

Assessment of Wind Power Potential as a Renewable Energy Source in Western Cape, South Africa

by

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Declaration

No part of this thesis has been submitted elsewhere for any other degree or qualification. The content of this thesis is all my own work unless referenced to the contrary in the text.

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Firstly, I want to thank my supervisor, Prof Adedayo Ademola Yusuff, and my co-supervisor, Prof Mbekisipho Twala, for being the best supervisor and co-supervisor any MTech student could ask for. Secondly, I want to thank my very close friends and family, especially Oliver Kambombo Tshibanda and Gaby Mujinga Tshibanda, who have supported me throughout this Magister Technologiae.

“He gives strength to the weary and increases the power of the weak. Even youths grow tired and weary, and young men stumble and fall; but those who hope in the LORD will renew their strength. They will soar on wings like eagles; they will run and not grow weary; they will walk and not grow faint”.

Isaiah.40:29-31

Abstract

The worldwide utilisation of renewable energy in electric power systems is growing rapidly due to concerns about the environment and the depletion of the sources of conventional power generation. The Weibull distribution is a widely used distribution, especially for modelling the random variable of the wind speed. In this respect, the authors present a comparative analysis of a number of methods used for estimating Weibull parameters. Results for a real – word database, are presented in a case study format. The techniques require historical Wind speed data, collected over a particular time interval, to establish the parameters of wind speed distribution for a specific location, namely Cape Town, South Africa. As can be observed, the scale and shape parameters, from the whole database and from seasonal values, have the best estimation in case of maximum likelihood (MLE). Thus, it can be summarised that the maximum likelihood (MLE) is the best method used to estimate the parameters for the two-parameter Weibull distributions taking into consideration the mean bias error (MBE) and the root mean squares error (RMSE) as measurements of comparison, while the maximum likelihood (MLE) method is the least accurate method.

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List of Abbreviations

AC:	Alternating current
AVR:	Automatic voltage regulator
BLDC:	Brushless direct current
CDF:	Cumulative distribution function
CF:	Capacity factor
DC:	Direct current
DFIG:	Doubly fed induction machine.
LSM:	Least squares method
MLE:	Maximum likelihood estimator
MOM:	Method of moments
PDF:	Probability density function
PMSG:	Permanent magnet synchronous generators
SG:	Synchronous generators
VAR:	Volt-ampere-reactive
VSI:	Voltage source inverter

List of Symbols

f_w : Weibull probability density function
 F_w : Weibull cumulative distribution function
 P_w : Wind power
 P_m : Mechanical power
 W_m : Turbine angular velocity
 P_t : Transmission output power
 η_m : Transmission efficiency
 P_e : Generator output power
 η_g : Generator efficiency
 C_{pR} : Coefficient of performance at the rated wind speed
 $(v_n)\eta_{mR}$: Transmission efficiency at rated power
 η_{gR} : Generator efficiency at rated power
 ρ : Air density
 A : Turbine area
 $C_{pR}\eta_{mR}\eta_{gR}$: Total efficiency of the turbine
 q : Transmission efficiency
 P_{mR} : Rated turbine shaft power
 P_{tR} : Rated mechanical power input.
 P_{eR} : Rated electrical power output.
 v_{in} : Cut-in speed.
 k : Shape parameter [dimensionless]
 c : Scale parameter [m/s]
 v : Wind speed [m/s]
 n : Number of measured values
 v_i : Set of the measured values.
 $\Gamma(\)$: Gamma function
 C_p : Coefficient of performance
 r_m : Maximum radius of the rotating turbine in meter
 v_n : Rated speed
 v_{ie} : Cut-off speed
 $P(\)$: Lower incomplete gamma function
 P_N : Normalised average power
 A_2, A_1, A_0 : Coefficients of power curve

Chapter 1

1. Introduction

1.1. Electric power generation from renewable energy sources

It is generally possible to harness wind energy to exploit the potential of wind power. However, the existing concerns in this field involve reducing the costs of the energy in kWh produced from wind. The cost of energy produced by wind turbines is close to that of the thermal power plants (TPP), thus leading to a cost of approximately 0.05–0.06 USD/kWh. Wind energy is currently widely used, with the maximum power of wind turbines ranging between 2.5 and 4 MW (new turbines reach powers of up to 7.5 MW) [1]. In recent years wind has become the fastest growing energy source. The majority of wind turbines generate more than 25% of their installed capacity although this percentage depends on both the wind profile site and type of wind turbine.

1.2. Electric power generation from wind sources

Wind power has proved to be a viable solution to the problem of non-renewable sources of energy or the generation of electric power from nuclear energy. It is believed that the technical potential of the world can provide 5 times more energy than was being consumed at the time of this study. The main advantage of wind energy is the zero emission of pollutants and greenhouse gases as fuels are not being burned. Unlike conventional power plants, where the cost of decommissioning may be several times higher than the cost of the power plant (as in the case of nuclear power plants), the cost of decommissioning wind turbines is minimal as, at the end of the normal period of operation, they are entirely recyclable. The main disadvantages of the production of electric power from wind energy is the high cost of production and the relatively reduced reliability of the turbines with further disadvantages being the visual pollution and the noise. Due to the intermittent characteristic of wind energy, the isolated use of wind turbines should include the provision of additional sources (batteries, diesel generators) as connection to the public network is conditional on the stability of the injected power parameters.

1.3. Wind characteristics

Wind represents the movement of air masses in the earth's atmosphere. This may be modelled as a thermodynamic cycle that extracts energy from the sun and distributes it to a heat conductive surface at a lower temperature. It follows that regions exist where the air pressure is either higher or lower than the average for a short term. Pressure differences are due to the irregular heating of air masses by solar radiation while, on account of the earth's rotation, air masses gather large quantities of kinetic energy. Wind energetic parameters include wind intensity, wind speed variation over time, extreme speeds, and wind direction.

Wind intensity is characterised by its effects on the environment; Thus, the main types of wind include:

- Calm (0–0.4 m/s)
- Gentle (0.4–5.8 m/s)
- Moderate (5.8–11 m/s)

- Strong (11–17 m/s)
- Storm (17–25 m/s)
- Hurricane ($v > 34$ m/s)

Wind speed variation over time is described by a vector quantity, characterised by a scalar quantity and the direction of the vector. The vector's size is important for both energy calculations and to evaluate stress on the endurance of the structures. The horizontal component of wind speed is applicable to measurement only. In weather stations, registrations are made at standard heights of 10 m to 50 m above ground level. As a result, stations around the world record weather data at regular intervals and store this data for the period of a century. Wind speed is measured with devices known as anemometers. However, wind speed data may be directly affected by the anemometer height, its exposure to surrounding buildings, hills, trees, the human factor that interprets wind speed values as well as the quality and maintenance of the anemometer. Comparisons between wind speeds are possible only if the area is prone to a wind energy potential and when the anemometer altitude and the surrounding obstacles are the same. Data may be used to provide an indication of the most suitable territory and regions in a country in terms of the potential for wind energy. Extreme speeds of a placement site are determined by the statistical methods deemed appropriate to rare phenomena. It is important be aware of such extreme speeds in view of the security of wind turbines against storms.

1.4.Wind turbine structure

There are two types of wind turbines, namely:

- vertical axis turbine
- horizontal axis turbine

Despite the axis orientation, the role of the turbine is to generate torque in order to drive the electric generator.

- **Vertical axis wind turbines**

The towers of vertical axis wind turbines are small, with a height ratio of 0.1 to 0.5 of the height of the rotor. This allows for placement of the entire energy conversion equipment (multiplier, generator) at the base of the wind turbine, thus making it possible to carry out maintenance of the operations. A device to guide the rotor is not required, as is the case with the horizontal axis wind turbine. However, since wind has a low velocity at ground level, this results in the reduced efficiency of the wind turbine. In addition, the turbines are required to be trained in order to start. This factor results in a greater orientation towards horizontal axis turbines as compared to vertical axis turbines. The most common vertical axis wind turbine structure is based on the principle of differential traction or the differential variation of incidence. As in the case of a Savonius rotor (Figure 2.1), the operation is based primarily on the differential traction principle. The effects of wind on each side of a curved body are characterised by different intensities with this resulting in torque which rotates the blades of the vertical axis wind turbine.

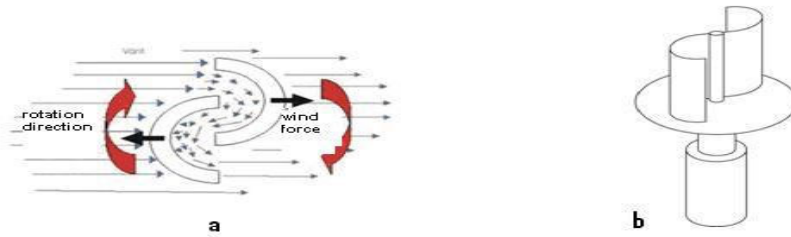


Figure 1. Basic diagram of the Savonius rotor (a) and the scheme of a Savonius rotor (b) [12].

Figure 2. Illustrates the image and scheme of a Darrieus rotor, which is based on the periodic variation of incidence. A profile placed in a stream of air, at different angles, is subjected to forces with varying intensity and direction. The total force generates a torque which rotates the unit.



Figure 2. Image of a Darrieus wind turbine (a) and the schema of the Darrieus rotor (b)[12].

- **Horizontal axis wind turbines**

The operation of the horizontal axis wind turbine is based on the windmill principle. The rotor of the turbine is, in most cases, three-bladed with a streamlined profile which results in an acceptable compromise between the power coefficient, cost and rotation speed of the wind gauge, as well as improving the aesthetic appearance of the turbine as compared to the two-bladed rotor.

Horizontal axis wind turbines are more widely used than vertical axis wind turbines as a result of their superior aerodynamic efficiency. In addition, they are not subjected to significant mechanical stress and are associated with low costs. This turbine type is depicted in Figure 2.3.



Figure 3. Image of a horizontal axis wind turbine [12].

There are two types of horizontal axis wind turbines:

- **Upwind:** The winds strike the face of the blades in the direction of the nacelle. The blades are rigid and are fitted with a device that turns the rotor in the wind direction.

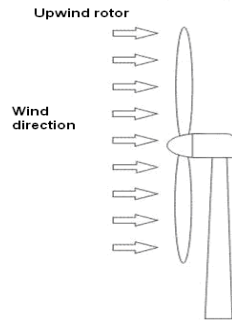


Figure 4. Schema of an upwind horizontal axis wind turbine [12].

- **Downwind:** The winds strike the back of the blades in the nacelle direction. The rotor is both flexible and self-guided.

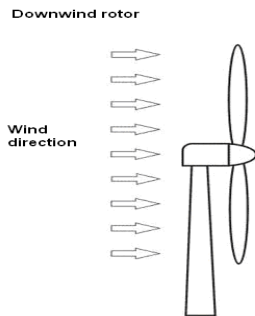


Figure 5. Schema of a downwind horizontal axis wind turbine [12].

The upstream wind turbine is the most popular as it is simple and offers the best results at high power. In addition, it does not require direction areas, has greater stability as compared to its downwind counterpart and the operation efforts are reduced. The horizontal axis wind turbine blades must always be oriented according to the direction and strength of the wind. Therefore, devices exist to set the nacelle in the wind direction and the blades according to the wind strength. At the time of this study there was significant interest in the horizontal axis propeller turbines in order to produce electricity on an industrial scale.

- **Horizontal axis turbine structure**

The horizontal axis turbine structure consists of the base (foundation), tower and nacelle which houses the blades, gearbox, generator, control and measuring instruments. These components are depicted in Figure 6.

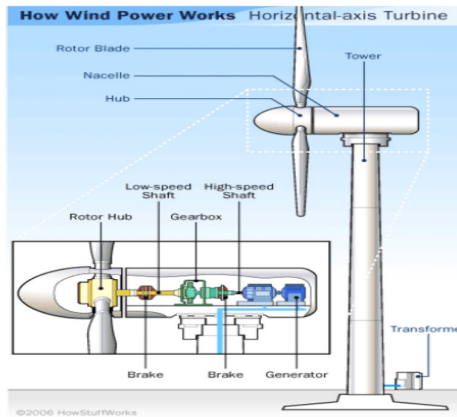


Figure 6. Horizontal axis turbine structure (<http://science.howstuffworks.com/windpower> (2012))

- a). Base (foundation)
- b). Tower
- c). Nacelle
 - Blade
 - Gearbox
 - Generator
 - Control instruments
 - Measuring instruments
 -

a). The foundation

The foundation represents the resistance structure of a wind turbine. As shown in Figure 7, it is a robust concrete base which supports the tower and the nacelle. A variety of foundations exist and are selected based on both the geotechnical conditions and the type of construction of the tower. The simplest form is the foot type, which is a foundation based on gravity and which exerts an overload in relation to the ground. This type may be used for different subgroups of soil and rock. The wind turbine foundations are generally octagonal – the foot may have a diameter from 15 to 20 m with an average depth of 1.2 to 1.8 m. It also contains a pedestal with a height of 2.4 to 2.7 m and diameter of 5.4 to 6 m.

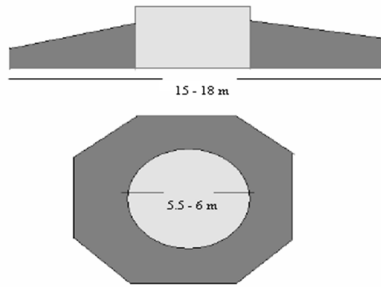


Figure 7. Wind turbine foundation [12].

b). The tower

The tower is the supporting structure of the nacelle and allows access into the wind turbine. It protects both individuals and the surrounding area against the propeller force and elevates the rotor above any obstacles that may reduce the wind velocity, and consequently, efficiency. Wind turbines may have tubular steel, lattice truss or concrete towers, depending on the size. A tubular tower is installed with the distribution grid of the electrical power produced by the wind turbine and contains access stairs to the nacelle.

A tubular tower is depicted in Figure 8 below.



Figure 8. Tubular tower [12].

It is important to select an appropriate height for the tower as a compromise usually must be made between the construction price and wind exposure. The height of the towers varies from 20 to 50 m and from 80 to 100 m. Low power turbines (under 1kW) are generally mounted on towers with a diameter several times larger than the rotor blade. Shorter towers would usually affect the power generated due to the low velocity of the wind which blows close to the earth's surface. The tower for medium or large turbines is slightly higher than the diameter of the rotor blades.

The basic types of towers used for almost all wind generator installations include:

- **Tubular pole towers**

Most large wind turbines are delivered with tubular cylindrical pole towers, which are manufactured in 20 to 30-metre-long sections, flanged at each end and bolted together on site.

The towers have a conical shape, with a base larger than the top in order to increase stability and economise on material.

- **Lattice truss towers**

Lattice truss towers are constructed using connected steel beams. The main advantage of such towers is the relatively low construction cost as lattice truss towers require half the material of the same stiffness used for tubular towers. However, the main disadvantage of lattice truss towers (although controversial) is the lack of visual appeal. This is, in fact, the reason why these types of towers are no longer used on a large scale.

- **Guyed towers**

Several small wind turbines use narrow poles supported by guy wires. The chief advantages of these towers include the weight and low cost although a disadvantage is that access around these types of towers is often difficult due to the wires. This renders them unsuitable for operation on wind farms. Moreover, these towers are often vandalised which compromises the safety aspect.

- **Hybrid towers**

The hybrid tower is a combination of the basic types of tower construction. In order to optimise the quantity of energy produced, when a group of turbines are installed on a wind farm, a specified amount of space must be maintained between the towers. Spacing is dependent on the terrain, location, wind direction, wind speed and size of the turbine. Optimum spacing is produced by aligning the towers in rows as above the prevailing wind direction. The distance between the rows is 8 to 15 times the rotor diameter while the distance between the towers is 2 to 4 times the propeller diameter, perpendicular to the prevailing wind direction. The average number of wind turbines on a wind farm may vary, even rising into the hundreds, depending on the power required. If land space is limited and/or expensive, an optimisation study including power and spacing specifications (Figure 9) must first be carried out to determine the maximum number of turbines that may be placed in order to obtain maximum energy.

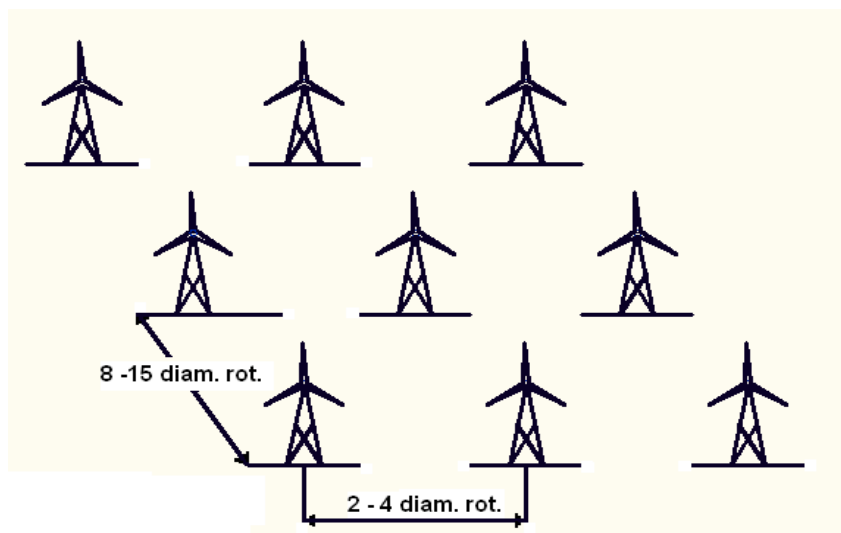


Figure 9. The optimal location of wind turbines on a wind farm [12].

Wind power plants represent groups of turbines which are placed in a certain territory in order to produce electricity from wind power. The optimum area in which to build a wind farm depends on the wind speed during an entire year in a certain region, as well as the altitude, terrain, and temperature. There are three types of locations, depending on the area in which a wind turbine is installed:

- **On the shore** – wind turbines are placed three kilometres or more in shore. The choice of the location depends on the effect of acceleration of an air mass above a certain obstacle (in this case, the shore). Since location errors may lead to a massive decrease in the quantity of electricity generated, studies are carried out for at least 1 year to determine the optimum area where wind turbines should be placed.
- **Near the shore** – wind turbines are placed a maximum of three kilometres in shore or 10 kilometres offshore. These turbines use the convection effect emanating from the difference in temperature between land and water.
- **In the open sea (offshore)** – wind turbines are placed more than 10 kilometres offshore. The turbines have a higher wind speed average and do not either generate noise or have an adverse visual effect. However, the disadvantages include the higher costs for construction, location, and maintenance (salty water has a strong corrosive effect on the turbine base). If the distances from the shore allow it, wind power plants may be connected to the electrical network.

- **Cost considerations**

A wind turbine tower costs almost 20% of the total price of the turbine installation. For a 50 m tower, the additional cost of another 10 m of height is approximately \$15 000. It is, reform, imperative that the towers be built optimally in relation to the final cost of the Energy to be generated. Lattice towers are cheaper to manufacture as they require half the quantity of steel normally used for a tubular tower.

- **Aerodynamic considerations**

It is generally, preferable to build a tall tower in areas with rough terrain as wind speed increases in proportion to the height above the ground. Lattice and guyed towers have the advantage of encountering less resistance to wind as compared to a massive tower.

- **Structural dynamics**

The rotors on short towers are subject to different wind speeds (thus lower bending moments) when a rotor blade passes through its top and bottom positions. This, therefore, increases the fatigue load on the turbine.

- **The height of the tower**

It is evident that it is not recommended to mount a 60 m diameter rotor on a tower which is less than 30 m high. In view of the costs of the rotor, generator and gearbox, it would be uneconomical to mount these onto small towers. Tall towers generate higher wind speeds and, hence, higher energy levels. As each metre of tower involves costs, the optimal height of a wind tower depends on:

- tower cost per metre of height
- wind speed variation above ground level (a higher degree of requires taller towers).

Manufacturers deliver towers that are of the equivalent height of the rotor diameter. A wind turbine has greater aesthetic appeal if the tower height is equal to the rotor diameter.

A wind farm consisting of 20 turbines of 500 kW each requires a land area of 1 to 2 km² although only 2% of this area is occupied by the turbines, together with access to roads. The remaining land area may be used for what was originally proposed. The results of optimisation are as follows:

- Larger turbines cost less compared to MW installations and occupy a smaller area (unless more turbines which the same installed power are used).

- The power fluctuations of large turbines cost more than the filtering operations of power while voltage fluctuations degrade the quality of the energy supplied, thus resulting in penalties from the network.

The optimisation method considers all the arguments above. In addition, it includes the effect of tower height, which is linked to the propeller diameter, available standard values, acquisition cost and wind speed. This results in several turbines of a specific power and area which minimises the cost of energy.

c). The nacelle

The nacelle is the housing which protects the generating components in a wind turbine and includes the transmission system, electric generator, control, safe, measurement systems and gearbox.

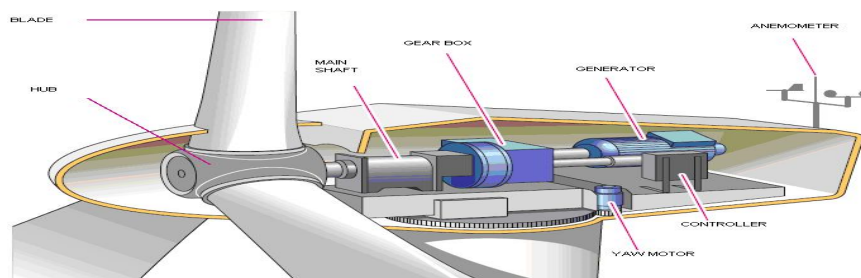


Figure 10. Nacelle components [12].

○ Turbine blade

The blade is the most distinctive component of a wind turbine. It is attached to the hub and it builds up the rotor. The rotor blade design uses the same technology as that applied in aeronautics in which composite materials assure similar levels of mechanical strength, flexibility, elasticity, and weight. The blade diameter (or area covered by the blade) is produced by the wind turbine power together with the width of the blades which determines the starting torque. This becomes greater as the blades become wider. For example, a power of 10 kW requires a blade diameter of 7 m, for a power of 0.2 MW the diameter is 27 m, and, for a power of 2 MW, the diameter may be as much as 72 m. The number of blades selected depends on the type of wind turbine. At the time of this study the three-blade rotor was the most common as it limits vibration, noise, and rotor fatigue in comparison to single or two-blade systems. The output of the two-blade rotor is 10% higher than that of the single-blade rotor and increases by 3% between the three-blade and two-blade systems. In addition, it is an efficient compromise between the cost and the rotation speed of the turbine with the aesthetic advantage of the three-blade system. In such a case, the cost of the blades represents a maximum of 10% of the total installation costs of a wind turbine.

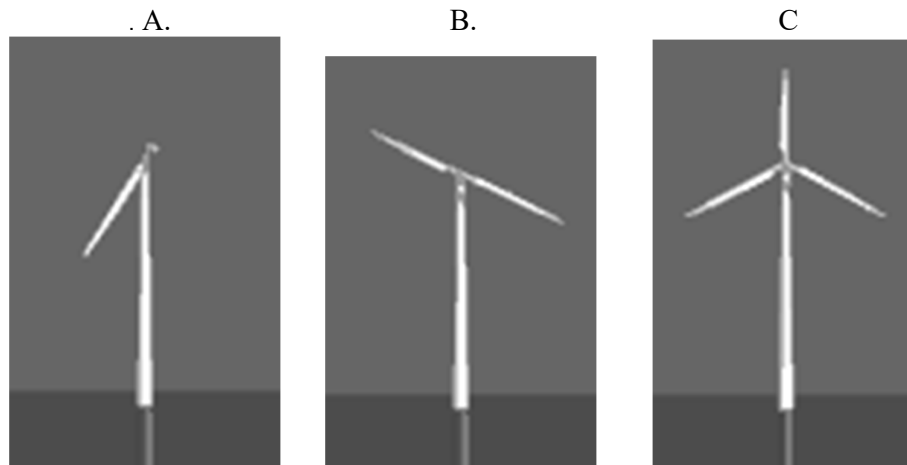


Figure 11. Types of horizontal axis wind turbine(A) single-blade;(B) two-blade and (C) three-blade [12].

- Transmission system

The blades are fixed to the hub which is a metallic structure which creates the possibility of adjusting the rotation of the blades. The rotor hub is fixed to the main shaft, providing support for the rotation mechanism of the blades or for its elements.

o Main shaft and secondary shaft

The main shaft is also known as a low-speed shaft because it rotates at low speeds of 20 to 40 rpm. The gearbox connects the low-speed shaft to the high-speed shaft which conducts and increases the rotational speed. The high-speed shaft drives the electric generator, a synchronous or asynchronous generator (an alternator or an induction generator), which has one or two pairs of poles. It is equipped with a brake disc (safety device) which limits the rotational speed during emergencies, for example, violent wind.

- o **The high-speed shaft** (also known as the secondary shaft or transmission shaft) drives the generator which conducts the movement from the gearbox to the generator. The speed of the shaft, as is the case with an electric generator, is 1200 to 1800 rpm.
- o **The gearbox** connects the low-speed shaft to the high-speed shaft and increases the rotational speed to the value required by the generator. It is attached to the main shaft.
- o **The brake** is a safety device which is attached directly to the high-speed shaft, between the gearbox and the electrical generator. The rotational speed of the turbine is maintained at a constant speed by adjusting the pitch angle of the blades during high-speed winds rather than triggering the high-speed shaft of the turbine. The brake is used only if the pitch system does not work properly or to lock the turbine into parking mode during maintenance operations.

The following three techniques are used to operate the braking system:

- Service braking – necessary to stop the rotor either manually during repairs and maintenance or automatically in case of wind overspeed.
- Running braking – used in case of abnormal operational conditions, for example, super saturation of the rotor or a voltage drop in the electric grid.
- When the system is not functioning, the turbine is prevented from starting performed by the emergency brake.

- **Control, safety and measuring systems.**

A wind turbine comprises several components which ensure the satisfactory operation of the wind turbine and prevent possible dangerous conditions from arising. The controller measures the following parameters as analogy signals:

- voltage, current and frequency in all three phases
- generator temperature, oil, and bearings
- temperature within the nacelle
- wind direction and speed.

The mechanical brake is a disc brake which is placed on the high-speed shaft of the gearbox. The role of the mechanical brake is to limit shaft speed in case of extreme wind speed. In addition, it is also used to stop rotation during maintenance to ensure the safety of the personnel.

- **Speed control system**

Development of wind turbine technology has expanding significantly over the 25 years preceding this study. Large turbines tend to be designed with variable speed, in-built pitch control and power electronics. On the other hand, small turbines are characterised by low control over both speed and operating systems. Speed control methods may be divided into the following categories:

- Pivot and the rotor orientation control – the rotor axis is controlled by the wind direction when the wind exceeds the limit for which the turbine was designed.
- Blade pitch control – changes the blade pitch angle with the wind velocity in order to regulate the rotor speed.
- Braking device – provides speed control when the wind speed exceeds the safety limit with the blades being locked in a position in order to initiate stop. The turbine must be restarted after the storm has ended.

Figure 12. The distribution of the control methods used in small turbines. Large turbines generally use power electronics in order to control speed.

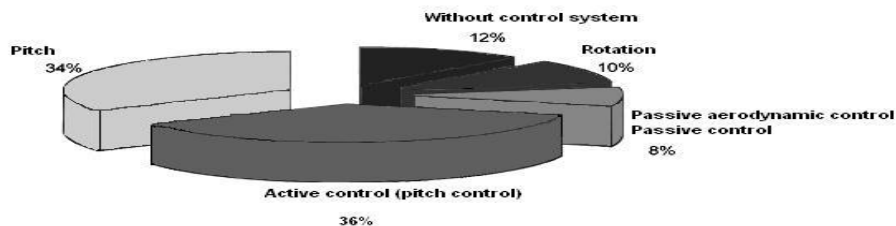


Figure 12. The distribution of speed control methods [12].

1.5.Problem statement

Wind power Potential based on renewable energy resourced to the distribution network may pose a challenge. Wind speed changes continuously and the statistical approach is considered a viable method to estimate its speed and frequency values. In this respect, wind speed probability density function plays an important role in electric power generation applications of wind turbines. An important requirement for effective wind power planning is an accurate estimation of wind speed distribution. Therefore, investigation of wind power generation should be carefully performed in accordance with wind speed probabilistic characters.

1.6.Aims and Objectives.

The aim of the study was to assess the potential of wind power as a renewable source of energy in Cape Town, South Africa. The study was, therefore, structured based on the following research objective:

Objective :

This involves a probabilistic approach of wind energy based on the wind speed frequency for long time intervals. Objective focuses on the direction of assess new techniques and methods for modelling and forecasting wind for the medium and long term. For this purpose, the approach will involve two directions, namely an investigation of main probabilistic distribution, which can be used to model the wind speed frequency and on evaluation of probability density function parameters, based on different wind profiles.

1.7. Importance/benefits of the study

It is anticipated that this research study would provide a new techniques and methods for modelling and forecasting wind sources in distribution systems.

1.8.Delimitation of the study

The scope of this project is limited to assessment of wind power potential for power electric, power generation and integration into the distribution grid only in the region in Cape Town, South Africa.

Chapter 2:

2. Literature Review

2.1. Renewable energy resource

The utilisation of renewable energy in electric power systems is growing rapidly due to environmental concerns as well as the concerns related to the depletion of conventional power generation sources. Wind power is a fast-growing electric generation technology. Wind generation offers enormous benefits to power systems, including an economical form of energy (in comparison to thermal generation), emission reduction, availability to large areas and relatively easy implementation of wind power farms [1]. However, wind generation also presents a series of difficulties to traditional power systems, namely, an uncontrolled level of power generation which is dependant solely on wind availability which may result in the poor predictability of wind generation (irregular fluctuation and intermittence of power generation) [9]. The most significant properties of wind generation include the frequency and magnitude of wind speed.

2.2. Characteristics of wind speed

Wind speed data is measured as average hourly values which are statistically analysed over a one-year period. The probability density distribution is derived from the values obtained with their distributional parameters being evaluated in accordance with acceptable statistical methods. The key factors of wind speed probability density function, height dependence and wind direction all play an important role in the evaluation of wind speed distribution. One of the primary characteristics of wind is that it is extremely variable with its properties varying from one location to another. Wind speed changes continuously. The statistical approach is considered a viable method to estimate the speed and frequency values of wind speed. In this respect, the wind speed probability density function plays an important role in the electric power generation applications of wind turbines.

2.2.1. The wind speed distribution

Several studies on wind energy have been published in scientific literature. These studies propose the use of a variety of probability density functions (e.g., normal, lognormal, gamma, Rayleigh, Weibull etc.) to describe wind speed distributions [9,10,11]. The common conclusion drawn in these studies is that the Weibull distribution with two parameters may be successfully utilised to describe the principal wind speed variation. The Weibull distribution is a two-parameter function, while the Rayleigh distribution uses one parameter only. Generally, analytical approximation uses the Weibull distribution function, obtaining useful information on the wind resource and using only the 2 parameters of the Weibull distribution.

The probability density function (PDF) of Weibull distribution may be expressed as follows:

$$f_w(v) = \frac{k}{c} \left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^k\right] \quad (2.1)$$

while the cumulative distribution function is (CDF):

$$F_W(v) = 1 - \exp\left[-\left(\frac{v}{c}\right)^k\right] \quad (2.2)$$

where: k is the shape parameter [dimensionless].

c is the scale parameter [m/s].

v is the wind speed [m/s].

In some statistical calculations the Weibull distribution function is characterised by a parameter, namely, the location parameter although, in general, it is considered zero in the statistical analyses of the wind speed. This hypothesis means neglecting the interval of the calm wind. In this case, the “bin” of the wind speed interval, $v = 0 \div 0.5 \text{ m/s}$, is associated with the time interval $\Delta T = 0 \text{ hours/years}$ and, thus, the frequency curve passes through the origin of the reference system. However, this hypothesis deforms both the real phenomena and the mean value of the wind speed, especially in respect of the low values of the wind speeds.

2.2.2. Wind speed variation depending on height.

On account of the measurements being taken at heights of 20 m to 120 m above the ground, it is important to consider the height, choice and type of wind turbine that should be installed. In order to complete the requisite database, an estimation of the wind speed variation at every 10 m height is required. Thus, a function is necessary to estimate the wind speed in terms of height which uses equations related to fluid mechanics. the relationship that describes wind speed $v(z)$ variation according to height z .

$$v(z) = \frac{v_f}{k} \left[\ln \frac{z}{z_0} - \xi \left(\frac{z}{L} \right) \right] \quad (2.3)$$

where: v_f – friction velocity; k – Karman constant ($= 0.4$), z_0 – is the surface roughness length and L – the scale factor [2, 3].

The function $\xi \left(\frac{z}{L} \right)$ is determined by the net solar radiation at the site.

This equation is valid on a short-term basis (1 minute) in relation to a monthly or yearly wind speed average. The surface roughness length z_0 will depend on the size of the space between the elements. Typical values are in the region of 0.01 cm for water, 1 cm for grass, 25 cm for tall grass and 1m to 4 m for trees [4]. In practice, however, z_0 is not precisely determined but is determined by measuring wind. This is valid for the dynamic friction v_f , which is related to both the air density and $\xi \left(\frac{z}{L} \right)$. The most common formula is related to the wind speed and is illustrated by (2.4):

$$\frac{v(z_2)}{v(z_1)} = \left(\frac{z_2}{z_1} \right)^\alpha \quad (2.4)$$

where z_1 – the height of measurement, approximately 10 m, z_2 – the height at which the wind speed is needed to determine an estimate and the α parameter, which is empirically determined. The aim of the equation is to observe the information on wind speed at 10 m to 150 m if there are no clear limits between the two. The exponent α varies with height, season of the year, nature of terrain, wind speed and temperature [5].

Several models have been proposed for the variation of α [6], using the linear logarithmic plot, as depicted in Figure 2.1.

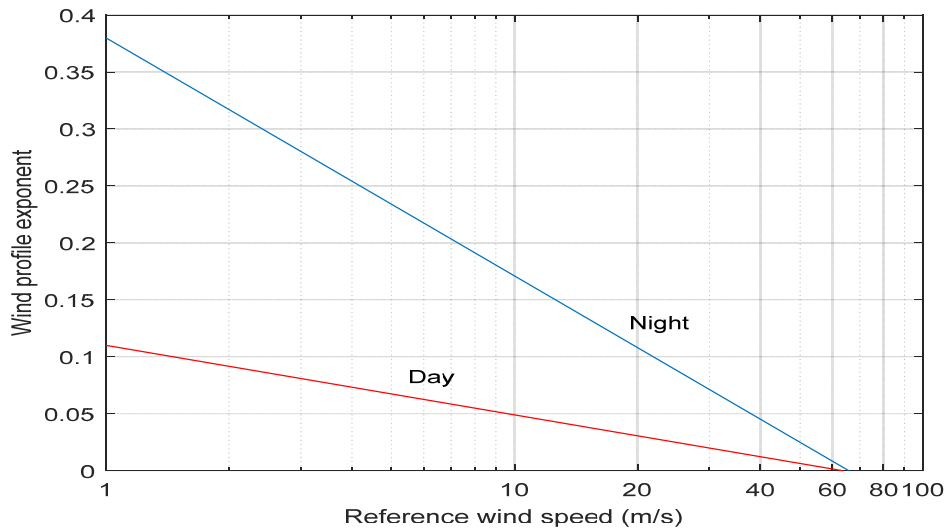


Figure 2.1. The variation of wind profile exponent α with wind speed $u(z_1) = v(z_1)$, [6].

A representative plot for a day and another plot for night are shown in Figure 2.1. Each plot varies with wind speed, according to the following equation (2.5):

$$\alpha = a - b \log_{10} v(z_1) \quad (2.5)$$

The coefficients a and b may be determined by a linear regression. Typical values are $a = 0.11$ and $b = 0.061$ during the day and $a = 0.38$ and $b = 0.209$ at night. Only one figure may be generated for each season of the year. However, temperature, wind direction and height, for example, may be plotted on separate figures. The average value of α is determined by multiple measurements for a more accurate approximation, namely, $\alpha = 1/7$. This average value should be used only if data from the specific location is not available.

The most commonly expressions for wind speed variation with the height, applies the law of wind profile power (based on the ground friction coefficient) and described by the following equation:

$$v(z)/v(z_r) = (z/z_r)^\alpha \quad (2.6)$$

In the above equation, $v(z)$ and $v(z_r)$ are the wind speeds at desired z and registered z_r heights, and α is the friction coefficient, which depends on both surface roughness and atmospheric stability [7]. Numerically, the friction coefficient ranges between 0.05 for smooth terrains and 0.5 for rough terrains, with the most frequently adopted value being 0.14. In accordance with equation (2.6), between the parameters of the Weibull distribution for different heights, there are the following relationships.

$$c(z) = c(z_r) \cdot (z/z_r)^\alpha; k(z) = k(z_r) \quad (2.7)$$

The above relationship is based on the expression of the Weibull distribution that is amended in accordance with [8] in respect of the wind speed and height relationship.

As per equation (2.7) above, shape parameter is a fixed property of the wind profile whereas the scale parameter may be modified in a narrow range by an adjustment of the desired height.

2.2.3. Wind speed statistics

The fact that the winds speed is constantly changing means that the wind speed is calculated using statistical methods. A widely used statistical method is the arithmetic mean. If there is a set of values v_i , such as the measured wind speeds, the mean of the set is defined as:

$$\bar{v} = \sum_{i=1}^n \frac{v_i}{n} \quad (2.8)$$

where: n is the number of measured values.

v_i is the set of the measured values.

The median is sometimes mentioned in the literature as a further possible method. If n has an odd value, the median is the middle number, and all numbers are arranged in order of size. On the other hand, if n has an even value, the numbers will be ranked in ascending order and the median is halfway between the middle two numbers. The arithmetic mean may be misinterpreted as a median, which would suggest that most values are either higher or lower than would have been preferable. However, when we consider a sampling interval in which it is not possible to arrange the data in an arithmetic progression, then both the median and the arithmetic mean may differ significantly. We may be interested in finding the differences for each deviation or each value from the mean, then arranging these deviations and finding their average value. In such a case the mean $v_i - \bar{v}$ becomes zero, which is not significant. Thus, positive values will be obtained by squaring each deviation. The variance of data may be defined as:

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^n (v_i - \bar{v})^2 \quad (2.9)$$

where the difference $n - 1$ is used more often than n .

Both the mean and the standard deviation vary in time, from one location to another. This may be of interest in order to arrange in ascending order and then to be able to choose the smallest, the median and the largest values. The smallest and the largest terms are rarely used in statistics because of the possibilities that such values may be widely separated from the rest of the values. In practice, the wind speed determined in percentage is often used, which allows us to consider cases where data may occasionally be chosen and determined as the wind speed probabilities.

$$p(v_i) = \frac{m_i}{n} \quad (2.10)$$

where the sum of all probabilities is 1.

We will define a cumulative distribution function of the measured wind speed probability $F(v_i)$, which will be less than or equal to v_i

$$F(v_i) = \sum_{j=1}^i p(v_j) \quad (2.11)$$

The cumulative distribution function may be:

$$F(-\infty) = 0, F(\infty) = 1.$$

We occasionally need the probability of the wind speed to be between certain limits or above a certain value and, in this case, $P(v_a \leq v_i \leq v_b)$ may be an extremely high number as defined:

$$P(v_a \leq v_i \leq v_b) = \sum_{j=a}^b p(v_j) \quad (2.12)$$

For theoretical reasons it is more convenient to model the wind speed variation using a continuous curve rather than having a table of given values. Accordingly, the probability of $p(v_j)$ value becomes the density function $f(v)$ while the density function $f(v)$ represents the probability that the wind speed is centred on v_i and the interval is 1 m/s. The discrete probabilities $p(v_i)$ have the same meaning if they were calculated using data which had been collected. The integral of the function over the whole interval is 1.

$$\int_0^{\infty} f(v)dv = 1 \quad (2.13)$$

The integration variable v is just a random variable representing the wind speed for the purpose of integration. When wind speed is considered as a continuous random variable, the cumulative distribution function has the properties $F(0) = 0$ and $F(\infty) = 1$. The function $F(0)$ value is not important if it is zero because wind speed may be zero, which is included in $F(0)$

In order to calculate wind speed, we usually also need the derived function $f(v)$, given below:

$$f(v) = \frac{dF(v)}{dv} \quad (2.14)$$

The mean value of the density function $f(v)$ is given by equation (2.2).

$$M[v] = \int_0^{\infty} vf(v)dv \quad (2.15)$$

Its variance is:

$$D[v] = \sigma^2 = \int_0^{\infty} (v - M(v))^2 f(v)dv \quad (2.16)$$

These equations are used to compute theoretical values of the mean and for a variety of statistical functions that are used in different applications.

Chapter 3

3. Theoretical Background

3.1. Wind Power Transmission and Generator Efficiencies

The shaft power output characteristics are generally not used directly but are coupled to a load through the transmission or other equipment. The load may be a pump, compressor, grinder, or electrical generator. In order to present a schema of the system, the load is considered as an electrical generator. Figure 3.1. starting with wind power P_w . Once power passes through the turbine, mechanical power P_m is produced at the turbine angular velocity ω_m which is supplied to the transmission. The transmission output power P_t is given by the type of turbine generator as well as transmission efficiency η_m .

$$P_t = \eta_m P_m \quad W \quad (3.1)$$

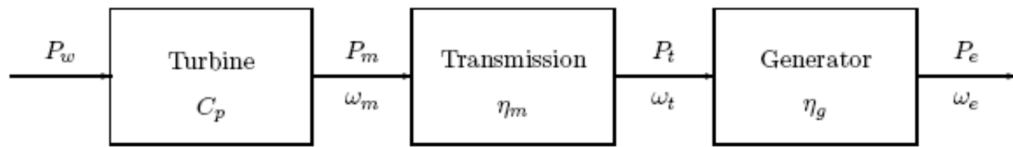


Figure 3.1. Wind electric system [40]. $P_e = P(v)$

Similarly, the generator output power P_e is given by the transmission output power and the generator efficiency η_g as follows:

$$P_e = \eta_g P_t \quad W \quad (3.2)$$

The equations above may be expressed as a single equation which represents the electrical power generated by the wind variation as:

$$P_e = C_p \eta_m \eta_g P_w \quad W \quad (3.3)$$

At rated wind speed, the electrical power output may be expressed as:

$$P_{eR} = C_{pR} \eta_{mR} \eta_{gR} \frac{\rho}{2} A v_n^3 \quad W \quad (3.4)$$

where C_{pR} is the coefficient of performance at the rated wind speed, $(v_n) \eta_{mR}$ is the transmission efficiency at rated power, η_{gR} is the generator efficiency at rated power, ρ is the air density and A is the turbine area. The quantity $C_{pR} \eta_{mR} \eta_{gR}$ is the total efficiency of the turbine with the symbol η_0 being used for this value:

$$\eta_0 = C_{pR}\eta_{mR}\eta_{gR} \quad (3.5)$$

The total efficiency at lower wind speeds must be known in order to determine the energy production of a turbine and, therefore, individual efficiencies must be determined. As the variation of C_p has already been examined, at this stage the generator and mechanical efficiencies should be considered. Transmission losses occur with each piece of equipment. At constant rotational speeds, the losses do not vary significantly in terms of the transmitted torque.

It is also reasonable to assume that the transmission loss is a fixed percentage with low values of the rated power as follows:

$$\eta_m = \frac{P_t}{P_m} = \frac{P_m - (0.02)qP_{mR}}{P_m} \quad (3.6)$$

where q is the transmission efficiency and P_{mR} – is the rated turbine shaft power.

This equation is presented in Figure 3.2. for one, two and three stages. The transmission efficiency is not effective for low power inputs. It is, therefore, preferable to select turbines which operate with transmission above the knee of the curve, as shown in Figure 3.2. below:

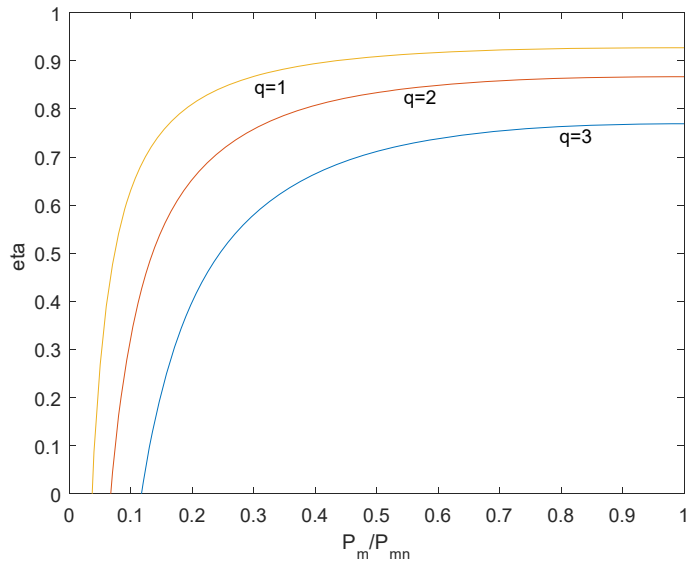


Figure 3.2. Transmission for 1, 2 and 3 stages with 2% loss per stage [13]

The generator losses may be specified in three categories, namely, hysteresis, eddy current losses and copper losses. Eddy current losses are determined by the operating voltage, frequency and windage, thus generating friction losses which vary with rotational speed. Copper losses vary with the square of the output current. These are obtained from the system losses which occur with hysteresis and eddy currents. Windage and bearing friction are considered fixed while copper losses are regarded as variable.

Losses are proportional to the surface area of the rotor while rated electrical power is proportional to volume. High efficiency of generators is possible up to loads of 0.85 for a 2 kW rating, 0.9 for a 20 kW rating, 0.93 for a 200 kW rating and 0.96 for a 2 MW rating with

efficiency continuing to ascend to 0.98. This efficiency and power variation depend on the efficiency variation of the turbine as well as transmission. The effects of rated power and actual power on generator efficiency may be combined as demonstrated in equation (3.7). [15]:

$$\eta_g = \frac{X - (0.5)Y(1-Y)(X^2 + 1)}{X} \quad (3.7)$$

where the parameters X and Y are given by:

$$X = \frac{P_t}{P_{tR}} \quad Y = 0.05 \left(\frac{10^6}{P_{eR}} \right)^{0.215} \quad (3.8)$$

where P_{tR} is the rated mechanical power input and P_{eR} the rated electrical power output.

Equations (3.4) is plotted in Figure 3.3. below for three rated generator sizes, including 20 kW, 200 kW and 2000 kW respectively. The curves are similar in shape to the transmission efficiency curves depicted in Figure 3.5. The output power of the electrical generator may be determined from C_p , η_m and η_g multiplying these values to determine the overall efficiency. A generator may supply 10 or 20% greater power than its rating for periods up to one hour if allowed to cool subsequent to this period. Wind is variable for long periods and any overload will be compensated for by periods of lighter load. Therefore, the average power delivered in one hour cannot exceed the rated power.

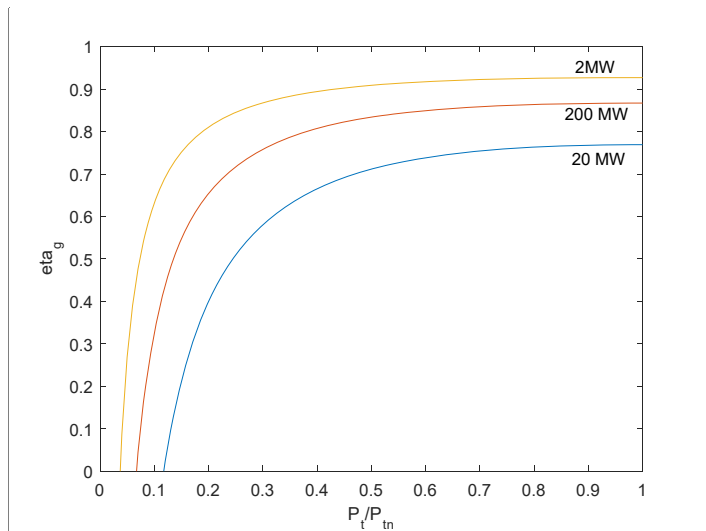


Figure 3.3. Generator efficiency for three different generators [13]

Shaft power and electrical power output increase almost linearly with wind speed up to maximum values. Wind power increases with the cube of wind speed which would seem to be appropriate as low efficiencies at low wind speeds are responsible for alignment of the power output curve. In Figure 3.3. the electrical power output rises above zero at a wind speed of approximately 5 m/s. This wind speed, at which electrical power production commences, is

referred to as the cut-in speed v_{in} . The turbine develops enough mechanical power to rotate itself at lower speeds with a cut in speed. Figure 3.3. presents a turbine at maximum operation conditions.

3.2. Output power of a wind turbine

The fraction of power extracted from the power in the wind by a practical wind turbine is usually denoted by the symbol C_p , which stands for the coefficient of performance. The actual mechanical power output may be expressed as:

$$P_m = C_p \left(\frac{1}{2} \rho A v^3 \right) = C_p P_w \quad W \quad (3.9)$$

The coefficient of performance is not a constant but varies with the wind speed, the rotational speed of the turbine, and the turbine blade parameters such as the angle of attack and pitch angle. The turbines operate with fixed pitch while the large horizontal axis turbines usually operate with variable pitch. The pitch is varied to hold C_p at its largest possible value up to the rated speed v_R of the turbine and is then varied to reduce C_p while P_w continues to increase with wind speed in order to maintain the output power at its rated value, P_{mR} .

This is depicted in Figure 3.4. below:

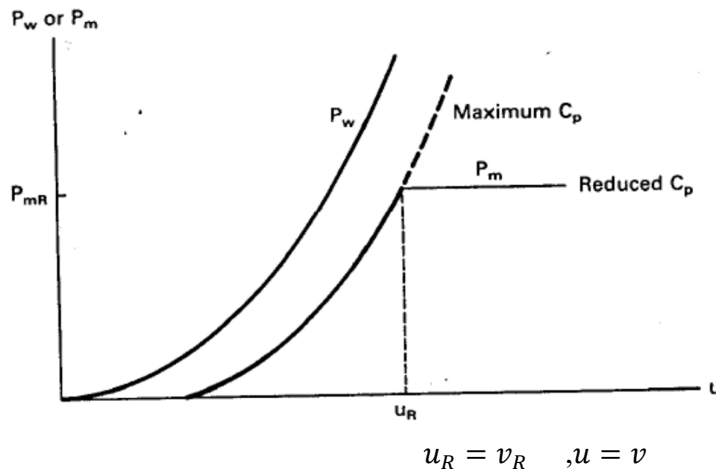


Figure 3.4. Shaft power output of a variable pitch turbine[37,38].

As it is not practical to hold C_p constant with pitch control because of manufacturing and control limitations, it will vary with wind speed even for a fixed rotational speed, variable pitch blade. A variation of C_p versus u is illustrated in Figure 3.5. The turbine starts producing power at a hub height wind speed of 6.3 m/s and a C_p of approximately 0.28 . A maximum C_p of 0.41 , defined as C_{pm} , occurs at 9 m/s . Designing the blades to have a maximum coefficient of performance below the rated wind speed helps to maximise the energy production of the turbine. The rated wind speed is 12.3 m/s at hub height while C_p has dropped to approximately 0.36 at this wind speed . The coefficient of performance at rated wind speed may be expressed as C_{pR} . Two curves for C_p are shown in Figure 3.5. for wind speeds above the rated wind speed with the upper curves showing the capability of the rotor and the lower curve showing C_p under actual operating conditions. The turbine is shut down at 20 m/s to

prevent damage from such high winds while the actual C_p is well under 0.1 when this wind speed is reached.

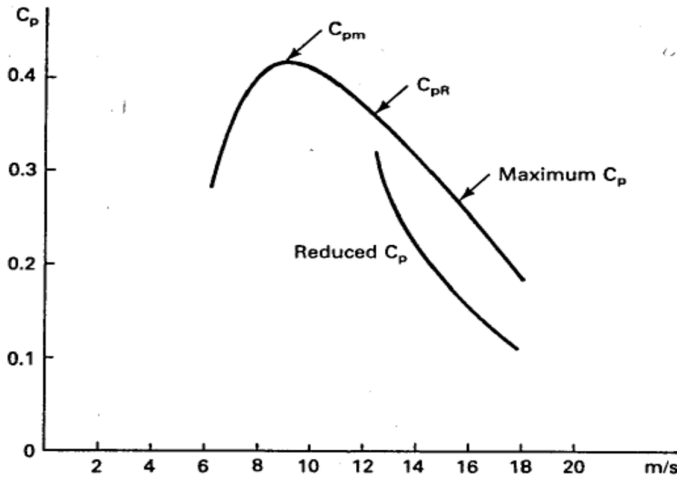


Figure 3.5. : Coefficient of performance versus wind speed [30,31].

The curve shown in Figure 3.5 is valid for one rotational speed only, in this case 17.5 *revolutions per minute* (r/min). When the rotational speed is changed, $r\omega_m$ changes and causes the angle of attack to change. This in turn changes C_p at a given wind speed. It is often convenient for design purposes to have a single curve for C_p , from which the effects of changing either rotational speed or wind speed can be determined. This means that the rotational speed and the wind speed must, somehow, be combined into a single variable before such a single curve may be drawn. Experiments show that this single variable is the ratio of the turbine tip speed $r\omega_m$ to the wind speed v . This tip speed ratio is defined as:

$$\lambda = \frac{r\omega_m}{v} \quad (3.10)$$

Where,

r_m is the maximum radius of the rotating turbine in m;

ω_m is the mechanical angular velocity of the turbine in rad/s;

v is the undisturbed wind speed in m/s.

The angular velocity ω_m is determined from the rotational speed $n(r/min)$ as expressed in the following equation:

$$\omega_m = \frac{2\pi n}{60} \text{ rad/s} \quad (3.11)$$

3.3. Electric generator for wind turbines

The electric generator converts the rotational mechanical energy of the high-speed shaft of the wind turbine to electric energy. The rotor windings rotate into the electromagnetic field generated by the stator and as a result, energy is produced by the windings. Electric generators which supply direct current (DC) in household applications for small wind turbines are available as well as a large variety of alternating current generators (AC). At the time of the study there were two types of generators for wind turbines with both constant speed and variable speed.

The generators may be asynchronous with a short-circuit rotor, wound rotor, or synchronous generators (SG) with separated excitation or permanent magnets. Some wind turbines with synchronous generators use electromagnets mounted on the rotor. These are supplied with direct current from the electric grid. Most turbines use generators with either 4 or 6 poles. The advantages of using these high-speed generators include the costs and small dimensions. The maximum torque a generator can support depends on the rotor volume. In order to achieve the required power, a generator which is slow but massive or one which is small with high-speed is available.

Asynchronous generators are classified as either one-phase or poly-phase generators. A single-phase generator is limited to 25 kW and produces alternating current at an appropriate voltage while poly-phase generators produce two or more alternating currents. Alternating current generators (AC) may be brushed or brushless, depending on the method used to transfer direct current to the generator field. The choice of electric generators used for wind turbines is usually based on the power which has been installed, site of the turbine and type of electrical load. The most common wind generators may be classified as brushless generators (BLDC), permanent magnet synchronous generators (PMSGs), induction generators (asynchronous) or special construction types of synchronous generators. Generally, BLDC generators are used for small wind turbines while asynchronous generators and synchronous generators are used for high-power turbines.

- Brushless generators – BLDC

During the 15 to 20 years preceding this study the advances in respect of BLDC generators has had led their being widely utilised in turbines with BLDC generators being widely used for small-power turbines (up to 15 kW) due to their simplicity of control, compactness, short cooling period, reduced noise, and increased reliability. The recent introduction of high-density energy magnets (rare-earth magnets, such as $Nd_2Fe_{14}B$ – neodymium-iron-boron) has enabled magnetic high flux densities. The absence of brushes and slip rings eliminates the need for regular maintenance and reduces any potential defect in these elements. The high frequency of the electric power converter eliminates any harmonics or distortions to a voltage or current waveform [14,15]. In view of its mechanical performance, the BLDC generator is characterised by powers of up to 15% higher than permanent magnet synchronous generators (PMSG). Brushless direct current generators (BLDC) are used primarily by low power wind turbines to generate energy. Table 3.1 presents the technical characteristics of a Skystream small wind turbine of 3.7 kW, equipped with a BLDC generator [16].

Table 3.1 Skystream technical specifications

Rated power	1.8 kW (2.4 kW max.)	Alternator	Brushless
Weight	170 IBM (77) kg	Yaw control	Passive
Rotor diameter	12 ft (3.72 m)	Grid feed	120/240 VAC50-60 Hz
Swept area	115,7 ft^2 (10.87 m^2)	Braking system	Regulation of electronic measuring
Rate speed	50-370 rpm	Rated wind speed	9 m/s
Wind speed value	9.7 - 63 m / s	Survival wind speed	63 m / s

The wind power speed characteristic is plotted in Figure 3.6.

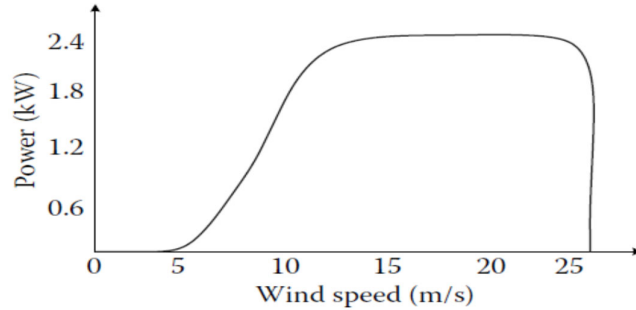


Figure 3.6. Performance graph of a Skystream turbine with a BLDC generator [16]

Figure 3.7 depicts a block diagram of the torque control of a wind turbine equipped with the BLDC type generator.

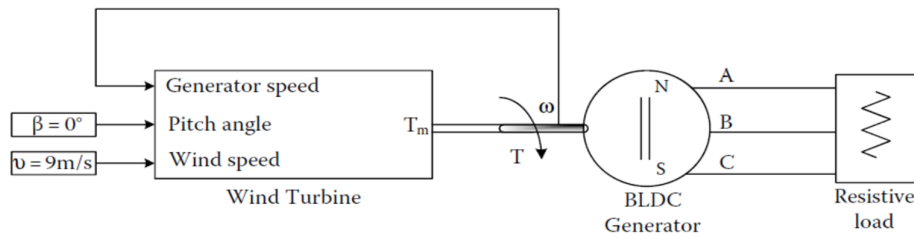


Figure 3.7. Block diagram of a wind turbine with a BLDC generator [16]

- **Permanent magnet synchronous generator**

Permanent magnet synchronous generators (PMSG) are suitable for wind power generators. This type of generator may be used in applications characterised by constant and/or variable wind speeds.

The generator shaft speed in radians per second is given in figure 3.8.

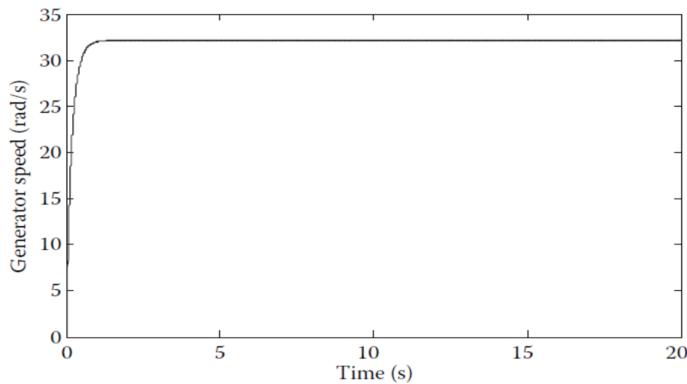


Figure 3.8. BLDC generator speed [16].

Figure 3.9. presents a schema where a permanent magnet generator (PMSG) is connected to the grid through both a rectifier and a three-phase power inverter. The power inverter controls the direct current voltage value and the input power factor value, respectively. However, a disadvantage of this system is that a PMSG is equipped with diode rectifiers which increase the current amplitudes and losses.

In addition, switching the diodes decreases the voltage by 5 to 10%. The grid side converter, interconnected with the network, may be used to control both the active and reactive power, which feeds through the network. The automatic voltage regulator (AVR) collects information on the wind turbine speed, DC voltage and current values.

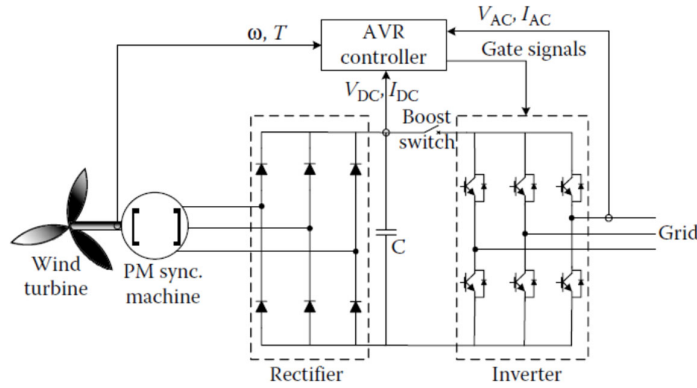


Figure 3.9. Block diagram of PMSG with rectifier and inverter [39].

Figure 3.10. illustrates a PMSG connected to a rectifier, where the rectifier is placed between the generator and the mean circuit and the inverter is connected to the grid. In this case, the converters are back-to-back [31]. This system allows their utilisation as an interface between the stator windings of the PMSG and the grid [17]. The turbine operates at its maximum efficiency while the variable speed of the PMSG may be controlled by a power converter.

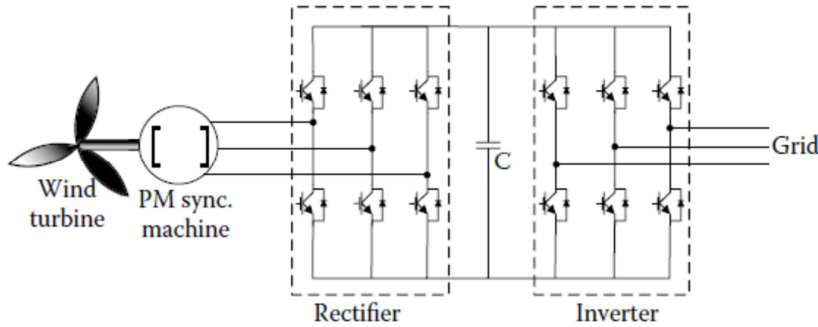


Figure 3.10. Block diagram of a PMSG with a back-to-back inverter [31].

As an alternative, a diode bridge may be used for DC/DC conversion and a voltage source inverter (VSI) may be used as an electronic interface between the generator and the grid [18]

–see Figure 3.11.

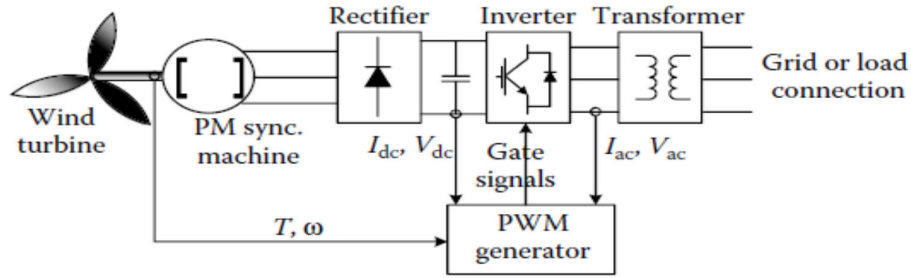


Figure 3.11. Block diagram of a simulated PMSG on a wind turbine [18].

In the main permanent magnet generators must be connected to the network by frequency converters as they operate on variable speed. This type of generator is available on the market as axial flux and transverse flux generators [19].

Axial flux generators are widely used in wind turbines which operate at low speeds. An axial flux permanent magnet generator comprises a magnetic core of the stator coil in a winding steel strip form and with permanent magnet disk shape rotors in the axial direction of the same axis, on both sides of the stator. Axial flux generators with slotted surface permanent magnets and opposite poles of the same or different magnetic polarities have the advantage of introducing static windings into the slots, air gap decreases, half the permanent magnet thickness and higher output.

A **radial flux generator** comprises an intertwined U-shaped laminated stator that leads to an optimum width magnet/pole pitch of 0.7/0.8. Levelling the generator and turbine rotor speed results in the determination of the number of pole pairs p and speed i , based on the power of the wind turbine.

The technical specifications of a Zephyros wind turbine with permanent magnet generators are presented in Table 3.2 [20].

Table 3.2. Zephyros wind turbine technical specifications

Rotor diameter	70,65 m
Rotor speed	Variable, nominal 23,5 rpm
Nominal power	2,0MW
Transmission	DD generator, single main bearing
Rated wind speed	13 m/s
Cut-in / cut-out wind speed	3 – 25m/s

Figure 3.12. presents the output power variation depending on wind speed of a Zephyros wind turbine.

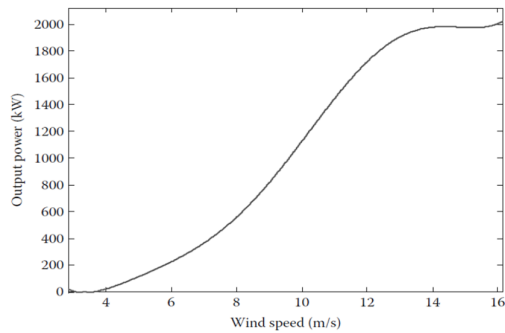


Figure 3.12. Wind speed versus output power curve of the Zephyros wind turbine [20].

- Induction machine

Three-phase, asynchronous, induction generators are generally used as electric motors. However, they may also be efficiently used as generators in circumstances where there is no access to the grid [20]. The majority of induction generators are used for wind turbines. It is important to note that electric power generators require an external reactive power source which will propel the excitation winding of the induction generator. However, this is not required for synchronous generators in similar applications [31-36]. The self-excitation phenomenon of an induction generator is easily performed using the magnetic saturation of the rotor. If the induction generator is driven by an engine, the residual magnetism from the rotor generates a small voltage which forms a capacitive current. In addition, it provides equilibrium for the increasing voltage. In order to cover the reactive power required for excitation windings, a static volt-Ampere-reactive (VAR) compensator (reactors or capacitors/condensers) is used [21, 22, 23].

- Doubly fed induction machine (**DFIG**)

The variable speed wind-turbine represents a modern concept that usually applies to a doubly fed induction machine (DFIG). This has the advantage of ensuring the efficient exploitation of wind power. The mechanical components of a DFIG wind turbine have a compact construction (lighter nacelle, smaller or absent gearbox, taller tower), together with more complex electric and electronic construction, which requires driving control algorithms. The mechanical rotational speed of the machine may be controlled by adjusting the current frequency to the rotor.

The two types of wind turbines are presented in Figure 3.13.

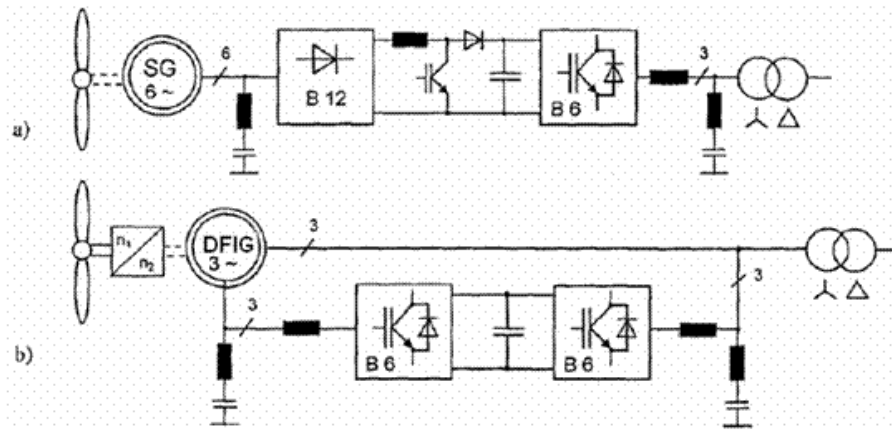


Figure 3.13. Wind turbines types a) synchronous generator – SG b) doubly fed induction machine – DFIG [40].

An efficient variable speed wind turbine uses an induction generator (DFIG – doubly fed induction generator), characterised by increasing quality, efficiency, and controllability in the wind turbine. Figure 3.14. illustrates the topology of a DFIG with AC/DC and DC/AC converters [24, 25]. These converters are placed in a four-quadrant AC/AC, using isolated gate bipolar transistors (IGBT) connected to the rotor windings. The speed and torque of the wound rotor induction generator may be controlled by regulating voltages from both the rotor and stator windings. The advantages of the DFIG include the following [28, 29]:

1. Main power is transferred through the stator windings of the generator which are connected directly to the grid. Approximately 65 to 75% of the total power is transmitted through the stator windings while the remaining 25% of the total power is transmitted by the rotor windings and is drawn out through the converters. Since the inverter rating is 25% of the total system power, the inverter cost and size are reduced.
2. While the generator losses are similar in both topologies (direct-in-line and DFIG), the inverter losses may be reduced from 3% to 0.75% as the inverter transfers 25% of the total power only. Thus, it is possible to obtain an efficiency improvement of approximately 2% to 3%.
3. DFIG topology offers control of the generator active and reactive power [26,27].

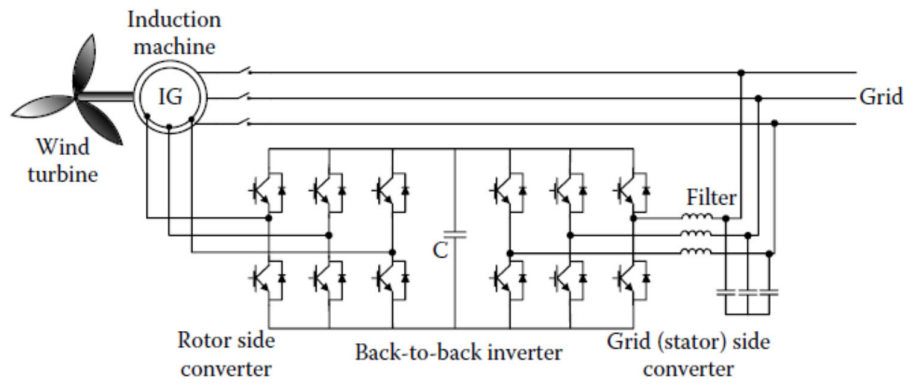


Figure 3.14. DFIG generator topology [41-45].

Chapter 4

4. Methodology

4.1. Random character modelling of wind speed

The wind speed probability density function plays an important role in electric power generation applications from wind sources. There are numerous studies cited in existing literature on wind energy which aimed to use a variety of probability functions (e.g., normal, lognormal, gamma, Rayleigh, Weibull etc) to describe the wind speed distribution [9,10,11]. There are, therefore, several density functions which are used to describe the frequency function with the most common being the Weibull and Rayleigh distributions. The Weibull distribution is a two-parameter function, while the Rayleigh distribution uses one parameter only. Generally, analytical approximation uses the Weibull distribution function, obtaining useful information on the wind resource and using only the 2 parameters of the Weibull distribution.

The probability density function (PDF) of Weibull distribution may be expressed as follows:

$$f_w(v) = \frac{k}{c} \left(\frac{v}{c}\right)^{k-1} \exp \left[- \left(\frac{v}{c}\right)^k \right] \quad (4.1)$$

while the cumulative distribution function is (CDF):

$$F_W(v) = 1 - \exp \left[- \left(\frac{v}{c}\right)^k \right] \quad (4.2)$$

where: k is the shape parameter [dimensionless];

c is the scale parameter [m/s];

v is the wind speed [m/s];

In some statistical calculations the Weibull distribution function is characterised by a parameter, namely, the location parameter although, in general, it is considered zero in the statistical analyses of the wind speed. This hypothesis means neglecting the interval of the calm wind. In this case, the “bin” of the wind speed interval, $v = 0 \div 0.5 \text{ m/s}$, is associated with the time interval $\Delta T = 0 \text{ hours/years}$ and, thus, the frequency curve passes through the origin of the reference system. However, this hypothesis deforms both the real phenomena and the mean value of the wind speed, especially in respect of the low values of the wind speeds.

4.2. Methods of parameter estimation

The accurate estimation of wind speed distribution is the most important requirement for both effective wind power planning and for the operation of a power system. It is essential that the assessment of wind generation is carefully investigated in accordance with the wind speed probabilistic character. The wind speed distribution may be determined only once its parameters have been established. The estimate parameters of the Weibull distribution may be determined using various estimation methods. Some methods are graphical, and some are

analytical [51,52,53]. However, each method has a criterion that produces an approximation of the most suitable method for the situation encountered in the analysis.

The most common analytical methods include the maximum likelihood estimator (MLE), the method of moments (MOM) and the least squares method (LSM). Since the estimates parameters play a major role in development of an electric power wind generator, it is important that different estimation methods are compared to fit the Weibull distribution parameters from the wind speed database. The techniques used to estimate the parameters of the Weibull distribution function are described in greater detail below.

4.2.1. The maximum likelihood estimator

The first method used is the maximum likelihood estimation (MLE). It is usually considered to be more robust while its results are more accurate. The basic principle underlying the MLE is to find the most likely values of the parameters for a given distribution that would best describe the data. Consider the probability density function (PDF):

$$f(x; _1, _2, \dots, _n) \quad (4.3)$$

where $_1, _2, \dots, _n$ are the distribution parameters that must be estimated.

After drawing k independent samples out of the distribution, we may search an estimation of the values of $_1, _2, \dots, _n$.

The maximum likelihood estimator (MLE) is an analytical method which is widely applied in engineering and mathematics problems.

$$L(c, k) = \prod_{i=1}^n f(v_i) \prod_{i=1}^n \frac{k}{c} \left(\frac{v_i}{c}\right)^{k-1} \exp\left[-\left(\frac{v_i}{c}\right)^k\right] \quad (4.4)$$

The final method of parameters evaluation involves determining the parameters of the partial derivatives of the likelihood function with respect to the parameters, setting the resulting equations equal to zero:

$$\frac{\partial \ln(L)}{\partial k} = \frac{n}{k} + \sum_{i=1}^n \ln v_i^k - \frac{1}{\alpha} \sum_{i=1}^n v_i^k \cdot \ln v_i^k = 0$$

$$\text{And} \quad (4.5)$$

$$\frac{\partial \ln(L)}{\partial c} = -\frac{n}{c} - \frac{1}{c^2} \sum_{i=1}^n v_i^k = 0$$

The values of c and k result from the simultaneous solving of both equations.

4.2.2. The method of moments (MOM)

The method of moments (MOM) is a further analytical method that establishes the distribution parameters. If wind speed data is known, the unknown parameters that depend on the Weibull distribution parameters are equalised with two empirical parameters, as in the following equations:

$$\bar{v} = \sum_{i=1}^n \frac{v_i}{n} \quad \sigma^2 = \frac{1}{n} \sum_{i=1}^n (v_i - \bar{v})^2 \quad (4.6)$$

The analytical expression of the mean and the variance of the Weibull distributions may be expressed as follows:

$$M(v) = \alpha\Gamma(1 + 1/k)$$

and

$$D^2(v) = \alpha^2 \cdot \left[\Gamma(1 + 2/k) - (\Gamma(1 + 1/k))^2 \right] \quad (4.7)$$

where $\Gamma ()$ is the gamma function.

It is possible to obtain the k parameter from the coefficient of variation and, after that, the c parameter may be established based on the first expression from the equation above.

4.2.3. Least squares method (LSM)

The least-squares method (LSM) is widely used in engineering problems to estimate the Weibull parameters. The method provides a linear relation between the two parameters with the two logarithms of the Weibull cumulative distribution function as the starting point – see below:

$$f(x) = \frac{kx^{k-1}}{\theta^k} \exp\left(-\left(\frac{x}{\theta}\right)^k\right), \quad x \geq 0 \quad (4.8)$$

This relationship represents a straight line, expressed as:

$$Y = \ln \ln \left[1 - \frac{1}{F_w(v)} \right], \quad X = \ln (v),$$

$$a = k \text{ and } b = -k \ln(c) \quad (4.9)$$

where X and Y are variables

a is the slope, and b is the intercept of the line on the y axis.

Using the simple linear regression, the c and k parameters result from the coefficient of polynomial linear fitting.

4.3. Distribution hypothesis tests

The estimation of statistical parameters depends on two random variables, one theoretical v and one empirical v' , which have different distributions. In this case, it must be shown whether there is a dependence or a difference between the two distributions, refer to the parameter value or to the unknown distribution function form.

In this case there are several comparison criteria, for example:

- Criteria comparing different characteristics (means, dispersion);
- Criteria comparing two frequency types;

- Graphical criteria.

Criteria that compare different characteristics as means, dispersions or quantiles include the following:

4.3.1. Moment criterion

The repartition moments depend on unknown parameters which equal the empirical moments:
Moment of order k:

$$m(v_k) = \frac{1}{n} \cdot \sum v_j^k \cdot f(v_k) \int_{-\infty}^{\infty} v^k dF_n(v) = \int_{-\infty}^{\infty} v^k \phi(v) dv \quad (4.10)$$

Centred empirical moment:

$$\mu_k = \frac{1}{n} \cdot \sum_1^n (v_j - \bar{v})^k \cdot f(v_k) \quad (4.11)$$

where

$$f(v_k) = p_k = \frac{n_k}{N} \quad N = \sum_1^n n_i \quad (4.12)$$

4.3.2. Quantile criterion

In this case there is the possibility that this criterion may be used to determine a variety of parameters, thus making it possible to calculate more quantiles.

The quantile of order n represents the real solutions of the equation.

$$F(v) = \frac{i}{n} \quad (4.13)$$

$i = 1, 2, \dots, n-1$

The quantiles divide the area of the distribution curve into equal parts.

It appears that the time of occurrence of the first failure depends on the time of failure if it is distributed as a function of normal asymmetric:

$$q_i = \frac{l_i}{N} \quad (4.14)$$

The quantiles are usually tabulated for various distributions.

4.3.3. Frequency criterion – Test χ^2

The absolute and global deviations are compared and

$$v_i - v_i' \quad (4.15)$$

If the deviations are small, a goodness of fit may be accepted.

It is calculated using the equation:

$$\chi^2 = \sum \frac{(v_i - v_i')}{v_i} \quad (4.16)$$

The following probability is:

$$P(\chi^2 > \chi_0^2) = \delta \quad (4.17)$$

Thus, resulting in: $\chi^2 < \chi_0^2$ or $\chi^2 > \chi_0^2$ where $\alpha = 1 - \delta$ is the confidence coefficient.

4.4. Estimation of the parameters of a wind turbine

4.4.1. Modelling of the output power generated by a wind turbine

The aim in this section is to determine the average power generated by a wind turbine based on the parameters of the wind speed distribution function, the operational parameters (v_{in}), (v_n), (v_{ie}) of a wind turbine to rated power $P_n = 1$.

These curves are either given by the turbine manufacturers or they are graphically represented using the output power data of the turbine and the wind speed from the studied area.

The power curve of a wind turbine is characterised by the following three speeds, namely, cut-in speed (v_{in}), rated speed (v_n) and cut-off speed (v_{ie}) – see Figure 4.1.

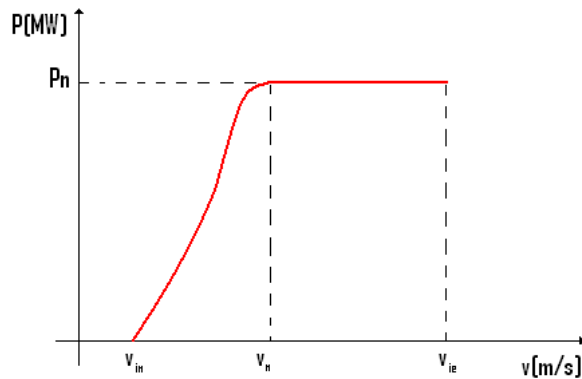


Figure 4.1. Power curve – wind speed [48].

The literature mentions several models which may be used to compute power and the capacity factor. However, two models only are discussed below:

The model usually determines the average power of a wind turbine based on wind speed while the power curve depends on both the output electrical power and the wind speed. The wind turbine starts generating power when the speed exceeds the cut-in speed (v_{in}). The output power increases with the wind speed between the cut-in speed v_{in} and the rated speed (v_n) after which the output power remains constant at the rated power level P_n . The cut-off wind speed v_{ie} is the maximum wind speed at which the turbine allows power generation, being usually limited by safety constraints.

1. The mathematical model is described as follows [45,46,47]:

$$P(v) = \begin{cases} P_n \cdot (A_2 v^2 + A_1 v + A_0) & \text{for } v_{in} < v < v_n \\ P_n & \text{for } v_n < v < v_{ie} \\ 0 & \text{otherwise} \end{cases} \quad (4.18)$$

where the coefficients A_1, A_2 and A_0 may be expressed as follows [49]:

$$A_2 = \frac{1}{(v_{in} - v_n)^2} \left[2 - 4 \left(\frac{v_{in} + v_n}{2v_n} \right)^3 \right] \quad (4.19)$$

$$A_1 = \frac{1}{(v_{in} - v_n)^2} \left[4(v_{in} + v_n) \left(\frac{v_{in} + v_n}{2v_n} \right)^3 - 3(v_{in} + v_n) \right]$$

$$A_0 = \frac{1}{(v_{in} - v_n)^2} \left[v_{in}(v_{in} + v_n) - 4v_{in}v_n \left(\frac{v_{in} + v_n}{2v_n} \right)^3 \right]$$

2. According to a second common model discussed in the literature, the output power produced by a wind turbine, P_n may be determined by using a linear polynomial function which describes the power variation on the minimum and rated speeds at which the turbine may operate.

The mathematical model is described as follows [48]:

$$P(v) = \begin{cases} 0 & (v < v_{in}) \\ a + bv^k & (v_{in} \leq v \leq v_n) \\ P_n & (v_n < v \leq v_{ie}) \\ 0 & (v > v_{ie}) \end{cases} \quad (4.20)$$

In the above expressions P_n - is the rated electrical power, v_{in} - is the cut-in wind speed, v_n - is the rated wind speed, v_{ie} - is the cut-off wind speed, and k is the Weibull shape parameter.

The coefficients a and b are expressed as follows:

$$a = \frac{P_n \cdot v_{in}^k}{v_{in}^k - v_n^k} \quad (4.21)$$

$$b = \frac{P_n}{v_n^k - v_{in}^k}$$

4.5. Probabilistic model of capacity factor

The average power output represents the so-called capacity factor (CF) of the wind turbine. The capacity factor is a measure of the productivity of a wind turbine (or any unit that produces electricity). It is possible to compare the real power output for a certain period (one year) with the total potential energy that could run at maximum capacity for the same period. The capacity factor is equal to the real power output compared to the energy that would have been produced if the turbine had functioned at a 100% capacity for the same period.

Based on the Weibull distribution function $f_w(v)$ equation (4.22) and on the characteristic wind power given by the first model expressed in (4.18), it is possible to determine the average power output produced by a wind turbine.

$$f_w(v) = \frac{k}{c} \left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^k\right] \quad (4.22)$$

Thus, starting from the first model, the expression of the average power output by the wind turbine is given by (4.22), which considers the characteristic wind power, which is non-zero, for example, for the range $v_{in} - v_n$ respectively $v_n - v_{in}$ are non-zero.

$$M(P) = \int_0^\infty P(v) \cdot f_w(v) dv \quad (4.23)$$

Using equation (4.18) we may expand the average power expression given by (4.22) with the resulting new ratio being described by equation (4.23).

$$M(P) = P_n \cdot \int_{v_{in}}^{v_n} (A_2 v^2 + A_1 v + A_0) f_w(v) dv + P_n \cdot \int_{v_n}^{v_{ie}} f_w(v) dv \quad (4.24)$$

It is noted that, in equation (4.23) the first integral may be written as a sum of three similar integrals I_k , where $k = 1, 2, 3$. The integral I_k may be expressed as follows only if variables $y = (v/c)^K$, $dy = K/c \cdot (v/c)^{K-1}$ respectively change to $v = c \cdot v^{1/K}$. This new form of the integral is given by equation (4.24) above.

$$\begin{aligned} I_k &= \int_{v_{in}}^{v_n} (A_k v^k) f_w(v) dv = \int_{v_{in}}^{v_n} (A_k v^k) \frac{K}{c} \left(\frac{v}{c}\right)^{K-1} \exp\left[-\left(\frac{v}{c}\right)^K\right] dv = \\ &\stackrel{v \leftrightarrow y}{=} A_k \int_{y_{in}}^{y_n} (c v^{1/K})^k \exp(-y) dy = c^k A_k \int_{y_{in}}^{y_n} (y^{(K/K+1)-1}) \exp(-y) dy \end{aligned} \quad (4.25)$$

In the previous integral of equation 4.25, this is commonly used in statistical analyses and, therefore, function Γ (gamma) has the following structure as expressed in equation (4.26).

$$\int_0^t y^{a-1} \exp(-y) dy = \Gamma(a) \cdot P(t, a) \quad (4.26)$$

where $\Gamma(\)$ is the gamma function and $P(\)$ is the lower incomplete gamma function.

The capacity factor may be calculated by integrating the normalised power curve multiplied by the wind speed distribution over all wind speed values [48]. Thus, the capacity factor may be derived based on both the power curve of wind turbine generator and the parameters of wind

speed distribution. Based on the quadratic model of the power curve and the Weibull distribution of wind speed, an analytical model for capacity factor evaluation has been developed – see equation (4.27).

$$CF = \sum_{k=0}^2 A_k \cdot c^k \cdot \Gamma\left(\frac{k}{K} + 1\right) \cdot \left[P\left(\left(\frac{v_n}{c}\right)^K, \frac{k}{K} + 1\right) - P\left(\left(\frac{v_{in}}{c}\right)^K, \frac{k}{K} + 1\right) \right] + \exp(-v_n/c) - \exp(-v_{ie}/c) \quad (4.27)$$

Starting from the second model of the output power from the wind turbine, namely, the model as expressed in equation (4.20), it is possible to determine a new expression of the average output power from a wind turbine. We have seen that the electrical power P produced by a wind turbine depends on the wind speed, the angular speed of the turbine, and the efficiency of the other moving mechanical parts.

Although the model could be more sophisticated in such a case suppose that power could be adequately described by equation (4.20). We combine the output power variation with the wind speed, with the wind speed variation, at the same place, to find the average electric power $M(P)$ that would be expected at a certain time from the turbine in question. The average power produced by a turbine is the most important parameter of a wind turbine because it determines the total energy and the total profit. This is a much better economic indicator than the nominal power that may be chosen a value too high.

$$M(P) = \int_0^{\infty} P(v)f(v)dv \quad (4.28)$$

where $f(v)$ is the Weibull distribution function expressed in (4.22).

$$f(v) = \frac{k}{c} \left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^k\right] \quad (4.29)$$

Substituting equations (4.20) and (4.29) in equation (4.28) we obtain:

$$M(P) = \int_0^{\infty} (a + bv^k)f(v)dv + P_n \int_{v_n}^{\infty} f(v)dv \quad (4.30)$$

In equation (4.30) there are two independent integrals which must be integrated. One integral integrates $v^k f(v)$ and the second integral integrates $f(v)$. The integration is possible only if the variables are changed:

$$x = \left(\frac{v}{c}\right)^k \quad (4.31)$$

Derivate dx may be written as:

$$dx = k \left(\frac{v}{c}\right)^{k-1} d\left(\frac{v}{c}\right) \quad (4.32)$$

and the two integrals above may be written as:

$$\int f(v)dv = \int e^{-x} dx = -e^{-x} \quad (4.33)$$

$$\int v^k f(v)dv \int c^k \left(\frac{v}{c}\right)^k f(v)dv = \int x e^{-x} dx = -c^k (x + 1) e^{-x} \quad (4.34)$$

When replacing the variables of the integral equation (4.30), they are reduced to a minimum number of terms, with the result being given in equation (4.35):

$$CF = \frac{\exp\left[-\left(\frac{v_{in}}{c}\right)^k - \exp\left[-\left(\frac{v_n}{c}\right)^k\right]\right]}{\left(\frac{v_n}{c}\right)^k - \left(\frac{v_{in}}{c}\right)^k} - \exp\left[-\left(\frac{v_{ie}}{c}\right)^k\right] \quad (4.35)$$

Equation (4.35), shows the effects of the cut-in, rated and cut-off speeds on the average power production of a turbine. For a given wind turbine with known c and k parameters, v_{in} , v_n and v_{ie} may be selected to maximise the average power and, thereby, maximise the total energy production. However, the relationship between v_{in} , v_n and v_{ie} must be considered. It is essential that the wind retain enough power at the cut-in speed in order to minimise any losses. In such a case, the efficiency of the generator and the gearbox are required to have losses at cut-in speeds smaller than 6.4% of the rated power while losses of 12.5% would indicate that this type of turbine is not efficient. It is expected that v_{in} would always fall between 0.4 and 0.5 v_n .

4.6. Selection of an appropriate wind turbine for the Western Cape

Commercial wind turbines typically demonstrate cut-off speeds of between 20 m/s and 25 m/s and rated wind speeds of between 10 m/s and 15 m/s [50]. If turbines operate at a wind speed range between the rated wind speed and the cut-off speed, the turbine control system could maintain constant a power output depending on the wind speed variation. However, this presents an engineering challenge with design difficulties which include building wind turbines that may subsist at wind speeds greater than 25 m/s. This means that the cut-off speed is typically $2v_n$, unless v_n is chosen at an unusually low level for a specific application. This study applied the rated wind speed v_n , which is an important component of wind turbine design. The selection determines the cut-in speed and dictates certain actions for the rotor. It is appropriate to select v_n in such a manner that the average power and the turbine area are as large as possible. Wind turbine investments consider the development of the turbine area and, therefore, maximising the average power will minimise the cost per unit of energy produced. If a rated speed which is too low is chosen, excessive energy will be lost at higher speed winds. On the other hand, if the rated speed is too high, the turbine will seldom operate at maximum capacity and will lose excessive energy at lower speed winds. The average power output would reach a maximum level at the peak value of the rated wind speed. This value could be determined by evaluating the equation $M(P)$ for various values of v_n and P_n . The choice of rated wind power is not dependent on the rated overall efficiency, air density or the turbine area which may be standardised. Thus, a normalised average power P_N is defined as:

$$P_N = \frac{M(P)}{\eta_0(\rho/2)Ac^3} = CF \left(\frac{v_n}{c}\right)^3 \quad (4.36)$$

where P_N –is the normalised average power.

Graphical plots of P_N are presented in Figure 4.2. for various values of the Weibull shape parameter and the modelling of the k parameter. Most turbines have cut-in speeds which are between 0.4 and 0.5 of the rated wind speeds, thus implying that the plots of P_N should bracket the designs of practical interest. Maximum power is reached at varying values of v_n/c for different values of k . The maximum power point for $v_{in} = 0.5 v_n$ varies from $v_n/c = 1.5$

to 2.5 while k decreases from 2.6 to 1.4. As the cut-in speed is low $0.4v_n$, the maximum power point varies from $v_n/c = 1.6$ to 3.0. If $k = 2$ at a particular site, the optimum value of v_n/c will be between 1.8 and 2.0. If the mean wind speed is 6 m/s, then the rated speed of the turbine should be approximately 12 m/s. This design choice powers wind regimes where $k = 2$. When wind speed is constant, k will be greater than 2. A rated speed of 11 m/s would be regarded as adequate for most sites, with mean wind speeds up to 6 m/s and a rated speed of 13 m/s or 14 m/s deemed appropriate for sites with highest wind speeds.

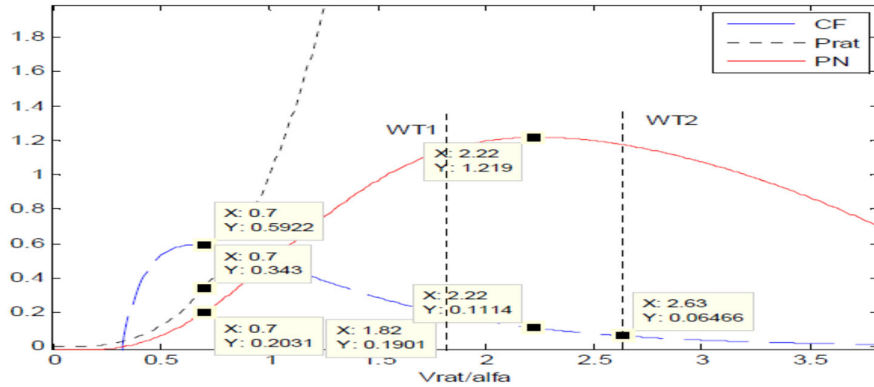


Figure 4.2. Normalised power versus normalised rated speed: [52].
 $Vrat/alfa = v_n/c$

Once v_n/c is selected to maximise the average power, the rated power may be established for a turbine with a given area and a rated overall efficiency located at an elevation with a known average air density. It is known that energy = (average power) (time). Therefore, the yearly energy production of such a turbine is:

$$W = M(P)(time) = (CF)P_n(time) \quad kWh \quad (4.37)$$

where 8784 is the number of hours for the year 2012.

Chapter 5

5. Results and discussion

Data on wind speed was taken and processed on the website:

<https://www.wunderground.com/history/daily/za/matrosfontein/FACT/date/2012-2-1>,

where we selected daily and hourly values of wind speed, temperature, dew point, humidity, visibility, wind direction, speed wind, speed gusts, rainfall, wind direction in degrees, etc. The data used in this case study was recorded for 2012, for the city of Cape Town, South Africa, and saved in the text files before being converted in Microsoft Office Excel files. In Microsoft Office Excel the data was processed in such a way that one year comprised a total of 8784 hours. After that the data obtained had been processed using a program in MATLAB software. The resulting data should help in proper selection of a wind turbine. After calculations had been performed using the program in MATLAB (Appendix 1), it was possible to determine wind velocity versus time for 8784 hours – see Figure 5.1.

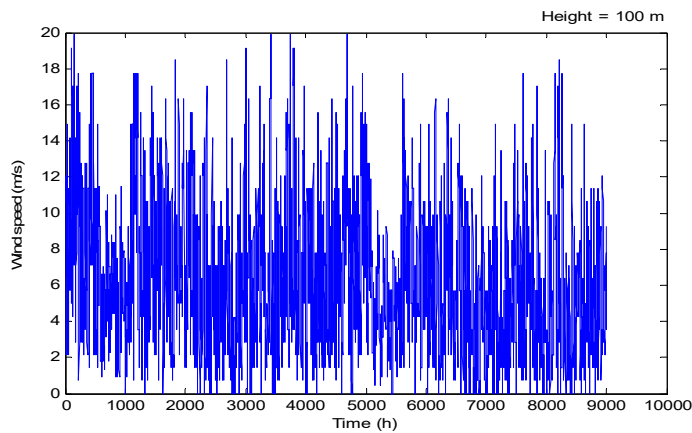


Figure 5.1. : Hourly wind speed database collected for Cape Town, South Africa

Using the same schemes as described in Appendix 1, it was possible to calculate the minimum, maximum, media, wind median and standard deviation for each month separately and then for the entire 2012.

The data obtained is presented in Table 5.1 below.

Table 5.1. Data from the calculation process performed in MATLAB.

Month	Calculation speed (v)					
	Min(v)	Max(v)	Media(v)	Mode(v)	Median(v)	Std deviation(v)
January	0,0278	14,4167	6,3400	6,6944	6,1667	2,7410
February	0,0278	13,8889	6,1181	4,6389	6,1667	2,7877
March	0,0278	12,8611	5,5894	5,6667	5,6667	2,6916
April	0,0278	13,3889	5,2333	4,1111	5,1389	2,6867
Mai	0,0278	12,3333	3,7157	2,5833	3,0833	2,2913
June	0,0278	13,8889	3,9271	2,0556	3,6111	2,6305
July	0,0278	14,4167	4,2292	1,5556	3,6111	2,6672
August	0,0278	14,4167	4,7162	2,5833	4,1111	3,0006
September	0,0278	11,8333	4,2552	2,0556	4,1111	2,4482
October	0,0278	14,9167	6,1732	3,0833	6,1667	3,1321
November	0,0278	15,4444	5,8110	4,6389	5,6667	3,0664
December	0,0278	14,9167	6,5350	6,1667	6,1667	2,9357
Total 8784 hours	0,0278	13,89353	5,220283	3,81945	4,972233	2,756583

Histogram frequencies of wind speed were drawn using hourly wind speed values – see Figure 5.2. The histogram may be approximated in statistical terms using the Weibull distribution, which was determined based on the parameters c and k .

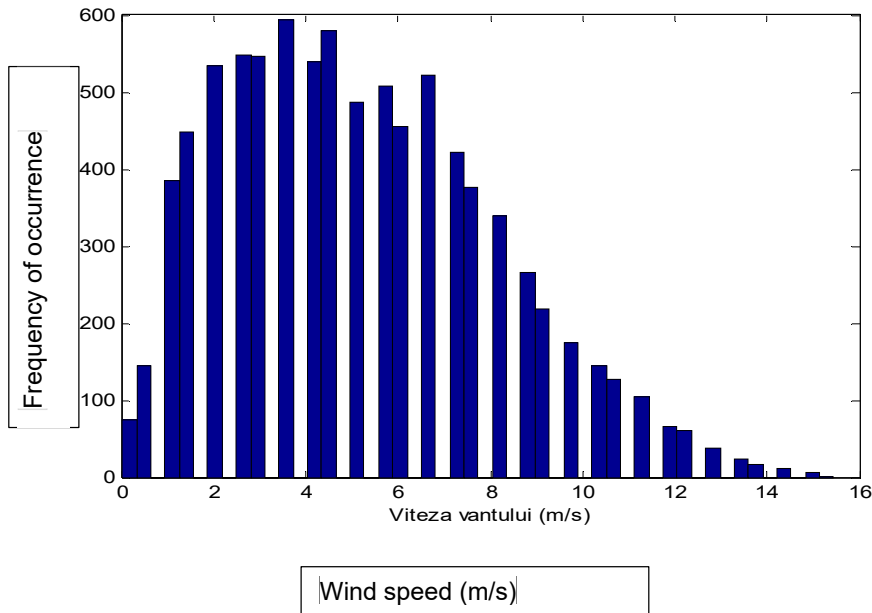


Figure 5.2: Histogram of frequency of wind speed

Case 1. Determination of the Weibull distribution parameters for the entire study period. Based on the values which were determined for the Weibull distribution parameters (c and k) for each month and for the entire study period.

The parameters c and k were determined using the three methods which were presented in Chapter 4 paragraph. 4.2, namely:

- maximum likelihood method (MLE)
- method of moments (MOM)
- least squares method (LSM).

The values of the two parameters (c and k) of the Weibull distribution, which resulted from the calculations using the three methods of estimation are presented in Table 5.2.

Table 5.2. Weibull distribution parameter values for the entire study period for height anemometer (10 m).

Month	MLE		MOM		LSM	
	c 1	k 1	c 2	k 2	c 3	k 3
January	7,13	2,44	7,15	2,47	7,00	1,91
February	6,89	2,31	6,90	2,33	6,80	1,89
March	6,28	2,15	6,31	2,19	6,08	1,70
April	5,90	2,02	5,91	2,04	5,72	1,81
Mai	4,12	1,60	4,16	1,67	3,80	1,39
June	4,32	1,47	4,36	1,52	4,00	1,35
July	4,69	1,57	4,72	1,63	4,34	1,40
August	5,24	1,59	5,26	1,61	5,04	1,43

September	4,76	1,76	4,78	1,80	4,51	1,53
October	6,93	2,02	6,97	2,07	6,67	1,64
November	6,56	1,99	6,56	1,98	6,38	1,95
December	7,36	2,35	7,37	2,37	7,18	1,87
Media	5,85	1,94	5,87	1,97	5,63	1,66
Total 8784 hours	5.85	1.79	5.87	1.84	5.44	1.56

The analysis of the wind profile which was conducted using the Weibull distribution revealed that c and k had the values $c = 5.85[m/s]$ and $k = 1.79[m/s]$ for measurements made at a height of 10 m. Since high power turbine height may vary between 60 and 120 m, for the purposes of this study a turbine height of 100 m was chosen.

$$\frac{c(100m)}{c(10m)} = \left(\frac{100m}{10m}\right)^c = \frac{c_{100}}{5.85 m/s} = \left(\frac{100m}{10m}\right)^{\frac{1}{7}} \Rightarrow c(100m) = 10^{\frac{1}{7}} \cdot 5.85 m/s \Rightarrow c(100m) = 8.12 m/s$$

Where the average value is determined by multiple measurements for a more accurate approximation, the exponent is assumed to have the value $c = \frac{1}{7}$.

Case 2. Determination of the Weibull distribution parameters for the four seasons

The Weibull distribution parameters were also determined for the 8784 hours wind speed for the study area for four seasons, namely, winter, spring, summer and autumn. The analysis presented in Table 5.3. revealed that the highest wind speed occurs in winter which leads to the production of significant quantities of electricity. In addition, Table 5.3. also revealed that wind speed is low during the summer and, hence, less electrical energy is produced as compared to the quantity produced in winter.

Table 5.3. Weibull distribution parameter values for the four seasons for height anemometer (10 m).

Season	MLE		MOM		LSM	
	c 1	k 1	c 2	k 2	c 3	k 3
winter	7,13	2,36	7,15	2,39	6,98	1,88
spring	5,42	1,87	5,45	1,81	5,09	1,87
summer	4,74	1,57	4,77	1,52	4,45	1,38
autumn	6,08	1,86	6,10	1,83	5,79	1,61

The Weibull distribution parameter values presented in Table 5.3. applied to height of 10 m, set the height of the anemometer. For the purposes of the study the Weibull distribution parameters were then recalculated for a turbine height of 100 m – see Table 5.4.

Table 5.4. Weibull distribution parameter for the four seasons for height anemometer (100 m)

Season	MLE	MOM	LSM

	<i>c</i> 1	<i>k</i> 1	<i>c</i> 2	<i>k</i> 2	<i>c</i> 3	<i>k</i> 3
winter	10,18571	2,36	10,21429	2,39	9,971429	1,88
spring	7,742857	1,87	7,785714	1,81	7,271429	1,87
summer	6,771429	1,57	6,814286	1,52	6,357143	1,38
autumn	8,685714	1,86	8,714286	1,83	8,271429	1,61

Two statistical tests, the mean bias error (MBE) and the root mean squares error (RMSE), were conducted on the data obtained from the Weibull estimation methods in order to determine which of the three methods was the more effective. After performing these two tests (Table 5.5) it was observed that the maximum likelihood (MLE) method achieved the best result.

Table 5.5. Results obtained from tests of effectiveness

Name of test	MLE	MOM	LSM
MBE	0.6817	0.9178	-2.3057
RMSE	2.2830	2.4964	5.0825

A total of 18 wind turbines produced by various manufactures (General Electric Energy, Vestas Wind Systems, Enercon and Nordex Energy AG) were analysed – see Table 5.6.

Table 5.6. presents the following parameters for each wind turbine:

- Rated (with values between 1.5 ÷ 3.3 [MW]);
- Start up speed (variation range is 2.5 ÷ 4 [m/s] ;
- Rated speed (variation being 11.5 to 14 [m/s] ;
- Closing speed (variation range is 20 to 28 [m/s] .

Table 5.6. List of wind turbines used for the study.

Case	Type of turbine	Rated power (MW)	start up speed (m/s)	Rated speed (m/s)	Closing speed (m/s)
1	1.5x - GE	1.5	3.5	11.5	20
2	1.5s - GE	1.5	3.5	14	25
3	1.5s - GE	1.5	4	12	25
4	N70 - 1.5	1.5	3	13	25
5	V100 - 1.8	1.8	3	12	20
6	V100 - 1.8	1.8	4	12	25
7	V100 - 2	2	3	12	20
8	V100 - 2	2	4	12	25
9	Enercon E - 82	2	2.5	13	28th
10	V80 - 2	2	4	14	25
11	V110 - 2	2	3	11.5	20
12	2.5xl - GE	2.5	3.5	12.5	25
13	N80 2.5	2.5	3	13	25

14	V100 - 2.6	2.6	3.5	14	23
15	V90 - 3	3	3.5	15	25
16	V112 - 3.3	3.3	3	13	25
17	V117 - 3.3	3.3	3	13	25
18	V126 - 3.3	3.3	3	12	22.5

Based on the mathematical models presented in Chapter 4, the value of the capacity factor (CF) was calculated according to the two models for each turbine part. The calculation algorithm in MATLAB is detailed in Appendix 2. For the entire period of the study the wind velocity was obtained, the analysed area for the Weibull distribution parameters, $c = 8.12[m/s]$ and $k = 1.79[m/s]$, at a height of 100 m. Based on the determination of the capacity factor (CF) for the 18 types of horizontal axis wind turbines the wind turbines were ranked in terms of their suitability for the study area – see Table 5.7.

Table 5.7. Compatibility of a wind turbine by wind profile for Weibull distribution parameters when $c = 8.12[m/s]$ and $k = 1.79[m/s]$ to a height of 100 m

Ranking	Case	Type of turbine	Rated power (MW)	start speed (m/s)	up Rated speed (m/s)	Speed off (m/s)	CF1 ($c_{100} = 8.12, k = 1.79$) (%)	CF2 ($c_{100} = 8.12, k = 1.79$) (%)
1	11	V110 - 2	2	3	11,5	20	33,7623	40,0290
2	1	1.5xle - GE	1,5	3,5	11,5	20	33,5541	38,6750
3	18	V126 - 3.3	3,3	3	12	22,5	31,7435	38,3824
4	3	1.5s - GE	1,5	4	12	25	31,4437	35,8238
5	6	V100 - 1.8	1,8	4	12	25	31,4437	35,8238
6	8	V100 - 2	2	4	12	25	31,4437	35,8238
7	5	V100 - 1.8	1,8	3	12	20	31,2865	37,9254
8	7	V100 - 2	2	3	12	20	31,2865	37,9254
9	12	2.5xl - GE	2,5	3,5	12,5	25	29,3964	35,2712
10	9	Enercon E - 82	2	2,5	13	28	27,4819	35,7806
11	4	N70 - 1.5	1,5	3	13	25	No. order	34,6313
12	13	N80 2.5	2,5	3	13	25	27,3571	34,6313
13	16	V112 - 3.3	3,3	3	13	25	27,3571	34,6313
14	17	V117 - 3.3	3,3	3	13	25	27,3571	34,6313
15	2	1.5sle - GE	1,5	3,5	14	25	23,2716	30,0141
16	14	V100 - 2.6	2,6	3,5	14	23	23,1692	29,9118
17	10	V80 - 2	2	4	14	25	23,1154	28,8093
18	15	V90 - 3	3	3,5	15	25	19,8285	26,9892

The analysis presented in Table 5.7. revealed that the V110 – 2 turbine was the most appropriate for the study area as, although the turbine was rated power 2 MW, it was first in the rankings, it could be selected while the second type of turbine, 1.5xle – GE, is 1.5 MW rated. The choice of a turbine depends primarily on the cost which is an extremely

important factor in the final price of the investment and, thus, it is important to find a balance between the price and the benefit from producing electricity. Ideally the turbine should operate for as long a period as possible during the year in order to harness the wind potential fully.

In Figure 5.3. the distribution of the 18 types of wind turbines based on the capacity factor (CF) is plotted, based on Table 5.7.

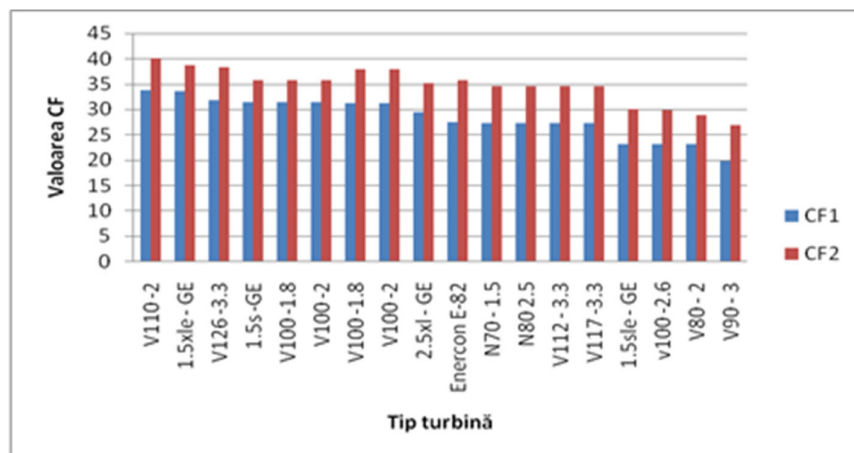


Figure 5.3. Distribution of wind turbines based on the CF, according to Table 5.7

The following is the parameters of the Weibull distribution for the 18 types of wind turbines are modified with this possibly leading to a change in the priorities governing the installation of wind turbines in wind region:

1. When $c = 12,22 [m/s]$ and $k = 1,79 [m/s]$;
2. When $c = 8,12 [m/s]$ and $k = 3,3 [m/s]$;
3. When $c = 12,22 [m/s]$ and $k = 3,3 [m/s]$.

Table 5.8. Compatibility of a wind turbine by wind profile for the Weibull distribution parameters when $c = 12.22[m/s]$ and $k = 1.79[m/s]$ to height of 100 m.

Ranking	Case	Type of turbine	Rated power (MW)	start speed (m/s)	up	Rated speed (m/s)	Speed off (m/s)	CF1 ($c_{100} = 12.22, k = 1.79$) (%)	CF2 ($c_{100} = 12.22, k = 1.79$) (%)
1	3	1.5s - GE	1,5	4		12	25	53,0504	56,5439
2	6	V100 - 1.8	1,8	4		12	25	53,0504	56,5439
3	8	V100 - 2	2	4		12	25	53,0504	56,5439
4	18	V126 -3.3	3,3	3		12	22,5	51,0261	56,0787
5	12	2.5xl - GE	2,5	3,5		12,5	25	50,9816	55,6694
6	9	Enercon E-82	2	2,5		13	28	50,4891	56,9740
7	11	V110 - 2	2	3		11,5	20	49,4531	54,1082
8	1	1.5xle - GE	1,5	3,5		11,5	20	49,3078	53,1968
9	4	N70 - 1.5	1,5	3		13	25	48,8859	54,6960
10	13	N80 2.5	2,5	3		13	25	48,8859	54,6960
11	16	V112 - 3.3	3,3	3		13	25	48,8859	54,6960
12	17	V117 - 3.3	3,3	3		13	25	48,8859	54,6960

13	5	V100 - 1.8	1,8	3	12	20	47,1605	52,2130
14	7	V100 - 2	2	3	12	20	47,1605	52,2130
15	2	1.5sle - GE	1,5	3,5	14	25	44,4831	50,2736
16	10	V80 - 2	2	4	14	25	44,3531	49,3744
17	14	V100 - 2.6	2,6	3,5	14	23	42,7159	48,5064
18	15	V90 - 3	3	3,5	15	25	40,3990	46,8500

Table 5.8. presents the analysis results for the turbines in the first three positions (1.5s - GE, V100 -1.8 and V100 – 2) that were found to have capacity factors (CF) with the same value, namely, 56,5439. These were followed by turbine V126 -3.3 in fourth position and 2.5xl – GE in fifth position. It may be observed that, in this case where $c = 12,22 [m/s]$, has high value as compared to the case analysed in Table 5.7. when $c = 8,12 [m/s]$. Thus, the wind speed is high in this case and, as a result, the profile of the wind turbines at an operating speed are different.

In Figure 5.4. the distribution of the 18 types of wind turbines according to the capacity factor (CF) is plotted, based on Table 5.8.

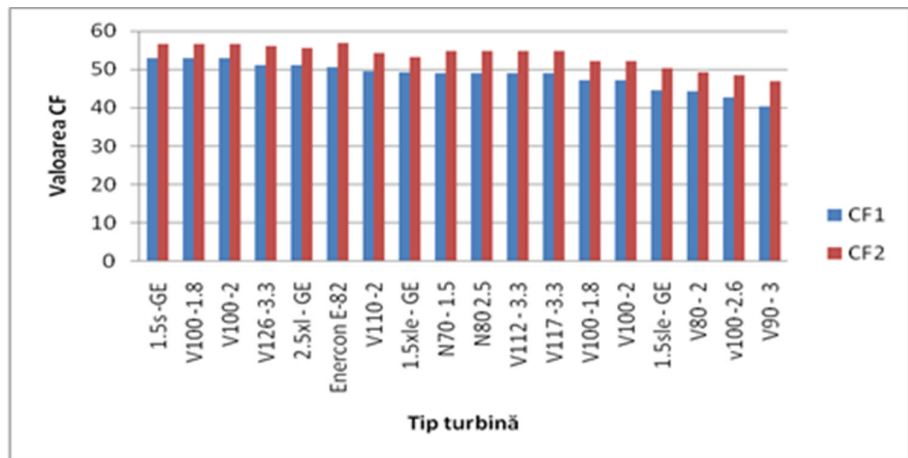


Figure 5.4. Distribution of wind turbines based on the CF, according to Table 5.8.

1. For the Weibull distribution parameter when $c = 8.12[m/s]$ and $k = 3.3[m/s]$, we obtained the order of the wind profile of the wind turbine, where we recalculate the capacity factor (CF) which resulted in the list presented in Table 5.9.

Table 5.9. Compatibility of a wind turbines by wind profile for the Weibull distribution parameters when $c = 8.12[m/s]$ and $k = 3.3[m/s]$ to a height of 100m.

Ranking	Case	Type of turbine	Rated power (MW)	start up speed (m/s)	Rated speed (m/s)	Speed off (m/s)	CF1 ($c_{100} = 8.12, k = 3.3$) (%)	CF2 ($c_{100} = 8.12, k = 3.3$) (%)
13	5	V100 - 1.8	1,8	3	12	20	47,1605	52,2130
14	7	V100 - 2	2	3	12	20	47,1605	52,2130
15	2	1.5sle - GE	1,5	3,5	14	25	44,4831	50,2736
16	10	V80 - 2	2	4	14	25	44,3531	49,3744
17	14	V100 - 2.6	2,6	3,5	14	23	42,7159	48,5064
18	15	V90 - 3	3	3,5	15	25	40,3990	46,8500

1	11	V110 -2	2	3	11,5	20	32,5398	29,5443
2	1	1.5xle - GE	1,5	3,5	11,5	20	32,2569	29,0178
3	5	V100 -1.8	1,8	3	12	20	28,9285	26,0828
4	7	V100 -2	2	3	12	20	28,9285	26,0828
5	18	V126 -3.3	3,3	3	12	22,5	28,9285	26,0828
6	3	1.5s -GE	1,5	4	12	25	28,3162	24,9513
7	6	V100 -1.8	1,8	4	12	25	28,3162	24,9513
8	8	V100 -2	2	4	12	25	28,3162	24,9513
9	12	2.5xl - GE	2,5	3,5	12,5	25	25,4514	22,5911
10	9	Enercon E-82	2	2,5	13	28	22,9756	20,6329
11	4	N70 - 1.5	1,5	3	13	25	22,8027	20,3569
12	13	N80 2.5	2,5	3	13	25	22,8027	20,3569
13	16	V112 - 3.3	3,3	3	13	25	22,8027	20,3569
14	17	V117 -3.3	3,3	3	13	25	22,8027	20,3569
15	2	1.5sle - GE	1,5	3,5	14	25	17,8246	15,6923
16	14	V100 -2.6	2,6	3,5	14	23	17,8246	15,6923
17	10	V80 - 2	2	4	14	25	17,5999	15,2475
18	15	V90 - 3	3	3,5	15	25	14,1147	12,4956

The analysis presented in Table 5.9. shows that, in this case, the first two values are very close, thus indicating that it would be feasible to choose either one of the two turbines ranked first and second.

In Figure 5.5. the distribution of the 18 types of wind turbines based on the capacity factor (CF) is plotted, based on Table 5.9.

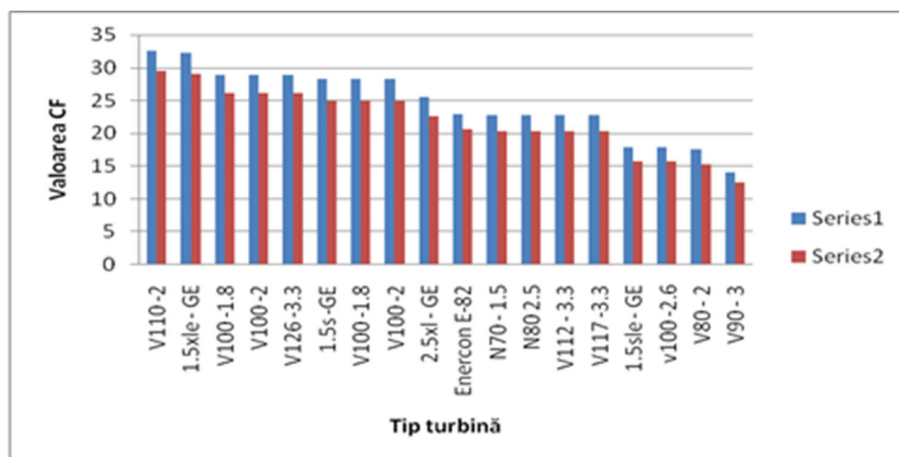


Figure 5.5.: Distribution of wind turbines based on the (CF), according to Table 5.9.

- The Weibull distribution parameters when $c = 12.22[m/s]$ and $k = 3.3[m/s]$, we obtained the order of the wind profile of the wind turbines, where we recalculate the capacity factor (CF) which resulted in the list presented in Table 5.10.

Table 5.10. Compatibility of a wind turbines by the wind profile for the Weibull distribution parameter when $c = 12.22[m/s]$ and $k = 3.3[m/s]$ to a height of 100m.

Ranking	Case	Type of turbine	Rated power (MW)	start up speed (m/s)	Rated speed (m/s)	Speed off (m/s)	CF1 ($c_{100} = 12.22, k = 3.3$) (%)	CF2 ($c_{100} = 12.22, k = 3.3$) (%)
1	11	V110 -2	2	3	11,5	20	32,5398	29,5443
2	1	1.5xle - GE	1,5	3,5	11,5	20	32,2569	29,0178
3	5	V100 -1.8	1,8	3	12	20	28,9285	26,0828
4	7	V100 -2	2	3	12	20	28,9285	26,0828
5	18	V126 -3.3	3,3	3	12	22,5	28,9285	26,0828
6	3	1.5s -GE	1,5	4	12	25	28,3162	24,9513
7	6	V100 -1.8	1,8	4	12	25	28,3162	24,9513
8	8	V100 -2	2	4	12	25	28,3162	24,9513
9	12	2.5xl - GE	2,5	3,5	12,5	25	25,4514	22,5911
10	9	Enercon E-82	2	2,5	13	28	22,9756	20,6329
11	4	N70 - 1.5	1,5	3	13	25	22,8027	20,3569
12	13	N80 2.5	2,5	3	13	25	22,8027	20,3569
13	16	V112 - 3.3	3,3	3	13	25	22,8027	20,3569
14	17	V117 -3.3	3,3	3	13	25	22,8027	20,3569
15	2	1.5sle - GE	1,5	3,5	14	25	17,8246	15,6923
16	14	V100 -2.6	2,6	3,5	14	23	17,8246	15,6923
17	10	V80 - 2	2	4	14	25	17,5999	15,2475
18	15	V90 - 3	3	3,5	15	25	14,1147	12,4956

1	11	V110 - 2	2	3	11,5	20	69,2506	67,1762
2	1	1.5xle - GE	1,5	3,5	11,5	20	69,1150	66,9280
3	18	V126 - 3.3	3,3	3	12	22,5	66,4567	64,2212
4	3	1.5s - GE	1,5	4	12	25	66,2088	63,7019
5	6	V100 - 1.8	1,8	4	12	25	66,2088	63,7019
6	8	V100 - 2	2	4	12	25	66,2088	63,7019
7	5	V100 - 1.8	1,8	3	12	20	65,9063	63,6708
8	7	V100 - 2	2	3	12	20	65,9063	63,6708
9	12	2.5xl - GE	2,5	3,5	12,5	25	62,9518	60,4790
10	9	Enercon E -82	2	2,5	13	28	59,8102	57,2800
11	4	N70 - 1.5	1,5	3	13	25	59,6548	57,1263
12	13	N80 2.5	2,5	3	13	25	59,6548	57,1263
13	16	V112 - 3.3	3,3	3	13	25	59,6548	57,1263
14	17	V117 - 3.3	3,3	3	13	25	59,6548	57,1263
15	2	1.5sle - GE	1,5	3,5	14	25	52,6443	49,8408
16	14	V100 - 2.6	2,6	3,5	14	23	52,6164	49,8129
17	10	V80 - 2	2	4	14	25	52,4806	49,5617
18	15	V90 - 3	3	3,5	15	25	46,0318	43,1069

It emerged from the analysis in Table 5.10. that, in this case, the CF values for the first two turbine types were very similar and that these turbines were the same as those in Table 5.9.

In Figure 5.6, the distribution of the 18 types of wind turbines based on the capacity factor (CF) is plotted, based on Table 5.10.

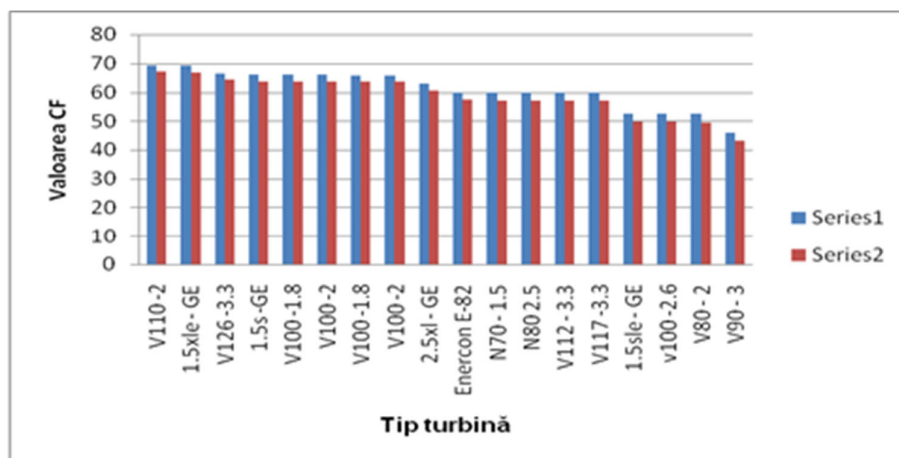


Figure 5.6. Distribution of wind turbines based on the CF, according to Table 5.10.

Case 2: For the four seasons

In this case the same types of turbines were selected as those in case 1

Table 5.11. Distribution of the wind turbines in winter

Ranking	Case	Type of turbine	Rated power (MW)	start up speed (m/s)	Rated speed (m/s)	Speed off (m/s)	CF 1 ($c_{100} = 10.18$, $k = 2.36$) (%)	CF2 ($c_{100} = 10.18$, $k = 2.36$) (%)

									(%)
1	11	V110 - 2	2		3	11,5	20	49,92	52,66
2	1	1.5xle - GE	1,5		3,5	11,5	20	49,73	51,88
3	18	V126 - 3.3	3,3		3	12	22,5	47,36	50,38
4	3	1.5s - GE	1,5		4	12	25	47,07	48,85
5	6	V100 -1.8	1,8		4	12	25	47,07	48,85
6	8	V100 - 2	2		4	12	25	47,07	48,85
7	5	V100 - 1.8	1,8		3	12	20	46,78	49,80
8	7	V100 - 2	2		3	12	20	46,78	49,80
9	12	2.5xl - GE	2,5		3,5	12,5	25	44,27	46,99
10	9	Enercon E - 82	2		2,5	13	28	41,65	45,62
11	4	N70 - 1.5	1,5		3	13	25	41,50	45,03
12	13	N80 2.5	2,5		3	13	25	41,50	45,03
13	16	V112 - 3.3	3,3		3	13	25	41,50	45,03
14	17	V117 - 3.3	3,3		3	13	25	41,50	45,03
15	2	1.5sle - GE	1,5		3,5	14	25	35,84	39,31
16	14	V100 - 2.6	2,6		3,5	14	23	35,76	39,23
17	10	V80 - 2	2		4	14	25	35,66	38,55
18	15	V90 - 3	3		3,5	15	25	30,87	34,76

In Figure 5.7. the distribution of the wind turbines depending on the capacity factor (CF) is plotted.

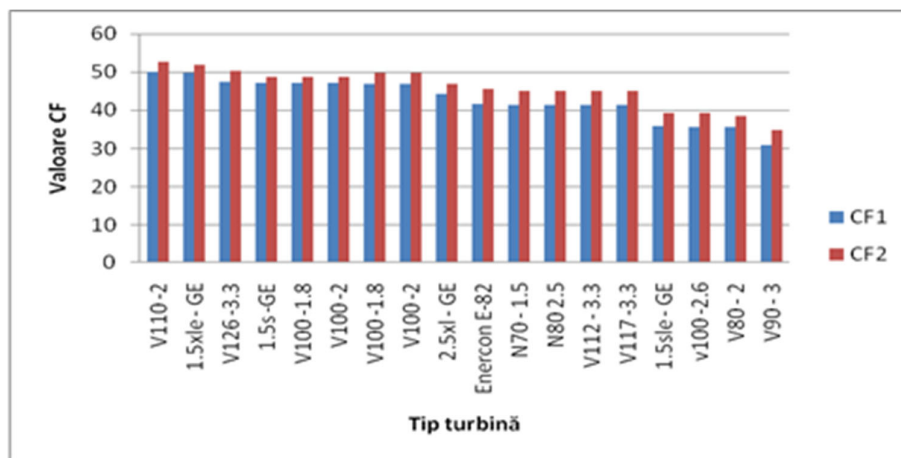


Figure 5.7. Distribution of wind turbines based on the CF, according to Table 5.11

Table 5.12. Distribution of the wind turbines in spring

Ranking	Case	Type turbine of	Rated power (MW)	start up speed (m/s)	Rated speed (m/s)	Speed off (m/s)	CF1 ($C_{100} = 10.18$, $k = 2.36$) (%)	CF2 ($C_{100} = 10.18$, $k = 2.36$) (%)
1	11	V110 - 2	2	3	11,5	20	31,11	37,14
2	1	1.5xle - GE	1,5	3,5	11,5	20	30,89	35,78
3	18	V126 - 3.3	3,3	3	12	22,5	28,82	35,19

4	5	V100 - 1.8	1,8	3	12	20	28,61	34,98
5	7	V100 - 2	2	3	12	20	28,61	34,98
6	3	1.5s - GE	1,5	4	12	25	28,40	32,53
7	6	V100 - 1.8	1,8	4	12	25	28,40	32,53
8	8	V100 - 2	2	4	12	25	28,40	32,53
9	12	2.5xl - GE	2,5	3,5	12,5	25	26,37	31,95
10	9	Enercon E - 82	2	2,5	13	28	24,44	32,37
11	4	N70 - 1.5	1,5	3	13	25	24,35	31,28
12	13	N80 2.5	2,5	3	13	25	24,35	31,28
13	16	V112 - 3.3	3,3	3	13	25	24,35	31,28
14	17	V117 - 3.3	3,3	3	13	25	24,35	31,28
15	2	1.5sle - GE	1,5	3,5	14	25	20,36	26,71
16	14	V100 - 2.6	2,6	3,5	14	23	20,33	26,67
17	10	V80 - 2	2	4	14	25	20,20	25,52
18	15	V90 - 3	3	3,5	15	25	17,07	23,76

In Figure 5.8. the distribution of the wind turbines depending on the capacity factor (CF) is plotted.

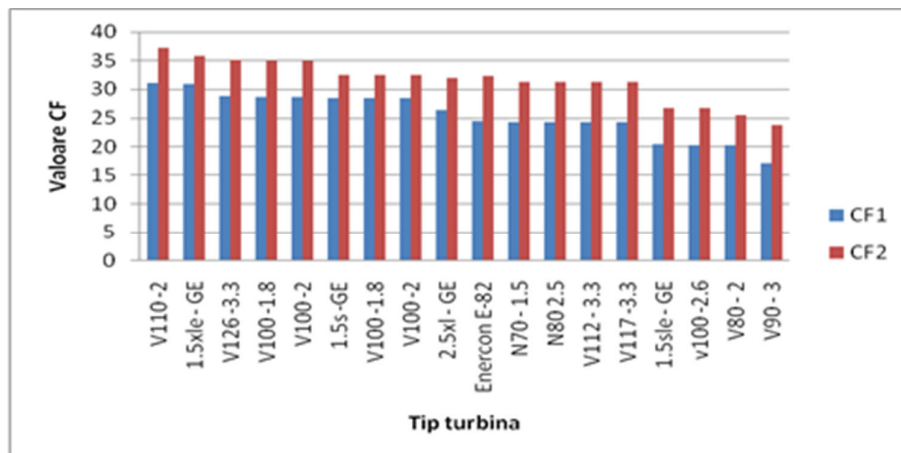


Figure 5.8. Distribution of wind turbines based on the CF, according to Table 5.12

Table 5.13. Distribution of wind turbines in summer.

Ranking	Case	Type turbine of	Rated power (MW)	Start up speed (m/s)	Rated speed (m/s)	Speed off (m/s)	CF1 ($c_{100} = 6.77$, $k = 1.57$) (%)	CF2 ($c_{100} = 6.77$, $k = 1.57$) (%)
1	11	V110 - 2	2	3	11,5	20	24,93	32,09
2	1	1.5xle - GE	1,5	3,5	11,5	20	24,72	30,50

3	18	V126 - 3.3	3,3	3	12	22,5	23,18	30,68
4	5	V100 - 1.8	1,8	3	12	20	22,90	30,40
5	7	V100 - 2	2	3	12	20	22,90	30,40
6	3	1.5s - GE	1,5	4	12	25	22,85	27,69
7	6	V100 - 1.8	1,8	4	12	25	22,85	27,69
8	8	V100 - 2	2	4	12	25	22,85	27,69
9	12	2.5xl - GE	2,5	3,5	12,5	25	21,26	27,71
10	9	Enercon E - 82	2	2,5	13	28	19,74	29,08
11	4	N70 - 1.5	1,5	3	13	25	19,66	27,69
12	13	N80 2.5	2,5	3	13	25	19,66	27,69
13	16	V112 - 3.3	3,3	3	13	25	19,66	27,69
14	17	V117 - 3.3	3,3	3	13	25	19,66	27,69
15	2	1.5sle - GE	1,5	3,5	14	25	16,49	23,66
16	14	V100 - 2.6	2,6	3,5	14	23	16,42	23,59
17	10	V80 - 2	2	4	14	25	16,35	22,32
18	15	V90 - 3	3	3,5	15	25	13,88	21,37

In Figure 5.9. the distribution of the wind turbines depending on the capacity factor (CF) is plotted.

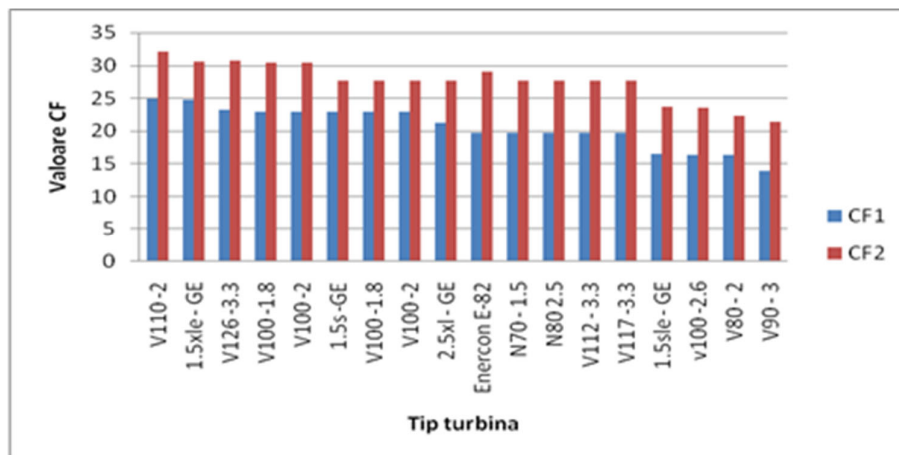


Figure 5.9. Distribution of the wind turbines based on the CF, according to Table 5.13

Table 5.14. Distribution of the wind turbines in autumn

Ranking	Case	Type of turbine	Rated power (MW)	start up speed (m/s)	Rated speed (m/s)	Speed off (m/s)	CF 1 ($c_{100} = 8.68$, $k = 1.86$) (%)	CF2 ($c_{100} = 8.68$, $k = 1.86$) (%)

1	11	V110 - 2	2	3	11,5	20	37,57	43,37
2	1	1.5xle - GE	1,5	3,5	11,5	20	37,36	42,12
3	18	V126 - 3.3	3,3	3	12	22,5	35,57	41,76
4	3	1.5s - GE	1,5	4	12	25	35,34	39,44
5	6	V100 - 1.8	1,8	4	12	25	35,34	39,44
6	8	V100 - 2	2	4	12	25	35,34	39,44
7	5	V100 - 1.8	1,8	3	12	20	34,96	41,15
8	7	V100 - 2	2	3	12	20	34,96	41,15
9	12	2.5xl - GE	2,5	3,5	12,5	25	33,14	38,67
10	9	Enercon E - 82	2	2,5	13	28	31,11	38,87
11	4	N70 - 1.5	1,5	3	13	25	30,95	37,80
12	13	N80 2.5	2,5	3	13	25	30,95	37,80
13	16	V112 - 3.3	3,3	3	13	25	30,95	37,80
14	17	V117 - 3.3	3,3	3	13	25	30,95	37,80
15	2	1.5sle - GE	1,5	3,5	14	25	26,55	33,00
16	14	V100 - 2.6	2,6	3,5	14	23	26,41	32,86
17	10	V80 - 2	2	4	14	25	26,39	31,86
18	15	V90 - 3	3	3,5	15	25	22,79	29,69

In Figure 5.10. the distribution of the wind turbines depending on the capacity factor (CF) is plotted.

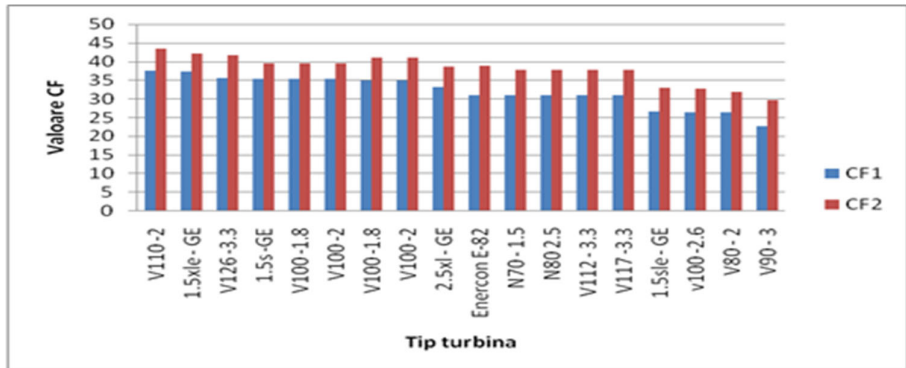


Figure 5.10. Distribution of wind turbines based on the CF, according to Table 5.14

From the analyses presented in Tables 5.11, 5.12, 5.13 and 5.14. it may be concluded that the first three types of turbines, namely, V110 – 2, 1.5xle – GE and V126 – 3.3 retained their positions in the rankings for the four seasons although with different values of the capacity factor (CF). Thus, it may be concluded that these three turbines were appropriate for the purposes of the study because they are resulting in the energy production being maximised.

The energy produced for the four seasons by the V110 – 2 turbines is expressed below:

Winter:

$$W = (CF) \cdot P_n \cdot t = \frac{49.92}{100} \cdot 2MW \cdot 2184h = 2181.5MWh$$

Spring:

$$W = (CF) \cdot P_n \cdot t = \frac{31.11}{100} \cdot 2MW \cdot 2208h = 1373.8MWh$$

Summer:

$$W = (CF) \cdot P_n \cdot t = \frac{24.93}{100} \cdot 2MW \cdot 2204h = 1100.9MWh$$

Autumn:

$$W = (CF) \cdot P_n \cdot t = \frac{37.52}{100} \cdot 2MW \cdot 2184h = 1638.8MWh$$

For the entire 2012

$$W = (CF) \cdot P_n \cdot t = \frac{33.76}{100} \cdot 2MW \cdot 8784h = 5930.9MWh$$

Summarised

As can be observed, the scale and shape parameters, from the whole database and from seasonal values, have the best estimation in case of maximum likelihood estimator (MLE). Thus, it can be summarised that the maximum likelihood estimator (MLE) is the best method used to estimate the parameters for the two-parameter Weibull distributions taking into consideration the mean bias error (MBE) and the root mean squares error (RMSE) as measurements of comparison, while the maximum likelihood estimator (MLE) method is the least accurate method.

Chapter 6

6. Conclusions and Recommendations

The potential of using wind power for generation of electricity could be great, nevertheless it is important to characterise wind power at a particular site in order to select the parameters of wind turbines. In practice, it is imperative to describe wind speed variation for an optimal systems design. The Wind variation for the typical site is commonly described in terms of the Weibull distribution. Therefore, it is vital to understand the best method of parameter evaluation that presents a minimal margin of error. In this respect, this study has been investigated to compare the results of three methods of parameter estimation, for the same database. Hourly wind speed data of the Cape Town region were statistically analysed. The probability density distributions were derived from this database and distributional parameters were evaluated. The computational results reveal that the method with the lowest values of statistical testing was maximum likelihood estimator (MLE), for a whole year database. However, in terms of accuracy, method of moments (MOM) and least squares method (LSM) also appeared to be suitable methods of assessing Weibull function since these parameters seemed to approximate those of maximum likelihood estimator (MLE). In view of the result that was obtained based Maximum-likelihood estimator (MLE), it is recommended that Maximum likelihood estimator method should be used for estimating potential of wind. Although this study focused on Cape Town, however, there are other sites that could be investigated. There seems to be high potential for generation of power from wind also in Port Elizabeth, KwaZulu- Natal, and other place apart from the coastal regions.

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7. Appendices.

7.1. Appendix 1

```

clc, clear all;
%%link to the data for the 12 months
F1=xlsread ('CapeJanuary bun!!.xls','H:H');
F2=xlsread ('CapeFebruary bun!!.xls','H:H');
F3=xlsread ('CapeMarch bun!!.xls','H:H');
F4=xlsread ('CapeApril bun!!.xls','H:H');
F5=xlsread ('CapeMay bun!!.xls','H:H');
F6=xlsread ('CapeJune bun!!.xls','H:H');
F7=xlsread ('CapeJuly bun!!.xls','H:H');
F8=xlsread ('CapeAugust bun!!.xls','H:H');
F9=xlsread ('CapeOctober bun!!.xls','H:H');
F10=xlsread ('CapeSeptember bun!!.xls','H:H');
F11=xlsread ('CapeNovember bun!!.xls','H:H');
F12=xlsread ('CapeDecember bun!!.xls','H:H');

%% The choice of wind speed data from the data file
windspeed1=[F1 (::8)]; % Km/h
windspeed2=(F2 (::8)); % Km/h
windspeed3=(F3 (::8)); % Km/h
windspeed4=(F4 (::8)); % Km/h
windspeed5=(F5 (::8)); % Km/h

```

```

windspeed6=(F6(:8)); % Km/h
windspeed7=(F7(:8)); % Km/h
windspeed8=(F8(:8)); % Km/h
windspeed9=(F9(:8)); % Km/h
windspeed10=(F10(:8)); % Km/h
windspeed11=(F11(:8)); % Km/h
windspeed12=(F12(:8)); % Km/h

%%wind speed for the entire period

wind speed 2012= [windspeed1', windspeed2', windspeed3', windspeed4',
windspeed5', windspeed6', windspeed7', windspeed8', windspeed9',
windspeed10', windspeed11', windspeed12']/3.6% $m/s$ 

Weibull distribution

%% Estimate the parameters
% Maximum likelihood method (Maximum likelihood estimator-MLE)
'1 c      k';
'scale  shape';
[a, b] =wblfit(windspeed2012);
c1=a (1);
k_1=a (2);

% Moments method (Method of moments-MOM)

'2c      k';
'2scale  shape';

M=mean(windspeed2012); D=std(windspeed2012,1);

CV=10;
k=1;
cvc=(D/M) ^2;
WHILE CV>CVC
k=k+0.00001;
cv=(gamma(1+2/k) - (gamma(1+1/k)) ^2)/(gamma(1+1/k)) ^2;
END
c2=(M/(gamma(1+1/k)));
k_2=k;

%Least Squares (Least squares method-LSM)
'3c      k';
'3scale  shape';
x=min(windspeed2012): max(windspeed2012);
n_elements = histc (windspeed2012, x);
c_elements=cumsum(n_elements);
PDF=n_elements/max(n_elements);
CDF=c_elements/max(c_elements);

X=log(x);
Y=log(-log(1-CDF))';
N=length(Y);
poly=polyfit (X (1: N-7), Y (1: N-7)',1);
c3=exp (-poly (2)/poly (1));
k_3=poly (1);
%% Distribution Hypothesis Testing

% Mean Bias Error-MBE

```

```

MBE 1=sum (CDF'-wblcdf (x, c1, k1))/sum (CDF);
MBE 2=sum (CDF'-wblcdf (x, c1, k1))/sum (CDF);
MBE 3=sum (CDF'-wblcdf (x, c1, k1))/sum (CDF);
MBE 4 = [MBE1, MBE2, MBE3] *100;

%Root mean squares error-RMSE

RMSE 1=sqrt (length (CDF)*(sum ((CDF'-wblcdf (x, c1, k1)). ^2)))/sum (CDF);
RMSE 2=sqrt (length (CDF)*(sum ((CDF'-wblcdf (x, c2, k2)). ^2)))/sum (CDF);
RMSE 3=sqrt (length (CDF)*(sum ((CDF'-wblcdf (x, c3, k3)). ^2)))/sum (CDF);
RMSE 4= [RMSE1, RMSE2, RMSE3] *100;
%% Calculation of the minimum, maximum, average, wind characteristics,
median
%% and standard deviation
'min(windspeed2012) \'; min(windspeed2012)
'max(windspeed2012)'; max(windspeed2012)
'mode(windspeed2012)'; mode(windspeed2012)
'median(windspeed2012)'; median(windspeed2012)
'std(windspeed2012)'; std(windspeed2012)

```

7.2. Appendix 2

```

clc, clear, close all

%%
%Determination of the capacity factor (CF)
a=8.22;
b=1.79;
k=b;
%%
FOR wt=1:18
P_N = [1.5,1.5,1.5,1.5,1.8,1.8,2,2,2,2,2,2.5,2.5,2.6,3,3.3,3.3,3.3];
v_in= [3.5,3.5,4,3,3,4,3,4,2.5,4,3,3.5,3,3.5,3.5,3,3,3];
v_n= [11.5,14,12,13,12,12,12,12,13,14,11.5,12.5,13,14,15,13,13,12];
v_ie= [20,25,25,25,20,25,20,25,28,25,20,25,25,23,25,25,25,22.5];

v_in=v_in(wt);
v_n=v_n(wt);
v_ie=v_ie(wt);

%%
k1=1/(c_in-v_ie) ^2;
k2=v_in+v_n;
k3 = ((v_in+v_n)/(2*v_n)) ^3;

```

```

A_0=K1*(v_in*k2-4*v_in*v_n*k3);
A_1=k1*(4*k2*k3-3*v_in-v_n);
A_2=k1*(2-4*k3);
%%
A (1) =A_0; A (2) =A_1; A (3) =A_2;

CFp=0;

FOR i=0:2

    CFp=CFp+A(1i+1) *a^(1i) *gamma(1i/b+1) *(gammainc((v_n/a) ^b), 1i/b+1)-
    gammainc((v_in/a) ^b),1i/b+1));

END

CFp= CFp - exp(-(v_ie/a) ^b) +exp(-(v_n/a) ^b);

CF1=CF1*100;
CFM1(wt)=CF1;
CFM 1 =CFM1
CF2=(exp(-(v_in/a) ^k) -exp(-(v_n/a) ^k))/(((v_n/a) ^k)-((v_in/a) ^k))-
exp((-v_ie/a) ^k));

CFM2(wt)=CF2*100;
CFM 2=CFM2
END

```