GRADE 10 LEARNERS' ACADEMIC EXPERIENCES OF LEARNING PARABOLIC FUNCTIONS IN SCHOOLS OF VHEMBE DISTRICT OF LIMPOPO PROVINCE

by

Takalani Lesley Mudau

a dissertation submitted in fulfilment of

Masters degree in Mathematics Education

at the

UNIVERSITY OF SOUTH AFRICA

SUPERVISOR: DR. TŠHEGOFATŠO MAKGAKGA

DECLARATION

Name: Mudau Takalani Lesley

Student number: 64088863

Degree: Masters in Mathematics Education

Grade 10 learners' academic experience of learning parabola functions in schools of Vhembe district of Limpopo province.

I confirmed that the dissertation, submitted for the qualification of Masters in education in Mathematics Education at the University of South Africa has not previously been submitted to this or any other university. I further declare that it is my own work, and that all the sources that I used have been recognised by complete references.

Signature: Date: 14 November 2022

DEDICATION

This dissertation is dedicated to my children, Odaho, Urangani and Wavhutali, you have brought love, peace and happiness to my family and my life. May God be with you for the rest of your lives. Without your presence in my life, I would not be able to cope.

Not forgetting my late mother and sister, Nyawasedza Mudau and Joyce Mudau who passed on while I am busy with my studies. Thank you for the support you have shown me. May your souls rest in eternal peace. My guardian wife, Mrs Lydia Mudau for being supportive, motivating and understanding throughout my journey.

May God bless you all.

ACKNOWLEDGEMENTS

Firstly, I would like to thank God for giving me the wisdom and strength to complete this study. The completion of this study has been a unique experience in which many people contributed greatly and in a unique way and are worth mentioning. My deepest appreciation and gratitude go to my supervisor, Dr Tšhegofatšo Makgakga, for his invaluable support, scholarly insight, encouragement, and guidance throughout this study.

A special word of thanks goes to the school principals, Mathematics teachers and learners who participated in this study.

I further acknowledge the support that I received from Prof Jabulani Nyoni who critiqued my work and my colleagues in Dr Makgakga's Microsoft Teams platform who were also there and contributed to the dialogues that have helped me shape my study.

I would also like to thank my ICT assistant Nemukula Mulivhuweni who assisted in the literature search and everything concerning technology.

ABSTRACT

The objective of this study was to determine Grade 10 learners' academic performance in learning parabola functions. Furthermore, the study sought to unearth errors that learners make when learning parabola functions and the possible causes of those errors. The study also examined how teachers support learners in addressing errors they experience when learning parabola functions. The zone of proximal development and scaffolding by Vygotsky and Newman's errors analysis theory of learning was used to frame the study. The study adopted the mixed-methods approach, and during the quantitative phase, a ten-item of achievement test was given to 90 learners to discover learners' academic performance. Three teachers were observed while teaching parabola functions to find out how to support learners when they are learning to solve parabola functions. During the qualitative phase which involved lesson observations and selected learners' semi-structured interviews learners' errors when learning parabola functions were further explored. Analysis of the learners' performance in the achievement test, learners' responses in the semi-structured interviews and lesson observations produced the study's findings. The findings of this study revealed that learners experienced challenges when they were learning parabola functions this is evidenced by their academic performance in parabola functions. The researcher recommends that further study be conducted on learners' performance and difficulties in understanding linear functions which form prior knowledge of understanding parabola functions.

Key terms: axis of symmetry; domain; error analysis; minimum points; maximum points; parabola functions; range; scaffolding; zone of proximal development

TABLE OF CONTENTS

ABBREVIATIONS AND ACRONYMS

ANA	Annual National Assessment
BR	Blank response
CAPS	Curriculum and Assessment Policy Statement
CD	Compact Disc
CR	Correct Response
DBE	Department of Basic Education
h_0	Null hypothesis
h_1	Alternative hypothesis
IPG	Interpreting parabola functions graphs
IR	Incorrect Response
InR	Incomplete Response
L1	Learner number one in numerical order
LI1,2,3	Learner interview number one in numerical sequence
OECD	Organization for Economic Cooperation and Development
PCK	Pedagogical Content Knowledge
PF1	Parabola Functions number 1
PF2	Parabola Functions number 2
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Education Quality
SBLOT1,2	School B Lesson Observation number 1 in numerical order
SPSS	Statistical Package for the Social Sciences
TIMSS	Trends in Mathematics and Science Study
QE	Quadratic Equations
SPG	Sketching Parabola functions Graphs
USB	Universal Serial Bus
WEF	World Economic Forum
WGU	Western Governors University
ZPD	Zone of Proximal Development

CLARIFICATION OF CONCEPTS

The following concepts were used in this research having the same meaning.

Parabolic functions and parabola functions

Students and learners

Teachers and educators/facilitator

Majority of learners and most learners

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1: OVERVIEW OF THE STUDY

1.1 INTRODUCTION AND BACKGROUND

Mathematics learners' performance is a challenge globally and South Africa is no exception. The 2015 Organisation of Economic Cooperation and Development (OECD) and the World Economic Forum (WEF) reveal South Africa Mathematics learners' poor performance (Mahlaba, 2020:2). Mabena (2021:1) concurs that in comparison to other countries, South Africa learners' performance in Mathematics is poor. The (ibid) further indicates that no significant progress was recorded in learners' performance in the Trends in Mathematics and Science Study (TIMSS) of 2007 and 2011. However, the 2015 TIMSS reported an improvement in Grade 9 Mathematics learners' performance. Despite this, South African learners remain in the last position in Mathematics performance (Reddy, 2016:3), performing at a national average score of 389 points and the country is ranked 38th out of 39 countries (Mabena, 2021:2).

The Southern and Eastern Africa Consortium for Monitoring Education (SACMEQ) 3 and 4 conducted in 2007 and 2017 show improvements in performance by South African learners (Department of Basic Education (DBE), 2017:27). The (ibid) goes further to indicate that Limpopo performed below the benchmark of 500 points in both these periods. In addition to testing learners and teachers in both numeracy and literacy, SACMEQ collects considerable background information on the schooling and home environment of learners (DBE, 2017:3). Roberts (2019:60) indicates that Mathematics performance in South Africa is very poor although the country invests extensively on education.

According to the DBE (2011:3), learners' performance in Mathematics is poor and the Limpopo province is amongst the worst. Ndebele (2016:16) concurs that results in Mathematics are poor and cite the 2014 Grade 3 Annual National Assessment (ANA) results which ranked Limpopo Province position last out of nine provinces. The ANA test is administered to find learners with problems and to notify the district officials, teachers and parents so that they can devise support strategies (DBE, 2014:9). In January 2017, Umalusi the quality assurance body indicated that the Grade 12 Mathematics pass rate is still not good (Roberts, 2017).

The national Mathematics Grade 12 performance between 2017 and 2021 is showing slight improvement with 51,9%; 58,0%; 54,6%; 53,8% and 57.6% respectively (DBE, 2021:5). The (ibid) further indicates that the 2021 diagnostic report reveal that most learners still achieve the average of 30% and 40% in Mathematics. The percentage of learners' performance in Mathematics in 2021 at 30% was 56.6% while 40% and above was 37.4%. In the same year, the Limpopo Province Mathematics learners' performance showed an improvement at position six out of nine provinces with 54,5%. Followed by Mpumalanga at 54,0%, Eastern Cape at 46,6% and KwaZulu- Natal with 54,2% (DBE, 2021). This highlights that Mathematics learners' performance in SA learners remains a challenge

District performance reflects that Vhembe West was position one with 58,6% in Grade 12 Mathematics learners' performance in the Limpopo Province, with other districts getting as little as 44,3% (DBE, 2021:11). However, some schools in the Vhembe West District like the school where the researcher is teaching still performed as low as 25,11%. The 2017-2021 DBE diagnostic reports indicate that one of the challenges is that learners do not understand how to solve parabolic function problems (DBE, 2021:102). According to the (DBE, 2011:12), at this level learners are supposed to be able to draw graphs, represent relationships between variables and use symbolic representations to represent parabola functions. Learners should be able to convert the given parabola functions to graphs, tables, words and formulae as stated in Curriculum Assessment Policy Statement (CAPS) (DBE, 2011:12). If learners fail to grasp parabola functions in Grade10, they'll grapple with the concept and interpretations in both Grades 11 and 12. The concept of parabola functions in Grades 10 and 11 carries a weight of 20% to 25% respectively. It also carries 25% of the total marks of Mathematics paper one and forms a basis of understanding polynomials of the third degree functions which weigh 15% in Grade 12 (DBE, 2014:12-14).

The 2021 DBE diagnostic report further indicates that learners are unable to find the x-intercept and turning point and confuse the domain and the range of parabola functions. The 2021 DBE diagnostic report concurs with Parent (2015:58) that learners are failing to determine x – and *y* –intercepts of the sketched parabola function. Parent (2015:10) maintains that the graph of parabola functions is sketched by letting $x = 0$ determine *y*-intercept, letting $y = 0$ so that equation ax^2 + bx + c = 0 and then use the fact that if (x -a)(x -b) = 0, then $x =$ a or x = b are x-intercepts and also know that the graph has an axis of symmetry at $x = \frac{-b}{2a}$ and one turning point which is given by $(\frac{-b}{2a}; f(\frac{-b}{2a})$. The (ibid) goes further to indicate that most learners perform poorly on these concepts.

Most studies conducted on parabola functions focus on Grade 11-12 learners' understanding (Zaslavsky,1997; Johnson, 2015; Bennng & Agyei, 2016; Rahaman & Mohammad, 2021). However, fewer studies have been conducted on the Grade 10 learners' academic achievement and errors in learning parabola functions. The challenges learners experience in Grade 10 must be explored so that these are not further experienced in Grades 11 and 12 and post matric. Against this background, the problem statement of the study is as follows.

1.2 PROBLEM OF STATEMENT

Learners' performance in parabola functions is a problem around the world and South Africa is no exception. Mutambara (2019:6) and Rahman (2021:13) reveal that learners encounter problems when solving parabola functions. In support, Nepomucena (2017:5) and Mutambara (2019:6) concur that learners experience challenges and difficulties when they are learning to solve parabola functions. Parabola functions appear to be one of the critical concepts in Mathematics and it features in different curriculums around the world (Nielsen, 2015:102).

According to Malahlela (2017:5), the application of the parabola function concepts goes beyond Mathematics and is incorporated into other disciplines in which functions are applied such as design sciences, economic sciences and geographical sciences. Therefore, it is important that when a learner exits secondary school, they have a mastery of the concept of parabola functions.

Teachers should utilise various approaches and methods like practical work, direct instruction, group work, explanatory problem solving, exposition and direct instruction to help alleviate learners' errors in the mastery of parabola functions (Ubah, 2018:848). Parent (2015:16) concurs that effort must be made to identify learners' errors to improve their performance in learning parabola functions. The author further contends that a lack of conceptual understanding in learning parabola functions is caused by teaching strategies and approaches mostly used in Mathematics classrooms.

In South Africa, Grade 10 learners have also shown difficulties and poor performance in parabola functions and the school where the researcher is teaching is no exception. Learners are unable to find the $x \; and \; y$ - intercepts of parabola functions and are unable to determine the range and domain in given parabola functions. Furthermore, learners do not know how to sketch the graph of parabola functions. These problems were identified in the analysis of learners' assessments such as class tests, classwork, homework and previous examination scripts. The researcher also held discussions with other Mathematics teachers in the neighbouring schools to determine their experiences on learners' performance in learning parabola functions. The teachers indicated that most learners cannot solve problems on parabola functions. As such, the researcher undertook to explore learners' experiences when learning parabola functions by determining their performances and errors.

1.3 AIM OF THE STUDY

This study aims to explore grade 10 learners' academic experiences when learning to solve parabola functions. The study aims to present possible ways that Grade 10 learners can employ to learn parabola functions to improve their performance and to prepare for Grades 11 and 12.

The following were the research objectives the study intended to attain:

- Determine the performance of Grade 10 learners' when solving problems in parabola functions.
- Identify errors that Grade 10 learners may possess when solving parabola functions.
- To determine the possible causes of errors learners may possess when solving parabola functions.
- To suggest the possible ways to help Grade 10 learners to learn parabola functions to improve their performance and alleviate the errors they possess.

1.4 RESEARCH QUESTIONS

The primary and secondary research questions of the study were outlined as follows:

1.4.1 Primary research question

- What are Grade 10 learners' academic experiences when learning to solve parabola functions?

1.4.2 Secondary research question

- What are Grade 10 learners' academic performance when solving parabola functions?
- What difficulties do Grade 10 learners face when solving parabola function problems?
- Why do learners commit these errors when solving parabola functions?
- How can learners' errors in Grade 10 parabola functions be addressed to improve learners' academic performance?

1.5 HYPOTHESIS

This study partly intended to test the following hypothesis on Grade 10 learners when solving parabola functions:

- Null hypothesis (h_0): Grade 10 learners' academic performance is poor when they are learning to solve parabola functions.
- Alternative hypothesis (h_1): Grade 10 learners perform well when they are learning to solve parabola functions.

1.6 RATIONALE OF THE STUDY

The researcher embarked on the study to obtain an insight into Grade 10 Mathematics learners' academic difficulties when learning to solve parabola functions. To attain this the study focused on Grade 10 learners' experiences when learning to solve parabola functions. The researcher investigated Grade 10 learners' academic experiences which constitute academic performance and difficulties in the form errors when learning parabola functions. The researcher recognised possible causes that result in learners' experiencing challenges when learning parabola functions. Furthermore, this study pointed out an effective learning approach to be utilised in assisting learners to master parabola functions. The rationale was to determine learners' performance in Mathematics and investigating parabola functions learning was the purpose.

The study also focused on influencing good practices that can be adopted in the classroom on the topic of parabola functions and graph sketching by helping learners to move from one parabola functions equation to the next i.e., $f(x) = x^2$ to $y = ax^2 + p$ then to $f(x) = ax^2 + bx + c = 0$, $f(x) = (x - x_1)(x - x_2)$ and then to $y = a(x - h)^2 + c$. Furthermore, the researcher intends to help learners to be able to use different strategies when solving parabola functions by finding out the causes of poor performance when learning Grade 10 parabola functions at the selected secondary schools in the Nzhelele East Circuit of the Vhembe District.

This study sought to benefit participants and the Limpopo Department of Education. The researcher aimed at collecting important information which may assist learners to understand solving problems when learning parabola functions. Moreover, the study sought to inform teachers of the difficulties Grade 10 learners experience when learning parabola functions and suggest different methodologies that could be employed to alleviate the challenge. The findings of this study might assist secondary schools and DBE officials in the Vhembe District to understand barriers that impact learners' poor performance in the learning of parabola functions.

1.7 RESEARCH METHODOLOGY

Jain (208:3) describes research methodology as solving a problem systematically. In the quest for the creation of knowledge, researchers use the research methodology to conduct research by following procedures that assist in the production of correct research data (Bilau, Witt & Lill, 2018:599). Sileyew (2019:10) describes research methodology as the design in which the researcher makes data selections and analysis procedures to make findings on a specific programme. Cohen (2018:185) points out that different research methodologies can be used when conducting research and these are qualitative, quantitative or mixed method forms of enquiry. Macmillan and Schumacher (2014:354) assert that qualitative research is research where the researcher collects data that is not numerical. Makgakga (2016:86) concurs that a qualitative study use data which are not in the form of numbers. Basias and Pollias (2018:92) indicate that quantitative research involves a systematic and empirical investigation of the phenomenon using numbers and the processing of numerical data. Whilst Crane, Henriques and Lusted (2018:12) describe mixed methods research as an approach in which the researcher combines qualitative and quantitative methods in one study.

The study employed mixed enquiry since the researcher wanted to look at both the qualitative and quantitative data. The researcher considered the mixed method to accommodate data in form of words and numbers (Macmillan et al., 2014:360). The researcher was also interested in understanding the perspective participants had on the parabola functions. The study was conducted in a classroom setting using a combination of qualitative and quantitative data to better understand the difficulties learners experience when learning to solve parabola functions (Boru, 2018:34). The participants were given chance to express different views and beliefs on their poor performance in Grade 10 parabola functions.

1.7.1 Research paradigm

According to McMillan and Schumacher (2014:354), research is a systematic investigation used to update the existing body of knowledge by gathering data and creating new information through analysis and interpretation. Accordingly, the paradigm is a viewpoint that a researcher holds about the world and is founded on a collection of common presumptions, beliefs, values and practices (Johnson & Christensen, 2012:32). Creswell (2018:17) asserts that a paradigm is a set of assumptions that direct behaviour in ontology, epistemology, axiology and methodology. Arguably, paradigms can intentionally or unconsciously determine the purpose and goals of the study Makgakga (2016:83).

There are various types of research paradigms, including the positivist paradigm, constructivist paradigm, transformational paradigm, and postcolonial paradigm, (Wagner, Kawulich & Garner, 2012:53). Both positivist and interpretative perspectives were used in this investigation. Pragmatism allowed the researcher to adopt a variety of techniques and viewpoints about parabolic functions. However, the study did not use a post-positivist, emancipatory or indigenous research paradigm because it does not seek to discover laws, destroy myths or promote transformation and social change.

1.7.2 Research approach

According to Taherdoost (2022:12), the research approach is a strategy and set of stages that progress from the general hypotheses to a specific approach to data collecting, analysis and interpretation. Huyler (2019:75) maintains that it encompasses the strategies and processes the study that the study covers, the steps from general hypotheses to specific techniques for data collecting, analysis and interpretation. To gather data for this study, both quantitative and qualitative methodologies were used. As information was acquired utilising both quantitative and qualitative data, the study also attempted to apply triangulation and an explanatory method. To make a stronger case, data were triangulated (McMillan & Schumacher, 2014).

1.7.3 Research design

Research design is a strategy outlining how the researcher will carry out the research program (Sileyew, 2020:15). According to McCombes (2021:13), the study design is a technique that aids in pointing out the direction to be taken when gathering data as well as the methods of analysing data. The four common qualitative research designs are case study, phenomenological study, grounded theory and content analysis Cohen (2018:137). Quantitative research design includes observational studies, correlational research and survey (Cohen, 2018:138).

The study followed an explanatory sequential design that sought to understand learners' academic experiences when learning parabola functions. The explanatory sequential design was chosen because the researcher first gathered quantitative data and thereafter qualitative data on the ground (at the schools where participants were experiencing the challenges). Sequential mixed method design responded to the gap identified in the quantitative study and the meanings learners constructed and the experiences they had in the learning of parabola functions (Creswell et al, 2018:360). The researcher was also interested in understanding the perspective participants had on the parabola functions. There was no attempt to influence the phenomenon of interest or accept the researcher's subjectivity because the study was conducted in a real-world setting (Yin, 2018:346).

1.7.4 Population and sampling

The population is the complete group of individuals within a context that the researcher may be interested in (Cohen, 2018:365). A sample is a selection of individuals, things, or things used for a measurement that is made from a larger population (Bhandari, 2020:5). Additionally, a sample is a subset of the population (Cohen, 2018:122). Shukla (2020:12) asserts that sampling is the process of choosing a population's representative portion or figuring out features or characteristics of the entire population.

Creswell (2018:48) indicates that sampling refers to the location where data are gathered, which can be a person, document, institution, place, or any other instance of data gathering. According to Whilst and Shukla (2020:14), there are many kinds of probability sampling, including stratified, cluster and simple random sampling. The (ibid) adds that there are various non-probability sampling techniques, such as convenience sampling, snowball sampling, quota sampling, purposive sampling, maximum variation sampling, critical case sampling and theoretical sampling. In light of this, purposive sampling was utilised in the study since it reflects the roles, opinions, knowledge and experiences that are crucial to the investigation (McMillan & Schumacher, 2014:365). 90 Grade 10 Mathematics learners from two high schools, along with three Mathematics teachers, made up the population. Both institutions are rural secondary schools located in the Vhembe District of the Limpopo Province. Additionally, both are public schools serving rural communities with a history of poverty. The learners' mother tongue is Tshivenda and they play and converse informally in their home language at home and school.

1.7.5 Instrumentation and data collections techniques

According to Thomas (2018:659), any device used in research to gather data, such as paper or computer questionnaires, devices that facilitates interviews are known as data collecting instruments. The data collection instruments in the study were made up of achievement tests, observation sheets and semi-structured interviews. According to Yin (2018:456), data collection is the process of gathering and measuring data on the variable of interest to provide answers to enquiries, formulate research questions, test the hypotheses and assess results. Creswell (2018:235) indicates that there are a variety of methods for gathering data, including interviews, observations, and document analysis.

Students' experiences in learning parabola functions were documented, as opposed to learner pass rate statistics, educator's files, or educator's profiles. The study used achievement assessments, semi-structural interviews and classroom observations to collect data. The purpose of the achievement test, semi-structured interviews and instructional observations was to describe the student's academic performance in learning parabola functions as well as any other barriers they encountered which prevented them from learning successfully.

Data collection took 12 weeks and was done by interviewing learners and observing teachers. To avoid disruption, the researcher became an outside observer and sat quietly at the back of the room. To create ownership of the research by the learner participants were interviewed in their classrooms. The interviews were conducted after school during the study to avoid period clashes and learners having to miss lessons.

In each case, it was very important to keep the discussion as short as possible to avoid participants getting tired during the interviewing process. The interviews lasted for 25 minutes and all interviews were recorded and field notes were taken.

1.7.6 Data analysis and interpretations

The act of analysing data means that raw data are converted into useful data (Cohen, 2018:219). Data were interpreted and subjected to various analyses to reveal the data's greater significance (Sepeng, 2010:75). In this study, the quantitative data were organised and categorised in terms of whether a response was incorrect, correct, incomplete or blank (Didis & Erbas, 2015:1141). Additionally, the descriptive statistics created from achievement-test frequency and SPSS were used to analyse quantitative data.

This study used many layers of Creswell's (2018:197) procedures to analyse the qualitative data. According to Cohen et al. (2018), numerous interview formats can be utilised to collect data from a range of sources. Semi-structured interviews conducted with students on how they learn parabola functions and classroom observations of parabola functions were used to gather qualitative data for the current study. Accordingly, the study topics and themes obtained from the data were transcribed and categorised. In addition, written texts were all used in the data collection process.

1.7.7 Validity and reliability

According to Cohen (2018:246), in qualitative research trustworthiness is about establishing credibility, transferability, conformability and dependability. However, in quantitative research trustworthiness is defined as the validity and reliability of the research. Researchers should apply multiple validity strategies to assess the accuracy of the findings (Creswell, 2018:51). Stahl (2020:4) states that trustworthiness should be based on credibility, dependability and reliability which are forms of external validity and conformability. According to Devault (2019:2), trustworthiness is the ability to show that the data support the outcomes. In this study, the researcher validated data by using semi-structured interview questions, observation schedules and test instruments the researcher to ensure trustworthiness.

Crystallisation is also suggested as a further measure of trustworthiness (Cohen et al., 2018). Crystallisation is utilised to assure the validity and reliability of the research's methodology and conclusions. In this study, the researcher verified the dependability of the quantitative results by expanding the qualitative paradigm with the use of triangulation to determine whether the test was producing the same results (Stahl, 2020:5). The validation of data is done by making notes of the original concepts, and fresh information learned in the field and member checks. The researcher used member checking and observers to compare the interpretations (Cohen et al., 2018:247). The researcher asked clear questions to ensure that participants' experiences and feelings are captured correctly, and to increase trustworthiness and credibility. The researcher observed how teachers teach parabola functions to get an understanding of the strategies used when teaching parabola functions. The researcher also interviewed learners to get an understanding of how learners learn parabola functions.

1.8 ETHICAL CONSIDERATIONS

Bhandari (2021:1) defines ethical considerations as a collection of principles that direct research procedures and designs. According to Moumita (2022:3), researchers have a responsibility to uphold the standards of behaviour that apply to most professions. Ethics is a subfield of philosophy that addresses human conduct and serves as a guide for social norms and standards of conduct (Fleming, 2018:210). According to Bhasin (2020:4), the area of philosophy that deals with the dynamic of moral judgment are known as ethics. Dube (2018:19) argues that ethical considerations are values, moral principles and obligations to ensure that participants are protected from all harm and dangers throughout the research process.

The researcher applied for an ethical clearance from the University of South Africa's Research Ethics Committee. Thereafter, requested authorisation to access the schools from the Vhembe district office. Further approval to carry out the study, was requested from the principals of the schools. The researcher also requested consent from Grade 10 Mathematics instructors/teachers and students. Participants' responsibilities and freedom to choose whether to participate or not were also discussed (Dube, 2018:5).

Participants were assured of the confidentiality of their personal information and the right to withdraw at any stage without consequences. The voluntary nature of participants' involvement was also communicated. After obtaining approval to carry out the research, informed consent forms were handed to the teachers and students. Students were also issued parental consent forms to ask parents and guardians for permission to involve the children in the study. The researcher avoided using the participants' names and instead employed pseudonyms (Fleming, 2018:214).

1.9 CHAPTERS OUTLINE

CHAPTER 1: INTRODUCTION AND BACKGROUND

This chapter discusses the background information, statement of the problem, research questions, goals and objectives, definitions of key terms and research hypotheses. It also gives an overview of the research methodology, ethical considerations and chapters outline.

CHAPTER 2: THEORETICAL FRAMEWORK

The theoretical framework used in this investigation is outlined in Chapter 2.

CHAPTER 3: LITERATURE REVIEW

The relevant literature that served as the study's guide is covered in this chapter.

CHAPTER 4: RESEARCH METHODOLOGY

The research methodology, research design, sample size and sampling techniques, data collection tools, process and data are discussed in this chapter. Also, discussed in Chapter 4 are measures for validity and reliability and ethical considerations.

CHAPTER 5: RESEARCH FINDINGS AND DISCUSSIONS

The research findings from an accomplishment achievement test together with semi-interviews with learners, lesson observations, discussions of the outcomes and results summary are presented in this chapter.

CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS

This chapter consists of the conclusions from the study, recommendations and suggestions of areas for further research.

1.9 CHAPTER SUMMARY

This chapter gave an overview of the background, problem statement, justification and objectives of the study. It also discussed the research techniques employed by the study and concluded with a summary of each chapter. The next chapter looks at the theoretical framework that guided the study.

CHAPTER 2: THEORETICAL FRAMEWORK

2.1 INTRODUCTION

The previous chapter discussed the background of the study which included the problem statement, the rationale for the study, the significance of the study and the chapter's overview. This chapter defines the general theory and theories in education and their importance in education. The chapter also outlines error analysis and justifies why it was adopted as the theoretical framework to understand Grade 10 learners' academic performance on learning parabola functions. Newman's (1983) error analysis is used to find out learners' difficulties in the form of errors when they are learning parabola functions. The chapter also discusses the need for teacher mediation by using scaffolding and zone of proximal development (ZPD) to help learners resolve their challenges (Vygotsky, 1978:86).

2. 2 UNPACKING THE NOTION OF THEORIES CONCEPT IN EDUCATION

The concept of theory has been widely used in the education field. According to Creswell (2018:86), a theory is a set of interrelated constructs, definitions and propositions that presents a systematic view of a phenomenon and specify relations among variables to explain a natural phenomenon. Furthermore, American Heritage Dictionary (2006:1429) describes the general theory as a set of rules made to explain a group of statements that has been continuously tested and are broadly accepted. Lefrancois (2019:3) argues that learning theories are descriptions that are based on belief systems and are supported by extensive research and are believed by a large number of people. Moreover, educational theories are often called a lens through which researchers can view the setting of the research (Lefrancois, 2019:3). Schunk (2020:5) further asserts that researchers use theories to frame their studies. Theories used in education have been studied, tested and debated for many years. The researcher's view is that educational theories seek to explain the experiences learners have when learning mathematics and which may influence their performances and mastering of mathematical concepts. The above definitions provide a deeper understanding of the theory chosen to guide the researcher to understand learners' experiences when learning parabola functions by basically looking at their errors and academic performance. Aubrey and Riley (2018:12) assert that some theories have been disputed and some have gone through the process of change and advancement.

Aubrey and Riley (2018) postulate that behaviourism, constructivism and humanism are the three important educational and learning theories. According to Becton (2022:7), behaviourism is a school of thought that views learning as a change in behaviour that can be predicted, measured and controlled. Constructivist argues that learning is an activity that takes place when learners engage actively in the process of constructing new knowledge incorporation with their prior knowledge (Becton, 2022:32). In contrast, the Humanists claim that teaching and learning should focus on the learners' needs and personal and emotional growth (Aubrey & Riley, 2018:7). In this study, the theory was used to guide and explain Grade 10 learners' academic performance and errors when learning Grade 10 parabola functions. Moreover, theory influenced the way the researcher analysed achievement tests and interpreted semi-structured interviews and lesson observation results.

2.2.1 The role of theories in educational research

According to Western Governors University (WGU) (2020:3), knowledge of theories is important in educational research and teaching practices. The (ibid) postulate that research is more grounded if it is supported by theoretical foundations in the field of education. The National Research Council (2002:28) concurs that knowledge of educational theories is of uttermost importance to both educational practitioners and researchers. More importantly, theories must play the role of helping researchers and teachers to recognise, understand and explain reasons learners experience difficulties and helps promote an environment that is conducive to improving teaching and learning. Therefore, teachers who incorporate theories in their practices develop effective instructional approaches and assessment skills which may minimise learners' misconceptions and errors resulting in improved learners' performance. Creswell (2018:137) asserts that a study that is theoretically linked to another contributes to extending the knowledge base, as opposed to one that is not linked. Makgakga (2016:18) concurs that the study which is not linked to theory is likely to be of low quality.

2.2.2 The importance of theories in teaching and learning

Teachers who understand the set of theories select teaching approaches that best suit the topic and the needs of their learners (Locky, 2021:3). Teachers' understanding of educational theories helps them to choose teaching and learning strategies that they can use to teach different mathematical content. Furthermore, teachers' understanding of educational theories improves their educational practices and expands their teaching strategies (Locky, 2022:4).

According to Becton (2022:25), behaviourism, cognitivism and constructivism are the main three theories used to create conducive teaching and learning environments. Behaviourism is a theoretical perspective of learning, which conceptualises learning as a process of changing one's behavioural patterns (Skinner, 1979:35). The philosophy underpinning behaviourism postulates that the outcomes of learning are an observable change in behaviour through a response to stimuli (Espanol, 2022:5). In the heart of behaviourists, learning is viewed as a change in the rate, frequency of occurrence, or form of behaviour or response which occurs primarily as a function of environmental factors (Schunk, 2020:45). The behaviourist theory stipulates that learning involves the formation of associations between stimuli and responses. In this view, learning is explained in terms of the observable phenomenon. Firstly, the behaviour is viewed as the result of a person's response to stimuli. Secondly, the stimuli can be manipulated to strengthen or reduce an individual's behaviour. Behaviourism theory of learning is interested in the effect of reinforcement, practice and external motivation on a network of associations and learned behaviours (UNESCO, 2022:7). Skinner (1979:225) highlights that the reinforcement can be positive or negative, and will lead to change in a person's behaviour.

In contrast to behaviourism, cognitive theories emphasise the acquisition of knowledge and skills. According to Schunk (2020:21), the formation of mental structures and the processing of information and beliefs to produce learning in individuals is a key aspect of cognitivism. Cognitive theorists view learning as an internal mental phenomenon deduced from what people say and do. The cognitive theories emphasise the importance of mental processes in learning. Although some strands in cognitive theories differ and disagree on the learning processes.

In this study, the researcher adopted Newman's error analysis learning theory linked with Vygotsky's ZPD and scaffolding. Newman error analysis learning theory, Vygotsky's ZPD, and scaffolding were chosen to predict, explain and control the research process of learners' experiences when they learn Grade 10 parabola functions at selected schools in the Vhembe District of the Limpopo Province.

2.2.3 The origin of Constructivist learning theory

The notion of constructivism is linked to Lev Vygotsky, John Dewey and Jean Piaget. The main two strands of constructivist perspectives are Piaget's constructivist perspective and Vygotsky's socio-constructivist perspective. Dewey (1987), Piaget (1977) and Vygotsky (1978) discarded the behaviourism perspective and made advancement to constructivism as they argued that traditional education plays a major in the transmission of knowledge from the teacher to the learner. However, these researchers indicated that traditional theories neglect individual autonomy (Dewey, 2013:17).

The term constructivism is also derived from Piaget's (1977) constructivist view of teaching and learning, which relates to the active construction of meaning, which was later extended by Vygotsky to include the social learning theory. McLeod (2019:19) indicates that the constructivist approach and socio-constructivist approach are the two theories of constructivism that indicate that learners' conceptions of knowledge result from their search for meaning and that during this process learners formulate and construct their own understanding and interpretations of their experiences.

Kulwicki (2021:45) posits that constructivism views learning as the reconstruction of knowledge in the learners' minds and attaching meaning to existing outside ideas. Germain (2019:28) concurs that constructivism learning theory views learning as a process where learners actively construct or build new ideas or concepts. Germain (2021:207) further asserts that constructivism views a learner as an individual constructing knowledge by himself or herself through a cognitive mind driven by self-regulation. Adams (2022:246) comments that constructivist learning theory helps us to understand the inherently complex process of learning in both people and animals. Piaget's (1966) constructivist learning theory focuses on different stages of child cognitive development such as the sensorimotor stage, preoperational stage, concrete operational stage and formal operational stage. These stages are also called individual construction of knowledge and are also referred to as individual constructive theory (Karsley, 2019:171). Piaget's views of constructivism are that, what is in the mind of a human being, is what is made or constructed by the human mind (Drew, 2022:17). Whilst, Vygotsky, a contemporary of Piaget, argues that social interactions are crucial for cognitive development (McLeod, 2019:76). For Piaget, knowledge construction takes place when knowledge is actively assimilated and accommodated into existing knowledge (Drew, 2022:17). However, Vygotsky places more emphasis on the social environment as a facilitator of development and learning (Schunk, 2020:46).

Jerome Bruner’s constructivist theory views learning as an active social process in which students construct new ideas or concepts based on current knowledge (Drew, 2022:19). The similarities between Piaget’s and Bruner’s constructivism are that social interaction results in learners' cognitive development. The difference between Piaget and Bruner theory is that Bruner's modes are not related in terms of which presupposes the one that precedes it. Bruner argues that what determines the level of intellectual development is the extent to which the child has been given appropriate instruction together with practice or experience (Kearsley, 2019:172). Concepts of parabola functions are well understood by the learners if they were presented and explained well by the educator who understands the concept.

Obi (2019:550) asserts that Dewey’s constructivism learning theory views the formation of knowledge as active participation that creates cognitive structures in their interactions with the environment. Cognitive interaction will occur as far as reality is structured through the cognitive structure created by the subject (Obi, 2019:550). Therefore, the cognitive structure must always be altered and adapted so that it can meet the demand of concept development necessary to understand the parabola function.

Hurst (2021:14) further argues that Von Glaserfeld's learning theory postulates that if learners were not adapted to the environment, learners would be unable to survive, and they could not learn and they would die because of their mistakes. For the biologist, however, there is an important difference because adaptation refers to the biological make-up, the genetically determined potential with which people are born; and learning is the process that allows us to build up skills in acting and thinking as a result of our own experience (Hurst, 2021:15). Newman’s error analysis originates from the research on the education of Mathematics language. This theory emphasises learners’ understanding of the words in the problem, symbols and making sense of what they have read.

In this study, Newman’s error analysis learning theory was used to guide the study because it focuses on how learners construct knowledge and the errors they experiences when learning parabola functions while Vygotsky’s, Dewey’s and Bruner’s learning theory focus only on how learners construct knowledge in a social environment. Von Glaserfeld talks about adaptation as an important process in the learning environment (Hurst, 2021), however, this is not the focus of this study. The researcher wants to find out what errors learners make when they learn to construct knowledge of parabola functions and (Newman, 1983:676) error analysis learning theory complemented by Vygotsky' (1978) ZPD and scaffolding will be used.

2.2.4 Newman errors analysis learning theory

The theory behind Newman's (1983:676) error analysis of cognitive development posits that humans naturally arrange their thinking processes into the simplest structures possible. Widawati (2020:1677) asserts that the Newman error model is often used to diagnose learners' errors in solving higher-order thinking skills in Mathematics. Newman (1977, 1983) further maintains that we have six stages such as reading errors, comprehension errors, transformation errors, process skills errors, encoding errors and carelessness errors that underline the difficult learners experience when solving mathematical problems. Learners experience reading errors when they misread the keywords in the questions for example when they fail to read mathematical symbols correctly and comprehension errors occur when learners can read the question but are unable to comprehend the requirements of the question (Rahman, 2021:3). Transformation errors occur when learners are unable to change the given mathematical information into tables and graphs while process skills errors occur when learners are unable to execute mathematical procedures correctly such as knowing mathematical operations but committed an error in calculations procedures (Rahman, 2020:3). The (ibid) further maintains that encoding errors occur when learners fail to write the final answer correctly and situations where students are rushed and do not focus on what the question is asking lead to miscalculations which result in carelessness errors. Rahman (2020:3) further asserts that carelessness occurs when learners understand the concept but fails to perform the calculations correctly.

In this study learners' experiences in learning parabola functions were looked at, and in the process, learners were allowed to use prior knowledge of solving quadratic equations to help them to understand finding the coordinate that they would use to draw the parabola functions. Newman error analysis model was used to analyse errors made by learners such as reading errors, comprehension errors, transformation errors, processing errors, encoding errors and carelessness errors.

2.2.4.1. Reading error

According to Retnawati (2020:165), reading error occurs when learners did not do what the question is asking and the meaning of the symbols used in the problem. In the learning of parabola functions, reading errors occur when learners are unable to read and understand the questions precisely. Reading errors also occur when learners are unable to read words, numbers and symbols that are used in the equations of parabola functions like $f(x) = x^2$ to $y = ax^2 + P$ and move to $f(x) = ax^2 + bx + c$ and then to $y = a(x - h\,)^2 + k$. Researcher used reading error analysis to check learners were not just reading the words in a question but find it difficult to recognize methematical symbols used in the parabola functions.

2.2.4.2 Comprehension error

Kristianto (2019:46) highlights that comprehension error occurs when learners are unable to write what is given in the problem. Comprehension error also occurs when learners are failing to follow correct procedures in applying the quadratic formula, completing the square, applying factored method and finding factors of the quadratic function. Comprehension error can be demonstrated by looking at the situation where learners are unable to use knowledge of the difference between two squares and other quadratic equations to find points on the Cartesian plan to follow the steps used in finding the roots of parabola functions. Comprehension error analysis can be used to check if learner understand mathematical symbols and problem used in in parabola functions to reduced difficulties in understanding parabola functions.

2.2.4.3 Transformation error

Transformation error occurs when learners are unable to change the mathematical problem into models such as equations, drawings, tables and graphs (Rahaman, 2018:123). Transformation errors can be demonstrated by situations where learners cannot be able to change give mathematical information into tables and graphs. Transformation error analysis can be used to check if learners can be able to convert given information into mathematical sentence, graphs, operations and symbols.

2.2.4.4 Process skills error

According to Kopfer (2022:2), process skills errors occur when learners cannot execute the required mathematical procedure correctly. For example, the learner knows mathematical operations to be used to solve the problem but committed errors in the calculation procedures. Process skills error can be used to analyse if learners were able to follow mathematical procedures and solve mathematical problems accurately.

2.2.1.5 Encoding errors

According to Trapsilasiwi (2020:3), encoding errors occur when learners are unable to use the information given to make knowledge discoveries and correlations among entities. Kopfer (2022:2) further asserts that encoding errors occur when learners are unable to use the information given to the model real world in the form of graphs. Encoding errors occur when learners are unable to think logically to find relationships between concepts and are not able to find alternative approaches to finding the correct answer. Encoding errors can be used to analysed if learners can write down the correct answer to a given problem using words, symbols, and numbers.

2.2.1.6 Carelessness errors

Carelessness error occurs when learners understand the concept but fail to get the correct answer because learners rush or did not focus on the problem to be solved and make miscalculations (Rahaman, 2020:4). Carelessness error occurs when learners understand the procedures to be done to find the $y -$ intercept, but because of doing the calculations quickly, learners end up making mistakes and get an incorrect answer.

2.3 THE SIGNIFICANCE OF NEWMAN ERROR ANALYSIS LEARNING THEORY

The purpose of using Newman's error analysis learning theory as a framework for this study is to use it to explore Grade 10 learners' experiences when they are learning parabola functions and hence the learners' achievement in mastering the concept. It also focuses on determining the teaching strategies that the teachers use to support learners when they are learning parabola functions. Furthermore, a focus is placed on determining the difficulties in the form of errors that learners experience when they are learning parabola functions and how the teacher can support the learners to overcome those challenges. According to this theory, the study sought to focus on cognitive processes looking at determining how learners commit errors when they are learning parabola functions. The theory also helped to guide focus on the variable that may emerge during the study such as errors in learners' academic achievement, mathematical reasoning, and pedagogical content knowledge (PCK).

2.4 SCAFFOLDING

The Newman's errors analysis is blended with the scaffolding of Vygotsky (1978:86) as the researcher intended to observe learners learns parabola functions. The notion of scaffolding was used because the researcher wants to see how learners were supported when they were learning parabola functions. Scaffolding was further used to help the researcher to analyse the data after classroom observation observing the ways in which learners were learning parabola functions.

Scaffolding is described as the role of the more capable peer in providing support to the learner's development in learning new materials (Aslam, 2018:2). In the same vein, Anggadewi (2018:210) also concurs that scaffolding provides structures that move a learner to the next level during learning and instructions. Reavis (2019:2) further asserts that researcher uses the concept of scaffolding as a metaphor to describe and explain the role of teachers or more capable peers, in guiding the learners learning and mental development. In this study of learners' experiences in learning parabola functions, scaffolding was used to close the gap of misconceptions and errors learners have when factorising parabola equations, finding the x and y-intercept, finding the axis of symmetry and sketching the graph of parabola functions.

Scaffolding, therefore, plays a role in effective learning, by allowing the teachers to use the knowledge gained to assist each learner in developing his/her knowledge and thinking (Anggadewi, 2018:212). Scaffolding as a teaching strategy facilitates a learner's ability to build on prior knowledge and to internalise new information through the support of the teacher (Aslam, 2018:2). Scaffolding will also help the teacher to link the learner's prior knowledge of linear function with the new knowledge of parabola functions supposed to be learned. When an educator uses scaffolding in teaching and learning, he becomes more of a mentor and facilitator of knowledge rather than the dominant content expert. When a learner's competencies increase independently or his level of understanding is increasing, scaffolding should be gradually decreased, and ultimately withdrawn (Reavis, 2019:3). At that moment, a learner should be able to solve problems independently, or have mastered the concepts (Chang, Chen, & Sung, 2002:19).

Taber (2018:155) commented the following about the scaffolding teaching strategy: Scaffolding motivates learners in relation to a given task, reduces learners' frustration in solving algebraic problems and models and indicates task expectations. The scaffolding teaching strategy requires the teacher to act as a facilitator of learning rather than a subject expert. Boonmoh (2019:2) also asserts that scaffolding instruction is necessary to be used in problem-based learning for learners' mental development. The teacher should design activities that the learners can solve independently which gives enough scaffold for the learners to close the gap of knowledge, concepts, and skills in solving parabola functions problems.

2.5 THE ZONE OF PROXIMAL DEVELOPMENT (ZPD)

According to Cherry (2021:1), ZPD is the range of abilities an individual can perform with the guidance of an expert, but cannot yet perform on their own. Vinney (2019:2) concurs that the ZPD is the gap between what learners can do independently and what they can potentially do with the assistance of a more knowledgeable partner. Kurt (2020:3) asserts that there is agreement among researchers that ZPD is at the heart of the concept of scaffolding. Scaffolding cannot work effectively without the ZPD. Vygotsky (1978:81) defines the ZPD as a distance between what learners can perform independently, and what they can perform with competent assistance. In other words, the more capable peer provides the scaffolds so that the learner can perform a task that they cannot perform; therefore, the educator helps the learner through the ZPD (Cherry, 2021:2). The ZPD is described by Vygotsky (1978:81) to be the distance between the actual development levels as determined by independent problem-solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers.

The ZPD plays a fundamental role in the broader understanding of how the foregoing interpretations might surface in the learners' knowledge of finding axis symmetry in the learning of the parabola functions. The term 'ZPD' is probably the most widely used and well-known idea associated with Vygotsky's scientific work. Furthermore, it is used with diverse kinds of learners, including the disadvantaged, the learning-disabled, the retarded, and gifted learners, as well as preschool learners (Smith, 1993:56). Yetman (2020:3) has indicated that failure to see the connections between the ZPD and the theory as a whole show that it is difficult to differentiate Vygotsky's concept from any instructional technique that systematically leads children with the help of an adult, through several steps in the process of learning some set of skills. The teachers who taught Grade 10 learners parabola functions gave attention to their learners' ZPD to find out the way learners factorized parabola equations to find the x and y-intercept.

Teachers must know their learners' prior knowledge, before introducing new knowledge to reach their goals during learning instruction (Taber, 2018:10).

The ZPD started from the socio-cultural theory of Vygotsky (1978:81). Cognitive development has socio-cultural origins, according to this theory. Furthermore, the individual's cognitive development has a relationship with social, cultural, and historical understanding (Thilly, 2021:4). Higher mental functions have their origins in social interactions with more experienced adults or peers (Vygotsky, 1978:86). In this study of learners' experience in learning parabola functions, the ZPD helped the researcher to discover how learners construct knowledge and how learners can be moved from actual level to developmental level.

Vygotsky has proposed a difference between what learners can do autonomously, and what they can perform with more knowledgeable peers or adults. Therefore, this concept constitutes the relationship between the two types of problem-solving behaviours of the learners (Main, 2021:5). The first behaviour is when the learners can solve a problem through social interaction, which is called the potential level of development. The second behaviour is when the learners can solve a problem by themselves, which is called the actual level of development (Cherry, 2021:4). Another focus of this study is to move learners from the actual level in solving parabola functions to the developmental level where learners can solve parabola functions without the assistance of the other or the educator.

Vinney (2019:3) further contends that learners learning through interaction with others can perform much better or achieve more than when they perform tasks independently without the support of more capable peers. Lastly, there is the collectivist which views the ZPD as the difference between what an individual can achieve through everyday actions, and what can be achieved by learners within collectives and the collectives themselves (Taber, 2018:9). In this study of learners' experiences in learning parabola functions learners learn in a classroom situation where they shared knowledge of parabola function using them to sketch and interpret the graph.

Vygotsky (1978:6) further maintains that the development of the mental functioning of a learner is initially social, and can undergo a process that has to become an internal function. Yetman (2020:3) concurs that teachers must interact with the learners after understanding their learners' prior knowledge before the learners internalise the content. Therefore, in this study of learners' experience in learning parabola functions, the teacher and learners worked collectively to modify learners, beliefs about drawing parabola functions based on their current understanding and on the new knowledge they construct.

2.6 CONSTRUCTIVIST LEARNING THEORY AND KNOWLEDGE CONSTRUCTION

Knowledge can be defined as a justified true belief that can be obtained through the process of sensory reasoning (Bolisani 2018:2). Mazorodze (2020:1) further indicates that knowledge can be obtained by a transfer from a person who has knowledge by teaching or by extracting it through exprience. When information is received by the learners, it gets interpreted and enriched, or supplemented, resulting in newly constructed knowledge (Bolisani, 2018:2). Koenig (2018:4) avers that the accuracy of transformed knowledge is composed of ambiguity and different interpretations. These processes of learning, knowledge construction and reorganisation result in what we call knowledge (Koenig, 2018:4).

Amster's (2021:2) definition of knowledge comes from Piaget's constructivist perspectives understanding of knowledge construction. Koenig (2018:4) argues that Piaget's constructivist learning theory sees a learner as an active participant in the process of constructing his or her own knowledge. Accordingly, Piaget's constructivist learning theory helped learners to actively construct their knowledge of factorising so that they can be able to find the x and y intercepts when they are drawing the graph parabola functions.

However, according to Piaget constructivist learning perception and knowledge formation, should be taken as a normal process of learning because it enables the construction of new legitimate knowledge (Gao, 2018:3). Gao (ibid) argues that it starts with the learner attempting to incorporate a new idea into an existing schema. Owing to the diversity of what needs to be learned, it is not always possible to link every new idea with an existing schema, resulting in assimilation or accommodation not taking place (Gao, 2018:4). This would result in a learner creating a new "box" by which he or she will try to memorise the new idea. Then learners are forced to attempt memorising the new idea which results in what is called rote learning isolated knowledge that is difficult to remember (Koenig, 2018:4).

This isolated knowledge then results in misconceptions, which in turn cause errors; this is called learning from a constructivist perspective. Piaget's constructivist theory also helped the researcher to discover the reason learners opt for memorisation when they are determining the axis of symmetry when they are drawing parabola functions. The implication of Piaget’s constructivism theory to learning has been a component of literature from many authors for example (Tomljecnovic, 2020:134). Purcell (2020:4) asserts that how learners construct knowledge needs to be taken as a normal part of the process of gaining knowledge and in fact may be a necessary step in the construction of knowledge (Aalto, 2022:7). Also important in the study is how learners think and construct knowledge when they are learning parabola functions.

2.7 CHAPTER SUMMARY

This chapter discussed the theoretical framework that framed the study of Grade 10 learners' academic experience in learning parabola functions. The researcher then gave the definitions and the importance of the general theory and theories in education. The chapter continued by discussing constructivist learning theory followed by how Jean Piaget's constructivist learning theory framed the study. The researchers further explained the significance of Newman errors analysis learning theory and then look at the notion of observational scaffolding. The study moved further to explain ZPD which can be used to close the gap learners have in learning parabola functions. The researcher ended the chapter by looking at constructivist learning theory and knowledge construction. The next chapter presents the literature review that guided the study.

CHAPTER 3: LITERATURE REVIEW

3.1 INTRODUCTION

The previous chapter discussed the theoretical framework which underpins the study. This chapter provides the literature review which discusses the issues learners experience when learning parabola functions which include learners' performance in parabola functions, the difficulties learners experience when learning parabolas and how teachers teach the parabola functions. This chapter also covers the literature reviewed that assisted the researcher to have a deeper understanding of the topic. It starts by defining what is a literature review and its purpose and discusses the primary and secondary sources in the literature review.

3.2 DEFINITION AND PURPOSE OF LITERATURE REVIEW

According to McCombes (2022:1), a literature review is a survey of scholarly sources on a specific topic. Valdes (2019:190) defines a literature review as a selection of both published and unpublished documents on a particular topic and a critical discussion and effective evaluation of these documents. Kowalczyk (2021:1) argues that a literature review is a process of establishing what is known and not yet known about a topic. The (ibid) goes further to say it involves identifying knowledge gaps on the topic. The information gained from the literature review helps researchers to execute research studies with accuracy. Streefkerk (2022:3) indicates that the literature review has two categories of sources: primary and secondary sources. He further describes the primary source as all sources that give the researcher direct evidence of the phenomena (Streffkerk, 2018:4). For instance, in this study, primary sources are the information about parabola functions obtained through learners' class tests, examination scripts, classwork and homework activities. The reason for using the primary source of the literature review was to identify what has been done and what is lacking in the delivery of Grade 10 parabola functions content.

According to Anon (2019), secondary sources consist of interpretations of the primary source information. Coe (2016:9) argues that secondary sources of literature are the broader knowledge about the phenomena understudy. Secondary sources of literature include professional journal articles, scholarly books, government documents, dissertations and electronic resources (Streefkerk, 2018:4). The knowledge gained from the secondary source helps researchers to gain a deeper understanding of the topic, in this instance challenges and difficulties faced by learners when learning parabola functions.

According to McCombes (2022:2), the objective of the literature review is to justify the planned study by analysing, criticising and finding gaps in the literature. Kim (2022:2) agrees that the objective of the literature review is to educate the researcher on the areas that need further research as well as offer context and rationale for the investigations. Kowalczyk (2021:1) states that the goal of the literature review is to inform the researcher as to whether other researchers have addressed and answered your research questions. It also alerts the researcher on the important issues and gaps the researcher might need to address and shows how others have handled the methodology and design in similar studies.

The researcher explored the following in the literature; how learners learn quadratics equations, definitions of parabola functions, the performance of learners in learning parabola, the difficulties learners experience in learning parabola, difficulties teachers experience in teaching parabola functions, the teacher teaching methods of parabolas and mathematical teaching proficiency.

3.3 LEARNERS' PERFORMANCE AND DIFFICULTIES IN LEARNING QUADRATIC EQUATIONS

Kabar (2018:112) defines a quadratic equation as an equation that can be represented in the form of $\boldsymbol{ax^2} + \boldsymbol{bx} + \boldsymbol{c}$, where $\boldsymbol{a} \neq \boldsymbol{0}$, and $\boldsymbol{a}, \boldsymbol{b}$, $\boldsymbol{c}$ are coefficients of the equations and $\boldsymbol{x}$ is the unknown. Furthermore, Quezada (2020:3) defines a quadratic equation as a polynomial whose highest power is squared of a variable. Mutambara (2020:21) asserts that quadratic equations may be set in the form of $\boldsymbol{ax^2} + \boldsymbol{bx} + \boldsymbol{c} = \boldsymbol{0}$ so that the equation can either be solved by factorisation, completing the square, using the quadratic formula and the graphical approach.

When solving quadratic equation by factorisation the expression $\boldsymbol{ax^2} + \boldsymbol{bx} + \boldsymbol{c}$ is set in the form $(\boldsymbol{hx} + \boldsymbol{x_1})(\boldsymbol{kx} + \boldsymbol{x_2})$ where the effect of multiplying by zero leads to $\boldsymbol{hx} + \boldsymbol{x_1} = \boldsymbol{0}$ and $\boldsymbol{kx} + \boldsymbol{x_2} = \boldsymbol{0}$ where the solutions of the equation are $\boldsymbol{x} = \frac{-\boldsymbol{x_1}}{\boldsymbol{h}}$ and $\frac{-\boldsymbol{x_2}}{\boldsymbol{k}}$ (Mutambara, 2020:21). Factorisation can be done when the terms can be factorised with ease but if they cannot be factorised with integers coefficients then completing the square or use of the quadratic formula must be carried out. Learners should be able to factorise the expression of the form $\boldsymbol{ax^2} + \boldsymbol{bx} + \boldsymbol{c}$ to find the product of the first and last terms which is acx^2. Kim ho (2022:22) alludes that to find two terms whose product is acx^2 and sum is bx is to replace bx and then factorise the resulting 4 terms by grouping. Mental structures required to perform the factorisation are the ability to multiply terms and simplify directed numbers correctly.

Factorisation should be understood as the reverse of expansion since the resulting terms must expand to the original expression (Hoon, 2018:25).

Completing the square leads to the quadratic formula. To solve $\boldsymbol{ax^2 + bx + c = 0}$ by completing the square (Hoon, 2019:26). Make the coefficient of x^2 a unity by dividing the whole equation by the coefficient of x^2, take the constant term to the right hand side leaving the terms in x^2 and x on the left, complete the square by adding the square of half the coefficient of x to both sides. The relation $(\boldsymbol{x + a})^2 = \boldsymbol{x^2 + 2ax + a^2}$ must be understood to see why half of the coefficient of x is added, after adding the square of half of the coefficient of x the expression with x^2, x and the square of half of the coefficient of x is factorised. Take the square roots of both sides and solve the resulting linear equations for x.

According to Nielsen (2015:15), if the method of completing the square is applied to the equation $\boldsymbol{ax^2 + 2ax + c = 0}$ the formula for solving quadratic equations is derived as follows:

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm\sqrt{b^2 - 4ac}}{2a}$$

To use the formula, the equation must be set to zero on one of the sides. Learners must transform the equation to the form $\boldsymbol{ax^2 + bx + c = 0}$. The value of $\boldsymbol{b^2 - 4ac}$ determines the nature of the roots. To use the formula, the learner must have the ability to substitute the formula correctly and perform the arithmetic calculations correctly.

Research conducted by Celik and Guzzel (2018:25) find that learners are unable to find factors of the parabola functions because they rely on procedural knowledge rather than conceptual knowledge. Hence, the teaching of parabola functions should focus on the development of the concept of parabola functions' conceptual knowledge rather than drilling learners to memorise procedures.

Learners should be able to factorise the expression of the form $\boldsymbol{ax^2 + bx + c}$ to find the product of the first and last terms which is acx^2. Kim ho (2022:24) posits that to find the two terms whose product is acx^2 and sum is bx is to replace bx and then factorise the resulting 4 terms by grouping. Mental structures required to perform the factorisation are the ability to multiply terms correctly and simplify directed numbers correctly. Factorisation should be understood as the reverse of expansion since the resulting terms must expand to the original expression (Hoon, 2019:31).

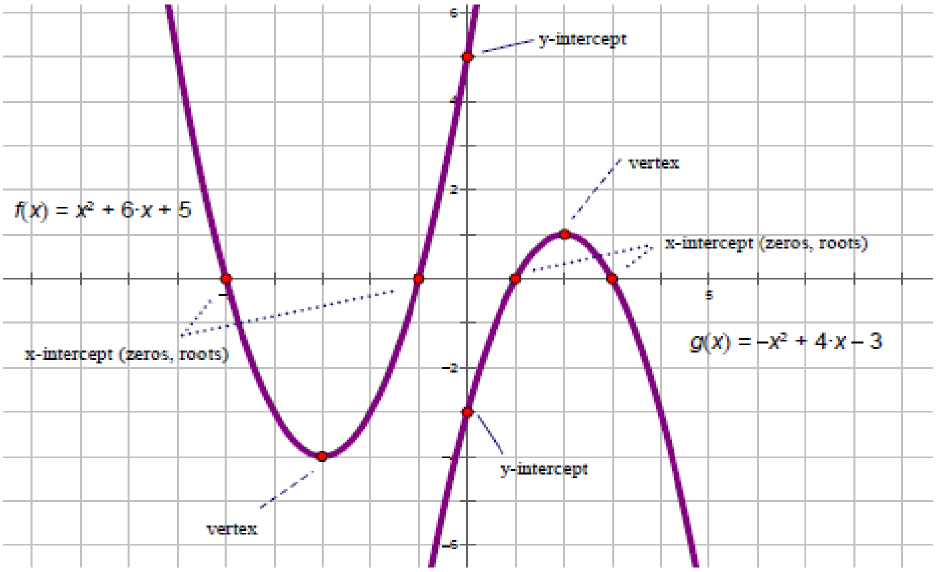
3.4 DEFINITION AND PROPERTIES OF PARABOLA FUNCTIONS

According to Ubah and Bansil (2018:848), a function is an expression that explains the connection between two variables in which the input variable always yields one output variable. Ndlovu (2019:53) views a function in Mathematics as a relationship in which any input value results in one different output value. Carballo (2022:13), on the other hand, claims that functions are classed based on the order of degree, which is the highest power of the variables or by the type of graph that the connection produces. Soto (2021:157) maintains that in function there is only one output for each input, such that each domain element is mapped to precisely one range element. The idea of function is more comprehensive since it encompasses linear functions, hyperbola functions, trigonometry functions and parabola functions, among others. As a result, the parabola function was the primary focus of this research.

Parent (2016:9**)** describes the graph of parabola function as a parabola which is recognised by its U-shaped formation. The (ibid) indicates that a parabola function is called a quadratic equation when the standard form equals zero which gives the form ax^2 + bx + c = $\mathbf{0}$. Ortiz (2020:21) argues that a quadratic equation is the equation of the form $ax^2 + bx + c = 0$ where *a, b* and *c* are constants and $a \neq 0$. The author further claims that the parabola functions are most defined in standard form as $f(x) = ax^2+bx + c$ where $a \neq 0$ (Ortiz, 2020:21). According to Ifunya (2018:848), the largest power of the variable in a parabola function is two, and it might have one, two, or no real roots. Mutambara (2010:21) and Uban (2018:848) show that there are different ways to express the parabola functions, such as expressing parabola function in factored form as $f(x)$ = $(x - x_1)$ $(x - x_2)$ and the vertex form $f(x)$ = $a\,(x - h)^2 + q$. In the same vein, Graf, Fife, Howell and Marquez (2018:3) show that the parabola function can be represented in the algebraic general standard form of: $y = ax^2 + bx + c$, vertex form of: $y = a(x - h)^2 + k$ and in factored form of: $y = a(x - r)(x - s)$, where r and s are the x-intercepts. Nielsen (2015:25) elucidates that a parabola function is called a quadratic equation when each of the forms is set equal to zero which gives the standard form of $ax^2 + bx + c = 0$. When we solve for the x values of the equation, we find the solution of the equation. Solving the equation is done by completing the square, the quadratic formula, factorising and graphing (Rueda, 2020:41). By graphing solutions are called roots, zeros or x-intercepts and are essential when the graph intersects with the x-axis. The vertex of the parabola is the turning point of the graph and is a minimum of $a > 0$ and a maximum of $a < 0$ (Mosquera & Uzuriaga, 2018:31).

Parent (2016:10) and Guzel (2018:127) show that learners struggle to comprehend the link between sketching the parabola function and utilising the graph to obtain the parabola's equation. To grasp the parabola function, learners must be taught how to interpret the graph and the various components of the graph this will help widen their comprehension of parabola functions in general. This will come in handy when learners get to the graphs of third degree polynomials. Understanding the quadratic functions and determining the minimum, maximum and limits, as well as when the function rises vs decreases and when the function is positive versus negative should assist students when exposed to higher order polynomials (Manzindu, 2016:21). The researcher discussed strategies needed by the learners to solve quadratic functions and hence the drawn and interpretation of parabola functions.

Figure 3.1: The parabola functions graph (Source: Parent, 2016:10)



The graph of a quadratic function is a curve called a parabola. When using the graph, learners must interpret the graph and read points from the graph such as the $\boldsymbol{x}$-intercepts which are solutions of the equation $\boldsymbol{ax} + \boldsymbol{bx} + \boldsymbol{c} = \mathbf{0}$ or the $\boldsymbol{x}$-coordinates of the intersection of the graph of $\boldsymbol{y} = \boldsymbol{ax}^{\mathbf{2}} + \boldsymbol{bx} + \boldsymbol{c}$ and the line $\boldsymbol{y} = \mathbf{1}$ which is the solutions of the equation $\boldsymbol{ax}^{\mathbf{2}} + \boldsymbol{bx} + \boldsymbol{c} = \mathbf{1}$ (Parent, 2016:10). The graph of parabola function opens up and has a minimum value when $\boldsymbol{a} > \mathbf{0}$ and opens down and has a maximum value when $\boldsymbol{a} < \mathbf{0}$ (Koram, 2019:164). Parent (2015:10) maintains that the graph can be used to find the maximum or minimum points and the line of symmetry and the graphical approach is used at the ordinary level and is also emphasised in ordinary level Mathematics textbooks. Ubah and Bansil (2018:848) concur that the intercept form of the parabola function can help learners to find the $\boldsymbol{x}$-intercepts of the parabola function and can also assist in finding the axis of symmetry of the parabola function. The parabola function can also be expressed in intercept form (factored form) if it occurs in the roots form of $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{a}(\boldsymbol{x} - \boldsymbol{x}_{\mathbf{1}})\,(\boldsymbol{x} - \boldsymbol{x}_{\mathbf{2}})$, where $\boldsymbol{x}_{\mathbf{1}}$ and $\boldsymbol{x}_{\mathbf{2}}$ are roots of the parabola function graph (Uban and Bansil 2018:848). The graph of a quadratic function can have one real root, two real roots or no real roots (Koram, 2019:164). Koram (2019:164) further maintains that the graph of a quadratic function of $\boldsymbol{y} = \boldsymbol{ax}^{\mathbf{2}} + \boldsymbol{bx} + \boldsymbol{c}$, where $\boldsymbol{a} \neq \mathbf{0}$, the $\boldsymbol{y}$- intercept is $\boldsymbol{a}(\mathbf{0}) + \boldsymbol{b}(\mathbf{0}) + \boldsymbol{c}$ or $\boldsymbol{c}$, the equation of the axis of symmetry is $\boldsymbol{x} = -\frac{\boldsymbol{b}}{\mathbf{2}\boldsymbol{a}}$ and the $\boldsymbol{x}$- coordinate of the vertex is $-\frac{\boldsymbol{b}}{\mathbf{2}\boldsymbol{a}}$. Nielsen (2015:11) concurs that the graph of the parabola has the expression of the vertex form of: $\boldsymbol{a}(\boldsymbol{x}-\boldsymbol{h})^2+\boldsymbol{k}$ and the vertex of the parabola will be at $(\boldsymbol{x},\boldsymbol{k})$.

3.5 LEARNERS' PERFORMANCE IN PARABOLA FUNCTION

The performance of learners in the learning of parabola function is poor. This is confirmed by Hoon, Singh and Halim (2018:82) on the knowledge of the function and quadratic function among secondary students in Selangor, which found that a substantial number of pupils had low achievement on the parabola function. Furthermore, Celik and Guzel's (2018:128) study on a clinical interview to uncover his thoughts on parabola function reveals learners' inadequate grasp of parabola function. While numerous research on parabola function has been conducted like research conducted Zaslavsky (1997), Nielsen (2016), Mutambara (2018) learners' performance in parabola function remains a difficulty; hence, the study of learners' experience in learning parabola function. Learners' poor performance in parabola function might be attributed to challenges encountered when studying parabola function.

3.6 DIFFICULTIES EXPERIENCED BY LEARNERS IN LEARNING PARABOLA FUNCTION

This study was influenced by learners' difficulty in learning parabola functions and plans to develop recommendations and solutions for how these obstacles can be resolved. Learners' challenges result in poor parabola function marks, which are caused by their lack of grasping graphical notions of parabola functions (Hoon, 2018:78). Parent (2015:15) adds that when learning to factor the quadratic function, students face several challenges.

3.6.1 Learners' difficulties in understanding graphs of parabola functions

Many studies show that students struggle to grasp the graph of parabola functions. Parent (2015:114) investigated how learners gain knowledge of the graph of the parabola function using a qualitative study technique that included think-aloud protocols where learners engaged in four quadratic function-based tasks. The study discovered that learners preferred the standard form to the vertex when solving problems on parabola functions and used it to confuse the $\boldsymbol{y}$-intercept of the standard form with the $\boldsymbol{y}$-coordinate of the vertex when the function was in vertex form. Nielsen

(2016:30) also did a study on learner experiences with comprehending and drawing parabola graphs and discovered that learners prefer drawing graphs from provided equations of functions rather than deriving an equation of function from the graph. These may be signs of learners lacking procedural fluency as well as mathematical proficiency. Zaslavsky (1997:25) observed that learners made assumptions about parabolas based on the graphs they saw and did not use their expertise in quadratic functions to assist them to comprehend the functions. According to Zaslavsky (2009:30), students' behaviours are characterised by presuming that the portion of the graph of parabola functions can represent the whole behaviour of the function. For example, learners might assume that if the $\boldsymbol{y}$-intercept is not shown on the graph, there is no $\boldsymbol{y}$-intercept. Students may further assume that a parabola has a vertical asymptote.

Mutambara (2019:9) in qualitative research of 24 preservice educators at a Zimbabwean teachers' college investigated the conceptual grasp of the quadratic function notion. The findings of the study indicated that most students were unable to locate the vertex of the graph of parabola functions. When students create the graph of parabola functions, they may encounter difficulties due to their prior knowledge of linear graphs. The symbols for linear and parabolic parameters are frequently interchangeable.

Linear functions are often represented as $\boldsymbol{y} = a\boldsymbol{x} + \boldsymbol{b}$ and quadratics as $\boldsymbol{y} = a\boldsymbol{x}^2 + \boldsymbol{bx} + \boldsymbol{c}$. Zaslavsky (2009:43) further explains that students' experiences with linear functions in the form of straight lines go back to the previous grade. So, they tend to rely excessively on linear principles. The (ibid) further explains that this leads some learners to try to apply their understanding of linear functions to parabola functions when graphing. For example, when learners want to create graphs of the parabola, they might try to find the slope of a parabola using the value of $\boldsymbol{b}$. The reason learners experience difficulties is a lack of conceptual knowledge, learners are unable to connect different concepts relating to parabola functions.

Furthermore, a study about learners' misconceptions of quadratic functions conducted by Ibeawuchi (2016:420) with a sample of 170 students from 17 classes in South Africa discover the following: most learners treat two different functions as equivalent for example the functions $\boldsymbol{f(x) = x^2 - 3x - 4}$ and $\boldsymbol{f(x) = 3x^2 - 9x - 12}$ and the reason given by the learners who had the difficulty is that after dividing by 3, the second function seems the same as the first and other similar functions which are multiples of the first and are treated as identical. However, two quadratic functions with differing leading coefficient values are distinct since they differ in all other coordinates except the x-intercepts and the vertex's x-coordinate. The students approached the functions as though they were dealing with quadratic problems. The issue stems from learners having completed solving quadratic equations and being taught this notion that analogous equations are the same and so they might act on smaller equations.

Students restrict the graph of the quadratic function to the visible region.

There is a tendency to read the graph of a quadratic function like a picture and therefore fail to understand that the graph crosses the y-axis if the graph shown does not show the y-intercept. Learners also fail to interpret the quadratic function as having an infinite domain. Similar findings were observed by Nielsen (2016:30).

The quadratic function is thought to be linear by students. The midpoint of two nearby points on the parabola is on a straight line with the two points, according to the pupils. The distance between the two points on the parabola seems to the learners to be a straight line; nonetheless, it is a curve since a parabola can pass through three collinear points regardless of how close they appear to be. Learners ascribing linearity also revealed itself in other ways, such as learners' inclination to unite the points they plotted while sketching a parabola with straight lines rather than curves. Over-attachment results from the fact that learners are taught linear functions initially, and they over-generalise their conjectures when they study quadratic functions. Ibeawuchi observed that Learners describe a special point by only one coordinate as they described the vertex by its x-coordinate only while it is a point and must be described by using a pair of coordinate axes. This concurs with Parent's (2015:114) examining how learners develop an understanding of the graph of the parabola function using think-aloud protocols where learners engaged in tasks focused on the quadratic function, the findings revealed that learners find it difficult to solve problems on quadratic functions (2016:420).

Didis and Erbas (2015:1142) studied the performance of 217 Turkish students in solving quadratic functions. The nature of students' difficulty in defining and solving parabola functions was determined through qualitative analysis of written replies and interview data. The results showed that learners experienced challenges in dealing with the parabola function. Celik (2017:6) further conducted a case study about learners' thinking about quadratic functions and discovered that learners did not know how to find the intersection of parabolic functions on the x-axis and y-axis. Furthermore, the study revealed that learners did not know when the parabola functions should open downward or upwards and did not know how to find the axes of symmetry of the parabola functions. Ayadin (2015:58) concurs that almost all learners in the study were unable to draw different parabola graphs when they were given algebraic statements. Furthermore, the (ibid) indicates that learners were unable to find the turning point of the parabola functions. These findings also concur with Nielsen (2016:30) and Parent (2015:114) on learners' understanding of parabola functions. These findings indicate that research still needs to be conducted on the difficulties learners experience when they are learning parabola functions.

3.6.2 Imposing linear structure on parabola functions

Because the symbols for the parameters of linear and quadratic equations are frequently the same, research shows that while solving quadratic equations, some learners try to apply their understanding of linear equations to quadratics (Nielsen, 2016:85). These findings are consistent with those of Ibeawuchi (2016:421) and Parent (2015:116), the authors discovered that learners struggle with treating two different functions as the same function; limiting the graph of quadratic functions to the visible region; attributing linearity to quadratic functions, and determining a special point by only one coordinate.

According to Malahlela (2017:88), learners apply rules learned from solving linear equations to quadratics, either incorrectly or to try to linearise quadratic equations. Working to isolate the variable by adding or removing words from both sides is an example of abusing these principles, as is dividing both sides by x in the example equation above. The difficulty is linked to the inability to locate components quickly. Didis and Erbas (2015:115) investigated the problem when the initial coefficient or constant term contains numerous variables. Tall and Lima (2014:35) noted that learners sought to convert quadratic problems into linear equations, and tend to use the quadratic formula as the only appropriate way to solving quadratic equations.

3.6.3 Difficulties in solving quadratic functions presented in different forms

According to Nielsen (2015:117), learners constantly struggle to solve quadratic functions that are presented in a different format than what they are used to. For example, a learner who can solve an equation in standard form in which $\boldsymbol{a}$= 1 and *b* and *c* are non-zero, such as $\boldsymbol{x^2}$ + 4$\boldsymbol{x}$ + 3 = $\mathbf{0}$, might struggle to solve equations such as $\boldsymbol{x^2+6x=0}$, in which the constant term is $\mathbf{0}$ and therefore not visible in the equation, and $\boldsymbol{x^2\ -x=8}$, in which the constant is on the other side of the equal sign (Didis, 2011:115). The (ibid) suggests that a quadratic equation in which the parameters *b* or *c* equal $\mathbf{0}$ (examples: $\boldsymbol{y=ax^2\ +c}$ *or* $\boldsymbol{y=ax^2\ +bx}$) does not seem like a quadratic to learners. They may assume that if a parameter does not appear in the equation, then it does not have a value, when in fact it has the value zero or one. For example, students might say that $\boldsymbol{y=ax^2+bx}$ $=\mathbf{0}$ does not have a $\boldsymbol{y}$-intercept because they believe *c* does not exist. In this example, *c* does exist and has a value of zero, and the parabola would have the $\boldsymbol{y}$-intercept at the point ($\mathbf{0}$,$\mathbf{0}$) (Zavlasky, 1997:45).

Learners have strong preferences for standard form, rather than vertex form, $\boldsymbol{y=a(x-h)^2+k}$ or factored form, $\boldsymbol{y=(x-a)(x-b)}$. Parent (2016:120) found that learners did unnecessary procedures as they transformed the parabola function standard form to the other forms. The (ibid) further demonstrates that learners struggle with solving quadratic equations using the factoring approach and discusses potential cognitive challenges associated with non-standard parabola function equations, finding positive and negative solutions to quadratic functions in the form where c is a constant and variable.

3.6.4 Difficulties in interpreting the graph of parabola functions

Learners experience difficulties when they are expected to interpret the graph of the parabola function. Ruli (2018:4) conclude that learners had difficulty comprehending the information included in the function graph. They also fail to identify the relationship between quadratic function and quadratic equations; they are unable to find the similarities between quadratic function and linear function; they cannot re-shape quadratic function within a given parameter and experience difficulties in solving quadratic function factorisation. Memmunum, Ayadin, Dinc, Coban and Sevindik (2015:57) indicate that majority of the learners are unable to answer the questions based on graph interpretations owing to their lack of knowledge of reading graphs. The findings are also supported by Graf, Fife, Howell and Marquez (2018:4), when they assert that learners are unable to find the vertex, minimum and maximum point of the given parabola function. The difficulties learners experience when learning parabola functions are the results of errors and misconceptions they have about parabola functions.

3.7 ERRORS AND MISCONCEPTION IN LEARNING PARABOLA FUNCTION

According to Voon, Julaihi, and Eng (2016:18), errors made by students when solving problems can be the result of carelessness, misinterpretation of symbols, a lack of relevant experience or knowledge related to the topic or concept; an inability to check the answer given; or the result of misconceptions. Misconceptions indicate a misunderstanding of the concepts, whereas mistakes reveal the erroneous application of the principles. According to Olivier (1989:13), mistakes are systematic incorrect replies that derive from underlying conceptual frameworks. Makonye and Fakude (2017:2) indicate that errors are slips and departures from correctness caused by mistakes that occur without any patterns. Knowledge is a spontaneous reaction that occurs in a learner's mind as a result of what they are experiencing or the information they are given (Makonye, 2016:2). Misconceptions and procedural knowledge emerge during this process of learning. Learning and knowledge formation and reorganisation processes unintentionally contribute to the accumulation of misunderstandings, which leads to learners committing errors (Malahlela, 2017:20). A misperception emerges as a result of the interplay between what the learners already know and what they are learning.

However, according to Luneta and Makonye (2016:42), learners' misunderstandings should be seen as a regular part of the learning process since they allow for the development of new genuine information to be handled effectively. According to Malahlela (2017:21), it begins with a learner seeking to assimilate a new thought into an old schema. However, it is not always possible to associate every new thought with an established schema. Therefore, a learner will create a new "box" in which s/he will attempt to memorise the new notion. The learner is then compelled to try to memorise the new notion, resulting in what is known as rote learning - isolated knowledge that is difficult to recall. This isolated information leads to misunderstandings which result in mistakes. In the study, learners' misconceptions and errors when factorising parabola functions and sketching the graphs were used as a basis to cultivate learners' understanding of parabola functions. Misunderstandings arise through overgeneralisation of past information, among other things, due to the subject's cumulative character, because any new learning is dependent on previous learning

(Lili, Julailhi & Howe, 2016:19). The (ibid) go further to indicate that for learning to occur there must be some type of generalisation. It just so happens that learners frequently overgeneralise, which leads to misunderstandings that cause them to make blunders (Malahlela, 2017:21).

Perceptions and recognition of learners as thinkers are reasonably obvious (Kazemi & Stipek, 2001:71). The authors argue that learners' errors and misunderstandings should be accepted as part of the process of developing knowledge. This was important for the study as it looked at how learners think and construct knowledge and how misconceptions and errors occur when they are learning parabola functions. The researcher drew on Makonye and Luneta (2016:120), and identified links between learners' mistakes and misunderstandings.

Misconceptions are difficult to rectify because learners do not easily absorb new concepts, but rather incorporate new ideas into the current schema (Makonye & Luneta, 2016:120). When learners' errors are evaluated and diagnosed, it is always important to uncover the core cause and choose the best strategy to rectify the problem (Makgakga, 2016:5). This study used the actual misconceptions and errors learners commit in learning parabola functions to help them move to the next level of parabola functions understanding.

According to Makonye and Stewell (2016:18), the processes that occur when learners create knowledge result in "procedural bugs," also known as cognitive conflict which teachers must exploit to grow learners' knowledge. While misunderstandings cannot be avoided, they must be allowed and accepted as part of the learning process. A teacher must use misconceptions as a chance to improve knowledge creation and learning. As a result, there is a significant interrelationship between cognitive growth, mistakes and misunderstandings in the development of knowledge (Malahlela, 2017:21). This study aimed to explore difficulties learners experience in learning parabola functions. As a result, Olivier, (1989:13); Kilpetrick, (2008:115); Luneta and Makonye, (2016:42); Makgakga, (2016:6) refers to the five strands of mathematical proficiency which are procedural errors, conceptual errors, generalisation errors, incomplete application and interference.

3.7.1 Procedural errors

These are mistakes that arise when a student fails to carry out the methods required to finish the given mathematical problem. Makgakga (2014:16) defines a procedural mistake as a learner's failure to employ appropriate processes in addressing a mathematical issue by following incorrect rules, techniques and procedures. According to Mamba (2012:17), procedural mistakes are errors that students make when attempting to solve a mathematical issue. Procedural errors can be identified when learners are failing to follow the necessary procedure of completing squares, using the quadratic formula.

3.7.2 Conceptual errors

According to Salihu (2017:42), conceptual errors occur when learners misinterpret mathematical rules, definitions or concepts. Mutambara et al. (2019:4) contend that learners who do not have a genuine knowledge of the basic ideas would struggle to acquire additional concepts. Furthermore, Priyani and Ekawati (2018:3) suggest that a student who does not comprehend a subject would use ineffective problem-solving strategies. It is the error that occurs when learners lack basic knowledge of determining the slope, x- and y-intercept of the parabola function.

3.7.3 Generalisation errors

According to Malahlela (2017:27), mathematics learning revolves around some forms of generalisation. Generalisation can occur in the form of over-generalisation of rules and over-generalisation of operations (Mazhindu, 2016:27). For example, learners fail to apply zero property and apply linear function rules to solve a parabola function.

3.7.4 Incomplete application of the rule

This occurs when a student successfully applies a rule but is unable to move to the next stage of the solution. This usually occurs when a learner can calculate the axes of symmetry effectively but unable to find the vertex parabola functions (Mutombo, 2015:14).

3.7.5 Interference

Graf et al. (2018:5) state that when new knowledge is presented learners find it difficult to assimilate and accommodate it to an existing schema and this results in learning sometimes becoming unsuccessful and this is when interference occurs. Interference occurs when learners fail to assimilate and accommodate new knowledge into existing schemas. Kaufman (2017:15) indicates that mistakes and misunderstandings are difficult to repair due to the presence of schemas learned at an earlier level. For example, a learner who was earlier taught that division always gives smaller and multiplications always give bigger numbers will find it difficult to accept that this is not possible for every calculation. Learners will also experience problems when they are using the formula of quadratic to determine intercepts on the x-axis. Errors and misconceptions learners experience can be addressed if learners can master Kilpatrick's five strands of mathematical proficiency.

3.8. MATHEMATICAL PROFICIENCY NEEDED IN LEARNING PARABOLA FUNCTIONS

Kilpatrick (2001:116) proposed five strands that learners should have to learn mathematics successfully using Mathematics proficiency (MP) and these are conceptual comprehension, procedural fluency, strategic competency, adaptive thinking and productive disposition.

3.8.1 Conceptual comprehension in learning parabola functions

The grasp of mathematical ideas, procedures and relations is referred to as conceptual understanding. According to Kilpatrick et al. (2001:139), conceptual comprehension refers to the linkages of the functional grasp of mathematical notions. Todorova (2016:3) defines conceptual understanding as a network of knowledge that develops in a relationship between existing and new information. Conceptual knowledge is based on facts, concepts or principles on which the notion is built (Makgakga (2016:14). According to Osterman and Brasting (2019:466), to reinforce the comprehension of mathematical concepts and conceptual understanding should be taught in mathematics classrooms. The (ibid) goes on to say that conceptual knowledge cannot exist in isolation, but must be combined with procedural knowledge. Ndlovu and Brijlall (2015:12) define conceptual knowledge as "the instrumental comprehension in which learners may apply computation rules to solve mathematical problems." Conceptual knowledge is more significant in mathematics learning since it helps learners to successfully use procedural information. Learners must have conceptual knowledge because, without it, they will be unable to engage in problem solving and would utilise incorrect processes to address issues. Mutambara (2019:1) goes on to say that learners with conceptual knowledge comprehend more than separate information and methods. Focusing on learners' conceptual grasp of mathematical topics increases pupils' likelihood of studying Mathematics freely and acquiring more abilities as needed (Makgakga, 2016:16).

The way learners think about a subject is determined by the cognitive framework they have formed (Battista, 2001; referenced in Makgakga, 2016:17). Osterman and Brasting (2019:466) argue that conceptual knowledge cannot work in isolation however it operates in partnership with procedural comprehensions. Celik and Guzel (2017:127) argue that learning is a cognitive action and teaching the concept of parabola functions must consider learners' prior knowledge as incorrect or incomplete, this understanding will enhance proper concepts construction and conceptual understanding.

To enhance comprehension of mathematical concepts, conceptual knowledge should be taught in mathematics (Osterman & Brasting, 2019:466). For example, learners should be taught how to find factors so that they can factorise effectively to find the x-intercept of parabola functions. Learners' who do not have conceptual understanding would be unable to come up with examples of the concepts, use the concept to form models, and diagrams, understand and utilised the concept's facts and define, contrast and compare concepts and related principles. Parabola functions' conceptual understanding will be attained when learners choose the best method of finding factors of quadratic functions between quadratic formulas, completing squares and factored form. Oysterman and Brasting (2019:467) argue that conceptual understanding should be used to reinforce procedural knowledge as it enables learners to move from one formula to another formula. For example, conceptual understanding can enable learners to move from quadratic equation to parabola functions equation: $\boldsymbol{f(x) = ax^2 + bx + c}$ to $\boldsymbol{ax^2 + bx + c = 0}$ and to move from one parabola functions to the next such as: $\boldsymbol{f(x) = x^2}$ to $\boldsymbol{y = ax^2 + p}$ then to $\boldsymbol{f(x) = ax^2 + bx + c = 0}$ and then to $\boldsymbol{y = a(x-h)^2 + k}$.

3.8.2 Procedural fluency in learning parabola functions

Procedural fluency is understanding when and how to use a procedure and being able to perform procedures accurately and effectively (Foster, 2017:122). Wojcik (2018:18) agrees that procedural fluency is the cognitive process of effectively and precisely following processes, which includes the use of procedures and formulas to solve problems. Procedural fluency is the ability of learners to perform processes flexibly, accurately, effectively and properly Kilpatrick et al. (2001:121). Hussain (2018:26) postulates that when learners are given a chance to use their strategies and procedures in solving mathematical problems, they develop procedural fluency.

McPherson (2016:4) asserts that procedural fluency is an important component that helps learners to choose the most suitable procedure. Kilpatrick (2001:122) concurs that a suitable level of procedural fluency is necessary for learners' to learn parabola functions effectively. Learners who have attained procedural fluency in learning parabola functions can demonstrate it by applying correct procedures in quadratic formulas and completing the square and factored methods when finding factors of quadratic functions. Furthermore, learners can be able to find points on the Cartesian plane to follow the steps applied in quadratic functions and steps used in finding the roots of parabola functions.

3.8.3 Strategic Competence in learning parabola functions

Strategic competency is the capacity to develop, express and solve mathematical problems (Kilpatrick et al., 2001:12; Syukriani, 2017:3) Syukriani (2017:3) goes further to say strategic competence entails the activities that occur in the learner's mind when formulating problems solving strategies, representing problems, understanding problems as well as choosing an appropriate answer to the problem. McGlamery (2017:3) agrees that strategic competency begins with the capacity to conceive mathematical issues, then depict the problem and begin to solve it. Hussain (2018:27), on the other hand, maintains that learners who have achieved strategic competency may use their own techniques to tackle challenges. The ability of students to formulate their own problems helps learners to represent the problem differently, verbally or graphically which enhances their understanding (Makgakga, 2016:67). Strategic competence enables learners to use various approaches or strategies to find the turning point, to find the zeros, to be able to switch from completing the square to factorisation and to understand how to find the range and the domain of parabola functions.

3.8.4 Adaptive Reasoning in learning parabola functions

Adaptive reasoning is the ability of the learner to utilise logical thought to make explanations, reflections and justifications (Kilpatrick, 2001:129). According to Awofala (2017:499) and Wibowo (2016:240), adaptive thinking, is the capacity to think rationally about the link between ideas and situations. It allows learners to evaluate various ways and defend results (Awofala, 2017:499). Rizki (2018:1) agrees that adaptive thinking comprises the ability to analyse, explain, justify and think rationally about the link between concepts and circumstances. In addition, McCluskey (2016:449) avers that adaptive reasoning is the ability to have logical thinking, reflection, explanations and justifications. It enables learners to understand why they use a particular method to find the zeros or to find the turning; it also enables learners to explain why they used the equations to find the turning points.

3.8.5 Productive disposition

Kilpatric et al. (2001:131) describe productive disposition as the stage where the learner sees Mathematics as sensible, useful and worthwhile which enhances self-efficacy. Productive disposition is the ability to perceive Mathematics as worthwhile (Woodward, 2018:161). According to Kusmaryono (2019:344), a mathematical disposition is a belief that Mathematics is logical, useful and valuable. Putra (2017:2) posits that productive disposition has to do with learners' persistent curiosity and flexible thinking when solving mathematical problems. Gael (2017:3) avers that developing a productive disposition toward Mathematics is key to being successful at school. A productive disposition enables learners to believe that Mathematics is a real subject and become motivated in solving mathematical problems. Kilpatrick et al. (2001,131) agree that creating a mathematical productive disposition in students allows them to make sense of Mathematics in the real world, realise the advantages of perseverance and feel the joys of making sense of Mathematics. For this study, learners' attainment of productive disposition assists learners to connect parabola functions to what is happening in the real world, such as the motion of high jumping athletes and the flight motion of a parachute, and it assists them to realise that Mathematics is real, which motivates to do the subject.

3.9 EFFECTIVE TEACHING STRATEGIES FOR TEACHING PARABOLA FUNCTION

There are numerous approaches to overcoming the learning challenges associated with quadratic functions. Rahmawat (2016:35) reveals that visual-based scaffolding may help overcome quadratic function problems. Parent (2016:126) indicates that one way to reduce learning hurdles is by re-formulating the current quadratic rule to be more easily recognised by learners.

According to Ruli et al. (2018:5) cited Schiro (2008), teaching parabola functions entail three basic operations: careful diagnosis and observation of students and individual needs and interests. Ensuring that the physical, social, emotional and intellectual environment in which learners can learn is conducive to learning; and facilitating learners' growth by intervening between them and the environment. Star (2015:35**)** posits that learners need to be challenged to use different strategies when solving parabola functions. For example, if you want learners to find the roots of quadratic functions expose them to the factored form, quadratic formula and completing a square method.

Benning (2016:26) investigated the impact of employing a spreadsheet in quadratic function education on the performance of senior high school students. The (ibid) has discovered that spreadsheet enhances learners' understanding of graphs of quadratic functions and help them to have a visual connection between the graphs of quadratic functions and the corresponding quadratic equations. These findings are consistent with Agye and Voogt's (2015:601) who assert that utilising a spreadsheet helps learners to check results, and identify connections between quadratic functions and graphs. To determine the influence of training quadratic functions with the use of GeoGebra Software is effective in improving learners' achievement and results of quadratic functions (Ovez, 2018:4).

3.10 KNOWLEDGE AND PEDAGOGICAL CONTENT IN MATHEMATICS

3.10.1 Understanding of how to teach parabola functions

Mathematics topic knowledge and the teacher's pedagogical content knowledge (PCK) both plays a role in learners’ understanding of parabola functions. According to Ma'rafi (2016:1), PCK is a teacher's knowledge that blends Mathematics content knowledge, pedagogy and how learners think, whereas Mathematics content knowledge refers to the teacher's mastery of the subject matter. Ekawati (2018:1) highlights that the nature, depth and organisation of teacher content knowledge and PCK influence the teacher’s presentation ideas and learners’ responses to questions. Shulman (1986:14, 1987:369) concur that this comprises the educator's knowledge base, such as general PCK, with specific reference to broad concepts and tactics of classroom management and subject matter organisation. Makgakga (2015:43) maintains that learners' knowledge of educational contexts, such as how classroom groups work, content knowledge, curriculum knowledge, a particular grasp of the materials and PCK, and their unique form of professional understanding plays an important role in learners' understanding. Studies, on teacher education undertaken with a focus on PCK, includes Gess-Newsome (1999:4); Verloop, Van Driel, and Meijer (2001:441). Among the seven knowledge domains, content and pedagogy are regarded as the primary knowledge domains (Makgakga, 2016:44).

According to (Makgakga, 2016:44), for Mathematics instructors the seven knowledge areas established by Shulman (1987:8) are divided into two categories. One type of domain is mostly subject-specific, which educators require in order to properly teach Mathematics in the classroom (Tsang & Rowland, 2005:11). This type of information is known as content knowledge. The other knowledge domain is not subject-specific, but generic and provides teachers with an opportunity to function professionally in the field of education (Tsang & Rowland, 2005:11).

3.10.2 The importance of Mathematics content knowledge in teaching parabola functions

According to Ubah (2018:848), for successful teaching and learning of parabola functions teachers need to have a strong grasp of topic knowledge. Deacon (2016:30) asserts that a teacher's understanding of Mathematics is connected to classroom instruction and impacts learners' academic progress. Shulman (1988:9) defines content knowledge as the quantity and organisation of information in the teacher's mind. According to Shulman (1988:9), content knowledge is the knowledge that comprises facts and concepts. As a result, successful teaching requires an awareness of what the teacher knows, how much they know, and what they should know (Ndlovu, Amin & Samuel, 2017).

Mathematics subject knowledge by the teacher means a comprehensive grasp of mathematics in depth, breadth, connectivity, and completeness (Raney, 2016:36). The (ibid) defines teachers' mathematical content knowledge as a deep mastery of fundamental mathematics. This defines the information that Mathematics teachers should have in order to teach. Schoenfeld and Kilpatrick (2008:323) argue that teachers must have mathematical topic knowledge in order to successfully educate learners. According to Hsi Wu (2017:10), instruction of mathematical content knowledge is specialised. The (ibid) further asserts that common content knowledge is carried by an adult who can solve mathematical problems using various techniques however, specialised content knowledge is the comprehension of mathematical material that is specific to education. Effective Mathematics teachers need specialised mathematical knowledge to teach learners. The Australian Association of Mathematics Teacher asserts that good Mathematics teachers ought to demonstrate the mathematics appropriate to the grade level (Shulman, 1986:14).

The teachers' knowledge of Mathematics can be demonstrated in various ways (Livy & Vale, 2011:23). Teachers with strong subject knowledge typically exhibit a mastery of mathematical principles which allows them to explain the concept of mathematical linkages (Livy & Vale, 2011:23). Teachers and learners can link mathematical principles during teaching and learning (Ball et al., 2009:399). Furthermore, mathematical linkages are significant when teachers apply their expertise, experiences, and mathematical knowledge to remedy low performance in parabola functions. Teachers who comprehend mathematical ideas have a vision and a fundamental comprehension of the sorts of questions that can be used to encourage learning.

Educators should also know when to help, when to be patient and when to enable learners to solve mathematical problems (Ball & Bass, 2009:96). According to Aksu (2016:36), an educator who lacks mathematical understanding is unable to address learners' challenges. On the other hand, knowing the subject matter thoroughly may not be beneficial for teaching mathematical ideas, educators need to make sense to their learners, understand how they work and find methods to make mathematical concepts more accessible to them (2016:36). According to the National Mathematics Advisory Panel Report (2008:9), primary teachers' Mathematics course do not predict their pupils' success. What is most important is having mathematical knowledge and thinking about how to apply it in a meaningful way in the field of mathematics education.

3.10.3 The role of pedagogical content knowledge in teaching parabola function

According to Ubah's (2018:32) research on pedagogical content knowledge (PCK), it is extremely significant in improving the quality of teaching and learner performance in Mathematics. Shulman (1987:9) defines PCK as the knowledge foundation that educators must have in order to teach successfully. The (ibid) went on to say that PCK is one of the most critical types of knowledge that teachers should have in order to teach successfully, simply understanding the subject is insufficient to teach it. According to Shulman (1987:14), PCK is dependent on a teacher's subject matter expertise, pedagogical knowledge, and how educators translate this information into various forms to help learners grasp the subject matter. Deacon (2016:25) claims that PCK is a discrete knowledge of the teacher’s cognition. PCK emphasises how teachers connect the subject matter knowledge to the pedagogical knowledge and integrated them into their reasoning.

Shulman (1986:9) defines PCK as “idea representations, illustrations, instances, explanations and demonstrations." It is the method of portraying and structuring the subject matter to make it understandable to others. Bowie and Reed (2016) report that PCK is the discrete knowledge domain of teaching that distinguishes the expert teacher. Furthermore, Ndlovu et al. (2017) argue that while pedagogical knowledge may apply to all teaching courses PCK is subject-specific. It is clear from the preceding paragraphs that teachers must have PCK in order to properly teach parabola functions. As a result, the study was interested in discovering the experiences that learners have when educators utilise PCK in the teaching and learning environment.

3.10.4 Components of PCK

According to Shulman (1987:5), the components of PCK include knowledge of the specific subject matter; knowledge of instructional tactics; knowledge of learners' conceptions; and an awareness of what makes studying a given topic difficult or easy for learners. Furthermore, Uba and Bansil (2018:67) many pre-service educators have erroneous and hazy beliefs about quadratic functions, and only a few students provide a mathematically solid explanation of the equation. A study conducted in Zimbabwe by Mutambara et al. (2018:49) discovered that student teachers were unable to describe the parabola function and were also unaware of the link between range and domain. The (ibid) advocate that teachers must develop instructional strategies that assist students to increase their knowledge of the quadratic function notion. More extensive explanations are required for student teachers who lack topic understanding and similar explanations are also a must for learners.

Celik and Guzel (2018:175) evaluated pre-service Mathematics instructors' PCK and discovered participants ignored potential learners' misunderstandings. In the same study, the framework for analysing PCK (Table 2.1) makes use of knowledge of teaching practices, knowledge of learners and information about the curriculum.

Table 3.1: Framework of PCK (Source: Bukova- Guzel, 2010:1875)

Knowledge of teaching strategies and multiple representations	Knowledge of the learner	Knowledge of the curriculum
- Using appropriate activities in instruction	- Having knowledge of the learners' prior knowledge	- Being aware of the elements of the mathematics curriculum (its conceptions, purposes, etc.)
- Using real-life examples and analogies in instruction	- Having knowledge of the possible difficulties the learners may experience during learning	- Being aware of the variety of instructional tools presented in the mathematics curriculum and how to use them
- Utilising different instructional strategies in the presentations	- Having knowledge of possible learner misconceptions	- Being aware of the instruments to assess the learners' learning and how to use them

This approach appears to solve difficulties from the first phase, where the educators were individually watched, as well as in the second phase, where the observation focused on the learner's experience while learning parabola functions. This knowledge aided the researcher in gaining insight into teachers' instructions and assisted the current researcher with the teachers' expertise on how to teach parabola functions.

Llyas (2016) and Ubah (2018) investigated Mathematics teachers' PCK by analysing how they taught parabola functions as well as their replies to students' questions. Their findings revealed that teachers lacked proper PCK tools for comprehending the features of the problems addressed, the solution production via various techniques, and knowledge of the learners' challenges. Investigations into pre-service teachers discovered that some are competent in some areas but not in others (Star, Carnongan, Foegen, Furgeson & Keating, 2015). Ramma (2014:34) discovered that there was no difference in pre-service and in-service teachers' mathematics skills and that all struggled to demonstrate topic knowledge and PCK. Budayasa and Juniati (2016) assessed the PCK ability of pre-service primary Mathematics teachers and it was discovered that a comprehensive mastery of Mathematics does not ensure effective teaching. Budayasa and Juniati (2016) propose that mathematics instructors be taught in terms of both mathematical knowledge and PCK. These studies found that pre-service and in-service educators have poor PCK. The educators' lack of PCK may hamper their ability to teach parabolic functions successfully. Shulman (1986:3). developed exercises for educators to acquire strong teaching techniques. These activities included mathematical knowledge generated by learners, mathematical knowledge understanding, transformation, instruction, assessment and reflection of teachers.

Transformation requires a combination of the preparation of the text materials and the representation of the teachers' ideas in the form of new analogies and metaphors for presenting the materials. Shulman (1986:7) asserts that teaching instructions comprise teacher activities such as classroom management, lesson presentations, interaction with the learners through questioning, forming and observing group work, and maintaining classroom discipline. In Shulman's activities. evaluation is another stage of pedagogical reasoning. It is the extension of the teaching instructions where the educators assess the comprehension and is concerned with the instructors' grasp of the topic.

Additionally, educators must understand what, when, and how to teach parabola functions in a variety of methods. When teaching parabola functions, educators' mastery of the subject helps the learners grasp the subject, allows them to apply numerous ways to solve mathematical problems and enjoy the learning experience. The second activity is transformation, in which educators must understand how to convert topic material into pedagogical knowledge that may supplement learners' abilities and backgrounds in mathematical learning (Shulman, 1992:15).

Evaluation allows teachers to assess and appraise their own performance and alter their teaching approaches as needed (Shulman, 1986:7). The (ibid) goes further to indicate that reflection is a critical part of the teaching cycle that is sometimes overlooked. Educators can reflect on their teaching practices to construct new information, enact new knowledge and critically analyse their instruction. Reflection is also crucial for students because it allows them to reconstruct, re-enact, and recall classroom experiences and successes (Ornstein & Thomas, 2000:14).

3.11 CHAPTER SUMMARY

The chapter discussed the literature that provides an understanding of the difficulties learners experience when they are learning parabola functions. The concept of parabola functions is the focus of the research, looking at in-depth knowledge of what are the difficulties learners encounter when they are learning parabola functions. Since the study was conducted on the student's comprehension of parabola functions, the topic of quadratic equations and parabola functions was also explored. Then learners' performance and difficulties when learners are learning parabola function were discussed. The chapter also discussed the mathematical proficiency needed by teachers to teach parabola functions effectively, educator pedagogical and content knowledge was further explored.

CHAPTER 4: METHODOLOGY OF RESEARCH

4.1 INTRODUCTION

The preceding chapter examined literature reviews, with an emphasis on the introduction thereof, their sources, their purpose, learners' performance and difficulties in learning quadratic equations, the definition of parabola functions and learners' performance in learning parabola functions. The researcher further espoused the difficulties in the form of errors experienced by learners when they were learning parabola functions and the mathematical proficiency needed in learning parabola functions effectively. The researcher also explored effective teaching techniques for learning parabola functions, as well as the significance of instructor topic knowledge and PCK in efficient parabola function teaching. This chapter discusses the mixed-method research technique used in this study. Research paradigms such as interpretative, positivist, post-colonial, and transformational paradigms are discussed, as well as the use of qualitative, quantitative, and hybrid approaches. In addition, data gathering tools were detailed, as were the data analysis methodologies employed. Following the description and purpose of the instruments employed, sampling, validity, and reliability were also evaluated. Following that, the ethical perspective chosen in performing this study is discussed.

4.2 RESEARCH PARADIGM

Bonache (2020:1) describes a research paradigm as the evaluation and collection of information on a particular subject to answer questions and generate new ideas. According to McMillan and Schumacher (2014:354), the research paradigm is a methodical investigation that collects data in order to produce new knowledge through analysis and interpretation. A paradigm is described as the conceptual prism through which the researcher views the methodological component of the research in order to identify the research methodologies to be employed and how data will be analysed (Kivunja, 2017:26). Creswell (2018:17) further asserts that a paradigm is a system of beliefs that directs activities in entomology, epistemology, axiology, and methodology.

According to Makgakga (2016:83), paradigm definitions imply that paradigms can either intentionally or unconsciously determine the reasoning and expectations of the investigation. There are several research paradigms, such as positivism, constructivism, transformational paradigms, and post-colonial paradigms (Wagner, Kawulich & Garner, 2012:53).

A positivist paradigm is defined as the study of human behaviour and activities, and it sees science as the method to discover the truth and comprehend the world in order to anticipate and govern it (William, 2020:23). According to Gemma (2018:44), positivism is typically connected with experiments and quantitative research that assume information is objective and devoid of the researcher's views and beliefs. In other words, the researcher does not meddle with the event under study. Positivist academics think that the universe is governed by constant and unchanging laws of causality and occurrence (Aliyu, Bello, Kasim & Martin, 2014:89).

According to Panhwar (2017:1), post-positivism comes forward in educational research as a result of the positivist paradigm's limitations of basing its findings on observable and empirical analytical facts. Moreover, Panhwar (2017:1) indicates that in post-positivism, the researcher adopts a mixed paradigm by combining positivism and interpretivist paradigms. According to Creswell (2018:7), post-positivism begins research with theory, then travels to the study site to collect evidence to support or reject a theory, and then revises the procedure so that more tests may be undertaken. As a result, in post-positivism, both qualitative and quantitative research methods are relevant (Tanlaka, 2019:11).

According to Kivunja (2018:33), the interpretivism paradigm postulates that reality is socially constructed and each individual constructs their knowledge according to their experiences and perceptions. The interpretivist paradigm is compatible with qualitative research and provides an alternative to both the positivist and post-positivist paradigms (Kelley, 2018:12). According to Creswell (2018:81), constructivism is considered qualitative research since the researcher asks larger questions so that participants can construct their own meaning through interaction with other individuals. A qualitative researcher interviews individuals using open-ended questions in order to gain a thorough grasp of the data.

According to Kaushik (2019:8), pragmatist scholars acknowledge that research may be conducted utilising single or numerous realities that are open for investigation. Furthermore, pragmatic researchers work against the positivist paradigm where the researchers use a single methodology to discover the truth (Kaushik, 2019:11). Creswell (2018:10) asserts that the pragmatic worldview is founded on the acts, events, and outcomes of the investigation rather than anything that existed previously. Parvez (2016:68) earlier pointed out that pragmatism is a philosophy that is underpinned by mixed method research which uses multiple approaches.

The positivist and interpretative perspectives were used in this investigation. The study, on the other hand, was pragmatic in that it sought to acknowledge Creswell's (2018:10) criteria. The researcher understand that pragmatism allows academics to employ numerous methodologies and perspectives on parabola functions. Because the study does not strive to uncover laws to dispel myths or to promote transformation and social change to historically oppressed people, this employ a post-positivist but not emancipatory, or indigenous research paradigm. The researcher's goal in this study was to investigate the academic performance and difficulties that students face when learning parabola functions and to suggest possible ways for the teacher to best address students' errors and misconceptions in order to improve students' academic performance in parabola functions. Achievement tests, semi-structured interviews, and lesson observation were used to determine learners' performance and difficulties when learning parabola functions, as well as how learners’ learns parabola functions, in order to suggest ways for educators to address errors and misconceptions and improve learners' academic performance.

Creswell's (2018:30) viewpoint that pragmatism permits researchers to employ diverse methodologies, distinct worldviews, and different assumptions, including different types of data collection and analysis that allow for mixed methods research, is accepted by the researcher. The researcher collected data using three methods: an achievement examination, a semi-structured interview, and classroom observations.

4.3 RESEARCH APPROACHES OF THE STUDY

A research technique is a methodical approach to issue resolution (Jain: 2018:34). According to Bilau, Witt, and Lill (2018:599), research methodology is the theory and analysis of doing research that has enhanced the technique used in creating research data and analysis toward knowledge development. Furthermore, Sinyosi (2015:33) claims that research methodology is the design in which the researcher selects data and performs analytic methods in order to create results on a given programme. According to McDonald (2015), numerous research approaches that may be employed while doing research include qualitative, quantitative, and mixed method kinds of inquiry.

4.3.1 Qualitative methods

According to Macmillan and Schumacher (2014:354), the qualitative research approach is naturalistic research in which the researcher joins the world of the participants as it is. Makgakga (2016:86) argues that a qualitative study is one that involves field research, naturalistic inquiry, and case study research.

This study employs field research and naturalistic inquiry principles in the natural environment of learners in rural schools, where a classroom is a natural setting where human behaviour and events occur. The qualitative research method aims at the in-depth investigation of a group or event (Vieira, 2019:1). Moreover, Rodriguez and Fernandez (2019:2) suggest that research can be conducted using qualitative methods for study development, explanatory, and descriptive to understand the problem which is being researched. The qualitative research approach was not used in this study since the qualitative method only employs words to explain data obtained, however this study wants to use both numbers and words to describe data. The study did not adopt a qualitative technique research investigation because the researcher wanted to combine both numbers and words as data of the research. The study also considers the fact that mixing of both number and words is more concerned with the meaning that individuals build, such as how people interpret the term and their experiences in the world (Macmillan et al., 2010:360). The researcher was also interested in learning about the participants' perspectives on parabola functions. The study was conducted in a real-life setting, with no attempt to influence the phenomenon of interest or accept the researcher's subjectivity. (Kobus, 2010:4). The participants were given the opportunity to voice their various perspectives and experiences with studying Grade 10 parabolic functions.

4.3.2 Interpretative approach

According to Quinlan (2018:1), the interpretative research approach relies on meaning making practice which involves inductive reasoning where the researcher conducts research from the experience in the fieldwork. Darby (2019:4) concurs that the interpretative research approach involves inductive reasoning in a real situation. The goal of this study was to understand how students perform academically and the obstacles they have when learning parabola functions, as well as to recommend solutions to correct mistakes and misconceptions in order to enhance their performance in parabola functions.

4.3.3 Qualitative approach

Creswell (2018:32) defines qualitative research as a method for examining and comprehending the meaning of people or groups of persons associated with a social or human problem. According to McMillan and Schumacher (2014:354), the qualitative research technique offers researchers with an in-depth description and comprehension of the human experience in a social situation. This researcher focuses on understanding learners' academic performance and the difficulties they experience when learning parabola functions and their views about parabola functions. Furthermore, Hammarberg, Kirkman, and Lacey (2016:497) claim that the qualitative research technique comprises semi-structured interviews to get perspectives on the specific primary issue. The study also investigated learners' views about their understanding of parabola functions and learners' descriptions of parabola functions gave the researcher a deeper understanding of the knowledge learners has on parabola functions.

4.3.4 Qualitative data

According to Pickell (2021:2), qualitative data is non-statistical data that is unstructured or semi-structured and can be generated through interviews, observations, text and documents. Mohajan (2018:18) mentions that qualitative researchers are interested in people's views, meaning, and experiences related to a certain issue. McMillan and Schumacher (2014:354) say that qualitative researchers must visit the field of study to gather data to answer research questions. This study aimed at collecting data by interacting with educators and learners on the difficulties learners experience when they are learning parabola functions. Because of COVID-19 and observations made when instructors teach parabola functions, the natural venue for this study was telephone interviews rather than face-to-face interviews. According to Creswell (2014:239), data collection in qualitative research comprises establishing the study's limits, gathering information through unstructured or semi-structured observations and interviews, documents, and visual resources, and developing the technique for recording information. The qualitative study includes the researcher visiting the research location to observe and collect field notes on the activities and behaviour of the participants (Creswell, 2018:47). The data collecting instrument utilised in this study is explained further in this chapter's research design section.

4.4 QUANTITATIVE METHODS

Bhandari (2020:2) describes quantitative research methods as involving collecting and analysing numerical data. Basias and Pollias (2018:92) further describe the quantitative method as a method that follows a positivist paradigm that involves a systematic and empirical investigation of the phenomenon using numbers and the processing of numerical data. Quantitative procedures include observations, measurements, and the number of facts observed objectively in research (Johnson & Christensen, 2012:225). Bhandari (2020:2) indicates that quantitative research methods consist of descriptive research, correlational research and experimental research.

4.4.1 Descriptive research method

In the descriptive research method, the researcher seeks an overall summary of the study variables (Bhandari 2020:2). McCombes (2019:2) further asserts that in the descriptive research method, the researcher does not control or manipulate variables but only observes and measures the variables. In this study, the researcher used the descriptive research method because the researcher wants to get descriptive views of learners' academic experience when they are learning parabola functions.

4.4.2 Correlational research method

In the correlational research method, the researcher investigates the relationship between the variables of the study. Cherry (2021:1) concurs that a correlational research method is a type of method that looks at the relationship between two or more variables.

4.4.3 Experimental research method

The experimental research approach investigates whether a cause-and-effect relationship exists between the variables (Cherry, 2022:2). Bevans (2019:2) previously agreed that in the experimental research approach, one or more variables are manipulated and their effects on one or more variables are measured.

4.5 MIXED METHODS

According to George (2021:2), mixed method research incorporates quantitative and qualitative research components. Creswell (2018:268) asserts that mixed method research involves both quantitative and qualitative data. The relevance of combining both types of methodologies within a single research stems from the assumption that neither qualitative nor quantitative methods are typically sufficient to compare the trends and features of a certain scenario (Cohen, 2018:31). Similarly, Crane, Henriques, and Lusted (2018:12) state that mixed methods research is a method in which the researcher mixes qualitative and quantitative approaches into a single study. This study employed mixed method research enquiry. The choice for a mixed method study is because the researcher intends to use both quantitative and qualitative approaches to make a stronger case. The researcher also considers the fact that mixed method research is more concerned with understanding the perspectives people have constructed, like how people perceive the world and the experiences they have in the world (Creswell, 2018:267). The researcher is also interested in learning about the participants' perspectives on parabola functions. The participants were given the opportunity to voice their various points of view and beliefs about their low performance in Grade 10 parabola functions. The researcher needed a deeper understanding of the need for and impact of an intervention programme by combining quantitative and qualitative data (Creswell, 2018:267).

4.5.1 MIXED-METHOD APPROACH

A mixed method research technique, according to Creswell (2018:235), is one in which the researcher gathers and analyses both qualitative and quantitative data within the same study. A mixed method research, according to George (2021:1), includes both quantitative and qualitative aspects to address research problems. The goal of this study was to investigate learners' academic performance and the issues Grade 10 students have when learning to solve parabola functions in order to offer solutions to correct mistakes and misconceptions and enhance learners' academic performance of parabola functions. A mixed methods approach was employed to collect quantitative and qualitative data for this study utilising achievement examinations, semi-structured interviews, and instructional observations. Data were triangulated to provide a better case for this study's explanatory quality (McMillan & Schumacher, 2014:364). The researcher confirmed equal representation of qualitative and quantitative data in this investigation to make stronger case.

4.6 MIXED METHODS RESEARCH DESIGNS

Cohen (2018:254) describes research design as a field of study within qualitative, quantitative, and mixed methodologies approaches that provides guidance for research techniques. Further, Makgakga (2016:95) defines research design as a strategy that outlines how the researcher intends to solve the study challenge. To get a thorough grasp of the research topic, this study employed a mixed-methods research technique that included the collecting, analysis, and blending of both quantitative and qualitative data inside a single study (Venkatesh, 2016).

4.6.1 Exploratory mixed methods design

According to Creswell (2018:256), an exploratory mixed method design is one that begins with gathering and analysing qualitative data and ends with quantitative data and final interpretations of the data set. Lall (2021:5) agrees that an exploratory design is one that begins with the gathering and analysis of qualitative data in the first stage, followed by quantitative data analysis that builds on the qualitative results in the second stage.

4.6.2 Convergent parallel mixed methods design

According to Demir and Pismek (2018:123), convergent parallel methods design is a research design in which the researcher collects quantitative and qualitative data concurrently in the same phase of the research process, weighing the approaches equally and evaluating the findings collectively. According to Berman (2017:5), a convergent parallel research design is one in which both quantitative and qualitative data strands are gathered and analysed concurrently or sequentially to answer research questions in a single study. Although this study included both qualitative and quantitative data, the quantitative and qualitative data were not gathered and analysed concurrently, but separately; hence, the convergent parallel techniques design was not used.

4.6.3 Explanatory design

According to Creswell (2018:224), the explanatory design is a sequential design in which the researcher begins by collecting quantitative data, analyses the data, and then uses the results to build on qualitative data (McMillan & Schumacher, 2014:365). The qualitative data elaborate on the preliminary quantitative findings of the first step (Cohen 2018:186). This technique intended that quantitative data should offer a broad overview of learners' performance and issues encountered while learning to solve parabola functions in order to inform the researcher to the sorts of questions they may ask when gathering qualitative data.

In the explanatory sequential design, quantitative and qualitative data are analysed individually (Creswell, 2018:224). The researcher is aware that explanatory design is the simplest way to apply, but it takes longer to implement due to its sequential structure. In this study, the researcher adopted an explanatory sequential design to gather and analyse quantitative data first, followed by qualitative data.

The researcher investigated learners' academic achievement as well as the problems they face when learning parabola functions in this study. The study focuses on Grade 10 learners' academic performance in parabola functions and the difficulties learners experience when they are learning parabola functions and the ways teachers can address errors and misconceptions to improve learners' academic performance in parabola functions. This study adheres to the pragmatist paradigm, and a mixed methods approach was employed to acquire a deeper understanding of academic performance and the obstacles that Grade 10 students have when studying parabola functions.

Explanatory research design is one that use qualitative data to aid in the explanation or elaboration of quantitative findings. In an exploratory research approach, quantitative data is used to evaluate and explain a link discovered in qualitative data. The purpose for utilising a sequential explanatory research method in this study is to include data from the test into data from semi-structured interviews and observations. Creswell (2016:23) agrees that the sequential explanatory study approach gathers and analyses numerical data first. According to Creswell, the purpose of this approach is to tell the researcher about the characteristics of the individuals who would be investigated in depth using qualitative methodologies. The sequential explanatory study approach, according to Venkatesh et al. (2016:45), provides for a more comprehensive analysis of the topic under inquiry.

The qualitative data gathering and analysis phases were followed by a quantitative phase (Venkatesh, Brown & Bala, 2016). This mixing would occur when the preliminary quantitative results were used to inform the gathering of secondary qualitative data (Creswell, 2018:145). In the first phase, Grade 10 students were given an achievement test to measure their academic performance on parabola functions. The second step includes semi-structured interviews and course observation to better understand the issues learners have when learning parabola functions in the form of mistakes and misunderstandings. Qualitative research assists the researcher in understanding the unique experiences of individuals from their point of view on the topic of parabola functions, and it produces data that cannot be obtained using statistical techniques or quantitative methods (Cohen, Manion & Morrison, 2018;78). After analysing data from achievement assessments and semi-structured interviews, lesson observations were undertaken.

4.7 STUDY POPULATION AND SAMPLING PROCEDURES

Majid (2018:5) defines a population as a collection of people who share qualities that the researcher is interested in. The study's target group included all Grade 10 Mathematics students and instructors from two schools in South Africa's Limpopo Province's Vhembe West District. These schools had 146 Grade 10 students and three Mathematics teachers. In the study, the researcher used 90 of 146 Grade 10 Mathematics students and all three teachers from two schools.

4.7.1 Schools' profiles

Schools in the research were assigned codes School A and B, with the capital letter A representing the numerical order of the school visited first. End-of-year mathematics results in School A and school B were represented by the table below:

Table 4.1 Presenting school A and B mathematics performance from 2019 to 2021

Year	School A mathematics Performance	School B mathematics Performance
2019	25%	33%
2020	11%	26%
2021	6%	20%

It was also possible to gather information about the teachers' qualifications and professional experience. Two of the three instructors who took part had a three-year diploma and an advanced certificate in teaching (ACE). The instructor in a pilot research group has a four-year teaching degree and more than 10 years of teaching experience.

Both schools are located in Limpopo Province's Vhembe West District. Furthermore, the schools are both public and formerly underprivileged. Tshivenda is the learners' native tongue; it is the language they use at home as well as when they play and converse informally at school.

4.7.2 The research sample

A sample, according to Boyd (2022:1), is a collection of persons, things, or stuff drawn from a larger group for measurement. The study sample was taken from two schools in the Vhembe West District of Limpopo Province, which nearly reflected the study's population. The study's population included 90 learners in Grade 10 who were studying Mathematics. A pilot study was developed and delivered to 45 learners and educators from one school. The research included 60 learners from School B, with an average of 30 learners per class. Two teachers taught these learners from three classes, one of them having more than 10 years of teaching experience. This sample included four classes of 30 students each. Teachers were included because their instructional approaches have an impact on their students' academic success (Bhandari, 2022:1).

4.7.3 Sampling procedures

According to Creswell (2018:48), sampling is a point of data collecting that might be a person, document, institution, venue, or any other instance of data gathering. Cohen

(2018:49) argues that a sample is a subset of the population. Boyd (2022:1) further indicated that sampling is the process of selecting a representative portion of a population or identifying parameters or characteristics of the entire population. McMillan and Schumacher (2014:365) added that a population is the total group of individuals in a context in whom a researcher may be interested. Moreover, Cohen (2018:224) avers that there are several forms of probability sampling, including simple random sampling, systematic sampling, stratified sampling, and cluster sampling. Furthermore, he claims that convenience sampling, snowball sampling, quota sampling, purposive sampling, maximum variation sampling, critical case sampling, and theoretical sampling are all examples of non-probability sampling (Cohen, 2018:225).

The participants in this study were chosen through purposive sampling. Purposive sampling employs homogenous sampling because it clearly identifies the roles, views, information, and experiences that are relevant to the investigation (McMillan & Schumacher, 2014:365). Crossman (2020:1) claims that purposive sampling is employed for in-depth topic analysis. The author also contends that purposive selection has the ability to pick a rich sample, which, when thoroughly investigated, may yield rich information pertaining to the topic of investigation (Crossman, 2020:1). The researcher employed purposive sampling because he reviewed the 2015-2019 diagnostic report and saw that certain schools performed really poorly. The researcher sought to investigate the causes of learners' low performance thus, the researcher purposefully chose these schools with bad performance. Homogeneous sampling was utilised to pick Grade 10 Mathematics students because students added essential perspectives, information, and experience to the research. The researcher wished to ascertain learners' knowledge and opinions in terms of their academic experience, as well as challenges in the form of mistakes and misconceptions when studying parabola functions.

Participants were learners from two schools in Limpopo Province's Vhembe West District. The study's sample included roughly 90 learners who majored in Mathematics in Grade 10. Learners from School A took part in a pilot research with 30 other learners and educators. In School B, 60 learners participated in the research, with an average of 30 learners in each in each class.

This research purposively selected seven educators who taught these classes. This study was conducted at the learner level, and educators were included since educators' instructional approaches may impact learners' academic success (Berndt, 2020:3).

The researcher also looked at how the educators' introduced the topics because this might affect learners' knowledge and accomplishment. A sample, according to Mugere (2016:109), is a collection of people, things, or items drawn from a larger group for measurement. Furthermore, Patton (2015:45) argues that this strategy is employed for in-depth problem research. The researcher purposely chose participants because they were learners who chose to pursue Mathematics in Grade 10, making them appropriate to be participants in this study. The learners were chosen based on the attributes they possessed that showed the issue under investigation.

The researcher also looked at how the instructor introduced the topics because this might have affected students' knowledge and accomplishment.

4.8 DATA-GENERATING INSTRUMENT FOR THE STUDY

A data collecting instrument is a device used in research to gather data, such as a paper questionnaire or a computer that helps with interviewing. It also refers to the complete process of building instruments while focusing on technical difficulties like dependability, validity, and trustworthiness (Cohen, 2018:68). Data collection, according to Greenberg (2018:98), is the act of obtaining and measuring information on the variable of interest in order to answer inquiries, present research questions, test hypotheses, and assess outcomes. According to Creswell (2018:274), we have several methods for gathering data, such as interviews, observations, and document or artefact studies.

4.8.1 Identification of data gathering instruments

The purpose of employing achievement assessments, classroom observations, and a semi-structured interview was to explain the experiences of learners and teachers in teaching and learning parabola functions in words and numbers. Quantitative data were collected with a pencil and paper, and students were required to write an achievement examination (see Appendix H). A semi-structured learner interview schedule was used to obtain qualitative data after learners completed the writing accomplishment examination. Cohen (2018:123) defines an interview as a discussion between two or more persons about a topic of interest. The class observation schedule (Appendix M), which was derived from Makgakga (2016:100), was also utilised to gather qualitative data. Observation, according to Cohen et al. (2018:123), allows the researcher to collect live data as it occurs naturally in social situations.

4.8.2 The function of data gathering instruments

Data gathering instruments may be divided into two types: those that are used to collect quantitative data and those that are used to collect qualitative data.

4.8.2 .1 Achievement test

The achievement test was used in the study to assess Grade 10 students' academic performance when learning about parabola functions. The achievement test, as previously stated, consisted of five parts of Grade 10 parabola functions, each with five questions, for a total of 24 questions. The sections included quadratic equations (QE) with two items; parabola function of the form $\boldsymbol{f(x) = x^2}$ (PF1) with six items, parabola function of the form $\boldsymbol{f(x) = -\frac{1}{2}x^2 - 3}$ (PF2) with seven items, interpreting the graph of parabola function of $\boldsymbol{g : y = ax^2 + p}$ and $\boldsymbol{h: y = bx^2 + q}$ with (IPG) four items, and sketching the graph of $\boldsymbol{h(x) = x^2 - 4}$ and $\boldsymbol{k(x) = -x^2 + 4}$ with (SPG) four items. These question items were proposed by South Africa's Curriculum and Assessment Policy Statement (CAPS, 2012:45).

The accomplishment examination was designed to assess students' academic performance as well as their challenges in solving parabola function issues. The challenges that students had in addressing parabola function issues were also taken into account. The sources of the problems were also investigated.

4.8.2.2 Semi-Structured interviews

Semi-structured interviews, in this study, were performed to learn about the participants' experiences and the meaning they create of those events through their descriptions and reflection on those accounts (Cohen, 2018:507). Semi-structured interviews, as previously indicated, were utilised to clarify and comprehend the difficulties raised by achievement test results. Furthermore, the semi-structured interviews were conducted to better understand the underlying thinking of Grade 10 students when solving Mathematics problems, including the notion of parabola functions. Semi-structured interviews were conducted with learners who had fared badly on the accomplishment examination in order to discover the mistakes and misconceptions they had throughout the test. In this study, the researcher interviewed ten students from Schools A and B for 25 minutes at a time.

The semi-structured interviews assisted the researcher in comprehending how students answered queries on parabola function principles. To acquire clarification on questions emerging from the examination of test findings, semi-structured interviews were conducted. For example, when students reported that they were not taught parabola functions in Grade 10, the researcher questioned them further. Furthermore, difficulties with equations and confusing methods employed in problem solution, as well as the rationale for not attempting to answer the questions, were explored. The semi-structured interviews accomplished the following goals:

- to clarify inaccurate test response;
- to probe learners' reasoning behind their reasoning of parabola functions; and
- to seek clarity on how learners, perceive and answer questions on parabola functions.

4.8.2.3 Observations from the classroom

Lesson observations were used to monitor how learners solved parabola function problems and engaged during a Grade 10 parabola function lesson. The researcher was interested in seeing how learners and teachers interacted in relation to what they stated in semi-structured interviews. The researcher noticed how learners unpacked parabola function ideas, such as how they employed parabola function words, how they followed processes for solving parabola functions during the class, and how learners were interested during their provided lessons. Furthermore, a class observation instrument was utilised to monitor how Grade 10 students gain mathematical knowledge on the topic of parabola functions. and how they demonstrated the ways of their reasoning as well as their thought processes of problem solving. The class observation schedule tool gathered information about how students calculated, understood, and solved parabola function issues. Eight lessons from Schools A and B were watched by the researcher. Each instructional observation session was 30 minutes long.

4.8.3 Developing Instruments for data collection

The tools were designed to assist the researcher in collecting data for the study in order to ascertain the following: (1) academic performance of Grade 10 students while calculating parabola functions; (2) problems in the form of mistakes and misunderstandings Grade 10 learners possess when solving parabola functions problem; (3) the best way in which learners' errors and misconceptions in Grade 10 parabola functions can be addressed to improve their academic performance.

4.8.3.1 Construction of the achievement test questions

According to Cohen (2018:563), an achievement test is based on what learners can do or know to find out the difficulties and problems they are encountering learning a concept. The achievement test was utilised in this research to analyse learners' academic performance as well as challenges in the form of mistakes and misconceptions that learners encounter when learning parabola functions.

The researcher created achievement test questions utilising standardised previous examination question papers from 2017 and 2018, as well as the Siyavula textbook, which covered the topic of parabola functions in Grade 10. The Limpopo Department of Education prepared and moderated the end-of-year examinations using standardised prior examination question papers. The question papers and Siyavula textbook fulfil the CAPS assessment requirements (CAPS, 2012:45).

These previous examination question papers and the Siyavula textbook were utilised by the researcher since they are set provincially and regulated by the Limpopo Department of Education and the National Department of Basic Education. Furthermore, the researcher was led in the creation of the instrument by the CAPS papers of Mathematics in Grade 10. According to the CAPS document, when creating a test on the topic of parabolas, the learner should be assessed in the following ways: (1) the learner must demonstrate knowledge of facts related to the concept of parabola functions; and (2) the learner must be able to use routine procedures in solving parabola functions problems. The CAPS document further states that the test must measure abilities, such as (1) finding the x and y intercepts; (2) analysing and interpreting the graph; and (3) determining the range and domain of parabola functions. Figure 4.1 depicts the format of the accomplishment examination used to cover the aforementioned abilities that influenced this study.

Table 4.2: The achievement test's design structure

Items	Significance
1. Quadratic equations	Equation factors can be found through factoring, completing a square, or using the quadratic formula.
2. Parabola function of $\boldsymbol{f(x) = x^2}$	Completing the table, plotting the points on the system of axes to form a graph and determining the range, domain and line in which $\boldsymbol{f(x)}$ is symmetrical and the value of $\boldsymbol{x}$ for which $\boldsymbol{f(x)}$ has a given value.
3. Parabola function representing the form: $\boldsymbol{f(x) = -\frac{1}{2}x^2 - 3}$	Determining the $\boldsymbol{y}-$ intercept, $\boldsymbol{x}-$ intercept, an axis of symmetry, and turning point of the graph, sketching the graph of the $\boldsymbol{f(x)}$ function, and finding the domain and the range of the functions.
4. Interpreting the parabola graph of: $\boldsymbol{g : y = ax^2 + p}$ and $\boldsymbol{h: y = bx^2 + q}$.	Finding the values of $\boldsymbol{a}$, $\boldsymbol{p}$, $\boldsymbol{b}$ and $\boldsymbol{q}$ and determining the value of $\boldsymbol{x}$ for which $\boldsymbol{g(x) \geq h(x)}$ and the values of $\boldsymbol{x}$ for which $\boldsymbol{g}$ is increasing.
5. Sketching the graphs of $\boldsymbol{h(x) = x^2 - 4}$ and $\boldsymbol{k(x) - x^2 + 4}$	Sketching the graph $\boldsymbol{h(x)}$ and $\boldsymbol{k(x)}$ on the same system of axes, describing the relationship of $\boldsymbol{h}$ and $\boldsymbol{k}$, reflection of $\boldsymbol{k(x)}$ about the line of $\boldsymbol{y}$ values.

The achievement test comprised five parts of Grade 10 parabola functions, each with five questions out of a possible 24. There were two items in the sections: quadratic equations and the parabola function of form $\boldsymbol{f(x) = x^2}$ with six items, the parabola function of form $\boldsymbol{f(x) = -\frac{1}{2}x^2 - 3}$ with seven items, interpreting the graph of parabola function of $\boldsymbol{g: y = ax^2 + p}$ and $\boldsymbol{h: y = bx^2 + q}$ with four items, and sketching the graph of $\boldsymbol{h(x) = x^2 - 4}$ and $\boldsymbol{k(x) = -x^2 + 4}$ with four items.

4.9.3.2 Element of the semi-structured interview schedule

Following a review of the learners' written replies to accomplishment, semi-structured interviews were prepared. Incorrect test responses were used to investigate the reasoning and mental processes that students used while responding to test questions. The researcher's study of test results assists him in identifying and categorising critical difficulties. Each question item's test analysis responses were classified into themes. These topics were then used to construct interview questions. Semi-structured interview questions were utilised to acquire insight into the mistakes, misunderstandings, and reasoning that resulted in the difficulty students had when addressing parabola function issues. Furthermore, learners were requested to submit explanations for the solutions they supplied to test questions, which helped the researcher identify the potential difficulties learners had when answering parabola function problems.

The following are some of the semi-structured interview questions utilised by the researcher (see Appendix E):

Question 1: How do you understand parabolas?

Question 2: Can you explain what $\boldsymbol{x} -$ and $\boldsymbol{y}$ $-$intercepts are in parabola functions?

Question 3: How did you find the x-intercept of parabola functions and why?

Question 4: How did you find the $\boldsymbol{y}$ $-$intercepts of parabola functions and why?

Question 5: Can you explain the axis of symmetry of parabola functions?

Question 6: How did you find the axis of symmetry of parabola functions and why?

Question 7: How do you understand the minimum and the maximum turning point of parabola functions?

Question 8: How did you determine the minimum and maximum turning points of parabola functions and why?

Question 9: How can you explain the domain and the range of parabola functions?

Question 10: How did you determine the domain and the range of this parabola?

Question 11: Why did you draw the graph this way?

4.8.3.3 Creating the lesson observation schedule

A non-participant observer, according to Cohen et al. (2018:543), takes a passive role and collects data through observation utilising a lesson observation plan. The lesson observation will differ from Sepeng's (2010:58) lesson observation (see Appendix G).

To observe, the researcher already had pre-established categories from the semi-structured interview analysis. The semi-structured interview data was then utilised to perform lesson observations in each class. The interviews, for example, yielded the following categories: (1) learning and teaching techniques; (2) misunderstandings and errors in learning parabolic functions; and (3) conceptual knowledge, procedural knowledge, and strategic competency. The following topics were addressed in the lesson observation schedule:

- Observing classroom procedures, such as the introduction of parabola functions, the teaching style, and dealing with learners' correct and erroneous replies;
- Observing learners' productive abilities, such as creating concepts for parabola functions;
- Observing evocative capabilities, such as whether learners can ask questions about parabola functions and analyse information about a specific graph notion;
- Observing evaluative abilities, such as if learners can: (1) assess their own work on the idea acquired; (2) recognise their own errors and misconceptions about parabola functions; and (3) utilise multiple ways to obtain the x and y-intercepts.
- Observing reflective capabilities, such as if learners can reflect on errors and misconceptions about parabola functions.

The lesson observation plan was designed to provide some space for the observer to fill when teaching and learning about parabola functions in Grade 10. The observed occurrences were graded on a five-point scale (see Appendix G). Sepeng's (2010:45) five-point grading scale was great, decent, medium, require more attention, and not suitable to the lesson. For a grading system, great equals five points, decent equals four points, and so on.

4.8.4 Considering the study’s validity and reliability

According to Surucu (2020:1), the data collecting tools had to pass validity and reliability testing before they could be used in the study. Devault (2019:2) defines quantitative and qualitative research trustworthiness as research validity and dependability. He goes on to say that trustworthiness in qualitative research is all about establishing credibility, transferability, confirmability, and dependability (Devault,2019:1). The study included both qualitative and quantitative methods, and both validity and reliability were taken into account. To examine the correctness of the findings, the researcher used a variety of validity methodologies (Creswell, 2018:51). Carroll (2021:1) maintains that trustworthiness is defined by credibility, dependability, and reliability, which is a type of external validity and confirmability, which is mostly a presentation issue. Crystallisation is another trustworthiness approach assessment, according to Cohen (2018:234). Crystallisation is utilised to assure the study's and its findings' reliability and dependability. As a result, the researcher assured the validity and reliability of the qualitative and quantitative study by employing unobtrusive measurements and employing easy multiple data collection procedures (Surucu, 2020:2). From the viewpoints of quantitative and qualitative research methodologies, the researcher tackled questions of scientific rigour for the current study.

4.8.4.1 Achievement test dependability

The accuracy of quantitative achievement test results can be determined by determining if the test consistently produces the same findings (Makgakga, 2016:131). The researcher validated the test's reliability by matching it to the parameters of parabola functions exclusively using the Grade 10-12 CAPS (CAPS, 2014). The examination was piloted in a school to assess students' performance and to identify the obstacles students have when learning parabola functions in order to reduce mistakes that can arise during research. The researcher confirmed the dependability of the quantitative data by determining whether a test consistently produced the same outcomes as his findings (Creswell, 2018:235). The dependability of the accomplishment test results was assessed using data captured in Microsoft Excel.

4.8.4.2 Ensuring the accuracy of achievement test

An instrument that measures what it is designed to measure demonstrates validity (Cohen et al., 2018:123). Valid data will be generated by an instrument that can measure what it is designed to measure. The current study addressed three types of validity: face validity, content validity, and criterion validity, and these scientific notions were related to the accomplishment examination. Face validity pertains to how the learners were presented with the test. By piloting the achievement test, the researcher determined if the assessment of learners' academic performance in parabola functions was worthwhile. The researcher verified that content validity was proven by designing a fair assessment that covered all the ideas that were meant to be addressed (Cohen et al., 2018:188). CAPS documents for Mathematics Grades 10-12 were used to achieve content validity (DBE, 2011). The researcher created a test using prior Mathematics examination question sheets that covered the cognitive levels specified by the Grade 10 Mathematics curriculum (DBE, 2011). The content validity ensured that the test effectively handled the topic of parabola functions.

The test was moderated by Mathematics educators' experts who used a test moderation form, including (1) a Vhembe West District subject specialist, (2) a Vhembe District head of the department (HoD) for high school Mathematics, and (3) an experienced Mathematics teacher from the researcher's school.
The teachers commented on various areas of the achievement test. Teachers have left the following comments: (1) the test should last an hour rather than 45 minutes; (2) questions should be checked for repetition; and (3) Grade 10 topic knowledge should be tested.

4.8.4.3 Ensuring reliability and validity in the qualitative part of the study

Creswell (2018:191) defines dependability as the capacity of an instrument to measure in the same manner each time it is used under the same conditions. Based on the interview methodology, the researcher employed semi-structured interview questions. By analysing confusing replies, the researcher obtained an in-depth insight of learners' opinions on parabola functions. This was accomplished by taking copious notes on the original concepts, additional material obtained in the field, and member checks. The researcher returned the data analysis to the respondents to ensure that what they reported was correct (Devault, 2019:2).

The researcher also employed participant validation by asking questions such, "How do I know that what I've documented is indeed what I've seen or happened?" and "Have I exactly comprehended the participant's experiences, feelings, and problems? Data validation was also ensured through classroom observation. The researcher employed an approach known as member checking, as well as more than one observer, and compared his comprehension to that of the other observers (Devault, 2019:3). The researcher specifically watched how teachers teach parabola functions in order to gain a knowledge of the tactics and approaches employed by teachers while teaching parabola functions. To meet the requirements of the observation schedule provided, the observation schedule was piloted in educational settings using Cohen observation method (Cohen, 2018:342).

4.9 PILOTING THE CURRENT STUDY

A pilot study, according to Hazzi and Maldaon (2018:24), is a short study used to evaluate research methodologies, data collecting tools, sample strategies, and other research approaches in preparation for the major study (Hazzi & Maldaon, 2018:56). All the research methodologies employed in the study were tested on a modest scale in the trials. Pilot testing was used to assess and verify the data gathering instruments for the use of certain phrases and keywords. A pilot study's goal was to establish which questions were clear and to identify issue areas that needed to be addressed. The pilot project was carried out in a specifically chosen school, which had been observed and had been delivering low end-of-year Mathematics scores for three years in Grade 12. The sample in the pilot study consisted of 30 learners and one teacher.

4.9.1 Pilot study performance evaluation

Before beginning the main study, the researcher piloted the achievement test to ensure that the instruments were useful. The researcher ensured that all participants sanitised, wore masks, and observed social distancing and video recorded to combat the spread of COVID-19. Piloting of the achievement test instrument as done in School A. To guarantee uniformity, the piloted test learners' writings were annotated using a memorandum. Data collected on achievement tests were analysed by the percentages learners got in each question using a computer and they were categorised using Didis and Erbas' (2015:46) written responses categories. The table was useful to analyse the learners’ achievement test:

Table 4.2: Rubric for analysing learners' response to the achievement test

	Correct response	Incorrect response	Partially correct	Blank response
Conceptual Knowledge				
Procedural Knowledge				

Adaptive reasoning				
Factual knowledge				

Piloting the study instruments included determining if the instrument provided the researcher with the information that it was designed to provide. Pilot studies were conducted to assist researchers in minimising mistakes that may arise in the major study. The tools were piloted by the researcher to confirm performance and challenges if learners were experiencing issues understanding parabola functions. The researcher was then able to make changes and revisions to the research tools before the main study.

4.9.2 Piloting the study's semi-structured interview

The semi-structured interview schedule was tested to see whether there were any elements that were confusing or ambiguous (Appendix E). The interview schedule was designed with open-ended questions on conceptual comprehension, procedural knowledge, and strategic competency on parabola functions in Grade 10 Mathematics classes as the primary focus. The pilot study assisted the researcher in checking for language usage, phrases, or wording in the interview items. The pilot study also assisted the researcher in determining the relevance of interview items in relation to the teaching and learning environments in the participating schools.

The researcher employed probing questions to get detailed information on the problems that learners have when learning parabola functions. Furthermore, the piloting procedure indicated the interview items that needed to be investigated further. The pilot research also provided an idea of how long each interview session should be.

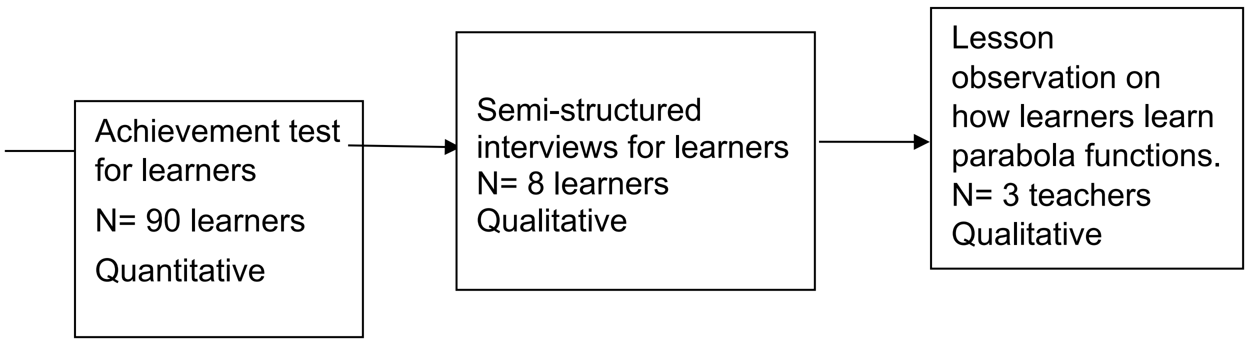
4.9.3 Piloting the study's lesson observation guide

A pilot study was also conducted in the lesson observation guide to check the relevance of the instrument and to discover the suitability of the data collection tool (Appendix G).

4.10. DATA COLLECTION PROCESS IN THE MAIN STUDY

According to Simplilearn (2022:1), data collection is the process of gathering and analysing accurate data from various sources to find answers to research problems, trends and probabilities to evaluate possible outcomes. The study used achievement tests, semi-structural interviews and lesson observation schedules to collect data (Creswell, 2018:274). Data collection, according to Simplilearn (2022:1), is the act of gathering and analysing correct data from numerous sources in order to find solutions to study issues, trends, and probabilities in order to evaluate prospective outcomes. To acquire data, the study employed achievement examinations, semi-structural interviews, and instructional observation schedules (Creswell, 2018:274). The following figure depicts the data collecting technique.

Figure 4.1: Data collection procedure



4.10.1 Achievement test data collection

Valenzuela (2022:1) describes an achievement test as a test that measures content knowledge or skills within a certain area. The researcher acquired quantitative data in this study by presenting an achievement test to 30 Grade 10 Mathematics students in their classrooms after school hours. The research procedure began after the researcher gave an introduction and explained the goal of the investigation. All students were given copies of the informed consent form, and an achievement examination was given to them.

4.10.2 Semi-structural interview data collection

A semi-structured interview, according to Cohen et al. (2018:511), comprises open-ended questions with language and sequencing adapted to each respondent, and the response is supplied with probes and prompts. The semi-structured interview data collection is built on an accomplishment examination that is linked at the study's intermediate stage (Venkatesh, et al., 2016:245). Qualitative data collection delved deeper into learners' written replies to gain a greater understanding of their academic experiences in learning parabola functions at the Grade 10 level. After analysing results from an accomplishment examination, semi-structured interviews were done in this study. To avoid learners' retention loss, the semi-structured interviews were held three days following the accomplishment examination (Didis & Erbas, 2015). During the achievement test, the researcher obtained the principal's phone number so that the semi-structured interviews could be performed over the phone rather than in person. All interviews were audio-recorded, and backup media, such as USB and CD, were utilised to prevent data loss. The semi-structured interview is used to identify mistakes, establish misunderstandings, clarify erroneous responses from achievement examinations, seek clarity in how learners articulate their solutions in parabola functions, and investigate learners' logic behind their procedures in solving parabola functions.

4.10.3 Lesson observation data collection

Halim (2018:1) describes classroom observation as an act of watching learners' and teachers' performance in the classroom. Following the completion of the semi-structured interviews with the learners, the researcher transitioned into a non-participant observer by performing classroom observations to better understand how the learners learn parabola functions. Since we are in a pandemic era, the researcher and participants sanitised themselves, wore masks, observed social distancing, and the researcher asked permission to sit at the back of the classroom to take notes. The researcher requested that participants must be video recorded but the researcher made sure that participants' faces were not shown to protect them. Furthermore, the researcher wrote letters to the principal, teachers, learners, and parents asking permission to video record the lesson so that they could be aware that the researcher was video recording the lesson (see Appendix I, J, K and L).

A non-participant observer, according to Cohen et al. (2018:543), takes a passive role and collects data through observation, utilising a lesson observation plan. Sepeng's (2010:45) lesson observation plan was adapted for this lesson observation schedule (see Appendix G). To observe, the researcher had pre-established categories from semi-structured interview analysis. The information gleaned from the semi-structured interviews was utilised to undertake lesson observations in each class. The researcher intended to perform four lesson observations in Class A and B at School B, for a total of eight lesson observations. During lesson observations, the researcher was interested in understanding how students learn parabola functions because students had previously told the researcher about their difficulties in learning parabola functions.

The researcher evaluated how learners unpacked parabola function topics such as quadratic formulae, x and y intercepts, axis of symmetry, domain, and range, as well as how learners interacted throughout their provided courses. Data from lesson observation helped the researcher complement what learners said in the semi-structured interviews. For example, learners might have said that they are not given activities in class in the semi-structured interview, then the researcher goes to class to observe how teachers give learners class activities. The class observation programme also addressed the following issues:

- Observing classroom procedures such as parabola function education and learning, learner learning styles, and the process of dealing with correct and erroneous learner replies.
- Observing learners' productive skills, such as defining parabola functions terms, solving problems, and applying and relating parabola functions concepts to real-life situations;
- Observing evocative skills, such as observing if learners can ask parabola functions-related questions and interpret new information on a given concept;
- Observing evaluative skills, such as observing if learners can evaluate their work on the concept learned; identify the error committed, and use alternative ways to solve parabola functions problems; and
- Observing reflective skills, such as observing if learners can reflect on errors and misconceptions, they have about parabola functions and on decision making in solving a specific problem.

The class observation plan was created with gaps for the observer to fill throughout the teaching of Grade 10 parabola functions. Sepeng's (2010:10) five-point rating scale, which includes excellent, good, mediocre, need more attention, and not suitable to the lesson, was employed. On the grading system, outstanding is worth five points, goodwill is worth four points, and so on. The lesson observation schedule tool was used to collect data on how students understand, analyse, solve, and compute issues with parabola functions. The researcher watched four lessons in Classes A and B, for a total of eight lessons seen over three weeks from School B, with each observation lasting 30 minutes. The researcher took field notes and then analysed the data.

Lesson observations were carried out at School A for two days during the piloting of the study in the classroom, with all learners, during the teaching and learning of parabola functions. For three weeks, lesson observation was carried out at School B in classes 1 and 2, with each class including all learners during the teaching and learning of parabola functions. Lesson observations were captured on video. According to Creswell (2018:307), qualitative data may be obtained utilising videotapes, computer messages, sounds, and movies. The researcher employed videotape to collect valuable information that might otherwise have been missed via observation.

4.11 THE ANALYSIS OF DATA

Data analysis suggests that the researcher was involved in segmenting and disassembling the data, sorting and organising data into useful data based on the information sources (Creswell, 2018:312). Data were analysed and interpreted through a process that included preparing the data for analysis, doing various analyses, delving further and deeper into understanding the data, portraying the data, and creating interpretations of the data's wider significance (Sepeng, 2010:75).

The researcher began by analysing data gathered during the pilot project, which included an achievement examination and semi-structured interviews that influenced the achievement test. A semi-structured interview influenced the lesson observations because the researcher wanted to confirm the data sets acquired from both the lesson observations and the semi-structured interviews. Furthermore, pilot data was analysed by looking at the obstacles learners have when learning parabola functions by observing how learners reply to questions. The researcher also examined the tools to discover any deficiencies. The researcher first analysed the quantitative data set, which is the learners' accomplishment test, and then the qualitative data set, which is the semi-structured interview and class observations, in this study.

4.11.1 The pilot study data analysis

A pilot study, according to Lowe (2019:117), is a tiny study carried out to pre-test various research methods in order to avoid the occurrence of flaws that could result in a loss of time and resources. A pilot study is a small-scale trial run of all the research techniques that will be used in the major investigation, before beginning the main investigation. Chidziva (2021:116) agrees that a pilot study should be carried out to determine the study's viability. A pilot study was conducted with forty-four out of sixty Grade 10 learners who were purposefully sampled from the participating school. The reason only forty-four learners participated in the study is because they were the only learners taking mathematics in Grade 10 who did not take part in the main study, and only to gain a descriptive understanding of learners' performance while learning parabola functions, thus, a pilot study was conducted. The researcher was also getting ready for the main study and evaluating its applicability so that adjustments could be made from the achievement examination, semi-structured interview questions, and lesson observation, if necessary, before starting the main study. According to Lowe (2019:117), pilot research must be carried out to determine whether the intended data collection tool is appropriate before it can be used in the main study. This indicates that the pilot study helps the researchers to improve or remove some aspects of the main study for achieving good findings. The pilot study revealed that Question 2.3 which asked learners to determine the domain and range of parabola functions was asking similar questions with Question 3.2, 3.3 and 5.4 and resulted in the researcher removing those questions so that learners would not be confused and to avoid repetition of questions.

Further, the researcher used the pilot study to find out if the test would give information on errors when learners are learning parabola functions.

4.11.2 Quantitative data analysis in the main study

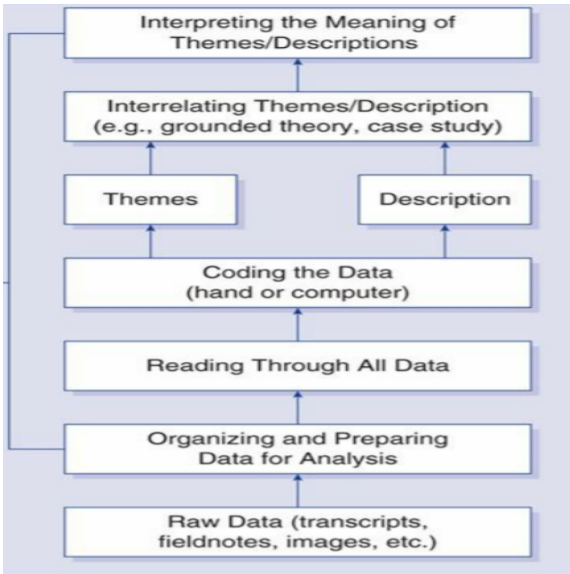
According to Eteng (2022:1), quantitative data analysis involves finding patterns, connections, and relationships from collected data. Furthermore, Warren (2022:1) indicated that quantitative data analysis involves categorising numerical data using various statistical method. As earlier indicated, this study followed an explanatory sequential mixed method design, the researcher analysed the quantitative data set in the form of learners' written achievement tests, as mentioned earlier, followed by the qualitative data set informed by the test results. The researcher organised and classified quantitative data in terms of accurate, wrong, incomplete, and blank responses (Didis & Erbas, 2015:1141). The quantitative data were organised and classified as correct response (CR), erroneous response (IR), incomplete response (InR), and blank response (BR) (Didis & Erbas, 2015:141). According to Didis and Erbas (2015:1141), incomplete replies are those that are not finished and are utilised to follow mathematically valid processes. The reasoning behind categorising the learners' replies was to create a descriptive picture of the learners' solutions to parabola function issues. The quantitative data set was analysed using Microsoft Office Excel spreadsheet, SPSS, frequencies table and graphs and by looking at the mean and data median.

Quantitative data analysis involves descriptive statistics calculated from 90 learners which involves the mean or the average, the median, the mode and the standard deviation. Johnson (2022:1) claims that the main study's data analysis entails the interpretation of information acquired through the application of analytical methodology and logical reasoning to identify patterns, correlations, and trends. As this study adheres to a sequential explanatory research strategy, the researcher began by analysing quantitative data before moving to qualitative data. Additionally, the written responses of the students were dissected item by item. Additionally, a memo was used to mark students' written responses, and it calculated the percentage scores for each question item. The performance of the students was then coded using Didis and Erbas et al. (2015:1141). In problem 1, students were required to factorise a quadratic equation in order to get the value of x given the equation. The researcher utilised aliases, such as L1, L2, and L3, where L stands for learner and the numbers correspond to the positions that learners had in the alphabet. In these problems, students were required to factorise quadratic equations using the quadratic formula, the factor method, or the square root method.

4.11.3 Qualitative data analysis in the main study

According to Warren (2020:2), quantitative data analysis is the act of acquiring, organising, and evaluating data in order to comprehend what it represents. Prior to analysing qualitative data, the researcher analysed quantitative data from the examination in order to construct semi-structured interview questions to identify learners' challenges in the form of mistakes and misconceptions when learning parabola functions. The semi-structured interview and lesson observation data were analysed. The researcher began the qualitative data analysis process by assigning learners codes such as LI1, LI2, LI3, and so on, with the letter L representing a learner, the letter I representing an interview, and the number next to the letter representing the numerical sequence in which interview sessions took place (Appendix F). LI10, for example, stood for learner interview number ten (10). The analysis of data from semi-structured interviews begins during the data gathering stage and continued until all interview material had been transcribed into themes, categories, and patterns. The study used Creswell's (2014:125) processes to analyse qualitative data from particular to generic, using many levels of analysis. The flow diagram depicts how the study's data was analysed (Creswell, 2018:246).

Figure 4.2: Flow diagram of qualitative data analysis



Qualitative data analysis involved the interpretation and writing of reports on semi-structured interviews conducted on learners' difficulties in learning parabola functions. By asking general and specialised questions, interview questions were matched to clarify some of the challenges learners encountered when learning parabola functions. The qualitative data were analysed utilising information provided by eight learners during the semi-structured interview, then investigated and organised according to the structured interview schedule themes, based on learners' responses (Didis & Erbas, 2015:1141). The researcher used Creswell's (2018:197) process to reduce and organise the content of qualitative data, making it more manageable and meaningful. Creswell's (2018:197) process states that the researcher must organise and prepare the data for analysis, which included scanning all observation and field notes, as well as interviews with the learners. Furthermore, the researcher examined all data to receive a broad feeling in order to select exceptional and intriguing learner interviews, and then read it again to grasp its significance. The researcher then sketched topics and classified them into relevant portions. These themes' descriptive phrasing was determined, and they were classified. As a result, data were numbered and shortened in order to reflect all categories. The data from each category was grouped together for preliminary analysis, and existing data was re-recorded if feasible. To organise interview data, the coding approach was employed.

Data analysis from lesson observation was classified as SBLOT1, SBLOT2, and SBLOT3, with SB indicating School B, LO representing lesson observation, T representing teacher, and the numerical number signifying the order in which the observation or school visit took place. For example, SBLOT3 stands for School B lesson observation for the teacher that took place in position number three sequentially (Appendix H). As a result, coding was done in accordance with Creswell (2018:197), who states that: codes that the researcher anticipates, based on current literature, codes that were unexpected at the start of the investigation, and finally, codes that encompass the theoretical viewpoint in the research. Data were analysed and interpreted through a process that included preparing the data for analysis, doing various analyses, delving further and deeper into understanding the data, portraying the data, and creating interpretations of the data's wider significance (Sepeng, 2010:75). Data were represented in this study by transcribing and analysing the data to make better sense of learners' semi-structured interviews and class observations.

4.12 METHODOLOGICAL APPROACH

Bouchrika (2021:1) describes the research methodological approach as a systematic method to resolve the research problem. The researcher used the table below as constructs from the theoretical framework as a lens to make sense of the data. Based on those constructs, quantitative and qualitative data that were gathered via achievement tests, semi-structured interviews, and instructional observation were discussed.

Table 4.3 Summary of constructs and descriptors from theoretical framework

Construct	Definition	Descriptors
Learning should focus on enabling learners to read important or main words in a given mathematical problem correctly. (Reading error)	The important information in the question and the meaning of symbols used in parabola function. Identifying keywords in the mathematical problem(s) given.	What is the question asking and the meaning of the symbols used in parabola functions? For example, learners should be able to understand the question precisely and the meaning of symbols in parabola functions like: $\boldsymbol{f(x) = x^2}$ to $\boldsymbol{y = ax^2 + p}$ then to $\boldsymbol{f(x) = ax^2 + bx + c = 0}$ and then to $\boldsymbol{y = a(x - h)^2 + k}$.
Learning should focus on developing learners to understand the meaning of mathematical problems well. (Comprehension errors)	Knowing to identify the known and unknown to solve the problem accurately and effectively.	Writing down what is given and not given in the question asked. Correct application procedure in quadratic formula, completing square and factored method when finding factors of quadratic functions. Finding points on the Cartesian plane, to follow the steps applied in quadratic functions, to follow steps used in finding the roots of parabola functions.
Learning should focus on developing learners' ability to change mathematical problems into mathematical models.	Knowing how to change mathematical problems into models, such as equations, drawings, graphs, or tables.	Learners can change mathematical problems into equations, drawing, tables, and graphs

(Transformation error)		
Learning should focus on developing learners to formulate, present, and decide on the appropriate rules or procedure and eliminate errors in applying the rules to solve parabola functions problems. (Process skills error)	Occurs in learner's mind by formulating problems solving strategies, representing problem mathematically, addressing the issue and selecting an effective solution. Knowing when and how to apply procedure to be able to perform procedures accurately and effectively.	When finding the zeros, the domain, the range, the turning point, and the axis of symmetry, learners can describe why they used a particular method. They can also explain why they used equations to get the turning points.
Give students many different opportunities to think logically, make reflections, explanations, justifications and writing the answer correctly. (Encoding).	Is the capacity of learners to reason logically about the connections between ideas and situations, able to consider alternative approaches and to justify any conclusion.	Learners are able to write the correct answers, show the truth of the answer and make conclusions when solving problems in parabola functions.
ZPD development Actual development level	What learners already learned and mastered	Demonstrate the understandings of linear functions and complete the table of input and output values and linked it with parabola functions.

Potential development level	What learner can achieve with the assistance of capable peers or teachers.	Educators provide support slowly to develop learners' knowledge of understanding of parabola functions and gradually. Learners show understanding by being able to perform a task without a support of a more capable peer.

4.13 ETHICAL CONSIDERATIONS

According to Dixon (2017:1), ethical considerations mainly refer to the common guidelines provided by committees to be followed when conducting research. Fleming and Zegwaard (2018:6) commented that researcher have a duty and obligation to follow the code of behaviour that regulates most professions. Anago academy (2022:1) defines ethics as a field of philosophy that deals with human behaviour and governs the norms and standards of human behaviour and connections with one another. According to Fleming and Zegwaard (2018:7), ethics is the discipline of philosophy that deals with the dynamic decision-making about what is immoral.

Moreover, Anago academy (2022:1) indicates ethics and ethical concerns in education are the act of exploiting existing literature or works of earlier writers or researchers without adequate citation and reference. According to Weinbaum (2019:5), ethical issues include values, moral principles, and duties, as well as the protection of children from any damage and threats during the study process. Dixon (2022:2) also claims that researchers have a responsibility to follow the code of conduct that regulates most professions.

To meet this ethical requirement, the researcher obtained an ethical clearance certificate from the University of South Africa's Research Ethics Committee in order to get admission to the institution. The researcher then approached Vhembe District and requested permission to visit the school. Following approval from the Vhembe

West District, the researcher addressed the principals of the two schools where the research would be done to gain permission to conduct research. The research was also shared with the Grade 10 Mathematics educators and students at the schools where the study was done.

The participants' roles and rights to participate or not participate were also outlined (Tashakkori & Teddlie, 2010:239). Participants were informed of their anonymity, that their participation was entirely voluntary, that they may withdraw from the study at any moment, and that their personal information would not be released. The researcher informed participants that their names would not be published; instead, pseudonyms were utilised (Bhandari, 2022:1). After receiving authorisation to conduct the research, informed consent was required and signed by the researcher, instructors, and students from two schools to formalise the agreement. Because of the detrimental influence on the research process and the results given, the researcher was aware of the bias that may interfere with the investigation.

4.14 SUMMARY OF THE CHAPTER

In this chapter, the researcher examined the study's research paradigms, qualitative, quantitative, and mixed method techniques, research design, data collection tools and tactics, and the comprehensive research methodology employed. Creswell (2018:239) guided the procedures employed. Data collection, validity, and dependability difficulties, as well as the ethical considerations that governed the process, were also discussed.

CHAPTER 5: PRESENTATION AND DISCUSSIONS OF FINDIGS

5.1 INTRODUCTION

The researcher covered the research methods in the preceding chapter. Research paradigms, methodology, and study design were discussed. Data analysis techniques, population and sampling, methodological approach, methodological norms, data analysis techniques, ethical considerations, and a chapter summary were also covered in the chapter. In this chapter, the quantitative and qualitative data on the pilot and main studies of Grade 10 students' academic experiences learning parabola functions in schools in the Vhembe District of Limpopo Province are analysed and discussed. The chapter is divided into three sub-sections: the achievement test analysis (quantitative), where learners' performance was statistically analysed using descriptive statistics analysis; semi-structured interviews with learners; and lessons observations (qualitative), where the researcher aimed to identify the root causes of students' misunderstandings and errors when they are learning parabola functions and a chapter summary is also presented.

As previously said, the objectives of these studies were to answer the following questions:

- What is Grade 10 learners' academic performance when solving parabola functions?
- What difficulties (if any) in the form of errors do Grade 10 learners face when solving parabola functions?
- Why do learners have errors when solving parabola functions?
- How can learners' errors in Grade 10 parabola functions be addressed to improve their performance?

Prior to delivering data analysis from the main investigation, the researcher first explains the data analysis from the pilot study.

5.2 SUMMARY OF FINDINGS FROM THE PILOT STUDY QUANTITATIVE DATA ANALYSIS

Additionally, the results of the pilot study indicated that the achievement test needed to be modified and adjusted. It was intended for the achievement test to last 45 minutes when it was given. The majority of learners, however, were unable to finish the achievement test in the allotted 45 minutes, and all learners finished the writing of the achievement test in an hour. The researcher, therefore, adjusted the duration of the writing achievement test to one hour. The research also noted that questions requiring learners to determine the domain and range were repeated and hence they were removed from Questions 3 and 5.

5.3 SUMMARY OF FINDINGS FROM THE PILOT STUDY QUALITATIVE DATA ANALYSIS

Semantic analysis was used to analyse the pilot study's qualitative data.

5.3.1 Summary of findings from semi-structured interviews in the pilot study

Conditions were altered during the pilot study of semi-structured interviews with 20 students in Grade 10 Mathematics to ensure social distance and make use of a classroom with enough ventilation. When learners were questioned about difficulties, they encountered studying parabola functions, they indicated that they did not understand some of the concepts like finding the x –intercepts and determining the domain and the range of parabola functions. All the learners who took part in the semi-structured interview answered the interview questions with ease and assurance. The semi-structured interview did not undergo any modifications from the researcher.

5.3.2 Summary of findings from lesson observations

Initially, the researcher planned to conduct ten lesson observations which were reduced to five through consultation with principals, educators and learners owing to COVID-19. The researcher found that learners were participating passively in the lesson of parabola functions with the teacher as the only source of knowledge.

5.4 DATA PRESENTATION AND ANALYSIS IN THE MAIN STUDY

Table 5.1 provides descriptive statistics on how well learners factored given quadratic equations using their past understanding of learning how to solve parabola functions.

Table 5.1 Descriptive statistic of learners' performance in factorising quadratic equations

Descriptive statistics	Descriptive Results	Number of learners(n)
Mean/ Average	0,54	90
Median	0,0	90

Mode	0,0	90
Standard Deviation	0,6	90

Descriptive statistics showed that the mean or average of achievement test was 0,54 and the median was 0 while the mode was 0. This statistic showed that the mean and the median were very similar to each other meaning that question 1 data set of learners' achievement were closer together, around zero. The mean and median have slightly positive difference, and this happens as a result of the mean and median having a slight difference in the distribution of learners' achievement in the test. The little positive difference between the mean and the median indicates that the data set is slightly positively skewed. The standard deviation was 0,6 which is very small and greater than the mean which further indicated that learners' achievement in the test were close together, at zero. The closeness of learners' marks was evidenced by most learners who got a minimum of zero (0) marks in the achievement test. The graph also confirms learners' poor performance in question items 1.1- 1.2 assessed learners on solving quadratic equations as prior knowledge of solving parabola functions.

Figure 5.1: Learners' response to question 1 items

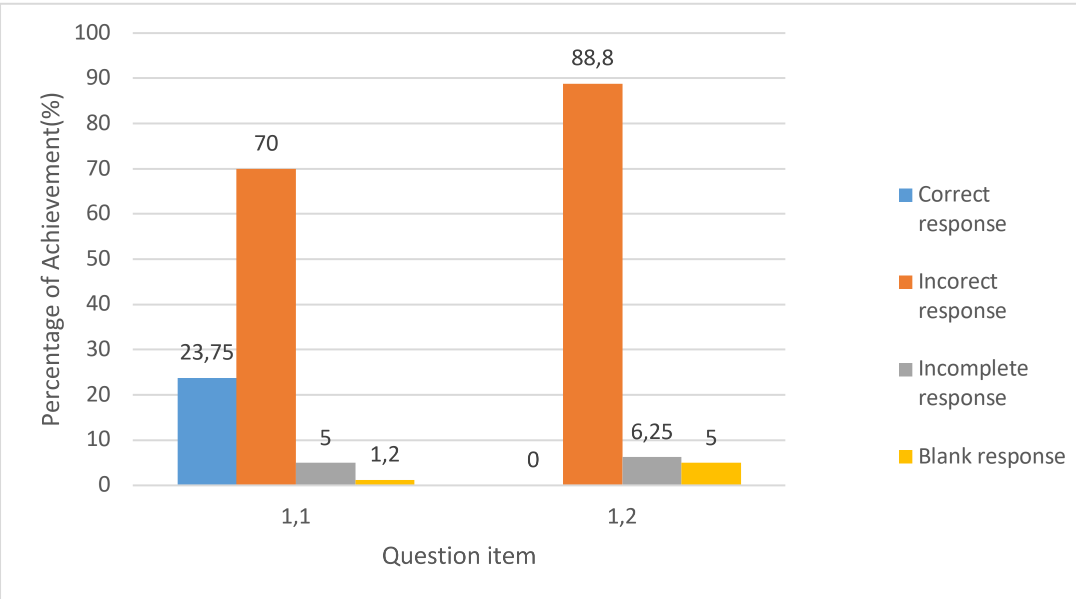


Figure 1.1 depicts that most of the learners showed high incorrect responses in question items 1.1 with 70% and 1.2 with 88% respectively. The results show 23.8% and 0.0% for correct response in question 1.1 and 1.2 respectively. This showed that learners lacked prior knowledge of solving quadratic equations of factorisation by

using factor form, using quadratic formula and completing the square. Learners who gave wrong answers or left blank responses received percentage scores of 5% and 1.2%, respectively.

This implies that the majority of students would be unable to find x −intercept of the parabola function because $x-$ intercepts are calculated by solving the quadratic equation. For instance, learner L3 and learner L4 did not solve the quadratic equation correctly. It implies a deficiency in intellectual comprehension and practical quadratic equation solving abilities (Kilpatrick, 2001:139). These low percentages indicate that students had a variety of difficulties comprehending how to solve quadratic equations using the factoring technique, the completing squares approach, or the quadratic formula. The reason learners are facing challenges is because they experience comprehension errors because learners were unable to understand what they were supposed to do (Newman, 1983:672). The following extract is incorrect responses given by learners solving quadratic equations.

Excerpt 5.1 of learners response in solving quadratic equations in question 1.2	**Solve the following quadratic equations**
Extract of incorrect responses by L3	$2x^2+x-6=0$ $2x^2+x=0+6$ $\frac{3x^2}{3}=\frac{6}{3}$ X $\sqrt{x^2}$ $\sqrt{2}$ X $x=1$ X 0
Extract of incorrect responses by L4	$2x^2+x-6=0$ $=3x^2-6=0$ $=\frac{3x^2}{3}=\frac{6}{3}$ $=\sqrt{x^2}=\sqrt{2}$ 0 $=x=2$

According to the excerpt 1.1 above, L3 and L4 in question 1.2 were required to use the factor method, the square method, or the quadratic formula to solve quadratic equations. The findings showed that 88,8% of respondents were unable to appropriately respond to the question. This is problematic since students were supposed to use the quadratic formula, the factor technique, or the square-root method to solve quadratic equations at their actual level, just as they did in Grade 9. According to Vygotsky (1978:81), learners at the actual level should be able to employ their prior knowledge of solving mathematical problems. It was anticipated that students would apply their prior knowledge to carry out the procedures used to solve quadratic equations. The excerpt further shows that L3 and L4 tried to solve quadratic by using the incorrect method of adding the additive inverse which is not the correct strategy of solving quadratic equations, learners were unable to identify like terms and unlike terms which made them to experience challenges when they are solving parabola functions. L3's written comments displayed a deficiency in procedural fluency and tactical knowledge (Kilpatrick et al., 2001:116) relating to solving of the quadratic equation. The excerpts demonstrates the learner's error of solving a quadratic equation. L3 and L4 were solving quadratic equations as if they are solving linear equations. Learners' solutions were good indications that they were treating quadratic equations as if they were dealing with linear equations. The finding is consistent with that of Ibeawuchi (2016:116) and Nielsen (2015:30), who found that when solving quadratic equations, students often try to transfer their knowledge of linear equations to quadratics because the symbols for the parameters in both types of equations are frequently the same. The errors learners experience results in learners having challenges in solving quadratic equations which may further hinder learners in understanding and determining the $\boldsymbol{x}$-intercept when learning parabola functions.

5.4.1 Learners' response to question 2 items

Learners were asked to demonstrate their understanding of drawing the graph of a particular parabola function in response to question 2. The questions were created to test students' abilities to plot and sketch a parabola function with the formula $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{x}^{\mathbf{2}}$, find the parabola function's domain and range, identify the line along which the graph of $\boldsymbol{f}(\boldsymbol{x})$ is symmetry, identify the value of $\boldsymbol{x}$ for which $\boldsymbol{f}(\boldsymbol{x}) = \mathbf{6}\frac{\mathbf{1}}{\mathbf{4}}$ and confirm the answer using a graph as well as indicate where the graph cuts the axes.

Table 5.3 shows descriptive statistics on learners' performance in sketching the graph of parabola functions.

Table 5.2 Descriptive statistic of learners' performance in sketching graph of parabola functions

Descriptive statistics	**Descriptive results**	**Number of learners(*n*)**
Mean/ Average	1,95	90
Median	1	90
Mode	1	90
Standard Deviation	1,66	90

Descriptive statistics on learners' performance in sketching a graph of a parabola indicates that the mean or average of the achievement test was 1,95 and the median was 1 while the mode was 1. This statistic showed that the mean and the median were very similar to each other meaning that in question 2, learners have a relatively smooth central distribution of data clustered towards the centre of the distribution curve. The mean and median have a slightly positive difference, and this happens as a result of the mean and median having slight difference in the distribution of learners' achievement in the achievement test. The little positive difference between the mean and the median shows that learners' achievement has been favourably skewed. The standard deviation was 1,6 which is very small and greater than the mean which further indicates that learners' achievement in the achievement test were clustered together around 1 as evidenced by the mode of the learners' achievement which is also 1. The reason learners' performance is poor is because they were experiencing (Newman et al., 1983:673) transformation errors because they were unable to change the given information of parabola functions into a sketched graph. Figure 5.2 represents question items 2.1- 2.6 learners' performance on finding the output values of a parabola functions, plotting the points of parabola functions, sketching the parabola functions graph and determining the domain and the range of the parabola functions.

Figure 5.2 Learners' response to questions 2 items

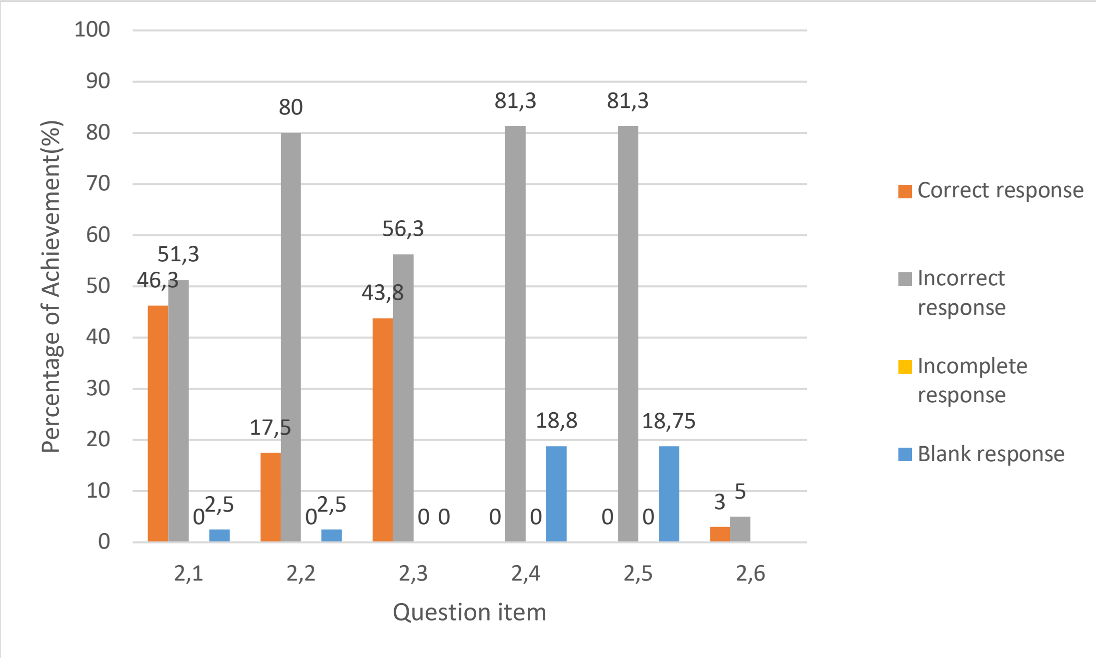
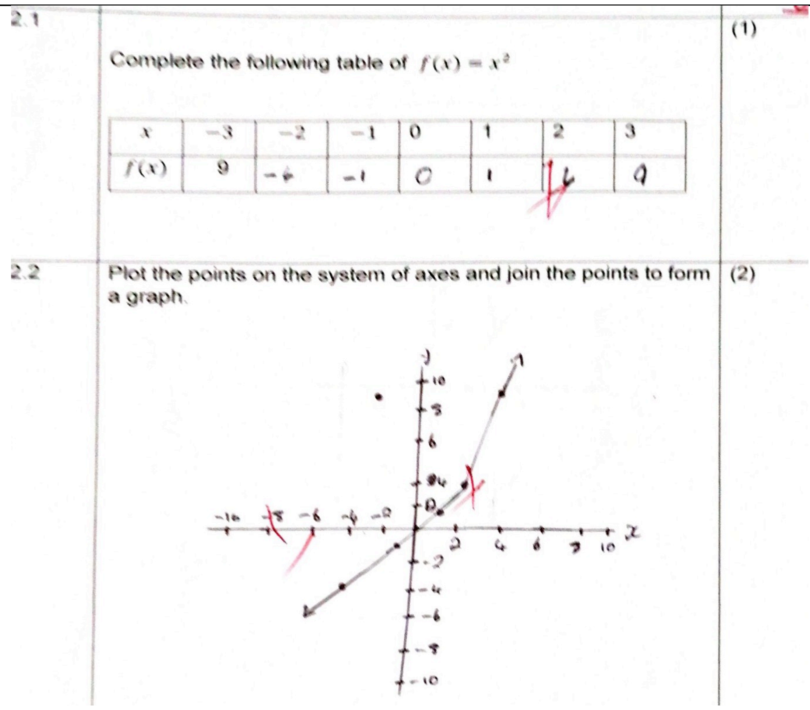


Figure 5.2 shows that most of the learners show high incorrect responses in question items 2.1- 2.5 with 51.3% in 2.1, 80% in 2.2, 56,3% in 2.3, 81.3% in 2.4 and 2.5 respectively. The results show that learners who managed to get correct responses in questions 2.1 were 46,3%, while in question 2.2 were 17,5% and 0% in question 2.3 with 18,5% and 18,75% of blank responses in questions 2,4 and 2,5 respectively. This demonstrated that students lacked understanding of how to draw a parabola graph by locating output values, plotting points on a Cartesian plane, and figuring out the domain and range of parabola functions. The reason learners were performing poorly is because they were lacking (Vygotsky et al., 1978:81) the actual level of completing tables to find the output. Learners were also committing (Newman et al., 1983:673) transformation error because they were unable to find the domain and range of parabola functions.

About 51% of the learners provided incorrect responses in question 2.1, 80% gave incorrect responses to question 2.2, in question 2.3, 56% of learners gave incorrect answers. Furthermore, 81% of learners provided incorrect answers in questions 2.4 and 2.5. Learners were unable to complete a table to find the output of the given parabola and this results in learners unable to sketch the correct parabola functions. This suggests that most learners were experiencing transformation error challenges; they were unable to change given information into parabola functions graph. It is evident from the excerpts of the response of L26 that most learners experienced various challenges when sketching the graphs of parabola functions in questions 2.1 and 2.2.

Excerpts 5.2 of learner's response when completing and sketching graph of parabola functions

Question 2.1 and 2.2	Completing the table and sketching the graph
Extract of incorrect response from L 26	

2.1 Complete the following table of $f(x) = x^2$ (1)

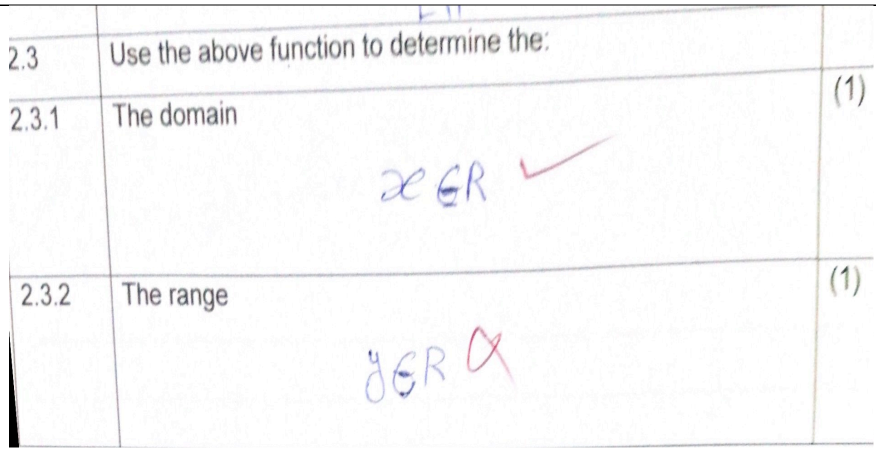
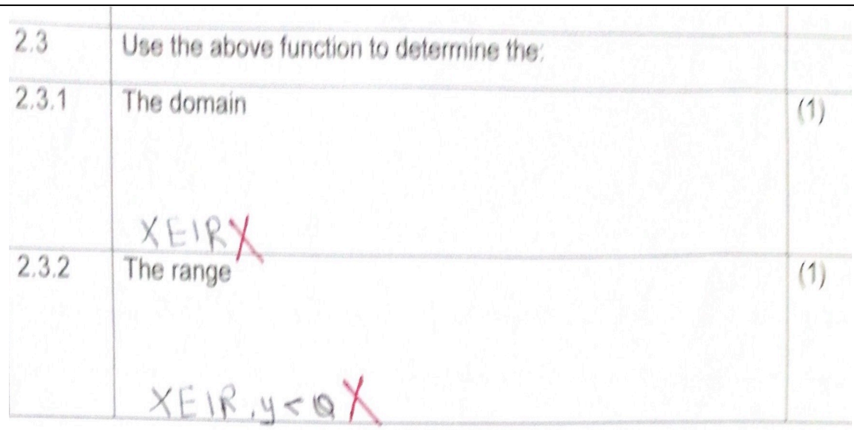
x	-3	-2	-1	0	1	2	3
$f(x)$	9	-4	-1	0	1	4	9

2.2 Plot the points on the system of axes and join the points to form a graph. (2)

According to Newman et al. (1983) and the excerpt 5.2, L26 appears to have been unaware of the proper method for calculating the output and outlining the parabola function graph, learners were experiencing Newman's transformation error. Additionally, the graph shows that 3% of the students left the question items 2.1 and 2.2 blank. Students were expected to substitute the given values of x to the given parabola function equation to find the output values which is the $f(x)$. This further indicates that learners were experiencing transformation errors (Newman et al., 1983:673).

According to figure 5.2, 56% of students have trouble identifying the domain and range of a sketched parabolic function. The excerpt 5.2 it demonstrates that although though L14 was able to identify a parabola function's domain, she had trouble determining its range, much like L42, who was unable to do so. According to Newman et al. (1983:673), these learners were experiencing Newman's transformation error because they were unable to find the range and the domain of the parabola function.

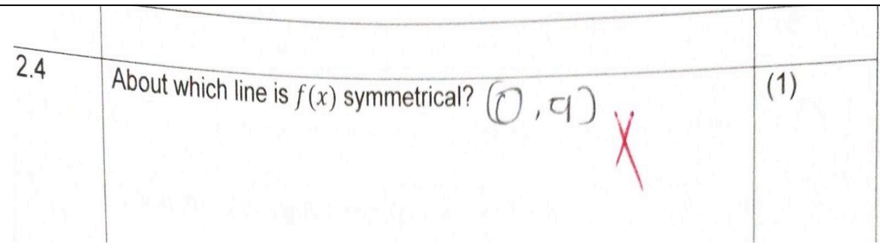
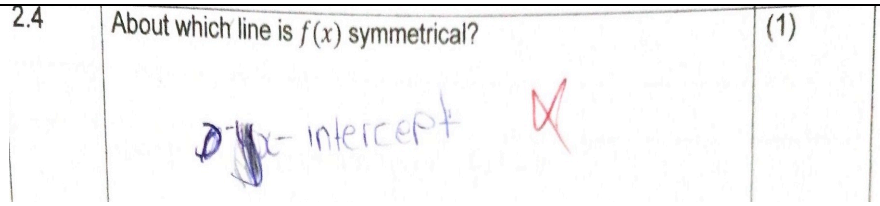
Excerpt 5.3 of learners' response using the parabola functions to find the domain and the range of the functions

Question 2.3	Using the function to determine the domain and the range
Excerpt of incorrect response of range from L 14	 2.3 Use the above function to determine the: 2.3.1 The domain (1) $x \in R$ 2.3.2 The range (1) $y \in R$
Excerpt of incorrect response of domain and range from L 42	 2.3 Use the above function to determine the: 2.3.1 The domain (1) $X \in IR$ 2.3.2 The range (1) $X \in IR, y < 0$

According to the excerpt 5.3, L14 and L42's wrong responses were caused by a lack of understanding of the function and rules of parabolas. Even though L14 partially answered the question correctly, the learner was unable to determine the range of the parabola functions that were sketched, which further demonstrates a conceptual knowledge of parabola functions that is lacking, as evidenced by Kilpatrick et al.

(2001:139). Furthermore, 81% of learners gave incorrect responses in question 2.4, while 19% of learners left blank responses in finding the line which is symmetrical about $f(x)$.

Excerpts 5.4 of learners response when determining the line which is symmetrical to $f(x)$

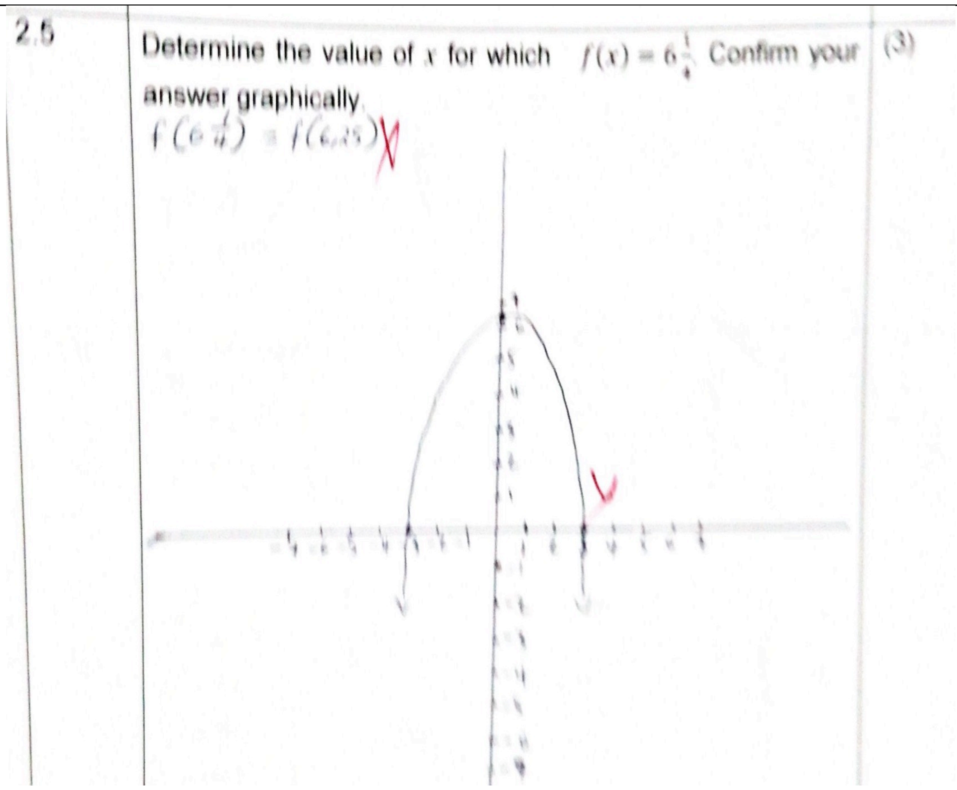
Question 2.4	About which line is $f(x)$ symmetrical?
Extract of incorrect response from L 54	 2.4 About which line is $f(x)$ symmetrical? (0 , 9) X (1)
Excerpt of incorrect response from L 82	 2.4 About which line is $f(x)$ symmetrical? (1) x- intercept X

The preceding sample displays faulty reasoning in L 82 and L 54, which appears to imply a lack of adequate conceptual knowledge in the mirror image or reflections that parabola functions form. It was observed that some learners think it is symmetrical about the line $x = 0$, failing to realise that $f(x)$ is symmetrical about the y −axis. Learners were experiencing these challenges because they lack (Kilpatrick et al., 2001:139) conceptual understanding; they were unable to compare and contrast concepts that were related.

Similarly, figure 5.2 shows that 81% of learners gave incorrect responses in question 2.5 and 19% of learners also left blank responses in determining the value of x for which $f\ (x) = 6\frac{1}{4}$ in question 2.5. The following excerpts demonstrate how learners L18 and L68 attempted to answer the given problem improperly. Students were supposed to understand that if $f(x) = x^2$ therefore, $x^2 = 6\frac{1}{4}$ which means that $x^2 =$

$\frac{25}{4}$, then solve for the value of x to get $x = 2,5$. The following is the extract showing how L18 and L66 had tried to answer 2.5 on calculating the value of x for which $f(x) = 6\frac{1}{4}$.

Excerpt 5.5 of learners' response when calculating the value of x for which $f(x) = 6\frac{1}{4}$.

Question 2.5	Calculate the value of x for which $f(x) = 6\frac{1}{4}$.
Excerpt of incorrect response from L 18	
Excerpt of incorrect response from L 66	

	2.5 Determine the value of x for which $f(x) = 6\frac{1}{4}$. Confirm your answer graphically. (3) $f(x) = 6\frac{1}{4}$ $x = 6\frac{1}{4}$ $(f) = 6\frac{1}{4}$ $= 3\frac{1}{2}$ X

The excerpts 5.5 clearly shows that learners had little or no understanding of the interpretation and solution of parabola functions. Overall, the quantitative analysis of question 2 items in Graph 5.2 reveals that the majority of learners were unable to provide solutions to questions about sketching the parabola function graph, determining the domain and range of the parabola function line which is symmetrical about the $\boldsymbol{f}(\boldsymbol{x})$ and the value of $\boldsymbol{x}$ for which $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{6}\frac{\boldsymbol{1}}{\boldsymbol{4}}$. Graph 5.2 also reveals that just 46% of students correctly answered item 2.1. It demonstrates that charting the co-ordinate of the graph of the parabola function was difficult for many students. According to Newman et al. (1983:135), these students were making compression errors because they did not know how to answer the question due to a lack of prior knowledge, which could have resulted in comprehension problems. The 18% of learners who provided erroneous responses on item 2.2 demonstrates that the majority of learners lacked understanding of plotting the graph of the parabola function. Furthermore, the extract of L30, L142, and L161 replies revealed that most students had a variety of difficulties when completing parabola function questions. According to Graph 5.3, only 43% of learners were able to discover the domain and range of the parabola functions, indicating that most learners struggle while solving parabola functions. In question 2.4 and 2.5 respectively, 0% of learners were unable to find the line which is symmetrical about $\boldsymbol{f}(\boldsymbol{x})$ and determining the value of $\boldsymbol{x}$ for which $f(\boldsymbol{x}) = \boldsymbol{6}\frac{\boldsymbol{1}}{\boldsymbol{4}}$. It is clear from Graph 5.3 that most learners experienced challenges in interpreting the graphs of parabola functions.

5.4.2 Learner responses to question 3 items

Question 3 was separated into three sub-items in which students were asked to draw the graph of parabola functions of $\boldsymbol{f(x) = -\frac{1}{2}\,x^2 - 3}$, by finding the $\boldsymbol{y}-$ intercept, $\boldsymbol{x}-$ intercept, axis of symmetry and turning point. The question items asked to learners to determine the $\boldsymbol{y}$ –intercept, $\boldsymbol{x}-$ intercept, the axis of symmetry, the turning point and then sketch the graph of parabola functions.

Table 5.4 shows descriptive statistics on learners' academic performance in determining the $\boldsymbol{x}-$ intercepts, the $\boldsymbol{y}$ –intercept and sketching the graph of a parabola function.

Table 5.4: Descriptive statistic of learners' performance in the sketching the graph of a parabola function from the given parabola functions

Descriptive statistics	**Descriptive value**	**Number of learners($\boldsymbol{n}$)**
Mean/ Average	1,46	90
Median	1	90
Mode	1	90
Standard Deviation	1,29	90

Descriptive statistics on learners' performance in sketching the graph of parabola when given parabola functions show that the mean or average of achievement test was 1,46 and the median was 1 while the mode was 1. This statistic showed that the mean and the median were very similar to each other, meaning that in question 3, learners have a relatively smooth central distribution of data clustered towards the centre of distribution curve. The mean and median have slightly positive differences and this happens as a result of the mean and median having slight differences in the distribution of learners' achievement in the achievement test. The achievement of learners has been slightly positively skewed, as seen by a slightly positive difference between the mean and the median. The standard deviation was 1,29 which is very small and greater than the mean which further indicated that learners' achievement in the achievement test were clustered together around 1, as evidenced by the mode of learners' achievement which is also 1.

The following graph represents question item 3.1- 3.3 learners' performance on determining the $\boldsymbol{x}-$ intercepts and the $\boldsymbol{y}$ –intercept of the parabola functions, finding the axis of symmetry of parabola functions, locating the turning point of parabola functions, and sketching the graph of parabola functions.

Figure 5.3: Learners' response to questions 3 items

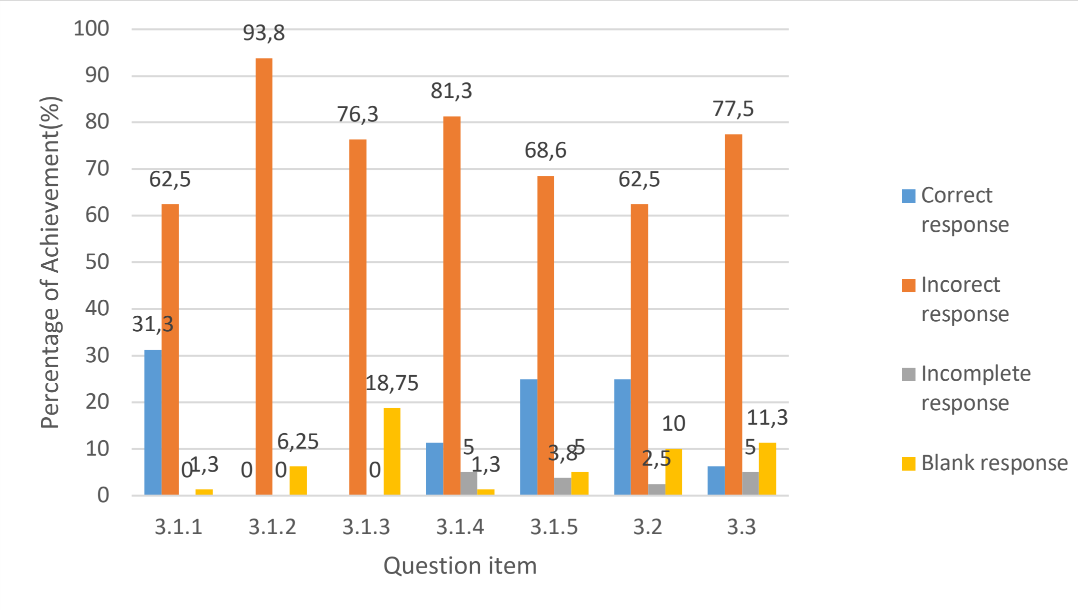


Figure 5.3 depicts that most learners showed high incorrect responses to question 3.1 with 93,8% question 3.2 with 62,5% and 3.3 with 77,5%. The results show 0% and 6.2% for correct responses in questions 3.1.2 and 3.3 respectively.

This further revealed that determining the $\boldsymbol{y}-$ intercept was a challenge to most learners because learners should have known that they determine the $\boldsymbol{y}$ −intercept by setting $\boldsymbol{x} = \mathbf{0}$ and do calculations to find the $\boldsymbol{y}$ −intercept. On item 3.1.1, 63% of learners offered wrong responses, while 1% supplied blank replies, indicating that most have difficulty in distinguishing the $\boldsymbol{y}$ −intercept of parabola function by setting $\boldsymbol{x} = \mathbf{0}$. According to the quantitative analysis of question 3 items in Graph 5.3, the majority of learners were unable to provide solutions to questions about specified parabola functions. Newman's (1983: 123), error analysis indicates that teaching should focus on developing learners to understand the meaning of mathematical problems well to enhance correct applications of procedures. It is clear from excerpts 5.5 of L54 and L74 responses that most students fail to use proper procedures when determining the $\boldsymbol{y}$ −intercept of parabola functions.

Excerpts 5.5 of L54 and L74 when they were determining the y −intercept

Question 3.1	Determine the y −intercept
Excerpt of incorrect response from L54	3.1.1 y − intercept y-intercept when $x = 0$ $0 = -\frac{1}{2}x^2 - 3$ $-\frac{1}{2}x^2 - 3 = 0$ $-\frac{1}{2}(0)^2 - 3 = 0$ $y = -2.$
Excerpt of incorrect response from L74	y − intercept y-intercept when $x = 0$ $F(0) = \frac{1}{2}0^2 - 3$ $= \frac{1}{2} - 3$

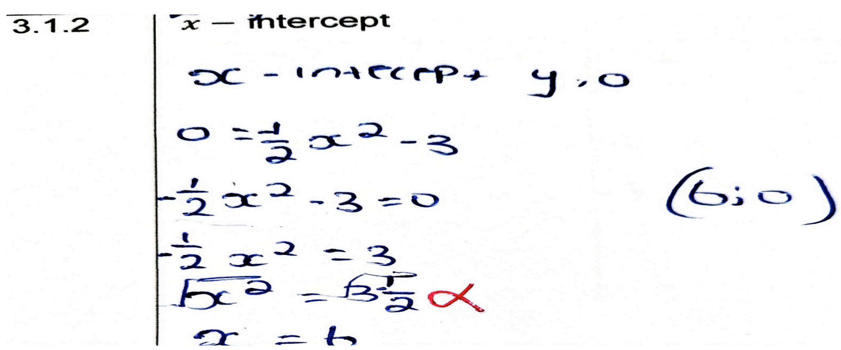
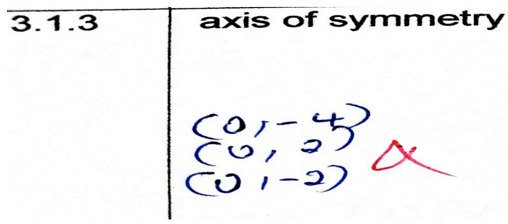
Excerpts 5.5 above show that learners' L54 and L74 snippets reveal the wrong answers supplied by learners to determine the y −intercept. It suggests that the L54 and L74 did not know the correct procedure to determine the y −intercept parabola functions. The challenges were further shown by L20 and L66 who left blank spaces when answering question 3.1.1 indicating that learners were lacking procedural fluency because they were failing to follow the procedure accurately to determine the y − intercept.

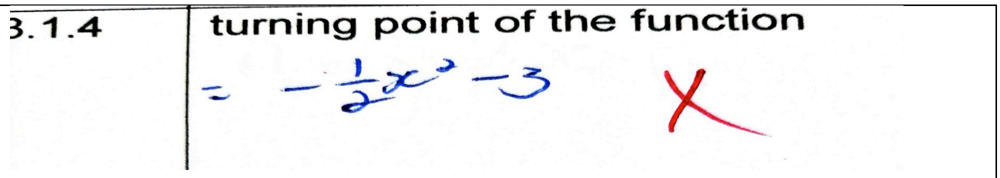
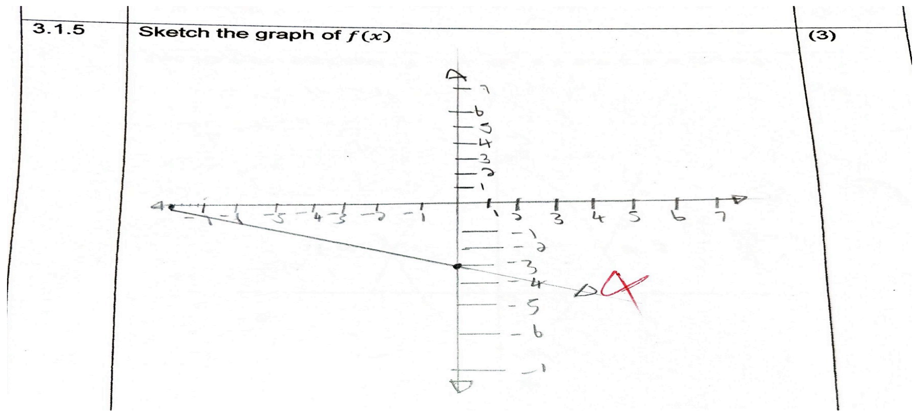
Excerpt 5.6 Showing learners' response on determining the y − intercept.

Excerpt of blank response from L20	3.1.1 y − intercept, $x = 0$
Excerpt of blank response from L66	3.1.1 y – intercept (1) [redacted] f(x)

Further, the excerpts 1.6 shows that learners were unable to detect the y- intercept of a given parabola function, no learner found the proper x- intercepts of a given parabola function, and 6% of learners left blank areas. In question 3.1.2, students were instructed to utilise the quadratic formula, factor technique, or complete the square to obtain the x-intercepts. This shows that the majority of students lacked the procedural fluency and strategic ability needed to cope with problems of this sort (Kilpatrick, 2001:3). It was also noteworthy that 69% of students did not understand that the axis of symmetry of parabola functions is the line of x=0. The preceding extracts from learners L45, L56, L12, and L32 reveal that learners did not comprehend the process of calculating x-intercepts of parabola functions, axis of symmetry, and turning point, which prevents them from sketching the right graph of a parabola function.

Excerpt 5.7 of learner's responses when they were determining the x −intercepts, axis of symmetry and the turning point of parabola functions.

<table>
<tr><td>Excerpt of incorrect response from L 45 on determining x − intercept.</td><td>
3.1.2 | x − intercept
x-intercept y=0
$0 = \frac{1}{2}x^2 - 3$
$-\frac{1}{2}x^2 - 3 = 0$
$-\frac{1}{2}x^2 = 3$
$\sqrt{x^2} = \sqrt{3\frac{}{2}}$
$x = 6$
(6;0)</td></tr>
<tr><td>Excerpt of incorrect response from L 56 on determining axis's of symmetry</td><td>
3.1.3 | axis of symmetry
(0;−4)
(0;2)
(0;−2)</td></tr>
<tr><td>Excerpt of incorrect response from</td><td></td></tr>
</table>

L 70 on determining turning point	
Excerpt of incorrect response from L 70 on sketching the graph	

It is evident from 5. 6excerpts that learners were lacking skills in determining the x -intercepts, axes of symmetry, the turning point and sketching the graph of parabola functions. To determine the y-intercept of the parabola functions, students should have placed x = 0 in the equation. For $x -$ intercepts, learners should have set $y = 0$ to determine the values of x which had no solution. To determine the axis of symmetry learners should have set $\boldsymbol{x} = -\frac{\boldsymbol{b}}{\boldsymbol{2a}}$ and known that in the parabola functions $\boldsymbol{f}(\boldsymbol{x}) = -\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{x}^{\mathbf{2}} - \mathbf{3}$, $\boldsymbol{b}$ is the coefficient of $\boldsymbol{x}$ which is zero and $\boldsymbol{a}$ is the coefficient of $\boldsymbol{x}^{\mathbf{2}}$ which is $-\frac{\mathbf{1}}{\mathbf{2}}$ which will give the axis of symmetry to be $x = \mathbf{0}$. According to Newman's error analysis, these learners were committing comprehension and transformation errors because they were unable to apply the correct procedures to determine the $\boldsymbol{x}$ and $\boldsymbol{y}$ –intercepts, to find the axis of symmetry and to change parabola functions equation into parabola functions graph. The reasons why learners experienced this challenge was because they were lacking strategic competence and procedural fluency (Kilpatrick et al., 2001:121).

5.4.3 Learners' written responses to question 4 item

Question 4 was separated into three sub-items in which students were asked to calculate the coefficient of $\boldsymbol{x}^{\mathbf{2}}$ and the constant term of parabola functions. The items in question 4 are about finding $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{p}$ and $\boldsymbol{q}$ in a given parabola functions graph of

$\boldsymbol{g}: \boldsymbol{y} = \boldsymbol{a}\boldsymbol{x}^2 + \boldsymbol{p}$ and $\boldsymbol{h}: \boldsymbol{y} = \boldsymbol{b}\boldsymbol{x}^2 + \boldsymbol{q}$ when given co-ordinates in the graphs, and determining the $\boldsymbol{x}$ value for which $\boldsymbol{g}(\boldsymbol{x}) \geq \boldsymbol{h}(\boldsymbol{x})$. The fourth item requires an understanding of the link between the graphs of parabola functions that are mirror images of one another. The questions tested students' thinking, computational, and procedural skills in addressing parabola function problems. Participants were supposed to comprehend the meaning of $\boldsymbol{a}, \boldsymbol{p}, \text{and } \boldsymbol{q}$ before answering the questions. Table 5.5 shows descriptive statistics on learners' performance in learning solving two given sketched parabola functions that are mirror images of each other.

Table 5.4 Descriptive statistic of learners' performance in learning solving sketched parabola that are mirror images of each other

Descriptive statistics	**Descriptive value**	**Number of learners($\boldsymbol{n}$)**
Mean/ Average	0,65	90
Median	0,0	90
Mode	0,0	90
Standard Deviation	1.30	90

In question 4, descriptive statistics on learners' performance in learning and solving the sketched graph of parabolas that are mirror images of each other shows that the mean or average of the achievement test was 0,65 and the median was zero (0) and the mode was also zero (0). This statistic again indicates that the mean and the median were very similar to each other, meaning that in question 4, learners' performance had a relatively smooth central distribution clustered towards the centre of the distribution curve. The mean and median have a slightly positive difference and this happens as a result of the mean and median have a slight difference in the distribution of learners' achievement in the achievement test. The achievement of learners has been slightly positively skewed, as seen by a slightly positive difference between the mean and the median. The standard deviation was 1,30 which is very small and greater than the mean which further indicating that learners' achievement in the achievement test were clustered together around zero (0) as evidenced by the mode of learner's achievement which is also zero (0).

In figure 5.4 out of 90 learners, 81%, 63 % and 61%, gave incorrect responses in questions 4.1, 4.2 and 4.3 respectively. Furthermore, 8%, 31% and 46% of the learners gave blank responses on question 4.1, 4.2 and 4.3 respectively. Moreover

4%, 3% and 4% gave incomplete responses in questions 4.1, 4.2 and 4.3 respectively. This shows that most learners experience difficulties in solving graphs of parabola functions that are mirror images of one another. Graph 5.4 demonstrates that most respondents submitted wrong answers to the question about parabola functions mirror image. Graph 5.4 represents question items 4.1- 4.3 assessed learners on interpreting two given sketched graphs of parabola functions.

Figure 5.4 Learners' response to question 4 items

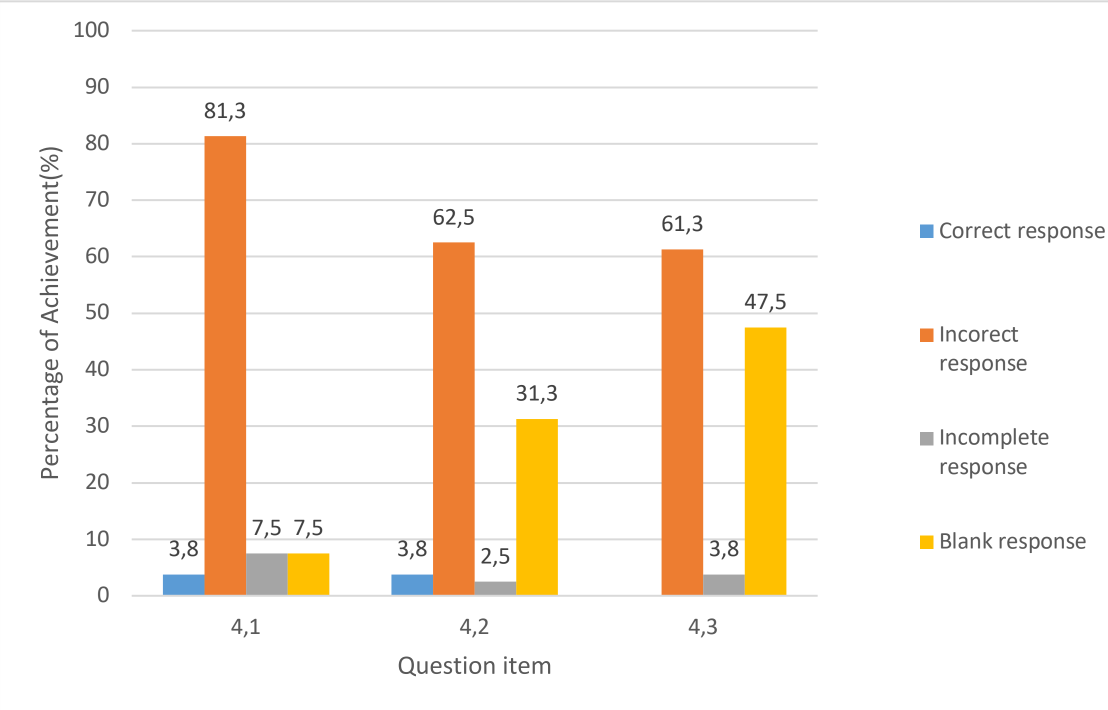


Figure 5.4 depicts that most learners showed high incorrect responses to questions 4.1 with 81,3% question 4.2 with 62,5% and 4.3 with 61,3%. The results show 3.8% and 0% for correct responses in questions 4.1, 4.2 and 4.3 respectively.

The overall quantitative analysis of question 4 items in Table 5.4 reveals that most learners were unable to provide solutions to questions relating to determining the value of a and p. Figure 5.4 shows that 4% of students correctly answered question 4.1. It demonstrates that many students struggled to discover the value of a and b. On question 4.1, 7.5% of learners supplied incomplete responses, while 81.3% offered erroneous responses. This implies that most students lacked knowledge of parabola functions, as well as reasoning and critical thinking skills. When answering parabola functions issues, it is clear from the extracts of responses from L64 and L77 that learners lack strategic ability and adaptable reasoning, as highlighted by Kilpatrick

(2001:129). Learners 67 and 77 did not understand the proper process for determining the value of a, nor did they understand that the value of the y-intercept is 9. They had a comprehension error, as demonstrated by Newman et al. (1983:124).

Excerpt 5.7 of learners' response when determining the values of a and p of a given parabola functions

Excerpt of incorrect response from L77 on finding the values of $\boldsymbol{a}$ and $\boldsymbol{p}$.	4.1 Find the values of a and p. $y = ax^2 + P$ $y = a(21) + 4$ $y = 21a + 4$ $y = 25a$ ✗
Excerpt of incorrect response from L67 on finding the values of $\boldsymbol{a}$ and $\boldsymbol{p}$.	4.1 Find the values of a and p. $= (4,7)^2 + (-6,7)$ $= \frac{1539}{100}$ $= 15,39$ ✗

It was further noted that question 4.2 was also poorly answered by most learners as question 4.1 (see, learners excerpt 1.6). Learners were supposed to have known again that $\boldsymbol{q}$ is the $\boldsymbol{y}$ −intercept which was $\boldsymbol{q} = \mathbf{23}$ and they were supposed to use one point on the graph to find the value of $\boldsymbol{b}$ which will be equal to $\boldsymbol{b} = -\mathbf{1}$. It is clear once more from the excerpt below of L43's comments that the learner lacked strategic competence and adaptive reasoning as indicated by Kilpatrick et al. (2001:130) when solving parabola functions problems. This student was unaware of the proper method for calculating the value of $\boldsymbol{b}$, he did not know that the value of $\boldsymbol{y}$ −intercept is $\mathbf{23}$. He was again experiencing compression errors as indicated Newman et al. (1983:124).

Excerpt 5.8 of learners' response when determining the values of b and q

Excerpt of incorrect response from L 43 on finding the values of b and q.	$y = bx^2 + q$ $q = 23$ $y = bx^2 + 23$ $y = b(4)^2 + 23$ $y = 16b + 23$ ✗

No learner managed to answer question 4.3 correctly and this indicates the learners' difficulty experience in answering questions requiring them to compare parabola functions of the same mirror image. The preceding extracts showed how learners L19 and L52 try to solve question 4.3 but fail to get the correct answer which also symbolised that learners' experienced difficulties when solving parabola function graphs of the mirror image.

Excerpt 5.9 of learners' response when determining the value of x for which g $(x) \geq h(x)$

Excerpt of incorrect response from L 19 on finding the values of b and q.	4.3 Find the value of x for which $g(x) \geq h(x)$. $x = 0{,}41 > 1{,}00$ ✗
Excerpt of incorrect response from L 52 on finding the values of b and q.	4.3 Find the value of x for which $g(x) \geq h(x)$. (2) $y = ax^2 + q \geq y = bx^2 + q$ ✗

According to excerpt 5.9, L52 did not know how to compare the mirror image's parabola functions. Learners were experiencing reading error (Newman et al., 1983:673). According to Newman's error analysis, learners should have read the question and understood the meaning of the symbols precisely. Furthermore, it shows that respondents performed badly due to a lack of comprehension of parabola functions, along with adaptive reasoning in parabola functions mirror image interpretations, per Kilpatrick et al. (2001:130).

5.4.4 Learner responses to question 5 items

This topic was separated into three sub-items in which students were asked to sketch and determine the relationship between parabola functions $h(\boldsymbol{x}) = \boldsymbol{x}^2 - \mathbf{4}$ and $\boldsymbol{k}(\boldsymbol{x}) = -\boldsymbol{x}^2 + \mathbf{4}$. The items in question 5 were to sketch the two parabola functions, to find the relationship between the two graphs, and to give the equation of $\boldsymbol{k}(\boldsymbol{x})$ reflected about the line $\boldsymbol{y} = \mathbf{4}$. Learners were challenged to use their understanding of mirror images of parabola functions once more, but this time with regard to sketching the graph of parabola functions, defining the relationship, and determining the domain and range of the two graphs. This question required a particular level of cognitive ability in relation to complex operations. To answer these questions, learners have to display problem-solving abilities. Item 5.1 asked students to perform the normal technique of determining the y and x-intercepts and then reason that the graph of $h(x)$ is the mirror image of $h(x)$. In addition, students were expected to demonstrate a deeper understanding of parabolic functions by demonstrating that $k(x)$ is the reflection of $h(x)$. In addition, question 5.3 required an understanding of reflection on the y-axis. Furthermore, question 5.4 required students to apply their understanding of parabola functions to determine the domain and range. Finally, the items challenged the students to justify their answers using parabola functions logic.

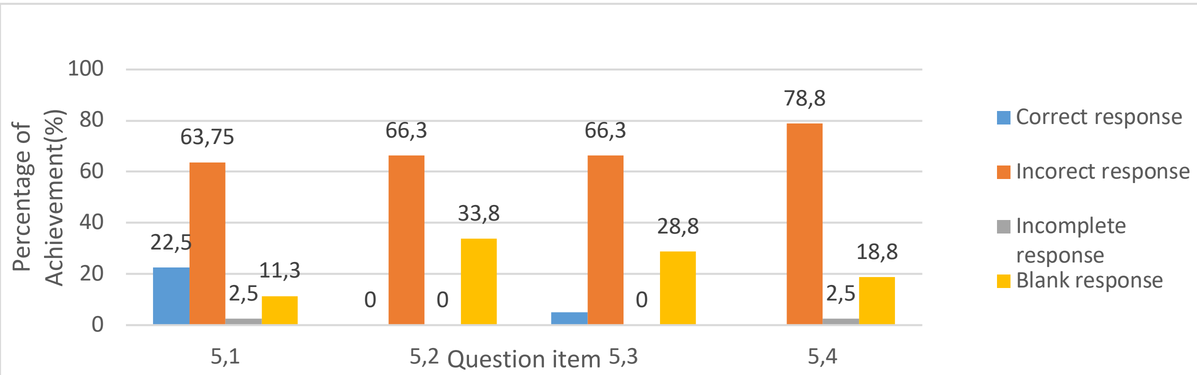
Table 5.6: Descriptive statistic of learner's performance in solving Quadratic functions

Descriptive statistics	**Descriptive value**	**Number of Learners(*n*)**
Mean/ Average	1,2	90
Median	0,0	90
Mode	0,0	90
Standard Deviation	1.69	90

In question 5, descriptive statistics on learners' performance in learning solving the sketched graph of parabola that are mirror images to one another shows that the mean or average of achievement test was 1,2 and the median was zero (0) and the mode was also zero (0). This statistic again shows that the mean and the median were very closer to each other and this indicates that in question 5, learners' performance had a relatively smoothed central distribution, clustered towards the centre of distribution curve. The mean and median have slightly positive difference and this happens as a result of the mean and median having slight differences in the distribution of learners' achievement in the achievement test. The achievement of learners has been slightly positively skewed, as evidenced by a slightly positive difference between the mean and the median. The standard deviation was 1,69 which is very small and greater than the mean which further indicates that learners' achievement in the achievement test were clustered together around zero (0) as evidence by the mode of learners' achievement which is also zero (0).

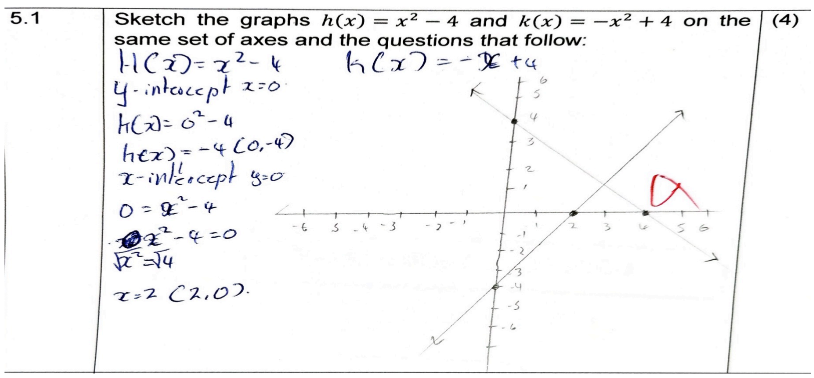
Graph 5.5 further represents question items 5.1- 5.4 on learners' performance in sketching two given parabola functions, analysing the two sketched graphs and describing the two sketched parabola functions.

Figure 5.5: Graph representing learners' response to question 5 items

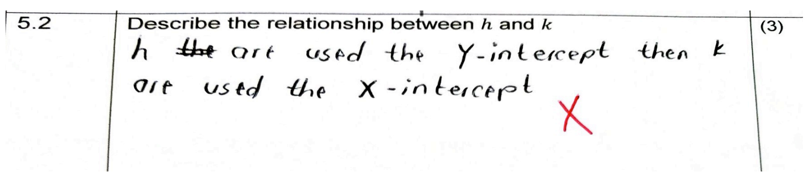


According to figure 5.5, 7% of students correctly answered item 5.1.1. It reveals that interpreting parabola that are mirror images was a challenge to many learners (67%). The graph depicts that most learners showed high incorrect responses in question 5.1 with 63,75% item 5.2 with 66,3%, question 5.3 with 66,3%, and 5.4 with 78,8%. The results show 22.5% and 0% for correct responses in questions 5.1, 5.2, 5.3 and 5.4 respectively. According to the data in Graph 5.5, 11%, 34%, 28%, and 19% of the students gave blank responses to question items 5.1, 5.2, 5.3, and 5.5. Of the 64% of the learners who gave incorrect responses in question 5.1, 3% gave an incomplete response which shows that learners were experiencing Newman's transformation errors in drawing parabolas that reflect each other (Newman, 1983:153). These preceding extracts of learners L2 show that learners did not understand the parabola functions of the mirror image.

Excerpt 5.10 showing learner's response on sketching parabola functions of mirror images on same set of axes

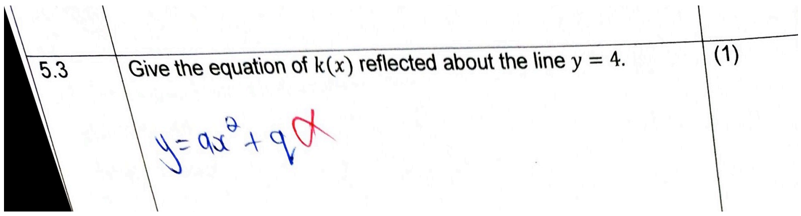
Excerpt of incorrect response from L2 on sketching the graph on same axis	 5.1 Sketch the graphs $h(x) = x^2 - 4$ and $k(x) = -x^2 + 4$ on the same set of axes and the questions that follow: (4) $H(x) = x^2 - 4$ $h(x) = -x + 4$ y-intercept $x=0$ $h(x) = 0^2 - 4$ $h(x) = -4$ $(0,-4)$ x-intercept $y=0$ $0 = x^2 - 4$ $x^2 - 4 = 0$ $\sqrt{x^2} = \sqrt{4}$ $x = 2$ $(2,0)$

Excerpt 5.10 shows that the learner was confusing linear functions and parabola functions and did not know the distinction between the two graphs. This can be the cause of the lack of procedural knowledge (Kilpatrick et al. 2001:122). The scholars noted that learners who lack procedural knowledge commit errors when they are moving from one function to other functions. The reason that this learner in figure 5.5 again shows that item 5.2 was answered badly with no learner managing to get the correct answer, 66% of learners gave incorrect responses and 34% gave blank responses. According to the preceding excerpt, L48 was unable to describe the relationship between $\boldsymbol{h}$ and $\boldsymbol{k}$.

Excerpt of incorrect response from L 48 on describing relationship between $\boldsymbol{h}$ and $\boldsymbol{k}$ of a given parabola functions.	 5.2 Describe the relationship between h and k (3) h ~~the~~ are used the Y-intercept then k are used the X-intercept

It was suggested that L48's incorrect responses were caused by a misunderstanding of the parabola functions mirror image graphs. Incorrect reasoning indicates that learners lack conceptual understanding of parabola functions mirror image graphs Furthermore, 66% of learners provided incorrect responses, while 29% provided blank responses for giving equation of $\boldsymbol{k(x)}$ reflected about the line $\boldsymbol{y = 4}$ in question 5.3. Data analysis in Graph 5.5 shows that only 5% of test respondents correctly answered question 5.3; the following extract provides a summary of learners' responses.

Excerpt 5.11 of learner response answering question on parabola functions reflecting another parabola functions

Excerpt of incorrect response from L9 on giving the equation of $\boldsymbol{k(x)}$ reflected about the line $\boldsymbol{y = 4}$.	 5.3 Give the equation of $k(x)$ reflected about the line $y = 4$. (1) $y = ax^2 + q$

The excerpt 5.11 of L 9 combined with L49 responses, show that learners lacked prior knowledge of parabola functions' mirror image calculation. The respondents appear to be unfamiliar with the meaning of parabola functions' mirror image. Finally, the results in figure 5.5 showed that participants received 3% for correct responses, 19% for blank responses, and 79% for incorrect responses.

It has been noted that learners were committing carelessness error and were leaving blank spaces and providing incomplete solutions when solving the parabola functions task test. Further, these responses indicate a lack of conceptual understanding of parabola function methods, as well as the procedural skills required for successful parabola function problem solving. The semi-structured interviews and lesson observations are thoroughly discussed in the following section of data analysis. Table 5.2 displays the themes that emerged from the analysis of the achievement test learners' work.

5.5 SUMMARY OF OVERALL RESULTS

According to George (2022:3), a summary of overall results should be done that focuses on descriptive statistics results to prove if the hypothesis was supported or not. The descriptive statistics results from the achievement test of 90 learners, shows that from question 1-5, the mean which is the average of the test was between 0.54 to 1.91 indicating that most of the learners were getting two marks from these questions which shows that learners are struggling to answer all questions in the achievement test.

The poor performance was further supported by the mode which is the most common value in each question. The researcher found that in question one, the mode was zero, in questions two and three the mode was one, while in question four and five the mode was again zero, indicating that most learners were getting one and zero from questions one to five which further confirm that learners' academic performance in learning parabola functions was poor. These results confirmed what Celik (2018:26) and Mutambara (2020:34) pointed out that learners' experience difficulties when they are learning parabola functions which result in poor performance.

Learners' academic poor performance in learning parabola functions were further indicated by standard deviations which shows the closeness or further apart of learners' marks to the average or a mean. The descriptive statistics revealed that the standard deviations were very low from question 1-5 ranging from 0.69 to 1.69 which indicates that learners' marks in the achievement were very close to the average. The researcher found that the mean of the achievement test ranged from 0.54 to 1.91, meaning again that most learners were getting between one to two marks in each question of the achievement test confirming learners' academic poor performance which prove the hypothesis to be correct. These results concur with Azizi's (2021:24) results who found that learners experience challenges when they are learning parabola functions which results in learners' academic poor performance.

5.6 THEMES EMANATING FROM ACHIEVEMENT TEST

The rationale of coming up with themes from the achievement test was to confirm the causes of errors through semi-structured interviews. Table 5.6 represents themes that emerged from the achievement assessment.

Table 5.6 Summary of the themes that emerge from achievement test

Themes	**Indicators and sub-themes**
Conceptual understanding	Unable to understand parabola functions, solve problems, unable to find output of the table, fail to sketch the parent graph of parabola functions, unable to plot points of parabola functions, misconceptions and errors.
Procedural fluency	Fail to follow procedure of finding the x and y intercepts. Unable to follow procedure to find the axis of symmetry. Fail to follow procedure of determining the domain and the range of parabola functions. Unable to follow procedure to determine the mirror image of parabola functions.
Adaptive reasoning	Unable to reason using Mathematics. Fails to interpret the graphs of parabola functions. Unable to determine variables of parabola functions.
Strategic competence	Fail to use different strategies when finding x-intercepts. Unable to used different strategies when finding the y- intercept. Failing to use different strategies when finding the axis of symmetry.
Problem solving	Poor problem-solving strategies (finding mirror image, interpreting the graph of parabola functions, and describing the relationships between the graphs of parabola functions.
Cognitive abilities	Low learner's levels of cognitive development. Higher order questions were poorly answered or left blank.

5.7 QUALITATIVE DATA ANALYSIS AND DISCUSSION

Cohen (2018:23) defines qualitative data analysis as the process of organising, accounting for, and explaining data. To gain a better understanding of the achievement test results, qualitative data from semi-structured interviews and lesson observations were gathered. The researcher sought to discover how learners perceived, expressed, and answered parabola function problems through semi-structured interviews with learners. Creswell's (2018:125) steps were followed in analysing the qualitative data from specific to general, involving multiple levels of analysis.

5.7.1 Semi-structured interviews' data analysis

A semi-structured interview, according to Doyle (2022:1), does not follow a formalised list of questions but instead asks more open-ended questions. The researcher collected data by taking notes and using video recording devices. The interviews were verbatim transcribed, and an excerpt was included in the semi-structured interview analysis. When a participant responded in a vernacular (local) language, the interviews were translated into English, in some cases. The interview questions focused on the themes that emerged from the achievement test analysis.

5.7.1.1 Semi-structured interviews' qualitative data analysis

The semi-structured interviews were analysed using a content analysis method.

5.7.1.1.1 Processes of content analysis

Doyle (2022:1) describes the content analysis process as a method which is used to analyse semi-structured interviews. The semi-structured interviews were analysed using content analysis. The content analysis aided in the sorting and summarisation of the data's informational content by items and common characteristics within the data (Souliotis, 2022:2). The researcher transcribed and sorted the responses of the participants to the questions. Important words and phrases were highlighted during this process, and notes were taken on what was common in each response. Following that, the common response was classified and coded. Cohen (2018:559) defines coding as the addition of a name or label to text that contains an idea. The meaning in the context was extracted using open coding, language, pattern description, and data trends. Emerging themes were also examined, and meaning was derived from them. Research objectives, study aims, and research questions were prioritised during the analysis and discussion of qualitative data. The following italicised questions were posed to students:

Conceptual knowledge: *How do you understand parabola functions?*

Challenges in parabola functions: *What do you believe is the source of the poor performance in parabola functions, and why?*

Parabola functions' learners support: *What do you think can be done to address these challenges in learning parabola functions?*

Errors in sketch parabola functions graphs: *What kind of difficulties do you experience when sketching the graph of parabola functions?*

Calculation methods: *Which methods do you find easiest to use when solving problems involving parabola functions on calculating* $\boldsymbol{x}$ *and* $\boldsymbol{y}$ *−intercepts?*

Procedural knowledge: *Which procedures did you employ in solving the axis of symmetry?*

Parabola functions concepts: *Which parabola function concepts do you find challenging to understand, support your statement?*

Curriculum concerns*: What can you say about the amount of time allotted to parabola function learning?*

In the following section, the researcher used the following interview excerpts as an example to present learner and researcher interaction, where the researcher is R and the learner is L.

During semi-structured interviews with Grade 10 Mathematics students, conditions were set up in accordance with COVID-19 protocols, such as social distancing, mask wearing, sanitising, and using well-ventilated rooms. When asked about the difficulties they encountered while learning parabola functions, the five students stated that they did not know how to find the x-intercepts, axis of symmetry, domain, and range of parabola functions.

When asked about the challenges they experience when interpreting the graphs of parabola functions, Learner L2 expressed that: "*sir, our teacher did not teach us to interpret the graphs; he only teaches us to plot the graphs, but we struggled in finding the intercepts on the* $\boldsymbol{x}$ *– axis.* On the same question, learner L7 responded that: "*Our teachers only teach us to draw the graph not to determine the axis of symmetry and to interpret the graph of parabola functions"*. The learners show that the teaching methods used to teach parabola functions appeared to have not been effective as they could not demonstrate an understanding of this concept. This research discovered that students were making Newman's (1983:3) errors such as reading errors, comprehension errors, transformation errors, process skills errors, and encoding errors. The participants in the main study were given the pseudonyms L4, L6, and L9. The following were conversations held with participants during the main study's semi-structure interviews to determine how learners made such mistakes.

Extracts of semi-structured interview of the researcher with learners on learners' understanding of parabola functions.

Results of the interviews with learners
Researcher: How do you understand parabola? L4: At parabola, I understand that when $\boldsymbol{a}$ is positive the graph is going to smile and when $\boldsymbol{a}$ is negative the graph will be frown. *Researcher: What is the $\boldsymbol{a}$ you are talking about?* L10: The $\boldsymbol{a}$ is found on $\boldsymbol{ax}^2 + \boldsymbol{q}$. *Researcher: What is the general formula of parabola function?* L14: I don't know, but our teacher told us that it is $\boldsymbol{ax^2} + \boldsymbol{q}$. 5.6.1.1.1.1 Question 1 analysis *Researcher: When I examined your script for question 1, I discovered that you have difficulty solving quadratic equations. Can you explain what comes to your mind when solving quadratic equations?* L15: I'm not sure I understand the factor method. My teacher told me that we solve quadratic functions using the factor method, which I find difficult to apply. L23: I struggle to understand how factor method can be applied, sir do we factorised quadratic equation by applying only factor? I do not believe that only one method can be used solving quadratic equations. L35: My teacher told me that we must first make the equation to standard and then apply factor method which I struggle to understand. L40: It is difficult to solve quadratic equations, the method used is not easy to understand. The following except 5.11 shows comprehension errors responses given by learners solving quadratic equations.

Question 1.1	Solve the following quadratic equations
Excerpt of incorrect response by L15	$x(x+3)$ $=(x)(x+3)$ $=(x)(x)+(x)(3)$ 0 $=x^2+3x$
Excerpt of incorrect response L35	$\Rightarrow x(x)+x(3)=0$ $\Rightarrow x^2+3x=0$ $\Rightarrow\sqrt{x^2}=\sqrt{3x}$ $\Rightarrow x=3$ $\Rightarrow x=3$ 0

The above excerpt 5.11 shows how learners' commit comprehension errors when answering question 1.1. Instead of equating the equation to zero, L15 and L35 multiplied the factors. In this regard, it suggests that learners were experiencing Newman's comprehension errors in solving quadratic equation because learners' experience difficulties applying the factors method or quadratic formulae to solve parabola functions. Based on this extract, there is evidence that learners understand the procedure of finding the x –intercepts but experience challenges of comprehension errors. Asmawati (2019:4) asserted compression errors learners were unable to apply the correct procedures when solving mathematical problems. The extract also implies that L15 and L35 were at a loss for what to do when confronted with the problem of solving quadratic equations due to comprehension errors. Therefore, learners experienced challenges in choosing relevant equations for solving quadratic equation due to comprehension errors (Branenburg, 2019:7).

Researcher: Tell me anything about input and output values in sketching the graphs of parabola functions.

L8: You determine the output values by substituting the given number of x for the quantity given on the y after that you draw graph.

L20: You find the output by using the x values given in the equation, but sir, it is difficult to find those values to sketch the graph.

L30: I do not understand them but just replace values on the table not understanding what is happening after that I sketch the graph

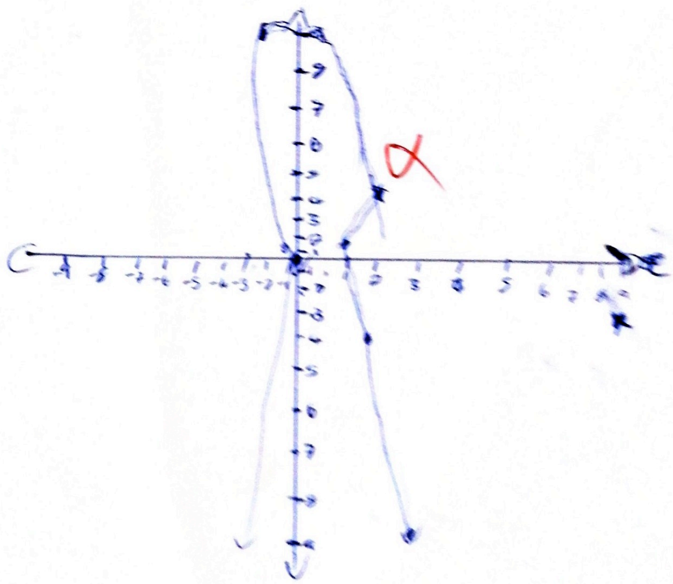
L40: Input values are values that put into the original equations to find the value of y then I sketch the graph.

Below is the excerpt 5.12 of L30's response, showing how they experience transformation errors when answering question 2.1.

2.1 Complete the following table of $f(x) = x^2$

x	-3	-2	-1	0	1	2	3
$f(x)$	9	4	1	0	1	-6	-9

2.2 Plot the points on the system of axes and join the points to form a graph.



Excerpt 5.12 suggest that L30 understands what should be done to find the input and output values in sketching the graph of parabola function, but learners were experiencing Newman's transformation error (Widiawati, 2020:1677). Furthermore, most learners gave blank responses to question items 2.1 and the same happened in question 2.2. They were expected to substitute given values of x to the given parabola function equation to find the out-put values which is the $f(x)$ and use the values they got to plot the point and to draw the graph. Transformation errors disable the learners to change mathematical information into tables, drawings and graphs according to Widiawati (2020:1677).

Researcher: How can you explain the domain and the range of a parabola function and how did you determine them in the achievement test?

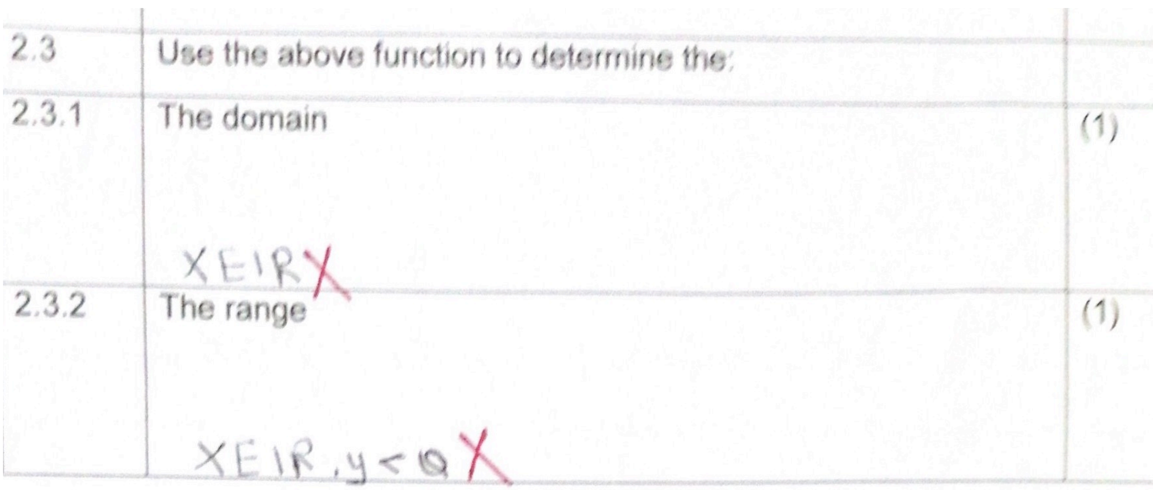
L6: I can define domain as x element of real number and the range is negative.

x element of real number and I did not calculate the range.

L10: Our teacher told us that we find the domain by looking to all real numbers, I find it difficult to calculate them.

L15: I understand domain and the range as x and y values.

Excerpt 5.13 below shows L15's response experiencing encoding errors on answering questions 2.3.



2.3	Use the above function to determine the:	
2.3.1	The domain XEIR	(1)
2.3.2	The range XEIR, y<0	(1)

In question 2.3, students were unable to explain why they chose the method they did to determine the domain and range of parabola functions. Learners were supposed to have known that the domains are the set of all x –co-ordinatees and the domain as a set of all $y-$ co-ordinates of a parabola function. Learners should have known that the domain is $x \in R$ and range as $\{y: y \in R, y \geq 0\}$. The above extract suggested that incorrect responses given by L15 were triggered by encoding errors in learning parabola functions. The extract demonstrates that L15 has encoding errors; the learner was unable to determine the domain and range of the sketched parabola functions. The learner further experiences a lack of conceptual understanding of parabola functions, as indicated by Kilpatrick et al. (2001:139).

Researcher: Can you explain the $x-$ and $-y$ intercept of parabola functions and how did you determine them when writing the achievement test

L4: When we want to find the y −intercept $x = 0$ and when we are finding y −

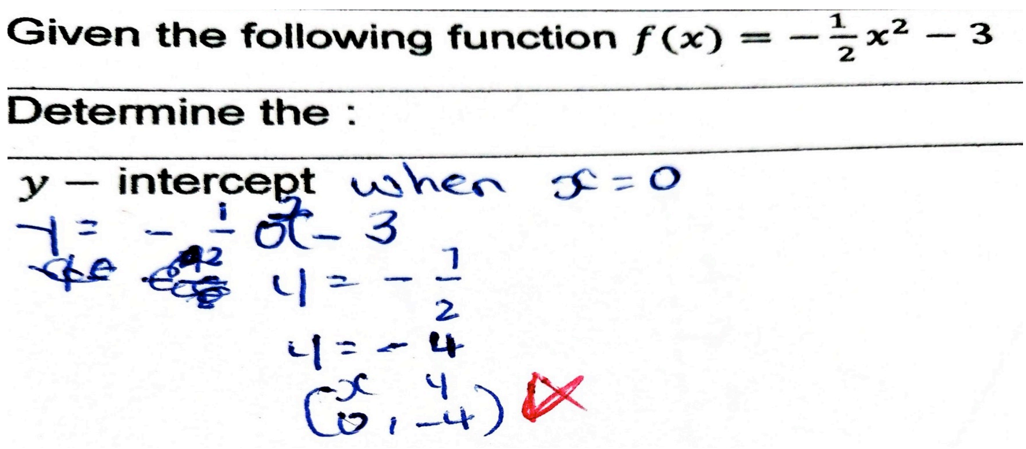
Intercept $x = 0$, then you solve the equation.

L12: I replace y with zero and then calculate it until it is finish

L24: I don't know, I just calculate

L40: Replace x values with zero and then solve equation to get the correct answer which is y −Intercept.

The excerpt 5.14 below represents L4 and L 24's response experiencing process skills errors in finding the y −intercept and $x-$ intercept respectively.


Given the following function $f(x) = -\frac{1}{2}x^2 - 3$

Determine the :

$y-$ intercept when $x = 0$

$y = -\frac{1}{2}x^2 - 3$

$y = -\frac{1}{2}$

$y = -4$

$(0, -4)$

3.1.2 $x -$ intercept

x - intercept $y , 0$

$0 = -\frac{1}{2}x^2 - 3$

$-\frac{1}{2}x^2 - 3 = 0$ $(6;0)$

$-\frac{1}{2}x^2 = 3$

$\sqrt{x^2} = \sqrt{3\frac{1}{2}}$ α

$x = 6$

The preceding excerpts 5.14 of learner L4's response shows that the learner understands the procedures supposed to be done to determine the $y -$ intercept. It suggests that L4 was experiencing process skills errors on determining the $y -$intercept parabola functions. The learner does have procedural fluency because he was able to follow the procedure accurately to determine the $y -$ intercept (Kilpatrick et al., 2001:121). Learners experience process skills errors as indicated by Widiawati (2020:7).

Learner 24 was supposed to have used the quadratic formula, factor method or completing the square to determine the $x -$ intercepts in question 3.1.2. This suggest that most learners were again experiencing process skills errors, but learners do have procedural fluency and strategic competence that can assist them in dealing with problem of this nature (Kilpatrick et al., 2001:122).

Researcher: Can you explain the axis of symmetry of a parabola function and how did you calculate the axis of symmetry?

L6: I don't know, we haven't taught.

L45: I did not calculate it because I don't know it.

L54: I do not know how to calculate the symmetry; our teacher did not teach us how to calculate the axis of symmetry.

The excerpt 5.15 below shows the reading errors' response on determining the axis of symmetry shown by L54.

3.1.3 axis of symmetry

$(0, -4)$

$(0, 2)$ α

$(0, -2)$

To determine the axis of symmetry learners should have set $\boldsymbol{x} = -\frac{\boldsymbol{b}}{\boldsymbol{2a}}$ and know that in the parabola functions $\boldsymbol{f}(\boldsymbol{x}) = -\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{x}^{\mathbf{2}} - \mathbf{3}$, $\boldsymbol{b}$ is the coefficient of $\boldsymbol{x}$ which is zero and $\boldsymbol{a}$ is the coefficient of $\boldsymbol{x}^{\mathbf{2}}$ which is $-\frac{\mathbf{1}}{\mathbf{2}}$ which will give the axis of symmetry to be the $x = \mathbf{0}$. According to Newman's error analysis, these learners were committing reading errors because they were unable to move from one equation to the next equation to determine the axis of symmetry. The reasons why learners were experience this challenge is because they were lacking (Kilpatrick et al., 2001:121) conceptual understanding and had reading errors.

Researcher: What challenges do you face when learning parabola functions?

L 6: Sir, I don't understand the way in which it is determine $\boldsymbol{x}$ –intercept, axis of symmetry, finding the domain, range and even interpreting those graphs.

Researcher: What can you say can be the solution in addressing the difficulties in solving parabola functions?

L 45: If my teacher can teach me all this topic I can understand.

L 12: If our teacher can teach us all topics rather than only sketching the graph, we can understand the topic.

Researcher: What do you think can be done to help you understand this concept of parabola functions?

L 24: Parabola functions should be given more time not one week

L 10: If this topic can be given enough time we can understand, sir we have done this topic by one week.

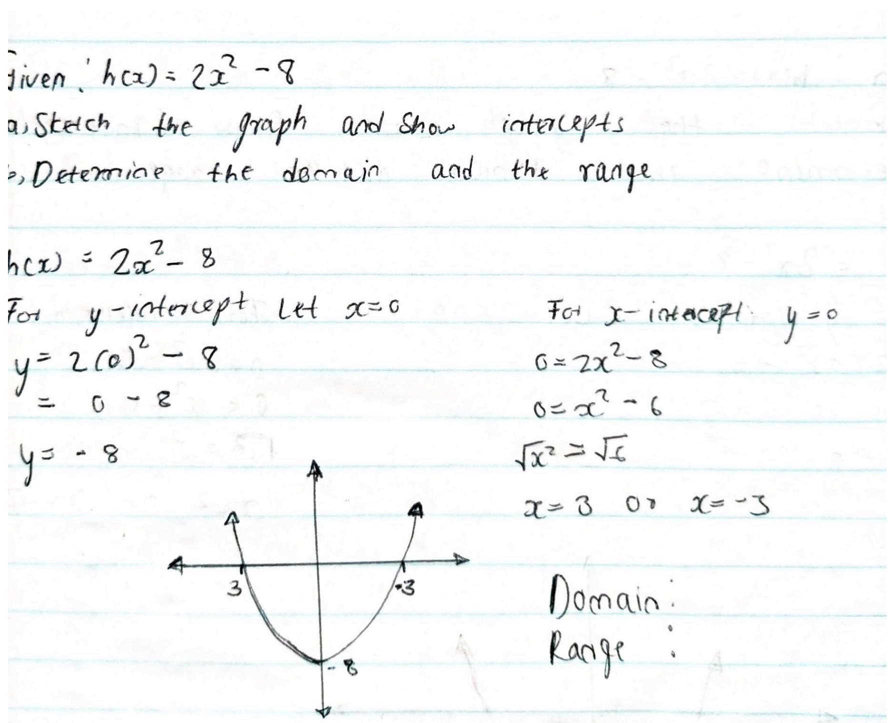
Data analysis from the semi-structured interviews revealed that the learners experience Newman's errors (1988:674) when they are learning parabola functions. Learners were unable to find the output values given the input values, unable to sketch the graph of parabola functions, unable find the $\boldsymbol{x}$ and $\boldsymbol{y}$ –intercepts of a parabola function, finding the range and determining the domain of parabola functions and determining the axis of symmetry of a parabola functions. The reason was due to the learners' prior experience. They lacked conceptual understanding, strategies competence, procedural fluency, reasoning skills, and problem-solving skills of learning parabola functions effectively, as well as Newman errors, such as comprehension error, reading error, transformation error, and process error (Newman et al., 1988:674). Furthermore, semi-structured interviews revealed that students were making compression Newman errors when calculating the x-intercept and drawing the graph of parabola functions (Newman et al., 1983:124) The semi-structured interview also revealed that students looked to teachers as their final source of knowledge. This is evident from L6’s response indicating that: she can understand if the teacher addresses all topics during their teaching. Furthermore, semi-structured interview responses revealed that the time allotted for the topic of parabola functions was insufficient for the learners to comprehend and master the material. The researcher continues with the study's data analysis by analysing lesson observation to confirm if what learners indicated in the semi-structured interviews was correct.

5.8 LESSON OBSERVATIONS

Lindorff (2018:2) defines lesson observation as "the method of directly observing teaching practice as it occurs in order to record learners' and educators' performance." The primary goal of the lesson observation was to confirm what learners said in semi-structured interviews and what the researcher learned from the achievement test analysis. Data from achievement tests and semi-structured interviews were insufficient to understand the difficulties learners faced when learning parabola functions. When collecting data, the researcher followed Sepeng's (2010:86) lesson observation schedule. Furthermore, the researcher observed lessons in Schools A and B for two weeks, following a schedule devised by the researcher, the teachers, and the students. The schedule was created in accordance with the Annual Teaching Plan (ATP), which stated that parabola functions should be assigned two weeks of teaching and learning time. The researcher observed both classes in school A, which was a pilot study, and school B, which was a main study, over a two-week period. Although teachers were not the focus of this study, the researcher observed how their students learned about parabola functions.

The researcher observed how teachers used questioning techniques to help students develop the conceptual and procedural understanding of parabola functions that they desired. The zone of proximal development and scaffolding described by Vygotsky et al. (1979:78) was not effectively applied to analyse the learners' accounts. It was critical to examine how teachers teach parabola in order to determine whether any of the difficulties encountered by students were caused by their teachers' methods of instruction. The researcher also observed students in both schools while they studied parabola functions. According to Vygotsky et al. (1978:89), an individual's mental functioning is derived from the support students receive from capable peers. The researcher utilised the observation instructions below to record how learners participated in the learning process, including how they asked questions and interacted with capable peers. The researcher saw the following abilities: productive abilities, evocative abilities, evaluative abilities, and reflective skills.

5.8.1 Learner observations on productive skills

Excerpt 5.16 of observations from the learner's workbooks.	 Given: $h(x) = 2x^2 - 8$ a) Sketch the graph and show intercepts b) Determine the domain and the range $h(x) = 2x^2 - 8$ For y-intercept let $x=0$ $y = 2(0)^2 - 8$ $= 0 - 8$ $y = -8$ For x-intercept $y=0$ $0 = 2x^2 - 8$ $0 = x^2 - 6$ $\sqrt{x^2} = \sqrt{6}$ $x = 3$ or $x = -3$ Domain: Range:

Learners were able to create notes on the notion of parabola functions, but they were unable to solve problems supplied in the exercises, indicating that the learning methodologies utilised did not assist learner knowledge acquisition. Furthermore, the study discovered that learners were unable to leverage their existing knowledge and expertise with parabola functions in creating their own solutions. Learners struggled to complete work on parabola functions independently, as seen by blank spots left by learners for identifying the domain and range of parabola functions. Further, learners were shown to be unable to determine x-intercepts and parabola functions. It was also discovered that these students were unable to read supplied graphs of parabola functions. The tasks emphasised the linkages between parabola functions and linear functions; nonetheless, learners were unable to apply their understanding of additive inverse to parabola function difficulties. In addition, learners' problem-solving reasoning and decision-making abilities were not improved, and learners made more errors while dealing with parabola functions.

5.8.2 Learner perceptions of evocative skills abilities

As evocative skills, researchers noticed learner interest and participation. Learners were engaging and immersed in the learning process; yet, when the teacher asked questions, the majority of the learners from both schools stayed mute. Teachers at both institutions used a conventional way of teaching in which they explained everything to the students, which might explain why students fared badly in the notion of parabola functions.

5.8.3 Learner observations on evaluative skills

Excerpt 5.17 of observations from the learner's workbooks.	Determine the equation of the parabola passing through the points (-2; -36) and (1; -12) $y = ax^2 + c$ Substitute the point (-2; -36) and substitute the point (1; -12) $\therefore -36 = a(-2)^2 + c$ and $-12 = a(1)^2 + c$ $-36 = 4a + c$ and $-12 = a + c$ ① and ② ②-① $-12 - (-36) = a + c - (4a + c)$ $-12 + 36 = 5a - 2c$ $\therefore a = -5$ and $c = -2$

The researcher observed that learners were unable to evaluate their own work when learning the topic of parabola functions due to Newman et al.'s (1983:674) encoding error. Some of the learners' answers, for example, had reading mistakes in which learners did not understand essential words presented correctly when calculating parabola functions. Some of learners' solutions observed contained transformation errors and misconceptions where learners wrote incorrect mathematical signs and failed to follow correct procedures of dealing with additive inverse. It was quite distressing to see that students were unable to identify the right methods for obtaining the equation of parabola functions when given two points. For example, learners were making errors in finding the value of $\boldsymbol{a}$ and $\boldsymbol{b}$ that could lead them in finding the parabola functions equation. Most learners were lacking conceptual comprehension as well as the practical expertise of working with parabola functions.

5.8.4 Learner observations on reflective skills

The researcher noticed that learners were continually making mistakes when studying parabola functions. Several frequent mistakes were identified throughout the process of learning parabola functions, notably, reading errors - learners were observed not reading the main information correctly and not using the given information to solve the problem. In most cases, the learners made comprehension errors where they were unable to understand the meaning of the questions. In other words, when studying parabola functions, students were unable to reflect and make decisions on how to solve issues. The study also discovered that the majority of the instructors' teaching styles were question and response methods. Further, the researcher noted that teachers were employing traditional methods of teaching, with teachers dominating much of the teaching and learning. Because learners were not given the opportunity to engage, the teaching tactics used resulted in their becoming passive recipients of knowledge and information. Lesson observations found that the teachers did not assist the students in unpacking the parabola functions' terminology and vocabulary. The study discovered that the majority of learners' learning methodologies did not aid in the building of parabola function knowledge. According to Vygotsky et al. (1978:123), engagement with capable peers increases understanding of topics presented. Lesson observations found that learners were not provided the opportunity to share their points of view in groups in order to enhance learning by able peers. Teachers, on the other hand, were found not using the Zone of Proximal Development (ZPD) while teaching parabola functions. According to Vygotsky et al. (1978:129), ZPD checks what the learners know and whether they can handle difficulties above their real development level if they are guided by more proficient peers.

Lesson observations found that learners were not offered assistance with activities that were difficult for them to master. For example, if students were unable to establish the domain and range of parabola functions, the session was terminated and the teacher advised them to conduct more study and assigned homework.

Before beginning Grade 10 parabola functions, teachers were supposed to assess students' past understanding. It was regrettable that some students were unable to relate their understanding of quadratic equations from Grade 9 to Grade 10 topics. The researcher also discovered that Grade 10 students had insufficient prior understanding of quadratic equation ideas in this investigation. Observations of lessons revealed that students were not provided assistance with exercises that were difficult for them to master. For example, if students were unable to determine the domain and range of parabola functions, the session was ended, and the teacher encouraged them to perform more research and provided homework. Teachers were instructed to examine pupils' prior grasp of parabola functions before commencing Grade 10 parabola functions. Some pupils were unable to link their grasp of quadratic equations from Grade 9 to Grade 10 themes, which was unfortunate. In this study, the researcher also revealed that Grade 10 pupils had insufficient prior comprehension of quadratic equation concepts.

According to Vygotsky et al. (1978:125), during effective teaching and learning, learners should ask questions and work hard to develop knowledge in order to find the answers for themselves. The researcher was astonished to hear that Grade 10 students did not complete their assignment. This indicates that learners were not motivated to study parabola functions. Furthermore, the researcher noted that the majority of the classwork assignments were met with blank replies from students. In the parabola functions course, the Grade 10 students did not ask any questions. Based on the observation of this session, it was discovered that there is a lack of productive disposition in the sense that learners do not take ownership of their work and are not committed to their learning. During parabola function lessons, the researcher observed the following challenges: learners were unable to solve parabola function problems, learners were unable to determine strategies, rules, and procedures to be followed to determine the x- intercepts, axis of symmetry, domain, and range of parabola function. Additionally, learners lacked understanding of parabola functions due to a lack of prior knowledge of quadratic equation knowledge, problem solving abilities, and thinking skills.

5.9 DISCUSSIONS OF THE STUDY'S MAIN FINDINGS

According to McCombes (2019:3), study findings are interpreted by focusing on theories, literature reviews, and research questions, as well as creating an argument in favour of your conclusions. The discussions of the main findings in the results analysis primarily focus on four themes: learner performance when learning parabola functions, challenges learners face when learning parabola functions, errors committed by learners when learning parabola functions, and teaching and learning resources used by learners when learning parabola functions.

5.9.1 Learners' performance when learning parabola functions

The study revealed that learners' academic performance when learning parabola functions were poor. This was obvious through descriptive statistics analysis, which demonstrated that the majority of students received zeros on the completion of the achievement test. According to the study, the factors that contributed to poor performance were difficulties learners encountered when learning parabola functions, such as understanding different methods that can be used to solve parabola functions and being unable to understand different strategies that can be used to solve parabola functions. Learners were unable to solve quadratic equations, find the output values and sketch the graphs of parabola functions, find the y and x-intercepts, determine the axis of symmetry, calculate minimum and maximum points, determine the domain and the range of parabola functions. This results support Hussain (2018:27) and Sabilah (2018:1), who said that learners who lack conceptual understanding, procedural fluency and strategic competence will experience difficulties when solving any mathematical problems, like parabola functions in the current study. The difficulty was further confirmed by the semi-structured interviews where learners were unable to describe parabola functions and failed to state any methods that can be used to solve parabola functions. This finding was further supported by lesson observations where learners were unable to describe parabola functions when asked by their teachers in the classroom, learners were unable to answer questions on how parabola functions can be solved.

5.9.2 Challenges faced by learners' when solving parabola functions

The researcher went on to say that students struggled on the achievement test because they could not solve quadratic equations, find the output values and sketch the graphs of parabola functions, find the x-intercept, find the axis of symmetry, find the minimum and maximum points, and determine the domain and range of parabola functions. This supports what Machaba (2021:87) and Ruli (2081:684) said in their articles that learners who lack prior knowledge of parabola functions, conceptual understanding and strategic competence are not able to solve parabola functions effectively.

The semi-structured interview conversations with learners indicated that learners were unable to calculate the x-intercepts, sketch the graph of parabola functions, establish the axis of symmetry and the domain and range, and sketch the graph of parabola functions.

The researcher discovered that most students could predict the y-intercept of parabola functions, but only a handful could determine the x-intercepts, axis of symmetry, minimum and maximum points. It is easier for learners who have conceptual comprehension, procedural fluency, and strategic competency to solve mathematics problems with parabola functions (Kilpatrick, 2001:116).

5.9.3 Learners' committing errors in solving parabola functions

Another aspect that contributed to learners' low performance on the accomplishment of the achievement test, according to this study, was that they made errors, such as comprehension errors, reading errors, transformation errors, encoding errors, and process skill errors. According to Newman (1988), in order for students to avoid making mistakes when solving mathematical problems, they must be able to read, interpret, convert, encode, and process the problem. This was also demonstrated in the classroom when students struggled to transfer the supplied parabola function information into graphs, indicating that they were making Newman's comprehension and transformation mistakes.

5.9.4 Teaching and learning resources

The researcher also noted that learners were relying on the teacher as their only source of knowledge which hinders them from being active participants in their learning and correcting errors they were committing which results in learners performed poorly in the achievement test. Learners took the textbook as their only source of knowledge. Learners should be exposed to different resources when acquiring knowledge, such as effective ways of using calculators, YouTube channels, where learners can acquire knowledge across broad perspectives about parabola functions, WhatsApp platform where learners can share knowledge amongst themselves and GeoGebra where learners can be able to manipulate parabola functions graphs will all help the learners (Ovez, 2018:3).

Learners should not be relying on teachers and textbooks as their only source. Learners relied on the instructor as the sole source of knowledge, which made it difficult for them to participate actively in their learning and to rectify errors they made. Through semi-structured interviews, it was revealed that learners only relied on their teachers and textbooks as their sources of knowledge. This was evident when most learners indicated that their teacher did not teach them about the axis of symmetry, minimum and maximum while others said that their textbooks do not have the topic - axis of symmetry. Through lesson observations, it was revealed that the majority of learners were passive recipients of knowledge, when the problem was posed to them, they were not willing to interrogate the problem instead they waited for the teacher to give them answers so that they could copy them. It was also observed that when learners were copying those answers, some learners were copying them incorrectly which further hindered their understanding of parabola functions.

5.10 CHAPTER SUMMARY

The researcher examined the data acquired from the achievement test, semi-structured interviews, and lesson observations in this chapter in order to evaluate the learners' academic performance in learning parabola functions. The study's data was analysed with reference to the research objectives and research questions. The data analysis process was divided into two stages: quantitative and qualitative. First, statistical approaches were used to analyse and explain achievement test results. Then, data from semi-structured interviews with learners were analysed and discussed. The examination of student interviews reveals that learners had difficulties in addressing parabola function issues. In addition, the researcher observed lessons to assess learners' productive skills, evocative skills, evaluative skills, and reflective abilities. The researcher observed that learners' encountered difficulties while learning parabola functions. It was also discovered that teachers' instructional methods hampered students' learning of parabola function ideas.

CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS

6.1 INTRODUCTION

The data analysis of Grade 10 learners' academic experiences and the difficulties they encounter when learning parabola functions are covered in the preceding chapter. This study sought to investigate the academic experiences of Grade 10 learners as they learned to solve parabola functions. The researcher's decision to undertake the study was motivated by the students' sub-par performance on the parabola functions unit in Grade 10 Mathematics. The researcher wanted to understand learners' academic experiences focusing on learners' performance and the errors they display when they were learning parabola functions. This chapter includes a summary of findings, an overview of the investigation, key findings, and conclusions, the researcher's perspective within the theoretical context of the study, as well as the study's limits, suggestions, and conclusions.

6.2 OVERVIEW OF THE STUDY

The researcher's observation of a persistent decline in mathematics performance in grade 12 served as the impetus for the investigation. Since the DBE implemented CAPS in 2012, learners' academic performance in mathematics has been poor. The study's literature evaluation, according to the researcher, suggests a connection between students' academic success and the teaching and learning of parabola functions. The researcher discussed a review of the literature in Chapter 2 in relation to the idea of parabola functions, learners' performance and challenges in learning parabola functions, errors in learning parabola functions, the mathematical proficiency required to learn parabola functions effectively, mathematical content knowledge, and PCK in teaching and learning of parabola functions. The study also used parabola functions in its teaching and learning to create a theoretical framework. The researcher outlined the research methodology in Chapter 4, which was centred on a sequential explanatory mixed-methods approach. The tools used for data gathering and analysis were also covered by the researcher.

The study's data collection process was divided into two stages: a quantitative phase and a qualitative phase. In the quantitative phase, achievement tests were used to obtain numerical data. In the second stage, semi-structured interviews and lesson observations were used to gather qualitative data. The categories of the learners' responses from Didis and Erbas (2015:1142) were used by the researcher to analyse quantitative data. Techniques from descriptive statistics were also used to improve the analysis of quantitative data. A content analysis procedure was used for the study of qualitative data. The researcher considers the purpose of the study and the research strategy in this final chapter. The implications and restrictions of the study's framework are described. Finally, the researcher makes suggestions for additional investigations.

6.3 SUMMARY OF RESULTS, STUDY MAIN FINDINGS AND CONCLUSIONS

The researcher studied learners' academic experiences when learning parabola functions at two schools by administering achievement assessments, semi-structured interviews, and classroom observations. The study included 90 Grade 10 learners from the rural schools as a sample and three teachers teaching mathematics in those two schools. Research conducted by Mutambara (2019:9) has shown that learners' performance in parabola functions was poor. Research conducted prior to this study has also shown that learners were underperforming in parabola function (Hoon 2018:78; Dids, Celk & Guzel, 2016:128; Nielsen, 2015:11). The assertion was confirmed by the poor performance in the achievement test. The achievement test results for the students indicated a mean percentage achievement of 4,5, a median of 4, a mode of 1, and a standard deviation of 5. Due to the learners' modest variances in how their marks were distributed on the test, the mean and median exhibit marginally positive differences. The standard deviation was 5 which is greater than the mean and indicates that most learners were achieving 1 mark in the test as evidenced by the mode which is 1. The results of the study showed that learners in Grade 10 were unable to solve problems involving parabola functions, solve quadratic equations, sketch the graph of a parabola function, identify the domain and range of a parabola function, locate the axis of symmetry of a parabola function, and locate the minimum and maximum point of a parabola function. These findings are consistent with the findings of Hoon's (2018:78) study which showed that students frequently find it difficult to comprehend the many ideas related to parabola functions. Similar results were obtained in the study by Didis and Erbas (2015:1142), which revealed that learners had trouble determining the minimum and maximum points, the axis of symmetry, and the domain and range of parabola functions.

Semi-structured interviews and lesson observation findings with the chosen learners revealed that some parabola function concepts, such as the axis of symmetry, turning point, interpreting examples of graphs, and determining the domain and range of parabola functions, are difficult for students to understand. Furthermore, the findings revealed that the minimum time allocated for the teaching of parabola functions in each school was not enough for covering all concepts hence suggest that schools adjust time allocated for teaching parabola functions.

The findings of this study also demonstrated that teachers' content and pedagogical competence were the root reasons of learners' difficulties, which is why not all ideas related to parabola functions were taught. The findings of this study are consistent with those of other investigations (Loh, 2019:2; Mutambara, 2019:35; Kabar, 2018:112; Uba & Bansil, 2018:848; Ruli, 2018:4; Deacon, 2016:30; Luneta & Makonye, 2016:2; Asksu, 2016:36; Mushipe, 2016:25; Manzindu, 2016:2; Ibueawachu, 2016:420; Shullman,1988:848). These studies indicate that Mathematics content knowledge and teacher PCK play a role in learners' understanding of mathematical concepts and hence the researcher suggest that further study may be conducted uncover how teacher content knowledge affects learners understanding of parabolic functions.

Research questions helped highlight the learners' academic performance when they were learning solving parabola functions. When learning solving parabola functions, learners performed poorly, according to an analysis of accomplishment test data. This result proves the hypothesis that Grade 10 learners' academic performance when they were learning solving parabola functions is poor.

The researcher also wanted to gain insight into the difficulties that learners' face as they learn to solve the concepts of parabola functions, specifically those that relate to sketching the graph of parabola functions, interpreting the graph of parabola functions, figuring out the x- and y-intercepts of parabola functions, figuring out the domain and range of parabola functions, figuring out the axis of symmetry, and describing the relationships of parabola functions. This research revealed that learners attempted to solve parabola functions using their intuitive knowledge and procedural strategies. Further research demonstrated that learners' have difficulties in applying methods for calculating linear functions to Grade 10 parabolic functions (Nielsen, 2015:91). The study also revealed that learners were unable to interpret the graph of parabola functions. Ruli (2018:684) reported the same results, which showed that learners were unable to interpret the data shown in parabola function graphs. The study found incorrect computation strategies, improper use of procedures, inability to identify the domain and range of parabola functions, and failure to describe the relationship on the provided sketched graph, all of which appear to have been brought on by Newman's (1983:68) reading and transformation errors, were some of the challenges that affected learners' poor performance. The same findings were observed in the studies by Parent (2016:114) and Guzzel (2018:25), which revealed that learners experience difficulties when learning to solve parabola functions in relation to solving and sketching the graph of parabola functions.

The researcher also discovered that supporting concepts of quadratic equations (which were supposed to be prior knowledge of learning parabola functions) were not effectively taught in the previous grades. Some learners point out that it is difficult to find the $x-$ intercept and axis of symmetry and determine the range, the minimum and the maximum points of parabola functions because they have not understood them well during teachers' instructions. The semi-structured interview results further indicated that learners were not ready to learn the topic of parabola functions; they considered parabola functions as a difficult topic.

Observations made during lessons showed that the teachers' techniques of instructing and learning did not help students understand how parabola functions work. Findings revealed that teachers only relied on the direct teaching method as the main teaching method which results in learners' experiencing challenges in understanding the concepts of parabola functions. While learners were merely passive recipients of the information, teachers were the primary sources of knowledge in the lessons. Minimal emphasis was given to a learner-centred approach through the direct questioning and answering method. The teaching and learning method exposed learners to rely on memorising some formulae with little grasp and mastery of concepts.

Further lesson observations showed that teachers were not effectively implementing Vygotsky's (1987:86) ZPD by helping students advance from their current understanding of linear functions to their potential understanding of parabolic functions. It was also observed that scaffolding was not effectively utilised as teachers were not seen to be supporting learners on how to solve quadratic equations.

6.5 THE RESEARCHER’S VOICE FRAMED BY THE STUDY’S THEORETICAL FRAMEWORK

The findings of this study provided information and insight on how learners should effectively learned parabola functions. Quantitative data analysis revealed that solving quadratic functions should be taught first to the learners before they are introduced to solving parabola functions (Mutambara et al., 2019:9). The earlier findings demonstrated that prior knowledge is crucial to learners' comprehension of new ideas. Before introducing the ideas of parabola functions, it is best to gradually introduce the teaching of quadratic equations. The results of this study show that students should solve quadratic problems at their current level of development and then progressively advance to their potential level of development, which aids in an effective knowledge of parabola functions. Moreover, the researcher observed that learners were passive receivers of knowledge rather than participating actively in the process of knowledge construction. It was further found by the researcher that students enter schools with incorrect knowledge of parabola functions which need to be addressed before teaching the topic of parabola functions. The informal knowledge learners held about parabolas explains why they commit errors when they are learning parabola functions. This finding concurs with Celik and Guzel’s (2017:25) that learners experience challenges when learning the concept of parabola functions owing to a lack of prior knowledge.

Before beginning to teach parabola functions, teachers may find it helpful to take into account their learners’ prior experience with quadratic functions. The study pointed out once more that most learners straggle to comprehend parabola functions because they lack prior knowledge of parabola function which led to learners' poor performance when they were learning parabola functions. To improve learners' strategies and ability in solving parabola functions, learning should employ a variety of parabola function-solving techniques. The researcher found that learners were making Newman's (1983:65) errors because they did not understand the various approaches utilised to locate the axis of symmetry of parabola functions. As a result, they were unable to locate the axis of symmetry of parabola functions. The study's findings once again demonstrated that teachers' lack of content knowledge and PCK of parabola functions was to blame for their learners' confusion about how to calculate the axis of symmetry. If teachers had good knowledge of the concept of parabola functions, learners would have a better comprehension of parabola functions' concepts which would translate to improved performance.

The following are major findings of this study of Grade 10 learners' academic experiences of learning parabola functions: learners come to class with incorrect knowledge of parabola functions; learners' knowledge of linear functions interfere with learners' knowledge of learning parabola functions; learners have inadequate conceptual and procedural knowledge in parabola functions; learners lack knowledge of finding the domain and the range of parabola functions, and do not have problem-solving skills.

6.6 LIMITATIONS OF THE STUDY

The researcher has noted and accepts the following limitations that resulted from this investigation: Only two schools were included in the research study's sample, which was too small. As a result, the conclusions of this research study cannot be applied generally. The study was restricted to Mathematics students in Grade 10. Rather than concentrating on more general Mathematics to enhance learners' performance, this study primarily looked at the topic of parabola functions.

6.7 RECOMMENDATIONS

The study's findings provided an overview of the steps that must be taken for students in Grade 10 Mathematics to successfully master parabola functions. According to the study, the learning process should begin by emphasising the prior knowledge that learners bring to the classroom, hence further research needs to be conducted to find out learners understanding of linear functions. The study also suggests that PCK and content knowledge of teachers of parabola functions should also be considered. The researcher suggests looking into the teaching strategies to help students learn parabola function ideas conceptually and practically hence researcher suggest further study to conducted on the approaches that can be used by the teachers to improve learners performances in learning parabola functions. Learners should be encouraged and given more exposure to the many approaches that are utilised to tackle parabola function problems in Grade 10. The current circumstance demanded that the time allotted in the CAPS (2012:45) document be exceeded due to the challenges faced by Grade 10 Mathematics students when tackling parabola function issues.

6.8 CONCLUSION

The academic experiences of Grade 10 Mathematics learners' when learning to solve parabola function problems were examined in this study. In the Limpopo Province of Vhembe District, two secondary schools participated in this study. An explanatory sequential mixed-method design was used by the researcher. For the purpose of gathering and analysing data, achievement assessments, semi-structured interviews, and instructional observations were used. According to the study's findings, students struggle to master parabola functions, as seen by their poor performance in completing parabola function tasks. Analysis of the achievement test revealed that reading errors, inadequate text comprehension, and the lack of a conceptual and practical grasp of parabola functions, as a result of comprehension problems, were the main contributors to the learners' difficulty. The study was able to pinpoint the issues where learners experience difficulty in comprehending parabola functions. It is necessary to take additional measures to overcome the learners' challenges with parabola functions.

The purpose of this study was to investigate how Grade 10 learners learned to solve parabola function problems in the classroom. The study was carried out as a result of the learners' poor mathematics academic performance. The researcher focused on parabola functions to examine how well learners performed academically and any errors they made while learning Grade 10 parabola functions.

The researcher advises that additional research be done on Grade 9 learners' knowledge of effective learning and comprehending solving quadratic equations, which is prior knowledge of solving parabola functions.

REFERENCES

Aalto E. & Mustonen, S. (2022). Design knowledge construction in pre-service teachers' collaborative planning talk, *Journal of Linguistics and education*, 40(3):67-87.

Abdullah, S. A., & Saleh, F. (2005). Difficulties in comprehending the concept of functions: Implication to instructions. In *CosMED International Conference, SEAMEO RECSAM*. Gelugor Pulau Pinang Malaysia.

Adams, P. (2022). Initial Teacher Education Partnership: bureaucracy, policy, and professional agency, unpacking the nexus. Springer.

Akaranga, S. I., & Makau, B. K. (2016). Ethical consideration and their application to research: a case of the University Nairobi. *Journal of Education Policy and Entrepreneurial Research*, ISSL: 2408- 770X, VOL.3.12. PP-9.

Aksu, Z., & Kul, U. (2016). Exploring mathematics teachers 'pedagogical knowledge in the context of knowledge of students, *Journal of education and practice*.

Alibali, M. (2006). Does visual scaffolding facilitate students Mathematics Learning? Evidence from early algebra. *Retrieved February*, *13*, p.2016. http://ies. ed.gov/funding/grantsearch/details.asp?

Aliyu, A. A., Bello, M. U., Kasim, R., & Martin, D. (2014). Positivist and Non-Positivist Paradigm in Social Science Research: Conflicting Paradigms or Perfect Partners? *Journal of Management and Sustainability,* 4(3): 79-95.

American Heritage Dictionary, (2006). American heritage dictionary 5th ed. New York: Turtleback.

Amineh, R. J., & Asl, H., D. (2018). Review of Constructivism and Social Constructivism,www.sematicscholar.org

Amsters, S. (2021). Knowledge Management (KM). *Journal for Educational in formation Management and governance* 2 (45) 67-86.

Anago Academy. (2022). What are the Ethical Consideration in research*, Journal of educational research*, www.enago.com.

Anggadewi, B. E. T. (2018). Scaffolding: How it works for students with learning difficulties, Department of Primary Teacher Education, Sanata Dharma University, Indonesia.

Aslam, N., Khanam, A., Gulnaz, H., & Akbar, H. (2018). Study of the impact of Scaffold Instructions on the Learning Achievemnts of Post-Graduate Students, Lahore College for Women University, www.researchgate.net.

Asmawati, A. Q. (2019). The influence of using team Game TOURNAMENT Towards Student Reading Comprehensions in Descriptive Text at the Regular EIGHT Grade of MTsN 2 Babdar Plumbing in the Academic Year 2018/2019, State Islamic University of Raden Itan Plumbing, Nigeria.

Aubrey, K., & Riley, A. (2018). Understanding and using Educational Theories, 2th edition, SAGE Publications Ltd.

Ayadin, B., Coben, M., Dinky, A. S., Memenum, D. S., & Sevindik, F. (2015). Failure and inability of high school students about quadratic equations and functions Uludag University, Turkey: *Journal of Education and Training Studies*, *3*(6), 50-60.

Awofala, A. O., & Sopekan, O.S. (2017). Mathematics Anxiety and Mathematics beliefs as Correlates of Early Childhood Pre-service Teachers' Numeracy Skills, *journal of the mathematical association of Nigeria, 10(2,),81-87.*

Azizi, I. (2021). The impact of Game- Based Modelling, and Collaborative Learning Methods on the Achievements, Motivations and Visual Mathematical Literacy Perceptions, *Journal of Educational Research*.eric.ed.gov.

Babbie, E., & Muoton, J. (2010). *The practice of social research*. (10th ed) Cape Town: Oxford University Press.

Babbie, E., & Muoton, J. (2013). *The practice of social research South Africa*. (2nd ed.) Cape Town: WSORTH Publishing company.

Ball, D. L., Thames, M. H., & Phelps, G. (2009). Content knowledge for teaching: what makes it special? *Journal of Teacher Education*, 59:389-407.

Bandi, D. (2016). Parliament of Republic of South Africa Portfolio Committee basic education, overview and analysis of SACMEQ study results, SACMEQ website.

Barbara, K., Garner, M., & Wagner, C. (2012). *Doing social research: A global context*, Croydon: McGraw-Hill Higher Education, CPI Group (UK) Ltd, CR04yy, ISBN-100077126408.

Basias, N., & Pollia, Y. (2018). *Qualitative and quantitative research in Business & Technology: Justifying a suitable research methodology*. University of Piraeus: ISSN: 2304-1013.

Becker C., Becker S., Cheridito, P., Jentzen A., & Ariel Neufeld A. (2019). Deep splitting method for parabolic PDEs. Department of Mathematics, ETH Zurich, Switzerland.

Becton, L. (2022). Ancomplete Guide to Schema Theory its Role in Education, Eudcation corner, www.educationcorner.com

Benning, I., & Agyei, D. D. (2016). Effect of using spread sheet in teaching quadratic functions on the performance of senior high school students. *International Journal of Education, Learning and Development,* 4(1), 11-29.

Berman, E. (2017). Domestic intelligence: New powers risks. Brennan Centre for Justice, New York University School of Law.

Berndt, A. E. (2020). Sampling Methods, *Journal of Human Lactation*, Volume 32, journas.sagepub.com.

Bevans, R. (2022). Multiple Leaner Regression: A quick Guide Example, *Journal Educational Research*, www.scribbr.com.

Bhandari, P. (2022). Ethical considerations in Research, Types and examples, www.scribbr.com.

Bhasin, H. (2020). What are Ethical Consideration in Research. *Journal for business educational research*, (23) 244-355.

Bilau, A. A., & Lill, T.E. (2018). Practice frame work for the management of post disaster housing reconstruction programmes, Department of Engineering and Architecture, Tallinn University of technology, Estonia.

Bolisani, E., & Bratiamu, (2019). The Elusive Defination of Knowledge, In: Ermegent Knowledge strategies, University of Padua, ideas.repec.org.

Bonache, J., & Festing, M. (2020). Research Paradigms in international Human Resource Management: An Epistemological Systematisation of field, *German Journal of Human Resource Management*. Zeitschrift fur Personal forschung.

Boonmoh, A., & Jumpakate, T. (2019). Using Scaffolded Instructions to improve Students' skills, Article for reflections, Volume 26, King Mongkut's University of technology, Thailand.

Boru, T. (2018). Research Methodology, Thesis for PHD, University of South Africa, Pretoria.

Bouchrika, I. (2021). How to Write Research Methodology: Overview, Tips, and Techniques, *Journal for scientific research*, 19(4),99-104.

Boyd, H. (2022). On the Structure and Mechanical Properties of in Vitro Salivary Pelliclles, Dissertation for Medical Science, Media-Tryck, University of Malmo.

Brandenburg, 2020. Piecewise quadratic growth during the 2019 novel coronavirus, Journal for educational research, 200(3)203638v2.

Bransford, J. D., Brown, A. L., & Cocking, R. R, (eds). (2000). *How people learn: brain, mind, experience and school.* Washington, C: National Academy Press.

Budayasa, M. I., & Juniata D. (2018). Pedagogical content knowledge: Teachers knowledge of students in learning mathematics on limit of function object, *Journal of physics*,

Burger, E., Chard, D., Hall, E., & Kannedy, P. (2007). *Algebra 2*. Orlando, FL: Hole, Renehart and Winston.

Carballo, R. R. (2022). Unimodular Gravity Vs General Gravity: A status Report, Dissertation of educational Science, University of Central Florida, USA.

Carroll, B. J., Feinberg, M., Smouse, P. E., Rawson, S. G., & Greden, J. F. (2018). The Carroll rating Scale for Depression, Development, Reliability and Validation. *Journal for Biotechnology Information*. Pubmed.ncbi.nim.nih.gov.

Celik, A. O., & Guzel E. B. (2017). Revealing Ozgur's thoughts of a quadratic function with a clinical interview: concepts and their underlying reasons. *International Journal of Research in Education and Sciences (IJRES),* 3(1), 12.

Celik, A. O., & Guzel E. B. (2018). Constructing axes of symmetry and Vertex for quadratic functions through quantitative reasoning: International Group for the Psychology of Education, Pamukkale university (Vol.5, p.127).

Chang, K. E., Sung, Y. T., & Chen, S. F. (2002). Learning through Computer-Based Concept Mapping with Scaffolding Aid. *Journal for computer assisted learning*, National Taiwan Normal University. Taiwan.

Cherry, K. (2021). What is the Zone of Proximal Development? Developmental Psychology, www.verywellmind.com.

Chidziva, J. (2021). Using Ubuntu Values to Enhance Disruptive Pedagogies for Effective and Meaningful Teaching of Grade 12 Euclidean Geometry, Thesis for Doctor of Philosophy in Education, University of South Africa. Pretoria.

Coe, A. (2016). Difference Between Primary and Secondary Sources, *Journal of Leadership Studies*, University of Phoenix, research.phoenix.edu.

Cohen, L, Manion, L., & Morrison, K. (2011). *Research methods in education* London: Routledge.

Cohen, L., Manion, L., & Morrison, K. (2013). *Research methods in education*. (7th ed.) London & New York: Routledge.

Cohen, L., Manion, L., & Morrison, K. (2018). Conducting Mixed Methods Research in education 7th ed. London & New York: Routle.

Crane, A., Henriques., I., & Husted, B.W. (2018). Quants and Poets: Advancing Methods and Methodologies in Business and Society Research. Journal for business and society editorial insights, 57(26) 000765031771812.

Creswell, J. W. (2014). *Research design: Qualitative, quantitative, & mixed methods approaches*. Sage: Singapore.

Creswell. J. W. (2018). Research design: qualitative, quantitative, & mixed methods Approaches. Sage: Singapore.

Crossman, A. (2020). Understanding Purposive Sampling: An Overview of the Methods and its Applications, *Journal for Creative Education*, www.thoughtco.com.

Dalal, A. K. & Priya, K. R. (2016). Introduction to qualitative research. In *Qualitative Research on Illness, Wellbeing and Self-Growth* (pp. 15-36). Routledge India.

Darby, J. L., Fugate, B. S., & Murry, J. B. (2019). Interpretive Research: A complementary Approach to Seeking Knowledge in Supply Chain Management, The international Journal of logistics Management, Volume 30, www.emerald.com.

Demir, S. B., & Pismek, N. (2018). A convergent Parallel Mixed-Methods study of controversial Issues in Social Studies classes: A clash of ideologies, Educational Sciences; Theory & Practices, Cumhuriyet University, Turkey.

Department of Basic Education. (2014). *Diagnostic report on Grade 12 National senior certificate examination*. Pretoria: Government Printers.

Department of Basic Education. (2014). Siyavula Mathematics Grade 10, Volunteers, Pretoria.

Department of Basic Education (DBE). (2021). Diagnostic report part 1content subjects, Grade 12 National senior certificate examination: Pretoria: Government Printers.

Department of Basic Education (DBE0. (2019) Diagnostic report part 1 content subjects, Grade 12 National senior certificate examination: Pretoria: Government Printers.

Department of Basic Education (DBE). (2017). The SACMEQ IV project in south Africa: A study of the conditions of schooling and the quality of education: Pretoria: Government Printers.

Department of Education. (2014). Curriculum and assessment policy statement (CAPS) grade 10-12, p1-298 Pretoria: Government Printers Department of Education. (2014). *Trends in education: Macro-indicators report 2009,* p. 79. Pretoria: Government Printers.

Devault, G. (2019). Establishing Trustworthiness in Qualitative Research*, journal for educational research*, 4(1/2).

Dewey, J. (1987). Experience and education*. journal of educational research*, New York, Macmillan.

Dewey, J. (2013). Making sense of educational Situation' through more than a century of Progressive Reform. Australia, Rotterdam/Boston/Taipei.

Diaz, V., Aravena, M., & Flores, G. (2020). Solving Problem Types Contextualized ot the quadratic functions and error Analysis: A case study, *journal of mathematics, science and technology education,* Universidad de Los Lagos. Chile.

Didis, M. G., & Erbas, A.K. (2015). Performance and difficulties of students in formulating and solving quadratic equations with one unknown. *Educational Sciences: Theory & Practice*, 15(4):1137-1150.

Dixon-Krauss, L. (1996). Vygotsky in the classroom: mediated literacy instruction and assessment. Reading: Addison Wesley Longman.

Dixon, N. (2017). Guide to managing ethical issues in quality improvement or clinical audit projects, Healthcare Quality Improvement partnership, London, UK.

Doly, A. (2022). What is a Semi- Structured Interview, Article for Scientific research, thebalancemoney.com?

Drew, C. (2022). What is Constructivism in Education? Piaget's Pros & Cons, journal *of educational research*,3(5) 753-756.

Dube, K., Gianella, S., & Smith, D. (2018). Ethical considerations for HIV cure-related research at the end of life, *journal article for educational health research*, 14: e1002470.

Dunn, W. E., & Lantolf, P. L. (1998). Vygotsky's zone of proximal development and Krashen's i+1: incommensurable constructs; incommensurable theories. *Language Learning*, 48(3).

Ellis, A. B., & Grinsted, P. (2008). Hidden lessons: How a focus on slope-like properties of quadratic functions encouraged unexpected generalizations. *The Journal of Mathematical Behaviour*, 27,277-296.

Ekawati, R., Lin, F., & Yang, K. (2018). The Enactment of Mathematics Content Knowledge and Mathematics Pedagogical Content Knowledge in Teaching Practice of ratio and proportion: A case of Two Primary. IOP Conference Series: Materials Science and Engineering, University of Niger Surabaya, Indonesia.

Eteng, O. (2022). Qualitative Data Analysis: Methods and Techniques Simplified*, journal of educational research,* hevodata.com.

Espanol, F. (2022). Most influential theories of learning, International Bureau of Education, Geneva, Switzerland.

Faulkner, V. N., & Cain, C. R. (2013). Improving the mathematical content knowledge of general and special educators: evaluating a professional development module that focuses on number sense. *Teacher Education and Special Education*, 36(2):115-131

Fleming, J., & Zegwaard, K. E. (2018). Methodologies, methods and ethical considerations for conducting research in work-integrated learning, Auckland University of technology, New Zealand.

Foster, C. (2017). Developing mathematical fluency: comparing exercises and rich tasks, *journal of educational studies in mathematics*, link.springer.com.

Fouka, G., & Mantzorou, M. (2011). What are the major ethical issues in conducting research? *Health Sciences Journal*: mantzorou@teiathh.gr.pp:3-14,E-ISSN:1791-809X.

Fredric, K. (2014). *Research ethic policy. Handbook of politics.* Rhodes University.

Gael, A. (2017). Productive disposition, The learning of Kaleidoscope, andrewgael.com.

Gamain, G. (2021). Why learning Theories in teaching, performing arts, podcasts.apple.com.

Gao, T., Chai, Y., & Liu, Y. (2018). A review of knowledge management about theoretical conception and design approaches. *international journal of crowd science,* Volume 2,232(13):2297-2308.

Gemma, R. (2018). Introduction to positivism, Interpretivist and critical theory. Nurse Research, 25(4), pp. 41- 49.

Gentles, S. E., Charles, C., Ploeg, J. M. & Mckibbon, A. K. (2015). Sampling in Qualitative research: Insights from an overview of the methods literature. *The Qualitative Report,* 20(15) 1772-1789.

George, T. (2022). How to Write a Research Results Section: Tips and Examples, Journal of Scientific Research? Scribbr.com.

Gess-Newsome, J. (1999). PCK: An introduction and orientation. (*In* J. Gess-Newsome and N. Lederman *eds*., Examining PCK: The construct and its implications for science education Boston: Kluwer. pp. 3-20).

Graf, E. A., Fife, J. H, Howell, H., & Marquez, E. (2018). The development of a quadratic functions learning progression and associated task shells, ETS research report series, Educational testing service, Princeton, NJ, 2330-8516

Graigle, M. (2011). *Overview of census data collection methods, processing, reporting and evaluation,* MT Department of commerce, US Census Bureau.

Greenberg, A. (2018). Overview of qualitative and quantitative data collection method, *Bengaluru, Karlsruhe*. https://www, cleverism.com/qualitative-data-collection – methods.

Grover, V. (2015). *Research approach an overview*, DAV College of Education, Abohar India, *ZENITH International Multi-Disciplinary Research Journal*, 5(8), pp.54-65.

Halim, S., Wahid, R., & Halim, T. (2017). Classroom Observation: A powerful Tool for Continuous Professional Development, International Journal on Language Research and Education, Volume 2, www.researchgate.net.

Hammarberg, K., Kirkman., & de Lacey, S. (2016). Qualitative research methods: When to use them and how to judge them, *National Centre for Biotechnology Information Journal*. Pubmed.ncbi.nlm.nih.gov.

Hartman, H. (2002). *Scaffolding & Cooperative Learning. Human Learning and Instruction* (pp. 23-69). New York: City College of City University of New York.

Hazzi, O., & Midaon, I. (2018). A Pilot Study: Vital Methodological Issues, *Vilnius tech journal,* gooleweblight.com.

His Wu, H. (2017). The content knowledge mathematics teachers need, research article for mathematics, University of California, Berkeley.

Hoon, T. S., Parmjit, S., & Abdul, H. U. K. (2018). Understanding of Function and quadratic Function among Secondary School Students in Selangor, Asian *journal of university education,* Volume 14. Eric.ed.gov.

Hurst, M. (2021). Social Constructivism and the Mediated Learning Experience, study.com.

Hussain, S. (2018) Framework for the evaluation of proficiency- Based Mathematics Instructional Material, *International journal of engineering & scientific research,* 21(3), pp, 395-421.

Ibeawuchi, E., & Ngoepe, M.G. (2016). Investigating grade 11 learners' misconceptions in understanding of quadratic functions in some South African's schools, *Journal for education,* University of South Africa, Pretoria.

Jain, K., & Jain, A. (2018). Introduction to research for medical student, volume-8, issue-3, print, 2249-555.

Johnson, D. (2022). What is Data Analysis, Research, types and Example, Educational *journal for research study*. Guru.com.

Johnson, N. L. (2015). Quadratic functions and solving quadratic equations: An analysis of students, thinking and reasoning. PhD dissertation. University of Washington. United States of America.

Johnson, R. B., & Christensen, L. B. (2015). *Educational research: Quantitative, qualitative, and mixed approaches.* Los Angeles CA: Sage Publications.

Kabar, M. G., & Erbas, A. K. (2018). Performance and difficulties of students in formulating and solving Quadratic Equations with one unknown, Article of Educational Science, Theory and Practice, www.researchgate.net.

Karseley, G., & Shneiderman, A. (2019). Engagement theory: a framework for technology-based teaching and learning in education. A functional review of research, Tchnol.com.

Kaufman, K. (2017). Decreasing math anxiety through teaching quadratic equations, Masters theses, University of New York, Brockport.

Kaushik, V., & Walsh, C. A. (2019). Pragmatism as a Research Paradigm and its Implications for Social Work Research, Social Sciences, University of Calgary, Canada.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The elementary school Journal, 102(1), 59-80.*

Kelley, M., Dowling, M., & Miller, M. (2018). The Search for Understanding: The role of of pragmatic Worldviews. Nurse Researcher,25940,9- 13.

Khau, M. (2014). Rural education and rural realities: Politics and rural research in Southern Africa, Nelson Mandela University.

Kim How, R. P. T., Zulnaidi, H., & Abdul Rahim, S. S. (2022). Teaching style preference and challenge faced by Malaysian teachers in teaching quadratic equations, *European journal of science and mathematics*, Education, University of Malaysia, Kuala Lumpur.

Kim, S. (2022). Literature Reviews: Starting your literature review, SETU Waterford Libraries,wit-ie.libguides.com

Kiren, C. (2007). Learning and teaching algebra at middle school through college levels: Building meaning for school and manipulation. Second handbook of research on Mathematics teaching and learning. Charlotte, NC, information Age Publishing.

Kiren, C. (2007). *Learning and teaching algebra at middle school through college levels: Building meaning for school and manipulation.* Second handbook of research on Mathematics teaching and learning. Charlotte, NC, information Age Publishing.

Kivunja, C., & Kuyini, A. B. (2017). Understanding and applying research paradigms in educational context, International Journal of Higher Education, University of New England, Australia.

Knuth, E. J. (2000). Understanding connections between equations and graphs. The Mathematics teacher, 93(1), 48-53

Kobus, M. (2010). *First step in research*. Pretoria: Van Schaik.

Koenig, M. E. (2018). What is KM? Knowledge Management Explained*, journal of educational management,* Long Island University, www.kmworld.com.

Kopfer, P. (2022). Teachers' perspectives on dealing with students' errors, Department of Business and economics Education, Goethe University Frankfurt. Frankfurt.

Koram, S. S. (2019). Key Concepts of Quadratic Functions and Inequalities*, journal of solid mechanism,* www.academia.edu.

Kortjents, I., & Moser, A. (2018). Practical guidance to qualitative research. Part 4: Trustworthiness and publishing, *European journal of general practice*, 24(1):1-5.

Kowalczyk, D., Skiddmore, S., & Scalla, S. (2021). What is research: purpose and example, Journal for Psychology Education, study.com.

Krauss, L. (1996). *Vygotsky in the classroom: Mediated literacy instruction and assessment.* Reading: Addison Wesley Longman.

Kristianto, E., & Suputro, D. D. R. (2019). Analysis of student s' error in proving the convergent sequence using Newman's error analysis procedure*. journal of physics*, IOP publishing.

Kulwicki, M. F. (2021). Constructivism: The long History from the Active Knower to the Active Learner, Curriculum, Assessment and teaching Transformation, University at Buffalo. www. Buffalo. Edu.

Kurt, S. (2020). Vygosky's Zone of Proximal Development and Scaffolding, *Educational technology journal,* educationaltechnology.net.

Kusmaryono, I., Suyinto H., & Dwidayati, N. (2019). The effect of Mathematical disposition and mathematical power formation: Review of Dispositional Mental Functions, *International journal of instruction*, e-ISSSN: 13308-1470.

La Banca, A. (2010). In search of scientific creativity: Retrieved 17 December 2011, from http// Problem finding, labanca net.

Lall, D. (2021). Mixed methods research: Why, When and How to use, *Indian journal of continuing nursing education,* Christian Medical College, India.

Leedy, P. D., & Oromrod J. E. (2010). *Practical research planning and design*. (9th ed.) New Jersey, Pearson Edition.

Lefrancois, G. R. (2019). Theory of human learning, Seventh ed, Cambridge University Press, theories-human-learning-mrs-gribbins-cat.

Lill, S., & Vale, C. (2016). First year pre-service teachers' mathematical content knowledge: Methods of solution for a ratio question. *Mathematics Education Research Journal* 13(2): 22-43.

Lindroff, A., & Sammons, P. (2018). Going beyond Structured Observations: Looking at classroom practice through a mixed method lens, *International journal on mathematics education.* Research gate.net.

Livy, S., & Vale, C. (2011). First year pre-service teachers' mathematical content knowledge: Methods of solution for a ratio question. *Mathematics education research Journal,* 13(2): 22-43.

LIyas, M. (2016). Teachers pedagogical content knowledge concerned to student's knowledge on quadratic functions, Conference Paper, Mathematics education view project, Universities Cokroaminoto, Palopo.

Locky, A., Conaghan, P., Bland, A., & Astin, F. (2021). Educational theory and its application to advanced life support course: a narrative review, Calderdale & Huddersfield NHS Trust. UK

Lowe, K. N. (2010). What is a Pilot Study, Journal for National of Medicine, bmed.ncbi.nlm.gov.

Luneta, K., & Makonye,P. J. (2013). Learners' Errors and misconceptions in introductory Differention: A Theoretical framework. US- China Education Review A, 3(12), 914-5.

Luneta, K. (2008). Error discourse in fundamental physics and mathematics: Perspectives of learner's misconceptions. 2008 International year book on teacher's education Brag: ICET.PP 385-400.

Mabena, M., Mokgosi, P. N, & Ramapela, S. S. (2021). Factors contributing to poor learner performance in mathematics: A case of selected schools in Mpumalanga province,Tshwane University of Technology, South Africa, 10.33225/pec/21.79.451.

Machaba, F., & Banjo (2022). Teachers' knowledge of instructional strategies when teaching Quadratic functions in Grade 10, Article for Mathematics Education. University OF South Africa. Pretoria.

Macmillan, J. H., & Schumacher, S. (2010). *Research in Education: Evidence-based enquiry*. Harlow: Pearson Education Limited.

MacMillan, J., & Schumacher, S. (2014). *Research in education*. (7th ed.) Harlow: Pearson Education Limited

Mahlaba, S. C. (2020). The state of South African Mathematics Education: Situating the hidden promise of multiple-solution tasks, *Journal of mathematics, science and technology Education*, North-West University, South Africa,1305-8223.

Majid, U. (2018). Research Fundamentals: Study Design, Population, and Sample Size, Research Fundamentals Editorial Series, URNCST Journal, University of Toronto. Toronto.

Makgakga, S. W. (2015). Errors and misconceptions in solving quadratic equations by completing a square. *Mathematics Education, (Online), (http://www. Amesa. org. za), diakses, 8.*

Makgakga, S. W. (2016). Twining two mathematics teachers teaching grade 11 algebra: A strategy for change in practice, PhD thesis. University of the North West.

Makonye, J. (2016). Learner mathematical errors in introductory differential calculus task: A study of misconception in the senior school certificate examinations, Doctor dissertation, University of Johannesburg, Johannesburg.

Makonye, J., & Stewell (2017). Exploring ‘Non- science’ Grade 11 Learners’ Error in solving Quadratic Equations*, Mediterranean journal of social science*, University of Witwatersrand, Johannesburg.

Malahlela, M. V. (2017). Using errors and misconceptions as a resource to teach functions to grade 11 learners, Masters Dissertation. University of Witwatersrand, Johannesburg.

Mamba, A. (2012). Learners’ errors when solving algebraic tasks: A case study of grade 12 mathematics examination papers in South Africa. Masters Dissertation, University of Johannesburg, Johannesburg.

Manzindu, L. (2016). Misconceptions learners encounter in solving quadratic equations: Med dissertation, Bindura University of science education, Zimbabwe

Marafi, M., & Rana, M.S. (2016). Refinery Waste: The Spent Hydro Processing Catalyst and its Recycling option, WIT transactions on Ecology and the Environment, Volume 202, pp. 219-230.

Mazumdar, M. (2022). Ethical Considerations in research every author should know, Focused Products and Solutions for your research Publication Journey.

Mbanjwa, H. T. (2014). The role of subject advisors in strengthening teacher instructional leadership practice. A case study of one educational district office in Kwazulu-Natal: University of KwaZulu Natal: Edgewood Campus (Dissertation Med).

McCluskey, C., Mulligan, J., & Mitchelmore, M. (2016). The role of reasoning in the Australian Curriculum: Mathematics, *Journal of mathematics education, university of Melbourne, Australia.*

McDonald, S. (2015). Conceptualizing longitudinal mixed method design, journal for specialist in group work, 42(1):87-101.

McGlamery, S., & Lentfer, V. (2017). Developing Strategic Competence in the service of inquiry Teaching: Assisting Pre-service Elementary Teachers to use Inquiry to Achieve Strategic Competence in Science Learning. *Journal of curriculum, teaching, learning and leadership in education*, University of Nebraska, Omaha.

McLeod, S. A. (2019). Bruner- Learning Theory in Education, scientific research publisher, htt:// www.simplypsychology.org/bruner.html.

McPherson, K. (2016). Procedural fluency from conceptual understanding, Thurgood Marshall College Fund, https//todaysmeet.com/ TQRP_CCSS Math.

Mutombo, L. P. D. (2015). Investigation of Learners’ ways working with algebraic graphs in high stake Mathematical Examination*, journal mathematics education*, etd.uwc.ac.za

Mohajan, H. K. (2018). Qualitative Research Methodology in Social Sciences and Related Subjects, *Journal of economic development, environment and people*, Premier University, Bangladesh, pp. 23-48.

Mosquera, M. C., & Uzuriga, V.L. (2018). Problem- based learning in Mathematics Education: Solution of quadratic equations by the area application method. *Journal of Mathematics Education*, (32) 231-330.

Moumita, M. (2022). Ethical considerations in research every author should know, *journal of educational research*, 3(42) 432-476.

Mugere, N. (2016). Sampling in research: University of Zimbabwe: *Research Gate Publications.* Doi: 10.4018/978- 5225- 0007-0 ch006.

Muller, S. (2016). *Why South African schools no choice but to drop maths pass mark?* University of Cape Town.

Mutambara, L. H. N., Tendere, J., & Chagwiza, C. J. (2019). Exploring the conceptual understanding of the quadratic function concept in teachers' colleges in Zimbabwe. *Journal of mathematics, science and technology education*, 16(2).

National Council of Teachers of Mathematics. NCTM. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: National Council of Teachers of Mathematics author.

National Council of Teachers of Mathematics. (NCTM). (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (NCTM). (2002). Principles and standards for school mathematics. Reston, VA: National council of Teachers of Mathematics.

National Mathematics Advisory Panel Report. (2008). Foundation for success. US Department of Education Report. Washington: DC. March 2006.

Ncube, M. (2016). Analysis of errors by learners in simplifying algebraic expression at Grade 9 level, University of South Africa, Masters of Mathematics Education.

Ndebele, N. (2017). *South African survey: South African institute of race relation: The power of ideas*, Johannesburg: Reports and Publications.

Ndlovu, N., Amin, A., & Samuel, D. (2015). Pre-service Teachers' Mental Constructions of concepts in matrix Algebra. African *journal of research in mathematics, science and technology education*, University of Kwazulu-Natal, Durban.

Ndlovu, N. (2019). Exploring learners' understanding of mathematical concepts necessary in the learning of grade 11 algebraic Functions: The case of three schools in uMgungundlovu, Dissertation for masters' degree, University of Kwazulu Natal, Durban.

Nepomucena, T.V., Da Silva, A.C., Jardim, D.F., & Da Silva, J.M. (2017). A study about teaching quadratic functions using mathematical models and free software, *journal of physics*, (IC- MSQUARE 2017) 28-31.

Newman, M. A. (1983). An Analysis of Six- Grade Learners' Errors on Written Mathematical tasks. http://users.tpg.com.au/arnold10.

Ngoepe, M.G. (2015). Learners' conceptions in understanding grade 4 fractions. South African Society for Education UNISA, Northwest Province.

Nielsen, J. L. (2015). Quadratic functions and solving quadratic equations: An analysis of students, thinking and reasoning. PHD dissertation. University of Washington. United States of America.

Obi, N. C; Ukpepi, C; & Ndifon, R. A. (2019). Constructivists' Theory and Science Education Classroom*, European journal of science research*.Volume154,University of Malabar. Canada.

Okur, M. 2013. Learning difficulties experienced by students and their misconceptions of the inverse function concept. Educational Research and Reviews, 8(12), 901–910.

Oliva, R.O. (2016). Trustworthiness, Counter Currents. ORG, Invisible paradical @gmail.com.

Olivier, A. (1989). *Presidential address delivered at the Thirteenth National Convention on Mathematics, Physical Science and Biology Education*, Pretoria, 3 – 7 July.

Ornstein, A. C., Thomas, J., & Lasley, I. (2000). *Strategies for effective teaching*. New York: McGraw-Hill.

Ortez, L. M. (2020). Introduction to Quadratic Functions, *Journal of Mathematics Education*, Satiro Francisco MNHS, www,sribd.com.

Osterman, P. (2019). Improving job quality for direct care workers, *journal for work and employment research*, MA 02139-4307.

Ovez, F. D. T. (2018). The impact of instructing quadratic functions with the use of GeoGebra software on students' achievement and level of reaching acquisitions, *International journal for education,* Balikesir University, Turkey.

Panhwar, H. A., Ansuri, T., Sanaullah, B.R., Shah, A, T., & Ali, A. (2017). Positivism paradigm: An Effective Paradigm for Social and Educational Research*. Academic journal article*, Vol. 45, quessta.com.

Parent, J. S. Z. (2015). Students understanding of quadratic functions: learning from student's voice, PhD thesis, the faculty of the graduate college, University of Vermont.

Patton, M. Q. (2015). Qualitative Research and Evaluation Methods, Journal of Educational Research, 4th ed. Sage, Thousand Oaks, CA.

Parvaiz, G. S., Mufti, O, & Wahab, M. (2016). Pragmatism for Mixed Method Research at Higher Education Level, Business and Economic review, Vol.8, pp.67-79.

Piaget, J. (1977). Epistemology and psychology of functions, Dordrecht, Netherlands, Reidel Company.

Piaget, J. (1966). Development and learning in R. Ripple and U., Rockcastle, Cornell University Press.

Pickell, D. (2021). Qualitative vs Quantitave Data: What 's the Difference*, Journal for statistical analysis*, www.g2.com.

Pickthorne, B. (1983). Error factors: A missing link between cognitive science and classroom practice. Instruction Science 11: 281-312.

Pilay, J. (2014). Ethical consideration in education involving children: Implication for education research in South Africa: *South Africa journals of child hood education*, University of Johannesburg, 2223-7674.

Priya, K. R. (2016). Quantitative research methodology in social sciences *journal for scientific research 01(02),* pp115- 178.

Priyani, H. A., & Ekawati, R. (2018). Error analysis of mathematical problems on TIMSS: A case of Indonesia secondary students, Journal of IOP Conference Series, Volume 296, Mater, Sci.Eng.

Purcell, K., Rainie, L., Buchanan, J., Friedrich, L., Jacklin, A., Chen, C., & Zickuhr, K. (2020). How teens Do Research in the Digital Word, Pew Research canter, www.pewresearch.org.

Putra, A. K., Budiyono & Slamet, I. (2017). Mathematical Disposition of Junior High School Students Viewed from learning Styles, AIP conference Proceedings, Sebelas University, Indonesia.

Quinlan, E., & Gantogtokh, O. (2018). Lesson in programme leadership from two cases designing new interdisciplinary master's programmes, Journal for Educational development, Australian National University, SEDA Special 39.

Rahman,T. A., & Mohamad, M.S. (2021). Quadratic functions in additional mathematics and mathematics: An analysis on students' errors, centre for foundation study, University Technology MARA. myjms.mohe.gov.my.

Rahmawati N. L. T., Suparta I. N., & Suweken G. (2016) Pembelajaran Dengan Visual Scaffolding Untuk Mengembangkan Kemampuan Siswadalam Menyelesaikan Masalah Terbuka Materi Fungsi Kuadrat *Prosiding Seminar Nasional MIPA 2016.*

Ramma, Y., Bholoa, A., Watts, M., & Ramasawmy, J. (2014). Pre-and in-service physics teachers' content and pedagogical content knowledge: implications and challenges. *American International Journal of research in science, technology, engineering & mathematics*, 8(1):30-34.

Raney, E. (2016). Cognitive characteristics: learners with mild disabilities. Needham Heights, MA: Alyn & Bacon, A Pearson Education Company. pp. 169-201.

Reavis, R. D., & Thomas, M, A. (2019). Building a Scaffolded Research Experience for Undergraduates, Sec Education Psychology, Earth ham College. United states.

Reddy, V. (2016). *TIMMS 2015 Highlights' of Mathematics and sciences achievement of Grade 9 South African learners, researching education, improving learning*. DBE.

Rentnawati, H, & Nurharyanto, D. W. (2020). The difficulties of the elementary school students in solving the mathematical narrative test items, *Journal prima edukasia 8(1):29-39.*

Riley, M. (2018). Emplacing the Research Encounter: Exploring Farm Life Histories, *journal of qualitative inquiry*, 16(8):651.

Rizik, H. T. N., Frentika, D., & Wijaya, A. (2018). Exploring Students adaptive reasoning skills and van Hiele levels of geometric thinking: a case study in *Geometry. journal of physics*, conf. ser.983 012148.

Roberts, A. K., & Spangenberg, E. D. (2020). Peer tutors' views on their role in motivating learners to learn mathematics. Pythagoras,41(1)1.520.

Roberts, N. (2017). The South African school maths story is well known, *Daily Maverick*. Mavereck.co.za/article/2017-01-08-school-maths-is-our-story.

Rodriguez, R. M., Rodriguez, J. B., & Fernandez, A. O. (2019). Case Study as a Research Method for Analysing MOOCs: presence and characteristics of those Case studies in the Main Scientific Databases, International Review of Research in Open and Distributed learning, Universidad de Granada, Spain.

Rueda, A. (2020). The Concept of Functions and Quadratic function grade 10 class, *Journal of mathematics education*. https//www.iejme.com.

Ruli, R. M., Prabawanto, S., & Mulyana, E. (2019). Didactical design research of quadratic function based on learning and learning trajectory, *Journal for physics.ser.*1157042060.

Ruli, R. M., Priatna, N., Sufyani, P., & Endang, M. (2018). Analysis learning obstacle on quadratic function topic*. International journal of information and education technology,* 8(9), 681-684.

Rushby, L. (2020). TIMSS 2019: Highlights of South African Grade 9 results in mathematics and science, University Cape, Town, Government publications librarian.

Sabilah, I., Siswono T. Y. E., & Masriyah. (2018). Students strategic competence toward open-Ended Problems Before and after the transition to Junior High School, *Journal of physics:* conference series.1108012017.

Salihu, F. O. (2017). An investigation grade 11 learners' errors when solving algebraic word problems in Gauteng, South Africa, Masters Dissertation, university of South Africa, Pretoria.

Schoenfeld, A. H., & Kilpatrick, J. (2008). Towards a theory of proficiency in teaching mathematics. (*In* Tirosh, D. & Wood, T. *eds.,* International handbook of mathematics teacher education, Volume 2: tools and processes in mathematics teacher education Rotterdam, Netherlands: Sense Publishers. pp. 321-354).

Schunk, D. H. (2020). Learning Theories: An Educational Perspective, 8th edition, Pearson, PGM1996609.htm.

Sepeng, J. P. (2010). Grade 9 second-language learners' in township schools: Issues of language and mathematics when solving word problems. Unpublished doctoral dissertation. Nelson Mandela Metropolitan University, Port Elizabeth.

Sepeng, P. (2014). Use of common-sense, language and reality in mathematics word problems solving*. African Journal of research in mathematics, science and technology education,* 16(1): 14-24.

Sepeng, P., & Webb, P. (2012). Exploring mathematical discussion in word problem-solving. Pythagoras, 33(1), pp.1-8.

Sepeng, P. (2013). Exploring mathematics classroom practices in South African multilingual settings. *Mediterranean journal of social sciences*, 4(6), p.62.

Shukla, P. (2020). Concept of Population and Sampling, *journal of educational technology*, 5(24) 345-444.

Shulman, L. S. (1987). The wisdom of practice: managing complexity in medicine and teaching. (*In* D.C. Berliner & B.V. Rosenshire *eds*., Talks to teachers: A festschrift for N.L. Gage. New York: Random House, pp. 369-386).

Shulman, L. S. (1988). Those who understand: Knowledge growth in teaching. *Educational researcher*, *15*(2), pp.4-14.

Simplilearn, (2022). What is Data Collection: Methods, Types, Tools, and Techniques, Journal Business Analytics, www.simplilearn.com.

Sinyosi, L. B. (2015). Factors affecting learners in mathematics at Nzhelele east circuit, MEd thesis. University of South Africa, Pretoria.

Siyepu, S, Tshabalala, T, & Ncube, A. C. (2013). *South Africa Journal of Education*: 2013, 33(2) Art, 714.

Sileyew,K.J. (2019). Research design and methodology, journal of metrics overview, 5(23): 85731.

Smith, E. L., Blakeslee, T. D., & Anderson, C. W. (2021). Teaching strategies associated with conceptual change learning in science*, Journal of research in science teaching*, Volume 30, onlinelibrary.wiley.com.

Soto, R. D. F., Castello, M. G. O., & Oxtez, P. (2021). Finding meaning in quadratic equations through a physical context: Falling Objects Motion*. Journal of physics conference*, researchgate.net.

Souliotis, D. (2022). The European Parliament in the news: A critical Analysis of the Coverage in the United Kingdom and Greece. *Journal of educational research,* Link. Springer.com.

Star, J. R., Caronongan, P., Foegen, A. M., Furgeson, J., & Keating B. (2015). Teaching strategies for improving Algebra Knowledge in the middle and High school student, digital respiratory, Educators practice guide, US, NEEE 2015-4010.

Streefkerk, R. (2022). Primary vs. Secondary Sources: Difference and Example, *Journal of american psychology*, www.scribbr.co.uk

Stylianides, A. J. & Stylianides, G. J. (2006). Content knowledge for mathematics teaching: a case of reasoning and proving. (*In* Novotná, J., Moraová, H., Krátká, M. & Stehlíková, N. *eds.,* Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education, 5:201-208. Prague: PME.

Surucu, L., & Maslakci, A. (2020). Validity and Reliability in Qualitative Research, *Journal of international business management studies*, Volume 3, bmij.org.

Syukriani, A., Juniati, D., & Yuli Eko Siswono, T. (2017). Strategic competence of senior secondary school students in solving mathematics problem based on cognitive style, AIP Conference proceedings 1867, 020033 (2017), https// doi.org/10.1063/1.4995136.

Taber, K. S., (2018). Scaffolding learning: principles for effective teaching, In M. Abend(Ed), Nova Science Publishers, New York.

Taherdoost, M. (2022). Review of Qualitative, Quantitative, and Mixed Method Research, Their Applications, Types, and Limitations, *journal of management science & engineering research,* 5(1):53-63.

Tanlaka, E. F., Ewashen, C, & King-Shier, K. (2019). Post positivist critical multiplism: Its value for nursing research, nursing open publish by John Wiley and Son Ltd university of Calgary, Canada

Tashakkori, A., & Teddlie, C. (2010). *Mixed methods in social behavioural research*. California: Sage Publications.

Tendere, J., & Mutambara, L. H. N. (2020). An Analysis of Errors and Misconceptions in the study of Quadratic Equations, *European journal of Mathematics and Science Education*, Volume 1, www.ejmse.com.

The World Economic Forum. (2016). Global Competitiveness Report 2015-16, September 2015, pp. 91-371. Switzerland: Davos Kloster.

Thilly, R. E. (2021). Major Domain in Child Development, Growth and Development, Phil Boorman, www.veryllfamilly.com.

Thomas, E., Bianco, A., Paoli, A., Palma, A. (2018). The relationship Between Stretching Typology and Stretching Duration: The effects on Range of Motion, *journal of biotechnology information*, 39(4):243-254.

Todorova, A. D. (2016). Procedural and conceptual understanding in undergraduate linear Algebra, Humboldt- University, Berlin.

Tomljeccnovic, Z., & Vorkapic, S. T. (2020). Constructivism in Visual Arts Classes, *Center for educational policy studies journal*, University of Rjecka. www.researchgate.net.

Trapsilasiwi, D., Murtkusuma, P. R., Oktavianingtyas, H. O.,Wiliandani,I., & Widodo, D.M.(2020). Analysis of Introverted Students' Error Based on Newman in Solving Arithmetic Sequences and Series Problems, Advances in Social Science, Education and Humanities Research. Volume 550, Jember University.

Troutsman, A. C. & Alberto, P. A. (1982). *Applied Behaviour analysis for Teachers: Influencing student Performance.* Columbus, OH: Murril Publishing.

Tsang, F. K. & Rowland, T. (2005). The subject-matter knowledge of Hong Kong primary school Mathematics teachers. Paper presented at the European conference on Educational Research, University College Dublin.

Tshabalala, T. & Ncube, A. C. (2013). Causes of poor performance of ordinary level pupils in Mathematics in rural secondary schools in Nyika District. *Nova Journal of Medical and Biological Sciences*, 1(1), 4-14.

Ubah, I. J. A., & Sarah Bansilal, S. (2018). Pre-service mathematics teachers' knowledge of mathematics for teaching quadratic functions University of Kwazulu-Natal, South Africa.

Unesco. (2022). Most influential theories of learning, International Bureau of Education, www.ibe.unesco.org

Vaiyavutjamai, P., & Clements, M. A. (2006). Effects of classroom instruction on students' understanding of quadratic equations*. Mathematics education research journal,* 18(1), 47-77.

Valdes, O. (2019). What is a Literature review, *Journal for humanities sciences*, www.thoughtcom.com.

Valenzuela, A. (2022). The harmful impact of the TAAS system of Testing in Taxas; Bneath the Accoutability Rhetoric. Distertion for Science Education, University of Texas, Austin.

Van Der Stuyf, R. R. (2002). Scaffolding as a teaching strategy. *Adolescent Learning and Development*, 52(3), pp.5-18.

Venkantesh, V., Brown, S. A., & Sulllivan, V.W. (2016). Guidelines for Conducting Mixed-Methods research: An Extension and illustration, *Journal of the association for information system,* University of Arkansas, New York.

Verloop N., Van Driel, J., & Meijer, P. (2001). Teacher knowledge and the knowledge base of teaching. *International Journal of educational research,* 35(5):441-461

Vieira, F., Flores, M. A., Sliva. J. L., & Almeida, J. (2019). Understanding and enhance change in post- Bologna Pre-service teacher Education, *International journal for educational research,* Milton Park, Routledge. pp. 45- 57.

Vinney, C. (2019). What is the Zone of Proximal Development? Defination and Example, Sciience, Tech, Maths and social Science, www.thoughtco,com.

Visser, M., Juan, A., & Feza, A. (2015). Home and school resources as a predators of Mathematics performance in South Africa. *South African Journal of Education*, volume 35. University of South Africa.

Voon Driel, J. H., & Berry, A. (2016). The teacher education knowledge base: pedagogical content knowledge. (*In* International encyclopaedia of education, 3rd ed., Vol. 7. Amsterdam: Elsevier. pp. 656-661).

Von Glaserfeld, E. (1984). An introduction to Radical Contructivism, New York, Norton and Co. Inc.

Vygotsky, L. S. (1998). The problem of age Hall, M, Trans. In Rieber, R.W. Ed., *The collected works of L. S. Vygotsky, Vol. 5. Child psychology.* New York: Plenum Press pp. 187-205). Original: 1933-1934.

Wagner, C., Kawulich, B.B., & Garner, M. (2012). Doing Social Research: A global context, McGraw-Hill Education, Berkshire. New York.

Warren, K. (2022). Qualitative vs Quantitative, A plain- Language Explanation with Examples, grandcoach.com.

Weinbaum, C., Landree, E., Blumenthal, M., & Piquado, T. (2019). Ethics in Scientific Resaearch: An Examination of Ethical Principle and Emerging Topics.Cyper*, International Journal for Scientific research*, www.researchgate.net.

Western Governors University, (2020). Teaching and Learning: Five Educational Learning Theories, www.wgu. edu .

Wibowo, T. (2016). Adaptive Reasoning Junior High School Students in Mathematics Problem Solving, Proceeding of 3 International conference on research Implementation and education of mathematics and science, Muhammadiyah University of Purworejo, Indonesia.

Widiatiwati, M., Surya, A., & Sukarno, S. I. (2020). Error Analysis of Fifth- Grade Students of Elementary School in Geometry Problem Solving Based on Newman's Procedure, Department of Elementary Teacher Education, Sebelas Maret University, Indonesia.

William, M. K., (2020). Positivism and Post- Positivism, Socialresearchmethods.net, Sydney, Australia, ABN 6616169021.

Wojcik, A. J. (2018). Developing Conceptual Understanding and Procedural Fluency in Algebra for High School students with intellectual Disability, Thesis for Doctor of Education Philosophy, UCV University Archives, Richmond.

Woodward, A., Beswick, K., & Oates, G. (2018). Positive Education and teaching for productive disposition in Mathematics, *Journal for mathematics education*, https//link.spriner.com.

Yetman, D. (2020). What is the Zone of Proximal Development? *Journal of educational application*. www.healthline.com.

Yin, R. K. (2018). Case study Research and Applications: Design and Methods, 6th ed, Thousand Oaks, CA, Sage. Zaslavsky, O. (1997). Conceptual obstacles in the learning of quadratic functions. *Focus on Learning Problems in Mathematics*, 19(1), 20-44.

APPENDICES

APPENDIX A: UNISA ethical clearance

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2020/07/08

Dear Mr TL Mudau

Decision: Ethics Approval from 2020/07/08 to 2023/07/08

Ref: **2020/07/08/64088863/01/AM**
Name: Mr TL Mudau
Student No.: 64088863

Researcher(s): Name: Mr TL Mudau
E-mail address: Mtakalani30@gmail.com
Telephone: 0825242814

Supervisor(s): Name: Dr T Makgakga
E-mail address: makgasw@unisa.ac.za
Telephone: 012 429 4293

Title of research:

Grade 10 learner's academic experiences in learning parabola functions in schools of Vhembe West district of Limpopo province.

Qualification: MEd Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2020/07/08 to 2023/07/08.

*The **medium risk** application was reviewed by the Ethics Review Committee on 2020/07/08 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.
2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

3. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
4. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
5. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
6. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
7. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
8. No field work activities may continue after the expiry date **2023/07/08**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2020/07/08/64088863/01/AM** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za

Prof PM Sebate
EXECUTIVE DEAN
Sebatpm@unisa.ac.za

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

APPENDIX B: Permission letter from Limpopo department of education

Ref: 2/2/2 Enq: Mabogo MG Tel No: 015 290 9365 E-mail:MabogoMG@edu.limpopo.gov.za

Mudau TL
P O Box706
VHUFULI
0971

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH

1. The above bears reference.
2. The Department wishes to inform you that your request to conduct research has been approved. Topic of the research proposal: **"GRADE 10 LEARNERS ACADEMIC EXPERIENCES IN LEARNING PARABOLA FUNCTIONS IN SCHOOLS OF VHEMBE DISTRICT OF LIMPOPO"**
3. The following conditions should be considered:

3.1 The research should not have any financial implications for Limpopo Department of Education.

3.2 Arrangements should be made with the Circuit Office and the School concerned.

3.3 The conduct of research should not in anyhow disrupt the academic programs at the schools.

3.4 The research should not be conducted during the time of Examinations especially the fourth term.

3.5 During the study, applicable research ethics should be adhered to; in particular the principle of voluntary participation (the people involved should be respected).

REQUEST FOR PERMISSION TO CONDUCT RESEARCH: MUDAU TL

Cnr. 113 Biccard & 24 Excelsior Street, POLOKWANE, 0700, Private Bag X9489, POLOKWANE, 0700
Tel: 015 290 7600, Fax: 015 297 6920/4220/4494

The heartland of southern Africa - development is about people!

3.6 Upon completion of research study, the researcher shall share the final product of the research with the Department.

4 Furthermore, you are expected to produce this letter at Schools/ Offices where you intend conducting your research as an evidence that you are permitted to conduct the research.

5 The department appreciates the contribution that you wish to make and wishes you success in your investigation.

Best wishes.

Dederen

Mrs Dederen KO
Acting Head of Department

22/09/2020
Date

APPENDIX C: Permission letter to district

Request for permission to conduct research at three schools in Vhembe West District

TITLE OF THE RESEARCH: GRADE 10 LEARNERS' ACADEMIC EXPERIENCE IN LEARNING PARABOLA FUNCTIONS IN SCHOOLS OF VHEMBE DISTRICT OF LIMPOPO PROVINCE

Date: 07/07/2020

The District Director

Department of Basic Education

Vhembe West District

Tel: 015 965 1895

Email: vhembewest.curriculum@gmail.com

Dear Sir

I, Mudau Takalani Lesley doing research under the supervision of Dr Makgakga T.P. a senior lecturer at the Department of Mathematics Education towards a M Ed at the University of South Africa. We have funding from DSF-POSTGRD for assistance in all travelling cost and tuition fee expenses. We are inviting you to participate in a study entitled learners experience in learning parabola function.

The study aims to explore difficulties or challenges Grade 10 learner's experiences when they are learning parabola functions in schools of Vhembe district of Limpopo Province.

Your district has been selected because the researcher is a teacher of Grade 10 mathematics in the district and he has noted that Grade 10 learners' academic perform is poor in solving parabola functions.

The study will entail observing educators while they are teaching parabola functions, after educator's observations, learners will write an achievement test which will be analyzed and then followed by structured interview.

The benefits of this study are:

Learners will improve their understanding and performance in the topic of parabola functions and when they move to the next Grade. To inform educators of the intervention strategies that can be implemented to improve the teaching of parabola functions.

To improve the morale of the learners and the teachers in the teaching and learning of parabola functions.

There are no Potential risks involved in this study. There will be no reimbursement or any incentives for participation in the research.

Feedback procedure will entail sending emails to the participant about the outcome of the research. For more information concerning this request, you can contact me at 066 044 1209 or mtakalani30@gmail.com or contact my supervisor Dr Makgakga T.P at 012 429 4293 or makgasw@unisa.ac.za.

Yours sincerely

____________________________ (insert signature of researcher)

Mudau Takalani Lesley

Researcher

APPENDIX D: Permission letter to the principal

P. O. Box 706
VHUFULI
0971
07/09/ 2020

The Principal
Miriyavhavha Sec School
P. O. Box 240
Nzhelele
0974

Dear Sir,

RE: PERMISSION TO CONDUCT A STUDY AT YOUR SCHOOL

I am a Masters of Education student specialising in Mathematics Education with UNISA. My dissertation supervisor is Dr Makgakga T.P. I am requesting to research your school on Grade 10 learners.

The title of my study is Grade 10 learner's academic experience in learning parabola functions in the school of Vhembe district of Limpopo Province. The study aims to explore the challenges and difficulties of Grade 10 learner's experiences when they are learning parabola functions. The intention is to come up with remedial measures which will either do away with or reduce these difficulties. This will improve the teaching and learning of Mathematics not only to Grade 10 learners but also to the other levels.

I intend to administer achievement test to 30 sampled Grade 10 mathematics learners and conduct a semi-structured interview to two learners after they have completed writing achievement test and observe the Grade 10 teacher. The participants will not be disadvantaged in any way. The right of participants to privacy, anonymity, confidentiality and respect for human dignity will be honoured during the research. Participation by learners is voluntary and anyone willing to withdraw can do so without penalty. The participation of learners has no foreseeable risks.

For more information concerning this request, you can contact me at 066 044 1209 or mtakalani30@gmail.com or contact my supervisor Dr Makgakga T.P at 012 429 4293 or makgasw@unisa.ac.za.

Yours faithfully

Takalani Lesley Mudau

APPENDIX E: Permission letter to the principal

Mudau T.L
P.O. BOX 706
VHUFULI
0971
10/09/2020

The Principal
Ndweleni Sec School
P/Bag X714
Nzhelele
0993

Dear Sir,

RE: PERMISSION TO CONDUCT A STUDY AT YOUR SCHOOL

I am a Masters of Education student specialising in Mathematics Education with UNISA. My dissertation supervisor is Dr Makgakga T.P. I am requesting to research your school on Grade 10 learners.

The title of my study is Grade 10 learner's academic experience in learning parabola functions in the school of Vhembe district of Limpopo Province. The study aims to explore the challenges and difficulties of Grade 10 learner's experiences when they are learning parabola functions. The intention is to come up with remedial measures which will either do away with or reduce these difficulties. This will improve the teaching and learning of Mathematics not only to Grade 10 learners but also to the other levels.

I intend to administer achievement test to 30 sampled Grade 10 mathematics learners and conduct a semi-structured interview to two learners after they have completed writing achievement test and observe the Grade 10 teacher. The participants will not be disadvantaged in any way. The right of participants to privacy, anonymity, confidentiality and respect for human dignity will be honoured during the research. Participation by learners is voluntary and anyone willing to withdraw can do so without penalty. The participation of learners has no foreseeable risks.

For more information concerning this request, you can contact me at 066 044 1209 or mtakalani30@gmail.com or contact my supervisor Dr Makgakga T.P at 012 429 4293 or makgasw@unisa.ac.za.

Yours faithfully

Takalani Lesley Mudau

APPENDIX F: Permission letter from principal

Nndweleni Secondary School

P/BagX714

NZHELELE

0993

Dear Sir/ Madam

PERMISSION TO CONDUCT A RESEARCH PROJECT AT NNDWELENI SECONDARY SCHOOL

It is with pleasure to inform you that we at Nndweleni Secondary School has accepted your request to conduct your research project at our school entitled: Grade 10 learners' academic performance of learning parabola functions in schools of Vhembe district of Limpopo province. We hope that the results of the research project will improve our learners' performance in Mathematics.

Yours in Education

The principal

Makhari M.A

APPENDIX G: Permission letter from educators

DEPARTMENT OF EDUCATION
NNDWELENI SECONDARY SCHOOL
05 OCT 2020
P/BAG X714 NZHELELE 0993
LIMPOPO PROVINCE

Miriyavhavha Tech High School

P/Bag X714

NZHELELE

0993

Dear Sir/ Madam

PERMISSION TO CONDUCT A RESEARCH PROJECT AT MIRIYAVHAVHA TECH HIGH SCHOOL

We as Mathematics educators as of Miriryavhavha Technical High Secondary have accepted your request to conduct your research at our school of the topic entitled: Grade 10 learners' academic performance of learning parabola functions in schools of Vhembe district of Limpopo province. The school trust that the findings of the research will improve learners' performance in Mathematics.

Kind regards

Nedzamba N.E (DH)

APPENDIX H: Achievement test

INTRODUCTION

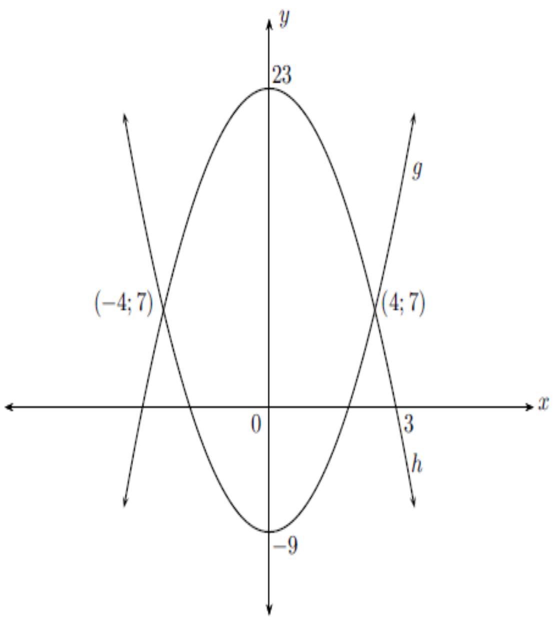
Achievement test below is part of the items learner's supposed to use for educational research on learner's understanding of parabola functions. It does not form part of a common assessment and it will not form part of grading purposes.

INSTRUCTIONS

1. Answer all questions in the space provided.
2. Show all working where necessary.
3. Non-programmable calculators may be used.
4. Answers must be rounded to the nearest one decimal place where necessary.
5. Write neatly and legibly.
6. Do not write your name on the answer sheet instead use pseudonyms.

Question number	Questions items	Marks allocation
	QUESTION 1	**5**
	Solve the following quadratic equations:	
1.1	$x(x+3)=0$	**(2)**
1.2	$3x^2+2x-1=0$	**(3)**
	QUESTION 2	**10**
2.1	**Complete the following table of $f(x)=x^2$** <table><tr><td>x</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>$f(x)$</td><td>9</td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table>	**(1)**

2.2	**Plot the points on the system of axes and join the points to form a graph.**	**(2)**
2.3	**Use the above function to determine the:**	
2.3.1	**The domain**	**(1)**
2.3.2	**The range**	**(1)**
2.4	**About which line is $f(x)$ symmetrical?**	**(1)**
2.5	**Determine the value of x for which $f(x) = 6\frac{1}{4}$. Confirm your answer graphically.**	**(3)**
2.6	**Where does the graph cut the axes?**	**(1)**
	QUESTION 3	**10**
3	**Given the following function $f(x) = -\frac{1}{2}x^2 - 3$**	
3.1	**Determine the :**	
3.1.1	**$y -$ intercept**	**(1)**
3.1.2	**$x -$ intercept**	**(2)**

3.1.3	**axis of symmetry**	**(1)**
3.1.4	**Turning point of the function**	**(1)**
3.1.5	**Sketch the graph of $f(x)$**	**(3)**
3.2	**What is the domain of the function?**	**(1)**
3.3	**Determine the range of the above function**	**(1)**
	QUESTION 4	**10**
4.	**Two parabola are drawn: $g : y = ax^2 + p$ and $h: y = bx^2 + q$.** 	

4.1	**Find the values of a and p.**	**(3)**
4.2	**Find the values of b and q.**	**(3)**
4.3	**Find the value of x for which $g(x) \geq h(x)$.**	**(2)**
4.4	**For which values of x is g increasing?**	**(2)**
	QUESTION 5	**10**
5.1	**Sketch the graphs $h(x) = x^2 - 4$ and $k(x) = -x^2 + 4$ on the same set of axes and the questions that follow:**	**(4)**
5.2	**Describe the relationship between h and k**	**(3)**
5.3	**Give the equation of $k(x)$ reflected about the line $y = 4$.**	**(1)**
5.4	**Give the domain and the range of h.**	**(2)**
	GRAND TOTAL	**45**

APPENDIX I: Achievement test marking guideline

QUESTION 1 **[5 Marks]**

1.1 $x(x+3)=0$

$x=0$ or $x+3=0$ ✓

$x=0$ or $x=-3$
✓

1.2 $2x+x-6$

$(2x-3)(x+2)=0$
✓

$2x=3$ or $x=-2$ ✓

$x=\frac{3}{2}$ or $x=-2$
✓

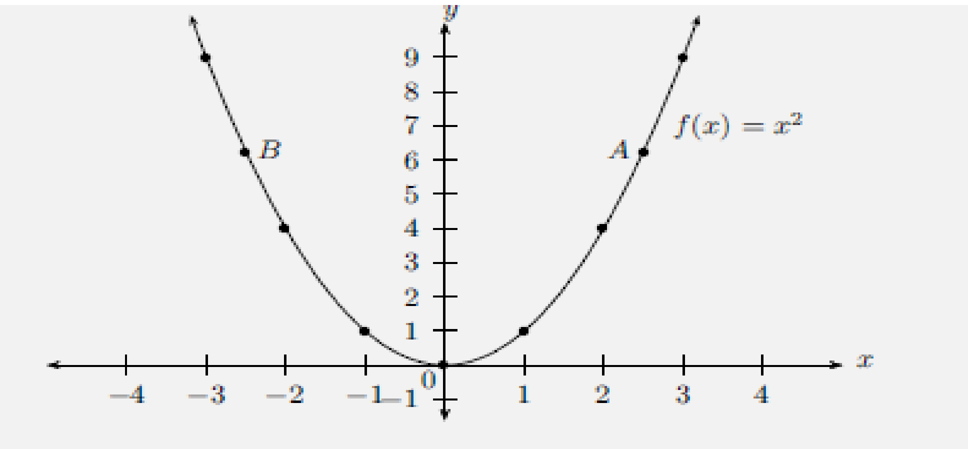
QUESTION 2 **[10 Marks]**

2.1

x	-3	-2	-1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9

✓

2.2



✓ ✓

2.3

2.3.1 Domain: x ∈ R. ✓

2.3.2 Range: {y: y ∈ R, y ≥ 0}. ✓

2.4 f is symmetrical about the y-axis. Therefore the axis of symmetry of f is the line x = 0. ✓

2.5 $f(x) = \frac{25}{4}$

$\frac{25}{4} = x^2$ ✓

$x = \pm\frac{5}{2}$ ✓

$= 2\frac{1}{2}$ ✓

2.6 At $x = 0$ ✓

<u>QUESTION 3</u> **[10 Marks]**

3.1.1 $f(x) = -\frac{1}{2}x^2 - 3$

$f(0) = -\frac{1}{2}(0) - 3$

$= -3$

y **–Intercept is (0, 3)** ✓

3.1.2 for y **–intercept let** $y = 0$:

$0 = -\frac{1}{2} - 3$

$3 = -\frac{1}{2}x^2$

$-2(3) = x^2$

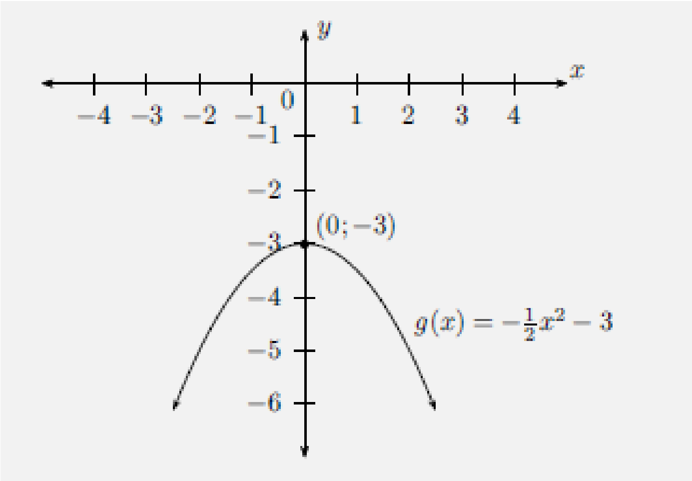
$-6 = x^2$ ✓

There is no real solution, therefore there are no x **–intercepts** ✓

3.1.3 The axis of symmetry is the line $x = 0$ ✓

3.1.4 The turning point is at (0, $-3)$ ✓

3.1.5



✓ ✓ ✓

3.2 Domain: $x \in R$ ✓

3.3 Range: $y \in (-\infty; -3)$ ✓

<u>QUESTION 4</u> **[10 Marks]**

4.1 p is the y −inetercept, therefore $p = 9$, To find a we use one of the points on the graph e.g (4;7).
✓

$$y = ax^2 - 9$$

$$7 = a(4^2) - 9 ✓$$

$$16a = 16$$

$$a = 1 ✓$$

4.2 q is the y-intercept, therefore q = 23 ✓
To find b, we use one of the points on the graph (e.g. (4; 7)):

y = bx^2 = 23

7 = b(4^2) + 23 ✓

16b = −16

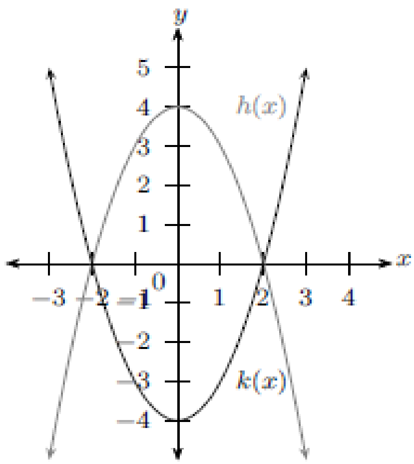
∴ b = −1 ✓

4.3 This is the point where g lies above h. From the graph we see that g lies above h.

when: x ≤ −4 or x ≥ 4 ✓

<u>QUESTION 5</u> **[10 Marks]**

5.1

13. 

✓✓✓✓

5.2

(a) h $(x) = x^2 - 4$ ✓

$k(x) = -x^2 + 4$ ✓

$= -(x^2 - 4)$

$-h(x)$ ✓

$k(x)$ **is therefore the reflection of** $h(x)$ **the** $x - axis$**.**

5.3 $y = x^2 + 4$ ✓

5.4 Domain h: $(-\infty; \infty)$ ✓

Range k: { $-4; \infty)$ ✓

TOTAL: 45 MARK

APPENDIX J: Evidence of learner achievement test

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

VHEMBE WEST DISTRICT

GRADE 10

08
45

Grade 10 learner's academic experiences in learning parabola Functions in schools of Vhembe district of Limpopo province.

MARKS:

4

TIME: 1 Hour

PSUEDO NAME;

ME: MSL 15

This question paper consists of 9 pages including the 2

INTRODUCTION

Achievement test below is part of the items learners supposed to use for educational research on learner's understanding of parabola functions. It does not form part of a common assessment and it will not form part of grading purposes.

INSTRUCTIONS

I. Answer all questions in the space provided.

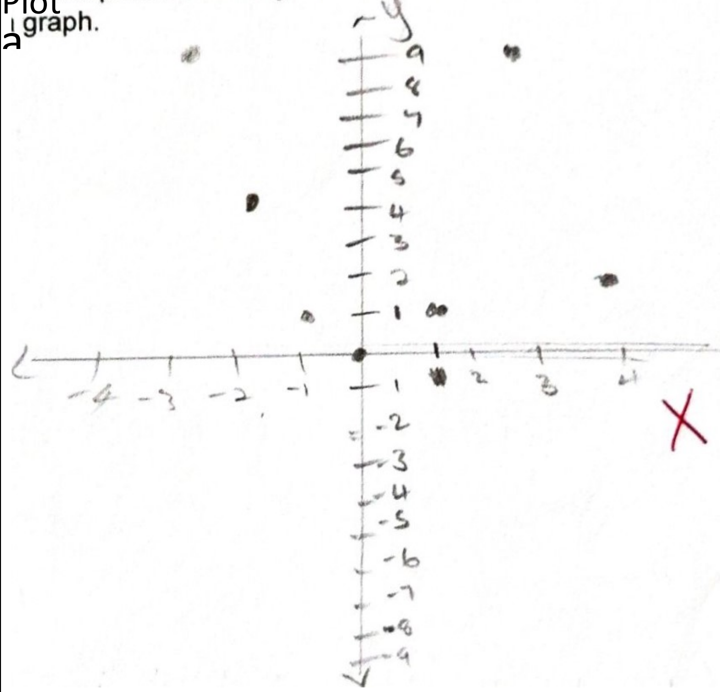
2. **Show all working where necessary.**
3. **Non-programmable calculators may be used.**
4. **Answers must be rounded to the nearest one decimal place where necessary.**
5. **Write neatly and legibly.**
6. **Do not write your name on the answer sheet instead use pseudonyms.**

Question number	Questions items	Marks allocation
	QUESTION 1	
	Solve the following quadratic equations:	
1.1	$x(x + 3) = 0$ $\cancel{x^2 + 3x = 0}$ $\cancel{\sqrt{x^2} + \sqrt{3x}}$ $\cancel{x +}$ $x(x) + x(3) = 0$ $x^2 + 3x = 0$ $(x + 1)(x + 3) = 0$	(2)

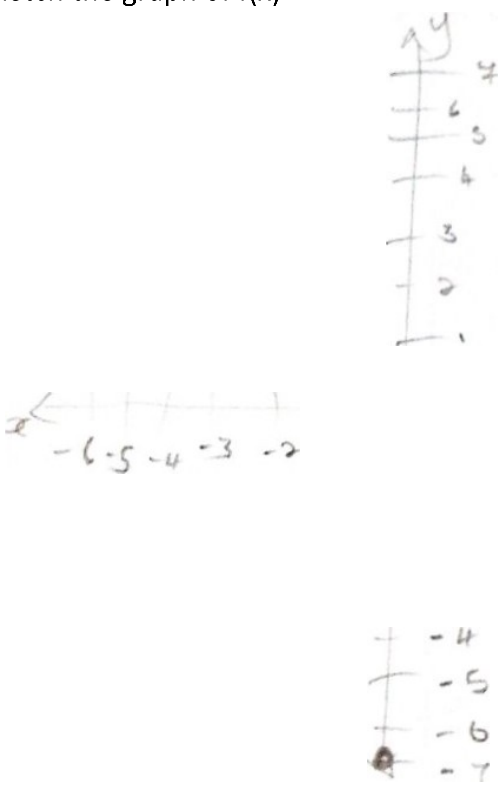
1.2	$2x^2 + x — 6 = 0$ $2x^2+x = 6+$ $\sqrt{2x^2}+\sqrt{x} = 6$ $2x + x = 6$ $\frac{3x}{3} = \frac{6}{3}$ pc ___ s	(3)

		10 2
	QUESTION 2	
2.1	Complete the following table of $f(x) = x^2$	(1)

x	-3	-2	-1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9

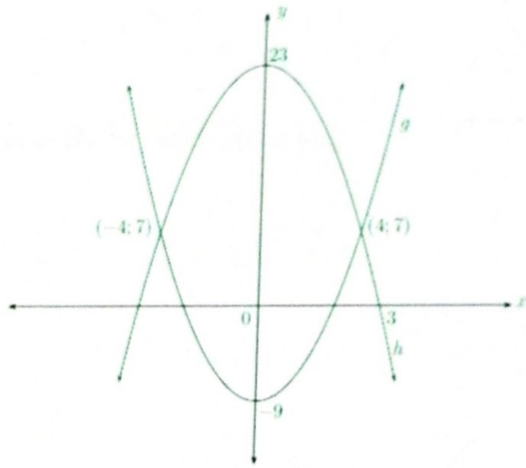
2.2	Plot the points on the system of axes and join the points to form a graph. 	(2)
2.3	**Use the above function to determine the.**	
23.1	**The domain** $x \in R$	(1)
23.2	**The range** $y \in R \geq 9$	(1)

2.4	**About which line is f(x) symmetrical?**	**(1)**
2.5 **2.6**	**Determine the value of x for which — 6 ^{2}. Confirm your answer graphically.**	**(3)**

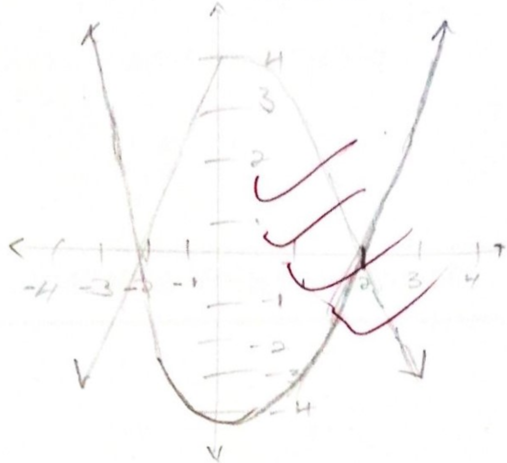
	Where does the graph cut the axes?	
3.1.5	Sketch the graph of f(x) 	(3)
3.2	What is the domain of the function? R	(1)
3.3	Determine the range of the above function R	(1)

5

	QUESTION 3	**10** 1
3	Given the following function $f(x) = -\frac{1}{2}x^2 - 3$	
3.1	Determine the :	
3.1.1	y – intercept $y = -\frac{1}{2}0^2 - 3$ $y = -\frac{1}{2} \times \frac{3}{1}$ $-1 - 6$ $y = -7$ ✗	(1)
3.1.2	x – intercept $0 = -\frac{1}{2}x^2 - 3$ $-\frac{1}{2}x^2 = 3 - 0$ $-\frac{1}{2}x^2 = \frac{3}{1}$ $\sqrt{-1x^2} + \sqrt{6x^2}$ $= 6x$ ✗	(2)
3.1.3	axis of symmetry -3 ✗	(1)
3.1.4	turning point of the function -3 ✗	(1)

	QUESTION 4	10 0
4.	Two parabola are drawn: $g: y = ax^2 + p$ and $h: y = bx^2 + q$.	
		
4.1	Find the values of a and p. $y = ax^2 + p$ $y = 3x^2 + 23$ $y = \sqrt{3x^2} + \sqrt{23}$ X	(3)
4.2	Find the values of b and q.	(3)

4.3	Find the value of x for which $g(x) \geq h(x)$.	(2)
	QUESTION 5	10 / 4

5.1	Sketch the graphs $h(x) = x^2 - 4$ and $k(x) = -x^2 + 4$ on the same set of axes and the questions that follow: 	(4) 4
5.2	Describe the relationship between h and k They are equal and they intercept at 2 X	(3)
5.3	Give the equation of $k(x)$ reflected about the line $y = 4$.	(1)
5.4	Give the domain and the range of h. Domain: $x \in \mathbb{R}$ Range $= y \in \mathbb{R} \leq 4$ -4	(2)
	GRAND TOTAL	**45**

APPENDIX K: Semi-structured interviews question guide

INTRODUCTION:

Thank you for taking your time to answer questions about parabola functions. I am researching on learner's experience in learning parabola functions.

Question 1: How do you understand parabola?

Question 2: Can you explain what $x-$ and $y-$intercepts are in parabola functions?

Question 3: How did you find the x-intercept of parabola functions and why?

Question 4: How did you find the $y-$ intercepts of parabola functions and why?

Question 5: Can you explain the axis of symmetry of a parabola function?

Question 6: How did you find the axis of symmetry of parabola functions and why?

Question 7: How do you understand the minimum and the maximum turning point of a parabola functions?

Question 8: How did you determine the minimum and maximum turning point of parabola functions and why?

Question 9: How can you explain the domain and the range of a parabola function?

Question 10: How did you determine the domain and the range of this parabola?

Question 11: Why did you draw the graph this way?

Thank you for taking part in this interview.

APPENDIX L: Semi-structured interviews schedule

The purpose of interview schedule is to explore the academic experience of learners in parabola functions.

Class: Grade 10 **Number of learners: 20**

School: B **Subject: Mathematics**

Learner Code[1]	**D1**	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**
1. LI1									
2. LI2									
3. LI3									
4. LI4									
5. LI5									
6. LI6									
7. LI7									
8. LI8									
9. LI9									
10. LI10									

1. Learners were given code name for identification.
2. The code D1 denotes Day 1, D2 means Day 2 and D3 implies Day 3 so on. D1 is the day a learner will be selected for the interviews.

APPENDIX M: Lesson observation

DEMOGRAPHIC DETAILS

Educator Highest Qualification: __________

Years of Subject Teaching Experience: __________

Subject: __________________ Date: __________________

Topic of Lesson: _____________ Lesson Duration: _________

Grade Observed: ___________ Number of learners in class: _

OBSERVING CLASSROOM PRACTICE

1. **How does teaching and learning of parabola functions take place? (Please list e.g.**

   Whole class)

   (a) ______________________________

   (b) _______________________________

   (c) ______________________________

   (d) _______________________________

2. **What teaching methodology or approach is being used?**

____

3. **How does teacher deals with correct and incorrect answers?**

____

OBSERVING TEACHING AND LEARNING IN THE CLASSROOM

The PEER system underlies the lessons in a classroom situation. It might not be possible to incorporate all of them in a particular lesson but each lesson will contain some aspects of this system. Please tick (□) your rating.

A	PRODUCTIVE SKILLS	**Excellent**	**Good**	**Average**	**Needs More Attention**	**Not Applicable to the lesson**
1.	**Learners can do the reading on the concept being taught.**					
2.	**Learners write notes on the concept taught.**					
3.	**Learners can solve problems given as exercises.**					
4.	**Learners can relate and apply the concept to real-life problems.**					
5.	**Learners can use their knowledge of and experience in the concept in formulating their responses.**					
6.	**Learners can accomplish work given on the concept independently**					
7.	**Learners can define and describe learned terms encountered when dealing with the concept.**					

8.	**Learners can follow the steps in solving exercises based on the concept.**					
9.	**Learners competently use technology (calculators) in areas where it is required in the concept.**					
1 0.	**Learners can deal with problems in real and abstract context using the concept.**					
1 1.	**Learners' ways of making decisions in problem-solving are enhanced.**					
B	EVOCATIVE SKILLS	**Excellent**	**Good**	**Average**	**Needs More Attention**	**Not Applicable to the lesson**
1.	**Learners ask questions for clarification.**					
2.	**Learners ask questions to consolidate their understanding of the concept**					
3.	**Learners are puzzled by certain areas of the concept and hence very inquisitive.**					
4.	**Learners can interpret new information on the concept.**					
5.	**Learners ask critical questions to ensure that the methods used are appropriate.**					

6.	**Learners use their referencing skills to acquire a better understanding of the concept.**					
C	EVALUATIVE SKILLS					
1.	**Learners are able to do self-assessment tasks in the concept learned.**					
2.	**Learners are capable of evaluating their own work on the concept.**					
3.	**Learners are able to evaluate procedures followed in problem solving in the concept.**					
4.	**Learners can identify errors committed when dealing with the concept.**					
5.	**Learners can discuss the pros and cons of using specific methods to solve problems.**					
6.	**Learners can identify incorrect ways of solving problems.**					
7.	**Learners have alternative ways to solve problems based on the concept.**					

D	REFLECTIVEIVE SKILLS	Excellent	Good	Average	Needs More Attention	Not Applicable to the lesson
1.	**Learners are constantly engulfed in the world of “exploration in errors.”**					
2.	**Learners reflect on errors committed to solving problems and work towards eliminating those errors.**					
3.	**Learners can respond to questions testing their comprehension of the learned concept.**					
4.	**Learners can select and use appropriate methods in solving problems.**					
5.	**Learners can hypothesize in problem-solving.**					
6.	**Learners can reflect on the decision they made in solving a particular problem.**					

APPENDIX N: Lesson observation schedule

	07H45-8H30	8H30-9H15	9H15-10H00	10H00-10H45	10H45-11H30	12H00-12H45	12H45-13H30	13H30-14H15
Monday	**SBLOT 1**				**SBLOT 2**			
Tuesday		**SBLOT 3**					**SBLOT 4**	
Wednesday				**SBLOT 5**				
Thursday			**SBLOT 6**					**SBLOT 7**
Friday						**SBLOT 8**		

The schools will be coded A, B for identification when analysing the results. The code SBLOT1 was used to denote that: SB stand for school B, LO stands for lesson observation and T1 stand for a teacher who will be in the numerical position 1 in the observation, SBLOT2 means school B lesson observation for teacher number two, and so on.

APPENDIX O: Informed consent for educators

Date: 07/07/2020

TITLE: GRADE 10 LEARNERS' ACADEMIC EXPERIENCES IN LEARNING PARABOLA FUNCTIONS IN SCHOOLS OF VHEMBE DISTRICT OF LIMPOPO PROVINCE

Dear Prospective Participant

My name is Mudau Takalani Lesley. I am conducting research under the supervision of Dr Makgakga T.P, a senior lecturer in the Department of Mathematics Education towards a M Ed at the University of South Africa. We have funding from DSF-POSTGRD for assistance in all travelling cost and tuition fee expenses. We are inviting you to participate in a study entitled learners experience in learning parabola function.

WHAT IS THE PURPOSE OF THE STUDY?

This study is expected to collect important information that could help learners to improve their understanding and performance in the topic of parabola functions.

To inform educators of the intervention strategies that can be implemented to improve the teaching of parabola functions.

To improve the morale of the learners and the teachers in the teaching and learning of parabola functions.

WHY AM I BEING INVITED TO PARTICIPATE?

You are invited because you are educator teaching Grade 10 Mathematics in the selected school in which research is going to take place.

I obtained your contact details from the principal of your school who get permission from the district office. The project will require only one educator teaching Grade 10 Mathematics to be observed.

WHAT IS THE NATURE OF MY PARTICIPATION IN THIS STUDY?

The study involves using video-tape and observing teachers while they are teaching a lesson on Grade 10 parabola functions, your interaction with learners will also be used in the findings of the report. I will also want your learners to write an achievement test which will be followed by a semi-structural interview.

The total duration for achievement test completion is 1 hour, the semi-structured interview is 30 minutes and lesson observations takes 45 minutes.

CAN I WITHDRAW FROM THIS STUDY EVEN AFTER HAVING AGREED TO PARTICIPATE?

Participating in this study is voluntary and you are under no obligation to consent to participation. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written consent form and you are free to withdraw at any time without giving a reason.

WHAT ARE THE POTENTIAL BENEFITS OF TAKING PART IN THIS STUDY?

There will be no reimbursement or any incentives for participation in the research however your content pedagogical knowledge will improve.

ARE THERE ANY NEGATIVE CONSEQUENCES FOR ME IF I PARTICIPATE IN THE RESEARCH PROJECT?

There are no negative consequences for participating in this research project no Potential risks are involved in this study.

WILL THE INFORMATION THAT I CONVEY TO THE RESEARCHER AND MY IDENTITY BE KEPT CONFIDENTIAL?

Your name and identity will be kept confidential all the times and in all academic writing about the research project. Your privacy will be maintained in all published and written data resulting from the study. If my research is published in national journals and presentation at national and international conferences, your identity will be kept anonymous. I will use pseudonyms instead of your names throughout the data collection, analyses process and presentation of research results.

HOW WILL THE RESEARCHER(S) PROTECT THE SECURITY OF DATA?

Hard copies of your answers will be stored by the researcher for a period of five years in a locked filing cabinet at home for future research or academic purposes; electronic information will be stored on a password-protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. Hard copies will be permanently burned and soft copies will be permanently deleted from the system five years after research has been conducted.

WILL I RECEIVE PAYMENT OR ANY INCENTIVES FOR PARTICIPATING IN THIS STUDY?

There will be no payment or any incentive received for participation in this research project.

HAS THE STUDY RECEIVED ETHICS APPROVAL?

This study has received written approval from the Research Ethics Review Committee of the CEDU, Unisa. A copy of the approval letter can be obtained from the researcher if you so wish.

HOW WILL I BE INFORMED OF THE FINDINGS/RESULTS OF THE RESEARCH?

If you would like to be informed of the final research findings, please contact Mr Mudau T.L on 066 044 1209 or email at mtakalani30@gmail.com. The findings are accessible for the participant within the period six (6) months.

Should you require any further information or want to contact the researcher about any aspect of this study, please contact Mr Mudau T.L at mtakalani30@gmail.com or phone him at 066 044 1209 or contact my supervisor Dr Makgakga S.W at 012 429 4293 or at makgasw@unisa.ac.za.

Thank you for taking the time to read this information sheet and for participating in this study.

Thank you.

____

(insert signature)

Mudau Takalani Lesley

APPENDIX P: Informed assent from learners

TITLE: GRADE 10 LEARNERS' ACADEMIC EXPERIENCE IN LEARNING PARABOLA FUNCTIONS IN SCHOOLS OF VHEMBE DISTRICT OF LIMPOPO PROVINCE

Dear Learner   Date 07/07/2020

I am doing a study on learner's experiences in learning parabola functions as part of my studies at the University of South Africa. Your principal has permitted me to do this study in your school. I would like to invite you to be a very special part of my study. I am doing this study so that I can find ways that your educators can use to make the teaching of parabola functions better. This may help you and many other learners of your age in different schools.

This letter is to explain to you what I would like you to do. There may be some words you do not know in this letter. You may ask me or any other adult to explain any of these words that you do not know or understand. You may take a copy of this letter home to think about my invitation and talk to your parents about this before you decide if you want to be in this study.

I would like to ask you to write an achievement test and thereafter participate in the semi-structural interview on questions based on parabola functions. You will spend an hour writing achievement test and thirty minutes in answering a semi-structured interview, and a total of one hour thirty minutes to complete this research project.

I will write a report on the study but I will not use your name in the report or say anything that will let other people know who you are. Participation is voluntary and you do not have to be part of this study if you don't want to take part. If you choose to be in the study, you may stop taking part at any time without penalty. You may tell me if you do not wish to answer any of my questions. No one will blame or criticise you. When I am finished with my study, I shall return to your school to give a short talk about some of the helpful and interesting things I found out in my study. I shall invite you to come and listen to my talk.

This study is expected to collect important information that could help learners to improve their understanding and performance in the topic of parabola functions.

To inform educators of the intervention strategies that can be implemented to improve the teaching of parabola functions.

To improve the morale of the learners and the teachers in the teaching and learning of parabola functions

There are no Potential risks are involved in this study.

You will not be reimbursed or receive any incentives for your participation in the research.

If you decide to be part of my study, you will be asked to sign the form on the next page. If you have any other questions about this study, you can talk to me or you can have your parent or another adult call me at 066 044 1209. Do not sign the form until you have all your questions answered and understand what I would like you to do.

Researcher: Mudau T.L Phone number: 066 044 1209

For more information concerning this request, you can contact me at 066 044 1209 or mtakalani30@gmail.com or contact my supervisor Dr Makgakga S.W at 012 429 4293 or makgasw@unisa.ac.za.

Do not sign the written assent form if you have any questions. Ask your questions first and ensure that someone answers those questions.

WRITTEN ASSENT

I have read this letter which asks me to be part of a study at my school. I have understood the information about my study and I know what I will be asked to do. I am willing to be in the study.

Learner's name (print): ---

Learner's signature:----------------------------------- Date: -----------------------

Witness's name (print): ---

Witness's signature: ----------------------------------- Date: ----------------------

(The witness is over 18 years old and present when signed.)

Parent/guardian's name (print): --

Parent/guardian's signature: ------------------------------ Date: ---------------

Researcher's name (print): ---

Researcher's signature: ------------------------------------- Date:--------------

APPENDIX Q: Educators consent to participate

(Return slip)

I, ___________________ (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read and the researcher has explained to me everything about the study and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty (if applicable).

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I agree with the recording of the lesson presentation.

I have received a signed copy of the informed consent agreement.

Participant Name & Surname (Please print) ---

Participant Signature: ----------------------------------- Date: ---------------------------

Researcher's Name & Surname (please print): --

Researcher's signature: -------------------------------- Date: ---------------------------

APPENDIX R: Parent consent for minor to participate in research project

Dear Parent

Your child is invited to participate in a study entitled Grade 10 learners' academic experience in learning parabola functions in schools of Vhembe district of Limpopo Province.

I am undertaking this study as part of my master's research at the University of South Africa. The purpose of the study is to explore the difficulties Grade 10 learner's experiences when they are learning Grade 10 parabola functions and the possible benefits of the study are the improvement of learner's knowledge in parabola functions and suggest possible teaching strategies that can be used to improve the teaching of parabola functions. I am asking permission to include your child in this study because he/she is doing mathematics in Grade 10. I expect to have many other children participating in the study.

If you allow your child to participate, I shall request him/her to:

- To write an achievement test based on the topic of parabola function on the 07/07/2020 at his/her school which will take one hour to complete.
- Take part in an interview which will focus on his/her understanding of parabola functions which will take place on 07/07/2020 at his/her school and takes thirty minutes.
- I am also asking permission to audio-tape your child during the interview session as a source of my research project data

Any information that is obtained in connection with this study and can be identified with your child will remain confidential and will only be disclosed with your permission. His/her responses will not be linked to his/her name or your name or the school's name in any written or verbal report based on this study. Such a report will be used for research purposes only.

There are no foreseeable risks to your child by participating in the study. Your child will receive no direct benefit from participating in the study; however, the possible benefits to education are knowledge improvement in the topic of parabola functions. Neither your child nor you will receive any type of payment for participating in this study.

Your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusal to participate will not affect him/her in any way. Similarly, you can agree to allow your child to be in the study now and change your mind later without any penalty.

The study will take place during regular classroom activities with the prior approval of the school and your child's teacher. However, if you do not want your child to participate, an alternative activity will be available like classwork or test to keep him/her busy.

In addition to your permission, your child must agree to participate in the study and you and your child will also be asked to sign the assent form which accompanies this letter. If your child does not wish to participate in the study, he or she will not be included and there will be no penalty. The information gathered from the study and your child's participation in the study will be stored securely on a password-locked computer in my locked office for five years after the study. Thereafter, records will be erased.

The benefits are learners will improve their understanding and performance in the topic of parabola.

APPENDIX S: Language editor certificate

Unit 3 West Square Business Park
407 West Avenue
Randburg
2194

12 November 2022

TO WHOM IT MAY CONCERN

This serves to confirm that I have edited and made the necessary corrections and emendations to the thesis:

GRADE 10 LEARNERS' ACADEMIC EXPERIENCES OF LEARNING PARABOLIC FUNCTIONS IN SCHOOLS OF VHEMBE DISTRICT OF LIMPOPO PROVINCE

by

TAKALANI LESLEY MUDAU

Sincerely

J Musi
Publisher, editor and translator

Professional EDITORS Guild
National

Professional Editors' Guild
PO Box3059
Tokai
Western Cape
7966

NPC 2015/060183/08
https://www.editors.org.za

STATEMENT DATE	28/02/2022
AMOUNT DUE	50.00CR
MEMBERSHIP NO	MUS001

MUSI - Joyce
PO Box 1550
Honeydew
Gauteng
2040

Tel: +27 84 513 3707• Fax: 086 532 6404• e-mail: caption@webmail.co.za • P O Box 1550 • Honeydew • 2040