THE INFLUENCE OF EXCEL MODELLING PROFESSIONAL DEVELOPMENT ON CONCEPTUAL UNDERSTANDING OF PERIODICITY OF TRIGONOMETRIC FUNCTIONS

by

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Submitted in accordance with the requirements for the degree of

DOCTOR OF PHILOSOPHY

in the subject of

MATHEMATICS, SCIENCE AND TECHNOLOGY
WITH SPECIALISATION IN MATHEMATICS EDUCATION

at the

UNIVERSITY OF SOUTH AFRICA

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January 2020

DECLARATION

I declare that the thesis “The influence of excel modelling professional development on conceptual understanding of periodicity of trigonometric functions” is my own work and has not been previously submitted to any institution of higher education. All sources cited or quoted have been duly indicated and acknowledged by means of complete references.

………………………………………………. January 2020

SIGNATURE DATE

(NHLANHLA LUPAHLA)

ABSTRACT

Utilising an embedded mixed methods research design, this study investigated the influence of an Excel-based modelling (EBM) teacher professional development on learners' conceptual understanding (LCU) of periodicity of trigonometric functions. A purposive sample of 11 Namibia Senior Secondary School Certificate, Higher Level (NSSCH) mathematics teachers and their 123 learners from a specific region in Northern Namibia, participated in the study. Large Hedges' g size effect values ($g \geq 0.8$) of EBM teacher professional development on teachers' TPACK self-efficacy (TSE) were confirmed. The semi-structured learner interviews, analysed using a computer aided qualitative data analysis (CAQDA) tool, established that learners found the EBM instruction to have high impact on their conceptualisation of periodicity of trigonometric functions.

The partial least squares structural equation modelling (PLS-SEM) technique was employed to model the relationships between TPACK constructs and their influence of TPACK on learners' conceptual understanding of periodicity of trigonometric functions. The results indicated that 66% of TPACK is attributed to teachers' Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK) and Pedagogical Content Knowledge (PCK). On the other hand, PLS-SEM showed that there was a significant positive relationship between TPACK and LCU, with TPACK accounting for 47% of variation in LCU ($p < 0.05$). There was, however, a weak correlation between teachers' TPACK self-efficacy and LCU ($r=0.25$).

Key words: Mathematical modelling, Excel-based modelling, Excel-based modelling problem-solving process, Inquiry-based learning, Conceptual understanding, Teacher self-efficacy.

DEDICATION

This thesis is dedicated to my late brother, Ndumiso Lupahla, and my late parents, Usher Bafana Lupahla and Laina Mpofu.

ACKNOWLEDGEMENTS

Firstly, I appreciate the Almighty God for giving me the will, strength and commitment to complete this study.

Secondly, I am grateful to my supervisor, Professor Nosisi Nellie Feza, who, despite moving from the University of South Africa to Central University of Technology - Free State - prior to the completion of my research, remained dedicated to guide and support me throughout the course study.

Similarly, I extend my gratitude to the entire staff members of the Institute for Science and Technology Education (ISTE), University of South Africa, for their constructive feedback in the ISTE postgraduate seminars and conferences where I presented periodic reports on the progress in my work.

I particularly want to extend my sincere gratitude to Professor Werner Blum, University of Kassel (Germany), who, as one of the experts attending the 9th Annual Unisa ISTE Conference on Mathematics, Science and Technology in 2018, availed his time to guide me through defining the modelling context of my study.

Lastly, I am thankful to the following people and institutions:

- My wife, Darlitha and my daughters, Nonhlanhla, Saneliso, Ayanda and Nompumelelo for their support and encouragement.
- All the teachers and learners who participated in the study.
- The parents who consented to their children’s participation.
- The Namibian Ministry of Education, Arts and Culture for giving me access to collect data for this study. I particularly want to acknowledge the leadership of the region and the schools where this study was conducted, for their unwavering support and cooperation during the implementation of the EBM teacher professional development programme.
- Professor Muchativugwa Liberty Hove, the language specialist, who edited my work.

May God bless you all.

ABBREVIATIONS AND ACRONYMS

ABC :	ActivityBased Costing
CAAPSA :	Computer-Aided Algebraic Problem Solving Assessment
CAEMA :	Computer-Aided Excel Modelling Assessment
CAQDA :	Computer-Aided Qualitative Data Analysis
CB-SEM :	Covariance-Based Structural Equation Modelling
CCSSM :	Common Core State Standards in Mathematics
CK :	Content Knowledge
CSCL :	Computer-Supported Collaborative Learning
DNEA :	Directorate of National Examinations and Assessment
EBM :	Excel-Based Modelling
EFL :	English as a Foreign Language
HED :	Higher Education Diploma
HOD :	Head of Department
IBL :	Inquiry-Based Learning
ICT :	Information and Communication Technology
LCU :	Learner Conceptual Understanding
LLO :	Learners’ Learning Outcomes
NCTM :	National Council of Teachers of Mathematics
NIED :	National Institute for Educational Development
NSSCH :	Namibia Secondary School Certificate (Higher Level)
NSSCO :	Namibia Secondary School Certificate (Ordinary Level)
OLS :	Ordinary Least Squares
PCK :	Pedagogical Content Knowledge

PK :	Pedagogical Knowledge
PLS-SEM :	Partial Least Squares Structural Equation Modelling
SPSS :	Statistical Package for the Social Sciences
STEM :	Science, Technology, Engineering and Mathematics
TCK :	Technological Content Knowledge
TIMSS :	Trends in International Mathematics and Science Study
TK :	Technological Knowledge
TPACK :	Technological Pedagogical Content Knowledge
TPK :	Technological Pedagogical Knowledge
TSE :	Teacher self-efficacy
VB-SEM :	Variance-Based Structural Equation Modelling

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CHAPTER ONE
INTRODUCTION AND BACKGROUND

1.1 INTRODUCTION

This study developed an Excel-based modelling (EBM) approach in the teaching of periodicity of trigonometric functions and mapped how the professional development training of teachers in this instructional practice exerted an impact on the development of their Technological Pedagogical Content Knowledge (TPACK) levels and as well as how it influenced learners' conceptual understanding (LCU) of trigonometric functions and graphs in the Namibian Senior Secondary Certificate Higher (NSSCH) level Mathematics curriculum. The study is a practical inquiry in which selected teachers were trained in the implementation of an Excel-based modelling instruction, with further evaluation of its influence on learning outcomes.

It is important to note that the study was conducted from 2014 to 2017, with a specific focus on the teaching and learning of trigonometric functions and graphs in the Namibian Senior Secondary Certificate, Higher (NSSCH) level mathematics curriculum. During this period, the Namibian basic education system consisted of seven (7) years of the Primary Phase (grades 1-7), three (3) years of Junior Secondary phase (grades 8-10) and two (2) years of Senior Secondary phase (grades 11-12), after which successful learners would proceed to higher and tertiary education.

Up to 2017, the Namibian senior secondary mathematics curriculum was differentiated in three tiers, namely:

(a) the Namibia Senior Secondary School Certificate core level (NSSCO-core),

(b) the Namibia Senior Secondary School Certificate extended level (NSSCO-extended), and

(c) the Namibia Senior Secondary School Certificate higher level (NSSCH) (Lupahla, 2014).

According to Lupahla (2014: 10), "the NSSCH Mathematics curriculum was an expansion of the NSSCO core and extended components, with the inclusion of an assessment of competencies in polynomials (remainder and factor theorems), identities, equations and inequalities, vectors in three dimensions, logarithmic and exponential functions, absolute value functions, trigonometric functions and calculus (differentiation and integration)". The content of the NSSCH curriculum is significantly more complex than the NSSCO curriculum and requires application of a higher level of problem-solving skills.

Table 1.1 shows the NSSCO and NSSCH grading systems in Mathematics. The grey cells do not have any grades assigned to the corresponding syllabus levels. The highest possible grade in the core component was C while the NSSCH grading uses a numerical scale from 1 to 4 as indicated. The point system used in the table was based on the admission scales used by the University of Namibia (UNAM) and the Namibia University of Science and Technology (NUST) for learners who wished to pursue further studies in higher education courses.

Table1. 1 The NSSC Grading System (Source: NIED, Ministry of Education, 2005)

NSSCO Level				NSSCH Level		
Symbol	% Core	%	Points	Symbol	% HIGCSE	Points
A*		85 – 100	8	1	75 – 100	9
A		75 – 84	7	2	60 – 74	8
B		65 – 74	6	3	50 – 59	7
C	75 – 100	55 – 64	5	4	40 – 49	6
D	62 – 74	45 – 54	4	U (Fail)	0 – 39	0
E	50 – 61	35 – 44	3			
F	42 – 49		2			
G	35 – 41		1			
U (Fail)	0 – 34		0			

Table 1.2. shows a summary of learners who sat for the NSSCO and NSSCH examinations in the period from 2010 to 2014 and their achievement outcomes. The table shows that on average only 3.4% of the learners took Mathematics at the NSSCH level over this five-year period. According to DNEA (2014), this suggests that not many learners and teachers were confident to tackle the NSSCH Mathematics content, thus raising the need to carry out intensive professional development programmes to upgrade teachers' capacities to deal with the challenges of the NSSCH mathematics content.

Table1. 2: Summary of NSSCO and NSSCH learners' Mathematics achievement from 2010 to 2014 (Source: http//www.dnea.gov.na (retrieved 08/06/2015)

Year	NSSC Mathematics Examination Level	Total Candidate Entries	Pass (A-C/1-3)	% Pass
2014	Ordinary	18752	3752	20.0%
	Higher	662	525	78.6%
2013	Ordinary	18957	3709	19.6%
	Higher	579	459	79.3%
2012	Ordinary	14924	3088	20.7%
	Higher	575	443	77.0%
2011	Ordinary	15249	3159	20.7%
	Higher	560	437	78.0%
2010	Ordinary	13704	2634	19.2%
	Higher	561	428	76.3%

Although the NSSCH candidates' achievement is generally higher than the NSSCO performance, it can be discerned from Table 1.2 that there has not been a dramatic improvement in learners' performance over the years from 2010 to 2014.
A redesigned basic education curriculum was launched in 2015, including adjustments to the number of years of study in different school phases. Eight (8) years of elementary education (pre-primary and grades 1-7), two (2) years of junior secondary education (grades 8-9) and two (2) years of senior secondary education are currently included in the updated curriculum (grades 10-11). Grade 11 is the first exit year of senior secondary school, and it is at this level that learners achieve a globally recognised National Senior Secondary Certificate Ordinary (NSSCO) level. Learners can then go on to grade 12, tertiary institutions, vocational education and training institutes, or the labour market (NIED, 2016).

Learners who proceed to grade 12 acquire an internationally recognised National Senior Secondary Certificate Higher (NSSCH) level, which gives them access to higher education institutions or the job market. A provision for a 13th grade is in place for learners who wish to take subjects at Advanced level. Despite these changes in the curriculum, the findings of this study remain significant and relevant to the revised curriculum, since the NSSCH mathematics content for trigonometric functions and graphs has not been altered in the revised grade 12 mathematics curriculum. The Namibian government makes effort to ensure that higher education institutions supply an adequate number of teachers to deal competently with the demands of the revised Basic Education Curriculum (NIED, 2016).

It is against this backdrop that this action research inquiry was conducted to complement the Namibian government's teacher training programmes. The study uses Guskey's (2000) model to evaluate the influence of the TPACK enhanced technology integration in the teaching of periodicity of trigonometric functions in the NSSCH curriculum.

1.2 BACKGROUND

The findings and recommendations from Lupahla (2014), who analysed the level of development of algebraic problem-solving skills of grade 12 learners in the Oshana region of Northern Namibia, inspired this study. Lupahla (2014) identified that learners struggled to solve non-routine algebraic problems due to a lack of conceptual understanding, a limited variety of problem-solving procedures, and difficulty with the terminology and phrases used in the problems.

Lupahla (2014: 191) also attributed these difficulties to a "lack of classroom training in the problem-solving process." The findings of this study also confirmed and validated that students performed better in tasks that included diagrams to illustrate the mathematical problem. Diagrams help simplify difficult situations and illustrate abstract notions (Kidman, 2002; Pantziara, Gagatsis & Pitta-Pantazi, 2004). In the Namibian Senior Secondary Certificate Higher (NSSCH) level curriculum, however, examiners confirm that learners fail to conceptualise the features of graphs and the relationships between the parameters of functions and their graphs (DNEA, 2013). This anomaly apparently contradicts the findings of Panziara and Pitta-Pantazi (2004) who assert that diagrams, in the form of graphs, make problems easier.

The researcher was therefore motivated to undertake the current study, focusing on areas of the grade 12 Mathematics curriculum that require integration of algebraic, geometric and graphic reasoning. The researcher's focus on the instructional content of trigonometric functions and graphs was informed by national examiners' reports issued by the Directorate of National Examinations and Assessment (DNEA).

The annual examiners' reports released by the Directorate of National Examinations and Assessment (DNEA) in Namibia from 2010 to 2016 suggest that both teachers and learners have difficulties in understanding trigonometric functions and their graphs in the Namibia Senior Secondary Certificate Higher (NSSCH) level Mathematics curriculum. Learners struggle to interpret the graphs of trigonometric functions (DNEA, 2013).

When given a question concerning a graph and an associated function, Knuth (2000) found that learners prefer to undertake sophisticated computations with the function to answer the question rather than reading the answer off the graph. According to Knuth (2000), learners only have a rudimentary comprehension of the link between graphs and functions. While learners frequently construct graphs from functions, he points out that they rarely practice determining functions from graphs. DNEA (2014) observes comparable difficulties among NSSCH mathematics students in grade 12, specifically their inability to connect the graphs of trigonometric functions to the solution of associated trigonometric equations.

The National Council of Teachers of Mathematics (NCTM, 1989) has argued for a change in the secondary school teaching of functions and graphs. According to NCTM (1989), emphasis should be on teaching graph reasoning and the use of computer-based graphing tools. Using graphs to improve conceptual understanding and reasoning could build a new learning environment that moves away from rote memorisation procedures based on traditional paper and pencil graph plotting procedures (NCTM, 1989).

Gebrekal (2007) also observed a number of obstacles in concept formation regarding the teaching and learning of functions in secondary school Mathematics in Eritrea. One of the obstacles was the difficulty to construct graphs of functions. The study suggested that by spending more time on the construction of graphs, learners did not get sufficient time to explore the nature and properties of functions and their graphs. The current study thus sought to develop a technology-enhanced teacher professional development programme and evaluate the implications of its implementation in the teaching of periodicity and derivation of symmetry properties of trigonometric functions and graphs in the NSSCH Mathematics curriculum.

1.3 MOTIVATION FOR THE STUDY

According to Beauchamp and Parkinson (2008), several countries have seen a decrease in the number of students studying Mathematics, as well as a decrease in the performance of those who do. Such failure was attributed to teachers' lack of crucial teaching competences and learners' lack of interest in the subject (Ottervanger, van den Akker, and de Feiter, 2007). This difficulty, according to Koehler and Mishra (2009), is caused by phlegmatic teaching and learning methods. This appears to indicate that effective teaching methods have a great potential for influencing the learning process. In recent years, the use of technology in education has been acknowledged as a significant instrument in supporting successful teaching and learning. Many studies (e.g., Tilya, 2008; Voogt, 2003) have shown the benefits of using technology in education to improve teaching and learning.

Choi-Koh (as cited in Kepceoglu & Yavuz, 2016) explored the patterns of one student's mathematical thought processes and detailed the nature of the trigonometry learning experiences that the student faced while engaged in individual investigations in an interactive technological environment. The use of technology aided the student in moving from the intuitive to the operational and ultimately the application stages of his thought processes (Choi-Koh, 2003).

Another study by Mafi and Lotfi (2012) assessed the impact of CATASCI, a trigonometry software program, on trigonometry students. The study concluded that computer-assisted mathematics education is more effective than traditional mathematics education in terms of student learning. Zengin et al. (2012), on the other hand, employed the dynamic mathematics software GeoGebra in their research to reach the same conclusion on the efficacy of technology use in mathematics courses.

The uniqueness of the current study is grounded on the following:

This study is the first to design and evaluate the outcomes of the implementation of a technology enhanced inquiry-based instructional programme that features a cross fertilisation of two theoretical frameworks, the TPACK framework and Guskey’s (2000) framework for evaluation of teacher professional development.

Hewitt (2008) suggested that there is a lack of critical perspectives among researchers who have contributed towards the TPACK theories. Chai et al., (2013) also noted that none of the reviewed theoretical papers reflexively challenges the TPACK framework. This study is thus the first to initiate a knowledge-building approach that could be applied to enhance teachers' TPACK in novel ways that would help the teachers build their own theories about ICT integration. In the current study, the teachers were exposed to a TPACK enhanced inquiry-based learning professional development programme in the teaching and learning of periodicity of trigonometric functions. The teachers then subsequently implemented the Excel-based modelling instruction with their learners and evaluated the teaching and learning outcomes, enabling them to generate new insights and observations that may reflexively challenge the current theories on the TPACK framework.

According to Argenbright (1993), the exploration of graphs with spreadsheets allows learners to encounter mathematics without having to rely upon an algebraic notation. The modelling of graphs reverses the traditional algebra-first approach and instead becomes a creative process to introduce algebraic concepts. It is against this backdrop that selected NSSCH Mathematics teachers were trained in the use of Excel-based modelling, followed by an evaluation of the impact of the implementation of the programme in teaching periodicity of trigonometric functions. The study further assesses the impact of using Excel-based modelling on the learners' conceptual understanding and problem-solving skill development in periodicity problems.

1.4 STATEMENT OF THE PROBLEM

While trigonometry is an important topic in the secondary school curriculum because it helps students develop cognitive strategies, studies have established that it is a difficult topic for students (Sarac and Aslan-Tutak, 2017). The trigonometry, which includes algebraic equations and formulas such as addition and sum-to-product formulas, makes mathematics complicated. Students have trouble interpreting related subjects because they do not grasp simple trigonometry principles (Steckroth, 2007). In their study of teachers' expectations and understanding of trigonometric concepts, Nabie et al. (2018) found that teachers thought trigonometry was abstract, complicated, and tedious to understand, and that they had little practical knowledge of simple trigonometric concepts. This seems to corroborate the DNEA reports on the apparent difficulties faced by both teachers and students in understanding the periodicity of trigonometric functions.

The largest obstacle in trigonometry is learners' lack of grasp of important concepts, according to research on the teaching and learning of trigonometry. Learners' grasp of trigonometric functions is often fragmented (Orhun, 2001; Brown, 2005), despite the fact that trigonometry is one of the most important courses in secondary school mathematics, requiring the integration of various algebraic, geometric, and graphical reasoning (Demir, 2012). According to research, novices have difficulty working with trigonometric function graphs (Kutluca & Baki, 2009). Brown (2005) verified that students were unable to make the basic connection between the unit circle and the sine function graph, that is, the relationship between a point on the unit circle and its representation on the sine graph. Orhun (2001) therefore advised teaching trigonometric functions through graphs.

Periodicity and its application in solving trigonometric equations is another aspect affecting Namibian higher level mathematics learners (DNEA, 2012). A study by Shama (1998) confirmed that learners made errors in conceptualising non-periodic functions as though they were periodic due to their fallacious conceptualisation of the process. According to Shama (1998) learners’ mistakes stem from an inability to perceive the graphs of functions as a whole, instead focusing on certain parts. Gebrekal (2007) also observed a number of obstacles in concept formation regarding the teaching and learning of functions in secondary school Mathematics in Eritrea. One of the obstacles was the difficulty of constructing graphs of functions. The study suggested that by spending more time on the construction of graphs, learners did not get sufficient time to explore the nature and properties of functions and their graphs.

DNEA’s reports from 2010 to 2016 have specifically cited that learners and perhaps teachers too, struggle to:

- Understand the connection between equations and graphs;
- Obtain the correct transformations of graphs of trigonometric functions;
- Find the amplitude and period and sketch and interpret graphs of the trigonometric functions of the form; $y = a \sin(bx) + c$, $y = a \cos(bx) + c$, and $y = a\tan(bx) + c$; where a, b and c are constant real values.

Despite the large number of trigonometry studies available, there is no study on teachers' expectations of their TPACK self-efficacy in trigonometric principles in the mathematics curriculum (Tuna, 2013). It is on the basis of these research-informed and practically observed problems that the current study developed a technology enhanced instructional intervention and teacher professional development programme and further evaluated its influence on the learners' conceptual understanding of periodicity of trigonoimetric functions.

1.5 RESEARCH QUESTIONS AND OBJECTIVES

1.5.1 RESEARCH QUESTIONS

The central research question of the study is:

How does Excel-based modelling (EBM) teacher professional development influence the learners' conceptual understanding of periodicity of trigonometric functions in the NSSCH curriculum?

In order to answer the central research question, the following research sub-questions are proposed:

RQ1: How do teachers' perceptions of their state of TPACK change after participating in the Excel-based modelling teacher professional development?

RQ2: How does the Excel-based modelling teacher professional development impact the teachers' self-efficacy in teaching periodicity of trigonometric functions?

RQ3: How effective were the teachers in the implementation of the Excel-based modelling instructional practice?

RQ4: What are the learners' perceptions of learning periodicity of trigonometric functions through the Excel-based modelling instruction?

RQ5: What is the influence of teachers' TPACK development on the learners' conceptual understanding of periodicity of trigonometric functions?

1.5.2 Objectives of the study

The overall objectives of the study are set to:

1. Determine to what extent Excel-based modelling training programme changes the perceptions of NSSCH Mathematics teachers about their level of Mathematics – TPACK development in trigonometric functions and graphs;

2. Determine how the Excel-based modelling professional development programme influences teacher self-efficacy in the teaching of trigonometric functions and graphs
3. Evaluate the teacher's effectiveness in the implementation of the Excel-based modelling approach in the teaching of periodicity of trigonometric functions and graphs in the NSSCH Mathematics curriculum;
4. Determine the learners' perceptions of learning periodicity of trigonometric functions through the Excel-based modelling instruction.
5. Determine how the development of teachers' TPACK influences the NSSCH learners' conceptual understanding of periodicity of trigonometric functions.

1.6 SIGNIFICANCE OF THE STUDY

Given that learners' academic achievement in the TPACK integrated lessons has not been reported by any of the reviewed studies (Chai et al., 2003), the current study endeavours to bring a new dimension to fill this gap in research. It is this new dimension of matching the teaching process to the learning response in TPACK integrated lessons that makes the current study unique.

Global research findings have demonstrated that the use of computer technologies enhances learners' conceptual understanding (Roblyer, 2006), improves learners' logical and analytical skills (Goos & Geiger, 1995, Hiebert et al., 2003; Cavanagh, 2006); and helps them to develop higher order thinking and problem-solving skills (Bailey, 1993; Boultoun-Lewis & English, 1998; Jonassen, 1999). None of these studies, however, have focused on TPACK integrated instruction, with specific attention to understanding learners' conception of learning. This is one of the gaps identified by Tsai et al., (2011), that the study strives to fill through exploring the learners' perceptions of learning trigonometric functions and graphs through Excel-based modelling.

According to Niess (2005), Excel-based modelling offers dynamic modelling capabilities that enhance the potential for engaging learners in higher-order thinking skills and exploration beyond initial solutions. Similarly, Niess, Sadri and Lee (2007), recognised the potential of Excel-based modelling for solving non-routine problems, motivating learners, and imparting opportunities for novice problem-solvers to extend phenomena to additional hypothetical situations. Niess et al. (2007) conducted a study to explore the potential of spreadsheets in education and reported that teachers who are able to design and enact Excel-based modelling lessons engage their learners in critical thinking to explore mathematical concepts and processes for accurate analysis. Similarly, Liang and Martin (2008) illustrated how to use interactive spreadsheets to enhance learners' understanding in complex problems involving application of calculus principles. Their study showed that spreadsheets can significantly simplify the interpretations of pure calculus principles.

According to Michelson (2006), functions are a significant part of Mathematics at upper Secondary School. Interactive Excel models allow learners to study the properties of graphs and the relations between the parameters of the functions and their graphs. In Namibia, the NSSCH examiners' reports have identified trigonometry as one of the sources of learners' poor performance and have inferred that perhaps teachers also struggle with the teaching of the content (DNEA, 2014). Trigonometry combines different algebraic, geometric and graphical concepts and procedures, thus this complex topic makes it challenging for learners to understand it conceptually. The current study developed interactive Excel-based applets through which learners can explore the connections between algebraic and graphical representations of trigonometric functions. Teaching graphs in a way that is connected to algebraic representations is recommended by a number of researchers (e.g. Van Dyke, 1994; Knuth, 2000).

"Spreadsheets facilitate a variety of learning styles which can be characterised by the terms: *open-ended, problem-oriented, constructivist, investigative, discovery oriented, active* and *learner-centred*. In addition, they are interactive; they give immediate feedback to changing data; they enable data, formulae and graphical output to be available on the screen at once; they give learners a large measure of control and ownership over their learning" (NCTM, 1989, p. 123).

It is the researcher's conviction that the Excel-based modelling teacher professional development programme and its implementation in the teaching of trigonometric functions and graphs offers learners the power of dynamic discovery, which is an important element of inquiry-based learning (IBL) in Mathematics and Science. It is against this background that the researcher undertook the current study to contribute new perspectives to the understanding of TPACK constructs, with particular reference to the teaching of functions and graphs, and impact on learners' conceptual understanding of periodicity of trigonometric functions.

The study is the first to develop a subject specific TPACK enhanced approach, and focusing on both the teaching and learning processes. A number of authors have highlighted that teachers and learners face problems in teaching and learning Mathematics in spite of financial and human inputs in improving the Mathematics performance in Namibia (Nambira, Kapenda, Tjipueja & Sichombe, 2009; MASTEP, 2002).

The Excel-based modelling instructional design for teaching trigonometric functions and graphs in the NSSCH curriculum could inculcate sound inquiry skills in both teachers and learners. It is this ultimate goal that this study strives to achieve - the transformation of mathematics teaching and learning using information and communication technology (ICT) enhanced pedagogy. It is the researcher's conviction that the findings of the study could ultimately strengthen current theories on the TPACK framework.

1.7 LIMITATIONS OF THE STUDY

Limitations are matters that arise in a study which are out of the researcher's control. (Simon & Goes, 2013). They limit the extent of a study, and sometimes affect the end results and conclusions that could be drawn. One limitation of this study is the low population of the learners doing higher level Mathematics in the whole country. Only 871 out of about 20 000 (4.36%) grade 12 Mathematics candidates nationally, registered for the higher level option in 2017 (DNEA, 2017). It was difficult for the researcher to cover all the regions because the learners doing higher level Mathematics are unevenly distributed across the country's fourteen (14) regions. The researcher therefore opted to select one specific region for this study due to its accessibility and the fact that learners in this region had been comparatively performing well over the previous five (5) years. The region's population of higher level candidates, represented 15.4% of the national population. The sample size of learners in the study represented 91.8% of the region's population and 14.1% of the national population. This study was therefore limited by the sample size inadequacy, hence the findings may not be a true reflection of the national performance trends on the subject of research.

1.8 DELIMITATIONS OF THE STUDY

Delimitations are features that come from constraints in the study's scope as well as deliberate exclusionary and inclusionary decisions made during the study plan (Simon & Goes, 2013). It is on this basis that the researcher used purposive sampling to minimise the effects of the limitations identified in the study on the results. All the schools offering higher level Mathematics in the region were considered for sampling. The sample of the teachers in the study was fairly representative of the overall characteristics of all the NSSCH teachers, considering their training, qualifications and background. The sample of teachers represented 100% of the grade 12 NSSCH teachers in the region and about 22% of the country's population of NSSCH Mathematics teachers.

Although the sample of teachers and learners was small, the research questions, objectives and variables of the study could still be adequately addressed on the basis of the theoretical perspectives upon which the study is conceptualised. The study can be replicated in any other region within the context of the TPACK conceptual framework. The small population and similar characteristics of higher level learners and teachers in the country also enhances the generalisability of the findings. The sampling procedure ensured that the selected learners were not significantly different from the rest of the learners in the national population.

1.9 DEFINITION OF TERMS

For the purposes of clarity, the following key terms are defined as follows:

1.9.1 Inquiry-Based Learning (IBL)

IBL is an approach to teaching and learning that places learners' questions, ideas and observations at the centre of the learning experience. The process involves open-ended investigations into a problem, requiring learners to engage in evidence-based reasoning and creative problem solving (Fielding, 2012). The inquiry based learning approach was adopted as the pedagogical content knowledge component for the teacher professional development programme.

1.9.2 Mathematical modelling

Many different interpretations of mathematical modelling have developed from various research orientations. Mathematical modelling, according to Burghes (as cited in Ferrucci, 2003), is a unifying theme for all mathematical applications. The essence of mathematical modelling, according to Mason and Davis (1991), is a movement between the physical situation being described and the exact mathematical representations of that model.

The mathematical representations are the derived algebraic representations of the mathematical relations exhibited in the displays, while the real world is represented by the Excel visual displays in this study. Mathematical modelling is defined in this study as the technique of describing phenomena using a combination of algebraic and graphical representations. Assumptions and variables are transformed into visual graphic models during the modelling process. If the original model is insufficient, the problem's assumptions must be updated, and the cycle must be repeated.

1.9.3 Excel-based modelling

The organised use of an Excel inquiry applet through which the user can manipulate the values of the parameters of a function thus enabling the user to explore the connections between graphs and their functions. Excel modelling facilitates the translation of the visualisations of the computer screen graphic displays (real world) to the algebraic representations that are derived through the modelling inquiry process (the mathematics).

1.9.4 Excel-based modelling problem solving process

This entails problem solving that uses visual Excel graphic models to establish the mathematical connections that exist between function variables. In the case of this study, the process involves investigating the mathematical connections through inductively exploring the graphic displays and their translation into algebraic representations, and how the algebraic representations are influenced by periodicity of trigonometric functions. Relevant conclusions are then drawn to allow for generalisation of the solution process.

1.9.5 Conceptual understanding

This entails an understanding of mathematical ideas, procedures, and relationships (Kilpatrick et al., 2001). Students' reasoning skills and conceptual understanding are enhanced by classroom standards that require students to justify and explain their ideas in order to make them clear (Kilpatrick et al., 2001).

1.9.6 Teacher self-efficacy

Self-efficacy is described as "beliefs in one's ability to plan and carry out the steps necessary to achieve specific goals" (Bandura, 1997, p. 3). Self-efficacy is measured in this study by the teachers' self-ratings of their TPACK capabilities on a scale of 1 (very low) to 5 (very high) (advanced). The change in mean ratings for each of the seven TPACK areas reflects a shift in teachers' self-efficacy in those categories.

1.10 OVERVIEW OF THE THESIS

The structure of the thesis is outlined in the following segment.

1.10.1 CHAPTER ONE: Introduction and background

This chapter provides an introduction to the research. This includes the background to the study, motivation for the study, statement of the problem, research questions and objectives, the significance of the study, limitations as well as delimitations of the study. The chapter further provides the definition of key terms used in the context of the study as well as an overview of the whole thesis.

1.10.2 CHAPTER TWO: Literature review

This chapter deals with the literature review, starting with an overview of the problem from a national and global perspective. The chapter also reviews findings of some previous studies on the teaching and learning of graphs and functions, and how the mathematics education community of practice has attempted to address the challenges. In particular, the chapter discusses the strengths of spreadsheet (Excel) modelling in the teaching of Mathematics, with specific focus on trigonometric functions and graphs. The last section of the chapter discusses the theoretical and conceptual frameworks used in this study to design the professional development programme for teachers and to evaluate its influence in the teaching of periodicity of trigonometric functions to a sample of Grade 12 higher level mathematics learners. The chapter further discusses the literature on the technological pedagogical and content knowledge (TPACK) framework, identifying its strengths and gaps, in order to ensure that the study has global significance towards potentially enriching the TPACK theories. Guskey’s (2000) model of evaluation of teacher professional development is also discussed. The basic modelling process is also discussed alongside the TIMSS

(2007) assessment framework, used for the evaluation of the learners' level of conceptual understanding of periodicity of trigonometric functions.

1.10.3 CHAPTER THREE: Research design and methodology

The chapter discusses in detail the research methodology employed in the study. It also presents the research design, the embedded intervention design, in which qualitative and quantitative data are collected simultaneously or sequentially, but with quantitative data playing a supportive role to the qualitative data (Creswell, 2012). The chapter presents the rationale for the choice of the research site, participants and in particular the sampling techniques. The chapter further outlines the data collection process and instruments. The validity and reliability of the instruments as well as ethical issues taken into consideration are also discussed in Chapter 3.

1.10.4 CHAPTER FOUR: Data presentation and analysis

This chapter presents the data and the analysis. The chapter also analyses data using Guskey's (2000) framework for evaluation of professional development and partial least squares structural equation modelling (PLS-SEM), the relationships between the TPACK constructs and the influence of teachers' TPACK on the learner's conceptual understanding of periodicity of trigonometric functions.

1.10.5 CHAPTER FIVE: Discussion of findings

This chapter presents the discussion of the findings in terms of the formulated research questions.

1.10.6 CHAPTER SIX: Summary, conclusions and recommendations

Chapter Six provides the conclusions drawn and recommendations of the study. The chapter further reflects on how the findings address the gaps in the TPACK theories and also suggests avenues for further study.

1.11 SUMMARY

This study's orientation was established in this chapter. The background to the study, motivation for the study, statement of the problem, research questions and objectives, the significance of the study, limitations as well as delimitations of the study were presented and discussed.

CHAPTER TWO
LITERATURE REVIEW

2.1 INTRODUCTION

This chapter presents a review of literature related to national and academic perspectives on the teaching and learning of trigonometric functions and graphs. It discusses the use of spreadsheets to support teaching and learning, the influence of graphing technology in enhancing visulaisation in mathematics, theoretical views of TPACK integration in the teaching and learning of trigonometric functions. The chapter culminates with a discussion of the theoretical frameworks as well as the conceptual framework generated from the two theoretical frameworks used in this study.

Prior to beginning any investigation, it is critical to identify and clarify what is already known in a certain domain of knowledge. This is vital for the study's relevance (Hart, 1998). To lay a solid foundation for the study approach, the researcher did a literature study (Webster & Watson, 2002). The results of the literature evaluation process should show that the planned research adds to the overall body of knowledge and contributes new knowledge (Sethi & King, 1998).

2.2 PERSPECTIVES ON TRIGONOMETRIC FUNCTIONS AND GRAPHS

2.2.1 National perspective

This part of the literature review clarifies the research problem in order get justify the significance of the study. The literature review includes an assessment of the NSSCH Mathematics syllabus content on functions and graphs, the NSSCH examination content on functions and graphs for a five-year period preceding the current study as well as the DNEA examiners' reports for the same period. The statement of the problem in section 1.3 was informed by the analysis of the NSSCH examination content and the DNEA examiners' reports from 2010 to 2014.

Table 2.1 shows the NSSCH mathematics syllabus content on functions and graphs, in which DNEA reported learners' poor performance, and was the focus of the action research inquiry dimension of the current study.

Table 2. 1: NSSCH Mathematics syllabus content cited by DNEA as among the areas of poor learner achievement from 2010 to 2014 (Source: NIED, Ministry of Education,

TOPIC	GENERAL OBJECTIVES	SPECIFIC OBJECTIVES
	Learners will:	Learners will be able to:
17. Trigonometry		
	- understand the meaning of a radian measure - know trigonometrical identities and use them to solve trigonometrical equations - understand amplitude and period of a trigonometrical function and apply to graphs	1 convert between radians and degrees and apply to equations and graphs 2 use the six trigonometrical functions for angles of any magnitude 3 recall the identities $\frac{\sin A}{\cos A} = \tan A$, $\frac{\cos A}{\sin A} = \cot A$, $\frac{1}{\sec A} = \cos A$, $\frac{1}{\operatorname{cosec} A} = \sin A$, $\sin^2 A + \cos^2 A = 1$, $\sec^2 A = 1 + \tan^2 A$, $\operatorname{cosec}^2 A = 1 + \cot^2 A$, and select and use one or more of these as appropriate to the context 4 find the amplitude and period and sketch and interpret graphs of the form $y = a\sin(bx) + c$, $y = a\cos(bx) + c$, and $y = a\tan(bx) + c$ 5 find solutions, in degrees or radians and within a specified interval, of the equations $\sin(nx) = k$, $\cos(nx) = k$, $\tan(nx) = k$, and of equations easily reducible to these forms, where n is a small integer or a simple fraction

Trigonometry is an important area of study with many important applications in engineering, astronomy, physics, architecture, and other quantitative disciplines. A firm understanding of trigonometric functions is a prerequisite for successful problem-solving processes in calculus and analysis. Hence, trigonometry has an important place in the mathematics curriculum in many countries even though its applications and teachability may vary from country to country at secondary school level (Delice & Roper, 2006).

2.2.2 Global perspective

According to Demir (2012), the complex nature of trigonometry is that it integrates concepts and procedures from algebra, geometry and trigonometry. For example, what makes trigonometric functions distinctive is that they cannot be computed directly through mere arithmetic algorithms, but require application of geometric and algebraic reasoning. The complex nature of trigonometric functions makes it difficult for secondary school learners to understand the topic conceptually (Demir, 2012).

Research on trigonometry is scant (Weber, 2005; Moore, 2010), and there appear to be only a few researchers exploring learners' levels of understanding of trigonometry under different instructional programmes. It is necessary for learners to develop a strong understanding of trigonometry concepts and to visualise the connections among the different contexts, namely; Triangle Trigonometry (trigonometry based on ratio definition in right angled triangles), Unit Circle Trigonometry (Trigonometric functions are defined as coordinates of points on the unit circle based on rotation angles), and Trigonometric Function Graphs (Trigonometric functions are defined in the domain of real numbers) (Demir, 2012).

Periodicity is an important concept which precedes applications of trigonometric functions in some physical real life phenomena such as simple harmonic motion and waves. The understanding of the meaning of periodicity, and connections between trigonometric graphs and the unit circle in regard to periodicity are all important, especially for successful solution of trigonometric equations.

Van Dormolen and Zaslavsky (2003) discussed the following two definitions of periodicity:

Definition A: A function *f(x)* is called periodic if there exists a non-zero number *n*, such that for every *x* that belongs to the domain of *f*, the following conditions are fulfilled:

(a) $x \pm n$ belongs to the domain of *f(x)*,

(b) $f(x \pm n) = f(x)$.

Definition G: A function *f(x)* is called periodic if there exists a non-trivial translation of the graph of *f(x)* along the horizontal axis such that the image coincides with the original (Demir, 2012).

Van Dormolen and Zaslavsky (2003) mentioned that these two definitions are equivalent, but definition A is analytic whereas definition G is global. They also distinguished them in that A is from a point-wise perspective while G is about the function as a whole.

Demir (2012) analysed these two perspectives of periodicity. According to Demir (2012), the first perspective indicates that learners view periodicity as a dynamic collection of elements. According to the second viewpoint, learners fail to visualise function graphs as a whole, preferring to concentrate on specific components. Students, for example, prefer to see the area enclosed by a line or a curve and the x-axis together as the period in a periodic graph between noncontinuity points, or the area enclosed by a line or a curve and the x-axis as the period in a periodic graph between noncontinuity points (Demir, 2012).

Learners are prone to regard the interval [0, π] as the sine function's period by looking at the closed curve in this interval as a whole and perceiving it as a repeating pattern in a graphical sense (Demir, 2012). Such misunderstandings can be avoided if students develop a good conceptual knowledge of periodicity (Demir, 2012). In this regard, Van Dormolen and Zaslavsky (2003)'s second definition G may be more accessible at the beginner level. Furthermore, linking periodicity to both trigonometric graphs and the unit circle may help students avoid making similar mistakes.

2.3 THE USE OF SPREADSHEETS TO SUPPORT TEACHING AND LEARNING

In this section, the researcher reviewed literature on spreadsheet applications from previous similar studies, particularly those illustrating the connections between trigonometric functions and their graphs. The purpose was to place the study in the context of existing work and provide justification for the relevance and significance of the current study.

Argenbright (2005) employed dynamic graphical displays to provide strong classroom demonstrations that improved mathematics learning while also giving students new ideas for incorporating appealing visual elements into their mathematics assignments and projects. Argenbright (2005), for example, demonstrated how to construct dynamic graphs of a function and its first two derivatives rather than static images.

The Excel environment makes it easier to understand and express functions in a variety of ways, such as from a formula, a table of data, or a graph. In addition, the Excel spreadsheet allows one to graphically show certain concepts and processes in a dynamic manner, which is difficult to do on paper or on a chalkboard. There is a lot of expertise and understanding about utilising spreadsheets to improve mathematics learning (Neuwirth, 2001).

There are some advantages in using the spreadsheets software for Science, Technology, Engineering and Mathematics (STEM) education:

- Spreadsheets provide an easily available graphical facility to show the results;
- It is possible to show the animations and real time changes in results;
- There is no need to write a big computer program for an algorithm, since the spreadsheet itself has some features which can be used;

- To test an algorithm in action, there is no need to write a complicated program. One can test his/her algorithm using Excel;
- Usually the spreadsheet is installed in newest computers and there is no need to buy or find a compiler;
- The Solver and Goal Seeking tools in Excel spreadsheet can perform optimization jobs.

The following are some of the benefits that spreadsheet software provides to both teachers and students, according to Chaamwe and Shumba (2016):

1) The broad usage and understanding of spreadsheet software reduces the expenses of purchasing, teaching, and learning the mechanics of a new software tool; 2) Such software is frequently taught and supported by personnel other than statisticians.

3) Teachers can create templates ahead of time for pupils to follow and complete certain calculations.

4) The spreadsheet calculating paradigm, with a few exceptions, allows for instantaneous updating of results when data is modified.

5) Because spreadsheets are such a versatile computing tool, they can easily be "coded" to execute calculations that aren't typical,

6) Spreadsheet software can now do many common statistical calculations, and

7) Spreadsheets are handy for inputting, updating, and modifying data before sending it for analysis to a traditional statistical program.

Spreadsheets may also be utilised to examine abstract notions in quantitative-based professions like accounting and finance, according to Dania et al. (2019). Understanding how students interpret the usage of spreadsheets in quantitative-based courses is critical, given the increased usage of computers in the classroom. However, Dania et al. (2019) pointed out that using spreadsheets needs a conceptual grasp of computers and that students and teachers may face a high learning curve (of spreadsheet software). In classes where spreadsheets are used often, students who have not been exposed to computers may feel left behind. Instructors who have never used computers or similar software may struggle to design courses using spreadsheets.

The TPACK framework was utilised in the EBM teacher professional development to guarantee that any technical gaps in the usage of Excel spreadsheets, if any, were addressed during the training prior to the implementation of the EBM lessons. It was also believed that teachers would be able to demonstrate to their learners how to use the EBM applets to investigate the relationships between trigonometric functions and their graphs, thus the EBM applets were made user-friendly.

Based on the advantages of using the spreadsheets in STEM education, the researcher chose Excel software to model graphs of trigonometric functions and solution of related trigonometric equations. The Excel modelling tools used in the study, were developed by the researcher and refined them during the pilot study phase.

Swetz (as cited in Ofori-Kusi, 2017), views modelling as a process that has its foundation on a system of conjectures. The process must begin with problem identification and lead to formulation of a hypothesis of the solution, collection and testing of data against the formulated hypothesis, and culminate in conclusions supported by mathematical rules. The modelling process is cyclic in nature because the process can re-start with the first step if the conclusions reached are not supported by mathematical rules.

The Excel-based modelling of trigonometric graphs (geometric representation) is expected to create a link to the algebraic representation of the graphs. Hopefully, such a platform should enhance the development of learners' ability to solve related problems with clear understanding of these mathematical representations.

Agyei and Voogt (2010) showed that the instructional use of spreadsheets to support visualisation of mathematical concepts enhanced the development of higher order thinking skills and mathematical concept formation in learners. In investigating trigonometric functions in the form $y = a\sin^2 x + b\sin x + c$, an interactive spreadsheet was used to prepare a graph of the function on the interval $0^0 \leq x \leq 360^0$. The y-values were determined using a formula with cell references for the parameters *a, b* and *c*. Learners got immediate visual screen output of the graphs, which enabled them to interpret how changes in the parameters affected the graphs. Instead of teachers communicating the concepts through “rules of thumb”, the learners were able to construct their own meanings of the relationship *between the graphs and their functions* (metacognition). The teachers facilitated the learners’ exploration process by changing the values of the parameters without having to resort to the manual drawing of the graphs on the board. This helped learners to explore many examples through observations and generalisation of patterns regarding the effect of the parameters on the trigonometric functions, thus conceptualising their mathematical knowledge. The current study uses Excel-based modelling as the technology component of the TPACK enhanced teacher professional development programme.

According to Cheverie (2012), spreadsheet applications support visualisation and organisation of non-numerical data. Furthermore, spreadsheets have the ability to make fast and accurate calculations through their inbuilt formula functions. These are some of the arguments and observations that motivated the researcher to use Excel-based modelling than the more popular contemporary graphing software. Allowing the teachers to understand the content of the programs and formulae that lead to visualised graphic outputs, could strengthen the teachers’ content knowledge of functions, for example the modelling of the **tangent** function was done in terms of the **sine** and **cosine** function, and this study was the first to develop an accurate and working model of the tangent function.

2.4 THE INFLUENCE OF GRAPHING TECHNOLOGY IN ENHANCING VISUALISATION IN MATHEMATICS

In terms of student learning, research has shown that using graphing technology improves students' spatial visualisation skills, critical thinking capacity, ability to establish connections across graphical, tabular, numerical, and algebraic representations, and overall mathematics accomplishment. Perhaps most crucially, O'Callaghan (1992) discovered that using graphing technology, pupils were able to greatly improve their general self-concept and attitude toward mathematics. They also seemed to appreciate the subject more because of the technology than the more analytic ones. In addition, O' Callaghan (1992) discovered that integrating graphing technology in the classroom helps close the achievement gap between high and low-achieving mathematics students.

Cunningham (1994) asserts that adding images to words enhances mathematical understanding, a view corroborated by Lupahla (2014). Lupahla (2014) investigated the algebraic problem-solving skills of 243 grade 12 Mathematics learners of a specific region in Namibia, and found that learners performed better in questions with diagrammatic (visual) illustrations. A similar view point is provided by Olsson (2017), who argued that students can rapidly and efficiently generate mathematical artefacts as visual representations like algebraic expressions and accompanying graphs using dynamic tools such as GeoGebra. These representations are dynamically related, which means that if one is modified, the others will change as well, providing opportunities to explore and analyse various facets and relationships between these objects.

On the other hand, technology allows for the quick creation and alteration of representations, exposing data trends and patterns (Vitale et al., 2015). Simulations will link graphs to complicated topics like climate change and traffic accidents (Adams & Shrum, 1990; McElhaney & Linn, 2011). Students may use technology to perform investigations of their own (Vitale et al., 2015). Roschelle et al. (2010) investigated a SimCalc simulation that compares a location versus time line graph with a jogging animation of characters. Students were instructed to make assumptions about what they expected to happen, after which they analysed and compared how a given function in one representation (e.g., a high, forward jogging speed) is represented in the alternate representation (e.g., a quick, forward jogging speed) (i.e., a steep positive slope).

When programs like SimCalc are used in conjunction with scaffolds to encourage students to make sense of the visual feedback, the authors argue that students are better able to connect the graph to the real world (Roschelle et al., 2010). Students can benefit from this technology with the help of both teachers and software. In research, simulations are used in design studies (Applebaum et al., 2017; McElhaney & Linn, 2011; Vitale et al., 2015) to offer visual input that is not present in traditional teaching.

Dermot and colleagues (2020) used meta-analysis to examine architecture and comparative experiments affecting 7699 students over a 35-year period. These researchers, which included graphing technology such as models, web resources such as graph utilities, and sensors, showed that graph technologies had a positive effect on both math learning and graphing. These researchers found that graphing technology improved mathematics learning by allowing students to conduct investigations that result in the generation of theories or predictions.

In particular, graphing technologies help students gain a better grasp of concepts and develop optimistic attitudes toward math (Adegoke, 2016). Furthermore, graphing technologies support teacher professional growth, especially in the teaching of mathematics (Jelatu, Sariyasa, & Ardana, 2018). They are also known to improve the understanding of mathematical concepts in geometry (Singh, 2018), trigonometry (Kepceoglu, 2016), and linear algebra (Mudaly & Fletcher, 2019). They've also been seen to help students understand geometric (Singh, 2018), trigonometric (Kepceoglu, 2016), and linear algebra concepts (Mudaly & Fletcher, 2019).

The basic modelling cycle, as illustrated in this study's conceptual model, also emphasizes the growth of learners' hypothesis generating skills through the discovery of relations between graphic and algebraic representations of trigonometric functions.

2.5 THEORETICAL FRAMEWORK

A theoretical framework serves as a roadmap for study (Grant & Osanloo, 2014). It is a framework that is built on current theory in a field of research and is related to or reflects a study's hypothesis. It serves as the foundation upon which a study is built. A theoretical framework serves the same purpose as a map or a travel itinerary (Sinclair, 2007; Fulton & Kramovich-Miller, 2010). The theoretical framework is made up of a theory's principles, constructions, concepts, and tenets (Grant & Osanloo, 2014).

The current study derived its theoretical framework from a combination of two theories; the technological pedagogical content knowledge (TPACK) framework, and Guskey's (2000) framework for evaluating teacher professional development programmes.

2.5.1 The TPACK framework

TPACK (technological pedagogical content knowledge) is a clear and useful framework for researchers working to understand technology integration in learning and teaching (Mishra and Koehler, 2006). TPACK is the interweaving of technology, pedagogy, and content. Figure 2.1 shows the seven TPACK components.

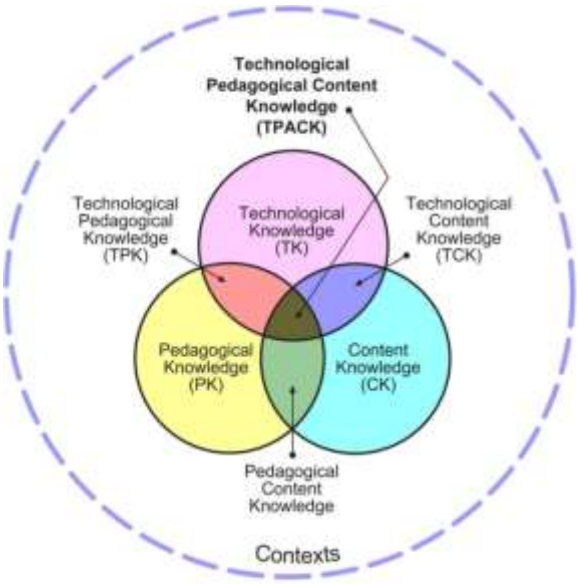


Figure 2. 1: The seven components of the TPACK framework (Mishra & Koehler, 2006).

The Seven components included in the TPACK framework, are defined as:

1. Technology knowledge (TK): Understanding of a variety of technologies, ranging from low-tech devices like pencil and paper to high-tech devices like the internet, digital video, interactive whiteboards, and software applications (Agyei, 2013). The required technology knowledge in the context of this study is the teachers' expertise of Excel-based visual modelling.

2. Content knowledge (CK): Knowledge of the topic area that teachers must be familiar with in order to properly teach (Agyei, 2013). The needed subject knowledge is the teachers' conceptual understanding of the links between trigonometric functions, their graphs, trigonometric ideas, operations, relations, and trigonometric equations.

3. Pedagogical knowledge (PK): "It is a collection of abilities that instructors must acquire in order to effectively manage and organize teaching and learning activities in order to achieve the desired learning outcomes. This expertise includes, but is not limited to, classroom management activities, the role of student motivation, lesson design, and evaluation of learning, as well as evaluation for learning" (Mishra & Koehler, 2006, p. 3). The teacher's pedagogical expertise manifests itself in their awareness of the benefits of the inquiry-based learning (IBL) approach, implementation of the IBL-pedagogy, assessment skills, and subject management.

4. Pedagogical content knowledge (PCK): PCK is defined by Shulman (as stated in Agyei, 2012) as knowledge about the teaching process (Shulman, 1986). Varied subject areas require different levels of pedagogical content knowledge, which combines both material and pedagogy with the purpose of improving teaching practices in specific topic areas. Teachers' pedagogical topic knowledge is measured by their ability to guide learners using Excel-based modelling to investigate the algebraic relationship between functions and their graphical representations in an inquiry-based learning environment.

5. Technological content knowledge (TCK): TCK is a knowledge of how technology and content interact and impact, complement, or inhibit one other (Mishra & Koehler, 2006). “Teachers must master more than the subject matter they teach; they must also have a thorough awareness of how the subject matter (or the types of representations that might be generated) may be altered by the use of certain technologies. Teachers must know which technologies are most suited for addressing subject-matter learning in their domains, as well as how content dictates or even transforms technology - or vice versa.” Mishra and Koehler (2006, p.3). TCK is defined as the capacity of instructors to utilise Excel to interactively model the links between the algebraic representation of functions and their graphs (Koh, 2013).

6. Technological pedagogical knowledge (TPK): TPK is defined by (Mishra & Koehler, 2006: 3) as "knowledge of how teaching and learning might alter when specific technologies are employed in certain ways." Consider how whiteboards may be utilised in the classroom. Whiteboards are commonly used in schools because they are stationary, visible to a large number of people, and easily modifiable. As a result, the whiteboard is typically situated in the front of the classroom and is managed by the teacher.” This location establishes a physical order in the classroom by dictating the positioning of tables and chairs, as well as structuring the nature of learner-teacher interaction, as students are typically only allowed to use it when the instructor requests it (Cohen, 2006).

The current study ensured that teachers could use a data projector so that they could demonstrate the effect of changing the parameters of a function by projecting the graphic display of the function onto a white screen. In this way, the teachers would facilitate a collaborative inquiry-based learning approach, also affording learners an opportunity to explore these connections by themselves, through the use of technology.

7. Technological pedagogical content knowledge (TPACK): Teachers who have TPACK behave with an intuitive awareness of how the three basic components of knowledge interact (CK, PK, and TK). The goal of the research was to determine the consequences of combining these three essential components through the teaching of functions and graphs (CK), inquiry-based learning (PK), and Excel-based modelling applets (TK).

2.5.1.1 TPACK as a framework for measuring teaching knowledge

The use of TPACK as a framework for assessing teaching knowledge might have an influence on the types of teacher professional development programs available (Lehiste, 2015). Using evaluations before and after a specific course or training program is one technique for measuring the growth of teachers' TPACK over time (Chai, Koh & Tsai, 2010; Hu & Fyfe, 2010; Hoffer & Grandegenett, 2012). These researchers found that pre-service teachers in a Singapore educational technology course made more significant advances in Content Knowledge (CK), Pedagogical Knowledge (PK), Technological Knowledge (TK), and most significantly in Technological Pedagogical Content Knowledge (TPACK) with relatively large effect sizes. In an educational technology course in Australia, Hu and Fyfe (2010) conducted a similar study. Teachers' self-efficacy to integrate their use of technology with curriculum and pedagogy grew dramatically, according to post-course survey data.

Chai, Koh, and Tsai (2013) conducted a survey of 74 journal articles that looked at ICT integration via the lens of technical pedagogical content knowledge (TPACK). The evaluation found that there are still a lot of ways to use the TPACK framework to bring about positive improvements in education. While ICT is becoming more common in classrooms and children are increasingly growing up with it, many instructors still find using ICT for teaching and learning a significantly difficult challenge (Shafer, 2008; So & Kim, 2009). As a result, TPACK is an essential theoretical framework that has aided in the direction of research concerning teachers' use of ICT (Brush & Saye, 2009; Kramarski & Michalsky, 2010).

More creation and research of technological environments based on TPACK, investigation of learners' learning conceptions with technology, and cross fertilisation of TPACK with other theoretical frameworks connected to the study of technology integration are all proposed by Chai et al., (2013). They also noticed that none of the research they assessed reported on students' academic progress in the TPACK integrated lessons. This is an obvious research gap that needs to be addressed. In this study, the researcher examines the impact of teachers' TPACK development on learners' conceptual understanding of trigonometric function periodicity.

In order to guide future research, Chai et al. (2013) developed a new depiction of the TPACK framework that incorporates the learners' perspectives of learning with technology. Marton, Dall'Alba, & Beaty, 1993; Tsai et al., 2011) define conception of learning as how learners view or interpret their experiences in a technology-enhanced learning environment toward specific topic.

Hoffer and Grandgenett (2012) mapped pre-service teachers' TPACK throughout the course of an 11-month training program and found that the students' TPACK grew significantly over time. Kurt, Mishra, and Kocoglu (2013) performed a survey of pre-service teachers in Turkey, and their findings revealed that their TK, TCK, TPK, and TPACK scores had increased statistically significantly.

Graham et al. (2009) investigated the TK, TCK, TPK, and TPACK of in-service teachers who took part in a US university's intense professional development program. The results showed that the participants had the most confidence in their TK at the start and conclusion of the course, followed by TPK, TPACK, and lastly TCK.

2.5.1.2 The relationship between TPACK domains

There is evidence that particular TPACK knowledge areas have an impact on teachers' overall TPACK views. Pedagogical knowledge (PK) and technological pedagogical knowledge (TPK) have the most influence on the development of TPACK, according to several research (Chai, Koh, Tsai, & Tan, 2011). TPK, TCK, and TPACK of pre-service teachers were shown to have high positive relationships, according to Koh and Sing (as cited in Lehiste, 2015). In a qualitative research, Koh and Divaharan (2011) discovered that pre-service teachers were mostly concerned with issues related to TPK.

A structural model of the TPACK framework formulated according to the conceptions of Mishra and Koehler (2006) is shown in Fig. 2.2.

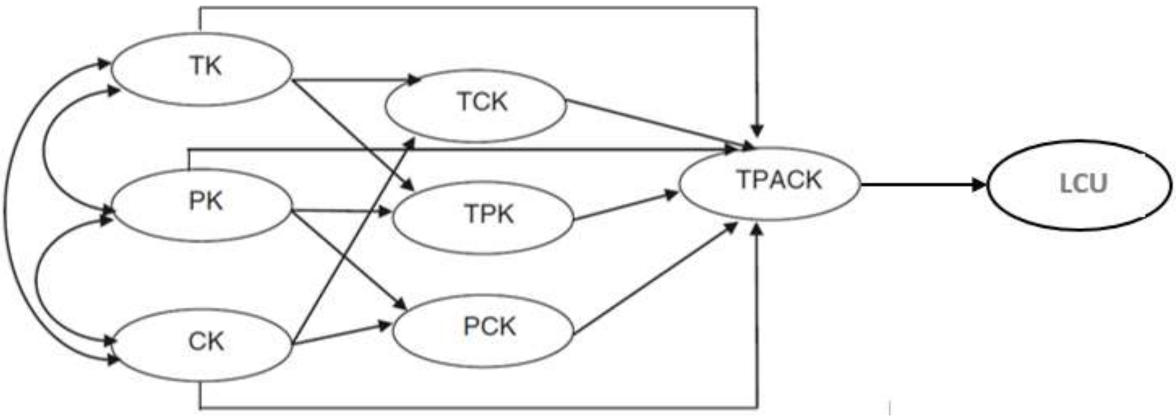


Figure 2. 2: Structural equation model of the TPACK framework and influence on LCU

According to Koh, Chai and Tsai (2013), this structural model hypothesizes two pathways to TPACK:

H1: TK, PK, and CK have direct and positive effects on teachers' TPACK.

In Hypothesis 1, TK, PK, and CK are defined as exogenous variables.

This hypothesis addresses Mishra and Koehler's (2006) postulation of TK, PK, and CK as the three main sources of TPACK.

H2: TK, PK, and CK have direct and positive effects on teachers' TCK, TPK, and PCK, which in turn have direct and positive effects on teachers' TPACK.

In addition, this study proposes the following hypothesis:

H3: TPACK has a positive effect on LCU

PCK, according to Shulman (1986), is separate from PK and CK. Nonetheless, based on Mishra and Koehler's (2006) description, the intermediate variables of PCK, TCK, and TPK cannot be unconnected to TK, PK, and CK. As a result, TPK, TCK, and PCK intermediate variables are characterised as endogenous variables with direct links to TPACK. In terms of the research of the development of the teachers' TPACK constructs and the effect of TPACK development on learners' conceptual understanding of periodicity of trigonometric functions, the suggested structural model by Koh, Chai, and Tsai (2013) also inspired the methodology of the current study.

2.5.1.3 Influence of development of teachers' TPACK on learners' conceptual understanding

ICT integration in teaching and learning has the potential to increase student conceptual comprehension and accomplishment in mathematics, according to several findings from mathematics education research (Chandra & Briskey, 2012; Tay, Lim, Lim, & Koh, 2012). Furthermore, the usage of ICT encourages students to collaborate more. As a result, in an ICT-enabled setting, students may freely investigate and comprehend key mathematical topics (Sang, Valcke, Braak, Tondeur, & Zhu, 2011; Crisan, 2004). As a result, it's critical to support professional development programs that provide teachers the tools they need to effectively use and integrate technology in mathematics classes.

A research by Mogari and Ogbonnaya (2014) examined the association between grade 11 students' trigonometric function achievement and their teachers' content knowledge (CK). The study's data was collected using cognitive tests administered to all participants, followed by a correlational and linear regression analysis. The Pearson product moment index revealed a statistically significant link between student accomplishment and teacher content knowledge (CK), with linear regression analysis revealing that instructor subject knowledge accounted for 76.8% of the variation in student achievement. As a result, the study concluded that teacher content knowledge is critical to improving student learning.

The current study is premised on the hypotheses by Koh et al. (2013) that TK, PK and CK have direct and positive effects on teachers' TCK, TPK, PCK and TPACK. A correlational analysis of the TPACK level of teachers' and learners' conceptual understanding of trigonometric functions, would therefore be a more holistic reflection of the influence of each exogenous variable on teachers' TPACK, hence the influence of each on learners' achievement.

2.5.2 Evaluation models for teacher professional development programmes

A variety of assessment methodologies are presently available for directing and analysing program development, professional development, and implementation success. Stufflebeam (2000, 2007), Stake (2000), Scriven (1994), Kirkpatrick & Kirkpatrick (2006), Guskey (1991, 2000, 2002), and others have all produced systematic assessment and evaluation frameworks that have been widely utilised to effect educational change. “The premise is that a school system's selected model is a useful instrument that can help them plan and evaluate their professional development initiatives. This assumption is rarely, if ever, backed by actual evidence, and is frequently based on customary practice. As a result, while schools may spend a lot of money planning and presenting professional development opportunities for their teachers, they often have little or no data to show that the criteria they choose for their training are reliable predictors of effectiveness” (Newman, 2010, p. 84).

2.5.2.1 Guskey’s framework for evaluation of teacher professional development

Professional development, according to Guskey (2000), is a set of procedures and activities aimed at improving educators' professional knowledge, abilities, and attitudes so that they can increase learners' learning. According to Guskey (2000), three major elements influence the quality of professional development: context (learning communities, leadership, and resources); process (data-driven, evaluation, research-based, design, learning, and collaboration); and content (research-based, design, learning, and collaboration) (equity, quality teaching and family involvement). In the selection of the study site, the researcher ensured that the selected region, schools and teachers adequately met these standards.

Guskey’s (2000) professional development framework emphasises the relationships assumed to exist between five levels; participants’ reactions, participants’ learning, organisational support and change, participants’ use of new knowledge and skills and learners’ learning outcomes. Guskey (2000) indicated that with each succeeding level, the process of gathering evaluation information gets slightly more complex. He suggested that each level builds on the preceding levels, with success at one level necessary for success at the higher levels. The purpose of the information gathered at each of the five levels is explained in the next section.

Level 1: Participants’ Reactions

At this stage, data is gathered to determine how participants felt about the material's quality and utility, as well as the format and delivery of the programme.

Level 2: Participants’ Learning

This level collects data to assess how well participants gained the desired knowledge and abilities as a result of their participation in the professional development program.

Level 3: Organisational Support and Change

This phase assesses the degree to which the program was supported by the organisations involved, beyond the level of the instructor. This aids the evaluator in comprehending the precise contextual aspects that may influence program performance.

Level 4: Participants' Use of New Knowledge and Skills

This level assesses how the professional growth has influenced the participants' practice. It verifies how well new skills and information are transferred to the classroom. The amount of time necessary for instructors to practice and reflect on new abilities is a challenge for evaluation at this level. In this study, the period between the exposure of teachers to the professional development programme and the time for them to practice and reflect on the outcomes was eighteen (18) weeks.

Level 5: Learners' Learning Outcomes

This final level in Guskey's (2000) framework is focused on understanding the impacts of the programme on learners' learning outcomes. The current study focused on assessing the development of learners' conceptual understanding in relation to the development of the teachers' TPACK, as a measure of impact of the professional development programme on learners' conceptual understanding of periodicity of trigonometric functions.

In the current study, the requirements for Level 3, organisational support and change, were provided for by engaging with all stakeholders, through the regional education directorate and school managers, to solicit that all schools put in place the specified necessary conditions and practices to support the programme. The researcher met with the regional educational management team, particularly the mathematics education officers, school principals, heads of departments and heads of subjects to define the minimum requirements for the success of the implementation of the programme by teachers at their respective schools. The researcher also ensured that the fidelity of organisational support was adequately monitored during implementation of the programme at the schools.

Table 2.2 summarises the context, process and content of the professional development programme for this study.

Table2. 2: Guskey's (2000) framework for evaluation of teacher professional development

Evaluation Level	Research question	How Will Information Be Gathered?	What is Measured or Assessed?	How Will Information Be Used?
1.Participants' Reactions	**RQ1:** How do teachers' perceptions of their state of TPACK change after participating in the Excel modelling programme?	• Analysis of mean differences between pre and post EBM TPACK survey questionnaire self-efficacy ratings) • Analysis of post EBM implementation evaluation questionnaire responses to items 11-20 (participants' reactions). • Analysis of group feedback on EBM enhanced collaborative problem solving experience	Initial satisfaction with the EBM experience	To improve program design and delivery
2.Participants' Learning	**RQ2:** How does the Excel modelling programme impact the teachers' self-efficacy in teaching periodicity of trigonometric functions?	Pre and Post EBM teacher professional development questionnaires	New knowledge and skills of participants	To improve program content, format, and organization
3.Organization Support & Change	Researcher, in collaboration with mathematics advisory services, school principals and heads of mathematics departments outlined and ensured all necessary support required for implementation of the Excel modelling instructional practice was provided to the teachers and learners.	Post EBM implementation questionnaire (Guskey Level 3 items)	The organization's advocacy, support, accommodation, facilitation, and recognition	• To document and improve organizational support • To inform future change efforts
4.Participant Use of New Knowledge and Skills	**RQ3:** How effective were the teachers in the implementation of the Excel based modelling instructional practice?	Lesson observations	Degree and quality of implementation of EBM instruction	To document and improve the implementation of program content
5. Student Learning Outcomes	**RQ4:** What are the learners' perceptions of learning periodicity of trigonometric functions through the Excel based modelling instruction? **RQ5:** What is the relationship between the level of development of teachers' TPACK and the learners' problem solving application of periodicity of trigonometric functions?	• Learners' assessment records (CAEMA tool for assessment of learners' collaborative problem solving process and summative test scores) • Structured interviews with learners • Direct observations (checklist lesson observation schedule)	Student learning outcomes: • Cognitive (Performance and Achievement: Learners' level of conceptual knowledge) • Affective (Attitudes and Dispositions)	• To focus and improve all aspects of program design, implementation, and follow-up • To demonstrate the overall impact of professional development

2.5.3 Measuring the effect size of teacher professional development interventions

The effect size of a phenomena on the population of interest may be characterized as a numerical reflection of its magnitude (Kelley and Preacher 2012). Small (d = 0.2), medium (d = 0.5), and high (d = 0.8) impact sizes were proposed by Cohen (1988). These naming standards, however, are only suggestions, not a definitive measure of impact magnitude (Cohen 1988; Brace et al. 2016).

To be valid, the spread of scores should be approximately normally distributed in a bell curve shape. This study calculated and employed effect sizes to investigate how the EBM professional development intervention changed the perceptions of mathematics teachers about their self-efficacy in the teaching of periodicity of trigonometric functions.

There are various types of effect sizes and ways to calculate them (Hedges, Shymansky, & Woodworth, 1989; Light & Pillemer, 1984; Rosenthal, 1984). The most often used effect size measures in behavioural sciences are Cohen's d, Hedges' g and Glass' delta.

2.5.3.1 Definition of Cohen's d and formula used to calculate effect size

According to Cohen (1988), Cohen's **d** is defined as the difference between means, ($M_1 – M_2$), divided by standard deviation (SD) of the scores of either of the two groups, provided that the variances of the two groups are homogenous. The difference between the means is thus positive if it indicates improvement and negative if it shows deterioration. In practice however the pooled standard deviation (SD^*_{pooled}) is usually used (Rosnow & Rosenthal, 1996).

Hence, for the purposes of this study Cohen's g is calculated using the formula:

$$Cohen's\ d = \frac{M_1 - M_2}{SD^*_{pooled}}$$

The pooled standard deviation is calculated using the formula**:**

$$SD^*_{pooled} = \sqrt{\frac{(SD_1^{\ 2} + SD_2^{\ 2})}{2}}$$

Where:

SD_1 = standard deviation for group 1

SD_2 = standard deviation for group 2

An alternative formula by Cohen (1988) is:

$$SD^*_{pooled} = \sqrt{\frac{(n_1 - 1)SD_1^{\ 2} + (n_2 - 1)SD_2^{\ 2}}{n_1 + n_2 - 2}}$$

Where:

n_1, is the sample size for the pre-EBM professional development group (group 1)

n_2, is the sample size for the post-EBM professional development group (group 2)

The equivalence between the two formulae can be noted in the substitution of values, n_1 =11 and n_2 =11, Cohen's (1988) alternative formula reduces to:

$$SD^*_{pooled} = \sqrt{\frac{(SD_1^{\ 2} + SD_2^{\ 2})}{2}}$$

2.5.3.2 Definition of Hedges' g and formula used to calculate effect size

Hedges' **g** is an inferential statistic. The main difference between Cohen's **d** and Hedges' **g** is that the latter is multiplied by a correction factor for small samples ($n \leq 20$). The formula for calculating Hedges' g is:

$$Hedges'g = Cohen's\ d \times \left(1 - \frac{3}{4(n_1 + n_2) - 9}\right)$$

2.5.3.3 Definition of Glass' d and formula used to calculate effect size

Glass's delta is defined as the mean difference between the experimental and control group divided by the standard deviation of the control group. The formula for calculating Glass's delta is:

$$Glass's\ delta = \frac{M_1 - M_2}{SD_{control}}$$

2.5.3.4 Interpretation of effect size

Cohen (1988) suggested that effect sizes can be categorised into small (d = 0.2), medium (d = 0.5), and large (d = 0.8). However, these naming conventions are merely suggestions, rather than an absolute indicator of effect size (Cohen 1988; Brace et al. 2016). To be valid, the spread of scores should be approximately normally distributed in a bell curve shape. This study calculated and employed effect sizes to investigate how the EBM professional development intervention changed the perceptions of mathematics teachers about their self-efficacy in the teaching of periodicity of trigonometric functions.

2.6 CONCEPTUAL FRAMEWORK

Researchers create conceptual frameworks all the time (Polit & Tatano, 2004). Conceptual frameworks, according to Ravitch and Carl (2016), are generative in that they represent the thinking throughout the whole research process. Typically, diagrams are used to clearly identify the components or variables of the study issue, with arrows indicating their interconnections. According to Latham (2017), the entire technique must agree with variables, their connections, and context. Researchers are free to use existing frameworks, but they must alter them to fit the nature of their research questions' environment (Fisher, 2007).

Using partial least squares structural equation modelling, the study's conceptual framework assesses the link between the level of development of TPACK components and the level of development of learners' conceptual knowledge. The conceptual understanding of the learners is examined in two ways: first, through a collaborative problem-solving process, and second, through individual learner performance on a written summative assessment test.

Figure 2.3 illustrates the conceptual framework of this study.

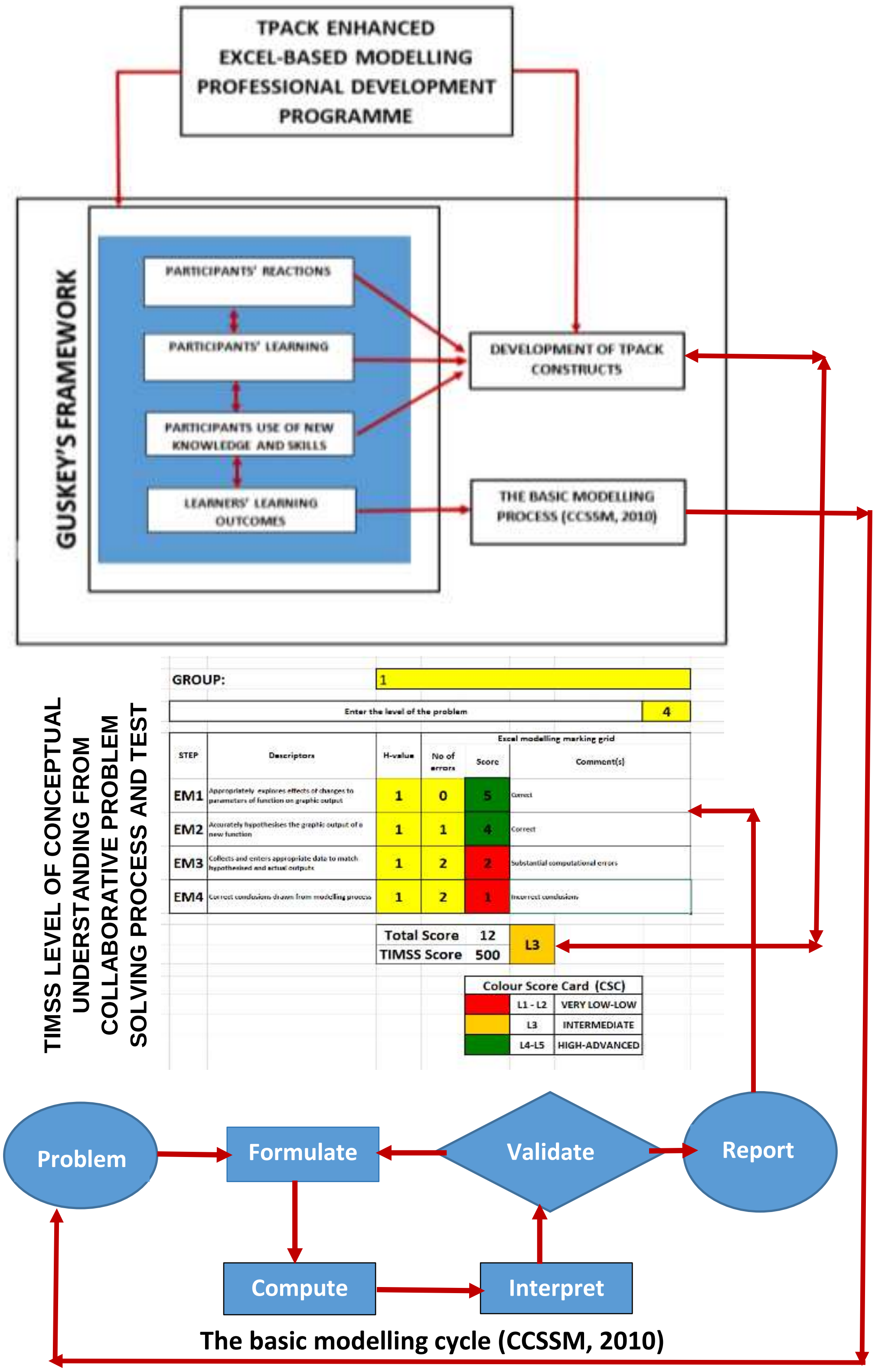


Figure 2. 3: The conceptual framework used for the study, based on the basic modelling process (CCSSM, 2010), TIMSS (2007) assessment framework, TPACK and Guskey's (2000) framework

2.6.1 The Excel-based modelling process as a system of knowledge acquisition

"Knowledge acquisition comprises the eliciting, modelling and encoding of domain knowledge. Eliciting knowledge means acquiring it from a domain expert. Modelling knowledge means structuring it into some form of knowledge representation. And, finally, encoding refers to the transfer of the modelled knowledge and its implementation in a working computer system. To ensure the usefulness of the system, the system is tested for usability through verification and validation of the knowledge base" (Häkansson, 2003, p. 1).

This thesis proposes a graphic representation and visual modelling approach to support the knowledge acquisition process. More specifically, it makes use of:

- Excel-based modelling which supports an inquiry approach that allows transferring knowledge between the learners/teachers and the computer model; by using graphic representations of the knowledge.
- An established graphic representation scheme in new areas of knowledge representation and presentation.

The Excel-based modelling process emphasises the visual representation and presentation of knowledge, aiming at the construction of useful and expressive conceptual models of knowledge (Häkansson, 2003).

2.6.2 The basic modelling cycle

Common Core State Standards in Mathematics (CCSSM) defines modelling as the process of choosing and using mathematical tools at hand to describe the world around us (CCSSM, 2010). In the most general sense, modelling involves identifying a question, posing it in a mathematical framework, solving the resulting problem and then interpreting the results in the context of the original question (see Figure 2.3). The basic modelling cycle consists of six steps;

1. **Problem identification:** Identifying variables in the situation and selecting those that represent essential features.
2. **Formulation of the model:** Creating and selecting geometric, graphical, tabular, algebraic or statistical representations that describe relationships between the variables.

3. **Compute:** Analysing and performing operations of these relationships to make observations and draw conclusions.
4. **Interpret:** Interpreting the results of the mathematics in terms of the original solution.
5. **Validate:** Validating the conclusions by comparing them with the given situation, and if not acceptable reformulate a different model.
6. **Report:** Reporting on the conclusions and the reasoning behind them.

For the purpose of this study, the six steps were reduced to four steps, since the formulation of the Excel models and the graphic outputs from the computations were already provided for in the development of the Excel-based modelling applets. The reduction in the number of steps also allows for easier assessment of achievement of learners at each step, during the problem-solving process. The four Excel-based modelling steps adapted for the current study are:

(1) Identify the problem,

(2) Hypothesise the solution,

(3) Collect data and test against hypothesis, and

(4) Draw conclusions.

For example, the following problems solved collaboratively by learners are used to illustrate the Excel-based modelling process in the context of this study. The accomplishment of learners through the four steps contributes towards the aggregated score used as a measure of the level of learners' conceptual understanding. The CAEMA tool was used to map the learners' achievement at each of the four modelling steps in the collaborative problem-solving process. The formative assessment (assessment for learning) was conducted to guide learners towards the development of their own metacognitive strategies. The collaborative problem-solving activities were aimed at equipping learners with their own language and tools for learning, allowing them to transfer and apply these skills of problem-solving into daily life; thus strengthening their ability to find answers or develop strategies for addressing problems with which they are not familiar.

Problem 1: Modelling the graph of a given trigonometric function *f(x)*

Sketch the graph of the function *f(x) = -4Cos 3x* for $x \in [0, 2\pi]$

Solution:

Em1: Identify the problem

Use an exploratory approach to investigate the effects of changes in *a, b and c* to the pattern of the of the family of graphs belonging to *f(x) = a Cos bx + c,* where a, b, c $\in \Re$ ($\Re$ is the set of real numbers).

Case 1: Investigate effect of changing a (b = 1, c = 0)

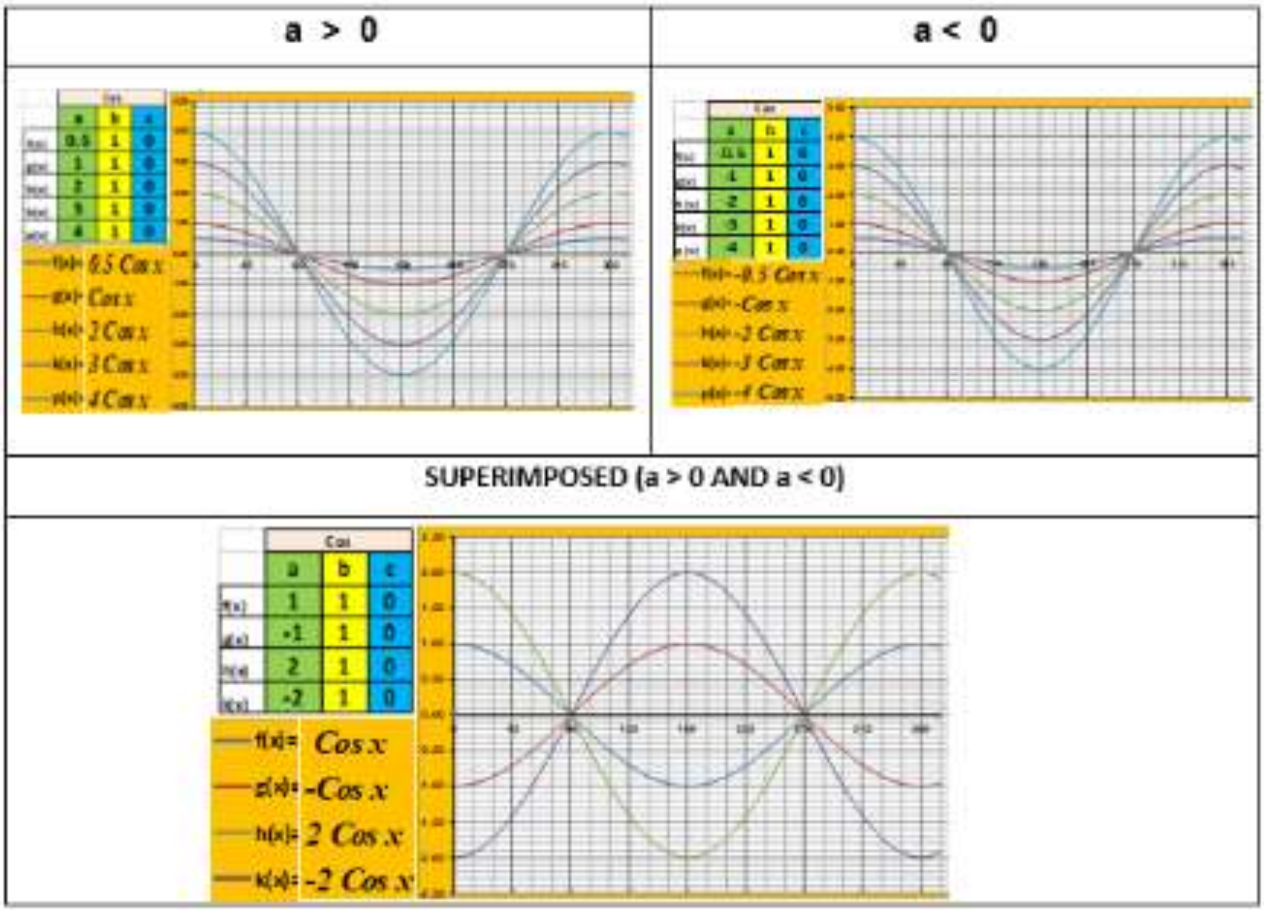
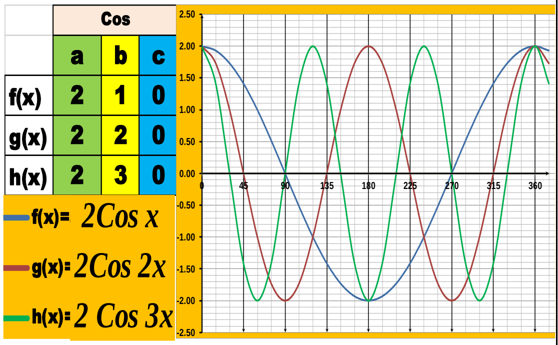


Figure 2. 4: Output of the modelling a family of graphs of *f(x) = a cos bx + c*

Case 2: Investigate effect of changing b (a = 2, c = 0)

b > 0



b < 0

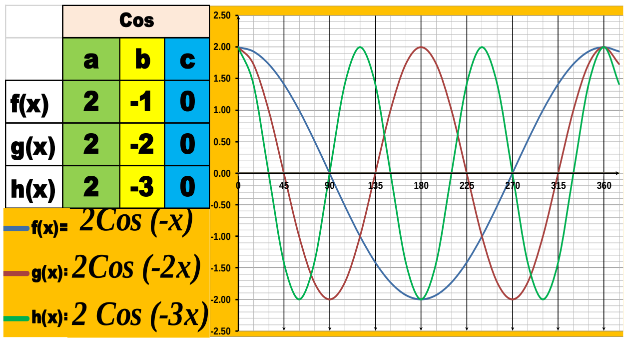
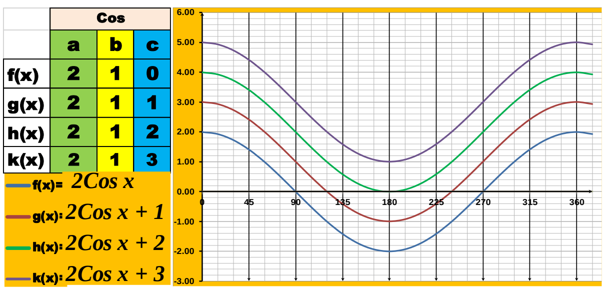


Figure 2. 5: Computer snippet of output of the modelling a family of graphs of *f(x) = 2 Cos (±b)x*

Case 3: Investigate effect of changing c (a = 2, b = 1)

c > 0



c < 0

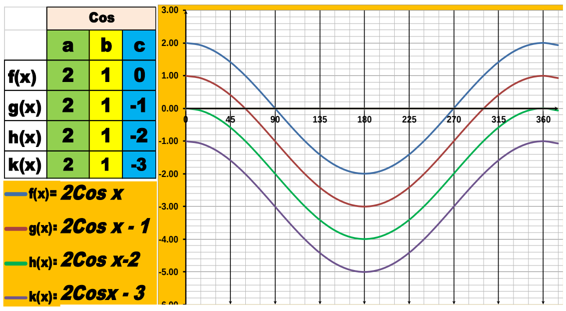


Figure 2. 6: Computer snippet of output of the modelling a family of graphs of *f(x) = 2Cosx ± c*

Em2: Hypothesise the solution

Observe the outcomes from Cases 1 to 3 and hypothesise the nature of the graph of f(x) = - 4 Cos 3x for $x \in [0, 2\pi]$

Case 1:

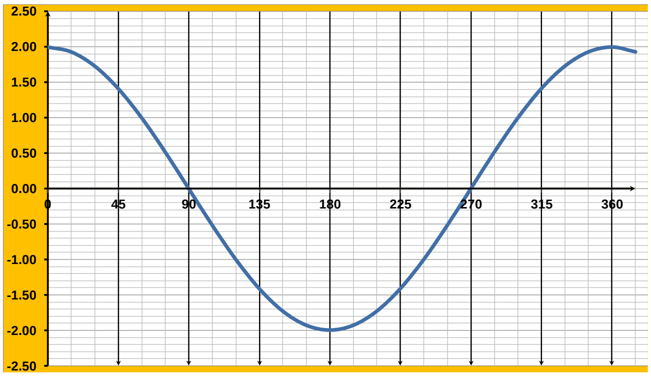
For a > 0 (*a* <0), the graph starts at the maximum (minimum), and monotonously decreases (increases) to the adjacent minimum (maximum) point. This means when a < 0, it results in a reflection of the graph of *f(x)* with a > 0, and vice versa.

Changing "***a***" affects the amplitude of the graph, i.e. half the difference between the minimum and maximum values of the range. Only periodic functions with a bounded range have an amplitude. The amplitude is essentially the radius of the range.

This suggests that for *f(x) = -4Cos3x*, the amplitude is 4. Since amplitude is length, we neglect the "−" sign.

Case 2: Changing "***b***" inversely affects the period of the function

For example for *f(x) = 2 Cos x*, b = 1, and the graph appears as shown below:



When the value of b = 1, the period of the cosine function is 360^0. The period of the basic cosine function is 360^0.

Figure 2. 7: Snippet of computer display of graph of *f(x) = 2 Cos x*

A similar exploration with changes in the values of bconfirmed that:

For *f(x) = 2 Cos 2x*, b = 2, the period is 180^0

For *f(x) = 2 Cos 3x*, b = 3, the period is 120^0

The relationship between the period and the value of **b** can be summarised in the Table 2.3.

Table 2. 3: Inverse proportionality between period and value of b in f(x)=aCosbx

Value of b	1	2	3
Period (degrees)	360	180	120

Hypothesis: The table represents inverse proportion in which the product of the corresponding values is constant = period of the basic cosine function = 360^0.

Mathematically this can be expressed as:

$$Period\ \ of\ basic\ \ function\ \ (P_{bf}) = Period\ \ of\ new\ \ function\ \ (P_{nf}) \times b$$

Hence $b = \dfrac{P_{bf}}{P_{nf}}$ with *f(x) = a Cos bx*

This suggests that for *f(x) = - 4 Cos 3x*;

Period is $\dfrac{360^0}{3} = 120^0$

Amplitude = 4

Since a < 0, then the graph starts from a minimum turning point (trough) and increases towards the maximum turning point (crest), before decreasing again towards the next minimum point, and so on.

Based on the exploratory observations on the effects of ***a*** and ***b*** on the graph of *f(x) = a Cos bx +c* and prior knowledge of trigonometry in a unit circle, a specific group of learners collaboratively hypothesised the following sketch:

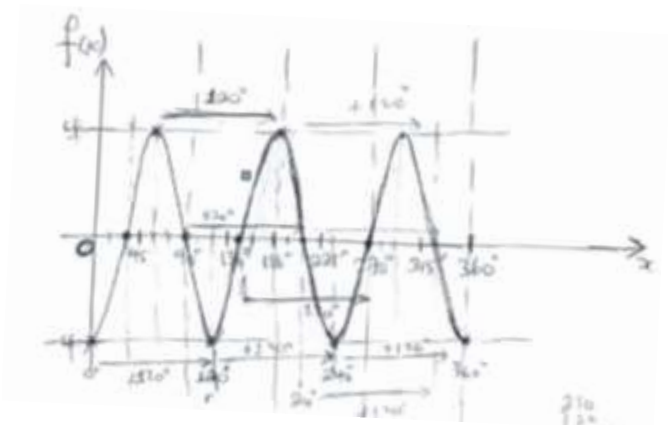


Figure 2. 8: Snippet of hypothesised sketch of *f(x) = - 4 Cos 3x*

Em3: Collect data and test against hypothesis

In this case the data collected are the values a = −4, b = 3 and the hypothesised sketch

The data is now entered into the Excel graphing template for *f(x) = a Cos bx + c*, which gives the following graphic output:

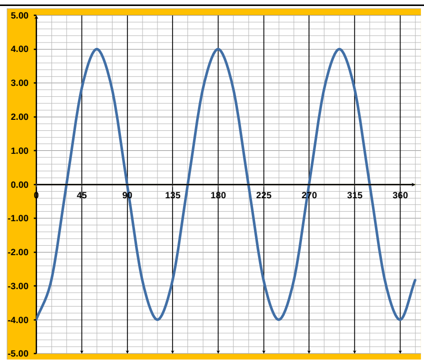
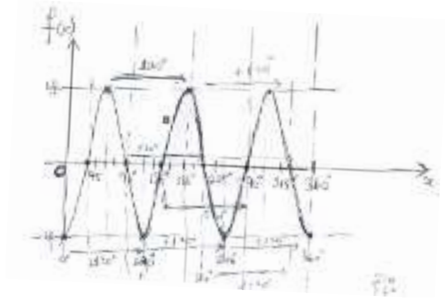
Excel Graph Output	Hypothesised Sketch
	

Figure 2. 9: Snippet of Excel output and hypothesised solution for *f(x) = - 4 Cos 3x*

Em4: Draw conclusions

Since the Excel Output and the hypothesised sketch match, then this shows that the hypothesised sketch is the correct representation for the function *f(x) = - 4 Cos 3x*.

Through this modelling process learners were able to construct their own knowledge of trigonometric functions and graphs, collecting data, making discoveries and testing those discoveries or making hypotheses and predictions about the problem (Osborne et al., 2008). This allows the students to search for information and learn on their own with the teacher's guidance.

Problem 2: Modelling the function of a given trigonometric graph

Determine the function that describes the graph given below:

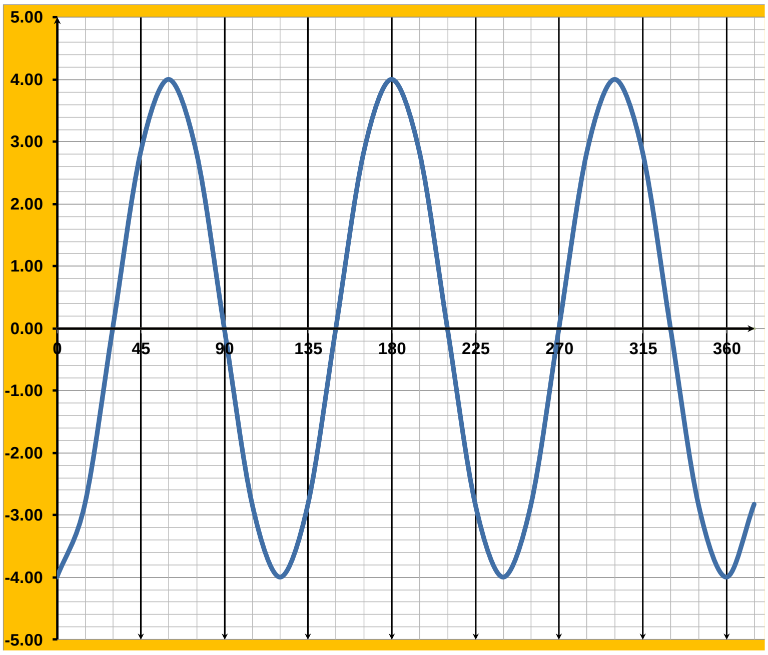


Figure 2. 10: Snippet of graph whose trigonometric function is to be determined

Solution:

This is the reverse process to modelling of the solution to problem 1.

Now the learners have to interpret the graph and derive the values of a (amplitude), b (the period of the graph), and $c = \frac{M + m}{2}$, where **M** is the maximum value and **m** the minimum value of the graph of the function *f(x) = a Cos bx + c*.

Problem 3: Modelling the solution to a trigonometric equation

Solve the trigonometric equation *-4 Cos 3x = 0,* for $x \in [0, \pi]$.

Solution:

Em1: Identify the problem

This requires reading and understanding what the problem actually requires us to find. In this case, the problem requires us to determine where the graph of *f(x) = - 4 Cos 3x* cuts the *x* - axis or the *x* - intercepts. This is prior knowledge that learners are assumed to have.

Em2: Hypothesise the solution

This consists of the learner searching for a solution strategy (strategising the solution of the problem). In this phase the learner embarks on an exploratory search of the possible solution strategies. The learner realises that the function *f(x) = - 4 Cos 3x* belongs to the family of trigonometric functions of the form; *f(x) = a Cos bx + c*, where *a, b, c* $\in \Re$ ($\Re$ is the set of real numbers). The Excel graphing technology used in this study is designed to input values of *a, b and c* and immediately display the corresponding graph for the input.

The following is the Excel display of the graph obtained through the input of a = - 4, b = 3 and c =0, for the function *f(x) = a Cos(bx) + c*.

BEFORE ENTERING INPUT VALUES **AFTER ENTERING INPUT VALUES**

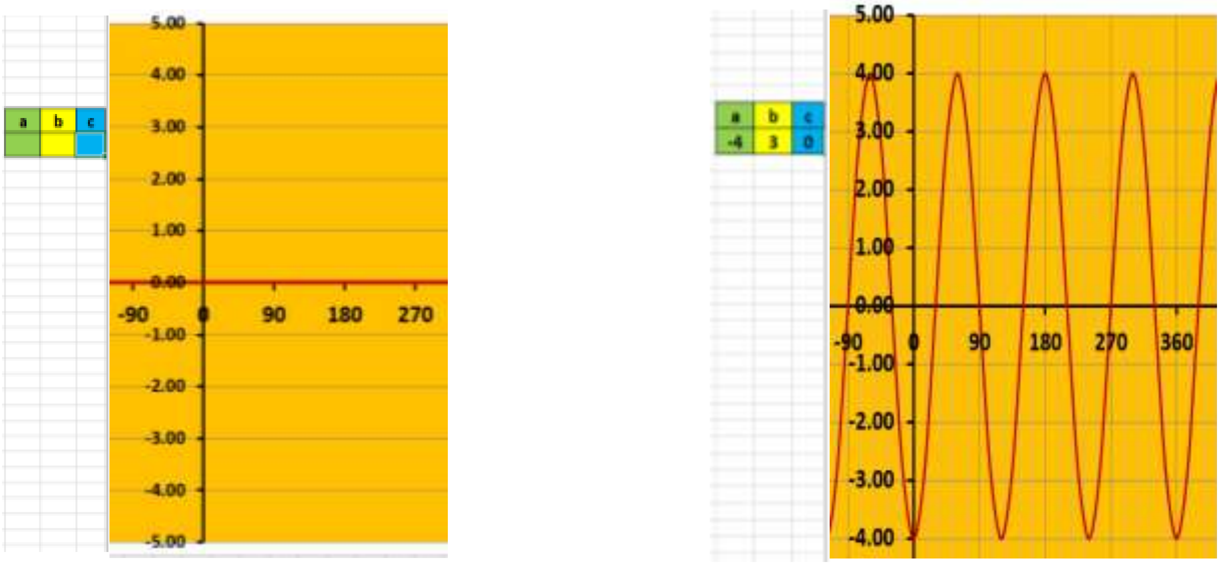


Figure 2. 11: Snippet of Excel graphic modelling output before and after entry of data

The hypothesised solution is given in the four possible values in the domain *x* ϵ [0,π] ;

$$x_1 = 30^\circ,\ x_2 = 90^\circ,\ x_3 = 150^\circ \quad \text{or} \quad x_1 = \frac{\pi}{6},\ x_2 = \frac{\pi}{2},\ x_3 = \frac{5\pi}{6}$$

Em3: Collect data and test against hypothesis

The data collected are the possible solutions $x_1 = 30^\circ,\ x_2 = 90^\circ,\ x_3 = 150^\circ$.

The data collected is tested in the statement "- 4 Cos 3x". If this data (solution) is correct, then on substituting each value of *x*, the statement must give a value of "0".

For $x_1 = 0^\circ$, $-4Cos3(30) = 0$ √

For $x_2 = 60^\circ$, $-4Cos3(90) = 0$ √

For $x_3 = 120^\circ$, $-4Cos3(150) = 0$ √

Em4: Draw conclusions

Since the values of x satisfy the hypothesised solution, the solution to the equation *– 4 Cos 3x = 0* for $x \in [0, \pi]$ is;

$x_1 = 30^\circ$, $x_2 = 90^\circ$, $x_3 = 150^\circ$ or $x_1 = \dfrac{\pi}{6}$, $x_2 = \dfrac{\pi}{2}$, $x_3 = \dfrac{5\pi}{6}$

The same process can be repeated for the **sine** and **tangent** functions. The inquiry process should be handled in a way that it facilitates the cognitive growth of Banchi and Bell's (2008) four levels of inquiry: confirmation, structured, guided and open inquiry. At the level of open inquiry, the learners should be encouraged to design and carry out investigations of their own, as well as communicating their own results.

2.6.4 Trends in International Mathematics and Science Study (TIMSS) achievement levels

Lupahla (2014, p.58) explains, "The TIMSS (2007) international evaluation of student achievement includes written examinations in mathematics and science, as well as a series of questionnaires that collect information on the educational and social elements of achievement." TIMSS (2007) recorded student accomplishment by cognitive domain for the first time, i.e., knowing, applying, and thinking. To summarise and explain student accomplishment at four locations on the mathematics and science scales, TIMSS (2007) employed scale anchoring. Scale anchoring is the process of choosing TIMSS scale benchmarks (scale points) to define student performance and then defining items that students scoring at the anchor points can answer correctly. The items are then organised into topic domains inside benchmarks for examination by math and science experts." The current study used a similar approach to classify the benchmark levels in the following categories:

2.6.4.1 Advanced International Benchmark (Score above 625)

Learners can organise information, make generalisations, solve non-routine periodicity problems in trigonometry, and draw and justify conclusions. They can:
Apply their knowledge of periodicity concepts and relationships between trigonometric functions and their graphs to solve related problems;

- Solve trigonometric equations and justify their solutions;
- Determine the new trigonometric function if the original function has undergone a phase shift;
- Interpret data from a variety of trigonometric graphs.

2.6.4.2 High International Benchmark (Score between 550 and 625)

Learners can apply their understanding and periodicity knowledge in a wide variety of relatively complex situations. They can:

- Determine trigonometric functions of the form; $f(x) = a \, Sin\,(bx)+c$, $g(x) = a \, Cos\,(bx)+c$, and $h(x) = a \, Tan\,(bx)+c$, for $-360^0 \le x \le 360^0$, from their respective graphs.
- Sketch the graphs of trigonometric functions of the form; $f(x) = a \, Sin\,(bx)+c$, $g(x) = a \, Cos\,(bx)+c$, and $h(x) = a \, Tan(bx)+c$, for $-360^0 \le x \le 360^0$.
- Determine the algebraic expressions of each of the graphs of; $f(x) = a \, Sin\,(bx)+c$, $g(x) = a \, Cos\,(bx)+c$, and $h(x) = a \, Tan(bx)+c$ after undergoing a phase shift.

2.6.4.3 Intermediate International Benchmark (Score between 475 and 550)

Learners can apply basic periodicity knowledge in straightforward situations. They can:

- Deduce the period and amplitude of basic trigonometric functions of the form; $f(x) = Sin\, x$, $g(x) = Cos\, x$, and $h(x) = Tan\, x$, for $-360^0 \le x \le 360^0$, from their respective graphs.
- Interpret graphs of basic trigonometric functions of the form; $f(x) = Sin\, x$, $g(x) = Cos\, x$, and $h(x) = Tan\, x$, for $-360^0 \le x \le 360^0$;

- Match complex trigonometric functions to their graphs and vice-versa;
- Demonstrate understanding of properties of basic trigonometric functions and graphs, including their transformations;

2.6.4.4 Low International Benchmark (Score between 400 and 475)

Learners have some basic periodicity knowledge. They can:

- Demonstrate understanding of the concepts of amplitude and period of basic trigonometric functions;
- Match basic trigonometric functions to their graphs and vice-versa.

2.6.4.5 Very Low International Benchmark (Score below 400)

Learners have made no attempt at all to solve problem or solution process is completely incorrect.

2.7 SUMMARY

This chapter presented the literature review on methods and techniques used to investigate the research problem identified at the onset in similar contexts. It justified the choices made and the felicity with which such studies validate, inform and provide guidance for a systematic enquiry in this current investigation. The next chapter discusses in detail the research methodology, the research design, research site and participants. The chapter further explains the data collection process and instruments used, as well as discussing the validity and reliability issues. Finally, the ethical issues taken into consideration during the process of conducting the research are discussed in the last section of Chapter 3.

CHAPTER THREE
RESEARCH METHODOLOGY

3.1 INTRODUCTION

This chapter presents the the research process and what was deliberately planned in the execution of the study. It provides information concerning the method that was used in undertaking this research as well as a justification for the use of this method. The chapter also describes the various stages of the research, which includes the selection of participants and the data collection and analysis process. A detailed discussion of the research ethics adhered to in the study is also offered in this methodology chapter.

The nature of the research question and the problem under investigation, according to Denzin and Lincoln (2005), determines the research approach or strategy. As a result, an investigation's research approach should be seen as a tool for answering the research questions. The purpose of this thesis was to establish how an Excel-based modelling teacher professional development program affected students' conceptual understanding of trigonometric function periodicity. The study also sought to establish how the Excel-based modelling teaching technique affected teachers' and learners' perceptions of trigonometric function periodicity. The key research topic that prompted this study was as follows:

How does Excel-based modelling instruction influence learners' conceptual understanding of periodicity of trigonometric functions in the NSSCH curriculum?

In order to answer the central research question, the following sub-questions were proposed:

RQ1: How do teachers' perceptions of their state of TPACK change after participating in the Excel-based modelling teacher professional development?

RQ2: How does the Excel-based modelling teacher professional development impact the teachers' self-efficacy in teaching periodicity of trigonometric functions?

RQ3: How effective were the teachers in the implementation of the Excel-based modelling instructional practice?

RQ4: What are the learners' perceptions of learning periodicity of trigonometric functions through the Excel-based modelling instruction?

RQ5: What is the influence of teachers' TPACK development on the learners' conceptual understanding of periodicity of trigonometric functions?

A mixed methods approach was decided upon as the methodology because this reinforces an understanding and interpretation of meanings as well as intentions underlying teaching and learning. Mixed methods research relies on both quantitative and qualitative strands (Creswell, 2009). The depth and detail of data collected in qualitative research provides a rich and detailed understanding; whereas quantitative research provides the ability to generate broad generalisations for a specific population (Patton, 2002).

3.1.1 Qualitative research

The collecting, analysis, and interpretation of detailed narrative and visual data to obtain insights into a specific phenomenon of interest, in this example, instructors' TPACK self-efficacy in teaching and learners' perceptions of EBM effects on their conceptual grasp of periodicity of trigonometric functions, is referred to as qualitative research. For example, the phrase "qualitative research" was defined by Strauss and Corbin (1990, p. 11) as "any sort of study that provides conclusions that are not reached by statistical techniques or other ways of quantification." According to Flick (2014, p. 542), "qualitative research is concerned in understanding subjective meaning or the social creation of topics, events, or practices by gathering non-standardised data and analysing texts and pictures rather than numbers and statistics," as stated by Strauss and Corbin (1990, p. 11).

Qualitative research, on the other hand, aims to uncover the numerous social meanings of individual experiences in order to develop a theory or pattern (Creswell, 2009). Qualitative research maintains a focus on the importance of the participant's perspective and how it informs the participant's own meanings (Creswell, 2009), maybe leading to a new observation or suggesting the possibility of future examination of potential predictors and critical components in another study (Creswell, 2009). (Yoshikawa, Weisner, Kalil, & Way, 2008). Qualitative approaches investigate phenomena utilising specific data in order to encourage interaction between people and events, which may reveal structural patterns and themes in the phenomenon (Cooper & Schindler, 2006).

3.1.1.1 Advantages of qualitative research

The typical advantages of qualitative research are:

1. Qualitative research generates a thorough account of participants' thoughts, views, and experiences, as well as interpreting the significance of their actions (Denzin, 1989).
2. Qualitative research (interpretivism) understands the human experience in specific contexts holistically.
3. The interpretivism research methodology is considered as an ideographic research, the study of particular cases or occurrences (Klein & Myers, 1999), and it has the ability to comprehend various people's voices, meanings, and experiences.
4. Qualitative research allows academics to learn about the participants' inner experiences and how meanings are created by and within cultures (Corbin & Strauss, 2008).
5. For data collection, qualitative research methods such as participant observation, semi-structured interviews, direct observation, and description of records are most typically utilised (Cohen, Manion, & Morrison, 2011). During data collection, the researchers interact directly with the participants, similar to how data is collected through interviews.
6. The qualitative research design (interactive method) has a more flexible structure than the quantitative since it may be built and reconstructed more easily (Maxwell, 2012). Thus, qualitative research methodologies may be used to provide detailed and suitable evaluations of a problem, and participants have enough flexibility to select what is consistent for them (Flick, 2011).

3.1.1.2 Disadvantages of qualitative research

1. According to Silverman (2010), qualitative research methods occasionally overlook contextual sensitivity in favour of focusing on meanings and experiences. For example, a phenomenological method aims to uncover, analyse, and comprehend the participants' experiences (Wilson, 2014; Tuohy et al.,2013).

2. The outcomes of a qualitative method may be viewed with scepticism by policymakers. In the United States, for example, quantitative approaches are usually given higher weight in research on teacher and student success (Ravitch, 2010). (Berg, 2009).
3. In terms of research methodology, a smaller sample size, such as that utilised in the current study, raises the question of generalisability to the entire research population (Harry & Lipsky, 2014; Thompson, 2011).
4. The case investigations take a long time, and there is only a limited method to generalise the conclusions to a wider population (Flick, 2011).

3.1.2 Quantitative research

Quantitative research, according to Bryman (2012, p. 35), is a technique that emphasises quantification in data gathering and analysis. This research approach tries to figure out the answers to queries like how many, how much, and to what degree (Rasinger, 2013). Quantitative approaches seek regularities in human lives, according to Payne and Payne (2004, p. 180), by dividing the social environment into empirical components called variables, which may be represented numerically as frequencies or rates, and whose relationships with one another may be studied using statistical techniques, and accessed through researcher-introduced stimuli and systematic measurement.

The quantitative method is appropriate for this study because a correlational comparison is made between TPACK variables and learners' conceptual understanding, using partial least squares regression analysis. Quantitative research provides the researcher with an opportunity to compare two or more variables and examine relationships or differences (Cozby, 2007). The EBM professional development and training, as evaluated by the PLS-SEM structural equation modelling of TPACK components and their effects on learners' conceptual comprehension, is a researcher-introduced stimulus in this work. Bryman (2012) also recognised positivism as the fundamental paradigm linked with quantitative research, in which knowledge is derived from empirical testing.

3.1.2.1 Advantages of quantitative research

1. The quantitative findings can certainly be applied to the entire population or a sub-group (Carr, 1994).
2. Apart from sampling, data processing takes less time since it employs statistical software such as SPSS (Connolly, 2007). In this study, for example, learners were assessed on their conceptual grasp of trigonometric function periodicity, and PLS-SEM was used to simulate how their conceptual understanding was impacted by instructors' TPACK growth.

3.1.2.2 Disadvantages of quantitative research

1. The quantitative (positivism) research paradigm ignores social phenomena's shared meanings (Denzin & Lincoln, 1998). It also misses the mark when it comes to deducing deeper meanings and explanations.
2. Another flaw of the quantitative research methodology is that it has a tendency to capture a phenomenon in a snapshot: it measures variables at a single point in time, regardless of whether the image occurred to capture one looking their finest or appearing particularly disorganised (Schofield, 2007).
3. Finally, because there is no direct link between researchers and participants when collecting data, the quantitative research paradigm neglects respondents' experiences and viewpoints in highly controlled settings (Ary, Jacobs, Sorensen, & Walker, 2013).

3.1.3 Rationale for mixed methods research

The motivation for employing a mixed methods research technique was, therefore, to build on the synergy and strength that exists between quantitative and qualitative research methods in order to gain a better understanding of the EBM teacher professional development on the learners’ conceptual understanding of periodicity of trigonometric functions.

This study was designed to both explore a detailed understanding of the experiences of teachers and learners with regard to their exposure to the Excel-based modelling instruction in the teaching and learning of periodicity of trigonometric functions and to establish patterns within the data with regard to the performance of teachers and of the learners they teach.

The qualitative research component of the study, examined both the teachers' and learners' reactions before, during, and after the implementation of the EBM professional development and teaching. The purpose was to understand the impact, if any, of EBM teacher professional development on learners' conceptual grasp of periodicity of trigonometric functions. On the other hand the PLS-SEM constituted the quantitative component of the embedded mixed methods design of the study. In this component, the influence of the participant teachers' level of knowledge and skills (TPACK) on the learners' level of conceptual understanding of periodicity of trigonometric functions was investigated.

3.2 RESEARCH PARADIGM

"As researchers, we must be able to grasp and communicate views about the nature of reality, what can be understood about it, and how we go about obtaining this knowledge," Rehman and Alharthi (2016, p.51) recommended. A paradigm is a fundamental belief system and theoretical framework that includes assumptions about ontology (concerned with what actually exists in the world about which humans can acquire knowledge), epistemology (how we know and the relationship between the knower and the known), methodology (the specific procedures or techniques used to identify, select, process, and analyse information about a topic), and methods (processes or **techniques** utilised in the collection of data or evidence for analysis in order to uncover new information or create better understanding of a topic). In other words, it is our method of comprehending and studying the world's reality." The study is grounded on two research paradigms: pragmatism and constructivism.

3.2.1 Pragmatism

McDermott (as cited in Stark, 2014: 98) submitted that, "at bottom, pragmatic epistemology is an attitude; one that does not make truth announcements, let alone pronouncements or manifestos, but rather is an experimental probing. Pragmatism has an inductive temper, yet it is far more aware of possible novelty and it is willing to treat ideas as explorers, ferreting out new ground on which to stand, even at the risk of being severely wrong." The Excel modelling teacher professional development programme was designed to integrate both an experimental and inductive action research inquiry, thus strongly befitting it as a pragmatic inquiry approach.

Cherryholmes (1992) suggests that pragmatic inquiry seeks to clarify meanings and looks to consequences. Pragmatism has often been identified in the mixed methods research literature as the appropriate paradigm for conducting mixed methods research (e.g. Howe, 1988; Tashakkori & Teddlie, 1998; Patton, 2002; Maxcy, 2003; Teddlie & Tashakkori, 2003, 2006; 2009; Johnson & Onwuegbuzie, 2004; Onwuegbuzie & Johnson, 2006; Johnson and Gray, 2010; Creswell & Plano Clark, 2011). It is therefore on this basis that the current study grounded itself, epistemologically, on the pragmatist paradigm.

3.2.2 Constructivism

The growth of active learning, also known as learning by doing, learning by experiencing, learning via action, learner-centred education, peer collaboration, and cooperative learning, captures the core of constructivism. In mathematics, constructivist pedagogy is based on the idea that learners may create knowledge via active involvement rather than passively listening to the teacher's classroom lecture (Richards, 1991).

The importance of constructivism in this study is that an Excel-based modelling inquiry method was used as an educational tool, allowing students to investigate the linkages between algebraic and visual representations of trigonometric functions on their own, allowing them to construct interpretations concerning periodicity and symmetry features of trigonometric functions.

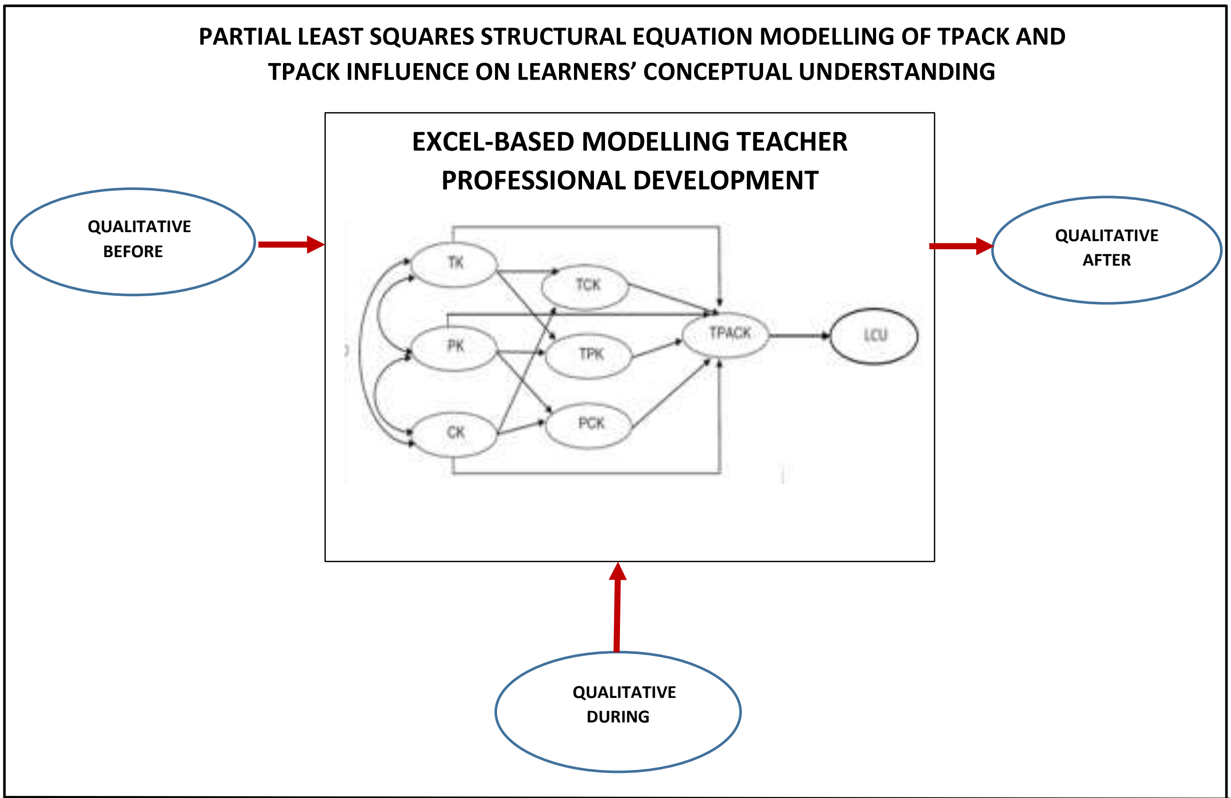
3.3 RESEARCH DESIGN

The study adopted an embedded intervention mixed methods research design. Specifically, the study merged the embedded-intervention design with Guskey’s (2000) model for evaluation of teacher professional development programme, with both qualitative and quantitative data collected to inform Guskey’s specific levels of professional development. A quantitative experimental intervention was embedded within a primarily qualitative methodology, as shown in Figure 3.1.

A mixed methods research design allows researchers to gather, analyse, and combine quantitative and qualitative approaches in a single study or a series of studies to better understand a research topic (Creswell & Plano Clark as cited in Ofori-Kusi, 2017). A mixed methods study allows for data collection, analysis, and mixing. The main premise is that combining quantitative and qualitative methodologies yields informed knowledge of the study problem and topic than using either technique alone. According to Greene (2007: xiii), “the mixed methods study offers for the ability to correct for inherent singular method shortcomings, capitalise on inherent method strengths, and counterbalance inescapable biases.”

3.3.1 Rationale for using the embedded intervention design

The embedded intervention design, also called the experimental intervention design (Creswell, 2015b), is characterised by the inclusion of an experiment or intervention trial embedded within a qualitative phase, which helps to minimise some of the problems associated with intervention studies. The embedded intervention design is a popular design for evaluation of an intervention or programme in an applied setting (e.g., in a school). Figure 3.1 illustrates the phases of the embedded intervention design employed in this study.



Qualitative Strand Before the Experiment:

- Define the objectives for the qualitative strand
- Gain insight of teachers' state of TPACK before exposure to intervention
- Rate teachers' baseline self-efficacy in teaching periodicity of trigonometric functions
- Use questionnaire and group interview/feedback to collect relevant data

Use the Qualitative Strand to:

- Refine professional development intervention
- Refine sampling procedures for participants
- Refine research questions for this phase

Qualitative Strand During the Experiment:

- Define the objectives for the qualitative strand
- Gather participants' reactions to the intervention
- Assess the state of organisational support
- Use lesson observation checklist to assess teachers during implementation of instruction
- Use CAEMA tool to assess learners' collaborative problem solving skills (conceptual understanding)

Use the Qualitative Strand to:

- Describe participants' experiences with the intervention
- Describe the intervention process
- Describe the treatment fidelity
- with the intervention
- Describe the intervention process
- Describe the treatment fidelity

Qualitative Strand After the Experiment:

- Define the objectives for the qualitative strand
- Use questionnaire, semi-structured interview, focus group discussion and CAEMA data to describe the relationship between the development of teachers' TPACK and learners' conceptual understanding of periodicity of trigonometric functions

Use the Qualitative Strand to:

- Explain why the outcomes occurred
- Describe how the teachers respond to the outcomes
- Describe how Excel based modelling influences the learners' conceptual understanding of periodicity of trigonometric functions
- TPACK and learners' conceptual understanding of periodicity of trigonometric functions

Figure 3. 1: The data collection process in the embedded intervention design

3.3.2 Quantitative Component

This research component focused on

(a) measuring the effect size of the intervention programme on the development of TPACK constructs of the participants

(b) investigating the structural relationships in the development of the TPACK constructs

(c) establishing the relationship between the teachers' TPACK development and the learners' conceptual understanding of periodicity of trigonometric functions.

The evaluation of the intervention focuses on the impact of the programme on both teachers and learners, using Guskey's (2000) model of evaluation of teacher professional development. The quantitative inquiry conducted paired t-tests to calculate the difference between pre-Excel modelling professional development and post-Excel modelling professional development in order to determine the likelihood that pre and post differences were not due to chance. Cohen's **d** (Hedges' **g**) was computed to determine the effect size of the Excel modelling professional development intervention between all knowledge domains of TPACK at $p<0.05$ level of significance for each pairing. The relationships between the teachers' TPACK constructs and how the teachers' TPACK influences the learners' conceptual understanding of periodicity of trigonometric functions were investigated, using partial least squares structural equation modelling (PLS-SEM), to test the formulated hypotheses.

The partial least squares structural modelling (PLS-SEM) technique was used to test the hypotheses for statistical significance, using the paired samples t-test values. PLS-SEM is a non-parametric method that is suitable for smaller sample sizes and non-normally distributed data. PLS-SEM is an empirically appropriate method to perform causal inference with formative constructs. Figure 3.2, illustrates the model used and the hypotheses that were tested to analyse the relationships between the latent variables. The latent variables shown as ovals were measured reflectively through multiple indicators, using a 5 point Likert scale based checklist lesson observation schedule. The statements associated with each indicator were rated on the Likert scale by the lesson observer teams, for each teacher, during the implementation of the EBM instructional programme. The model further analysed the influence of TPACK development on learners' conceptual understanding of periodicity of trigonometric functions.

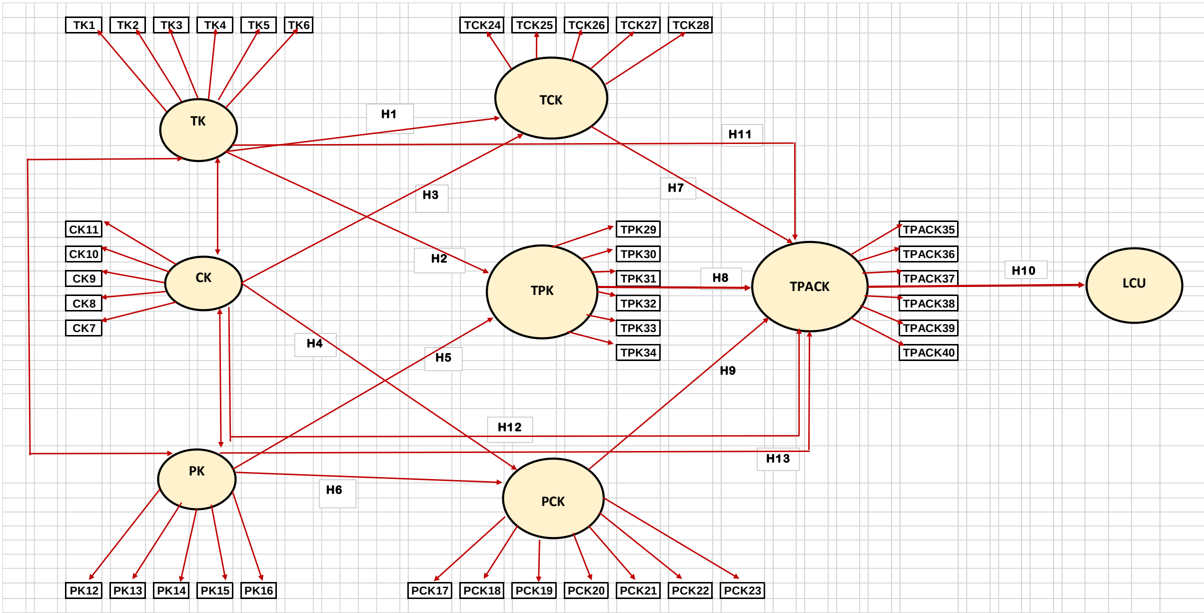


Figure 3. 2: The model used to explore the TPACK relations and influence on learners' conceptual understanding

In the model, LCU represents learners' conceptual understanding. The summative assessment test score (SA) was used to determine the learners' level of conceptual understanding.

The model hypothesised thirteen causal associations at, $p < 0.05$ level of significance as follows:

H1: There is a positive relationship for Technological Knowledge (TK) on Technological Content Knowledge (TCK)

H2: There is a positive relationship for Technological Knowledge (TK) on Technological Pedagogical Knowledge (TPK)

H3: There is a positive relationship for Content Knowledge (CK) on Technological Content Knowledge (TCK)

H4: There is a positive relationship for Content Knowledge (CK) on Pedagogical Content Knowledge (PCK)

H5: There is a positive relationship for Pedagogical Knowledge (PK) on Technological Pedagogical Knowledge (TPK)

H6: There is a positive relationship for Pedagogical Knowledge (PK) on Pedagogical Content Knowledge (PCK)

H7: There is a positive relationship for Technological Content Knowledge (TCK) on Technological and Pedagogical Content Knowledge (TPACK)

H8: There is a positive relationship for Technological Pedagogical Knowledge (TPK) Technological and Pedagogical Content Knowledge (TPACK)

H9: There is a positive relationship for Technological Content Knowledge (TCK) on Technological and Pedagogical Content Knowledge (TPACK)

H10: There is a positive relationship for Technological Pedagogical Content Knowledge (TPACK) on Learners' Conceptual Understanding (LCU)

H11: There is a direct positive relationship for TK on TPACK

H12: There is a direct positive relationship for CK on TPACK

H13: There is a direct positive relationship for PK on TPACK

3.3.2.1 Rationale for using PLS-SEM

Structural equation modelling can be categorised into covariance-based SEM (CB-SEM) and variance-based SEM (VB-SEM). Variance-based SEM is also known as Partial Least Squares Path modelling or Partial Least Squares Structural Equation Modelling (PLS-SEM).

In general, CB-SEM analysis deals with testing theory whereas PLS-SEM is focused more on performing predictive-causal investigation in empirical studies. While the CB-SEM assumes a true model in estimation (confirmatory), PLS-SEM first estimates the constructs scores and then estimates the statistical significance of path coefficients in the structural model. Specifically, the PLS-SEM algorithm is inclined towards an exploratory study in a limited information context. Nonetheless, PLS-SEM can also be used as a confirmatory analysis to create new measures or paths in an incremental study, such as the case of the current study in which an additional path from the TPACK construct to the Learner Conceptual Understanding (LCU) variable was added.

In summary, CB-SEM is based on goodness-of-fit (GOF) whereas PLS-SEM is concerned about the predictive power. Secondly, sample size and data characteristics could also influence the choice of SEM approaches (Chin, 2010). A large sample size and normality of data are pre-requisite to perform CB-SEM. By contrast, PLS-SEM is a non-parametric method that is suitable for smaller sample sizes and/or non-normally distributed data (Goh & Rasli, 2013; Berrone, Makri & Gomez-Meija, 2008; Echambadi, Campbell & Agarwal, 2006). Finally, PLS-SEM is the appropriate method to perform causal inference with formative constructs. It is against this backdrop that the current study employs the PLS-SEM method in the quantitative experimental intervention phase. Table 3.1 presents a summary of differences between both methods.

Table 3. 1: The difference between covariance-based SEM and variance-based SEM

Dimension	Covariance-based-SEM	PLS-SEM
Algorithm	The algorithm attempts to generate estimates for the latent constructs in the structural paths and the corresponding measurement loadings by maximising the covariance of any connected two items in the structural paths so that it is similar to the covariance obtained from actual sample data.	The algorithm involves two important processes. First, the algorithm attempts to generate estimated score of latent constructs based on the connected items. Second, the algorithm generates PLS estimates based on the immediate blocks of a particular construct in the structural path.
Implication	Focus on covariance of all items in the proposed model based on the goodness-of-fitness and chi-square statistic.	Focus on maximising of variances of dependent variables. PLS-SEM is a predictive-oriented approach.
Distributional Assumptions	CB-SEM is a parametric approach which assumes there are identical distributions in observations and these observations are independent.	No distributional assumption is made. PLS-SEM is essentially a non-parametric approach.
Confirmatory/Exploratory Studies	CB-SEM utilises full information, i.e., maximum likelihood, under the assumption of a “true” model. Thus, the CB-SEM focuses on confirmatory analysis	PLS-SEM can be used in an exploratory study, which is a limited information approach (i.e., the theoretical knowledge is relatively limited).
Sample size	Relatively large sample size required in analysis. The required sample size is based on the Cohen statistical power analysis	The sample size requirement can be based on the OLS regression rule, which is 20 cases per dependent variable
Construct	CB-SEM is limited to the use of reflective construct	Reflective and formative construct can be used

Source: Derived from Chin (2010)

3.3.3 Qualitative Component

Due to the low number of teachers at the NSSCH Mathematics curriculum level, the sample size of the current study does not meet the minimum sample size requirements based on the sampling method outlined in Table 3.4. It was therefore necessary to corroborate the quantitative data through learner interviews and teachers' feedback on the EBM assisted collaborative problem solving process, to gain a deeper understanding of the participants' experiences (both teachers and learners) with the EBM intervention. Figure 3.1 details the rationale for each of the three phases of the qualitative strands in the embedded experimental intervention design. The qualitative strand before the intervention was undertaken in order to gain insight into the teachers' state of TPACK prior to exposure to the EBM professional development intervention. The group feedback from teachers was necessary to explore teachers' experiences and expectations in order to refine the professional development intervention as well as the research questions and sampling procedures for the participants. The qualitative phase during the intervention was designed to assess the participants' reactions to the professional development intervention (Guskey, 2000) as well as monitor the fidelity of implementation of the EBM instructional method. The qualitative strand after the experiment served as a triangulation strategy. Triangulation is the use of one or more methods of data collection in a study of some aspects of human behaviour (Cohen, Manion & Morrison, 2007). It enhances concurrent validity and establishes corroboration of data gathered throughout the entire research process (Campbell & Fiske, 1959).

3.3.4 Description of the research site and participants

Figure 3.3 below shows the site where the research was conducted, in Northern Namibia. The region covers an area of 26 604.8 km^2 and it is the 9th largest of the fourteen regions, with a population of 243 166. The Omusati region has a total of thirty (30) senior secondary schools, of which at the time of conducting the research, only eight (8) offered the Namibia Senior Secondary Certificate Higher (NSSCH) Level Mathematics. The total number of grade 12 learners registered for the NSSCH Mathematics curriculum end of year examinations in 2017, when the data was collected, were 134, representing about 15.8% of the national population of Grade 12 NSSCH Mathematics learners. This demonstrates that only a very small population of Grade 12 learners take Mathematics at Higher Level.

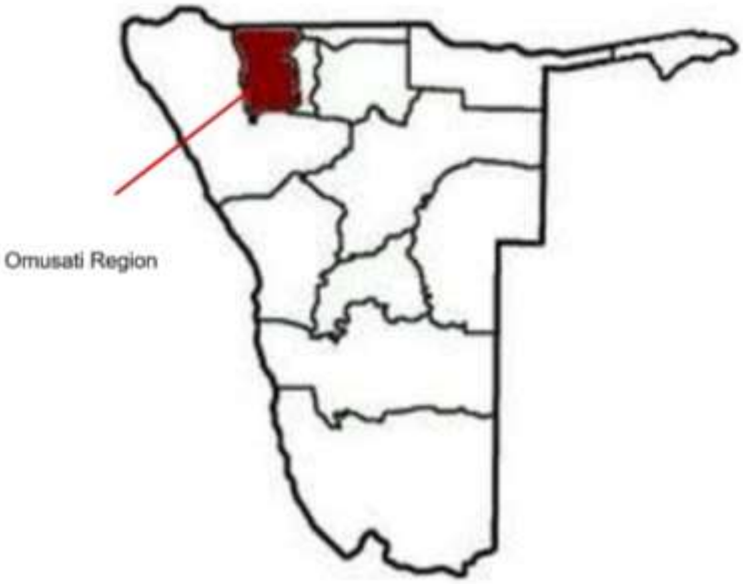


Figure 3. 3: Map of Namibia showing the Omusati Region in Northern Namibia

The participants in the research were 123 Grade 12 NSSCH Mathematics learners from eight secondary schools, 11 grade 12 NSSCH mathematics teachers and 1 Mathematics Education Officer. Table 3.2 shows the distribution of participants at the various stages of the research.

Table 3. 2: Number of participants at key phases of the study

Stage	Number of learners	No of Grade 12 NSSCH teachers	No. of Trainers & Observers	Total number of participants
Teacher professional development	0	11	2	13
Implementation of Excel modelling Intervention	123	11	2	136
Evaluation of Excel modelling intervention	123	11	26	160

Table 3. 3: Timeline of data collection

Period	Activity
10 April 2017	Pre-intervention baseline observations by researcher (teachers' collaborative problem-solving activity, teacher questionnaire, group discussion)
10 April – 14 April 2017	Teacher professional development
1 June – 22 June 2017	Implementation of Excel-modelling instructional method at schools and lesson observations
June 2017	Post-intervention observations by participants (Learners' summative assessment test, Learner questionnaire, Teacher questionnaire, Learner interviews)

3.4 POPULATION, SAMPLING AND SAMPLING TECHNIQUE

3.4.1 Population of the study

There are fourteen (14) educational regions in Namibia. The population of the study was constituted by the number of schools that offered NSSCH Mathematics at grade 12 level in the 2017 academic year in Omusati region. The number of schools that offered grade 12 NSSCH Mathematics in Omusati region in 2017 were eight (8). The total number of learners enrolled for the NSSCH Mathematics level were 134. The total number of teachers teaching Grade 12 NSSCH Mathematics from the 8 schools were eleven (11), with three schools having two separate groups each taught by a different teacher.

3.4.2 Sampling of the region

The NSSCH Mathematics curriculum is taken by very few schools across the country due to its complexity for many teachers and learners (DNEA, 2016). Nationally, only 871 learners were enrolled to take the NSSCH examination in 2017. Some regions did not have a single learner enrolled for Mathematics at this level. The Omusati region was selected because of its outstanding performance in NSSCH Mathematics examinations over the previous five (5) years. The easy accessibility of the schools is another factor why the researcher opted to conduct the study in Omusati region.

3.4.3 Sampling of schools

All the eight schools that offered Grade 12 NSSCH mathematics were selected to participate in the study. The schools all met the selection criteria requiring that all the

NSSCH mathematics teachers possess relevant qualifications and experience in teaching at this level. The schools' environment met the pre-requisite standards of context (learning communities, leadership, and resources); process (data-driven, evaluation, research-based, design, learning, and collaboration) and content (equity, quality teaching and family involvement), to influence the quality of professional development.

3.4.4 Sampling of teachers

The sampling criterion was that the participating teachers should possess a minimum of a Higher Education Diploma (HED) in Mathematics Education, and should have taught at the NSCCH level for at least three years. Participants were all the 11 NSSCH Mathematics teachers since they met these requirements. The sampled teachers thus represented 100% of the population of Grade 12 NSSCH teachers in the particular region.

3.4.5 Sampling of learners

All the 134 learners enrolled for the Grade 12 NSSCH Mathematics curriculum in the 2017 academic year were selected to participate in the study. However, the actual number of learners that participated was 123, representing 91.8% of the regional population of the NSSCH learners. 11 learners were either absent from some of the Excel-based modelling lessons or did not complete some of the assessment tasks.

3.4.6 Sampling techniques used in the study and rationale

Due to the low number of learners taking the NSSCH mathematics curriculum, and the low number of teachers teaching at this level, the researcher chose to employ purposive sampling. In purposive sampling, the standard used to choose participants and sites is that they are information rich (Patton, 2002). Purposive sampling thus applies to both individuals and sites. Within the purposive sampling technique, the intensity sampling strategy was used to guarantee that the chosen site and participants provided information rich cases within a conducive environment for the evaluation of the influence of the Excel-based modelling intervention. The rationale for using the intensity sampling strategy is that the Omusati NSSCH mathematics teachers and learners, despite the reported challenges by DNEA (2014) in dealing with the application of periodicity of trigonometric functions in problem-solving, the region has generally demonstrated a high level of mathematical achievement, suggesting highly effective teachers and highly talented learners.

One of the most fundamental issues in PLS-SEM is that of minimum sample size estimation. The sample size necessary to yield stable results in PLS-SEM depends on the complexity of the model as well as other contextual factors (Jackson, 2003). The researcher analysed the adequacy of the teachers' sample size with reference to literature and other previous studies in the PLS-SEM domain. The following were some notable recommendations for the selection of a minimum sample size.

3.4.6.1 Minimum R-squared method

This method builds on Cohen's (1988) power tables for least squares regression, and lists minimum required sample sizes based on three elements. The first element of the minimum R-squared method is the maximum number of arrows pointing at a latent variable in a model. The second is the significance level used for hypothesis testing. The third is the minimum R^2 in the model. Table 3.4, presented by Hair et al. (2014), shows a simplified version using a significance level of 0.05 and assumes that power is set at 0.8.

Table 3. 4: Sample size recommendation in PLS-SEM for a statistical power of 0.8

Maximum Number of Arrows Pointing at a Construct	Significance Level											
	1%				5%				10%			
	Minimum R^2				Minimum R^2				Minimum R^2			
	0.10	0.25	0.50	0.75	0.10	0.25	0.50	0.75	0.10	0.25	0.50	0.75
2	158	75	47	38	110	52	33	26	88	41	26	21
3	176	84	53	42	124	59	38	30	100	48	30	25
4	191	91	58	46	137	65	42	33	111	53	34	27
5	205	98	62	50	147	70	45	36	120	58	37	30
6	217	103	66	53	157	75	48	39	128	62	40	32
7	228	109	69	56	166	80	51	41	136	66	42	35
8	238	114	73	59	174	84	54	44	143	69	45	37
9	247	119	76	62	181	88	57	46	150	73	47	39
10	256	123	79	64	189	91	59	48	156	76	49	41

Based on the context of the current study, the recommended minimum sample size should have been 157, given that the structural equation modelling of the TPACK constructs involves a maximum of six arrows pointing at the TPACK variable and level of significance 0.05 with a minimum R^2 value of 0.10. Given the available population of only eleven (11) NSSCH Mathematics teachers, it was therefore not feasible to use a sample size of 157 teachers.

3.4.6.2 The 10-times rule method

The most commonly used minimum sample size estimation in PLS-SEM is the "10-times rule" method (Hair et al., 2011; Peng & Lai, 2012). Among the variations of this method, the most common is that the sample size should be greater than 10-times the maximum number of links pointing towards any latent variable in the model (Goodhue et al., 2012). The recommended sample size would have been more than 60 teachers, again a situation not feasible with the small population of NSSCH teachers.

3.4.6.3 Sample size estimation used in previous similar studies

At this point, the researcher's dilemma was that in the absence of an alternative acceptable statistical justification to use a sample size of 11 teachers, the design of the study would have to change or more participants be incorporated, which would mean restarting the data collection phase.

Through more literature search the researcher encountered a published journal article by Tenenhaus et al. (2005) in which they obtained significant results using PLS-SEM methodology to study the relationships between hedonic judgments and product characteristics with a sample size of six (6). Using ordinary least squares (OLS) regression models on the same data also yielded the same significant results. The results of this study were further validated by sensory data experts. Tenenhaus et al. (2005), argued that when the effects of an intervention are strong, PLS-SEM methodology does not require a sample of many individuals.

Another study by Anderson, Hesford and Young (2002) used PLS-SEM as the data analysis technique for examining factors of the successful implementation of activity based costing (ABC) in a Thai telecommunications company, with a sample size of 18 participants. This case study also had significant results in which seven success factors were confirmed as having contributed to the successful implementation of ABC.

In a correlational study of the relationship between grade 11 students' achievement in trigonometric functions and their teachers' content knowledge, Mogari and Ogbonnaya (2014), used a sample of 19 teachers and their respective 418 grade 11 learners in an education district in North West province, South Africa. In a similar way to the PLS-SEM technique used by the current study, Mogari and Ogbonnaya (2014) successfully used linear regression analysis to determine the R square values for analysing the students' achievement variation. The researcher therefore argues that, given sampling limitations using the first two sampling methods, the study could adopt the methodology used by Tenenhaus et al. (2005) and Anderson et al. (2002), without compromising the significance of the results. The researcher, however, recommends that the study be repeated using a larger sample in future.

3.5 DATA COLLECTION PROCESS

The data collection process followed the embedded intervention design shown in Figure 3.1 and was guided by the research questions of the study and aligned to the conceptual framework shown in Figure 2.3. It is essential to understand that the research process followed the following stages:

Stage 1: Baseline assessment of NSSC mathematics teachers' state of TPACK prior to undergoing the Excel-based modelling professional development

Stage 2: Teachers were trained in the Excel-based modelling approach in the teaching of periodicity of trigonometric functions and solution process in related trigonometry problems

Stage 3: An assessment of teachers' self-efficacy and confidence level in the teaching of periodicity of trigonometric function was conducted after the professional development intervention

Stage 4: Teachers implemented the Excel modelling instructional method at their respective schools with NSSCH learners while regular observations were done by the researcher, the Mathematics Education Officer and peer teachers. This phase also involved the assessment of learners' class activities and general response to the programme implementation. A summative assessment of learners' conceptual understanding of periodicity was conducted.

Stage 5: Post-intervention feedback by participants included gathering teachers and learners' experiences about the Excel-based modelling instruction. Learners were also interviewed to gain a deeper understanding of their experiences of learning through the Excel-based modelling (EBM) approach.

3.5.1 Pre-EBM teacher professional development baseline observations by researcher

An understanding of the teachers' baseline level of TPACK was established before the professional development training intervention. This was done through a baseline pre-EBM teacher professional development TPACK survey questionnaire (See Appendix B).

3.5.1.1 Teachers' Collaborative Problem-Solving Activity

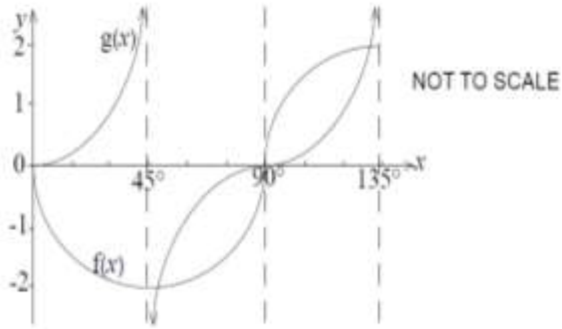
A baseline activity sheet comprising 7 questions selected from previous NSSCH Mathematics Grade 12 examination questions was distributed to the teachers prior to being exposed to the EBM professional development. The researcher presumed that since all the teachers had three or more years of teaching experience, they would already have attempted answering all the questions in the activity sheet during their previous examination questions based revision sessions. The teachers were divided into three (3) groups and asked to develop a marking scheme for the activities. The teachers had to work out the solutions to the activities, present their solution strategies, as well as relate to their previous experiences, and report on any challenges that were faced by the teachers or learners in answering the questions. A computer aided tool, the Computer Aided Algebraic Problem Solving Assessment (CAAPSA) tool, adopted from Lupahla (2014), was used to map the teachers' problem-solving process, in Polya's (1957) model. The CAAPSA tool was originally designed by Lupahla (2014: iv), "to map the thinking process in learners' algebraic problem-solving process from their written work, using Polya's (1957) framework" (p. iv). Polya (1957) set out his summary of the core verbal steps in problem-solving thus:

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back.

The ability of teachers to execute Polya's four steps was mapped through their observed written solution process and group presentations. The rest of the participants were afforded an opportunity to ask the presenters follow up questions for clarity, if any. In order to illustrate the execution of CAAPSA in assessing the periodicity based problem-solving process in Polya's model, consider the following problem in Figure 3.4.

Functions f and g are defined by

$$f: x \mapsto a \sin bx, \quad \text{for } 0^\circ \le x \le 135^\circ \text{ and}$$
$$g: x \mapsto \tan cx, \quad \text{for } 0^\circ \le x \le 135^\circ.$$



(a) Find the values of a, b and c. [3]

Figure 3. 4: Snippet of Problem 1 from teachers' activity sheet

The researcher found the CAAPSA tool convenient to adapt for the assessment of periodicity problems due to the extent of algebraic reasoning involved in the solution process. The group presentations of the solution process clarified the thinking process when there was none available from written solutions. In the adaptation of the CAAPSA tool, the following were the assessment descriptors used at each of Polya's four stages.

A rubric was constructed to classify each problem by weight of assessment objectives, from level 1 (lowest) to level 4 (highest). Table 3.5 shows the descriptors of assessment objectives and weighting of each problem.

Table 3. 5: Table of descriptors for weighting of periodicity problems

LEVEL	OBJECTIVES
1	Identify the fundamental properties of the functions: *f(x) = Sin x,* *g(x) = Cos x, and h(x) = Tan x* Define period and amplitude
2	Understand the effects of changing the values of a, b, and c on the graphs of: *f(x) = a Sin(bx)+c,* *g(x) = a Cos(bx)+c, and* *h(x) = a Tan(bx)+c* Obtain the correct transformations of graphs of trigonometric functions.
3	Sketch the graphs of the functions of the form: *f(x) = a Sin (bx)+c, g(x) = a Cos (bx)+c, and h(x) = a Tan (bx)+c, for* $-360^0 \leq x \leq 360^0$.
4	Application of knowledge of periodicity to find amplitude and period and sketch and interpret graphs of the form; $y = a \sin(bx) + c$, $y = a \cos(bx) + c$, and $y = a \tan(bx) + c$ *Solve trigonometric equations of the form; a*trig function (bx) + c = 0*

Figure 3.5 shows a snippet of the solution process by Group 1. Follow up questions to the group's presentation were posed with the responses provided through the participation of the whole group.

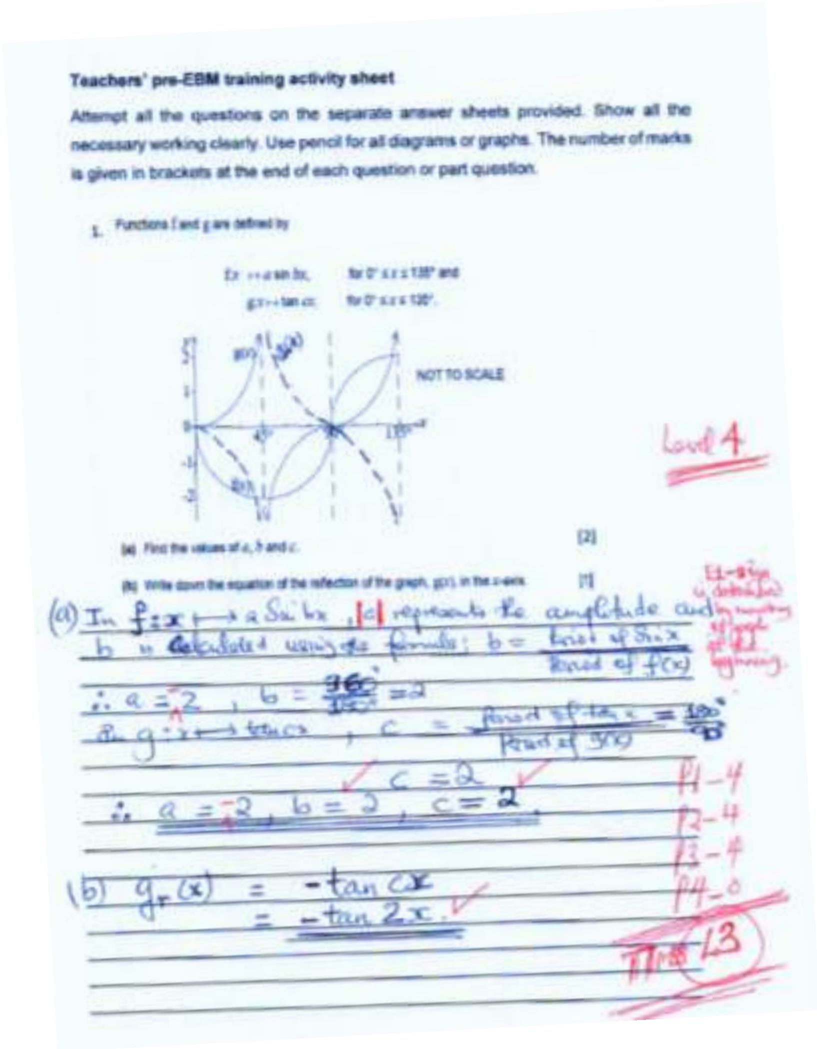


Figure 3. 5: Snippet of sample solution process by one group of teachers in the pilot study phase

Figure 3.6 shows a Group 1 teacher presenting the group's solution process to Problem 1, to the rest of the participants, after which follow up questions were posed. The participants then collectively engaged in an open discussion to reach a consensus on the correct solution process.

Figure 3. 6: A teacher from Group 1, presents their solution strategy while being observed by the rest of the participants

3.5.1.2 The CAAPSA tool

The CAAPSA tool was previously used by Lupahla (2014: 54) "to assess Grade 12 learners' algebraic problem-solving skills in Polya's model, by computing the number of errors (conceptual, procedural or computational) at each stage. The CAAPSA tool then retains CAAPSA levels (1 to 5) at each of Polya's steps, thus allowing the researcher to identify the stages at which learners encounter difficulties". The next section discusses the CAAPSA processing in the context of how it was used to assess the teachers' collaborative problem solving process.

3.5.1.2.1 The CAAPSA processing

All input data is entered in the yellow cells on the *Excel* worksheet as shown in Figure 3.7. For the assessment of the solution process to periodicity problems involving functions or graphs of functions of the form; *f(x) = a*trig function (bx) + c,* the study adapted the algebraic problem solving steps proposed by Lupahla (2014: 82), namely;

Polya Step 1: Deduce the amplitude and the period from a function or graph. If there is a reasonable attempt to correctly deduce the amplitude and period, then H = 1; otherwise H = 0. The processing stops. If H = 1, count and enter the number of errors;

Polya Step 2: Deduce the values of a, b and c, from the period, amplitude and y-intercept of the graph. If there is a reasonable attempt to determine the correct values of a, b and c, then H = 1, otherwise H = 0. The processing stops. If H = 1, count and enter the number of errors in the step;

Polya Step 3: If there is a reasonable attempt to apply an appropriate solution strategy with the values deduced in step 2, then H = 1, otherwise H = 0. The processing stops. If H = 1, count and enter the number of errors in the solution process;

Polya Step 4: If there is a reasonable attempt to check the solution then H = 1, otherwise H = 0. The processing stops. If H = 1, count and enter the number of errors in the checking. If the answer in step 3 is correct and no checking has been attempted, then H=1 and number of errors = 0.

In each of the cases above "H" represents a correct attempt to execute Polya's step. The decision on whether an attempt to execute Polya's step is correct or not is determined by the descriptors in the CAAPSA marking tool. If the attempt is reasonable, then H=1, even if the actual execution of the step has errors. If the attempt is completely wrong, then H = 0.

3.5.1.2.2 The CAAPSA output

Figure 3.7 shows the CAAPSA output for the assessment of the solution strategy used by Group 1 teachers. The assessment is based on Polya's (1957) problem-solving steps and TIMSS (2007) benchmarks.

GROUP:	1
Enter the level of the problem	4

STEP	Descriptors	H-value	Excel modelling marking grid		
			No of errors	Score	Comment(s)
P1	Identifies key data required for solution process	1	1	4	It is not a but lal that represents amplitude, consequently sign is not taken into account
P2	Identifies appropriate solution strategy (e.g. formula)	1	0	4	Correct
P3	Correctly employs chosen strategy	1	0	4	Correct
P4	Checks the accuracy of solution in the context of the problem	0	0	0	No checking attempt seen. Group only realises error during group prsentations and discussions.

Total Score	12	L3
TIMSS Score	500	

Colour Score Card (CSC)		
(red)	L1 - L2	VERY LOW-LOW
(orange)	L3	INTERMEDIATE
(green)	L4-L5	HIGH-ADVANCED

Figure 3. 7: CAAPSA output of solution process by group 1 teachers.

The results were used to identify any gaps, other than the ones in the examiners' reports that the EBM professional development training would have to address. The researcher subsequently used this data to ensure that the TPACK based teacher professional development intervention was relevant to the teachers' professional development needs.

3.5.1.3 Pre-EBM teacher professional development TPACK baseline survey questionnaire

The questionnaire was used to measure TPACK self-efficacy perceptions of teachers, prior to exposure to the EBM professional development intervention. The data obtained was used to determine the effect size of the EBM professional development intervention through comparison to post-intervention self-efficacy perceptions of teachers. Effect sizes are the most important outcome of empirical studies.

Researchers want to know whether an intervention or experimental manipulation has an effect or how big the effect is (Lakens, 2013). Hedges' **g** formula for sample sizes less than 20 was used to compute the effect size. The method is explained in detail later under the data analysis section.

3.5.2 The Excel-based modelling teacher professional development programme and observed outcomes

The designing of the Excel-based modelling instructional method was underpinned by the basic modelling process (CCSM, 2010) and the TPACK framework. The pedagogical component of the TPACK framework emphasised the constructivist inquiry based learning approach through which learners could construct their own meanings of the connections between graphs and functions of trigonometric functions, particularly in terms of their periodicity.

To fulfil this purpose, selected teachers underwent one week of EBM professional development, during the second week of April 2017, to enable them to use the developed Excel-based modelling applets in the teaching and learning of periodicity of trigonometric functions and graphs in the NSSCH curriculum. The teachers were taken through the design, content and implementation of Excel modelling applets for the teaching of periodicity of trigonometric functions and graphs in the NSSCH Mathematics curriculum. For example, Figure 3.8 shows an Excel applet display of one of the activities conducted in the modelling of the graphs of the **sine** and **cosine** functions.

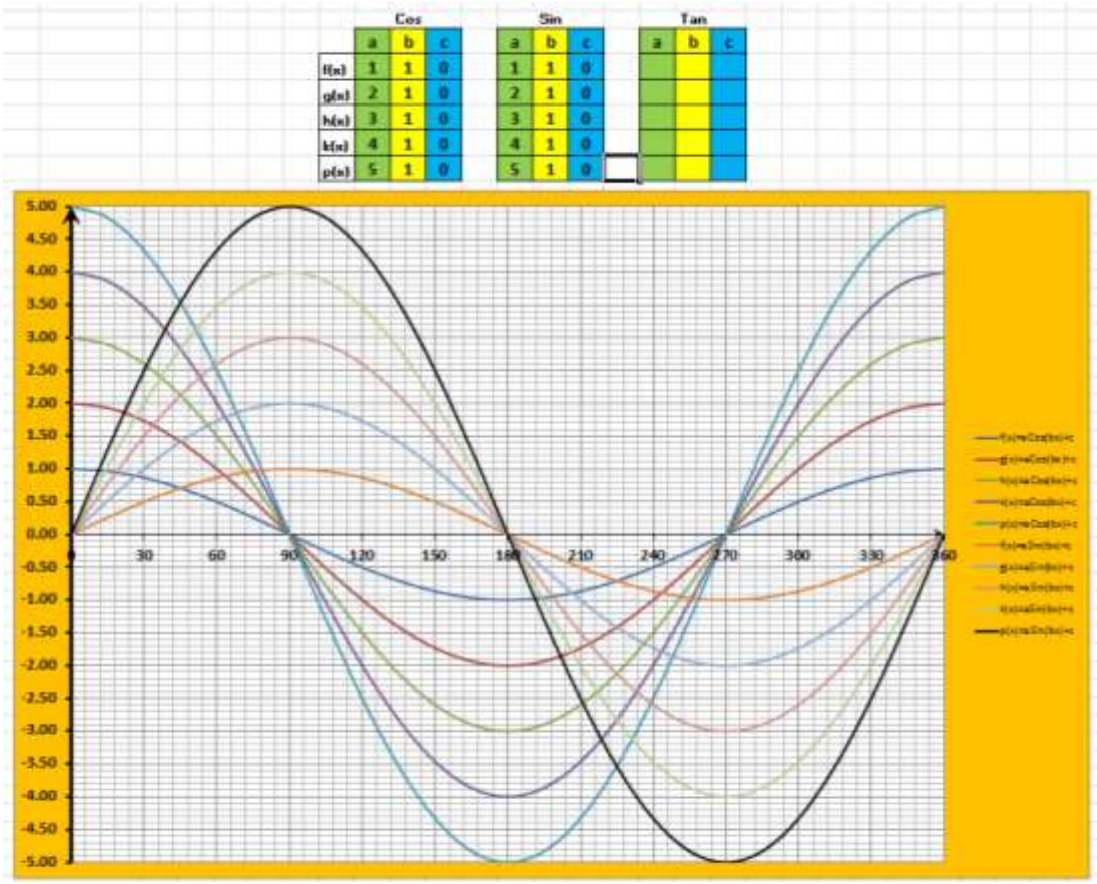


Figure 3. 8: A snippet of the Excel display of the modelling of the Sine and Cosine functions

The input variables in the models of the **cosine** and **sine** functions in the form *f(x) = a*trigfunction (bx) + c* are the values of *a, b* and *c* for the function *f(x), g(x), h(x), k(x) and p(x)*. Displaying the **sine** and **cosine** function graphs simultaneously on the same axes allows for more observations and comparisons of the characteristics of the functions. The learners can deduce how the periodicity and/or amplitude of the graphs are affected by manipulating the values of ***a*** and ***b***. The effect of changing ***c***, which results in the graph being translated vertically, can also be deduced, thus creating an active process in which learners construct new concepts based upon their own experiences.

The exploration of the tangent function was done separately because of the unique characteristics of asymptotes. Figure 3.9 shows a typical display of the **tangent** function used in the EBM exploration. The two functions f(x) = tan x and h(x) = 2 tan 2x + 1 are modelled on the same axes to facilitate the inquiry into the effects of changes in the values of **a, b** and **c** on the graph of y = a*tan (bx) + c.

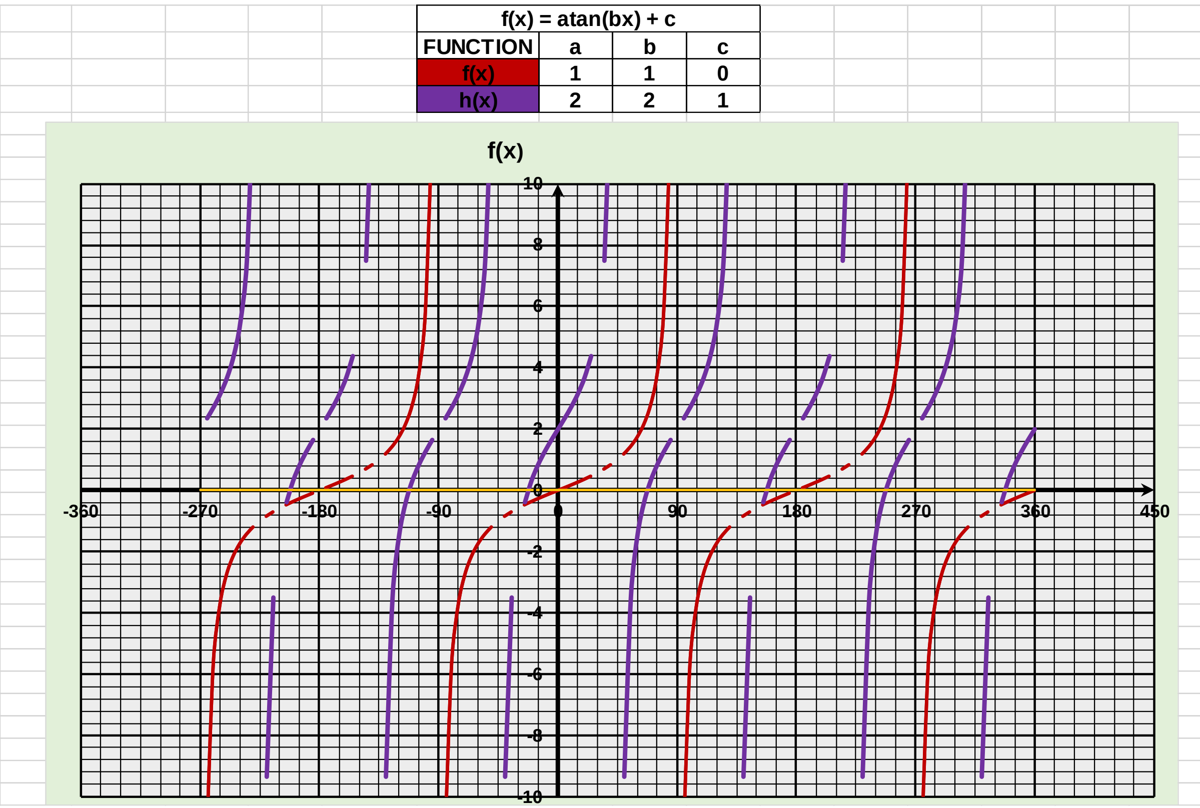


Figure 3. 9: A snippet from the Excel display for the modelling of the Tangent function

3.5.3 Implementation of the EBM instruction and observed outcomes

The teachers implemented the intervention, using the lesson plans that were developed and refined during the professional development training. Six (6) lessons of one (1) hour duration each, inclusive of assessment activities, were taught by each teacher during the afternoon study periods. The lessons and assessment activities were based on common lesson plans developed and shared in soft copy during the course of the professional development training programme. Teachers were given room to flexibly implement their own styles and additional class assessment activities as befitting to their circumstances. The fidelity of implementation of the EBM instruction by the teachers was regularly monitored and reported by their peer teachers, subject heads, heads of department, the Mathematics Education Officer and the Researcher. A reporting instrument (checklist lesson observation schedule) was provided for all lessons observed.

3.5.4 Post-intervention observations by researcher

3.5.4.1 Post-EBM instruction teaching and learning outcomes

The evaluation was done using Guskey's framework of evaluation of teacher professional development, hence the data gathered according to Guskey's five levels of professional development, as described in Table 2.2. PLS-SEM was used to model the relationship between TPACK constructs and the relationship between teachers' TPACK development and LCU.

3.6 INSTRUMENTS

The study employed the following instruments:

3.6.1. Pre-EBM teacher professional development TPACK baseline survey questionnaire

The questionnaire was designed to measure the teachers' self-perceived level of development of the seven TPACK constructs (see Appendix B). The design process of the questionnaire took cognisance of the fact that the optimal number of items that should be associated with latent variables should not be fewer than three items per variable (Ding et al., 1995; Tomás et al., 2000).

The instrument was a structured questionnaire that used a 5 point Likert scale in which the respondent strongly agrees (SA), agrees (A), is uncertain (U), disagrees (D) or strongly disagrees (SD) with the statements about the phenomena under investigation. The questionnaire had 70 questionnaire items, 10 in each of with the 7 TPACK knowledge domains.

3.6.2 Teacher collaborative problem-solving activity sheet

The activity sheet consisted of 7 semi-structured past examination questions in the mathematics content of trigonometric functions and graphs (see Appendix A). The purpose was to gain insight into the teachers' problem-solving challenges or misconceptions in the domain of trigonometric functions and graphs. This was done to ensure relevance and appropriateness of the professional development intervention. The teachers were grouped into three groups, Group 1 (3 teachers), Group 2 (4 teachers) and Group 3 (4 teachers). To enhance effective cooperative interactions, the teachers were randomly assigned to the groups, ensuring that the groups were of mixed gender and none of the teachers in each group were from the same school. The groups were given 1 hour to attempt answering all the questions and developing a marking scheme. The teachers' problem-solving process was assessed using a computer assisted problem solving assessment tool, in which the process was scored using Polya's problem solving framework. The numeric scores were then used to validate the teachers' perceived level of content knowledge (CK).

The groups were given an opportunity to reflect back on their solutions and collectively share alternate solution strategies to some of the challenging questions they had encountered in the activity sheet. In the process, the researcher obtained additional data to validate the teachers' perceived levels of content knowledge (CK) and pedagogical knowledge (PK).

3.6.3 Checklist lesson observation schedule

A 50 item checklist list lesson observation schedule was developed to evaluate teachers TPACK levels in practice as well as assess learning outcomes (see Appendix E). The purpose was to monitor the fidelity of implementation of the EBM instructional method, to map the state of teachers' TPACK throughout the EBM implementation stage, as well as obtain data on learners' learning outcomes. The data on learners' learning outcomes was used in Guskey's (2000) Level 5, for evaluation of the impact of the teacher professional development programme. A total of five (5) lessons per teacher were observed by panels of 2 observers composed of peer mathematics teachers or head of department, the region's mathematics education officer responsible for monitoring the NSSCH mathematics curriculum standards and/or the researcher (see table 4.21).

3.6.4 Post-EBM teacher professional development TPACK survey questionnaire

This questionnaire was identical to the pre-intervention TPACK survey questionnaire, with the sole purpose of using both the pre and post survey scores to measure the effect size of the professional development programme (See Appendix D). This was done in order to assess the teachers' confidence levels before moving to the implementation phase of the EBM instruction with their learners at their respective schools.

3.6.5 Post EBM instruction teacher evaluation questionnaire

A 50 item EBM teacher professional development survey questionnaire was developed and administered to all the 11 participating teachers, after the implementation of the EBM instructional method (see Appendix H). The purpose of this instrument was to map the teachers' perceptions of the impact of the EBM teacher professional development outcomes, using Guskey's (2000) evaluation model for teacher professional development. Furthermore, the questionnaire data would provide information to validate the extent to which the school systems to provided the projected organisational support and change (See Tables 4.17 and 4.18). In the model, Guskey (2000) proposed five levels of evaluating professional development namely; ***participants' reactions***, ***participants' learning***, ***organisation support (school support) and change***, ***participants' use of new knowledge and skills*** and the ***learners' learning outcomes*** (see Table 2.2).

The questionnaire is a structured one that uses a 5 point Likert scale in which the respondent strongly agrees (SA), agrees (A), is uncertain (U), disagrees (D) or strongly disagrees (SD) with a number of statements about the phenomena under investigation. The questionnaire had 50 questionnaire items.

3.6.6 Post-EBM instruction learner interview schedule

After the implementation of the programme, a purposively selected sample of learners from each school were interviewed to establish and verify their learning experiences with the EBM approach. This was done to serve as a data triangulation strategy, in Guskey's (2000) evaluation model.

3.6.7 The Computer Aided Excel Modelling Assessment (CAEMA) tool

The design of the CAEMA tool was influenced by the basic problem-solving model and the adopted four step problem solving process. The CAEMA tool is a modification of the CAAPSA tool, to assess the Excel-based modelling process, using the four modelling steps adopted from Swetz and Hartzler (1991); (1) Identify the problem, (2) Hypothesise the solution, (3) Collect data and test against hypothesis, and (4) Draw conclusions.

As part of the formative assessment of learners, during the implementation of the EBM instruction, learners engaged in a collaborative problem-solving activity, using the EBM applets to support their inquiry process. The CAEMA tool was used by the teachers and/or lesson observers, to map the learners' problem-solving process using Swetz and Hartzler's (1991) modelling steps. The following diagrammatic illustration depicts the CAEMA processing. The main components and content of the CAEMA tool are based on the steps of a computer processor, input, processing, and output (graphic screen display).

The first step requires the user to enter the level of the problem being solved, as determined by the criteria shown in Table 3.6.

Table 3. 6: The descriptors for the four levels of periodicity problems and associated teacher/learner activities

LEVEL	OBJECTIVES	TEACHERS' ACTIVITIES	LEARNERS' ACTIVITIES
1	• Identify the fundamental properties of the functions: *f(x) = Sin x, g(x) = Cos x, and h(x) = Tan x* • Define period and amplitude	• Facilitate the demonstration of the Excel modelling of: *f(x) = Sin x, g(x) = Cos x, and h(x) = Tan x, for* $0^0 \leq x \leq 360^0$. • Explain the concepts of period and amplitude for each graph	• Model the graphs and deduce the properties of each function • Match given functions to their graphs and vice-versa
2	• Understand the effects of changing the values of a, b, and c on the graphs of: *f(x) = a Sin(bx)+c, g(x) = a Cos(bx)+c, and h(x) = a Tan(bx)+c* • Obtain the correct transformations of graphs of trigonometric functions.	• Facilitate the demonstration of the Excel modelling of: *f(x) = a Sin(bx)+c, g(x) = a Cos(bx)+c, and h(x) = a Tan(bx)+c, for* $0^0 \leq x \leq 360^0$. • Give practice activities for learners to predict the effects of a,b,c on the graphs of the given trigonometric functions	• Use Excel modelling to explore the effects of changing *a,b,c* on the graphs of: *f(x) = a Sin(bx)+c, g(x) = a Cos(bx)+c, and h(x) = a Tan(bx)+c, for* $0^0 \leq x \leq 360^0$. • Learners predict the graphic outputs for different values of *a,b,c* and then verify their predictions through
3	• Sketch the graphs of the functions of the form: *f(x) = a Sin (bx)+c, g(x) = a Cos (bx)+c, and h(x) = a Tan (bx)+c, for* $-360^0 \leq x \leq 360^0$.	• Give practice activities for learners to sketch the graphs of the given trigonometric functions: *y = a sin(bx) + c, y = a cos(bx) + c, and y = a tan(bx) + c;*	• Learners sketch the graphs and then verify their solutions through matching their sketches to the modelling process graphic outputs for the given functions.
4	• Application of knowledge of periodicity to find amplitude and period and sketch and interpret graphs of the form: *y = a* sin(*bx*) + *c*, *y = a* cos(*bx*) + *c*, and *y = a* tan(*bx*) + *c*	• Assess learners' conceptual understanding of periodicity through past examination based questions • Use the CAEMA tool to assess the learner's problem solving process • Give feedback to learners	• Learners individually solve periodicity problems • Learners collaboratively discuss their solutions through the aid of Excel modelling.

3.6.7.1 Rubric for scoring the excel based modelling process

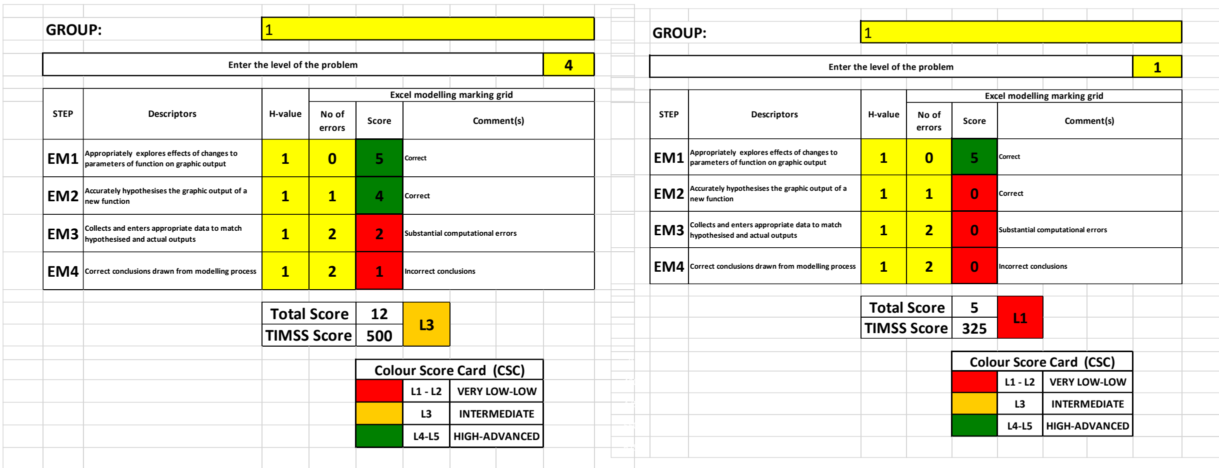
Once the level of the problem has been assigned, the errors committed by the learners at each modelling step are entered in accordance with the descriptors in the rubric in Table 3.7. Table 3.7 shows the rubric used to assess the EBM problem-solving process using the Computer Assisted Excel-based Modelling Assessment (CAEMA) tool. The Excel-based modelling problem-solving process was scored at each of the four steps, using an analytic scoring rubric, adapted from Charles, Lester and O’Daffer (1987).

Table 3. 7: Indicators adapted from Charles, Lester and O'Daffer's (1987) analytic scoring rubric, for scoring the Excel-based modelling problem solving process

Step	Stages of Solving	Score	Characteristics	Indicators
1	Identifying the problem	0	Complete misunderstanding of the problem	No attempt to solve the problem, no exploratory data inputs identified or entered
		1	Very limited identification of the problem	Incorrect input data identified or entered and used to explore input-output relationships (IOR)
		2	Minimal identification of the problem	Minimal but unsuccessful attempt to identify and enter required input data and explore IOR
		3	Intermediate identification of the problem	Some reasonable effort to enter correct input data and explore IOR
		4	High level of identification of the problem	Almost all input data correctly identified and entered to explore IOR
		5	Advanced level of identification of the problem	Correct and complete input data identified and entered to explore IOR
2	Hypothesise the solution	0	Complete lack of hypothesis	No attempt to establish relationship between input data and screen display
		1	Very limited effort to hypothesise the solution	Very limited effort to establish relationships in context of problem situation
		2	Partially hypothesised solution based on part of the problem identified	Minimal effort to establish correct input and output relationships in context of problem situation
		3	Appropriate hypothesis based on the identified problem but with some errors	Some reasonable effort to establish relationships in context of problem situation
		4	Hypothesis could have led to a correct solution if input data had been correct	Almost all relationships established in context of problem situation
		5	Appropriate hypothesis proposed	Complete and correct IOR established in context of problem situation.
3	Collect output data and test against hypothesis	0	Data collection and hypothesis testing incorrectly executed or not executed at all	No attempt or totally incorrect execution of hypothesis testing
		1	Minimal effort to collect and test output data against hypothesis	Minimal attempt to execute hypothesis testing
		2	Some reasonable but inadequate data collected and used for hypothesis testing	Reasonable but inadequate attempt to correctly execute hypothesis testing
		3	Partially correct hypothesis testing but with major omissions or errors	Partially correct execution of hypothesis testing
		4	Hypothesis testing executed correctly but with some output data entry errors	Almost a correct and complete execution of hypothesis testing
		5	Correct and complete execution of data entries	Correct and complete execution of hypothesis testing
4	Draw conclusions/ generalisations	0	No evidence of attempt to draw any relevant conclusions/generalisations	No attempt to evaluate solution
		1	Minimal evidence of conclusion or generalisation	Incorrect evaluation of solution
		2	An inadequate attempt to draw relevant conclusions from solution process	Inadequate attempt to evaluate solution
		3	A partial attempt to draw relevant conclusions from solution process	Partially correct evaluation of solution
		4	An almost adequate establishment of relevant conclusions from solution process	An almost adequate evaluation attempt
		5	Adequate and relevant conclusions drawn from solution process	Correct and complete evaluation of the solution process

3.6.7.2 The CAEMA processing

The CAEMA tool is designed such that the score allocation at each step is given a weight corresponding to the level of the problem. For example, the same problem would yield different assessment outputs if assigned different levels, as shown in Figure 3.10 below.



GROUP: 1

Enter the level of the problem: 4

STEP	Descriptors	H-value	No of errors	Score	Comment(s)
		Excel modelling marking grid			
EM1	Appropriately explores effects of changes to parameters of function on graphic output	1	0	5	Correct
EM2	Accurately hypothesises the graphic output of a new function	1	1	4	Correct
EM3	Collects and enters appropriate data to match hypothesised and actual outputs	1	2	2	Substantial computational errors
EM4	Correct conclusions drawn from modelling process	1	2	1	Incorrect conclusions

Total Score	12	L3
TIMSS Score	500	

Colour Score Card (CSC)	
L1 - L2	VERY LOW-LOW
L3	INTERMEDIATE
L4-L5	HIGH-ADVANCED

GROUP: 1

Enter the level of the problem: 1

STEP	Descriptors	H-value	No of errors	Score	Comment(s)
		Excel modelling marking grid			
EM1	Appropriately explores effects of changes to parameters of function on graphic output	1	0	5	Correct
EM2	Accurately hypothesises the graphic output of a new function	1	1	0	Correct
EM3	Collects and enters appropriate data to match hypothesised and actual outputs	1	2	0	Substantial computational errors
EM4	Correct conclusions drawn from modelling process	1	2	0	Incorrect conclusions

Total Score	5	L1
TIMSS Score	325	

Colour Score Card (CSC)	
L1 - L2	VERY LOW-LOW
L3	INTERMEDIATE
L4-L5	HIGH-ADVANCED

Figure 3. 10: Outputs of CAEMA assessment of same problem assigned different weight levels

The assessment of the level 1 problem yields TIMSS Score 325 and TIMSS Level 1, whereas at Level 4, the assessment yields TIMSS score 500 and TIMSS level 3. This is because the CAEMA factors in the level of competencies being assessed and adjusts the scale accordingly, such that higher level competencies have more weight than lower level competencies.

3.6.8 Summative assessment test for learners (Conceptual Understanding)

This was a structured 8 item test in which learners analysed the characteristics of eight (8) separate trigonometric graphs and determined the corresponding algebraic representations of their functions. The summative assessment test score was used as a measure of the learners' conceptual understanding in the partial least squares equation model.

3.7 PLS-SEM METHODOLOGY

The study used a variance based structural equation modelling technique to test the direct and indirect effects or mediated relationships between the TPACK constructs and the direct effect of teachers' TPACK on learners' conceptual understanding. The research used the latent construct scores to analyse the predictive relevance as suggested by Hair et al. (2011). The data was analysed using a two-step approach recommended by Hair et al. (2013); the first step consists of evaluation of the measurement model; while the second step involves testing the reliability and validity of the measures. This is followed by an assessment of the structural relationship of the model. Hence, the research combined the weights of items for each TPACK construct through PLS algorithm to generate the latent variable scores.

3.8 VALIDITY AND RELIABILITY OF INSTRUMENTS

To determine the validity and reliability of the data collection instruments of the study, a pilot study was conducted with a different group of 11 teachers and 60 learners at the Rössing Foundation Centre, where the researcher worked as a Mathematics Education Officer. The 11 teachers attended a vacation school teacher professional development programme for 10 days in April 2016, while the implementation of the instructional programme was done in August 2016. The participants in the pilot study were thus outside of the sample of the main study.

The content validity and internal consistency reliability of the instruments were determined during the pilot study. The questionnaires used to collect data were adapted from previous pilot studies, in which they were demonstrated for content validity, meaning they fairly and comprehensively cover the domain that they purport to cover (Carmines & Zeller, 1970).

To ensure construct validity, the questionnaires were benchmarked against the ones used by Schmidt et al. (2009), in a study in which they developed and validated a TPACK survey instrument to assess TPACK for preservice teachers, using factor analysis. Construct validity refers to the degree to which the items on an instrument relate to the relevant theoretical construct (Kane 2001; DeVon et al. 2007).

The lesson observation checklist was adapted from the national teacher evaluation documents, mathematics advisory services and standards for teacher assessment. The learners' summative test was benchmarked against the curriculum objectives and assessment guide for conceptual understanding level questions in periodicity of trigonometric functions. The current study employed both qualitative and quantitative methods as a triangulation strategy to ensure the validity of the findings.

3.8.1 The pre-EBM intervention TPACK baseline survey questionnaire

3.8.1.1 Content and Construct validity

A panel of five experts from the Ministry of Education, Arts and Culture curriculum advisory services were also asked to review the relevance of each questionnaire item on a 4 point Likert scale;1 = not relevant, 2 = somewhat relevant, 3 = relevant, 4 = very relevant. For each item, the item content validity index (I-CVI) was computed as the number of experts giving a rating of 3 or 4, divided by the total number of experts. For example, an item rated 3 or 4 by four out of five experts has I-CVI of 0.80. It is advised that I-CVI should be 1.00 in case of five or fewer judges and in case of six or more judges; I-CVI should not be less than 0.78 (Lynn, 1986; Polit & Beck, 2006; Rubio, Berg Weger, Tebb, Lee, & Rauch, 2003). For each item, in the TPACK baseline survey questionnaire, all the five (5) experts gave values of 3 or 4, hence the item content validity index (I-CVI) = 1.

3.8.1.2 Internal consistency reliability

The questionnaire was piloted on the group of 11 teachers and 60 learners from regions other than the specific region where the main study was conducted. The questionnaire responses were subjected to a test of reliability of scale using the Cronbach reliability analysis in the SPSS programme. The SPSS output yielded a high Cronbach alpha value of 0.956.

Table 3.8 shows the SPSS screen output for the results.

Table 3. 8: SPSS screen shot of analysis of Cronbach Alpha

Case Processing Summary

		N	%
Cases	Valid	11	100.0
	Excluded[a]	0	.0
	Total	11	100.0

a. Listwise deletion based on all variables in the procedure.

Reliability Statistics

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
.956	.959	70

3.8.2 Post EBM teacher professional development TPACK survey questionnaire

3.8.2.1 Content and Construct validity

The post intervention TPACK survey questionnaire was also adapted from Schmidt et al. (2009), to ensure construct validity. For content validity, a panel of five experts from the Ministry of Education, Arts and Culture curriculum advisory services were also asked to review the relevance of each questionnaire item on a 4 point Likert scale; 1 = not relevant, 2 = somewhat relevant, 3 = relevant, 4 = very relevant. For each item, all the five (5) experts gave values of 3 or 4, hence the item content validity index (I-CVI) = 1.

3.8.2.2 Internal consistency reliability

The questionnaire was administered to the 11 teachers in the pilot group and tested for internal consistency reliability using Cronbach's coefficient alpha value calculated using the SPSS package and yielded a high value of 0.896.

3.8.3 Checklist lesson observation schedule

3.8.3.1 Content and Construct validity

To ensure construct validity, national indicators on teacher evaluation were used to develop the lesson observation checklist. A panel of five experts from the Ministry of

Education were also asked to review the relevance of each checklist item on a 4 point Likert scale;1 = not relevant, 2 = somewhat relevant, 3 = relevant, 4 = very relevant. For each item, all the five (5) experts gave values of 3 or 4, hence the item content validity index (I-CVI) = 1 was established and confirmed.

3.8.3.2 Internal consistency reliability

The lesson observation checklist was tested for internal consistency reliability using Cronbach's coefficient alpha value calculated using the SPSS package after observing some of the lessons taught by the teachers. The SPSS package yielded a Cronbach alpha value of 0.878.

3.8.3.3 Inter-rater reliability

The inter-rater reliability (IRR) was used to determine the relative similarity between the ratings of the two raters in each lesson observed. The rationale for using IRR is that even if the two evaluators have little or no agreement, their allocated scores could still have high IRR. In other words, one rater might consistently award lower scores while the other consistently awards higher scores, but concurring on the relative ranking of the same teachers. The lesson observation scores from each lesson and observer team were entered into a an Excel spreadsheet in which the individual item ratings would immediately be processed to determine the mean rating and level for each TPACK construct and the learners' learning outcomes. The observation outcomes were accepted as reliable for $r > 0.5$ for each category.

3.8.4 Post-EBM instruction teacher survey questionnaire (Guskey)

3.8.4.1 Content and Construct validity

To ensure construct validity, the questionnaire was developed as an adaptation from previous studies, in which construct validity was guaranteed through factor analysis. A panel of five experts from the Ministry of Education were also asked to review the relevance of each questionnaire statement on a 4 point Likert scale; 1 = not relevant, 2 = somewhat relevant, 3 = relevant, 4 = very relevant. For each item, all the five (5) experts gave values of 3 or 4 for each of the items, hence the item content validity index (I-CVI) = 1 was established and confirmed.

3.8.4.2 Internal consistency reliability

The questionnaire was tested for internal consistency reliability using Cronbach's coefficient alpha value (internal consistency method), calculated using the SPSS package. The Cronbach alpha value was calculated and yielded a high value of 0.935371.

3.8.5 Reliability of the learners' summative test

3.8.5.1 Content and Construct validity

There are several abilities which are exhibited by a learner with conceptual understanding (CU). Kilpatrick et al. (2001) states that learners with CU are able to use their mathematical conceptual knowledge for explaining new mathematical constructs. This ability helps students in expanding their knowledge. Moreover, Kilpatrick et al. (2001) submits that students with CU are able to use several representations and communicate their ideas. In addition, students with CU are able to choose a representation that is suitable for a specific mathematical situation. This ability helps students in communicating their ideas efficiently and effectively. In order to ensure construct validity, three indicators were used to develop the test items to measure students' CU in this study:

1. Being able to match graphic representations of trigonometric functions to the corresponding basic functions of **sine**, **cosine** or **tangent**.
2. Being able to deduce amplitude and period from a graphic representation of a trigonometric function.
3. Knowing how to write an algebraic representation of a graph of a trigonometric function.

A panel of five experts from the Ministry of Education, Arts and Culture was asked to review the relevance of each question in measuring conceptual understanding of periodicity of trigonometric functions. The experts rated the relevance of each question on a 4 point Likert scale; 1 = not relevant, 2 = somewhat relevant, 3 = relevant, 4 = very relevant. For each question, all the five (5) experts scored values of 3 or 4 for each of the questions, hence the item content validity index (I-CVI) = 1.

3.8.5.2 Internal consistency reliability

The 8-item test was marked for all the 60 learners in the pilot group. The Spearman Brown split-half technique was used to determine the reliability of the test. The test was divided into two subtests, by selecting odd items for one subtest and even items for the other subtest each participant's score was then computed on the two halves such that each participant would have a score for the odd items and the even items. The two sets of scores were analysed for split-half reliability in SPSS and yielded a Spearman Brown coefficient = 0.819, showing that the test items were consistent with what they were intended to measure.

Table 3.9 shows the SPSS output for the split-half reliability analysis of the summative assessment test (conceptual understanding) on periodicity of trigonometric functions.

Table 3. 25: SPPS output internal consistency reliability of summative test items

Reliability Statistics

Cronbach's Alpha	Part 1	Value	1.000
		N of Items	1[a]
	Part 2	Value	1.000
		N of Items	1[b]
	Total N of Items		2
Correlation Between Forms			.693
Spearman-Brown Coefficient	Equal Length		.819
	Unequal Length		.819
Guttman Split-Half Coefficient			.817

a. The items are: HALF 1

b. The items are: HALF 2

3.8.6 Validity and reliability of the CAEMA tool

The validity and reliability of the CAEMA tool was already established in a study conducted by Lupahla (2014), to map the algebraic problem solving skills of Grade 12 learners in a specific region of Namibia. The study used a version of the CAEMA tool, referred to as the Computer Aided Algebraic Problem Assessment (CAAPSA) tool. The CAAPSA tool was designed to use Polya's (1957) framework to map the thinking processes in learners' algebraic problem solving from their written work (Lupahla, 2014).

3.8.7 Validity and reliability of the problems in the collaborative problem solving activity

The problems used in the problem solving activity were adapted from the national NSSCH past examination question papers, hence both construct and content validity were ensured. Besides, a panel of five mathematics subject experts from the Ministry of Education were asked to review the relevance of each problem in measuring the curriculum competencies, on a 4 point Likert scale; 1 = not relevant, 2 = somewhat relevant, 3 = relevant, 4 = very relevant. For each problem, all the five (5) experts gave values of 4, hence the construct validity of the problems was confirmed with index (I-CVI) = 1.

3.8.8 Validity the post-EBM instruction learner semi-structured interview schedule

The questions were reviewed by a panel of five experts from the Ministry of Education. They were asked to review the relevance of each question in probing for appropriate feedback on the learners' EBM experience, in line with Guskey's level 5 indicators (see Appendix G and Table 2.2). Using a 4 point Likert scale; 1 = not relevant, 2 = somewhat relevant, 3 = relevant, 4 = very relevant, they all confirmed a construct validity (I-CVI) = 1.

3.9 ETHICAL CONSIDERATIONS

Creswell (2014: 583) advises that "since mixed methods research combines both quantitative and qualitative research, ethical considerations need to attend to typical ethical issues that surface in both forms of inquiry. Qualitative issues relate to obtaining permission, protecting anonymity of respondents, not disrupting sites, and communicating the purpose of the study, avoiding conflicted interests in data collection, respecting indigenous cultures, not disclosing sensitive information, and masking the identities of participants".

To adhere to the University of South Africa's ethical clearance requirements, the researcher applied for ethical clearance through the University's Ethics Review Committee. The researcher had been informed by the supervisor to proceed with the research on the assurance that the ethical clearance had been granted and the certificate would be forwarded through the researcher's e-mail. The researcher brought it to the attention of the Ethics Review Committee (ERC), in June 2018, that the ethical clearance certificate had not yet been forwarded, after which the ERC could not retrieve any records in this regard, hence took the decision attached as Appendix O.

Prior to the researcher applying to the University's Ethics Review Committee, permission had been requested and granted by the Permanent Secretary of the

Ministry of Education for the researcher to conduct the study (See Appendix P and Appendix Q).

A detailed description of the ethical considerations ensured by the study appears in sections 3.91 to 3.99.

3.9.1 Negotiating access

The researcher applied to the Permanent Secretary of Education in the Government of Namibia for permission to conduct the research in sampled schools. Permission was granted and copied to the Regional Director of Education in the specific region where the study was carried out (See Appendix Q). The researcher, through the participating teachers and school managers, explained to the sampled learners, the aim of the study and emphasised that their participation was completely voluntary.

3.9.2 No harm to participants

The researcher ensured that no harm befell the participants in the study. The participants were informed, through the consent form, that if at any time they experienced any adverse reactions, they could withdraw without any consequences pursuant to their withdrawal.

3.9.3 Privacy and Anonymity

Individuals participating in a research study expect their privacy to be safeguarded. For the sake of anonymity, the schools, the names of learners and teachers who participated in the study were coded into pseudonyms and alphabetic codes for this purpose. Codes were assigned to schools, teachers and learners to preserve anonymity. For example S1T04 refers to teacher 4 of school 1, S6T05, teacher 5 of school 6, and so on. The schools were coded from school 1 (S1) to school 8 (S8). Learners were coded from S1L1 to S8L18, referring to learner 1 from school 1 up to learner 18 from school 8. For example, a school 4 (S4) only had four learners, hence the learners in that school were coded S4L1, S4L2, S4L3 and S4L4 respectively.

The researcher ensured that no identifying information about the participants was revealed in written or other communication without the participants' consent.

3.9.4 Confidentiality

The researcher assumed the responsibility of keeping the information obtained from the data collection and analysis confidential. The researcher ensured confidentiality of more sensitive information that was obtained from the learners, teachers and others, for example lesson observers, who might be in a vulnerable position.

3.9.5 Informed Consent

Letters of consent to participate were distributed to the parents and guardians of all sampled learners. All the parents responded positively and consented to the participation of their charges (see Appendix M). Equally, the purpose of the research was explained to the participating teachers, who also consented to their participation in writing (see Appendix N).

3.9.6 Establishing rapport

The researcher spent a whole one and half hour session, prior to commencement of the training, discussing general issues on the challenges experienced by the teachers in the teaching and learning of trigonometric functions and graphs. The purpose of this induction activity was to get the teachers to relax and establish genuine rapport with the researcher. Since the EBM instruction would be implemented by the teachers themselves with their respective learners, the assumption was that the rapport between the teachers and their learners had been firmly established.

3.9.7 Avoiding intrusiveness

All activities were planned, timetabled and communicated to all participants and stakeholders well in advance. The researcher ensured that other timetabled school activities were not interrupted since the researcher negotiated with the regional authorities that the EBM instruction takes place during the afternoon study sessions.

3.9.8 Inappropriate Behaviour

The researcher at all times maintained a professional relationship in interacting with all the participants.

3.9.9 Data Interpretation

The researcher, collected, presented and interpreted the data honestly, and avoided data manipulation to achieve preferred outcomes.

3.10 SUMMARY AND CONCLUSION OF CHAPTER THREE

In chapter three the researcher described the research design used in this study to evaluate the effects of the Excel-based modelling professional development programme and instructional method, as described in section 3.5.2. Among the issues explained were the research paradigm, the research design, sampling techniques, and data collection procedures. The chapter concluded with an explanation of the ethical guidelines that the researcher used during the data collection procedures.

The next chapter explains the qualitative and quantitative data analyses and findings of the study, namely findings from the pre-intervention, EBM professional development phase, post-EBM professional development phase, implementation of the EBM instruction, and post-EBM implementation phase, using Guskey's (2000) framework of evaluation of teacher professional development. The post-EBM implementation phase also assessed the learners' conceptual understanding in periodicity of trigonometric functions to establish if there was any significant relationship between teachers' TPACK and learners' conceptual understanding.

CHAPTER FOUR
DATA PRESENTATION AND ANALYSIS

4.1 INTRODUCTION

This chapter presents and analyses the data generated to respond to the research questions. Analysing data in a mixed-method research study is potentially the most complex step because the researcher has to be adept at analysing both the quantitative and qualitative strands, as well as integrating the results into a coherent and meaningful way that yields strong meta-inferences (Onwuegbuzie & Combs, 2010).

To achieve coherence in the presentation and analysis of data, the process followed the sequence in Guskey's framework of evaluation of teacher professional development. Qualitative and quantitative data are presented and analysed simultaneously based on the research questions addressed at each level of Guskey's framework. Table 4.1 shows the format in which the data analysis was conducted.

Table 4. 1: Summary of data analysis sequence and procedures used

Evaluation Level	Research question	DATA ANALYSIS PROCEDURES	
		QUALITATIVE	QUANTITATIVE
1.Participants' Reactions	**RQ1:** How do teachers' perceptions of their state of TPACK change after participating in the Excel modelling programme?	• Analysis of post EBM implementation evaluation questionnaire • Teachers' feedback on EBM enhanced collaborative problem solving experience	Comparison of mean differences in pre-EBM and post-EBM TPACK self-efficacy ratings to determine whether there is change in TPACK self-efficacy.
2.Participants' Learning	**RQ2:** How does the Excel modelling programme impact the teachers' self-efficacy in teaching periodicity of trigonometric functions?		Calculate effect size (Cohen's d) from pre and post EBM TPACK survey questionnaire using descriptive statistics.
3.Organization Support & Change	Researcher, in collaboration with mathematics advisory services, school principals and heads of mathematics departments outlined and ensured all necessary support required for implementation of the Excel modelling instructional practice was provided to the teachers and learners.	Analysis of Post EBM implementation questionnaire responses to items 21 to 30 using descriptive statistics for ensuring fidelity of EBM implementation.	
4.Participant Use of New Knowledge and Skills	**RQ3:** How effective were the teachers in the implementation of the Excel based modelling instructional practice?	Analysis of checklist lesson observation outcomes	Analysis of lesson observation ratings of EBM instruction per teacher using descriptive statistics
5.Student Learning Outcomes	RQ4: What are the learners' perceptions of learning periodicity of trigonometric functions through the Excel based modelling instruction?	• Structured interviews with learners (transcription and coding) • Direct observations (checklist lesson	• Descriptive analysis of learners' collaborative problem solving process (CAEMA) • Analysis of learners' summative test scores (LCU)
	RQ5: What is the influence of teachers' TPACK development on the learners' conceptual understanding of periodicity of trigonometric functions?		• PLS-SEM inferential analysis according to proposed model

4.2 ANALYSIS OF OUTCOMES OF GUSKEY'S LEVELS OF PROFESSIONAL DEVELOPMENT

4.2.1 ANALYSIS OF TEACHERS' REACTIONS (GUSKEY LEVEL 1)

In order to fully map the reactions of the teachers after exposure to the EBM programme, the researcher analysed the teachers' state of TPACK before and after the EBM professional development intervention. Pre-intervention analysis focused on the CAAPSA output in the collaborative problem-solving process, group feedback, pre and post-EBM TPACK survey questionnaire outcomes. The pre-EBM observations were aimed at establishing the baseline status of teachers' TPACK. The post-EBM analysis used descriptive statistics to measure mean differences in TPACK self efficacy before and after the EBM professional development.

4.2.1.1 Pre-EBM intervention data analysis

4.2.1.1.1 Analysis of the teachers' collaborative problem solving process

The teachers' collaborative problem-solving activity sheet consisted of seven (7) problems selected from previous national NSSCH mathematics examination papers. The collaborative problem-solving process and outcomes were analysed using the CAAPSA tool and Polya's (1957) problem-solving heuristics.

Problem 1

All the three groups attempted problem 1, and the outcomes are reported in the following section, with the corresponding CAAPSA output and transcription of the group's explanation of the solution process.

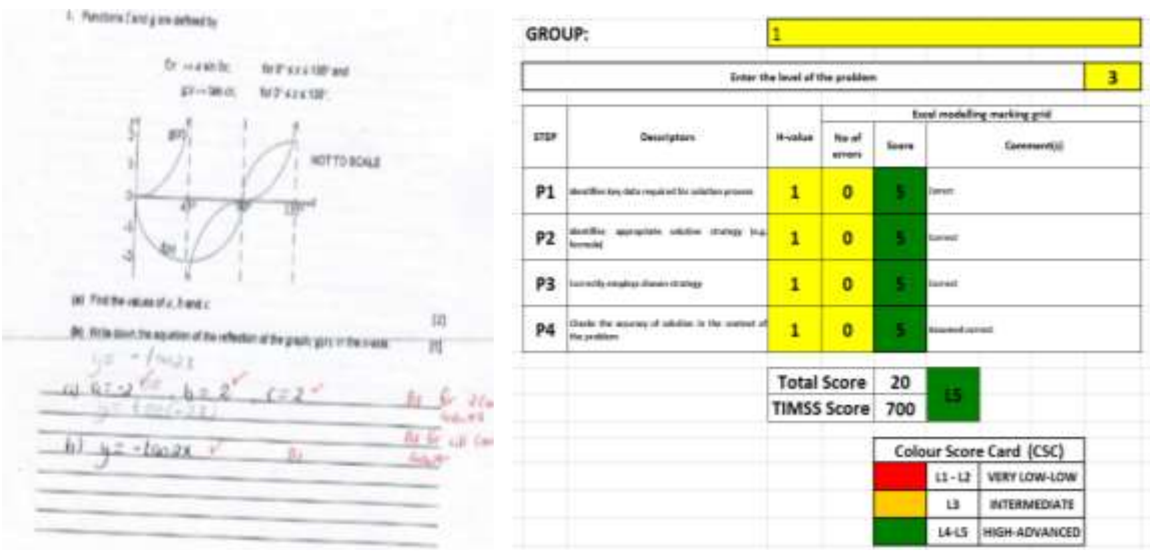


Figure 4. 1: Group 1 solution to problem 1 and the corresponding CAAPSA output for solution process

The evaluation of the written solution process using Polya's model indicated Level 5, which is an advanced level of teachers' content knowledge (CK). The group presented its solution process clearly, and no questions arose for further discussion.

The evaluation of Group 2's written solution process, using Polya's model indicated Level 3, which is an intermediate level of teachers' content knowledge (CK). The group presented its solution process, and the participants reflected on the solution process as follows:

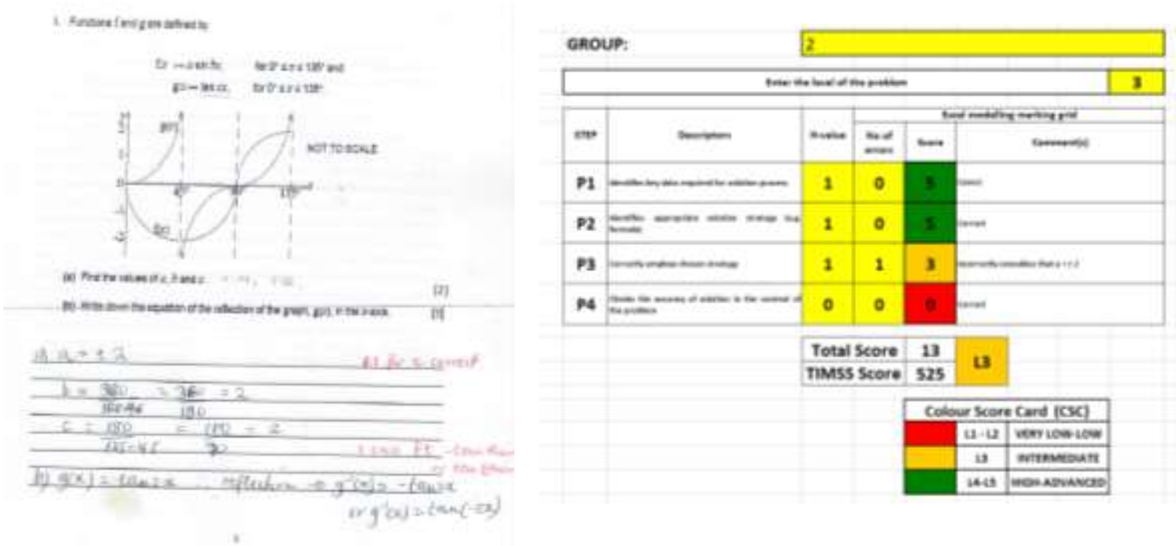


Figure 4. 2: Group 2 solution to problem 1 and the corresponding CAAPSA output for solution process

Participant 7 (P7): *I am not sure why you have a = ± 2. Are you sure about that?*

Group 1: *Yes, a = ± 2, because the amplitude can be measured going up or down.*

P3: *The amplitude is always positive, but the sign of **a** is determined by whether the graph starts going down or up. The graph of f(x) starts by going down, so the sign of **a** must be negative.*

All: Okay….

P9: *Is this true with all the trigonometric functions' graphs….if it starts by going down then "**a**" is negative and if it starts by going up "**a**" is positive?*

P3: *Hhhmmm!... I think so…*

P1: *But if we go to the negative side, the graph starts going up, so you should specify that this is true if you are going to the right…*

P5: *How can you check if **a = ± 2** is correct or not?*

P11: *(Walking to the flip chart board in front of the classroom and then talking to the whole group)… What if we substitute, say x = 45^o and substitute into f(x) = 2sin2x, then check whether this will give us the y-coordinate for the minimum turning point (45^o ; -2) on the graph? If we do this for f(x) = 2sin2x, we get f(x) = 2sin2(45$^{o)}$ = 2sin90^o = 2, so **a =2** can't be correct…if we take f(x) = -2sin2x, we get f(x) = -2sin2(45$^{o)}$ = -2sin90^o = -2, so **a =-2** is correct.*

Through the group discussion, the participants, including group 2 members, realised that **a = 2**, was not correct.

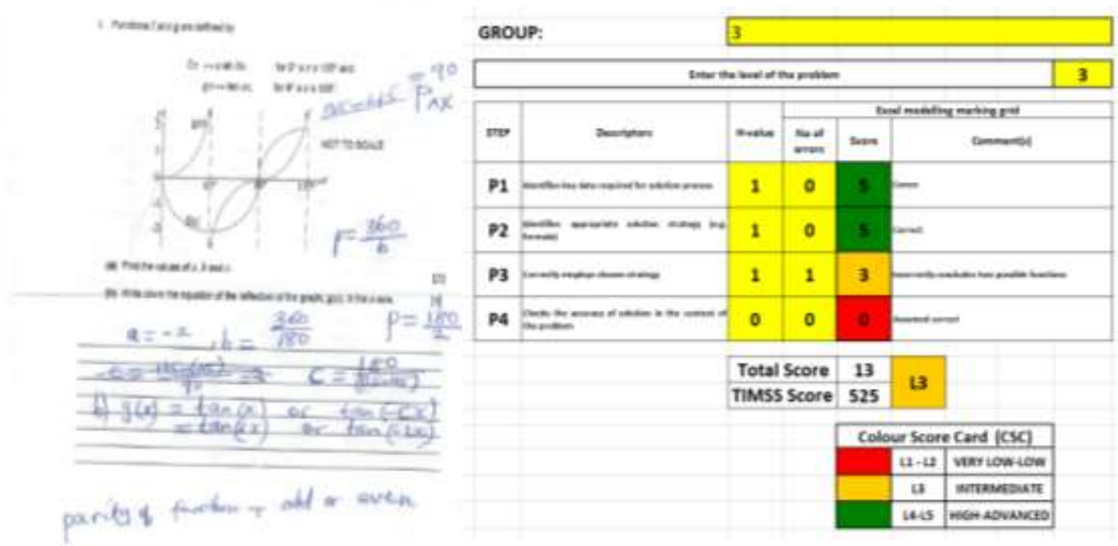


Figure 4. 3: Group 3 solution to problem 1 and the corresponding CAAPSA output for solution process

The value of a = -2 is correct thus in step 1 of Polya, H=1 and number of errors = 0.

The value of b = 2 is also correct, including the working $b = \frac{360^0}{180^0}$. The value of c is also correctly worked out from $c = \frac{180^0}{(135^0 - 45^0)}$ = 2. However, the group incorrectly concludes that there are two possible equations of the reflection of g(x) ; g_r(x) = tan(2x) or g_r(x) = tan(-2x), hence in step 3, H=1 and because of the incorrect alternative g_r(x) = tan(2x), number of errors =1. In step 4, H=0 because the group does not attempt to check their solutions. Had the solutions been correct, CAAPSA would assume that H=1 with no errors.

Similarly, all the other problems 2 to 7 were assessed using CAAPSA, and the CAAPSA outputs per group are shown in Tables 4.2 to 4.4.

Table 4. 2: Summary of collaborative problem solving outcomes for Group 1

GROUP	1									
POLYA'S STEP	PROBLEM 1 LEVEL 3	PROBLEM 2 LEVEL 4	PROBLEM 3 LEVEL 4	PROBLEM 4 LEVEL 4	PROBLEM 5 LEVEL 4	PROBLEM 6 LEVEL 4	PROBLEM 7 LEVEL 4	MEAN	SD	VARIANCE
1	5	5	5	4	5	5	5	4.857142857	0.377964473	0.142857143
2	5	5	5	4	5	5	5	4.857142857	0.377964473	0.142857143
3	5	5	4	3	5	5	4	4.428571429	0.786795792	0.619047619
4	5	5	0	0	5	5	0	2.857142857	2.672612419	7.142857143
MEAN	5	5	3.5	2.75	5	5	3.5	4.25		

In Polya's steps 1 and 2, the mean success rate was 97.14%, 88.57% in step 3 and 57.14% in step 4. The group lacked strategies to evaluate their solutions. The group's overall problem-solving success rate was 85%.

Table 4. 3: Summary of collaborative problem solving outcomes for Group 2

GROUP	2									
POLYA'S STEP	PROBLEM 1 LEVEL 3	PROBLEM 2 LEVEL 4	PROBLEM 3 LEVEL 4	PROBLEM 4 LEVEL 4	PROBLEM 5 LEVEL 4	PROBLEM 6 LEVEL 4	PROBLEM 7 LEVEL 4	MEAN	SD	VARIANCE
1	5	5	5	5	5	5	5	5	0	0
2	5	5	5	5	4	4	5	4.714285714	0.487950036	0.238095238
3	3	5	4	1	2	2	4	3	1.414213562	2
4	0	5	0	0	0	0	0	0.714285714	1.889822365	3.571428571
MEAN	3.25	5	3.5	2.75	2.75	2.75	3.5	3.357142857		

The success rate in Polya's step 1 was 100%. In step 2 the group registered 94.29% success rate, 60% in step 3 and 14.29% in step 4. Group 2 also lacked strategies to verify the accuracy of their solutions. The group's overall problem-solving success rate was 67.14%.

Table 4. 4: Summary of collaborative problem solving outcomes for Group 3

GROUP	3									
POLYA'S STEP	PROBLEM 1 LEVEL 3	PROBLEM 2 LEVEL 4	PROBLEM 3 LEVEL 4	PROBLEM 4 LEVEL 4	PROBLEM 5 LEVEL 4	PROBLEM 6 LEVEL 4	PROBLEM 7 LEVEL 4	MEAN	SD	VARIANCE
1	5	5	5	5	5	5	5	5	0	0
2	5	5	5	5	5	5	5	5	0	0
3	3	5	4	1	3	1	3	2.857142857	1.463850109	2.142857143
4	0	5	0	0	0	0	0	0.714285714	1.889822365	3.571428571
MEAN	3.25	5	3.5	2.75	3.25	2.75	3.25	3.392857143		

The success rate in Polya's steps 1 and 2 was 100%. In step 3, the group registered 57.14% success rate, 60% and 14.29% in step 4. Group 3 lacks strategies to evaluate the solution process. This group's overall problem-solving success rate was 67.86%.

Table 4.5 shows the mean success rate for the whole group of 11 teachers' based on their three group levels of achievement in Polya's problem-solving process. This table therefore depicts an estimated baseline level of content knowledge of the group prior to the EBM professional development intervention.

Table 4. 5: Summary of collaborative problem-solving outcomes for the whole sample

GROUP	OVERALL									
POLYA'S STEP	PROBLEM 1 LEVEL 3	PROBLEM 2 LEVEL 4	PROBLEM 3 LEVEL 4	PROBLEM 4 LEVEL 4	PROBLEM 5 LEVEL 4	PROBLEM 6 LEVEL 4	PROBLEM 7 LEVEL 4	MEAN	SD	VARIANCE
1	5	5	5	4.66666667	5	5	5	4.952380952	0.125988158	0.015873016
2	5	5	5	4.66666667	4.66666667	4.66666667	5	4.857142857	0.178174161	0.031746032
3	3.66666667	5	4	1.66666667	3.33333333	2.66666667	3.66666667	3.428571429	1.049061057	1.100529101
4	1.66666667	5	0	0	1.66666667	1.66666667	0	1.428571429	1.781741613	3.174603175
MEAN	3.83333333	5	3.5	2.75	3.66666667	3.5	3.41666667	3.666666667		

The success rate in Polya's step 1 was 99.05% , step 2 (97.14%), step 3 (68.57%), and 28.57% in step 4. The sample's overall problem-solving success rate was 73.33% or below Level 4 on a scale of 0 (very low) to 5 (advanced).

Teachers were most successful in the solution process of problem 2, with a raw CAAPSA score (CS) of 5. They struggled, however, with problem 4, with a score of 2.75. Their solution process in problems 1, 3, 5, 6, and 7 was satisfactory with CAAPSA scores ranging from 3.4 to 3.8.

4.2.1.1.2 Analysis of pre-EBM professional development correlations between TPACK constructs

The self-reflected mean levels per construct were calculated for each teacher and summarised as shown in Table 4.6. After determining the levels of TPACK constructs per teacher, further analysis of correlations between these constructs was done to get the existing state of TPACK of the teachers, in terms of which constructs presented weak, moderate and strong correlation prior to the EBM intervention. Table 4.7 summarises the baseline state of teachers' TPACK in terms of these correlations, calculated using Pearson's Product Correlation Moment.

An analysis of the correlations between the TPACK constructs prior and after EBM professional development was done according to the description below by Mulder (as cited in Lupahla, 2014):

1.00 – perfect correlation
0.80 to 0.99 – very high correlation
0.60 to 0.79 – high correlation
0.40 to 0.59 – moderate correlation
0.20 to 0.39 – low correlation

0.01 to 0.19 – very low correlation

0.00 – no correlation

Table 4. 6: Summary of teachers' mean levels of TPACK self-efficacy prior to EBM professional development

#	Teacher Code	LEVEL OF TPACK COMPONENTS BEFORE EBM TRAINING						
		TK	CK	PK	PCK	TCK	TPK	TPACK
1	S1T04	3.30	2.70	3.70	3.83	1.80	3.00	3.10
2	S1T11	2.60	3.20	3.80	3.75	1.30	2.60	2.50
3	S2T09	2.60	2.70	4.10	3.42	2.00	3.10	3.00
4	S2T10	2.90	3.30	3.40	3.50	3.10	2.50	2.00
5	S3T08	2.40	3.90	4.00	3.25	2.50	2.50	3.50
6	S4T07	2.80	3.70	3.90	3.67	2.70	3.60	3.70
7	S5T06	2.70	4.20	4.00	4.08	3.30	3.30	3.90
8	S6T03	2.80	3.80	3.80	3.92	3.10	3.90	3.80
9	S6T05	3.30	3.30	4.20	3.92	2.30	3.50	3.50
10	S7T02	3.10	3.60	3.80	3.92	2.30	3.20	3.90
11	S8T01	3.40	4.20	4.00	4.17	3.80	3.90	4.10
	Mean	2.900	3.509	3.882	3.766	2.564	3.191	3.364

Table 4.7: Summary of correlations among TPACK constructs prior to EBM professional development

CORRELATIONS BETWEEN TPACK DOMAINS							
DOMAIN	TK	CK	PK	PCK	TCK	TPK	TPACK
TK	1.000	-0.052	-0.028	0.657	0.215	0.482	0.249
CK		1.000	0.186	0.399	0.763	0.400	0.646
PK			1.000	0.143	-0.011	0.409	0.584
PCK				1.000	0.376	0.694	0.551
TCK					1.000	0.516	0.455
TPK						1.000	0.751
TPACK							1.000
SCORES							

There was a low correlation between PK and TCK (-0.011), TK and PK (-0.028), TK and CK (-0.052), PK and PCK (0.143), CK and PK (0.186)., TK and TCK (0.215) and TK and TPACK (0.249). Moderate correlation was verified between CK and TPK (0.400), PK and TPK (0.409), TCK and TPACK (0.455), TK and TPK (0.482), TCK and TPK (0.516), TK and CK (0.547), PCK and TPACK (0.551), and between PK and TPACK (0.584). High to very high correlations were recorded between TK and PCK (0.657), CK and TPACK (0.646), PCK and TPK (0.694), TPK and TPACK (0.751), CK and TCK (0.763).

4.2.1.2 Analysis of the participants reactions during the EBM professional development intervention

The researcher introduced the EBM tools by first analysing the three basic functions, **sin**, **cos** and **tan** and their periodicity properties. The impact of changing the values of **a**, **b** and **c** for the function form: *f(x) = a*trigfunction (bx) + c* was explored by comparatively looking at the changes to the resultant graphs of **sin**, **cos** and **tan**. The

EBM tools were then used to revisit the collaborative problem solving activity through modelling of the problems and the solution process in Excel. The observed connections between algebraic and graphical representations of trigonometric functions were discussed by the group. Each participant had a laptop in which the researcher had uploaded all the EBM tools prior to the commencement of the training. Participants were made familiar with the folder and how to navigate through each tool. The participants were later asked to give feedback on how the EBM applets support their problem solving process.

4.2.1.2.1 Analysis of the modelling outcomes of the basic trigonometric functions

This was the first Excel-based modelling tool for the function *f(x) = a*sin (bx) + c.*

4.2.1.2.2 Inquiry process steps for the sine function and outcomes

Inquiry process steps:

Step 1: *Analyse the screen and note the input cells for the required values of a, b and c for the function, f(x) = a*sin (bx) + c. At this point the screen is blank (no graphic display).*

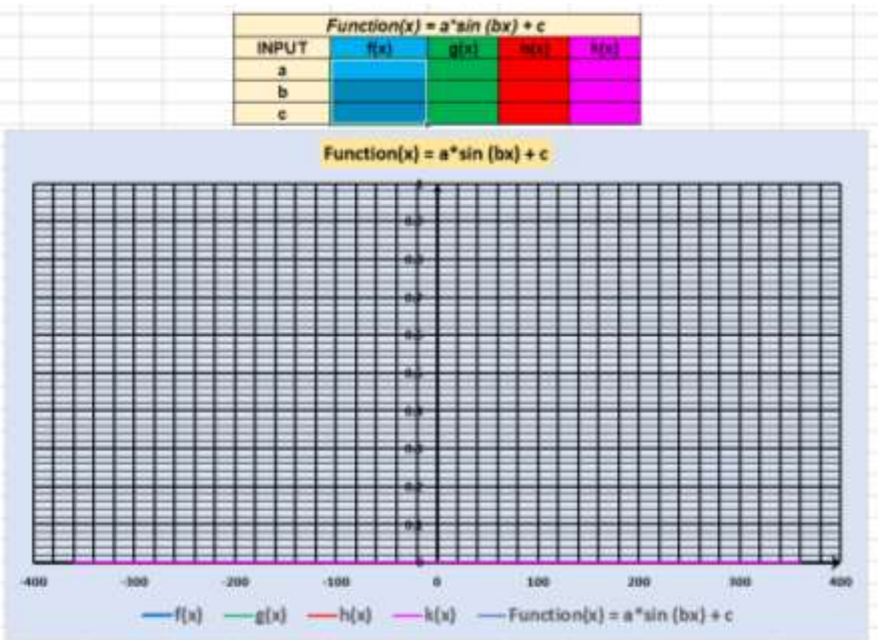


Figure 4.4: Screen output before processing starts

The colour coding has been matched to the graphic outputs, i.e. blue colour for f(x), green colour for g(x), red for h(x) and purple for k(x), as shown in Figure 4.5 below.

Step 2: For f(x), enter values a = 1, b = 1 and c = 0 and describe the characteristics of the graph of *f(x) = a*sin (bx) + c.*

Outcome

The participants correctly entered the values a = b = 1 and c = 0, and obtained the display, shown in Figure 4.5.

Function(x) = a*sin (bx) + c				
INPUT	f(x)	g(x)	h(x)	k(x)
a	1			
b	1			
c	0			

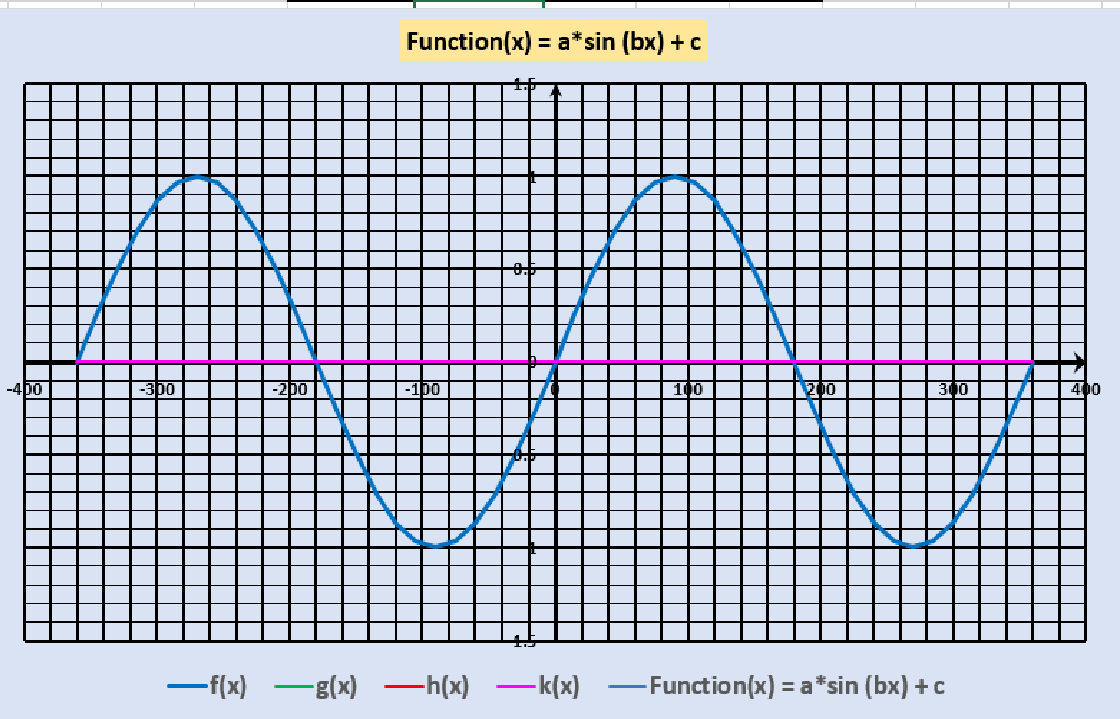


Figure 4. 5: Screen output after entering values of a, b and c

4.2.1.2.3 Analysis of group feedback on the modelling of the sine function

The following is a summary of the feedback discussion between the researcher and the participants on their observations and conclusions from the inquiry process in the exploration of the **sine** function.

Researcher (R): *How would you deduce the amplitude and the period from the graph?*

Participant 1(P1): *(Scratching her head)… I can find the period by looking at the distance in the x-axis (sic) from the origin (0;0) to the next zero after the point (0;0) has* *traversed through the crest and trough of the curve towards the right. The next such zero is (360;0). So the period is 360^0.*

Researcher(R): *Correct, …is there anyone else with a different approach to getting the period…is it necessarily to the right that one should move to determine the period? I know your learners are already familiar with this concept from their Physics topic on waves…*

Participant 7(P7): *Periodic functions repeat the same pattern every period…If you use tracing paper (demonstrating on flipchart board)… and trace the first part of the function from -360^0 to 0^0, the pattern starts repeating again from $0^0$to 360^0, so the period is 360^0… (whole group claps for participant 7)…*

Researcher (R): *How do we deduce the amplitude?*

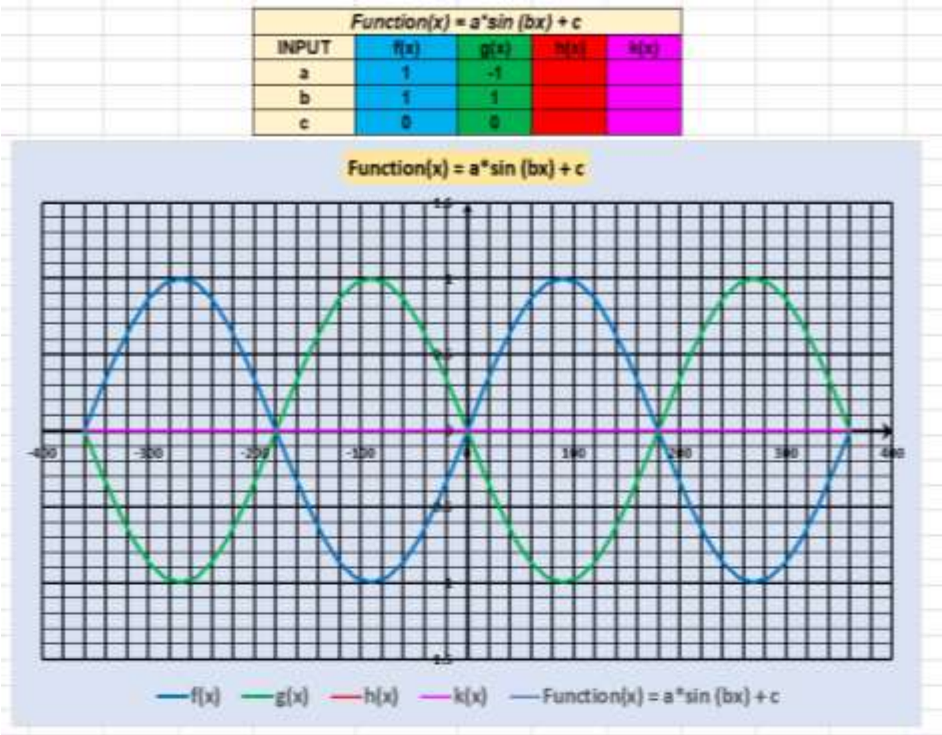
Participant 8 (P8): *The amplitude is the height from the centre line to the peak …or to the trough… or we can measure the height from highest to lowest points and divide that by 2. So, in this case the amplitude is 1.*

Researcher (R): *Very good, participant 8…what then is the value of* ***a*** *in f(x) = a*sin (bx) + c, and how would you let your learners deduce this?*

Participant 2 (P2): *The value of* ***a*** *is 1… but sir can we make the value of* ***a****= -1 and see what happens?*

Researcher (R): *Yes, go ahead and fill in a = -1 for g(x) , but keep the values of b and c the same, what happens?*

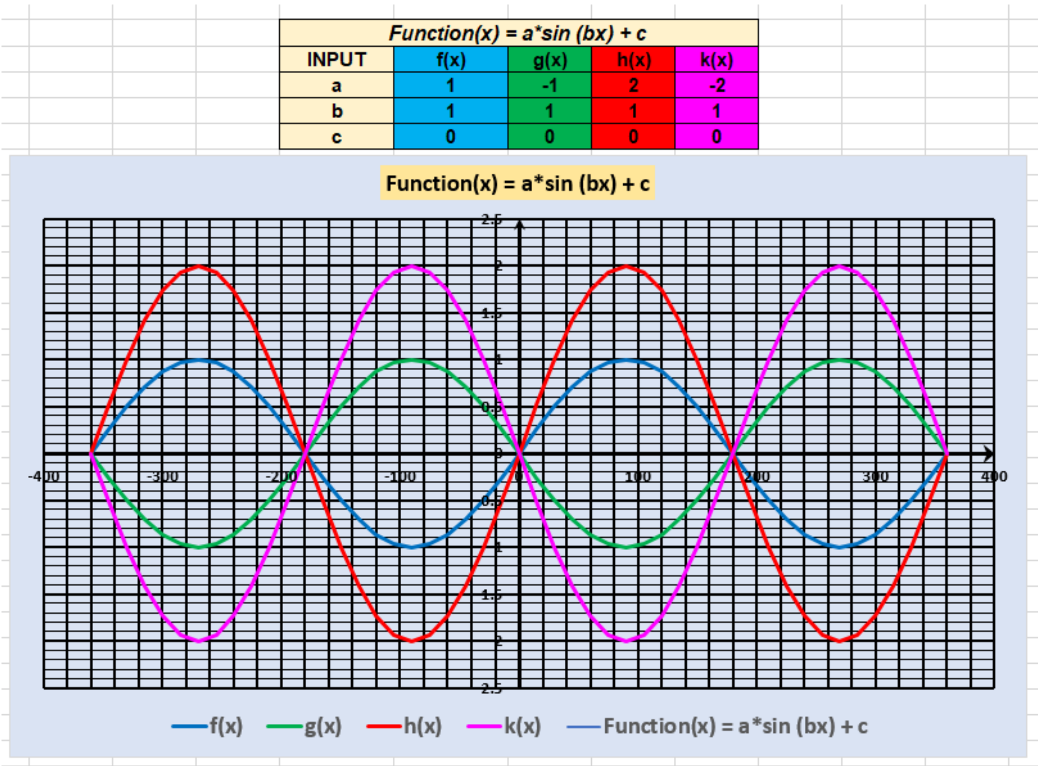
At this stage the participants fill in a = -1 in g(x) and get the following output in Figure 4.6.



Function(x) = a*sin (bx) + c				
INPUT	f(x)	g(x)	h(x)	k(x)
a	1	-1		
b	1	1		
c	0	0		

Figure 4. 6: Screen output after entering values of a, b and c

The participants correctly deduced that g(x) becomes a reflection of f(x), and predicted correctly the outputs for h(x) =2sinx and k(x) =-2sinx, as shown in Figure 4.7.



Function(x) = a*sin (bx) + c				
INPUT	f(x)	g(x)	h(x)	k(x)
a	1	-1	2	-2
b	1	1	1	1
c	0	0	0	0

Figure 4.7: Screen outputs for f(x), g(x), h(x) and k(x) on the same axes

Step 3: For f(x) and g(x) enter the values a = 1, b =1, c = 0 and a = 1, b = 2 , c = 0 , respectively and compare and explain the characteristics of the resultant graphs of *g(x) = sin 2x.*

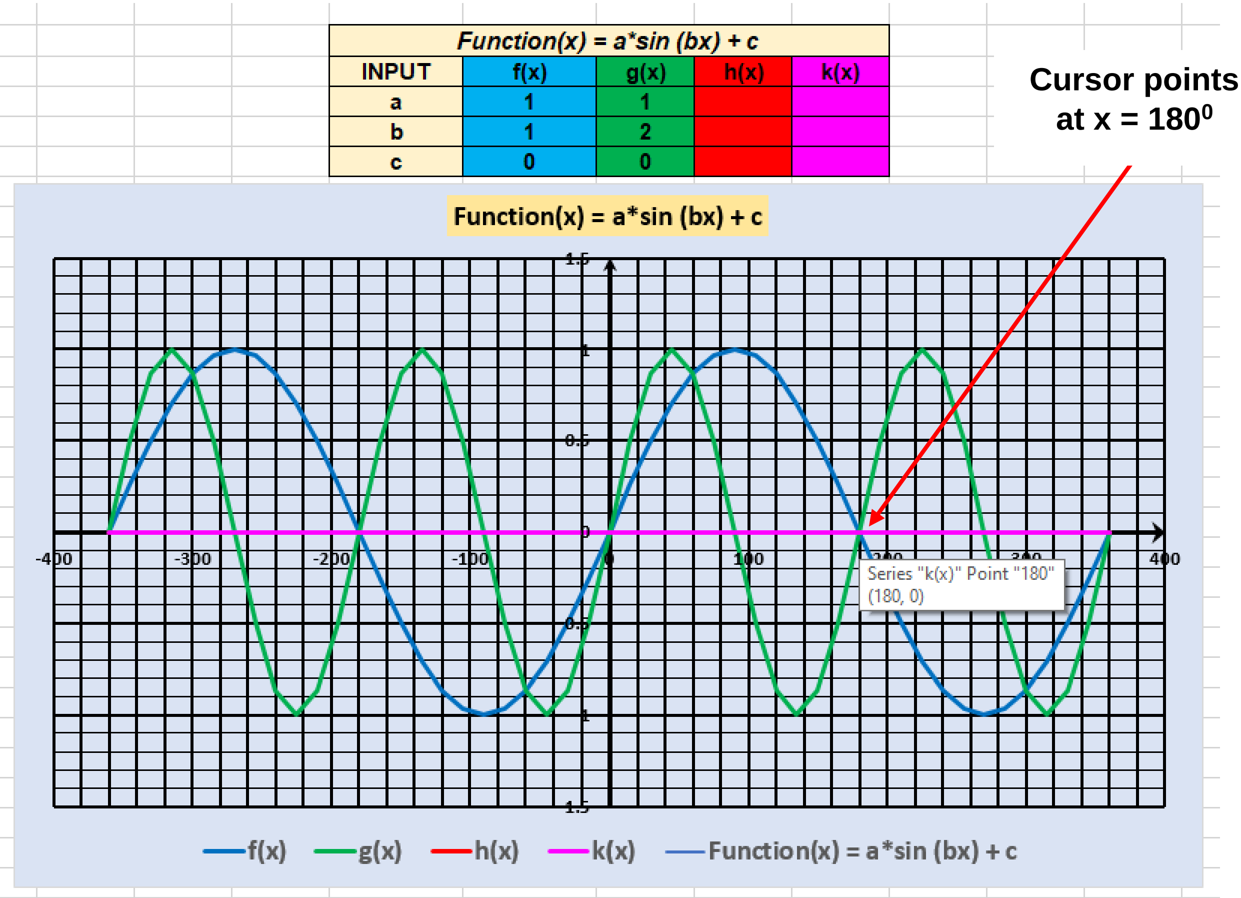


Figure 4.8: The function f(x) = sin x and g(x) = sin 2x, modelled on the same axes

The participants collectively deduced the following relationships:

There is an inverse proportionality relationship between period and the value of b, with the constant of proportionality equal to the period of the basic sine function = 360^0, as illustrated in Table 4.8 below.

Table 4. 8: Illustration of inverse proportion between the values of b and the period

Value of b	1	2
Period (degrees)	360	180

$$Period \ \ of \ basic \ \ function \ \ (P_{bf}) = Period \ \ of \ new \ \ function \ \ (P_{nf}) \times b$$

Hence $b = \dfrac{P_{bf}}{P_{nf}}$ with $f(x) = a \sin bx$

This suggests that for $f(x) = \sin 4x$;

Period is $\dfrac{360^0}{4} = 90^0$

By modelling, the participants verified that this was a correct conclusion, based on the display obtained in Figure 4.9.

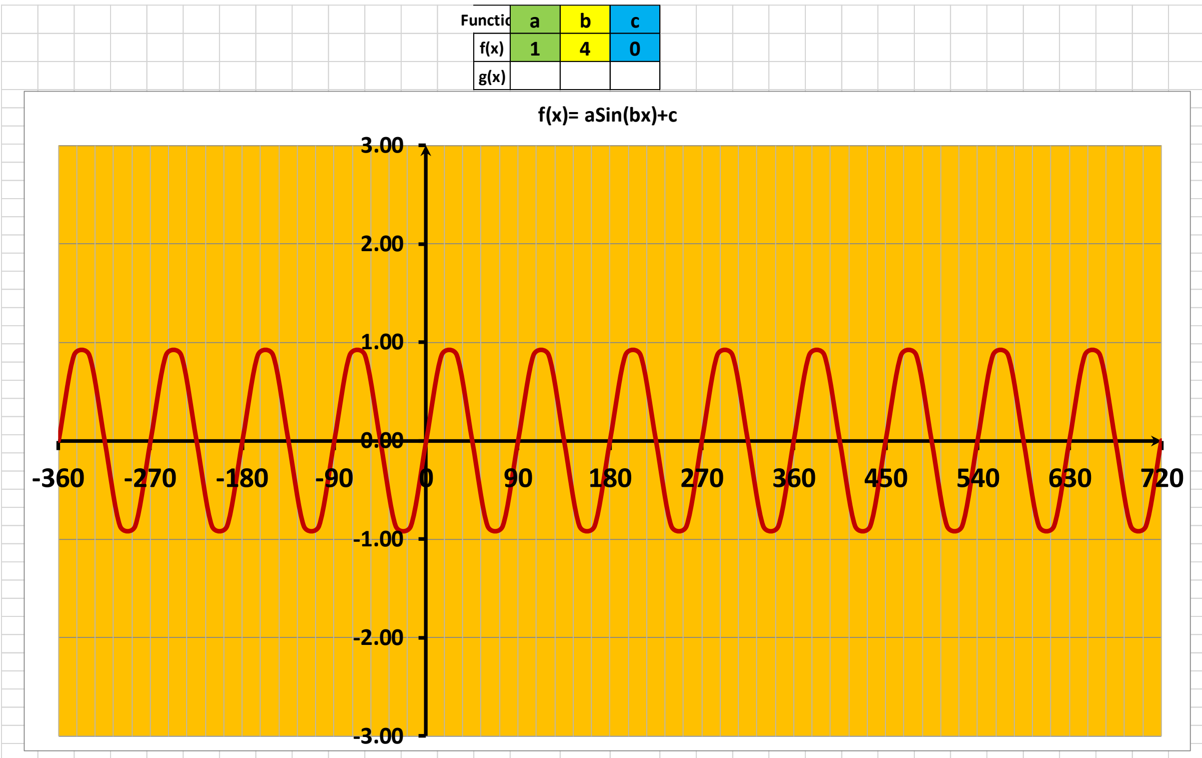


Figure 4. 9: Model of f(x) = sin 4x, showing that the product of the period of f(x) and b is 360^0

Step 4: For f(x), g(x) and h(x) enter the values a = 1, b =1, c = 0, a = 1, b = 2 , c = 2 and a = 1, b = -3, respectively and compare and explain the characteristics of the resultant graphs of *f(x) = sin x, g(x) = sinx + 1 and h(x) = sinx - 3.*

The participants executed the inputs for this step and obtained the output in Figure 4.10. The group discussion led to correct conclusions about the effect of changing c resulting in a vertical translation of the graphs. The participants could still relate the resultant graphs to their functions in terms of deducing the values of a, b and c.

*Function(x) = a*sin (bx) + c*				
INPUT	f(x)	g(x)	h(x)	k(x)
a	1	1	1	
b	1	1	1	
c	0	2	-3	

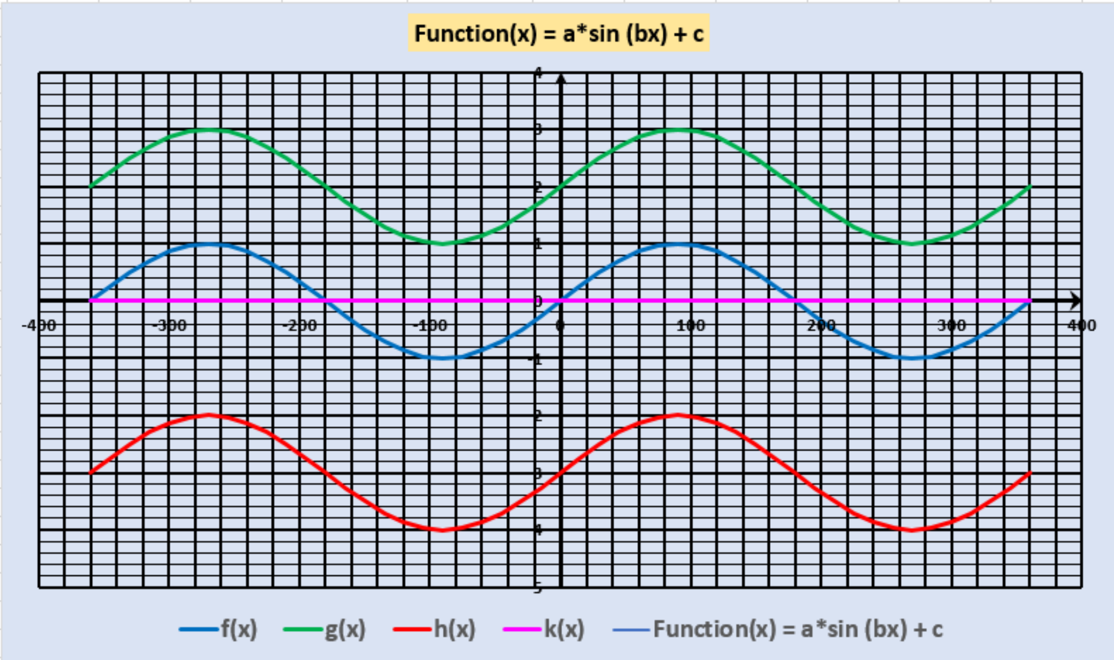


Figure 4. 10: Models of g(x) and h(x) as vertical translations of f(x) through vertical displacements of +2 units and -3 units respectively

Step 5: Predict the characteristics of the graph of $f(x) = -2\sin\frac{x}{2} + \frac{3}{2}$ for $-360^0 \leq x \leq 360^0$.

The group unanimously agreed on a particular sketch, drawn on a flipchart board. Figure 4.11 shows the sketch that was hypothesised by the group. The discussion went on as follows:

P11: *We need to get the values of a, b and c. What is the value of a?*

P6: *(after raising hand and being recognised)… a = -2, which means the amplitude is 2, but the graph starts descending to the right towards the trough.*

P11: *What is the value of b?*

P9: *b is 2… (scratching head)… no…no… (then withdraws his participation)…*

P4: *b is ½…or 0.5 because x/2 is the same as ½ (x)…*

P11: *… and the value of c?*

P10: *c is 3/2, which means the graph is translated upwards through 1½ units…*

P11: *So a = -2, b = 2, and c = 1½, do you agree?*

Whole group:… Yes!

The group then agreed on the hypothesised sketch.

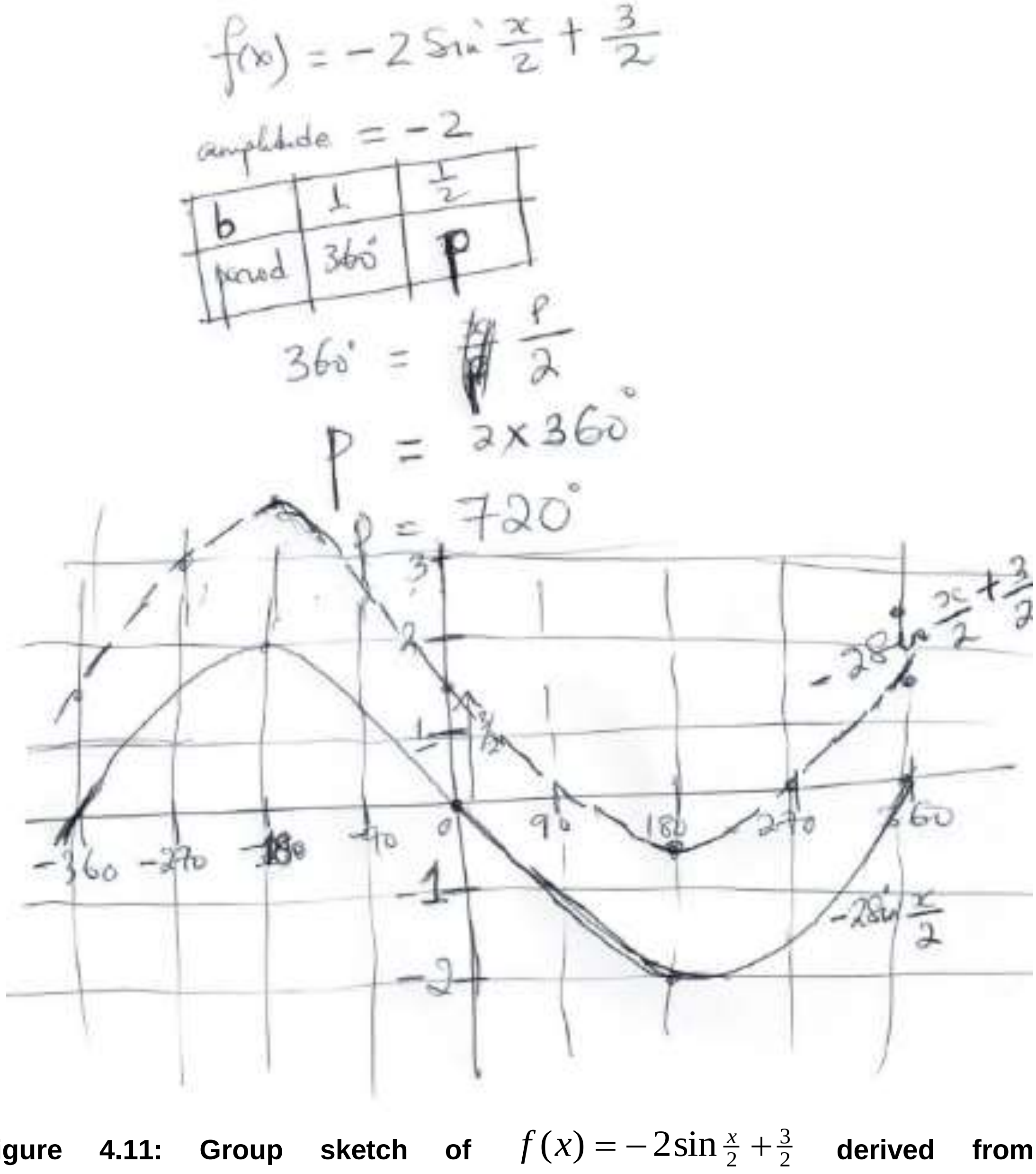


Figure 4.11: Group sketch of $f(x) = -2\sin\frac{x}{2} + \frac{3}{2}$ **derived from** $f(x) = -2\sin\frac{x}{2}$

After this hypothesised sketch, the group, then did verification by entering the values of a, b and c into the provided Excel applet and compared the computer output to their sketch. Figure 4.12 shows the computer output and the hypothesised sketch.

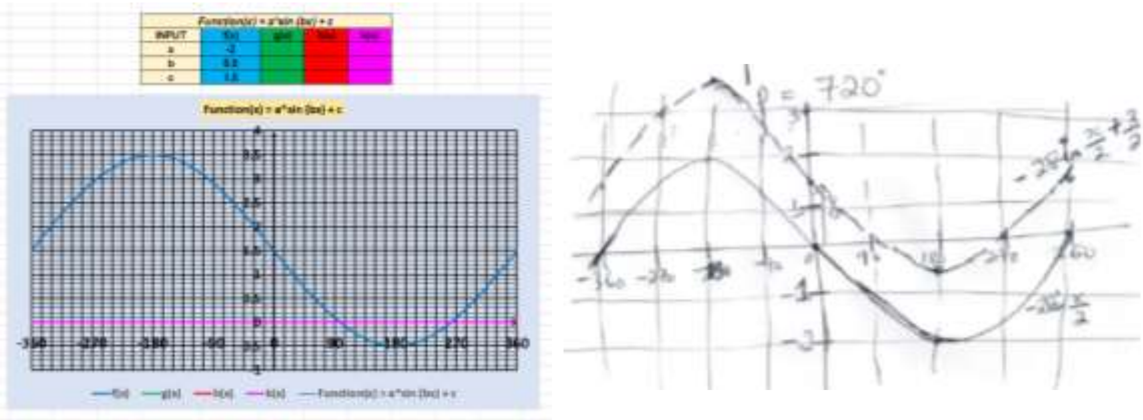


Figure 4.12: Comparison of Excel model and hypothesised graphs of $f(x) = -2\sin\frac{x}{2} + \frac{3}{2}$

4.2.1.2.3 Inquiry process steps for the Cosine function f(x) = a*cos (bx) + c and outcomes

The graphs matched. The teachers were then asked to develop lesson plans based on the activity involving the sine function.

The cosine function was explored in a similar process, allowing participants to deduce the connections between the various graphs and algebraic expressions for *f(x) = a*cos (bx) + c.*

After exploring the connections between the graphs and algebraic expressions for f(x) = a*cos (bx) + c, the teachers were asked to sketch on the same axes the graphs of f(x) = -2cosx and g(x) = -3cos2x for $0^0 \leq x \leq 360^0$, after which they were to model their hypothesised graphs in Excel and compare the outputs. This activity was based on Problem 3 of the pre-EBM teacher collaborative problem-solving activity, with the aim of probing teachers' reactions to the substitution of the routine pen and paper problem-solving process to the EBM problem-solving process. The teachers worked collaboratively to produce the following sketch, in Figure 4.13.

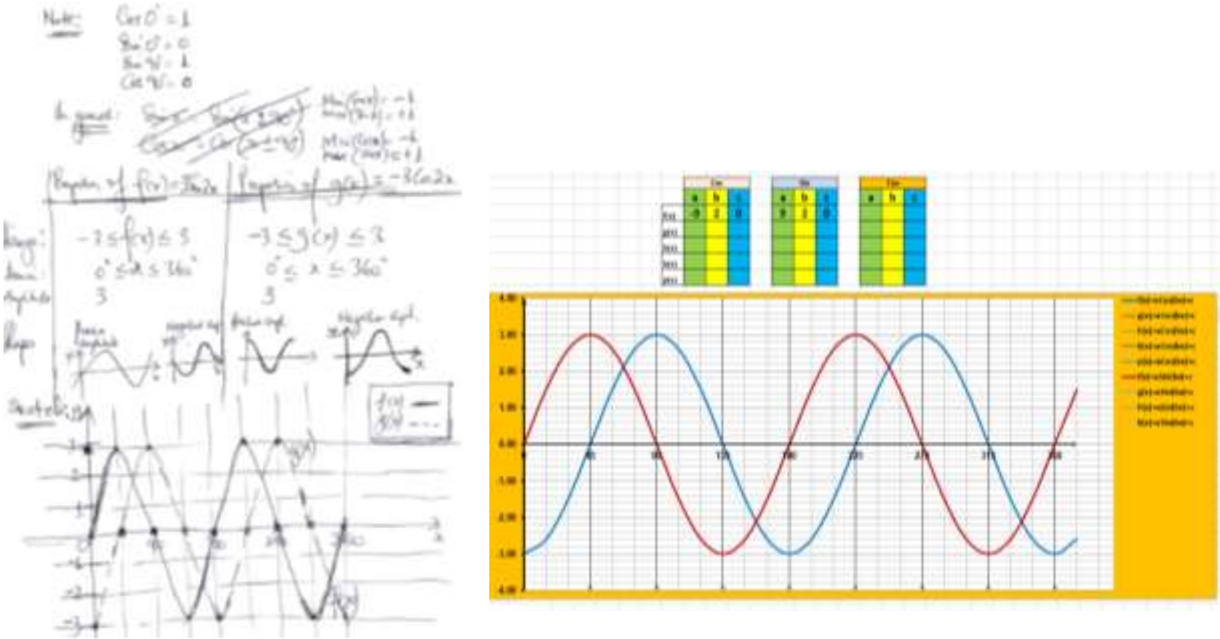


Figure 4. 13: Comparison of predicted and modelled functions

The teachers resorted to the characteristic properties of the sine and cosine functions, particularly, the amplitude, range and the typical shapes due to changes in the sign of the amplitude value.

4.2.1.2.4 Analysis of group feedback after the modelling of the cosine function

Through further discussions, with reference to the graphs that were projected as shown in Figure 4.14, the teachers concluded as follows:

Participant 8 (P8): *(pointing to the cosine graphs)…If the y-intercept of the cosine graph is negative then the graph has the form g(x) = -acosbx+c, but f(x) = +acosbx+c if the y-intercept is positive, but I am not sure about the other graphs.*

Participant 7 (P7): *I do not think it's the easiest way for the learners to understand… look! (pointing at three pairs of graphs in Figure 4.14, that were modelled and projected on a white screen), for any graph…if the first part of f(x) > 0…going right…then the value of "a" in f(x) = a*trigfunction(bx)+c is positive.*

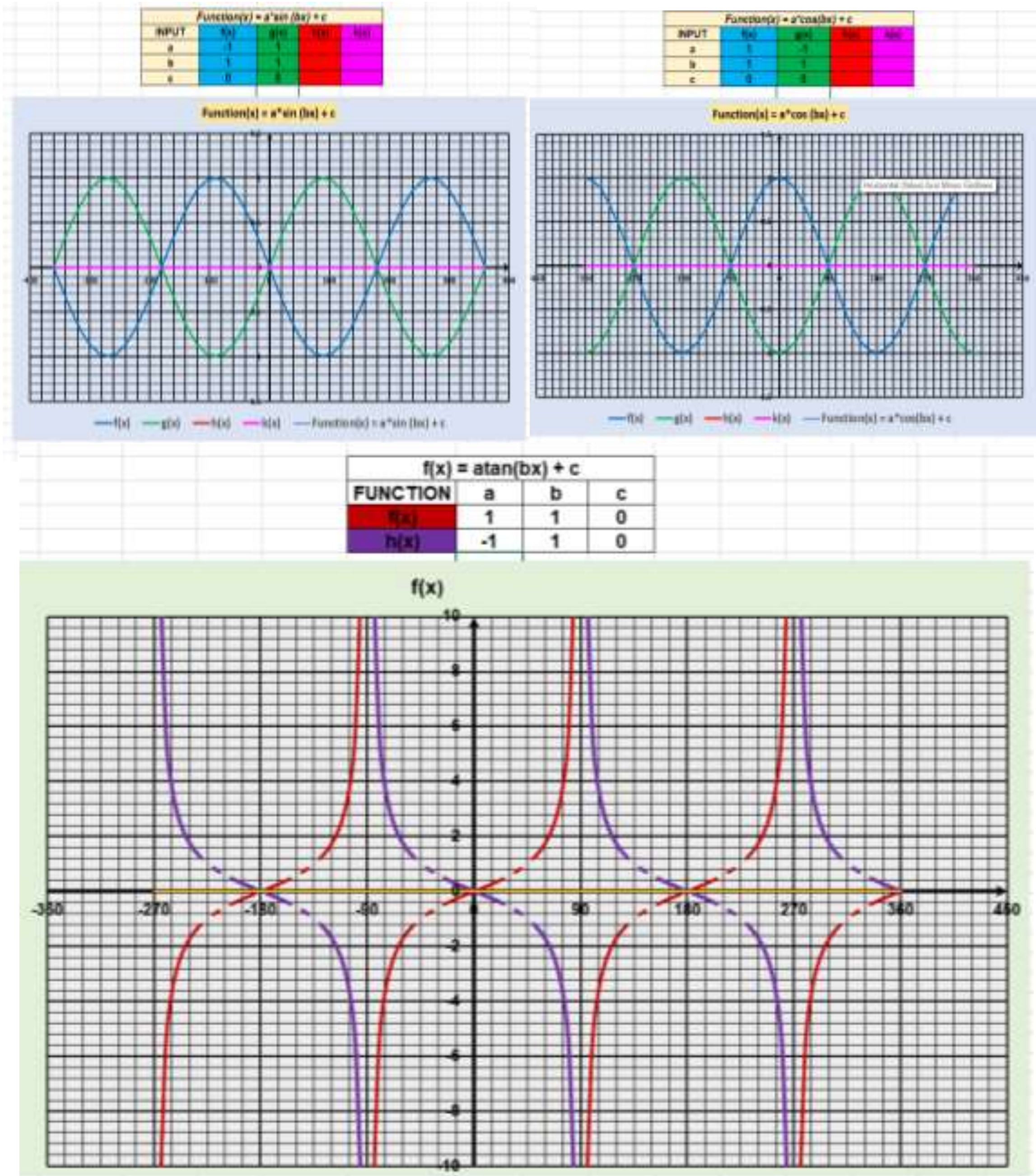


Figure 4. 14: : EBM outputs of sine, cosine and tangent functions for a>0 and a <0

The researcher then projected the graphs of sine, cosine and tangent, in Figures 4.15, 4.16 and 4.17 respectively, to counter the validity of the statements of Participants 8 and 7. The teachers were then asked to refine their statements to explicate and refine the projected cases.

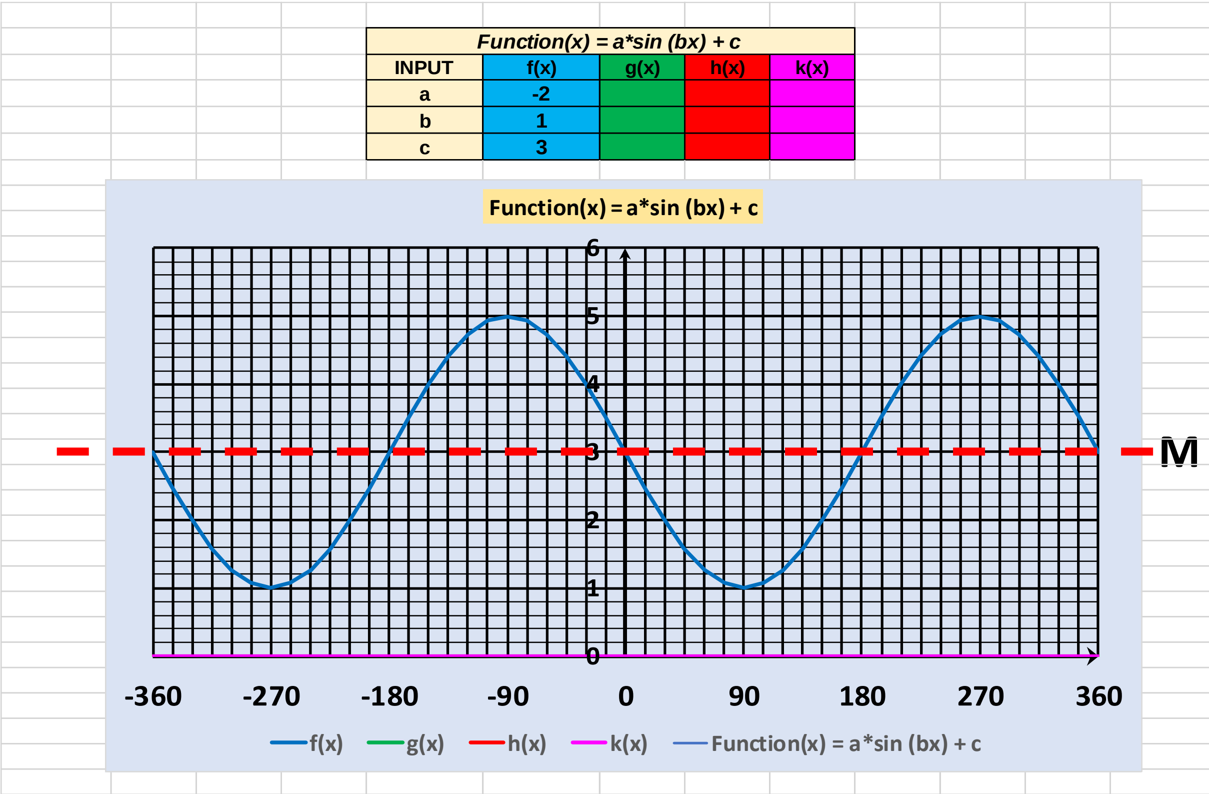


Figure 4. 15: EBM output of f(x) = -2*Sin(x)+3

Researcher (R): According to Participant 7, the value of "a" in *f(x) = a*trigfunction(bx)+c is positive if the first part of f(x) is also positive to the right. But look, here, the first part of (f(x) is positive but the value of a = -2…how can we improve this statement to include this case?*

Participant 3 (P3): *We should consider the line that cuts the graph horizontally through the middle (pointing to the graph and shows the line M)… so the statement should be …if the first part of the graph of f(x) is above the line M, then the value of "a" is positive… So you see here the graph is below M, that's why "a" is negative (the whole group claps hands in agreement with P3).*

Researcher (R): *Let's look at the Cosine graph…then we will summarise our conclusions at the end.*

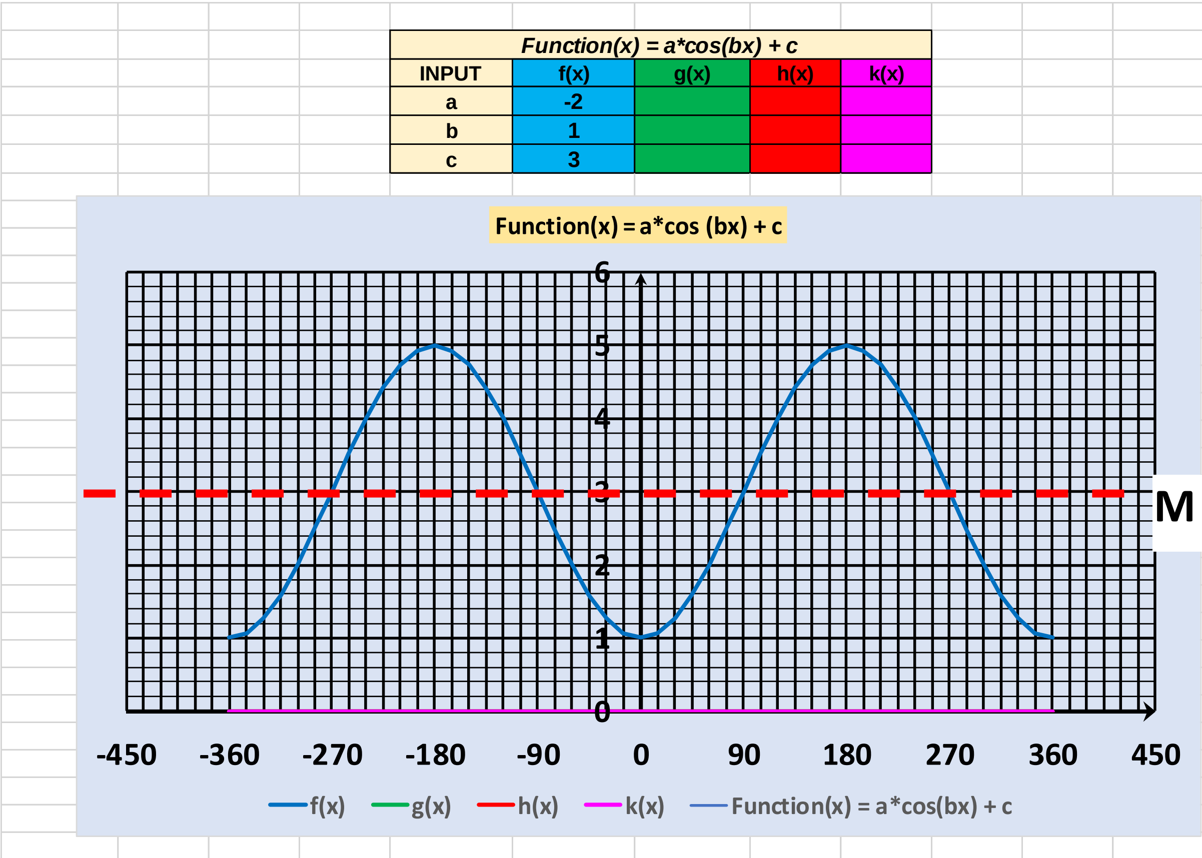


Figure 4. 16: EBM output of f(x) = -2*Cos(x)+3

Researcher (R): *Participant 8 said if the y-intercept of the cosine graph is negative then "a" is negative and vice versa. But look here, the y-intercept = 1 (positive), while the value of "a" is negative…what do you have to say about this?*

Participant 1 (P1): *We also change the statement to say…if the y-intercept is below the line M then "a" is negative and vice versa… if the y-intercept is above the line M then "a" is positive…*

Participant 4 (P4): *What if the y-intercept is on the line M?*

Researcher (R): *Let's all think carefully about P4's question…what do you think?*

Participant 9 (P): *This can only happen if we have the basic cosine function…then the line M will be the x-axis… (the whole group agrees).*

Researcher (R): *Take note that if we use P7's statement, then we should investigate if the first part of the graph to the right (or the first quadrant)… is* *below or above the line M. Here we see that the graph is below the line M, so the value of "a" is negative…do we agree?*

Whole group: *(very excitedly fidgeting)…Yes!*

4.2.1.2.5 *Inquiry process steps for the tangent function and outcomes*

The following graph of *f(x) = tanx - 4* was projected by the researcher. Teachers were then engaged in analysing the graph in relation to previous conclusion of participant 7, (P7) who had concluded that if f(x) > 0 in the first quadrant, then the value of "a" in f(x) = a*trigfunction(bx)+c is positive. By engaging the teachers through a guided inquiry process, the researcher used this example to demonstrate that this conclusion was not correct.

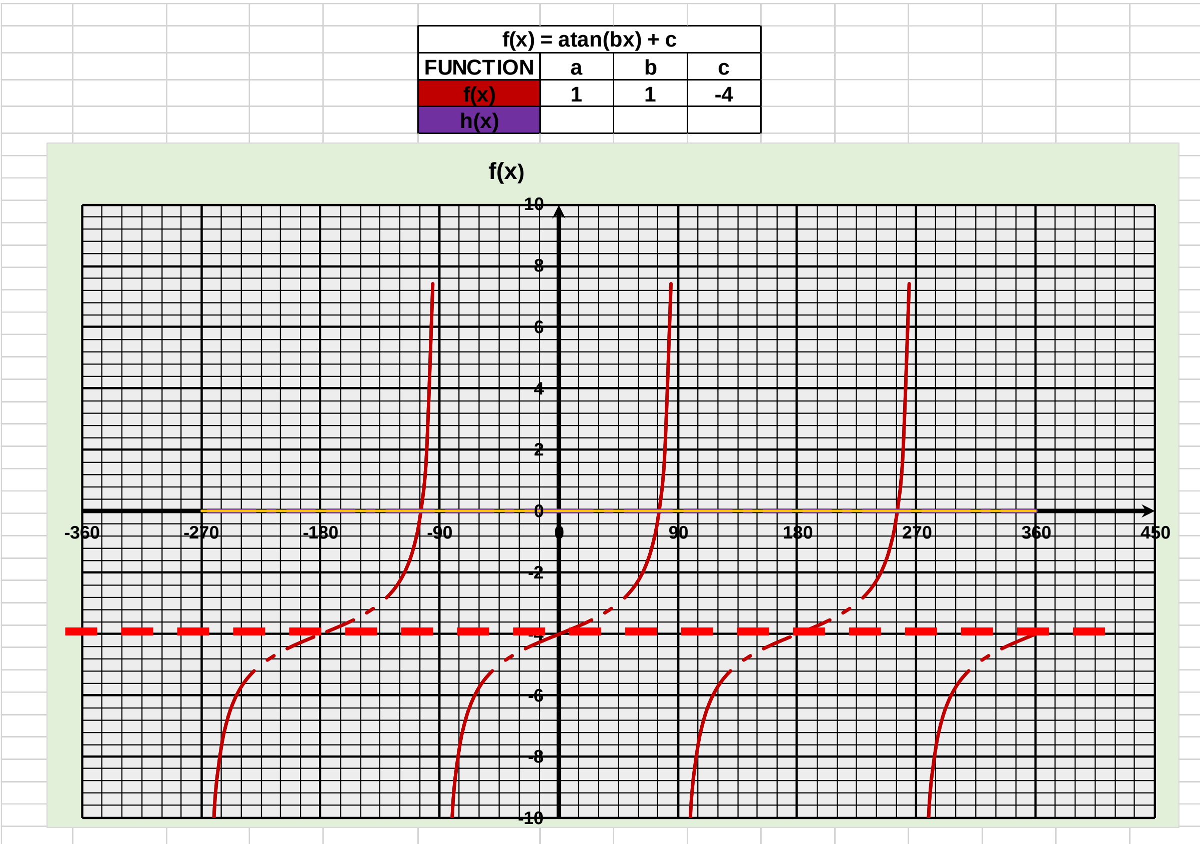


Figure 4. 17: EBM output of f(x) = tan(x) - 4

The following discussion captured the participants' conceptions on the characteristics of the tangent function and graph.

Researcher (R): *This is a counter example to the statement that if the function f(x) is positive in the first quadrant, then the value of "a" is positive, but here we can see that* *the value of "a" is positive whereas the function f(x) is negative in the first quadrant…how can we refine our statement to include this case?*

Participant 5 (P5): *We need to refer to the midline M… (frowning and holding her cheeks between her hands)… where is the midline here?*

Researcher (R): *Very good P5…where is the midline?*

Participant 7 (P7): *(jumps from chair and heads straight to the white board)… look! In all the graphs…the value of "c" is the y-intercept of the midline M…whooah! (punching the air)… So the midline for the tangent function is the horizontal line through y=-4…. (and draws the line M on the graph).*

Researcher (R): *So what would be the refined statement for determining the value of "a" from the tangent graph?*

Participant 5 (P5): *If the graph of tangent in the first quadrant is below the midline M, then "a" is negative and vice versa". In this case the graph is above the line M, so "a" is positive.*

Researcher (R): *Good! …Now let us summarise our criteria for finding the values of "a", "b" and "c" in the function f(x) = a*trigfunction(bx)+c.*

The group concluded as follows:

For the sine and cosine functions, the value of "a" is obtained from the amplitude, which is the distance from the midline M to the maximum or minimum points of the function. If the function is above (below) the midline M in the first quadrant, then the value of "a" is positive (negative).

$b = \frac{P_{bf}}{P_{nf}}$ where P_{bf} is the period of the basic function and P_{nf} is the period of the displayed function.

The value of "c" is the y-intercept of the midline M.

It turned out that the conclusions of the group were absolutely correct. Teachers observed that the EBM tools provided them with an opportunity to independently explore more unconventional connections between graphs and algebraic expressions, including analysing the characteristics of graphs in relation to changes of signs to **a** and **b** in f(x) = a*trigfunction (bx) + c.

The tangent function initially posed some deep-seated challenges due to the nature of the graph having asymptotes, it was difficult to programme the applets to display the corresponding changes in the asymptotes for different values of "b". However, having laid the foundation with the sine and cosine functions, it was possible to create a modelling applet for the tangent function that allowed the research participants to explore the impact of changes in the values a, b and c. Figure 4.18 shows the EBM output display for the functions f(x)= tan x and h(x) = 2tanx to explore how the value of a = 2 could be determined from the graph of h(x) = 2tanx.

y = atan(bx) + c			
FUNCTION	a	b	c
f(x)	1	1	0
h(x)	2	1	0

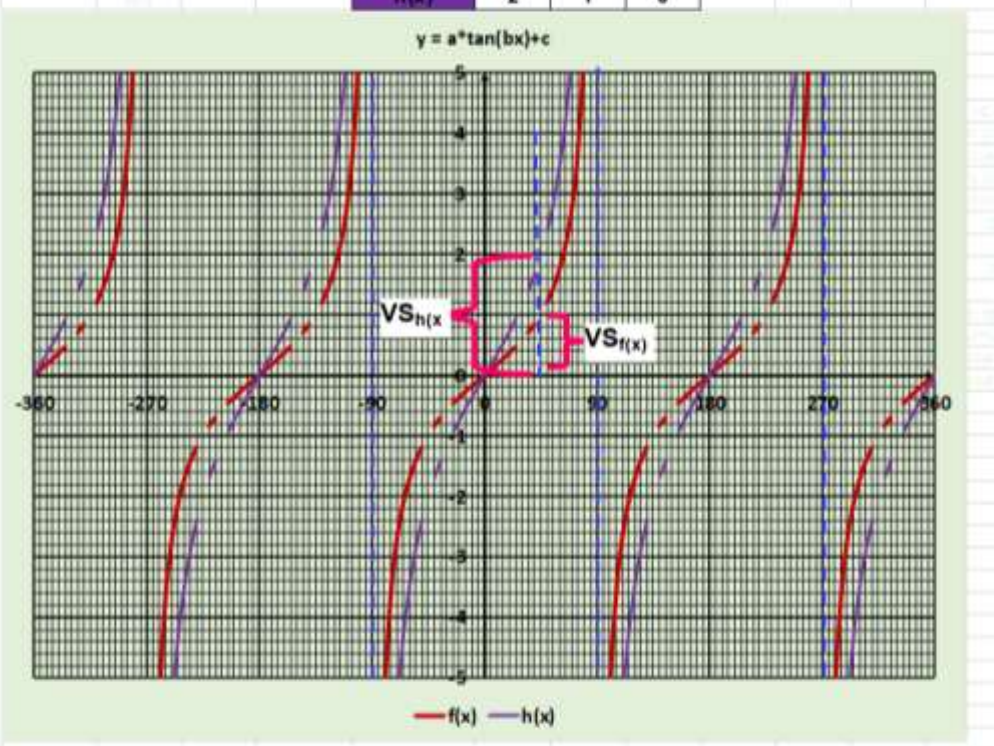


Figure 4. 18: Modelling applet to explore how the function y = a*tan(bx)+c can be deduced from the graph

Guided by one of the prepared lesson plans , the following steps were explained as a procedure to determine the value of “a” (See Lesson plan 4, Appendix C4).

Step 1: Determine the midline of the graph

The midline M of the graph is always through the y-intercept of the graph. In this case, for both f(x) and h(x), the midline is the x-axis through y = 0. The y-intercept is also the value of c.

Step 2: Determine the value of b

In this case for both f(x) and h(x), $b = \frac{Period \quad of \quad basic \quad function}{Period \quad of \quad displayed \quad function} = \frac{180^0}{180^0} = 1$

Step 3: Determine the value of a

The vertical line through half-way between x=0 and the next asymptote ($x = 45^0$) helps us to find the vertical stretch (VS) of the graph. In this case for f(x), the vertical stretch for f(x) is $VS_{f(x)} = 1$. For h(x), the vertical stretch is $VS_{h(x)} = 2$.

For each function, its vertical stretch represents the value of **a**, with the sign taken as the value of the tangent line through the point of the graph on the vertical stretch. In this case the values of **a** are positive for both functions.

Step 4: Write the function

f(x) = 1. tan (1.x) + 0 = tan x

h(x) = 2.tan (1.x) + 0 = 2tanx

The graph in Figure 4.19 was then displayed for the teachers to deduce the function y=a*tan(bx)+c.

y = atan(bx) + c			
FUNCTION	a	b	c
f(x)	-0.25	2	1
h(x)			

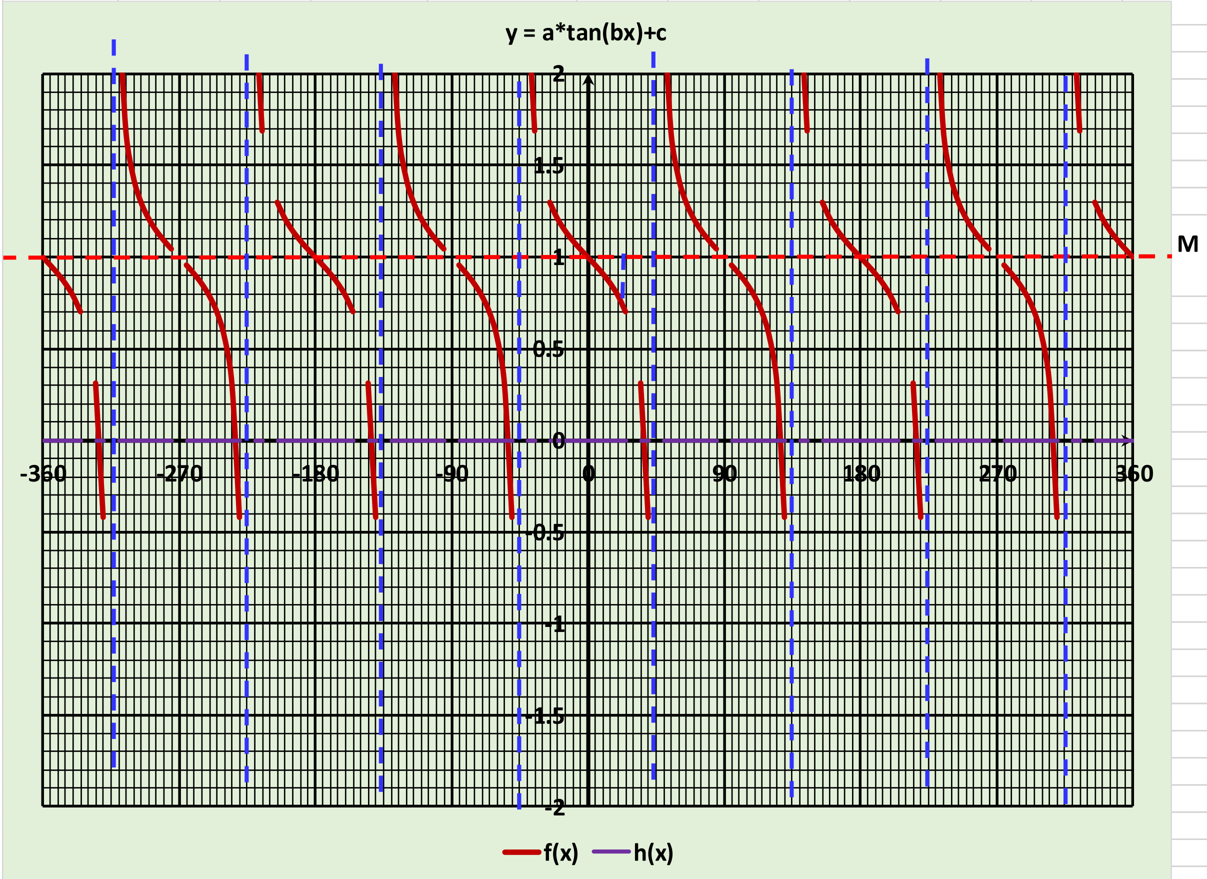


Figure 4. 19: Graphic display of function y = a *tan(bx)+c given as a collaborative teacher activity to determine values of a, b and c.

The participating teachers correctly followed the solution process:

Step 1: Determine the midline of the graph

The midline M of the graph is always through the y-intercept of the graph. In this case, the midline is the x-axis through y = 1 = c.

Step 2: Determine the value of b

In this case for both f(x) and h(x), $b = \dfrac{Period \quad of \quad basic \quad function}{Period \quad of \quad displayed \quad function} = \dfrac{180^0}{90^0}$ = 2

Step 3: Determine the value of **a**

The vertical line through half-way between x=0 and the next asymptote ($x = 22.5^0$) helps us to find the vertical stretch (VS) of the graph. In this case, the vertical stretch is VS = 0.25.

In this case the value of **a** is negative.

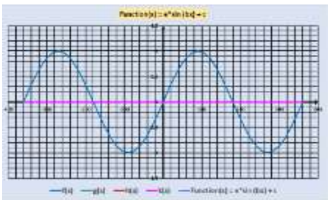
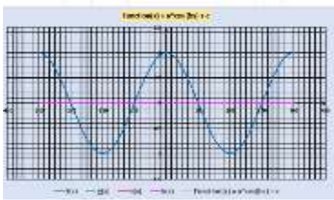
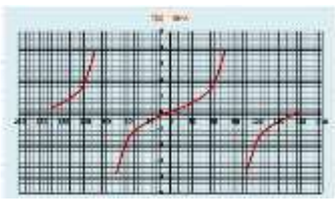
Step 4: Write the function

$$y = -\frac{1}{4}\tan(2x) + 1$$

4.2.3.7 Analysis of symmetry and parity of sine, cosine and tangent functions

Table 4.9 shows the participants'' observed outcomes from the exploration of symmetry and parity of trigonometric functions.

Table 4. 9: Summary of exploratory outcomes of relationship between symmetry and parity of trigonometric functions

Function	EBM graph output	Line symmetry	Rotational symmetry	Parity
Sine		No line symmetry	Rotational symmetry of order 2, about origin (Origin Symmetry)	sin(-x) = -sin(x) Odd
Cosine		Symmetric about the y-axis	No rotational symmetry	cos(-x) = cos(x) Even
Tangent		No line symmetry	Origin symmetry	tan(-x) = -tan(x) Odd

4.2.1.2.6 Inquiry outcomes on conceptions of symmetry and parity of trigonometric functions

The following discussion captures the participants' conceptions of symmetry and parity of trigonometric functions.

Researcher (R): *Can we draw any relationship between the symmetry and parity of the three trigonometric functions?*

Participant 3 (P3): *The even function has line symmetry about the y-axis...the odd functions do not have line symmetry…they have origin symmetry…*

Participant 2 (P2): *Can we say…even functions have line symmetry, and odd functions have origin symmetry?*

Researcher (R): *Interesting observation, let us go and explore further…and perhaps use a more analytical method, by relating the sine, cosine and tangent functions to the unit circle.*

The teachers were then given the following activity in Figure 4.20 and 3 minutes to analyse the graphs and respond to the question.

The graphs below depict four trigonometric functions. Identify which of the graphs are $f(x) = \sin(x)$, $g(x) = \cos(x)$, and $h(x) = \tan(x)$. Explain how you know.

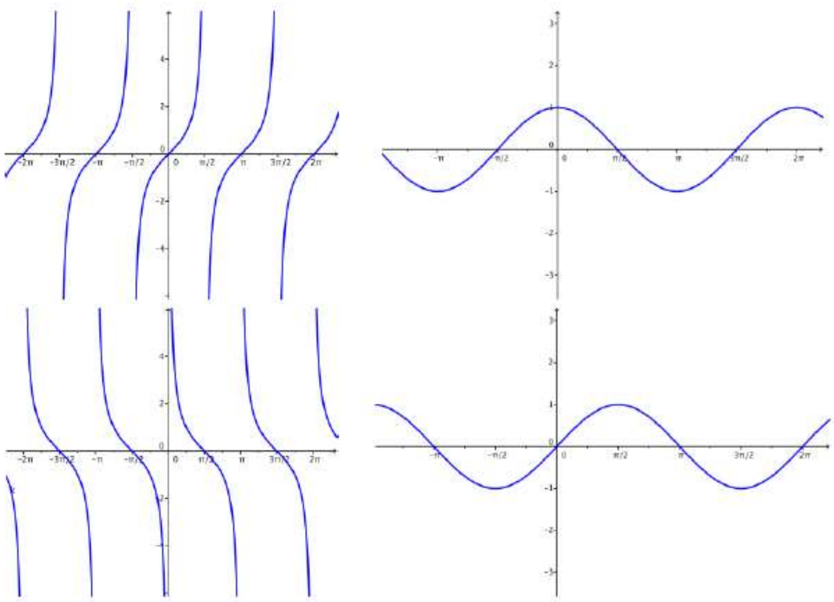


Figure 4. 20: Teacher assessment activity on conceptual understanding of periodicity of trigonometric functions

The following discussion summarises the participants' rationale for their chosen responses in identifying the graphs.

***Researcher (R)**: …(After 3 minutes)…right let us discuss the solution together…participant 10…what do you think of the first graph?*

***Participant 10 (P10):** The first graph is h(x) = tan(x)…because the range is all real values, it is not restricted like cos(x) and sin(x)…... and also tan(0) = 1…(the other participants agree).*

***Researcher (R):** What function matches the second graph?*

***Participant 5 (P5):** The second graph must be g(x) = cos(x), because…the range is between -1 and 1, and cos(0) = 0…also* $Cos(\frac{n}{2}) = -Cos(\frac{n}{2})$ *as we have seen in the parity of the cosine function…(the group agrees).*

***Researcher (R):** Now, let us analyse the third graph…what do you think?*

***Participant 11 (P11):** It is h(x) = tan(x)…maybe rotated….*

***Participant 2 (P2):** But … in this graph tan(0) is not equal to 0…so it can't….(the whole group looks confused)*

***Researcher (R):** Clearly, based on your observations…this graph is not for the tangent function…but let us leave it for now…we can discuss it at some point later…Can we identify the function for the last graph?*

***Participant 6 (P6):**… (laughing)… obviously f(x) = sin(x), because the range is between 1 and -1 , sin(0) = 0 and sin(ɳ) = sin(-ɳ)… (they all agree).*

The discussion helped the teachers relate the periodicity of the graphs of the sine, cosine and tangent functions to their symmetry and parity.

4.2.2 ANALYSIS OF TEACHERS' LEARNING (GUSKEY LEVEL 2)

Descriptive statistics (Cohen's ***d*** and Hedges' ***g)*** were also used to measure effect size of the EBM professional development intervention, with a follow up qualitative analysis of the post –EBM group feedback on their perceived changes in TPACK self-efficacy after exposure to the EBM professional development. Participant teachers were asked to reflect on how the EBM applets exerted an impact on their self-efficacy to solve the problems in the collaborative problem solving activity. On the other hand, the effect size of the EBM professional development was calculated to corroborate how the teachers' self-efficacy perceptions changed after exposure to the EBM programme.

4.2.2.1 Influence of EBM professional development intervention on TPACK self-efficacy

The data analysis in this section focused on the teachers' state of TPACK after the EBM professional development intervention. The correlations between the post-EBM professional development TPACK constructs were analysed to establish whether there are any strong associations (non-causal) between the variables. Paired samples t-test statistics was determined to investigate the level of statistical significance ($p<0.05$) for which the observed changes in TPACK constructs could be attributed to the EBM intervention. The effect size of the intervention was determined through the calculation of Cohen's **d** Index (CI), by comparing the group means of the teachers'self reported levels of pre-EBM and post-EBM TPACK constructs. For each construct, the null and alternate hypotheses were stated as:

H_0: There is no difference in mean pre and post levels of TPACK construct.

H_1: There is a difference in mean pre and post levels of TPACK construct.

If $p < 0.05$, then the null hypothesis of no difference is rejected.

4.2.2.1.1 Analysis of post-EBM intervention correlations between TPACK constructs

Table 4.10 shows the teachers' self-reported levels of TPACK constructs after undergoing the EBM professional development programme.

Table 4. 10: Summary of teachers' mean levels of TPACK self-efficacy after EBM professional development

#	Teacher Code	LEVEL OF TPACK COMPONENTS AFTER TRAINING						
		TK	CK	PK	PCK	TCK	TPK	TPACK
1	S1T04	3.90	4.00	4.40	4.00	4.00	3.80	4.00
2	S1T11	3.90	3.90	4.10	4.00	4.00	3.70	4.00
3	S2T09	3.60	4.00	4.10	4.08	3.90	4.00	4.00
4	S2T10	4.00	4.00	4.00	4.00	4.00	4.00	4.00
5	S3T08	4.10	4.00	4.00	4.00	4.00	3.90	4.00
6	S4T07	4.00	4.00	4.00	4.00	4.00	4.00	4.00
7	S5T06	4.30	4.40	4.00	3.92	4.00	4.00	4.10
8	S6T03	4.00	4.00	4.10	4.00	4.00	4.00	4.00
9	S6T05	3.70	4.00	4.40	4.33	4.10	4.00	4.10
10	S7T02	3.80	4.00	4.00	4.00	4.00	4.00	4.00
11	S8T01	4.30	4.80	4.60	5.00	5.00	5.00	5.00
	Mean	3.964	4.100	4.155	4.121	4.091	4.036	4.109

Table 4.11 shows the correlations between the TPACK constructs after the EBM professional development intervention.

Table 4.11: Summary of correlations among TPACK constructs after EBM teacher professional development

CORRELATIONS BETWEEN TPACK DOMAINS							
DOMAIN	**TK**	**CK**	**PK**	**PCK**	**TCK**	**TPK**	**TPACK**
TK	**1.000**	**0.703**	**0.068**	**0.283**	**0.516**	**0.480**	**0.523**
CK		**1.000**	**0.536**	**0.774**	**0.868**	**0.890**	**0.913**
PK			**1.000**	**0.815**	**0.738**	**0.604**	**0.721**
PCK				**1.000**	**0.957**	**0.915**	**0.952**
TCK					**1.000**	**0.942**	**0.991**
TPK						**1.000**	**0.956**
TPACK							**1.000**

The correlation was very high between TCK and TPACK (0.991), followed by PCK and TCK (0.957), TPK and TPACK (0.956), TCK and TPK (0.942), PCK and TPK (0.915) and CK and TPACK (0.913). Very low correlations still existed between TK and PK (0.068), TK and PCK (0.283) and TK and TPK (0.480).

The following Tables 4.12 to 4.14 show the SPSS output for the analysis of the t-test paired samples statistics for the 7 pairs of the TPACK pre and post EBM TPACK survey ratings, conducted with confidence level 0.95 and p-value = 0.05.

Table 4. 12: Paired Samples Statistics of the seven TPACK constructs

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	TKpost	3.9636	11	.22033	.06643
	TKpre	2.9000	11	.33166	.10000
Pair 2	CKpost	4.1000	11	.26458	.07977
	CKpre	3.5091	11	.52241	.15751
Pair 3	PKpost	4.1545	11	.21149	.06377
	PKpre	3.8818	11	.21826	.06581
Pair 4	PCKpost	4.1209	11	.30998	.09346
	PCKpre	3.7664	11	.28342	.08545
Pair 5	TCKpost	4.0909	11	.30481	.09190
	TCKpre	2.5636	11	.72839	.21962
Pair 6	TPKpost	4.0364	11	.33548	.10115
	TPKpre	3.1909	11	.51275	.15460
Pair 7	TPACKpost	4.1091	11	.29818	.08990
	TPACKpre	3.3636	11	.65310	.19692

Table 4. 13: Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	TKpost & TKpre	11	-.014	.968
Pair 2	CKpost & CKpre	11	.622	.041
Pair 3	PKpost & PKpre	11	.284	.398
Pair 4	PCKpost & PCKpre	11	.457	.158
Pair 5	TCKpost & TCKpre	11	.570	.067
Pair 6	TPKpost & TPKpre	11	.578	.063
Pair 7	TPACKpost & TPACKpre	11	.413	.207

Table 4. 14: Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	TKpost - TKpre	1.06364	.40068	.12081	.79446	1.33282	8.804	10	.000
Pair 2	CKpost - CKpre	.59091	.41341	.12465	.31318	.86864	4.741	10	.001
Pair 3	PKpost - PKpre	.27273	.25726	.07757	.09990	.44556	3.516	10	.006
Pair 4	PCKpost - PCKpre	.35455	.31001	.09347	.14628	.56281	3.793	10	.004
Pair 5	TCKpost - TCKpre	1.52727	.60843	.18345	1.11853	1.93602	8.325	10	.000
Pair 6	TPKpost - TPKpre	.84545	.42039	.12675	.56303	1.12788	6.670	10	.000
Pair 7	TPACKpost - TPACKpre	.74545	.59559	.17958	.34533	1.14558	4.151	10	.002

For easier analysis, the key data from the three SPSS output tables was integrated into one table. Table 4.15 shows an amalgamated version comparing the pre and post EBM survey ratings.

Table 4. 15: Comparison of pre and post EBM teacher professional development TPACK self-efficacy

Knowledge Domains	Pre-EBM survey		Post-EBM survey		Post-Pre Mean	p
	Mean	SD	Mean	SD		
TK	**2.90**	**0.33**	**3.96**	**0.22**	**1.06**	***0.000**
CK	**3.51**	**0.52**	**4.10**	**0.26**	**0.59**	***0.001**
PK	**3.88**	**0.22**	**4.15**	**0.21**	**0.27**	***0.006**
PCK	**3.77**	**0.28**	**4.12**	**0.31**	**0.35**	***0.004**
TCK	**2.56**	**0.73**	**4.09**	**0.30**	**1.53**	***0.000**
TPK	**3.19**	**0.51**	**4.04**	**0.34**	**0.85**	***0.000**
TPACK	**3.36**	**0.65**	**4.11**	**0.30**	**0.75**	***0.002**
***p < 0.05**						

The results from the pre-EBM survey indicated that the participants began the training programme with the greatest level of confidence in their PK, followed by PCK, CK and TK. The teachers were least confident about their TCK, TPK and TPACK. At the end of the EBM intervention, the teachers had the greatest level of confidence in their PK, followed by PCK and CK. The teachers still showed least confidence in their TK, TCK, TPACK and TPK.

The comparison of self-assessment outcomes revealed significant development of the teachers' technological pedagogical knowledge (TPK) and technological pedagogical content knowledge (TPACK). There was only limited growth in participants' PK and PCK, which possibly emanates from the fact that PK and PCK were not specifically taught during the programme, given that the teachers in the sample were selected on the basis of their successes in the NSSCH Mathematics curriculum implementation over the previous three years. Paired samples t-tests conducted between all TPACK constructs indicated the p-value <0.05 level of significance for each pairing.

From Table 4.15, it can be observed that all p values are less than 0.05, meaning that there is a very small probability that the increase in the level of TPACK constructs occurred by chance, under the null hypothesis of no difference. Hence, the null hypothesis is rejected for all TPACK constructs. Consequently, the conclusion is that there is strong evidence that the EBM professional development intervention significantly improves the teachers' TPACK levels.

4.2.2.1.2 Quantitative analysis of effect size of EBM intervention on TPACK self-efficacy

In order to qualify the use of Cohen's ***d*** and Hedges' ***g***, the analysis of independence of association between the Pre-EBM and Post-EBM data on the seven (7) TPACK constructs was conducted. In addition, the normality of the data was explored using two popular approaches for demonstrating normality, namely; (a) formal normality tests (analysis of kurtosis, skewness and comparison of mean and median) and, (b) graphical analysis by Q-Q plot (quantile-quantile plot). The Q-Q plot is accepted as an effective visualisation tool for assessing the empirical probability distribution of a random variable, against any hypothesised theoretical distribution (Kei-Wei et al., 2019; Razali & Wah, 2011). Q-Q plot compares two probability distributions by plotting theoretical quantile (horizontal axis) against empirical quantile (vertical axis). If the data sets are normally distributed, the Q-Q plot will exhibit a pattern similar to a diagonal positive straight line.

The analysis of independence of association between the pre-EBM and post-EBM self efficacy data was done using the chi-square statistic, with the null hypothesis (H_0) formulated as:

H_0: There is no significant association between the teachers' Pre-EBM and Post-EBM self-efficacy ratings.

The analysis yielded a Chi-square value (X^2) of 0.651 and critical value of 18.307. Since 0.651 < 18.307, H_0 is not rejected. Therefore, it was demonstrated that the Pre-EBM and Post-EBM data sets were independent (See Appendix J).

The current study first used descriptive statistics (analysis of kurtosis, skewness, mean and median) to establish if the data satisfied the normality requirement, based on the following requirements:

1. $-2 < kurtosis \ \ valkue < 2$
2. $-2 < skewness \ \ valkue < 2$
3. $Mean \approx \ \ Median$

The results satisfied the above three conditions for normality. However, normality cannot be concluded from the mere fact that data satisfy descriptive statistics for normality, hence it was necessary to further apply the Q-Q plot test for normality. Both the pre-EBM and post-EBM self-efficacy ratings of TPACK level of development followed a normal distribution. (See Appendix L1 and Appendix L2).

Although Hedges' **g** and Cohen's **d** are similar in that they both have an upward bias in results of approximately up to 4%, for sample sizes less than or equal to 20, Hedges' **g** is considered a better measure of effect size. For this reason, Hedges' **g** is sometimes referred to as the **corrected effect size** (Glen, 2016).

Previous studies conducted with small sample sizes of less than or equal to twenty (20) participants (Ryan, 1995), also recommended the use of Hedges' **g** for computations with both large and small samples. It was observed that the effect size estimated using Hedge's **g** was always smaller than the equivalent Cohen's **g** estimation, because Hedges' **g** corrects for upward bias that arises in Cohen's **d**, when applied to small samples.

It is against this backdrop that Hedges' **g** was found the most appropriate for the analysis of effect size. The study, further ensured that the data satisfied the relevant assumptions for computation of Cohen's **d** and Hedges' **g**, namely; that (a) the data are sampled independently of one another; (b) the data are sampled from normally distributed populations; and (c) there is homogeneity of variance for all groups under study (Kelley, 2005). Since SPSS does not automatically calculate Cohen's **d**, Cohen's **d** was calculated in an Excel spreadsheet using the formula $d = \frac{Mean}{SD}$ or $d = \frac{t}{\sqrt{N}}$

Table 4.16 shows the output for the processing of Cohen's **d** and Hedges' **g** values.

Table 4. 16: The Cohen's d values calculated using paired differences (N=11)

PAIRED DIFFERENCES				p	Cohen's d	Hedges' g
Knowledge Domains	Mean	SD	t			
TK	1.06364	0.40068	8.804	*0.000	2.655	2.55
CK	0.59091	0.41341	4.741	*0.001	1.429	1.38
PK	0.27273	0.25726	3.516	*0.006	1.060	1.02
PCK	0.35455	0.31001	3.793	*0.004	1.144	1.10
TCK	1.52727	0.60843	8.325	*0.000	2.510	2.41
TPK	0.84545	0.42039	6.67	*0.000	2.011	1.93
TPACK	0.745	0.59559	5.151	*0.002	1.252	1.20
*p < 0.05						

The interpretation of Hedges' **g** was as follows:

Small: g < 0.50

Medium: 0.50 ≤ g < 0.80

Large: g ≥ 0.80

All the Hedges' **g** values were large (**g** ≥ 0.8), indicating that the EBM professional development programme had a significant influence on the growth of all teachers' TPACK constructs. The Hedges' **g** was largest for TK (2. 55), TCK (2.41), and TPK (1.93). This means that the EBM intervention had the largest effect to changes in teachers' TK, TCK and TPK. The Hedges' **g** was relatively small for PK (1.02), PCK (1.10), TPACK (1.20) and CK (1.38). This means that the EBM intervention had a relatively smaller effect to changes in teachers' PK, PCK, TPACK and CK.

4.2.2.1.3 Qualitative analysis of influence of EBM intervention on teachers' self efficacy in teaching periodicity of trigonometric functions

The post EBM implementation questionnaire, consisting of 50 items, 10 from each of Guskey's five levels of professional development, was used to capture data on teachers' perceived level of self-efficacy in teaching periodicity of trigonometric functions. The second section of the questionnaire particularly focused on evaluating the participants' reactions to the professional development programme as well as to establish their perceived level of TPACK self-efficacy after exposure to the EBM professional development. Table 4.17 is a summary of the responses by the 11 participants, showing the mean self-efficacy rating for each statement, on a scale of 1(lowest) to 5 (highest).

Table 4. 17: The impact of the EBM training on teachers' self efficacy

Item	Participants' learning (1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)	MEAN
11	The training improved my understanding of the subject content and helped me to be better prepared in my teaching	**4.636**
12	I now understand functions and graphs well enough to employ multiple strategies in solving related problems in the NSSCH Mathematics curriculum.	**4.182**
13	I now understand better the connection between equations and the graphs of the functions in the equations (e.g. trigonmetric equations and the graphs of the trigonmetric functions)	**4.364**
14	I am confident that I can now deliver the learning content on graphs and functions like an expert in the subject	**4.364**
15	This training increased my knowledge and skills in the teaching of functions and graphs	**4.545**
16	The mathematics content knowledge I need to teach my learners in the topic of functions and graphs in the NSSCH Mathematics curriculum has increased	**4.455**
17	I now understand better the graphical intepretation of trigonometric functions	**3.727**
18	My level of confidence in the use of Excel modelling has increased	**4.000**
19	I am convinced that the Excel modelling approach will make my teaching of functions and graphs easier	**4.091**
20	The Excel modelling instruction led me to reflect more critically on my teaching	**4.182**

The participants had the strongest perception that the training had improved their understanding of the subject content and preparedness in the teaching of trigonometric functions and graphs. Surprisingly, the rating to item 18 shows a relatively low level of confidence in the use of the EBM instructional technique. While the training was done collaboratively, the teachers had to implement the EBM instruction at their respective schools individually with their learners. Perhaps this contributed to the low perceptions about the ease of use and usefulness of the EBM tools, hence their perceived low self-efficacy.

4.2.3 ANALYSIS OF ORGANISATION SUPPORT AND CHANGE (GUSKEY LEVEL 3)

Data was gathered though the items 31-40 of section 3 (organisation support and change), of the post EBM implementation questionnaire. The table below presents the participants' mean rating of each statement.

Table 4. 18: Organisation support and change

Item	School support and change (1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)	MEAN
21	The school management was aware that I was trying out a new teaching approach	**3.909**
22	The school made provisiion for all the necessary resources I needed to implement the Excel modelling instruction with my learners	**3.818**
23	I informed my colleagues so that they could be part of my lessons to observe the implementation of the new approach	**3.636**
24	My supervisor was available and willing to assist me with any challenges I encountered during the implementation of the new approach	**3.727**
25	My trainer regularly visited the school to support me with the programme implementation	**3.909**
26	I could easily access my trainer whenever I needed advice during implementation	**4.273**
27	My trainer was always available and willing to support me whenever I encountered challenges during the implementation of the EBM instruction	**4.091**
28	I regularly met with my trainer to review the progress during the implementation of the new instructional approach	**3.818**
29	My trainer observed some of the lessons I taught and constructively discussed with me the outcomes of the lessons	**3.636**
30	I had clear guidelines on conducting a self-review and evaluation of my EBM lesson outcomes	**3.636**

There was a satisfactory level of organisational support in the implementation of the new EBM instructional approach. In general, all schools made adequate provision of basic resources that the teachers needed to implement the EBM instructional techniques.

4.2.4 ANALYSIS OF PARTICIPANTS' USE OF NEW KNOWLEDGE AND SKILLS (GUSKEY LEVEL 4)

4.2.4.1 Analysis of lesson observation outcomes

The fidelity of implementation of the lessons was monitored through lesson observations. The EBM instruction implementation plan was based on seven lesson plans that were developed by the group, in accordance with the defined competencies for periodicity of trigonometric functions and related concepts in the NSSCH Mathematics curriculum. The first five lessons were exploratory, with teachers guiding the process of discovering the connections between the graphic and algebraic representations of trigonometric functions. In these five lessons, the teachers used a laptop and data projector to enable the visualisation of the graphs that resulted from different inputs of a, b and c in the functions; *a*trigfunction (bx)* + *c*. A sixth practical lesson of 2 hours was dedicated to learners' collaborative problem-solving activities in which they used the EBM tools to independently explore the solutions. The learners were given the same problems as those administered in the teachers' baseline collaborative problem-solving activity. The seventh lesson was the administration of the summative assessment test for learners. The sample lesson plans and test are attached as annexures.

The teachers used the CAEMA tool to map and score each step of the learners' collaborative modelling process during lesson 6. The objective was to test the validity of the CAEMA tool by comparing the formative assessment scores to the summative test scores for learners' conceptual understanding. In a particular study of 138 students in 6 elementary school classrooms, researchers found a positive link between tracker scores and end-of-year assessment scores. The study found a moderate and significant correlation (0.357) between tracker scores and diagnostic student assessment scores. In other words, if a formative assessment score was high, so was the summative assessment score. Consequently, preliminary evidence supported using tracker metrics to monitor students' conceptual understanding. The relationship also held true using OLS (ordinary least squares) regression to estimate controlling for student characteristics. The tracker score was the second largest predictor of student achievement.

Similarly, it would be expected that if the CAEMA tool is valid, then there should be a strong correlation between the learner groups’ mean formative and summative assessment scores. Table 4.19 shows the lesson observation schedule and observer teams that were used over an observation period of two weeks.

Table 4. 19: Schedule and observation teams that were used for lesson observation

		DATE OF LESSON OBSERVATION					6 (CAEMA)	7 (TEST)
LESSON		1	2	3	4	5		
TIME	TR CODE	01/06	06/06	08/06	13/06	15/06	19/06	22/06
14:00-15:00	S1T04	TEAM 1	TEAM 4	TEAM 4	TEAM 2	TEAM 3	TEACHERS USE CAEMA TO MAP THEIR LEARNERS’LEVEL OF CONCEPTUAL UNDERSTANDING	ALL LEARNERS WRITE FROM 14:00 – 15:00
15:30-16:30	S1T11	TEAM 1	TEAM 4	TEAM 4	TEAM 2	TEAM 3		
14:00-15:00	S2T09	TEAM 2	TEAM 4	TEAM 3	TEAM 1	TEAM 3		
15:30-16:30	S2T10	TEAM 2	TEAM 4	TEAM 3	TEAM 1	TEAM 3		
14:00-15:00	S3T08	TEAM 3	TEAM 4	TEAM 3	TEAM 4	TEAM 1		
14:00-15:00	S4T07	TEAM 4	TEAM 4	TEAM 2	TEAM 4	TEAM 2		
14:00-15:00	S5T06	TEAM 4	TEAM 1	TEAM 4	TEAM 3	TEAM 3		
14:00-15:00	S6T03	TEAM 4	TEAM 2	TEAM 4	TEAM 3	TEAM 3		
15:30-16:30	S6T05	TEAM 4	TEAM 2	TEAM 4	TEAM 4	TEAM 3		
14:00-15:00	S7T02	TEAM 4	TEAM 3	TEAM 4	TEAM 4	TEAM 3		
14:00-15:00	S8T01	TEAM 4	TEAM 4	TEAM 4		TEAM 3		
15:30-16:30				TEAM 2	TEAM 1			

OBSERVER TEAMS:

TEAM 1	TEAM 2	TEAM 3	TEAM 4
RESEARCHER	EDUCATION OFFICER	HOD	SUBJECT HEAD
PEER TEACHER	PEER TEACHER	PEER TEACHER	PEER TEACHER

Each school was visited by the researcher (Team 1) or education officer (Team 2), at least once, and the rest of the visits were by Teams 3 and 4. Teams 3 and 4 and all identified peer teachers were constituted of staff members based at the respective schools, and were given prior orientation in the use of the checklist observation instrument. Inter-rater reliability was used to ensure that the observers were not subjective. Each observer was provided with a customised Excel observation template in which they could directly enter their assessment scores giving an immediate output of ratings, on a scale of 1(very low) to 5 (Advanced), for each TPACK construct and learners' observed learning outcomes. Figure 4.21 shows a snippet of the computer output of one of the lessons observed for teacher S1T04. The red and amber colour coding reflect areas that need improvement.

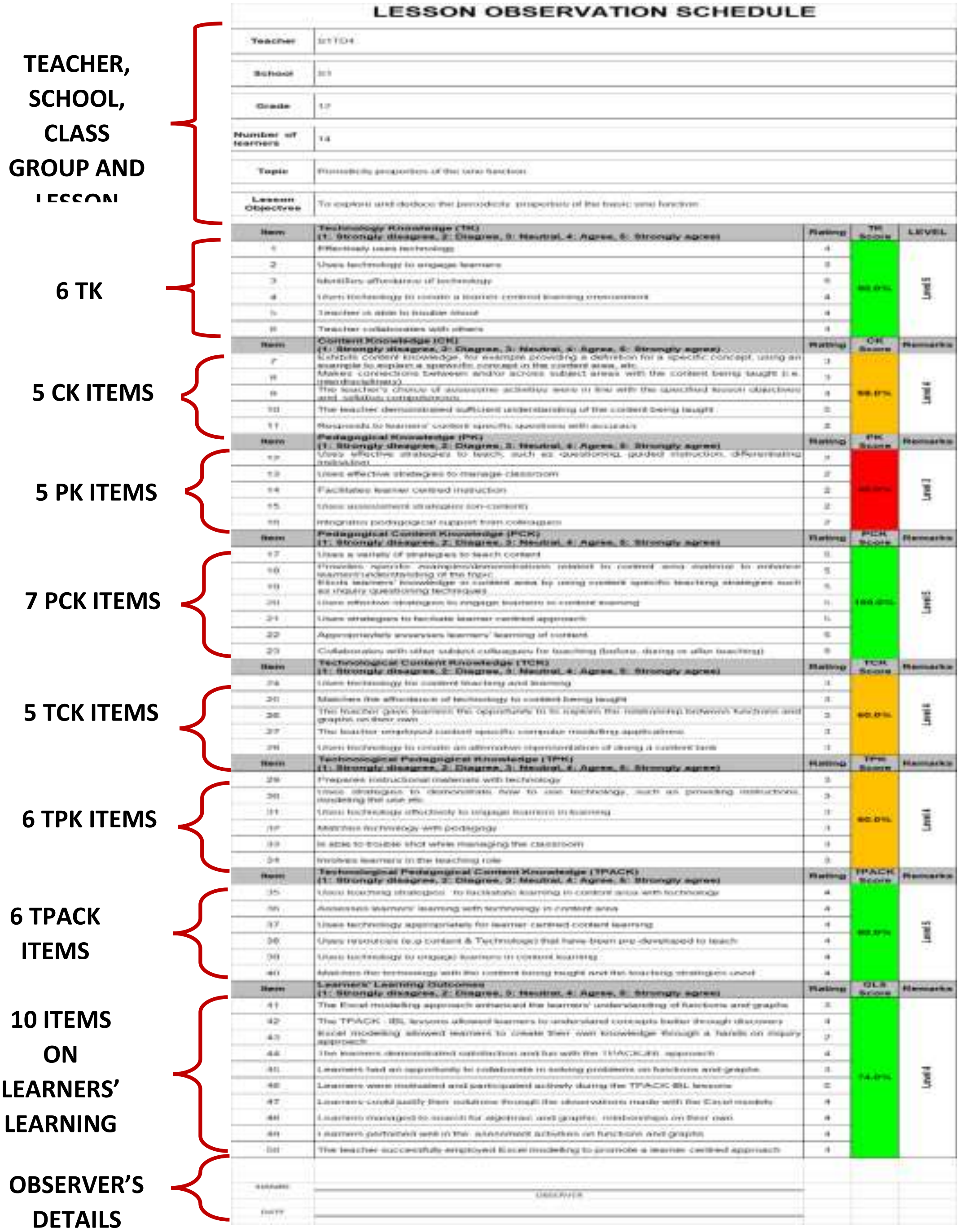


Figure 4. 21: Snippet of Excel output of lesson outcomes for teacher S1T04

Table 4.20 below summarises the lesson observation outcomes for each teacher, based on the observed lessons that met the inter-rater correlation of r > 0.5. All the pairs of lesson observation reports met the criteria for r > 0.5.

Table 4. 20: Summary of mean lesson observation ratings, based on observed TPACK constructs and learners' learning outcomes (LLO)

TEACHER CODE	TK	CK	PK	PCK	TCK	TPK	TPACK	LLO
S1T04	3.83	4.00	4.20	4.29	4.40	4.17	4.00	3.84
S1T11	3.50	4.00	3.80	4.00	4.40	4.00	4.00	3.84
S2T09	3.33	3.60	3.80	4.14	4.00	4.17	3.00	3.75
S2T10	4.00	4.00	4.00	4.00	4.20	4.00	4.00	3.90
S3T08	3.50	4.20	3.80	4.29	3.80	4.17	3.00	3.54
S4T07	3.67	4.00	3.60	4.14	4.00	3.83	4.00	3.85
S5T06	4.17	4.20	4.60	4.43	4.00	4.17	5.00	3.96
S6T03	3.67	4.40	3.80	4.14	4.00	4.00	3.00	3.71
S6T05	3.67	4.00	3.80	3.86	4.40	4.17	4.00	4.04
S7T02	3.67	4.00	3.80	4.14	4.00	4.00	4.00	3.83
S8T01	4.17	4.80	4.20	4.57	4.80	4.50	4.00	4.19
MEAN	3.74	4.11	3.95	4.18	4.18	4.11	3.82	3.86

The evaluation of the teachers' demonstrated state of TPACK, through lesson observations, showed the teachers' technology knowledge (TK) to be lowest (3.742), followed by PK (3.945). Surprisingly, the pedagogical content knowledge (PCK) and technological content knowledge (TCK) were highest at 4.182 each. This seems to suggest that TK and CK together build up higher levels of TCK, provided the teachers' have sufficient content knowledge (CK).

The learners' level of conceptual understanding was rated high at 4.018. Figure 4.22 below shows a bar chart for the mean distribution of ratings of the seven TPACK constructs and the learners' learning outcomes.

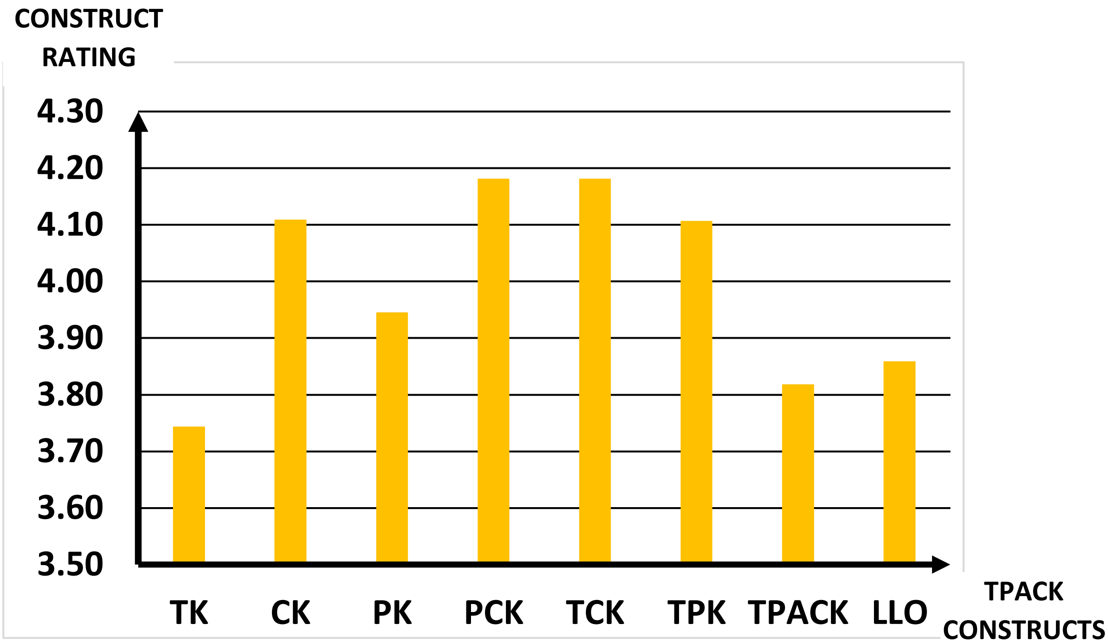


Figure 4. 22: Distribution of mean ratings of variables observed during lesson observations

The distribution shows a range of 3.742 to 4.182 (0.420), with most of the variables rated above 4.00, thus showing a relatively high rating of lesson outcomes.

4.2.4.2 Comparison of teacher self-efficacy and lesson observation ratings

The rationale for comparing the teachers' self reported efficacy ratings and the lesson observation ratings for each TPACK construct was a data triangulation strategy. The Pearson Product Moment correlation coefficient between the mean scores of TPACK constructs from Table 4.12 and 4.22 was calculated. Table 4.21 shows the desctiptive stastistics for the matched scores.

Table 4.21: Correlation between post-EBM professional development self-reported and observed teachers' TPACK knowledge

TPACK CONSTRUCT	MEAN TPACK RATING		SD
	SELF EFFICACY	LESSON OBSERVATION	
TK	3.96	3.74	0.16
CK	4.10	4.11	0.01
PK	4.15	3.95	0.14
PCK	4.12	4.18	0.04
TCK	4.09	4.18	0.06
TPK	4.04	4.11	0.05
TPACK	4.11	3.82	0.20
MEAN	4.08	4.01	
PEARSON CORRELATION		0.42	

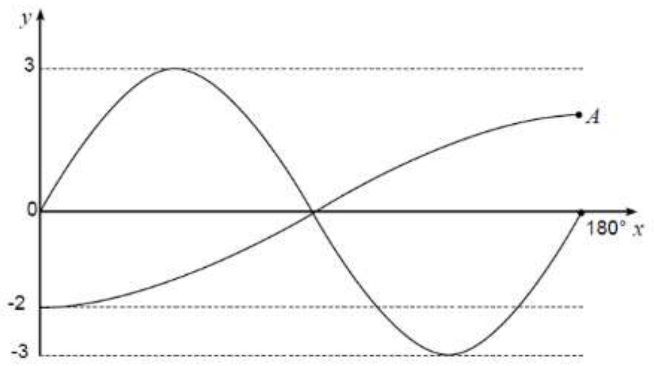
The mean teacher self-efficacy and lesson observation ratings were almost identical, with mean ratings of 4.08 and 4.01 respectively. The Pearson product moment correlation factor was moderate with r = 0.42.

4.2.5 ANALYSIS OF LEARNERS' LEARNING OUTCOMES (GUSKEY LEVEL 5)

4.2.5.1 CAEMA processing of learners' collaborative problem solving activity

The learners' collaborative problem solving activity was largely in the form of oral presentations and manipulation of the EBM tools to verify their problem-solving processes and outcomes. For example, the following is a description of the learners' solution process to problem 3 (see Figure 4.23), as observed and described by the reseacher.

3.



The diagram shows the graphs of $y = a \sin bx$ and $y = c \cos dx$ for $0° \leq x \leq 180°$
(0,-2) and A are stationary points.

(a)	Determine the values of a, b, c and d.	[4]
(b)	Find the coordinates of the point A on the diagram.	[2]
(c)	Find the period of $y = a \sin bx$.	[1]
(d)	Determine the range of $y = c \cos dx$.	[1]
(e)	Write down the new equation of y = a sin bx, when the y-axis is shifted 45° to the right.	[2]

Figure 4. 23: Problem 3 of learners' collaborative problem-solving activity

Solution process:

Learners modelled the basic sine and cosine functions as a point of departure for identifying the two graphs. Figure 4.24 shows the graphic outputs of y = sin x and y= cos x.

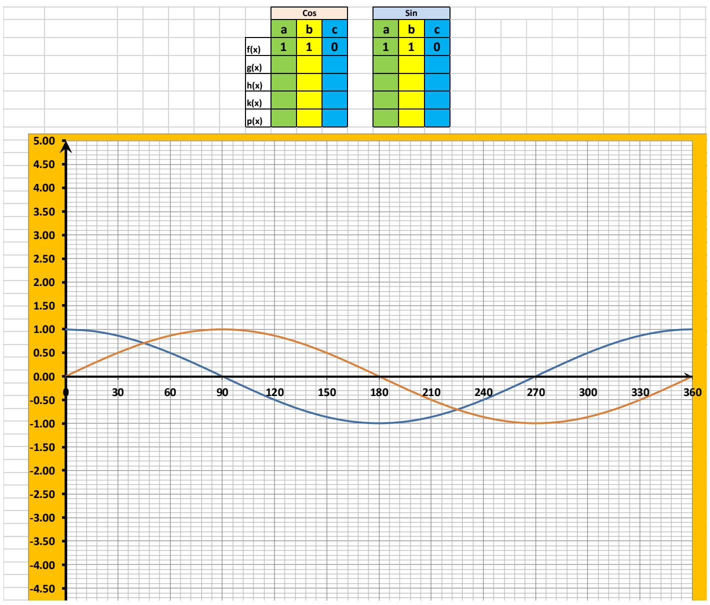


Figure 4. 24: Graphs of y = sin x and y= cos x

The learners were now the agents manipulating the EBM tools with a data projector that was provided to view the computer screen and graphic outputs for discussion.

S8L3: *(Explaining what he has done with EBM tools to get the visual displays in Figure 4.24)… in order for us to know which one is sine and which one is cosine, we need to show the basic functions first…that means a = 1, b=1, c = 1 and d =1…so you can see that the one on top (in the activity sheet) is sine, because sin 0 = 0, so the one below is the cosine graph… (then gives chance to another learner to continue with discussion)…*

S8L15: *But we can see that the graphs are still not the same…let's start with sine…the amplitude is 3, so a =3, and the period is 180^0 so b =2… (replacing values of a and b with 3 and 2 respectively and projecting the graph output in Figure 4.25).*

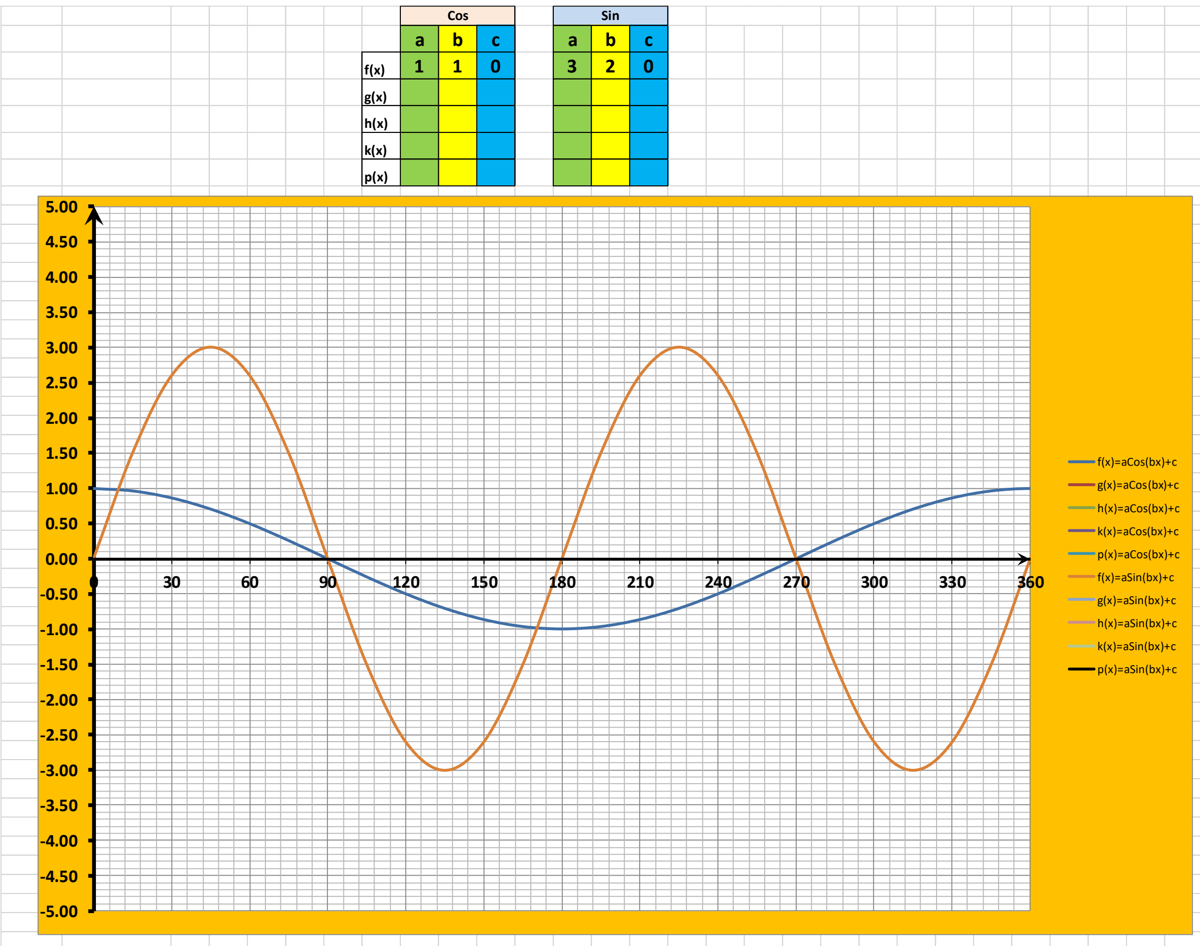


Figure 4. 25: Output after making a = 3 and b = 2 for the sine function

The learners agreed that the sine function now matched the one in the activity sheet. However, they still needed to match the cosine function, hence they immediately engaged further in discussions.

S8L7: *The amplitude for the cosine function is 2, so we make c = 2 …but I am not sure about the period… (interjection by another learner)…*

S8L11: *We started at the minimum at y=-2, so the period should be up to the next minimum. At A… is maximum, and there is a distance equal to* 180^0*…the next minimum is therefore at* 360^0*…that is the period…so d = 2.*

S8L7: *(entering c =2 and d = 2 in the input cells for the cosine function)… and obtains the output in Figure 4.26)…*

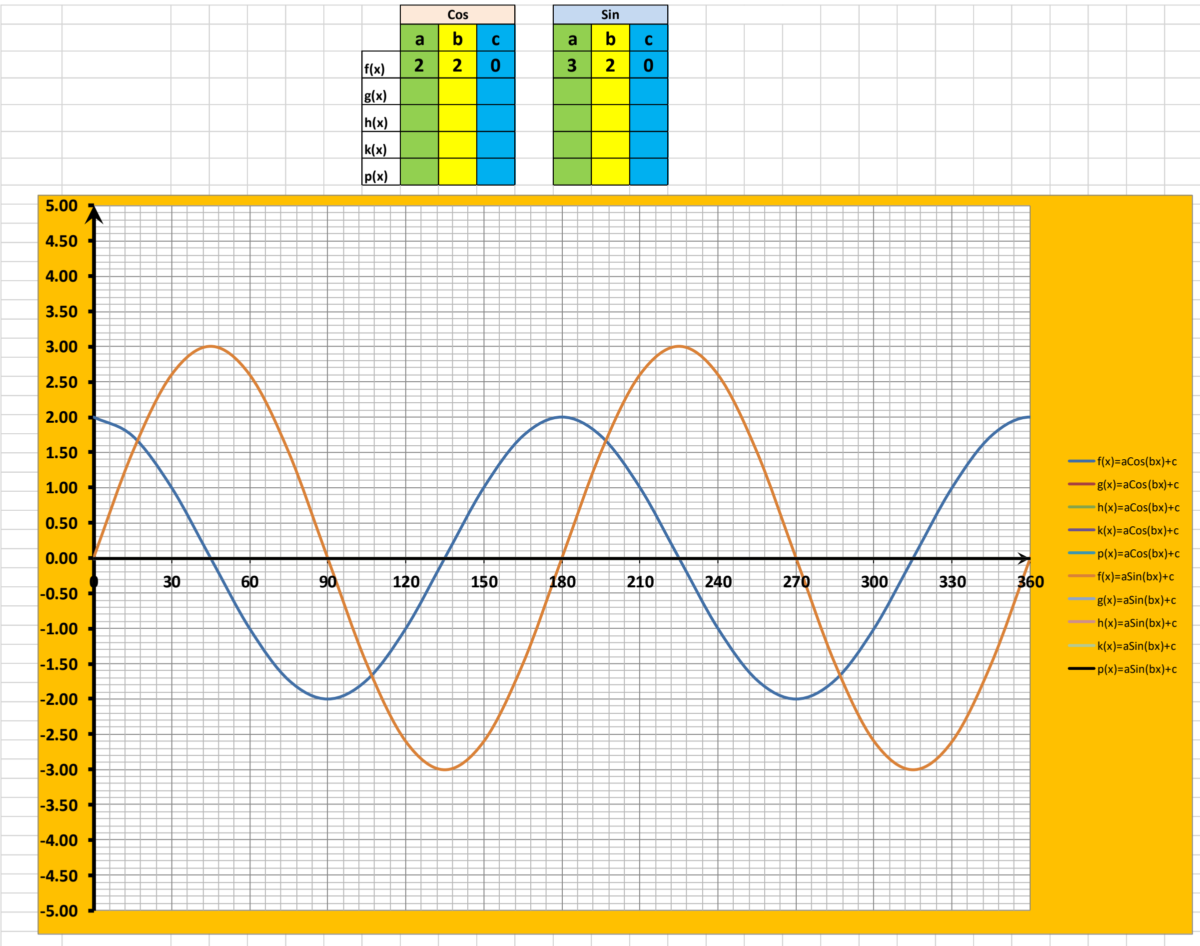


Figure 4. 26: Output after making c = 2 and d= 2

The learners noticed that the cosine graph did not match with the one in the activity sheet, and decided to explore this observation by changing the signs of c and d, and obtained the following output in Figure 4.27. The teacher assisted the learners in adjusting the domain of the functions to $0^0 \leq x \leq 180^0$.

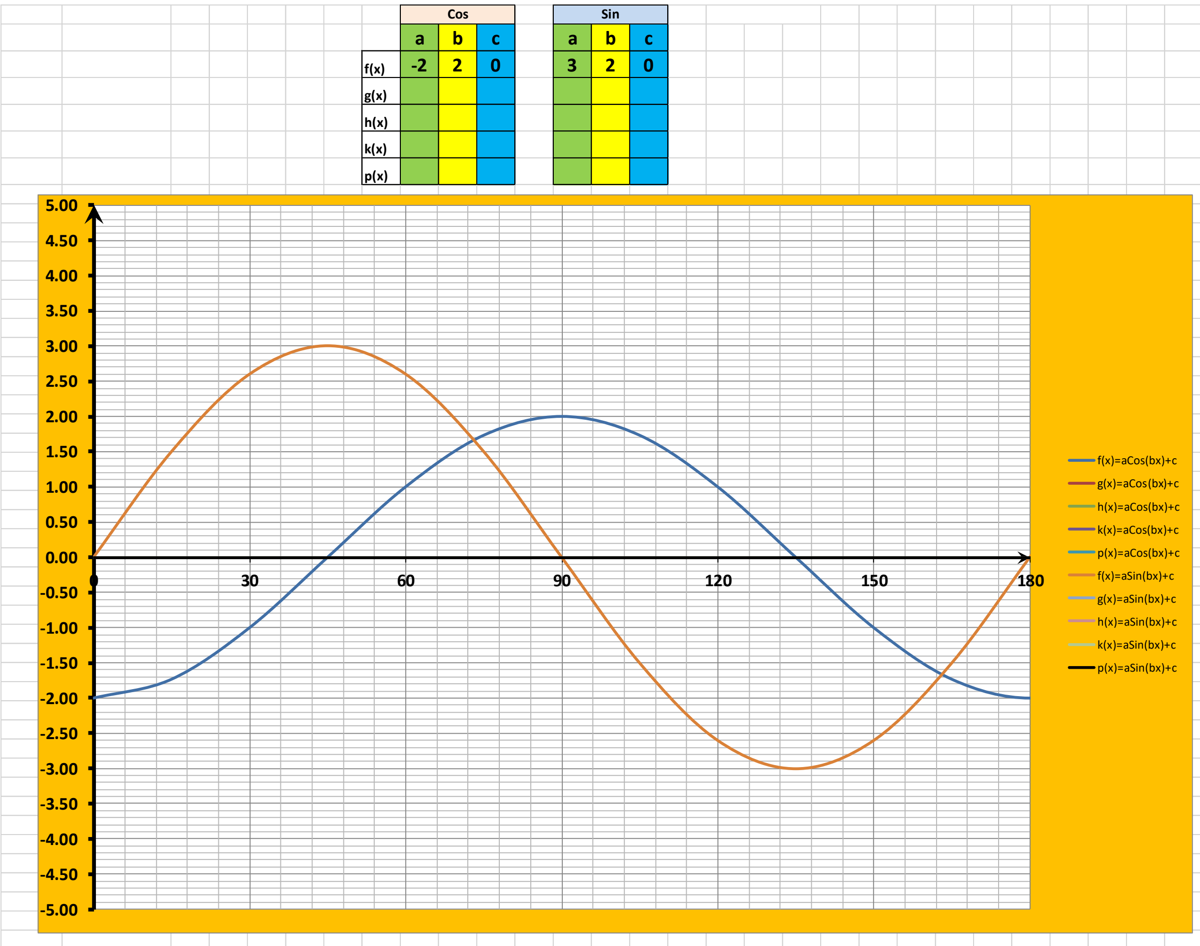


Figure 4. 27: Display for a = 3, b= 2, c= -2 and d =2

S8L1*: (notices something is not right)… But the period of the cosine function here is not 360^0…look (walking to the screen and tracing the graph)… from minimum to minimum is 180^0…and not 360^0… we must make d = 1.*

Learner S8L7 makes the suggested changes with the consent of the rest of the group and obtained the new display in Figure 4.28.

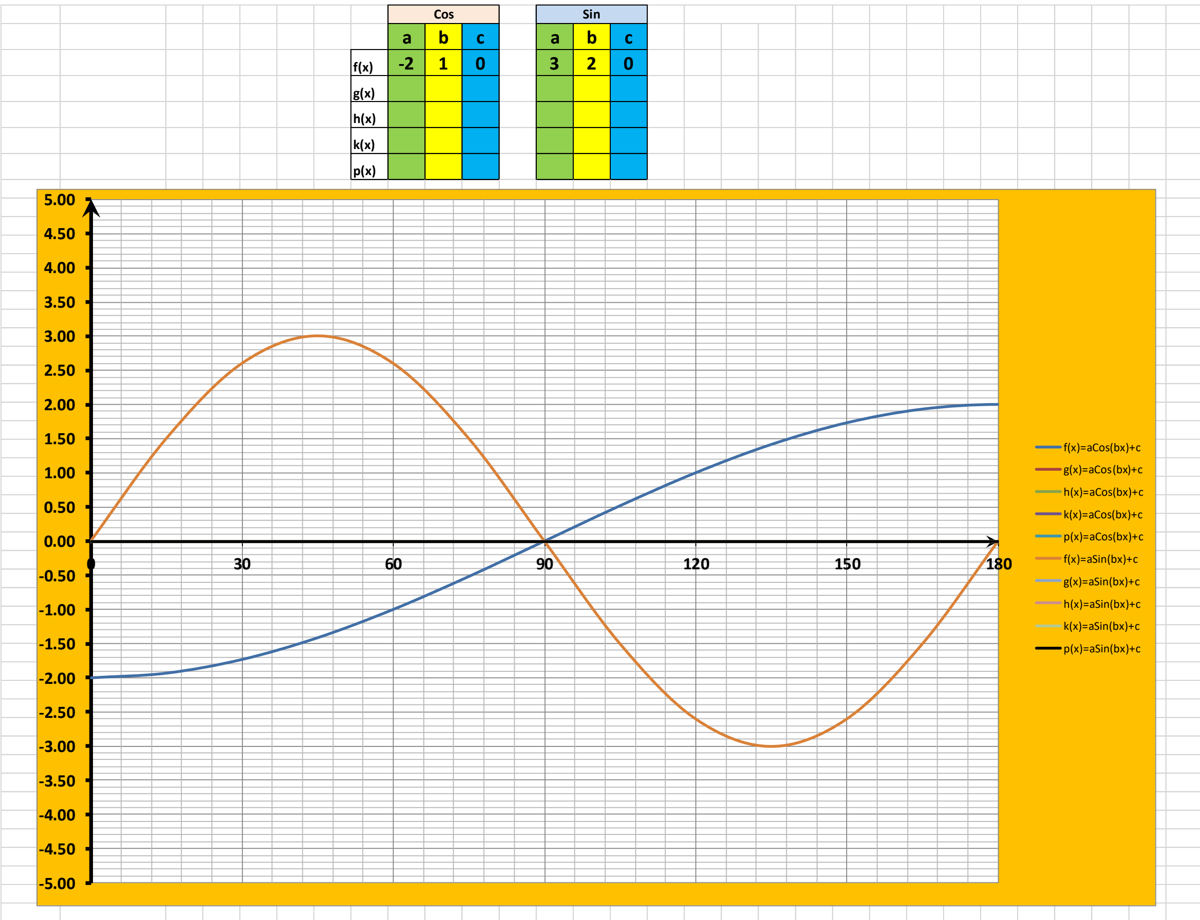


Figure 4.28: Final output matching the hypothesised sketches to the graphs of y = a sin bx and y = c cos dx

The learners all agreed that this was the correct model for the functions y = a sin bx and y = c cos dx.

The learners successfully solved parts (b), (c) and (d), but struggled with (e). The learners could not understand the new situation, and the teacher had to clarify by showing the movement of the y-axis and the implication of this shift on the domain of the new graph, as illustrated in Figure 4.29. Learners then continued to discuss the possible solution.

S8L13: *Now the new function is a cosine function because at x = 0, the function is not 0. The amplitude is still 3…and the period is still 180^0. So the new function is y = 3 cos 2x.*

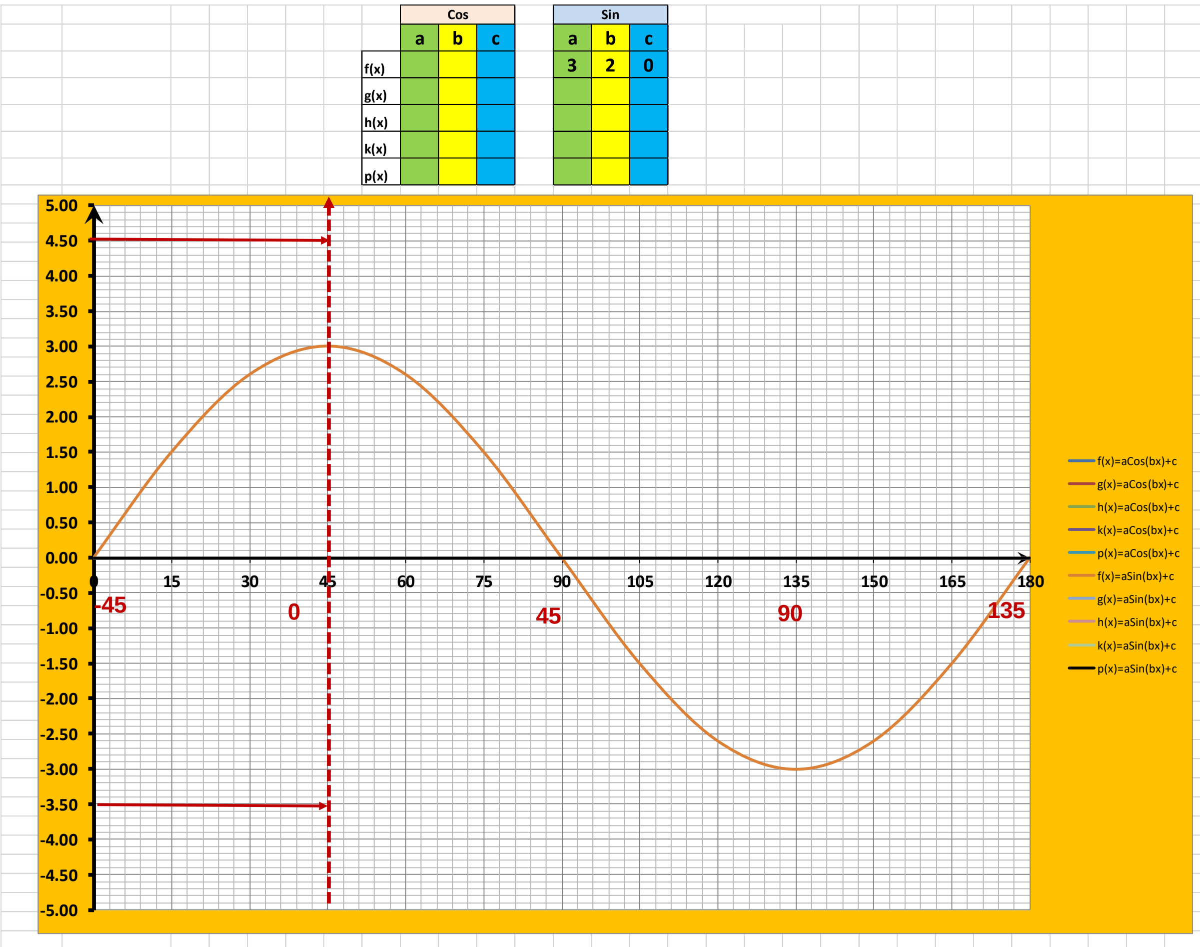


Figure 4. 29: Illustration of resultant graph after shifting the y-axis 45^0 to the right

The CAEMA output for this collaborative problem-solving process is shown in Figure 4.30, followed by an explanation of how the number of errors at each step were determined.

GROUP:	S8 (L1-L18)

Enter the level of the problem	4

STEP	Descriptors	H-value	Excel modelling marking grid		
			No of errors	Score	Comment(s)
EM1	Appropriately explores effects of changes to parameters of function on graphic output	1	0	5	Correct
EM2	Accurately hypothesises the graphic output of a new function	1	0	5	Although there was initially a sign error to the value of c, it was rectified after collaborative discusions.
EM3	Collects and enters appropriate data to match hypothesised and actual outputs	1	1	4	Learners could neither interpret nor visualise the scenario of part (e)
EM4	Correct conclusions drawn from modelling process	1	0	4	Correct conclusions obtained after teacher's assistance.

Total Score	18	L5
TIMSS Score	650	

Colour Score Card (CSC)		
(red)	L1 - L2	VERY LOW-LOW
(amber)	L3	INTERMEDIATE
(green)	L4-L5	HIGH-ADVANCED

Figure 4. 30: CAEMA output of collaborative solution process

The CAEMA tool requires a lot of concentration and time to use while observing the learners' work, sometimes requiring the user to render overt assistance to the learners. For this reason, the researcher could not use the CAEMA tool in all class groups, but relied on outcomes obtained by teachers, based on the training they had received during the EBM professional development. Each group's CAEMA scores were aggregated as a triangulation method to demonstrate concurrent validity in the measure of learners' conceptual understanding in comparison to the outcomes from the summative test. For example, in the CAEMA output above, the total score was 18 out of a possible 20, hence the conceptual understanding is rated at 90%. After the solution of all the 7 problems, the scores were aggregated to give a group percentage rating of conceptual understanding. The aggregated CAEMA scores were then compared to the aggregated summative test scores, using bivariate Pearson correlation analysis in SPSS. Table 4.22 shows the TIMSS levels obtained from the observed learning outcomes, using the CAEMA tool to assess the learners' EBM-assisted collaborative problem solving process.

Table 4. 22: CAEMA levels of observed learning outcomes for each class group

Teacher Code	S1T04	S1T11	S2T09	S2T10	S3T08	S4T07	S5T06	S6T03	S6T05	S7T02	S8T01
CAEMA TIMSS LEVEL	4	4	3	4	3	4	5	3	4	4	4

Table 4.23 shows the SPSS output for the correlation between the CAEMA and test scores.

Table 4. 23: Output of correlation between CAEMA and test scores for assessment of learners' conceptual understanding

Correlations			
		VAR00001	VAR00002
VAR00001	Pearson Correlation	1	.656*
	Sig. (2-tailed)		.028
	N	11	11
VAR00002	Pearson Correlation	.656*	1
	Sig. (2-tailed)	.028	
	N	11	11
* Correlation is significant at the 0.05 level (2-tailed).			

The fairly strong correlation (0.656) was significant at the 0.05 level ($p = 0.028$), showing that there was concurrent validity between the CAEMA and test measures of learners' conceptual undrerstanding.

4.2.5.2 Analysis of learners' summative test scores

The analysis of the learners' summative test scores gives a clearer and more meaningful indication of the learners' learning outcomes. Table 4.24 below is a summary of the learners' scores per school per teacher.

Table 4. 24: Summary of learners' summative assessment per school per teacher

POSSIBLE	10	4	14	14	20	19	5	10	10	10	18	134
ACTUAL	7	4	12	11	17	19	5	10	10	10	18	123
TEACHER CODE	S3T08	S4T07	S2T09	S2T10	S1T04	S1T11	S5T06	S6T05	S6T03	S7T02	S8T01	
1	79.2	100	87.5	83.3	95.8	83.3	95.8	83.3	54.2	79.2	83.3	
2	54.2	58.3	58.3	70.8	91.7	83.3	83.3	91.7	91.7	70.8	83.3	
3	95.8	83.3	83.3	70.8	70.8	70.8	66.7	79.2	66.7	79.2	79.2	
4	33.3	66.7	91.7	66.7	83.3	75	58.3	66.7	54.2	66.7	70.8	
5	70.8		70.8	87.5	79.2	91.7	91.7	100	83.3	83.3	70.8	
6	79.2		79.2	83.3	87.5	87.5		70.8	54.2	87.5	95.8	
7	83.3		54.2	83.3	70.8	66.7		79.2	100	66.7	95.8	
8			70.8	83.3	79.2	58.3		95.8	58.3	83.3	79.2	
9			66.7	83.3	58.3	66.7		70.8	95.8	70.8	91.7	
10			87.5	79.2	66.7	100		70.8	83.3	79.2	95.8	
11			70.8	66.7	83.3	95.8					70.8	
12			79.2		79.2	70.8					75	
13					83.3	58.3					83.3	
14					95.8	87.5					79.2	
15					66.7	70.8					79.2	
16					54.2	79.2					95.8	
17					58.3	58.3					91.7	
18						100					87.5	
19						54.2						
20												
MEAN SCORE	70.83	77.08	75.00	78.02	76.71	76.75	79.16	80.83	74.17	76.67	83.79	

The summative test scores were used as a reflection of the learners' level of conceptual understanding. The correlation between the teachers' observed TPACK (OT), from the lesson evaluation outcomes strongly correlated with learners' conceptual understanding (r=0.66). Although the correlation between teacher self-efficacy (TSE) and observed TPACK (OT) is moderate (r=0.4), the correlation between TSE and LCU was low (r=0.25) (See Appendix F). While the findings might suggest a strong relationship between OT and LCU, in reality, the existence of a strong correlation does not imply a causal relationship.

The analysis therefore proceeded to extrapolate the Partial Least Squares Structural Equation Modelling (PLS-SEM) involving inferential statistics that could test specific hypotheses of relationships between TPACK constructs and learners conceptual understanding of periodicity of trigonometry functions. Section 4.7 gives a detailed account of the PLS-SEM analysis and results.

4.2.5.3 Analysis of learners' interview responses on their EBM experience

In the absence of a control group to conduct a comparative rating of teachers' lesson observation outcomes, it is difficult to infer that the rating of teachers exposed to the EBM professional development would be higher than those who were not. Consequently, the researcher sought more clarity from the learners themselves on how they compared their traditional "chalk and talk" to their EBM learning experiences. A purposively selected sample of 11 learners, one from each class group per school, were interviewed to get this estimation on their EBM learning experiences. The sampling was done in a way that learners of all TIMSS level scores in the summative test were included. Computer-assisted qualitative data analysis (CAQDA) was employed through an Excel analysis tool, whose processing was premised on Guskey's (2000) model. The researcher assigned identification labels, **GL5**, **GL5^{-}** and **GL5^{R}** to the transcribed interview responses, associated with Guskey's level 5 on student learning outcomes. A **GL5** label was assigned to a positive statement, a **GL5^{-}** was assigned to a negative statement, while a **GL5^{R}** was assigned to a repeated statement, hence not considered in the CAQDA count. Only the number of positive and negative peceptions, respectively represented by the number of **GL5** and **GL5^{-}** labels, were counted and entered into the CAQDA template as shown in Figure 4.31. A total of 187 perceptions of the 11 interview respondents on learners' learning outcomes were recorded. The resultant processing assigned CAQDA level 5 (advanced) to the overall learners' perceptions of experienced learning outcomes. The following figure shows the processing output after entering a total number of 66 **GL5** labels of which 8 were negative perceptions, especially on the challenges of manipulating and interpretating the EBM applet for the tangent function.

Figure 4.31 shows the output of the computer-assisted qualitative data analysis on the learners' self-reported levels of conceptual understanding.

LEARNER CODES:		S1L1, S1L20, S2L10, S2L18, S3L4, S4L2, S5L5, S6L5, S6L7, S7L4, S8L12				
Guskey Level	Descriptors	Learners' perceived learning outcomes				
		H-value	Enter total number of mutually exclusive statements with GL5 labels	Enter number of negative statements with GL5 labels	CAQDA Rating	Comment(s)
GL5	Overall learners' perceptions of their learning outcomes	1	66	8	5	ADVANCED

Colour Score Card (CSC)		
(red)	L1 - L2	VERY LOW - LOW
(amber)	L3	MODERATE
(green)	L4-L5	HIGH - ADVANCED

Figure 4. 31: CAQDA processing output of learners' self evaluation of their level of conceptual understanding after EBM exposure

The Excel-based CAQDA processing functions in the same principle of "penalties for errors", that is used in the CAAPSA tool. The following samples of snippets illustrate how CAQDA was used to translate the learners' qualitative (narrative) responses on their EBM experiences and level of conceptual understanding to a numeric scale from 1 (very low) to 5 (advanced). For each snippet, the researcher scrutinised the responses to ensure that none of the statements repeated what was already stated.

Figures 4.32 to 4.34 are samples of snippets of learners' interview responses on their Excel-based modelling (EBM) learning experiences.

1. Briefly explain how the Excel modelling influenced your understanding of the topic of trigonometric functions and graphs in the Higher Level Mathematics curriculum?

Excel had really positively influenced my understanding of the topic of graphs. I learned how the graphs are affected by amplitudes and the period. I had learned that if I changed the amplitude to be a negative value in a sine graph, the graph will turn upside down. And ... the graph is affected by the period and oscillation. Excel has improved my knowledge on Trigonometry graphs GL5 GL5 GL5 GL5

2. Did the teacher demonstrate the Excel modelling or did you also get an opportunity to explore on your own, the connections between functions and graphs?

In the beginning, she demonstrated and then showed us how to use it. In the end she gave us time to sit with the programme and we used it on our own during the study period. It was easy to use because you only have to change the number on the keyboard, then you see the visual picture of the graph just changing right before your eyes. GL5

3. How do you see your preparedness for the final NSSCH Mathematics examination, in particular with respect to the questions on trigonometric functions?

It is going well and I am more confident in this topic. GL5

4. Did you at any point work as a group and if so, briefly explain the activities you did?

Yes, we worked on many tasks as a group, helping each other and discussing our solutions.

5. Would you recommend this type of learning with technology to other learners and why?

I would very much recommend it because it's better to see what you are learning. People get a better understanding of something they see visually than just theory. It is a way of visualising Maths.

6. What were the challenges you encountered in the TPACK-IBL lessons?

N/A.

7. What else do you think is important for us to know which is important in supporting learners to learn with technology?

Just try and do more, especially on logarithms, such programs really help us in our school work.

Figure 4. 32: Snippet of analysis of learner S1L1's interview responses on EBM learning experience

There were six (6) recorded statements, namely:

1. *"Excel had positively influenced my understanding on the topic of graphs"*
2. *"I learned how the graphs are affected by the amplitudes, the period"*
3. *If I changed the amplitude to be a negative value in a sine graph, the graph will turn upside down"*
4. *"Excel has improved my knowledge on trigonometry graphs"*
5. *"It was easy to use because you only have to change the number on the keyboard, then you see the visual picture of the graph just changing right before your eyes"*
6. *"I am more confident in this topic"*

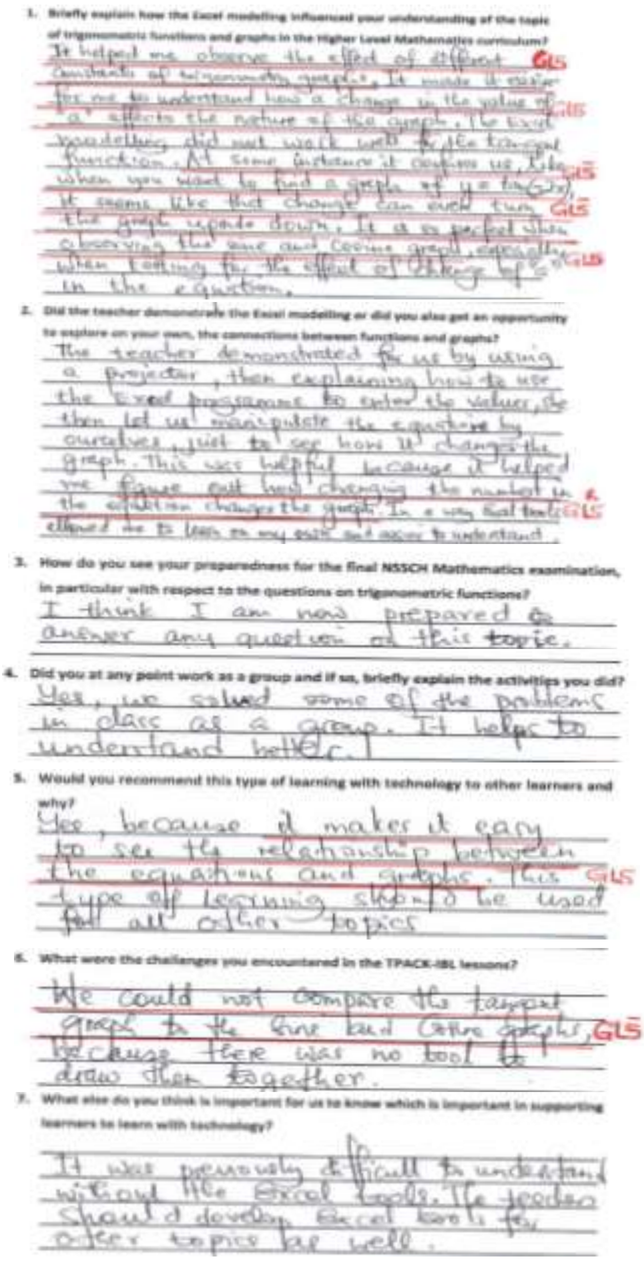
Although the second and third statements are related, none is a repetition of the other. The third statement amplifies the assertion of the second statement, by giving specific details with respect to how graphs were affected by changes in the sign of ***a*** in *a *trigfunction(bx)+c*. The third statement in itself demonstrates the learner's depth of conceptual understanding. On the other hand, the fourth statement is a repetition of the first statement, hence is assigned the label $GL5^R$ and consequently not counted. As a result, in this learner's response we count three positive GL5 statements. The fifth statement reflects the learner's ease of use of the technology, and that the hands-on exploratory approach offered better insight into the mathematical connections involved. The last statement reflects a perception that EBM increases learner confidence in the problem-solving process. This particular learner attained TIMSS level 5 (Advanced) in the summative test.

The next figure 4.33 illustrates an example of responses of a learner S1L20 that have negative **GL5** statements.

There are eight (8) relevant recorded statements, namely:

1. *"It helped me observe the effect of different constants of trigonometry"*
2. *"It made it easier for me to understand how a change in the value of* ***a*** *affect the nature of the graph"*
3. *"The Excel modelling did not work for the tangent function"*
4. *"At some instances, it confuses us, like when you want to find a graph of y = tan(-2x), it seems like that change even turns the graph upside down"*

5. *“It is so perfect when observing the sine and cosine graph….”*
6. *“It helped me figure out how changing the numbers in the equations changes the graph”*
7. *“It makes it easy to see the relationship between the equations and graphs”*
8. *“We could not compare the tangent graph to the sine and cosine graphs”*



1. Briefly explain how the Excel modelling influenced your understanding of the topic of trigonometric functions and graphs in the Higher Level Mathematics curriculum?

It helped me observe the effect of different constants of trigonometry graphs. It made it easier for me to understand how a change in the value of "a" affects the nature of the graph. The Excel modelling did not work well for the tangent function. At some instance it confuses us, like when you want to find a graph of y = tan(2x), it seems like that change can even turn the graph upside down. It is so perfect when observing the sine and cosine graph, especially when looking for the effect of change of [illegible] in the equation. GLS GLS GLS GLS GLS

2. Did the teacher demonstrate the Excel modelling or did you also get an opportunity to explore on your own, the connections between functions and graphs?

The teacher demonstrated for us by using a projector, then explaining how to use the Excel programme to enter the values, she then let us manipulate the equations by ourselves, just to see how it changes the graph. This was helpful because it helped me figure out how changing the number in the equation changes the graph. In a way that tool allowed me to learn on my own and easier to understand. GLS

3. How do you see your preparedness for the final NSSCH Mathematics examination, in particular with respect to the questions on trigonometric functions?

I think I am now prepared to answer any question of this topic.

4. Did you at any point work as a group and if so, briefly explain the activities you did?

Yes, we solved some of the problems in class as a group. It helps to understand better.

5. Would you recommend this type of learning with technology to other learners and why?

Yes, because it makes it easy to see the relationship between the equations and graphs. This type of learning should be used for all other topics GLS

6. What were the challenges you encountered in the TPACK-IBL lessons?

We could not compare the tangent graph to the sine and cosine graphs, because there was no tool to draw them together. GLS

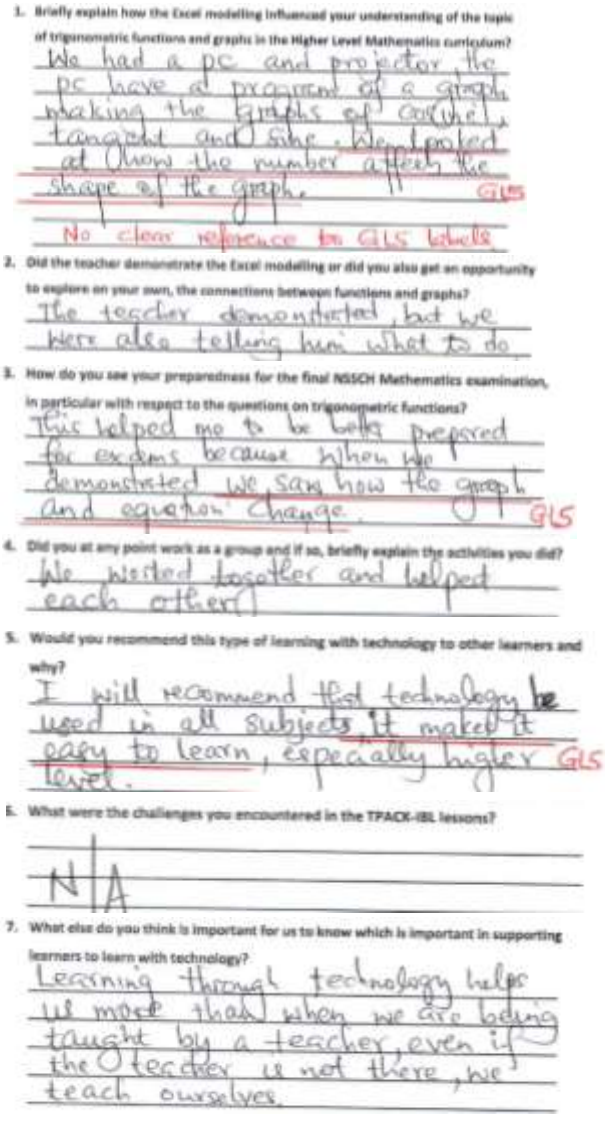
7. What else do you think is important for us to know which is important in supporting learners to learn with technology?

It was previously difficult to understand without the Excel tools. The teacher should develop Excel tools for other topics as well.

Figure 4.33: Snippet of analysis of learner S1L20’s interview responses on EBM learning experience

The third, fourth and eighth statements are negative perceptions of the EBM approach, hence each is labelled GL5-. Counting, there is a total of 8 statements, of which three are negative, four positive and one repeat. When entering the perception count into the CAQDA tool, only seven labels are considered, with the exclusion of the repeat label.

Figure 4.34 shows learner S3L4's responses. This particular learner had the lowest score of 33.3% (TIMSS level 2) in the summative test.


1. Briefly explain how the Excel modelling influenced your understanding of the topic of trigonometric functions and graphs in the Higher Level Mathematics curriculum?

We had a pc and projector, the pc have a programme of a graph making the graphs of cosine, tangent and sine. We looked at how the number affects the shape of the graph. GL5

No clear reference to GL5 labels

2. Did the teacher demonstrate the Excel modelling or did you also get an opportunity to explore on your own, the connections between functions and graphs?

The teacher demonstrated, but we were also telling him what to do.

3. How do you see your preparedness for the final NSSCH Mathematics examination, in particular with respect to the questions on trigonometric functions?

This helped me to be better prepared for exams because when he demonstrated we saw how the graph and equation change. GL5

4. Did you at any point work as a group and if so, briefly explain the activities you did?

We worked together and helped each other.

5. Would you recommend this type of learning with technology to other learners and why?

I will recommend that technology be used in all subjects. It makes it easy to learn, especially higher level. GL5

6. What were the challenges you encountered in the TPACK-IBL lessons?

N/A

7. What else do you think is important for us to know which is important in supporting learners to learn with technology?

Learning through technology helps us more than when we are being taught by a teacher, even if the teacher is not there, we teach ourselves.

Figure 4. 34: Snippet of analysis of learner S1L2's questionnaire response on EBM learning experience

The three (3) relevant recorded statements in the learner's interview response were:

1. *"We looked at how the number affects the shape of the graph"*
2. *"We saw how the graph and equation change"*
3. *"It makes it easy to learn"*

This learner attained TIMSS level 2 (Low) in the summative test, and seems to also struggle to narrate his experiences of learning with the EBM instructional approach.

The qualitative analysis was also used as a triangulation strategy to compare the learning outcomes from the lesson observation reports, to learners' own perceptions of their EBM experience. Generally, the learners' perception is that the EBM approach was helpful in enhancing their understanding of the connections between graphs and fucnctions as shown by the high CAQDA level obtained in Figure 4.31.

4.2.5.4 ANALYSIS OF PLS-SEM OUTCOMES

The analysis of the relationships between the TPACK latent variables, and relationship between independent and dependent variables in the formulated hypotheses H1 to H13 was evaluated using the Partial Least Squares Structural Equation Modelling (PLS-SEM) approach. The advantage of using PLS-SEM is that indicators with categories, ordinal scales, intervals, or ratios can be used on the same model, and does not require a large number of samples. The PLS-SEM approach is suitable for prediction purposes, but can also be used to explain whether there is a relationship between latent variables (Ghozalli, 2014; Lattan & Ramli; Wong, 2013). The data analysis commenced by summing the total score on the teachers' post-EBM implementation TPACK survey questionnaire and answers on the learners' summative test scores for each indicator of research variables. Furthermore, the relationship between the teachers' TPACK level and learners' conceptual understanding scores was established to explain the influence of the Excel-based modelling professional development of periodicity of trigonometric functions. Internal reliability of the seven constructs was first established through high Cronbach alpha values for all constructs: TK ($\alpha = 0.707$), CK ($\alpha = 0.926$), PK ($\alpha = 0.777$), PCK ($\alpha = 0.956$), TCK ($\alpha = 0.978$), TPK ($\alpha = 0.959$), and TPACK ($\alpha = 0.979$).

4.2.5.4.1 Assessment of measurement model

The measurement model assessed the reliability and validity of the measures. Hair et al. (2010) suggest that the commonalities should be greater than 0.45 for the measures to be considered valid. Except for two loadings, all the outer loadings in the current model yielded commonalities between 0.546 and 0.999, as recommended by Hair et al. (2010).

4.2.5.4.2 Assesement of structural model

The research evaluates the structural model based on the structural path coefficients. Figure 4.35 illustrates the variance (R^2) in the endogenous variables and the path coefficients for the direct and indirect relationship between TPACK constructs. The R^2 scores assess the strength of the predictive model.

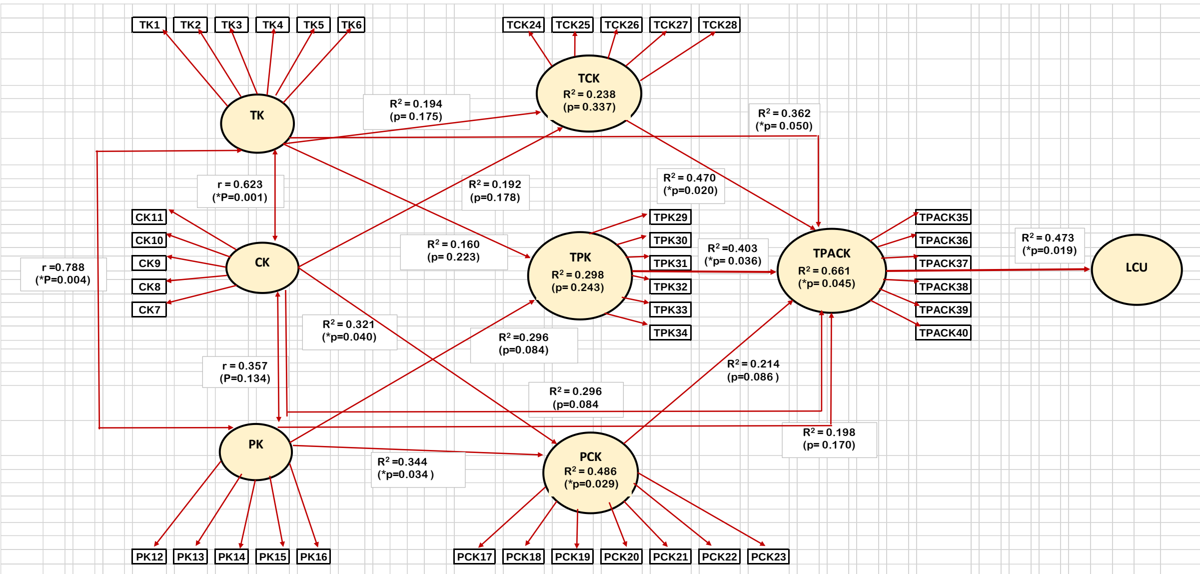


Figure 4. 35: Structural model displaying the paths between the constructs

Based on the outcomes of the PLS-SEM model paths significance levels and R^2 values, for the formulated hypotheses we conclude as follows:

H1: There is a significant relationship between Technological Knowledge (TK) and Technological Content Knowledge (TCK)

$H1_0$: There is no significant relationship between TK and TCK

Conclusion: Since p = 0.175 > 0.05, we accept the null hypothesis, which means TK does not significantly influence the development of TCK.

H2: There is a positive relationship for Technological Knowledge (TK) on Technological Pedagogical Knowledge (TPK)

$H2_0$: There is no significant relationship between TK and TPK.

Conclusion: We conclude that there is no positive relationship between TK and TPK, because p= 0.223>0.05.

H3: There is a positive relationship for Content Knowledge (CK) on Technological Content Knowledge (TCK)

$H3_0$: There is no positive relationship between CK and TK

Conclusion: Since $p = 0.178 > 0.05$, we accept the null hypothesis, which means there is no positive relationship between CK and TK.

H4: There is a positive relationship for Content Knowledge (CK) on Pedagogical Content Knowledge (PCK)

$H4_0$: There is no significant relationship between CK and PCK

Conclusion: Since $p = 0.040 < 0.05$, we reject the null hypoythesis, which means there is a significant relationship between CK and PCK, with CK responsible for about 32% of the variation in PCK.

H5: There is a positive relationship for Pedagogical Knowledge (PK) on Technological Pedagogical Knowledge (TPK)

$H5_0$: There is no significant relationship between PK and TPK

Conclusion: Since $p = 0.084 > 0.05$, we accept the null hypothesis, implying that there is no sigfnificant relationship between PK and TPK.

H6: There is a positive relationship for Pedagogical Knowledge (PK) on Pedagogical Content Knowledge (PCK)

$H6_0$: There is no significant relationship between PK and PCK

Conclusion: Since $p = 0.034 < 0.05$, we reject the null hypothesis. Therefore we conclude that there is a strong relationship between PK and PCK with PK responsible for about 34% change in TPACK.

H7: There is a positive relationship for Technological Content Knowledge (TCK) on Technological and Pedagogical Content Knowledge (TPACK)

$H7_0$: There is no significant relationship between TCK and TPACK

Conclusion: We conclude that there is a significant relationship beytween TCK and TPACK, since $p = 0.020 < 0.05$. We also conclude that TPK contributes 47% of the change in TPACK.

H8: There is a positive relationship for Technological Pedagogical Knowledge (TPK) Technological and Pedagogical Content Knowledge (TPACK)

$H8_0$: There is no significant relationship between TPK and TPACK

Conclusion: Since $p = 0.036 < 0.05$, we reject the null hypothesis, meaning that there is a positive relationship between TPK and TPACK, with TPK responsible for 40% of the growth in TPACK.

H9: There is a positive relationship for Technological Content Knowledge (PCK) on Technological and Pedagogical Content Knowledge (TPACK)

$H9_0$: There is no significant relationship between PCK and TPACK

Conclusion: Since $p = 0.086 > 0.05$, we accept the null hypothesis. Therefore, we conclude that there is no significant relationship between PCK and TPACK.

H10: There is a positive relationship for Technological and Pedagogical Content Knowledge (TPACK) on Learners' Conceptual Understanding (LCU)

$H10_0$: There is no significant relationship between TPACK and LCU

Conclusion: Since $p = 0.019 < 0.05$, there is a significant relationship between TPACK and LCU with TPACK accounting for 47% of variation in LCU.

H11: There is a direct positive relationship for TK on TPACK

$H11_0$: There is no significant relationship between TK and TPACK

Conclusion: Since $p = 0.05$, we conclude that there is a significant relationship between TK and TPACK with TK accounting for 36% growth in TPACK.

H12: There is a direct positive relationship for CK on TPACK

$H12_0$: There is no significant relationship between CK and TPACK

Conclusion: Since $p = 0.084 > 0.05$, there is no direct relationship for CK on TPACK.

H13: There is a direct positive relationship for PK on TPACK

$H13_0$: There is no significant relationship between PK and TPACK

Conclusion: Since $p = 0.170 > 0.05$, we conclude that there is no direct relationship between PK and TPACK.

4.3 Summary

In Figure 4.35, CK ($R^2 = 0.321$) and PK ($R^2 = 0.344$) positively and directly affect PCK and explain 49% of TCK variation. In addition, direct and positive effects of TCK ($R^2 = 0.470$), TPK ($R^2 = 0.403$) and PCK ($R^2 = 0.293$) on TPACK are observed, accounting for 66% of the change in TPACK. TCK is the variable that affects TPACK the most. According to this result, TCK plays critical role in influencing teachers' technology integration. TPK is the second variable that has the greatest effect on TPACK variation after TCK. These two findings suggest that teachers' ability to integrate technological knowledge with content and pedagogical knowledge has an impact on TPACK development. PCK has less effect on on the teachers' TPACK development compared to the other two constructs. The next chapter offers a comprehensive discussion of the results in terms of the research questions formulated at the beginning of this study.

CHAPTER FIVE
DISCUSSION OF THE RESULTS

This section discusses the results in terms of the research questions formulated at the beginning of this study. The discussion of the results focuses on the influence of the Excel-based modelling (EBM) teacher professional development on the enhancement of the learners' conceptual understanding of periodicity of trigonometric functions.

5.1 HOW DO TEACHERS' PERCEPTIONS OF THEIR STATE OF TPACK CHANGE AFTER PARTICIPATING IN THE EXCEL-BASED MODELLING TEACHER PROFESSIONAL DEVELOPMENT?

The main focus of this component of the study was to examine the changes in teacher efficacy after exposure to the EBM professional development, and to identify teachers' attributions of these variations in their perceptions of self-efficacy. This discussion of the results connects to the research questions formulated at the beginning of this study. The results clearly show how teachers perceived the Excel-based modelling (EBM) teacher professional development as having changed their self-efficacy, especially in the solution of mathematical problems in the collaborative problem-solving activity.

The discussion of the results corresponding to this research question is based on data obtained from Table 4.15 and Guskey's level 2 items of the 50 item post-EBM implementation evaluation questionnaire (Table 4.17), and group feedback on their experience with the EBM enhanced collaborative problem solving process.

5.1.1 Comparison of differences between pre and post EBM teacher professional development self-efficacy mean ratings

The results from Table 4.15 show that the highest growth, calculated as the difference between the mean pre and post EBM intervention teachers' self-efficacy rating, was in TCK (1.53), followed by TK (1.06), and TPK (0.85).

The growth in TK, TCK and TPK self-efficacy makes sense as the intervention focused more on the use of technology due to limitations in time. This observation is supported by Hammond et al. (2011), who concluded that technological knowledge is undoubtedly one of the foundations of ICT integration. Graham et al. (2009) also suggested that raising the teachers' technological skills equally increases the likehood of them boosting their confidence in other knowledge domains.

Perceived self-efficacy with respect to use of technology has been confirmed as a critical factor in decisions about technology integration in teachers' practices (Hill, Smith, & Mann, 1987). This is corroborated by Gibson and Dembo (1984), who developed an instrument to measure teachers' sense of efficacy for teaching. Teachers' self-efficacy beliefs have been linked to classroom achievement gains (Dembo & Gibson, 1985) and have suggested a positive relationship to change in individual teacher practice (Smylie, 1988), ratings of lesson presentation, classroom management and questioning (Saklofske, Michayluk, & Randhawa, 1988) and teacher success in implementing innovative programmes (Stein & Wang, 1988).

Taken together, the current study and the studies reviewed above point towards teachers' self-efficacy beliefs as useful indicators of the likelihood of success at technology integration. Certainly they provide sufficient reason to undertake further investigations in this area and to consider what approaches to teacher education and professional development might be effective in enhancing self-efficacy for teaching with technology.

The results from Table 4.15 indicate strong evidence that the teachers perceived overall growth in their TPACK levels, with all the TPACK mean differences (md) recording positive values, ranging from 0.27 to 1.53.

The highest reported self-efficay growth was in TCK (md=1.53), TK (md=1.06), TPK (md=0.85) and TPACK (md=0.75). The mean ratings of participants' learning on the post-EBM instruction evaluation questionnaire apparently validate the perceived growth in TK, TCK, TPK and TPACK, given that all the questionnaire statements in Table 4.17 are linked to the EBM technology experience, and are all highly rated between 4.182 (Agree) to 4.636 (Strongly Agree). The findings of this study are corroborated by an Estonian study on the impact of a professional development programme on in-service teachers' TPACK (Lehiste, 2015), which indicated the most significant growth in TK, TPK TPACK. In both studies TK and TPK are registered the most significant growth.

Although there are differences in the third constructs between the two studies, it is noticeable that both TCK and TPACK are technological forms of knowledge. The difference in the growth of the third constructs in the current study and the Estonian study (TCK and TPACK respectively) could be attributed to the fact that whereas the current study involved qualified and experienced teachers who had already been in service for at least three years, the latter was a study based on novice teachers, whose pedagogical knowledge (PK) was still developing.

Further, it is commonly acknowledged that teachers' self-confidence and self-efficacy influence their use of technology and that individuals who are intentional about allocating time for technology use have positive self-confidence and self-efficacy (Oral, 2008; Rugayah, Hashim & Wan, 2004). It is necessary to study teachers' and prospective teachers' judgments about their efficacy capacities in addition to analysing international standards for developing teachers' and prospective teachers' TPACK efficacy.

5.1.2 Group feedback on EBM enhanced collaborative problem solving process

The teachers, in their group feedback after the EBM teacher professional development, indicated that they could easily solve the problems that they had struggled with in the collaborative problem solving process, for example problem 4. They attributed their increased self-efficacy in the problem solving process to the fact that EBM enabled them to discover important relationships between the trigonometric graphs and their algebraic representations.

Furthermore, the participants' feedback on the EBM enhanced collaborative problem solving process indicates that the Excel-based modelling teacher professional development facilitated increased understanding and representation of trigonometric functions in various ways, confirming O'Callagan's (1992) observation that spreadsheet graphing applications help improve visualisation skills and the ability to make connections between graphical and algebraic representations (See section 4.2.3.2).

For example, the group feedback on the modelling of the sine function (section 4.2.3.2), through a collaborative EBM inquiry based approach, portrays that the EBM graphing applets enabled them to successfully deduce the values of $a = 1, b = 1,$ and $c = 0$, from the graphic representation of $f(x) = \sin x$. When the group was asked how they would deduce the amplitude and the period from the graph, they correctly established the algebraic, geometric and graphic representations linked to periodicity properties of the given graph. This suggests that the EBM applets enhanced their integration of algebraic, geometric and graphic reasoning. Through exploration with different values of a, b, and c, they discovered more relationships, for example, making a = -1, yields to a reflection of f(x) in the x-axis. The EBM exploratory approach thus facilitated the development of self-directed inquiry skills.

Ostensibly, some previous studies have also established that although there are several factors that contribute to an educator's lack of self-efficacy, one must certainly be a lack of awareness and expertise to overcome learners' unique learning difficulties (Mizell, 2008). As a result, a correlation between professional development and teacher self-efficacy may exist. Teacher self-efficacy, according to Bray-Clark and Bates (2003), is a primary source of teacher effectiveness, and should be a central focus of teacher professional development initiatives.

Mizell (2008, p.6), reported the following about the impact of professional development on teacher efficacy:

> Professional development that is of high quality is an important tool for elevating the teachers' self-efficacy. The greater their self-efficacy, the more they know and the more they can apply that expertise to learners' real world learning challenges.

There are, however, different viewpoints on the level of professional development provided to teachers by school systems. Research shows that when teachers are provided with high quality professional development opportunities and actively engage in them, their self-efficacy increases (Ross & Bruce, 2007). Correspondingly, the collaborative problem-solving feedback by teachers confirms that the EBM teacher professional development enhanced teachers' modelling skills in the four Excel-based modelling steps used in the collaborative problem solving activity, namely; (1) Identify the problem, (2) Hypothesise the solution, (3) Collect data and test against hypothesis, and (4) Draw conclusions (See section 2.6.2).

5.1.3 Feedback on Guskey Level 2 questionnaire items

These preliminary results had several limitations which could affect the objectivity of their interpretation. Firstly, the duration of the EBM teacher professional development programme was limited to one (1) week. The TPACK constructs may not necessarily develop at the same time and in the same way, hence TPACK should be continuously examined at various phases, throughout the duration of the training programme. Secondly, self-assessment reports may be subjective as participants tend to respond in ways that reflect positively on their knowledge and skills. Finally, since all of the participants were experienced higher level mathematics teachers, they might have been more confident in their level of content and pedagogical knowledge, especially as most of the teachers rated their content and pedagogical knowledge very highly.

5.2 HOW DOES THE EXCEL-BASED MODELLING TEACHER PROFESSIONAL DEVELOPMENT IMPACT THE TEACHERS' SELF-EFFICACY IN TEACHING PERIODICITY OF TRIGONOMETRIC FUNCTIONS?

The discussion of the results corresponding to this research question is based on data obtained from the pre- and post-EBM teacher professional development survey questionnaires (Appendices B and D). The effect size of the EBM intervention was determined using Cohen's **d** and Hedge's **g** formulae for calculating effect sizes of interventions.

The large Hedges' **g** values from Table 4.16 show a significant growth of teachers' TPACK self-efficacy after the EBM teacher professional development. The largest effect sizes were observed in teachers' TK (g=2.55), TCK (g = 2.41), and TPK (g = 1.93). The least growth was observed in PK (g=1.02), PCK (g=1.10), TPACK (g=1.20) and CK (1.38).

Consistent with Bandura (1997), the results of this study show that through additional training to supplement the teaching experience, the efficacy of teachers as a whole has undergone positive changes. This finding is consistent with some previous studies (Brousseau, Book and Byers, 1988; Ghaith and Yaghi, 1997; Green-Wood, Oléjnik and Parkay, 1990; Klassen and Chiu, 2010; Wolters and Daugherty, 2007).

Tate (2009) and Douglas et al. (2004) support the notion that teachers feel more confident with integrating technology into their classroom practices after participating in professional development. According to self-efficacy theory, mastery experience could be the source of self-efficacy, which includes teachers' mastery of cognitive content and pedagogical knowledge (e.g., Bautista & Boone 2015).

Therefore, the findings suggest that the EBM professional development programme provided mathematics teachers with teaching experiences that strengthened their self-efficacy. Teachers with high self-efficacy levels are more open to new ideas, show greater willingness to try new teaching methods, design and organise their classes better, and are more enthusiastic and satisfied with their teaching (Allinder, 1994; Ashton, 1985; Bamburg, 2004; Guskey, 1998; Tschannen-Moran &Woolfolk Hoy, 2001).

The findings of this study are important in supporting the advocacy towards strengthening teacher professional development initiatives, given that teachers' self-efficacy is related to their performance, commitment, persistence, and motivation in implementing reform-oriented instructional practices (Gabriele & Joram 2007; Ross & Bruce 2007). Teachers with high self-efficacy are more likely to have higher pedagogical competencies of adopting effective teaching strategies (Goddard et al. 2004). From this perspective, the higher the teachers' perceived self-efficacy, the higher their self-directedness and motivation.

5.3 HOW EFFECTIVE WERE THE TEACHERS IN THE IMPLEMENTATION OF THE EXCEL-BASED MODELLING INSTRUCTIONAL PRACTICE?

The data related to this research question was gathered through the checklist lesson observation schedule. The rationale for using lesson observation scores rather than self-reported scores was to avert the limitations concommitant with teacher subjectivity, discussed in section 5.1. The results from Table 4.21 show a high correlation between the self-reported ratings and lesson observation ratings ($r = 0.6$), hence the lesson observation scores could be taken as a reliable measure of teacher efficacy in the implementation of the EBM instruction.

The lesson observation scores from Table 4.21 indicate that the teachers' highest TPACK knowledge rating was in PCK (4.182) and TCK (4.182), followed by CK (4.109) and TPK (4.106). The fact that PCK and TCK are jointly the highest, above the rest of the constructs, suggests that the technology integration process should be grounded on both pedagogical and content knowledge.

Lately, there has been a shift of technology integration models from technology-focused models to pedagogy-focused ones. It is understood that while technology-focused models aim to enhance teachers' knowledge and skills for the use of technology, pedagogy-focused models aim to link teachers' knowledge with pedagogical knowledge throughout their classroom practices (Baran & Uygun, 2016; Yurdakul, 2011). The results of the current study suggest that successful technology integration depends on the robust development of teachers' knowledge and skills in all the three basic domains of content, pedagogy and technology.

5.4 WHAT ARE THE LEARNERS' PERCEPTIONS OF LEARNING PERIODICITY OF TRIGONOMETRIC FUNCTIONS THROUGH THE EXCEL-BASED MODELLING INSTRUCTION?

The data linked to this research question was gathered from the the semi-structured learner interviews. Based on the Computer Aided Qualitative Data Analysis (CAQDA) output in Figure 4.31, the EBM instruction strongly influenced the learners' conceptual understanding of periodicity of trigonometric functions (Level 5).

Learners were convinced that the use of the EBM inquiry-based learning approach facilitated an environment in which they collaboratively constructed their own knowledge about the phenomena studied. The EBM inquiry provided a platform to explore the relationships between the algebraic and graphic representations of trigonometric functions. The learners thus managed to create meanings about periodicity of trigonometric functions and explore other crtical properties like symmetry.

The learners' rationale for these positive perceptions are supported by Zengin et al. (2012), who concluded that computer assisted mathematics education is more effective in learners' learning than traditional approaches. The Excel-based modelling approach, according to the majority of the learners, helped them to:

1. Recognise the gaps in their knowledge of periodicity of trigonometric functions;
2. Control their own learning (metacognition) by obtaining immediate feedback in the exploration of the connections between graphic and algebraic representations of trigonometric functions; and
3. Increase their motivation, as the process provided a new learning experience suffused with fun.

According to Güven (2000), learners were provided an opportunity to make assumptions, test and explore hypotheses and relations through the EBM modelling process. The result is similar to the findings of the experimental study by Zengin et al. (2012), in which they used GeoGebra in the teaching of the concept of periodicity of trigonometric functions. According to the results of this study, with the aid of technology, it was easier for learners to navigate between abstract, visual and concrete concepts of trigonometric functions than they had been in the traditional approaches.

The analysis of the learners' interview responses, transcribed in snippets in Figures 4.31 to 4.34, all contain common statements like, "EBM made it easy to learn periodicity of trigonometric graphs", "EBM made it easy to see the relationship between the equations and graphs", and "EBM improved my knowledge on trigonometry graphs". In addition, the CAQDA output of the learners' self-evaluation of their level of conceptual understanding after exposure to EBM instruction was TIMSS level 5 (Advanced).

The responses of the learners to question 2 of the interview schedule (See Figures 4.32 and 4.34), fit the description of inquiry-based learning (IBL) as the approach that was used for instruction. Innately, the learners attribute their self-directed learning skills to IBL, through the use of the EBM exploratory applets. The learners' perceptions are supported by Trna et al. (2012), who described the inquiry learning model as learning that requires learners to solve problems through investigation activities that increase learners' comprehension of mathematical concepts, operations and relations independently.

Sanjaya (2006, on the other hand, sees inquiry learning as a set of learning activities that emphasise the process of thinking objectively and analytically in order to seek and find their won solution to the problem at hand. The answers of learners S1L1 and S1L20 to question 2 support this viewpoint (see Figures 4.32 and 4.33). Also, according to Sanjaya (2006), inquiry learning is based on the premise that humans have an inherent desire to discover their own knowledge. The primary goal of inquiry learning is to assist learners in developing academic discipline and thinking skills through asking questions and seeking answers through a heightened sense of curiosity.

Learners are taught to share their ideas and discover their own insights that can be used to solve problems using the inquiry learning process. The use of the EBM inquiry learning method should reduce the learners' reliance on the teachers relative to understanding the periodicity of trigonometric functions, and increase their participatory involvement in class (Soewarso, 2000).

In a related review, Kepceoglu and Yavuz (2016) evaluated the influence of trigonometric functions and found that learners who participated in GeoGebra-assisted lessons outperformed those who received conventional instruction. An explanation for this difference, according to Ross et al. (2011), may be that deep comprehension of trigonometry inaugurates the ability to navigate between abstract, visual and concrete representations of mathematical objects, and learners are especially challenged by their inability to formulate and transpose algebraic expressions.

Furthermore, Ibrahim and Llyas (2016) investigated the impact of GeoGebra on the teaching of periodicity concepts in trigonometric functions. A quasi-experimental study was performed, in which 36 tenth-grade high school learners took part. Learners in both the experimental group and control groups were given five questions to answer after 15 days of instruction, which were evaluated descriptively. According to the study's results, GeoGebra-assisted mathematics instruction was verified as more successful than conventional methods of teaching mathematics.

Similarly, the results of the current study indicate that when learners are given the opportunity to explore mathematical concepts, operations and relationships on their own, they develop their own understanding. The importance of using the EBM method is that it improves learners' practical understanding of trigonometric function periodicity. The technology-supported inquiry methodology, which integrates algebraic, geometric and graphic logic, can be effectively applied to any other topic in the mathematics curriculum.

5.5 WHAT IS THE INFLUENCE OF TEACHERS' TPACK DEVELOPMENT ON THE LEARNERS' CONCEPTUAL UNDERSTANDING OF PERIODICITY OF TRIGONOMETRIC FUNCTIONS?

The outcomes of the Partial Least Squares Structural Equation Modelling (PLS-SEM) technique provided the basis for the discussion corresponding to this research question. As a result of the analysis of the data obtained, a model explaining TPACK at 66% level was created. In e other studies that investigated the relationships between

TPACK sub-dimensions, TPACK's level of explanation is between 54% and 76% (Günbatar et al., 2017; Çelik et al., 2014; Övez & Akyüz, 2013).

5.5.1 Relationship between the level of development of teachers' TPACK domains

In this model, CK and PK directly and positively influence PCK. The findings of the study demonstrate that the positive and direct influence of CK and PK on PCK, simultaneously enhances the teachers' PCK. As stated by Shulman (1986), teaching requires the use of of content knowledge and pedagogical knowledge together. CK directly accounts for about 32% of PCK while PK accounts for 34%. It is important, however, to note that CK and PK simultaneously account for 49% of PCK variation, which is in line with LeBlanc et al.'s (2017) assertion that the development of teachers' knowledge in these two areas and their integration in teaching is an important undertaking.

On the other hand, there is a significantly strong relationship between the level of development of TK and PK with a correlation coefficient of $r = 0.788$ ($p= 0.04$). There is also a significantly strong relationship between TK and CK ($r= 0.623$ and $p=0.01$). In contrast there is no strong relationship between CK and PK ($r=0.357$). This result suggests that independently having sufficient knowledge in CK and PK domains is not adequate for PCK, and the integration of these two domains should be specifically buttressed through professional development programmes.

Some findings from previous researches confirm that TPK, TCK and PCK are directly and positively affected by their constituent knowledge domains (Chai, Koh, Tsai, & Tan, 2011; Çelik et al., 2014; Övez & Akyüz, 2013; Savaş, 2011). Contrary to these findings, in the case of this study, this is only true for PCK. Furthermore, Harris and Hoffer (2011) suggest that TPK, TCK and PCK of teachers directly and positively affect their TPACK. Surprisingly, according to the results in this study, the direct effect of PCK on TPACK is insignificant. Instead, it is only the teachers' TCK and TPK that directly and positively affect their TPACK. This is substantiated by the results from Table 4.15 and Table 4.16, which suggest the change in PK (post – pre mean = 0.27) and the effect of EBM intervention on PK (Hedge's $\boldsymbol{g} = 1.02$) were the least. This could be explained by the fact that the EBM professional development emphasised on the technology construct, given that all the participating teachers had at least three (3) years experience of teaching at the NSSCH level.

A scale developed by Kiray (2016a) was used to measure TPACK self-efficacy perceptions of teachers and the data obtained in the study were analysed by structural equation modelling. The direct and positive effects of Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK) and Pedagogical Content Knowledge (PCK) from external variables consisting of binary knowledge domains account for 65% of the change in TPACK. This is an interesting measurement outcome which is almost identical to the current study results in which 66% of TPACK is attributed to TCK, TPK and PCK.

TPK was the variable that affected TPACK the most. According to this finding, TPK has a critical importance in teachers' technology integration. Another important finding in this study is that teachers' CK directly and positively influences TCK and PCK, and this effect is greater than the effect of TK and PK. When considered in the context of the results of this research, a gradual model covering CK and PCK instead of a direct technology-based approach to professional development programmes could be proposed and developed to increase TPACK self-efficacy of teachers.

However, the findings of the study fail to establish a strong link between teacher efficacy and learners' conceptual understanding of periodicity of trigonometric functions. This is observed from the low correlation confirmed between teacher self-efficacy and learners' conceptual understanding ($r = 0.25$) (See Appendix F). This is contrary to some previous studies, for example, Clayson and Sheffet (2006), Mojavezi and Tamiz (2012), who concluded that there was a positive influence of teacher self efficacy on learner achievement. Whereas the EBM teacher professional development chronicled hight effect sizes on teachers' self-efficacy, there was no evidence to support this viewpoint.

5.5.2 Influence of development of teachers' TPACK on the learners' conceptual understanding of periodicity of trigonometric functions

The influence of Excel-based modelling professional development on learners' conceptual understanding of periodicity of trigonometric functions was measured through modelling the relationship between the teachers' TPACK development and learners’ achievement in the summative test. The learners’ percentage scores were translated to the TIMSS scale of 1 to 5, as used for the TPACK ratings. There was a significant positive influence of TPACK on learners’ conceptual understanding (LCU) at p = 0.019. The teachers’ TPACK had an influence of about 47% on learners’ conceptual understanding, suggesting that improvement in teachers’ TPACK could significantly contribute towards an improvement in learners’ academic achievement.

The influence of Excel-based modelling professional development on learners’ conceptual understanding of periodicity of trigonometric functions was measured from three sources of data, namely, the CAEMA tool and through modelling the relationship between the teachers’ TPACK development and learners’ percentage scores in the summative test. There was a significant positive influence of TPACK on learners’ conceptual understanding (LCU) at p = 0.019. The teachers’ TPACK had an influence of about 50% on learners’ conceptual understanding, suggesting that improvement in teachers’ TPACK could significantly contribute towards an improvement in learners’ academic achievement.

The findings are consistent with those of Mogari and Ogbonnaya (2014), who found that teacher content knowledge of trigonometric functions is linked to learner achievement in a significantly positive way. Despite the fact that their analysis only measured the effect of one construct, CK, in the TPACK context, the results can be extrapolated to support the findings of this study since CK accounts for about 30% of TPACK variation. Tchoshanov (2011) backed up the findings of the report, claiming that a teacher whose experience is limited to mathematical procedures would have less opportunities to affect learners’ progress.

Drijvers (2016) describes an overview of three review studies that provided information on the effect of using technology in mathematics education by disclosing effect sizes, which sheds more light on the impact of TPACK growth on learners' conceptual understanding of mathematics. Li and Ma (2010) conducted an analysis of 46 research studies on the use of computer technology in mathematics education in primary and secondary school classrooms, estimating 85 effect sizes in total. Positive effect sizes were found to be statistically significant. In experiments that used a constructivist approach to teaching, effect sizes were larger. This perspective is consonant with the findings of the current study's constructivist EBM instruction, which found that TPACK was responsible for about half of the learners' conceptual understanding.

Rakes, Valentine, McGatha, and Ronau (2010) recorded 109 effect sizes in their second review analysis, which focused on algebra in particular. The authors concluded that approaches that centred on conceptual understanding had impact sizes that were roughly twice as large as those that focused on procedural understanding. This indicates that technology had a greater capacity for enhancing learners' conceptual understanding than it was for procedural goals.

Cheung and Slavin (2013) used 74 effect sizes from primary and secondary school mathematics research in their third analysis report. On the contrary, they discovered that, despite anticipated improvements from the introduction of advanced technologies, improved ICT infrastructure, and increased pedagogical experience, educational technology's overall effectiveness did not improve over time.

Overall, the use of technology in mathematics education seems to have a strong positive impact, but with a limited effect scale. Given that any creative educational intervention has a positive impact in and of itself (Higgins, Xiao, & Katsipataki, 2012), these findings do not provide overwhelming evidence for the efficacy of digital resources in mathematics education.

CHAPTER SIX
SUMMARY OF THE STUDY, CONCLUSIONS AND RECOMMENDATIONS

This chapter provides the conclusion to the entire thesis. It incorporates a summary of key findings and outlines some limitations, proffers recommendations and ultimately proposes specific areas for further study.

6.1 SUMMARY OF THE STUDY

The research was premised on the poor conceptualisation of periodicity of trigonometric functions by both teachers and learners in the NSSCH mathematics curriculum in Namibia (DNEA, 2014). Other international studies have also reported similar conceptual obstacles in the teaching and learning of trigonometric functions in secondary school mathematics (Gebrekal, 2007; Shama, 1998; Orhun, 2001).

Against this background, the researcher developed an Excel-based modelling (EBM) professional development programme for teachers to implement as a possible remedy to the current poor conceptualisation of periodicity of trigonometric functions amongst teachers and students. The EBM was designed to support the visualisation of the connections between trigonometric functions and their graphs through visual exploration of their periodicity, symmetry and parity properties.

Chai et al., (2013) suggested more development and research of technological environments based on TPACK, study of learners' learning with technology and cross fertilisation of TPACK with other theoretical frameworks related to technology integration. Learners' academic achievement in the TPACK integrated lessons has not been reported by any of the studies they reviewed, which is a clear gap that needs urgent attention. Although Mogari and Ogbonnaya (2014) investigated the relationship between grade 11 students' achievement in trigonometric functions and their teachers' content knowledge (CK), they did so without isolating the other possible exogenous variables, specifically pedagogical knowledge (PK) and technological knowledge (TK), which might have accounted for significant variations in learners' achievement. The uniqueness of this current study is grounded on its design and methodology to address these gaps.

An embedded mixed methods research design and approach was used to map the influence of the implementation of the EBM instruction on learners' conceptual understanding of periodicity of trigonometric functions. The TPACK theory and Guskey’s (2000) model of evaluation of teacher professional are the theoretical frameworks that influenced the course of the study. The Excel-based professional development focused on simultaneously strengthening the teachers’ content knowledge (CK), pedagogical knowledge (PK) and technological knowledge (TK) through demonstration lessons and feedback on collaborative problem-solving activities.

The implementation of the EBM instruction was done at 8 secondary schools in a specific region of Northern Namibia. The participants were a sample of 11 NSSCH grade 12 mathematics teachers and their 123 learners. The EBM influence on learners’ conceptual understanding was measured through a correlational analysis of the association between TPACK and learners’ test scores in a summative test. The partial least squares equation modelling (PLS-SEM) technique was employed to estimate and verify the influence of TPACK on learners’ conceptual understanding.

The results showed that TPACK positively influenced learners’ conceptual understanding and accounted for approximately 50% variation in learners’ conceptual knowledge of trigonometric functions. The learners, in their interview responses, also suggested that the EBM instructional approach motivated them towards the whole learning process as they managed to construct and express their own meanings of the patent mathematical connections by themselves. They could easily navigate between the geometric and algebraic representations of trigonometric functions. The Cohen’s *d* index was used to measure the size of the EBM professional development intervention on teachers’ self-efficacy, and indicated that the EBM professional development programme had a significant positive influence on the growth of all teachers’ TPACK constructs.

6.2 CONCLUSIONS

This study demonstrated significant associations between the level of development of specific TPACK constructs and their influence on the development of learners’ conceptual understanding. Research on learners’ conception of learning with technology is a gap that previous studies had not addressed at the time of conducting this study. In particular, Chai et al. (2013) suggested more development and research of technological environments based on TPACK; study of students’ learning conception with technology; and cross fertilisation of TPACK with other theoretical frameworks related to the study of technology integration. The researcher is convinced that the current study contributes significant insights and findings to fill this gap.

The findings of this study show that TPK, TCK and PCK had the most influence on TPACK variation. The CAQDA output on the conceptions of both learners and teachers in teaching and learning through EBM technology suggests that TPACK integrated lessons are more effective in enhancing learners' conceptual understanding than the traditional pen and paper approach. The PLS-SEM results indicate a strong association between teachers' TPACK development with 47% of variation in learners' conceptual understanding that is directly attributed to TPACK. From the open-ended learner interview outcomes, learners expressed the conviction that the EBM approach facilitated the development of their metacognitive skills, an important aspect of the pragmatic and constructivist paradigms upon which the study was grounded. The results of the qualitative findings strongly supported the PLS-SEM quantitative findings.

The learners' collaborative problem-solving in the classroom activities, supported by visualisation of the mathematical applets through the use of the Excel-based applets, were effective in developing their conceptual understanding of periodicity of trigonometric functions. Arcavi (2003: 223) argues:

> Visualisation has a powerful complementary role for mathematics students in three aspects: as (a) support and illustration of essentially symbolic results, (b) a possible way of resolving conflict between (correct) symbolic solutions and (incorrect) intuitions, and (c) as a way to help us re-engage with and recover conceptual underpinnings which may be easily by-passed by formal solutions.

This observation by Arcavi (2003) is also supported by Stahl, Koschmann and Suthers (2006), who insist that Computer Supported Collaborative Learning (CSCL) transforms learning from being only a matter of accepting fixed facts, to the dynamic, on-going, and complex interactions primarily taking place within communities of practice. CSCL approaches to visualisation vary from mirroring systems, which display basic actions to collaborators, metacognitive tools, which represent the state of interaction via a set of key indicators, and coaching systems, which offer advice based on an interpretation of those indicators (Soller & Jermann, 2005).

6.3 RECOMMENDATIONS

6.3.1 Recommendations for effective teacher professional development programmes

The impact of TPACK enhanced instruction and the development of learners' conceptual understanding was demonstrated in this research. Given that Rakes et al. (2010) also found a significant link between TPACK and learners' conceptual understanding, it is recommended that school support systems adopt the EBM teacher professional development.

More emphasis should be placed on the development and implementation of teacher professional development programmes in the area of TPACK enhanced Collaborative Learning. TPACK enhanced collaborative problem-solving is built on the premise that collaborative knowledge construction and problem-solving can be assisted effectively by technology. This premise was not always obvious to instructors and researchers since early theories emphasised conditioning behaviours and/or strengthening memory traces through repeated associations and practices by individual learners. Researchers have since confirmed that learners' active engagement with learning materials and strategy use is critical to successful learning (Chi, 2009; O'Donnell & King, 1999). There is an equal need to buttress the active development of learners' conceptual understanding of trigonometry through technology-integrated lessons that support visualisation of mathematical connections.

6.3.2 Recommendations for further research

Chai et al. (2013) reviewed 74 journal papers that investigated ICT integration using the TPACK framework. Their study verified that "TPACK was a mushrooming area of research with more application in the North American region. The papers that were reviewed employed varied and sophisticated research methods that have yielded positive results in enhancing teachers' capability to integrate ICT for instructional practice. However, there are still many potential gaps that the TPACK framework could be employed to facilitate deeper change in education" (Chai et al., 2013, p. 31). In particular, Chai et al. (2013) suggested more development and research of technological environments based on TPACK; study of students' learning conception with technology; and cross fertilisation of TPACK with other theoretical frameworks related to the study of technology integration.

A more recent study by Harris et al. (2017) analysed TPACK research and development over a 12 year period, spanning from 2004 to 2016. Approximately 1,200 publications that utilised the TPACK construct, impacting the practice of postsecondary faculty, administrators, and others pursuing meaningful educational integration of technology, were generated over this period. Harris et al. (2017), noted in their analysis that TPACK's rapid dissemination has generated multiple points of divergence, which in turn need further study; especially the construct's accurate measurement and validation; how to assist pre-service and in-service teachers' TPACK development; contextual influences upon teachers' TPACK; and the relationship of TPACK-based knowledge to teachers' decision-making and action.

The current study fills some of these identified gaps by using a cross fertilisation of the TPACK theory with Guskey's (2000) framewok of evaluation of professional development. Although the findings are supported by some of the research from previous studies, for example, the observed positive influence of TPK, TCK and PCK on TPACK which accounts for 66% of TPACK variation is identical to the findings of Kiray (2016a). However, there are diverging views on which of these constructs has the greatest influence on TPACK. Due to the small sample size used by the current study, the researcher proposes that the methodology used here be adopted in a quasi-experimental study, using a larger sample size in order to establish the extent of the influence TPACK explored.

6.3.3 Limitations of the study

The sample size of the study did not allow for the employment of the quasi-experimental design, which would have yielded a more robust explanation of the causal associations between the TPACK latent variables and the influence of the EBM intervention on learners' conceptual understanding of periodicity of functions. It is against this backdrop that the researcher recommends a similar study in future, using a larger sample size and a quasi-experimental design.

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LIST OF APPENDICES:

APPENDIX A: THE EBM PROFESSIONAL DEVELOPMENT PROGRAMME

Professional Development Programme (Approximately 40 hours)

DATE: 10 TO 14 APRIL 2017

DAY 1

Monday 10 April:

TIME	ACTIVITY	VENUE	FACILITATORS
08:30 – 09:00	Registration	Outapi Mall Boardroom	Mathematics Education Officer
09:00 – 10:00	Introduction of participants and welcoming remarks	Outapi Mall Boardroom	Mathematics Education Officer
10:00 – 10:30	TEA BREAK		
10:30 – 11:30	Group reflections on challenges in teaching and learning of trigonometric functions and graphs	Outapi Mall Boardroom	Researcher
11:30 – 13:00	Completion of pre EBM professional development TPACK-baseline survey questionnaire by teachers	Outapi Mall Boardroom	Researcher
13:00 – 14:00	LUNCH BREAK		
14:00 – 16:00	Collaborative problem solving activity (CPSA) and group presentations of solution of CPSA.	Outapi Mall Boardroom	Mathematics Education Officer
16:00	END OF DAY		

DAY 2

Tuesday 11 April:

TIME	ACTIVITY	VENUE	FACILITATORS
08:30 – 09:00	Recap of previous day's activities	Outapi Mall Boardroom	Researcher
09:00 – 10:00	Group reflection on challenges in the collaborative problem solving activity	Outapi Mall Boardroom	Mathematics Education Officer
10:00 – 10:30	TEA BREAK		
10:30 – 12:00	Introduction to EBM content and applets	Outapi Mall Boardroom	Researcher
12:00 – 13:00	Demonstration of application of EBM applets to explore the characteristics of the basic trigonometric functions	Outapi Mall Boardroom	Researcher
13:00 – 14:00	LUNCH BREAK		
14:00 – 16:00	Collective development of lesson plan 1 and trial run of EBM resources (Exploration of sinx, cosx and tanx)	Outapi Mall Boardroom	Researcher & MEO
16:00	END OF DAY		

DAY 3

Wednesday 12 April:

TIME	ACTIVITY	VENUE	FACILITATORS
08:30 – 09:00	Recap of previous day's activities	Outapi Mall Boardroom	Mathematics Education Officer
09:00 – 10:00	Demonstration of technology enhanced Inquiry Based Learning Pedagogy in teaching the sine function	Outapi Mall Boardroom	Researcher
10:00 – 10:30	TEA BREAK		
10:30 – 12:00	Collective development of lesson plan 2 and trial run of EBM resources (Detailed exploration of Sine)	Outapi Mall Boardroom	Researcher
12:00 – 13:00	Collective development of lesson plan 3 and trial run of EBM resources (Detailed exploration of Cosine)	Outapi Mall Boardroom	Researcher
13:00 – 14:00	LUNCH BREAK		
14:00 – 16:00	Collective analysis of EBM applets and exploration with tangent function	Outapi Mall Boardroom	Researcher & MEO
16:00	END OF DAY		

DAY 4

Thursday 13 April:

TIME	ACTIVITY	VENUE	FACILITATORS
08:30 – 09:00	Recap of previous day's activities	Outapi Mall Boardroom	Researcher & MEO
09:00 – 10:00	Collective development of lesson plan 4 and trial run of EBM resources (Detailed exploration of Tangent)	Outapi Mall Boardroom	Researcher & MEO
10:00 – 10:30	TEA BREAK		
10:30 – 12:00	Collective analysis of how EBM applets could be used to explore solutions to the collaborative problem solving activity sheet	Outapi Mall Boardroom	Researcher & MEO
12:00 – 13:00	Demonstration session on EBM use to support problem solving process	Outapi Mall Boardroom	Researcher
13:00 – 14:00	LUNCH BREAK		
14:00 – 16:00	Sketching graphs including graphs with shifts, and deducing equations of graphs with the aid of EBM technology	Outapi Mall Boardroom	Researcher & MEO
16:00	END OF DAY		

DAY 5

Friday 14 April:

TIME	ACTIVITY	VENUE	FACILITATORS
08:00 – 08:30	Recap of previous day's activities	Outapi Mall Boardroom	Researcher & MEO
08:30 – 09:00	Consolidation videos from Khan Academy	Outapi Mall Boardroom	Researcher
09:00 – 10:30	Collective development of lesson plan 5 and trial run of EBM resources (Deducing equations from graphs)	Outapi Mall Boardroom	Researcher & MEO
10:30 – 11:00	TEA BREAK		
11:00 – 12:00	Completion of post EBM professional development TPACK- survey questionnaire by teachers	Outapi Mall Boardroom	Researcher
12:00 – 13:00	Photo session and closing remarks	Outapi Mall Boardroom	Mathematics Education Officer
13:00 – 14:00	LUNCH BREAK		
13:00	END OF DAY		

Session 1: Collaborative problem solving activity (2 hours)

Objectives:

To evaluate teachers' baseline state of CK and PK

Activities:

Teachers are divided into 3 groups of 4, 4 and 3 participants and allocated 1 hour to solve 7 trigonometry problems in the teacher activity sheet

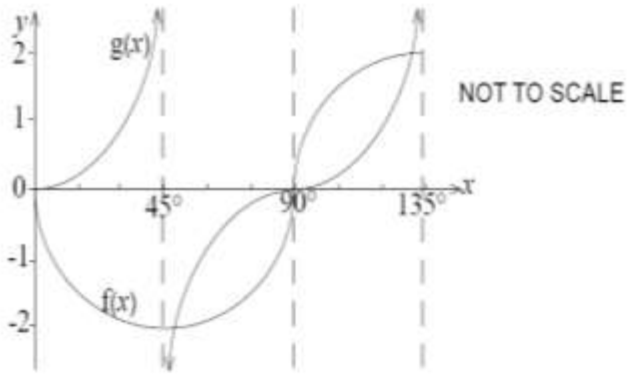
Groups are allocated 1 hour to present their solutions emphasising on the pedagogy they would employ to deliver the content in their lessons.

Teachers' pre-EBM training activity sheet

Attempt all the questions on the separate answer sheets provided. Show all the necessary working clearly. Use pencil for all diagrams or graphs. The number of marks is given in brackets at the end of each question or part question.

1. Functions f and g are defined by

$$f:x \mapsto a \sin bx, \quad \text{for } 0^\circ \leq x \leq 135^\circ \text{ and}$$
$$g:x \mapsto \tan cx, \quad \text{for } 0^\circ \leq x \leq 135^\circ.$$



(a) Find the values of a, b and c. [2]

(b) Write down the equation of the reflection of the graph, $g(x)$, in the x-axis. [1]

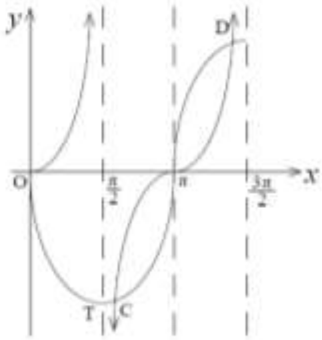
2.

Functions, f and g are defined by

$$f: x \mapsto -2\sin x,\quad 0 \le x \le \frac{3\pi}{2},$$

$$g: x \mapsto \tan x,\quad 0 \le x \le \frac{3\pi}{2}.$$

The diagram shows the curves $y = f(x)$ and $y = g(x)$ intersecting at O, C and D.



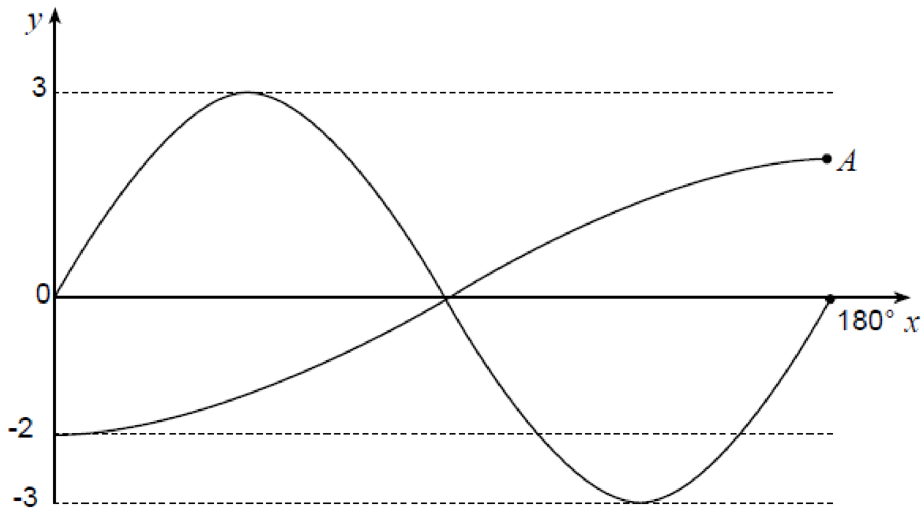
(a) Find the coordinates of T, the turning point of the curve $y = f(x)$. [1]

(b) Write down the period of g. [1]

(c) Write down the range of f. [1]

(d) If C has coordinates $\left(\frac{2}{3}\pi, -\sqrt{3}\right)$, write down the coordinates of D. [2]

3.



The diagram shows the graphs of $y = a \sin bx$ and $y = c \cos dx$ for $0° \leq x \leq 180°$

(0,-2) and A are stationary points.

(a) Determine the values of a, b, c and d. [4]

(b) Find the coordinates of the point A on the diagram. [2]

(c) Find the period of $y = a \sin bx$. [1]

(d) Determine the range of $y = c \cos dx$. [1]

(e) Write down the new equation of $y = a \sin bx$, when the y-axis is shifted 45° to the right. [2]

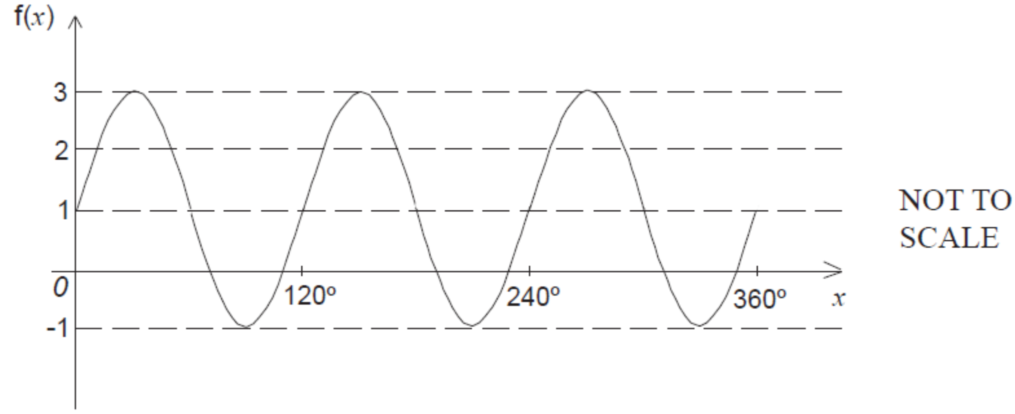
4. (a) The functions f and g are such that $f(x) = \sin x + 1$, for $-90^0 \le x \le 270^0$,

$g(x) = \cos 2x$, for $-90^0 \le x \le 270^0$. Draw sketch graphs of $y = f(x)$ and $y = g(x)$ on the same system of axes. [4]

(b) Find the range of values where $f(x) \times g(x) < 0$ for $-90^0 \le x \le 0^0$ [1]

5.

(a)



The diagram shows part of a trigonometric function defined by

$$f: x \longrightarrow a \sin(bx) + c, \quad \text{for } 0° \leq x \leq 360°.$$

(i) Find the values of the positive integers a, b and c. [3]

(ii) Find the smallest value of x for which $f(x) = 0$. [2]

(b) Given that $p = 4\sin x - 3\cos x$ and $q = 4\cos x + 3\sin x$,

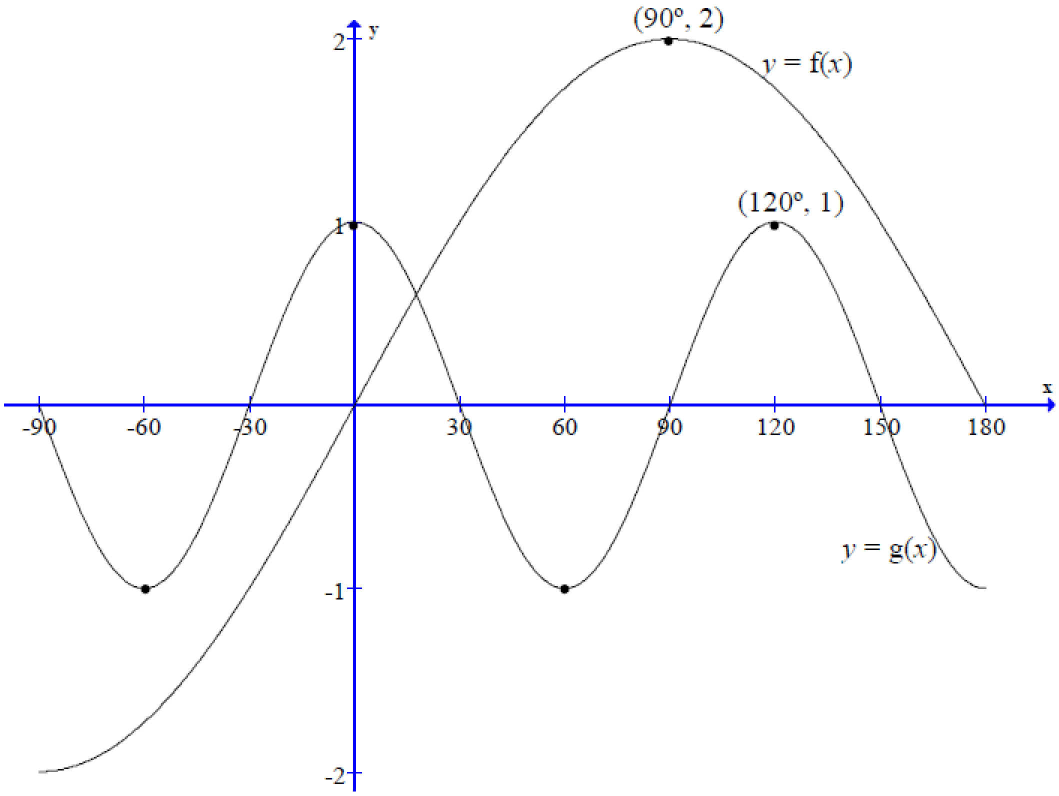
(i) find the value of the acute angle x, in radians, for which $p = q$, [3]

(ii) show that $p^2 + q^2$ is constant for all values of x. [3]

6 (i) On the same diagram, sketch and label the graphs of $f(x) = -2\sin x$ and $g(x) = \cos 2x$ for the interval $0 \le x \le \pi$. [4]

(ii) Hence state the number of solutions of the equation $f(x) = g(x)$ in the interval $0 \le x \le \pi$. [1]

7. The diagram shows the graphs of two functions f and g, for $-90° \leq x \leq 180°$.



(a) The function f is given by f(x) = a sin bx, where a, b are constants.

(i) Find the value of a and b for $-90° \leq x \leq 180°$. [2]

(ii) Find the range of the function f for $-90° \leq x \leq 180°$. [1]

(b) The function g is given by g(x) = p cosqx, where p and q are constants.

(i) Find the value of p and q in the given interval $-90° \leq x \leq 180°$. [2]

(ii) Find the period of the function g in the given interval $-90° \leq x \leq 180°$. [1]

(c) Use the diagram to find the range of x-values for which the product

$f(x).g(x) < 0$ in the given interval $-90° \leq x \leq 180°$. [3]

Sessions 2 - 4: Group reflection on the challenges in the collaborative problem solving activity and introduction to EBM applets (2 hours 30 minutes)

Objectives:

To gather more data on teachers' gaps in their CK and PK after the collaborative problem solving process

Introduce teachers to the EBM applets and the formulae used to programme

Develop capacity to use EBM tools to explore periodicity and symmetry of trigonometric functions

Activities:

Teachers continue working in their 3 groups

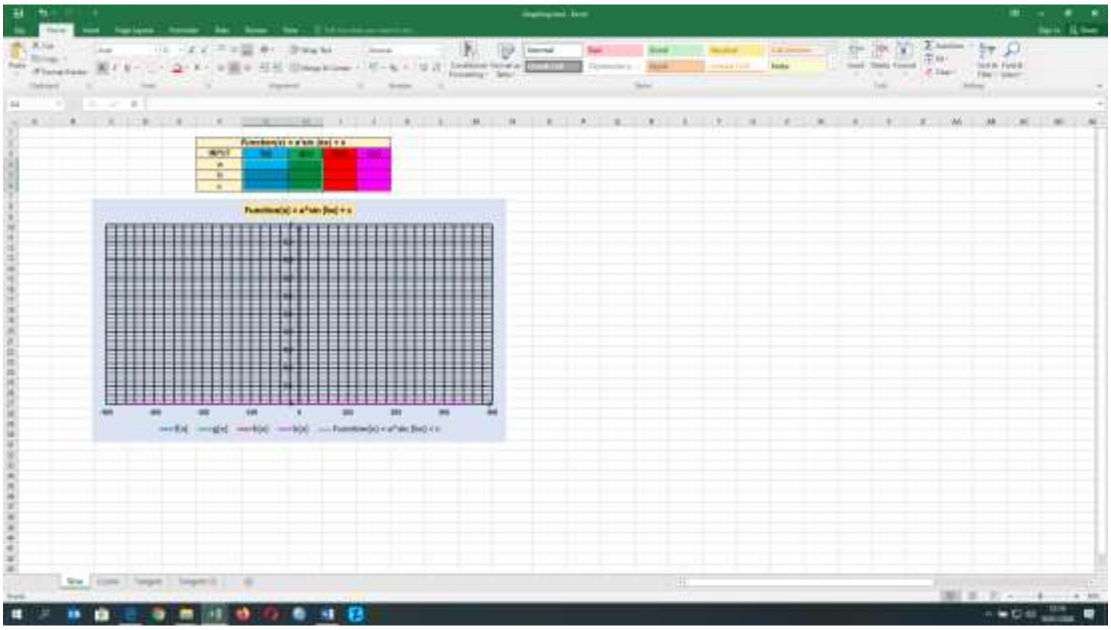
Each teacher's laptop is uploaded with the EBM applets

The EBM applets are individually projected onto the screen and their content explained by researcher while participants view from their laptops

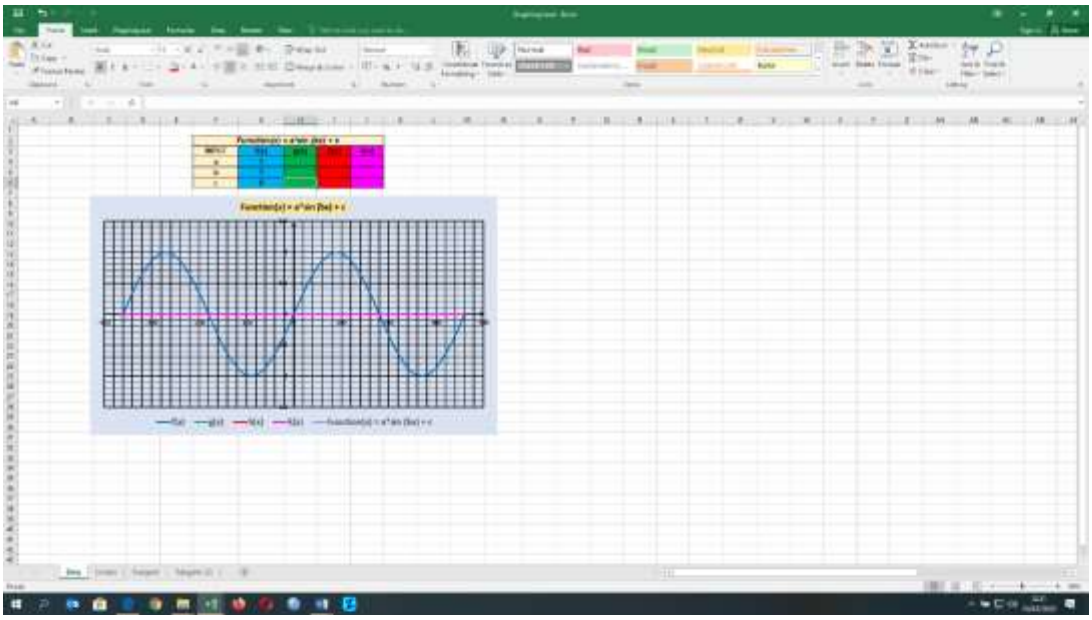
Demonstration of application of applets to explore characteristics of basic trigonometric functions is done collectively

Practical activities:

Open the Sine applet



Enter the values of a = 1, b=1 and c=0



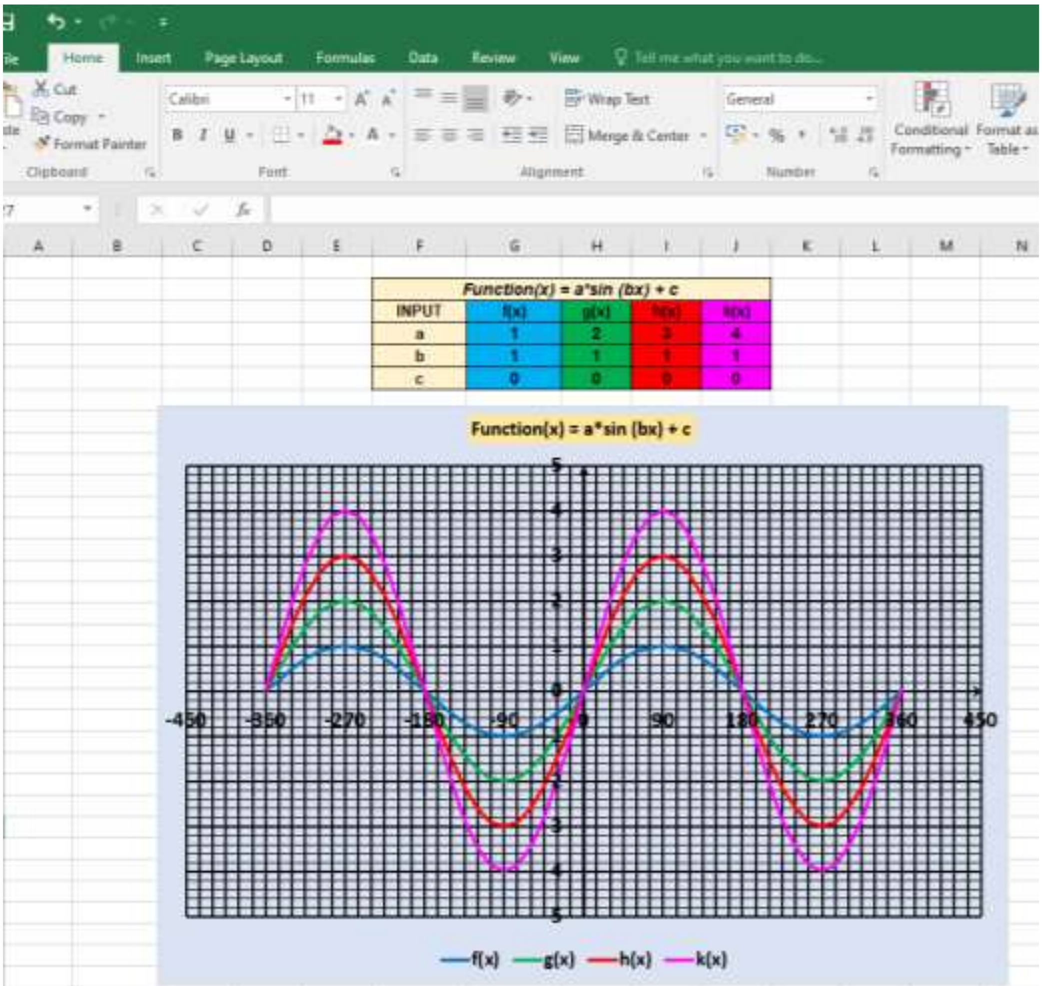
Write down the function f(x)

Answer: f(x) = sinx

When a=b=1 and c = 0 , we obtain the basic function of f(x) =a*sin(bx) + c

Identify the amplitude and period of f(x) = sinx

Explore the effects of changing the value of a, with b=1 and c = 0, and complete the table below:



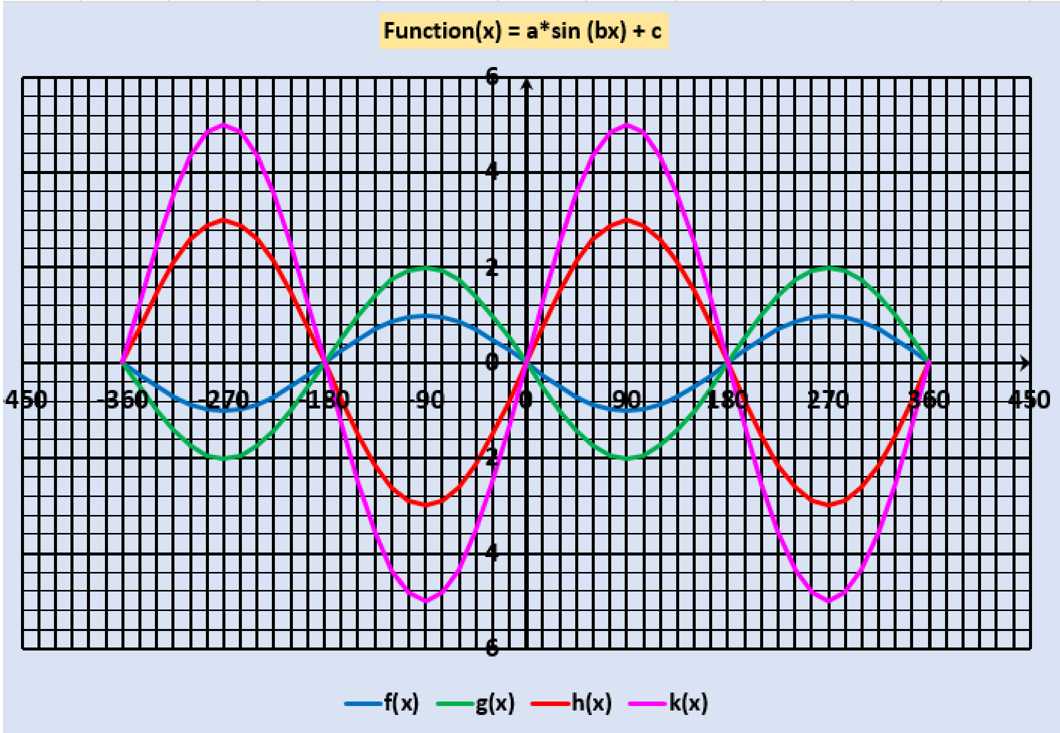
What do you conclude?

Without using EBM applets, deduce the amplitude and period for the functions in the table below:

a	b	c	Function	Amplitude	Period
-1	1	0	f(x) = -sinx		
2	-1	0	g(x) = 2sin(-x)		
-3	-1	0	h(x) = -3sin(-x)		
5	1	0	k(x) = 5sinx		

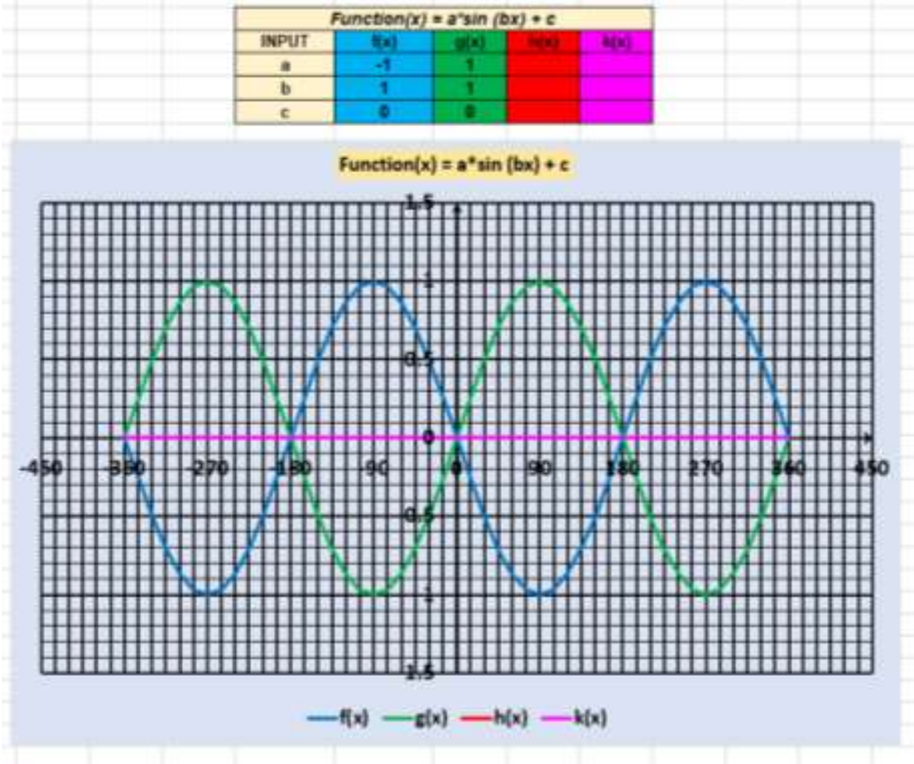
Use EBM sine applets to graph the above functions and verify whether you get the same values of amplitude and period as you obtained in the table.

*Function(x) = a*sin (bx) + c*				
INPUT	f(x)	g(x)	h(x)	k(x)
a	-1	2	-3	5
b	1	-1	-1	1
c	0	0	0	0



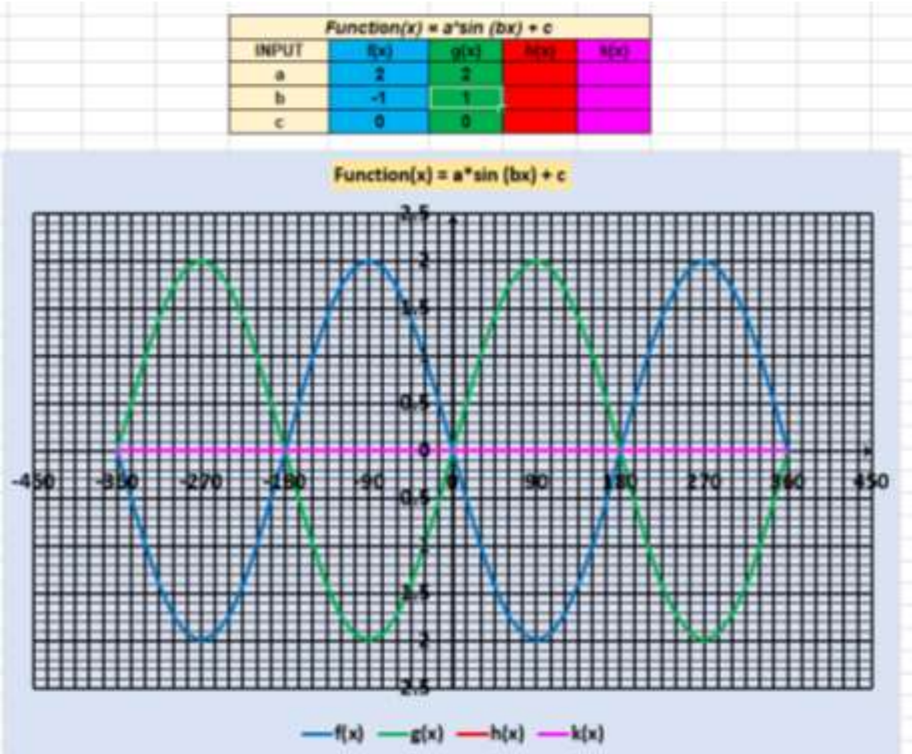
On the same applet draw corresponding colour pairs of functions and discuss your observations.

y = -sinx and y= sinx and

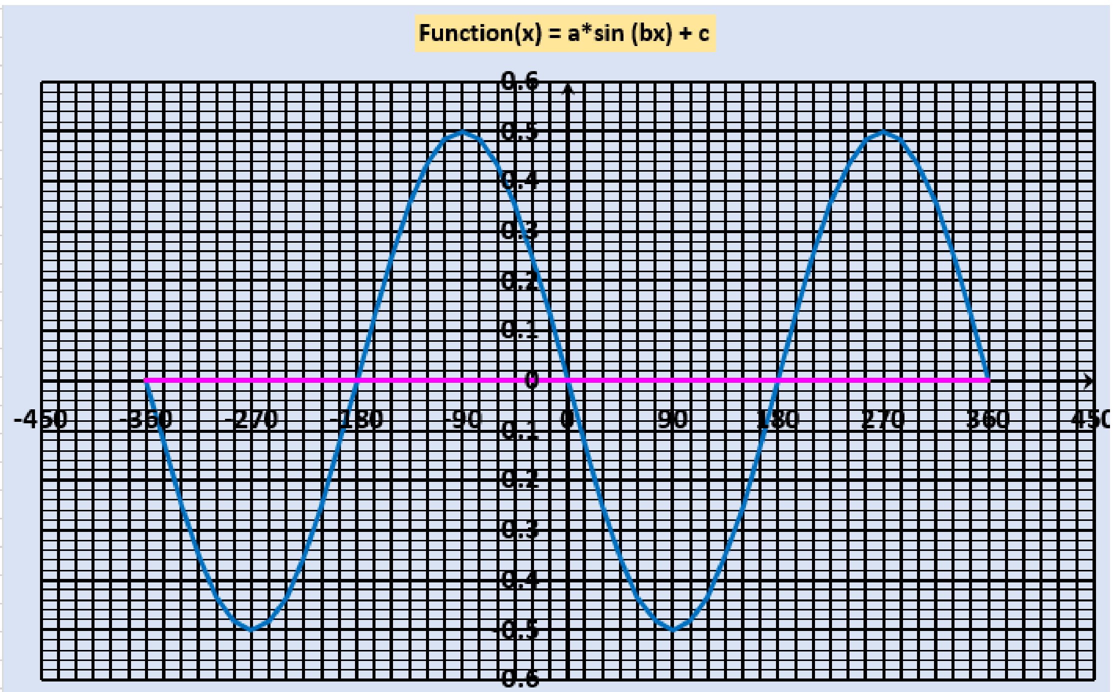


Key:

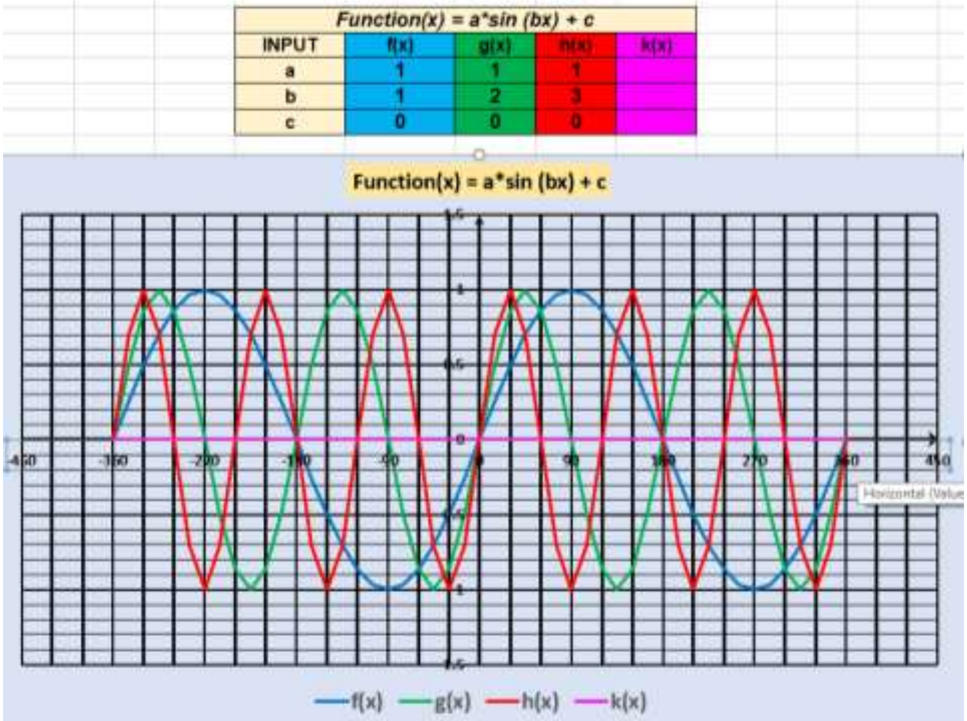
y = 2sin(-x) and y= 2sin(x)



Determine the sine function, f(x) = asin(bx)+c, for the graph shown below.

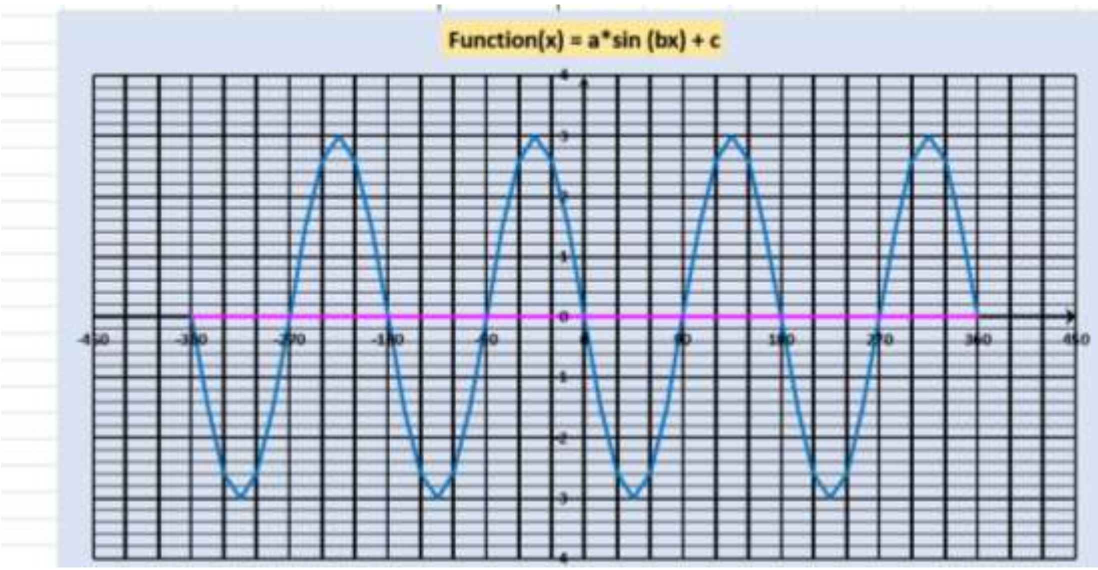


Enter the values of a = 1, b = 1 and c=0. On the same applet, maintain the same values of a=1 and c=0, and enter 3 different values of b=2, b=3 and b=4, and complete the table associated to the graphs.



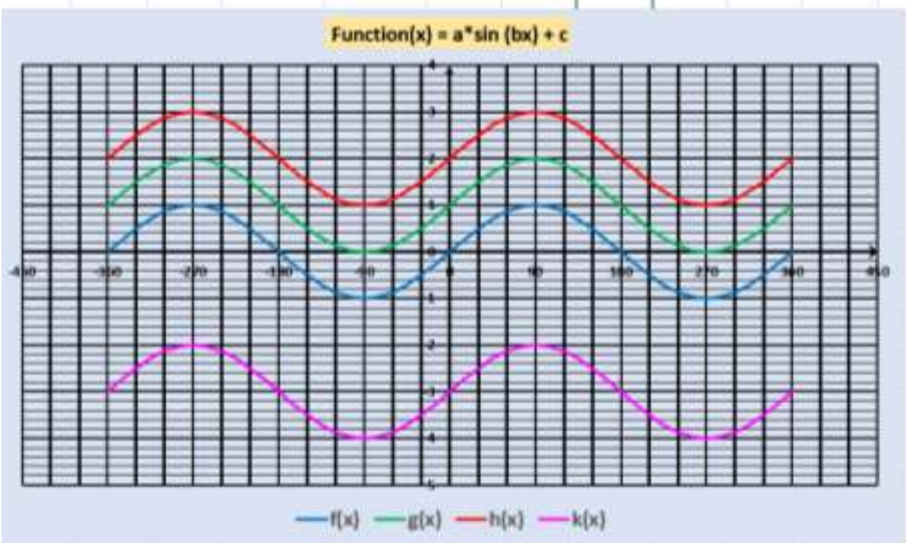
a	b	c	Function	Amplitude	Period
1	1	0	f(x) = sinx	1	360^0
1	2	0	g(x) = sin2x	1	180^0
1	3	0	h(x) = sin3x	1	120^0

Determine the function f(x), for the graph below

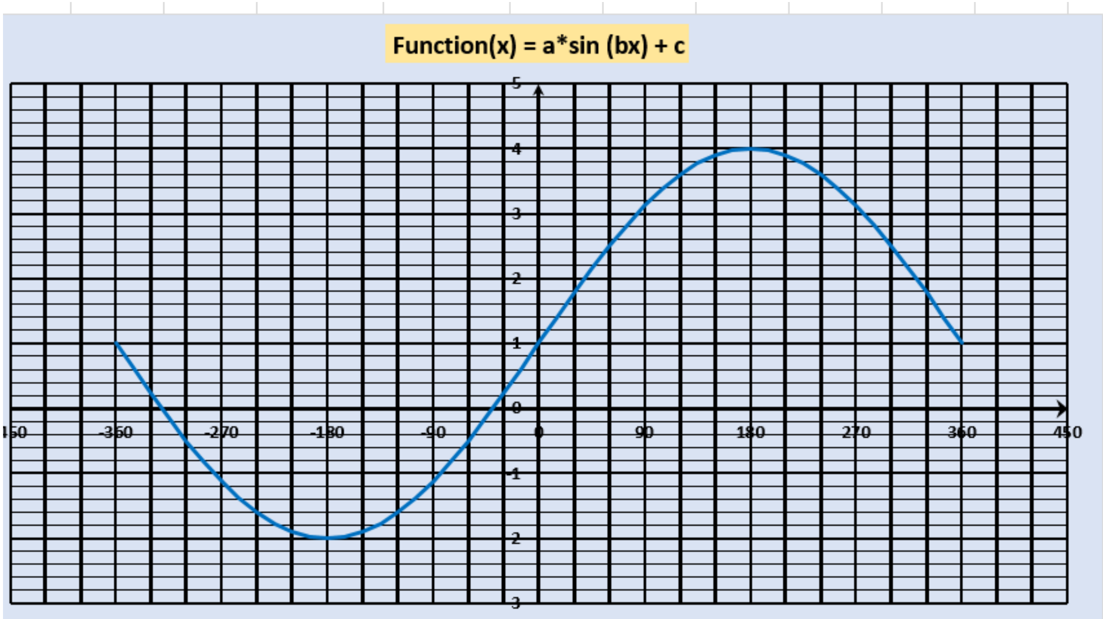


For f(x) = asin bx +c, fix a=b=1, and change c=0, c=1, c=2, c=-3, and discuss your observations

Function(x) = a*sin (bx) + c				
INPUT	f(x)	g(x)	h(x)	k(x)
a	1	1	1	1
b	1	1	1	1
c	0	1	2	-3



Determine the function shown in the graph below

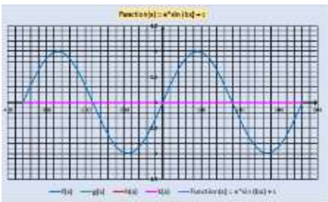
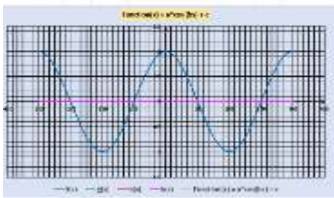
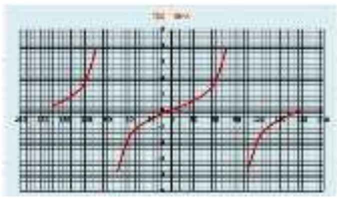


In a similar way, explore the cosine and tangent functions

Develop a lesson plan on how you would teach the periodicity and symmetry properties of each function.

Develop a lesson plan on how you would determine the equation of each of the trigonometric functions from their graphs. Incorporate any other relevant supporting e-resources to enhance the learners' conceptual understanding.

Analyse the symmetry and parity of trigonometric functions to conclude the following:

Function	EBM graph output	Line symmetry	Rotational symmetry	Parity
Sine		No line symmetry	Rotational symmetry of order 2, about origin (Origin Symmetry)	sin(-x) = -sin(x) Odd
Cosine		Symmetric about the y-axis	No rotational symmetry	cos(-x)= cos(x) Even
Tangent		No line symmetry	Origin symmetry	tan(-x) = -tan(x) Odd

Session 5: Applying EBM tools to support the solution process of the 7 problems in the teachers' activity sheet (1 hours 30 minutes)

Objectives:

To consolidate EBM modelling skills and application of CK, TK and PK

To consolidate teachers' TPACK for periodicity of trigonometric functions

Activities:

Teachers continue working in their 3 groups

Teacher groups take turns to demonstrate EBM strategies to support problem solving process

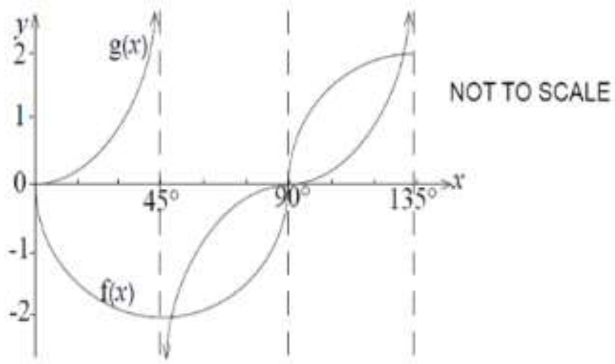
For example:

For problem 1:

1. Functions f and g are defined by

$$\text{f}:x \mapsto a \sin bx, \qquad \text{for } 0° \le x \le 135° \text{ and}$$

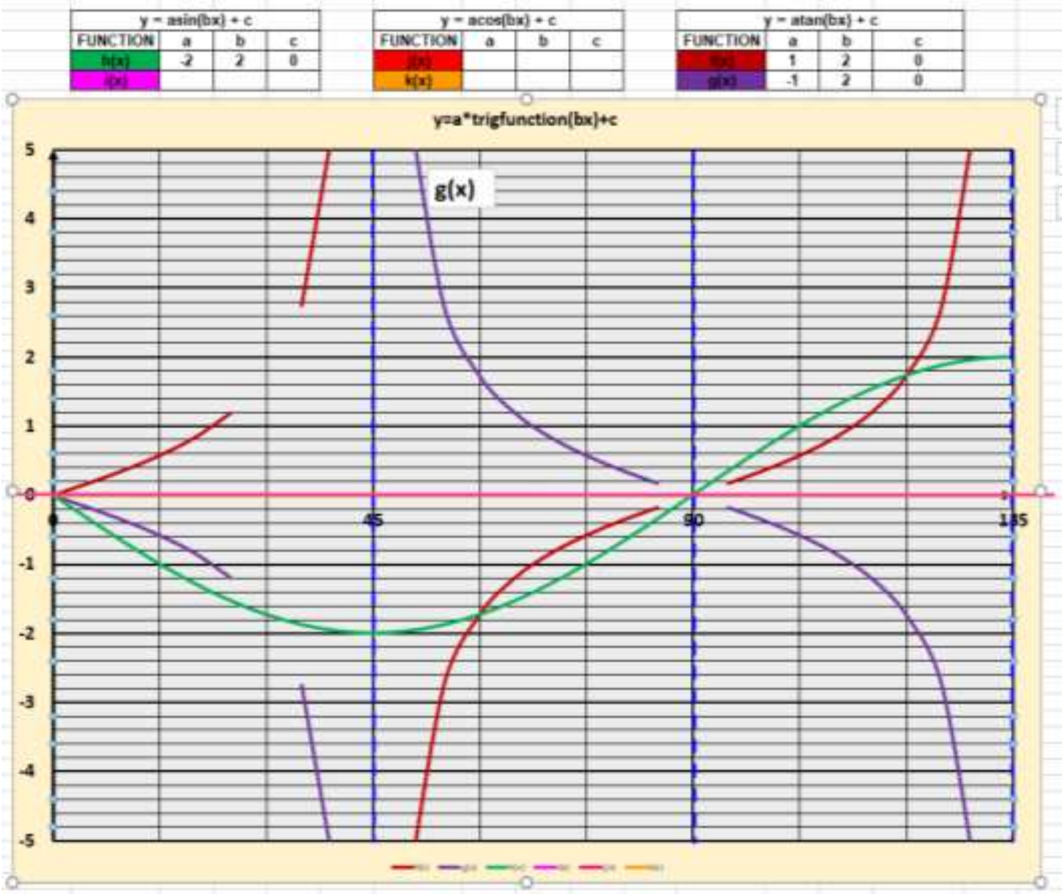
$$\text{g}:x \mapsto \tan cx, \qquad \text{for } 0° \le x \le 135°.$$



(a) Find the values of a, b and c. [2]

(b) Write down the equation of the reflection of the graph, g(x), in the x-axis. [1]

The modelled solution using EBM applets would be:



The model and values used justifies the solutions:

a = -2, b = 2, c = 2

g(x) = -tan2x

APPENDIX B: PRE - EBM PROFESSIONAL DEVELOPMENT TPACK SURVEY QUESTIONNAIRE

Objective: To evaluate level of development of NSSCH Mathematics teachers' technological pedagogical content knowledge before professional development intervention

Teacher Code: ________________________________

The purpose of this questionnaire is to gain information regarding the level of development of technology, pedagogical practices and content knowledge of NSSCH Mathematics teachers.

Please mark with a cross (X) the correct responses or provide an answer where indicated.

All the information obtained from the questionnaire shall be treated as confidential.

Highest academic qualification: ____

Highest professional qualification: ______________________________________

Number of years teaching NSSCH Mathematics: ___________________________

Item	**Technology Knowledge (TK)** **(1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
1	I have had sufficient opportunities to work with different technologies.					
2	I can create a variety of graphs and charts in Excel					
3	I know about basic computer hardware and their functions					
4	I knowing about basic computer software and their functions					
5	I know how to create formulas in Excel					
6	I know how to use the protection feature in Excel to prevent data entry to a specified range of cells					
7	I keep up with important new technologies.					
8	I have the technical skills to use computers effectively.					
9	I can communicate through Internet tools (e.g., e-mail, MSN Messenger)					
10	I am able to use a presentation program (e.g., MS Powerpoint)					
Item	**Content Knowledge (CK)** **(1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
11	I understand functions and graphs well enough to employ multiple strategies in solving related problems in the NSSCH Mathematics curriculum.					
12	I understand the connection between equations and the graph of the functions in the equations (e.g. trigonmetric equations and the graphs of the trigonmetric functions)					
13	I have knowledge in developing class activities, investigations and projects in mathematics					
14	I follow recent developments and applications in mathematics					
15	I collect and follow up-to-date resources (ex, books, journals) in mathematics					
16	I have the mathematics content knowledge I need to teach my learners in the topic of functions and graphs in the NSSCH Mathematics curriculum					
17	I continue to develop my understanding of mathematics.					
18	I have various ways and strategies of developing my understanding of teaching mathematics					
19	I deliver my mathematics learning content like an expert in the subject					
20	I have sufficient knowledge about teaching mathematics concepts					

Item	**Pedagogical Knowledge (PK)** **(1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
21	I know how to organize and maintain classroom management.					
22	I can adapt my teaching based-upon what learners currently understand or do not understand.					
23	I can use a wide range of teaching approaches in a classroom setting (collaborative learning, direct instruction, inquiry learning, problem/project based learning etc.).					
24	I am able to stretch my learners' thinking by creating challenging tasks for them.					
25	I am able to guide my learners to adopt appropriate learning strategies.					
26	I am able to help my learners to monitor their own learning.					
27	I am able to help my learners to reflect on their learning strategies.					
28	I am able to use different evaluation methods and techniques					
29	I am able to plan group activities for my learners.					
30	I am able to guide my learners to discuss effectively during group work.					
Item	**Pedagogical Content Knowledge (PCK)** **(1: Strongly disagree, 2: Diagree, 3: Neutral, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
31	I know how to select effective teaching approaches to guide student thinking and learning in mathematics.					
32	I can adjust my teaching to make it more inclusive					
33	I know how to develop efficient lessons that will help to ensure that all topics are completed in the required time.					
34	I can develop evaluation tests and surveys in my mathematics teaching practice					
35	I can prepare a good lesson plan including class activities and homework					
36	I am able to meet objectives described in my lesson plan					
37	I can make connections among related concepts in mathematics					
38	I can make connections between mathematics and other related subjects					
39	I know how to select effective teaching approaches to guide my learners to discover concepts in mathematics.					
40	I can help my students to understand the content knowledge of mathematics through various ways.					

Item	**Technological Content Knowledge (TCK)** **(1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
41	I can choose technologies that enhance the content for a lesson on functions and graphs.					
42	I know about technologies that I can use for understanding the relationship between functions and their graphs					
43	I can using content-specific computer applications					
44	I can use technologies to help me to reach curriculum objectives easily in my lesson					
45	I can prepare a lesson plan requiring use of inquiry based instructional technologies					
46	I can develop class activities and projects involving use of instructional technologies					
47	I know about technologies that I can use to promote mathematical inquiry					
48	I know about the technologies that I have to use for the research of content of functions and graphs					
49	I am able to lead learners to discover concepts and mathematical relationships through the use of technology					
50	I can use appropriate technologies (e.g. multimedia resources, simulation) to represent the content of my teaching content.					
Item	**Technological Pedagogical Knowledge (TPK)** **(1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
51	I can adapt the use of the technologies that I am learning about to different mathematics teaching activities.					
52	I can choose technologies that enhance students' learning for a lesson.					
53	I am able to use technology to introduce my students to real world scenarios.					
54	I think deeply about how technology could influence the teaching approaches I use in my classroom					
55	I can create opportunities for learners to use digital technology for individualised learning.					
56	I can create computer based activities that provide immediate feedback to learners					
57	I am able to facilitate my students to use technology to find more information on their own.					
58	I am able to facilitate my learners to use technology to plan and monitor their own learning.					
59	I think critiacally about how to use technology in my classroom					
60	I am able to facilitate my learners to use technology to construct different forms of knowledge representation.					

Item	**Technological Pedagogical Content Knowledge (TPACK)** **(1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
61	I can create technology-enhanced lessons that are learner centred					
62	I can provide leadership in helping others to coordinate the use of content, technologies and teaching approaches at my school and/or region.					
63	I can integrate appropriate instructional methods and technologies into the teaching of functions and graphs					
64	I can select contemporary strategies and technologies that help me to teach the content of functions and graphs effectively					
65	I can teach successfully by combining my content, pedagogy, and technology knowledge					
66	I can teach lessons that appropriately combine mathematics problem solving , technologies and teaching approaches.					
67	I think critically about how to use technology in my classroom.					
68	I can select technologies to use in my classroom that enhance what I teach, how I teach and what learners learn.					
69	I take a leadership role among my colleagues in the integration of content, pedagogy, and technology knowledge					
70	I can use strategies that combine content, technologies and effective teaching approaches in my classroom.					

APPENDIX C1: LESSON PLAN 1

Topic: Introduction of key concepts and EBM analysis of periodicity of basic trigonometric functions

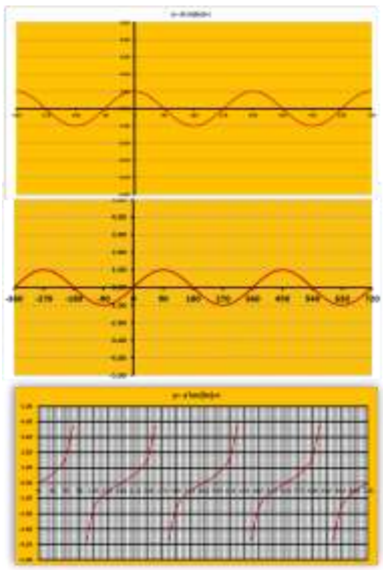
Objectives:

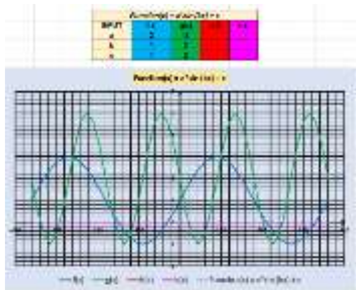
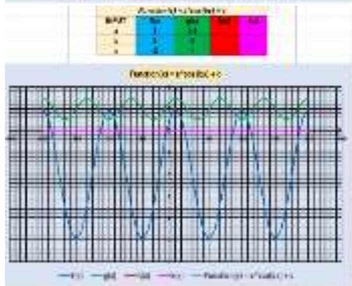
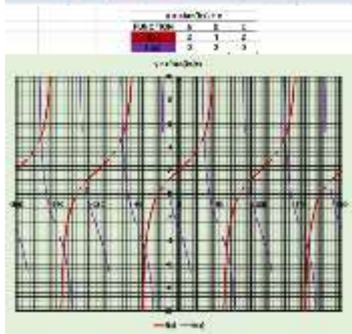
Demonstrate to learners, with the aid of a data projector, how the Excel modelling applets are used to graph the functions of the form; $y = a\sin(bx) + c$, $y = a\cos(bx) + c$, and $y = a\tan(bx) + c$

With the aid of the projected graphs, define the key terms related to periodicity of trigonometric functions (midline, amplitude, period)

For different values of a, b and c, learners should be able to identify the midline, hence deduce the amplitude and period.

DATE:_________ DURATION:1 hour TOPIC: Introduction of key concepts on periodicity

TIME	STAGE	TEACHER'S ACTIVITIES	LEARNERS' ACTIVITIES
10 minutes	INTRODUCTION	Use EBM applets tools to demonstrate how the Excel modelling applets are used to graph the functions of the form; $y = \sin x$, $y = \cos x$, *and* $y = \tan x$ (i.e a =1, b= 1 and c = 0) Project the graphs onto a white screen hung in front of the class 	Learners should work in groups to enter the parameters a=1, b=1 and c=0 into the applets for $y = a\sin(bx) + c$, $y = a\cos(bx) + c$, *and* $y = a\tan(bx) + c$ Learners should describe the characteristics of each resultant function $y = \sin x$, $y = \cos x$, *and* $y = \tan x$ in their own words

<table>
<tr><th>TIME</th><th>STAGE</th><th>TEACHER'S ACTIVITIES</th><th>LEARNERS' ACTIVITIES</th></tr>
<tr>
<td>40 minutes</td>
<td>DEVELOPMENT</td>
<td>
The Midline: is the horizontal line that passes exactly in the middle between the maximum and minimum turning points
Amplitude: is the vertical distance between the midline and one of the turning points
Period: is the distance between two consecutive maximum points or minimum points.
Note: For a tangent function, we could define the period as the distance between consecutive zeros or asymptotes of the function.
Assessment activity:
Use selected graphs of y = a sin(bx) + c, y = a cos(bx) + c, and y = a tan(bx) + c, for learners to identify the midline, amplitude and period
<table>
<tr><th>Function</th><th>Midline</th><th>Amplitude</th><th>Period</th></tr>
<tr><td>y=2sinx+1</td><td>y=1</td><td>2</td><td>360^0</td></tr>
<tr><td>y=-3sin2x+2</td><td>y=2</td><td>3</td><td>180^0</td></tr>
<tr><td>y=3cos2x-2</td><td>y=-2</td><td>3</td><td>180^0</td></tr>
<tr><td>y=0.5cos3x+1</td><td>y=1</td><td>0.5</td><td>120^0</td></tr>
<tr><td>y=2tanx+2</td><td>y=2</td><td>None</td><td>180^0</td></tr>
<tr><td>y=-3tan2x-3</td><td>y=-3</td><td>None</td><td>90^0</td></tr>
</table>
</td>
<td>
Learners observe projected graphs of y = a sin(bx) + c, y = a cos(bx) + c, and y = a tan(bx) + c, and deduce the values of a, b, and c.
Learners should be able to view the following graphic displays and deduce the midline, amplitude and period



</td>
</tr>
</table>

TIME	STAGE	TEACHER'S ACTIVITIES	LEARNERS' ACTIVITIES
10 minutes	CONCLUSION	For lesson summary, show video: https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:trig/x2ec2f6f830c9fb89:amp-mid-period/v/midline-amplitude-period **Homework:** Learners should be given access to the EBM applets to graph different pairs of functions of $y = a\sin(bx) + c$, $y = a\cos(bx) + c$, *and* $y = a\tan(bx) + c$ and deduce the amplitude and period.	Learners watch video Learners to collaboratively complete homework during their study period with support of EBM applets. Learners to present their work in the next class

SUMMARY OF OBSERVED TEACHING AND LEARNING OUTCOMES:

Key achievements in the lesson

Challenges encountered in the lesson

General comments

APPENDIX C2: LESSON PLAN 2

Topic: Periodicity, symmetry and parity of the sine function

Objectives:

Explore the effects of changes in the values of “a”, “b” and “c” to the graph of $y = a \sin(bx) + c$

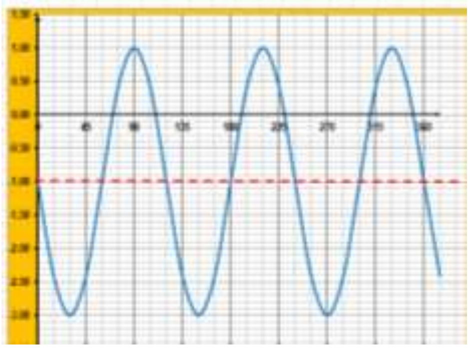
Analyse the symmetry properties of the sine function

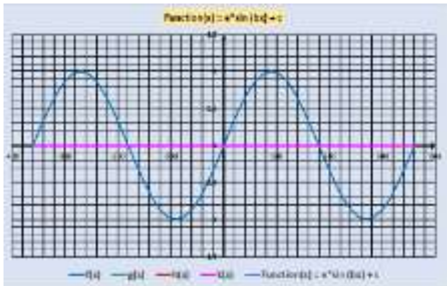
Deduce the parity of the sine function

Predict the outputs of sine graphs for different values of “a”, “b” and “c”

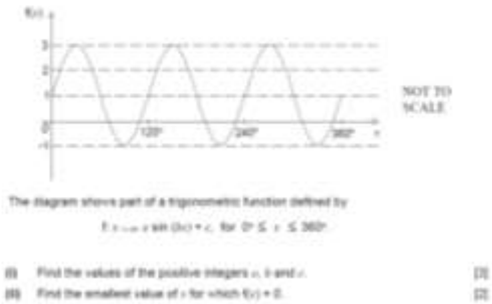
Sketch the graphs of given sine functions

DATE:__________ DURATION:1 hour TOPIC: Periodicity, symmetry and parity of sine

TIME	STAGE	TEACHER'S ACTIVITIES	LEARNERS' ACTIVITIES
10 minutes	INTRODUCTION	Engage the learners to discuss previous class homework and summarise key findings with respect to the effects of changing the values of "a", "b" and "c" on the amplitude, period and midline of y=a*trigfunction(bx)+c Ask learners to predict (sketch) the graph of y=-2sin3x-1 for $0^0 \leq x \leq 180^0$ Use EBM applets to illustrate the graph of y=-2sin3x-1 after learners have given their sketches	Learners present key findings from previous homework activity and use EBM applets to justify their conclusions Learners sketch the graph of y=-2sin3x-1 without the aid of EBM tools Learners use EBM applet for sine function to enter values a =-2, b=3 and c=-1 and compare their sketches to the EBM output of the graph of y=-2sin3x-1, shown below.  *The dotted red horizontal is the midline y = -1.

TIME	STAGE	TEACHER'S ACTIVITIES	LEARNERS' ACTIVITIES
40 minutes	DEVELOPMENT	Use EBM applets tools to facilitate exploration of how the changes to "a", "b" and "c" affect the midline, amplitude and period of y = *asin(bx)* + *c* Use the EBM applets to facilitate exploration of how the changes to the signs of "a" and "b" affect the graph of y = *asin(bx)* + *c* Engage learners to use the EBM tools to explore the line and rotational symmetry of the function y = *sinx* Engage the learners to use the EBM tools to deduce the odd parity of the sine function; sin(-x) = -sin(x)	Learners should collaboratively discuss the outcomes of the exploration of periodicity, symmetry and parity of the sine function. Learners should observe the graph of y = sinx Learners should be able to deduce that the function y = *sinx* , shown below:  Has no line symmetry Has rotational symmetry of order 2 about origin Has odd parity, i.e. sin(-x) = -sin(x)

<table>
<tr><th>TIME</th><th>STAGE</th><th>TEACHER’S ACTIVITIES</th><th>LEARNERS’ ACTIVITIES</th></tr>
<tr><td>10 minutes</td><td>CONCLUSION</td><td>Recap key points of the lesson and homework orientation

Homework:

</td><td>Learners are given access to EBM applets to assist them in exploring the solution to the homework</td></tr>
</table>

SUMMARY OF OBSERVED TEACHING AND LEARNING OUTCOMES:

Key achievements in the lesson

Challenges encountered in the lesson

General comments

APPENDIX C3: LESSON PLAN 3

Topic: Periodicity, symmetry and parity of the cosine function

Objectives:

Explore the effects of changes in the values of “a”, “b” and “c” to the graph of y = *acos(bx)+c*

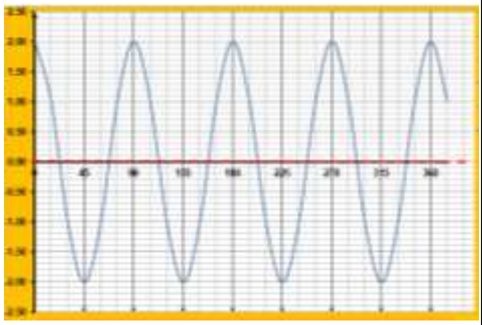
Analyse the symmetry properties of the cosine function

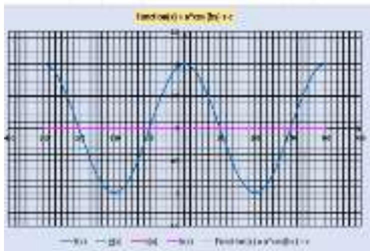
Deduce the parity of the cosine function

Predict the outputs of cosine graphs for different values of “a”, “b” and “c”

Sketch the graphs of given cosine functions

DATE:_________ DURATION:1 hour TOPIC: Periodicity, symmetry and parity of cosine

TIME	STAGE	TEACHER'S ACTIVITIES	LEARNERS' ACTIVITIES
10 minutes	INTRODUCTION	Engage the learners to collaboratively discuss solution to previous lesson homework. Ask learners to predict (sketch) the graph of y=2cos4x for $0^0 \le x \le 360^0$ Use EBM applets to illustrate the graph of y=2cos4x after learners have given their sketches	Learners present key findings from previous homework activity and use EBM applets to justify their conclusions Learners sketch the graph of y=2cos4x without the aid of EBM tools Learners use EBM applet for cosine function to enter values a =2, b=4 and c=0 and compare their sketches to the EBM output of the graph of y=2cos4x, shown below.  *The dotted red horizontal is the midline y = -1.

TIME	STAGE	TEACHER'S ACTIVITIES	LEARNERS' ACTIVITIES
40 minutes	DEVELOPMENT	Use EBM applets tools to facilitate exploration of how the changes to "a", "b" and "c" affect the midline, amplitude and period of y = *acos(bx) + c* Use the EBM applets to facilitate exploration of how the changes to the signs of "a" and "b" affect the graph of y = *acos(bx) + c* Engage learners to use the EBM tools to explore the line and rotational symmetry of the function y = *cosx* Engage the learners to use the EBM tools to deduce the even parity of the cosine function; cos(-x) = cos(x)	Learners should collaboratively discuss the outcomes of the exploration of periodicity, symmetry and parity of the sine function. Learners should observe the graph of y = sinx Learners should be able to deduce that the function y = *cosx*, shown below:  Is symmetric about the y-axis Has no rotational symmetry Has even parity, i.e. cos(-x) = cos(x)

TIME	STAGE	TEACHER'S ACTIVITIES	LEARNERS' ACTIVITIES
10 minutes	CONCLUSION	Recap key points of the lesson and homework orientation Homework: **(a)** The functions f and g are such that $f(x) = \sin x + 1$, for $-90° \leq x \leq 270°$, $g(x) = \cos 2x$, for $-90° \leq x \leq 270°$. Draw sketch graphs of $y = f(x)$ and $y = g(x)$ on the same system of axes. [4] **(b)** Find the range of values where $f(x) \cdot g(x) < 0$ for $-90° \leq x \leq 0°$ [1] Expected EBM output:  $g(x) = \cos 2x$ $f(x) = \sin x + 1$	Learners should attempt answering the homework activity without the aid of EBM applets Learners will present their solution in the next lesson

SUMMARY OF OBSERVED TEACHING AND LEARNING OUTCOMES:

Key achievements in the lesson

Challenges encountered in the lesson

General comments

APPENDIX C4: LESSON PLAN 4

Topic: Periodicity, symmetry and parity of the tangent function

Objectives:

Explore the effects of changes in the values of “a”, “b” and “c” to the graph of y = $a\tan(bx)+c$

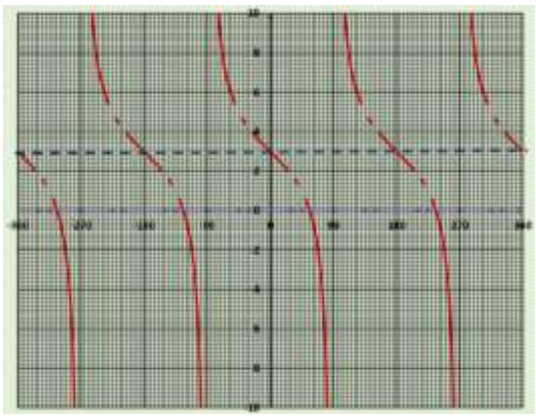
Analyse the symmetry properties of the tangent function

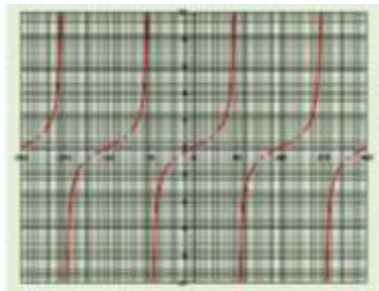
Deduce the parity of the tangent function

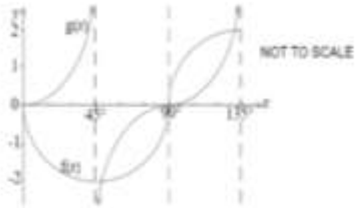
Predict the outputs of tangent graphs for different values of “a”, “b” and “c”

Sketch the graphs of tangent functions

DATE:_________ DURATION:1 hour TOPIC: Periodicity, symmetry and parity of tangent

TIME	STAGE	TEACHER'S ACTIVITIES	LEARNERS' ACTIVITIES
10 minutes	INTRODUCTION	Engage the learners to collaboratively discuss solution to previous lesson homework. Ask learners to predict (sketch) the graph of y=-2tanx+3 for $-360^0 \leq x \leq 360^0$ Use EBM applets to illustrate the graph of y=-2tanx+3 after learners have given their sketches	Learners present key findings from previous homework activity and use EBM applets to justify their conclusions Learners sketch the graph of y=-2tanx+3 without the aid of EBM tools Learners use EBM applet for cosine function to enter values a =-2, b=1 and c=3 and compare their sketches to the EBM output of the graph of y=-2tanx+3, shown below.  *The dotted horizontal is the midline y = 3.

TIME	STAGE	TEACHER’S ACTIVITIES	LEARNERS’ ACTIVITIES
40 minutes	DEVELOPMENT	Use EBM applets tools to facilitate exploration of how the changes to “a”, “b” and “c” affect the midline, amplitude and period of y = *atan(bx)* + *c* Use the EBM applets to facilitate exploration of how the changes to the signs of “a” and “b” affect the graph of y = *atan(bx)* + *c* Engage learners to use the EBM tools to explore the line and rotational symmetry of the function y = *tanx* Engage the learners to use the EBM tools to deduce the odd parity of the tangent function; tan(-x) = -tan(x)	Learners should collaboratively discuss the outcomes of the exploration of periodicity, symmetry and parity of the sine function. Learners should observe the graph of y = tanx Learners should be able to deduce that the function y = *tanx*, shown below:  No line symmetry Has rotational symmetry of order 2 about origin (origin symmetry) Has odd parity, i.e. tan(-x) = -tan(x)

TIME	STAGE	TEACHER'S ACTIVITIES	LEARNERS' ACTIVITIES
10 minutes	CONCLUSION	Recap key points of the lesson and homework orientation Homework: Functions f and g are defined by $f: x \mapsto a \sin bx$, for $0° \le x \le 135°$ and $g: x \mapsto \tan cx$, for $0° \le x \le 135°$.  (a) Find the values of a, b and c. [2] (b) Write down the equation of the reflection of the graph, $g(x)$, in the x-axis. [1]	Learners are given access to EBM applets to assist them in exploring the solution to the homework Learners will present their solution in the next lesson

SUMMARY OF OBSERVED TEACHING AND LEARNING OUTCOMES:

Key achievements in the lesson

Challenges encountered in the lesson

General comments

APPENDIX C5: LESSON PLAN 5

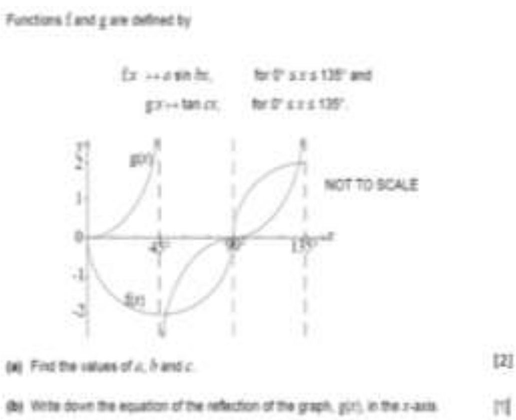
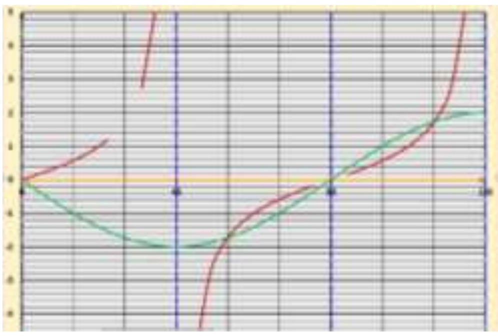
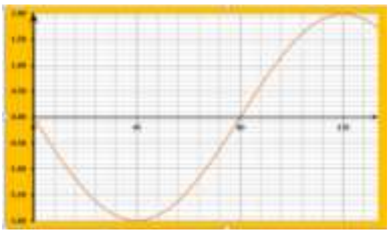
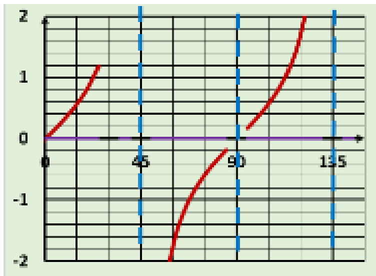
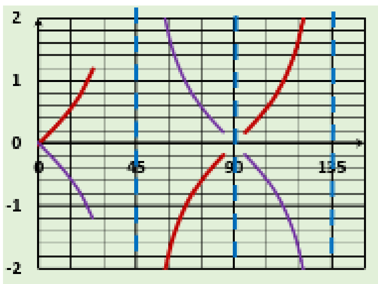
Topic: Deriving a trigonometric function from its graph

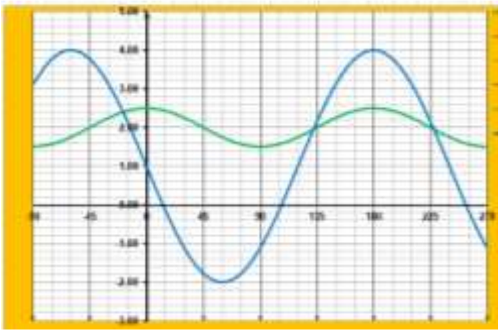
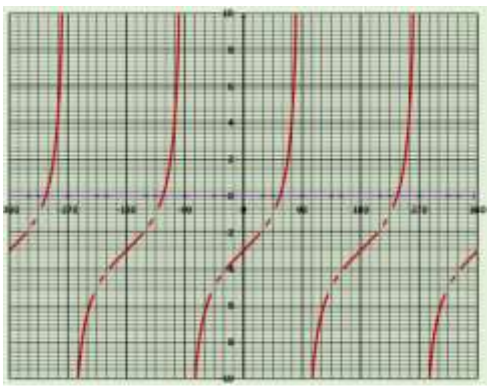
Objectives:

Learners should determine the trigonometric functions corresponding to given graphs

Learners should solve periodicity problems in the teachers' pre-EBM training activity sheet

DATE:_________ DURATION:1 hour TOPIC: Obtaining a function from its graph

TIME	STAGE	TEACHER'S ACTIVITIES	LEARNERS' ACTIVITIES
10 minutes	INTRODUCTION	Engage learners to discuss the homework solution, with the aid of EBM applets  Functions f and g are defined by $f: x \mapsto a \sin bx$, for $0° \le x \le 135°$ and $g: x \mapsto \tan cx$, for $0° \le x \le 135°$. (a) Find the values of a, b and c. [2] (b) Write down the equation of the reflection of the graph, g(x), in the x-axis. [1] EBM reproduction of f(x) and g(x) with a = -2, b = 2 and c = 2 	Learners should observe the following EBM output for *f(x) = -2sin2x* for verification and realise that for a=-2 and b=2, the graphs  match. Learners should observe the following EBM output for *g(x)* = *tan2x* for verification and realise that for c=2, the graphs match.  Learners should observe the following EBM output for *g(x)* = *-tan2x* for verification and realise that the equation is *y=-tan2x* or *y=tan(-2x)* from the odd parity of *y=tanx.* 

TIME	STAGE	TEACHER'S ACTIVITIES	LEARNERS' ACTIVITIES
40 minutes	DEVELOPMENT	Recap how to determine the functions of y=asin(bx)+c and y=acos(bx)+c, by deducing the midline, amplitude and period. Explain with the aid of the video https://youtu.be/vz1QVNL_0Bs how to determine the function of the graph of y=atan(bx)+c and y=acos(bx)+c Explain with the aid of the video https://youtu.be/x_yn02gwnPA how to determine the function of the graph of y=atan(bx)+c Activity 1: Find the equation of the graphs below:  Activity 2: Find the equation of the graph below: 	Learners watch first video and answer activity 1 to determine equations of sine and cosine graphs. Learners watch second video and answer activity 2 on finding the equation of a tangent graph Learners may use EBM applets to verify their solutions

TIME	STAGE	TEACHER’S ACTIVITIES	LEARNERS’ ACTIVITIES
10 minutes	CONCLUSION	Recap key points of the lesson and homework orientation Homework: Learners should, with the aid of EBM tools, collaboratively attempt all the questions in the teachers’ pre-EBM training activity sheet	Learners are given access to EBM applets to assist them in exploring the solution to the homework Learners will present their solution process in the next lesson

SUMMARY OF OBSERVED TEACHING AND LEARNING OUTCOMES:

Key achievements in the lesson

Challenges encountered in the lesson

General comments

APPENDIX C6: LESSON PLAN 6

Topic: EBM supported collaborative problem solving

Objectives:

Learners should collaboratively solve the problems in the teachers' pre-EBM training activity sheet

Learners should present their solution process to be assessed using the CAEMA tool

DATE:_________ DURATION:1 hour TOPIC: Problem solving

TIME	STAGE	TEACHER'S ACTIVITIES	LEARNERS' ACTIVITIES
5 minutes	INTRODUCTION	Engage learners to find out progress on the collaborative problem solving activity given as homework in previous class	
50 minutes	DEVELOPMENT	If activity is completed, ask different learners to present their solution process, if not learners should complete activity in class. Teacher should use the CAEMA tool to assess the collaborative solution process	Learners collaboratively complete activity in class and present their solutions, with the aid of EBM applets as a verification tool
5 minutes	CONCLUSION	Inform learners to prepare for the next activity which will be a written summative test to assess their understanding of periodicity and related concepts	Learners should continue exploring EBM applets in preparation for the test

SUMMARY OF OBSERVED TEACHING AND LEARNING OUTCOMES:

Key achievements in the lesson

Challenges encountered in the lesson

General comments

APPENDIX D: POST - EBM TEACHER PROFESSIONAL DEVELOPMENT TPACK SURVEY QUESTIONNAIRE

Objective: To evaluate level of development of NSSCH Mathematics teachers' technological pedagogical content knowledge after professional development intervention.

Teacher Code: ______________________________

The purpose of this questionnaire is to gain information regarding the level of development of technology, pedagogical practices and content knowledge of NSSCH Mathematics teachers.

Please mark with a cross (X) the correct responses or provide an answer where indicated.

All the information obtained from the questionnaire shall be treated as confidential.

Highest academic qualification: ____

Highest professional qualification: ______________________________________

Number of years teaching NSSCH Mathematics: ____________________________

Item	**Technology Knowledge (TK)** **(1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
1	I have had sufficient opportunities to work with different technologies.					
2	I can create a variety of graphs and charts in Excel					
3	I know about basic computer hardware and their functions					
4	I knowing about basic computer software and their functions					
5	I know how to create formulas in Excel					
6	I know how to use the protection feature in Excel to prevent data entry to a specified range of cells					
7	I keep up with important new technologies.					
8	I have the technical skills to use computers effectively.					
9	I can communicate through Internet tools (e.g., e-mail, MSN Messenger)					
10	I am able to use a presentation program (e.g., MS Powerpoint)					
Item	**Content Knowledge (CK)** **(1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
11	I understand functions and graphs well enough to employ multiple strategies in solving related problems in the NSSCH Mathematics curriculum.					
12	I understand the connection between equations and the graph of the functions in the equations (e.g. trigonmetric equations and the graphs of the trigonmetric functions)					
13	I have knowledge in developing class activities, investigations and projects in mathematics					
14	I follow recent developments and applications in mathematics					
15	I collect and follow up-to-date resources (ex, books, journals) in mathematics					
16	I have the mathematics content knowledge I need to teach my learners in the topic of functions and graphs in the NSSCH Mathematics curriculum					
17	I continue to develop my understanding of mathematics.					
18	I have various ways and strategies of developing my understanding of teaching mathematics					
19	I deliver my mathematics learning content like an expert in the subject					
20	I have sufficient knowledge about teaching mathematics concepts					

Item	**Pedagogical Knowledge (PK)** **(1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
21	I know how to organize and maintain classroom management.					
22	I can adapt my teaching based-upon what learners currently understand or do not understand.					
23	I can use a wide range of teaching approaches in a classroom setting (collaborative learning, direct instruction, inquiry learning, problem/project based learning etc.).					
24	I am able to stretch my learners' thinking by creating challenging tasks for them.					
25	I am able to guide my learners to adopt appropriate learning strategies.					
26	I am able to help my learners to monitor their own learning.					
27	I am able to help my learners to reflect on their learning strategies.					
28	I am able to use different evaluation methods and techniques					
29	I am able to plan group activities for my learners.					
30	I am able to guide my learners to discuss effectively during group work.					
Item	**Pedagogical Content Knowledge (PCK)** **(1: Strongly disagree, 2: Diagree, 3: Neutral, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
31	I know how to select effective teaching approaches to guide student thinking and learning in mathematics.					
32	I can adjust my teaching to make it more inclusive					
33	I know how to develop efficient lessons that will help to ensure that all topics are completed in the required time.					
34	I can develop evaluation tests and surveys in my mathematics teaching practice					
35	I can prepare a good lesson plan including class activities and homework					
36	I am able to meet objectives described in my lesson plan					
37	I can make connections among related concepts in mathematics					
38	I can make connections between mathematics and other related subjects					
39	I know how to select effective teaching approaches to guide my learners to discover concepts in mathematics.					
40	I can help my students to understand the content knowledge of mathematics through various ways.					

Item	**Technological Content Knowledge (TCK)** **(1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
41	I can choose technologies that enhance the content for a lesson on functions and graphs.					
42	I know about technologies that I can use for understanding the relationship between functions and their graphs					
43	I can using content-specific computer applications					
44	I can use technologies to help me to reach curriculum objectives easily in my lesson					
45	I can prepare a lesson plan requiring use of inquiry based instructional technologies					
46	I can develop class activities and projects involving use of instructional technologies					
47	I know about technologies that I can use to promote mathematical inquiry					
48	I know about the technologies that I have to use for the research of content of functions and graphs					
49	I am able to lead learners to discover concepts and mathematical relationships through the use of technology					
50	I can use appropriate technologies (e.g. multimedia resources, simulation) to represent the content of my teaching content.					
Item	**Technological Pedagogical Knowledge (TPK)** **(1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
51	I can adapt the use of the technologies that I am learning about to different mathematics teaching activities.					
52	I can choose technologies that enhance students' learning for a lesson.					
53	I am able to use technology to introduce my students to real world scenarios.					
54	I think deeply about how technology could influence the teaching approaches I use in my classroom					
55	I can create opportunities for learners to use digital technology for individualised learning.					
56	I can create computer based activities that provide immediate feedback to learners					
57	I am able to facilitate my students to use technology to find more information on their own.					
58	I am able to facilitate my learners to use technology to plan and monitor their own learning.					
59	I think critiacally about how to use technology in my classroom					
60	I am able to facilitate my learners to use technology to construct different forms of knowledge representation.					

Item	**Technological Pedagogical Content Knowledge (TPACK)** **(1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
61	I can create technology-enhanced lessons that are learner centred					
62	I can provide leadership in helping others to coordinate the use of content, technologies and teaching approaches at my school and/or region.					
63	I can integrate appropriate instructional methods and technologies into the teaching of functions and graphs					
64	I can select contemporary strategies and technologies that help me to teach the content of functions and graphs effectively					
65	I can teach successfully by combining my content, pedagogy, and technology knowledge					
66	I can teach lessons that appropriately combine mathematics problem solving , technologies and teaching approaches.					
67	I think critically about how to use technology in my classroom.					
68	I can select technologies to use in my classroom that enhance what I teach, how I teach and what learners learn.					
69	I take a leadership role among my colleagues in the integration of content, pedagogy, and technology knowledge					
70	I can use strategies that combine content, technologies and effective teaching approaches in my classroom.					

APPENDIX E: COMPUTER ASSISTED LESSON OBSERVATION TEMPLATE

LESSON OBSERVATION SCHEDULE

Teacher	
School	
Grade	
Number of learners	
Topic	
Lesson Objectves	

Item	Technology Knowledge (TK) (1: Strongly disagree, 2: Diagree, 3: Neutral, 4: Agree, 5: Strongly agree)	Rating	TK Score	Remarks
1	Effectively uses technology	1	70.0%	Level 5
2	Uses technology to engage learners	3		
3	Identifies affordance of technology	5		
4	Uses technology to create a learner centred learning environment	4		
5	Teacher is able to trouble shoot	4		
6	Teacher collaborates with others	4		
Item	**Content Knowledge (CK) (1: Strongly disagree, 2: Diagree, 3: Neutral, 4: Agree, 5: Strongly agree)**	**Rating**	**CK Score**	**Remarks**
7	Exhibits content knowledge, for example providing a definition for a specific concept, using an example to explain a spewcific concept in the content area, etc.	3	68.0%	Level 4
8	Makes connections between and/or across subject areas with the content being taught (i.e. interdisciplinary)	3		
9	The teacher's choice of assessme activities were in line with the specified lesson objectives and syllabus competencies	4		
10	The teacher demonstrated sufficient understanding of the content being taught	5		
11	Responds to learners' content specific questions with accuracy	2		
Item	**Pedagogical Knowledge (PK) (1: Strongly disagree, 2: Diagree, 3: Neutral, 4: Agree, 5: Strongly agree)**	**Rating**	**PK Score**	**Remarks**
12	Uses effective strategies to teach, such as questioning, guided instruction, differentiating instruction	2	40.0%	Level 2
13	Uses effective strategies to manage classroom	2		
14	Facilitates learner centred instruction	2		
15	Uses assessment strategies (on-content)	2		
16	Integrates pedagogical support from colleagues	2		

Item	Pedagogical Content Knowledge (PCK) (1: Strongly disagree, 2: Diagree, 3: Neutral, 4: Agree, 5: Strongly agree)	Rating	PCK Score	Remarks
17	Uses a variety of strategies to teach content	5	100.0%	Level 5
18	Provides specific examples/demonstrations related to content area material to enhance learners'understanding of the topic	5		
19	Elicits learners' knowledge in content area by using content specific teaching strategies such as inquiry questioning techniques	5		
20	Uses effective strategies to engage learners in content learning	5		
21	Uses strategies to faciliate learner centred approach	5		
22	Appropriaytely assesses learners' learning of content	5		
23	Collaborates with other subject colleagues for teaching (before, during or after teaching)	5		
Item	**Technological Content Knowledge (TCK) (1: Strongly disagree, 2: Diagree, 3: Neutral, 4: Agree, 5: Strongly agree)**	**Rating**	**TCK Score**	**Remarks**
24	Uses technology for content teaching and learning	3	60.0%	Level 4
25	Matches the affordance of technology to content being taught	3		
26	The teacher gave learners the opportunity to to explore the relationship between functions and graphs on their own	3		
27	The teacher employed content specific computer modelling applications	3		
28	Uses technology to create an alternative representation of doing a content task	3		
Item	**Technological Pedagogical Knowledge (TPK) (1: Strongly disagree, 2: Diagree, 3: Neutral, 4: Agree, 5: Strongly agree)**	**Rating**	**TPK Score**	**Remarks**
29	Prepares instructional materials with technology	3	60.0%	Level 4
30	Uses strategies to demonstrate how to use technology, such as providing instructions, modeling the use etc.	3		
31	Uses technology effectively to engage learners in learning	3		
32	Matches technology with pedagogy	3		
33	Is able to trouble shot while managing the classroom	3		
34	Involves learners in the teaching role	3		
Item	**Technological Pedagogical Content Knowledge (TPACK) (1: Strongly disagree, 2: Diagree, 3: Neutral, 4: Agree, 5: Strongly agree)**	**Rating**	**TPACK Score**	**Remarks**
35	Uses teaching strategies to faciliatate learning in content area with technology	4	80.0%	Level 5
36	Assesses learners' learning with technology in content area	4		
37	Uses technology appropriately for learner centred content learning	4		
38	Uses resources (e.g content & Technology) that have been pre-developed to teach	4		
39	Uses technology to engage learners in content learning	4		
40	Matches the technology with the content being taught and the teaching strategies used	4		

Item	Learners' Learning Outcomes (1: Strongly disagree, 2: Diagree, 3: Neutral, 4: Agree, 5: Strongly agree)	Rating	GL5 Score	Remarks
41	The Excel modelling approach enhanced the learners' understanding of functions and graphs	3	74.0%	Level 5
42	The TPACK - IBL lessons allowed learners to understand concepts better through discovery	4		
43	Excel modelling allowed learners to create their own knowledge through a hands-on inquiry approach	2		
44	The learners demonstrated satisfaction and fun with the TPACK-IBL approach	4		
45	Learners had an opportunity to collaborate in solving problems on functions and graphs.	3		
46	Learners were motivated and participated actively during the TPACK-IBL lessons	5		
47	Learners could justify their solutions through the observations made with the Excel models	4		
48	Learners managed to search for algebraic and graphic relationships on their own	4		
49	Learners perfomed well in the asessment activities on functions and graphs	4		
50	The teacher successfully employed Excel modelling to promote a learner-centred approach	4		

APPENDIX F: ANALYSIS OF CORRELATIONS BETWEEN TPACK SELF-EFFICACY (TSE), OBSERVED TPACK (OT) AND LEARNERS CONCEPTUAL UNDERSTNDING (LCU)

CORRELATION BETWEEN TPACK SELF EFFICACY (TSE) AND OBSERVED TPACK (OT)

TEACHER CODE	TSE	OT
S1T04	3.90	4.00
S1T11	3.90	4.00
S2T09	3.60	3.00
S2T10	4.00	4.00
S3T08	4.10	3.00
S4T07	4.00	4.00
S5T06	4.30	5.00
S6T03	4.00	3.00
S6T05	3.70	4.00
S7T02	3.80	4.00
S8T01	4.30	4.00
PEARSON CORRELATION		0.40

CORRELATION BETWEEN TPACK SELF EFFICACY ((TSE) AND LCU

TEACHER CODE	TSE	LCU
S1T04	3.90	76.71
S1T11	3.90	76.75
S2T09	3.60	75.00
S2T10	4.00	78.02
S3T08	4.10	70.83
S4T07	4.00	77.08
S5T06	4.30	79.16
S6T03	4.00	74.17
S6T05	3.70	80.83
S7T02	3.80	76.67
S8T01	4.30	83.79
PEARSON CORRELATION		0.25

CORRELATION BETWEEN OBSERVED TPACK AND LCU

TEACHER CODE	OT	LCU
S1T04	4.00	76.71
S1T11	4.00	76.75
S2T09	3.00	75.00
S2T10	4.00	78.02
S3T08	3.00	70.83
S4T07	4.00	77.08
S5T06	5.00	79.16
S6T03	3.00	74.17
S6T05	4.00	80.83
S7T02	4.00	76.67
S8T01	4.00	83.79
PEARSON CORRELATION		0.66

APPENDIX G: POST EBM-INSTRUCTION INTERVIEW SCHEDULE FOR LEARNERS *(video-taped)*

Objective: The interview consists of post observation questions, based on the observed teaching and learning outcomes, for further clarification.

Briefly explain how the Excel modelling influenced your understanding of the topic of trigonometric functions and graphs in the Higher Level Mathematics curriculum?

Did the teacher demonstrate the Excel modelling or did you also get an opportunity to explore on your own, the connections between functions and graphs?

How do you see your preparedness for the final NSSCH Mathematics examination, in particular with respect to the questions on trigonometric functions?

Did you at any point work as a group and if so, briefly explain the activities you did?

Would you recommend this type of learning with technology to other learners and why?

What were the challenges you encountered in the TPACK-IBL lessons?

What else do you think is important for us to know which is important in supporting learners to learn with technology?

APPENDIX H : TEACHER POST - EBM INSTRUCTION EVALUATION QUESTIONNAIRE

Objective: To evaluate the EBM teacher professional development programme using Guskey's (2000) model

Teacher Code: ________________________________

The purpose of this questionnaire is to gain information regarding your evaluation of the TPACK-IBL professional development training and implementation.

Please mark with a cross (X) the correct responses or provide an answer where indicated.

All the information obtained from the questionnaire shall be treated as confidential.

Item	**Participants' Reactions (Level 1)** **(1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
1	I have been more inspired towards using Excel modelling in more areas of the Mathematics curriculum					
2	The trainer gave us enough time to ask questions and answered them well					
3	The length/duration of the workshop was appropriate					
4	The timing of the workshop was appropriate					
5	The information and activities presented were relevant and useful					
6	The objectives of the training programme were clear and relevant to my professional development needs					
7	The Excel modelling skills were relevant to my work					
8	The materials/resources/handouts provided were useful					
9	The trainer was knowledgeable, approachable and helpful during the sessions					
10	The group discussions were fruitful and productive					

Item	**Participants' learning (Level 2)** **(1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
11	The training programmme improved my understanding of the subject content and helped me to be better prepared in my teaching					
12	I now understand functions and graphs well enough to employ multiple strategies in solving related problems in the NSSCH Mathematics curriculum					
13	I now understand better the connection between equations and the graphs of the functions					
14	I am confident that I can now deliver the learning content on graphs and functions like an expert in the subject					
15	The training increased my knowledge and skills in the teaching of fcunctions and graphs					
16	The mathematics content knowledge I need to teach my learners in the topic of functions and graphs in the NSSCH mathematics curriculum has increased					
17	I now understand better the graphical interpretation of trigonometric functions					
18	My level of confidence in the use of Excel modelling has increased					
19	I am convinced that the Excel modelling approach will make my teaching of functions and graphs easier					
20	The Excel modelling approach led me to reflect more critically on my teaching					

Item	**School support and change (Level 3)** **(1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
21	The school management was aware that I was trying out a new teaching approach					
22	The school made provision for all the necessary resources to implement the Excel modelling instruction with my learners					
23	I informed my colleagues so that they could be part of my lessons to observe the implementation of the Excel modelling approach					
24	My supervisor was available and willing to assist me with any challenges I encountered during the implementation of the Excel modelling approach					
25	My trainer and officers from the regional office regularly visited the school to support me with the Excel modelling programme implementation					
26	I could easily access my trainer whenever I needed advice during implementation of the Excel modelling instruction					
27	My trainer was always available and willing to support me whenever I encountered challenges during the implemeantation of the TPACK-IBL instruction					
28	I regularly met with my trainer to review the progress in the implementation of the new instructional approach					
29	My trainer observed some of the lessons I taught and constructively discussed with me the outcomes of the lessons					
30	I had clear guidelines on conducting a self-review and evaluation of my TPACK-IBL lesson outcomes					
Item	**Participants use of new knowledge and skills (Level 4)** **(1: Strongly disagree, 2: Diagree, 3: Neutral, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
31	I confidently used the Excel modelling approach to teach the content of trigonometric functions and graphs					
32	The Excel modelling instruction enhanced my learners' understanding of the relationship between trigonometric functions and their graphs					
33	I am confident that in future I will be able to develop my own Excel modelling tools in the teaching of various topics					
34	I observed possible ways through which the implementation of the TPACK-IBL approach can be improved					
35	I effectively applied the knowledge and skills from the TPACK-IBL professional development programme					
36	I am confident that I have acquired sufficient knowledge and skills to enable me to train other NSSCH teachers in the TPACK-IBL instruction					
37	The Excel tools enhanced my broader understanding of the connections between trigonometric functions, graphs and equations					
38	Through Excel modelling, I am now able to explore more relationships bwteen the graphic and and algebraic representations of trigonometric functions					
39	I was able to guide learners to discover concepts and algebraic relationships between trigonometric functions and their graphs					
40	I am able to demonstrate the use of Excel tools to understand the interpretation of graphs of trigonometric functions					

Item	**Learners' learning outcomes** **(1: Strongly disagree, 2: Diagree, 3: Uncertain, 4: Agree, 5: Strongly agree)**	SD	D	U	A	SA
41	The Excel modelling approach enhanced my learners' understanding of functions and graphs and helped them to be better prepared for examinations					
42	The TPACK-IBL lessons allowed my learners to understand concepts better through discovery					
43	Excel modelling of functions and graphs allowed learners to create their own knowledge through a hands-on inquiry approach					
44	The learners demonstrated satisfaction and fun with the TPACK-IBL approach					
45	Learners had an opportunity to collaborate in solving problems on functions and graphs					
46	Learners were motivated and participated actively during the TPACK-IBL lessons					
47	Learners could justify their solutions through the observations made with the Excel models					
48	Learners successfully managed to serach for algebraic and graphic relationships on their own					
49	Learners perfomed well in the assessment activities on functions and graphs					
50	Excel modelling promoted a learner centred approach in the teaching and learning of functions and graphs					

Any other comments on your TPACK – IBL experience:

APPENDIX I: LEARNERS' SUMMATIVE TEST

PERIODICITY OF TRIGONOMETRIC FUNCTIONS

DURATION: 1 HOUR

SCHOOL NAME: ___________________________________

LEARNER CODE: __________ DATE:____________________

Instructions:

Attempt to answer all the questions in the spaces provided.

If any working is needed, use the spaces provided.

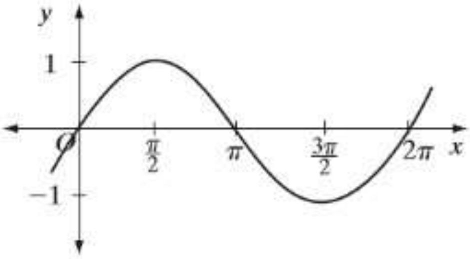
The total marks for the test is 24. Each question carries 3 marks.

Graphic calculators are not allowed.

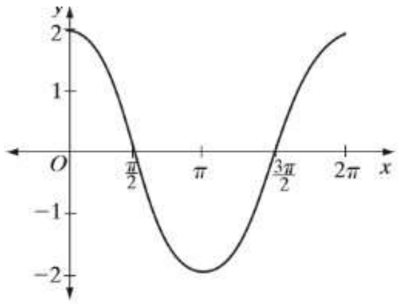
This question paper consists of 3 pages

For each of the following write the equation of the graph on the solid line underlined below each graph.

(b)

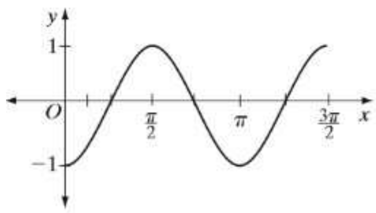


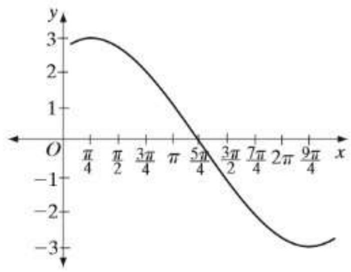
(a)



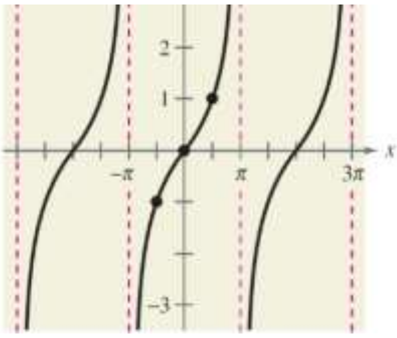
(c)

(d)

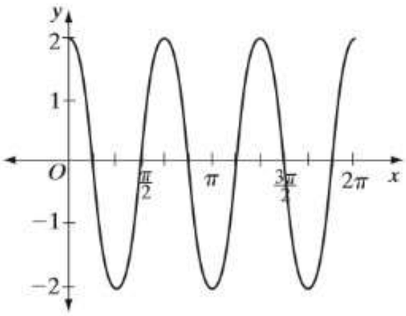




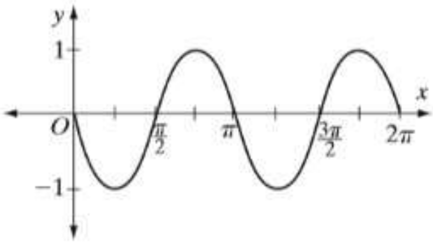
(e)


2
1
−3
−π
π
3π
x

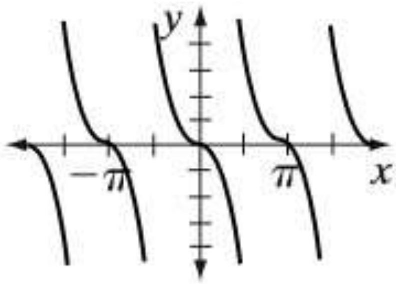
(f)


y
2
1
−1
−2
O
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
x

(g)


y
1
−1
O
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
x

(h)


y
−π
π
x

APPENDIX J: CHI-SQUARE TEST FOR INDEPENDENCE OF PRE AND POST-EBM SELF-EFFICACY RATINGS OF TPACK CONSTRUCTS (p = 0.050)

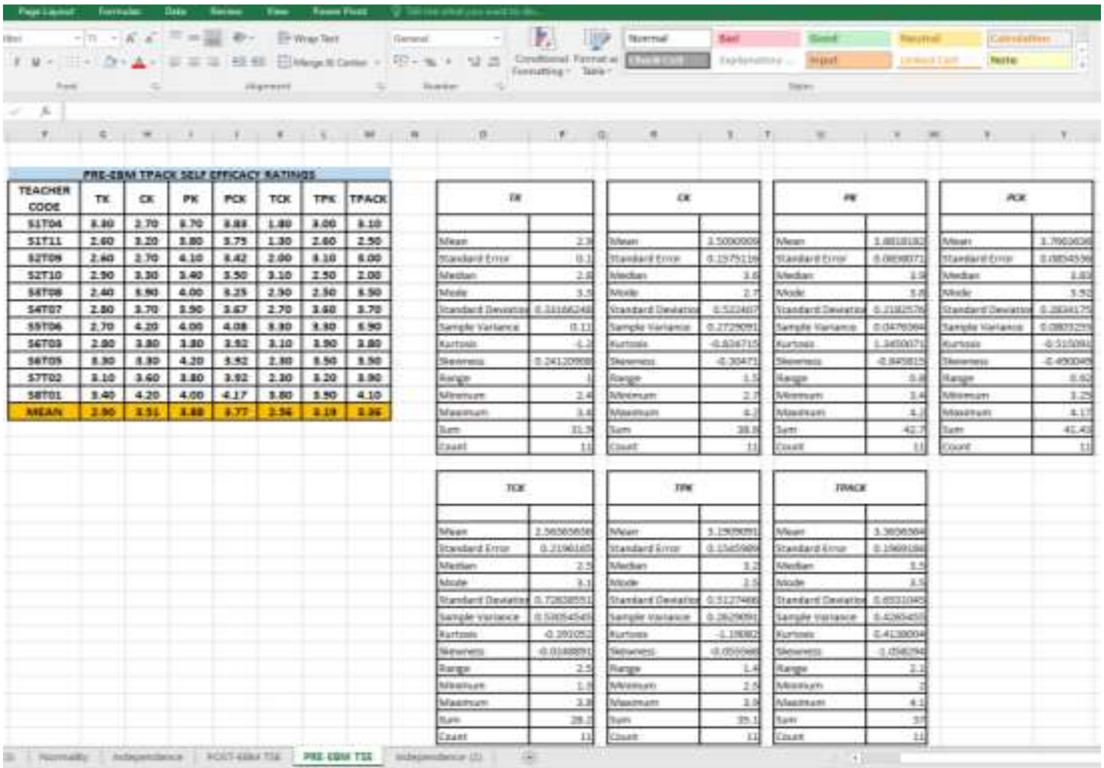
TEACHER CODE	PRE-EBM	POST-EBM	TOTAL
S1T04	3.10	4.00	7.10
S1T11	2.50	4.00	6.50
S2T09	3.00	4.00	7.00
S2T10	2.00	4.00	6.00
S3T08	3.50	4.00	7.50
S4T07	3.70	4.00	7.70
S5T06	3.90	4.10	8.00
S6T03	3.80	4.00	7.80
S6T05	3.50	4.10	7.60
S7T02	3.90	4.00	7.90
S8T01	4.10	5.00	9.10
TOTAL	37.00	45.20	82.20

O-E	PRE-EBM	POST-EBM
S1T04	0.00288	0.00235
S1T11	0.06197	0.05072
S2T09	0.00722	0.00591
S2T10	0.18181	0.14883
S3T08	0.00456	0.00373
S4T07	0.01581	0.01294
S5T06	0.02483	0.02033
S6T03	0.02380	0.01948
S6T05	0.00183	0.00150
S7T02	0.03329	0.02725
S8T01	0.00000	0.00000

EXPECTED	PRE-EBM	POST-EBM
S1T04	3.20	3.90
S1T11	2.93	3.57
S2T09	3.15	3.85
S2T10	2.70	3.30
S3T08	3.38	4.12
S4T07	3.47	4.23
S5T06	3.60	4.40
S6T03	3.51	4.29
S6T05	3.42	4.18
S7T02	3.56	4.34
S8T01	4.10	5.00

Significance	0.999976755
Pearson Chi square	0.65103
Df	10
Significance	0.999976755
Critical value	18.307

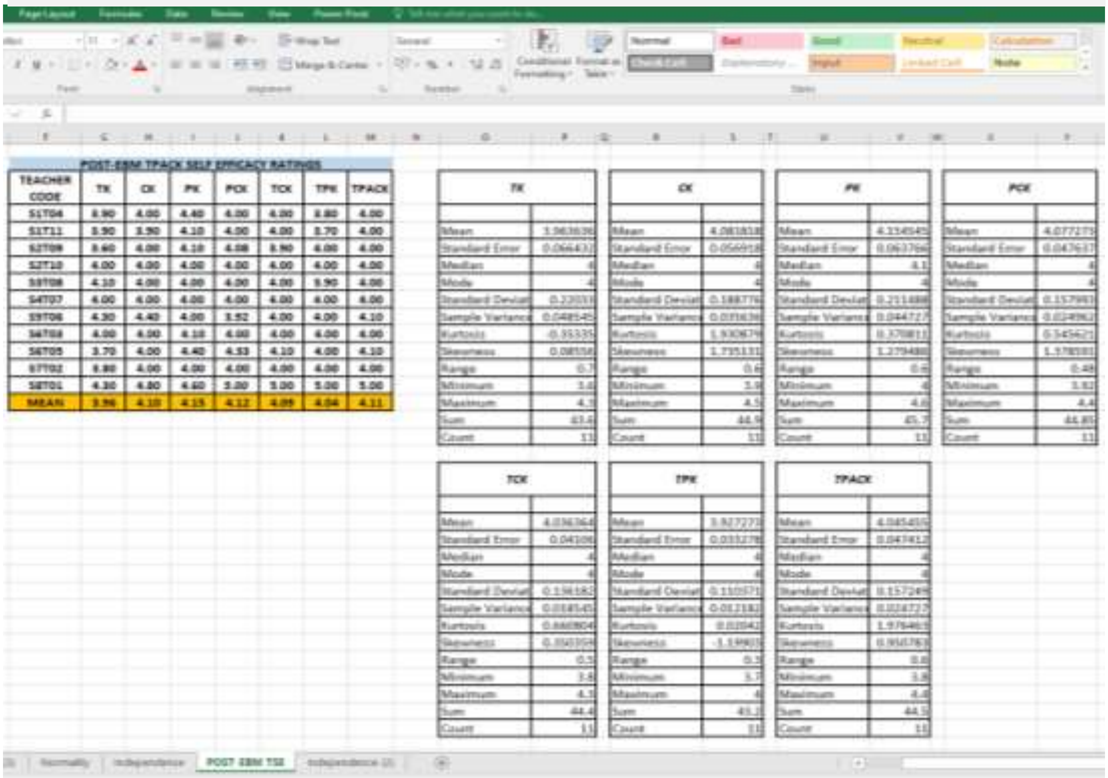
APPENDIX K: DESCRIPTIVE STATISTICS FOR ANALYSIS OF NORMALITY OF PRE AND POST-EBM SELF-EFFICACY



PRE-EBM TPACK SELF EFFICACY RATINGS

TEACHER CODE	TK	CK	PK	PCK	TCK	TPK	TPACK
S1T04	3.30	2.70	3.70	3.83	1.80	3.00	3.10
S1T11	2.60	3.20	3.80	3.75	1.30	2.60	2.50
S2T09	2.60	2.70	4.10	3.42	2.00	3.10	3.00
S2T10	2.90	3.30	3.40	3.50	3.10	2.50	2.00
S3T08	2.40	3.90	4.00	3.25	2.90	2.30	3.50
S4T07	2.80	3.70	3.90	3.67	2.70	3.60	3.70
S5T06	2.70	4.20	4.00	4.08	3.30	3.30	3.90
S6T03	2.80	3.80	3.80	3.92	3.10	3.90	3.80
S6T05	3.30	3.30	4.20	3.92	2.30	3.50	3.50
S7T02	3.10	3.60	3.80	3.92	2.30	3.20	3.90
S8T01	3.40	4.20	4.00	4.17	3.80	3.50	4.10
MEAN	2.90	3.51	3.88	3.77	2.56	3.19	3.36

	TK	CK	PK	PCK	TCK	TPK	TPACK
Mean	2.9	3.50909090	3.88181818	3.7663636	2.56363636	3.19090909	3.36363636
Standard Error	0.1	0.1575116	0.0658071	0.0854536	0.2196165	0.1545989	0.1969184
Median	2.8	3.6	3.9	3.83	2.5	3.2	3.5
Mode	3.3	2.7	3.8	3.92	3.1	3.5	3.5
Standard Deviation	0.33166248	0.522407	0.2182576	0.2834175	0.72838551	0.5127466	0.6531045
Sample Variance	0.11	0.2729091	0.0476364	0.0803255	0.53054545	0.2629091	0.4265455
Kurtosis	-1.2	-0.826715	1.3450071	-0.515091	-0.391052	-1.19082	0.4138004
Skewness	0.24120908	-0.30471	-0.845815	-0.490045	-0.0148891	-0.055948	-1.058294
Range	1	1.5	0.8	0.92	2.5	1.4	2.1
Minimum	2.4	2.7	3.4	3.25	1.3	2.5	2
Maximum	3.4	4.2	4.2	4.17	3.8	3.9	4.1
Sum	31.9	38.6	42.7	41.43	28.2	35.1	37
Count	11	11	11	11	11	11	11

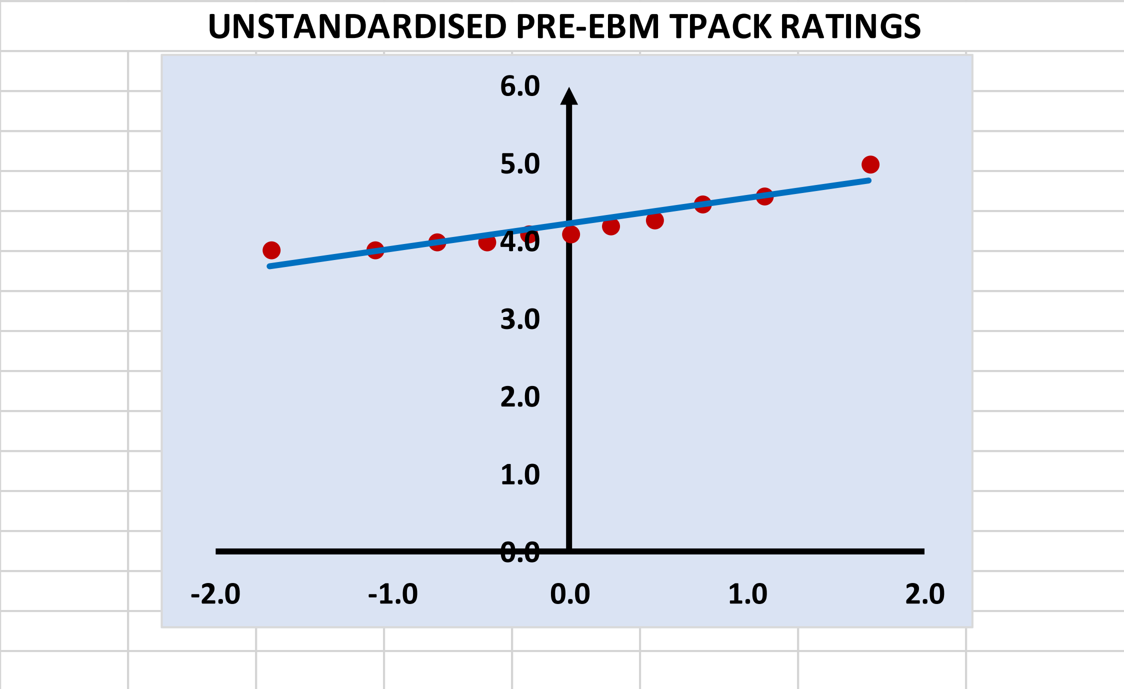


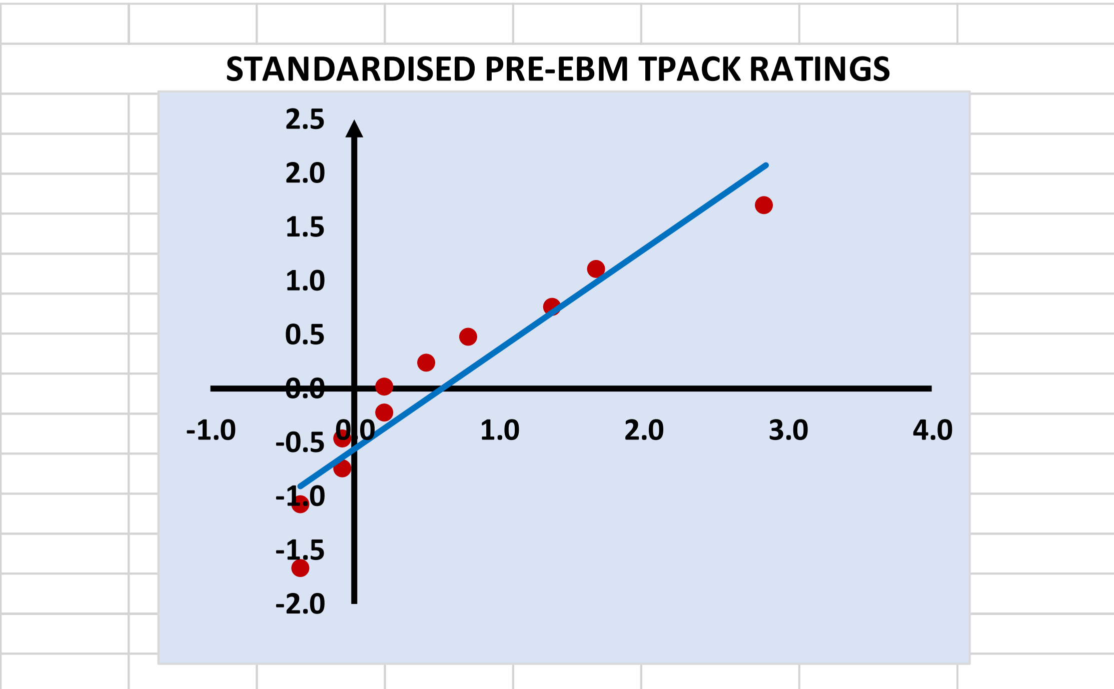
POST-EBM TPACK SELF EFFICACY RATINGS

TEACHER CODE	TK	CK	PK	PCK	TCK	TPK	TPACK
S1T04	3.90	4.00	4.40	4.00	4.00	3.80	4.00
S1T11	3.90	3.90	4.10	4.00	4.00	3.70	4.00
S2T09	3.60	4.00	4.10	4.08	3.90	4.00	4.00
S2T10	4.00	4.00	4.00	4.00	4.00	4.00	4.00
S3T08	4.10	4.00	4.00	4.00	4.00	3.90	4.00
S4T07	4.00	4.00	4.00	4.00	4.00	4.00	4.00
S5T06	4.30	4.40	4.00	3.92	4.00	4.00	4.10
S6T03	4.00	4.00	4.10	4.00	4.00	4.00	4.00
S6T05	3.70	4.00	4.40	4.33	4.10	4.00	4.10
S7T02	3.80	4.00	4.00	4.00	4.00	4.00	4.00
S8T01	4.30	4.80	4.60	5.00	5.00	5.00	5.00
MEAN	3.96	4.10	4.15	4.12	4.09	4.04	4.11

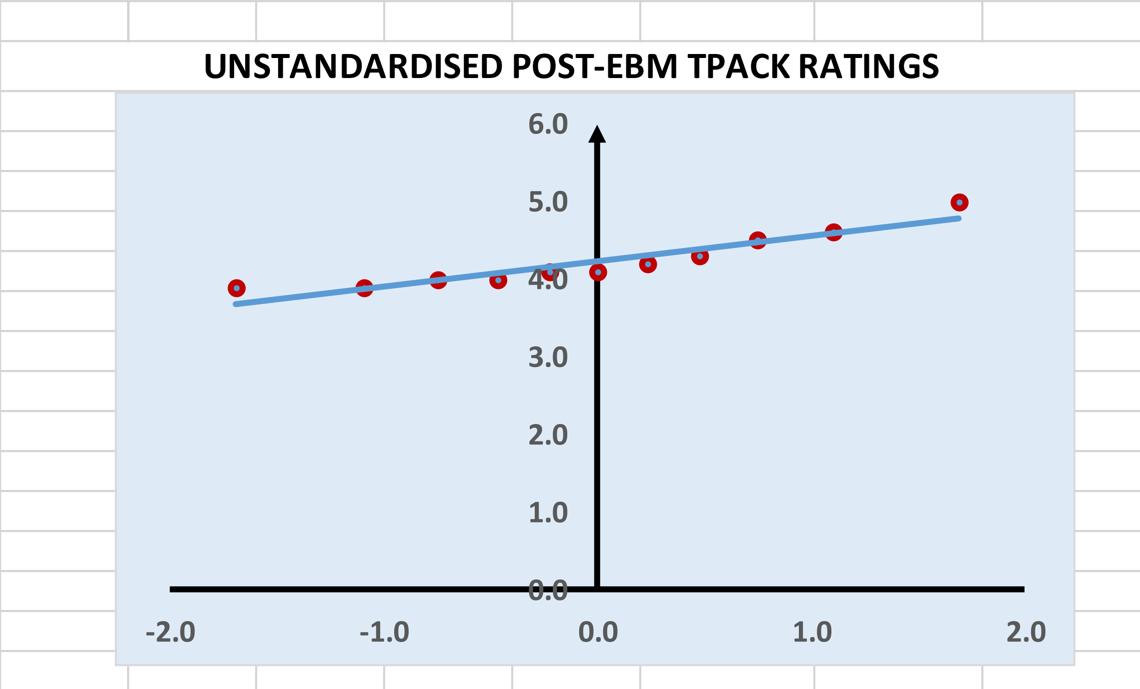
	TK	CK	PK	PCK	TCK	TPK	TPACK
Mean	3.963636	4.081818	4.154545	4.077273	4.036364	3.927273	4.045455
Standard Error	0.066432	0.056918	0.063766	0.047637	0.04106	0.033278	0.047412
Median	4	4	4.1	4	4	4	4
Mode	4	4	4	4	4	4	4
Standard Deviation	0.22033	0.188776	0.211488	0.157993	0.136182	0.110371	0.157249
Sample Variance	0.048545	0.035636	0.044727	0.024962	0.018545	0.012182	0.024727
Kurtosis	-0.35335	1.930679	0.370811	0.545621	0.660804	0.02042	1.976461
Skewness	0.08556	1.735131	1.279488	1.378591	0.350359	-1.19903	0.950763
Range	0.7	0.6	0.6	0.48	0.5	0.3	0.6
Minimum	3.6	3.9	4	3.92	3.8	3.7	3.8
Maximum	4.3	4.5	4.6	4.4	4.3	4	4.4
Sum	43.6	44.9	45.7	44.85	44.4	43.2	44.5
Count	11	11	11	11	11	11	11

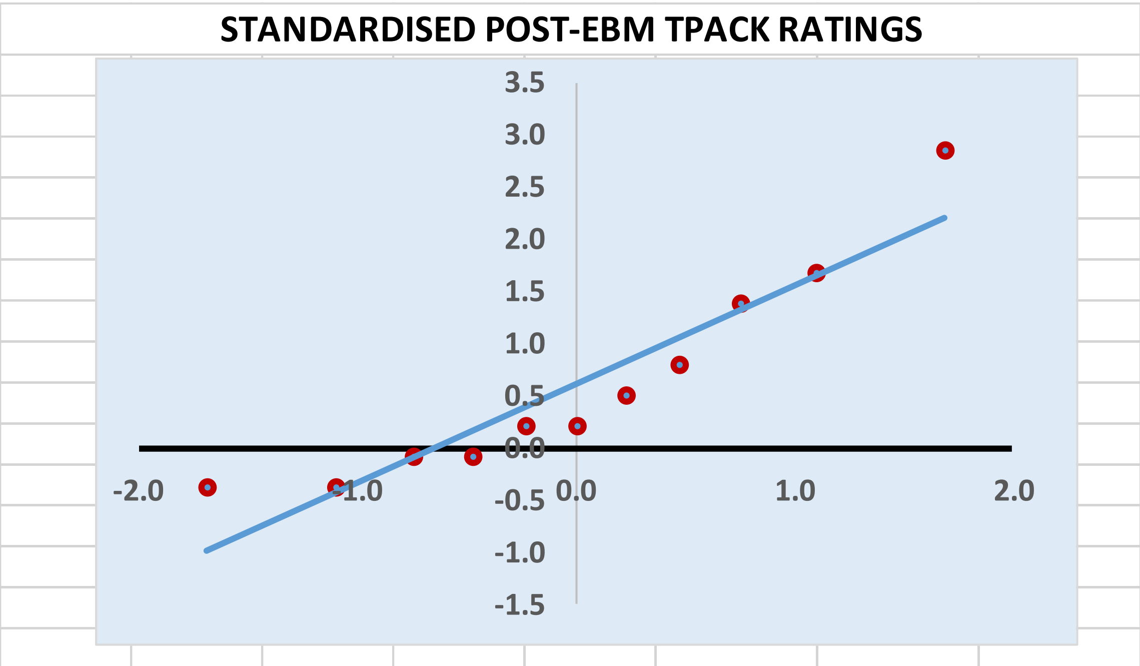
APPENDIX L1: NORMAL QUANTILE-QUANTILE PLOTS FOR TEST OF NORMALITY OF DATA FROM PRE-EBM RATING OF TPACK SELF EFFICACY





APPENDIX L2: NORMAL QUANTILE-QUANTILE PLOTS FOR TEST OF NORMALITY OF DATA FROM POST-EBM RATING OF TPACK SELF-EFFICACY





APPENDIX L3: SPSS OUTPUT FOR PLS-SEM ANALYSIS

Your temporary usage period for IBM SPSS Statistics will expire in 5853 days.

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS R ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT TCK
  /METHOD=ENTER TK.
```

Regression

Notes

Output Created		22-DEC-2019 16:28:38
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.

	Cases Used	Statistics are based on cases with no missing values for any variable used.
Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT TCK /METHOD=ENTER TK.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.11
	Memory Required	2640 bytes
	Additional Memory Required for Residual Plots	0 bytes

[DataSet0]

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	TK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.441	.194	.105	.27363

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.163	1	.163	2.171	.175
	Residual	.674	9	.075		
	Total	.836	10			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	2.437	1.187		2.053	.070
	TK	.466	.316	.441	1.473	.175

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS R ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT TPK
  /METHOD=ENTER TK.
```

Regression

Notes

Output Created		22-DEC-2019 16:29:44
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any variable used.

Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT TPK /METHOD=ENTER TK.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.02
	Memory Required	2640 bytes
	Additional Memory Required for Residual Plots	0 bytes

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	TK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.400	.160	.067	.16643

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.048	1	.048	1.716	.223
	Residual	.249	9	.028		
	Total	.297	10			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	3.164	.722		4.382	.002
	TK	.252	.192	.400	1.310	.223

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS R ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT TCK
  /METHOD=ENTER CK.
```

Regression

Notes

Output Created		22-DEC-2019 16:30:40
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any variable used.

Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT TCK /METHOD=ENTER CK.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.02
	Memory Required	2640 bytes
	Additional Memory Required for Residual Plots	0 bytes

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	CK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.438	.192	.102	.27406

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.160	1	.160	2.135	.178
	Residual	.676	9	.075		
	Total	.836	10			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	2.456	1.184		2.074	.068
	CK	.420	.287	.438	1.461	.178

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS R ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT PCK
  /METHOD=ENTER CK.
```

Regression

Notes

Output Created		22-DEC-2019 16:31:48
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any variable used.

Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT PCK /METHOD=ENTER CK.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.02
	Memory Required	2640 bytes
	Additional Memory Required for Residual Plots	0 bytes

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	CK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.624	.389	.321	.16733

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.160	1	.160	5.727	.040
	Residual	.252	9	.028		
	Total	.412	10			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	2.456	.723		3.397	.008
	CK	.420	.175	.624	2.393	.040

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS R ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT TPK
  /METHOD=ENTER PK.
```

Regression

Notes

Output Created		22-DEC-2019 16:32:57
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any variable used.

Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT TPK /METHOD=ENTER PK.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.01
	Memory Required	2640 bytes
	Additional Memory Required for Residual Plots	0 bytes

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	PK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.544	.296	.218	.15236

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.088	1	.088	3.786	.084
	Residual	.209	9	.023		
	Total	.297	10			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	2.805	.671		4.183	.002
	PK	.330	.170	.544	1.946	.084

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS R ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT PCK
  /METHOD=ENTER PK.
```

Regression

Notes

Output Created		22-DEC-2019 16:33:41
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any variable used.

Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT PCK /METHOD=ENTER PK.
Resources	Processor Time	00:00:00.02
	Elapsed Time	00:00:00.01
	Memory Required	2640 bytes
	Additional Memory Required for Residual Plots	0 bytes

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	PK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.640	.409	.344	.16452

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.169	1	.169	6.234	.034
	Residual	.244	9	.027		
	Total	.412	10			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	2.378	.724		3.284	.009
	PK	.457	.183	.640	2.497	.034

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS R ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT TCK
  /METHOD=ENTER TK CK.
```

Regression

Notes

Output Created		22-DEC-2019 16:34:41
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any variable used.

Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT TCK /METHOD=ENTER TK CK.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.02
	Memory Required	3088 bytes
	Additional Memory Required for Residual Plots	0 bytes

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	CK, TK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.488	.238	.047	.28226

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.199	2	.099	1.249	.337
	Residual	.637	8	.080		
	Total	.836	10			

Coefficients

Model		Unstandardized Coefficients B	Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	2.043	1.356		1.507	.170
	TK	.290	.417	.275	.696	.506
	CK	.256	.378	.267	.676	.518

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS R ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT TPK
  /METHOD=ENTER TK PK.
```

Regression

Notes

Output Created		22-DEC-2019 16:35:42
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any variable used.

Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT TPK /METHOD=ENTER TK PK.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.02
	Memory Required	3088 bytes
	Additional Memory Required for Residual Plots	0 bytes

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	PK, TK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.546	.298	.123	.16136

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.089	2	.044	1.700	.243
	Residual	.208	8	.026		
	Total	.297	10			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	2.841	.746		3.809	.005
	TK	-.047	.303	-.075	-.156	.880
	PK	.366	.292	.603	1.254	.245

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS R ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT PCK
  /METHOD=ENTER CK PK.
```

Regression

Notes

Output Created		22-DEC-2019 16:36:21
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any variable used.

Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT PCK /METHOD=ENTER CK PK.
Resources	Processor Time	00:00:00.05
	Elapsed Time	00:00:00.02
	Memory Required	3088 bytes
	Additional Memory Required for Residual Plots	0 bytes

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	PK, CK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.767	.588	.486	.14565

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.243	2	.121	5.719	.029
	Residual	.170	8	.021		
	Total	.412	10			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	1.580	.771		2.049	.075
	CK	.305	.164	.453	1.866	.099
	PK	.342	.174	.478	1.970	.084

REGRESSION

/MISSING LISTWISE

```
/STATISTICS COEFF OUTS R ANOVA
/CRITERIA=PIN(.05) POUT(.10)
/NOORIGIN
/DEPENDENT TPACK
/METHOD=ENTER TCK.
```

Regression

Notes

Output Created		22-DEC-2019 16:37:24
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any variable used.

Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT TPACK /METHOD=ENTER TCK.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.02
	Memory Required	2640 bytes
	Additional Memory Required for Residual Plots	0 bytes

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	TCK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.686	.470	.411	.27565

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.606	1	.606	7.981	.020
	Residual	.684	9	.076		
	Total	1.290	10			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	.530	1.263		.420	.685
	TCK	.852	.301	.686	2.825	.020

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS R ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT TPACK
  /METHOD=ENTER TPK.
```

Regression

Notes

Output Created		22-DEC-2019 16:38:08
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any variable used.

Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT TPACK /METHOD=ENTER TPK.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.02
	Memory Required	2640 bytes
	Additional Memory Required for Residual Plots	0 bytes

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	TPK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.635	.403	.336	.29262

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.520	1	.520	6.068	.036
	Residual	.771	9	.086		
	Total	1.290	10			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	-1.344	2.208		-.609	.558
	TPK	1.323	.537	.635	2.463	.036

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS R ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT TPACK
  /METHOD=ENTER PCK.
```

Regression

Notes

Output Created		22-DEC-2019 16:38:50
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any variable used.

Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT TPACK /METHOD=ENTER PCK.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.02
	Memory Required	2640 bytes
	Additional Memory Required for Residual Plots	0 bytes

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	PCK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.541	.293	.214	.31846

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.378	1	.378	3.723	.086
	Residual	.913	9	.101		
	Total	1.290	10			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	.089	2.076		.043	.967
	PCK	.957	.496	.541	1.930	.086

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS R ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT TPACK
  /METHOD=ENTER PCK TCK TPK.
```

Regression

Notes

Output Created		22-DEC-2019 16:39:57
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any variable used.

Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT TPACK /METHOD=ENTER PCK TCK TPK.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.02
	Memory Required	3616 bytes
	Additional Memory Required for Residual Plots	0 bytes

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	TPK, TCK, PCK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.813	.661	.515	.25014

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.852	3	.284	4.540	.045
	Residual	.438	7	.063		
	Total	1.290	10			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	-2.393	1.991		-1.202	.268
	PCK	.772	.525	.436	1.471	.185
	TCK	.757	.353	.609	2.147	.069
	TPK	.023	.748	.011	.030	.977

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS R ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT LLO
  /METHOD=ENTER TPACK.
```

Regression

Notes

Output Created		22-DEC-2019 16:40:58
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any variable used.

Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT LLO /METHOD=ENTER TPACK.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.02
	Memory Required	2640 bytes
	Additional Memory Required for Residual Plots	0 bytes

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	TPACK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.688	.473	.414	.13070

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.138	1	.138	8.077	.019
	Residual	.154	9	.017		
	Total	.292	10			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	2.521	.472		5.338	.000
	TPACK	.327	.115	.688	2.842	.019

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS R ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT TPACK
  /METHOD=ENTER TK.
```

Regression

Notes

Output Created		22-DEC-2019 16:42:18
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any variable used.

Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT TPACK /METHOD=ENTER TK.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.02
	Memory Required	2640 bytes
	Additional Memory Required for Residual Plots	0 bytes

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	TK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.602	.362	.291	.30238

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.467	1	.467	5.111	.050
	Residual	.823	9	.091		
	Total	1.290	10			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	1.132	1.312		.863	.411
	TK	.790	.350	.602	2.261	.050

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS R ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT TPACK
  /METHOD=ENTER CK.
```

Regression

Notes

Output Created		22-DEC-2019 16:42:58
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any variable used.

Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT TPACK /METHOD=ENTER CK.
Resources	Processor Time	00:00:00.02
	Elapsed Time	00:00:00.03
	Memory Required	2640 bytes
	Additional Memory Required for Residual Plots	0 bytes

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	CK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.544	.296	.217	.31777

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.381	1	.381	3.778	.084
	Residual	.909	9	.101		
	Total	1.290	10			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	1.429	1.373		1.041	.325
	CK	.648	.333	.544	1.944	.084

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF OUTS R ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT TPACK
  /METHOD=ENTER PK.
```

Regression

Notes

Output Created		22-DEC-2019 16:43:43
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on cases with no missing values for any variable used.

Syntax		REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT TPACK /METHOD=ENTER PK.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.02
	Memory Required	2640 bytes
	Additional Memory Required for Residual Plots	0 bytes

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	PK	.	Enter

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.445	.198	.109	.33901

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.256	1	.256	2.227	.170
	Residual	1.034	9	.115		
	Total	1.290	10			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	1.869	1.492		1.253	.242
	PK	.563	.377	.445	1.492	.170

CORRELATIONS

/VARIABLES=TK CK

/PRINT=TWOTAIL NOSIG

/MISSING=PAIRWISE.

Correlations

Notes

Output Created		22-DEC-2019 16:44:45
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics for each pair of variables are based on all the cases with valid data for that pair.
Syntax		CORRELATIONS /VARIABLES=TK CK /PRINT=TWOTAIL NOSIG /MISSING=PAIRWISE.
Resources	Processor Time	00:00:00.03
	Elapsed Time	00:00:00.02

Correlations

		TK	CK
TK	Pearson Correlation	1	.623
	Sig. (2-tailed)		.041
	N	11	11
CK	Pearson Correlation	.623	1
	Sig. (2-tailed)	.041	
	N	11	11

```
CORRELATIONS
  /VARIABLES=CK PK
  /PRINT=TWOTAIL NOSIG
  /MISSING=PAIRWISE.
```

Correlations

Notes

Output Created		22-DEC-2019 16:45:44
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>

	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics for each pair of variables are based on all the cases with valid data for that pair.
Syntax		CORRELATIONS /VARIABLES=CK PK /PRINT=TWOTAIL NOSIG /MISSING=PAIRWISE.
Resources	Processor Time	00:00:00.02
	Elapsed Time	00:00:00.01

Correlations

		CK	PK
CK	Pearson Correlation	1	.357
	Sig. (2-tailed)		.282
	N	11	11
PK	Pearson Correlation	.357	1

	Sig. (2-tailed)	.282	
	N	11	11

CORRELATIONS

/VARIABLES=TK PK

/PRINT=TWOTAIL NOSIG

/MISSING=PAIRWISE.

Correlations

Notes

Output Created		22-DEC-2019 16:46:57
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics for each pair of variables are based on all the cases with valid data for that pair.

Syntax		CORRELATIONS /VARIABLES=TK PK /PRINT=TWOTAIL NOSIG /MISSING=PAIRWISE.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.00

Correlations

		TK	PK
TK	Pearson Correlation	1	.788
	Sig. (2-tailed)		.004
	N	11	11
PK	Pearson Correlation	.788	1
	Sig. (2-tailed)	.004	
	N	11	11

```
CORRELATIONS
  /VARIABLES=CK PK
  /PRINT=TWOTAIL SIG
  /MISSING=PAIRWISE.
```

Correlations

Notes

Output Created		22-DEC-2019 16:48:30
Comments		
Input	Active Dataset	DataSet0
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	11
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics for each pair of variables are based on all the cases with valid data for that pair.
Syntax		CORRELATIONS /VARIABLES=CK PK /PRINT=TWOTAIL SIG /MISSING=PAIRWISE.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.00

Correlations

		CK	PK
CK	Pearson Correlation	1	.357
	Sig. (2-tailed)		.282

	N	11	11
PK	Pearson Correlation	.357	1
	Sig. (2-tailed)	.282	
	N	11	11

APPENDIX M: PARENTS INFORMED CONSENT FORM

Enquiries: Mr. N. Lupahla
Tel: 065-240259
Cell: 0812780772
e-mail: nhlanhla.lupahla@riotinto.com

09 October 2015

To: The Parent/Guardian of:

____

____

____

Dear Sir/Madam

RE: Request for permission to select your child as a subject of my research at schools in Omusati, Oshana, Ohangwena and Oshikoto Regions in the 2016 academic year.

I am a PhD (Maths, Science and Technology Education) student studying with the University of South Africa (UNISA). I am doing a research to investigate the implications of professional development of higher level mathematics teachers in a technology enhanced teaching approach. I have opted to carry out my study with schools in Omusati, Oshana, Ohangwena and Oshikoto regions.

My study will use a sample of 60 higher level learners from selected schools in each region.

This study will hopefully provide opportunities to learn more about the challenges, barriers and successes in the process of teaching and learning in the mathematics classrooms.

The findings of my research have the potential to:
Promote academic advancement of teachers and learners

Facilitate learners to acquire problem solving skills and explore the world around them.

I have identified your child as a potential participant in the study; hence request your permission for his/her participation. The learner is free to withdraw from the research any time without any negative consequences.

Please complete and return the attached acceptance form.

Yours Sincerely

____

Nhlanhla Lupahla
Institute of Science and Technology Education
University of South Africa
Mobile No: +264-812780772
Supervisor: Professor N. N. Feza

CONSENT OF PARENT

I,

____,

(Full names of parent/legal guardian)

the parent/legal guardian of

____

(Full names of child)

grade 12 learner at

____

acknowledge

(Name of School)

receipt of request to allow my child to be the subject of the research being conducted by NHLANHLA LUPAHLA as explained in the letter.

I | agree to allow | | do not agree to allow | my child to be subject of the research

(Tick in the appropriate box)

and that Mr Nhlanhla Lupahla may unconditionally:

- observe my child
- keep samples of photocopies of his/her work/assessment records
- take photographs/videos to use in the research report
- conduct structured interviews/ focus group discussions on my child's experiences in the ICT enhanced learning approach

All the information collected shall be treated as confidential and the dignity and well-being of the researched learners shall be ensured. The learner is free to withdraw from the research any time without any negative consequences.

Kindly sign and return this letter to the school on or before the 20th of February 2016.
………………………………………………………………

Signed at____ on this___________day of
(Parent/Guardian's physical location/address) (Date)

____________________, 2016.
(Month)

APPENDIX N: TEACHER INFORMED CONSENT FORM

Enquiries: Mr. N. Lupahla
Tel: 065-240259
Cell: 0812780772
e-mail: nhlanhla.lupahla@riotinto.com

09 October 2015

To: NSSCH Mathematics Teacher

____
____
____

Dear Sir/Madam

RE: Request for your participation in my research at schools in Omusati, Oshana, Ohangwena and Oshikoto Regions in the 2016 academic year.

I am a PhD (Maths, Science and Technology Education) student studying with the University of South Africa (UNISA). I am doing a research to investigate the implications of professional development of higher level mathematics teachers in a technology enhanced teaching approach. I have opted to carry out my study with schools in Omusati, Oshana, Ohangwena and Oshikoto regions.

My study will use a sample of 4 higher level teachers from selected schools in these regions.

This study will hopefully provide opportunities to learn more about the challenges, barriers and successes in the process of teaching and learning in the mathematics classrooms.

The findings of my research have the potential to:

- Promote academic advancement of teachers and learners

- Facilitate learners to acquire problem solving skills and explore the world around them.

I have identified you as a potential participant in the study; hence request your permission to participate. You are free to withdraw from the research any time without any negative consequences.

Please complete and return the attached acceptance form.

Yours Sincerely

____

Nhlanhla Lupahla
Institute of Science and Technology Education
University of South Africa
Mobile No: +264-812780772
Supervisor: Professor N. N. Feza

<u>CONSENT OF PARTICIPANT</u>

<u>Research Title: Excel modelling in the teaching of functions and graphs in the Namibian Higher Level Mathematics Curriculum.</u>

I, ……………………………………………………………. (Participant's name) consent to participate in the research conducted by Nhlanhla Lupahla (Researcher's name) as it has been described to me in the information sheet. I understand that this is confidential data and it will be used to develop: (a) teaching programmes in senior secondary school mathematics, integrating computer technology and (b) a professional development module for mathematics teachers. I consent for the data to be used in this manner.

Signed Date

…………………………………… ………/……/……..…

APPENDIX O: ETHICAL CLEARANCE DECISION

From: Nosisi Feza [mailto:nosisi.piyose@gmail.com]
Sent: 28 June 2018 13:53
To: Nhlanhla Lupahla <Nhlanhla.Lupahla@mheti.gov.na>
Subject: Fwd: FW: Ethical clearance certificate (32341008)

Dear Mr Luphahla

Please find below the university's response about your ethics.

Regards
---------- Forwarded message ---------
From: Padayachee, Keshnee <Padayk@unisa.ac.za>
Date: Thu, 28 Jun 2018, 13:39
Subject: FW: Ethical clearance certificate (32341008)
To: Nosisi Feza <nosisi.piyose@gmail.com>
Cc: Lotriet, Hugo <lotrihh@unisa.ac.za>, Da Veiga, Adele <dveiga@unisa.ac.za>, Havenga, Michele <Havenmk@unisa.ac.za>, Mogari, David <Mogarld@unisa.ac.za>, Visagie, Retha <visagrg@unisa.ac.za>

Dear Professor Feza,

I have consulted Dr Retha Visagie, Manager of Research Integrity in the Research Support Directorate (UNISA) regarding Mr Nhlanhla Lupahla (student number: 32341008).

According to policy, we cannot grant clearance retrospectively. In this situation issuing a research ethics certificate would compromise the Policy on Research Ethics and could pose a risk to the Ethics Review Committee (ERC).

It is suggested that Mr Lupahla submits a copy of his thesis to the ERC (prior to submission for examination).

A sub-committee composing of three members will consider whether the research was conducted in accordance to the ethical standards set out in the Policy on Research Ethics, with the exception of producing a legitimate ethics clearance certificate.

To this end, the thesis must have a section that clearly describes the ethical considerations in detail for assessment purposes. Authenticity and honesty in reporting is critical.

The attached document could be useful in this regard. Refer the student to Section 4, RESEARCH GOVERNANCE: AN ETHIC OF ACCOUNTABILITY.

The ERC will then issue a letter stipulating that according to their assessment the thesis provides sufficient information that the researcher acted in accordance with the ethical standards set out in the Policy on Research Ethics based on an independent assessment conducted.

I trust this clarifies the matter.

Kind Regards,

Keshnee Padayachee
Associate Professor
Institute for Science and Technology Education (ISTE)
College of Graduate Studies
Building Robert Sobukwe | Office 4-429
Tel: +27 12 337 6191| E-mail: **padayk@unisa.ac.za**

UNISA university of south africa

UNISA CSET ETHICS REVIEW COMMITTEE

25 February 2020

Dear Mr Lupahla

ERC Reference #: Not applicable	
Name: Nhlanhla Lupahla	
Student #: 32341008	

Decision:
Research Ethics Notification Letter

Researcher(s): Nhlanhla Lupahla
Tel: +264-812780772
nhlanhla.lupahla@riotinto.com

Supervisor (s): Prof. N.N Feza
nosisi.piyose@gmail.com

Working title of research:

THE INFLUENCE OF EXCEL MODELLING PROFESSIONAL DEVELOPMENT ON CONCEPTUAL UNDERSTANDING OF PERIODICITY OF TRIGONOMETRIC FUNCTIONS

Qualification: PhD in Mathematics Education

This letter serves as notification that the research undertaken in the thesis entitled: "The Influence of Excel Modelling Professional Development on Conceptual Understanding of Periodicity of Trigonometric Functions" was conducted in the absence of an approved research ethics certificate. The research study was conducted from 2015 to 2019 with fieldwork being conducted during April 2017. According to policy, the university cannot grant research ethics clearance retrospectively.

The researcher submitted his thesis to the CSET ERC for review to establish if the researcher acted in accordance with the ethical standards set out in the Policy on Research Ethics based on an independent assessment conducted.

The research study is regarded as a low risk study. Human participants were involved directly, namely teachers and minors (Grade 12 learners) from Namibia. The topic is regarded as uncontroversial, non-sensitive information was collected, questionnaire data collected was anonymised and no personal information was collected using the questionnaires. Video recordings and photos were taken during fieldwork, which included personal information. Approval was obtained from the gatekeeper and consent was obtained from parents. Participation in the study was voluntary and participants could withdraw at any time.

The following aspects must be taken note off:

1. The assent from minors was obtained verbally, while signed parental consent was obtained.
2. The study outcome appears to be beneficial. The study appeared to have no evidence that the participants were harmed by the research instruments provided. There were no intimate details required from the participants. The benefits of the study outweigh the potential risks.
3. Confidentiality is protected when the subject's identity cannot be linked with personal responses. From the data record, there appears to be no identifying markers that compromise participants' confidentiality. The researcher informed the participants on their right to confidentiality verbally.
4. Participants were given an opportunity to withdraw from the study without penalty.
5. Some minors were videotaped, these videos must be destroyed to protect the privacy rights of the minors. The researcher has indicated that the videos are not uploaded on any platform.
6. There are no confidentiality agreements with the study assistants (trainers and observers (n=26)), however, the researcher declared that the"involvement of the lesson observer teams was provided for by the school management of each participating school as part of the pre-negotiated arrangements"
7. The informed consent forms did not address a number of the required consent aspects, namely: the right to confidentiality and anonymity, expected duration of participation, potential risks and measures, compensation, reimbursement, periods for which records will be kept, confidentiality and secure storage of data, types of individual or organisation to which the university usually discloses information, how privacy will be protected in publications of information, how feedback will be provided, consent for third party sharing, description of procedure to be applied in

dealing with the minors, statement that minor's identity will not be revealed, a statement that the minor should discuss participation with the parents prior to signing the form, a statement that the parent(s)/guardian(s) of the minor will receive a copy of the signed form and invitation to ask questions. The researcher has indicated that a thirty minute oral discussion, outlining some of these aspects, was conducted with the teachers and minors prior to commencing with the fieldwork.

Yours sincerely,

Signature
Chair of ISTE ERC : Prof K. Padayachee
E-mail: padayk@unisa.ac.za
Tel: (012) 337 - 6191

Signature
Chair of CSET ERC : Prof A. Da Veiga
E-mail:
Tel: (012) 429-xxx

Signature
Executive Dean: Prof B. Mamba
E-mail:
Tel: (012) 429-xxx

STUDENT DECLARATION

The researcher agrees that:

1. The researcher(s) confirms that the research project adhered to the values and principles expressed in the UNISA Policy on Research Ethics.
2. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the CSET ERC Committee.
3. The researcher(s) conducted the study according to the methods and procedures set out in the thesis.
4. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing, accompanied by a progress report.
5. The researcher ensured that the research project adhered to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
6. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
7. At the end of five years of safely storing the research data, the audio/video storages of the activities of the implementation of the EBM professional development sessions, lesson observation outcomes and interviews will be incinerated. Hardcopy documents, such as completed questionnaires and samples of learners' classwork and assessment records will be shred.
8. No video material, recordings or photos of teachers or children may be posted in the public domain.
9. Any photos used in the thesis must be anonymised.
10. Derivative works published from this thesis will have to stipulate that UNISA did not provide research ethical clearance and rather clearance was obtained from the gatekeepers and consent from parents.

11. Feedback should be provided to the participants.

Student Signature:

Date: 27/02/2020

Supervisors Signature:

Date: 02/03/2020

APPENDIX P: LETTER OF REQUEST FOR PERMISSION TO CONDUCT RESEARCH

The Rössing Foundation
Ondangwa Education Centre
P O Box 479
Ondangwa

3 December 2014

The Permanent Secretary
Ministry of Education
P Bag 13186
Windhoek

Dear Sir/Madam

Re: <u>Request for permission to conduct academic research with NSSC Higher Level Mathematics teachers at schools in Oshikoto, Oshana, Ohangwena and Omusati regions in the 2015 and 2016 academic years.</u>

I am a PhD (Mathematics, Science and Technology Education) student studying with the University of South Africa (UNISA), student number 3234-100-8. I am doing a research to investigate the implications of professional development of higher level mathematics teachers in a technology enhanced inquiry-based (TEIB) teaching approach in the teaching and learning of functions and graphs in the NSSC Higher Level curriculum.

My sample size will be 16 teachers (4 teachers from each region). This study will be a follow up on the findings and recommendations from my MSc research which assessed the level of development of the algebraic problem solving skills of 210 Grade 12 learners from Oshana region. The study found that learners failed to deal with non-routine problems because of inadequate conceptual understanding, limited range of solution strategies and difficulty with words and phrases in the given problems. The study also attributed these challenges to lack of classroom training in the problem solving process.

Given the poor performance of Higher Level mathematics learners (also cited in the DNEA, NSSC (H) examiners' reports of 2012 and 2013) in the interpretation of graphs of functions (Polynomial, Absolute Value, Trigonometric, Exponential, and Logarithmic functions), the study will focus on designing and implementing a technology-enhanced inquiry based instruction, using Excel spreadsheets to enhance the exploration and construction of meanings of graphical representations of these various types functions by the teachers and learners themselves. The study seeks to understand, through teachers' self-reflection, how the teaching and learning process changes when we shift from a traditional teacher-led classroom to a TEIB environment.

Through exploration, the study will hopefully provide opportunities to understand more about the challenges, barriers and successes in the process of teaching and learning in the mathematics classroom. In addition, it is hoped the research will provide practical insight for other mathematics teachers using or intending to use a technology-supported, inquiry-based learning environment. The study might also provide a baseline for planning professional development opportunities for secondary mathematics teachers, particularly within the technological pedagogical content knowledge (TPACK) framework.

I therefore request your office to grant me permission to conduct the research in the 2015 and 2016 academic years. During this period, I will also assess the impact of the programme and make appropriate recommendations based on my findings.

Yours faithfully

Nhlanhla Lupahla (0812780772)
E-mail : nhlanhla.lupahla@riotinto.com

APPENDIX Q: PERMISSION TO CONDUCT RESEARCH

REPUBLIC OF NAMIBIA

MINISTRY OF EDUCATION

Enquiries:	Mr C. Muchila	Private Bag 13186,
E-mail:	Cavin.Muchila@moe.gov.na	WINDHOEK
Tel:	+264 61 2933297	Namibia
Fax:	+264 61 2933922	

File no: 11/1/1

Date: 05 December 2014

To: Nhlanhla Lupahla
Rössing Foundation
P. O Box 479
Ondangwa, Namibia

Dear: Mr N. Lupahla

SUBJECT: PERMISSION TO CONDUCT ACADEMIC RESEARCH IN OSHIKOTO, OSHANA, OHANGWENA AND OMUSATI REGION

Your correspondence regarding the subject above, seeking permission to conduct academic research in the schools of Oshikoto, Oshana, Ohangwena and Omusati Regions has reference.

Kindly be informed that the Ministry does not have any objection to your request of conducting academic research at identified schools in the regions concerned.

You are, however, kindly advised to contact the Regional Council Offices, Directorates of Education, for authorisation to go into the schools and for proper information coordination.

Also take note that your activities should not interfere with the normal school programmes. Participation by either teachers or learners should be on a voluntary basis. Should you involve minors in your activities, consent for participation should first be obtained from the parents/guardians of the minor(s).

By copy of this letter the Regional Education Directors are made aware of your request.

Sincerely yours

9/12/14

09 DEC 2014

Mr. Alfred Ilukena
PERMANENT SECRETARY
cc: Director of Education: Oshikoto, Oshana, Ohangwena and Omusati

All official correspondence must be addressed to the Permanent Secretary

APPENDIX R: CERTIFICATE OF EDITING

Office: 0183892451

FACULTY OF EDUCATION

Cell: 0729116600

Date: 29th May, 2021

TO WHOM IT MAY CONCERN

CERTIFICATE OF EDITING

I, **Muchativugwa Liberty Hove**, confirm and certify that I have read and edited the entire thesis, **THE INFLUENCE OF AN EXCEL-BASED MODELLING TEACHER PROFESSIONAL DEVELOPMENT ON THE LEARNERS' CONCEPTUAL UNDERSTANDING OF PERIODICITY OF TRIGONOMETRIC FUNCTIONS**, by **NHLANHLA LUPAHLA**, submitted in accordance with the requirements for the degree of **DOCTOR OF PHILOSOPHY**, in the subject of **MATHEMATICS, SCIENCE AND TECHNOLOGY EDUCATION**, at the **UNIVERSITY OF SOUTH AFRICA**.

Nhlanhla Lupahla was supervised by **Professor N.N. Feza**.

I hold a PhD in English Language and Literature in English and am qualified to edit such a thesis for cohesion and coherence. The views expressed herein, however, remain those of the researcher/s.

Yours sincerely

Professor M.L. Hove (PhD, MA, PGDE, PGCE, BA Honours – English)

APPENDIX S: TURNITIN ORIGINALITY REPORT

THE INFLUENCE OF AN EXCEL-BASED MODELLING TEACHER PROFESSIONAL DEVELOPMENT ON THE LEARNERS' CONCEPTUAL UNDERSTANDING OF PERIODICITY OF TRIGONOMETRIC FUNCTIONS

ORIGINALITY REPORT

15%	10%	2%	7%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

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2	uir.unisa.ac.za Internet Source	2%
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