kotsi ya ho tsetela naheng e itseng le boholo ba ho hloka botsitso bo ka tswalang tahlehelo ya ditjhelete dinaheng tse fumanweng di le sehlopheng sena sa matsete a tshwanang ka dintho tse itseng le ho sebetsa ka ho tshwana mmarakeng, tseo e leng Aforika Borwa, Ghana, Egepeta le Botswana dilemong tsa 2010–2020. Dimotlolo tsa GARCH (1,1), EGARCH, TGARCH, GARCH-In-Mean, FIGARCH, FIEGARCH, DCC MGARCH le VAR di sebedisitswe ho tshwanela datha. Lesedi le fumanweng la phuputsong le fana ka lesedi le ka sebedisetswang ho etsa potefolio le ho bala sekgahla sa poello e lebelletsweng aseteng kapa letseteng. Lesedi le fumanweng le ka boela la ba bohlokwa hore ho be le kutlwisiso e kgolo ya diphetoho tsa tlhophiso ya dipoello tsa LPE, tse fanang ka tlhophiso ya ntshetsopele ya disebediswa tsa tlhahlobo e ntjha ya sehlopha sena sa diasete.

Lesedi le fumanweng la dintlha tse tharo tse ka sehloohong e ne e le le latelang: La pele, Matsete a LPE a dinaha tse fuputswang a bontshitse dikameho tse entsweng hore di shebahale ka mokgwa o itseng oo e seng wa tlhaho wa boholo ba tlhokeho ya botsitso bo amanang le boholo ba phetoho e etsahalang e fetohang le nako, boholo ba tlhokeho ya botsitso bo eketseha haholo ha ditheko di theoha ho feta ha ditheko di phahama ka palo e lekanang le kgonahalo e kgolo ya datha e fapaneng le tse ding, empa ha ho a ba le ntho e bontshang kamano e mpe pakeng tsa poello le tlhokeho ya botsitso bo amanang le boholo ba phetoho e etsahalang kapa phetoho e sa lebellwang tlhophisong ya datha eo ho sebetswang ka yona. Phuputso e bontshitse hore matsete a LPE a fetotse tsela eo a tsamayang ka yona hantle le ho ba le ntlha ya datha e itseng e fapanang haholo le dintlha tse ding. Dipoello tsa matsete a LPE a Aforika Borwa di ne di batla di le kabo e tlwaelehileng ha tsa dinaha tse ding di bontshitse dikabo tse sa tlwaelehang, e leng ntho e tlwaelehileng ka datha ya ditjhelete. Egepeta e bontshitse mehlala ya ditheko tse hlokang botsitso tse namang ka mahlakore a mangata, mme tatellano ya ditheko tse hlokang botsitso tsa Ghana e bontshitse hore matsete a LPE a na le sekgahla se phahameng sa poello eo o ka e lebellang diaseteng tse nang le kotsi e kgolo se lekanang le boemo ba

kotsi e sa rarollwang. Datha ya Botswana e ne e tsitsitse le ho fumanwa e sa lokela ho ho etswa diteko tse ding tse sebedisang motlolo wa GARCH. Ntlheng ya bobedi, ho ne ho se na bopaki ba ho se lekalekane ha datha boemong ba nako e telele ba diphetoho tse sa tsitsang le tse sa lebellwang ba datha eo ho fuputswang ka yona. Ho sa kgathalatsehe hore matsete ana a na le tshusumetso efe ditseleng tse fapaneng tseo ho hlahiswang le ho tsamaisa dintho ka yona, dikarabelo tsa oona diketsahalong dife kapa dife tse sa lebellwang tse bang le kameho ya tshohanyetso e ntle kapa e mpe, di fumanwe id tshwana. Hona ho bontsha hore matsete a kentsweng dikhamphaning tse sa hwebeng mebarakeng ena ya Aforika a diabo a ba le ditlhophiso tse tshwanang ka mora nako e telele. Kahoo, le ha matsete ana e le diasete tse fanang ka moputso o tsitsitseng ka nako e telele, tlhahlobo ya diaterese tsa batsetedi e ke ke ya sebedisetswa ho thibela kotsi ka ho aba matsete ditumellanong tse fapaneng tsa ditjhelete. Ntlheng ya boraro, phuputso e fane ka bopaki ba hore dintho tse amang tlhahiso ya naha e itseng di na le tshusumetso e fokolang matseteng a LPE a mabapi le datha e fuputswang, ho hanana le mohopolo wa hore dintho tse nang le kotsi ya ho tsetela naheng e itseng ke ditjeo tsa tlhahiso ya ntho e nngwe mebarakeng ena.

Phuputso e na le dintho tse ngata tseo e di etsang. Ya pele, phuputso e bile ya pele ya ho hlahloba dintho tse ikgethang tse amanang le dintlha tse hlaloswang ka dipalo boemong ba diphetoho tse sa tsitsang le tse sa lebellwang matseteng a LPE mebarakeng e kgethilweng ya Aforika. Ya bobedi, phuputso e tlatsetsa lesedi le fumanweng patlisisong ya nako e fetileng ka ho ithuta le ho fihlela diqeto tse utlwahalang ka datha, ka hore e hloka tlhahlobo ya dintho tse eso fuputswe, tse sebetsang ka ho tshwana mmarakeng, leseding, mehopolong le makaleng a thuto e itseng, ke hore, LPE mebarakeng ya Aforika. Ditlhophiso tsa sehlopha sena sa matsete a sebetsang ka ho tshwana mmarakeng se fapana le dihlopha tsa boholoholo tsa matsete a sebetsang ka ho tshwana mmarakeng. Ya ho qetela, phuputso e tlatsetsa lesedi dingolweng tsa LPE ka ho fana ka kutlwisiso ya bohlokwa e mabapi le LPE boemong ba Aforika e le sehlopha sa matsete a qalang ho ba teng a sebetsang ka ho tshwana mmarakeng le dintho tse thusang ho etsa tjhelete ya ho ntshetsa moruo pele.

MANTSWE A BOHLOKWA:

Matsete a LPE, Ho ithuta le ho fihlela diqeto tse utlwahalang ka datha, diphetoho boholong ba tlhokeho ya botsitso bo amanang le boholo ba phetoho e etsahalang, Aforika, dintho tse amang naha e itseng, dimotlolo tsa GARCH, dimotlolo tsa VAR, dikameho tse mpe moruong wa naha e nngwe, ditemoso tsa boleng ba ntho e itseng, mebaraka ya moruo wa dinaha tse holang

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ACRONYMS

AIC	Akaike's Information Criterion
ACF	Autocorrelation function
PACF	Partial Autocorrelation function
ARCH	Auto-Regressive Conditional Heteroscedasticity
ARIMA	Auto-Regressive Integrated Moving Average
CAPM	Capital Asset Pricing Model
CCC-GARCH	Constant Conditional Correlation GARCH model
DCC-GARCH	Dynamic Conditional Correlation GARCH
EGARCH	Exponential Auto-Regressive Conditional Heteroscedasticity
FIEGARCH	Fractionally Integrated Exponential GARCH
FIGARCH	Fractionally Integrated GARCH
GARCH	Generalized Auto-Regressive Conditional Heteroscedasticity
GED	Generalised Error Distribution
LPE	Listed Private Equity
NAV	Net Asset Value
NIC	News Impact Curve
SBIC	Schwartz's Bayesian Information Criterion
TGARCH	Threshold Auto-Regressive Conditional Heteroscedasticity
VAR	Vector Auto-Regressive
VECM	Vector Error Correction Model

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1 CHAPTER ONE: INTRODUCTION AND BACKGROUND

1.1 Chapter Introduction

Statistical properties and volatility dynamics are a strategic part of investment decisions and provide a structured approach to modelling and valuations of financial assets. Volatility of returns gives way to the many models that are used in predicting price movements and preceding valuation models that follow. Whilst studies have been done on volatility modelling, little attention has been given to listed private equity investments (LPE) (Tegtmeier,2023).

Studies show that private equity investments have mainly used traditional techniques such as the CAPM in evaluating investments. Even though there have been several studies on private equity investments in Africa, the discourse of what explains the returns of this investment asset class remains vague. Consensus exists within the body of knowledge that the traditional models provided within the body of knowledge continue to be incapable of predicting future price movements. Studies by Andrei, Cujean, and Wilson (2022), Jiang, Wel, and Zhu (2018), Acheampong and Swanzy (2015), Michaelides and Spanos (2016), and Roll (1997) provide evidence of the inconclusiveness of the models in asset pricing despite providing the theoretical foundations that serve as the starting point for valuations. Even though this problem is universal and affects all investment asset classes, it is believed to be more severe in emerging and developing markets than in developed markets (Damodaran, 2018). Contemporary literature demonstrates a shift in emphasis from the CAPM to the identification of other risk factors influencing equity returns. This provides evidence of the inconclusiveness of CAPM in asset pricing, despite providing the theoretical foundations that serve as the starting point for valuations. Given the fact that LPE investments are unknown to investors and academics (Dopke and Tegtmeir, 2018) and there is a growing interest to conceptualise private equity investments, the study uses content analysis to review the different theorisations on valuations provided in the academic space so as to provide an understanding of this asset class.

A study by Harasheh, Amaduzzi and Darwish (2020) notes that market models perform better functionalities than dividend discount models. The study borrows the notion and utilises market models to examine the statistical properties of LPE investments in selected markets in Africa. To do this, the study utilises GARCH models, developed by Bollersleve and Taylor (1986) and other GARCH extensions such as GARCH-M, IGARCH, EGARCH (Nelson,1991), TGARCH and FIGARCH models to examine the volatility dynamics, both in the short run and long run. The findings contribute to the understanding of volatility dynamics of LPE investments in the selected African markets and provides a basis for further studies on LPEs in African markets.

Research using financial time series modelling has focused on estimating time varying volatility, establishing the presence of non-constant and time dependent volatility of traditional asset classes. Though studies on alternative asset classes have been done, these have focused on real estates (Milcheva,2022), commodities (Bonato,2019), hedge funds (Li, Li and Tee, 2020) with little attention being paid to LPEs. Studies on LPE investments were done by Tegtmeier (2023, 2021), Bachmann, Tegtmeier, Gebhardt and Steinborn (2019), Bilo, Christophers, Degosciu and Zimmermann (2005) and Brown and Kraeussl (2012). These studies looked at listed private equity investments, but no analysis has been done for African economic geography which has fragmented growth patterns to other continents. This study is the first study to examine the volatility dynamics of LPE's in Africa.

The statistical properties of an asset's returns are generally explained in terms of volatility clustering, leptokurtosis and non-linear dependency characteristics. Previous studies on financial assets note that financial assets are generally not normally distributed and exhibits non-constant variance. Studies by Cont (2001), Corbay and Rad (1994), and Kat and Brooks (2001) examined statistical properties of financial assets and notes that financial assets generally exhibit non-constant variance. Private equity investments differ from traditional asset classes and they exhibit characteristics which make them part of a diversifiable part of a portfolio.

Damodaran (2018), Damodaran (2016), Fritzen (2012), and Naumosky (2012) have demonstrated that current academic models are more applicable to developed markets with stable economies than to emerging and developing markets. They argue that these models ignore country-specific risk factors in favour of internal factors of the operating firms, industry-specific factors, and competitors. Contrary to mature markets, little is known about the country-specific risk factors that determine the structure of private equity returns in Africa's emerging markets. Thus, a scholarly question arises: how do country-specific risk factors affect returns generated in Africa? It is therefore crucial to analyse the response of investment returns to country risk factors in Africa, not only for examining the valuation aspects but also for elucidating the volatility dynamics.

1.2 Background of the study

Private equity is defined as a form of capital that is provided to start-up and distressed unlisted firms and can be used to improve balance sheet positions, as a cushion for working capital, strategic acquisitions, and to finance new projects (Gatauwa, 2022; Donahue and Timmerman, 2021; Raftopoulos and MacAdam, 2019; Gudiskis and Urbsiene, 2015). They can also be investments made through privately negotiated transactions by private firms into public firms through leveraged buyouts that result in delisting (AMG, 2017). On the other hand, venture capital is a form of private equity financing that is used during the infancy stage of business development (Reiner, 2013; Portman and Mlambo, 2013; Dess and Yin, 2010). In other terms, venture capital investments are private equity investments in early-stage companies.

Research points to the fact that this investment has gone wrong because there have been problems in identifying the differentiating factors between successful and unsuccessful investments. According to Campbell (2012), although the private equity industry is expanding in Africa, the returns of this alternative investment asset class are not well understood. Studies that have looked at the impact of country-specific factors and returns on investments include Song, Tang, Wang, and Ma (2022) Huy, Dat, and Anh (2020); Pan and Mishra (2018); Khan, Tantisantiwong, Fifield, and Power (2015); Attari, Irfan, and Safdar (2013); Osamwoyi and Osagie (2012). Studies by Mpofu (2011), Nasseh and Strauss (2000), Tatom (2002), Elsas, as well as El-Shaer and Theissen (2003) analysed the impact of macroeconomic variables on stock returns. Important to note is the fact that these studies looked at traditional asset classes; the assumptions of CAPM were criticised for failing to account for the relevance of other macroeconomic variables. Mpofu's (2011) findings revealed a significant relationship between stock returns and beta, but no strong evidence of the CAPM relationship in risk and returns.

The theory of financial economics and its applications are increasingly at odds. In empirical tests, the CAPM model, a theoretical pillar of modern finance, fails. The view that the CAPM does not hold is largely shaped by economists' consensus that beta does not explain expected returns. The CAPM, on the other hand, remains the most widely used among investors and firms to this day. To add to the controversy, the CAPM does not apply on specific occasions, such as responding to news or at night (Andrei, Cujean, and Wilson, 2022). Why do economists continue to dismiss a theory that practitioners refuse to abandon? In light of this argument, the study uses technical analysis to model the returns of private equity firms to validate the assertion and assess which model can accurately predict the expected returns of the asset class.

Based on an analysis of traditional asset classes, Damodaran (2020), Frank and Shen (2016), Estrada (2001), Erb, Harvey, and Viskanta (1996), and Bekaert and Harvey (2002) developed alternative methods for estimating the cost of equity. The difficulty lies in the dearth of research that critiques these hypotheses. This study, therefore, attempts to bridge the gap by empirically examining one segment of alternative investments – listed firms that invest in private equity – in order to validate the hypotheses and assess their plausibility for LPE investments in developing markets.

Africa exhibits different economic growth patterns, driven by differences in resources, political stability, etc. According to the African Economic Outlook (2016), Southern Africa was the worst performer in 2016, owing to low oil and metal prices. The World Bank also indicated that Sub-Saharan Africa's growth rate was continuously going down, from 4.6% in 2014 to 3.4% in 2015. It is worth noting that on the African continent, countries such as Kenya and Ethiopia are rapidly developing. Given the fragmented nature of economic development in Africa, it then becomes imperative to contextualise this asset class by way of examining the valuation models to assess if they speak to the systematic risk elements of this asset class and if they possess statistical properties that are different from other asset classes.

Studies on the drivers of the private equity industry have been done mostly in Europe (Groh & Liochtenstein, 2009; Proharovs & Pavlyuk, 2013; Bernorth & Colavecchio, 2014), USA (Gompers & Lerner, 1998; Jeng & Wells, 2000), in emerging Asian economies (Joshi & Subrahmanya, 2014; Ukaebgu, 2014), and in Africa by Errais and Gritly (2022) and Nkam, Akume and Sama (2020). There remains a void in terms of research output in Africa, mainly to build knowledge, seek the truth, and propel policy interventions.

Private equity as an investment tool influences other disciplines of economics. It can use the microeconomic mirror to examine the behaviour of investors. Studies such as Robertson (2017), Cabral-Cardoso, Cortez, and Lopes (2016), and Felix and Pires (2013) looked at the characteristics of investors in private equity. This asset class can be viewed from a macroeconomic angle, which looks at the impact of private equity on other subject matter. Studies such as Ames, Stiebale, and Wright (2016) looked at the impact of private equity on firms' patenting activity; Bernstein, Lerner, and Sorensen (2016) looked at private equity and industry performance; and

Davis, Haltiwanger, and Hadley (2014) looked at private equity, jobs, and productivity, to mention but a few examples of such studies.

The spectrum of model valuation approaches ranges from conventional to modern, from behavioural finance to machine learning methodologies. Although technical analysis has been utilised for more than a century, there has been a recent increase in scholarly interest in its application to financial analysis. According to Beneki, Alexandros, Nikoloas, and Stephanos (2019), financial markets may not be as efficient as once believed. Technical analysis is gaining prominence because traders find it much easier to use than fundamental analysis, which requires a greater understanding of internal and external factors and is time-consuming due to the laborious process of analysing such large amounts of data.

There are numerous empirical examinations of the return and volatility behaviour of developed markets. In the last three decades, the emphasis has shifted from developing and emerging markets to globalisation, which facilitates integration thus creating more global investment opportunities. According to Trivedi et al. (2021), empirical research is required to analyse the risk dynamics of developing and emerging economies in order to inform investors. The concepts of statistical modelling of returns have garnered a great deal of interest among researchers, but the majority of these outputs seek to explain asset returns for traditional asset classes better than alternative asset classes.

The statistical models that support asset prices are the basis for risk valuation and asset pricing. Thus, the quality of risk measures and the validity of prices are highly dependent on how well the statistical model captures the behaviour of the underlying asset. According to Koo and Kim (2022), flaws in statistical models can be catastrophic for more than just financial speculators. The market for private equity benefits from statistical model-based project evaluations. GARCH models have been widely utilised to model return and volatility dynamics because they can easily capture the majority of empirically discovered financial data properties (Song, Tang, Wang & Ma, 2022; Setiawan et al., 2021; Wang, Tsai & Li, 2019). Some contemporary models, such as econo-physics approaches, utilise complex mathematical computations and capture quantum properties such as discreteness, indeterminacy, entanglement, etc. The study was guided by the principle of parsimony, which instructs us to select the simplest model that fits the available evidence. In other words, the optimal model is the one with the fewest parameters and evolutionary changes.

According to Rahman and Bristy (2016), “country risk” refers to unique economic, political, and financial risks that lead to unexpected investment losses in a country. Further to this, Lee and Lee (2018) defined country-specific factors as the probability that macroeconomic conditions like exchange rates, inflation, economic growth, government regulations, and political stability affect investments. From these two definitions, it implies that country risk factors are divided into economic, political, and financial risks. The study used economic risk parameters — inflation and Gross Domestic Product (GDP) — to interrogate the structural relationships that exist among these and listed private equity returns.

The empirical premise of the study is that investments in developing economies require a higher rate of return; as countries integrate, country-specific factors are reduced, and systematic risk elements in the cost of capital are eliminated through diversification. Liquidity constraints plague developing markets, particularly private equity investments. This market segment's (LPEs) availability of market prices enables more accurate performance measurements, thereby facilitating the examination of the long-term effects of asset pricing models.

The construction of statistical models is not limited to asset pricing analysis. To address issues such as how various assets from various transactions affect the market, one may also need market microstructure knowledge (Tu & Liao, 2020). How do news releases influence price fluctuations? To answer these types of questions, a statistical analysis is also necessary. This study's objective is to examine the returns on LPE investments in selected African markets, model the volatility and examine their interactions with country-specific factors.

This study was informed by several outputs in the academic space. Studies by Portman and Mlambo (2013), Gudiskies and Urbsiene (2015), Campbell (2012), Leautier (2017) on private equity investments in Africa suggest that future studies should investigate the return characteristics of these investments. Gurdikis and Urbsierne suggested that studies develop models that investigate return characteristic: Campbell notes that it is imperative to understand the dynamics of this asset to make it more successful, and Portman and Mlambo (2013) suggested the need for studies that examine the decreasing importance of valuation tools for private equity investments. This necessitated the need to develop an empirical analysis that looks into the return characteristics, volatility dynamics and their relationship with country-driven risk parameters. The analysis would seek to answer the academic question: how can investors view private equity in emerging markets in Africa?

1.3 Research problem

Highly segmented markets, informal information channels, concentrated ownership of resources and limited access to capital markets all set the market far from theoretical concept valuations, rendering them inapplicable (Damodaran, 2020) in African countries. Market imperfections and unavailability of data in these markets tend to violate the assumptions' theoretical foundations of traditional valuation models. This notion is supported by Gimple (2010) who argues that market inefficiencies in developing economies complicate the theoretical foundations of traditional valuation frameworks.

In addition, information risk on published data is so rampant in these countries. When preparing financial asset valuations, past financial performance is key in forecasting, and once manipulated, the firm's true financial position is compromised. Data manipulation is also found in developed markets but the risk is very high and aggravated by lack of accounting standards, unqualified personnel and information asymmetry between buyers and sellers in emerging and developing markets (Lee and Lee, 2018). Besides, the market is also characterised by limited trading; therefore, adapting valuation models elsewhere deceives the valuation exercise. The increased participation in private equity investments and its economic importance as a financing tool for development calls for valuation techniques that speak to the fragmented growth patterns and forward looking. Therefore, the most pragmatic approach is to analyse the return properties of LPE investments by evaluating their statistical properties and volatility dynamics, their individual patterns and how the interact with one another. A fundamental goal of economics is to understand how variables reacts to each other, not only at the first moment but through to the fourth moment of analysis. This then means that altering one variable influences the volatility of another variable. The study, therefore, adopted GARCH models to analyse the interactions of the return series of the LPE investments in selected markets in Africa.

The study also notes that the driving factors behind private equity investments in Africa have received scant attention in both theoretical and empirical valuation studies. Therefore, there is no econometric model that explains the structural behaviour of private equity returns in this region in terms of country-specific factors. Studies by Damodaran (2020, 2016, 2012, 2003), Naumoski (2011) and Fritzen (2012) provide significant debates on the applicability of country-specific risk factors to investment valuation tools. They imply that these factors are essential for addressing the valuation problem in emerging markets. According to Naumoski (2011), it is more difficult to estimate the cost of finance in emerging markets than in developed markets. The study questioned the viability of comparing the returns on comparable investments in emerging markets and developed markets without adjusting for country risk. The conclusion was that country risk is more significant in emerging markets than in developed markets, so there should be some form of risk premium compensation. It was proposed that country-specific risk factors be incorporated into the required rate of return by incorporating a country risk premium in comparison to similar investments in mature markets. If we shift our focus to Africa, can we conclude with confidence that this asset class is resilient to country risk premiums?

Damodaran (2003) highlighted that country risk premiums should only be estimated when the investor is not globally diversified and when risk is correlated across markets. It should be noted that the type of risk that is relevant for estimating the cost of equity is systematic or undiversifiable risk. So, if the additional risk of investing in Nigeria or South Africa can be diversified away, then there should not be any risk premium. Damodaran (2011) highlighted that country risk should be specific to a country; country-specific factors determine the premium that investors require. This raises an academic question: how is volatility generated, and what drives it in Africa?

1.4 Research Objectives

This study complements previous research that examined the returns on private equity in emerging markets. Prior studies have paid little attention to listed private equity investments, which are slowly proliferating in Africa. LPE firms enable stock market investors to participate in a variety of portfolios of unlisted firms that would otherwise be available only to institutional investors (LPEC, 2022).

Thus, there is a need for statistical modelling of investments and the identification of country-specific factors that have a substantial impact on the returns of listed private equity investments. A statistical analysis of the behaviour of these variables offers additional insight into the pricing and valuation of this asset class. This study employs technical analysis, time series modelling, and econometric modelling to examine the expected returns of the asset class and their relationship to country-specific factors, whereas previous studies benchmarked their work on behavioural finance using econometric instruments.

The objective of this study is to provide an answer to the question, "How do we characterise the distribution of private equity returns in Africa?" This is accomplished by analysing the volatility of these investments over time and the structural relationship with country-specific variables in order to develop portfolio construction strategies. In conclusion, a plausible set of objectives is established to best answer the given primary research question. These include:

1. To examine the statistical properties of Listed Private Equity returns in selected markets in Africa
2. To model private equity returns and examine the volatility dynamics using GARCH models
3. To examine the impact of country-specific factors on returns for listed private equity investments in Africa

1.5 Justification of the study

This study contributes to the current body of knowledge by providing a basis on which future research on LPE investments in African markets can reference. This study provides an understanding of the volatility dynamics of selected African markets, which the existing literature has not yet filled. According to Tegtmeier (2023), LPE investments are a new area and unknown; hence, this study contributes to the growing body on literature of private equity investments.

In addition, LPEs are a portion of private equity investments that are relatively unknown in the academic and investments space. This study provides a data-driven approach to the relevance of this asset class to country-specific risk factors thereby informing investors pertinent issues in the pricing of these assets and portfolio construction.

Lastly, the study extends previous empirical findings on statistical properties and volatility spillover effects amongst LPE investments in different markets. This helps governments and policy makers in creating interventions that promote the growth in this asset class.

1.6 Limitations of the study

In conducting this study, a myriad of obstacles had to be overcome. There were some obstacles that the researcher was able to circumvent, while others remained unresolved; therefore, the researcher humbly submits the issues raised to future research for resolution.

To the best of the researcher's knowledge, this is the first study to examine the modelling of LPEs in African markets, or, put differently, the first study to examine the performance of this asset class in this market. As much as the study provides valuable insights into understanding the asset class and paves the way for future research, it was hampered by a lack of academic literature on LPEs in Africa or other economies with comparable economic geographies. The study employed non-academic articles as references for discussions that academic articles lacked.

The second limitation of the study involved the variables. This study confirms that LPEs in Africa do not participate exclusively in private equity, but rather operate as LPE fund managers, where the listed company participates in private equity companies and has interests in other sectors, including banks, insurance, and construction, to name a few. This type of LPE indicates that its characteristics are also influenced by other counter segments. The study's research is therefore characterised by this LPE type. Private firms that heavily invest in private equity investments in Africa fall outside the scope of this study.

The third limitation relates to the fact that integrity and dependability of the selected data relies heavily on the data sources. The study utilised share price, GDP, and inflation data. The study utilised data from reputable sources and a method for validating the data's authenticity. Historical share prices were obtained by subscribing to https://www.african-markets.com/en/stock-markets/egx/listed-companies and then validated using data from www.yahoo.finance, a public source. The World Bank, one of the most reputable institutions, provided the GDP and inflation data, and validations were performed using other country-specific factors data sources.

Fourth, the study encountered difficulties with data adequacy. In Africa, the concept of publicly traded companies investing in private equity is still in its infancy (I Soumare et al., 2021); consequently, the study had limited options for obtaining a sample. The researcher had to utilise the limited data available from countries with such firms. In a sense, according to all the research, the countries selected for the study were the only ones with these investments. This implies that the findings are restricted to the selected nations. Tegtmeier (2021,2023) demonstrated that the expanding asset class of LPEs is still unknown in both the academic and real-world contexts of emerging markets.

Fifth, the study's daily and monthly returns do not occur as frequently as the GDP and inflation indices. The purpose of the study was to examine the interactions between these variables; therefore, converting high-frequency data to low-frequency data was one way to link the variables. This has implications because, according to

Ndlovu, 2019, conversions tend to degrade the error term's behaviour. To accomplish this, the study utilised the raw data for the opening and closing stock prices. Thus, the error terms become available for a period of one year.

The study's final limitation relates to its scope. The scope of the study was from 2010 to 2020. This implies that the conclusion of the study is limited to the time period of the study, despite the fact that the selection of the time period was dependent on the availability of data and the stated problem.

1.7 Definition of key terms

1.7.1 Emerging Markets

According to Klonowski (2013), an "emerging market" is the one that has a responsible public finance system with low debt levels, minimal reliance on foreign exports, improved governance regimes, and rapid economic growth. The International Monetary Fund classifies the world into advanced, emerging, and developing by grouping them according to levels of economic integration, income, and export levels[1].

Mazzi (2013) classified emerging markets as those with rudimentary financial markets or those that are developed. MSCI Index Research (2014) concurs in that it classifies them as countries that have some characteristics of a developed market but are yet to meet the standards of high capita income and developed financial markets.

From an investor's perspective, emerging markets are those economies that can withstand economic turmoil. This is mainly due to the fact that they have a strong resource base and can support a manufacturing industry. Economies such as the

BRICS and MINT came into being because they benefited from investors' geographical preferences and potential investments in infrastructure, power generation, and natural resources.

According to Raftopoulos and MacAdam (2019), Africa has gained more tolerance to risk compared to the past; hence, it has received more funds than emerging counterparts like Hungary, Poland, etc. A growing economy creates space for investments and fosters entrepreneurship. Long-term high growth is inextricably linked to significant positive trends (Hirsch, 2017).

According to EMPEA (2017), South Africa, Nigeria, and Egypt are the continent's emerging countries based on their growth rates. The study also highlights that Kenya has been dominant in private equity investments, accounting for close to half of the total deals in East Africa. Having mentioned that, the FTSE Country Classification Process 2017 came up with a index that incorporates all of these. Hence, it forms the basis for the population in this study.

1.7.2 Country-specific risk factors

According to Damodaran (2020), the term " country-specific risk factors" refers to the distinct economic political and financial dangers that can result in unanticipated financial losses from investments made in a particular nation. In addition, Lee and Lee (2018) provided a definition of nation risk factors as the possibility that macroeconomic variables such as exchange rates, inflation, economic growth patterns such GDP, and political stability have an effect on investments. The purpose of this study is to determine whether these country-specific elements have an effect on the returns created by private equity investments, which in turn tend to influence the valuation tools that are supplied in in the existing body of knowledge.

This study adopts the Accelerator theory developed by Keynesin 1936 which assumes that investments are driven by changes in the national income. Increased demand of a product in a market is met by either increasing production that leads to more profits. Increased profits then create growth that consequently attracts more investors (Fritzen, 2012). An economy that experiences increased production levels generally receives a larger budget for spending coming from taxable income as firms generate high returns. Given this notion, understanding the factors that facilitate economic growth becomes important as beyond that comes in foreign direct investment in form of private equity investments.

The debate for economic growth has been examined empirically in so many studies dating back to the work of Schumpeter in 1911 who advocated for a finance led growth concept (Olowofeso, 2015). The understanding of facilitators of the same has changed overtime. Dating back to Robert Solow's first generic growth model in 1956, divergent views have emerged on how economic development is determined.

During the period up to the 1980s, most studies argue that economic development is based on input factors like labour, capital and productivity, and even today there are academics who also support the notion. The understanding is that economic growth creates space for investment opportunities and the two are determined by the accumulation of input factors. This growth model was initiated by Solow in 1956 and is classified as the neo-classic view.

According to Fritzen (2012), studies in the 1990s show that economic growth bears no relationship with input factors and does not exclusively explain economic growth patterns but other institutional factors. Studies done by Dunning, Kimb and Park (2006), Moss, Ramachandran and Standley (2007), and Arbache (2008) all pointed that input factors cannot sorely explain economic growth patterns but noted that government interventions can contribute by creating institutions that can best improve resource allocation. This can be done, for instance, by improving infrastructural development, creating stable exchange rate regimes and barriers of trading, and working on education and employment levels. In Africa, it can be noted that most economies cannot function independently hence there exists a strong correlation between economic growth and the factors highlighted.

This study evaluates the impact of country-specific risk factors on the volatility of private equity investments in order to gain a better understanding of the exogenous factors that influence the volatility of this asset class. The divergent channels of country risk may halt differences in direction, intensity, and significance, which is required for valuation, asset pricing, and portfolio construction.

1.8 Structure of the thesis

Chapter 2: The chapter reviews contemporary literature regarding private equity investments in Africa, the private equity model and listed private equity investments. Discussions on the relevance of valuation tools are brought into context and the country-specific factors that drive private equity investments are examined. Their impacts on investment and valuation are elaborated. The chapter sets the tone for the research approach by providing the basis for traditional models and why they are being superseded by conventional methods which were then used to investigate the research question.

Chapter 3: The chapter sets out the theoretical framework and justification of the study, and critically examines the research questions. The instruments for testing the research questions are also examined. The research problem is examined to gravitate towards a hypothesis that can be scientifically and statistically measurable. This chapter is an extension of Chapter 2 as it provides a different view of analysing expected returns given in the academic space.

Chapter 4: The chapter reviews the methodological framework and provides a blueprint of how the study is conducted. The review explains why a preferred choice was chosen scientifically. A justification of the methodology is also postulated . The research philosophy is provided and justification for tools for the research enquiry presented.

Chapter 5: The chapter presents the empirical findings on the statistical properties and volatility dynamics of LPE firms under study. Firstly, the chapter looks at an analysis of the trends of the LPE investments in Africa's return series. The second part looks at the descriptive statistics and basic diagnostic tests that the variables are subjected to. Third, the chapter presents a country-by-country analysis of short-run volatility and different GARCH models that were fitted into the daily log-return data series. Fourth, the study looks at the long run volatility dynamics of this asset class.

Chapter 6: The chapter presents findings on the interaction of volatilities of LPE investments under study with other variables in order to establish their structural relationships. Structural relationships of variables are a key analytical tool that explains how volatility is generated and what drives it. This chapter also presents findings relating to spillover effects between LPE returns for South Africa, Egypt and Ghana and their associated country specific factors.

Chapter 7: The chapter details the conclusion of the study regarding statistical modelling of LPEs for the countries under study. The research problem, research question, hypothesis and methodology are summarised. The chapter brings back the framework of the study and links it to the hypothesis and findings thereof in terms of how hypotheses formulated in the study were rejected or failed to be rejected. The contribution to new knowledge is spelt out, and suggestions for further research are given.

2 CHAPTER TWO: LITERATURE REVIEW

2.1 Chapter Introduction

This chapter examines recent research on private equity investments in Africa, the private equity model, and publicly traded private equity investments. Context is provided for discussions regarding the applicability of valuation tools, and the country-specific factors that drive private equity investments are examined. Their effects on investment and valuation are discussed in detail. The chapter sets the tone for the research methodology by explaining why traditional models are being supplanted by conventional methods, which were then used to investigate the research question.

2.2 History of Private Equity

Private equity investments, also known as venture capital investments, are an alternative asset class that has existed for over 100 years and has been successfully used in America to bring about innovations that are in existence today (Field, 2022; Klonowski, 2013; Gompers, 1994). The earliest venture capital firm to be incorporated was American Research and Development (ARD) in 1946. ARD would sell shares to private investors, who would then offload them to other investors on a publicly traded closed-end fund (Gompers & Lerner, 2001). As a result, the firm made strides in the industry, including 1947 investments that were made in the High Voltage Engineering Company, a firm that established a cancer treatment making use of X-ray technology. Another successful investment made by ARD was the Digital Equipment Company (DEC), where a $70 000 investment grew to $355 million (ibid.). This success story stimulated most investors, and it became the concept standard for all venture capitalists at that time as they wanted to finance the next DEC.

In 1958, a new setup called a limited partnership was formed, which gave a time period of approximately 10 years for the partnership to return assets to the investors. In this case, the investors will then have shares allocated to them in the new public firm. They will decide the period in which to liquidate their investment so as to realise returns on their investments (ibid.).

In 1978, a new form of regulation was introduced. The first was the decrease in capital gains tax, and the second was a modification of pension fund regulations (Porteba 1989). This facilitated the growth of venture capital and private equity investments as pension funds could invest up to 10% in this asset class. According to Campbell (2012), these changes enabled firms to receive funding, which resulted in a significant growth in technological development.

According to Tegtmeier (2023) private equity and venture capital continue to play an important role in economic development and technological growth. Gompers and Lerner (2001) noted that with increased funding, a situation can arise in which there will be an oversupply of capital chasing too few deals. Despite such setbacks, private equity has gained popularity worldwide. Africa, being the last to adopt this asset class, has had more funds channelled because of its high growth potential. Mckean and Hobson (2013) noted that there has been a growing appetite for private equity investments in Africa as investors look at the opportunities arising from resource-rich and consumer-driven economies. In a way, global financialisation enabled Africa to gain access to this investment tool through emerging technologies and a high growth potential.

A foremost observed benefit of private equity is its ability to stimulate economic growth by way of building small, innovative firms into corporates. Apple Computers, Seagate, Federal Express, and Microsoft are among the most notable venture capital firms that have contributed to the growth of America (Sahlman, 2022; Jeng and Wells, 2000; Gompers, 1994). Evidence on the positive economic impact by private equity is shown through the works of Davis et al (2021), LPEC (2022), Pomerance and McCarthy (2018), Gudiskis and Urbsiene (2015), Popov and Roseboom (2009), and Fehn and Fuchs (2003).

2.3 The Private Equity Model

Researchers define private equity in various ways depending on the nature of the research. Dopke et al (2018) define private equity as private investments made at any stage of the firm's business lifecycle. According to Leeds and Sunderlands (2003), private equity is a financial tool that targets start-ups and late-stage investments. Soumare et al (2021) define private equity as a type of alternative funding in which investment funds and private investors invest directly in private companies or participate in buyouts of publicly traded firms.

Moon (2006) views it as financing at a later stage of the firm's life cycle. Hence, we can conclude that private equity is a form of financing provided at any crucial point in the development of a business. Venture capital specifically refers to funds offered to start-ups or early-stage firms. Whereas venture capital investments refer to the financing of early-stage companies, private equity refers to the financing of a broad range of companies, from early-stage to high-growth stage to distressed companies. Thus, venture capital investments are a subset of private equity investments.

These forms of investment may involve buying new shares as a way of providing fresh capital to a firm, or they may take the form of buying shares from existing shareholders (Soumare et al, 2021). A general partner is one who has unlimited personal liability and has the right to participate in the company's management. A limited partner has total liability, which is limited to the capital that would have been provided. Limited partners may be partnerships, corporations, or funds.

Institutional investors avoid making direct investments in private firms because they lack the skills and resources needed to monitor and structure the investments. Instead, they make their investments through private equity funds.

Private equity managers raise funds from either institutional or individual investors and invest the funds in businesses that will eventually become private companies. They conduct due diligence in sourcing investment opportunities and then actively manage the investments through monitoring the daily trading activities of the company, having voting control, and putting in place control mechanisms that reward success and penalise failure. Gains are realised by floating those investments through an initial public offering (IPO).

There are different ways in which private equity investments are structured. These are as follows:

1. Venture capital: These are investments made to firms in early development stage or start-ups where the form of financing is mostly seed capital. It is prominent in new marketing concepts, new products without a track record, during application of new technology to expand the business etc. (Tegtmeier, 2021)

2. Growth capital: This is a form of investment in mature firms that are seeking capital either to expand into new markets to restructure the operations or to finance a merger.
3. Leveraged buyouts: This type of private investment involves incorporating debt financing that is collateralised against the firm's assets (ibid).
4. Mezzanine capital: These investments consist of half equity and half of either secured or unsecured debt. These investors would normally require a higher premium for increased risk than other types of investments (Megginson, 2004).
5. Distressed capital: These are investments in financially distressed firms (ibid).

2.4 Listed Private Equity Investments

There are different types of LPE to consider in the private equity market. Firstly, by far the most popular in Africa is the LPE fund manager. In this LPE, the managing partner is a listed firm that has controlling stakes in private equity portfolio firms. This listed company participates in private equity companies and has interests in other segments such as banks, insurance, and construction, to mention but a few. This type of LPE is the most common in Africa and enjoys the flexibility of the direct and indirect LPE features. In the rest of the world, the LPE fund manager is uncommon, but in Africa, it is slowly mushrooming (Soumare et al 2021).

A direct LPE participates directly in investments by businesses. To do this, it uses its own funds and sometimes participates passively by way of engaging limited partnerships. In the former, the direct LPE manages the selection and valuation process of the investments, and in the latter, it is sometimes involved in passive investments.

An indirect LPE is made up of firms that do not actively participate in private equity deals. They delegate decision-making authority to a general partner. In some cases, an external private equity partner is appointed to manage the portfolio of limited partnerships. This portfolio consists of a diversified pool whose cost of minimum funding is quite high; hence, the investments leverage a network of relationships, which is characteristic of funds of funds (Tegtmeier, 2021).

A study by Tegtmeier (2021) on global listed private equity investments notes that this asset class is relatively unknown globally, and the study was the first to provide empirical research on global listed private equity investments. This study focuses on the emerging world of LPEs in Africa, which happen to have the characteristics of LPE fund managers. Döpke et al (2018) conducted another study on global LPEs and discovered that global risk factors are not a pricing factor on LPEs globally.

By and large, LPEs in general provide shareholders with a diversified portfolio that exhibits more liquidity exposure with no fixed investment horizon. It also offers better access to investments as it does not offer restrictions in terms of funding to seed in. Apart from that, LPEs are more transparent than traditional private equity firms as disclosure requirements are mandatory (LPX Group, 2022). Besides offering a high degree of flexibility in terms of diversification, LPEs have been noted to outperform other asset classes. Several studies, including Pomerance and McCarthy (2018), Sharma (2018), Sulaiman (2018), and Brown and Kaplan (2019), have deconstructed the private equity puzzle by examining general returns and valuation. A common understanding is that if they operate efficiently, private equity investments can grow economies in terms of their industrialisation path. A study by Soumare, Kanga, Tyson, and Raga (2021) critiqued the lack of government efforts in promoting capital markets that allowed an increase in market participants for LPEs in Africa.

Studies by Tegtmeier (2021), Döpke and Tegtmeier (2018), Xiu, Sun, Chen, and Li (2016), Yuan, Zhao, and Wang (2016), Portman and Mlambo (2013), Reiner (2013), and Bilo et al (2005) lamented the paucity of research in this area and suggested that academics develop a set of plausible suppositions or theories to be used to explicate and help stakeholders in the private equity market better understand the industry.

Hence, the practical way is to explore the link between the returns of private equity, their relationship with country risk factors, and their effect on valuation. The immediate inquisitiveness is to analyse the current models put forward in the academic space and assess their relevance to the academic puzzle of assessing the statistical relevance of the identified variables.

2.5 Development of Private Equity in Emerging Markets

Emerging markets have gained traction in private equity investments since the global financial crisis. This is driven by an expanding middle class, urbanisation, increased population wealth, and significant domestic infrastructural investment. According to Klownosky (2012), the movement of funds from the developed world to the emerging world was driven by increased fiscal discipline, better corporate governance, and less reliance on foreign exports. Empirical studies on the impact of private equity on economic growth have shown that the private equity industry is significant in fostering economic development. This is shown through the works of Gudiskis and Ursiene (2015), and Hellmann and Puri (2000).

Emerging markets are those that can withstand economic turmoil (Bliss, 2012). They are better able to adapt to downfalls and can create investment opportunities globally. Markets such as India and China have leveraged capital injections into geographical markets that required large capital outlays, such as power generation, infrastructure development, and natural resource exploitation.

The development of private equity in emerging markets is demonstrated by an upsurge of fundraising and investment activities. According to EMPEA (2018), funds raised and capital employed in emerging markets increased by 11% and 21%, respectively, year-on-year. According to Klownosky (2012), private equity's financial performance in emerging markets has been positive, driven by multiple expansions and growths as well as the investee firm's improved operational and financial growth. Between 2007 and 2010, private equity fundraising increased from $3.2 billion to $23.5 billion. Private equity stimulates economic development by promoting innovation (Marti and Balboa, 2001; Kumar and Orleck, 2002; Popov and Roseboom, 2009), improving portfolio company productivity (Ernst and Yong, 2012), lowering portfolio company default rates (Kaplan and Stromberg, 2009), encouraging firm start-ups (Samila and Soreson, 2011), and positively influencing the rate of new business creation (Popov and Roseboom, 2009).

It has been noted that while global investors have mainly targeted China, Brazil, and India, some emerging countries like South Africa, Nigeria, Turkey, and Indonesia are also fast gaining popularity (Campbell, 2012; Wilton, 2012). Despite high economic growth patterns, emerging countries are also establishing industries that are historically favoured by private equity investors. For instance, India and Russia dominate software development, and China is known for renewable energy and some of the clean tech market segments. Asia is a leading centre for pharmaceutical research and development. These industries developed due to the availability of a well-educated workforce that worked for low wages compared to the same workforce based in the developed world. India created a favourable private equity investment climate by reducing capital gains tax, instituting FDI protection laws, increasing disclosure requirements for IPOs, allowing investors 49% equity in Indian firms, creating proprietary rights, and passing laws governing bankruptcy (Klonowsky, 2013). As a result, India and China accounted for 47% of all private equity investments in emerging markets.

According to Wilton (2012) and Klownosky (2013), Nigeria and Columbia, which are known to be risky investment destinations, received more investments than other emerging markets such as Poland because they offer higher risk tolerance levels than the latter. Private equity, like any other investment tool, has faced evolutionary and transitional challenges. Returns for this asset class have also been volatile and highly variable, especially on foreign direct investments (ibid).

Regardless of all these challenges, investors have had a high appetite to invest in this economic space (EMPEA, 2018). This is because of the perceived sentiment that these countries will soon improve on institutional development and exit opportunities. Given this, global investors must ensure that they develop suitable deal flows, improve operating experience by understanding local norms and investment climate, and invest in a pool of fund management experts.

2.6 The African Private Equity Landscape

It has been observed that the characteristics of private equity in Africa differ from those on other continents. According to Lapavitsas (2011), Africa was the last continent to adopt this asset class because it could only capitalise on the advantages that arose as a result of technological advancements. The increased use of technology has made room for individual investors and crowd funding as sources of capital to create a road map for private equity investments in Africa. Campbell (2012) supports this view and notes that private equity and hedge funds were adopted in Africa in the year 2000, as well as the adoption of technologies such as data switches and 4G, which increase market information flow, thereby fostering economic growth patterns.

The unprecedented movement of funds towards Africa in 2014 was followed by an era of low economic growth patterns and unstable currencies in many countries in Africa. Despite this, optimism about Africa's growth prospects still prevails. According to Klonowsky (2013), a growing middle class and improved communications infrastructure coupled with increasing foreign direct investments from China have contributed to this growth-fuelled trend.

Apart from that, Ashiagbor (2014) noted that there has been an increased flow of domestic pension funds that are now being channelled to private equity investments in Africa. Sagna and Sagna (2012) supported this notion in retrospect by noting that pension funds can be utilised to create a sustainable financial model for African development. It then follows that economies receiving such funds will move on a high growth trajectory, which is different from those economies that do not have such facilities, creating economic disparities in Africa. Campbell (2012:135) highlights that "Africa is experiencing a generational shift that is moving towards an educated and relatively young workforce." This, in a way, increases cash flows for pension funds, consequently increasing funds channelled to private equity. A study by Robertson (2017) also noted that even in Asia, most institutional investors who fund private equity and hedge funds are pension funds.

As pointed out in earlier discussions, during the period of the global financial crisis, investments significantly suffered in terms of returns. As a result of Africa's economic recovery, the continent's focus shifted from investments in energy and railways to technologically oriented sectors such as healthcare, communications and media, agribusiness, and real estate investments to accommodate the continent's expanding population. This was done in order to accommodate Africa's need to provide housing for its growing population. Africa was able to reposition itself on the path toward economic growth as a direct consequence of this shift in the focus that was placed on investments.

In addition, there are a number of cities that are home to financial markets that are more substantial than those found in other places. In South Africa and Nigeria, big cities with a growing middle-aged population tend to offer more access to credit facilities and insurance services to local firms (Leautier, 2017). As a result, the opportunity for private equity business is expanding to the point where governments in the countries involved are offering active participation. Due to geographical differences, there is often a considerable degree of variety to be seen in the financialisation of the various countries.

According to Campbell (2012), depending on the country that is being examined, Africa has inequalities in terms of the political and macroeconomic factors that make up the continent. In addition to this, the study points out that, in contrast to Europe, the United States of America and Asia, regional risks do not affect Africa to the same degree as they do on these other continents. This increases spatial differences hence economies are viewed differently.

The returns on private equity investments vary depending on such factors as when the investments are made, how they are made, and where they are made. Additionally, the rate of return may also be affected by the kind of investments that are being made (Leautier, 2017). Klausner (2013) provided support for this concept when he noted that investors tend to hold time-sensitive investments. Because of this, it became necessary to develop a model that was capable of quantifying risk parameters and as a result, to provide an answer to the following question: what are the interactions between private equity investments and country-specific factors in Africa? This question is raised because it is seen as necessary to interrogate the dynamics of these interactions to provide more information about the behaviour of this asset class.

Africa exhibits different economic growth patterns, driven by differences in resources, political stability, etc. According to the African Economic Outlook (2016), Southern Africa was the worst performer in 2016, owing to low oil and metal prices. The World Bank also indicated that Sub-Saharan Africa’s growth rate was continuously going down, from 4.6% in 2014 to 3.4% in 2015. It is worth noting that on the African continent, countries such as Kenya and Ethiopia are rapidly developing.

Based on the anecdotal evidence presented, it is critical for stakeholders in alternative investments to understand the geographical and spatial factors that influence these investments in Africa in order to minimise risk and maximise return.

2.7 Traditional Valuation Methods on Private Equity Investments

A key problem in finance has been measuring investments in terms of their risk and return characteristics. According to Fritzen (2012), contemporary investment tools were developed with the aim of evaluating firms whose focus was on internal factors. The most common measure for private equity returns is the internal rate of return (IRR), which shows the effective rate of return that makes the present value of all cashflows zero. According to the East Africa Venture Capital Association (EAVCA) (2017), the limitation of IRR is that it assumes that cashflows have the same rate of return as the initial investment. The discounted cash flow technique, which discounts the firm's cash flows with the cost of capital, is another technique used. The weighted average cost of capital looks at the debt-to-equity mix and compares it with that of the competitors. In addition, the total value to paid-in capital (TVPI) measures the performance of a fund in terms of the multiple of the initial investment that can be accrued if the assets from the investment can be sold and added to distributions that were received in the fund. Important to note is that these models employ the risk premium, which forms the required return from the investment and is derived from the capital asset pricing model. Risk premiums are a key element in every risk-return model for investment, and they hence form an important component when estimating the cost of capital and the cost of equity in finance.

According to Balboa and Matri (2003), private equity investments follow a cyclical process known as the "private equity investment cycle." The fundraising stages are: the investment stage, the investment optimization stage, and the divestment stage. The investment stage involves the screening of investment opportunities by the private equity investors. The stage requires that maximum due diligence be done by the investor as the decisions taken at this stage directly affect the actual return that will be realised at exit (Balboa and Matri, 2003). It is at this stage that target firm evaluation models (qualitative and quantitative) are employed. Some of these models include book value, Tobin's Q, scorecard valuation, discounted cash flow, IRR, cash-on-cash return, comparable transactions, market comparables, and the venture capital method.

The book value approach looks only at the net tangible assets of the target firm, making the technique less relevant when evaluating start-ups as they can have more of their assets in intangible form. The scorecard valuation technique mainly focuses on the qualitative aspects of the target firm, such as the management team (with 95% of venture capital firms focusing on this management team according to Gompers et al. (2016)), competition, fit with the fund, and the products offered (Kaplan and Stroomberg, 2004; Gompers et al., 2016). Tobin's Q approach focuses on the ratio of the replacement cost to the firm's market value as the main factor that drives investments (Fritzen, 2012). According to Ghara and Godwin (2010), investors will only place funds in an investment where the perceived marginal increase in market value exceeds the replacement cost.

The comparable transaction method is a relative target firm evaluation model that is based on approximately similar precedent transactions and key firm performance multiples (Gompers et al., 2016). The market comparable valuation technique makes use of the market capitalisation of listed companies comparable to the target firm in an attempt to value the target firm. The IRR is one of the most widely used firm evaluation techniques, and it shows the effective rate of return that equates the net present value of all cashflows to zero. According to the East Africa Venture Capital Association (EAVCA) (2017), the limitation of IRR is that it assumes that cashflows have the same rate of return as the initial investment.

These models help private equity investors screen investments, focusing mainly on firm-specific factors. Each of the models has its own limitations, so no particular model can be used in isolation. Though the models touch on both qualitative and quantitative fundamentals that affect firm-specific risk, they do not incorporate country-specific risk.

2.8 Applicability of the Traditional Valuation Techniques in Africa

A key tool in any valuation technique, be it the Discounted Cashflow Approach, the Internal Rate of Return, the Net Asset Value approach (NAV), is the cost of equity. This is based on the Capital Asset Pricing Model (CAPM), borne out of the Harry Markowitz assumption of risk and return that only undiversifiable or systematic risk is compensated. Studies such as Mpofu (2011) examined the significance of systematic risk and discovered that CAPM beta fails to explain stock returns. Major anomalies stem from momentum effects (Jagadesh and Titman, 1993), book-to-price earnings (Basu, 1973, 1983), firm leverage (Bhandari, 1988), and reversal effects (DeBondt and Thaler, 1985, 1987).

Contemporary literature provides a shift in emphasis from the CAPM and builds economic theory towards identifying other risk factors influencing equity returns. Acheampong and Swanzy (2015), Roll (1997), Michaelides and Spanos (2016), Jiang et al. (2018), and Brown (2013) all criticised CAPM and provided evidence of its inconsistency in asset pricing despite presenting the theoretical underpinnings that form the point of departure in valuations.

Studies by Pandey and Sengal (2015), Sutrisna and Nasri's (2019) analysis on the size effect, Fatima et al.'s (2017) study on price ratios and stock returns, Apergis and Rehman's (2018) investor sentiments, Obrimal et al.'s (2015) market efficiency, amongst others, also provide indications that there are other factors that help explain cross-sectional returns than just the Markowitz framework.

In addition, studies based on geographical settings, such as those done by Carter, Miller and Ward (2017), Obrimah et al (2015) amongst others, raise conclusions that CAPM does not apply in some economic geographical settings. Lee, Cheng, and Chong (2015) in Malaysia proved the relevance of CAPM in that setting. Nonetheless, Klonowsky (2013) notes that investments are determined by economic geographical factors or spatial factors that drive investments. Damodaran (2016, 2012) and Fritzen (2012) highlight that CAPM assumptions do apply in mature markets but cannot be used as a valuation tool for emerging markets, more so in Africa, which tend to exhibit fragmented growth patterns and more country specific factors.

Markets are highly segmented in developing countries, and there are informal information channels, concentrated ownership of resources, and limited access to capital markets, all of which set the market far from the theoretical concept assumed for CAPM, rendering it inapplicable. Market imperfections and the unavailability of data in these markets tend to undermine the assumptions' theoretical foundations. This notion is supported by Gimple and Borges (2010), who argue that market inefficiencies in these economies complicate the theoretical foundations of traditional valuation frameworks. This is because the information risk associated with published data is so rampant in those countries. In valuation, past financial performance is key to forecasting, and once manipulated, the true financial position of the firm is compromised. Manipulation of data is also found in developed markets, but in emerging and developing markets, the risk is very high and aggravated by a lack of accounting standards, qualified personnel, and information asymmetry between buyers and sellers (Lee and Lee, 2018).

Any private equity firm intending to invest abroad is interested in future cash flows and the cost of equity. As has been alluded to before, investments in emerging markets are riskier, and more so given that private equity investments are also riskier than traditional asset classes. The million-dollar task is based on an estimation of the marginal compensation required by equity holders. It is important to note that the relevant risk in this case is an irreversible or market risk. So, if the risk of investing in Zambia can be diversified, then there should not be an additional premium to be compensated. If it cannot be eliminated through diversification, then an additional country risk premium has to be estimated (Damodaran, 2016). For a private equity investor who is globally diversified, this means there is potential for the complete elimination of systematic risk. For investors focusing solely on Africa, this reduces the likelihood of diversifying away this risk, necessitating an estimate of the additional compensation for this risk. For a globally diversified investor, it is also necessary to assess the level of risk correlation within countries. If there is a positive correlation, it indicates that a portion of the country's risk is non-diversifiable or systematic (Naumosky, 2011).

2.9 CAPM Valuation Modifications

Because this is a well-studied topic, several alternative models have been propagated in an attempt to make the models more relevant. These competing perspectives analysed the CAPM model and universally accepted the shortfalls associated with the model but differed in the way in which these problems could be solved. Understanding the shortcomings of the valuation models put forward helps map the way for new models that can be developed and clearly elaborates the shortcomings that need to be addressed.

The country risk premium approach uses the spread between the yields on the home country bond and the yield on the foreign denominated bond to determine investors' risk expectations in their home country. Additional returns that compensate for this risk are then determined (Gimpel & Borges, 2010). The main problem with this methodology is that the depth and breadth of financial markets in emerging and developing economies are very poor (Lee and Lee, 2018). There might be little or no liquid foreign-denominated bond existing in that market, or even if it were there, it might not match the same term structure as the domestic one. Comparing bonds with different maturity profiles leads to wrong valuation estimates. Apart from that, Damodaran (2020) argues that when using spreads on a bond, it is wiser to consider the average spread over a period of time than the one prevailing at the moment. This approach, however, works only when the economy does not experience structural shifts.

In addition, this approach does not consider the diversification effect of spreading investments. The study assumes that the yield is an indicator of expectations that investors have that the government will honour debts, which is not always the case, especially in the developing world like Africa, as was supported by Topal (2016).

Lessard's model adjusted the CAPM by first looking at the risk premium that investors would require for a similar project in the U.S., and having done that, the premium was multiplied by the country beta of the domestic country. This beta was the sensitivity of the country to the variability of U.S. equity returns. This element was the beta measure that investors required in a given country. This approach mainly depends on two issues: how to arrive at the country beta, and the reliability of the U.S. as the proxy. Some countries do not have a sovereign rating but can still use either the current default spread on credit default swaps or sovereign bonds. A study by Bekaert, Harvey, Lundblad, and Siegel (2014) showed that only the political risk element explained the movement of funds within countries' equity markets; hence, that should be the risk relevant for country risk estimations.

This methodology depends on the reliability of the United States as a proxy. After the global financial crisis, emerging markets recovered much quicker than the developed world, which weakened the U.S. as a proxy (Gimpel and Borges, 2010). Hence, applying the same to Africa gives a totally wrong benchmark.

In the Godfrey-Espinosa Model, the authors recommended the use of total risk rather than systematic risk elements alone in estimating the risk premium. The analysis was predicated on the assumption that markets are highly correlated and that investments with diversification potentials are low risk. The implication, then, was that leaving out the global portfolio led to unreliable estimates. In general, this method would make sense when a globally undiversified investor incorporates country risk in estimating the risk premium. Deligonul (2020) provides perspectives for delineating total risk and concludes that asset-specific risk explains cross-sectional variance in developed markets. Contrary to popular belief, studies on emerging markets have revealed that these economies may have negative country beta when compared to developed markets. This could be due to the small correlations exhibited by these markets in relation to developed markets, hence the model replaced country risk beta with the ratio of equity volatility of the local market to that of the U.S. so as to arrive at a correlation of one. Critics of this model also highlight the duplication of the country risk premium as it is already included in market volatility. Erb (1995) concludes that 40% of equity volatility is explained by economic and political factors, referring to real estate investments. According to Harasheh, Amaduzzi, and Darwish (2020), market models outperform dividend discount models in terms of equity volatility.

Furthermore, this model violates CAPM assumptions in that beta is replaced with volatility risk under the assumption that 40% of country risk premium explains local volatility. However, critics of this assumption point out that emerging markets are volatile in nature, rendering the findings irrelevant.

In the Goldman Sachs Model, Mariscal and Hargis in 1999 introduced more company-specific variables into the Godfrey-Espinosa model (Damodaran, 2016). The model raises the various risk drivers that influence the risk premium. Global investor expectations, country-specific factors, and firm-specific factors are incorporated in the model. Harvey (2001) notes that this approach is subjective and has technical problems. Firstly, it does not provide guidelines on how to estimate firm-specific risk premiums. Secondly, it captures the firm-specific risk separately, and the same is also embedded in the estimations for the local beta. Apart from that, he also notes that adjusting beta with volatility factors has no economic foundation, a notion supported by Damodaran (2020, 2016), Frank and Shen (2016), Fritzen (2012), and Klownoski (2013).

The Global CAPM was developed to derive equity returns for any global investor, regardless of their country of location. The basic assumption is that markets are integrated, so investors carry the same risk-return profile everywhere in the world. In addition, the model also assumes that there are no restrictions on moving funds from country to country and minimal transaction costs.

The weakness in this notion is that emerging markets carry with them many financial barriers that deter global market integration. Bakaert and Harvey (2002) noted a gradual positive relationship between emerging and developed markets, indicating that the G-CAPM will become more relevant in the near future. The G-CAPM is also supported by Damodaran (2020), who notes that using locally derived factors for emerging and developing markets is useless due to market inefficiencies.

Advocates of the local capital asset pricing model believe that markets are segmented and thus exposed to country-specific factors that can be diversified. Pereiro (2001) suggests that the required rate of return for equity stock should be based on local market risk factors where the local risk-free rate is added to the country risk factor and to the local beta's sensitivity of the assets under valuation in the local index.

Godfrey and Espinosa (1996) noted the presence of duplication on country risk, which is already embedded in the market premium, which results in an overestimation of the discount rate. Pereiro (2001) also came up with the adjusted hybrid CAPM, in which uses a combination of global and local beta.

In the Solomon-Smith-Barney Model, Zenner and Akaydin (2002) extended a G-CAPM under the assumption that local risk factors do not work due to market inefficiencies. Based on this notion, they adjusted the country risk premium in line with the risk characteristics of specific projects to allow the riskiness of the project to determine the premium. It therefore follows that, much as global risk factors are useful, adding a country risk premium in accordance with the riskiness of specific projects may bring the models closer to reality.

The Erb-Harvey-Viskanta Model was proposed so as to deviate from the CAPM model and avoid using traditional beta measurement. The model uses country ratings published by institutional magazines, which serve as proxies for political and other country risk parameters. The idea is that country ratings are forward-looking and hence can accommodate the volatile patterns obtained in emerging markets better than relying on historical beta generates a noble idea, but it would require statistical evidence to be rational. Yuan, Zhao and Wang, 2016 note that in the academic world, few empirical papers have accounted for the statistical properties of ratings. Empirical studies help to validate hypotheses and expand the knowledge base in the academic world; hence, every theoretical construct should be accompanied by a wide array of empirical evidence.

In addition, the Erb-Harvey-Viskanta Model proves that valuation approaches do not bear the same economic foundations as CAPM can also yield viable results. It accommodates economies that are inefficient and do not have equity markets. In addition, the ratings are not real-time and unreliable for forward-looking events. This notion is also supported by Estrada (2008), who notes that overreliance on credit ratings is highly subjective.

In the Downside Risk Model, Estrada highlights how the assumptions of the CAPM approach are easily violated in emerging markets and how that can be solved. The argument is that the variance of returns is dubious and applies to normally distributed values. Hence, the solution is to replace the variance with the semi-variance as it offers smaller fluctuations in returns. Estrada (2002) conducted studies to provide evidence in support of the approach, and the conclusion was that emerging markets are better captured by downside volatility. Klonowsky (2013) notes that emerging and developing markets have fragmented markets that do not support the same theoretical constructs as those of developed markets. The model may be theoretically correct, but an empirical test to determine whether applying it will produce the accuracy and usefulness of valuation models in the developing world is needed.

It is evident that there is no consensus among valuation models regarding the most applicable technique. However, it is commonly argued that the CAPM is incapable of explaining the returns of emerging and developing markets. Notably, all the aforementioned studies utilised traditional asset classes and did not mention alternative assets such as private equity investments. Private equity firms that are publicly traded behave differently than traditional stocks in that they are extremely risky and illiquid. In this regard, they respond differently than other asset classes; therefore, it would be beneficial to identify the interaction between their returns and country-specific risk factors prior to proposing an appropriate valuation technique. As alluded to by Harasheh, Amaduzzi, and Darwish (2020), market models perform better than dividend discount models in terms of equity volatility.

Studies on the valuation of alternative asset classes have primarily focused on the valuation of real estate, excluding private equity investments. Tojani, Morano, and Ntalianis (2018), Belloti (2017), Doumpos, Papastamos, and Andritsos (2020) examined the valuation framework for the real estate industry, whereas Yeh and Hsu (2018), Lo, Shih, Wang, and Yu (2019), Harasheh, Amaduzzi, and Darwish (2020), and Qoyum et al (2020) examined the empirical applications of valuation models. These studies may have produced theoretically accurate analyses, but the applicability to the private equity asset class is a primary concern.

The global CAPM would be appropriate if markets were integrated and investors held a diversified market portfolio. Risk and expected returns are anticipated to remain unchanged. Due to the highly segmented nature of African countries, country-specific factors must be incorporated for valuation purposes; given that private equity is a highly illiquid financial instrument, the risk factor must also be incorporated in order to comprehend the dynamic structure of returns. According to Damodaran (2020), the equity risk premium is a function of economic risk, information flow, the cost of liquidity, catastrophic risk, government policies, risk aversion and expected returns. Due to the highly segmented nature of African countries, country-specific factors must be incorporated for valuation purposes. According to Damodaran (2020), the equity risk premium is a function of economic risk, information flow, the cost of liquidity, catastrophic risk, government policies, and risk aversion. Klonowsky (2013) found that private equity investments in developing and emerging markets are influenced more by their macroeconomic conditions than in mature markets, it follows that the risk premium for emerging and developing markets cannot be used in risk return models for anyone holding assets abroad or operating in a foreign market. Using the arguments on CAPM adjustments, and the works of Dubitsky (2020) who questioned the reliability of credit ratings, Ilmanen, Chandra and McQuinn (2019) who demystified the concept of illiquid assets typical of private equity, Rudin, Mao, Zhang and Fink (2019) who distilled the concept of idiosyncratic risk in private equity investments, as well as Pomerance and McCarthy (2018), who observed that market portfolios are failing, it is important to quantify the identified risk characteristics and examine their possible interactions to arrive at the idea valuation approach. Figure 2-1 is a conceptual framework of the theoretical arguments brought forward in the chapter.

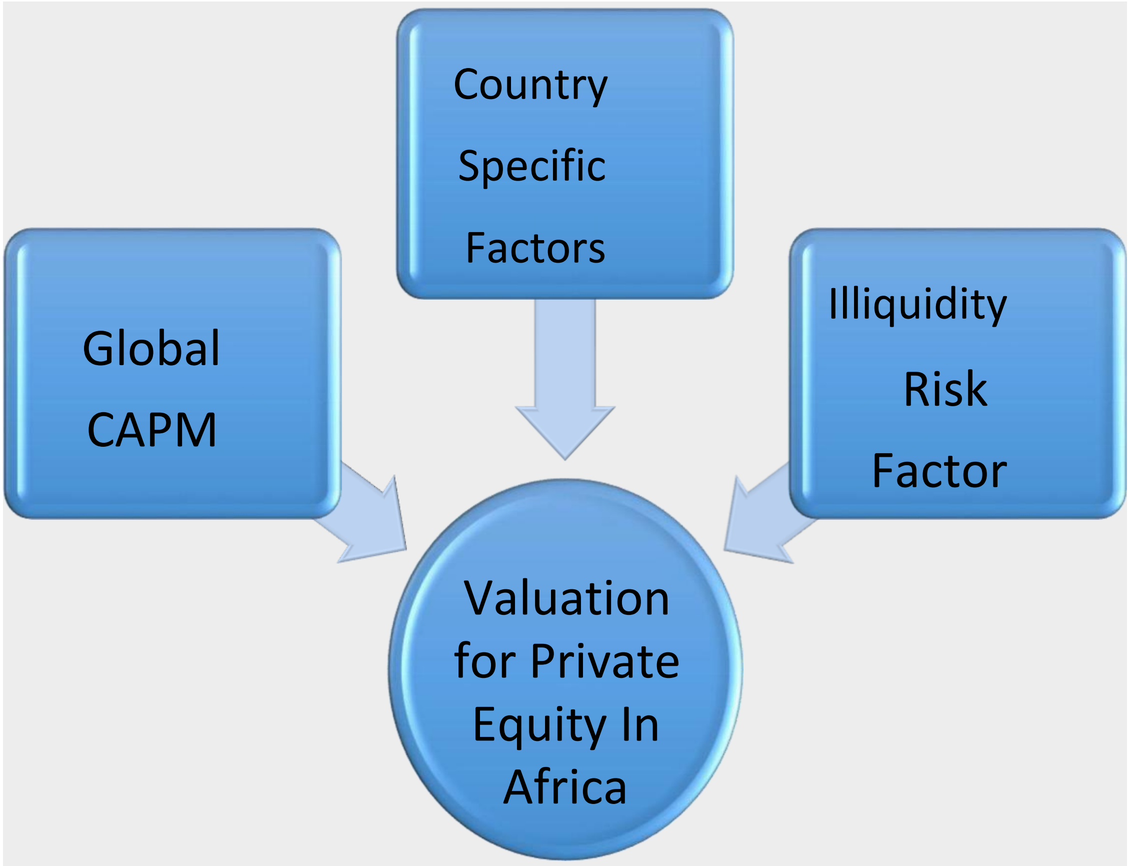


Figure 2-1: Conceptual illustration

Source: Researcher

The model demonstrates that valuation is a function of the global CAPM, the country beta, and the illiquidity coefficient. The implication is that country risk is relevant in equity premium valuations for an investor holding private equity funds abroad or operating the investment in a foreign market, contingent upon whether the market is open or segmented, whether the investor is globally diversifiable or not, and whether they subscribe to the one-factor model or the multifactor model. Private equity as an alternative investment is expected to behave differently than traditional asset classes. The question then arises as to whether the returns can be replicated by investing in traditional asset classes. According to Brown and Kaplan (2019), since the global financial crisis, private equity investments have generated higher returns than public equity investments. Czasonis, Kritzman, and Turkington (2019) note that private equity managers are constrained by the notion of no-arbitrage pricing. As a result, the study indicates that private equity managers are more likely to use own discretion in valuations, resulting in a great deal of bias. In addition, Rudin, Mao, Zhang, and Fink (2019) demonstrate that the volatility and equity beta of private equity are lower than those of public equity and that risk properties are more stable. In as much as these studies are empirical, they do not address the concerns of the developing world, which consist of segmented markets and fragmented economic growth patterns, as supported by Klonowski (2019). From this analysis, investments in segmented or developing economies require a higher rate of return. On the other hand, as country-specific factors are diversified, a decline in the required rate of return should be anticipated in any country that integrates. According to Damodaran (2020), an analysis of the equity risk premium for emerging and developing markets may not be relevant for an investor planning to invest abroad.

The aforementioned assertion is supported by empirical evidence but the majority of studies were conducted on traditional asset classes, which have different risk characteristics than alternative asset classes, and the studies that examined this asset class did not provide an empirical analysis of the structural behaviour of the returns in this asset class. The majority of studies on the drivers of the private equity industry (in terms of total assets under management, total transaction value, and private equity fundraising and investments) have been conducted in Europe

(Ljumovic, 2020; Hoyan, Hutson and Nevich, 2017; Precup, 2019; Menegolo, 2019), the United States (Jarmuzek and Rosenov, 2019; Sunny, 2019), Asia (Sannajust and Chevalier, 2018; MacMadi, 2020; Neerza, Ndlwana and Botha, 2018; Ndlwana and Botha, 2019)

Investments in developing economies are anticipated to have a higher required rate of return; as countries integrate, their cost of capital decreases as country-specific factors are eliminated through diversification. Liquidity constraints plague developing markets, particularly private equity investments. LPEs tend to leverage on capital markets to manage this problem. In addition, the market segment's availability of market prices enables a more accurate performance measurement, thereby facilitating the examination of the long-term effects of asset pricing models.

It is therefore necessary to examine the relationship between country-specific risk factors and private equity investments. The valuation and pricing of this asset class can be better understood through a statistical analysis and modelling the data series to establish their relationship.

As investments become more global, the approach to valuation issues becomes more complex. Given the contradictory arguments presented by different researchers, it is evident that country risk is a priced factor. Africa can utilise geographical factors to attract private equity investments that contribute to sustainable development. In Africa, markets are imperfect, and information is scarce. This weakens the theoretical foundations of CAPM by making them more complicated. This suggests that a model capable of capturing the entire risk spectrum is more applicable than the CAPM. Capital markets in developing nations are highly segmented, preventing the use of global market beta as a risk indicator.

Existing literature offers various justifications for determining the relevant market beta, but the majority of studies point to a negative correlation between the cost of capital and risk in these markets. This suggests that a country with a lower rating would have higher expected investment returns.

2.10 Chapter Summary

Unresolved issues regarding country risk factors and private equity returns in Africa have been examined. First, the study examined private equity investments and focused on those that are publicly traded. This unit of private equity has not been investigated in the existing body of knowledge; therefore, this study fills the gap by shedding light on the behaviour of LPEs. Second, the study revealed fundamental insights regarding the CAPM model and anomalies that prompted the development of other models such as the five factor, momentum factor, and dividend factor models. It has been determined that the CAPM yields an expected return that is too low to be reasonable in developing countries. Capital markets in developing nations are highly segmented, preventing the use of global market beta as a risk indicator. Existing literature offers various justifications for determining the relevant market beta and expected returns, but the majority of studies point to a negative correlation between returns and risk. This suggests that a country with a lower rating would have higher expected investment returns, substantiating the relevance of country risk in this market. The study clarifies the hypothesis that valuation of private equity investments in Africa is a function of the Global CAPM, country risk factors, and premium demands resulting from the illiquidity of the asset class. In addition, from the perspective of investment analysis, it is crucial to establish the structural contemporaneous interaction between country risk factors and returns for private equity investments in order to comprehend how volatility is generated within and outside the system of country risk factors. In contrast to previous studies that examined cross-sectional returns for all investments, this study focused on LPE investments as a development financing tool in Africa.

The following chapter examines the study's theoretical framework, justifies the research methodology, and examines the research questions.

3 CHAPTER THREE: THEORETICAL FRAMEWORK AND HYPOTHESIS DEVELOPMENT

3.1 Chapter Introduction

The preceding chapters analysed and discussed both theoretical and empirical issues pertaining to valuation in general, as well as ongoing debates regarding the applicability of valuation tools to emerging and developing markets. Market models were noted to perform better than discount models. Consequently, this chapter contextualises market models as a means of evaluating LPEs in Africa.

This chapter's objective is to examine the research problem from a theoretical and empirical perspective and to develop testable hypotheses that will guide the empirical work. The chapter is organised in the following manner: section 3.2 presents the theoretical framework; section 3.3 critically examines the objectives and formulates the research problem into testable hypotheses to facilitate empirical work; and section 3.4 provides the conclusion.

3.2 Theoretical Framework

The existence of a theoretical framework that guides research is imperative in that it uplifts the standards of the study and provides a path through which research is directed. In this way, studies are grounded in theoretical constructs providing for more meaningful findings that are acceptable in the subject matter (Imenda, 2014; Cresswell, 2014). Besides, theoretical frameworks stimulate research by way of providing a path to the research enquiry. According to Adom, Husesein and Adu

Agyem (2018), the absence of a theoretical framework that guides the study results in obscured aims, objectives and significance of the study.

Harasheh, Amaduzzi and Darwish (2020) note that market models perform better equity volatility modelling than dividend discount models. Therefore, the majority of financial irregularities cannot be accounted for using conventional models. Behavioural finance makes it simple to understand why a person makes a certain decision, but it is more difficult to understand how future decisions will be made. According to Gabauer (2020), the theory and practice of finance revolves around the volatility of the financial markets. Although it was first stated that the observed stock market volatility was inconsistent with the predictions of the present value models, which were fairly popular in the past, this issue is actually not wholly new and has emerged in a systematic way. In addition, models with stochastic discounts have a timeless variation that is unreasonably upwards making it hard to explain disparities. Traditionalists or financial economists can be broadly categorised as stock market researchers who use Gaussian random walk-based statistical approaches to analyse particular stock market events (Kapusuzoglu and Ceylan, 2018). The study proposes GARCH models to simulate the returns of LPE investments mainly because of their merit for analytical tractability.

As explained in Chapter 2, in an attempt to solve for the shortfalls in traditional valuation techniques, academics have tried to come up with ways of adjusting the same to make it more relevant to emerging markets. Studies by Damodaran (2011; 2012; 2016), Harvey (2001), Bakaert and Harvey (2002), Zanner and Akaydin (2002), Gimpel and Borges (2010) provide adjustments to CAPM anomalies. In line with that, there have been evolving differences over the risk factors that are relevant for valuation purposes. Through the works of Obrimah et al (2015), Michealides and Spanos (2016), Jing, Wei and Zhu (2018) and Balakrishnan (2016), we gain an insight that there exists additional factors besides CAPM beta that explain cross-sectional returns of equity. Unknown in all these findings is whether they also affect private equity in the same way traditional asset classes. That said, this study tries to unravel the revelation that the independence of asset returns as a product of economic rationality is followed by its statistical properties in the return distribution. Therefore, the study utilises the GARCH models to bring about the returns characteristics of listed private equity (LPE's) investment as this helps map the way for new models that can be developed and clearly elaborates the shortcomings that need to be addressed.

Even though statistical models have existed since the early 1900s, it was not until the early 1980s that they began to gain traction in the finance sector. The ARCH model, introduced by Engle in 1982, and the GARCH model, introduced by Bollersleeve in 1986, made possible advances in financial econometric modelling. The models became popular in finance due to their ability to depict volatility clustering and mean reversion properties in financial time series. According to Trivedi et al. (2021), GARCH models can capture large volumes of the data's time varying volatility and still provide close to accurate estimates. The GARCH model also solved the problem of lack of clarity in analysing stock market returns by advancing leverage effects analytics that had been propagated by Black's theory in 1976. In this theory, the understanding is that a fall in a firm's equity tends to cushion its debt-to-equity ratio thereby raising its volatility. This notion is quite key in understanding the predictions of volatility and impacts on investment behaviour.

Another important aspect that was necessary in understanding financial markets is the notion of market efficiency.

To determine whether the EMH holds water, researchers have tried to explain the stock market behaviour. Research has been conducted to determine the effect of stock price volatility on economic fundamentals and information adequacy. A study by Moussa et al., (2018) on determinants of stock market volatility noted the presence of economic variables and absence of effects of market indices. Whilst the study hinges on the modelling of LPEs and the interaction of the returns with the country-specific factors, it is also imperative to provide an assessment of how the findings help inform the current valuation tools in the academic space.

In financial time series data, the variance is not constant as it can be seen to have some periods that are volatile than others. When the variance is not constant the process is said to be heteroscedastic which equates to a larger magnitude in the residuals. The heteroscedastic residuals are also noted to be auto correlated as spikes in volatility are not randomly placed in time (Tsay, 2013). Because of this phenomenon that is exhibited by financial time series data, the framework assists in finding volatility measures that are able to predict volatility using residuals. What is important to note about the data that is handled is that it has a long memory, hence, the ARCH and GARCH framework applies weights to observations. The latest observations have more weights than that of time past. In the ARCH model, weights that are applied on the residuals are the best parameters for the equation, yet GARCH only adds on the time factor to almost the same problem as described.

GARCH models have been established to explain the empirical regularities in financial data. Most financial time series returns data exhibit common characteristics such as non-stationarity and that there is little autocorrelation that is inherent in the return series. For data points in a series, squaring them makes them serially correlated and breeds a non-linear relationship in the series. The return series tend to exhibit volatility clustering; the series also exhibits leptokurtosis driven by the fact that financial time series data is not normal, so it exhibits thick tails. In addition, prices in financial assets have a habit of being negatively correlated with changes in volatility. Finally, that volatility of different securities tends to move together (Tsay, 2013).

Modelling volatility in financial markets is vital because it is often perceived as a noteworthy element for the evaluation of assets, the quantification of risk, investment decision-making, the valuation of stocks, and monetary policy pronouncements. Stock market volatility is virtually time varying. It is widely accepted that volatility changes in financial market are predictable. Various models have been employed by wide-ranging empirical studies for future volatility forecasting and measuring the certainty of volatility forecasts. Amongst them are Silva (2022), Dixit and Agrawal (2019), Amudha and Muthukamu (2018), and Sen, Mehtab, and Dutta (2021).

According to Gimpel and Borges (2010), a study by Bekaert and Harvey in 1997 examined volatility amongst emerging markets and noted that volatility is difficult to model in emerging markets and that as markets integrate, the volatility is strongly influenced by global factors, whereas in segmented markets, volatility was mainly driven by local factors. In their study, they also concluded that market liberalisation had an insignificant impact on volatility.

Another study by Fabozzi (2004) on identifying the best model for modelling volatility for returns and spillover effects noted that in China the GARCH (1,1) and TGARCH models best captured the changing aspects of volatility and other important aspects of risk management.

Financial assets are typically characterised by instability. Since the 1980s, this concept has been considered a fact. There is now a field within finance known as financial modelling due to the increased volatility in the market. The purpose of this research is to obtain a deeper comprehension of the dynamic systems that explain the volatility of LPE investments in the economies under consideration. In financial modelling contexts, things like volatility, returns, and fat-tailed distributions are common features.

There are a number of models available for predicting market volatility. According to Sen, Mehtab, and Dutta (2021), it is highly unlikely that any model used to analyse financial time series data is also flawed. However, not all models are created equal. To know whether a model is good, an analysis is made of how well it fits the data and how closely it matches reality. Stochastic volatility and GARCH models are two types of volatility models. The GARCH model describes volatility as a deterministic function, whereas the stochastic volatility model models it as a random, unobservable process. Since GARCH-type models can be easily analysed, they have gained popularity in volatility forecasting (Koo & Kim, 2022).

An easy method for detecting volatility clustering is to capture changing variance using ARCH and GARCH models. This is because the models are autoregressive, meaning that there is a positive correlation between the current risk and the risk for the previous lag. In addition, the volatility is conditional, meaning that the volatility for the following year is conditional on the information available in the current period. Finally, the data was tested to check for heteroscedasticity and passed the test;

hence, the GARCH model was the best technique to model the data as it depicted non-constant volatility. Put another way, the time series exhibited time-varying variance, which made it a better candidate for GARCH models than ARMA models. Besides, the GARCH models specifically refer to the fact that they can account for different factors in varying markets, making them a good choice for capturing the spatial factors in the markets under study.

Another technique that can capture volatility clustering is the exponential weighted moving average approach, which gives more weight to current observations than to past observations and has a decay factor that ensures that today's variance is positively correlated with yesterday's volatility. This metric was also used in the statistical analysis of the LPEs to complement the GARCH models, as it is actually a subset of GARCH (1,1).

The entire study has been divided into three sections. The initial section examines the statistical characteristics of the data series. Herein, the return distributions and their characteristics in terms of the mean, variance, skewness, kurtosis, and normality are examined in greater detail. Additionally, tests for stationarity and ARCH effects are performed to assess the suitability of the data series for GARCH modelling. Second, the study applies the various GARCH models to the data series to evaluate the stylised characteristics of the investigated data. In this section, all models that capture both short-term and long-term memory characteristics of the data are utilised. The study concludes by examining the relationship between listed private equity firms and country-specific data, using GDP and inflation as surrogates for country-specific factors. By utilising VAR models and impulse response functions, the study examines the structural relationship between the variables. When necessary, diagnostic tests are performed in every case. Therefore, the study is meticulously crafted with time series modelling and econometric modelling, cradled in the arms of behavioural finance. With this framework, the study addresses the primary research question: how and why is volatility generated for LPEs? Figure 3.1 summarises the study's framework in accordance with the research objectives.

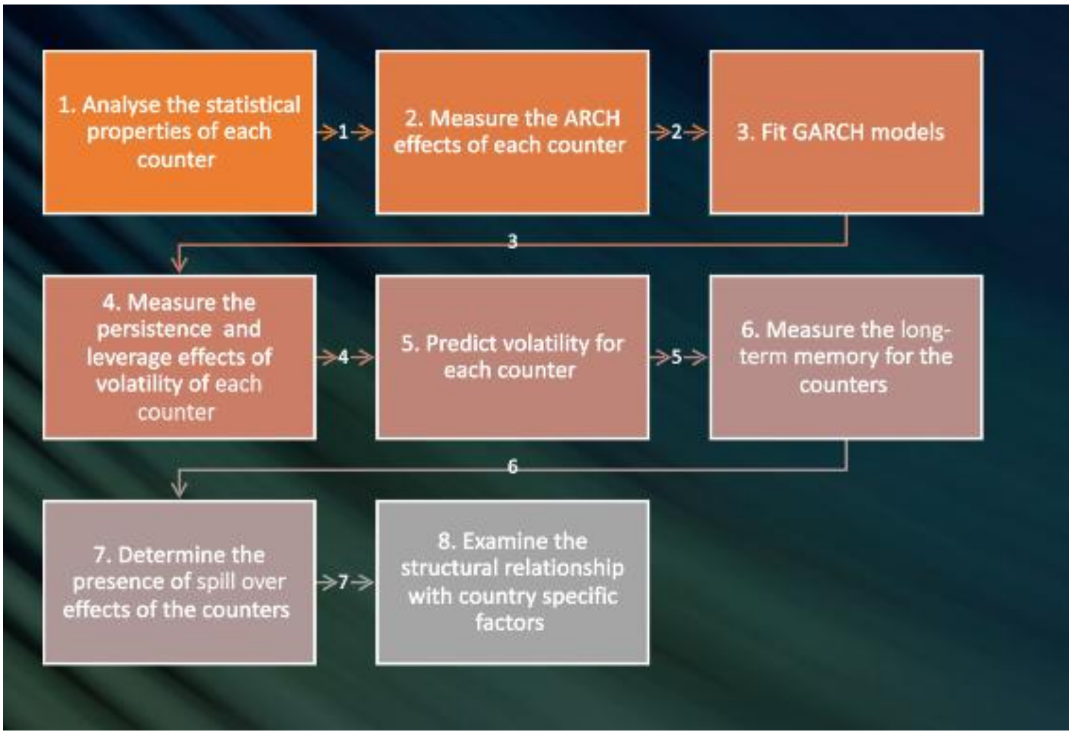


Figure 3-1: Framework of the study

Source: Researcher

3.3 Hypothesis Development

This section contextualises the objectives and the arguments surrounding the objectives in order to highlight the main research question: how is volatility generated and what drives it?

3.3.1 Objective 1: To examine the statistical properties of LPE returns in selected markets in Africa

According to Danielsson (2011), three statistical features explain the behaviour of an asset's returns: volatility clustering, leptokurtosis, and non-linear dependency. Volatility clustering is the occurrence of low returns in a period that previously saw high returns. This effect is typically caused by the filtering of new information that drives price movements, which occurs in clusters rather than consistently across time. Leptokurtosis, on the other hand, is characterised by return distributions with fat tails and an excess peak near the mean. Non-linear dependence explains the link between multivariate financial data; for instance, non-linear dependence between diverse asset classes can be noticed during financial crises, when several assets are likely to move together in response to certain market pressures (ibid.). This study's statistical analysis was mainly centred on these three aspects.

In the 1980s, a new category of statistical models for asset returns came into being with the goal of modelling the non-linear dependence that can be observed in asset return series. These models were developed in order to predict the future value of assets. For example, the volatility clustering effect, which Mandelbrot first identified in 1963, was reinterpreted probabilistically, marking a significant achievement in this decade. In the period 2010-2020, the observations of previous authors about the weird behaviour of volatility have been viewed in a significantly different way. Instead of being seen as an expression of infinite variance (a situation where the probability that the next random number is far away from the mean is very high), the effect of volatility clustering was interpreted as a temporal non-linear dependence coming from the conditional variance (Bhowmik & Wang, 2018). Nonetheless, the GARCH model can capture these characteristics due to the reasons outlined in the previous section.

In order to dissect the volatility of the return distribution, descriptive statistics are analysed. The data would have been transformed in a way that is meaningful to the user; for example, the descriptive statistics in this study should enable an investor or other user to make meaningful comparisons between the counters. In addition, the significance of this section lies in identifying patterns within the data in order to effectively present the other objectives of this study. Financial variables were statistically analysed starting from the first moment to the fourth moment of distribution.

The mean is the first step in describing a probability distribution function and is a metric that is used to determine the central value or position of the function, best known as a measure of central tendency. The mean is generally considered to be the most reliable measure of central tendency (Tsay, 2013), but the median and mode are also viable options. The mean is the first moment of analytics in statistics. The arithmetic mean, μ is statistically expressed as follows:

$$\mu = E(X) = \frac{1}{n}\sum_{i=1}^{n} x_i \quad \text{...(1)}$$

Where: μ is the mean n= number of observations x_i= expected values of x in this case, the returns.

The dispersion is another metric that may assist us in elaborating a description of a probability distribution function. The variance is the measure of dispersion that is commonly used. The variance generally measures the spread around the mean and represents the second moment of analysis. It is provided as follows:

$\delta^2 = Var(X) = \sum_i (x_i - E(X))^2 \, P(X = x_i)$……………………………………………..(2)

There are two possible ways in which a probability distribution might be organised around its mean: symmetrically or asymmetrically. The skewness of a distribution is a common metric that is used to describe its degree of asymmetry. When measuring skewness, a value that is negative indicates that the distribution is skewed to the left, while a value that is positive suggests the opposite. A normal distribution has a skewness of zero. The skewness of a distribution is best known as the third central moment of distribution and is expressed as follows:

$\frac{E((X-\mu)^3)}{(Var(X))^{\frac{3}{2}}}$ ………

…………………………………………………………(3)

Measuring the concentration of alternative outcomes in a probability distribution function's tail may reveal more information about the function (Hall & Asteriou, 2016). The most extreme possible values are located in the "tails" of a probability distribution function. In applications dealing with finances, these "tails" are the ones that provide information on the possibility of a financial collapse. The kurtosis is a measurement that takes both the peak and the tail into account. The term "kurtosis" refers to the fourth central moment of distribution and is expressed as:

$\frac{E((X-\mu)^4)}{(Var(X))^2}$………………………………………………………………………………(4)

It is well documented that the kurtosis of a normal distribution is 3. As a result, a leptokurtic distribution has a kurtosis greater than 3 and has a higher probability mass in the tails. Evidence abounds to support the claim that the kurtosis of a normal distribution is equal to 3 (ibid.). Hence, it is important to note that a distribution with a kurtosis larger than 3 is considered leptokurtic because it has a bigger probability mass in the tails.

That said, both in theory and practise, financial data has been widely known as not normally distributed. Due to the stochastic nature of the data series, achieving asymmetry is highly unlikely. Private equity investments are known to be different from traditional asset classes in that they are illiquid and highly risky. This study endeavours to bring evidence as to whether they exhibit the same characteristics as other financial assets. Hence, the study was posited on the assumption of non-normality, and the objective is hypothesised as follows:

- Hypothesis 1: The asset returns data is not normally distributed.

To achieve this, the study utilised the Jarque-Bera test for normality. The test statistic compares the skewness and kurtosis of the series to the values that would be expected from a normal distribution and quantifies the difference between the two. Under the assumption of a normal distribution, the Jarque-Bera statistic should be distributed as χ^2, and its calculation should have two degrees of freedom. The following is the formula for the Jarque-Bera:

$$JB = \frac{T-K}{6}[\frac{S^2}{4} * (K-3)^2] \qquad (5)$$

Where *T* is the number of observations, *S* is skewness and *K* is kurtosis (Bera, 1981).

The greater the Jarque-Bera value is, the less likely it is that the provided series was selected from a normal distribution (Tsay, 2013). This likelihood decreases as the size of the value increases. The null hypothesis for the Jarque-Bera test is that the series follows a normal distribution, while the test statistic for the Jarque-Bera test follows a chi-square distribution with two degrees of freedom. Since samples drawn from a normal distribution have an anticipated skewness of 0 and an expected excess kurtosis of 3, the null hypothesis is a joint hypothesis of the skewness being zero and the excess kurtosis being 3. This is because the null hypothesis states that the excess kurtosis will be 3. According to its definition, the Jarque-Bera statistic will be increased by any departure from the norm. The p-value that is supplied should be larger than 0.05 for the null hypothesis of normality to be accepted, say at the 5% level of significance. It is expected that the study fails to accept the null hypothesis of the Jarque-Bera test as the data is not normally distributed.

3.3.2 Objective 2: To model private equity returns and establish volatility dynamics using GARCH models

Since the beginning of human history, people have been fascinated by the future and have attempted to predict what lies ahead. Everyone, from the farmer attempting to forecast the weather to the king desiring to know where and with what weapons enemies will attack, is interested in predicting the future. Everyone attempts to predict the immediate and distant future. Numerous techniques have been developed in an effort to predict volatility, as forecasting has always been a subject of intense interest and study in the field of finance. However, the problem in financial markets is that the majority of volatility anomalies cannot be explained by conventional models. According to Kapusuzoglu and Ceylan (2018), while it is simple for behavioural finance to explain why a person has made a particular decision, it has proven more difficult to predict that individual's future behaviour. Nonetheless, this is an entirely new problem because it has developed in a systematic manner. It is argued and observed that the volatility of the stock market contradicts the predictions of long-standing present value models. In both theory and practice, the study of the volatility dynamics of financial assets is a central concept.

According to studies, global asset pricing models are incapable of explaining cross-sectional returns in developing and emerging markets. Through the works of Acheampong and Swanzy (2015), Michaelides and Spanos (2016), Jiang et al. (2018), and Carter, Miller and Ward (2017), the academic space is informed of the critical factors for asset pricing in emerging markets. This study examines the volatility dynamics of LPDs in Africa to validate Klonowsky's (2012) assertion that investments are determined by economic geographical factors or spatial factors that drive investments. Various models, including the GARCH model, have been developed to measure leverage effects because of the lack of understanding of stock market returns. In theory, the volatility of financial markets can be predicted in large part by its own volatility. Aside from this, the findings on volatility do influence economic forecasts, have the potential to spread anxiety, and in some instances can deter investors from taking advantage of favourable market dictates. In order to adequately answer the primary research question, the following were applied to the data:

Testing for ARCH effects

To apply GARCH models to the LPEs return series, the presence of stationarity and ARCH effects in the residuals is tested. To do this, the Ljung Box test, which is one of the methods for testing for the absence of serial correlation, is used. It was developed by Box and Pierce in 1970. The hypothesis is as follows:

- Hypothesis 2: There exists ARCH effect in the data

Tests statistics in both cases are shown in the methodology chapter.

Modelling data using GARCH models

The GARCH model and its extensions are used to model all data that passes through the ARCH effects. According to Gyamerah (2019), the GARCH is an extension of the ARCH model that integrates the moving average with the autoregressive model. The study restricted all GARCH models to 1.1 for tractability. The standard GARCH model developed by Bollerslev in 1986, the Threshold GARCH developed by Zakoian (1994), the GARCH-in-Mean developed by Engle, Lilien, and Robins (1987), the Exponential GARCH developed by Nelson in 1991, and the FIGARCH model that was developed by Baillie Bollerslev and Mikkelsen (1996) were all hypothesised as follows:

- Hypothesis 3: The model coefficients are statistically significant

The study's goal is to develop fundamental understandings of stylised effects associated with the data under consideration's LPEs. In the preceding chapters, leverage effects, risk-return characteristics, and spillover effects, among others, are investigated and explained.

Testing for spillover effects for the data under study

The degree of covariation or volatility of stock prices is often taken into consideration when making judgements on the allocation of a portfolio's assets. Atenga and Mougoué (2021) notes that it is vital, in addition to analysing the volatilities of these investments, to investigate the occurrence of spillover effects. If the volatility of one market influences the volatility of the other, the existence of any direct volatility transmissions (spillover effects). It is, therefore, important to decompose volatility across data series in order to gain an understanding of the key drivers of volatility in LPE investments of some African markets. The findings offer valuable information to an investor concerning the opportunities for diversification. From the discussions highlighted, African markets are highly segmented, therefore we can hypothesis that:

- Hypothesis 4: There exists spillover effects amongst LPE in countries under study

In testing the hypothesis, parameters are established by using heteroscedasticity that the financial variables exhibit in the using a GARCH models, using GARCH DCC models. The DCC models are well designed to capture correlational clustering and spillovers in financial time series analysis (Amudha & Muthukamu, 2018).

3.3.3 Objective 3: To establish the impact of country-specific factors on returns for listed Private Equity investments in Africa

As a country develops, its market moves from segmented to integrated, and country-specific factors are eliminated through diversification, reducing the volatility of financial assets in the economy (Erb, Harvey & Viscanti, 1996). Hence, this market tends to have returns that are negatively correlated with developing markets. So, investing in a developing country offers diversification potential. Private equity in Africa is becoming popular, as witnessed by an increase in fundraising activities (EMPEA, 2018). It is expected that the resulting capital flows result in the integration of these capital markets. The impact of country risk is gradually being reduced as risk sharing and capital flows improve. The objective is therefore distilled into the following 3 testable hypotheses:

- Hypothesis 5: There exists structural contemporaneous interaction within country specific factors and returns.

The hypothesis given is consistent with Erb, Harvey and Viscanta (1995). The hypothesis is posited on the assumption that country-risk measures are correlated to expected stock returns in emerging and developing markets (ibid). The global price of risk is less than that of the local market because the world market portfolio is less volatile than the local market. In developing countries, there is high volatility with correspondingly high future returns, markets are segmented from the developing world and these returns are influence more by local information. Hence, we expect that increases of this local information would result in a corresponding increase in returns. We can, therefore, hypothesise that country-specific factors are a priced factor in asset pricing and valuations especially in private equity investments in Africa where funding mostly comes from developed nations; a marginal increase in country risk factors should have a corresponding increase in returns (according to CAPM propositions) for private equity investments in Africa as follows:

- Hypothesis 6: Country specific factors are a priced factor in LPE valuations in the data under study

Literature provides justification for the need for statistical evidence describing the relationship between country risk and returns in Africa. According to De Wet (2005), establishing only the initial responses of variables may not provide a more accurate description of their characteristics. A response to one variable influences the volatility of the other variable; therefore, a second-moment analysis is required.

Although it has been acknowledged that there is a relationship between country risk factors and returns, the extent to which these financial variables interact remains unresolved. According to Erb, Harvey, and Viskanta's (1995) study on country-risk measures, there is a strong correlation between equity valuation and country-risk measures. Given that private equity investments are illiquid and more volatile than traditional asset classes, the research identifies an unexplored possibility of determining the structural relationship between these risk factors and private equity investments, which are more volatile than traditional asset classes. The study used the VAR model to examine the impact of country-specific factors on LPE investments. VAR makes the data well suited for assessing interdependencies that exist among variables (Marvelous, ,2017). The model consists of the LPE returns, the GDP and Inflation. Impulse response functions and the variance decomposition was also used to complete the analysis. The impulse response functions trace impact to shocks of one endogenous variable to the other in the VAR system, whilst the variance decomposition splits the variation in an endogenous variable to shocks in the VAR system (ibid). As a result, the variance decomposition informs us about the impact of each innovation in influencing the variables in the VAR.

3.4 Chapter Conclusion

In this chapter, the study's theoretical framework and justification were outlined, and the research questions analysed. In addition, the methodologies used to test these research questions were examined. The research problem was examined in order to develop a scientifically and statistically testable hypothesis. This chapter is a continuation of the preceding one because it provides a unique perspective on analysing the expected returns provided in an academic setting. The chapter elaborated on the notion from the previous chapter that investments in Africa are a function of country risk factors and premium demands resulting from the illiquid nature of the asset class; therefore, it is equally important to establish the structural contemporaneous interaction between country risk factors and returns for private equity investments in order to provide an understanding of how volatility is generated within and outside the system of country-specific factors. Unlike previous studies that examined cross-sectional investment returns, this study focused on listed private equity investments as a development financing tool in Africa. The study, therefore, worked on the following hypotheses:

- Hypothesis 1: The LPE returns data are not normally distributed.
- Hypothesis 2: There exists ARCH effect in the data
- Hypothesis 3: The model coefficients are statistically significant
- Hypothesis 4: There exists spillover effects among LPEs in countries under study
- Hypothesis 5: There exists structural contemporaneous interaction within country-specific factors and returns.
- Hypothesis 6: Country-specific factors are a priced factor in LPE valuations in African markets

The next chapter examines methodological considerations.

4 CHAPTER FOUR: REVIEW OF METHODOLOGICAL APPROACH

4.1 Chapter Introduction

Volatility modelling of financial assets is crucial because it is frequently regarded as a significant factor in asset valuations, risk quantifications, investment decision-making, stock valuation, and the announcement of monetary policy. The stock market's volatility varies over time. It is widely accepted that fluctuations in market volatility are predictable. For the purpose of projecting volatility and quantifying the degree of certainty associated with such forecasts, empirical studies on a broad range of topics have employed a variety of models.

There are a variety of modelling techniques, including Random Forests, Quantum-physics approaches like the Hilbert space, ARMA models, GARCH models, Machine learning, Logit models, and Probit models, to name a few. Financial economists utilise statistical models based on the Gaussian random walk and are considered traditionalists. In utilising statistical models, researchers analyse specific stock market events. In any case, researchers are guided by the principle of parsimony, which states that the preferred model is the one that provides the simplest scientific explanation consistent with the evidence presented. The previous chapter provided a chronology of the subject matter and contextualised pertinent issues. In Africa, it was deemed necessary to model the volatility of LPEs and establish the relevance of country-specific factors in valuation models.

This chapter reviews the associated methodological issues and is structured as follows: The first section discusses the research philosophy; the second, the research design; the third, a review of methodological choices discussing GARCH models used d in the study; the fourth, diagnostic tests and residuals analysis; and the fifth, spillover effects and m-values. The chapter concludes by analysing the justification for the utilised time period.

4.2 Research Philosophy

According to Creswell and Creswell (2018), research philosophy is a framework of values and beliefs used in carrying out research. As a result, the research philosophy is discussed prior to descriptions of specific methodologies used in the study in order to best clarify the structure of inquiry and the methodological approach surrounding the study.

4.2.1 Positivist approach

The study was carried out in accordance with Comtee's positivist philosophy, which argued in 1853 that there is no other factual knowledge or comprehension other than that which is based on observed evidence or facts. According to Mishra Bhushan and Shashi (2017), positivism derives from the natural sciences because it tests hypotheses derived from established theory by measuring observable social realities. The principal belief is that knowledge is valid, objective, and universal in the sense that generalisable theoretical models can be developed (Saunders, Lewis & Thornhill, 2012). The purpose of the study was to gain a better understanding of private equity investments by analysing the short-run behaviour and structural relationships in the first and second moments with country-specific factors in Africa and recommending appropriate investment strategies. Consequently, this study employed formal logic to validate or assess empirical knowledge.

Furthermore, the study intended to investigate the returns and volatility dynamics of private equity investment activity, as well as how they responded to plausible changes in country risk factors .The results of this empirical investigation have implications for investment decisions; therefore, the use of a positivist paradigm, which entails the use of consistent logical approaches, allows for the elimination of bias in order to achieve the greatest possible objectivity by deriving results from observed facts (Gujarati & Porter, 2009).

Tran, Minh, and Tuam (2020) hypothesise that the truth of a theoretical proposition can only be established after the certainty of an empirical fact has been determined. Therefore, the verification process enables the determination of all certainties regarding the study's underlying theory. Using empirical data to model returns on private equity investments in Africa will enable the establishment of the model's truth value. Given the research questions at hand, this doctrine will pave the way for an approach that combines deductive and inductive reasoning.

This study aims to model the volatility of listed private equity returns, statistically examine the structural relationship between them, and determine how they interact with certain country risk factors. The researcher employed the co-relational research design, which measures the degree of association between variables through statistical analysis. The causal relationship between listed private equity and country-specific factors in Africa has not been determined, leaving investors without a conclusive understanding of how and why this asset class's volatility is generated. According to Creswell and Creswell (2018), this design is appropriate when the researcher has a priori knowledge of the topic and seeks additional information regarding the direction of causality. This research utilised secondary data derived from publicly available information to provide results that provide substantial insight into a particular investment option.

4.2.2 Inductive and deductive approaches

The objective of attempting to explain the volatility of private equity returns and then identifying possible country-specific variables that influence returns in these selected countries entails that utilise existing knowledge to generate new theory by identifying the variables. This validates its status as an inductive method. According to Saunders, Lewis, and Thornhill (2012), an inductive approach is one that employs known premises to generate untested conclusions. In this instance, secondary data was utilised to formulate a theory.

In addition, the objective of analysing the structural relationship between country-specific factors and returns on private equity investments necessitates a deductive methodological approach in which the data was used to test the existing theories. The study then built an empirically tested framework.

The purpose of this study is to investigate the investment behaviour of this asset class, which can be used to evaluate potential investments and portfolio construction strategies in Africa's emerging economies. Inferring from a sample to the economy as a whole indicates that the model is based on inductive reasoning.

4.3 Choice of Analysis

In order to forecast financial data, a statistical model is required. In the study, financial time series were presented in the form of daily log returns, and the key concept was a model that could analyse the dynamical structure or behaviour of the log returns over time. Engle, who was awarded the Nobel Memorial Prize in

Economic Sciences in 2003, introduced ARCH ("Autoregressive Conditional Heteroscedasticity") models in 1982. Then, Bollerslev (1986) proposed GARCH models as a generalisation of the ARCH process. The primary benefit of GARCH models is that they can capture a number of important characteristics of financial time series.

One could also consider time series using the Autoregressive Moving Average (ARIMA) method developed by Box and Jenkins. This model's fundamental assumption is that it cannot be applied to data that is not normally distributed. In practice, it is impossible to achieve symmetry when dealing with financial data; therefore, the study utilised the data to identify the presence of significant ARCH effects. Significant ARCH effects indicate that the variance is non-constant and varies over time (Shanthi & Thamilsevan, 2019). In such cases, volatility is not modelled using ARIMA but rather GARCH models.

4.4 Research Design

This study utilised secondary data in the form of stock prices and macroeconomic variables. The study focused primarily on volatility models that incorporated time-varying conditional moments and persistence asymmetries, patterns of volatility clustering and mean reversion characteristics. Consequently, important variables considered were the stock prices of publicly traded companies that invest in private equity. Daily and monthly closing prices for 2010 to 2020 were obtained from the Africanmarkets.com and Yahoo Finance websites.

The study began by analysing and selecting all the counters for the various exchanges that invested in private equity investments. In some countries, private equity was still in its infancy; therefore, the data set for newly listed companies that invest in the private equity space could generate meaningful conclusions. However, the study selected only companies that had been listed on an exchange since 2010. Notably, the returns utilised were lognormal returns, and missing data was assumed to be the average of the previous and subsequent prices.

In Africa, countries South Africa, Ghana, Botswana and Egypt were found to have the type of data that was adequate for the research. Firstly, they have listed companies that have vested interests in private equity investments (LPEs). Secondly, even though other countries do have the LPEs on the exchanges, the companies are still in infancy and do not have sufficient data to model the return series. In general, modelling data requires a minimum data point of at least 30. The smaller the data, the less significant the results (Tsay, 2013). The study, therefore, selected the four countries on the basis that the counters had adequate data for modelling (10 years).

Daily and monthly data observations covered the period from 01 January 2010 to 01 September 2020. Log returns were used in the study because they have the characteristic of stationarity and mean reversion which was essential for building stable statistical and econometric models (ibid).

The data was first subjected to descriptive analysis via trend analysis and descriptive statistics. This was done to analyse and compare the patterns of the data over time in order to establish a connection between the behaviour over time and the economic environment, as well as to examine the measures of central tendency. The mean, median, maximum and minimum returns, standard deviation, skewness, kurtosis, and Jacqui-Bera statistics were used to analyse the data. The descriptive analysis was conducted using EViews 12 with a confidence interval of 99% and NumXL with a confidence interval of 90%.

The study investigated volatility dynamics using the TGARCH, EGARCH, GARCH in Mean, and GARCH (1,1) models, and a country-by-country analysis was conducted to determine the model that best fits each country's investment. The models were chosen based on the RMSE, the MAPE, and the Theil Inequality Coefficient. The study employed Nyblom's parameter stability test and the news impact curve as diagnostic tests to ensure model stability in all instances.

FIGARCH models were used to investigate the investment's long-term memory because GARCH models are traditionally designed to capture the short-run volatility time dependence behaviour of the series while ignoring the long-term behaviour. The diagnostic tests were based on the analysis of the news impact curve.

The study concluded by analysing the structural relationships of LPE investments and identifying their spillover effects and the structural relationship between returns and country-specific factors. To accomplish this, the research employed MGARCH models employing DCC and VAR methodologies.

4.5 Review of Methodological Choices

This section examines the various methodological approaches for estimating volatility and the contemporary issues surrounding them. Since the inception of the GARCH family of models, numerous researchers have utilised the outputs to establish the volatility patterns of stock prices. Modelling volatility in financial markets is crucial because it is frequently regarded as a significant factor in asset valuation, risk quantification, investment decision-making, stock valuation, and monetary policy formulation (Gujarati & Porter 2009). The volatility of the stock market is virtually time varying. It is widely accepted that fluctuations in market volatility are predictable.

4.5.1 Volatility estimation models

Since the beginning of the 20th century, models have been created to describe the behaviour of asset returns. The field of econometrics was founded on the modelling of the mean until 1980. The handling of time series involved modelling the actual values of the series. In the middle of the 1980s, the importance of modelling volatility and its effects on the mean became apparent. In brief, analysis has shifted from first order to second-order moments. Financial variables interact with one another at both the second and mean moments (Shanthi & Thamilsevan, 2019). This suggests that changes in the variance of a time series have an effect on changes in the time factor that follows it, which may have an effect on the volatility of other related variables.

For a better understanding of returns, forecasting, building portfolios, and other investment options, it is important to investigate how they move and how they relate to each other.

Engle developed the popular GARCH models in 1982 after discovering insights about the volatility clustering that surrounds financial time series. In this context, large changes in volatility are correlated with large changes, whereas small changes have the opposite effect. Bollerslev (1986) later discovered that the variance of Engle's ARCH effects is divisible into conditional and unconditional variance, allowing the unconditional variance to be modelled with its innovations. Several recent empirical studies by Wang, Xiang, Lei and Zhou (2022), Silva (2022), Sen, Mehtab, and Dutta (2021), Engelhardt et al. (2021), and Stefan, Daniel, and Camelia

(2021) have employed a variety of GARCH models for predicting future volatility and assessing the accuracy of volatility forecasts. Other studies have also criticised this GARCH model by Bollersleve (1986), citing its weaknesses in terms of the non-negativity assumptions, and have improved upon it by creating hybrid models that are able to capture the strengths of GARCH and draw upon the strengths of other models to optimise prediction accuracy. Such studies include, to name a few, Siti and Kasypi (2021), Koo and Kim (2022), and Kim and Lee (2018).

Despite their limitations, the ARCH and GARCH models have become indispensable in the analysis of financial time series. These models are quite useful when the objective of the analysis is to estimate future returns and explain the behaviour of those returns. The behaviour of time series, according to Kumar and Biswal (2019), is determined by three statistical properties: volatility clustering, leptokurtosis, and non-linear dependence. When periods of high volatility and periods of low volatility are separated, volatility clustering occurs. a situation in which high volatility today is followed by high volatility tomorrow and the next period, and vice versa whenever volatility is low. This could be explained by the fact that information in finance that tends to influence price fluctuations tends to arrive in irregularly spaced batches (Sen, Mehtab & Dutta, 2021).

When distributions in a series have fat tails and high peaks at the mean, they are said to be leptokurtic, and when returns for different assets move in the same direction due to market forces (Dixit & Agrawal, 2019), they are said to be skewed. This is typically the case during financial crises and is even more prevalent in multivariate time series data. Throughout the years, GARCH models have evolved into numerous distinct types, some of which are discussed in the next section.

4.5.2 Auto regressive Conditional Heteroscedastic (ARCH) models

In the words of Engle (1982:987), the ARCH model is best described as "*mean zero, serially uncorrelated processes with non-constant variances conditional on the past but constant unconditional variances."* The ARCH process means that the series in question has a time-varying variance that depends on lagged values (heteroscedastic autocorrelation). The positives of the ARCH models are that they can generate accurate models for financial forecasting, but the condition for doing so is that high ARCH orders have to be selected in order to achieve that. This notion was supported in this study by the findings analysis chapter. This is a daunting task to achieve, and, with the principle of parsimony taking precedence, the study chose GARCH models as low-order parameters can easily achieve the same result as over parameterised ARCH models. Apart from that, higher-order estimation also violates the non-negativity assumptions that underpin these models.

4.5.3 Generalized Auto-Regressive Conditional Heteroscedasticity (GARCH) Models

As alluded to in earlier discussions, GARCH models provide a solution to ARCH's deficiencies; hence, they became the more popular model than ARCH. The GARCH (*q, p*) is given by the term;

$$\sigma_t^2 = w + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{i=1}^{P} \beta_i \sigma_{t-1}^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Where q represents the order of ε_t^2 and p represents the order of σ_t^2.

The GARCH (1,1) specification, a time varying conditional volatility, is a function of its own past lag one term plus the past innovations. The conditional variance equation in GARCH (1,1) is modelled as

$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ ……………………………………………………………..(7)

Where;

$\varepsilon_{t-1}^2 - are\ returns\ with\ mean\ (\mu)\ of\ zero\ and\ \sigma\ of\ 1$

$\alpha_1 - Alpha$ $\beta_1 - Beta$ α_0- *constant*

$\alpha_0, \alpha_1\ and\ \beta_1$ are coefficients of the model with the following conditions:

$w > 0;\ \alpha_1 > 0; \beta_1 > 0: \alpha_1 + \beta_1$ <1. This condition shows that all parameters $\alpha_0, \alpha_1\ and\ \beta_1$ are non-negative. The stationary condition of $\alpha_1 + \beta_1$ <1 should hold to ensure weakly stationarity of the GARCH process. α_1 indicates short run persistency of shocks while β_1 implies the long-run persistency.

Silva (2022), Li, Xing, Huang, and Li (2022), and Bhowmik and Wang (2019) modelled volatility using GARCH models and found that GARCH models produce accurate predictions of stock volatility in the future.

Song (2022) did comparative analyses of the GARCH models and deep learning on machine learning and noted that the forecasting effect of deep learning on a machine learning methodology does outperform GARCH models. However, according to studies by Sen, Mehtab, and Dutta (both of 2021), Shanthi and Thamilsevan (both of 2019), Kapusuzoglu and Ceylan (both of 2018), and others, the GARCH remains a good approach to modelling volatility.

GARCH models exhibit stylised facts such as heavy tails, asymmetric returns, absence of correlations, and volatility clustering (Kumar & Biswal, 2019); apart from that, if the time interval in which we are estimating returns is increased, the unconditional returns get closer to a normal distribution. These stylised facts were unravelled in studies by Wang, Xiang, Lei and Zhou (2022), and Wang, Tsai, and Li

(2019), who noted that due to problems associated with GARCH (p, q) models, several extensions of the model were proposed. One way of circumventing non-negativity challenges is to use artificial constraints to force negative coefficients to be non-negative. In addition, GARCH models do not account for leverage effects, nor do they allow for direct interaction between the conditional variance and the mean (Amudhaw & Muthukamu, 2018); hence, other GARCH extensions have been developed to manage restrictions associated with the basic GARCH model. The table 4-1 shows a summary of some GARCH models and their main features.

Table 4-1: Summary of GARCH models and their main features

Year developed	Model	Major characteristics
1986	GARCH	The model is persimonious, requires few parameters and has few restrictions on coefficients
1988	VECH-GARCH	This model is expressed in terms of a vectorised conditional variance matrix
1995	BEKK-GARCH	This is a multivariate model that estimates the conditional mean function and conditional volatility function of high dimensional relationships which are used to test volatility spillovers between the multi-market segments when studying more than one variable or want to study the spillover effect.
1990	CCC-GARCH	An n-dimensional GARCH model was proposed that comprises univariate GARCH processes related to one another with a constant conditional matrix, hence the name Constant Conditional Correlation GARCH
1991	EGARCH	This model presents the risk aversion properties of the return series. It takes the mean to be a function of the conditional volatility of the return series (Koutmos D, 2012). Similar to the TGARCH in that it captures the leverage effects of shocks in a financial market.
1994	TGARCH	The main target of the TGARCH model is to capture asymmetries in terms of negative and positive shocks.
1996	FIGARCH & FIEGARCH	These models possess a long-term memory nature that allows for modelling volatility of variables. The models allow for a slow hyperbolic decay rate for

		lagged innovations in the conditional variance function.
2001	DCCGARCH	The DCC is used to study the interdependency of one variable to another. It examines the covariance between two or more variables
2012	Realised GARCH	A model that specifies the properties of returns to realsied measures. It facilitates the modelling of the returns and future volatility refered to as the leverage effect.
2013	GARCH MIDAZ	The methodology uses mixed data sampling to model data and is suitable for long-term relationships

Source: Researcher summary compilation

4.5.3.1 Exponential GARCH (EGARCH) models

The EGARCH model was put forward by Nelson in 1991 in an attempt to solve the problem of the basic GARCH model being unable to capture asymmetric effects on conditional variance. A recent study by Bakry (2022) on COVID impacts demonstrated that news pronouncements have an asymmetric effect depending on geographical locations. Other studies, such as Wang et al. (2022), Siti and Kasypi (2021), and Varughese and Mathew (2017) conducted analyses based on this approach and concluded that EGARCH models are a good fit for analysing investments.

The EGARCH (*q,p*) is as follows:

$$\log(\sigma_t^2) = w + \sum_{i=1}^{q}[\alpha_i\varepsilon_{t-i} + \lambda_i(\alpha_i|\varepsilon_{t-i}| + E|\varepsilon_{t-i}|)] + \sum_{i=1}^{p}\beta_i \log(\sigma_{t-1}^2) \ldots\ldots\ldots\ldots\ldots(8)$$

Where the EGARCH (1, 1) is:

$$\log(\sigma_t^2) = w + \alpha_1\left[\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}\right] + \lambda_1\left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}\right] + \beta_1 \log(\sigma_{t-1}^2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

- ε_t are returns with zero mean and unit variance
- $w,\ \alpha_1,\ \beta_1, \lambda_1$ – model coefficients
- $\alpha_1\varepsilon_{t-1}$ is a sign or asymmetry effect
- $\lambda_1(\alpha_1|\varepsilon_{t-1}| + E|\varepsilon_{t-1}|)$ is a magnitude effect

If the ARCH coefficient term is close to zero whilst the GARCH coefficient term is close to 1, then it means that GARCH effects are stronger than those for ARCH suggesting volatility effects have more persistence than past shocks impacts. If the sum of the two coefficients are close to 1, then it indicates the presence of a long memory process.

Since the logarithms are always positive, then constraints have naturally been dealt with. If the $\lambda_1 < 0$ then it implies that the negative shocks generate a larger volatility than positive shocks. And when $\lambda_1 > 0$ the positive news generate larger volatility than negative shocks.

Amudha and Muthukamu (2018) established the importance of leverage effects on investment appraisals and noted that generally the effects of bad news on the volatility of a share far more outweigh the effects of good news as investors are more worried of bad news than good.

4.5.3.2 Threshold (TGARCH) GARCH models

The TGARCH model is also called the GJR model, named after the people who developed the model: Glosten, Jagannathan, and Runkle (1993). This model enables us to differentiate the conditional variance for good and bad news or shocks. The GRJ GARCH (q, p) model takes the following form:

$$\sigma_t^2 = w + \sum_{i=1}^{q}(\alpha_i + \lambda_i I_{t-i})\varepsilon_{t-1}^2 + \sum_{i=1}^{P}\beta_i\sigma_{t-1}^2 \quad \text{..(10)}$$

Where

$$I_{t-i} = \begin{cases} 1, if\ \varepsilon_{t-i} < 0 \\ 0, Otherwise \end{cases}$$

TGARCH (1,1) model:

$$\sigma_t^2 = w + (\alpha_1 + \lambda_1 I_{t-1})\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \quad \text{.......(11)}$$

Where $I_{t-1} = \begin{cases} 1, if\ \varepsilon_{t-1} < 0 \\ 0, Otherwise \end{cases};$

$w,\ \alpha_1,\ \beta_1,\ \lambda_1$ are coefficients for the constant, alpha, beta and the asymmetric term respectively. The non-negativity conditions are such that $w > 0, \alpha_1 > 0,\ \beta_1 \geq 0,$ and $\alpha_1 + \lambda_1 \geq 0$. In addition, the model is still admissible even if $\lambda_1 < 0$ provided that $\alpha_1 + \lambda_1 \geq 0$.

Ever since the inception of the GARCH models, researchers have used the technique to model data in finance. Budi Setiawan et al. (2021) did a comparative study on the volatility dynamics of stock markets in emerging and developing markets for two crisis periods, the COVID-19 period and the Global Financial Crisis (GFC), and noted that the COVID-19 period had negative stock market returns of a greater magnitude than the GFC. Lum and Islam (2016) used the GARCH models to investigate the Australian markets and noted that the stocks do have asymmetric distributions and that fitting MGARCH models helped improve the forecasting power of the data. In contrast, a study by Shanthi and Thamilsevan (2019) on Indian markets showed that the data has periods in which asymmetric effects are not found, but TGARCH models proved the most suitable way of modelling the data. Amongst studies that investigated volatility persistence is that by Fakhfekh (2021) which concluded that volatility across all indices in Tunisia was lowest in the construction and food and beverage sectors, which had insignificant asymmetric and persistent effects, while the finance sector, the automobile sector, and the insurance sector displayed significant asymmetric effects. These findings are then important in accounting for volatility and constructing hedging strategies.

GARCH models have also been used in studies by Silva (2022), Chronopoulos et al. (2018), Li, Xing, Huang and Li (2022), Dixit and Agrawal (2019), and Trivedi et al. (2021), to name a few. There are also other studies that have infused the GARCH models with machine learning (Song et al., 2022; Koo & Kim, 2022), which noted that the hybrid models perform better than them operating individually.

4.6 Modelling for Long Run Volatility: FIGARCH Models

ARCH and GARCH models help explain the behaviour of data by giving room for conditional variance to respond to past behaviours over time. Studies have shown that volatility series do possess a long memory, which affects their future volatility over a long time horizon (Chen et al., 2022). Considering this, Baillie Bollerslev and Mikkelsen (1996) developed the FIGARCH model. According to Baillie et al. (2007), "long memory" is the presence of a slow hyperbolic decay in autocorrelations and impulse response weight. The long-memory nature of the fractional GARCH family of models allows them to be better volatility modelling techniques compared to other heteroscedastic models as they are better able to improve the accuracy of forecasts and provide efficiency in parameter estimations. Hence, the study explored them as investors are worried about long-run dependency in portfolio selection and when looking at estimations of value at risk in financial risk management. Other studies, such as Babyemi et al. (2022), Bawa et al. (2020), and Haque and Farzana (2021) have confirmed that the FIGARCH model produces the best fit and is relevant for modelling long memory processes. The FIGARCH model takes the following form:

$$\sigma_t^2 = w[1 - \beta\ (L\ )^{-1} + \{1 - (1 - \beta(L)^{-1}\ \phi(L)(1 - L)^d\}\mu_t^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(12)}$$

Or

$$\sigma_t^2 = w[1 - \beta(L)^{-1} + \lambda(L)\mu_t^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(13)}$$

Where *L* is the lag operator and *d* refers to the fractional difference parameter that measures the degree of long memory behaviour.

Despite the fact that the FIGARCH model is better able to capture the long memory process, Hsieh, Chung, and Lin (1999) point out that the differencing part at times does have structural problems, which then results in biased parameter estimations; hence, he proposed a FIGARCH model (a fractionally integrated exponential GARCH), in which the exponential tries to capture the asymmetrical issues that point to biased estimates. Other extensions of the FIGARCH are the FIAPARCH and plenty other hybrid model extensions such as VACD-FIGARCH, DCC-FIGARCH, and ARFIMA-FIGARCH.

4.7 Diagnostic Tests

The study exposed the data to some diagnostic tests. Since GARCH models are non-linear models, the study utilised the quasi-maximum likelihood (Q-MLE) approach to estimate the most likely values that the parameters can take. In general, the Q-MLE is known to be consistent, in contrast to maximum likelihood estimation models, which are used when dealing with linear data.

According to Tsay (2013), the Q-MLE technique possesses a normal limiting distribution and offers asymptotic standard errors that are valid under non-normality. Under this framework, the formula that the log likelihood function would take is given by the following representation:

$$Q - MLE = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t-1}^{T} log_t^2 - \frac{1}{2}\sum_{t=1}^{T}\frac{\varepsilon_t^2}{\sigma_t^2} \ldots\ldots\ldots\ldots (14)$$

4.7.1 Tests for normality

For each mean and standard deviation combination, a theoretical normal distribution can be fitted. Hence, when testing for normality, the data has to be fitted into a normal distribution to see how it performs and whether it can conform. Probability plots help us compare two data sets in terms of their distribution. The Quantile-to-Quantile plot can be used to test for normality and assists in detecting skewness and kurtosis in the data, which can be tested by coming up with a histogram for the return data and using the Jacquie-Bera statistic for normality. The probability-to-probability (P-P) plot uses the cumulative distribution to measure how well the theoretical distribution fits given data. The theoretical distribution can be normal, lognormal, exponential, beta, gamma, etc. P-P plots magnify deviations in the middle.

4.7.2 ARCH LM Test

A collection of random variables is said to be heteroscedastic if there are subpopulations with different variability. This is an occurrence that is not desirable in the application of regression analysis and the analysis of variance, as it can make the research wrongly specified. The ARCH LM test is a Lagrange multiplier test for detecting ARCH effects in the residuals. This particular heteroscedastic requirement was inspired by the observation that in many financial time series, the size of residuals appeared to be related to the size of recent residuals. The ARCH LM statistic is calculated by testing the null hypothesis that there is no ARCH of order *q* in the residuals. Then the following regression is run:

$$\varepsilon_t^2 = \beta_0 + \left(\sum_{j=1}^{q}\beta_j\,\varepsilon_{t-s}^2\right) + v_t \ldots\ldots\ldots\ldots (15)$$

4.7.3 Forecasting power tests

Finally, the predictive power of the model was used to analyse how accurate the forecasted values were in relation to what is observed. Numerous studies have employed varied performance measures to assess the forecasting performance of the different models that are used in volatility modelling. For this study, the Mean Absolute Error was used, and it was calculated in the following manner:

$$MAE = \frac{1}{T} \sum_{t=1}^{T} \left(\left| \widehat{\sigma_t^2} - \sigma_t^2 \right| \right) \ldots\ldots (16)$$

4.8 Error Distributions

As previously stated, financial data is not normally distributed and frequently exhibits "fat tail" characteristics. For series that exhibit fat tails, normal distribution cannot adequately account for that characteristic. This forces analysts to make assumptions about the distribution of these returns. This is the major reason behind the preference of students' scores as an error distribution in GARCH models as compared to the normal distribution, the negative inverse normal, and the generalised error distribution (GED). Bollersley (1987) invented the students' T distribution, while W Gosset invented the GED in 1908 and popularised by Nelson (1991). Some empirical studies such as Wang, Chan, and Choy (2011), and Feng and Shi (2017) demonstrated that the Students' T distribution and the GED fits were best for GARCH modelling in financial time series. The data was subjected to three GARCH model error constructs in order to select the ideal error distribution for the study: the Gaussian normal distribution, the students' T distribution, and the generalised error distribution (GED), resulting in the selection of the most preferred model. The GED proved to be the most preferred model as it had the highest log likelihood and the lowest Schwartz IC; hence qualifying enough to be taken for further evaluation.

The probability density function for a normal distribution is given as follows:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(17)}$$

Where σ is the standard deviation whilst μ is the mean of the distribution

If the $\mu = 0$ and $\sigma = 1$ then a normal distribution is obtained and the data is symmetric around the mean. The Students' Tprobability density function is given by:

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\nu\pi}} (1 + \frac{x^2}{\nu})^{-\frac{\nu+1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .. \text{(18)}$$

where v is the number of degrees of freedom where $v > 0$ and the mean is 0 for $v > 1$, otherwise undefined and the variance is given by $\frac{\nu}{\nu-2}$ for $v > 2$, otherwise undefined. The T-distribution has heavier tails which means that the probability of values falling far from its mean is higher than that of the normal distribution and is also symmetric and bell-shaped like the normal distribution (Lingbing F & Yanlin S, 2017).

4.9 Residuals Analysis

The study exposed all the GARCH models under study to several stability tests. The first is the Nyblom stability test, which was developed by Nyblom in 1989. The study used the Nyblom stability tests to measure parameter stability in both the mean and variance components of the GARCH model. The null hypothesis of this test is that the parameters are stable, i.e., they do not change with time. The results of this test are presented and discussed in the findings chapter.

The other test conducted is the autocorrelation test. The ACF tool describes how well the present value of the series is related to its past values. The study used the autocorrelation function (ACF), as it provides values of autocorrelation of the series with its lagged values, to check how it fares with the unit root test. The Ljung-Box Q statistic was used in the study. This is a statistical test that assesses whether any group of autocorrelations in a financial time series are significantly different from zero. It, therefore, tests the overall randomness of a number of lags that are dependent on the choice of the researcher when using the statistical software EViews, which was used for analysis in this research and is generally called a portmanteau test. On testing whether a series has significant autocorrelations, the Q test is calculated by:

$$Q_k = T\,(T+2)\sum_{i=1}^{k}\frac{r_i^2}{T-i} \quad \text{(19)}$$

Where *T* represents number of samples, *k* stands for the number of lags and r_i is the *i*th correlation. The null hypothesis is that there are no autocorrelations.

In addition, the study conducted the sign bias test, also called the Engle and Ng test. This test was proposed by Engle and Ng in 1993 to analyse the significance of leverage effects on the residuals of the model. The study questioned the presence of positive or negative shocks on future volatility and whether the magnitude of the shock also affected future volatility, in order to provide a good justification for estimating some asymmetric GARCH models.

Of the three tests that exist in this test, the study utilised the one that investigates both size and sign bias simultaneously. The regression is as follows:

$$u_t^2 = \phi_0 + \phi_1 S_{(t-1)}^{-} u_{(t-1)} + \phi_2 S_{(t-1)}^{\mp} \phi_3 S_{(t-1)}^{-} u_{(t-1)} + v_t \quad \text{(20)}$$

Where u_t^2 denotes the squared residuals of a GARCH model fitted to the returns is a constant as a dummy variable that takes the value of 1 if < 0 and zero if otherwise. The results of these tests are shown in the findings chapter.

4.10 Tests for Spillover Effects: MGARCH Models

When one univariate time series has an impact on another univariate time series, it is known as multivariate analysis. A multivariate series has more than one time-dependent variable. There may be many variables on which the series might depend. The relationship between the volatilities and co-volatilities of several univariate variables or markets is investigated using a multivariate GARCH model.

The study investigated the MGARCH models to determine whether listed private equity investments in Africa are interrelated, given that portfolio allocation decisions are influenced by the degree of covariation of stock prices or volatility following a shock.The study was looking for spillover effects, specifically whether the impact of positive and negative news is the same in markets of the same size.

The different types of MGARCH models are the constant conditional correlational GARCH (CCC GARCH), the dynamic conditional correlational GARCH (DCC GARCH), the varying conditional correlational GARCH (VCC GARCH), and the Baba Engle Kraft Kroner GARCH (BEKK GARCH).

Katzke (2013) used DCC and BEKK models and noted that uncertainties in both local and global markets significantly influenced the short-run dynamics of sectoral returns in South Africa. Another study on 15 world indices using MGARCH models (BEKK) by Sing et al. (2008) found positive spillover effects affecting Indian markets, primarily Asian markets, and the US market, while Indian markets have a negative impact on the US and Pakistan markets. Using the daily returns of the Gulf equity markets, Rao (2008) used MGARCH and VAR models to note significant spillover effects and persistence in these markets. Other similar studies include Matei, Rovira, and Agell (2019), and Peng et al. (2017). Hence, the study found it necessary to identify spillover effects among listed private equity investments in Africa.

The BEKK-GARCH model is a multivariate model that estimates the conditional mean function and conditional volatility function of a high-dimensional relationship that is used to test volatility spillovers between the multi-market segments when studying more than one variable or wanting to study the spillover effect. It helps in analysing the effect of volatility on stock returns estimated by the maximum likelihood. The model is specified as follows:

$H(t) = CC' + A\,u(t-1)u\,(t-1)'A + B'H(t-1)B + D'v\,(t-1)'D$ ……………….(21)

It is the time varying variance and covariance matrix of time series variables. u denotes the matrix of the residuals from the mean equation. A, B, C and D are the mean coefficient matrices.

$v(t-1) = u\,(t-1) \circ 1_{U<0}\,(u\,(t-1))$ ……………………………………… ……(22)

Where: $u(t-1) = [u_{1,t-1}u_{2,t-1}]'$

$v(t-1) = [v_{1,t-1}\;\; v_{2,t-1}$

$\circ$ denotes the Hadamard product

More specifically, the variance of the first asset returns can be written as:

$\sigma_{1,t}^2 =$

$$C(1,1)^2 + A(1,1)^2\, U_{1,t-1} + 2\, A(1,1)A\,(2,1)u_{1,t-1} + A(2,1)^2 u_{2,t-1}^2 + B(1,1)^2\sigma_{1,t-1} + 2B(1,1)B(2,1)\sigma_{12,t-1} + B(2,1)^2\sigma_{2,t-1}^2 + D(1,1)^2 v_{1,t-1}^2 + 2D(1,1)D(2,1)v_{1,t-1}v_{2,t-1} + D(2,1)^2\, V_{t-1}^2 \ldots\ldots\ldots\ldots(23)$$

All squared terms always positively affect the asset return variance in the next period. If this term is positive, it means a positive change in asset return covariance will increase the first asset return variance in the next period. If the value is negative, it means that, assuming the positive two-asset return covariance and an increase in the three-asset return covariance; it varies slightly, or rather weekly.

4.11 Tests for Structural Relationships: Vector Auto Regressive Models

The VAR model was used in the study to investigate the interaction of listed private equity returns with country risk factors (GDP and inflation). The term "autoregressive" generally means the presence of lagged values of the dependent values on the right-hand side of the equation (Gautam & Kanoujiya, 2022). A vector is a system that contains a vector of two or more variables. The VAR model was found appropriate in this case and is commonly used for forecasting systems of inter-related time series as well as for analysing the dynamic disturbances in the system of variables. Wang, Xiang, and Zhang (2022), Su, Du, Shahzad, and Long (2020), Lucheroni, Boland, and Ragno (2019), and BenSada, Litimi, and Abdallah (2018) use this approach and provide empirical evidence to investors on portfolio construction.

The VAR model is constructed only if the variables are integrated in order one, which means they are stationary after the first differencing. If the variables are co-integrated, construct both short-run and long-run models, VAR and VEC models, respectively. If the variables are not co-integrated, construct only shorn-run VAR models. All the variables in the VAR system are endogenous; there are no exogenous variables. The study utilised a VAR (1) model using three variables: the volatility of returns, the GDP, and inflation. The structure is such that each variable is a function of itself and past lags of the other variables. The study measure three different time series variables denoted $x_{t,1}$, $x_{t,2}$ and $x_{t,3}$. The Autoregressive model of order 1 known and VAR (1) is specified as in equation 24;

$$x_{t,1} = \alpha_1 + \phi_{11}x_{t-1,1} + \phi_{12}x_{t-1,2} + \phi_{13}x_{t-1,3} + w_{t,1}$$

$$x_{t,2} = \alpha_2 + \phi_{21}x_{t-1,1} + \phi_{22}x_{t-1,2} + \phi_{23}x_{t-1,3} + w_{t,2}$$

$$x_{t,3} = \alpha_3 + \phi_{31}x_{t-1,1} + \phi_{32}x_{t-1,2} + \phi_{33}x_{t-1,3} + w_{t,3} \qquad (24)$$

Where:

$x_{t,1}$, $x_{t,2}$and $x_{t,3}$ are the dependent variables which are dependent on the lagged variables $x_{t-1,1}$, $x_{t-1,2}$and $x_{t-1,3}$

α_1, $\alpha_2$and α_3 are the constant terms.

$w_{t,1}$, $w_{t,2}$ and $w_{t,3}$are error terms

ϕ_s are the regression coefficients of the lagged variables

The regression coefficients of the lagged variables for the model come from the matrix of the autoregressive parameter represented by the matrix

$$\begin{bmatrix} x_{t,1} \\ x_{t,2} \\ x_{t,3} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} + \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \phi_{1,3} \\ \phi_{2,1} & \phi_{2,2} & \phi_{2,3} \\ \phi_{3,1} & \phi_{3,2} & \phi_{3,3} \end{bmatrix} \begin{bmatrix} x_{t-1,1} \\ x_{t-1,2} \\ x_{t-1,3} \end{bmatrix} + \begin{bmatrix} w_{t,1} \\ w_{t,2} \\ w_{t,3} \end{bmatrix} \qquad (25)$$

The dependent variable is a function of its lagged values and the lagged values of other variables in the model. The VAR model is specified in levels, hence the VAR in differences would be mis-specified. The data was tested to see if the variables were integrated in order one, and that condition was met satisfactorily, allowing the data to be used in the model. The results of this analysis are shown in the next chapter.

4.12 Impulse Response Functions

Impulse response functions help to identify the impact of one unit of a shock on variable A to variable B (De Wet, 2005). To identify impulse responses, some restrictions called decompositions are applied. The study used the Cholesky decomposition, which is a built-in function in EViews. Impulse response functions were found a necessary tool in tracing that impact of country risk factors to the log returns of private equity investments. This tool was used in the study to examine how the movement in variables reacted to structural innovations from outside the system.

The impulse response function is an essential tool in empirical causality analysis. Studies such as Long and Herrera (2021) as well as Boppart, Krusell, and Mitman (2018) have used this tool to analyse the effects on present and future values of endogenous variables of one standard deviation of shocks to one of the innovations. The findings of this study based on impulse response functions are shown in the next chapter.

4.13 Justification of Time Period used in the Study

The study used the time period 2010–2020 for all listed companies that invest in private equity investments. This was informed first and foremost by the research problem, which culminated in the research objectives. The study endeavoured to model the behaviour of the returns of this asset class in an effort to ascertain what generates the volatility and the driving factors. This means the data had to be analysed over time; therefore, time series analysis was needed. Time series analysis ordinarily requires a large data set; hence, a large data set was required (Tsay, 2013).

In addition, according to Klownosky (2012), ever since the global financial crisis, emerging markets have gained force in private equity investments. This is driven by an expanding middle class, urbanisation, increased population wealth, and significant domestic infrastructural investment. This argument follows that the data being sought to analyse the performance of this investment was from 2008 going forward. This study then standardised the data set by identifying the most common data period that matched the investments with the largest time span in existence. Table 4-2 provides a summary of the key models used in the study.

Table 4-2: Summary of key models used

Outcome	Key models used
Analyse the statistical properties of each counter	Descriptive analysis and trend analysis, normality tests (Jaque-Bera test), autocorrelation tests (unit root tests using the ADF test)]
Measure the ARCH effects	Ljung Box Test and the Lagrange multiplier test
Fit the GARCH models, measure persistence and leverage effect	GARCH (1,1); EGARCH; TGARCH and GARCH-IN-MEAN
Model stability tests	Nyblom parameter stability tests, News Impact test and the Sign Bias test
Predict volatility of each counter	RMSE, MAPE and Theil Inequality Coefficient
Measure the long-term **memory for the counters**	FIGARCH models
Determine the presence spillover effects of the counters	MGARCH DCC model

Examine the structural relationship of counters with country-specific factors	GDP and Inflation, VAR model and impulse response functions test

Source: Researcher Compilation

4.14 Conclusion

The chapter has examined methodological issues associated with statistical modelling of private equity investments in Africa. The research was viewed through the positivist lenses of both inductive and deductive reasoning. Using time series analysis and GARCH models, under the guidance of the principle of parsimony, to answer research questions was determined to be the most appropriate method. In addition, the MGARCH model was proposed to identify spillover effects from volatility. It was noted that it is possible to determine the volatility generated by a particular variable, i.e., whether it is the result of an exogenous structural shock or endogenous interactions between variables. In order to accomplish this, the study utilised a VAR model, which is capable of variance decomposition. The concept of impulse response was introduced to determine how second moments react to shocks in variable structural innovations. These structural innovations are crucial because they determine the future behaviour of assets' second moments. Without this knowledge, significant miscalculations of portfolio variances are possible, leading to poor investment decisions. It is essential to account for this asset behaviour when interpreting asset price changes and predicting the future paths of their variances and correlations. This study employed the first methodology to model LPEs in African economies, primarily because it is the first study to do so.

5 CHAPTER FIVE: EMPIRICAL FINDINGS AND INTERPRETATION: STATISTICAL PROPERTIES AND VOLATILITY DYNAMICS OF LPE INVESTMENTS

5.1 Chapter Introduction

This chapter presents the empirical findings; all interpretations are presented in accordance with the methodology discussed in the preceding chapter. The chapter begins with an analysis of the return series of listed private equity investments in Africa. The second section examines the variables' descriptive statistics and fundamental diagnostic tests. A presentation of the country-by-country analysis of short-term volatility and the various GARCH models that were fit to the daily log-return data series follows. Prior to concluding the chapter, the study investigates the volatility dynamics of this asset class over the long term.

5.2 Time Series Analysis of the Private Equity Return Series

This section examines the trends of daily log returns and monthly log returns for listed private equity returns. It is important to note that the different time series analysed for the various nations displayed both common trends and subtle differences.

From 3 January 2010 to 23 September 2020, South Africa exhibited clear upward trends with marked and stable volatility. According to SAVCA (2016), the industry has experienced an average annual growth rate of 11.8% since 1999. During the period, South Africa's raw data exhibits an upward trajectory followed by pronounced decreasing trends. The flat series from September 2006 to September 2013 is clearly related to the global financial crisis.

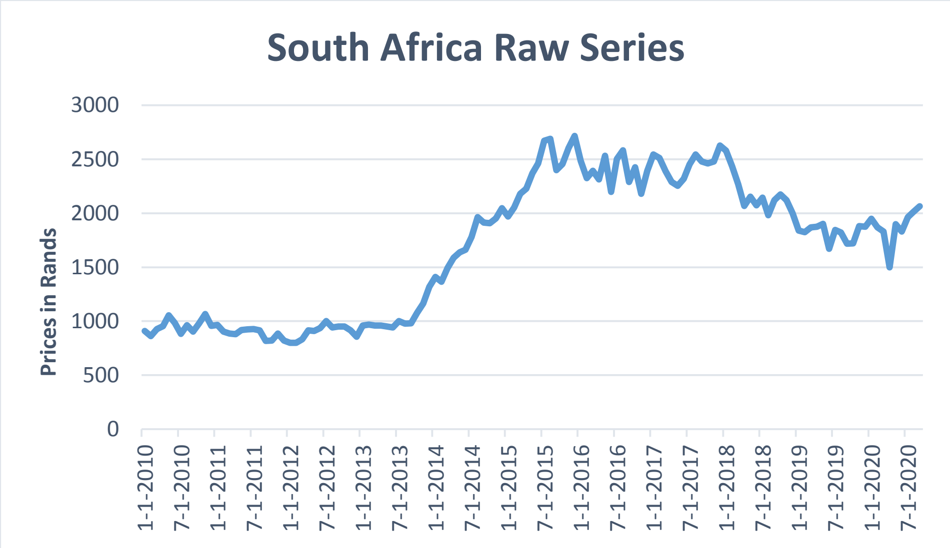


Figure 5-1: Trend analysis South African LPE Data

Source: Researcher Compilation

Despite a relatively rapid recovery of the stock, as evidenced by an increase from 2013 to April 2015, this is mitigated by a steady decline of the stock in the subsequent period. In spite of this, there has been a notable increase in the share price that may be exacerbated by negative repercussions of the financial crisis, which have not only impeded a speedy recovery but also rendered its restoration nearly impossible in the short term. Since the financial crisis of 2008, the South African government has enacted regulations to promote the growth of the private equity industry, mandating pension funds to invest 10% of their portfolios, up from 2.5%, and introducing tax incentives in 2014.

Despite the fact that South Africa continues to experience low economic growth, the economic climate for private equity is largely favourable (Nkam, Akume & Sama, 2020). Between 2007 and 2014, the sector accounted for 76% of the transaction volumes in South Africa and 92% of the value of transactions in Southern Africa.

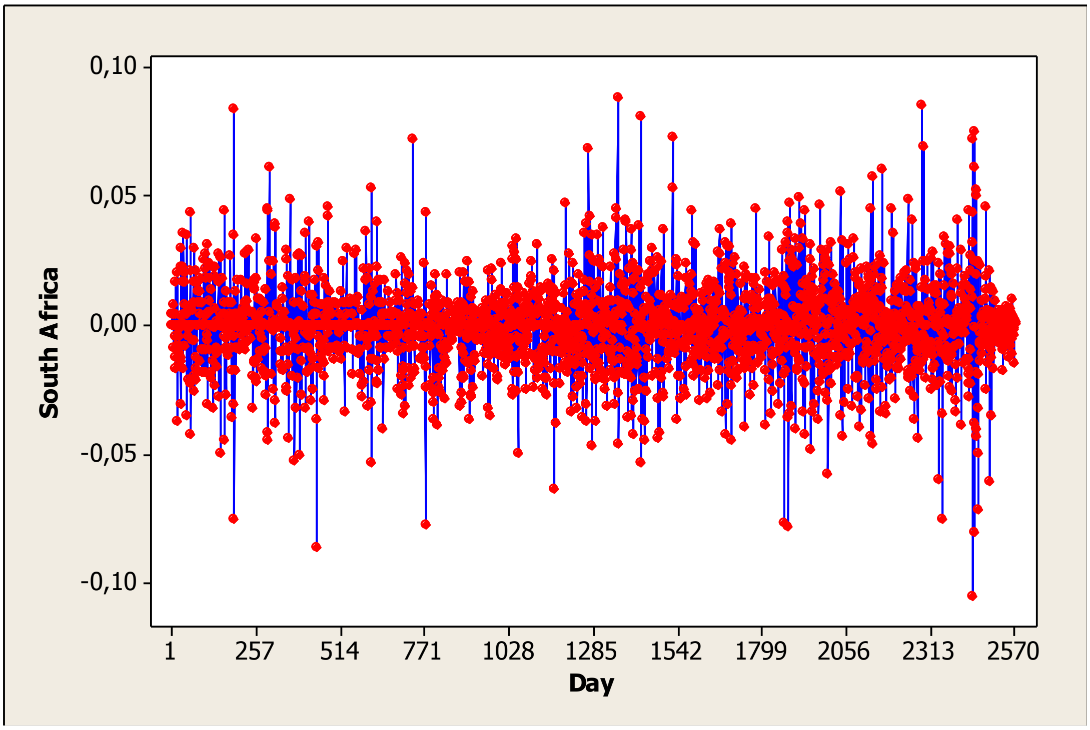


Figure 5-2: Daily log returns for South Africa

Source: Researcher Compilation

The log return plot of the South African return series in Figure 5-2 exhibits significant mean reversion, as indicated by log returns reverting to zero. Investors can utilise this characteristic as a timing strategy in identifying the buy-and-sell horizons. Though there are no statistically discernible movements in the log series, the notion of mean reversion is clearly depicted.

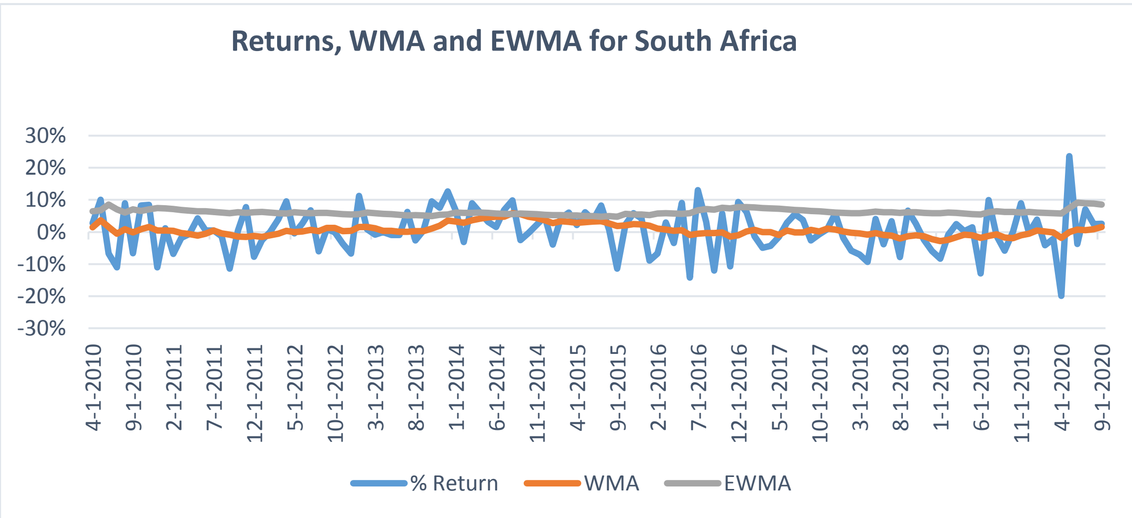


Figure 5-3: Log returns, WMA and EWMA for South Africa

Source: Researcher Compilation

By using the weighted moving average as a proxy for the process's marginal mean and the exponentially weighted moving average as a proxy for marginal standard deviation, as shown in Figure 5-3, the study noted that the daily log returns exhibit a stable mean over time and the volatility is somewhat bound between 5% and 7% per trading day. The daily log series and raw data series for Botswana are shown in the Figures 5-4 and 5-5:

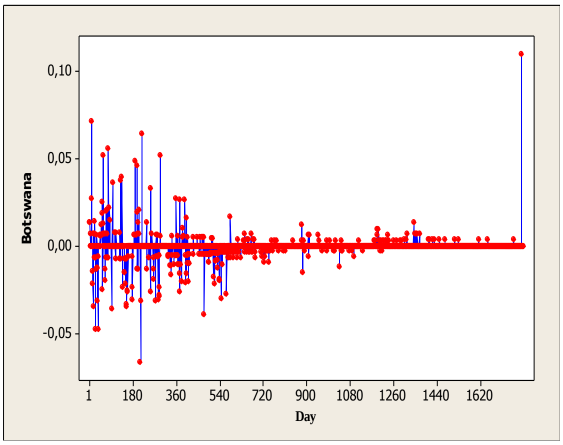


Figure 5-4: Botswana log returns

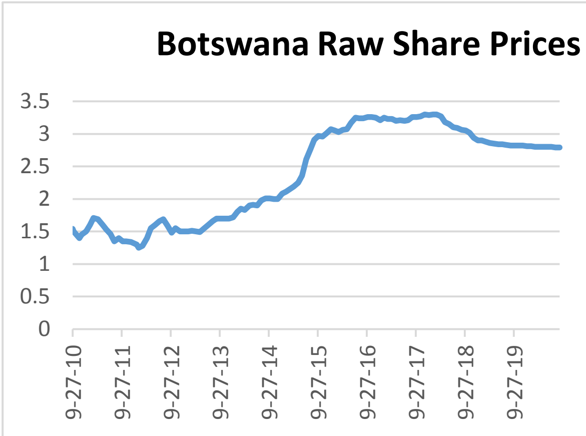


Figure 5-5: Botswana raw share prices

Source: Researcher Compilation

The Botswana log and raw data series show gradual upward trends from the beginning of the series period to January 2013, followed by very low volatility. This is attributed to downturn-oriented events in the global market and a decrease in private equity activity in the country (AEO, 2014). Volatility clustering is depicted in the series, and evidence of mean reversion characterises the log series in the period to 2015. The idea behind the mean reversion property is that prices that deviate from the long-term norm will naturally revert to their previously understood state. Botswana has historically enjoyed strong and stable growth since the inception of its independence, coupled with prudent macroeconomic policies. Botswana, the poorest economy at independence, used its diamond resource to catapult its economy to the upper middle class, aided by increased demand for diamonds and the ease of restrictions on its mobility. As much as other equity counters show significant growth over time, listed private equity counters display a flat and deteriorating trajectory. According to CEDA's Annual Report 2009, firms under the Venture Capital Fund portfolio came under stress mainly due to the recession and slow uptake of the investment vehicle in the country.

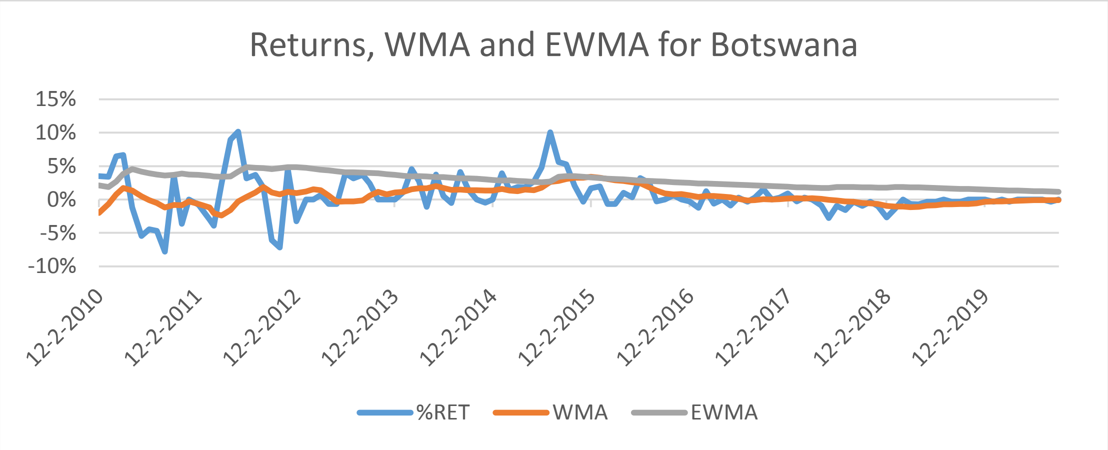


Figure 5-6: Log Returns, WMA and EWMA for Botswana

Source: Researcher Compilation

The weighted moving average and the exponentially weighted moving average indicated that the long-term mean for the return series was converging toward zero, indicating reversion to the mean. This is shown in Figure 5-6. The log return series and raw data series for Egypt are given in Figures 5-7 and 5-8 respectively.

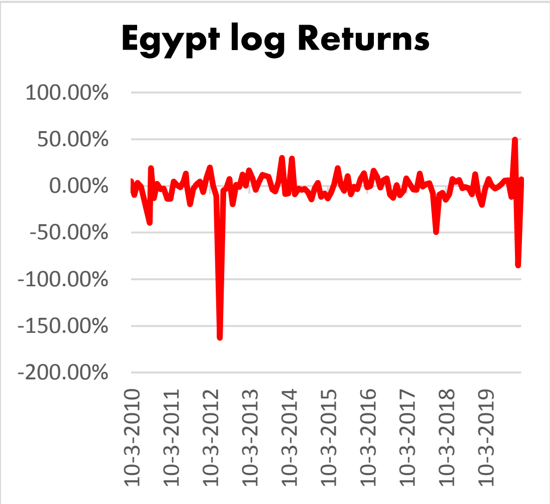


Figure 5-7: Egypt Log returns

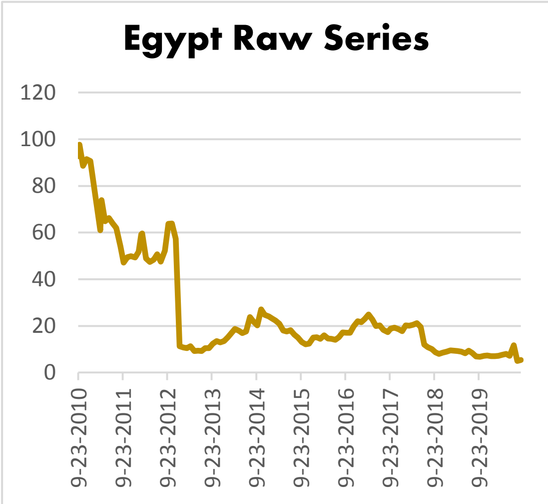
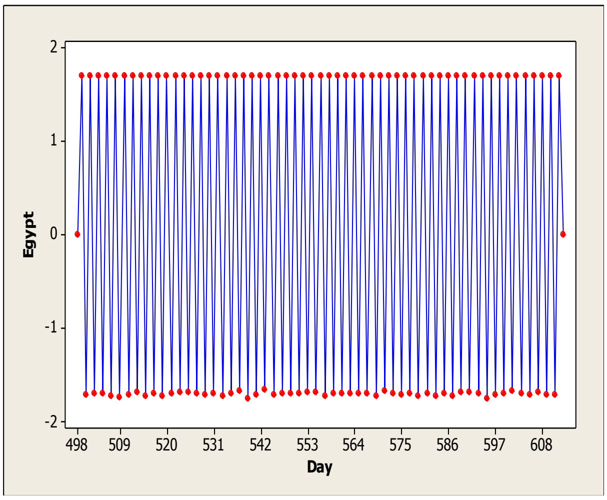


Figure 5-8: Egypt raw series

Source: Researcher's Compilation

The Egyptian series is divided into three episodes, covering the time period from the 25 January to 30 November 2011, before, during, and after the Egyptian revolution (Paciello, 2011). This period was marked by numerous uprisings, including protests, riots, and strikes, which halted a great deal of uncertainty regarding stocks and investments. As a result, the series stabilised after the elections, but the series is characterised by mean reversion and very low volatility around the mean. There are no seasonal characteristics to be extracted from the series.



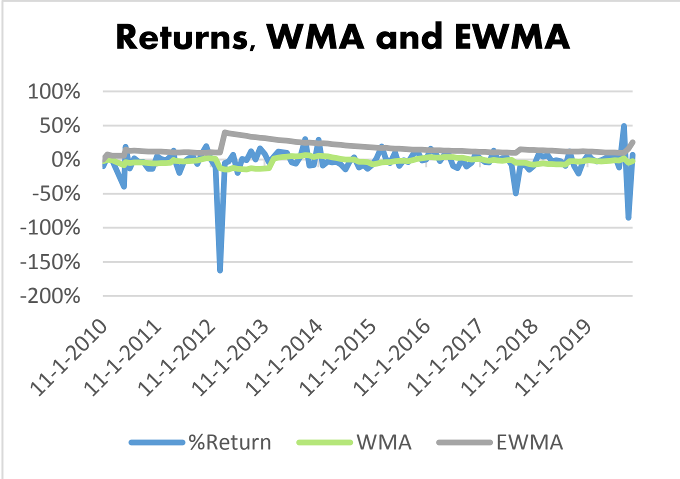
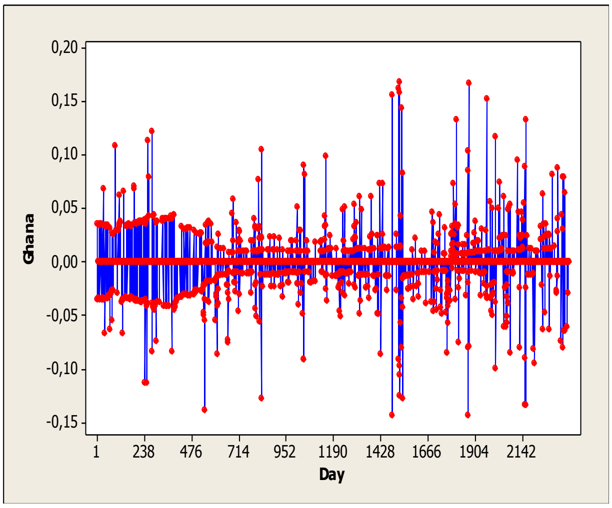


Figure 5-9: Egypt series for period 25 January 2011 to 30 November 2011

Figure 5-10: Returns, WMA and EWMA for Egypt

Source: Researcher Compilation

The long run mean of the returns, the weighted moving average, and the exponentially weighted moving average, indicated a stability of return as shown in Figure 5-10. The daily log returns and raw series for Ghana are shown in Figure 5-11 and 5-12 respectively, and indicate a mean reversion stable volatility across time.



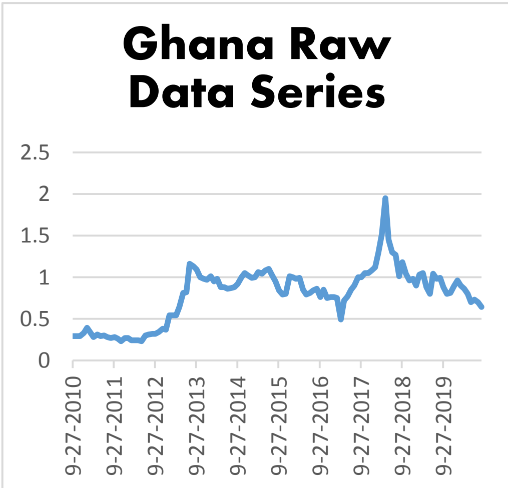


Figure 5-11: Ghana log returns series

Figure 5-12: Ghana raw data series

Source: Researcher Compilation

Ghana's log return series was characterised by statistically discernible upward and downward trends that are short-lived and gradually moving downward, indicating low volatility. Though private equity started in 1991, it proliferated after the global financial crisis in Ghana. Hence, the data series shown does not indicate the effects of this era, which was characterised by mean reversion and no traces of seasonality. Ghana is a major player in West Africa's private equity growth, and its investments are mainly done through Pan-African and regional funds (Botchway & Akobour, 2020). While some dedicated funds are closing, the lack of Ghana-specific fundraising reflects investor preference for regional vehicles rather than a decrease or increase in fundraising for Ghana. This is exhibited by the downward trend and little to no volatility in the series. Divakaran et al. (2018) demonstrated that in Ghana, the regulatory framework creates confusion in the system such that financial institutions are reluctant to invest in private equity. Figure 5-13 shows the log returns, the weighted moving average, and the exponentially moving weighted average. The log returns display mean reversion properties, and there was general stability in the weighted moving average as well as the exponentially weighted moving average.

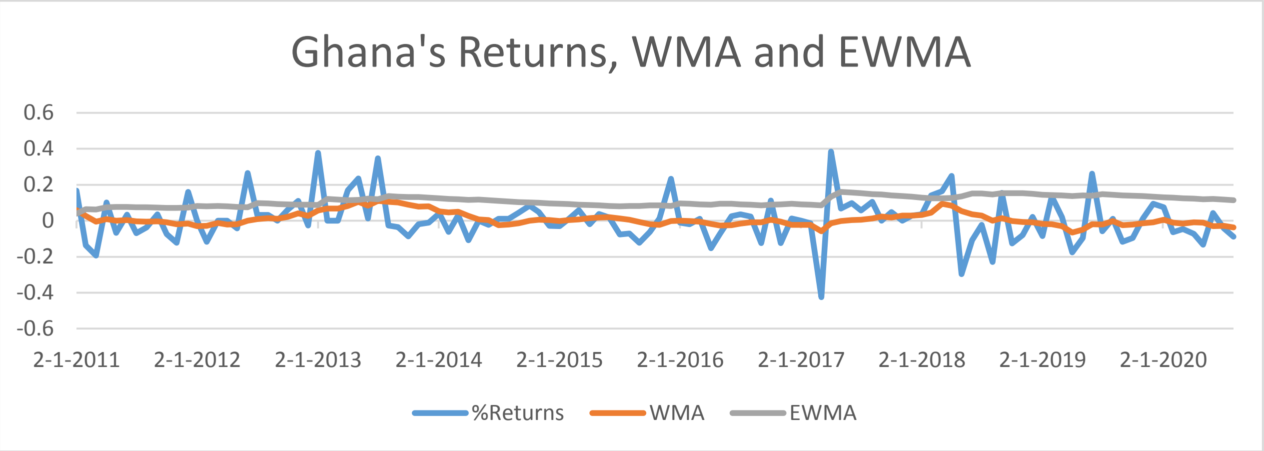


Figure 5-13: Ghana's returns, WMA and EWMA

Source: Researcher Compilation

5.2.1 Descriptive statistics of the study

This section discusses the results for the descriptive statistical properties of private equity returns at both the 1% and 5% confidence intervals. The discussion is based on the mean, maximum, minimum, standard deviation, skewness, kurtosis, and normality tests at 1% and at 5%.

Table 5-1: Descriptive statistics for LPE returns at 5% level of significance

	Botswana	**Egypt**	**Ghana**	**South Africa**
Mean	0.00495	-0.0237	0.0066	0.0664
Median	0	-0.00918	0	0.00583
Maximum	0.101783	0.495037	0.38485	0.23639
Minimum	-0.07833	-1.62703	-0.42567	-0.19885
Standard deviation	0.029668	0.204452	0.12	0.06616
Skewness	0.3635	-4.58527	0.37409	-0.11457
Excess Kurtosis	2.124687	33.65053	2.41535	0.70526
Jarque-Bera	248261.4	31849.09	16234.44	1570.193
Probability	0.000	0.000	0.000	0.000
Observations	1798	2442	2377	2573

Note: $p<0.05$*, $p<0.01$**, $p<0.001$***

Source: Researcher Compilation

Table 5-2: Descriptive statistics for LPE returns at 1% level of significance

	Botswana	Egypt	Ghana	South Africa	Panel
Mean	-0.000269	0.00120	-0.000371	-0.0003	8.59E-05
Maximum	0.109751	2.100644	0.168623	0.088697	2.100644
Minimum	-0.066691	-2.056563	-0.143101	-0.1053	-2.056563
Standard deviation	0.007185	0.377279	0.023961	0.017059	0.195069
Skewness	2.652756	0.030935	0.413913	0.027549	0.076598
Kurtosis	60.3209	20.69207	15.77615	6.826637	76.89223
Jarque-Bera	248261.4	31849.09	16234.44	1570.193	2090758
Probability	0.001	0.001	0.001	0.001	0.001
Observations	1798	2442	2377	2573	9190

Note: $p<0.05$*, $p<0.01$**, $p<0.001$***

Source: Researcher Compilation

Tables 5-1 and 5-2 highlight the statistical properties of Botswana, Egypt, Ghana, and South Africa's private equity returns. At both confidence intervals, the average private equity return demonstrates both positive and negative returns. Botswana, Ghana, and Egypt private equity returns are on average low and negative compared to South Africa, and none of them are significantly different from zero at the 5% significance level, whereas Egypt's tends to outweigh the others at the 1% significance level. In contrast, the combined results of the four countries produced a profit. The findings demonstrate that investments in private equity can either increase or decrease capital depending on the investment's financial environment. Observing the maximum returns achieved during the period, it can be concluded that there is a high potential for achieving high returns in the four countries, with Egypt dominating other nations at both confidence intervals. The minimum returns indicate, however, that Egypt's private equity returns had the lowest returns during the sample period. Egypt and Botswana carry the highest and lowest 1% confidence interval risks, respectively, according to the standard deviation findings. It is observed that the standard deviation of the panel is less than that of Egypt, suggesting there is room for diversification and risk management. At the 1% significance level, the private equity returns for the four countries are positively skewed, indicating that there are more positive values in the tails of the distribution. Egypt and South Africa are negatively skewed at 5%, indicating that investors can anticipate frequent small gains and a few large losses.

Kurtosis results at a significance level of 1% indicate that the values are greater than 3, indicating that private equity returns are leptokurtic and have a distribution that is more skewed than normal. However, at 5%, South Africa, Ghana, and Botswana’s Kurtosis is less than 3, indicating lighter tails than the normal distribution and a reduced likelihood of significant price fluctuations. Normality tests of private equity returns using the Jarque-Bera tests indicate a non-normal distribution because the probability is less than 0.05 for all countries; however, at a 5% level of significance, a non-normal distribution is indicated by a p-value of 0% for Botswana, Egypt, and Ghana, and the result is close to normality for South Africa with a p-value of 30.11%. Additional tests discussed in the next section contributed to the study's analysis of the statistical properties of the private equity investments under consideration. In addition, 50% of the distribution for South Africa falls between 3.45% and 5.76%; Egypt between 7.51% and 5.9%; Ghana between -0.06508% and -0.045536%; and for Botswana between -0.65% and 1.85%. This indicates that South Africa's returns are positive and superior to those of other countries.

The study also examined the presence of significant ARCH effects and white noise to determine which model best fits the data. Conceptually, significant ARCH effects indicate that the variance is not constant but rather varies over time. The economic justifications for stochastic volatility models' theoretical constructs that replicate the volatility clustering effect in financial time series are not explained. These topics are discussed in greater detail later in the chapter.

5.2.2 Unit root tests result for private equity returns

This section focuses on the unit root tests for private equity returns using the augmented Dickey-Fuller (ADF). The ADF is tested for the private equity return series in three unique cases: intercept, intercept and trend, and no intercept and trend.

Table 5-3: Unit root tests results for private equity returns

Series	**Intercept**	**Intercept and trend**	**None**
Botswana	-25.64989***	-25.81679***	-18.09236***
Egypt	-29.60824***	-29.60339***	-29.56734***
Ghana	-47.61762***	-47.64541***	-47.61558***
South Africa	-55.35755***	-55.35273***	-55.34965***
Panel	446.851***	1030.97***	365.766***

Note: $p<0.05$*, $p<0.01$**, $p<0.001$***

Source: Researcher Compilation

Table 5.3 summarises the unit root tests for the private equity returns series. The ADF null hypothesis is that there is a unit root. The findings reveal that there is no unit root in at 0.05, 0.01 and 0.001 level of tests for the countries and panel series. It follows that the private equity returns exhibit stationarity at a level under the three categories. A summary of EViews output for Botswana is shown in the appendices.

The augmented Dickey–Fuller (ADF) statistic, used in the test, is a negative number. The more negative it is, the stronger the rejection of the hypothesis that there is a unit root at some level of confidence. If the calculated test statistic is less (more negative) than the critical value, then the null hypothesis is rejected and no unit root is present. In essence, a time series is non-stationary if it exhibits a unit root and exemplifies a random walk series. The study concluded that there is no unit root for private equity returns in Botswana, Egypt, Ghana, and South Africa for a model with no constant and no trend, a model with constant, and a model with constant and trend. Hence, the data was stationary.

5.2.3 Further analysis for stationarity

The study supplemented the Dicker-Fuller test for stationarity to determine whether the statistical properties of the variables in the study change or remain constant over time. The expectation is not that each data point's value must be identical, but rather that the overall behaviour of the data should remain constant. The autocorrelation function (ACF) was used in the study because it provided values of autocorrelation of the series with its lagged values to determine how it differed from the unit root test. Simply put, the ACF tool describes the relationship between the present value of a series and its past values.

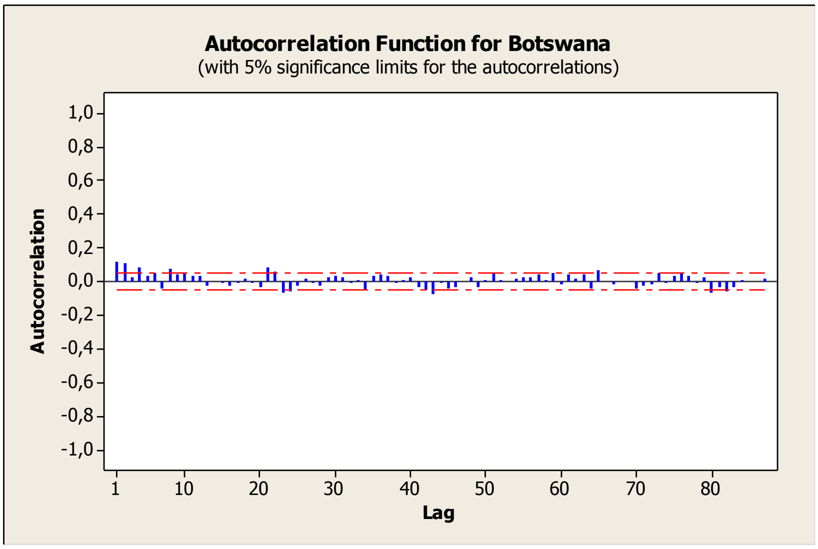


Figure 5-14: Autocorrelation function for Botswana

Source: Researcher Compilation

The ACF for Botswana showed that the private equity returns in Botswana were stationary at a 5% significance level. This means we expect one out of 20 to fall outside the critical limits at the 5% level of significance. The returns were uncorrelated as the sample autocorrelation function of the returns exhibited an insignificant value, very similar to asset returns, with at most lags indicating the absence of linear serial dependence in the returns.

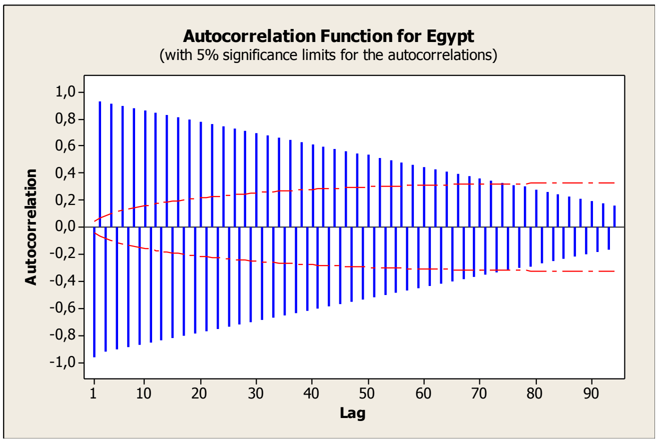


Figure 5-15: Autocorrelation function for Egypt

Source: Researcher Compilation

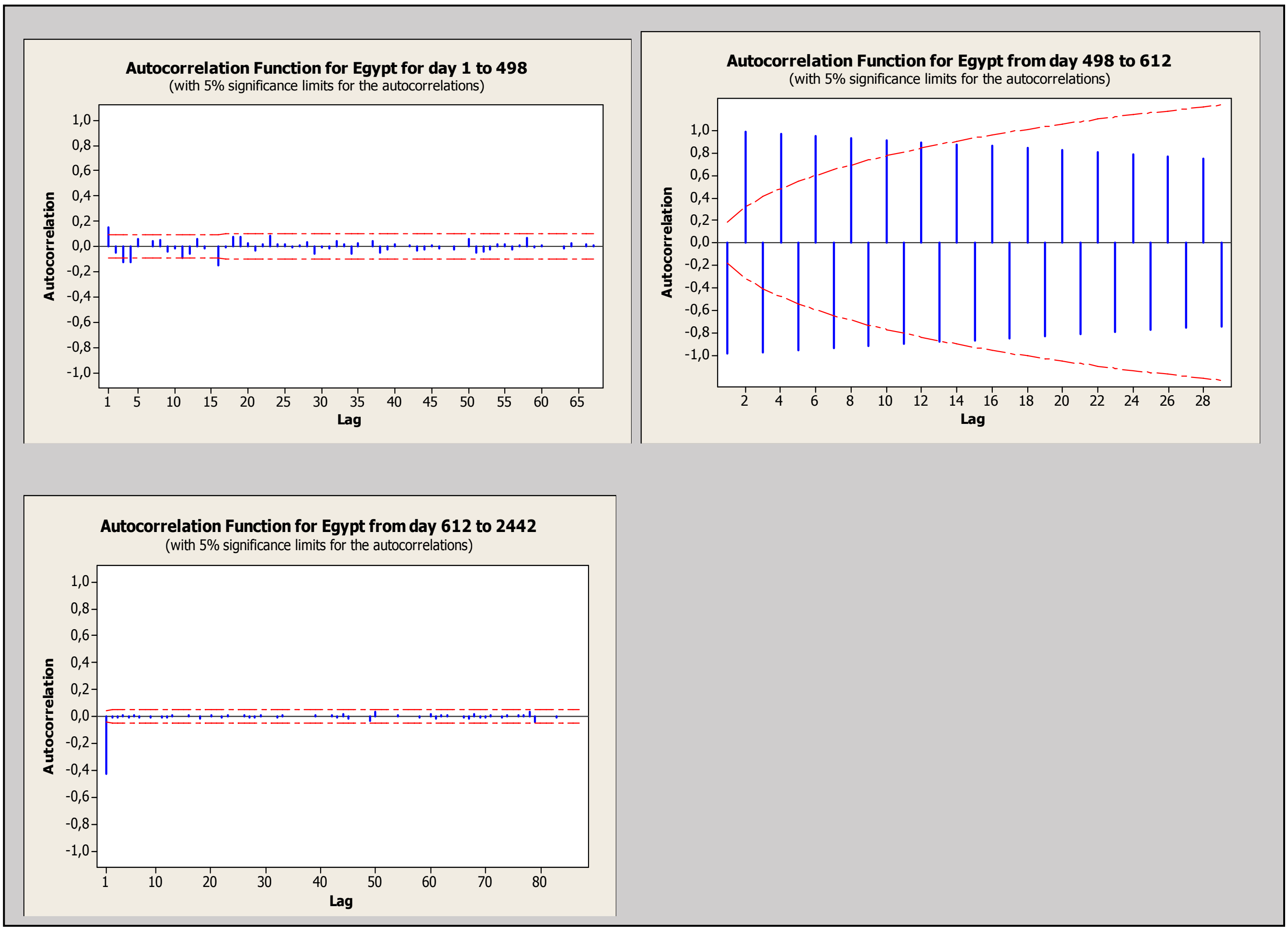


Figure 5-16: Autocorrelation Function for Egypt (Days 1 - 2442)

Source: Researcher Compilation

As shown in Figure 5-16, the ACF correlogram for Egypt is not decaying rapidly, and therefore the LPE returns were not stationary at the 5% level of significance for the period indicated. This series correlation is termed "persistence" or "inertia", and is mainly due to the high volatility clustering from days 499 to 611. Persistence can drastically reduce the degrees of freedom in time series modelling (AR, MA, and ARMA models). In the test for statistical significance, presence of persistence complicates the test as it reduces the number of independent observations. Hence, the Egyptian series was cognisant of the period which was exhibiting this non-stationarity.

The findings revealed that private equity returns were stationary from day 1 to 498. There were significant positive and negative correlations with periods of high volatility clustering between periods 498 and 612, to which we can attribute the problem of non-stationarity. This was caused by the Egyptian Revolution era, which caused a lot of instabilities in the market.

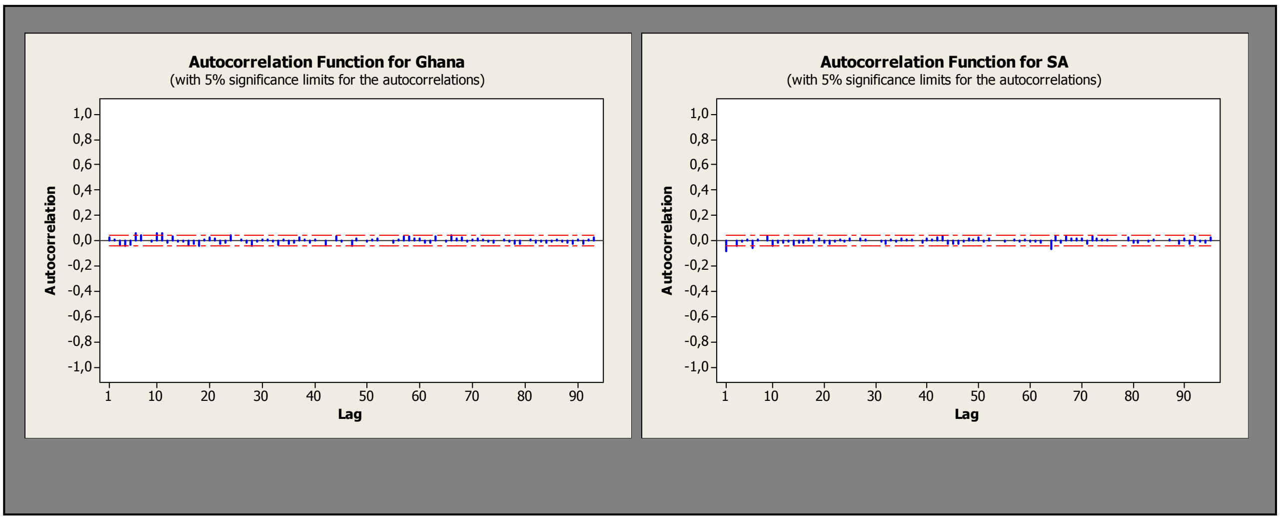


Figure 5-17: Autocorrelation function for Ghana and South Africa

The private equity returns were stationary from day 498 onwards, as indicated in Figure 5-18. When a market shock happens, the volatility in prices of financial assets increases significantly. Now, the phenomenon of volatility clustering suggests that the impact of this market shock will be felt for some time in the future. This means that EWMA volatility reacts faster to the market shock, hence explaining the turbulence during this period. The ACF correlograms show that the private equity returns of Ghana and South Africa were stationary at a 5% level of significance.

5.2.4 Normality test results for LPE returns

Further tests for normality were also done to complement the Jaque-Bera test. The study used the Anderson-Darling test because, according to a study by Razali and Wah (2011) on comparative tests for normality, this test is one of the best in terms of normality. If the data is perfectly normal, the data points on the probability plot will form a straight line. The Anderson-Darling test is a modification of the Cramer-von Mises (CVM) test that gives more weight to the tails of the distribution.

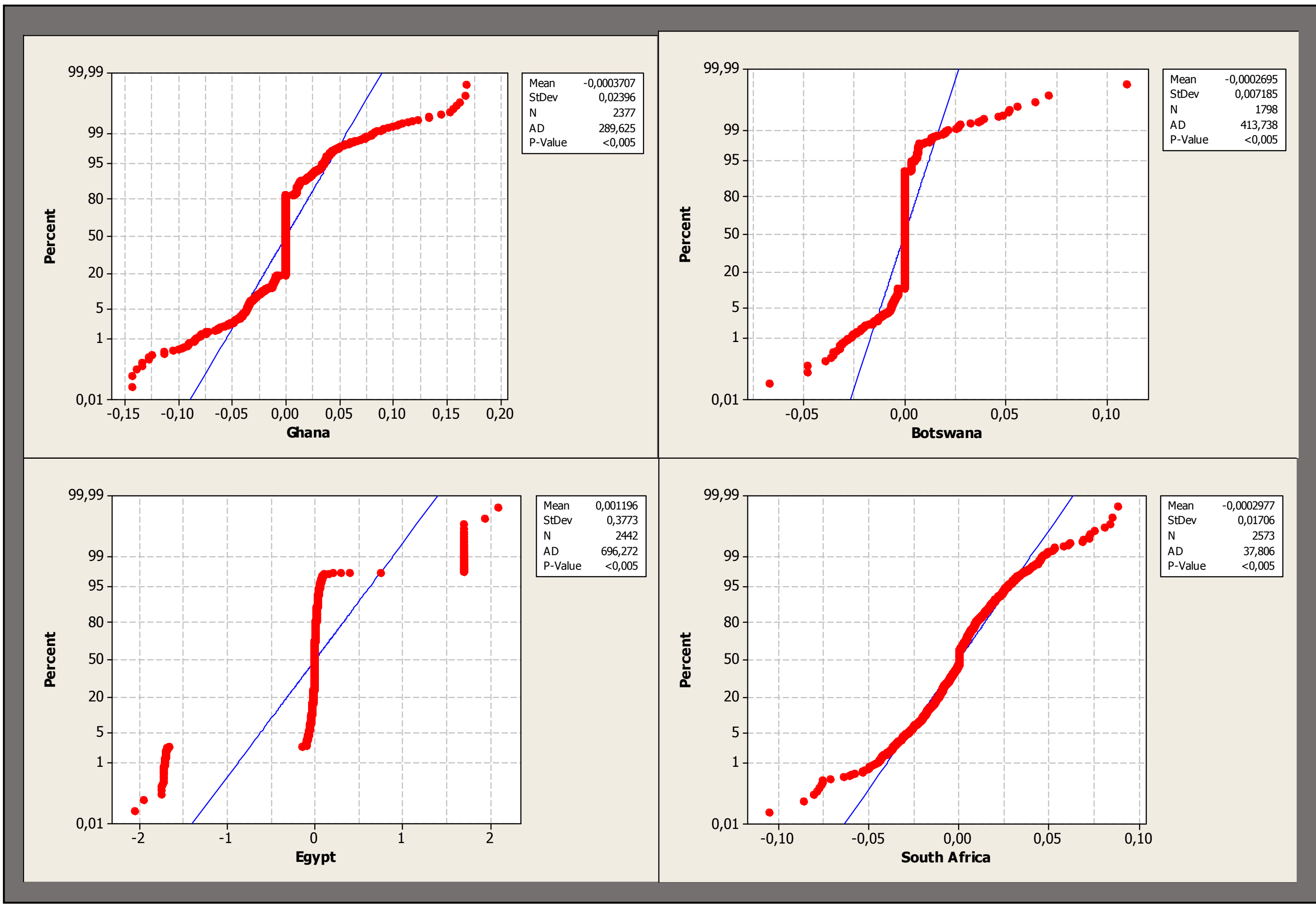


Figure 5-18: Normality tests for Ghana, Botswana, Egypt and South Africa

The normality tests and p-values demonstrated that private equity returns in Botswana, Egypt, Ghana, and South Africa were not normally distributed, as confirmed by the Jaque-Bera test statistic. Nonetheless, South Africa was close to normal, as evidenced. In any case, the phenomenon that financial data is not normally distributed is well known (Tsay, 2013). Due to the violation of the normality assumption, the ARIMA and SARIMA time series models cannot be used for univariate time series analysis, necessitating the use of the ARCH and GARCH methods.

All private equity returns are positively skewed with excess kurtosis, according to the study's descriptive statistics (a normal distribution assumes a constant mean and variance). The returns on South Africa’s private equity were closest to the norm. As explained, differences in skewness and kurtosis reflect the various economic geographies. Private equity returns for Egypt exhibit high volatility clustering during the period from day 498 to day 612, which has implications for financial time series modelling. Consequently, the study proceeded to test for the ARCH effect on the data series for the four countries in order to evaluate their suitability for the GARCH family of models.

The generalised autoregressive conditional heteroscedasticity (GARCH) process is a technique for estimating market volatility. The model is utilised by financial institutions to calculate the return volatility of stocks, bonds, and other investment vehicles.

The time series was mean reverting, the data non-stationary, and it was not normally distributed, hence the use of GARCH modelling was supported. GARCH models capture three variables: long-run variance, lagged volatility, and square of lagged returns. These three variables are assigned different weights that total 1. The stochastic and conditional GARCH models are the most applicable for modelling private equity returns. The section that follows looks into the volatility dynamics of listed private equity investments in Africa.

5.3 Volatility Dynamics of LPE Firms in Africa

This section uses the statistical approaches discussed in the methodological section to present and analyse objective 2, the volatility dynamics of private equity investments in Africa. A country-by-country analysis was done starting with the data series that had heteroscedastic properties and therefore fit for the GARCH models as discussed in the section 5.2. The GARCH models were fitted to the volatility of the selected daily log returns. This empirical work starts with the GARCH selection, progresses to the different GARCH models that were fitted, peaks with the examination of spillover effects between the countries, and concludes with the contribution of the findings to the study. In all cases, model fitting, measures for the persistence of volatility, volatility predictions, robustness tests, and model validations were done.

5.3.1 GARCH Model Diagnosis: South African Listed Private Equity Investments

A statistical model is required to forecast financial data. The study presents financial time series in the form of daily log returns, with the critical concept being a model that can analyse the dynamical structure or behaviour of the log returns over time. Engle, the winner of the 2003 Nobel Memorial Prize in Economic Sciences, introduced ARCH models in 1982. Then, in 1986, Bollersleve proposed GARCH models as a generalisation of the ARCH process. The main benefit of GARCH models is that they can capture several important properties of financial time series.

Time series can also be analysed using Box Jenkins' Autoregressive Moving Average (ARIMA) methodology. The model's critical assumption is that it cannot be used for data that is not normally distributed. In practice, it is impossible to achieve symmetry when dealing with financial data; thus, the study used the data to identify the presence of significant ARCH effects. When there are significant ARCH effects, it indicates that the variance is not constant and changes over time (Brooks, 2002). In such cases we do not use ARIMA but rather GARCH models to model volatility. The study tested for ARCH effects on the South African log return series, and the results were as follows:

Table 5-4: ARCH effects test for South Africa

	DW stat test	Arch LM test
ALSI	2.033911	88.50033 [0.000] ***

***, ** and * indicate significance at the 1%, 5% and 10% levels respectively.

The presence of ARCH tests on the log return series for South Africa's private equity return series was detected after testing for their presence on the log return series as a way of justifying the GARCH estimation, as shown in Table 5-4. The effects were tested using the residuals obtained from the Ordinary Least Squares regression of the mean equation. According to the findings in Table 5-4 and others in the Appendix, both at the 5% and 1% confidence levels, the coefficient's p-value was lower than the critical one for all the daily return series, indicating that the H_0 is rejected and therefore there is heteroscedasticity and evidence of the ARCH effect.

The hypotheses for the models are the following:

- H_0: The model coefficients are not statistically significant.

- H_1: The model coefficients are statistically significant.

Therefore, the null hypothesis was rejected for the series under the GED error distribution, indicating that the GARCH is a sufficient model.

Furthermore, as shown in Figure 5-19, volatility varies with time, indicating the presence of conditional volatility (heteroscedasticity), which is common with financial time series data.

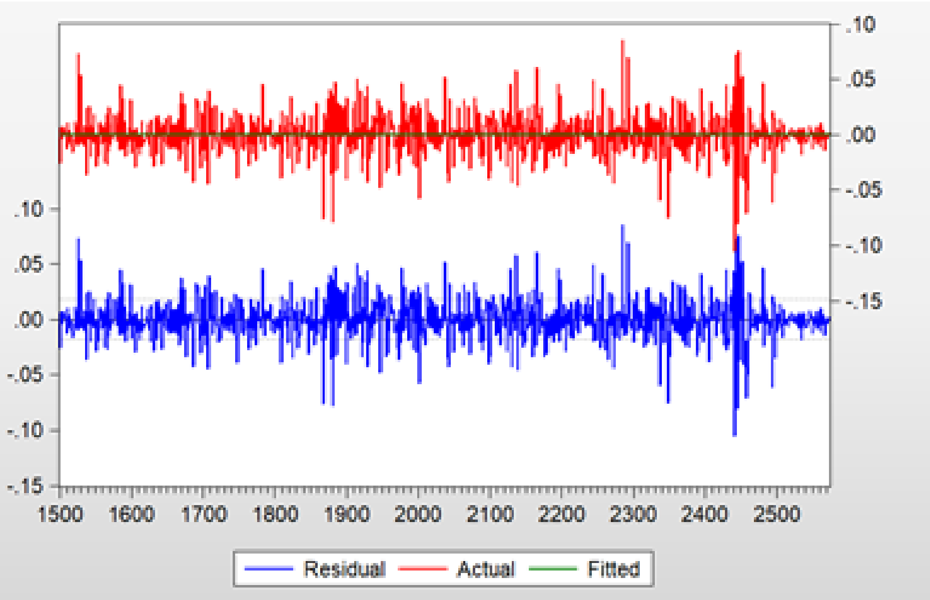


Figure 5-19: Residual, fitted and actual values

Source: Researchers Compilation

5.3.1.1 Model selection

In justifying the type of GARCH estimation, the study favoured a model with the fewest number of parameters, according to Tsay (2013), which had significant ARCH and GARCH coefficients, a high adjusted R-square, a high log-likelihood ratio, the lowest Schwartz Information criterion (it imposes the heaviest penalties for loss of degrees of freedom), no heteroscedasticity and no autocorrelation, and no heteroscedasticity (correlogram). The study did not assume a normality condition because GARCH models are naturally skewed to the left or right and have fat tails. Consequently, the study subjected the data to three GARCH model error constructs, namely the Gaussian normal distribution, the student-t distribution, and the Generalised error distribution (GED), and chose the most preferred model as shown in Table 5-5. The GED qualified as it is the one with the highest log likelihood and the lowest Schwartz Information criterion.

Table 5-5: Parameter estimates for GARCH (1,1) for South Africa

	Normal distribution	Student's T	GED
Significant Coefficient	ALL	ALL	ALL
ARCH Significant	YES	YES	YES
GARCH Significant	YES	YES	YES
Log Likelihood	6941.86	7119.91	7194.44
Adj R^2	0.006670	0.004970	-0.000670
Schwartz IC	-5.38276	-5.518162	-5.576195
Heteroscedasticity (residuals)	NO	NO	NO
Autocorrelation (residual)	NO	NO	NO

Source: Researcher Compilation

5.3.1.2 GARCH, MGARCH, TGARCH and the EGARCH Results

This section presents the results of the different models that were assessed: the GARCH, MGARCH, TGARCH, and EGARCH. These models were analysed using the Gaussian framework, after which forecasting was done to determine whether the model correctly predicts market movements.

GARCH estimations for South Africa's listed private equity investments showed that the average stock return and its past value do not significantly predict its current series. Both coefficients were not statistically significant. The returns were generally too low, so one concluded that there was evidence of market efficiency. This is reflected in the Table 5-6:

Table 5-6: GARCH (1,1) Parameters for South Africa LPE

GARCH (1.1)	Estimate	Std Error	T-statistic	P-Value
Omega	2.94E05	6.65E-06	4.425486	0.0000
Alpha	0.191285	0.031699	6.034402	0.0000
Beta	0.726152	0.38267	18.97581	0.0000
α +β	0.917437			

Source: Researcher Compilation

The constant, the GARCH term, and the ARCH term are all positive and statistically significant. The time-varying volatility of the GARCH model includes a constant of 0.00000294 plus its past error of 0.72615 and a component that depends on its past errors 0.1913, thereby meeting the stability conditions. Both the ARCH term and the GARCH term are non-negative and significant, hence satisfying the model assumption. The ARCH term coefficient is tending towards 0 and the GARCH term is tending towards 1, showing that the GARCH effects are stronger than ARCH effects and suggesting that volatility effects have more persistence than past shock impacts. The fact that the two coefficients are close to 1 indicates that there may be a long memory process in the volatility and provides evidence of volatility clustering (Sen et al., 2021; Tsay, 2013). The t-values of alpha and beta are each greater than the table value of 1.98, and combining these with the presence of fat tails leads to the conclusion that volatility clustering is quite persistent in South African listed private equity markets. The impact curve for the GARCH (1,1) series is clearly symmetrical, as shown in Figure 5-20, and thus meets the conditions for the GARCH (1,1) as a symmetrical function.

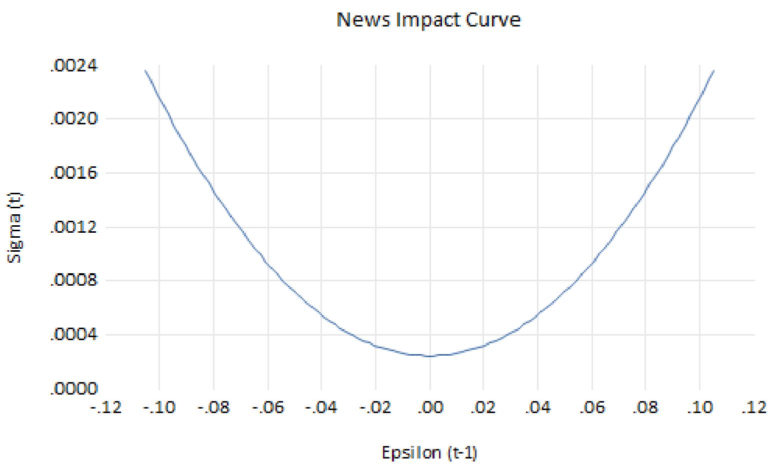


Figure 5-20: News Impact Curve for South Africa's GARCH (1,1)

Source: Researcher Compilation

5.3.1.2.1 Diagnostic test based on the Engle and Ng test

The Engle and Ng tests were used in the study to determine whether there was a sign bias. The study questioned the presence of positive or negative shocks on future volatility and whether the magnitude of the shock also affected future volatility, to provide a good justification for estimating some asymmetric GARCH models. Of the three tests, the study utilised the one that investigates both size and sign bias simultaneously. The regression is as follows:

$$u_t^2 = \phi_0 + \phi_1 S_{(t-1)}^{-} u_{(t-1)} + \phi_2 S_{(t-1)}^{\mp} \phi_3 S_{(t-1)}^{-} u_{(t-1)} + v_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(26)$$

Where u_t^2 denotes the squared residuals of a GARCH model, that takes the value of 1 if the residuals are greater than zero and takes the value 0 when it is otherwise. . Figure 5-21 shows the output for the Engle and NG test on EViews 9.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000133	2.44E-05	5.457393	0.0000
DUMMY1	0.000115	3.72E-05	3.088437	0.0020
DUMMY1*GARCH11RES(-1)	-0.005038	0.001607	-3.134551	0.0017
DUMMY2*GARCH11RES(-1)	0.012466	0.001472	8.467390	0.0000

Figure 5-21: Engle and NG test

The coefficient for S_{t-1}^{-}, ϕ_1 is significant at 1% hence a strong indicator for sign bias and the coefficients for $S_{t-1}^{-}u_{t-1}$ and $S_{t-1}^{+}u_{t-1}$ are all significant at 1% indicating a strong size bias. These results serve as justification for estimating GARCH models that allow for asymmetric volatility.

5.3.1.3 GARCH-in-Mean model

The study utilised the GARCH-M model in analysing the risk return trade-off in time varying volatility for these different private equity markets. Investors require a premium as compensation for holding risky assets. If the risk is captured by the volatility or by the conditional variance, then the conditional variance may enter the conditional mean function of $Y_{t.}$. The GARCH-M model allows the conditional mean to depend on its own conditional variance. It models a time varying risk premium to explain asset returns. That is:

$$Y_t = c + \Sigma\, h_t + u_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

Therefore, the GARCH-M (p,q) model is stated at

$$h_t = \varphi + \sum_{k-1}^{p} \theta_k\ h_{t-k} + \sum_{t=1}^{q} b_1\, u_{t-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

And from the findings the following results were obtained:

Table 5-7: GARCH-in-Mean model parameter estimates

Variable	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	-0.000665	0.039654	-0.016770	0.9866
C	1.05E-05	0.000613	0.017177	0.9863
Variance Equation				
C	3.16E-05	7.64E-06	4.137456	0.0000
RESID(-1)^2	0.206679	0.036966	5.591061	0.0000
GARCH(-1)	0.718596	0.041601	17.27349	0.0000

Source: Researcher Compilation

The variance term GARCH is not statistically significant in the mean equation as shown in Table 5-7, but its inclusion substantially increases the significance of the GARCH term in the variance equation. The probability of 0.0% clearly shows that by including it has improved the GARCH term in the variance equation (71.9%). This implies that there is evidence that the risk premium is significant to hedge South African listed private equity firms for an investor considering the variance in making investment decision. There exists a relationship between these returns and volatility; the risky asset is worth holding as the risky assets. Studies by Floros (2006) and Atenga and Mougoue (2021) established the relationship between volatility and returns for African countries, and of interest is that their findings show that volatility dynamics are place sensitive, and given the heterogeneity of investor appetite, it is also important to assess the asymmetrical tendencies of investments to the news.

TGARCH Estimations

The study hypothesised the impact of good and bad news on private equity investments may be asymmetric. When good (bad) news hit a financial market, assets tend to enter a state of tranquillity (turbulence) and volatility increase (decreases). The main target of the TGARCH model is to capture asymmetries in terms of negative and positive shocks. To do that, it simply adds into the variance equation a multiplicative dummy variable to check whether there are statistically significant differences when shocks are negative. The conditional TGARCH (1,1) model is stated as

$$h_t = \varphi + \theta_1 h_{t-1} + b_1 \mu_{t-1}^2 + \Upsilon_1 u_{t-1}^2 D_{t-1} \quad (29)$$

Where D_t takes the value of 1 (shocking news) for u_t^2 less than 0 or otherwise, therefore good news and shocking news have a different impact.

Table 5-8 depicts the results for the TGARCH estimations and shows that the coefficient of the asymmetric term is negative (-0.021329) and statistically insignificant indicating the absence of leverage effects. This implies that for listed private equity investments in South Africa, the notion that volatility increases more when there is a price fall than a price rise does not hold water. However, the statistic cannot be relied upon as it is weak.

Table 5-8: TGARCH Analysis for South Africa

	Coefficient	**Std Error**	**T-Statistic**	**Significance**
Mean	-3.44E -07	0.000215	-0.001602	0.9987
Constant	2.97E-05	6.74E-06	4.416205	0.0000
ARCH	0.190848	0.036870	5.176227	0.0000
GARCH	0.736410	0.037698	19.53426	0.0000
Asymmetry	-0.021329	0.048548	-0.439334	0.6604
GED	0.983515	0.031392	31.32971	0.0000

Source: EViews

Ahmed and Suliman (2011:121) revealed that a 'negative sign indicates that negative news tends to have more impact on volatility than good news in that negative shocks imply that a higher next period conditional variance than positive shocks of the same sign'. Since the statistical significance of this leverage effect is weak, TGARCH cannot be used to test the leverage effect.

5.3.1.4 EGARCH Estimates

Similar to a TGARCH, the exponential GARCH model was developed by Nelson (1991) to capture the leverage effect of shocks (policies, information, news, incidents and events) on the financial market. It allows for the testing of asymmetries. To do this, the log of the variance series is used. The key distinguishing characteristic that separates this model from other GARCH and ARCH models is its conditional variance equation which is specified as:

$$\ln\left(\delta_y^2\right) = \omega + \beta \ln(\delta_{t-1}^2) + \gamma \frac{u_{t-1}}{\sqrt{\delta_{t-1}^2}} + \left[\alpha \frac{|\mu_{t-1}|}{\sqrt{\delta_{t-1}^2}} - \sqrt{\frac{2}{\Pi}}\right] \ldots\ldots\ldots\ldots(30)$$

The exponential nature of the EGARCH model implies that conditional variance is always positive, regardless of parameter values; therefore, γ measures the asymmetric effect. If γ is statistically significant and has a negative sign, this implies that a fall in returns results in greater volatility than an increase in returns of the same magnitude (leverage effect).

Table 5-9: EGARCH parameter estimates for South Africa

	Coefficient	Std Error	T-Statistic	Significance
Constant	-0.995803	0.198078	-5.027317	0.0000
ARCH	0.016883	0.028098	0.600857	0.5479
GARCH	0.903755	0.022376	10.38946	0.0000
Asymmetr	0.29637	0.040972	7.233616	0.0000
GED	0.892290	0.031287	31.32971	0.0000

Source: Researcher Compilation

From the findings in table 5-9,

$$\log \delta_t^2 = -0.995803 + 0.296377(|E_{t-1}| - E|E_{t-1}|) + 0.903755 log\sigma_{t-1}^2 \ldots\ldots\ldots\ldots\ldots (31)$$

This model's leverage parameter is positive, indicating that EGARCH failed to capture the existence of leverage effects as well. As the value is statistically significant, it can be concluded that the effect is asymmetric. This value (0.296377) indicates the magnitude of a shock to the variance effects on the future volatility of the returns of the listed private equity series. The news impact curve illustrates how conditional volatility reacts to past shocks. Figure 5-24 demonstrates that the impact of news, whether positive or negative, is asymmetric supporting the conclusion that positive news carries more weight than negative news for private equity investments in this region. This indicates that investments in private equity in this region can serve as defensive assets in times of turmoil. Studies by Zhang et al. (2022), Curato and Sanfelici (2015), and Amudha and Muthukamu (2018), to name a few, established the significance of leverage effects on investment valuations and noted that the effects of bad news on the volatility of a share generally outweigh the effects of good news because investors are more concerned about bad news than good news. The depiction of this on the news dissemination curve in Figure 5-22 supports this notion.

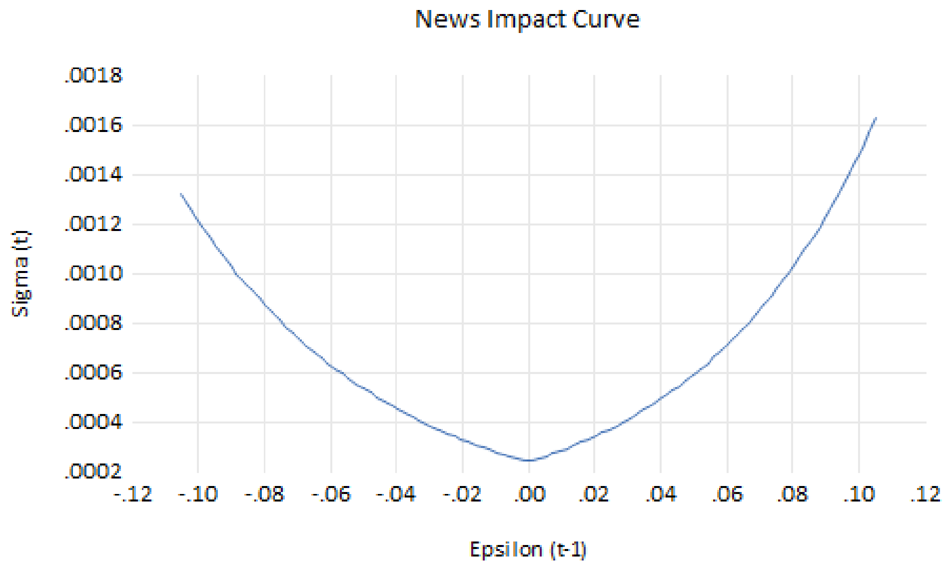


Figure 5-22: News Impact Curve for South Africa LPE

Source: Researchers Compilation

5.3.1.5 Residual tests analysis

To check if the model that has been fitted to the data was correctly specified and captured all the stylised facts that financial time series exhibits, post estimation testing for the ARCH effect was conducted. The ARCH LM test and correlogram analysis was employed to analyse the residuals.

Table 5-10: Autocorrelations of residuals for GARCH (1,1) for South African LPEs

Order	AC	PAC	Q-Stat	P-Value
1	-0.01	-0.015	0.5809	0.446
3	-0.03	-0.035	4.2765	0.233
6	-0.00	-0.009	7.0191	0.319
9	-0.00	0.005	8.2253	0.512
12	-0.00	-0.001	10.263	0.593
15	-0.01	0.009	11.201	0.738
18	-0.01	0.011	11.490	0.778
21	-0.02	-0.026	15.409	0.802
24	-0.01	-0.016	16.244	0.879
27	-0.01	-0.018	17.247	0.925
30	-0.01	-0.011	18.993	0.940
33	0.016	0.014	21.335	0.941
36	0.012	0.013	23.513	0.946

Source: Researcher Compilation

Table 5-10 shows the results that were attained on analysis of the residuals when testing for the presence of ARCH effects. From the findings relating to GARCH (1,1), and others in the appendix for other models, evidence suggests that there is no autocorrelation in the residuals as all residual coefficients are statistically insignificant, hence the model passes the residuals test both at one lag and at 36 lags. The Chi square values for 1 lag and 36 lags are 0.9535 and 0.9503 respectively. As the values can be seen to be greater than 0.05, we failed to reject the null hypothesis and concluded that there were no ARCH effects in the residuals found. This goes without saying that the stylised facts that exhibited in the South African listed private equity investments have been modelled accurately in the study.

5.3.1.5.1 Stability tests for the data

The study used the Nyblom stability tests which gauges parameter stability in both the mean and variance components of the GARCH model. The null hypothesis of this test is that the parameters are stable i.e they do not change through time. The findings from GARCH (1,1) Nyblom stability test based on EViews statistical package are attached in the appendix. These results indicated the inability to reject the null hypothesis both for the individual coefficients and for the joint test of all the coefficients based on the reported critical values for all the models under study.

Much as the use of t-statistics is significant when it exceeds the value 2.0, the following table provided a summary of individual stability tests at different significance levels to confirm that indeed the parameters used in the study were stable and provided evidence of absence of structural breaks in the series for the alternative asset class in South Africa. According to Chronopoulos et al. (2018), identifying structural breaks in the series is important as it provides true mechanisms of the mechanisms driving changes in the data.

Table 5-11: Mptotic critical values for Nyblom stability tests

	Significance Level					
Degrees of Freedom (m+1)	1%	2.5%	5%	7.5%	10%	20%
1	0.748	0.593	0.470	0.398	0.353	0.243
2	1.07	0.898	0.749	0.670	0.610	0.469
3	1.35	1.16	1.01	0.913	0.846	0.679
4	1.6	1.39	1.24	1.14	1.07	0.883
5	1.88	1.63	1.47	1.36	1.28	1.08
6	2.12	1.89	1.68	1.58	1.49	1.28
7	2.35	2.10	1.90	1.78	1.69	1.46
8	2.59	2.33	2.11	1.99	1.89	1.66
9	2.82	2.55	2.32	2.19	2.10	1.85
10	3.05	2.76	2.54	2.40	2.29	2.03

Source: Hansen (1990)

The study also analysed the sign bias test to examine the misspecification of the conditional variance in the models. The sign bias test investigates whether positive or negative shocks have differing impacts upon future volatility; it also investigates whether the magnitude of the shock also affects future volatility. The Engle test conducted earlier on confirmed the presence of both sign and size bias in the listed private equity series for South Africa. From Table 5-12 the high p-values from the different models indicate that the model is correctly specified. The positive sign bias and the negative sign bias all show high p-values at 1% whilst the sign bias was statistically significant at 5%. The joint test was statistically insignificant at for GARCH (1,1), EGARCH and GARCH-in-Mean showing indications for correct model specifications. This implies that the study failed to reject the null hypothesis that the volatility model was fairly correct.

Table 5-12: Volatility Specification based on News Impact Curve for South Africa LPE's

Volatility Specification based on News Impact Curve				
	GARCH (1,1)	EGARCH	TGARCH	GARCH M
Sign bias	1.953717 (0.0508)	2.948447 (0.0032)	3.441503 (0.0006)	2.042902 (0.0412)
Negative Sign Bias	1.548179 (0.1216)	1.820630 (0.0688)	2.395350 (0.0167)	1.745931 (0.0809)
Positive Sign Bias	0.250465 (0.8022)	0.807746 (0.4193)	0.538886 (0.5900)	0.047916 (0.9618)
Joint Test	5.089220 (0.1657)	8.846064 (0.0316)	12.70937 (0.0054)	5.712904 (0.1267)

Source: Researcher Compilation

5.3.1.5.2 Descriptive statistics for the error term

In order to check whether the assumptions are not violated and validate the functionality of the models, the study carried out a residual diagnostic. Three aspects of residuals from the fitted GARCH models were tested. The first was that the standardised residuals from GARCH models should approach normality. Figure 5.22 augmented the Jaque-Bera normality test in the table to provide the histogram of residuals, hence providing a visual tool. The second was that standardised squared residuals should not be autocorrelated and third, the ARCH effects should not be seen to exist in the residuals. The ARCH LM test was done on the residuals to diagnose the ARCH effects.

As shown in Figure 5.25, the histogram results for the normality assumption of the errors of all the GARCH models understudy reflect normality. The GARCH-in-Mean displays a bimodal distribution which is close to normality, whilst the error of GARCH (1,1) showed a distribution close to normality. GARCH (1,1) also showed insignificant negative skewness and slightly more peakness (leptokurtic) than the histogram for GARCH-in-Mean. TGARCH and EGARCH displayed more or less the same features; heavier on the left which can be attributed to news distribution discussed in earlier sections. In addition, the Lung-Jun Box test showed that the residuals were independently distributed.

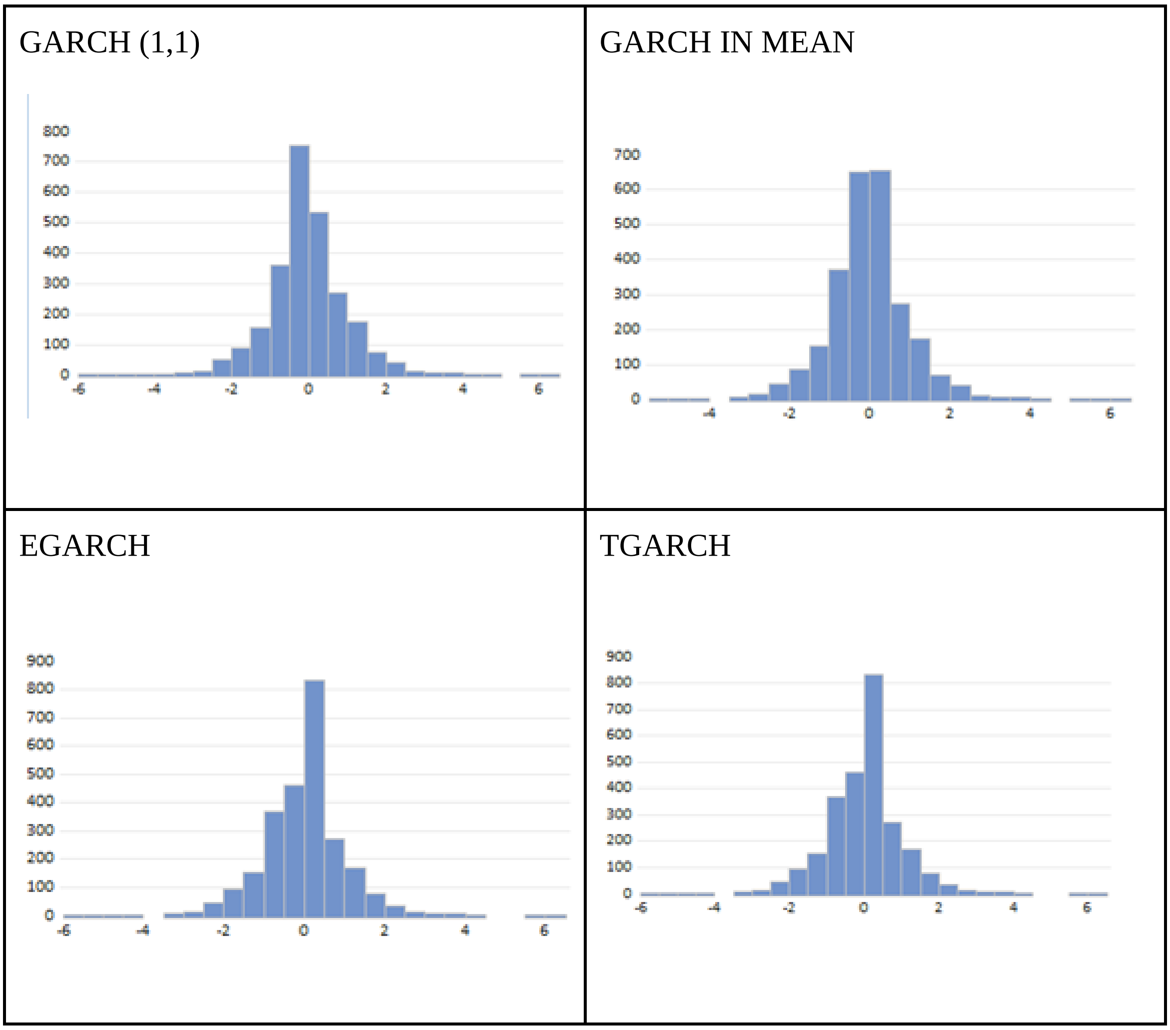


Figure 5-23: Distribution of error terms

Source: Researcher compilation

Table 5-13 shows that the residuals indicate heavy fat tails exhibiting leptokurtic distribution for all models and the residuals were moderately skewed with GARCH-in-Mean displaying the lowest standard deviation. The Jaque-Bera confirmed the normality of the data and overly the study concluded that the residual diagnosis meets all the assumptions of GARCH modelling.

Table 5-13: Analysis of residuals

Model	**GARCH (1,1)**	**GARCH IN MEAN**	**EGARCH**	**TGARCH**
Mean	-0.025618	-0.024603	-0.024576	-0.024024
Median	-0,000294	-6.87e-05	8.99e05	2.22e-05
Maximum	6.471199	6.353857	55.829941	6.401422
Minimum	-5.615690	-5.492868	-5.606189	-5.582855
Std. Dev	1.012038	0.989581	0.995076	1.003575
Skewness	0.061928	0.063470	0.026529	0.068865
Kurtosis	6.912999	6.936201	6.487507	6.845057
Jaque-Bera	1642.533	1662.779	1303.738	1587.051
Probability	0.000000	0.000000	0.000000	0.000000
Observations	2572	2572	2573	2573

Source: Researcher Compilation

5.3.1.6 Model selection and volatility forecasting performance

The study assessed models in terms of their ability to forecast future returns using the lowest values of error terms. To obtain the best forecasting model, the statistics from that volatility model should have the lowest Schwartz Information criteria, the lowest Root Mean Square Error (RMSE), a low Mean Absolute percentage error, and a low Theil's Inequality Coefficient. The Table 5-14 shows the output for forecasting parameters of the models used.

Table 5-14: Forecast performance of estimated models

MODEL	**Forecasting Horizon**	**RMSE**	**MAPE**	**Theil Inequality Coefficient**	**Overall Ranking**
GARCH (1,1)	30 Days	0.005761^{2}	199.6044^{1}	0.999367^{1}	1
EGARCH	30 Days	0.05760^{1}	199.8463^{3}	0.999718^{3}	2
TGARCH	30 Days	0.05761^{2}	199.8463^{3}	0.999931^{4}	4
GARCH in Mean	30 Days	0.005761^{2}	199.6988^{2}	0.999613^{2}	3
Ranking		EGARCH	GARCH (1,1)	GARCH (1,1)	

Forecast Sample: Superscript denotes the rank of the model

Source: Researcher Compilation

From the findings, the GARCH (1,1) model outperformed other models whilst the TGARCH model was lowest in forecasting conditional volatility of South African listed private equity investments. According to Dixit and Agrawal (2019), parsimonious models tend to perform better than other complex non-linear models. Figure 5.24 shows the sample volatility forecasts for the models under study.

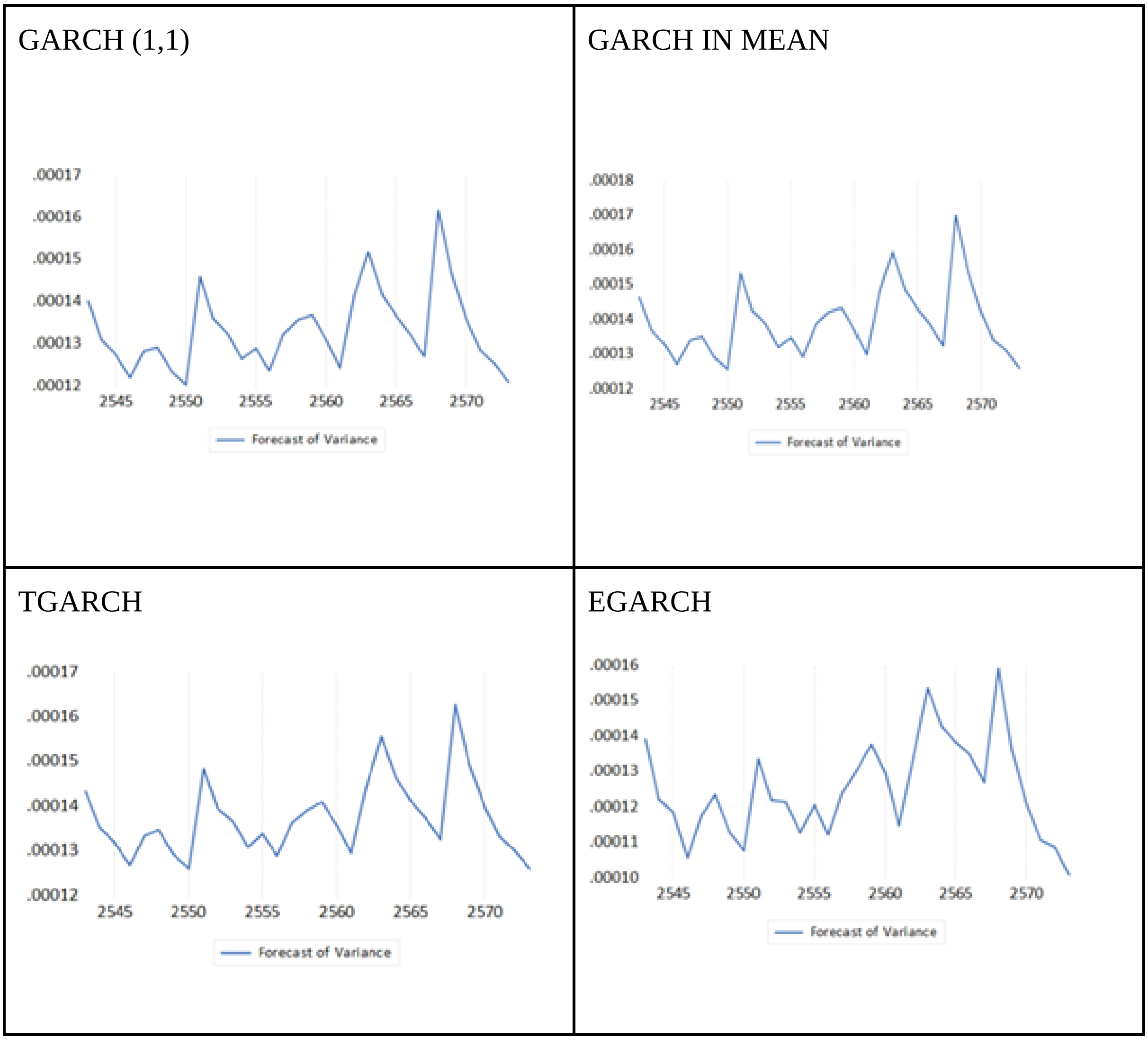


Figure 5-24: Sample volatility forecasts

Source: Researcher Compilation

It is a representation of how the in-sample observations look when using the static forecast on EViews. From the diagram, the return on assets is relatively stable but it is evident that the volatility of the index is high, trending down towards the end of the series. The study also predicted that volatility may also occur in the year 2020. The absence of leverage effects was seen and evidence of the prolificacy of private equity investments in the region and associated government efforts in supporting the same as explained in the early discussions of the descriptive analysis. In all cases, Theil Inequality coefficient shows that the series was a good fit as the value that depicts a good fit has to be equal to 1 or less than one but close to zero.

5.3.1.6.1 Exponentially Weighted Moving Average (EWMA)

GARCH models have decay features similar to EWMA but possess long-term average parameter that drags volatility back to a longer run value. When using in sample forecasting, GARCH forecasts can give an excellent fit better than EWMA methods, but it has issues of overfitting which may not give a good fit. Against this background, the study also utilised the EWMA method to validate GARCH forecast for the volatility of the South African listed private equity firms. The EWMA approach takes an average of volatility for the previous days assigning more weight to recent observations and less weight to past observations. The weights decline exponentially. A decay factor of 0.94 is used by NumXl and is also commonly used in modelling as it captures volatility clustering. On the other end, in as much as it is good at capturing conditional features, it is the worst at capturing autoregressive features. Figure 5.25 shows the output from NumXl foresting volatility for 24 months. This forecast is as much as the GARCH dynamic forecast trend.

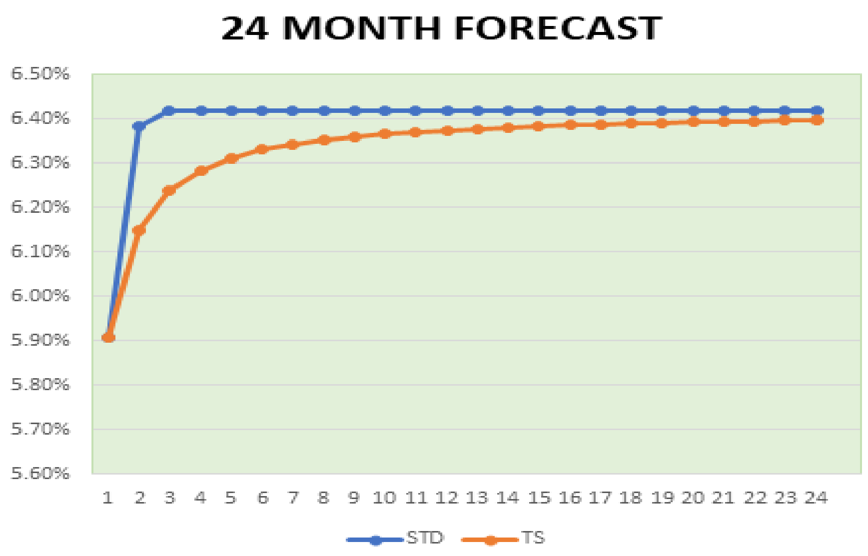


Figure 5-25: Forecast using Numxl for South Africa data series

Source: Researcher Compilation

5.3.1.7 Summary: Volatility dynamics for South Africa

In view of the geometric specifications presented and analysis conducted, it can be concluded that South African private equity investments exhibit stylised characteristics such as volatility clustering, asymmetric effects, leptokurtic distributions and found no evidence of leverage effects and structural breaks. The parameter estimates under study were found to meet stability conditions and they suggested elevated levels of persistence in the conditional volatility of the series. This suggests that this asset class is a defensive asset and a good hold during turbulent times. By and large, the GARCH (1,1) model was seen to be the best predictive model than other competing models in the study. The GARCH models looked at so far were designed to capture the short run dynamics of the investments, hence the study also explored analysis based on long memory dynamics of this alternative investment asset class, to cement the argument. The next section examines the volatility dynamics in Ghana.

5.3.2 Analysis of Ghanaian Listed Private Equity Firms

The study also analysed Ghana's listed firms that are into private equity investments. As alluded in earlier chapters, Ghana is one of the top destinations for private equity, rated to be in the top four most attractive countries in Africa. The investment has received a lot of government and corporate support, forming a Private Equity Association. Market investors and speculators need information to analyse the gains and losses from investments. Analysing volatility is helpful as it informs investors of a measure of the risk involved in holding an asset. Hence, this study analysed this asset class by examining its volatility dynamics and spillover effects with other markets.

The study tested for the heterogeneity in the series by testing for the presence of ARCH effects. As shown by the Table 5-15, the study noticed that the Chi-square is statistically significant at 1% and 5% much as the LM ARCH component; hence, the null hypothesis was rejected and it was concluded that there exist ARCH effects. Therefore, GARCH models were found enough to mitigate heteroscedasticity.

Table 5-15: Tests for ARCH effects

	DW stat test	Arch LM test
ALSI	2.097577	237.5925 [0.000] ***

***, ** and * indicate significance at the 1%, 5% and 10% levels respectively.

5.3.2.1 Diagnostic tests / Model Selection

The study examined the three parameters: normal distribution, Student's T and GED using the elements listed in Table 5-16 to identify the best model parameter to use in the GARCH estimations. The study, therefore, utilised the Normal distribution in all symmetric and asymmetric GARCH estimates.

Table 5-16: Diagnostic Tests model parameters

Criteria	**Model Parameters**			**Best Model**
	Normal Distribution	Student's T Distribution	GED	
Significant Coefficient	ALL	ONE	NONE	Normal Distribution
ARCH Significant	YES	YES	NO	Normal Distribution
GARCH Significant	YES	YES	NO	Student's T
Log likelihood	5806.086	7989.355	24337.69	
Akaike	-4.883069	-6.719995	-20.48122	GED
Schwartz IC	-40870920	-6.705417	-20.46664	Normal & Student's T
Heteroscedasticity (residuals)	NO	NO	YES	Normal & Student's T
Autocorrelation (residuals)	NO	NO	YES	

Source: Researcher Compilation

5.3.2.2 GARCH (1,1) Parameter estimates

By employing a similar method to Jatin Trivedi et al (2021), the GARCH (1,1) model was fitted to the series. Table 5-17 shows the parameter estimates.

Table 5-17: GARCH (1,1) parameter estimates for Ghana LPE

GARCH (1,1)	Estimate	Std Error	T-statistic	P-Value
Omega	7.76E-05	3.64E-06	21.28393	0.0000
Alpha	0.182215	0.011165	16.31954	0.0000
Beta	0.687635	0.012550	54.79103	0.0000
α +β	0.86985			

Source: Researcher Compilation

Both the GARCH and ARCH terms are statistically significant. The GARCH term is tending towards 1 whilst the ARCH term is tending towards 0 which is evident that the GARCH effects are stronger than those for ARCH; suggesting that volatility effects have more persistence than past shock impacts. The fact that the two coefficients are close to 1 indicates that there may be a long memory process in the volatility. This also shows that this GARCH model is a better forecasting model on periods of high volatility. The news curve diagnostic plot on Figure 5-26 how volatility responds to a shock in past events. The curve is clearly symmetrical.

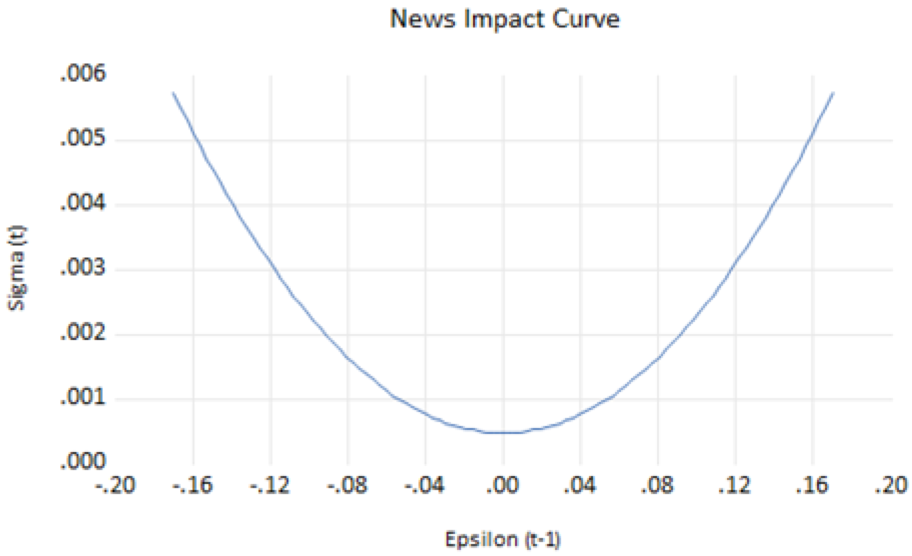


Figure 5-26: News impact curve diagram for Ghana LPE investments

Source: Researcher Compilation

GARCH-in-Mean

The propensity of this asset class as a good investment to hold was investigated using the GARCH-in-MEAN. Table 5-18 shows outputs from both standard deviation and variance.

Table 5-18: GARCH-in-Mean parameter estimates for Ghana

	Coefficient	Std Error	T-Statistic	Significance
SQRT(GARCH)	0.008847	0.080645	0.109700	0.9126
Constant	7.78E-05	3.73E-06	20.83363	0.0000
ARCH	0.182579	0.011122	16.41622	0.0000
GARCH	0.686857	0.012743	53.90244	0.0000

Source: Researcher Compilation

The GARCH coefficients in both the mean and variance equations were positive and statistically significant in all cases. Hence, providing evidence that the risk premium for Ghana LPE investments is significant to hedge. In other words, it is a risky asset worth including in a portfolio.

Diagnostic test based on the Engle and Ng test

In general, the leverage effect for financial assets exists when volatility increases more following a large price fall than for a large price rise of the same magnitude. When a stock price falls, the value of a company's equity falls which increases its leverage. Since the value of debt, relative to equity is now higher, the company is now riskier for investors; consequently, volatility increases to mirror this rise in risk. Table 5-19 shows the output for the tests.

Table 5-19: Diagnostic test based on the Engle and Ng test

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000318	5.20E-05	6.113694	0.0000
DUMMY1	-0.000403	0.000166	-2.426712	0.0153
DUMMY1*GARCHRESID(...	-0.034042	0.004083	-8.336817	0.0000
DUMMY2*GARCHRESID(...	0.025655	0.002729	9.401497	0.0000

Source: Researcher's compilation

The coefficient for S_{t-1}^{-}, φ_1 has a p-value of 0.0153 which demonstrates that it is significant at the 5% level. This is a strong indicator of sign bias. The coefficients for $S_{t-1}^{-}\,\mu_{t-1}$ and $S_{t-1}^{+}\,\mu_{t-1}$ are both significant values with p-values of 0.0000. This is a strong indicator of size bias. The evidence presented serves a good justification for estimating GARCH models which allow for asymmetric volatility in the returns for Ghanaian listed private equity investments. Therefore, the study proceeded to do EGARCH and TGARCH model estimations.

5.3.2.3 TGARCH model parameters

Table 5-20 shows the results from the analysis done on EViews 12 for TGARCH using Ghanaian listed Private Equity series.

Table 5-20: TGARCH model parameters for Ghana LPE

	Coefficient	Std Error	T-Statistic	Significance
Constant	7.76E-05	364E-06	21.33649	0.0000
ARCH	0.183851	0.014876	12.35874	0.0000
GARCH	0.687391	0.012499	54.99516	0.0000
Asymmetry	-0.002909	0.017548	-0.165784	0.8683

Source: Researcher Compilation

The coefficient for γ is negative showing that leverage effects were not present. However, the p-value was not statistically significant; therefore, only a weak argument can be made on the absence of leverage effects in arriving at an investment decision. The other terms in the conditional variance equation are significant and positive. A large decrease in the share price will cause the returns of the asset to be volatile in the immediate future period than a large price increase of an identical magnitude.

5.3.2.4 EGARCH model parameters

The study also utilised the EGARCH because it allows for the variance to react differently depending on the sign or size of the shocks it receives. An insight on the sign of the shock has an influence on the future volatility of an asset's returns. Persistence of volatility and how past volatility helps predict future volatility is an important variable in portfolio construction. The findings in the equation 32 were achieved using EViews 12 statistical package.

$$\ln\left(\delta_y^2\right) = -1.308225 + 0.848462 \ln(\delta_{t-1}^2) + 0.006906 \frac{u_{t-1}}{\sqrt{\delta_{t-1}^2}} + 0.293 \left[\frac{\mu_{t-1}}{\sqrt{\delta_{t-1}^2}} - \sqrt{\frac{2}{\Pi}}\right] ..(32)$$

The effect on future volatility in terms of the magnitude of the shock to the variance was 0.006906 and highly significant at 1%. This shows there is a positive relation between the past variance and the current variance, bigger the magnitude of the shock to the variance, the higher the volatility.

The asymmetric term was positive which indicates no evidence of the leverage effect. The notion that bad news will increase volatility more than good news of the same size does not appear to hold water on this stock. This is cemented by Figure 5-27 below. The β coefficient was positive and statistically significant at 1%; therefore, past volatility helps predict future volatility on Ghanaian returns for listed private equity firms.

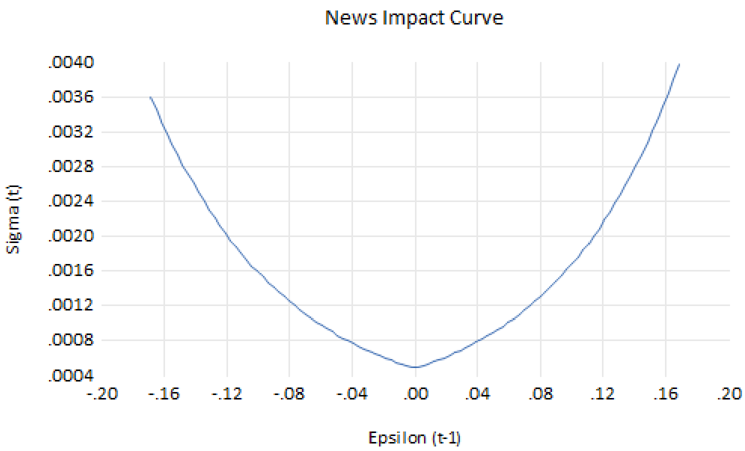


Figure 5-27: News impact curve for Ghana

Source: Researcher Compilation

5.3.2.5 Residuals Diagnosis

The study conducted ARCH tests on the residuals for all models under study, as a way of proving that enough evidence has been established that the models that had been fitted to the data were correctly specified. The absence of ARCH effects was tested from the residuals obtained from the Ordinary Least Squares regression (OLS) of the mean equation. Table 5-21 shows findings from the ARCH test.

Table 5-21: Arch tests

	DW Stat test	Arch LM test
ALSI	1.722343	0.086501 [0.7686] ***

***, ** and * indicate significance at the 1%, 5% and 10% levels respectively.

The DW test and the p-value of the ARCH LM statistic (0.7686) showed that the residuals of the GARCH (1,1) model did not exhibit ARCH behaviour, evidence that the model was well specified. In addition, the study tested for serial correlation using both squared residuals and residuals shown in Table 5-22.

Table 5-22: ACF and PACF for Ghana

Order	AC	PAC	Q-Stat	P-Value
1	0.006	0.006	0.0867	0.768
3	-0.018	-0.018	1.3232	0.724
6	-0.001	-0.001	1.4051	0.966
9	-0.009	-0.009	1.7990	0.944

12	0.015	0.015	2.6961	0.997
15	0.019	0.020	3.7555	0.999
18	0.004	0.005	4.4872	0.999
21	-0.001	-0.001	5.0661	1.000
24	-0.002	-0.002	5.2307	1.000
27	0.002	-0.003	6.1972	1.000
30	-0.008	0.010	6.6331	1.000
33	0.077	0.079	22.519	0.915
36	0.008	0.010	27.974	0.828

Source: Researcher Compilation

The findings show that both the ACF and the PACF lay within the confidence interval and that the p-values were well above 5% level of significance in the 36 lags that were specified. The fact that the p-values were not significant is enough evidence for no autocorrelation on the residuals. Therefore, the model passed both the heteroscedasticity and the residuals test.

A look at the standardised residuals in Figure 5-28 indicate that the volatility was seen to be varying with time showing the existence of conditional volatility (heteroscedasticity).

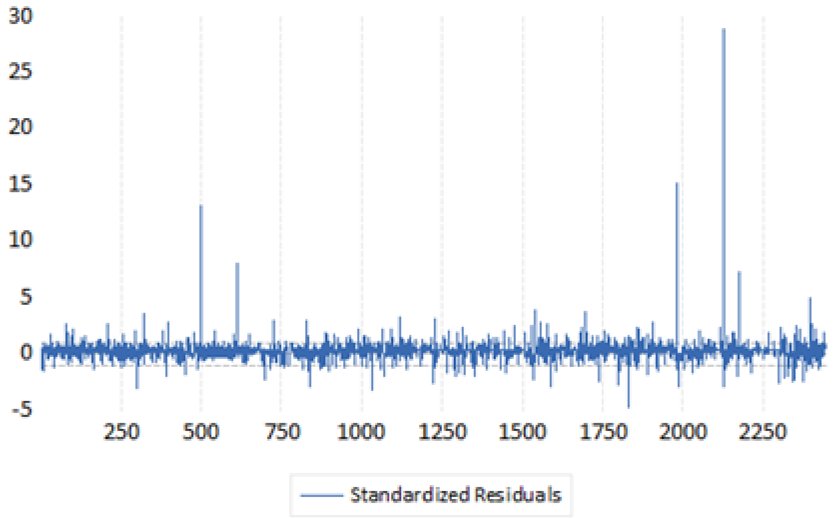


Figure 5-28: Standardised residuals of the error terms for Ghana

Source: Researcher Compilation

5.3.2.6 Descriptive analysis of the error terms

A brief descriptive analysis of the error terms shows that all models under study embodied positive skewness with GARCH (1,1) and GARCH-in-Mean slightly mirroring normal skewness. A closer examination of the residual plot also shows that residuals exhibited signs of normality, hence the GARCH models were considered the best fit for the Ghana LPE investments.

Table 5-23: Descriptive statistics for the error terms for Ghana

	GARCH (1,1)	GARCH-IN-MEAN	EGARCH	TGARCH
Mean	0.007087	-0.008779	0.0022369	-0.013236
Median	0.022338	-6.10e-05	7.96e-13	-5.94e-10
Maximum	9.938668	7.908163	9.524101	13.61582
Minimum	-6.903644	-6.121448	-5.738767	-9.077227
Std. Dev	1.000353	0.938097	0.888823	1.266515
Skewness	0.520547	0.566150	1.486659	0.615367
Kurtosis	16.91984	15.47489	27.07155	20.47406
Jaque-Bera	19289.74	15540.13	58239.73	30391.66
Probability	0.000000	0.000000	0.000000	0.000000
Observations	2376	2377	2376	2377

Source: Researcher Compilation

Although all models exhibited leptokurtic distribution, the GARCH-in-Mean was more pronounced than GARCH (1,1), followed by the other two asymmetric GARCH models. The huge spread between the maximum and the minimum explains the shape the distribution of the error terms.

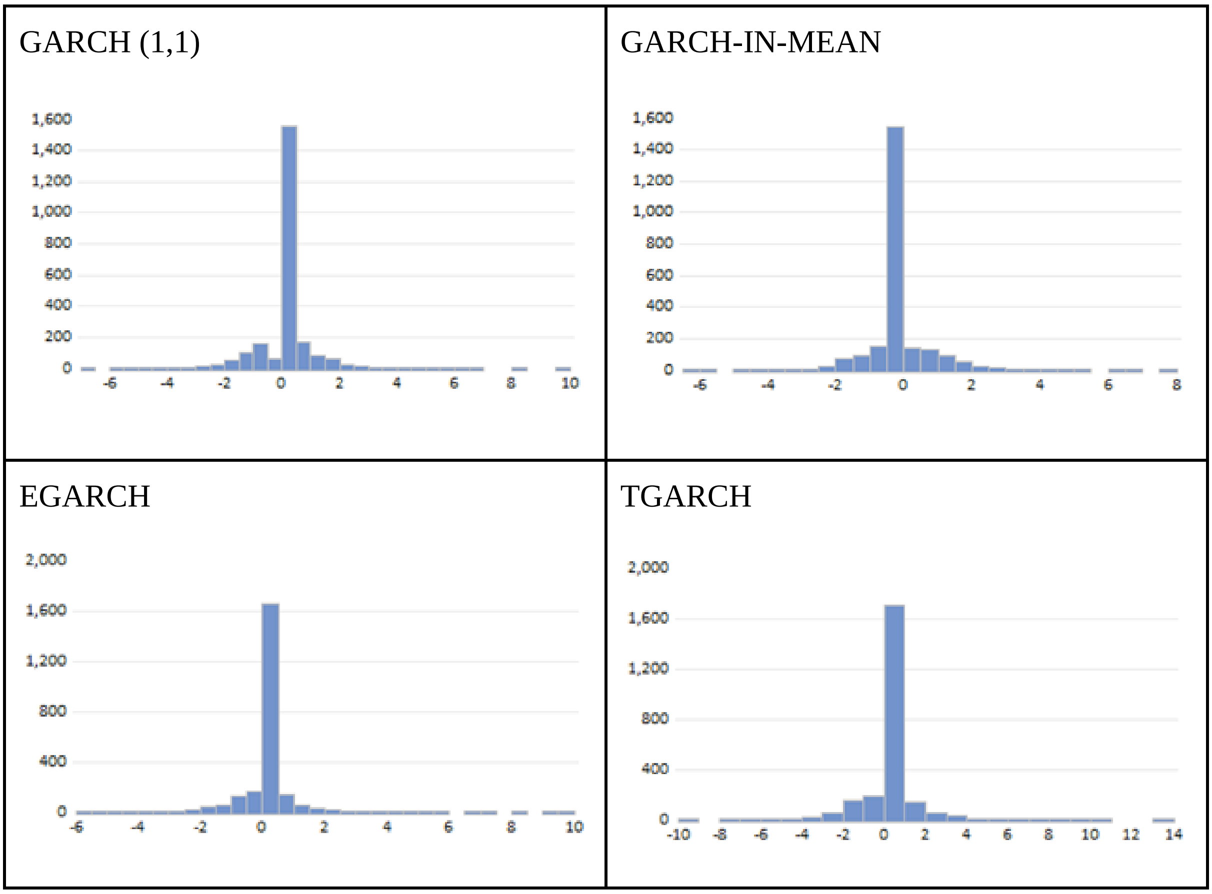


Figure 5-29: Error term diagnostics for Ghana

Source: Researcher Compilation

5.3.2.7 Parameter stability tests

The Nyblom parameter stability test provides information on the stability of the estimated parameters of a model by identifying structural breaks. Failure to identify structural breaks often results in inaccurate model forecasts and therefore invalid conclusions (McKnight et al., 2022). The study conducted a Nyblom parameter stability test. The estimated parameters in Figure 5-30 show that the values are under 5% of the critical value, therefore confirming that the parameters were stable and there were no structural breaks. The confidence ellipse also validated the analysis, and the estimated parameters were all within the unit circle (see appendix).

Nyblom Parameter Stability Test
Null Hypothesis: Parameters are stable

Variable	Statistic	1% Crit.	5% Crit.	10% Crit.
C	0.276254	0.748	0.470	0.353
GHANA(-1)	0.387184	0.748	0.470	0.353
C	0.367603	0.748	0.470	0.353
RESID(-1)^2	0.100954	0.748	0.470	0.353
GARCH(-1)	0.279899	0.748	0.470	0.353
Joint	1.325932	1.880	1.470	1.280

*Critical values from Hansen 1990

Figure 5-30: Parameter Stability Test for Ghana

Source: Researcher Compilation

5.3.2.8 Model selection

The study assessed the models forecasting power by making use of the Log likelihood test and the AIC test as shown in Table 5-24.

Table 5-24: Model selection for Ghana LPE's

Parameter	GARCH (1,1)	GARCH in Mean	EGARCH	TGARCH
α_0	7.76 E-05	7.78 E-05	-1.308225	7.76E-.05
α_1	0.182215	0.182579	0.006906	0.183851
β	0.687635	0.686857	0.848462	0.687391
$\alpha+\beta$	0.86985	0.869436	0.855368	0.871242
γ	-		0.293098	-
AIC	-4.883096	-4.8822475	-4.870297	-4.882473
Log likelihood	5806.086	5807.821	5791.913	5807.819

Source: Researcher Compilation

A lower AIC accompanied by a high log-likelihood points to the fact that the selected model fits the dataset better than any other model. From Table 5-26, the GARCH-in-Mean is the best model in terms of data fitting. These findings coincide with the findings from Maqsood et al. (2017) wherein there is a positive correlation between volatility and expected returns on the Nairobi Stock exchange.

5.3.2.9 Forecasts for Ghana LPE Series

According to Tsayi (2013), a good model of volatility should capture the conditionality feature (volatility clusters) and autoregressive features where volatility exhibits mean reversion. GARCH models generally capture both the clustering and the mean reversion components. Its parameters have decay components that erode the volatility back to a longer run value.

This analysis concludes with a discussion of the predictive accuracy of the selected models. The MSE value indicates the precision with which the model predicts future parameters and estimates model coefficients. The graphs in Figure 5-31 depict forecasts for the month of December 2020. A one-month period was chosen because a longer period would produce an unreliable confidence interval with a very wide range of values. As the researcher projects further into the future, it is natural for his or her level of confidence in the forecast values to diminish (Koo & Kim,2022).

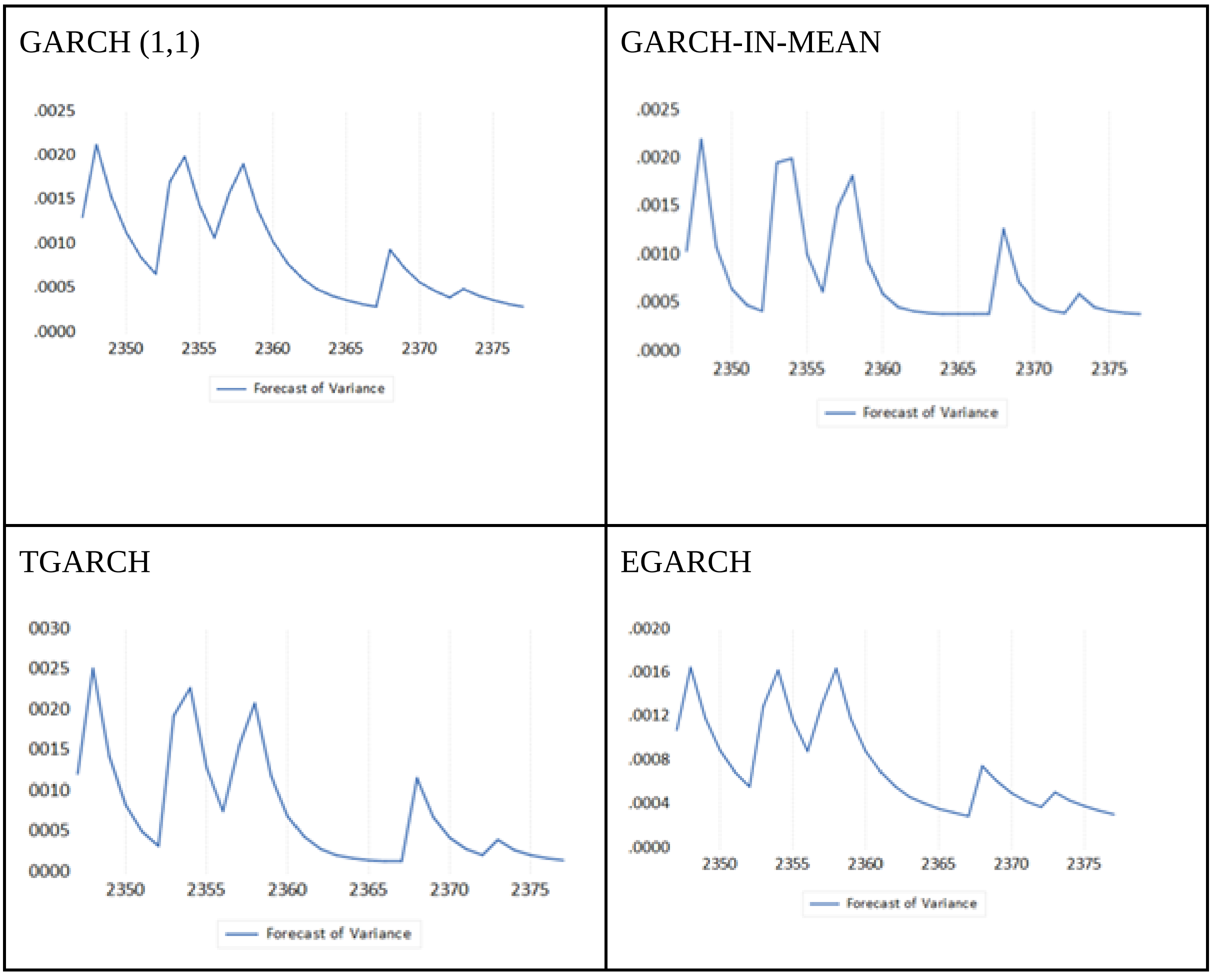


Figure 5-31: Forecasts for models for LPE investments in Ghana

Source: Researcher Compilation

In order to construct a reliable model, a high level of precision is required to facilitate decision making. The parameters used to evaluate the functionality of the models and ultimately select the best one are listed in the Table 5-25. The model that exhibited the lowest value of error measurements in this series was the asymmetric GARCH relative to all other asymmetric models. Similar to South African findings, the GARCH (1,1) model was parsimonious and generally widely accepted to other relatively complex linear models.

Table 5-25: Model selection parameters for Ghana

MODEL	RMSE	MAE	Theil Inequality Coefficient	Symmetric MAPE	Ranking
GARCH (1,1)	0.030913	0.014316	0.983531	199.3801	1
EGARCH	0.031279	0.014754	0.986421	199.9099	3
TGARCH	0.031018	0.014300	1.000000	200.0000	2
GARCH M	0.031026	0.014754	0.999393	199.9978	4
Best Model	GARCH (1,1)	TGARCH	GARCH (1,1)	GARCH (1,1)	

Source: Researcher Compilation

The least model in forecasting the conditional volatility for Ghana Private Equity series was the GARCH-in-Mean model; the TGARCH model outperformed the EGARCH models in forecasting the conditional variance although it failed to explain leverage effects of the series. Although these models have nuanced differences in terms of performance, the consensus was that the return on assets was stable but showed intense volatility. The return on assets was stable and there was turbulence throughout the course of the series; hence, we still expect some turbulence for the following year.

5.3.2.10 Summary of volatility dynamics for listed private equity investments in Ghana

This study's findings fil the research gap of demystifying the volatility dynamics of LPE investments in Ghana. They suggest that these investments carry with them a risk premium that is commensurate with the inherent risk. No evidence of leverage effects was noted. Given the attention the asset class is receiving from the government of Ghana, the investment can help optimise the portfolio on the part of risky assets. All models selected passed the tests for assumptions, and the stability and stationarity tests, hence assumed to be the best models to fit data series for Ghanaian listed private equity investments. The next section looks at the volatility dynamic of the Egyptian LPEs.

5.3.3 Egypt's Analysis

5.3.3.1 Introduction

Despite positive economic growth trends and favourable global waves which naturally saw increased capital flights of private equity to Egypt, this industry is also dominated by locals that actively pull their key competences, adopt international best practices to leverage on local consumer demands. Private equity investments in Egypt are a catalyst for consolidating and globalising their portfolio companies (AEO, 2018). Listed firms that are actively involved in private equity have increased their competencies and expanded operations globally. This study relies on the heteroscedastic features exhibited in the series for these investments to explore the volatility dynamics of this asset class. Tests for the existence of ARCH effects was followed by data fitting into different GARCH models starting with GARCH (1,1), followed by asymmetric GARCH models. The study also presents the diagnostic tests done and wraps up with a model forecasting and the conclusion provides a summary of the findings obtained.

5.3.3.2 Test for ARCH effects

The study tested for the presence of ARCH effects as shown in Table 5-28, which shows the series has constant volatility. The DW test provides evidence of the presence of ARCH effects. The f-statistic is highly significant at 1% and 5% (20.31098 > 1.2438); hence, we reject the null hypothesis and conclude that there exist ARCH effects. Volatility does not depend on time; hence GARCH analysis was fit for the data.

Table 5-26: ARCH effects

	DW stat test	Arch LM test
ALSI	2.068200	20.31098 [0.000] ***

***, ** and * indicate significance at the 1%, 5% and 10% levels respectively.

Source: Researcher Compilation

5.3.3.3 GARCH (1,1) Parameter estimates for Egypt

The findings for the GARCH (1,1) are indicated on the Table 5-27.

Table 5-27: GARCH (1,1) parameter estimates for Egypt

Parameter	Coefficient	Standard Error	P-Value
Omega	0.000392	2.16E-05	0.0000
Alpha	7.332084	0.95060	0.0000
Beta	0.012977	0.002085	0.0000
α+β	7.345061		

Source: Researcher Compilation

The findings show that the equation is as,

$$\sigma_t^2 = 0.000392 + 7.332084E_{t-1}^2 + 0.012977\sigma_{t-1}^2$$

Since $\alpha + \beta > 1$ $i.e$ $7.332084 + 0.012977 = 7.345061$, this indicates that the volatility is increasing overtime therefore jeopardising the stationarity assumption of the GARCH model. This nullifies the validity of GARCH (1,1) in estimating volatility dynamics of Egyptian listed private equity investments.

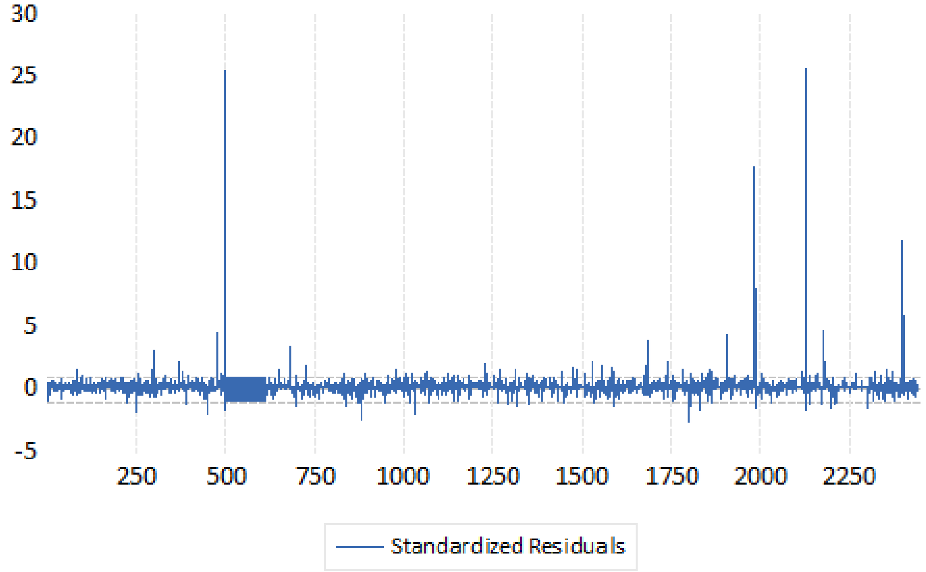


Figure 5-32: Standardised residuals of Egyptian LPE investments

Source: Researcher Compilation

A closer analysis of the standardised residuals indicate that they are stable overtime; but there exist some jump diffusions in the series. The Egypt series depicts episodes of jump diffusions which need to be captured when forecasting the series. Even if the model passed assumptions tests, tests for asymmetry were conducted to ascertain whether the series was fit for asymmetric GARCH models. The Engle and Ng test findings are shown on Table 5-28.

Table 5-28: Engle and Ng test

Parameter	t-Statistic	P-Value
Sign Bias	-1.209803	0.2265
Negative-Bias	-1.631795	0.1029
Positive-Bias	1.307621	0.1911
Joint- Bias	6.156555	0.1045

Source: Researcher Compilation

From the findings in Table 5-28, Egypt's log returns indicate statistically insignificant sign bias; negative and positive size bias much as the joint bias indicating that the GARCH (1,1) and its extensions for asymmetry were applicable in the series. From this analysis, the study went on to explore other GARCH extensions on the series that captured volatility asymmetries. Generally, for some financial asset stocks in particular, volatility increased more following a large price fall than for a large price rise of the same magnitude. One explanation for this behaviour was given in the context of stock returns which states that when a stock price falls, the value of a company's equity falls which increases its leverage. Since the value of debt, relative to equity is now higher, the company is now riskier for investors. Consequently, volatility increases to mirror this rise in risk. The study explored the GARCH extensions with a view to analyse the impacts of such news on volatility. These are discussed underneath.

5.3.3.4 Asymmetric volatility models

Table 5-29 shows a summary of the findings for all the asymmetric GARCH models that were explored.

Table 5-29: Parameter estimates for asymmetric GARCH models for Egypt

MODEL	Parameter	Coefficient	Standard Error	P-Value
TGARCH	Omega	0.000339	1.16E-05	0.0000
	Alpha	19.88285	0.139948	0.0000
	Beta	0.137887	0.004196	0.0000
	γ	-19.37913	0.113390	0.0000
EGARCH	Omega	3.880744	0.032128	0.0000
	Alpha	1.600417	0.009285	0.0000
	Beta	0.531204	0.004882	0.0000
	γ	2.250772	0.014660	0.0000
GARCH in Mean	Omega	0.025034	0.000211	0.0000
	Alpha	0.770869	0.175115	0.0000
	Beta	0.007745	5.14E-13	0.0000

Source: Researcher Compilation

Findings from the TGARCH model showed that the asymmetric term was negative and statistically significant indicating that the impact of negative shock on volatility was significantly lower than the impact of positive shock. This is depicted by the graph in Figure 5-33.

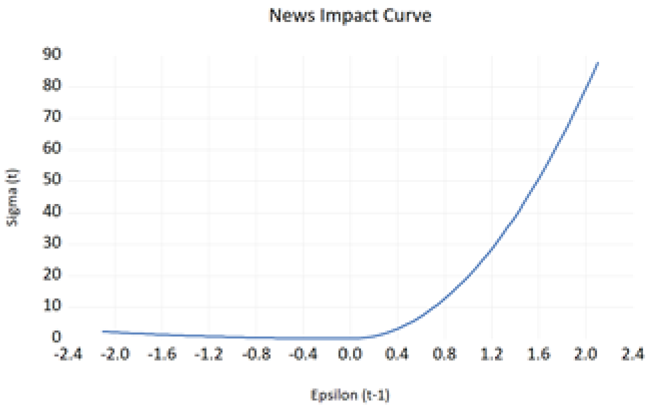


Figure 5-33: News impact curve analysis for Egypt LPE's

Source: Researcher Compilation

The EGARCH term indicates that the volatility persistence was moderate (0.531204), and that the shock has a significant impact on volatility. The sign effect indicates that there was a direct relationship between the error terms and volatility. The asymmetric term was positive, implying that good news weigh more than bad news. The findings were cemented by the TGARCH news impact curve.

A closer examination of the TGARCH and EGARCH, however, reveals that the sum of the alpha and beta coefficients was greater than 1, which violates the stationarity conditions. These models cannot be used to analyse the volatility dynamics of listed private equity investments in Egypt for this reason. Insofar as these two asymmetric models are unable to produce conclusive results, the study concluded that the asymmetric effects of news on conditional volatility did not exist for this asset class of alternative investments in Egypt.

On the other hand, the GARCH-in-Mean variance term was statistically significant and positive, whereas the GARCH mean term was insignificant, indicating an inverse relationship between risk and return for this stock, making it an excellent defensive stock. This model suggests that ARCH effects were greater than GARCH effects, indicating that volatility persistence was evident. The ratio of shock persistence to volatility equals 0.777.

Based on the analysis presented, it is clear that the Egyptian listed private equity investments are best represented by the GARCH mean from the GARCH family, as all other models failed to meet the stationarity assumptions. Consequently, only the GARCH-in-Mean was tested further in the study to confirm its model fitting capabilities.

5.3.3.5 Diagnostic tests

The study conducted post estimation tests for ARCH effects as was done for South Africa and Ghana, and found evidence of no presence of ARCH effects. The table for the ARCH tests for residuals is shown on the appendix. Besides this, the study tested for autocorrelation of residuals. The ACF and PACF indicated the absence of serial correlation based on the analysis of p-values in the 36 lags specified by the study; consequently, the null hypothesis for the Ljung-Box Q-statistic cannot be rejected and sufficient evidence indicates that the residuals were not serially correlated. Table 5-30 displays the results of residual tests for the error term.

Table 5-30: Autocorrelations of residuals for GARCH (1,1) for LPEs in Egypt

Order	AC	PAC	Q-Stat	P-Value
1	0.011	0.011	0.2704	0.603
3	0.005	0.005	0.3605	0.948
6	0.003	0.003	0.4421	0.998
9	0.004	0.004	0.5527	1.000
12	0.003	0.003	0.6273	1.000
15	0.003	0.003	0.6974	1.000
18	0.003	0.003	0.7551	1.000
↓	↓	↓	↓	↓

Source: Researcher Compilation

To ensure that the models capture the intended behaviour, the study made use of the Nyblom stability tests, the sign bias test and the confidence ellipse test. This tests provided a means of assessing the structural change in a time series. Tests for all models indicated that the models are all correctly specified hence the data fitting was correct for all the models (see appendix). In addition, the study noted that the sign bias parameter, the negative sign bias and the positive sign bias test were closer to 0; hence, the model was correctly specified. Additionally, a confidence ellipse test was conducted and provided evidence that the unit circle was within the confidence bounce hence the model was stationary. The Nyblom parameter stability test yielded the following result:

Nyblom Parameter Stability Test
Null Hypothesis: Parameters are stable

Variable	Statistic	1% Crit.	5% Crit.	10% Crit.
C	0.116051	0.748	0.470	0.353
C	0.274103	0.748	0.470	0.353
RESID(-1)^2	0.130296	0.748	0.470	0.353
GARCH(-1)	0.129585	0.748	0.470	0.353
Joint	63.61463	1.600	1.240	1.070

*Critical values from Hansen 1990

Figure 5-34: Nyblom Parameter stability output for Egypt LPEs

Source: Researcher Compilation

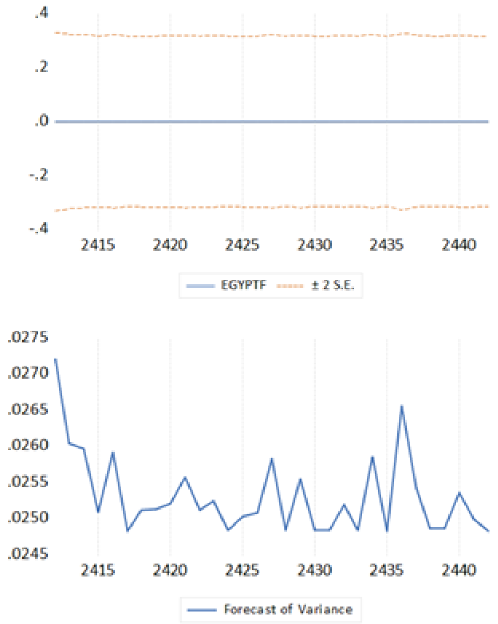
Figure 5.34 provides evidence that the parameter statistics were all below 0.47, which was the 5% critical value hence confirming that the parameter estimates were stable.

5.3.3.6 Descriptive statistics for the error term

Further tests on the EGARCH model were done to analyse the descriptive statistics of the error term. A close analysis of the distributional properties of the error terms showed that the error terms were positively distributed with excess kurtosis (see appendix). The diagrammatic plot indicates signs of excess kurtosis revolving around the mean. This shows that the residuals passed the diagnostic tests for normality.

5.3.3.7 Volatility Forecasting

The study utilised a modified sample to forecast data series for Egyptian listed private equity investments using a 30-day forecasting horizon. As a rule of thumb, model functionality is best assessed by analysing the parameters. The AIC, the RMSE and the Theil's Inequality coefficient was used to assess the model.



Forecast: EGYPTF	
Actual: EGYPT	
Forecast sample: 2412 2442	
Included observations: 31	
Root Mean Squared Error	0.023319
Mean Absolute Error	0.019100
Mean Abs. Percent Error	NA
Theil Inequality Coef. 0.964511	
Bias Proportion	0.001238
Variance Proportion	0.998663
Covariance Proportion	0.000099
Theil U2 Coefficient	NA
Symmetric MAPE	177.5756

Figure 5-35: Volatility Forecasts for Egypt

Source: Researcher Compilation

Figure 5.35 shows the forecasts for Egyptian LPEs. It is evident that the volatility of the index was very high. The Theil Inequality coefficient depicts a good fit as it was closer to 1. The MAE value of 0.0191 was approximately closer to 0; hence, the model that was fitted was a good fit to the series. The value for the return series was 0.9645 which describes the goodness of fit for the data. From the graph, the study deduced that volatility may occur the following year and the return on assets would be stable overtime.

Volatility persistence, clustering and leverage effects were examined for listed private equity investments in Egypt. The study found no evidence of the leverage effects; however, traces of volatility clustering associated with some jump diffusions were noticed. Although GARCH models are widely recommended as best volatility forecasting techniques, they were not so appropriate for modelling this alternative asset class. The GARCH-in-Mean model was found appropriate for modelling this time series. The study noted a negative relationship between returns and volatility, and observed that volatility surprises tend to follow correspondingly upward revisions of reward process. Hence, the asset is a good hold asset. The TGARCH and EGARCH failed to meet stationarity conditions; consequently, news on stock was found not prevalent in this asset class.

5.4 Modelling for Long Run Volatility

This section examines the long-term memory of the private equity investment series in Africa. Traditionally, the GARCH models that have been examined thus far are intended to capture the short-run volatility time dependence behaviour of the series without taking into account the long-term behaviour. As has been outlined in previous discussions, the focus of this study was on volatility models that rely on time-varying conditional moments and persistence asymmetries. Patterns of volatility clustering and mean reversion characteristics. ARCH and GARCH models aid in the explanation of data behaviour by allowing conditional variance to respond to past behaviours over time. Studies have demonstrated that volatility series have a long memory that influences future volatility over an extended time horizon. Considering this, Baillie Bollerslev and Mikkelsen (1996) developed the FIGARCH model. Long memory is characterised by a slow hyperbolic decay in autocorrelations and impulse response weight, according to Baillie et al (2007). Long memory fractional GARCH models are superior to other heteroscedastic models in terms of their ability to improve forecast accuracy and provide efficient parameter estimations due to their long memory properties. As investors are concerned about long-term dependence in portfolio selection and when estimating value at risk in financial risk management, the study investigated these variables. The results for the South Africa FIGARCH output are depicted in the Table 5-31.

Table 5-31: Parameter estimates for FIGARCH models

Country	**South Africa**		**Ghana**		**Egypt**	
Parameter	FIGARCH	FIEGARCH	FIGARCH	FIEGARCH	FIGARCH	FIEGARCH
ARCH term	0.274288 (0.2182)	-	0.86717 (0.0000)	-	0.329022 (0.2040)	-
GARCH term	0.496308 (0.0900)	-	0.68963 (0.0000)	-	0.005910 (0.9741)	-
d	0.392659 (0.0001)	-1.163099 (0.00000)	0.00763 (0.449)	-4.9741 (0.0000)	0.520088 (0.0000)	-0.719029 (0.0000)
α (ARCH)		-0.476656 (0.00000)		0.98605 (0.0000)		-1.008334 (0.0000)
β (GARCH)		0.851798 (0.00000)		-0.04491 (0.0000)		0.985105 (0.00000
Θ_1		0.315854 (0.00000)		0.34709 (0.0000)		0.950257 (0.0000)
Θ_2		0.024708 (0.1581)		0.04914 (0.0000)		-0.462182 (0.0000)
AIC	-5.588152	-5410256	-4.882392	-4.8717	-0.276708	-4.003491
Residual ARCH effect	NO	NO	NO	NO	NO	NO

Note: P-value is given in parenthesis. All models estimated on EViews.d are a fractional difference parameter that measures the degree of long memory behaviour:

α- FIGARCH ARCH term; β – FIGARCH GARCH term; Θ_1 –leverage effects; Θ_2- asymmetric term

Source: Researcher Compilation

From the findings, all parameters were positive which satisfied the FIGARCH assumption. Both GARCH and ARCH terms were statistically insignificant for South Africa and Egypt, and significant for Ghana. The sum of GARCH and ARCH terms was less than 1 providing evidence of stationarity of the long memory process. The estimated d parameter lay between 0 and 1 presenting evidence of stability of the process and its t-statistics indicates a long-term dependence which points to the presence of fractional integration in the series.

The estimated *d* parameter in the Ghana series was close to 1 indicating that the decaying impact was quite insignificant; hence, it exhibits the same characteristics as the GARCH (1,1), and as such, the long-term memory of the data series was not significant. The asymmetric term was statistically significant; thus leverage effects do exist. In addition, tests for residuals showed no evidence of ARCH effects; hence, all the models passed the test for existence of ARCH effects.

5.4.1 Diagnostic tests for FIGARCH and FIEGARCH of listed private equity returns in Africa

The study conducted diagnostic tests with the understanding that in order for these forecasting models to be ideal, they must satisfy stationarity conditions, have normally distributed residuals, stable parameters, and a goodness of fit (Muhammad & Nusrat, 2021). Table 5-32 summarises the findings from the correlogram of squared residuals analysis of the correlation results of the squared standardised residuals.

Table 5-32: Serial correlation test of squared residuals

Country	FIGARCH	FIEGARCH	Comment
South Africa	Q_5 = 5.0019 (0.416) Q_{10} =5.8716 (0.826) Q_{20} =11.573 (0.930) Q_{30}= 17.122(0.967)	Q_5 = 2.8927(0.717) Q_{10} = 3.5786(0.964) Q_{20} =7.8830(0.993) Q_{30}= 14.645(0.992)	Evidence of no auto-correlation of the residuals
Ghana	Q_5 = 1.3793(0.927) Q_{10} = 1.8954(0.997) Q_{20} =4.9814(1.000) Q_{30}=6.5666(1.000)	Q_5 = 2.1866(0.823) Q_{10} = 3.4038(0.970) Q_{20} =6.5852(0.998) Q_{30}= 9.40193(1.000)	Evidence of absence of auto correlation of the residuals for all lags specified
Egypt	Q_5 = 0.4072(0.995) Q_{10} = 0.4361(1.000) Q_{20} =0.4658(1.000) Q_{30}= 0.4903(1.000)	Q_5 = 1.2051(0.944) Q_{10} = 1.2263(1.000) Q_{20} =1.327(1.000) Q_{30}= 1.4026(1.000)	All lags provide evidence of absence of auto correlation of residuals

Note: P-value is given in parenthesis; $Q_{(n)}$ is the n^{th} lag Ljung-Box test statistics

Source: Researcher Compilation

The findings from the post estimation correlogram in Table 5-34 show the parameters for the 5^{th} lag, 10^{th} lag, the 20^{th} lag and the 30^{th} lag for the FIGARCH residuals that the researcher had specified. The p-values of the residuals are all greater than 5% showing that the residuals are not serially autocorrelated, proving that the LPE returns were modelled accurately at 5% confidence interval. Apart from that, the study conducted the Nyblom parameter stability test with the view of assessing the stability of parameters and checking for the presence of structural breaks. Studies done by Tsiaras et al. (2022) and Bawa (2020) showed evidence of structural breaks using this test in their respective series. Table 5-33 shows the results obtained from the test:

Table 5-33: Nyblom parameter stability tests

Country and Model	Nyblom Parameter Stability	Variables	Nyblom Statistic	Comment
SOUTH AFRICA FIGARCH	1% Critical Value =0.748 5% Critical Value=0.47 10% Critical Value= 0.353	Constant ARCH GARCH D parameter	1.548456 2.646421 2.816126 1.584766	All the estimated coefficients are unstable, providing evidence of structural breaks.
FIEGARCH	1% Critical Value =0.748 5% Critical Value=0.47 10% Critical Value= 0.353	Constant Omega Alpha Beta Theta 1 Theta 2 D	0.083654 0.162792 0.143999 0.156030 0.576030 0.125057 0.094943	All the estimated coefficients are stable
GHANA FIGARCH	1% Critical Value =0.748 5% Critical Value=0.47 10% Critical Value= 0.353	Constant ARCH GARCH D- parameter	0.358384 0.090148 0.280252 0.331041	All the estimated coefficients are stable

FIEGARCH	1% Critical Value =0.748 5% Critical Value=0.47 10% Critical Value= 0.353	Constant Omega Alpha Beta Theta 1 Theta 2 D	0.373953 0.807907 0.246469 0.370156 0.430767 0.217682 0.486881	All coefficients except for Omega and the d-parameter are unstable providing indications for structural breaks
EGYPT **FIGARCH**	1% Critical Value =0.748 5% Critical Value=0.47 10% Critical Value= 0.353	Constant ARCH GARCH D- parameter	23.16752 4.034751 2.386360 6.880559	All coefficients are unstable pointing towards evidence of structural breaks
FIEGARCH	1% Critical Value =0.748 5% Critical Value=0.47 10% Critical Value= 0.353	Constant Omega Alpha Beta Theta 1 Theta 2 D	0.147048 0.141732 0.108072 0.321307 2.035492 1.133064 0.642485	All coefficients except the Theta are stable

Source: Researcher Compilation

According to the evidence presented in Table 5-33, both the FIGARCH and FIEGARCH data series for Egypt exhibit structural breaks. This is bolstered by the analysis performed earlier in the descriptive analysis revealing regime shifts and breakpoints. South Africa's FIEGARCH coefficients indicated that all estimated coefficients were stable, whereas the FIGARCH coefficients were unstable, indicating that the FIGARCH model contained structural breaks. Therefore, the FIEGARCH model was the most appropriate forecasting model for listed South African private equity investments, as it is stable.

In addition, the Ghana series revealed that for FIEGARCH, all estimated coefficients were unstable with the exception of omega and the d-parameter, whereas for FIGARCH, all parameters were stable. This, therefore, indicates that the FIGARCH model was ideal for modelling private equity investments in Ghana.

In addition, diagnostic tests based on the News Impact Curve (NIC) were conducted for the two models for which it was assumed that negative and positive shocks have an asymmetric effect on the volatility generated in a data series for a FIEGARCH model. In their FIGARCH model estimations, Paul and Birthal (2021) and Benzai, Aouad and Djellouli (2022) used NIC as a model diagnosis tool. Therefore, the applicability of the FIEGARCH and FIGARCH models for estimating the future volatility of listed private equity investments was examined. Table 5-34 displays the results of the analysis conducted using EViews 12 statistical software.

Table 5-34: Model diagnostic tests parameters

Country	Method	Sign-Bias t-Test	Negative Size Bias t-Test	Positive Size Bias t-Test	Joint Test
South Africa	FIGARCH	3.326966 (0.0009)	2.835082 (0.0046)	-0.028268 (0.9770)	13.73865 (0.0033)
	FIEGARCH	2.510211 (0.0121)	1.461969 (0.6839)	0.407207 (0.6839)	4.101614 (0.2510)
Ghana	FIGARCH	0.050500 (0.9597)	-0.152183 (0.8791)	-0.134597 (0.8929)	0.110139 (0.9906)
	FIEGARCH	0.091351 (0.9272)	-0.676305 (0.4989)	-0.225126 (0.8219)	0.886175 (0.8288)
Egypt	FIGARCH	1.101484 (0.2708)	-1.353517 (0.1760)	1.172165 (0.2412)	4.101614 (0.2510)
	FIEGARCH	0.818864 (0.4129)	0.212269 (0.8319)	0.246058 (0.8057)	0.832458 (0.8417)

Source: Researcher Compilation

The findings illustrate that except for FIGARCH model for South Africa, all other models yielded statistically insignificant t-statistics for the data series. This implies that the presence of data asymmetries was not found in the long run volatility dynamics of the private equity investments in Africa. Despite different economic systems from which these investments operate, their individual responses to shocks whether positive or negative was homogenous. This may signal that private equity investments in Africa tend to have homogenous dispositions in the long run.

McKnight et al. (2022) attest to the need for market spillovers to be examined as they assist in building investment strategies and policy making. of these investments.

5.5 Chapter Conclusion

Modelling and forecasting volatility is essential because investors require data to analyse the erratic behaviour of financial assets, not only in the initial moments, but also in the second, third, and fourth moments. The chapter was broken up into three sections. The first section examined the statistical properties of the counters under consideration and determined, based on descriptive statistics, that all private equity returns are positively skewed and have excess kurtosis. The returns on South African private equity are closest to the norm. As explained, differences in skewness and kurtosis reflect the various economic geographies. Clustering of high volatility in the time series of Egypt's private equity returns has implications for financial time series modelling. Consequently, the study proceeded to test for the ARCH effect on the data series for the four countries in order to evaluate their suitability for the GARCH family of models. Botswana's data failed the ARCH test, so they were ineligible for GARCH modelling. To model the data series, the study recommended alternative techniques, such as quantum finance methodologies.

South African LPEs exhibit features such as volatility clustering, asymmetric effects, and leptokurtic distributions, but there was no indication of leverage effects or structural breaks. The examined parameter estimates were found to satisfy stability conditions and to indicate elevated levels of persistence in the conditional volatility of the series. This indicated that this asset class was a defensive asset and a prudent investment during difficult times. The GARCH (1,1) model was determined to be the most accurate predictive model in the study, outperforming its rivals.

The findings in Ghana indicated that these investments carried a risk premium proportional to their inherent risks. No evidence of leverage effects was found. All selected models passed the tests for assumptions, stability, and stationarity, and were therefore presumed to be the optimal models for fitting data series for Ghana LPEs.

There was no evidence of leverage effects in the Egypt LPE data, but there were traces of volatility clustering associated with some jump diffusions. The GARCH-in-Mean model was deemed suitable for data modelling. In addition, the study discovered that there was a negative correlation between returns and volatility, and that volatility surprises tend to follow upward revisions of the reward process. The TGARCH and EGARCH models failed to meet stationary conditions, indicating that stock news were not prevalent in this asset class. As a result, the asset is a defensive asset.

The third and final section examined the long-term memory behaviour of the LPEs under study, and no evidence of data asymmetries in the long-term volatility dynamics of the African LPEs under study was found. Although these investments operate in different economic systems, their individual responses to positive or negative shocks are consistent.

The subsequent chapter examines volatility spillover effects and the interaction between LPE return volatility and country-specific factors.

6 CHAPTER SIX: EMPIRICAL FINDINGS AND INTERPRETATION: SPILLOVER EFFECTS AND STRUCTURAL RELATIONSHIPS OF LPE INVESTMENTS, EVIDENCE FROM SELECTED MARKETS IN AFRICA

6.1 Chapter Introduction

This chapter employs the statistical methods described in the preceding chapters to examine the interaction of private equity investment volatility with other variables to determine their structural relationships. The structural relationships of the variables are a crucial analytical tool that explains how and why volatility is generated. Following a shock, portfolio allocation decisions are typically influenced by the degree of covariation or volatility of stock prices. Multivariate GARCH models facilitate the comprehension of the relationship between the volatilities and co-volatilities of multiple univariate variables. This chapter examines the existence of spillover effects and the structural relationships between listed private equity returns for South Africa, Egypt, and Ghana, as well as their respective country-specific factors.

The MGARCH model allows for accurate estimations of the current relationship between the data series. The study decomposed volatility across data series in order to gain a comprehensive understanding of the key drivers of volatility in listed private equity investments in certain African markets. The primary research questions addressed whether the volatility of one market influences the volatility of the other, along with whether the volatility of private equity is transmitted directly or indirectly, and if there is a contemporaneous interaction between country risk factors and private equity returns. This chapter's findings provide insights regarding diversification and describe the type of systematic risk to which the economies under study are exposed.

The study proposes the DCC-GARCH model, which has the flexibility of a univariate model but not the complexity of a multivariate series (Gabauer, 2020), to establish the relationship between more than one variable, in this case the structural relationship between private equity investments in South Africa, Ghana, and Egypt. This model directly parameterises the conditional correlations.

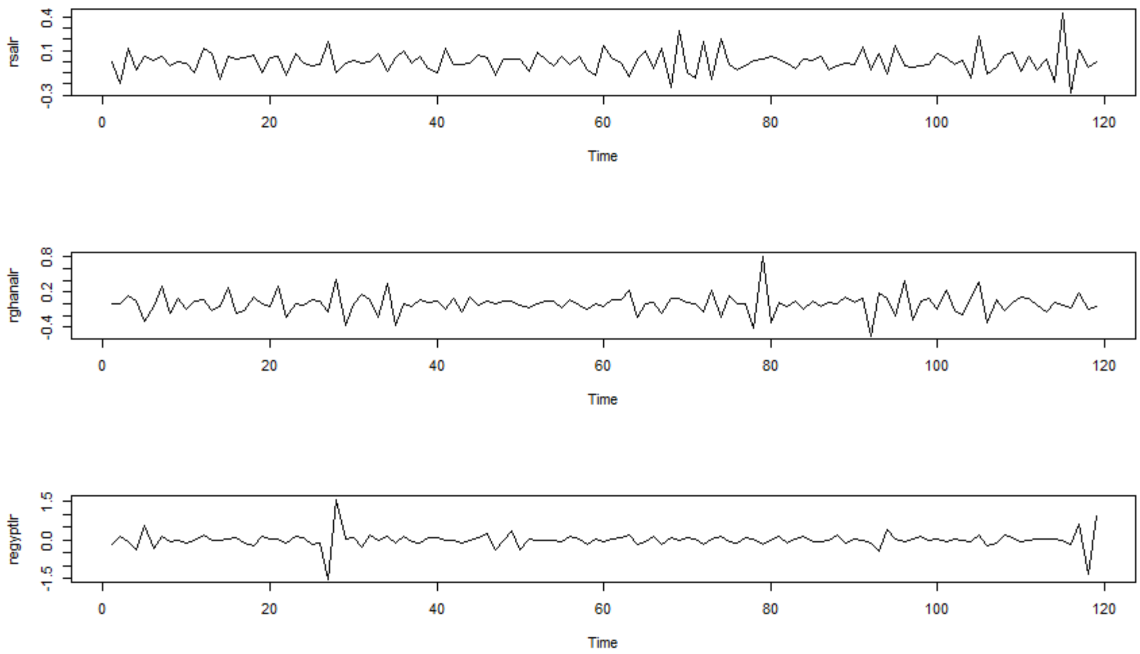


Figure 6-1: Time series plot for each variable

Source: Researcher Compilation

An examination of the time series plots for the returns of private equity in Figure 6-1 shows the returns are moving in the same direction up to period 60, and from there, South Africa exhibits more volatility patterns, followed by Ghana, and that they are all moving in the same direction. Egypt displays more stable returns across time. It is nevertheless important to examine their correlation plots and fit the DCC to examine the magnitude of the correlation and common linkages and assess any possibilities of one market influencing the other.

6.2 Model Fit

Table 6-1 shows the findings from the DCC parameter estimations:

Table 6-1: DCC parameter estimates

Market	**Parameter**	**Estimate**	**Standard Error**	**P-value**	**Persistence**
DCC conditional correlation parameters	α_1	0.0000	0.046969	1.0000	$\alpha_1 + \beta_1$
	α_2	0.0000	0.045425	1.0000	$\alpha_2 + \beta_2$
	β_1	0.418944	6.636860	0.949688	0.418944
	β_2	0.488962	6.804701	0.942716	0.488962

Source: Researcher Compilation

Evidence from the DCC estimates shows that there is no short-term spillover effect, as the estimates are not statistically significant for all country combinations. The DCC beta shows that there is no long-term persistence of all investment cross-country combinations. The α and β estimates do satisfy the condition of $\alpha + \beta < 1$ which indicates that the conditional variance is mean reverting towards the equilibrium level; however, the notion cannot be relied upon due to statistical insignificance demonstrated by p-value. These findings point to the fact that private equity investments in Africa are a good place to diversify, as the systematic risk elements show no evidence of spillover effects. A similar study by Princ (2010) on the Prague Stock Exchange showed a unidirectional impact of foreign markets affecting the Czech market. Table 6-2 shows the correlation matrix for the investment combinations.

Table 6-2: Correlation matrix for each variable

	Rghanalr	Rsalr	regyptlr
rghanalr	1.00000000	-0.07000886	0.07716451
rsalr	-0.07000886	1.00000000	-0.04668158
regyptlr	0.07716451	-0.04668158	1.00000000

Source: Researcher Compilation

The evidence illustrates that the investment combinations for Ghana and South Africa as well as South Africa and Egypt are negatively correlated whilst that of Ghana and Egypt are positively correlated on the last day of the analysis. The correlation plot shown in Figure 6.2 shows that the correlation for South Africa and Ghana follows an upward negative correlation trend, hence the diversification effect is getting stronger overtime, whereas for Egypt and South Africa, the effect is trending downward and is generally weak overtime. On the other end, Ghana and Egypt are positively correlated, showing that combining the assets does not create value, and the strength is high over time, indicating its weakness for portfolio creation, as shown in Figure 6-2.

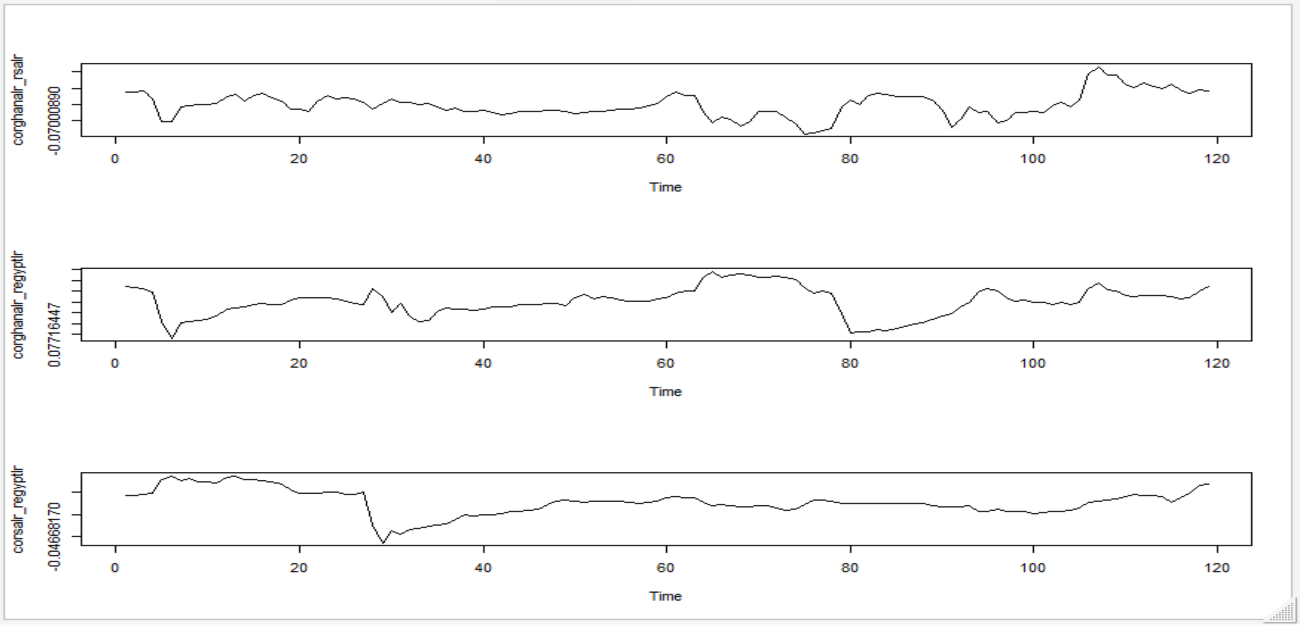


Figure 6-2: Correlation plots

Source: Researcher Compilation

A closer look at the covariance analysis shows that the correlation effects subsists. The negative covariance is prevalent for South Africa and Ghana as well as for Egypt and South Africa, as shown in Figure 6-3. Trends over time also show that the covariance for Ghana and Egypt and that for Ghana and South Africa is more volatile over time compared to the South Africa and Egypt series, and they are all moving towards negative terms, providing more room for diversification benefits.

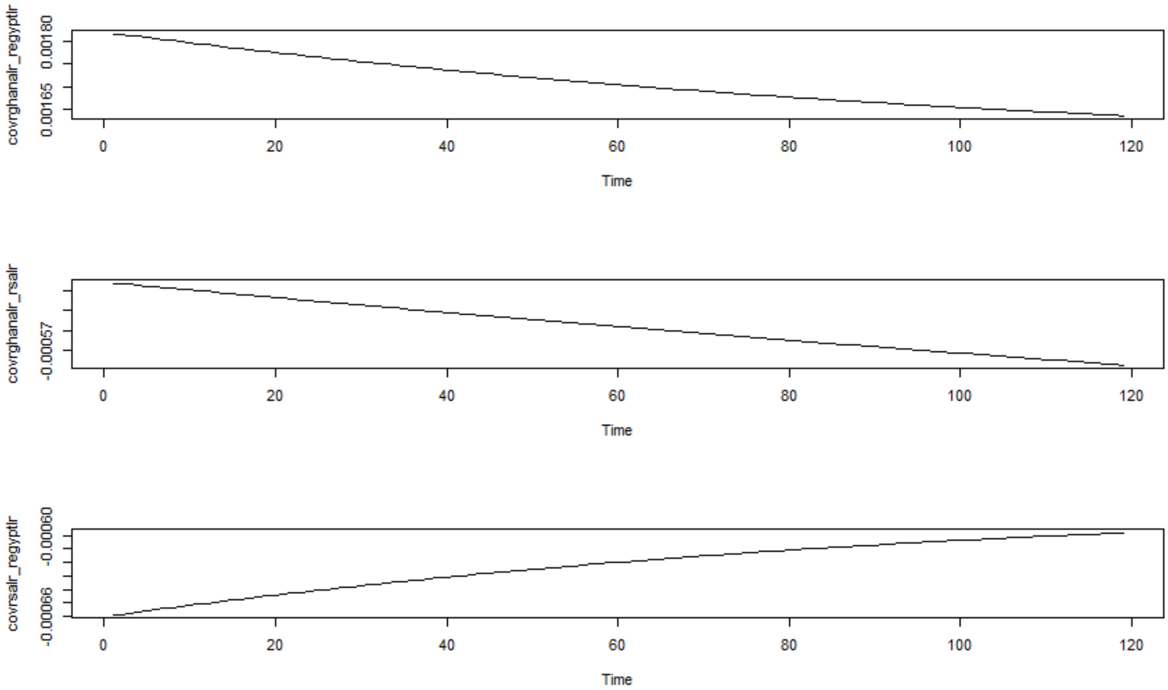


Figure 6-3: Covariance plots

Source: Researcher Compilation

Further analysis for the conditional mean, conditional covariance, and conditional correlations, all of which are provided in the appendix, point to the notion that there is no evidence of interdependence of one equity variable to the other and that the relationship between them is more negative than positive, implying that the relationship is good in terms of portfolio construction. The study, therefore, fails to reject the hypothesis that there exists no spillover effects amongst LPEs in the African markets under study. Lastly, Figure 6-4 shows the conditional quantiles for the model, which assesses the impact of a covariate on a quantile of the outcome conditional on specific values of other covariates, and the volatility is relatively stable over time.

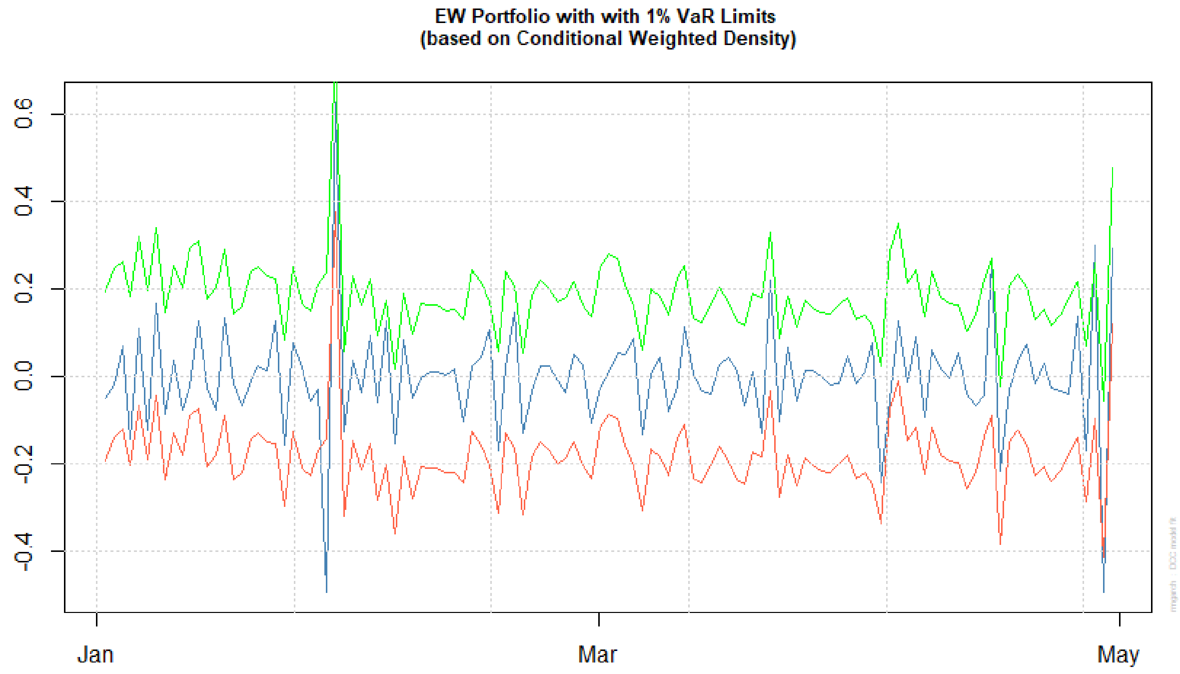


Figure 6-4: Portfolio plot with conditional density VAR limits

Source: Researcher Compilation

6.3 Structural Relationship of Private Equity Investments with Country Risk Factors

The previous analysis provided more insights on the interaction between intercountry private equity returns to analyse how they can be used in portfolio construction. As pointed out in earlier chapters, accounting for structural behaviour in assets is important in interpreting changes in asset prices and in predicting the future paths of their variance and correlations (Karunanayake, 2011). Hence, the study used the VAR model to investigate the interaction of these returns with their country risk factors (GDP and inflation), and examined any significant impacts from shocks using the variance decomposition. The VAR model was found appropriate in this case and is commonly used for forecasting systems of inter-related time series and for analysing the dynamic disturbances on the system of variables. The VAR approach bypasses the need for structural modelling by treating every variable as endogenous in the model as a function of the lagged values of all the variables in the system. The data at hand was tested to determine whether the variables were integrated of order one and that condition was satisfactorily met, qualifying the data to the model. In achieving the objective, the study did a descriptive analysis of the data, specified the model, performed unit root tests, determined the optimal lag, ran the VAR model including its interpretations, performed diagnostic tests and variance decomposition, analysed out the impulse response dynamics and finally examined the relevance of this analysis to portfolio construction.

6.4 Tests for Stationarity

The study tested for stationarity to guard against the possibility of obtaining and interpreting spurious model results. Table 6-3 shows results from the Augmented Dicker Fuller test.

Table 6-3: Tests for stationarity

Country	**Variable**	**@level**	**@ 1^ST Differencing**	**2^nd Differencing**
Egypt	GDP	-4.022197 (0.0106)	-8.053597 (0.0000)	No need to test
	RTN	-1.634842 (0.4382)	-3.99181 (0.5435)	-7.688940 (0.0001)
	INF	-2.762155 (0.0906)	-4.193337 (0.0089)	No need to test

Ghana	GDP	-3.023563 (0.0588)	-5.286051 (0.0016)	No need to test
	INF	-2.342398 (0.1744)	-4.020338 (0.0118)	-5.825923 (0.0009)
	RTN	-3.466666 (0.0333)	-2.436228 (0.1530)	No need to test
South Africa	GDP	-2.115985 (0.2419)	-3.400800 (0.0329)	-4.796941 (0.0041)
	INFL	-2.631625 (0.1118)	-6.857931 (0.0004)	No need to test
	RTN	-3.583744 (0.0243)	-3.964343 (0.0163)	No need to test

Source: Researcher Compilation

According to the Augmented Dicker Fuller test, Egypt’s GDP and inflation, Ghana’s GDP and returns, as well as South Africa’s inflation and returns were all stationary at first differencing whilst the rest were stationary at second differencing. Log returns are useful in modelling this data than raw data because it is at first differencing that the data’s mean, variance and covariance is made constant over time. Figure 6.5 provides confirmation that the data is stationary at first differencing; the rest are shown in the appendix.

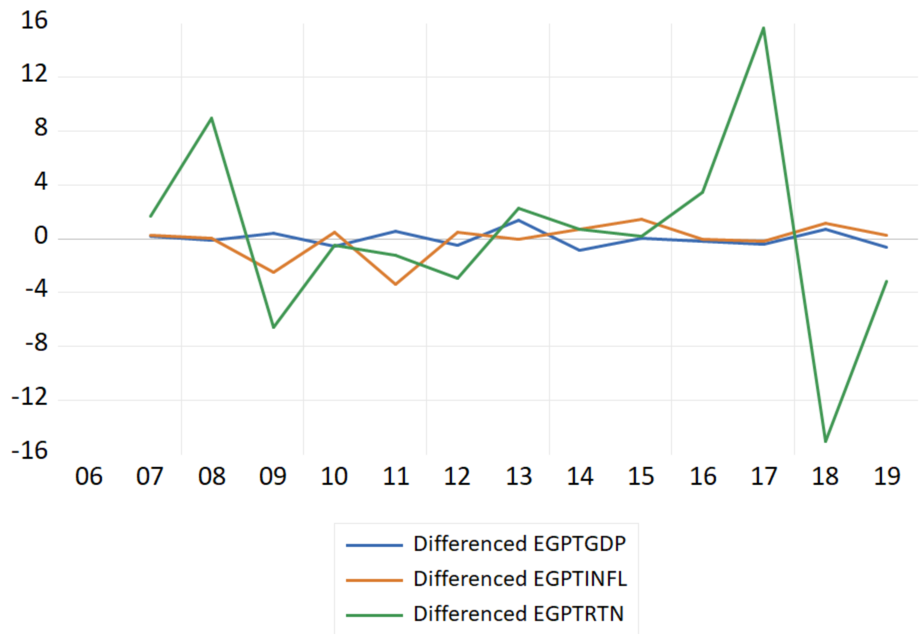


Figure 6-5: Stationarity tests of the variables

Source: Researcher Compilation

Figure 6-5 show that the data is stationary and provides an indication of the co-integration picture of these variables. Indications are that the returns and inflation seem to be following each other, hence co-integration tests were made to assess the magnitude of interaction and the long-term relationship of the variables. Observations emanating from the analysis indicate that for Ghana and Egypt, the trace is greater than the 5% critical value hence the analysis demonstrates that there exist contemporaneous interactions in the models for South Africa and Egypt. Data for Ghana shows that there are no co-integrations hence no contemporaneous interactions amongst variables as the trace value is greater than the critical value. The findings are summarised in Table 6-4.

Table 6-4: Trace Value vs Critical values for the data

Country	Hypothesis	Trace Value	5% Critical Value	P-Value
Egypt	1	35.35746	29.79707	0.0103
	2	12.966.06	15.49471	0.1161
	3	3.717853	3.841165	0.0538
South Africa	1	28.69317	29.79707	0.06666
	2	8.963757	15.49471	0.3687
	3	1.182718	3.841465	0.2768
Ghana	1	4..78973	29.79707	0.0019
	2	17.58995	15.49471	0.0238
	3	7.306548	3.841465	0.0069

Source: Researcher Compilation

Overly, these findings fail to reject the study hypothesis that there exists structural contemporaneous interactions within country risk factors and returns. Using the Max-Eigen statistic, the study notes that the null hypothesis bears similar conclusions with the Trace statistic at 0.05%. Literature provided justification for the need for statistical evidence describing the relationship between country risk and returns in Africa. According to De Wet (2005), establishing merely the initial reactions of variables may not provide a more precise definition of their properties. Considering that a response to one variable affects the volatility of another, further analysis on the interactions of the LPE returns and country-specific factors using a VAR model to decompose the variance and impulse response functions to analyse the impact of country-specific factors on LPE's under study` was necessary to conclude the finding of whether country risk factors are a priced factor in private equity valuation models.

6.4.1 Model specification

The study adopted a 3-variable VAR model as in equation 24 and 25. From the equations, the dependent variable is a function of its lagged values and the lagged values of other variables in the model as shown in equation 33. This provided an example for South Africa, wherein the same was done for Ghana and Egypt's volatility of returns, GDP and inflation.

$$\begin{bmatrix} sartn \\ sagdp \\ sainf \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} + \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \phi_{1,3} \\ \phi_{2,1} & \phi_{2,2} & \phi_{2,3} \\ \phi_{3,1} & \phi_{3,2} & \phi_{3,3} \end{bmatrix} \begin{bmatrix} sartn_{t-1,1} \\ sagdp_{t-1,2} \\ sainfl_{t-1,3} \end{bmatrix} + \begin{bmatrix} w_{t,1} \\ w_{t,2} \\ w_{t,3} \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots \text{(33)}$$

All variables have equal lags of one and VAR was specified in levels to avoid model mis-specifications. Generally, the VAR model is estimated by Ordinary Least Squares (OLS). In deciding the maximum lags (k in the equation), the data points were considered, keeping in mind the fact that too many lags will weaken the degrees of freedom and lead to statistically insignificant coefficients as well as multicollinearity problems whilst too few lags will result in specification errors. The criteria for this selection is discussed hereunder.

6.4.2 Selection of Lag

A VAR of lag order 1 was estimated to obtain consistent estimates for the reduced-form residuals. The lag length was estimated using Schwartz Information (SC) criterion, the Hannan-Quinn Information criteria (HQ) and the Alkaike Information criteria (AIC). One lag was selected based on the fact that it is at one lag that the residuals are stationary. All three criteria indicate a lag length of one for the VAR. The results are presented in Table 6-6:

Table 6-5: Test statistics and choice criteria for selecting the order of the VAR model

Lag	AIC	SC	HQ
0	-16.92937	-16.59712	-16.60797
1	**-19.26119**	**-19.23731**	**-19.01362**
2	-19.23037	-19.57583	-19.76687
3	-19.74713	-16.62493	19.74090
4	-19.8418	-16.91532	-19.95282

Source: Researcher Compilation

6.4.3 Model results

From the findings presented in Table 6-6, the study notes that a one-lag length is the one that can achieve stationarity for residuals; this was then adopted for building the VAR model so as to identify the interactions between country risk factors and private equity returns. Table 6-6 shows a summary of South Africa's results using EViews12 for VAR and variance decomposition of South Africa private equity log returns.

Table 6-6: VAR and variance decomposition South Africa

	SAGDP	**SAINF**	**SARTNS**
SAGDP (-1)	0.417741 (0.030327) [1.37745]	-4.922761 (2.90146) [-1.69665]	0.215495 (2.42697) [0.08879]

SAINF (-1)	-0.000374 (0.002190) [-0.01710]		0.256718 (0.20949) [1.22546]	0.646742 (0.01752) [3.69087]
SARTNS (-1)	-0.028088 (0.02511) [-1.11876]		-0.461135 (0.24020) [-1.91982]	0.356940 (0.02009) [1.77656]
Variance Decomposition for South Africa for SARTNS				
Period	**Standard Error**	**SAGDP**	**SARTNS**	**SAINF**
1	1.150631	0.203333	98.97697	0.019698
2	1.514623	0.120448	64.91934	34.96021
3	1.677083	6.582546	54.22047	38.99698

Source: Researcher Compilation; ( ) standard errors [ ] T-values

The T-statistic in Table 6-6 demonstrates that inflation has a significant impact on returns. The historical realisations of inflation are associated with an average increase of 64,7%, all else being equal. Other variables have weak t-statistics, indicating that they are insufficiently predictive of the South African LPEs. In the short term, 99.976% of the forecast error variance in private equity returns is explained by the private equity returns themselves, as indicated by the forecast error variance. This indicates that other variables in the model have little effect on LPE returns; they are exogenous. Long-term inflation influences return by 38.99% and GDP by 6.58%; therefore, inflation is a strong predictor of country risk for LPE investments under study, as indicated by VAR.

Table 6-7: VAR and variance decomposition for Ghana

	GNARTN		GNAGDP	GNAINF
GNARTN (-1)	0.402942 (0.35024) [1.15048]		-0.019667 (0.02173) [-0.90511]	-0.155532 (0.27508) [-0.56541]
GNAGDP (-1)	4.259216 (4.64082) [0.91777]		-0.023193 (0.28792) [-0.08056]	-5.773632 (3.64492) [-1.58402]
GNAINFL (-1)	-0.404701 (0.43204) [-0.93671]		0.022171 (0.02680) [0.82714]	0.320894 (0.33933) [0.94567]
Variance Decomposition for Ghana for GNARTNS				
Period	**Standard Error**	**GNRTN**	**GNAGDP**	**GNAINF**
1	3.932733	100.00	0.00000	0.00000
2	4.740054	90.25453	3.762697	5.982776
3	4.945206	86.40464	6.776232	6.819133

Source: Researcher Compilation ;() standard errors [ ] t-values

All variables in Ghana exert a small influence on the dependent variable (ceteriz paribus), as determined by VAR t-values shown in tTble 6-7. In general, if the t-statistic of a variable is greater than or equal to 1.96, the conclusion is that the lagged period value of a variable is significant in determining either its own present value or that of another endogenous variable (Tsay, 2013). The variance decomposition provides information on the relative importance of each random innovation in affecting the variables in VAR. As shown by the variance decomposition, in the short term, the Ghana LPE returns are highly endogenous because they are completely influenced by their own variable; in period 1, a 100% of the forecast error variance in the returns is explained by the returns themselves. However, this influence weakens over time. The GDP and inflation both have a 0% impact on equity returns over the short term. Whilst the study is the first to examine LPEs using this approach, previous studies such as Din (2020) and Omran and Bilan (2021) used the VAR for analysing the dynamic impact of random disturbances on a system of variables and noted variables determining either their own present value or that of other endogenous variables. Table 6-8 shows that GDP and inflation have little influence on the activities of LPE investments. This could be because this asset class benefits from Pan-African deals and regional funds (Botchway and Akobour, 2020).

Table 6-8: VAR and variance decomposition for Egypt

	EGPTRTN		**EGPTGDP**	**EGPTINF**
EGPTRTN (-1)	0.197834 (0.31776) [0.62260]		0.001375 (0.01915) [0.07182]	0.031941 (0.06716) [0.47559]
EGPTGDP (-1)	1.659502 (5.19394) [0.31951]		-0.487091 (0.31297) [-1.55636]	1.059970 (1.09780) [0.96554]
EGPTINFL (-1)	0.821920 (1.14111) [0.72028]		-0.142553 (0.06876) [-2.07322]	0.780413 (0.24119) [3.23573]
Variance Decomposition for EGYPT for EGPTRTNS				
Period	**Standard Error**	**GHRTN**	**GHGDP**	**GHINF**
1	6.290203	100.00	0.00000	0.00000
2	6.501906	96.88756	0.402157	2.710283

3	6.559769	95.97277	0.395287	3.931941

Source: Researcher Compilation ;() standard errors [ ] t-values

In the short run, the forecast error variance of LPE returns in Egypt is entirely self-explanatory, and in the long run, it remains strongly endogenous. In addition, the influence of inflation and GDP on LPE investments is very weak as indicated by the t-statistic of the variables in Table 6-8; only past realisations of inflation have strong significant influence and is associated with 78% increase in current GDP. Although the response is counterintuitive as investments are generally expected to respond to economic growth patterns, Egyptian LPEs are also devoted to country-specific factors.

This goes to say that LPE investments in the African markets under study are highly endogenous, as they strongly influence their own variable in both the short and long run, as evidenced by the findings. In addition, the country risk factors have a negligible effect on LPE investments.

Thus, the hypothesis that country-specific factors are a priced factor in LPE valuations in the African markets under study is refuted. This result is consistent with the findings of Dopke, Jorg, and Tegtmeyer (2018), who investigated global LPEs and determined that global risk factors are not a pricing factor for global LPEs.

6.4.4 Model diagnostic tests

The study conducted diagnostic tests for the VAR models to ensure that the model is stable. Firstly, residuals tests were done to check their stationarity. Generally, for all the models the residuals were stationary, meeting the model requirements. The fluctuations for Egypt are too wide implying high volatility for the period 2008 to 2014 for the returns and country risk factors. The index for South African inflation experienced some seasons of very high and low volatility between 2012 and 2018. Residuals for GDP for Ghana increased during the period 2012 to 2018.

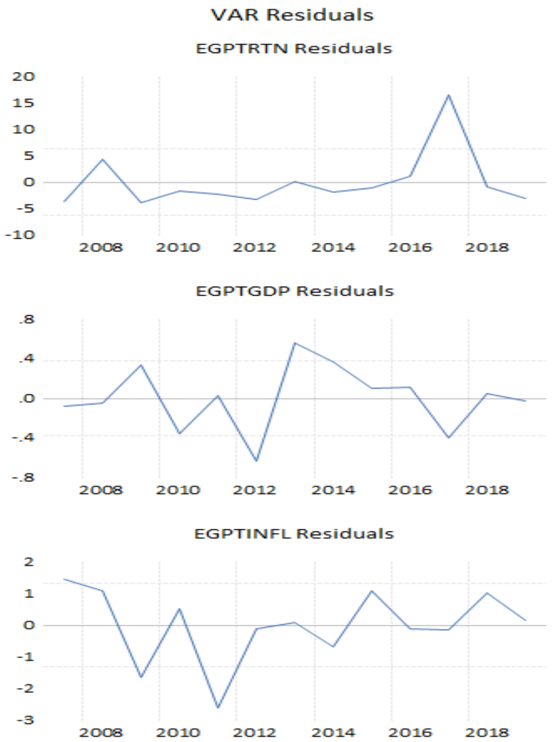


Figure 6-6: VAR residuals

Source: Researcher Compilation

In addition, the study tested for autocorrelation under the hypothesis of no autocorrelation and findings show that the residuals are not serially correlated as shown in Figure 6-7.

VAR Residual Serial Correlation LM Tests
Date: 10/10/22 Time: 21:36
Sample: 2006 2019
Included observations: 13

Null hypothesis: No serial correlation at lag h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	4.502407	9	0.8754	0.439190	(9, 9.9)	0.8841

Null hypothesis: No serial correlation at lags 1 to h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	4.502407	9	0.8754	0.439190	(9, 9.9)	0.8841

*Edgeworth expansion corrected likelihood ratio statistic.

Figure 6-7: Residuals autocorrelation tests

Tests for normality show that the Jarque-Bera jointly, the errors in the VAR system are normally distributed as shown in Figure 6-8.

Component	Jarque-B...	df	Prob.
1	2.857585	2	0.2396
2	1.483981	2	0.4762
3	0.655998	2	0.7204
Joint	4.997564	6	0.5441

Figure 6-8: Tests for normality showing the Jarque-Bera

The study also checked for homoscedasticity and noted the presence of homoscedasticity in the error terms; hence, the model passes the diagnostic test.

Joint test:

Chi-sq	df	Prob.
39.94083	36	0.2993

Figure 6-9: Homoscedasticity tests

6.5 Impulse Response Functions

The study also examined how second moments of the variables react to shocks in the structural innovations of a variable by way of bringing in impulse responses. Impulse response functions were found a necessary tool in tracing that impact of country risk factors to the log returns of private equity investments. The characteristics of a variable are easily seen by examining the effects of shocks to variables in the system. As explained before, structural innovations apply one standard deviation of the variables and observations across time are noted and their effects on portfolio risk management are assessed.

The impact of shocks on returns and how GDP and inflation respond for South Africa are shown in Figure 6-10. The rest are displayed in the appendices.

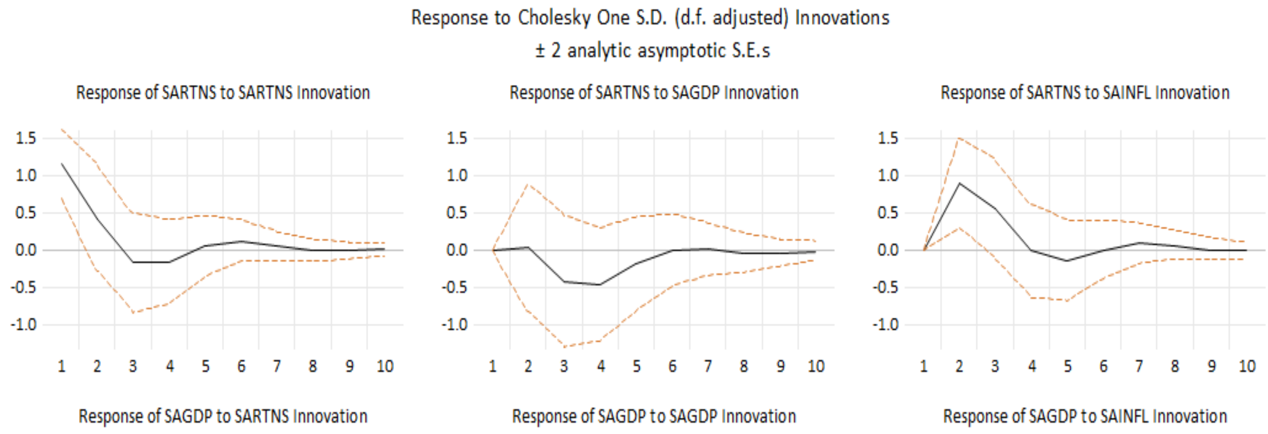


Figure 6-10: Impulse response functions

All the impulse response functions lie within the 95% confidence interval for all the variables under study. This is a necessary condition for this analysis. The response of private equity returns to a one standard deviation shock to GDP initially has no noticeable impact on periods 1 and 2. From period 2, the response declines to reach a stable state in period 3. From period 4, the log returns rise gradually before levelling off in period 6. This means that South African private equity log returns are negative in the short and long run.

A one-standard deviation shock to log returns initially increases the inflation from period one to period two. This positive response sharply declines until the fourth period, when it hits its steady state value, from where it remains in the negative region up to period six. From period six onward, the shocks stabilise. This implies that shocks to log returns have an asymmetric impact both in the short run and the long run.

For Ghana and Egypt, the study notices that the extent to which variables also react to other variables is significant. This could be explained by the fact that other than the asset, performance is greatly influenced by structural innovations inside the endogenous system. For Ghana, inflation reacts greatly to shocks from outside while being less affected by structural innovations from outside. Hence this analysis fails to reject the hypothesis that there exist structural contemporaneous interactions within country specific factors and returns for LPEs in African markets.

From the findings on the impulse response functions, they study notices interactions between country-specific factors and returns generated by LPE investments in African markets, contrary to the findings on VAR which noticed a similar relation for only one country under study, Ghana. However, the study also notices that though there are some levels of interaction, country-specific factors are not a priced factor due to the evidence presented in the findings.

6.6 Chapter Conclusion

This chapter investigated the existence of spillover effects and the structural relationships between LPE returns for South Africa, Egypt, and Ghana as well as their associated country-specific factors. The study's empirical findings revealed no evidence of long-term persistence of all investment cross-country combinations. In addition, the findings showed that there was no evidence of one equity variable being dependent on another, and the relationship was more negative than positive, making the investments suitable for portfolio construction. Further analysis on the structural dynamics of LPE returns and country-specific factors showed that the LPEs under study were highly endogenous, whereas the country-specific factors were exogenous, as they had a weak influence on the LPEs under study; hence, the hypothesis that country risk factors affect returns for LPE investments in the countries under study was rejected. However, the findings from the impulse response functions indicated the presence of an asymmetric impact both in the long run and in the short run, albeit of a lower magnitude. The next chapter presents the conclusions from the study, the recommendations, and areas for further research.

7 CHAPTER SEVEN: CONCLUSION, RECOMMENDATIONS AND SUGGESTIONS FOR FUTURE RESEACH

7.1 Chapter Introduction

This chapter presents the study's conclusion on the topic, statistical modelling of LPE investments in select African markets. Prior studies on private equity investments paid no attention to the dynamics of LPE investments' volatility. This study provides information that aids academics and investors in gaining a better understanding of the emerging asset class of LPE investments in Africa. Thus, the study investigates the volatility of returns for listed private equity investments in selected African markets in order to evaluate the volatility contained within and across these markets. The specific objective of the study was to shed light on the statistical properties of these investments and assess the volatility transmission dynamics within individual investments and various markets in order to provide a foundation for the development of new valuation tools. The study concluded by examining the interaction between the volatility of these investments and country-specific factors in each of the examined nations.

The study made several significant contributions to understanding the transmission dynamics of volatility across the markets under consideration. To the best of the knowledge available, this is the first study to investigate the statistical properties and volatility dynamics of LPEs using time series and econometric modelling. This thesis extends previous findings on volatility modelling using GARCH by investigating an unexplored market in the body of knowledge, the LPEs at large, which has different dispositions than alternative asset classes. The study contributes significantly to the LPE literature and provides valuable insights for a better understanding of LPEs in

Africa as a new asset class. This is because LPEs are a relatively unknown form of private equity among investors and academics (Tegtmeier, 2021).

Secondly, the study extends previous empirical findings on volatility spillover effects and interactions between volatility transmissions and country-specific factors across the selected nations. The study largely contributes to the ongoing discussion regarding whether country-specific factors influence expected returns sufficiently to warrant inclusion in valuation tools.

Finally, the study contributes to the ongoing discussion on valuation tools and their relevance in emerging markets. The new knowledge provides a data-driven approach towards the debate by providing an empirical analysis of the relevance of country-specific factors in valuation models for emerging and developing markets, more so for alternative investments, LPEs in particular.

7.2 Summary of the Study

The increased interest in conceptualising LPE investments in Africa necessitated a reassessment of academic valuation theories to better comprehend this asset class. These theories or discourses have not considered alternative investment asset classes, which are illiquid and have long-term horizons; therefore, an analysis of their statistical properties is key in investment strategy and policy formulations.

The findings from literature made it clear that country-specific characteristics do have a role in the pricing of assets for private equity investments in Africa. This conclusion was reached as a direct result of the debates that were presented in the study. In light of the deficiencies that have been pointed out, the research concluded that it is necessary to investigate the structural link between LPE returns and country-specific factors -- the study utilised inflation and GDP. The theoretical void in the academic arena was addressed by the establishment of the relationship between country-specific factors and projected returns from LPE investments. A diagnosis of the problem is key before providing a solution on valuation formulations. That said, the study hypothesised that country-specific factors are a priced factor in LPE investments.

In addition, the study noted that LPE investments in African markets have not been examined. This research gap, together with the fact there has been a problem in differentiating markers of successful investments amongst private equity investments, attributed to problems in valuation tools prompted this study, mainly to unravel the myth associated with this private equity segment, and to address the research question on how we characterise LPEs in Africa. To achieve this, the study distilled the research questions to testable hypothesis around the statistical properties, volatility modelling dynamics and structural relationships with country specific factors.

There is a series of empirical tests of return and volatility behaviour for LPEs in developed markets and researchers are of the opinion that the studies are still very few. No study has analysed the statistical properties of this asset class in Africa. Studies for developed markets, and as shown in the study, have utilised the NAV approach; hence, this study fills the void of examining markets that tend to exhibit fragmented economic growth patterns to assess their volatility patterns. Also the study embraced statistical modelling and econometric modelling to bring about the answer to the main research question: How is volatility generated for LPEs in Africa and what drives it?

Financial time series data has turbulent periods. When the variance is not constant the process is said to be heteroscedastic which equates to a larger magnitude in the residuals. The residuals that are heteroscedastic are also noted to be autocorrelated as spikes in volatility are not randomly placed in time. Residuals allow GARCH models to forecast volatility in financial time series data. GARCH models can accurately predict enormous quantities of time-varying volatility. The GARCH model was used to capture non-normality in financial data. ARMA models collect normally distributed data, which was found not suitable in the study, given the fact that the series under study is financial data.

The study analysed the statistical characteristics of the data series. In this session, the return distributions and their characteristics in terms of the mean, variance, skewness, kurtosis tests were done as well as tests for normality. The study utilised GARCH models to focus on time varying conditional moments, persistence asymmetries, volatility clustering patterns and mean reversion properties. ARCH and GARCH models help explain the behaviour of data by giving room for conditional variance to respond to past behaviours overtime. The study analysed the short-run behaviour by making using of GARCH (1,1), TGARCH, GARCH in Mean and EGARCH models; and the long-term behaviour of volatility by utilising FIGARCH and FIEGARCH models, guided by the principle of parsimony.

Finally, the study examined the existence of spillover effects using MGARCH DCC models and the final unit of analysis examined the structural relationship between LPE returns and country-specific factors using VAR models to conclude the research hypothesis.

The research studied LPEs in segments of Africa for which market pricing were available. In contrast to prior research on private equity, this study examined the characteristics of the counter's heterogeneity in terms of returns, volatility, spillover effects as well as their relationship to country-specific variables. LPEs make up a desirable asset class that is seeing rapid expansion in the market. From a practical standpoint, this empirical research gives information to investors that favour unlisted private equity investments. The primary advantages for this category include its ability to leverage liquidity from the exchange and the availability of performance metrics for comparing one investment to another. Figure 7-1 provides a summary of the research and the section thereafter provides an analogy of the research findings and results of the hypothesis tested. Being the first study to examine statistical properties and volatility dynamics of the countries under study, the findings all contribute to new knowledge in the academic space. Further studies will build on these findings going forward.

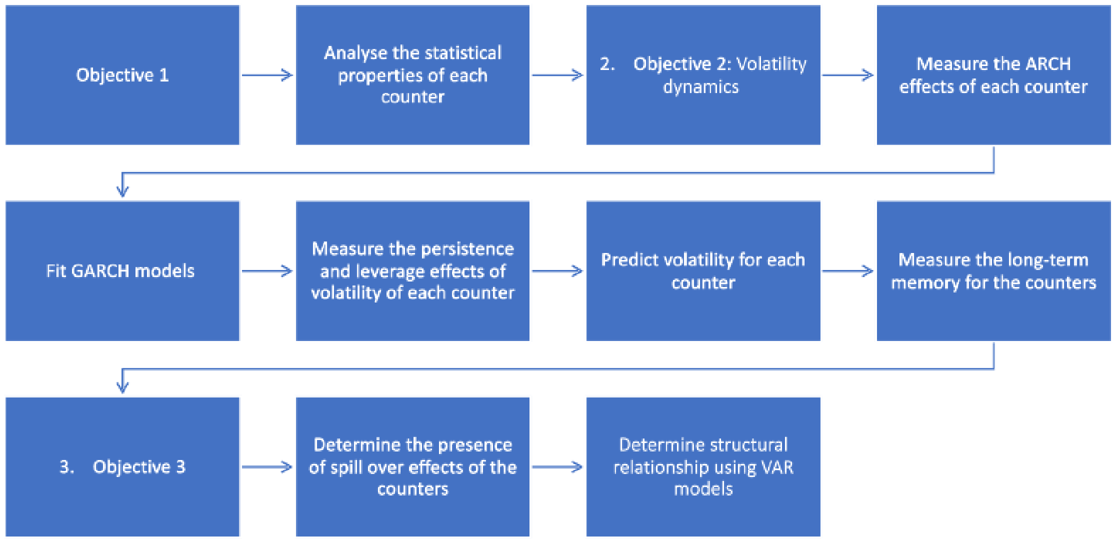


Figure 7-1 Summary of the study

Source: Researcher compilation

7.3 Statistical Properties for Listed Private Equity Returns

An examination of statistical properties answered the first objective of the study. Descriptive statistics were examined and an analysis from the first moments of distribution to the fourth moments were done in order to dissect the volatility dynamics. The study hypothesised that the data is not normally distributed and that it has leptokurtic distributions.

The mean private equity returns show both positive and negative returns at both confidence intervals. On average the private equity returns from Botswana, Ghana and Egypt are low and negative as compared to South Africa; Egypt tends to outweigh them all. On the contrary, the combined results for the four countries yielded a positive return.

The findings highlight that investment in private equity could increase or destroy capital depending on the financial landscape being invested. Looking at the maximum returns achieved over the period, it is observed that there is a great potential to achieve high returns in the four countries, with Egypt dominating other countries.

However, the minimum returns show that Egypt's private equity returns had the lowest returns over the sample period. Based on the standard deviation findings, Egypt and Botswana carry the highest and lowest risks respectively. It is noted that the panel standard deviation is lower than Egypt which suggests that there is potential for diversifying and managing risk. The private equity returns for the four countries are positively skewed, revealing that there are more positive values on the distribution tails.

Egypt and South Africa are negatively skewed indicating that investors may expect frequent small gains and a few large losses. Kurtosis results show that the values are leptokurtic and have a distribution that is highly peaked than that of a normal distribution. However, using lower probabilities of estimation South Africa, Ghana and Botswana kurtosis point towards lower possibility for significant price changes.

The study, therefore, fails to reject the hypothesis that the data has leptokurtic distributions. Normality tests of private equity returns using the Jarque-Bera tests illustrate a non-normal distribution. From the tests, the study also fails to reject the hypothesis that the data is not normally distributed. This is similar to the finding by Alagidede (2009) who did a study on African stock markets and found the presence of non-normality, leptokurtic distributions against those for developed markets. This confirms that LPE investments have similar characteristics to traditional asset classes. In addition, a closer look at the return distributions show that South Africa's returns are characterised by positive and better returns than other countries under study. This could have been made possible by the fact that South Africa now has a mature private equity industry than other countries in the study.

Due to the aforementioned factors, the study was justified in using GARCH modelling since all-time series are mean reverting; data is non-stationary and is not normally distributed except for Botswana.

7.4 Volatility dynamics for LPE Investments in Selected Markets in Africa

7.4.1 Volatility dynamics for South Africa

Based on the geometric specifications presented and the analysis performed, the study concludes that South African private equity investments exhibit stylised characteristics such as volatility clustering, asymmetric effects and leptokurtic distributions, with no evidence of leverage effects or structural breaks. Tests for ARCH effects failed to reject the hypothesis that the model has ARCH effects and therefore fit for GARCH modelling. The parameter estimates under consideration were also found to meet stability conditions and indicated elevated levels of persistence in the series' conditional volatility. This suggests that LPEs in South Africa are a defensive asset and a good long-term hold. All model parameters were statistically significant hence the study failed to reject the hypothesis that the model coefficients are statistically significant.

By and large, the GARCH (1,1) model was found to be the best predictive model in the study, outperforming the other competing models. In particular, GARCH estimations for South Africa's listed private equity investments show that the previous price cannot predict tomorrow's price. The average stock return and its past value does not significantly predict its current series. Both coefficients are not statistically significant. The returns are generally too low; thus, one can conclude that there is evidence of market efficiency. Evidence from the alpha and beta terms show that volatility clustering is quite persistent in South African listed private equity markets.

Findings from the GARCH-in-Mean show that there exists a relationship between returns and volatility; the risky asset is worthy to hold as there exist commensurate returns as explained in the findings chapter. Zhang and Yang (2018) analysed the risk return relationships of investments using GARCH-in-Mean and noted the same results for Asian countries.

The impact of news – be it good or bad – was seen to be asymmetric, cementing the conclusion that in this region private equity investments good news outweigh bad news. This indicates that private equity investments in this region can make defensive assets during turbulent times. Evidence from the data presented on LPEs in South Africa show that the GARCH (1,1) model outperformed other models whilst the TGARCH model is least in forecasting conditional volatility of South African LPEs.

7.4.2 Volatility dynamics for Ghana

This study's findings fill a research gap by demystifying the volatility dynamics of LPE investments in Ghana. Accordingly, these investments carry a risk premium that is proportional to the inherent risk. There was no evidence of leverage effects. Given the government of Ghana's attention to the asset class, the investment can help optimise the portfolio's risky assets. All models chosen passed the assumptions, stability and stationarity tests and thus assumed to be best models for fitting data series for Ghana listed private equity investments. The study fails to reject the hypothesis that the model coefficients are statistically significant. In addition, the data was initially subjected to an ARCH test and the findings failed to reject the hypothesis that data does contain ARCH effects, which demonstrates the presence of heteroscedasticity, a condition for GARCH modelling.

The GARCH term tends towards 1 whilst the ARCH term is tends towards 0 which is evidence that the GARCH effects are stronger than those for ARCH; suggesting that volatility effects have more persistence than past shock impacts. The fact that the two coefficients are close to one indicates that there may be a long memory process in the volatility. This also demonstrates that the GARCH model performs better in periods of high volatility. The news curve diagnostic shows how volatility responds to a shock in past. The curve is clearly symmetrical. The asymmetric term is positive which indicates no evidence of the leverage effect. The notion that bad news will increase volatility more than good news of the same size does not appear to hold water on this stock. The findings also show that past volatility helps predict future volatility on Ghanaian returns on listed private equity firms.

The model that exhibited the lowest value of error measurements in this series is the asymmetric GARCH relative to all other asymmetric models. Similar to the findings for South Africa, the GARCH (1,1) model is parsimonious and generally widely accepted to other relatively complex linear models. Apart from that, the least model in forecasting the conditional volatility for Ghana Private Equity series is the GARCH in Mean model; the TGARCH model outperformed the EGARCH models in forecasting the conditional variance although it failed to explain leverage effects of the series.

7.4.3 Volatility dynamics for Egyptian LPEs

The effects of volatility persistence, clustering and leverage on LPE investments in Egypt were investigated. The study found no evidence of leverage effects, but it did notice traces of volatility clustering associated with some jump diffusions. GARCH models, while widely recommended as the best volatility forecasting techniques, were ill-suited to modelling this alternative asset class. For this time series, the GARCH-in-Mean model was found to be appropriate. According to the findings, there is a negative relationship between returns and volatility surprises tend to follow correspondingly upward revisions of the reward process. As a result, the asset is a good long-term investment. The TGARCH and EGARCH failed to meet stationarity conditions, indicating that stock news was not prevalent in this asset class. The study failed to reject the hypothesis that the model coefficients are statistically significant.

A closer analysis of the standardised residuals indicate that they are stable overtime; but there exist some jump diffusions in the series. The Egypt series depicts episodes of jump diffusions which need to be captured when forecasting the series. Even if the model passed assumptions tests, tests for asymmetry were conducted to ascertain whether the series was fit for asymmetric GARCH models. The study also failed to reject the hypothesis that there are ARCH effects in the data series.

Findings from the TGARCH model show that the asymmetric term is negative and statistically significant indicating that the impact of negative shock on volatility is significantly lower than the impact of positive shock. On the other end, that GARCH-in-Mean provided evidence that there is an inverse relationship between risk and return for this stock, hence a good defensive stock. From the analysis brought forth, it is evident that the Egyptian listed private equity investments are better modelled by the GARCH-in-Mean model.

7.5 Modelling for Long Run Volatility

Because the examined GARCH models are intended to capture the short-term dynamics of investments, the study expanded its analysis to include FIGARCH models, which are based on the long-term dynamics of the series. According to the evidence presented, both the FIGARCH and FIEGARCH data series for Egypt contain structural breaks. This is supplemented by a previous descriptive analysis that revealed regime shifts and breakpoints. The South African FIEGARCH coefficients indicate that all estimated coefficients are unstable, indicating that the FIGARCH model contains structural flaws. Due to its stability, the FIEGARCH model is the most suitable forecasting model for listed South African private equity investments.

All models, except for the FIGARCH model for South Africa, produce statistically insignificant results for the data series. Hence the study rejects the hypothesis that the model coefficients are statistically significant. This indicates that data asymmetries are absent from the long-term dynamics of volatility for African private equity investments. Although these investments operate in different economic systems, their individual responses to positive or negative shocks are identical. This could suggest that over time, LPE investments in the market under study tend to have similar dispositions in the long term. Studies by McKnight et al. (2022) demonstrate the importance of examining market spillovers for the development of investment strategies and policy decisions. pertaining to these investments

7.6 Impact of Country-specific Factors and Returns for LPE Investments in Africa

The economic justifications for stochastic volatility models' theoretical constructs that replicate the volatility clustering effect in financial time series are not explained in the academic space. This objective sought to determine the relationship between country-specific factors and the returns generated by the LPE investments under consideration. The finding help bring about an understanding as to whether country-specific factors are a priced factor in valuations for LPEs in Africa and provide an insight on valuation models developments. Before looking at this relationship, the study initially investigated the existence of any spillover effects on the LPE return series for the countries under study.

DCC estimates indicate that there are no short-term spillover effects as the estimates are not statistically significant for all country combinations, and DCC beta indicates that there is no long-term persistence of all cross-country investments. These findings suggest that private equity investments in Africa are a good place to diversify because there is no evidence of spillover effects from systematic risk factors. The study therefore fails to reject the hypothesis that there is no spillover effects in the data series.

The data series show that Ghana and South Africa are negatively correlated, as are South Africa and Egypt, whereas Ghana and Egypt are positively correlated. Using correlation plots, the correlation between South Africa and Ghana indicates that the diversification effect is strengthening over time, whereas the correlation between Egypt and South Africa tends downwards and is generally weak. On the other hand, Ghana and Egypt are positively correlated, which indicates that combining the assets does not create value and that the strength is high over time, indicating a weakness in portfolio creation.

The findings for short-run structural relationships using VAR models indicate that, for South Africa, past inflation realisations are significant enough to predict returns, whereas GDP is insufficient to predict the return series. As demonstrated by the VAR results, inflation is a strong long-run predictor of the country-specific factor for private equity investments, whereas other variables are highly exogenous. Hence, the study fails to reject the hypothesis that there exists structural contemporaneous interactions within country-specific factors and returns generated by LPE investments in Africa.

In the short run, the forecast error variance of private equity returns in Egypt is entirely self-explanatory, and in the long run, it remains strongly endogenous. In addition, the returns have a negligible impact on the country's risk factors over the short and long term.

All variables in Ghana exert little influence on the dependent variable, as determined by VAR. In the short term, Ghana private equity returns are highly endogenous and gradually weaken over time. The long-term impact of GDP and inflation on equity returns is minimal.

Private equity investments in Africa are highly endogenous as they strongly influence their own variable in both the short and long run. Consequently, the study fails to reject the hypothesis that country-specific factors are a priced factor in LPE valuations in Africa. Dopke, Jorg, and Tegtmeier (2018) investigated Global LPEs and determined that global risk factors are not a factor in the pricing of global LPEs. Therefore, the study validates this finding, in the selected markets of Africa.

The finding also provides an insightful contribution towards Damodaran (2020,2018), Naumoski (2012) and all researchers who have examined the relevance of country-specific factors using behavioural finance techniques.

7.7 Contribution of the Study to the Body of Knowledge

Significant contributions were made to the statistical properties and modelling dynamics of LPEs in the countries under study. Being the first study to explore LPEs in African markets, the key findings of the study translate to new knowledge, both in the academic space and for practical investment decisions.

The study first demystified unknown statistical properties of these investments' returns on the body of knowledge. No study has quantified the statistical properties of Africa's LPE markets. The study determined that investments in private equity could either increase or decrease capital depending on the investment's financial environment. Observing the maximum returns realised during the period, it can be concluded that there is a great potential for achieving high returns in the countries, with Egypt dominating others. Egypt and South Africa are negatively skewed, indicating that investors can anticipate numerous small gains and a few significant losses. The results of kurtosis analysis indicate that private equity returns are leptokurtic and have a distribution that is more skewed than a normal distribution. This information has crucial implications for investment choices.

The study also contributes to our understanding of the volatility dynamics of these investments. Nkam, Akume and Sama (2020) and Errais and Gritly (2022), examined the driving factors of private equity investments in African countries, but did not examine LPE investments in Africa. This study fills this void by examining this sector of private equity investments. It employed GARCH models to investigate the volatility dynamics of listed private equity investments in certain African markets. The short-term volatility of the examined countries' data series revealed subtle distinctions. Private equity investments in South Africa were observed to exhibit stylised characteristics such as volatility clustering, asymmetric effects, leptokurtic distributions, and the absence of leverage effects and structural breaks. GARCH (1,1) was the best model for predicting returns, and the risk-return relationship was found to be robust. Volatility in Ghana was observed to carry a risk premium proportional to inherent risk, with no leverage effects. The TGARCH model outperformed other models in forecasting Ghana return series volatility. It was observed that the Egyptian private equity series exhibited volatility clustering and jump diffusions. It was observed that the GARCH-in-Mean model outperformed other models. An analysis of the long-term volatility of all countries revealed that there were no data asymmetries in the series and that individual responses to shocks were similar, indicating that the long-term volatility characteristics were homogeneous. This also corroborates with the findings by Alagidede (2009) on African financial stock markets.

In conclusion, the study contributes to the ongoing discussion on valuation tools. By analysing the structural relationship between private equity investments and country-specific factors, the study concludes that country risk factors are also highly exogenous, as they have a negligible impact on private equity investments. Consequently, their relevance in valuation tools for certain markets is negligible. As such, they are not a priced factor in valuations for this investment. Damodaran (2020; 2016; 2012; 2003), Naumoski (2011), and Fritzen (2012) provide significant debates on the applicability of country-specific risk factors to investment valuation tools. This study contributes to bridging this gap by highlighting the insignificance of country risk factors to listed private equity firms, consistent with the study on globally listed LPEs by Dopke, Jorg, and Tegtmeier (2018) and a study done by Tegtmeier (2023).

7.8 Recommendations of the Study

Based on the findings and conclusions of the research, the study makes recommendations to two stakeholder groups. The study begins with specific suggestions for investors and portfolio managers. Secondly, it provides economists and policymakers with policy recommendations.

7.8.1 Recommendation to investors

Although these investments operate in distinct economic systems, their individual responses to shocks are identical over time. This suggests that the dispositions of LPE investments in the African markets under study are consistent over time. This supports the notion that long-term investments these African markets are not location sensitive. In addition, the findings on long-term volatility dynamics indicate that LPEs in the African markets under study are a good way to diversify because there is no evidence of spillover effects from systematic risk factors. The implications thereof is that active investors with a short-term investment horizon can geographically diversify their investments, whereas investors with a long-term investment horizon are not location sensitive. Campbell (2012), Klausner (2013), and Leautier (2016) have all found that private equity investments in Africa are location sensitive; therefore, this study confirms that these investments are only location sensitive in the short term and that diversification does not create value over the long term.

In addition, the study notes that the private equity returns for all countries analysed are positively skewed, indicating that there are a greater number of positive values at the tails of the distribution. In addition, a closer examination of the return distributions reveals that South Africa's returns are characterised by positive and superior returns compared to those of other countries studied. According to the study, there is a negative correlation between returns and volatility surprises which tends to follow the upward revisions of the reward process. The preceding implies that investors can include this asset class in their portfolio construction because it is bullish.

In addition, the effect and impact of news was found to be asymmetric, confirming the conclusion that LPE investments in the African markets under study. Therefore, good news weigh more than bad news. Consequently, these investments can serve as a defensive asset during times of turmoil.

The study found evidence of market efficiency in South Africa, indicating that active portfolio managers have no opportunities to generate excess returns over the long term. Therefore, the study advises managers to abandon active management strategies for listed private equity investments in this country and instead convert the assets into passive investments. Steyn (2019) and Heymans and Santana (2018), among others, investigated the efficiency of the Johannesburg Stock Exchange, and their findings supported the notion that some of the smaller and, in some cases, younger indices are not always efficient.

7.8.2 Recommendations for economists and policy makers

In South Africa, inflation is a strong predictor of private equity investments, whereas GDP is insufficient for predicting volatility, according to the study's findings, which are based on VAR results. Therefore, policy analysts should monitor inflation levels in order to expand the secondary market for private equity.

In Ghana, the short-term returns on listed private equity investments are highly endogenous and the long-term returns are weakly influenced by inflation and GDP, whereas in Egypt, the country-specific factors under study have no effect on the return characteristics of listed private equity investments. These findings contribute to the ongoing discussion regarding the valuation relevance of country-specific factors. This implies that Ghana-specific factors have no bearing on the value of these investments and therefore should not be incorporated.

The study concludes by recommending that policymakers in African nations create enabling environments for the growth of the private equity industry in secondary markets, as these markets are highly endogenous and can thrive regardless of economic factors. Moreover, private equity investments are crucial for the growth of industries and start-ups as a tool for financing economic development.

7.8.3 Recommendation for researchers

The study suggests that academics and researchers investigate the new and expanding market for LPE investments. According to the researcher's knowledge, previous studies have analysed global LPE investments, and a great deal of research can be conducted on various markets.

7.9 Suggestions for Future Studies

This is the first study to debunk the statistical properties of publicly traded private equity investments. The study suggests additional research be conducted to elucidate additional information regarding this asset class. This could be accomplished by analysing stock seasonality patterns, market efficiency characteristics, and conducting a similar study to validate the findings. This is important in the alternative asset class investment world, as well as for gaining a better understanding of the asset class as a source of funding for economic development in any country.

Furthermore, future studies could explore other statistical models like machine learning and some econo-physics methodologies to gain more insights on the behaviour of this asset class segment. In addition, further studies could use ARMA models and examine the volatility dynamics for Botswana LPE as they failed the tests for GARCH modelling.

In addition, the study was limited by the availability of data over a 10-year span, resulting in a small sample size. As more private equity firms become more active on the secondary market, the study recommends including more emerging markets to improve the generalizability of the results if more data becomes available. Future research should incorporate as many country-specific variables as possible to increase the policymaking utility of the results for Africa's emerging markets. If policymakers are aware of the particular country-specific factors that need to be improved, it is simple for them to find ways to enhance the business environment in order to increase private equity investments in secondary markets. Future research can also employ other methodologies, such as machine learning, to investigate the statistical properties and volatility dynamics of Africa's listed private equity firms.

7.10 Chapter summary

The study concludes that African LPEs exhibit volatility clustering, asymmetric effects and leptokurtic distributions with no leverage effects and structural breaks. These investments have no long-term volatility asymmetries. These assets' shock reactions are consistent across economic systems. This suggests that African LPEs have long-term homogenous dispositions; therefore, geographical selection cannot diversify returns even if they are defensive assets. The study showed that LPEs are highly endogenous because they strongly influence themselves both in the short and long term, and country-specific factors are highly exogenous and have a weak influence on the LPEs of the data under study, disproving the hypothesis that country-specific factors are a priced element in emerging markets. These findings have fundamental implications on valuation, portfolio construction and asset pricing activities for investors, researchers, academics and policy makers.

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9 APPENDIX 1: Ethics Approval Certificate

UNISA university of south africa

UNISA DEPARTMENT OF FINANCE, RISK MANAGEMENT AND BANKING ETHICS REVIEW COMMITTEE

Date: 14 FEBRUARY 2019

Dear Ms CM Murape

ERC Ref #2019/CEMS/FRMB/002
Name : Ms CM Murape
Student #: 62187287

Decision: Ethics Approval from 01 March 2019 to 28 February 2024

Researcher(s): Name Ms CM Murape

E-mail address cmurape16@gmail.com, telephone +263773665242

Supervisor (s): Name Prof RT Mpofu
E-mail address mpofurt@unisa.ac.za, telephone 012 429 4808

Working title of research:

Statistical modelling of private equity investment returns in selected emerging markets in Africa

Qualification: PHD

Thank you for the application for research ethics clearance by the Unisa DFRB Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 01 March 2019 to 28 February 2024

*The Negligible **risk application** was **reviewed** by the DFRB Ethics Review Committee on 14 February 2019 in compliance with the Unisa Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment*

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The proposed research may now commence with the provisions that:

1. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.
2. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the DFRB Committee.
3. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
4. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing, accompanied by a progress report.
5. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
6. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data require additional ethics clearance.
7. No fieldwork activities may continue after the expiry date (2024). Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number 2019/CEMS/FRMB/002 *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Yours sincerely,

Signature

Chair of DFRB ERC : Prof K Tsaurai

E-mail: tsaurk@unisa.ac.za

Tel: (012) 429-2140

Signature

Executive Dean: Prof T Mogale

E-mail:mogalemt@unisa.ac.za

Tel: (012) 429-4805

URERC 25.04.17 - Decision template (V2) - Approve

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10 APPENDIX 2: Language Editing Certificate

Ke Nna Publishing Services

This certificate serves to confirm that the thesis, "STATISTICAL MODELLING OF PRIVATE EQUITY INVESTMENTS IN SELECTED EMERGING MARKETS IN AFRICA: EVIDENCE FROM GARCH AND VAR MODELS" was edited by Ms Sehlodimela. She is contracted by the University of South Africa's College of Economic and Management Sciences to provide academic editing services.

The services provided include:

1. Ensuring accuracy in grammar and punctuation to improve readability and clarity;
2. Consistency and structural enhancements to aid in creating a cohesive article that has a logical flow and appropriate tone; and
3. Formatting in alignment with the stipulated style guide

For any enquiries relating to the above, see below contacts:

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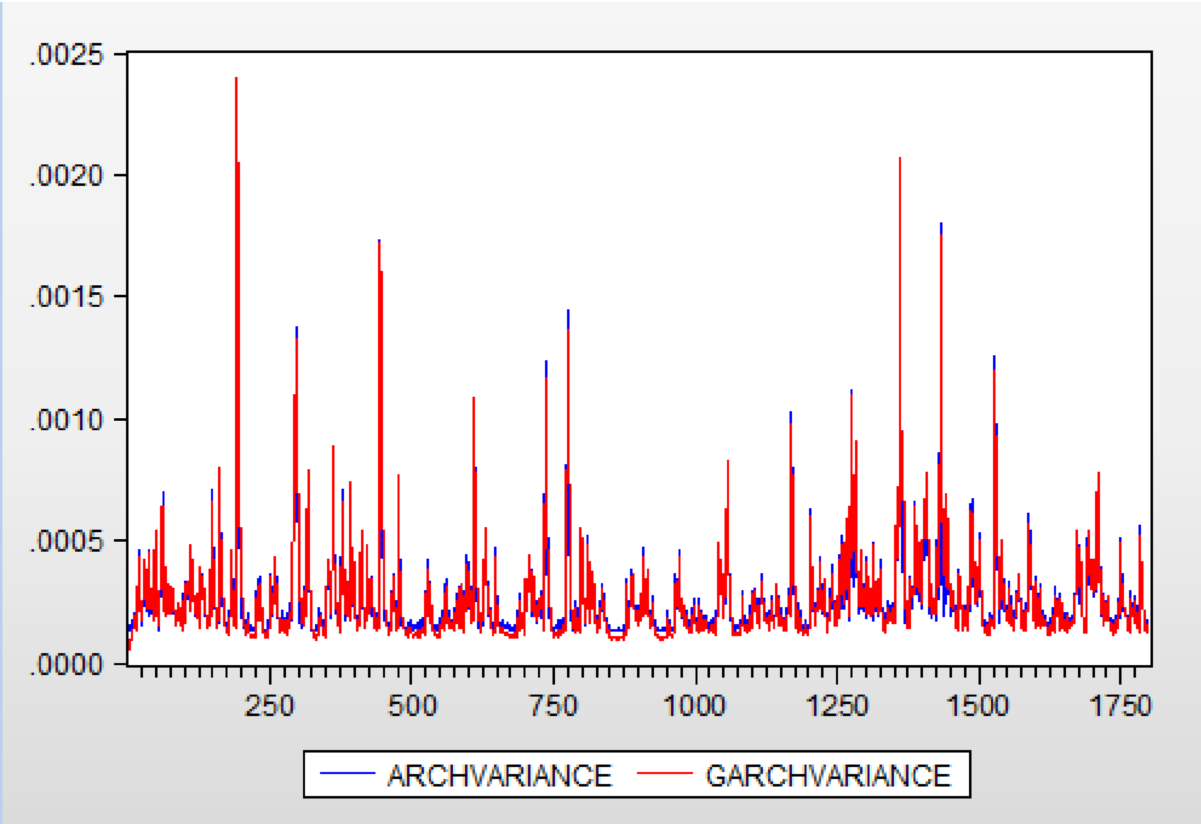
11 APPENDIX 3: SOUTH AFRICA’S Data Analysis

a. Testing for ARCH effects

Heteroskedasticity Test: ARCH

F-statistic	88.50033	Prob. F(1,2569)	0.0000
Obs*R-squared	85.61968	Prob. Chi-Square(1)	0.0000

b. ARCH-GARCH Variance plot



c. Engle and NG Test

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000133	2.44E-05	5.457393	0.0000
DUMMY1	0.000115	3.72E-05	3.088437	0.0020
DUMMY1*GARCH11RES(-1)	-0.005038	0.001607	-3.134551	0.0017
DUMMY2*GARCH11RES(-1)	0.012466	0.001472	8.467390	0.0000

d. EGARCH Model output

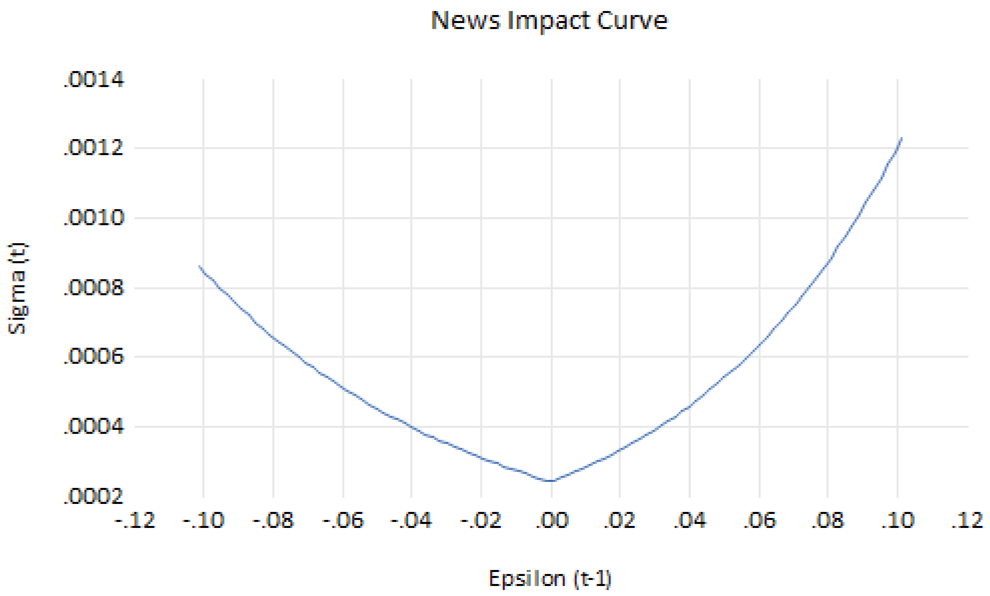
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-1.40E-06	0.000165	-0.008496	0.9932
SA(-1)	9.07E-06	0.012214	0.000742	0.9994
	Variance Equation			
C(3)	-0.995803	0.198078	-5.027317	0.0000
C(4)	0.296377	0.040972	7.233616	0.0000
C(5)	0.016883	0.028098	0.600857	0.5479
C(6)	0.903755	0.022376	40.38946	0.0000
GED PARAMETER	0.892290	0.031287	28.51988	0.0000

e. GARCH-in-Mean output

Dependent Variable: SA
Method: ML ARCH - Generalized error distribution (GED) (Marquardt / EViews legacy)
Date: 03/26/22 Time: 10:50
Sample: 1 2573
Included observations: 2573
Failure to improve Likelihood after 20 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	-0.000665	0.039654	-0.016770	0.9866
C	1.05E-05	0.000613	0.017177	0.9863
	Variance Equation			
C	3.16E-05	7.64E-06	4.137456	0.0000
RESID(-1)^2	0.206679	0.036966	5.591061	0.0000
GARCH(-1)	0.718596	0.041601	17.27349	0.0000
GED PARAMETER	0.867833	0.029346	29.57279	0.0000
R-squared	-0.000315	Mean dependent var		-0.000298
Adjusted R-squared	-0.000704	S.D. dependent var		0.017059
S.E. of regression	0.017065	Akaike info criterion		-5.599720
Sum squared resid	0.748753	Schwarz criterion		-5.586072
Log likelihood	7210.040	Hannan-Quinn criter.		-5.594772
Durbin-Watson stat	2.174779			

f. News impact Curve for EGARCH



g. Residuals test analysis EGARCH

Heteroskedasticity Test: ARCH

F-statistic	0.003391	Prob. F(1,1794)	0.9536
Obs*R-squared	0.003395	Prob. Chi-Square(1)	0.9535

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 02/04/22 Time: 03:27
Sample (adjusted): 3 1798
Included observations: 1796 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.077766	0.063706	16.91772	0.0000
WGT_RESID^2(-1)	-0.001375	0.023610	-0.058236	0.9536

h. Serial correlation using both squared residuals and residuals

Date: 02/04/22 Time: 03:40
Sample: 1 1798
Included observations: 1797

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.001	-0.001	0.0034	0.953
		2	-0.019	-0.019	0.6717	0.715
		3	-0.033	-0.033	2.6294	0.452
		4	-0.005	-0.006	2.6837	0.612
		5	-0.016	-0.017	3.1527	0.676
		6	-0.011	-0.012	3.3526	0.763
		7	-0.017	-0.018	3.8736	0.794
		8	-0.025	-0.027	5.0279	0.755
		9	0.015	0.014	5.4536	0.793
		10	-0.005	-0.007	5.4948	0.856
		11	-0.007	-0.009	5.5780	0.900
		12	-0.016	-0.016	6.0390	0.914
		13	-0.034	-0.036	8.0792	0.838
		14	-0.019	-0.021	8.7509	0.847
		15	-0.013	-0.017	9.0785	0.873
		16	0.027	0.023	10.384	0.846
		17	-0.010	-0.013	10.581	0.878
		18	0.004	0.001	10.613	0.910
		19	-0.001	-0.003	10.617	0.936
		20	-0.007	-0.010	10.704	0.954
		21	-0.016	-0.017	11.142	0.960
		22	-0.004	-0.006	11.178	0.972
		23	0.031	0.030	12.949	0.953
		24	0.001	0.000	12.951	0.967
		25	-0.001	-0.004	12.953	0.977
		26	-0.016	-0.016	13.419	0.980
		27	-0.015	-0.018	13.849	0.983
		28	-0.047	-0.049	17.842	0.930
		29	-0.001	-0.003	17.846	0.947
		30	0.001	-0.002	17.847	0.961
		31	-0.014	-0.017	18.210	0.967
		32	0.040	0.036	21.181	0.928

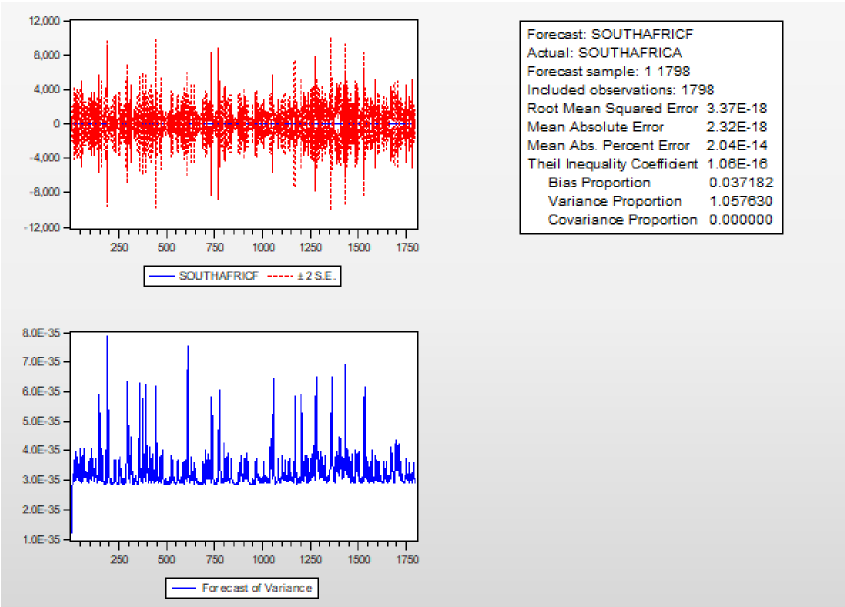
i. Nyblom parameter stability test

Nyblom Parameter Stability Test
Null Hypothesis: Parameters are stable

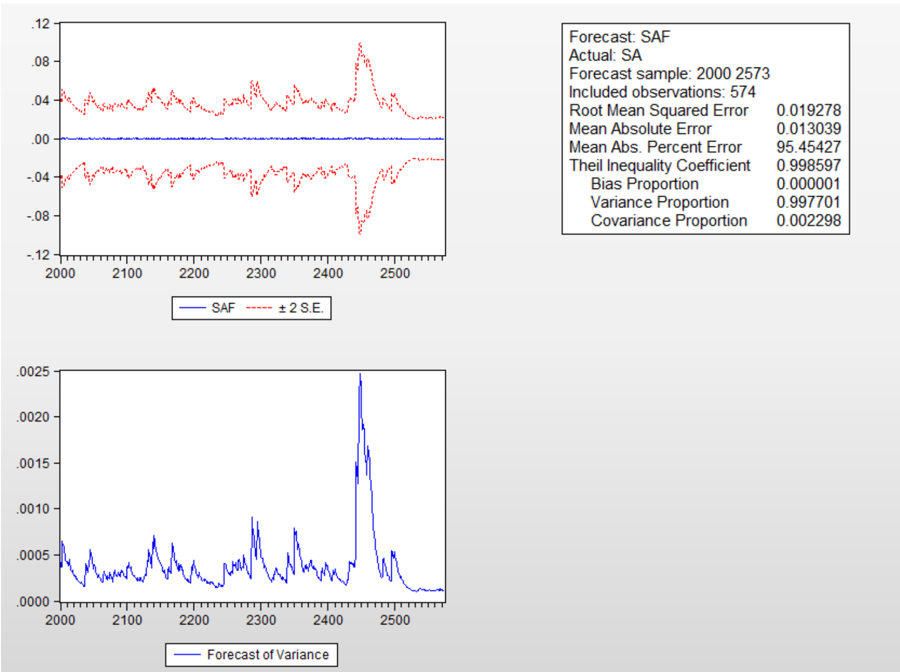
Variable	Statistic	1% Crit.	5% Crit.	10% Crit.
C	0.209236	0.748	0.470	0.353
C	0.141453	0.748	0.470	0.353
RESID(-1)^2	0.075257	0.748	0.470	0.353
GARCH(-1)	0.100925	0.748	0.470	0.353
Joint	0.672647	1.600	1.240	1.070

*Critical values from Hansen 1990

j. Forecast using the full sample



k. Forecast using a modified sample



l. Descriptive analysis at 5% using NUMXL

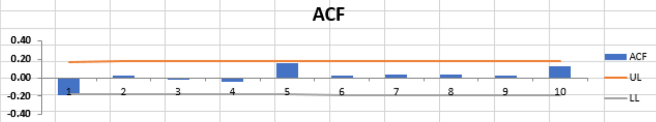
Summary Statistics	
Average:	0.64%
Standard Deviation:	6.62%
Skew:	-0.11
Excess Kurtosis:	0.71
Median:	0.58%
Minimum:	-19.89%
Maximum:	23.64%
1st Quartile:	-3.45%
3rd Quartile:	5.76%

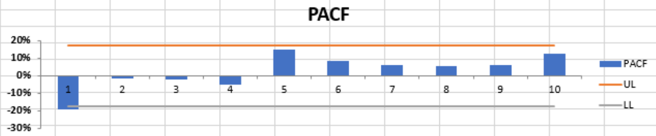
Significance Test		5.00%
Target	P-Value	SIG?
0.000	13.79%	FALSE
0.000	30.26%	FALSE
0.000	9.21%	FALSE

Test	P-Value	SIG?
White-Noise?	12.76%	TRUE
Normal Distributed?	30.11%	TRUE
ARCH Effect?	2.12%	TRUE

Correlogram Analysis

Lag	ACF	UL	LL	PACF	UL	LL
1	-18.84%	17.32%	-17.32%	-18.84%	17.32%	-17.32%
2	2.32%	17.93%	-17.93%	-1.27%	17.32%	-17.32%
3	-2.28%	17.94%	-17.94%	-2.16%	17.32%	-17.32%
4	-4.15%	17.95%	-17.95%	-5.14%	17.32%	-17.32%
5	16.15%	17.97%	-17.97%	15.00%	17.32%	-17.32%
6	2.29%	18.41%	-18.41%	8.54%	17.32%	-17.32%
7	3.90%	18.41%	-18.41%	6.00%	17.32%	-17.32%
8	2.88%	18.44%	-18.44%	5.73%	17.32%	-17.32%
9	2.82%	18.45%	-18.45%	6.19%	17.32%	-17.32%
10	11.99%	18.46%	-18.46%	12.74%	17.32%	-17.32%





m. Volatility dynamics at 5% using NUMXL

GARCH(1,1)

Param	Value
μ	0.01
α_0	0.00
α_1	0.15
β_1	0.15

Goodness-of-fit

LLF	AIC	CHECK
169.32	-332.63	1

Residuals (standardized) Analysis

	AVG	STDEV	Skew	Kurtosis	Noise?	Normal?	ARCH?
	-0.02	0.99	-0.45	0.21	TRUE	TRUE	FALSE
Target	0.00	1.00	0.00	0.00			
SIG?	FALSE	FALSE	TRUE	FALSE			

EGARCH(1,1)

Param	Value
μ	0.00
α_0	-5.37
α_1	0.31
γ_1	-0.47
β_1	0.07

Goodness-of-fit

LLF	AIC	CHECK
170.05	-330.09	1

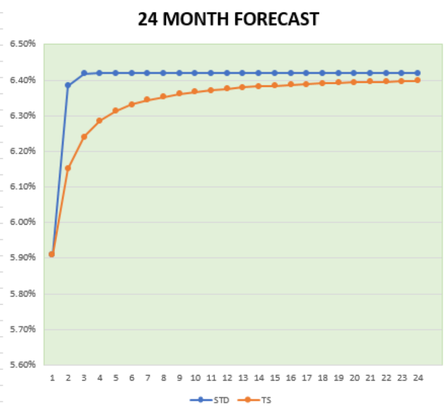
VL
6.42%

Residuals (standardized) Analysis

	AVG	STDEV	Skew	Kurtosis	Noise?	Normal?	ARCH?
	0.03	1.00	-0.50	0.14	TRUE	TRUE	FALSE
Target	0.00	1.00	0.00	0.00			
SIG?	FALSE	FALSE	TRUE	FALSE			

Step	Mean	STD	TS	UL	LL
1	0.29%	5.91%	5.91%	11.87%	-11.29%
2	0.29%	6.38%	6.15%	12.80%	-12.23%
3	0.29%	6.42%	6.24%	12.86%	-12.29%
4	0.29%	6.42%	6.29%	12.87%	-12.30%
5	0.29%	6.42%	6.31%	12.87%	-12.30%
6	0.29%	6.42%	6.33%	12.87%	-12.30%
7	0.29%	6.42%	6.34%	12.87%	-12.30%
8	0.29%	6.42%	6.35%	12.87%	-12.30%
9	0.29%	6.42%	6.36%	12.87%	-12.30%
10	0.29%	6.42%	6.37%	12.87%	-12.30%
11	0.29%	6.42%	6.37%	12.87%	-12.30%
12	0.29%	6.42%	6.38%	12.87%	-12.30%
13	0.29%	6.42%	6.38%	12.87%	-12.30%
14	0.29%	6.42%	6.38%	12.87%	-12.30%
15	0.29%	6.42%	6.38%	12.87%	-12.30%
16	0.29%	6.42%	6.39%	12.87%	-12.30%
17	0.29%	6.42%	6.39%	12.87%	-12.30%
18	0.29%	6.42%	6.39%	12.87%	-12.30%
19	0.29%	6.42%	6.39%	12.87%	-12.30%
20	0.29%	6.42%	6.39%	12.87%	-12.30%
21	0.29%	6.42%	6.39%	12.87%	-12.30%
22	0.29%	6.42%	6.40%	12.87%	-12.30%
23	0.29%	6.42%	6.40%	12.87%	-12.30%
24	0.29%	6.42%	6.40%	12.87%	-12.30%

Step	STD	TS
1	5.91%	5.91%
2	6.38%	6.15%
3	6.42%	6.24%
4	6.42%	6.29%
5	6.42%	6.31%
6	6.42%	6.33%
7	6.42%	6.34%
8	6.42%	6.35%
9	6.42%	6.36%
10	6.42%	6.37%
11	6.42%	6.37%
12	6.42%	6.38%
13	6.42%	6.38%
14	6.42%	6.38%
15	6.42%	6.38%
16	6.42%	6.39%
17	6.42%	6.39%
18	6.42%	6.39%
19	6.42%	6.39%
20	6.42%	6.39%
21	6.42%	6.39%
22	6.42%	6.40%
23	6.42%	6.40%
24	6.42%	6.40%



12 APPENDIX 4: Egypt GARCH analysis

a. Testing for ARCH effects

Heteroskedasticity Test: ARCH

F-statistic	20.31098	Prob. F(1,2438)	0.0000
Obs*R-squared	20.15969	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 03/27/22 Time: 12:39
Sample (adjusted): 3 2442
Included observations: 2440 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.009898	0.003516	2.815101	0.0049
RESID^2(-1)	0.090896	0.020169	4.506770	0.0000

b. GARCH (1,1)

Dependent Variable: EGYPT
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 03/27/22 Time: 12:42
Sample (adjusted): 2 2442
Included observations: 2441 after adjustments
Convergence achieved after 216 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

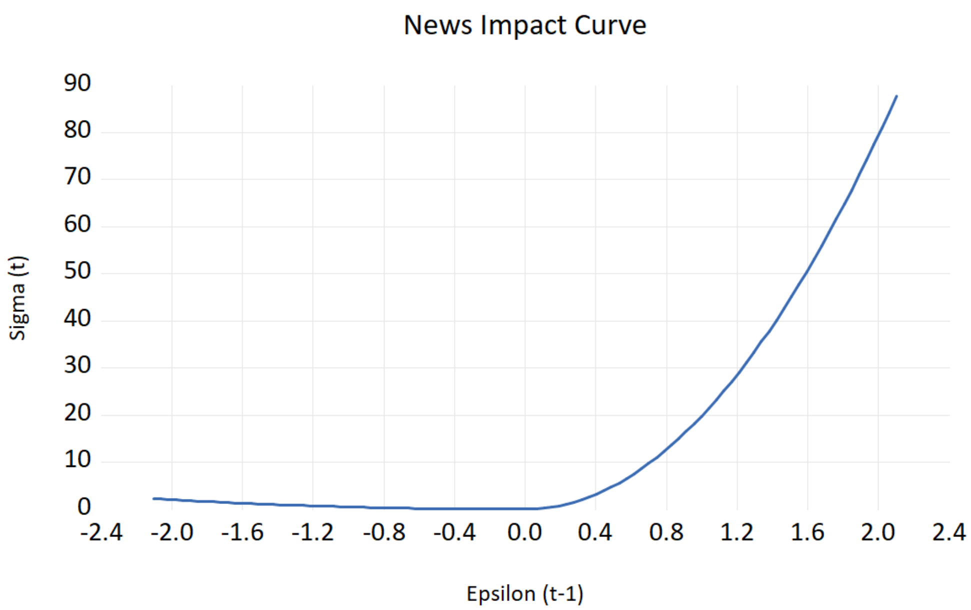
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.011110	0.000190	-58.52844	0.0000
EGYPT(-1)	-0.957449	0.000663	-1444.495	0.0000
		Variance Equation		
C	0.000392	2.16E-05	18.16994	0.0000
RESID(-1)^2	7.332084	0.095060	77.13152	0.0000
GARCH(-1)	0.012977	0.002085	6.225073	0.0000

c. TGARCH

Dependent Variable: EGYPT
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 03/30/22 Time: 10:59
Sample (adjusted): 2 2442
Included observations: 2441 after adjustments
Convergence achieved after 111 iterations
MA Backcast: 1
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0)
+ C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	-0.990013	0.000199	-4962.764	0.0000
MA(1)	0.588471	0.006808	86.43386	0.0000
Variance Equation				
C	0.000339	1.16E-05	29.23667	0.0000
RESID(-1)^2	19.88285	0.139948	142.0727	0.0000
RESID(-1)^2*(RESID(-1)<0)	-19.37913	0.113390	-170.9076	0.0000
GARCH(-1)	0.137887	0.004196	32.86524	0.0000

d. News Impact Curve- TGARCH



e. EGARCH

Dependent Variable: EGYPT
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 03/30/22 Time: 11:11
Sample (adjusted): 2 2442
Included observations: 2441 after adjustments
Convergence achieved after 46 iterations
MA Backcast: 1
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	-0.986257	0.000237	-4167.193	0.0000
MA(1)	0.637264	0.006365	100.1246	0.0000
Variance Equation				
C(3)	-3.880744	0.032128	-120.7884	0.0000
C(4)	2.250772	0.014660	153.5324	0.0000
C(5)	1.600417	0.009285	172.3700	0.0000
C(6)	0.531204	0.004882	108.8041	0.0000

f. Residuals Test – ARCH

Heteroskedasticity Test: ARCH

F-statistic	0.269738	Prob. F(1,2439)	0.6036
Obs*R-squared	0.269930	Prob. Chi-Square(1)	0.6034

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 03/27/22 Time: 13:10
Sample (adjusted): 2 2442
Included observations: 2441 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.292074	0.107864	2.707802	0.0068
WGT_RESID^2(-1)	0.010516	0.020247	0.519364	0.6036

g. ACF Test

Date: 03/27/22 Time: 13:14
Sample: 1 2442
Included observations: 2442

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
|	|	1	0.011	0.011	0.2704	0.603
|	|	2	0.003	0.003	0.2986	0.861
|	|	3	0.005	0.005	0.3605	0.948
|	|	4	0.003	0.003	0.3878	0.983
|	|	5	0.003	0.003	0.4139	0.995
|	|	6	0.003	0.003	0.4421	0.998
|	|	7	0.003	0.003	0.4698	1.000
|	|	8	0.004	0.004	0.5094	1.000
|	|	9	0.004	0.004	0.5527	1.000
|	|	10	0.004	0.003	0.5833	1.000
|	|	11	0.003	0.003	0.6064	1.000
|	|	12	0.003	0.003	0.6273	1.000
|	|	13	0.003	0.003	0.6517	1.000
|	|	14	0.003	0.003	0.6749	1.000
|	|	15	0.003	0.003	0.6964	1.000
|	|	16	0.003	0.003	0.7149	1.000
|	|	17	0.003	0.003	0.7345	1.000
|	|	18	0.003	0.003	0.7551	1.000

*Probabilities may not be valid for this equation specification.

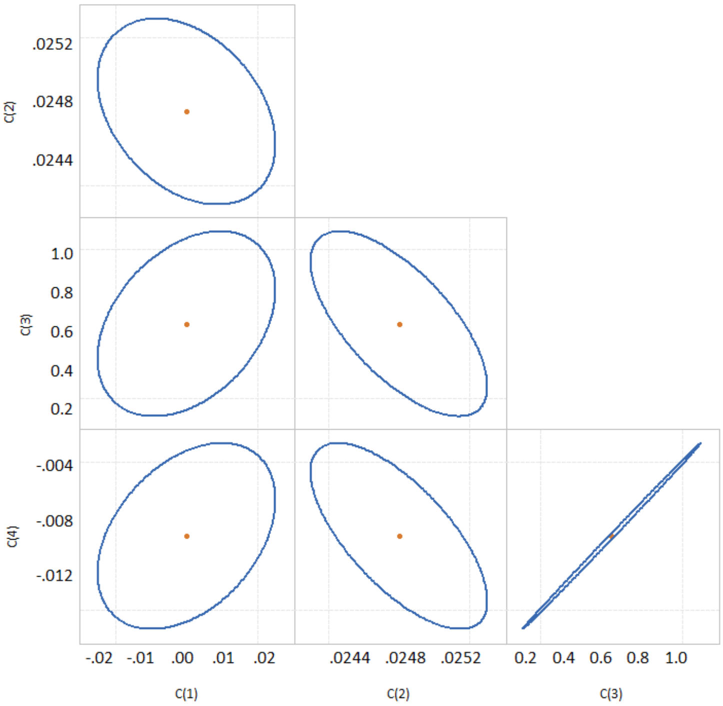
h. Nyblom Stability Test

Nyblom Parameter Stability Test
Null Hypothesis: Parameters are stable

Variable	Statistic	1% Crit.	5% Crit.	10% Crit.
C	0.116051	0.748	0.470	0.353
C	0.274103	0.748	0.470	0.353
RESID(-1)^2	0.130296	0.748	0.470	0.353
GARCH(-1)	0.129585	0.748	0.470	0.353
Joint	63.61463	1.600	1.240	1.070

*Critical values from Hansen 1990

i. Confidence ellipse test



j. GARCH summary using NumXl at 5% significance level using NumXl

GARCH(1,1)

Param	Value
μ	-0.02
α_0	0.04
α_1	0.00
β_1	0.00

Goodness-of-fit

LLF	AIC	CHECK
20.29	-34.59	1

Residuals (standardized) Analysis

	AVG	STDEV	Skew	Kurtosis	Noise?	Normal?	ARCH?
	0.00	0.99	-4.59	33.72	TRUE	FALSE	FALSE
Target	0.00	1.00	0.00	0.00			
SIG?	FALSE	FALSE	TRUE	TRUE			

EGARCH(1,1)

Param	Value
μ	0.00
α_0	-7.31
α_1	0.64
γ_1	-0.06
β_1	-0.76

Goodness-of-fit

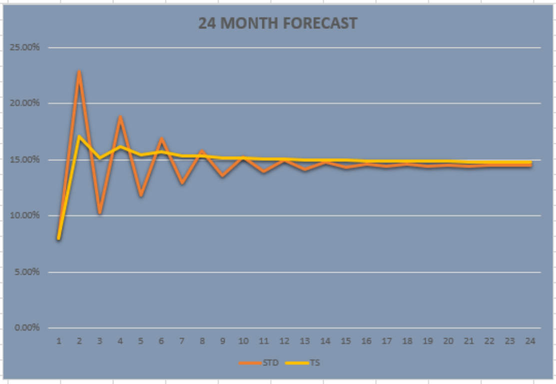
LLF	AIC	CHECK
64.95	-119.90	1
VL		
0.145081176		

Residuals (standardized) Analysis

	AVG	STDEV	Skew	Kurtosis	Noise?	Normal?	ARCH?
	-0.08	0.99	-0.15	3.55	TRUE	FALSE	FALSE
Target	0.00	1.00	0.00	0.00			
SIG?	FALSE	FALSE	FALSE	TRUE			

Step	Mean	STD	TS	UL	LL
1	-0.36%	7.96%	7.96%	15.25%	-15.97%
2	-0.36%	22.89%	17.13%	44.49%	-45.22%
3	-0.36%	10.26%	15.19%	19.75%	-20.47%
4	-0.36%	18.88%	16.19%	36.64%	-37.36%
5	-0.36%	11.88%	15.43%	22.92%	-23.64%
6	-0.36%	16.89%	15.68%	32.74%	-33.47%
7	-0.36%	12.92%	15.32%	24.97%	-25.69%
8	-0.36%	15.84%	15.38%	30.68%	-31.41%
9	-0.36%	13.57%	15.19%	26.24%	-26.96%
10	-0.36%	15.26%	15.20%	29.55%	-30.28%
11	-0.36%	13.96%	15.09%	27.00%	-27.72%
12	-0.36%	14.94%	15.08%	28.92%	-29.64%
13	-0.36%	14.19%	15.01%	27.45%	-28.17%
14	-0.36%	14.76%	14.99%	28.56%	-29.28%
15	-0.36%	14.32%	14.95%	27.71%	-28.43%
16	-0.36%	14.65%	14.93%	28.35%	-29.08%
17	-0.36%	14.40%	14.90%	27.86%	-28.59%
18	-0.36%	14.59%	14.88%	28.23%	-28.96%
19	-0.36%	14.45%	14.86%	27.95%	-28.68%
20	-0.36%	14.56%	14.85%	28.17%	-28.89%
21	-0.36%	14.47%	14.83%	28.00%	-28.73%
22	-0.36%	14.54%	14.82%	28.13%	-28.85%
23	-0.36%	14.49%	14.80%	28.03%	-28.76%
24	-0.36%	14.52%	14.79%	28.10%	-28.83%

Step	STD	TS
1	7.96%	7.96%
2	22.89%	17.13%
3	10.26%	15.19%
4	18.88%	16.19%
5	11.88%	15.43%
6	16.89%	15.68%
7	12.92%	15.32%
8	15.84%	15.38%
9	13.57%	15.19%
10	15.26%	15.20%
11	13.96%	15.09%
12	14.94%	15.08%
13	14.19%	15.01%
14	14.76%	14.99%
15	14.32%	14.95%
16	14.65%	14.93%
17	14.40%	14.90%
18	14.59%	14.88%
19	14.45%	14.86%
20	14.56%	14.85%
21	14.47%	14.83%
22	14.54%	14.82%
23	14.49%	14.80%
24	14.52%	14.79%



13 APPENDIX 5: Ghana GARCH analysis

a. Testing for ARCH effects

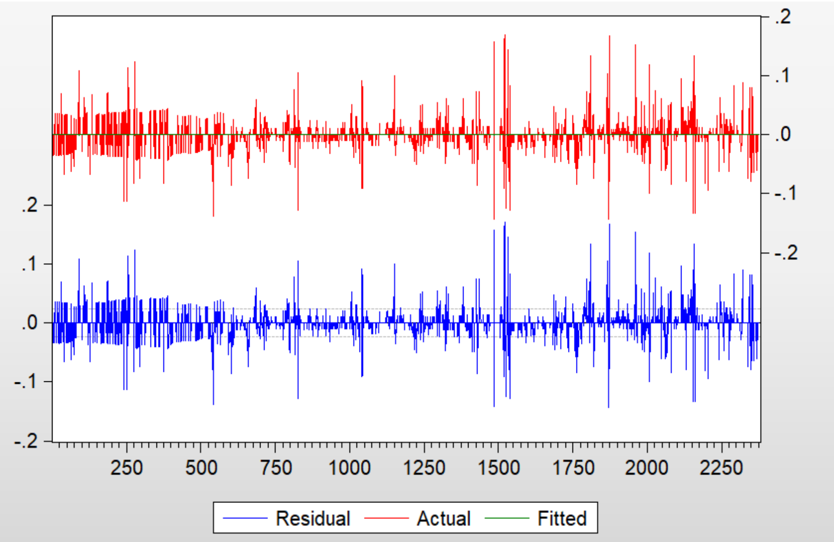
Heteroskedasticity Test: ARCH

F-statistic	237.5925	Prob. F(1,2373)	0.0000
Obs*R-squared	216.1510	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 02/25/22 Time: 23:05
Sample (adjusted): 3 2377
Included observations: 2375 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000400	4.45E-05	8.981313	0.0000
RESID^2(-1)	0.301678	0.019572	15.41403	0.0000

b. Residuals plot

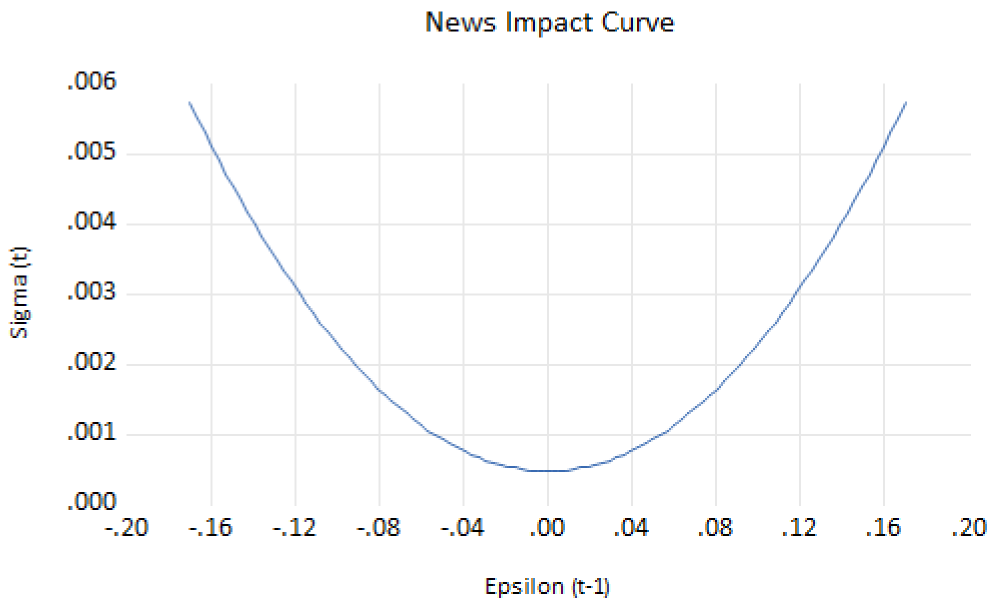


c. GARCH (1.1)

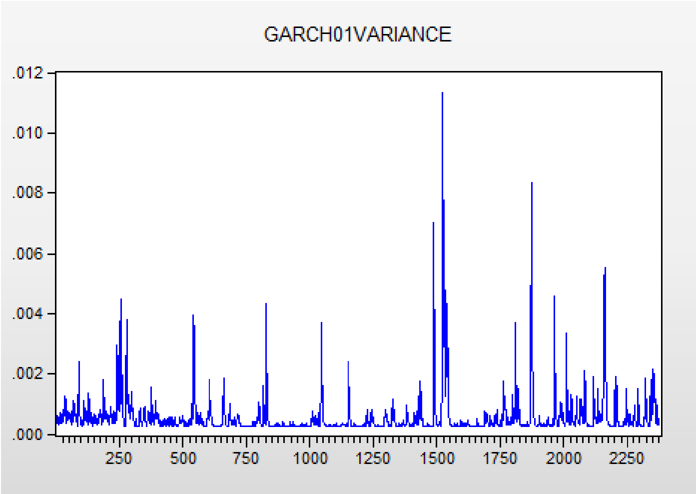
Dependent Variable: GHANA
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 02/25/22 Time: 22:56
Sample (adjusted): 2 2377
Included observations: 2376 after adjustments
Convergence achieved after 34 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000412	0.000398	-1.037515	0.2995
GHANA(-1)	-0.004287	0.024894	-0.172193	0.8633
		Variance Equation		
C	7.76E-05	3.64E-06	21.28393	0.0000
RESID(-1)^2	0.182215	0.011165	16.31954	0.0000
GARCH(-1)	0.687635	0.012550	54.79103	0.0000

d. News Impact Curve- GARCH (1,1)



e. Conditional variance graph



f. GARCH in MEAN- Standard Deviation

Dependent Variable: GHANA
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 04/19/22 Time: 11:52
Sample: 1 2377
Included observations: 2377
Convergence achieved after 31 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.008847	0.080645	0.109700	0.9126
C	-0.000565	0.001644	-0.343833	0.7310
Variance Equation				
C	7.78E-05	3.73E-06	20.83363	0.0000
RESID(-1)^2	0.182579	0.011122	16.41622	0.0000
GARCH(-1)	0.686857	0.012743	53.90244	0.0000

g. Engle and Ng test

Dependent Variable: U
Method: Least Squares
Date: 02/27/22 Time: 03:55
Sample (adjusted): 3 2377
Included observations: 2375 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000318	5.20E-05	6.113694	0.0000
DUMMY1	-0.000403	0.000166	-2.426712	0.0153
DUMMY1*GARCHRESID(...	-0.034042	0.004083	-8.336817	0.0000
DUMMY2*GARCHRESID(...	0.025655	0.002729	9.401497	0.0000

R-squared	0.067810	Mean dependent var	0.000573
Adjusted R-squared	0.066631	S.D. dependent var	0.002209
S.E. of regression	0.002134	Akaike info criterion	-9.460119
Sum squared resid	0.010796	Schwarz criterion	-9.450397
Log likelihood	11237.89	Hannan-Quinn criter.	-9.456580
F-statistic	57.49129	Durbin-Watson stat	1.920448
Prob(F-statistic)	0.000000		

h. TGARCH

Dependent Variable: GHANA
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 03/26/22 Time: 21:03
Sample: 1 2377
Included observations: 2377
Convergence achieved after 30 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0)
+ C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000388	0.000430	-0.901959	0.3671
		Variance Equation		
C	7.76E-05	3.64E-06	21.33649	0.0000
RESID(-1)^2	0.183851	0.014876	12.35874	0.0000
RESID(-1)^2*(RESID(-1)<0)	-0.002909	0.017548	-0.165784	0.8683
GARCH(-1)	0.687391	0.012499	54.99516	0.0000

i. EGARCH

Dependent Variable: GHANA
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 02/28/22 Time: 04:00
Sample (adjusted): 2 2377
Included observations: 2376 after adjustments
Convergence achieved after 66 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000245	0.000365	0.672479	0.5013
GHANA(-1)	0.021687	0.023515	0.922297	0.3564
		Variance Equation		
C(3)	-1.308225	0.055602	-23.52833	0.0000
C(4)	0.293098	0.010974	26.70946	0.0000
C(5)	0.006906	0.008704	0.793349	0.4276
C(6)	0.848462	0.007027	120.7443	0.0000

j. Residuals Diagnosis- ARCH Test

Heteroskedasticity Test: ARCH

F-statistic	0.086501	Prob. F(1,2373)	0.7687
Obs*R-squared	0.086571	Prob. Chi-Square(1)	0.7686

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 02/27/22 Time: 02:30
Sample (adjusted): 3 2377
Included observations: 2375 after adjustments

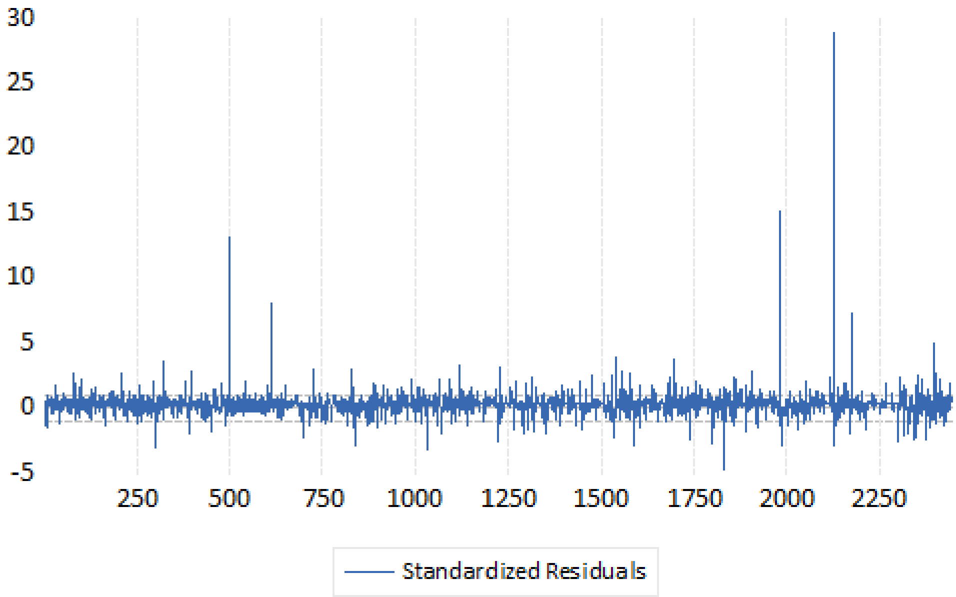
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.993678	0.084518	11.75699	0.0000
WGT_RESID^2(-1)	0.006037	0.020528	0.294111	0.7687

k. The serial correlation test using both squared residuals and residuals

Date: 02/04/22 Time: 03:40
Sample: 1 1798
Included observations: 1797

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.001	-0.001	0.0034	0.953
		2	-0.019	-0.019	0.6717	0.715
		3	-0.033	-0.033	2.6294	0.452
		4	-0.005	-0.006	2.6837	0.612
		5	-0.016	-0.017	3.1527	0.676
		6	-0.011	-0.012	3.3526	0.763
		7	-0.017	-0.018	3.8736	0.794
		8	-0.025	-0.027	5.0279	0.755
		9	0.015	0.014	5.4536	0.793
		10	-0.005	-0.007	5.4948	0.856
		11	-0.007	-0.009	5.5780	0.900
		12	-0.016	-0.016	6.0390	0.914
		13	-0.034	-0.036	8.0792	0.838
		14	-0.019	-0.021	8.7509	0.847
		15	-0.013	-0.017	9.0785	0.873
		16	0.027	0.023	10.384	0.846
		17	-0.010	-0.013	10.581	0.878
		18	0.004	0.001	10.613	0.910
		19	-0.001	-0.003	10.617	0.936
		20	-0.007	-0.010	10.704	0.954
		21	-0.016	-0.017	11.142	0.960
		22	-0.004	-0.006	11.178	0.972
		23	0.031	0.030	12.949	0.953
		24	0.001	0.000	12.951	0.967
		25	-0.001	-0.004	12.953	0.977
		26	-0.016	-0.016	13.419	0.980
		27	-0.015	-0.018	13.849	0.983
		28	-0.047	-0.049	17.842	0.930
		29	-0.001	-0.003	17.846	0.947
		30	0.001	-0.002	17.847	0.961
		31	-0.014	-0.017	18.210	0.967
		32	0.040	0.036	21.181	0.928

I. Residuals Plot



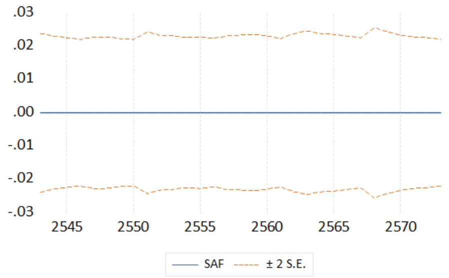
14 APPENDIX 6: GARCH forecasts for all counters

a. Summary

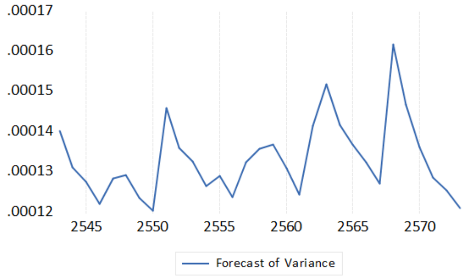
COUNTER	MODEL	Forecasting Horizon	RMSE	MAE	Theil Inequality Coefficient
South Africa	GARCH (1,1)	30 Days	0.005761	0.004410	0.999367
	EGARCH	30 Days	0.05760	0.004410	0.999718
	TGARCH	30 Days	0.05761	0.004410	0.999931
	GARCH in mean	30 Days	0.005761	0.004410	0.999613
Ghana	GARCH (1,1)	30 Days	0.030913	0.014316	0.983531
	EGARCH	30 Days	0.031279	0.014754	0.986421
	TGARCH	30 Days	0.031018	0.014300	1.000000
	GARCH In mean	30 Days	0.031026	0.014754	0.999393
Egypt	GARCH(1,1)	30 Days	0.037668	0.031801	0.758643
	EGARCH	30 Days	0.027028	0.022077	0.824990
	TGARCH	30 Days	0.027699	0.022624	0.816262
	GARCH in mean	30 Days	0.023319	0.019100	0.964511

b. Forecasts using static modelling:South Africa

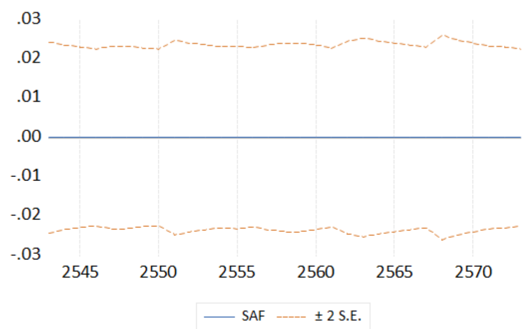
i. Garch (1,1)



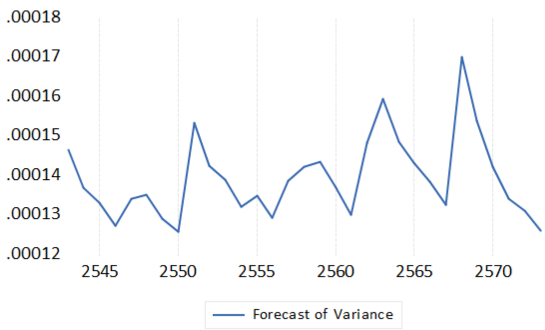
Forecast: SAF	
Actual: SA	
Forecast sample: 2543 2573	
Included observations: 31	
Root Mean Squared Error	0.005761
Mean Absolute Error	0.004410
Mean Abs. Percent Error	NA
Theil Inequality Coef.	0.999367
Bias Proportion	0.025704
Variance Proportion	0.973967
Covariance Proportion	0.000329
Theil U2 Coefficient	NA
Symmetric MAPE	199.6044



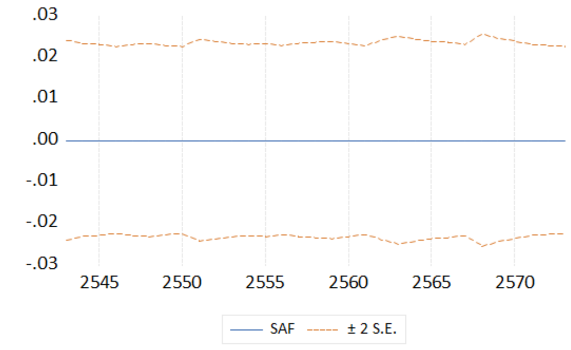
ii. GARCH-in-mean



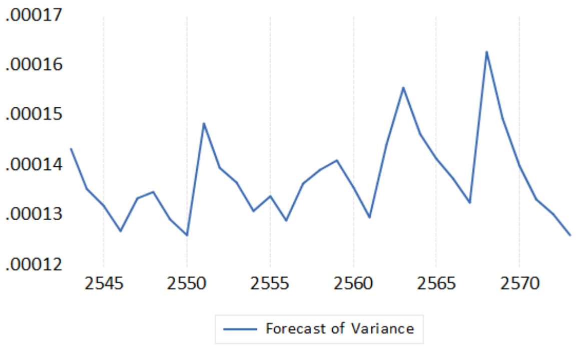
Forecast: SAF	
Actual: SA	
Forecast sample: 2543 2573	
Included observations: 31	
Root Mean Squared Error	0.005761
Mean Absolute Error	0.004410
Mean Abs. Percent Error	NA
Theil Inequality Coef.	0.999613
Bias Proportion	0.025634
Variance Proportion	0.974260
Covariance Proportion	0.000106
Theil U2 Coefficient	NA
Symmetric MAPE	199.6988



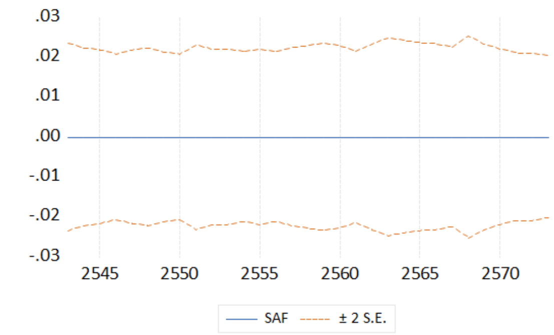
iii. TGARCH



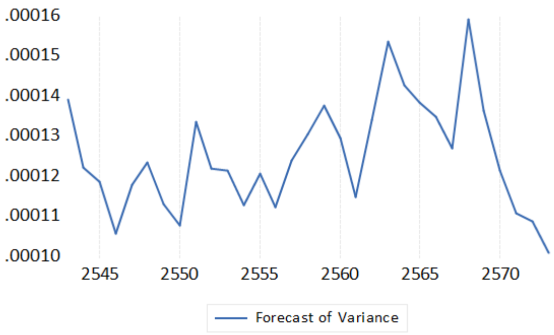
Forecast: SAF	
Actual: SA	
Forecast sample: 2543 2573	
Included observations: 31	
Root Mean Squared Error	0.005761
Mean Absolute Error	0.004410
Mean Abs. Percent Error	NA
Theil Inequality Coef.	0.999931
Bias Proportion	0.025471
Variance Proportion	0.974529
Covariance Proportion	0.000000
Theil U2 Coefficient	NA
Symmetric MAPE	199.9628



iv. EGARCH

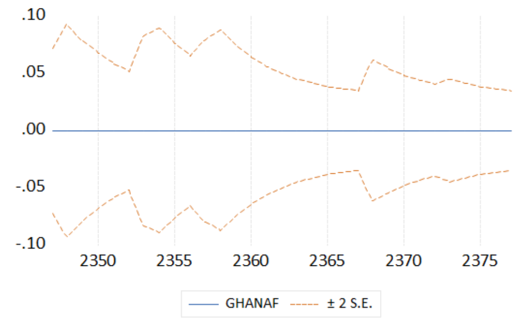


Forecast: SAF	
Actual: SA	
Forecast sample: 2543 2573	
Included observations: 31	
Root Mean Squared Error	0.005760
Mean Absolute Error	0.004410
Mean Abs. Percent Error	NA
Theil Inequality Coef.	0.999718
Bias Proportion	0.025413
Variance Proportion	0.974566
Covariance Proportion	0.000021
Theil U2 Coefficient	NA
Symmetric MAPE	199.8463

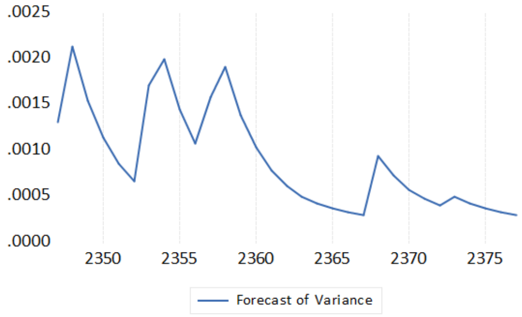


c. Forecasts using static modelling: Ghana

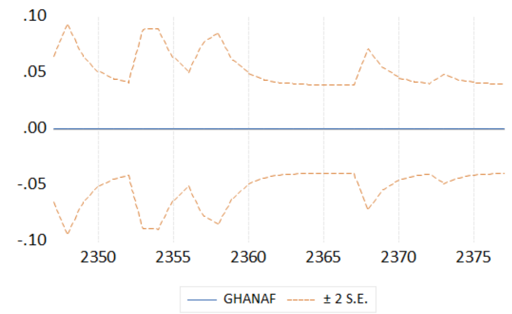
i. GARCH (1,1)



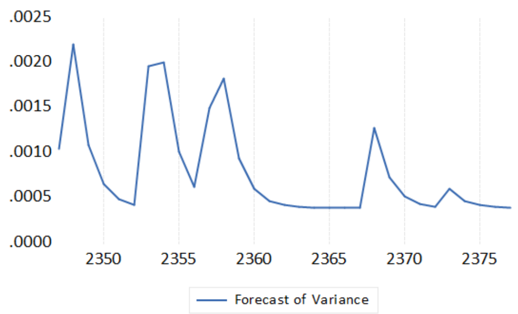
Forecast: GHANAF	
Actual: GHANA	
Forecast sample: 2347 2377	
Included observations: 31	
Root Mean Squared Error	0.030913
Mean Absolute Error	0.014518
Mean Abs. Percent Error	NA
Theil Inequality Coef.	0.983531
Bias Proportion	0.021966
Variance Proportion	0.972547
Covariance Proportion	0.005486
Theil U2 Coefficient	NA
Symmetric MAPE	199.3801



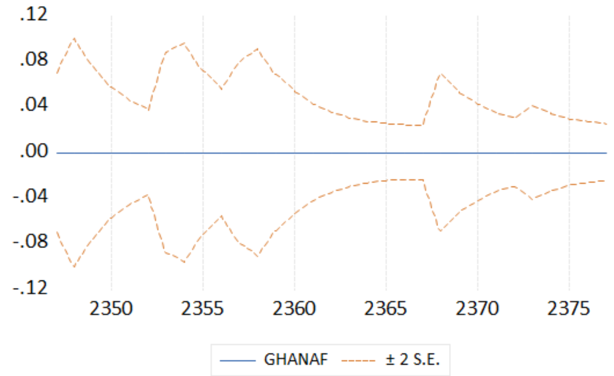
ii. GARCH-in-Mean



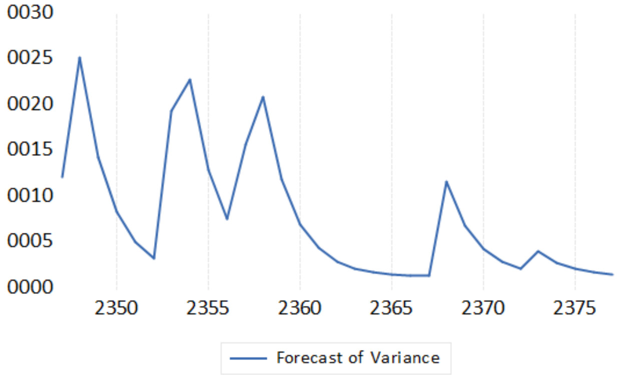
Forecast: GHANAF	
Actual: GHANA	
Forecast sample: 2347 2377	
Included observations: 31	
Root Mean Squared Error	0.031026
Mean Absolute Error	0.014316
Mean Abs. Percent Error	NA
Theil Inequality Coef.	0.999393
Bias Proportion	0.025860
Variance Proportion	0.972360
Covariance Proportion	0.001781
Theil U2 Coefficient	NA
Symmetric MAPE	199.9978



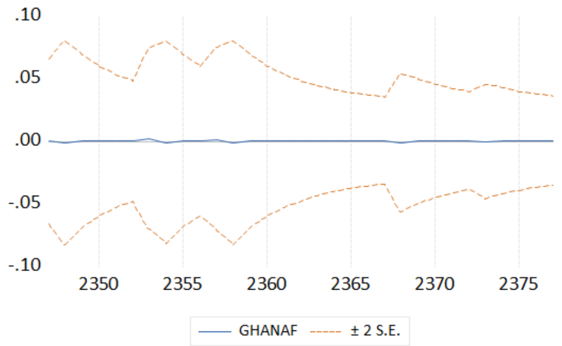
iii. TGARCH



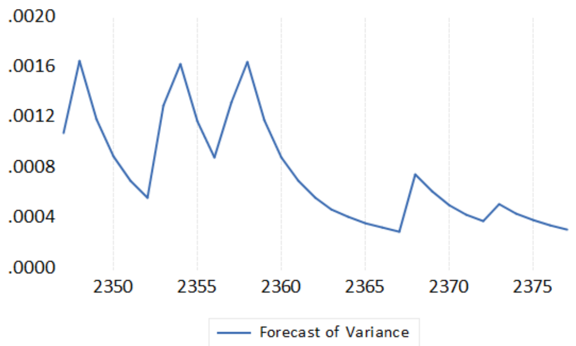
Forecast: GHANAF	
Actual: GHANA	
Forecast sample: 2347 2377	
Included observations: 31	
Root Mean Squared Error	0.031018
Mean Absolute Error	0.014300
Mean Abs. Percent Error	NA
Theil Inequality Coef.	1.000000
Bias Proportion	0.025701
Variance Proportion	0.974299
Covariance Proportion	0.000000
Theil U2 Coefficient	NA
Symmetric MAPE	200.0000



iv. EGARCH

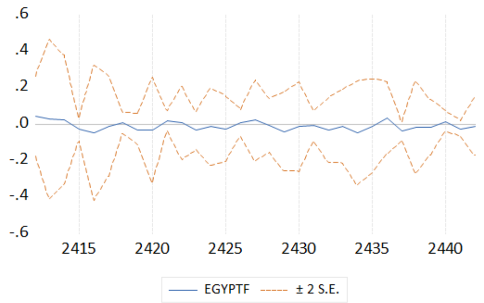


Forecast: GHANAF	
Actual: GHANA	
Forecast sample: 2347 2377	
Included observations: 31	
Root Mean Squared Error	0.031279
Mean Absolute Error	0.014754
Mean Abs. Percent Error	NA
Theil Inequality Coef.	0.986421
Bias Proportion	0.026718
Variance Proportion	0.916119
Covariance Proportion	0.057163
Theil U2 Coefficient	NA
Symmetric MAPE	199.9099

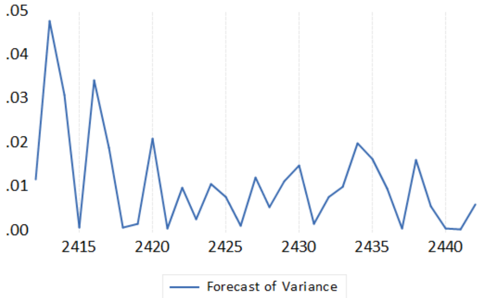


d. Forecasts using static modelling: Egypt

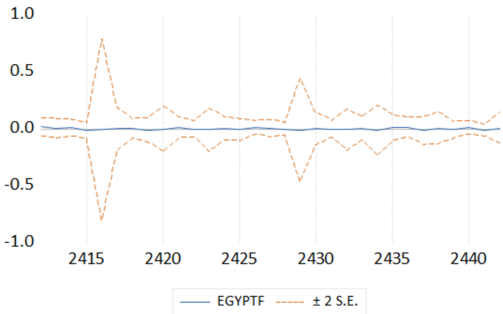
i. GARCH (1,1)



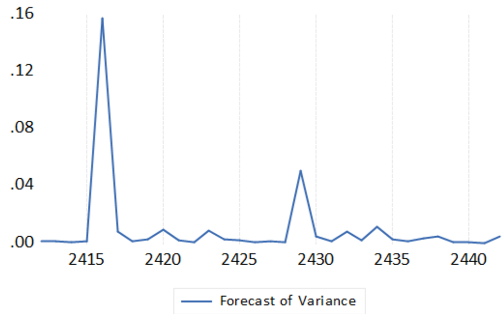
Forecast: EGYPTF	
Actual: EGYPT	
Forecast sample: 2412 2442	
Included observations: 31	
Root Mean Squared Error	0.037668
Mean Absolute Error	0.031801
Mean Abs. Percent Error	NA
Theil Inequality Coef.	0.758643
Bias Proportion	0.103714
Variance Proportion	0.000450
Covariance Proportion	0.895836
Theil U2 Coefficient	NA
Symmetric MAPE	150.6894



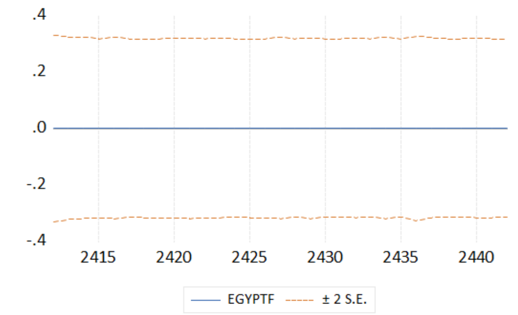
ii. EGARCH



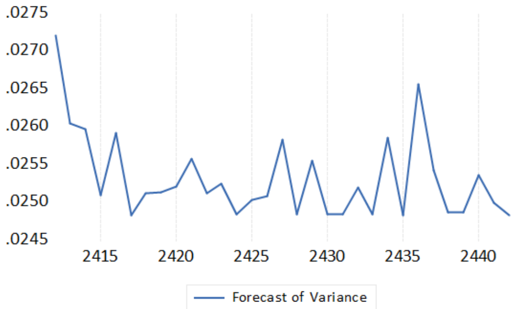
Forecast: EGYPTF	
Actual: EGYPT	
Forecast sample: 2412 2442	
Included observations: 31	
Root Mean Squared Error	0.027028
Mean Absolute Error	0.022077
Mean Abs. Percent Error	NA
Theil Inequality Coef.	0.824990
Bias Proportion	0.002687
Variance Proportion	0.264798
Covariance Proportion	0.732515
Theil U2 Coefficient	NA
Symmetric MAPE	161.1432



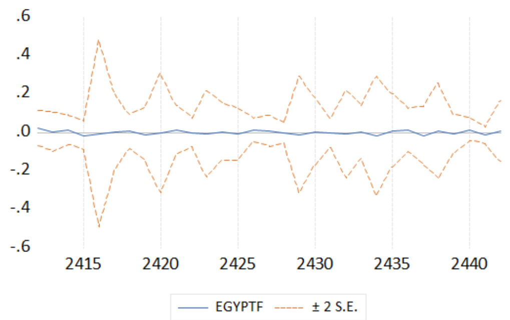
iii. GARCH-in-Mean



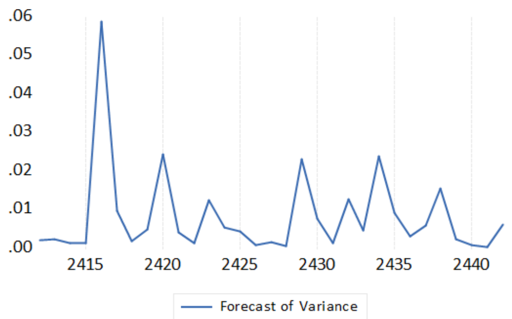
Forecast: EGYPTF	
Actual: EGYPT	
Forecast sample: 2412 2442	
Included observations: 31	
Root Mean Squared Error	0.023319
Mean Absolute Error	0.019100
Mean Abs. Percent Error	NA
Theil Inequality Coef.	0.964511
Bias Proportion	0.001238
Variance Proportion	0.998663
Covariance Proportion	0.000099
Theil U2 Coefficient	NA
Symmetric MAPE	177.5756



iv. TGARCH



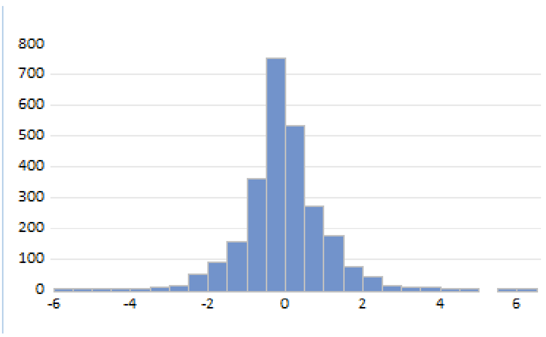
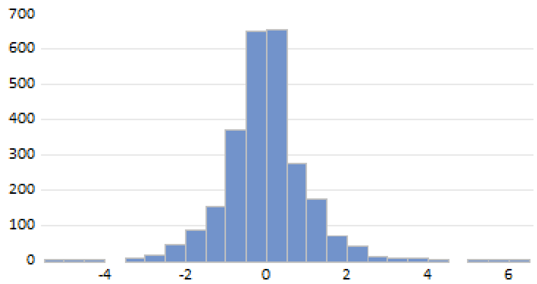
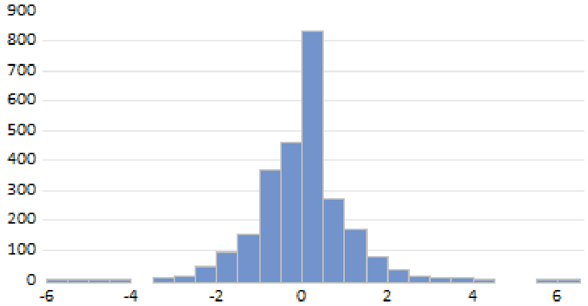
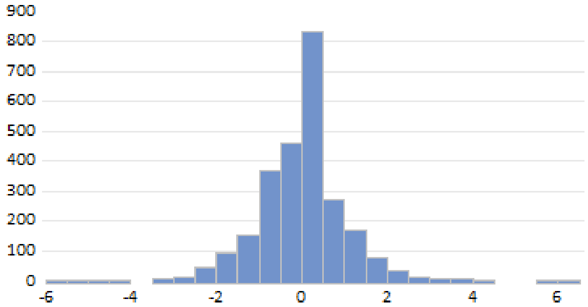
Forecast: EGYPTF	
Actual: EGYPT	
Forecast sample: 2412 2442	
Included observations: 31	
Root Mean Squared Error	0.027699
Mean Absolute Error	0.022624
Mean Abs. Percent Error	NA
Theil Inequality Coef.	0.816262
Bias Proportion	0.002502
Variance Proportion	0.211433
Covariance Proportion	0.786064
Theil U2 Coefficient	NA
Symmetric MAPE	158.9923



15 APPENDIX 7: Descriptive statistics for the error term

a. South Africa

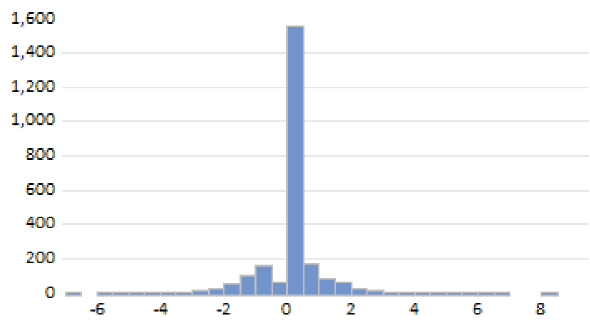
Model	GARCH (1,1)	GARCH IN MEAN	EGARCH	TGARCH
Mean	-0.025618	-0.024603	-0.024576	-0.024024
Median	-0,000294	-6.87e-05	8.99e05	2.22e-05
Maximum	6.471199	6.353857	55.829941	6.401422
Minimum	-5.615690	-5.492868	-5.606189	-5.582855
Std. Dev	1.012038	0.989581	0.995076	1.003575
Skewness	0.061928	0.063470	0.026529	0.068865
Kurtosis	6.912999	6.936201	6.487507	6.845057
Jaque-Bera	1642.533	1662.779	1303.738	1587.051
Probability	0.000000	0.000000	0.000000	0.000000
Observations	2572	2572	2573	2573

GARCH (1,1)	GARCH IN MEAN
	
EGARCH	TGARCH
	

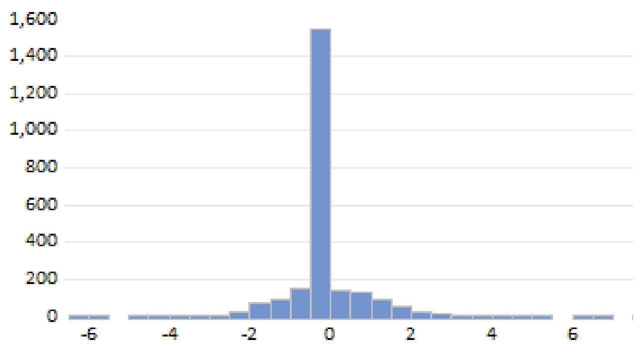
b. Ghana

Model	GARCH (1,1)	GARCH IN MEAN	EGARCH	TGARCH
Mean	0.007087	-0.008779	0.0022369	-0.013236
Median	0.022338	-6.10e-05	7.96e-13	-5.94e-10
Maximum	9.938668	7.908163	9.524101	13.61582
Minimum	-6.903644	-6.121448	-5.738767	-9.077227
Std. Dev	1.000353	0.938097	0.888823	1.266515
Skewness	0.520547	0.566150	1.486659	0.615367
Kurtosis	16.91984	15.47489	27.07155	20.47406
Jaque-Bera	19289.74	15540.13	58239.73	30391.66
Probability	0.000000	0.000000	0.000000	0.000000
Observations	2376	2377	2376	2377

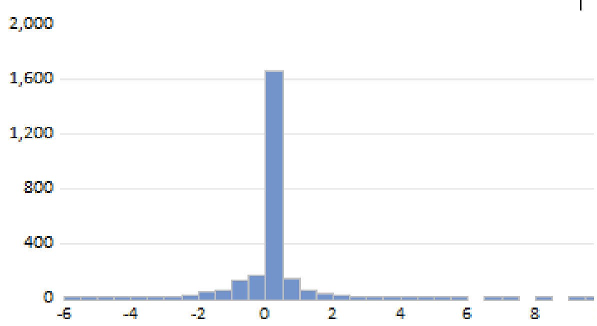
GARCH (1,1)



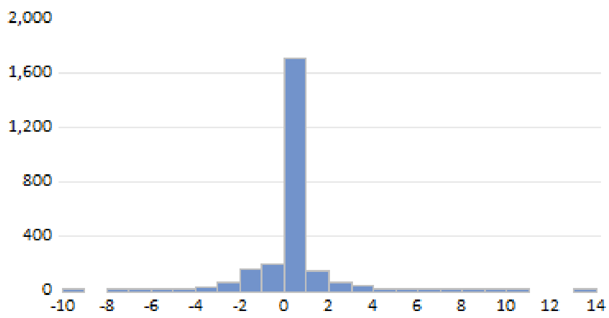
GARCH IN MEAN



EGARCH



TGARCH

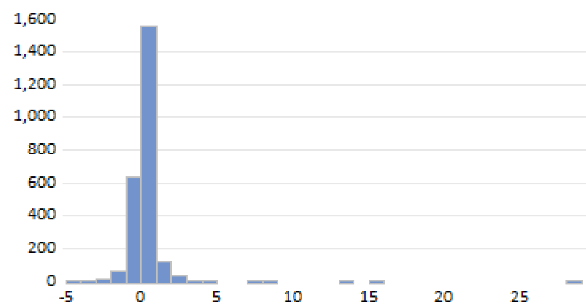


c. Egypt

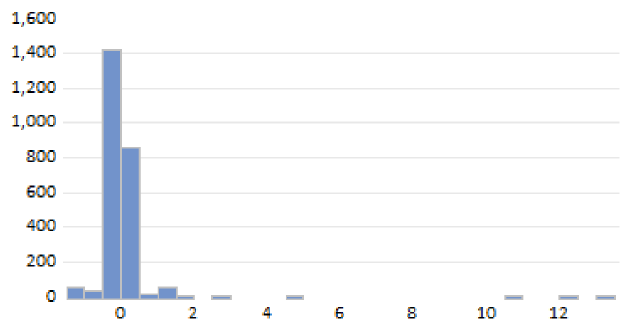
Model	GARCH (1,1)	GARCH IN MEAN	EGARCH	TGARCH
Mean	0.213517	0.012572	0.039480	0.032577
Median	0.231222	-0.005177	0.004343	0.007976
Maximum	28.86782	13.28125	18.90045	19.91096
Minimum	-4.709627	-1.160929	-5.722189	-4.904380
Std. Dev	0.980255	0.529935	1.000325	1.001174

Skewness	12.93865	15.13956	6.370534	7.457878
Kurtosis	338.5372	347.8349	112.4998	135.8495
Jaque-Bera	11518959	12192490	126015	1817678
Probability	0.000000	0.000000	0.000000	0.000000
Observations	2441	2442	2441	2441

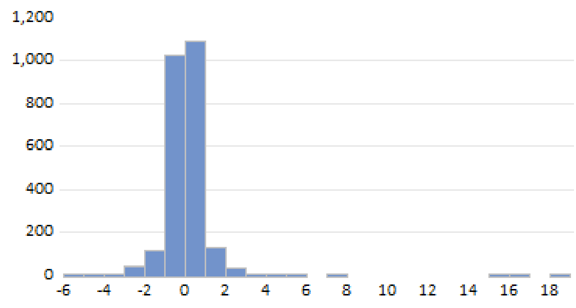
GARCH (1,1)



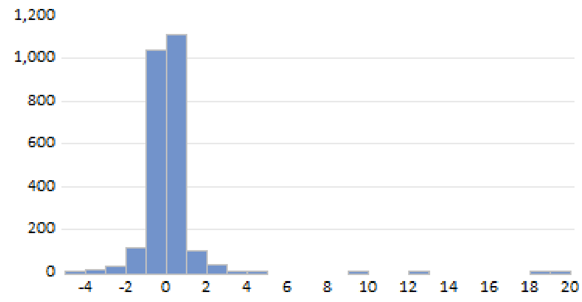
GARCH IN MEAN



EGARCH



TGARCH



16 APPENDIX 8: Long run Volatility models

a. FIEGARCH Output

i. South Africa

Dependent Variable: SA
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 04/09/22 Time: 00:36
Sample: 1 2573
Included observations: 2573
Convergence achieved after 21 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0)
+ C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000228	0.000293	-0.777995	0.4366
		Variance Equation		
OMEGA	-7.651425	0.124591	-61.41246	0.0000
ALPHA	-0.476656	0.081940	-5.817170	0.0000
BETA	0.851798	0.032943	25.85692	0.0000
THETA1	0.315854	0.029411	10.73928	0.0000
THETA2	0.024708	0.017506	1.411367	0.1581
D	-1.163099	0.279272	-4.164753	0.0000

ii. Ghana

Dependent Variable: GHANA
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 04/09/22 Time: 00:37
Sample: 1 2377
Included observations: 2377
Convergence achieved after 44 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.001608	0.000366	-4.399096	0.0000
		Variance Equation		
OMEGA	-5.977969	0.091884	-65.05977	0.0000
ALPHA	0.986058	0.071247	13.84006	0.0000
BETA	-0.449104	0.068101	-6.594692	0.0000
THETA1	0.347093	0.018309	18.95734	0.0000
THETA2	0.049148	0.010039	4.895741	0.0000
D	-0.497414	0.049067	-10.13753	0.0000

iii. Egypt

Dependent Variable: EGYPT
Method: ML ARCH - Student's t distribution (Marquardt / EViews legacy)
Date: 04/11/22 Time: 07:40
Sample (adjusted): 2 2442
Included observations: 2441 after adjustments
Failure to improve Likelihood after 11 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000267	0.000231	1.158723	0.2466
EGYPT(-1)	-0.999668	0.001025	-975.2190	0.0000
		Variance Equation		
OMEGA	-6.412219	0.029049	-220.7361	0.0000
ALPHA	-1.008334	0.002010	-501.7229	0.0000
BETA	0.985105	0.003368	292.4823	0.0000
THETA1	0.950257	0.050281	18.89881	0.0000
THETA2	-0.462182	0.047523	-9.725359	0.0000
D	-0.719029	0.104207	-6.899974	0.0000

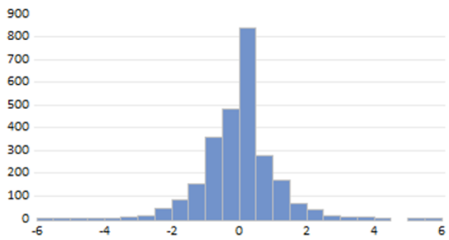
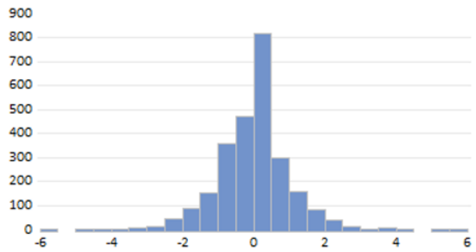
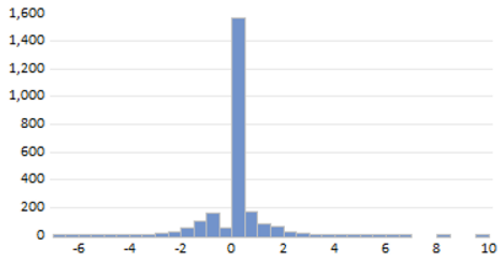
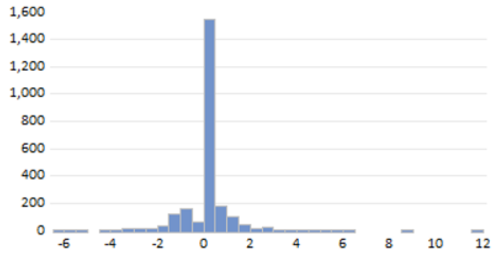
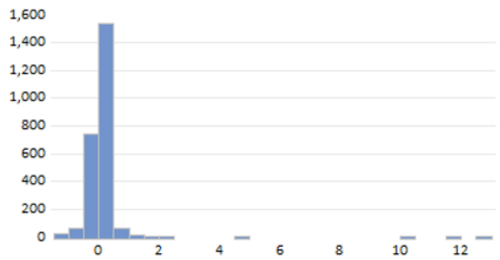
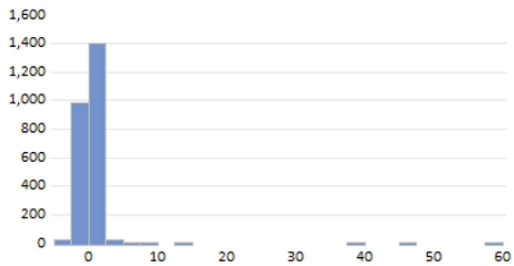
b. FIGARCH Output

i. Ghana

Dependent Variable: GHANA
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 04/08/22 Time: 20:15
Sample: 1 2377
Included observations: 2377
Convergence achieved after 35 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000391	0.000404	-0.967235	0.3334
		Variance Equation		
C(2)	7.51E-05	4.42E-06	17.01126	0.0000
RESID(-1)^2	0.867170	0.009100	95.28824	0.0000
GARCH(-1)	0.689693	0.012841	53.71119	0.0000
D	0.007212	0.009543	0.755718	0.4498

c. Residuals Normality Tests Results- FIGARCH and FIEGARCH

	FIGARCH	FIEGARCH
SOUTH AFRICA		
GHANA		
EGYPT		

d. Serial correlation test of squared residuals

1. South Africa

Sample: 1 2573
Included observations: 2573

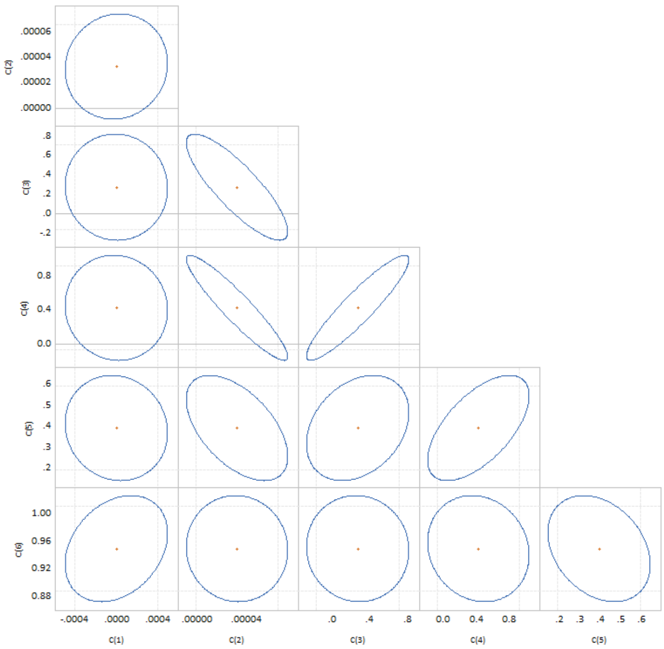
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.024	-0.024	1.4685	0.226
		2	-0.004	-0.005	1.5100	0.470
		3	-0.027	-0.027	3.3682	0.338
		4	-0.011	-0.013	3.6986	0.448
		5	-0.022	-0.023	5.0019	0.416
		6	-0.000	-0.002	5.0019	0.544
		7	-0.014	-0.015	5.4755	0.602
		8	0.012	0.010	5.8620	0.663
		9	0.001	0.001	5.8658	0.753
		10	0.001	0.000	5.8716	0.826
		11	0.022	0.022	7.0959	0.791
		12	-0.003	-0.002	7.1191	0.850
		13	-0.005	-0.004	7.1829	0.892
		14	-0.019	-0.018	8.1030	0.884
		15	-0.014	-0.014	8.6259	0.896
		16	0.010	0.010	8.8991	0.918
		17	-0.014	-0.015	9.3984	0.927
		18	-0.012	-0.014	9.8028	0.938
		19	-0.016	-0.018	10.465	0.941
		20	0.021	0.018	11.573	0.930
		21	-0.028	-0.028	13.552	0.888
		22	-0.010	-0.013	13.801	0.908
		23	0.005	0.005	13.868	0.931
		24	-0.013	-0.015	14.302	0.940
		25	-0.005	-0.006	14.375	0.955
		26	-0.012	-0.013	14.720	0.962
		27	-0.018	-0.019	15.519	0.962
		28	-0.020	-0.022	16.532	0.957
		29	0.015	0.013	17.122	0.960
		30	-0.011	-0.011	17.411	0.967
		31	-0.009	-0.014	17.634	0.974
		32	0.021	0.020	18.823	0.969
		33	0.010	0.009	19.094	0.974
		34	-0.002	-0.002	19.110	0.981
		35	0.025	0.026	20.799	0.973
		36	0.013	0.014	21.250	0.976

ii. Ghana

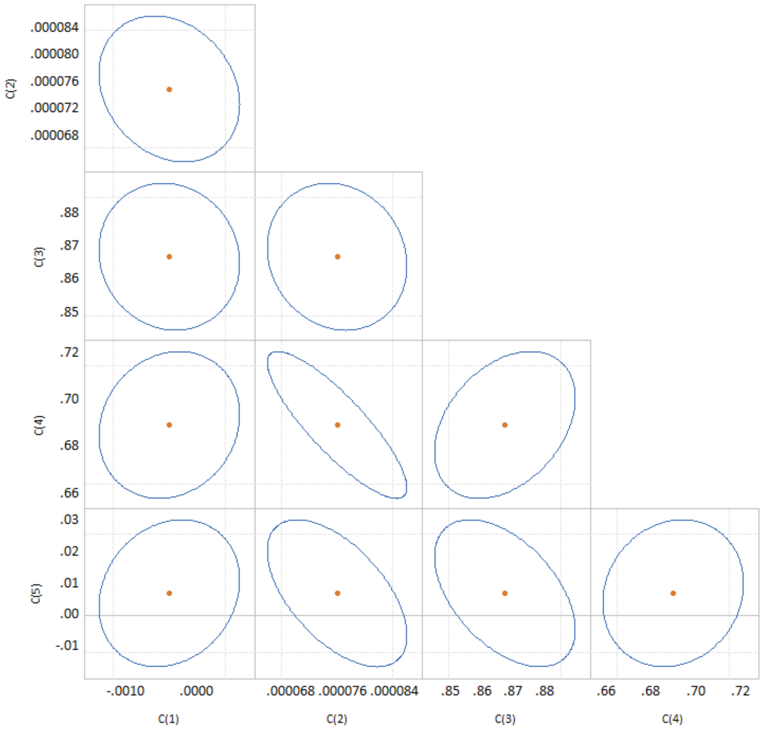
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.006	0.006	0.0831	0.773
		2	0.013	0.013	0.5147	0.773
		3	-0.018	-0.018	1.3067	0.728
		4	-0.005	-0.005	1.3622	0.851
		5	-0.003	-0.002	1.3793	0.927
		6	-0.000	-0.001	1.3797	0.967
		7	0.001	0.001	1.3823	0.986
		8	-0.009	-0.009	1.5587	0.992
		9	-0.009	-0.009	1.7650	0.995
		10	-0.007	-0.007	1.8954	0.997
		11	-0.011	-0.010	2.1592	0.998
		12	0.015	0.015	2.6753	0.997
		13	-0.002	-0.002	2.6841	0.999
		14	-0.000	-0.001	2.6842	1.000
		15	0.019	0.020	3.5605	0.999
		16	0.007	0.007	3.6909	0.999
		17	-0.017	-0.018	4.3894	0.999
		18	0.004	0.005	4.4356	1.000
		19	-0.005	-0.005	4.4968	1.000
		20	0.014	0.014	4.9814	1.000
		21	-0.001	-0.001	4.9839	1.000
		22	-0.008	-0.009	5.1362	1.000
		23	-0.002	-0.001	5.1485	1.000
		24	-0.002	-0.002	5.1606	1.000
		25	0.000	0.000	5.1611	1.000
		26	-0.020	-0.020	6.1128	1.000
		27	-0.002	-0.003	6.1250	1.000
		28	-0.003	-0.002	6.1430	1.000
		29	-0.008	-0.008	6.3043	1.000
		30	0.010	0.010	6.5666	1.000
		31	-0.020	-0.021	7.5624	1.000
		32	-0.015	-0.015	8.0982	1.000
		33	0.078	0.079	22.673	0.912
		34	-0.003	-0.004	22.689	0.930
		35	0.047	0.044	28.094	0.790
		36	0.008	0.009	28.244	0.818

e. Parameter stability tests- Confidence ellipse

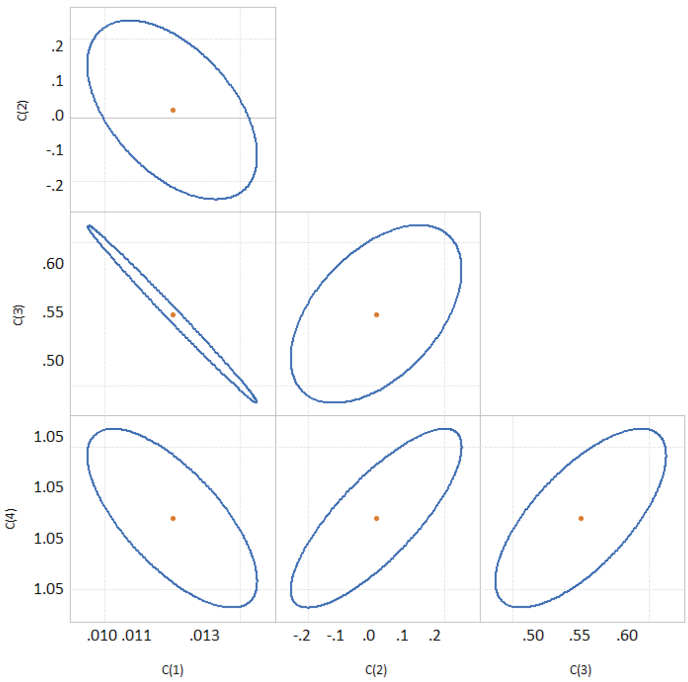
i. South Africa-FIGARCH



ii. Ghana- FIGARCH

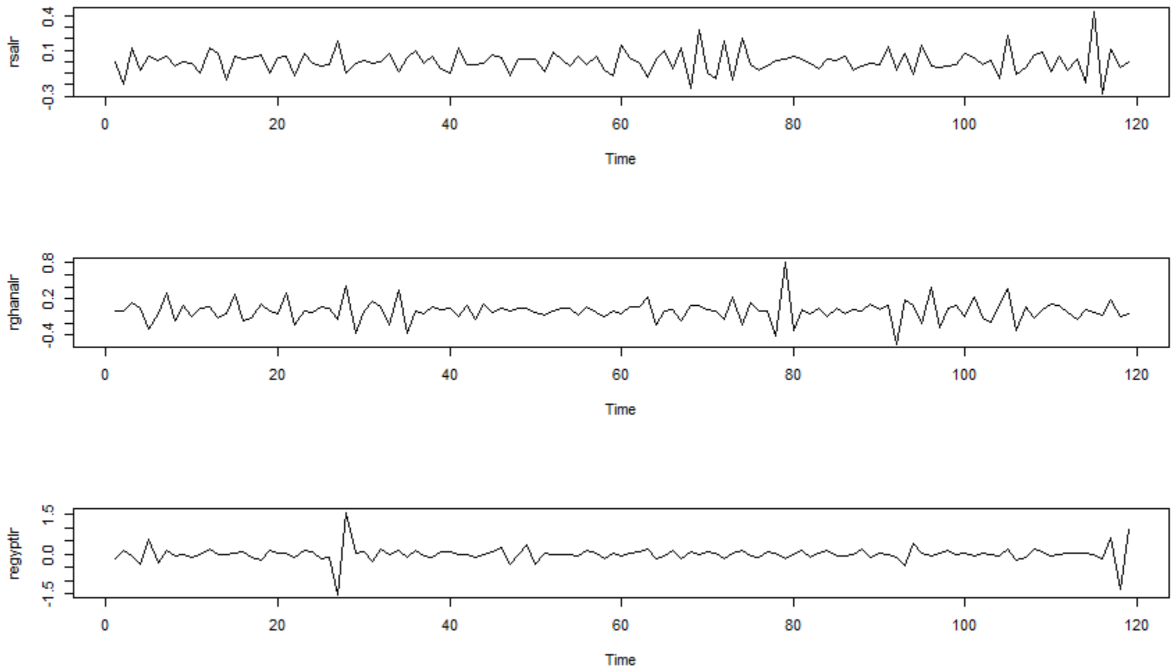


iii. Egypt- FIGARCH

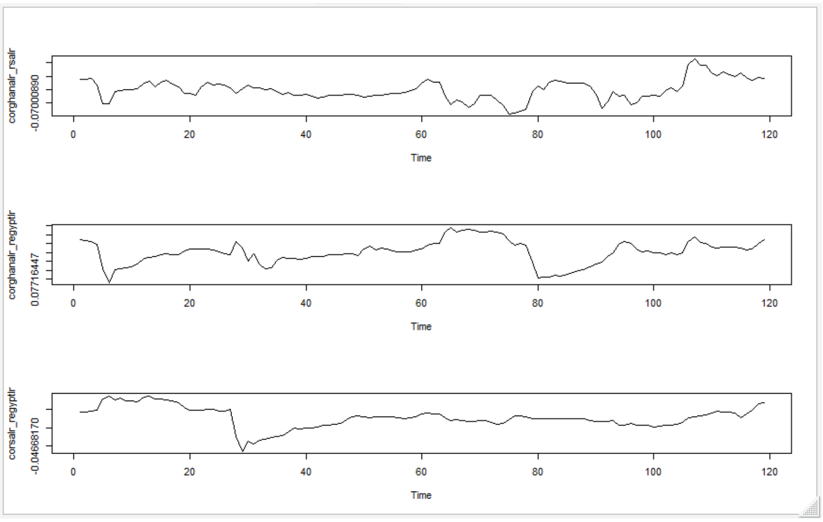


17 Appendix 9: DCC Model outputs from R statistical software

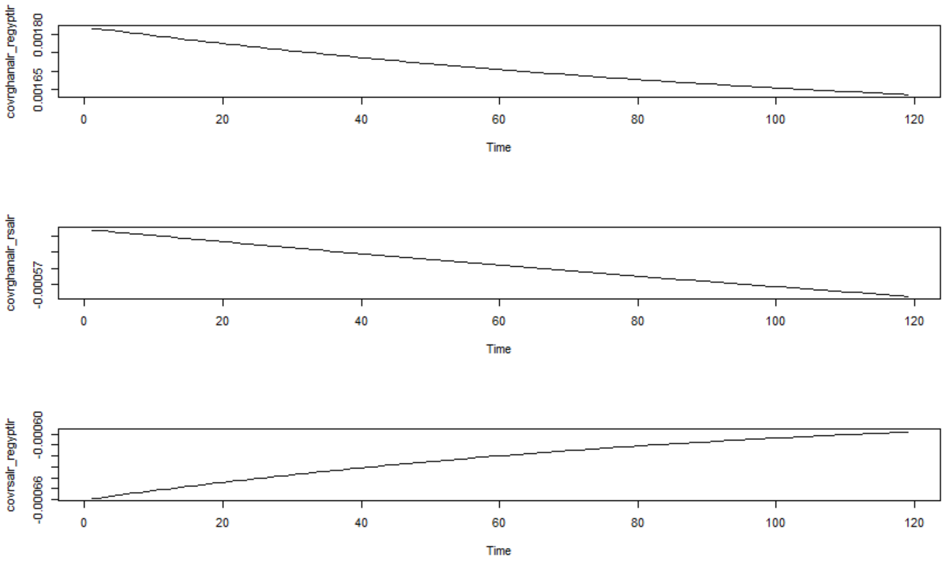
a. Time series plots for the LPE



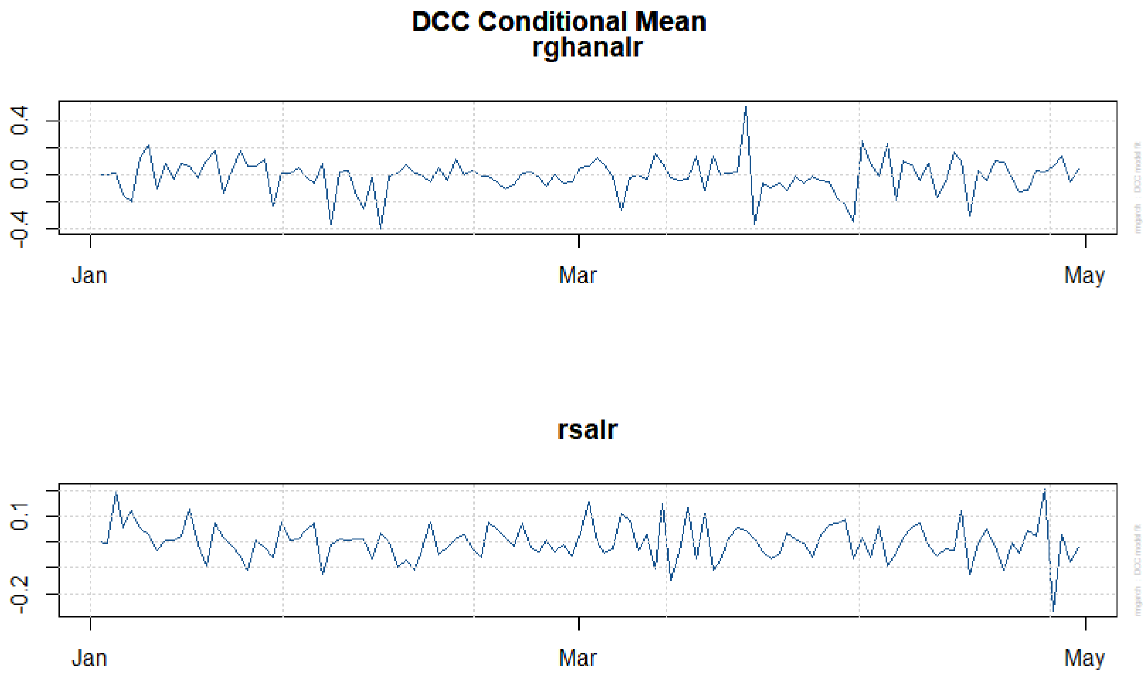
b. Correlation output



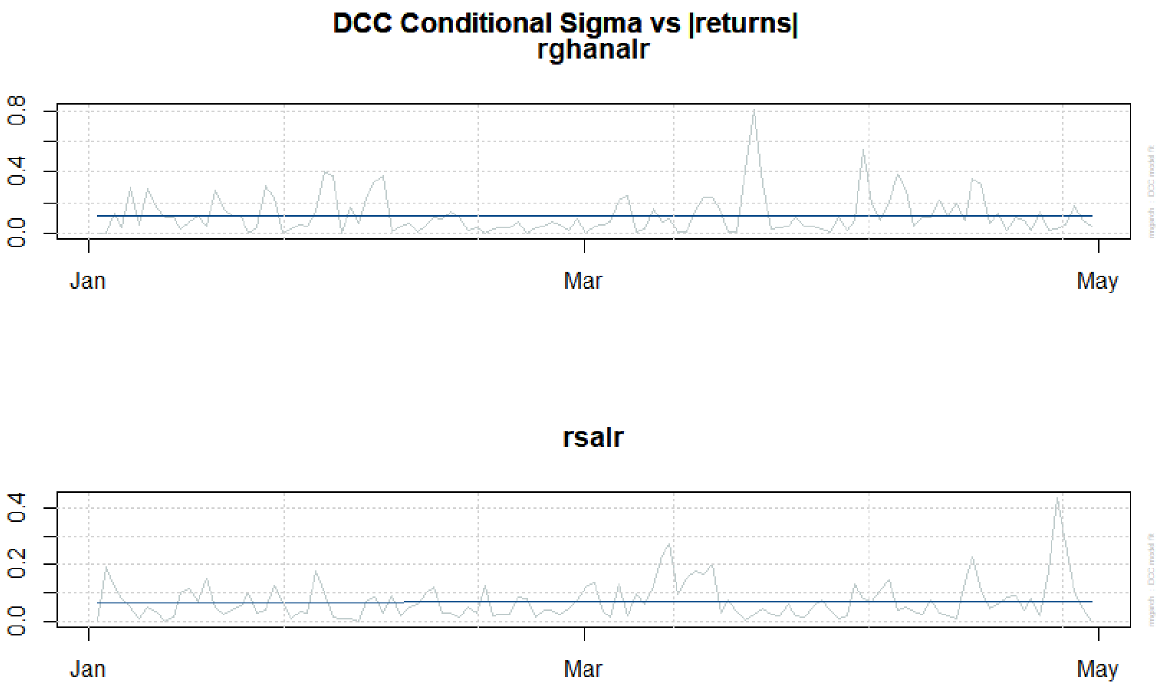
c. Covariances



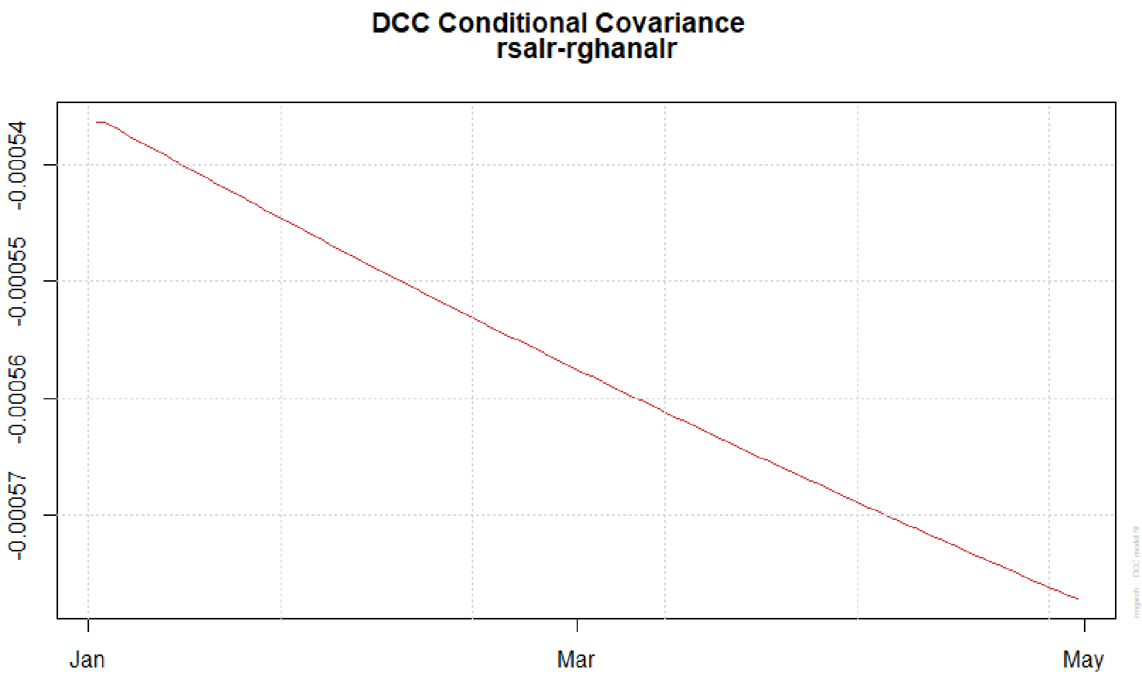
d. Conditional Mean



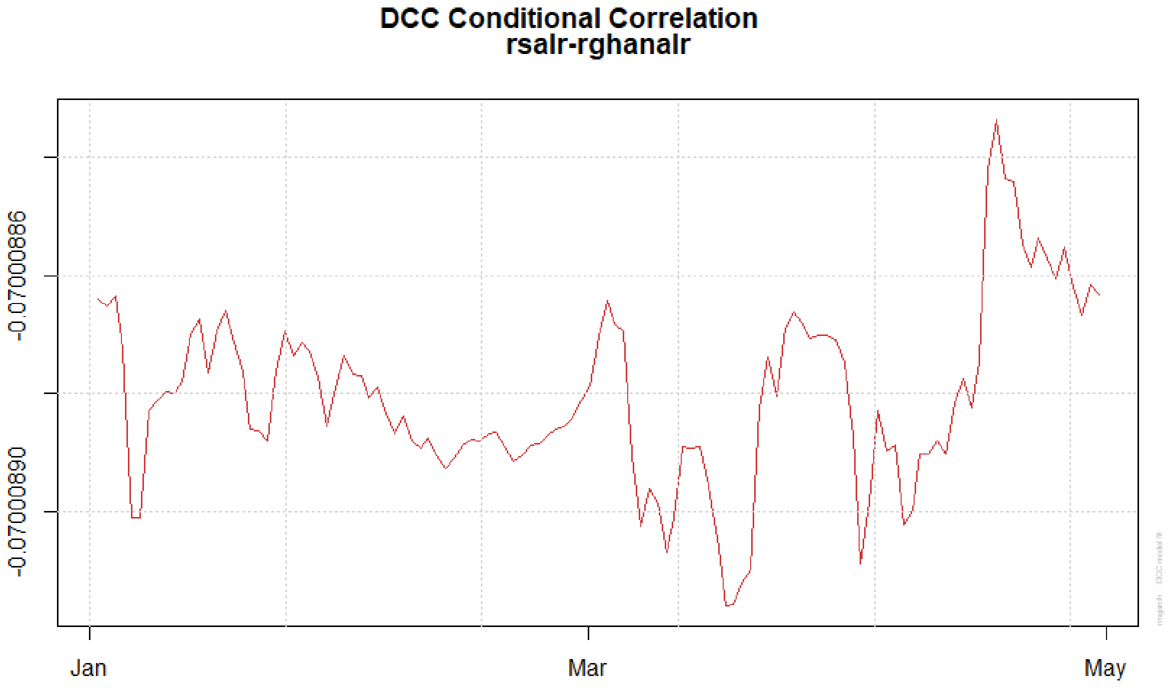
e. Conditional Sigma (vs realised absolute returns)



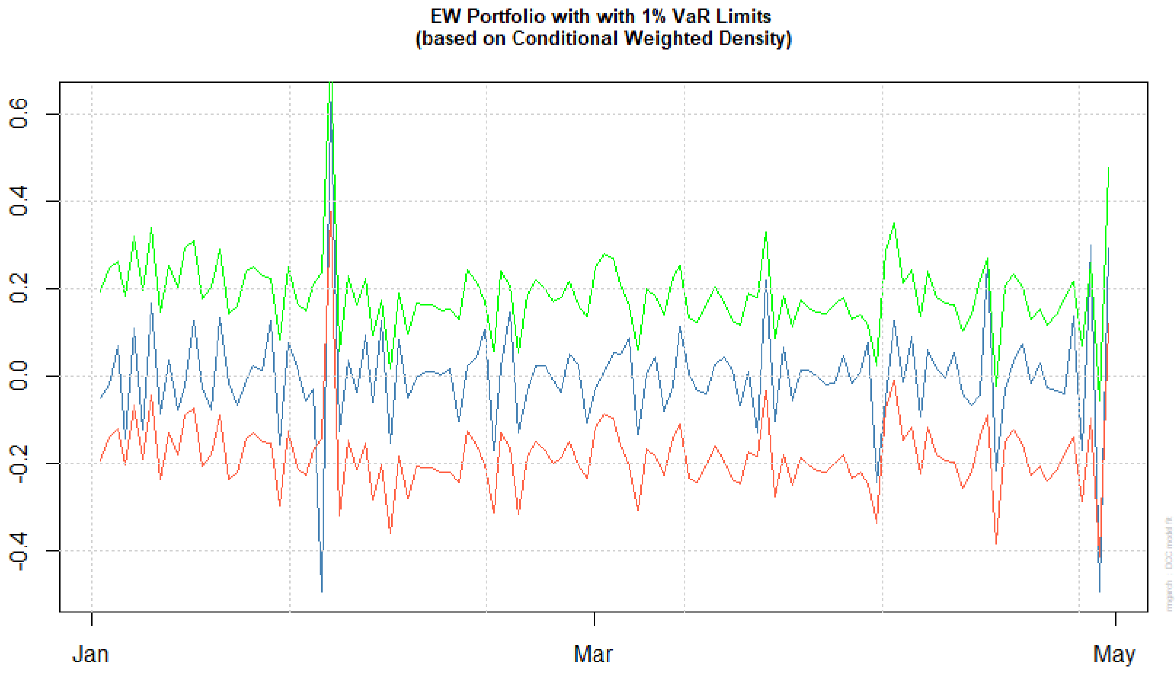
f. Conditional covariance



g. Conditional correlation



h. EW Portfolio Plot with conditional density VaR limits



i. Model code (R software)

```
library(readr)
equity <- read_csv("PrivateEquity.csv")
View(equity)

#Installing necessary packages
install.packages("tseries")
install.packages("rugarch")
install.packages("rmgarch")

#Loading installed packages

library(tseries)
library(rugarch)
library(rmgarch)

#Checking stationerity of variables with Augmented Dickey-Fuller Test

adf.test(equity$salr)
adf.test(equity$ghanalr)
adf.test(equity$egyptlr)

#Stationerity check with Box-Ljung test
Box.test(equity$salr, lag=12, type="Ljung-Box")
Box.test(equity$ghanalr, lag=12, type="Ljung-Box")
Box.test(equity$egyptlr, lag=12, type="Ljung-Box")

#time series plot

ts.plot(equity$salr)
ts.plot(equity$ghanalr)
ts.plot(equity$egyptlr)

#making data stationery
rsalr=diff(equity$salr)
rghanalr=diff(equity$ghanalr)
regyptlr=diff(equity$egyptlr)


ts.plot(rsalr)
ts.plot(rghanalr)
ts.plot(regyptlr)

#Step 1: Creating the univariate normal Garch for each series

model1 = ugarchspec(mean.model = list(armaOrder = c(0,0)),
                          variance.model = list(model = "sGARCH"),
                          distribution.model =  'norm' )

#dcc specification - GARCH(1,1) for conditional correlations
modelspec = dccspec(uspec = multispec( replicate(2, model1)),
                    dccOrder=c(1,1),
                    distribution="mvnorm")
modelspec

#Estimate the DCC model
```

```
modelfit= dccfit(modelspec, data = data.frame(rghanalr,rsalr))
modelfit

#Estimation of the correlation
correlation=rcor(modelfit)
dim(correlation)

correlation[, ,dim(correlation)[3]]


#DCC Graphs

plot(modelfit) 2
```

18 APPENDIX 10: VAR MODELLING (EVIEWS)

a. Egypt

i. VAR Estimates

Vector Autoregression Estimates
Date: 09/25/22 Time: 04:37
Sample (adjusted): 2007 2019
Included observations: 13 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	EGPTRTN	EGPTGDP	EGPTINFL
EGPTRTN(-1)	0.197834	0.001375	0.031941
	(0.31776)	(0.01915)	(0.06716)
	[0.62260]	[0.07182]	[0.47559]
EGPTGDP(-1)	1.659502	-0.487091	1.059970
	(5.19394)	(0.31297)	(1.09780)
	[0.31951]	[-1.55636]	[0.96554]
EGPTINFL(-1)	0.821920	-0.142553	0.780413
	(1.14111)	(0.06876)	(0.24119)
	[0.72028]	[-2.07322]	[3.23573]
C	5.905267	1.309538	-0.000312
	(8.04202)	(0.48458)	(1.69977)
	[0.73430]	[2.70240]	[-0.00018]

ii. Residuals analysis- VAR LM Test

VAR Residual Serial Correlation LM Tests
Date: 09/25/22 Time: 04:39
Sample: 2006 2019
Included observations: 13

Null hypothesis: No serial correlation at lag h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	9.068632	9	0.4310	1.064276	(9, 9.9)	0.4588
2	6.263128	9	0.7133	0.655347	(9, 9.9)	0.7314

Null hypothesis: No serial correlation at lags 1 to h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	9.068632	9	0.4310	1.064276	(9, 9.9)	0.4588
2	13.26606	18	0.7755	0.410581	(18, 3.3)	0.9065

*Edgeworth expansion corrected likelihood ratio statistic.

iii. VAR Residual Heteroskadasticity test

VAR Residual Heteroskedasticity Tests (Levels and Squares)
Date: 09/25/22 Time: 04:41
Sample: 2006 2019
Included observations: 13

Joint test:

Chi-sq	df	Prob.
39.60496	36	0.3123

Individual components:

Dependent	R-squared	F(6,6)	Prob.	Chi-sq(6)	Prob.
res1*res1	0.192062	0.237719	0.9480	2.496804	0.8688
res2*res2	0.771927	3.384555	0.0818	10.03505	0.1232
res3*res3	0.724066	2.624057	0.1327	9.412860	0.1517
res2*res1	0.237736	0.311881	0.9090	3.090564	0.7974
res3*res1	0.434442	0.768166	0.6215	5.647751	0.4638
res3*res2	0.716770	2.530704	0.1416	9.318015	0.1565

iv. Variance decomposition

Variance Decomposition of EGPTRTN:

Period	S.E.	EGPTRTN	EGPTGDP	EGPTINFL
1	6.290203	100.0000	0.000000	0.000000
2	6.501906	96.88756	0.402157	2.710283
3	6.559769	95.67277	0.395287	3.931941

Variance Decomposition of EGPTGDP:

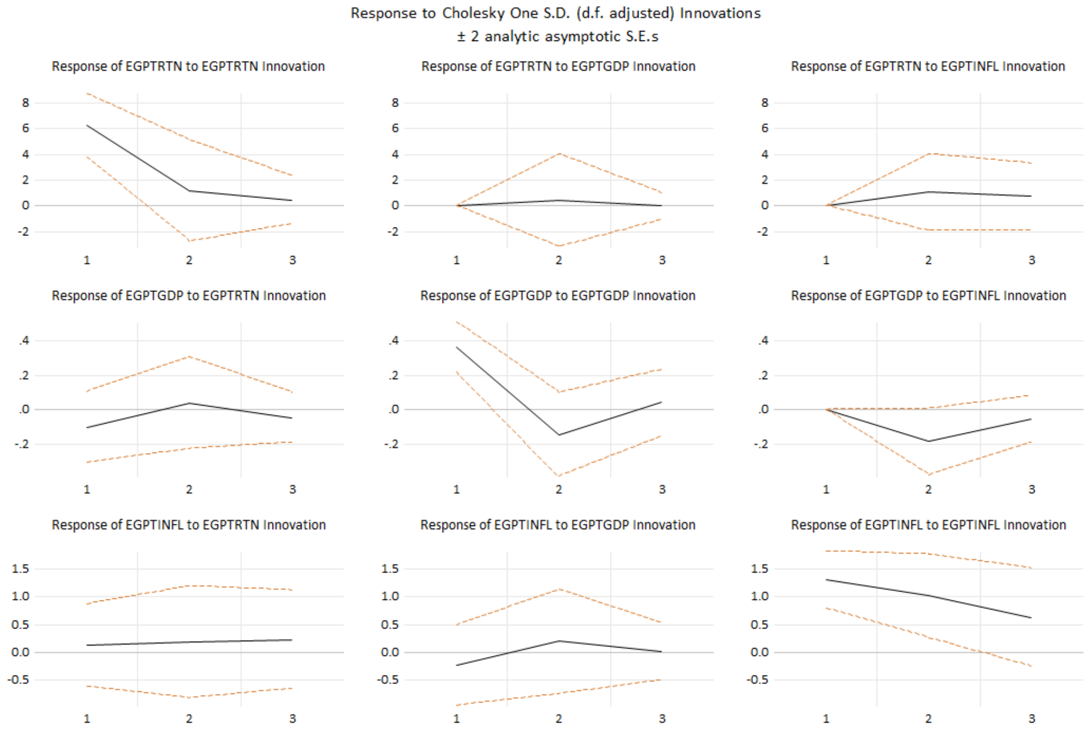
Period	S.E.	EGPTRTN	EGPTGDP	EGPTINFL
1	0.379025	7.164339	92.83566	0.000000
2	0.447825	5.931575	76.88246	17.18597
3	0.455146	6.731110	75.27617	17.99272

Variance Decomposition of EGPTINFL:

Period	S.E.	EGPTRTN	EGPTGDP	EGPTINFL
1	1.329503	0.904380	3.142717	95.95290
2	1.696674	1.836578	3.363467	94.79995
3	1.824716	3.176831	2.918587	93.90458

Cholesky One S.D. (d.f. adjusted)
Cholesky ordering: EGPTRTN EGPTGDP EGPTINFL

v. Impulse Response functions



b. GHANA VAR Modelling

i. Ghana VAR estimate output

Vector Autoregression Estimates
Date: 09/25/22 Time: 03:55
Sample (adjusted): 2007 2019
Included observations: 13 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	GNARTN	GNAGDP	GNAINFL
GNARTN(-1)	0.402942	-0.019667	-0.155532
	(0.35024)	(0.02173)	(0.27508)
	[1.15048]	[-0.90511]	[-0.56541]
GNAGDP(-1)	4.259216	-0.023193	-5.773632
	(4.64082)	(0.28792)	(3.64492)
	[0.91777]	[-0.08056]	[-1.58402]
GNAINFL(-1)	-0.404701	0.022171	0.320894
	(0.43204)	(0.02680)	(0.33933)
	[-0.93671]	[0.82714]	[0.94567]
C	9.015710	0.350884	7.852671
	(6.72023)	(0.41693)	(5.27809)
	[1.34158]	[0.84159]	[1.48779]

ii. Residuals LM TESTS

VAR Residual Serial Correlation LM Tests
Date: 09/25/22 Time: 03:58
Sample: 2006 2019
Included observations: 13

Null hypothesis: No serial correlation at lag h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	6.657367	9	0.6727	0.707768	(9, 9.9)	0.6928

Null hypothesis: No serial correlation at lags 1 to h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	6.657367	9	0.6727	0.707768	(9, 9.9)	0.6928

*Edgeworth expansion corrected likelihood ratio statistic.

iii. Heteroskedasticity Test

VAR Residual Heteroskedasticity Tests (Levels and Squares)
Date: 09/25/22 Time: 04:01
Sample: 2006 2019
Included observations: 13

Joint test:

Chi-sq	df	Prob.
37.07734	36	0.4191

Individual components:

Dependent	R-squared	F(6,6)	Prob.	Chi-sq(6)	Prob.
res1*res1	0.593580	1.460511	0.3286	7.716545	0.2596
res2*res2	0.562381	1.285092	0.3842	7.310951	0.2930
res3*res3	0.562348	1.284918	0.3843	7.310518	0.2931
res2*res1	0.413356	0.704611	0.6592	5.373627	0.4969
res3*res1	0.775644	3.457197	0.0783	10.08337	0.1212
res3*res2	0.145325	0.170035	0.9756	1.889225	0.9296

iv. VAR Decomposition

Variance Decomposition of GNARTN:				
Period	S.E.	GNARTN	GNAGDP	GNAINFL
1	3.932733	100.0000	0.000000	0.000000
2	4.740054	90.25453	3.762697	5.982776
3	4.945206	86.40464	6.776232	6.819133
Variance Decomposition of GNAGDP:				
Period	S.E.	GNARTN	GNAGDP	GNAINFL
1	0.243990	2.207069	97.79293	0.000000
2	0.272384	16.09540	78.46702	5.437579
3	0.287455	19.87557	73.13655	6.987878
Variance Decomposition of GNAINFL:				
Period	S.E.	GNARTN	GNAGDP	GNAINFL
1	3.088785	13.22559	0.749396	86.02502
2	3.672928	19.69934	13.19793	67.10273
3	3.719732	19.31979	15.17028	65.50993
Cholesky One S.D. (d.f. adjusted)				
Cholesky ordering: GNARTN GNAGDP GNAINFL				