EXPLORING LEARNERS’ UNDERSTANDING OF TRIGONOMETRIC FUNCTIONS USING GEOGEBRA SOFTWARE: A CASE OF GRADE 11 MATHEMATICS LEARNERS AT A SCHOOL IN TSHWANE SOUTH DISTRICT

by

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I declare that the above dissertation is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

SIGNATURE | 16 May 2020
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ABSTRACT

The purpose of this study was to explore the learners' understanding of trigonometric functions using GeoGebra software. A qualitative case study approach was used with six Grade 11 learners at a school in Tshwane South District. The data were collected during a seven-day period using multiple methods: a diagnostic test, worksheets, a smart recorder, a trigonometric functions test, one-on-one interviews and focus-group discussions. The findings showed clearly that the use of GeoGebra enhanced Grade 11 learners' understanding of trigonometric functions. The findings also showed that the use of GeoGebra helped the learners to understand the basic trigonometric functions graphs. This enabled them to sketch freely without using the point-by-point method. Based on this study, it is recommended that GeoGebra should be made available to all Grade 11 learners. This will encourage them use the software out of regular school hours.

Die doel van hierdie studiegids was om die leerder se kennis van die gebruik van GeoGebra sagteware, ten opsigte van trigonometriese funksies te ondersoek. ‘n Kwalitatiewe benadering is gevolg met ses Graad 11 leerders by ‘n skool in die Tshwane Suid Distrik. Die data is ingewin oor ‘n periode van sewe dae, deur gebruik te maak van verskeie metodes: ‘n diagnostiese toets, werkkaarte, ‘n “smart” opname, ‘n trigonometriese funksies toets, een-tot-een onderhoude en fokusgroepe waar besprekings plaasgevind het. Die data-analise wys duidelik dat die gebruik van GeoGebra, die Graad 11 leerders gehelp het om trigonometriese funksies beter te verstaan. Die uitkomste wys ook duidelik dat GeoGebra die leerders gehelp het met ‘n beter begrip van die basiese trigonometriese funksies. Dit het hulle gehelp om vryhand sketse te doen en nie noodwendig die punt-tot-punt metode nie. Gebaseer op hierdie studie beveel ons aan dat GeoGebra beskikbaar gemaak moet word aan aale Graad 11 leerders. Dit sal leerders motiveer om ook die sagteware op hulle eie buite skoolure te gebruik.

Injongo yocwaningo lolu bekuyikuhlola ukuzwisisa kwabantwana amagrafu e-trigonometry ma bewafunda nge softhiwe ye GeoGebra. Abantwana abayisithupha abakubanga letshumi lanye abaphuma kusigodi se Tshwane South babambiqhaza

kulolucwaningo lwendlela ye ‘qualitative case study’. Imininingo iqoqwe kumalanga ayisikhombisa kusetshenziswa indlela ezilandelayo: ukuhlolwa kwe-diagnostic, amaphepha okusebenzela, isingxoxo zamunye ngamunye lezingxoxo leqembu. Iziphumo ezinkulu zikhombe ngokucacile ukuthi ukusetshenziswa kwesofthiwe yeGeoGebra kukhulise ukuzwisisa kwamagrafu e-trigonometry ngabafundi bebanga letshumi lanye.Iziphumo njalo zibonise ukuthi abafundi bazwisisa izinto eziyisiseko ngala magrafu ma bewafunda ngesofthiwe ye GeoGebra. Lokhu kunike abafundi amandla okudwebadweba lamagrafu ngokushesha ngokukhululeka. Kusekelwa ngalezi ziphumo, kunconyiwe ukuthi abafundi bafumane isofthiwe yeGeoGebra ngaso sonke isikhathi. Lokhu kuzabakhuthaza ukuthi basebenzise lesofthiwe noma bengaphandle kwesikolo.

KEY WORDS

Trigonometric functions; Trigonometric functions graphs; Graphs; GeoGebra; Technology integration; Technology; Constructivist; Prior knowledge; Collaboation; Scaffolding; Visualisation; Generalisation; Understanding; Learner centred; Parameter; Smartboard; Smart recorder; ICT; Learners

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My siblings for their belief in my abilities.

The learner participants and the host educator.

LIST OF ACRONYMS AND ABBREVIATIONS USED IN THE STUDY

CAI	Computer Assisted Instruction
CAPS	Curriculum Assessment Policy Statement
CD	Compact Disc
CPU	Central Processing Unit
DBE	Department of Basic Education
DoE	Department of Education
FET	Further Education and Training
GDE	Gauteng Department of Education
ICT	Information and Communication Technology
L	Learner
LAB	Lesson Activity Builder
MEC	Member of Executive Committee
NCS	National Curriculum Statement
NSC	National Senior Certificate
OBE	Outcome Based Education
PMS	Performance Management System
R	Researcher
SBA	School Based Assessment
SONA	State of the Nation Address
STW	Super Teach Worksheets
TPCK /TPACK	Technological Pedagogical Content Knowledge
TV	Television
UNISA	University of South Africa

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CHAPTER 1
GENERAL INTRODUCTION

1.1 INTRODUCTION

The focus of this research was to explore Grade 11 learners' understanding of trigonometric functions using GeoGebra software. This introductory chapter presents the background of the problem, statement of the problem, purpose of the study, research question, methodological considerations, significance of the study, definition of key terms and organisation of the study.

1.2 BACKGROUND OF THE STUDY

The integration of technology in Mathematics Education started long ago. The thrust of this research is to enrich literature in understanding trigonometric functions. Kaput (2007) reports that ICT tools are suited for secondary school learners. The tools being referred to here are desktops, laptops, tablets, and software. Such advanced tools help learners to understand abstract mathematical concepts. Powers and Blubaugh (2005) purport that the proper integration of technology has positive effects in Mathematics Education. The purpose of this study was to explore Grade 11 learners' understanding of trigonometric functions using GeoGebra software.

The South African education system White Paper on e-Education (2004) summoned schools to amplify and to enrich curriculum implementation and delivery using Information Communication Technology (ICT) in the classroom. The Gauteng Department of Education (GDE) has previously met these requirements through the Gauteng Online Project. To make schools an ICT-enabled space and keep abreast of technological advancement, the Department in 2014 came up with a policy on the use of tablet devices and 3G/Wi-Fi connectivity that were to be supplied to schools in terms of the Gauteng Province e-Learning solutions programme. The two main goals of the strategy were the provision of access by schools to educational content and guidance in the utilisation of tablets in the classroom; and establish monitoring and reporting mechanisms to facilitate the effective usage of ICTs (GDE, 2014).

The year 2015 saw the Gauteng province MEC of education injecting ICT resources in the form of smartboards, tablets, and laptops in all non-fee paying secondary schools. This study used the smartboard that has GeoGebra software. The provision of these ICT resources was graced and supported by the former president of the country who launched the Education leg of Operation Phakisa which aims to transform education by appropriately integrating ICT (Emerging technologies, 2015). Former President Zuma admitted the government's low rate in providing ICT resources and integrating them in the classroom. The current president, Mr Ramaphosa, in his first 2019 State of the Nation Address (SONA), tasked the Department of Basic Education (DBE) with the improvement of the education system through the development of Fourth Industrial Revolution (4IR) skills and competences needed now and in the future through Operation Phakisa (DBE Director General’s Provincial Engagements, 2019). However, although secondary schools in Gauteng now have access to ICT resources such as smart boards, laptops, tablets and software, their integration in the teaching and learning of mathematical concepts is still elusive. Perhaps, the current research on exploring Grade 11 learners’ understanding of trigonometric functions using GeoGebra software could motivate mathematics educators to work towards the task at hand.

Demir (2012), De Villiers and Jugmohan (2012), Martin-Fernandez, Ruiz-Hidalgo and Rico (2019), Ozudogru (2017) and Pfeifer (2017) report sporadicity in studies on trigonometry and trigonometric functions. So far Brown (2005) and Weber (2005) are some of the few scholars who have undertaken research on learners’ understanding of trigonometric functions. This poses a gap in research about trigonometric functions, since persistent challenges on learners are cited by the Grade 12 examiners through their Diagnostic Reports (DBE Diagnostic Reports 2011, 2012, 2013, 2014, 2015, 2016, 2017 and 2018).

Although trigonometric functions include some algebra and geometry, this study has focused on the graphical aspect, that is, the visual representation. In the Grade 10 to 12 Mathematics Curriculum Assessment Policy Statement (CAPS) document, functions are taken as graphs. The DBE National Senior Certificate Examination Diagnostic Reports (2011 to 2018) portray low performance in trigonometric functions. The average percentage performance for the questions on trigonometric functions for the past seven years is as follows, according to the respective Diagnostic Reports:

2012 (10%); 2013 (34,8%); 2014 (37%); 2015 (42%); 2016 (38%); 2017 (35%) and 2018 (41%). Figure 1.1 shows that trigonometric functions lie in the bottom three low performing topics in 2018. The same pattern is portrayed in 2011 to 2017 (see Appendix S).

All the reports for the previous years have published numerous difficulties experienced by learners in this topic. From the researcher's teaching experience, the difficulties lie in the learners' versatility with parameters *a*, *k*, *p* and *q* in the following trigonometric function graphs (Curriculum Assessment Policy Statement (CAPS) Grades 10 to 12):

- $y = a\sin x$; $y = a\cos x$; and $y = a\tan x$
- $y = \sin(kx)$; $y = \cos(kx)$; and $y = \tan(kx)$
- $y = \sin(x + p)$; $y = \cos(x + p)$; and $y = \tan(x + p)$
- $y = \sin x + q$; $y = \cos x + q$; and $y = \tan x + q$

Demir (2012) asserts that there is need to research trigonometry of real numbers using various teaching and learning techniques. This has prompted the current research to integrate GeoGebra in the learning of the effects of parameters on trigonometric functions. The study by Demir (2012) also reveals that the study of the behaviour of graphs of trigonometric functions under varying parameters has not been considered much. There is relevance in Demir's (2012) study because he argues that trigonometry is important in many academic fields. The low performance in trigonometric functions in the South African FET band exists as a huge crack in Mathematics Education that can be traced from yesteryears to the present day. It needs immediate intervention to halt it from continuing into the coming years.

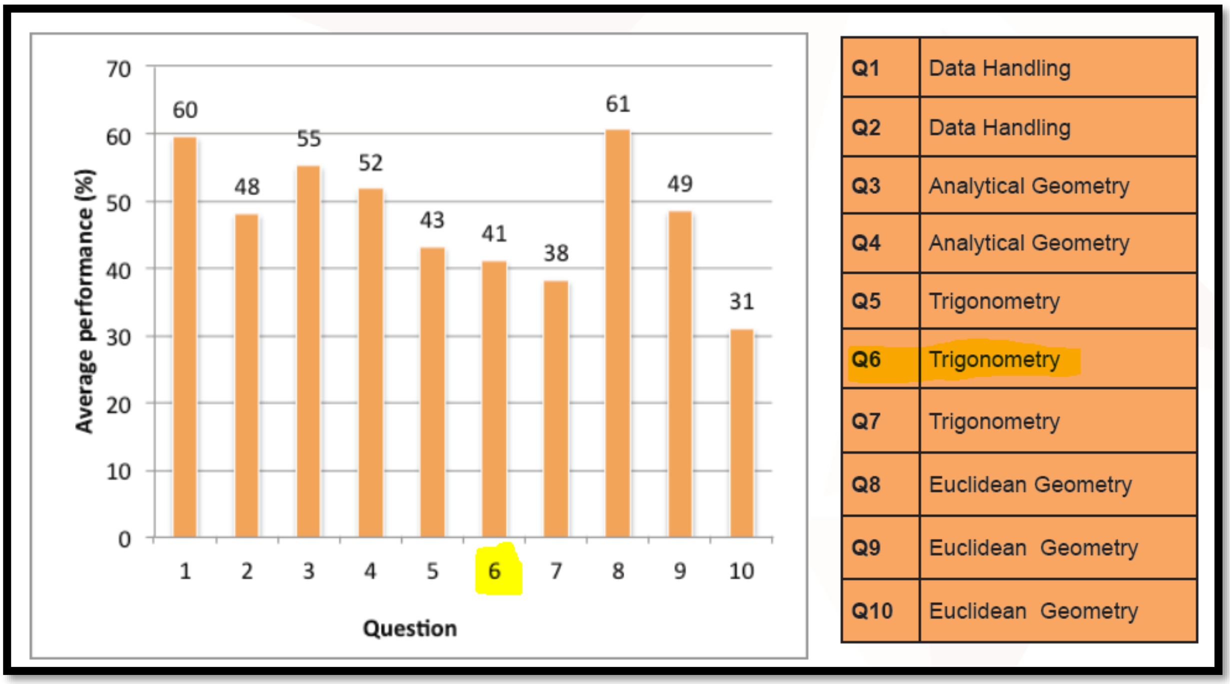


Figure 1.1 Average percentage performance per question for 2018 Paper 2 (Trigonometric functions – Question 6) (DBE Diagnostic Report, 2018:143)

Sun and Pyzdrowski (2009) assert that some learners say that there is no need for further knowing graphs since they have no relevance in their daily living. It appears the use of traditional methods has not overcome the difficulties faced by learners in understanding of trigonometric functions (Naidoo and Govender, 2014). In this study the researcher explored Grade 11 learners' understanding of trigonometric functions using technology, in particular, GeoGebra software that is installed in smartboards.

1.3 STATEMENT OF THE PROBLEM

The average percentage performance per question shown in Section 1.2 reveals that the learners' achievement in trigonometric functions is in need of improvement. The CAPS requires learners to be able to draw and interpret trigonometric function graphs showing the effects of the parameters *a*, *q*, *k* and *p,* at most two parameters at a time (FET CAPS 2010:32). Drawing from the researcher's experience in teaching trigonometric functions at Grades 10-12 level, many learners experience challenges in drawing graphical representations of trigonometric functions and interpreting the effects of the parameters. During school visits done over a period of 20 months, the researcher, as a District Subject Advisor, has seen disparity in the way mathematics educators address this topic. Some educators in Tshwane South District cover this topic in a day or two instead of 10 days in Grade 10 and five days in Grade 11 (according to the Annual Teaching Plan), whilst others avoid it totally.

De Villiers and Jugmohan (2012:1) noted that “many learners appear to have little understanding of the underlying trigonometric principles and thus resort to memorising and applying procedures and rules, while their procedural success masks underlying conceptual gaps or difficulties”. Mathematics is rated as one of the gateway subjects in the DBE (see Section 2.3 in Chapter 2) and trigonometry requires learners to simultaneously reason algebraically, geometrically and graphically. The topic carries 43-50 marks in the Grade 12 second paper. As Demir (2012) sees it, learners have a fragmented understanding of trigonometric functions and this is brought about by the chalk and talk methods of teaching. The use of GeoGebra in learning trigonometric functions, therefore, remains a necessity in order to enhance learners’ understanding.

1.4 PURPOSE OF THE STUDY

The intention of this study was to explore Grade 11 learners’ understanding of trigonometric functions using GeoGebra software. Specifically, the study explored how learners used GeoGebra to understand trigonometric functions, through the lens of the constructivist and understanding theories. Learner-centred approach activities were used that allowed learners to interact with GeoGebra while the researcher remained the facilitator. Additionally, the study sought to provide recommendations and implications to mathematics education practitioners in terms of using GeoGebra to enhance the understanding of trigonometric functions.

1.5 RESEARCH QUESTION AND SUB-QUESTIONS

The following main question guided the researcher’s enquiry:

How does the use of GeoGebra software enhance Grade 11 learners’ understanding of trigonometric functions?

The following sub-questions assisted in answering the main question:

(i) How do GeoGebra environments help learners in understanding trigonometric functions?

(ii) How is learners’ understanding of trigonometric functions after interaction with GeoGebra?

(iii) What are learners’ experiences and views on the use of GeoGebra in exploring trigonometric functions?

1.6 SIGNIFICANCE OF THE STUDY

It is critical for mathematics education practitioners to find solutions to low performance by learners in trigonometric functions. There are many studies that have been conducted in integration of technology in teaching and learning in general. Studies by De Villiers and Jugmohan (2012) and Pfeifer (2017) revealed that there is a negligible number of studies that have been conducted in South Africa on the teaching and learning of trigonometry. The current research is the first in the GDE to explore the integration of GeoGebra software in the Grade 11 trigonometric functions in the smartboard era. The study, therefore, contributes to the limited literature and empirical research that exists with respect to smartboards, smart recorders, integration of GeoGebra and the GDE ICT rollout. The findings of this research reveal a solution to problems that learners face in understanding trigonometric functions. Furthermore, the study’s implications, findings and recommendations are useful to mathematics subject advisors, teachers, learners, curriculum developers and curriculum designers to appreciate and value the use of GeoGebra in enhancing understanding of mathematical concepts. The study also motivates teachers in incorporating the use of technology into their teaching thereby improving the performance of learners. The findings of this study are of interest to mathematics education academics as well as provoking them to pursue further research on the integration of GeoGebra in trigonometric functions and other topics in mathematics.

1.7 METHODOLOGICAL CONSIDERATIONS

This research employed a qualitative case study methodology whose philosophical underpinnings are those of constructivist or interpretive paradigm. Data were collected at a school in Tshwane South District through tests (diagnostic and trigonometric functions test), worksheets, one-on-one interviews and focus-group interviews. A pilot study was conducted at a similar environment 21 days before the main research. In this research, learners interacted with GeoGebra as they tackled activities on the worksheets. Their experiences in using GeoGebra as a learning and thinking tool became the main source of data for the research. The data were analysed as described in Chapter 3.

1.8 ASSUMPTIONS

This study assumes that learners’ responses during one-on-one interviews and focus-group interviews were honest and truthful. Additionally, it was assumed that the learners’ performance in the worksheet activities and tests was not affected by exhaustion that may be brought by the extended school day after 14:30.

1.9 DEFINITION OF KEY TERMS

Functions - functions are used to create mathematical models of relationships between two variables. By using the defining equation of the function and knowing the properties of the function, one will be able to decide the shape or the intercepts with the axes of the graph of function (Abbott, Botsane, Bouman, Bruce, du Toit, Muthige, Pillay, Schalekamp and Smith, 2012).

GeoGebra - is viewed by Kepceoglu and Yavuz (2016), as a dynamic mathematics software for all levels of education that brings together geometry, algebra, spreadsheets, graphing, statistics and calculus in one easy-to-use package. The current study exploits GeoGebra’s provision to see graphical, numerical and algebraic representations of mathematical object on the same screen (Hohenwarter and Preiner, 2007).

Parameter - a variable that restricts or gives a particular form or shape to the equation it characterises. Trigonometric graphs are defined by parametric equations which group the graphs into families. When values are assigned to the parameters and substituted into the equation, it becomes a specific equation with a specific shape that is restricted by the values. Parameters do not change the type of graph, but the characteristics of a particular graph within a family of graphs (Bradley, Campbell and McPetrie, 2012).

Technology/digital media/ICTs - are terms "used to include devices such as computers, digital cameras, TVs, video or CD players, MP3 players, overhead and data projectors, electronic whiteboards, cell phones, memory devices and printers. It also includes programmes or software that can be used with the equipment, as well as the use of email and internet services and utilisation of computer" (GDE, 2011:56). The three terms are used synonymously throughout the study.

Technology integration - the use of ICTs, that is, computers, smartboards, and other technical tools (for example, calculators, data projectors, software, internet) in teaching and learning.

1.10 ORGANISATION OF THE STUDY

This dissertation consists of five chapters, as follows:

Chapter 1 provides background of the study, statement of the problem, purpose of the study, research question, significance of the study and a brief research methodology. The chapter also laid out assumptions and provided the reader with definition of key terms and concepts.

Chapter 2 reviews the literature of the study, that is, articles; journals; books; information from websites; DBE and GDE policy documents, publications, circulars, and memos. The chapter also details Constructivism and understanding theories as the theoretical frameworks that inform the study.

Chapter 3 presents the qualitative research methodology. The philosophical underpinnings guiding the study are discussed and justified in this chapter. Research design, research setting, sample and population, piloting, instruments used to collect data, data collection procedures and analysis strategies, quality criteria and ethical considerations are also explained.

Chapter 4 presents results of data collected through worksheets, tests and interviews of the research and discusses the findings.

Chapter 5 summarises all the chapters, presents major findings of the study, the limitations and delimitations of the study, the implications and recommendations for further research.

1.11 CONCLUSION

This chapter has provided the background to the study, as well as giving the statement of the problem, the purpose of the study, the research question and sub-questions, the significance of the study, some methodological considerations and the assumptions. Definitions of key terms have been given. The dissertation has argued that trends in learners' performances in trigonometric functions portray low and worrying levels that call for immediate solutions. The purpose of this study was to explore Grade 11 learners' understanding of trigonometric functions using GeoGebra software through a case study method. The following chapter presents a literature review that provides a focus for the research and delineates the scope of the study.

CHAPTER 2
LITERATURE REVIEW

2.1 INTRODUCTION

This chapter reviews related literature under the following sub-headings: history of digital media in mathematics education; the importance of integrating technology in mathematics education; constraints in the integration of technology in mathematics education; the importance of trigonometry in the curriculum; the integration of technology in trigonometry and trigonometric functions; the smartboard; GeoGebra and theoretical framework. The purpose of this study was to explore Grade 11 learners' understanding of trigonometric functions using GeoGebra software and this research comes at a time when Demir (2012); De Villiers and Jugmohan (2012); Martin-Fernandez et al (2019); Pfeiffer (2017) and Ozudogru (2017) are reporting a dearth of research literature in the learning of trigonometric functions. In this chapter, the researcher explored previous research in order to understand methodology and research techniques that helped in answering the research question and to identify gaps.

2.2 HISTORY OF DIGITAL MEDIA IN MATHEMATICS EDUCATION

The South African mathematics curriculum follows in the footsteps of giants like Australia, USA, Singapore, Japan, and Cuba. The curriculum thrives on blending mathematical content and practices from such international communities to meet its domestic needs (Motshekga, 2018). For instance, Lesson Study (originating from Japan), Mathematics Learning Communities of Practice (with Cuban origins) are currently being implanted into the South African mathematics curriculum and practice through Mathematics Education Chairs Initiatives. The researcher found it essential to deliberate (briefly) on the history of digital media or technology in the mathematics education community abroad, in South Africa, in Gauteng, in Tshwane South District. This is done in an endeavour to trace the path followed by technology into our mathematics classrooms where it is supposed to enhance learners' understanding of concepts such as trigonometric functions. Dejic and Mihajlovic (2014) insist that such histories in mathematics education increase learners' and educators' motivation; build positive attitude towards mathematics, better understanding and development of mathematical concepts.

Uses of technology have expanded to encompass tasks such as “performing numerical computations, inventory control, and point of sale transactions, manufacturing operations, word processing and a variety of chores in the home” (Harrison and Harrison, 1983:1). Harrison and Harrison (1983) report that the first educational application of computers in America was designed by Patrick Suppes in the mid-1960s. These applications in the classroom were mainly drill and practice programmes in primary school mathematics. This era provoked the continued refining of technology integration in American classrooms. A study by Alexander, Knezek, Christensen and Tyler-Wood (2019) in the United States was conducted as a response to a decline in the learners’ performance in mathematics, sciences and technology. The findings of the study revealed that learners developed positive attitudes towards these subjects after being taught using technology.

Compared with the rest of the world, Britain started early and developed acceptance and integration of technology into schools further than any other country in 1986 (Makandidze, 2004). While there were many more computers in American schools by 1986 their use was relatively limited and the programs that American teachers were using were largely confined to drill and practice in English and Mathematics. Goos (2010) reports that in Australia, technology integration in mathematics classes started in the 1970s with simple four- function calculators.

> "*Since then, computers equipped with increasingly sophisticated software, graphics calculators that have morphed into 'all purpose' hand-held device integrating graphical, symbolic manipulation, statistical and dynamic geometry packages, and web-based applications offering virtual learning environments have changed the Mathematics teaching and learning terrain”* (Goos, 2010:67).

The general history of technology in mathematics education includes research done by Drijvers, Kieran, Mariotti, Ainley, Andresen, Chan and Meagher in 2009. The findings of the study report that the main frame computer was developed in 1942 followed by the first four-function calculator in 1967. The microcomputer was invented in 1978 followed by the graphing calculator in 1985. Drijvers et al (2009) note that since the 1960s, mathematics education practitioners in schools and tertiary institutions started to appreciate the importance of integrating technology. This resulted in dramatic changes from those years up to now both in technology development and in the way it is used in mathematics education to teach and learn concepts.

The study by Drijvers et al (2009) goes on to say that computer assisted instruction (CAI) was one of the applications to follow drill and practice in mathematics education. CAI is perceived as learner centred since it allows learners to master subject matter at their own pace. In the 1970s, Papert (a mathematics educationist) was informed by Piaget's theories to coin programming in Logo and BASIC languages. Piaget's theories also prescribe constructivist learning, a theory that informed the current study. The programmes aimed to foster problem solving skills in learners (Drijvers et al, 2009).

The arrival of the microcomputer in 1978, according to Drivjers et al (2009), prompted the development of more specialised pieces of software aimed at mathematics learning, such as CABRI Geometry, Function Probe, spreadsheets and computer algebra systems. Despite the availability of such resources, the integration of technology had not pervaded most mathematics classrooms by the 1990s.

Joseph (2012) purports that in South Africa computers found their way into the classroom in the 1980s with the aim to modernise education. In the same vein, Gumbo (2013) argues that the "main reason technology education was introduced in the South African National curriculum and other countries in the world was the recognition of the need to produce engineers, technicians and artisans needed in the modern society as well as the need to develop a technologically literate society for the modern world." The DoE introduced an ICT Education policy in 2004 that prescribes the integration of ICTs in education so as to develop higher order thinking skills in learners (Ndlovu and Lawrence, 2012).
The Gauteng Department of Education (GDE) has seen the MEC recognising the importance of integrating technology in schools. This resulted in tablets for learners, laptops for educators and smart boards being delivered to non-fee-paying schools and the aim was to have paperless and chalkboard free classrooms at the beginning of

term 3 of 2015. However, this has not been achieved to expected levels due to constraints discussed in Section 2.6.

The South African Department of Basic Education launched the ‘Siyavula Digital Maths and Science Practice Programme’ in 2018 (Benson, 2018). This is an online programme that allows learners in the FET band to practice solving problems in any topic in the CAPS. There are 113 selected secondary schools in Gauteng that can access the programme free of charge when using MTN or Vodacom sim cards. This is a drill and practice programme like those used by America and Britain in the yester-years. The South African mathematics classroom remains in need of integration programmes (such as GeoGebra) that help learners to understand mathematical concepts.

Mathematics District Subject Advisors in the GDE have undergone training in the use of the smartboard, clickers, google forms, downloaded videos and Siyavula from 2015 to date. Educators from schools that received smartboards and whose schools are in the Siyavula project were also trained in the same period. In 2019 the GDE Performance Management and Development Directorate launched a training course for both educators and District Subject Advisors titled End User Computing Learnership Induction. The course (that is set to run from January to October 2020) is aimed at equipping these educationists with skills in integrating technology in learning and teaching. Unfortunately, the course contents do not cater for aspects that use computer tools for subject matter learning. Attendants are being trained on Word, Excel, Database, PowerPoint, and Outlook. Presently, the DBE has already planned to introduce Robotics in its Curriculum in a bid to meet the 4IR demands.

2.3 THE IMPORTANCE OF INTEGRATING TECHNOLOGY IN MATHEMATICS EDUCATION

The presence of ICTs in mathematics classrooms is a crucial issue and still raises many unanswered questions. The Mathematics Teaching and Learning Framework for South Africa: Teaching Mathematics for Understanding (2018) has reiterated the need to incorporate technology in mathematics classrooms in order to promote conceptual understanding of mathematical concepts. In any one mathematical course, the proper integration of technology reinforces mathematics education (Dick and Hollebrands, 2011). Technology, as the Center for Technology Learning (2007) sees it, can ease learning thereby saving learners from doing laborious computations. The notion held is that technology has the power of representing mathematical concepts in varied ways that enhance learners' understanding. For instance, GeoGebra links the algebraic and graphical forms of functions such as trigonometric functions. Nkhwamule (2013) reports that Botswana integrated ICTs, particularly computers, into its mathematics education system to foster learners' interest in learning Mathematics. The findings of the study revealed that the enrolment in mathematics classes increased even if the improvement on performance was negligible. In the same vein, Bingimlas' (2009) study reports that ICT integration proved to be effective in learning mathematical concepts and exposing learners to the information age.

Static drawings on the chalkboard and paper do not encourage learners to grasp mathematical concepts. Piaget (1970) discovered that children first develop ideas concretely and later progress to abstractions. Technology integration is based on Piaget's ideas (Constructivism) since it enables learners to understand abstract mathematics concepts (Center for Technology Learning,2007). In a South African mathematics class, an abstract topic like trigonometric functions can be made easier by integrating technology. Researchers like Roschelle, Pea, Hoadley, Gordin and Means (2000) reported technology as enabling teachers to easily build upon learners' prior knowledge and skills; emphasize the connections among mathematical concepts; connect abstractions to the real-world settings; address common misunderstanding and introduce more advanced ideas.

The introduction of technology in the South African curriculum was in quest of nurturing higher order thinking skills amongst learners (DoE, 2004). Performance in mathematics (especially in trigonometric functions, see Diagnostic reports 2011 to 2018) in South Africa is below expectations yet the subject is tagged as a gateway. In general, gateway subjects enable one to purse a broad range of options after Grade12. According to Daniels (2013) mathematics is a gateway to science, medicine, commerce, engineering and other vital parts of the economy. Learners develop interest and understanding in mathematics when taught using technology as revealed in the focus-group discussions in the current study. This is supported by the National Council of Teachers of Mathematics (2011) who found that teachers can afford learners to pass mathematics through the proper use of technology. In addition, Makandidze (2004) maintains that technology can conceivably take over certain of the teacher's laborious tasks. Furthermore, Sheehan and Nillas (2010) argue that learners perform better in mathematical generalisations when they learn using technology tools such as GeoGebra software. The current research used the benefits of using technology as a leverage to explore Grade 11 learners' understanding of trigonometric functions using GeoGebra software.

2.4 CONSTRAINTS IN THE INTEGRATION OF TECHNOLOGY IN MATHEMATICS EDUCATION

The focus of the current study was to explore Grade 11 learners' understanding of trigonometric functions using GeoGebra software (in a South African classroom in Tshwane South District). Kissane and Kemp (2009) contend that current technological advancements have tremendious potential in exploring many trigonometry concepts such as trigonometric functions, but the terrain is not level. The researcher, therefore assumes that anything that constrains the integration of technology in mathematics classrooms militates against efforts to promote the understanding of trigonometric functions. This section attempts to discuss constraints or challenges of integrating technology as reported by scholars and from the researcher's experience in the South African curriculum.

Balanskat, Blamire and Kafala (2006) argue that education systems continue to encounter challenges during the process of bringing technology into the classroom.

This argument is supported by Bingimlas (2008:235) who states that "Due to ICTs' importance in society and possibly in the future of education, identifying the possible obstacles to the integration of these technologies in schools would be an important step in improving the quality of teaching and learning". The Centre for Implementing Technology in Education research team (2015) in America identified common challenges facing schools and districts with respect to integrating technology in mathematics education. The challenges included financial limitations, time factors and technological pedagogical content knowledge (TPACK) training for teachers. The Gauteng Department of Education in 2015 failed to have all Grade 12 classes paperless by the beginning of third term due to financial constraints in erecting infrastructure and provision of tablets and smartboards. Schools in Gauteng have limited ICT resources such as hardware, software and connectivity. It appears that governments do not prioritise technology integration programmes whenever they have financial constraints. In 2019, it emerged that most Gauteng District Subject Advisors' laptops were malfunctioning, and they needed to be replaced. The GDE is currently not able to provide new laptops due to financial challenges. Again, the Mathematics Teaching and Learning Framework for South Africa: Teaching Mathematics for Understanding (2018:83) insists that: "ICT is not a prerequisite for teaching for understanding." The framework seeks to concentrate on equipping educators with content knowledge which is contradictory to the DoE's White paper on e-Education (2004). Such a stance is demotivating educators in utilising tools found in laptops and smartboards in the teaching and learning of topics like trigonometric functions. Again, the research finding reported in the Mathematics Teaching and Learning Framework for South Africa: Teaching Mathematics for Understanding (2018) reveals that there are Grade 12 learners who sit for their final Mathematics examinations without calculators. This indicates that the use of technology in the learning of Mathematics in South Africa is still a challenge.

Inadequate contact time for teaching Mathematics was indicated as constraining teacher's integration of technology (Nkhwalume, 2013; Centre for Implementing Technology in Education, 2015). Nkhwalume's (2013) study claimed that most of the teacher's time is spent on administrative tasks such as the Performance Management System (PMS) and the rest to assessing students work. Such a scenario exists in South African schools where an overflow of paperwork is drowning teachers.

Nkhwalume's (2013) study reveals that lack of administrative support at school levels also militates against technology integration. From the researcher's experience, computer integrated lessons need more time than that allocated by school timetables while curriculum heads are not willing to extend the periods. Such challenges are dampening the teachers' ambitions to integrate technology.

Mathematics teaching is characterised by complexity, as it is framed by the classroom interactions, the tasks assigned to the student and the overall social contexts (Skott, 2010). Mali, Biza, Kaskadamis, Potare and Sakonidis (2013) are of the view that in technology-based mathematics lessons the situation becomes even more complex as the nature of tools and management issues complicate learner-teacher interaction in moving from the technological to the mathematical objects. Monaghan (2004) points out that learners tend to concentrate on technological details at the expense of Mathematics. This is also being experienced since 2015 with Gauteng learners when most of them held tablets for the first time in their lives. They were carried away by many features found in technology and this hampered teaching and learning of subject matter. It is difficult to have all learners on the same page of technology or tablets since some will be busy with games. These challenges might increase instead of decreasing the conflicts and contradictions present in the class activity being done, thus weakening and blurring the mathematical meaning construction (Mali et al, 2003).

Bingimlas (2009) cited the lack of TPACK in teachers as one of the main challenges crippling technology integration. Koehler, Mishra, Kereluik, Shin and Graham (2014) purport that the TPACK framework describes the kinds of knowledge needed by a teacher for effective technology integration. They insist that TPACK theory concerns itself with teachers' knowledge, creativity and skills in properly integrating technology to teach subject content. Teachers keep aloof from bringing technology in the classroom once they are not conversant with the digital tools. Poor computer skills and differing levels of computer literacy levels are viewed by many researchers as a constraint to integrating computers in mathematics teaching and learning (Koehler et al, 2014 and Catherall, 2005). During the smartboard training of Grade 12 teachers in Gauteng in 2015, a handful of teachers confessed that they were unable to use computers. Most educators visited in Tshwane South District are seen to be using smartboards in the same way they used the chalkboard, just writing subject matter using their fingers. Even when teaching trigonometric functions they use the dot-plot with smartboard pens or their fingers instead of using GeoGebra in the smartboard. This is due to lack of TPACK amongst these educators. These educators are expected to use smartboards to facilitate learning of mathematical concepts. Again, District Subject Advisors are not competent in the use of smartboards due to the fact that since they laid hands on the gadget during training, they do not have access to it for further practice. This challenge could be addressed by having at least one smartboard in each district office in Gauteng. Furthermore, the in-service courses (see Section 2.2) offered to educators and District Subject Advisors are too general and shallow to equip them for subject content specific TPACK. There is need for GDE to revisit its training of school and office-based educators by infusing aspects that address the understanding of mathematical concepts like trigonometric functions.

Catheral (2005) identifies technical support problems as one of the on-going constraints in technology-integrated teaching and learning. This is being experienced by the first schools to have smartboards in Gauteng Province. Tshwane South District teachers undergoing smartboard training had to be moved from one school to the other in Tembisa following the malfunction of the technology (smartboards) in these schools. It was not clear who was responsible for repairing the technology. All the smartboards in Tshwane South District schools are mounted with 3G sim cards, but most of them remain not connected because schools cannot fund their connectivity. In Botswana teachers complained that technical problems such as failing to connect to the internet, waiting long periods for websites to open, printers not printing, malfunctioning computers, and teachers having to use old computers, discouraged many from using computers for teaching and learning (Nkhwalume, 2013). In South Africa, load shedding that occurs regularly during school time also hampers technology integration.

2.5 THE IMPORTANCE OF TRIGONOMETRY IN THE CURRICULUM

Trigonometry is a topic that has pervaded any one curriculum because of its prerequisivity to other topics in mathematics and it is applicable to many important career courses in tertiary education and other spheres of life. Therefore, a discussion of literature which indicates its importance is provided here to mathematics education stakeholders as motivation for the study. Moreover, the importance of this topic (in the curriculum) adds value to the current study that explored learners' understanding of trigonometric functions using GeoGebra. Learners will be motivated to learn and explore trigonometry if educators make them aware of the future benefits of doing this topic.

Learning trigonometry formats learners' brains by incorporating memorisation of concepts and problem solving abilities (Hill, 2015), thereby preparing them for further topics like complex numbers, differentiation, integration, differential equations, fourier series, Laplace transformation, matrices, plane analytical geometry and systems of equations (Bourne, 2018). The Mathematics Teaching and Learning Framework for South Africa: Teaching Mathematics for Understanding (2018) reports that even learners who pass Grade 12 mathematics still struggle at university. The researcher argues that lack of conceptual learning and understanding of topics such as trigonometric functions is the result of such a trend.

The application of trigonometry dates back to the erection of pyramids by the ancient Egyptians. In the 4IR, trigonometry is applied in the following fields: medicine (tangent in calculating lengths of plates to support open fracture arms; sine and cosine graphs are used to model breath rates and to detect anaemia, asphyxia and hypoxia); surveying (where tangent of an angle is used to calculate the distance across rivers and gorges); astronomy (where the sine rule is used to find the distance between planets which helps in positioning satellites, telescopes and space stations); architecture (where trigonometric functions are used to design domes, suspension bridges and support beams); and music production (where the cosine and sine graphs are used by sound engineers to measure sound waves) (Hill,2015). Ferrao (2018) also reports that trigonometric functions are used in the production of video games that youngsters are fond of nowadays. Perhaps South African learners could be motivated to understand trigonometric functions with the aim of designing and developing video games in future. Career courses like marine, mechanical and flight engineering, criminology (where the location of the shooter can be detected), navigation and electronics are additionally some of the daily life applications of trigonometry (Ashbox, 2019 and Ferrao, 2018).

It is clear that understanding trigonometric functions by learners will prepare them for the present and future that requires them to solve complex ecomomic and social problems. Mastering this topic therefore remains important in the South African terrain where there are scarce skills. Educators should appreciate and make learners aware of the importance of trigonometry in real life because this is prescibed by the FET CAPS. Examiners in FET CAPS should include questions that require learners to list applications of trigonometric functions in the same way physical science examiners ask for the applications of topics like the Doppler Effect. This will help learners realise the importance of the topic in life and motivate them to learn it with understanding. The current study did not delve into the application of trigonometric functions because of time and resource limitations and this leaves the aspect open for further research.

2.6 THE INTEGRATION OF TECHNOLOGY IN TRIGONOMETRY AND TRIGONOMETRIC FUNCTIONS

Ozudogru (2017:1) notes, “Despite the fact that there is determined difficulties about learning trigonometry, research literature in this subject is sparse.” Trigonometry features in all secondary school curricula throughout the world because of its importance as discussed in Section 2.5. In the South African FET curriculum, trigonometry is allocated a major share of 43 to 53 marks towards the final examination or assessment and this renders understanding of this topic by learners important. In a study by Kendal and Stacey (1996) involving Grade 10 learners, four learners out of 178 obtained zeros in the trigonometry test. The researchers recommended the abandonment of traditional methods in teaching the topic. In the South African FET band learners perform poorly in trigonometry functions (DoE and DBE Diagnostic Reports 2011 to 2018). Perhaps the use of GeoGebra in learning the topic could improve the performance since Venter and Barnes (2008), De Villiers and Jugmohan (2012) and Scott-Wilson (2009) posit that teaching and learning in South African mathematics classrooms is dominantly traditional and teacher-centred.

Kissane and Kemp (2009) of Murdoch University in Australia studied teaching and learning trigonometry using technology (graphic calculators and graphing applets on computers). They report that the use of technology enabled learners to draw graphs of trigonometric functions quicker and to understand the basic features of different functions. In addition, the study found that graphing applets on computers enhanced the learners' understanding of periodicity and amplitude thereby enabling them to sketch the graphs freely. The South African curriculum does not allow graphic calculators in their examinations. The instruction in this regard for both Papers 1 and 2 reads: "Use an approved scientific calculator (non-programmable and non-graphical calculator)" (Gauteng Department of Education Preparatory Examination Mathematics Paper 1 2018:2). Graphics calculators are rarely found amongst learners because of this reason and their expensiveness. Van Woudenberg's (2017) study reported GeoGebra as a better replacement of the graphing calculator because of its potentialities (see also Section 2.8.2). The study revealed that learners became more positive about GeoGebra when they used the software more often and recommended its use in examinations when all learners could access it through their individual laptops and tablets. This finding agrees with outcome of discussions at the DBE – NRF Community of Practice in Mathematics and Science Conference in 2018 where one delegate suggested that learners should be allowed to use technology in examinations and a new way of assessing in that regard should be crafted.

Ng and Hu (2006) investigated the impact of using web-based simulation on learning (understanding and sketching) trigonometric graphs. Constructivist theories were used as the theoretical framework where individual and social construction of knowledge was employed. The participants of the study were 29 Grade 9 students from a Singapore school, who worked in groups of not more than four and engaged in online discussions as well after school. Activity sheets (that participants answered with the help of the technology), a quiz that tested learners on sketching graphs and an oral examination (that probed learners on what had been written in the quiz) were used to collect data. These instruments and approach (respectively) helped the researcher in the designing and administering of worksheets, tests and one-on-one interviews with Grade 11 learners at a school in Tshwane South District. The learners' performance in Ng and Hu's (2006) study was analysed question by question (and this is how the researcher analysed tests that were administered in the current study), and the findings showed an improvement in sketching graphs and describing transformations. The current research used offline GeoGebra software in the smartboard with Grade 11 mathematics learners as participants, at the time when sketching and describing of trigonometric functions is a central requirement in the CAPS.

In the CAPS era, the common approach to trigonometric functions is the chalk and talk approach. When sketching $y = a\sin k(x - p) + q$ on the chalkboard, two to three sketches are drawn on the same set of axes to represent the concept of transformation. This is laborious and time-consuming given the 30 to 45-minute periods in most schools. Again, the natural flow of building the idea of transformation is lost during the process. These factors could contribute to the poor performance in trigonometric functions. One can contend that these are traditional absolutist approaches that militate against the success of improving FET learners' performance in trigonometric functions. This is supported by the Mathematics Teaching and Learning Framework for South Africa: Teaching Mathematics for Understanding (2018) that underlines the existence of ineffective teaching and learning practices in South African mathematics classrooms.

Hertel and Cullen (2011) investigated pre-service secondary teachers' understanding of trigonometric functions using a dynamic geometrical environment at a university in the United States of America. The research aimed to help students to build a robust and connected understanding of trigonometric functions and explored the role that dynamic geometric environments can play in its development. The current study that sought to explore Grade 11 learners' understanding of trigonometric functions using GeoGebra employed group work and collaboration that were also used by Hertel and Cullen (2011) during sketching of trigonometric functions using the dynamic geometry software. The study revealed a statistically significant growth from pre-test to post-test attributed to the use of the software. In the South African context, training pre-service teachers in such use of the software in all universities could enhance the teaching and learning of the trigonometric functions. Again, the environments provided by GeoGebra in the current research positively supported Grade 11 learners' exploration and understanding of trigonometric functions in the same way as Hertel and Cullen's (2011) study did.

In the Netherlands, Demir (2012) carried out a study entitled "Students concept development and understanding of sine and cosine functions.'' It involved a mathematics class of 24 students aged 16-17. The study examined students' concept development and understanding of the sine and cosine function. The new approach based on the implemented learning trajectory was found to be more effective than the traditional approach in terms of promoting a connected understanding of trigonometric functions, not only in terms of angles but also on the domain of real numbers. Similar to Demir's (2012) study, the current study collected some of its data through a diagnostic test, worksheets, trigonometric test and interviews. The researcher employed small groups of Grade 11 learners during worksheet activities whilst Demir (2012) had learners working in pairs and their discussions audio recorded. The current study could not audio record group discussions due to limited time and resources. Learners in the current study used GeoGebra software in the smartboard to tackle their worksheets whereas those in Demir (2012) engaged GeoGebra applets in desktops found in the school laboratory. Demir (2012) reported limited literature and recommended further research in trigonometry and this prompted the current study to go a step further to employ group work and focus-group discussions in a South African classroom. Again, the current study addresses gaps in the literature by exploring Grade 11 learners' understanding of sine, cosine and tangent functions since Demir's (2012) study was limited to sine and cosine. The integration of GeoGebra in this study saw Grade 11 learners being able to draw and interpret trigonometric graphs without struggling.

A study by De Villiers and Jugmohan (2012) in Kwa Zulu Natal examined six Grade 10 learners' conceptual understanding of the sine function (at introductory level) using Sketchpad. Like GeoGebra that was used in the current study, Sketchpad is also a dynamic geometrical software package. De Villiers and Jugmohan (2012) used interviews and questionnaires to collect data with the aim of ascertaining the extent to which Sketchpad afforded learners' conceptual understanding of the sine function and to examine the quality of learning that had taken place. The study revealed that integrating Sketchpad enhanced learners' understanding of the basic properties of the sine function. One-on-one interviews in the current study also probed learners' written work for understanding in the same way De Villiers and Jugmohan (2012) did. The study at hand did not only explore the sine function but also cosine and tangent functions, post introductory level with Grade 11 learners as participants using GeoGebra software. The current study employed the constructivist perspective and understanding theory as its theoretical framework while De Villiers and Jugmohan (2012) used only the constructivist perspective. Demir's (2012) study is similar to that of De Villiers and Jogmohan (2012) in that it started at introductory levels of a function but went further to the sketching of sine and cosine functions. Both studies defined the frontiers of knowledge in exploring Grade 11 learners' understanding of trigonometric functions using GeoGebra software.

Agyei (2013) studied the effect of using interactive spreadsheet as a demonstrative tool in the teaching and learning of Mathematics concepts and reported that learners got first-hand information on the role played by each part of the equation by observing how changes in the parameters had immediate feedback on graphs of trigonometric functions. Eight student teachers and 135 learners participated in the study where data were collected using self-reports, product evaluation and observations. The learners altered the values of parameters while observing and recording changes in the graphs on their worksheets. The demonstrations and interactions with the tool helped the learners to visualise and understand the effects of varying parameters on trigonometric graphs. Agyei (2013) shows that the environment also enables learners and teachers to explore many examples without having to draw them physically on the chalkboard, a strength that is also possessed by GeoGebra software that was used in the current research. The current study employed procedures parallel to those of Agyei (2013) using GeoGebra as a tool instead of interactive spreadsheet. Both studies explore and exploit the idea of varying parameters of trigonometric functions using software. GeoGebra software is more readily available to schools that have smartboards (especially those that benefited from the Gauteng Education MEC's rollout programme) and may be more easily downloaded into laptops and tablets than interactive spreadsheets. The Grade 11 learners in the current study were hands on with the GeoGebra software like the learners in Agyei's (2013) study. Such approaches are synonymous with constructivist principles that prescribe learning by doing, a practice that promotes understanding of mathematical concepts.

Naidoo and Govender (2014) explored the implications of the use of online GeoGebra software in teaching trigonometric graphs to 25 Grade 10 learners from the same institution. Two worksheets (the first answered without GeoGebra and the second tackled while interacting with GeoGebra), semi-structured interviews and observations were used for collecting data. The use of worksheets and interviews by Naidoo and Govender (2014) influenced the current study in engaging similar instruments. The study reported that GeoGebra enhanced the learners' understanding of the graphs. The researchers argue that the GeoGebra programme allows immediate feedback. This is said to have motivated learners and boosted their confidence in handling trigonometric graphs. Naidoo and Govender's (2014) study reports that GeoGebra enables learners to visualise the behaviour of trigonometric functions as they change parameters, a finding that is shared by researchers such as Fahrudin and Pramudya (2019). Unlike Naidoo and Govender's (2014) work that depended on online GeoGebra, the current study used the GeoGebra that is already in the smartboard. Again, this study was confined to Grade 11 learners.

Separate studies by Rahman and Puteh (2015) and by Fahrudin and Pramudya (2019) report that learners' understanding of trigonometric material is still poor and learners regard trigonometry as abstract and difficult to understand when taught using the lecture method. Furthermore, the two studies agree that the use of GeoGebra can motivate struggling (less gifted) learners in understanding trigonometric concepts in a simple and interesting way. It follows that GeoGebra caters for South African mathematics classes that are mostly made up of learners of varying performance levels. The Grade 11 learners who participated in the current study were of mixed abilities and their individual levels of giftedness were not considered during the study. Fahrudin and Pramudya (2019) also revealed that GeoGebra provides visualisation of trigonometric material thereby facilitating the understanding of concepts such as periodicity of trigonometric functions.

A study by Irawan, Mukhlash and Adzkiya (2019) in Indonesia was triggered by the fact that learners were not motivated to learn trigonometry because they did not understand the concepts. According to the study, the main contribution to not understanding was that teachers rarely used visualisation. Irawan et al (2019) noted that visualisation is important in that it provides a clearer picture of a concept. The study then engaged 20 learners and used GeoGebra for visualisation from drawing simple graphs to drawing graphs of trigonometric functions with animation and other features. Unlike Irawan et al (2019), learners in the current study used GeoGebra to draw graphs of trigonometric functions without any animations because of the researcher's limitations in the animation fraternity. The results of Irawan et al's (2019) study showed that 85% of the learners felt happier, understood better and reaped benefits after trigonometric learning by visualisation. Irawan et al's (2019) findings corroborate with those of De Villiers and Jugmohan's (2012) study which reported that the dynamic viewing of trigonometric functions when using Sketchpad enhanced learners' conceptualisation. The outcomes from these studies informed the researcher's interpretation of some results in the current study.

Pfeiffer (2017) studied the building of knowledge in circle geometry, trigonometric functions and other functions using GeoGebra. The participants were 48 Stellenbosch University students enrolled for a Mathematics and science bridging course. The students worked individually and in groups during the interaction with GeoGebra. A mixed methods exploratory case study was used where data was collected using pretest-posttest (quantitative), pre- and post – intervention questionnaires, observations, in-depth and focus-group interviews. Different learning trajectories made up of tasks that were designed under the guidance of the social constructivist perspective in learning and other theories were also employed in the collection of data. The analysis of data was more biased towards the qualitative than the quantitative. Pfeiffer (2017) used data and theory triangulation to promote validity of the study. The results showed that GeoGebra enhanced students' understanding of transformation functions, circle geometry and general solutions of trigonometric equations. Participants also suggested ways in which GeoGebra could be used to improve their understanding of concepts. Pfeiffer's (2017) study assisted the researcher with the skills and approaches in administering worksheets, conducting one-on-one interviews and focus-group discussions, and working towards the validity of the current study. Learning trigonometric functions with GeoGebra in Pfeiffer's (2017) study proved to be fun and enabled learners to understand and tackle abstract tasks better.

Scott-Wilson (2009) conducted an action research study that used a Geometer Sketchpad with 16 Grade 11 learners (who participated on voluntary basis) to study learning of graphs and their properties. Social constructivist and situated cognitive approach to learning formed the theoretical framework of Scott-Wilson's (2009) study. A structured questionnaire, learner journals, learners' written work and focus-group discussions were employed in data collection. It emerged from Scott-Wilson's (2009) study that the repeated use of Geometer's Sketchpad extended learners' thinking and it enabled learners to skilfully handle graphs. During focus-group discussions, learners in Scott-Wilson's (2009) study revealed that the technology environment created reminded them about computer games and not learning of graphs. In contradiction to the results of studies discussed earlier, visualisation had very little impact on conceptualisation of graphs by participants in Scott-Wilson's (2009) study.

2.7 THE SMARTBOARD

A smartboard is an interactive whiteboard that can be connected to one or more laptops, PCs, tablets, or other electronic devices (Really-Learn-English, 2017). The Tots-n-Tech E-Newsletter (2011) maintains that the smartboard was developed in 1991 by a company called SMART Technologies. Muhanna and Nejem (2013) hold that the smartboard interactive whiteboard is made up of a computer with smartboard software, a projector and the smartboard interactive whiteboard itself. From the researcher's experience, the smartboard works by the touch of a finger or the provided pens. It allows teachers and learners to interact with lessons on the board using their fingers or a pointer to draw, write, drag, sort and match. The smartboard is almost the same size as the standard traditional chalkboard. Figure 2.2 shows a smartboard in use in the researcher's former classroom.

The smartboards that are being supplied to schools by the Gauteng Department of Education have varied software including GeoGebra software. The GeoGebra software used to be accessed online and downloaded into laptops (Naidoo and Govender, 2014). It is now advantageous to mathematics teachers and learners to have the software conveniently embedded in the smartboard. Most CAPS approved FET textbooks exist in soft copy in these smartboards after being uploaded by the Gauteng Department of Education.

The smartboard has numerous uses in the field of teaching and learning. Wikimedia foundation (2017) adds the notion that the Smart Notebook software is included with the smartboard and allows users to compile notes, images, and other media into virtual notebooks which can be projected and edited using the smartboard itself. In the same vein, Martinelli (2016) purports the following as what could be done with a smartboard: creating lessons using built-in templates enshrined in the SMART's Lesson Activity

Builder (LAB); allowing learners to submit answers using computers, tablets or smartphones; teaching algebra and geometry with virtual manipulatives; adding a smiley face to learner board work; handing out copies without having to use the dreaded copy machine; evenly dividing shapes to teach fractions; writing anything; finding out who has contributed what in group projects; figuring out which learners "get" it; facilitating some professional development requirements; connecting with one's tech tutor when one gets stuck. The last use is not possible in Gauteng schools because the smartboards are not connected to the internet. The study at hand confines itself to the use of GeoGebra software in the teaching and learning of trigonometric graphs. The researcher benefited from the smartboard's potential to record (using a smart recorder imbedded in it) and save lessons in the form of videos.

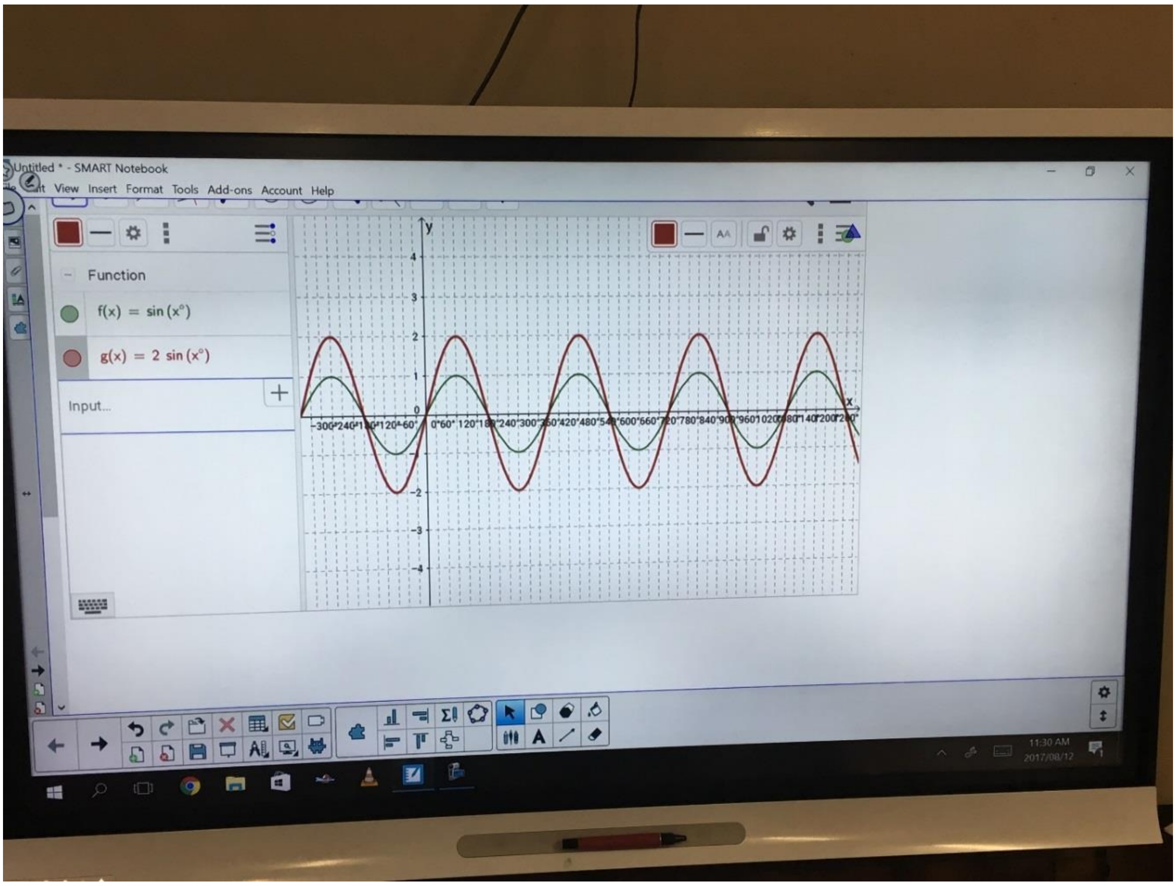


Figure 2.2: The smartboard photographed by Lancelot Makandidze on 12 August 2017

2.7.1 The smartboard in mathematics education

Muhanna and Nejem (2013) in Jordan studied the attitudes of mathematics teachers toward using a smartboard in teaching mathematics and to determine the effect of gender, experience, and qualification of teachers on their attitudes. The study found that the mathematics teachers had positive attitudes towards the use of a smartboard in teaching mathematics. There was no statistically significant difference due to gender, but there were statistically significant differences due to experience and qualification. It also emerged from the study that the interactive quality of a smartboard lends itself to a degree of learner participation not offered by other presentation methods.

A study that sought to investigate the effect of using a smartboard on mathematics achievement and retention of seventh grade learners was carried out by Nejem and Muhanna (2014) in Jordan. The outcome of the research showed a positive effect of using smartboard on learners' achievement and retention in mathematics. There was evidence that the following five reasons contributed towards this positive effect: (i) mathematics teachers are able to do many things on the smartboard to make learning mathematics more interesting such as making slide shows, using internet, draw pictures and executing mathematical games; (ii) learners feel comfortable using a smartboard and they are listing, hearing and are engaged during a lesson using a smartboard, (iii) using a smartboard helps learners to have more fun and be motivated in their mathematics lessons; (iv) visual representation on the smartboard helps learners to understand and remember mathematical information; (v) using a smartboard in teaching mathematics helps learners to participate more in class discussions, enables them to stay on task better, helps them to express their thoughts better, and enables them to be more creative. In South African schools, most learners drop mathematics for mathematical literacy, especially in township and rural schools. According to the NSC Diagnostic Report (2017) 313 030 and 245 103 sat for mathematical literacy and mathematics respectively. Perhaps integrating GeoGebra in learning trigonometric functions will attract and retain more learners in mathematics.

2.7.2 Limitations of the smartboard

The smartboard has its own limitations. The hardware and software are expensive. Each device is estimated to cost R100 000.00. This has seen the Gauteng Department of Education staggering the supply in phases, starting from Grade 12 in 2014. Presently supplies have gone as far as Grade 11, but not all schools have received the hardware.

The security of smartboards is a challenge in all schools in Gauteng. Most schools have lost their smartboards to thieves. Either the whole smartboard is stolen or only the CPU at the back of the gadget. It remains impossible to replace the stolen equipment because schools cannot afford to insure them. The school where this study was conducted had 3 of its 34 smartboards stolen to date whilst several schools have been left with nothing. The use of smartboards also suffers when there are power cuts and load shedding.

Really-Learn-English (2017:3) has revealed the following warnings about the use of the smartboard: A smartboard can't make you a better teacher – a bad teacher with a smartboard is still a bad teacher, and a good teacher doesn't need one to be a good teacher; don't use a smartboard just because it is there - think about the usefulness of what you are trying to use it for; if you ask yourself, 'What is a smartboard going to add to this activity?' and can answer the question in a positive way, go ahead and use it.

2.8 GEOGEBRA

2.8.1 What is GeoGebra?

GeoGebra is defined by Hohenwarter and Fuchs (2004) as interactive geometry software that offers algebraic possibilities like entering equations directly. From the researcher's experience, GeoGebra instantly displays corresponding graphs when algebraic equations and trigonometric functions equations are typed in the input bar. Reis (2010) maintains that GeoGebra was coined at the University of Cambridge Education Institute and it has received many awards and is being used by many education systems across the globe. The software has found its way into South African classrooms as of late. All the smartboards in Tshwane South District schools visited by the researcher have GeoGebra software.

2.8.2 Potentialities of GeoGebra

Hohenwarter and Fuchs (2004) report that the basic objects in GeoGebra are points, vectors, segments, polygons, straight lines, all conic sections and functions in *x*. The notion held is that GeoGebra enables dynamic constructions to be done, moved, edited and saved. Mathematics Education practitioners who are deeply knowledgeable in the software can produce applets. Objects can be drawn by simply entering related geometric data into the software (Hohenwarter and Fuchs, 2004). Hohenwarter and Fuchs (2004) purport that educators can use GeoGebra for lesson planning, preparations and presentations. In the current study, the researcher used GeoGebra to prepare worksheets and tests.

Numerous potentialities of GeoGebra like discovery learning, teaching fractions, learning circles, development of worksheets, creative thinking, periodicity of trigonometric functions, general student achievement and automated theorem proving have been noted by scholars. Tran, Nguyen, Bui and Phan's (2014) study in Vietnam revealed that GeoGebra software's dynamism enables it to be used in investigative tasks and assignments. Such tasks allow learners to construct their own knowledge using the tool. This corroborates with constructivist learning environments (employed in the current study) that promote understanding of mathematical concepts.

In general, South African learners are challenged by handling of fractions as indicated by their responses to assessments in the topic. A study by Thambi and Eu (2013) in Malaysia sought to examine the effect of teaching fractions using GeoGebra. The results of the study showed that learners who were taught using GeoGebra outperformed those using the traditional method and this revealed that GeoGebra enabled learners to visualise fractions better. The researcher has used fractional parameters during the exploration of Grade 11 learners' understanding of trigonometric functions using GeoGebra indicating that fractions pervade many important topics in mathematics. Performance in fractions could be improved in the South African curriculum by exploiting this GeoGebra potential as early as primary school where the topic is introduced.

Shadaan and Eu (2013) investigated learners' understanding in learning circles using GeoGebra in a Malaysian secondary school. Findings of this study indicated that learners understood circles better when they learnt using GeoGebra. Learners in the experimental group outperformed those in the control group. In South African secondary schools GeoGebra can be used to teach Circle Geometry. The study by Suratno (2016) concluded that participants who used GeoGebra worksheets developed better mathematical discovery than those taught by the traditional approach.

Yildiz, Baltaci and Demir (2017) carried out a study that investigated learning of analytical geometry and student teachers' thinking skills. The study revealed positivity on the use of GeoGebra in both aspects that were investigated. Kepceoglu and Yavuz (2016) in Turkey conducted a study on periodicity to high school learners. The outcome indicated that GeoGebra's use triumphs over traditional methods. Again, Arbain and Nurbiha's (2015) research showed that GeoGebra improved learner performance and positivity towards mathematics as well as providing diversity in learner activities.

Botana, Hohenwarter, Janicic, Kovacs, Petrvic, Rocio and Weitzhofer (2015) report that GeoGebra has the potential to prove multiple theorems instantly and more functions of the software are yet to be brought to light. The CAPS FET has Euclidean Geometry that is challenging to both educators and learners. GeoGebra could be used to prove riders. This should provoke research in the use of the software in this topic by South African mathematics education practitioners.

From the on-going discussion (and some studies discussed in Sections 2.6) and, it is clear that GeoGebra has numerous potentialities that can be utilised to teach concepts in mathematics. The current study sought to exploit GeoGebra in the learning of trigonometric functions in environments informed by constructivist and understanding theories (see Section 3.8.2).

2.9 THEORETICAL FRAMEWORK

A theoretical framework is a group of related ideas that provides guidance to a research project or a business endeavour (Business Dictionary, 2015). According to Labaree (2009), the theoretical framework is the structure that can hold or support a theory of research study. The notion held is that a theoretical framework establishes the roots of a study in theoretical and conceptual terms. Constructivist and understanding theories formed the current study's theoretical framework. The two theories informed the planning and sequencing of data collection; designing of data collection instruments; interpreting and understanding data; interpreting and understanding the relevance of findings. The researcher employed the two theories for triangulation purposes in which they supported each other although the constructivist perspective dominates the other (see Chapter 3 Section 3.9.1). The current study's literature review also included studies by scholars such as Ng and Hu (2006), De Villiers and Jugmohan (2012), Scott-Wilson (2009), Forster (1999), Agyei(2013), Pfeifer(2017) and Demir (2012) that involved the constructivist theory in the pursuit of learning trigonometric functions using technology. The following sub-sections discuss the two theories in detail, their application and relevance in previous studies and in the current study.

2.9.1 Constructivism

In general, constructivist theory is rooted in a number of disciplines like philosophy, anthropology, psychology, sociology, and education. Dewey, Piaget, Vygotsky, Bruner, and Glasersfield are the proponents of this theory whose backbone is the use of experience in active knowledge construction (Paily, 2013). Prior knowledge; pertinent environments (like digitalised ones); collaboration (like group work); varied presentations of concepts (using tools like GeoGebra); visualisation; generalisation; individual and social construction of knowledge and understanding; scaffolding (that is synonymous to ZPD by peers and technology) and reflection are some of the elements that define Constructivism and have appeared to be important and relevant in this study (Discroll, as cited in Amarin and Ghishan, 2013; Naidoo and Govender, 2012; Pfeiffer, 2017; Hoover, 1996).

Hamdani (2013) notes that constructivism insists individual construction of knowledge (insisted by Piaget) where learners are not passive receivers of knowledge. Vygotsky is the founder of social constructivism where he emphasizes the importance of the interaction with others such as peer, teachers and parents to build knowledge (Hamdani, 2013). This is supported by Amarin and Ghishan (2013) who purport that constructivism values social interaction in the learning process as promoted by Vygotsky with the assumption that knowledge is constructed by learners as they attempt to make sense of their experiences. Social interaction and construction of knowledge was utilised by Grade 11 learners in group work during worksheet activities (see Chapter 3 Section 3.8.2 and Chapter 4 Section 4.3).

The focus of this study was to explore Grade 11 learners' understanding of trigonometric functions using GeoGebra software. Constructivist ideas guided the construction, designing and sequencing of data collection instruments. For instance, the administering of a diagnostic test (before worksheet activities) that aimed to establish Grade 11 learners' prior or assumed knowledge was in line with constructivist principles that see such knowledge as a prerequisite to construction of new knowledge. This is supported by Cometto (2008) who posits that constructivism emphasises on the use of assumed knowledge in the construction of new knowledge. Again, the researcher explained and interpreted results from tests, worksheets and interviews mostly (not wholly) using tenets of the constructivist perspective and understanding theories in pursuit of answering the research question.

2.9.2 A Theory of Understanding

'A Theory of Understanding' was coined by a philosopher called David Chart in 2000 and it sees understanding as a philosophical and psychological phenomenon. The theory asserts that people understand something when they can predict what it will do under a wide range of possible conditions, and that explanations are statements that improve understanding. It follows that learners will show understanding of a concept by being able to tackle tasks asked in a different way and apply what has been learnt (in world settings, for example). In a mathematics classroom, it means that learners should not be taught in order to reproduce worked examples in a test, homework or examination because such approaches or assessments will not be measuring learners' understanding.

Chart's (2017) assertions are opposed to rote learning and understanding is viewed as the possession of mental models which provide the ability to simulate things and situations. Mental models, according to Chart (2017), are mental constructs that are similar to physical models. It appears understanding is a result of building mental models and these models built in the learners' minds help them to explain subject matter. The notion held is that mental models form the psychological part of this theory whilst explanation is the philosophical. Chart (2017:2) posits that: “By building many mental models of things that we might encounter, we open up the possibility of understanding situations that we have never previously encountered.” This statement means that learners reach higher levels of abstraction through the experience of building mental models of mathematical concepts.

In the current research, the researcher was guided by Chart’s (2017) theory of understanding in creating an environment that allowed Grade 11 learners to interact with GeoGebra to construct mental models in trigonometric functions. Explanation (the philosophical element of the theory of understanding) which is an extension of understanding assisted the researcher in interpreting the explanations given by learners in response to worksheet activities, tests and one-on-one interviews. Based on the Chart’s (2017) theory of understanding principles, reproduction of questions from the diagnostic test through worksheets to the trigonometric test was minimised in order to afford learners an opportunity to deal with new situations or questions. Furthermore, the design of question 4 of the trigonometric test (where learners were expected to apply what they had learnt using GeoGebra to obtain equations of given sketches) was informed by Chart’s (2017) theory of understanding that insists that learners should be able to cope with unexpected contingencies when they understand a concept fully.

2.9.3 Constructivism and understanding

The purpose of this study was to explore Grade 11 learners’ understanding of trigonometric functions using GeoGebra software whilst Demir (2012) argued that it was challenging to locate a suitable framework for understanding of trigonometry in mathematics education literature. From the vast literature explored by the researcher, understanding appears to be inherent in Constructivism. Mathematics is a science. Shiland(1999) contends that Constructivism is an essential ingredient in the understanding of sciences. In the same vein, Graffam (2003) asserts that blending theories from both constructivist and understanding camps is suited for mixed abilities learners (like the Grade 11 learners at a school in Tshwane South) who participated in this study. In fact, Hamdunah and Imelwaty (2019) report that constructivist approach is effective on conceptual understanding and its use will enable learners to understand mathematics material well and be able to improve their mathematical abilities. The blend of constructivism and understanding theories was employed throughout the study particularly in the designing of worksheet activities; tests questions; interpretation and understanding of data and findings. The whole study is informed by a conglomerate of the two theories since integration of technology is important and relevant to the understanding of mathematics concepts (Conley, 2004; Setyawan, Kristanto and Ishartono,2018). Guidance of learners by the researcher (as a facilitator), working of participant learners in groups, varying of parameters in graphs using GeoGebra (as a cognitive, mind or learning tool) by learners are all synonymous with constructivists and understanding principles. The two theories have been used as a reinforcement to each other and in turn to the study.

The researcher demonstrated the operation of GeoGebra (guidance and facilitation) to learners before interaction and continued to support them whenever they faced challenges with the software during worksheet activities (see Chapter 3 Section 3.8.2). Bada and Olusegun (2015) insist that the educator's primary role, in a Construcivist approach, is to create and maintain a collaborative and problem solving environment. In addition, Applefield, Huber and Moallem (2000) maintain that a costructivist teacher should stimulate thinking in learners that results in meaningful learning, deeper understanding and transfer of learning to real world contexts; and the teacher accomplishes this through incorporating strategies that encourage knowledge construction through primarily social learning processes, in which students develop their own understanding through interactions with peers and the teacher. During interactions with peers, learners explain and justify their answers to one another thereby promoting understanding according to Chart's (2017) theory and this is what transpired during worksheet activities.

Learners in this research worked in small groups as they interacted with GeoGebra to tackle the activities in the worksheets. This is collaborative elaboration (group work) in social constructivist learning that results in learners building understanding together that would not be possible if they worked individually (Van Meter and Stevens, 2000; Greeno, Collins and Resnick, 1996 cited in Bii, Mukwa and Too, 2019). Again, group work made up of small numbers of heterogeneous learners where the educator remains the facilitator are some of the basic characteristics of constructivist learning

environments (Tam, 2000 cited in Bada and Olusegun, 2015). The Grade 11 learners thrived in constructivist environments (see Chapter 3 Section 3.8.2) that allowed them to discuss and cooperate with peers to solve issues, question each other, and work through providing evidence and justification to the answers to worksheets.
Bada and Olusegun (2015) purport that constructivists believe that learning is affected by the context in which an idea is taught. In this study Geogebra in conglomeration with group work (worksheet activities) created an environment that richily exposed learners to trigonometric functions. Bada and Olusegun (2015) also hold that learners construct their own understanding and knowledge through experience and reflection on those experiences. In this regard learners reflected on their experiences (in interacting with GeoGebra) when they left the smartboard to sit down in groups to answer questions on the graphs that they had drawn using GeoGebra. The learners in this study were also afforded the opportunity to learn from their hands on experience with GeoGebra use in trigonometric functions. This notion is supported by Hurst (1998, cited in Teehan, 2019) who argues that the person doing the work is the one doing the learning.

2.9.4 Constructivism, technology and understanding

Sabzian, Gilakjani and Sodouri (2013) assert that using technology in the classroom as a means of instruction would be useful to the teachers if they are supported by appropriate educational theories and models. It is further contended that the use of ICT tools has to be informed by Constructivism. The notion held is that Constructivism and technology use should be braided and blended together to ensure effective learning. In the current research, the researcher designed learners' worksheet activities (that were completed through interaction with GeoGebra software) under the guidance of constructivist tenets. The set up allowed Grade 11 learners to explore trigonometric functions concepts in a GeoGebra environment informed by the constructivist perspective.

Vygotsky also emphasizes the need for tools such as language and the computer to mediate knowledge construction (Hamdani, 2013). This is supported by Alvine (2000) who argues that learners should learn concepts using technology tools rather than to learn about technology. In Constructivism, these tools for learning are called cognitive or mind tools. Calculators, spreadsheets, communication software are some of the examples of such tools (Sabzian et al, 2013). MacClintock (1992, cited in Sabzian et al, 2013) noted that constructivist learning environments should see content to be covered informing the tools to be used and not vice versa. Sabzian et al (2013:689) give the following arguments on tools:

> *“the role of a mind tool is to broaden the students’ cognitive functioning while they are learning and to take on the students on the tasks while making knowledge that they have not managed to acquire otherwise. Mind tools make the students capable of becoming critical thinkers. By making use of cognitive tools, learners are also engaged in knowledge creation rather than knowledge reproduction. Learners utilise the available software to use technology to both make and show knowledge”*

In this research GeoGebra software is the cognitive, mind or thinking tool that was used for constructing knowledge in trigonometric functions. In accordance (and in corroboration) with Chart’s (2017) theory of understanding GeoGebra is the tool that Grade 11 learners used to construct their mental models in trigonometric functions.

The purpose of this study was to explore Grade 11 learners’ understanding of trigonometric functions using GeoGebra software. The exploration was seen through the lenses of constructivist and understanding theories. A synthesis of many researchers’ and scholars’ literature, findings and views reveal that technology environments that are informed by these theories promote learners’ understanding through interweaved (but not limited to) prior knowledge, collaboration, scaffolding, generalisation, visualisation. Grade 11 learners who participated in the current study worked in groups where collaboration thrived as the learners interacted with GeoGebra during worksheet activities. Wu, Farrell and Singley’s (2002) report that technology environments support collaborative engagements where learners critique and correct one another’s solutions before coming up with a refined solution for the group. This is supported by Maroske (2015) who reports that such solutions are a result of shared understanding. Maroske’s (2015) study prescribes groups of mixed abilities (like those employed in this study with Grade 11 learners) where tasks have clear outcomes because they promote high levels of collaboration.

Collaborative-technology environments in the learning of mathematics concepts support peer scaffolding (Lombardi, 2017; Maroske, 2015) and technology scaffolding (Bakker, Smit and Wegerif, 2015; Wu et al 2002). Scaffolding is a process that helps a learner to understand a mathematical concept and be able to solve problems in the aspect that the learner would not have been able to understand or solve without the assistance of peers, technology, and teachers. This is supported by Gibbons (2002, cited in Bakker et al, 2015) who perceives scaffolding as a temporary, intentional responsive support that assists learners to move towards new skills, concepts of levels of understanding. Furthermore, the concept scaffolding originates and is inherent in Vygotsksy's Zone of Proximal Development (ZPD) whose idea is of the zone of what the learner can do when supported (or even scaffolded) by an adult or knowledgeable other (Bakker et al, 2015; Lombardi, 2017). In addition, Maroske's (2015) study reports that learners scaffold the understanding of peers during interactions by building on one another's ideas and through explanations. Explanations, according to Chart's (2017) theory of understanding, improve understanding. Again, studies by Bakker et al (2015) and Wu et al (2002) showed that technology scaffolding for learners working in groups fosters conceptual understanding in mathematics. In this research, GeoGebra scaffolded Grade 11 learners during worksheet activities while the learners scaffolded one another within a group.

Ellis (2011) studied generalising promoting actions in quadratic functions where Grade 8 learners worked in groups and their group discussions were video recorded. The results of the study revealed that collaborative work during interaction amongst learners enabled them to create and adjust generalisations in the functions although no technology had been used. Geraniou, Mavrikis, Hoyles and Noss (2010) used a computer software tool called eXpresser to create a learning environment that supported mathematical generalisation in number patterns. The approach in the study was also collaborative group work where generalisation flourished through discussing, justifying and defending of constructions and rules. The researcher found these studies by Ellis (2011) and Geraniou et al (2010) important because Grade 11 learners in the current study entered numerical values in GeoGebra software when drawing trigonometric functions and there was a section in the worksheets that required learners to come up with a generalisation in algebraic form.

It appears environments in which learners interact with technology while working collaboratively in groups promote learning through collaborative visualisation (Wu et al, 2002), where learners conceptualise mathematical ideas from engaging in visual arguments (Caligaris, Rodriguez and Laugero, 2015). Arcavi (2003, cited in Nghifimule and Schafer, 2018:58) views visualisation as: "The ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas, and advancing understandings". Many studies (Agyei, 2013; Fahrudin and Pramudya, 2019; Irawan et al, 2019; Naidoo and Govender, 2014; Scott-Wilson, 2019; Thambi and Eu, 2013) discussed earlier have reported on the strength of technology through visualisation. The researcher found Forster's (1999) study also interesting because its methodology and findings are supportive to the current study.

Forster (1999) conducted a study on 'Applying Constructivist Theory to Practice in a Technology-Based Learning Environment' where 68 Grade 12 learners of mixed abilities and four educators participated. Matrices, exponential functions and descriptive statistics were the topics that were found to be suitable for technology integration in the study. The technology employed by Forster (1999) in the study were graphics calculators and computer spreadsheets. Assessment scores from worksheets (that were collaboratively done in groups where learners chose their working partners), class tests (that were written individually) and questionnaires were used to elicit data from learners whilst teachers provided it in the form of verbal and written feedback. Forster (1999) interviewed three learners to ascertain their responses to written work. Her data interpretation was guided by continuous literature review whilst discussions were informed by Constructivism and the findings were that: technology (like GeoGebra) has the potential to offer learners dynamic displays and visualisations; learners did better in worksheets than class tests because of the possibility that they used mechanical means to tackle questions in the worksheets; the other possible reason for low attainment was that learners did not have prior knowledge on which to build new knowledge; active involvement was found to be necessary for understanding. Forster's (1999) finding on visualisation concurs with

Malabar and Pountney's (2002) paper which argues that visual software (like GeoGebra used in the current study) creates environments that enable learners to build their own knowledge and improve their understanding of more abstract concepts. It appears Forster's (1999) study could have been better if a diagnostic test was administered like in the current study. This is essential in aligning prior knowledge with new activities to be employed for the study at hand.

2.10 CONCLUSION

This chapter has reviewed related literature under the following sub-headings: history of digital media in mathematics education; the importance of integrating technology in mathematics education; constraints in the integration of technology in mathematics education; the importance of trigonometry in the curriculum; the integration of technology in trigonometry and trigonometric functions; the smartboard; GeoGebra and theoretical framework. The Mathematics Teaching and Learning Framework for South Africa's (2018) dimensions are all aimed at teaching mathematics with understanding. The literature reviewed emphasises that technology integration enhances understanding of mathematical concepts. A handful of research reports discussed in the reviewed literature portray the technology integration in trigonometric functions in general while others limited themselves to sine and cosine functions (Kissane and Kemp,2010; Ng and Hu, 2006; Demir, 2012; De Villiers and Jugmohan, 2012). This study explored the power of GeoGebra in learners' understanding of the effects of parameters a, k, p and q on sine, cosine and tangent functions.

The poor performance in trigonometric functions has been published in the NSC Diagnostic Reports from 2011 to 2018 without hands-on remedies being sought. This study, therefore, serves as a solution to the long recurring problem in trigonometric functions. The studies on the use of GeoGebra done in some parts of South Africa depended on the software package as an online resource (Naidoo and Govender, 2014). This study is conducted timeously, in the Tshwane South District context, where GeoGebra exists in smartboards that pervaded mathematics classrooms in 2015. It is held that this study is the first of its kind in the smartboard era of Tshwane South District. GeoGebra is now an accessible resource whose potentialities should be exploited in the exploration of trigonometric functions.

Constructivist and understanding theories reviewed in this chapter, prepared the ground for application of these ideas into this study. The design of learners' activities in this study is informed by both Constructivism and understanding theories. The theories contend that learners actively construct new knowledge and understanding through their experiences, construction of mental models and explanation (Hamdani, 2013; Bhowmik, 2014; Skemp, 1976; Chart, 2017). Again, this is in line with learner-centred classrooms, which is one of the dimensions of the Mathematics Teaching and Learning Framework for South Africa (2018).

This study follows qualitative tradition that prefers literature review to be distributed over the dissertation as a whole, so that it reads like an on-going conversation between research and scholarly theory (Kent University, 2016). The notion held is that all the chapters in this study are buttressed by literature and theories. The chapter that follows gives the research methodology used in this study, that is supported by the literature and framework discussed in this chapter.

CHAPTER 3
RESEARCH METHODOLOGY

3.1 INTRODUCTION

The previous chapter presented the literature review and highlighted theoretical frameworks that guide the research methodology. The motive of this chapter is to present the methodology that answered the following main research question**:**
How does the use of GeoGebra software enhance Grade 11 learners' understanding of trigonometric functions?

The following sub-questions helped to answer the main question:

(i) How do GeoGebra environments help learners in understanding trigonometric functions?
(ii) How is learners' understanding of trigonometric functions after interaction with GeoGebra?
(iii) What are learners' experiences and views on the use of GeoGebra in exploring trigonometric functions?

First, the research paradigm is discussed. Second, the research approach and the research design are described and justified. The chapter also describes the research setting, the population and the sample, data collection and analysis procedures. Lastly, trustworthiness and ethical considerations are dealt with.

3.2 PARADIGM

Willis (2007, cited in Taylor and Medina, 2013) defines a paradigm as a comprehensive belief system or a world view that guides research and practice in a field. Paradigms are a means of conducting rigorous researches that help educationists to refine their practices (Taylor and Medina, 2013). Positivist and post-positivist are traditional paradigms whilst the interpretive, the critical and the postmodern are relatively new paradigms. All paradigms are on a par and they are powerful when applied in the proper context.

The philosophical underpinnings of this case study of Grade 11 mathematics learners at a school in Tshwane South District are those of the constructivist, which is also called the interpretive paradigm. This is supported by Baxter and Jack (2008), Golafshani (2003) and Adom, Yeboah and Ankrah (2016) who posit that case studies that are qualitative in nature affiliate to the constructivist or interpretive paradigm. In addition, Baxter and Jack (2008) assert that this philosophical underpinning sees the truth as reciprocal to the way we view social settings. This paradigm sees participants as informants to researchers and this relationship is vital (Crabtree and Miller, 1999). It is while informing the researchers that the participants are able to describe their views of reality and this enables the researchers to understand the participants' actions better. The researcher used unstructured one-on-one and focus-group interviews that allowed Grade 11 learners to ascertain what they had written and to express their views and experiences on the use of GeoGebra and understanding of trigonometric graphs.

The constructivist or interpretive paradigm collects data mostly through interviews, participant observation, pictures, photographs, diaries and documentation. This study aligns itself with this paradigm since the researcher took screen shots of the smartboard presentations; allowed the smart recorder to record learners' activities using GeoGebra; took photographs of learners' work to support analysis. Again, Golafshani (2003) contends that the constructivist or interpretive paradigm is characterised by the use of numerous ways of collecting data to ensure validity and reliability (trustworthiness). Similarly, the current research employed tests, worksheets and interviews to collect data.

3.3 RESEARCH APPROACH

This study used a qualitative research approach. According to MacMillan and Schumacher (2014), qualitative research is an in-depth study using face-to-face or observation techniques to collect data from people in their natural settings. In the same vein, Creswell (2012) asserts that qualitative research concerns itself with small groups of humans. Again, qualitative research is capable of thoroughly examining problems in education that exist in different settings and obtain relevant recommendations

(Anderson, 2010). Six Grade 11 learners (a small number of individuals) participated in this study.

This study thrives on the following strengths of qualitative research: open ended questions can be used in interviews where follow ups can be done; data obtained directly from people is richer than quantitative; findings can be replicated to other contexts (Anderson, 2010).

The qualitative research approach has its own limitations which are, according to Anderson (2010): research quality is prone to bias because it relies on the researcher's competences; can be laborious when large volumes of data are involved; it is sometimes not as well understood and accepted as quantitative research within the scientific community; concealing identification of sites and individuals can be a challenge. The researcher minimised the limitations of this study by piloting the instruments used and through triangulation. Codes were also used in place of the participants' names.

3.4 RESEARCH DESIGN

The researcher employed qualitative case study design which Baxter and Jack (2008) see as a methodology that equips researchers with potentialities of investigating complicated cases in a given setup. A case study, according to the University of Southern California (2006), is an in-depth study of a particular research problem rather than a sweeping statistical survey or comprehensive comparative inquiry. It follows that this design allows large scale fields or populations to be divided into smaller cases that are manageable or feasible to study. This design is suited for evaluating theories and models and resonates well with exploring of unknown phenomenon (University of Southern California, 2006). The focus in this study was Grade 11 learners and through their participation, the researcher explored in depth how the use of GeoGebra software in learning trigonometric functions enhanced their understanding.

MacMillan and Schumacher (2014) contend that a case study is qualitative research that examines in depth a bounded system, or a case, over time, employing multiple sources of data found in the setting. The researcher employed tests, worksheets, a smart recorder and interviews in the collection of data and used constructivist and understanding theories as its lenses, a process that ensures rigour (Baxter and Jack, 2008). The study limited itself to Grade 11 learners in a school in Tshwane South District and this renders it a single case study design that is exploratory in nature. Yin (cited in Baxter and Jack, 2008) sees an exploratory case study as a type that is used to explore those situations in which the intervention being evaluated has no clear, single set of outcomes.

3.5 RESEARCH SETTING

A research setting is perceived as the site of participants (a natural setting) where the qualitative researcher goes to conduct a research (Creswell, 2003, cited in Hossain, 2011). The current study was conducted at a public non-fee secondary school (grades 8 to 12) in Tshwane South District (Pretoria city, Gauteng Province) in South Africa. The secondary school, which has an enrolment around 1 600, is a previously disadvantaged school located in a township comprising of lower to middle class residents. Learners access the school either by foot, public transport or scholar transport hired by GDE. The transport for the learners leaves soon after the last period (around 14:30) and this posed challenges in learners' participation in the current study. Classes in the school are of mixed ability, meaning that they are made up of promoted and progressed learners since the Department of Basic Education prohibits screening. The researcher chose the school for the following reasons: there is limited research of this nature in such schools yet they are in majority of South Africa (Pfeiffer, 2017); easy access; availability of smartboards; learners benefited academically from participating in the study.

3.6 SAMPLE AND POPULATION

3.6.1 The sampling method

This study used purposeful or purposive sampling. MacMillan and Schumacher (2014) see purposeful sampling as a type of sampling that allows choosing small groups or individuals who are likely to be knowledgeable and informative about the phenomenon of interest; selecting cases without needing or desiring to generalise to all such cases. In the same vein, Creswell (2012) argues that purposeful qualitative sampling is when we target humans or settings that are rich with data that can help in examining the problem at hand. The researcher was interested in the understanding of trigonometric functions by Grade 11 learners at a school in Tshwane South District. The school is one of the full ICT secondary schools in the district. Full ICT schools have smart boards in all classrooms whilst the rest of the schools have smartboards in a few Grades 11 and 12 classrooms. The researcher was assured of access to a smartboard due to their abundance in the school. The Grade 11 learners who participated in the study were a homogeneous group in terms of their level of mathematics by grade (Creswell, 2012) and the researcher believed that they will fulfil the needs of the study (Rugg, 2013)

3.6.2 Participant selection

There was only one Grade 11 mathematics class at the school with 29 learners and the invitation to participate was extended to all of them. Participation in this study was voluntary. The researcher took the class list and randomly labelled names L1 to L29 before meeting the prospective volunteers. Each volunteer was later informed of his or her code. This was done to conceal the identification of participants throughout the research process. The naming also facilitated tracking of learners' responses in the instruments used. Attendance by some of those few learners who had volunteered was erratic due to transport issues and other commitments like School Based Assessments (SBAs) completion, and this prompted the researcher to consider, concentrate and focus on those learners (L7, L12, L17, L19, L24 and L26) whose attendance was at least five out of seven days (see Table 3.1). The six participants comprised two boys and four girls whose ages ranged from 16 to 18. The researcher could not stop other learners who randomly came to the sessions (due to the attraction to smartboard activities) for ethical reasons. This resulted in varying group combinations on selected days as shown on Table 3.4.

Table 3.1: Participants attendance

		Day and Attendance							
Learner	**Gender**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**Total Attendance**
L2	female	absent	Present	absent	absent	absent	absent	absent	1
L4	male	absent	Present	absent	absent	absent	absent	absent	1
L5	male	absent	Present	absent	absent	absent	absent	absent	1
L6	female	absent	Present	absent	present	present	absent	absent	3
L7	female	present	Present	present	present	present	present	absent	6
L8	female	absent	Present	absent	absent	absent	absent	absent	1
L12	female	present	Present	present	present	present	present	present	7
L13	female	absent	Present	absent	absent	absent	absent	absent	1
L14	male	absent	Absent	present	absent	present	absent	absent	2
L15	male	absent	present	present	absent	absent	absent	absent	2
L16	female	absent	present	present	absent	absent	absent	absent	2
L17	female	absent	present	absent	present	present	present	present	5
L18	female	absent	present	absent	absent	absent	absent	absent	1
L19	male	present	present	absent	present	present	present	present	6
L20	male	absent	present	absent	absent	absent	absent	absent	1
L22	male	absent	present	absent	absent	absent	absent	absent	1
L24	male	present	present	absent	present	present	present	present	6
L26	female	present	present	absent	present	present	present	present	6
L29	female	absent	present	absent	absent	absent	absent	absent	1
Activity		**Diagnostic Test**	**Worksheets 1,2 and 3**	**Worksheet 4**	**Worksheet 5**	**Worksheet 6**	**Trigonometric Test**	**Unstructured one-on-one interviews and Focus Group Interviews**	

3.7 PILOTING

The researcher piloted the study three weeks before the main enquiry began. Piloting was done to determine feasibility of the full-scale study and to pave way for amendments to instruments and/or design of the future study. The pilot study was a compressed trial run or rehearsal that sought to test the instruments and afford the researcher some experience on interviewing. In addition, the results of the pilot study `were not analysed systematically since the researcher aimed to have a glimpse of every stage of the main study. This is in line with scholars like De Villiers and Jugmohan (2012) and Uddin (2011) who piloted their instruments (interviews and activity instruments; pre and post exercises, observations) and made adaptations or alterations without analysing data and publishing results. Bell (2005) argues that all data-gathering instruments should be piloted to test how long it takes recipients to complete them, to check that all questions and instructions are clear and to enable the researcher to remove any items which do not yield usable data.

The instruments used in this study were piloted at a neighbouring secondary school, where nine Grade 11 mathematics learners participated on voluntary basis. The school was chosen because its learners and those of the main site have the same characteristics since they come from the same catchment area. Permission to conduct the rehearsal was obtained through the letter from GDE. The pilot study was done in four days (in the afternoon after normal lessons had been completed) due to limited time allocated to the researcher by the school. Participants were named TB1 to TB9 to conceal their identity. Learners sat for the diagnostic test on the first day, did worksheets 1 to 3 in the following two days and the fourth day was used for the interviews. Two laptops with GeoGebra software were used for worksheet activities since the school did not have a smartboard and the participants worked in two groups of four and five. The computers offered an environment like that of a smartboard with GeoGebra software.

Cadete (2017:2) argues that adjustments are unavoidable when piloting has been completed. After conducting the piloting, the researcher found that the main study was going to be feasible with a few amendments or changes in each of the instruments. The diagnostic and the trigonometric tests had their durations changed to 45 and 60 minutes respectively, in response to the time taken by the pilot group. Again, the researcher was informed by participants that the space for learner responses to questions was small and it was adjusted after piloting. The space had to be increased to promote neat and clear graphs.

The researcher reduced the number of activities per worksheet in the main study since groups were sharing one smartboard. This enabled all the group work to be completed within a session allocated for a day in the full-scale study. The researcher was compelled to train the participants in the main study thoroughly on the operation of GeoGebra so that they could do activities in the worksheets without being challenged by the software.

The researcher was signalled, by reactions in the pilot study, to establish rapport and use simple English when interviewing. This emanated from most learners who declined being interviewed citing that they were not good in English. Bell (2005) maintains that interview schedules need to be tested because there is need for a researcher to practice asking questions and recording responses. After replaying and replaying the tape-recorded interviews (for two learners), the researcher learnt that he asked leading questions when following up answers from interviewees and this was rectified. The tape-recorded interviews were played to the supervisor who in turn suggested some improvements. The interview questions were refined, and questioning skills improved.

3.8 INSTRUMENTS, DATA COLLECTION AND ANALYSIS PROCEDURES

The study used a diagnostic test, worksheets, a trigonometric functions test, one-on-one unstructured interviews and a single focus-group interview (respectively) as its main data collection instruments. The researcher developed and administered all the data collection instruments. The instruments were authenticated and approved by the researcher's supervisor and the UNISA College of Education Ethics Review Committee.

Table 3.2: Data collection schedule

Day	Activity
1	Diagnostic test
2	Lesson 1[worksheets on parameters *a* and *q*]
3	Lesson 2 [worksheets on the combination of *a* and *q* parameters]
4	Lesson 3 [worksheet on parameter *p*]
5	Lesson 4 [worksheet on parameter k and worksheets on functions combining at most two of the parameters *a, k, p* and *q*]
6	Trigonometric functions test
7	Interviews: one-on-one followed by focus-group interviews

The collection of data was done in seven days after school as indicated in Table 3.2. The methods of data collection in qualitative research are expanding to encompass active participation by participants such as worksheet activities used in the current study (Creswell, 2003, cited in Hossain, 2011). This is in line with constructivist theories that prescribe that learners should be active participants in construction of their own understanding. In addition, (Creswell, 2003, cited in Hossain, 2011) argues that a case study (like the current study allows a wide range of diverse methods of data analysis strategies. The following sub-sections discuss each instrument, how it was used to collect data and how the data was analysed.

3.8.1 Diagnostic test, data collection and analysis strategies

The diagnostic test (see Appendix O) was written by five participants (L7, L12, L19, L24 and L26) within 45 minutes, nine days before worksheet activities commenced. McGee (2018) contends that diagnostic testing in education happens before instruction ever begins. In addition, Shim, Shakawi and Azizan (2017: 364) hold that "One way to gather information about students' basic skill is through the use of diagnostic test. The diagnostic test in education is a preliminary assessment mainly used to detect students' strengths and weaknesses in learning. It allows educators to cater their teaching style and content to suit to the students' basic knowledge." The purpose of the diagnostic test was to assess and evaluate the participants' assumed or prior knowledge and skills on Grade 10 trigonometric functions in line with constructivist elements, and to triangulate it with other data sources. Table 3.3 presents information regarding the aim of each test item. The test items were created according to the Grade 10 CAPS requirements.

Table 3.3: Content Tasks in the Diagnostic Test

Question	**Task**
1	Plotting trigonometric functions involving parameters *a* and *q*.
2	Identifying amplitude and period from given trigonometric function(s).
3	Stating the range of given trigonometric functions.
4	Stating the relationship between sine graph and its transformations

The administering of the diagnostic test was informed by the constructivist perspective that insists on the building of new knowledge on prior knowledge (see Cometto, 2008 in Chapter 2 section 2.9.1). The constructivist perspective argues that prior knowledge points to relevant knowledge learners may already have and to knowledge which may be necessary in order to support them in accessing the new topic (Kinchin, 1998; Ishii, 2003; Sjoberg, 2007; Project Maths Development Team, 2009). The Grade 10 knowledge tested in the diagnostic test served as assumed (prior or previous) knowledge on which to build Grade 11 trigonometric functions aspects. Learners' responses and performance in the content tasks guided the researcher on what to include in the first worksheet(s) to ensure a smooth linkage between the two grades' content. This corroborates with one of the basic guiding principles of constructivist learning which states that new knowledge can only be built on a structure developed from prior knowledge (Hein, 2007, cited in Mogashoa, 2014).

The diagnostic test was analysed qualitatively question by question. The learners' responses to the test were triangulated with other data collection methods used in this study. For instance, learners were interviewed about what they had written in the diagnostic test during focus-group interviews. The learners' responses in the diagnostic test (like drawing of graphs) were compared with those in the worksheets and in the Trigonometric test in order to trace how GeoGebra enhanced the learners' understanding of trigonometric functions.

3.8.2 Worksheets, data collection and analysis strategies

Super Teacher Worksheets (STW) (2016) defines worksheets as simple printable teaching resources that, when combined with good teaching, can help learners learn important concepts. STW (2016) further notes that smartboards, iPads, computers, chalkboards, and worksheets are all important tools of our teaching and learning trade. It is held that worksheets do not have to be mundane drill-and-practice rituals for learners. Instead, they should be engaging, interactive, creative, hands-on, fun, and useful tools for learners and teachers. This capability to engage, interact and hands-on offered by worksheets promotes a dynamic process where learners can easily construct their knowledge and understanding as prescribed by Constructivism (Gilakjani, Leong and Ismail, 2013). The researcher developed and designed the worksheets using GeoGebra software.

The researcher sought to explore Grade 11 learners' understanding of trigonometric functions using GeoGebra software. The purpose of the worksheet activities was to enable learners to deeply explore trigonometric functions concepts using GeoGebra through drawing graphs and answering questions involving parameters *a, k, p* and *q*. It was therefore paramount for the researcher to create a learning environment (using worksheets and GeoGebra) that allowed learners to construct their knowledge and understanding, in corroboration with constructivist and understanding theories. The worksheet activities enabled learners to interact or interface with GeoGebra. Furthermore, the worksheet activities (tackled using GeoGebra) served as hands-on and minds-on engagements, where learning by doing took centre stage and this is in line with constructivist learning practices (Adom, Yeboah and Ankrah, 2016). According to CAPS, Grade 11 trigonometric functions aspects should be built on Grade 10 ones. Having established the learners' prior knowledge, the researcher altered and designed Worksheets 1 to 3 to incorporate Grade 10 work in parameters *a* and *q* (see Appendix I to K). This was done to allow learners to engage GeoGebra in Grade 10 work that they already knew and simultaneously closing gaps identified in the diagnostic test before exploring Grade 11 aspects. The step is in agreement with Hoover (1996) and Adon et al (2016) who insist that facilitators or researchers employing constructivist learning approaches should establish prior knowledge in order to be able to design learning environments that will exploit inconsistencies between learners' current understandings and the new aspects to be learnt. Again, the GeoGebra environments used in this study are viewed as constructivist in nature by Roschelle, Pea, Hoadley, Gordin and Means (2000) who argue that technology enables learners to smoothly build upon their previous knowledge and skills.

On the second day of data collection (nine days after administering the diagnostic test), group work on worksheets activities began (see Appendices I to N for Worksheets 1 to 6 respectively). The researcher (as a facilitator according to the constructivist perspective) guided the learners (in their respective groups) on how to project trigonometric functions on the smartboard using GeoGebra and how to operate the smartboard in general through demonstration and explanation. In particular, the learners were trained on: x and y- axis scales and their units; inserting equations into the input bar; zooming in and out; recording their GeoGebra work with a smart recorder and saving it.The learners were further supported by the researcher in smartboard operations throughout the preliminary sessions. Towards the end of the sessions, learners were using GeoGebra with minimum help from the researcher. This is in line with Shelly, Cashman, Gunter and Gunter (2008) who recommend that learners should have the ability to use computers, technology, and digital media because this provides learners with a sound foundation of operation and application skills that can be transferred to current, new and emerging technologies. In the focus-group discussions learners revealed that being enlightened on the settings and operations of GeoGebra (by the researcher) helped them a lot in efficiently and effectively using the software in tackling worksheets activities. Again, viewing through the constructivist lens the role of a facilitator taken by the researcher during worksheets activities promoted learner-centredness and self-regulation in the use of GeoGebra (Naidoo and Govender, 2014). In this context, GeoGebra is a cultural and learning tool for the learners' cognitive development (McLeod, 2018). Viewing from Chart's (2017) Theory of understanding, GeoGebra was a tool that learners used to build their mental models with respect to trigonometric functions concepts.

The learners worked on the worksheets (Worksheets 1 to 6) in groups of their choices of not more than six learners each (see Table 3.4). During worksheet activities sessions, several Grade 11 learners were attracted by the interaction with software, but their attendance was erratic, and the researcher could not stop them from attending for ethical reasons. Since learners joined groups of their choices the naming of the groups changed each day, as shown in Table 3.4.

Table 3.4: Groups that worked on worksheets

Day	**Worksheet number(s)**	**Groups and members**
2	1,2 and 3	Group 1 [**L7**, L8, L13, **L19**, L20, L29] Group 2 [L4, L5, L15, L16, **L24**, **L26**] Group 3 [L2, L6, **L12**, **L17**, L18, L22]
3	4	Group 1B [**L7**, **L12**, L14, L15, L16]
4	5	Group A1 [**L7**, **L19**, **L26**] Group A2 [L6, **L12**, **L17**, **L24**]
5	6	Group A1 [**L7**, L14, **L19**, **L26**] Group A2 [L6, **L12**, **L17**, **L24**]

The smart recorder captured all the proceedings done by each group as they interacted with GeoGebra and the work was saved in the smartboard and later copied into the USB by the researcher before storing it in the laptop for easy access during analysis and for audit trail purposes. Three worksheets (Worksheet 1, 2 and 3) were done by each group on the second day of data collection, Worksheet 4 on the third day, Worksheet 5 on the fourth day and the last Worksheet 6 on the fifth day. The groups rotated to use the one smartboard that was available. The first session ran from 14:30 to 17:45. The amount of work could not fit within the two hours' time schedule allocated to the researcher by the school. This prompted the researcher to scale down the number of worksheets done per session, as well as the number of tasks per worksheet for the days that followed.

Macleod and McLeod (2016) purport that in mathematics education the pendulum has moved from a focus on the individual learner and hence it is worthwhile to consider how learners learn individually and in groups. This means that mathematics learning environments should accommodate individual and social construction of knowledge, which is a basic tenet of constructivist learning. Grade 11 learners in the current study interacted with one another and with GeoGebra software as they solved problems on the worksheets. Antonelli (2013) purports that a worksheet provides an important scaffold for learners as they construct their understanding. The environment that the researcher created for data collection thrived on a conglomeration of worksheets, GeoGebra and group work where varied scaffoldings prevailed (see section 2.9.4 in Chapter 2). This approach follows on the footsteps of studies by Weber (2005); Ng and Hu (2006); Demir (2012); Agyei (2013); Naidoo and Govender (2014); Brijlall and Niranjan (2015); Pfeiffer (2017); Suratno (2016); Forster (1999) that used a blend of worksheets and technology in the collection of data in mathematics classrooms.

The worksheets were analysed worksheet by worksheet and this analysis, supported by smart recordings, sought to explore how GeoGebra enhanced learners understanding of trigonometric functions. Money in Action (2008) contends that worksheet activities may be used as assessment tasks and if learners complete them successfully, they have met the relevant assessment standard. According to the CAPS (2010), Grade 11 learners should be able to show understanding of the effects of parameters *a*, *k*, *p* and *q* on trigonometric functions. Each worksheet had the first part that required learners to draw graphs using GeoGebra and the second part where learners had to leave the smartboard and answer questions on paper in the spaces provided. The second part needed learners to reflect on their interaction with GeoGebra.

The learners (L7, L12, L17, L19, L24 and L26) in bold as indicated in Table 3.4 were the targeted participants in the current study due to their high frequency of attendance (see Table 3.1). The analysis focused on these targeted learners where their understanding of trigonometric functions was delved within their respective group worksheet activities, comprising of smart recorded activities and hand-written work. A follow up on worksheet activities written work was done through one-on-one interviews. The learners' responses and understanding (performance) in the worksheet activities were triangulated with those in the diagnostic test, trigonometric functions test and one-on-one interviews. The researcher employed the constructivist learning and understanding theories in understanding and interpreting results. Again, learners' experiences with GeoGebra were probed during focus-group interviews.

3.8.3 Trigonometric functions test, data collection and analysis strategies

The researcher administered the trigonometric functions test a day after the last worksheet (Worksheet 6) was done. The participants (L7, L12, L17, L19, L24 and L26) sat for the test. The test was made up of four questions and was completed in an hour (see Appendices Q and R). Table 3.5 presents the aspects contained in the test. The test was guided by CAPS and textbooks by Mouton (2012) and Abbott et al (2012). The purpose of the trigonometric test (after learners had exploited the potentialities of GeoGebra software) was to: explore and evaluate the learners' learning gains and understanding of trigonometric functions; elicit learners' individual knowledge, skills and or abilities to sketch and interpret trigonometric functions. Gall, Gall and Borg (2005:314) highlight the following on tests: "While more typical of quantitative research, tests can serve a useful purpose in qualitative research."

The trigonometric functions test acted as a summative evaluation while the worksheets provided evaluation of the learners' understanding at formative level. Following similar lines, researchers such as Uddin (2011) and Naidoo and Govender (2014) used two worksheets to evaluate learners' understanding of trigonometric graphs before and after the learners interacted with GeoGebra. In the current research, the analysis of the data collected through these instruments (diagnostic test, worksheets, trigonometric functions test and one-on-one interviews) helped the researcher to explore in depth (through assessment) the learners' understanding of trigonometric functions. This is in line with Hiebert and Carpenter (1992, cited in Barmby, Harries, Higgins and Suggate, 2007:41) who observed that a profile of learners' understanding can be generated and inferred from the learners' responses to a variety tasks.

Table 3.5: Content tasks of the trigonometric functions test

Question	Task
1	Sketching trigonometric functions with parameters *a*, *k*, *p* and *q*
2	Using a given graph to answer questions on its behaviour and characteristics (interpreting a trigonometric function).
3	Interpreting the equation of a trigonometric function.
4	Determining the equation of given sketches.

The trigonometic test was analysed question by question. Maxwell (2010) insists that the use of numbers does not make a study mixed method, instead numeric data makes qualitative studies more scientific. Numeric data in the form of scores in the trigonometric test questions and parameter values assisted the researcher to identify patterns that emerged during analysis (Sandelowski, 2001 and Maxwell, 2010). Furthermore, the existence of numbers in the data facilitated generalisation within participant learners (Maxwell, 2010). The learners' responses in the test were also compared to those in the diagnostic test and worksheets in order to establish understanding brought by the use of GeoGebra as a learning tool. Furthermore, a follow up on the responses to the Trigonometric test was done through one-on-one interviews as a way of triangulating. Understanding and interpreting data from the trigonometric funcions test mainly assited the researcher to respond to the second sub-research question.

3.8.4 Unstructured one-on-one interviews, data collection and analysis strategies

Interviews are a common form of data collection in case study research and they allow the researcher to attain rich, personalised information (Hancock and Algozzine, 2011). Mertler and Charles (2011) perceive interviews as conversations between the researcher and participants in the study. Clarifications on responses can be sought through follow up questions. Furthermore, an interview can obtain more useful information than a questionnaire (Mertler and Charles, 2011).

The researcher opted for the one-on-one unstructured interviews because the questions to be asked depended on how each learner answered questions on trigonometric functions. The Center for Innovation in Research and Teaching (2015) assert that unstructured interviews have no standard set of questions and are often used to explore an idea. It is also held that open-ended questions can be used in unstructured interviews. Unstructured interviews are strong in that they are more in-depth than other interviews. They allow the interviewer to follow up; they are less rigid and have room for open responses (Center for Innovation in Research and Teaching, 2015).

The disadvantage of unstructured interviews is that they are more time consuming and have less consistency in data collection (Center for Innovation in Research and Teaching, 2015). In this study the interviews were kept reasonably short, an average of 15 minutes per interviewee. The researcher drafted a framework or schedule of questions on which the interview was guided. Establishing such a framework beforehand greatly simplifies recording and analysis of data (Bell, 2005).

A day after writing the trigonometric test, the researcher scanned through the learners' solutions to the worksheets and the trigonometric test. Five learners (L12, L17, L19, L24 and L26) present on the day, volunteered and were interviewed according to their varying responses or solutions to the written questions. The sixth learner, L7, could not attend the interviews because of the commitments in other learning areas in that afternoon. The interview sessions with learners were tape-recorded using a voice recorder which is a technique supported by Hancock and Algozzine (2011). Furthermore, tape-recorded interviews can be replayed several times thereby allowing coding and summarising of utterances.

Kerlinger (cited in Cohen, Manion and Morrison, 2004) argues that interviews may be used in conjunction with other methods in a research undertaking to go deeper into the motivations of respondents and their reasons for responding as they do. The learners' understanding of trigonometric functions was being probed in the current research. The researcher relied on constructivist and understanding theories to understand and interpret data from one-on-one interviews. The ability to explain how and why functions behaved the way they appeared on the completed worksheets and written tests was evidence that learners had understood. According to Dewey and in Gestalt psychology, understanding is an act (Seirpinska, 1990). It is also noted that explanation, for Dewey, means to understand. This is line with Chart's (2017) Theory of Understanding discussed in Chapter 2, section 2.9.2. Furthermore, Seirpinska (1990) reports that understanding and explaining are even more deeply reconciled in the conception of interpretation of discourse or text. The ability to describe and interpret the effects of the parameters *a, k, p* and *q* on trigonometric functions by learners, during the one-on-one interviews, was an indication of understanding. This eliminated learners who would have memorised the rules without understanding.

The main purpose of interviews was to probe further, the learners' understanding of the tasks done in the worksheets and in the trigonometric functions test. The learners were interviewed on the responses that they made on paper. The researcher asked questions such as: "May you describe the graph that you drew on question 2.1 in the trigonometric test …" This was done to authenticate each learner's written work and/or understanding. Such an approach was also used by Ng and Hu (2006). Barmby et al (2007) argue that providing learners with opportunities to explain their reasoning in what they have written can be used for the purposes of eliciting the learners' understanding. In the same vein, Stiff (2001) contends that learners in a constructivist environment should be given an opportunity to explain their mathematics. From Chart's (2017) Theory of Understanding learners used mind models that they constructed during interaction with GeoGebra to explain their understanding of trigonometric functions during one-on-one interviews. So, each learner was interviewed on the work done on worksheets and on the trigonometric functions tests. The tape-recorded responses were typed into text and transcribed verbatim, separated and categorised according to the worksheets and trigonometric functions test respectively. Written assessments were triangulated with transcriptions from one-on-one interviews. In the excerpts, R= question by researcher and L = response by learner. Korstjen and Moser (2018) contend that excerpts from interviews add to the rigour of a study since they establish thick description.

3.8.5 Focus-group interviews, data collection and analysis strategies

Focus-group interviews were conducted mainly to support and explain findings from the other preceding research methods. In fact, Flaim and Speckart (2016) contend that focus-group data can help a research defend its results more convincingly. The interview probed learners' experiences in learning using GeoGebra a process that is supported by Mather (2003). This study follows on the footsteps of Kleve (2009) who used focus-group interviews in mathematics educational research, together with other methods, for the purposes of obtaining information from participants as well as to validate the whole research and its findings.

Rudiana, Sabandar and Subali (2018) see a focus-group as a small gathering of persons who have a common interest or characteristic, assembled by a moderator (interviewer or facilitator), who uses the group and its interactions to gain information about a particular issue. Discussions by participants during focus-group interviews produces rich data (Kitzinger, 1995). Mostly, participants in focus-group discussions have common experience which in this case is learning trigonometric functions using GeoGebra. The learners in this study were in the same Grade 11 class and worked in groups during lessons where they interacted with GeoGebra. They were therefore comfortable to air their opinions, views and experiences on the use of GeoGebra towards their understanding of trigonometric functions.

The following are the strengths of using focus-group interviews: they promote high validity; they provoke fresh ideas during discussions; there is an opportunity to follow trails further than with a questionnaire, as the interviewer responds to points raised by members of the group; they are easy to analyse; they are compatible with other methods of collecting data; more ideas flow during discussions than during individual interviews; participants have an opportunity to justify their responses (Rudiana et al, 2018; Mather, 2003; Kleve, 2009; Kitzinger, 1995).

Informed by their strengths, the researcher conducted the focus-group interviews after the one-on-one interviews. The process followed the pattern recommended by Krueger and Casey (2002) which is: welcome; overview of topic; ground rules and first question. All the learners who underwent one-on-one interviews (L12, L17, L19, L24 and L26) also participated in the discussion on voluntary basis. A good group interview should have 5 to 10 participants (Krueger and Casey, 2002 and Rudiana et al, 2018). The researcher and the learners sat round a table and learners took chances to respond to discussion questions. The questions asked by the researcher were mostly aligned to the research question. The discussions were tape-recorded. Learners were tagged with their code names and the researcher instructed them to address one another using those codes for anonymity purposes in the tape-recorded discussion. The learners were also requested to say their code names each time they started speaking. This facilitated transcription.

The process of data analysis in focus-group interviews began by categorising and organising data in search for patterns, critical themes and meanings that emerged from the data. Furthermore, McMillan and Schumacher (2014: 347) insist that “In other words, qualitative researchers create a picture from the pieces of information obtained. The process is like a funnel. In the beginning, the data may seem unconnected and too extensive to make much sense, but as the researcher works with the data, progressively more specific findings are generated.” The interviewees’ tape-recorded voices were typed into texts which were then transcribed verbatim. The transcribed texts then became the data that were analysed.

3.9 QUALITY CRITERIA

3.9.1 Trustworthiness

This section discusses the rigour and trustworthiness of the study and its findings. Oates (2006, cited in Ponelis, 2015) posits that such a discussion is vital to convince readers and examiners that the study is of a high quality. Trustworthiness (which encompasses both reliability and validity in qualitative enquiries according to Golafshani, 2003) refers to the soundness of a study and is based on credibility, dependability, confirmability and transferability (Lauckner, Paterson and Kruper, 2012; Anney, 2014; Shenton, 2004). Each of the four quality criteria has several techniques or strategies to ensure it, but the researcher concentrated on those applicable and suitable for the current study.

3.9.1.1 Credibility

Credibility, according to Lauckner et al (2012), is the extent to which findings accurately describe or capture the phenomenon being studied. In addition, Statistics Solutions (2019) view credibility as the most important aspect in trustworthiness since it is concerned with the researcher clearly linking the study's findings with reality in an attempt to demonstrate the truth of the study's findings. Credibility could be established through prolonged and varied field experience, time sampling, reflexivity, peer examination, interview technique, triangulation, establishing authority of researcher and structural coherence (Korstjens and Moser, 2018; Agostinho, 2004). In this study credibility was demonstrated through triangulation and prolonged engagements with participants.

Gall et al (2005) views triangulation as the process of using multiple data collection methods, data sources, analysts, or theories to check case study findings. In addition, Olsen (2004) defines triangulation as the mixing of data or methods so that diverse viewpoints or stand points cast light upon a topic. Golafshani (2003); Fusch, Fusch and Ness (2018); Mohajan (2017); Gunawan (2015) and Cohen, Manion and Morrison (2004) agree that triangulation promotes validity and reliability (hence trustworthiness) of data and results in qualitative research, thereby mitigating the participants' and the researcher's biases. Furthermore, StatisticsSolutions (2019) purport that triangulation exists in four types which are: methods or methodological triangulation; triangulation of sources; analyst triangulation and theoretical triangulation. The researcher employed methodological and theory triangulation in exploring how the use of GeoGebra enhanced Grade 11 learners' understanding of trigonometric functions.

Denzin (1970 and 1978, cited in Fusch et al, 2018) argues that methodological triangulation exists as either within-method (multiple sources of data found within one design) or between-method (also known as across method that sources data from quantitative and qualitative techniques). This study used the within-method triangulation where data was collected using tests, interviews, worksheets and a smart recorder. Data from these multiple sources enabled the researcher to understand deeply how GeoGebra enhanced Grade 11 learners' understanding of trigonometric functions.

As discussed in Chapter 2 (see Section 2.9), this study was theoretically triangulated by utilising the constructivist and understanding theories. Fusch et al (2018) view theory triangulation as the application (by the researcher) of different theories and alternative theories to a data set. This view corroborates the view of Turner and Turner (2009) who see theory triangulation as involving the use of more than one theoretical framework in the interpretation of data. The theories in use may be related or having opposing viewpoints (Turner and Turner, 2009), and in this study constructivist and understanding theories supported each other. Constructivist and understanding theories assisted the researcher in understanding and interpreting data, examining and explaining findings.

Prolonged engagement refers to adequate time spent in a research site resulting in the establishment of rapport between the researcher and participants, and familiarity with the environment (Pandey and Patnaik, 2014). The researcher achieved this by spending more than seven days with learners (starting from the completion of assent to participate forms until the end of data collection). Learners showed motivation and enjoyment during engagements and they acknowledged this in the focus-group interviews (see Chapter 4 Section 4.5.1). Again, the researcher was familiar to the school's environment since the school is visited regularly during curriculum monitoring and support.

3.9.1.2 Dependability

Lauckner et al (2012) perceive dependability as the ability of the study to account for variability over time. In addition, Korstjens and Moser (2018) maintain that dependability also prescribes interpretations that emanate from authentic data. Anney (2014) argues that dependability is characterised by peer examination; stepwise replication; audit trail; triangulation and a code-recode strategy. The researcher established dependability through triangulation (that has already been discussed under credibility) and audit trail.

Morrow (2005) views audit trail as a detailed chronology of research activities and processes. In the current study, audit trail was achieved by saving and safely storing:

one-on-one and focus-group interview recordings and their corresponding transcriptions; smart recordings (videos) of learners' worksheet activities using GeoGebra; learners' marked scripts for diagnostic test, worksheets and trigonometric functions test; Ethics approval certificate from the UNISA College of Education Ethics Review Committee; signed assent to participate in this study by learners; GDE permission to conduct research. Such a step allows an auditor to sample suspicious findings and trace them back to raw data as purported by Lincoln and Guba (1985). This is supported by Miller (1997) who argues that the systematic record keeping required to have an audit done enhances the rigour of the study. The researcher retained and kept abreast of the research proposal approved by the UNISA Masters and Doctorate Graduate office and the drafts of the dissertation, which is in line with audit trail according to Miller (1997). This facilitated the reframing and refining the dissertation by assisting the researcher in identifying weak areas or sections where improvements could be made. For instance, the proposal and the first draft of the dissertation had only one major research question and the final write up now has three sub-questions. Again, the researcher accomplished audit trail through providing a transparent research path (Korstjens and Moser, 2018) by detailing and explaining data collection processes, data understanding and interpretation strategies for the study (Dodge, 2011) (see Chapter 3 Section 3.8).

3.9.1.3 Confirmability

Confirmability is an aspect of trustworthiness that deals with the extent to which the process of collecting data and coming to conclusions is clear and can be followed by another (Lauckner et al, 2012). The researcher established confirmability by demonstrating dependability since both seek to ensure that data and findings presented by the researcher were not derived from fabricated data sources (Anney, 2014). Lincoln and Guba (1985) have also noted that the two (confirmability and dependability) are synonymous.

3.9.1.4 Transferability

Transferability ensures trustworthiness of a study by ascertaining the likelihood that the findings of a study will have meaning in other similar situations (Lauckner et al, 2012). In the same vein, Bitsch (2005, cited in Anney, 2014) views transferability as synonymous to generalizability (in quantitative research) and could be achieved through purposeful sampling and thick description. In this study, Grade 11 learners at a school in Tshwane South District were purposefully sampled because they were the informants with characteristics that assisted the researcher to answer the research question. The approach accords with Devault (2019) who argues that purposeful sampling maximises specific data relative to the context in which it is collected. This engagement of purposeful sampling in establishing transferability is supported by Lincoln and Guba (1985).

Thick description entails describing the context and or setting in which a research was carried out in terms of location, sample strategy, socio-economic and demographic situations (Korstjens and Moser, 2018; Lincoln and Guba, 1985). The researcher demonstrated thick description in the details given in Chapter 3 Sections 3.5 and 3.6.

3.9.2 Validity of instruments

Validity for instruments that take the form of written assessments is achieved through content validity. Creswell (2012:162) posits that content validity concerns itself with 'whether the scores from the instrument show that the test's content relates to what the test is intended to measure.' In this research content validity of the diagnostic test, worksheets and the trigonometric functions test were ensured through moderation and validation by two senior Mathematics educators in the District. One of the educators is a Lead educator, a PLC Cluster leader (involved in the setting and moderation of SBA tests and examinations) in the subject and has more than ten years of experience in marking NSC Mathematics examinations. The other educator has a total of 21 years of teaching experience, 10 years in the University of Cambridge Local Examinations Syndicate syllabus and 11 years in the CAPS. Again, all the instruments used in this study were also scrutinised by the researcher's supervisor, authenticated by UNISA College of Education Ethics Review Committee and piloted in the neighbouring school by the researcher three weeks before the main study began. This process is recommended by Shillingburg (2016) who insists that content experts should be engaged in evaluating how well assessment instruments represent the content taught and in ensuring a stronger validity evidence. In the same vein, Mohajan (2017) purports that there are no statistical means to ascertain content validity besides the judgement by experts in the field who are capable of amending and or discarding unclear, obscure, ineffective and non-functioning questions.

3.10 ETHICAL CONSIDERATIONS

The researcher obtained ethical clearance from UNISA College of Education Ethics Review Committee (see Appendix A) and obtained permission to conduct the study from the Gauteng Department of Education's Education Research and Knowledge Management (see Appendices B and H). MacMillan and Schumacher (2014) contend that research ethics are focused on what is morally proper and improper when engaged with participants or when accessing archival data. Consent was sought from the District Director; the principal of the school; the learners and the parents of the learners (see Appendices C to F respectively). The letters requesting assent from the learners stated that participation was non-compulsory, and withdrawal from participation was within the learners' rights. This is supported by MacMillan and Schumacher (2014) who argue that the rights and welfare of participants should be protected when conducting research. Again, participants used codes instead of their real names for anonymity's sake.

3.11 CONCLUSION

In this chapter the constructivist or interpretive paradigm is the philosophical underpinnings of the study's research methodology. The study is a qualitative case study that is exploratory in nature. The pilot study was conducted, and it indicated adjustments that were supposed to be done to the research instruments. Six Grade 11 learners at a full ICT school in Tshwane South District participated in the study. A diagnostic test was used to check on the learners' assumed knowledge. The trigonometric functions test was written after worksheet sessions were finished. The one-on-one and focus-group interviews were employed to further explore the learners' understanding of trigonometric functions using GeoGebra. The worksheets and the two tests were analysed qualitatively question by question whilst interviews were analysed after transcription. Constructivist and understanding theories were employed in understanding and interpreting data and findings. The methodology chapter ended by discussing quality criteria and ethical considerations. Chapter 4 presents the data analysis and results.

CHAPTER 4
DATA PRESENTATION, ANALYSIS AND FINDINGS

4.1 INTRODUCTION

This research focused on exploring Grade 11 learners' understanding of trigonometric functions using GeoGebra software. The purpose of this chapter is to analyse and interpret data collected from Grade 11 learners with a view to answering the following primary research question:

How does the use of GeoGebra enhance Grade 11 learners' understanding of trigonometric functions?

This is achieved by addressing the following three sub-questions:

(i) How do GeoGebra environments help learners in understanding trigonometric functions?
(ii) How is learners' understanding of trigonometric functions after interaction with GeoGebra?
(iii) What are learners' experiences and views on the use of GeoGebra in exploring trigonometric functions?

The research design used is a qualitative case study in which six Grade 11 volunteer learners (L7, L12, L17, L19, L24 and L26), made up of four girls and two boys participated. Purposeful sampling was used to choose the Grade 11 learners. Data were collected using diagnostic test, worksheets, trigonometric functions test, one-on-one interviews and focus-group discussions.

The data from the five data collection instruments were understood and interpreted mostly through the threaded and woven elements of the constructivist perspective (learning from experience; prior knowledge; collaboration; visualisation; generalisation; scaffolding) and understanding theories (building mental models that help learners to explain subject matter; explanations to one another that improve understanding). Although the constructivist and understanding theories were an anchor in this study

(Grant and Osnaloo, 2016), the researcher took precautions against overly reliance on the theories that could have jeopardised other emergent findings from the data (Colins and Stockton, 2018). This notion is supported by Grant and Osnaloo (2016) who argue that there is no one perfect or right theory for a dissertation. The content aspects in trigonometric functions, according to CAPS requirements, also assisted the researcher in effectively understanding and interpreting data and findings in the light of these theories. Brown (1992, cited in Pfeiffer, 2017) notes that it remains a challenge for researchers to identify data elements that are in line with chosen theoretical frameworks when analysing qualitative data.

The learners' results in the two tests and worksheets were obtained by counting the number of correct (scored 1) and incorrect (scored 0) answers in each question. L is the code name for a learner and R is for the researcher. The data collected from one-on-one interviews were blended and triangulated with the question-by-question analysis of the trigonometric test and worksheets respectively, since the interviews sought to follow up on the learners' responses to these assessments. Focus-group interviews data were analysed by categorising and organising them into themes and meanings that are mostly aligned to the constructivist perspective and understanding theories. Triangulating data from the tests, worksheets and interviews aimed to establish trustworthiness. Verbatim quotations in the study also provided trustworthiness by strengthening credibility (Corden and Sainsbury, 2006). The smart recordings also helped the researcher gain a deep insight into how the learners were interacting with GeoGebra.

The following sections: analysis of diagnostic test; how GeoGebra environments help learners in understanding trigonometric functions; learners' understanding of trigonometric functions after interacting with GeoGebra; and learners' experiences and views on the use of GeoGebra in exploring trigonometric functions present the understanding (analysis) and interpretation of findings that were gleaned from the five data sources of this study. The presentation of the analysis follows a synthesis of organisational models found in previously published studies of similar nature by De Villiers and Jugmohan (2012); Demir (2012); Ng and Hu (2006); Agyei (2013) and Pfeiffer (2017).

4.2 ANALYSIS OF DIAGNOSTIC TEST

The diagnostic test was written by five learners (L7, L12, L19, L24 and L26) within 45 minutes, nine days before lessons commenced. The aim of the diagnostic test was to assess the participants' assumed or prior knowledge and skills on Grade 10 trigonometric functions before interaction with GeoGebra. The administering of diagnostic test was informed by the constructivist perspective that argues that prior knowledge points to relevant knowledge learners may already have and to knowledge which may be necessary in order to support them in accessing the new topic (Kinchin, 1998; Ishii, 2003; Sjoberg, 2007; Project Maths Development Team, 2009). The learners' responses to the test were triangulated with other data collection methods used in this study. For instance, learners' performances were tracked from the diagnostic test through worksheets to the trigonometric functions test and the learners were interviewed about what they wrote in the diagnostic test during focus-group interviews. The learners' work in the diagnostic test assisted the researcher in identifying learning gains during and after interaction with GeoGebra. Again, the level of performance ascertained in the diagnostic test informed the sequencing of worksheet activities. The test was analysed question by question, qualitatively. The diagnostic test and its memorandum are in Appendices O and P respectively.

.

Question 1

This question required learners to plot graphs of trigonometric functions (tangent, sine and cosine) involving the parameters *a* and *q*, as prescribed by Grade 10 CAPS. Three (L12, L19 and L24) out of five learners managed to plot the cosine graph and its transformation correctly (see Figures 4.1 to 4.3). From Figures 4.1 to 4.4 (c) it shows that learners' graphs are not smooth. There is a possibility that learners understood or perceived the graphical representation of trigonometric functions as joining of coordinates from table of values from their calculators.

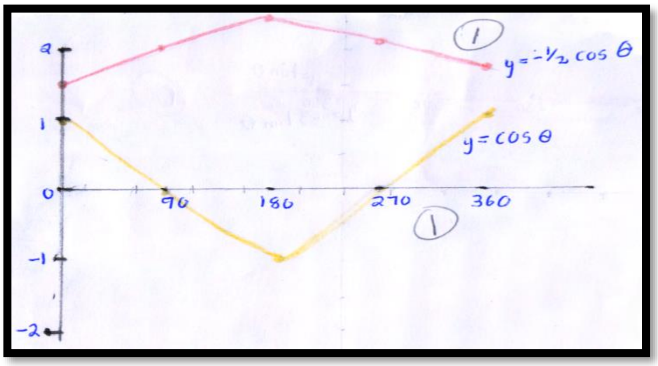


Figure 4.1 L24's answer to 1 (a) in the diagnostic test

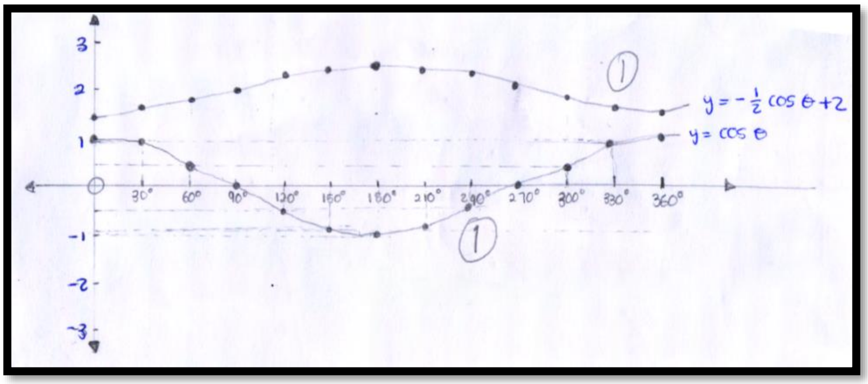


Figure 4.2 L12's answer to 1 (a) in the diagnostic test

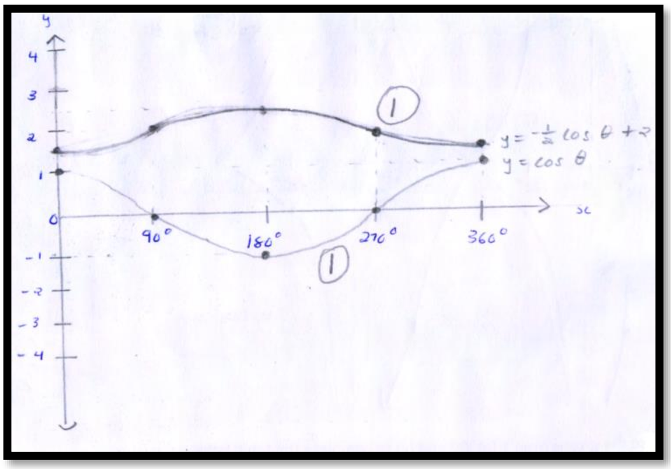


Figure 4.3 L19's answer to 1 (a) in the diagnostic test

For 1(b), two learners (L19 and L24) were able to come up with the sine graph but failed to plot its transformation. One learner (L7) could not draw both the sine and its transformed image (see Figure 4.4b). Interestingly two learners (L12 and L26) plotted both sine graphs correctly. Two participants (L12 and L19) correctly plotted the tangent graphs in the test whilst three could not plot them.

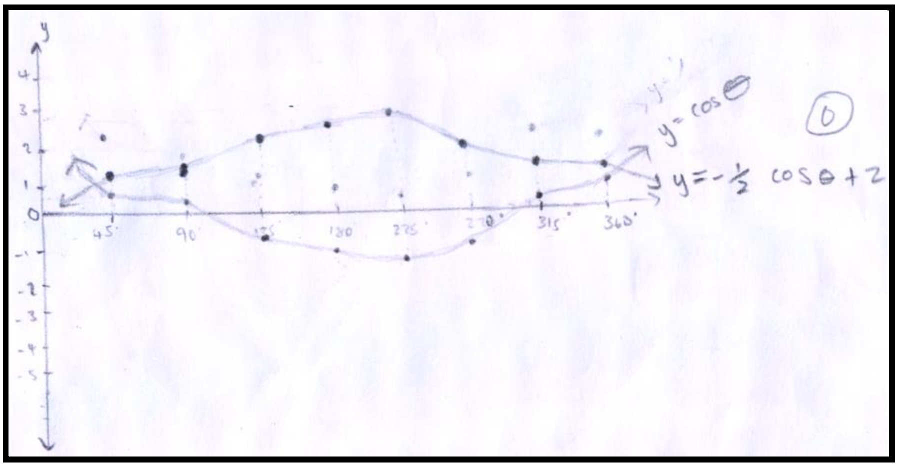


Figure 4.4a L7's answer to 1 (a) in the diagnostic test (trying to draw cosine graphs)

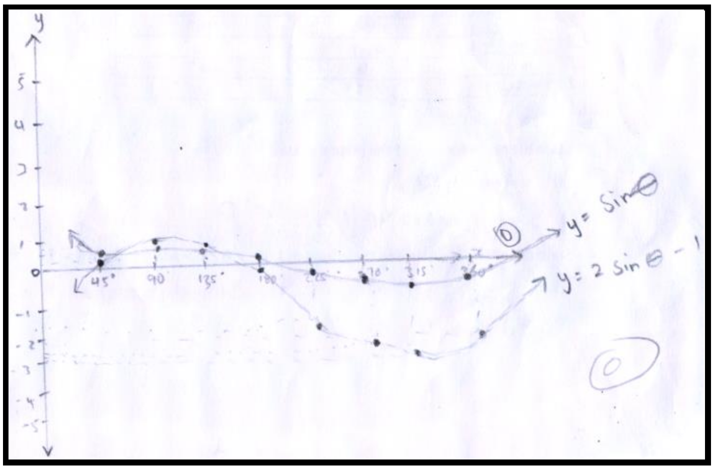


Figure 4.4b L7's answer to 1 (b) in the diagnostic test (trying to draw sine graphs)

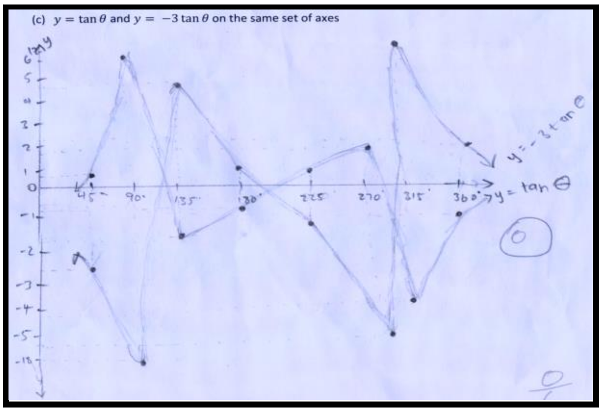


Figure 4.4c L7's answer to 1 (c) in the diagnostic test (trying to draw tangent graphs)

Considering all of L7's work in this question, (see Figure 4.4 a to c), it indicates that the learner struggled to draw basic sine, cosine and tangent graphs. Figure 4.4 (c) shows that L7 did not have a picture of the tangent graph and just joined the points from the calculator.

Question 2

Here learners were required to determine the amplitude and period of given graphical equations. All the learners attempted the question, but only one learner (L19) obtained correct solutions for the whole question. L12 provided correct amplitudes but was unable to determine the correct periods. L26 obtained 4 periods correct but was unable to determine any of the amplitudes. L24 managed only one problem (amplitude) and L7 provided incorrect responses to all the problems. It is essential for learners to master amplitude and periodicity concepts at Grade 10 level since this will help them build on parameters like *p and k* in Grade 11.

Question 3

This question expected learners to determine the range of given graphical equations. Three (L12, L19 and L24) out of five managed to score 100% on this question whilst two learners (L7 and L26) obtained 0%.

Question 4

This question expected learners to be able to determine the relationship of the graphs drawn in 1(b). Only two learners (L12 and L19) were able to present the correct relationship between these graphs. The skill which is tested here is one of the core learning outcomes in the FET trigonometric functions section. One possible reason for the learners' challenges could be that they did not adequately explore the basic features of trigonometric functions in Grade 10.

4.2.1 Summary remarks

The learners' responses to the diagnostic test showed that they had basic knowledge of trigonometric functions that would serve them well in Grade 11, despite the gaps emerged from the analysis. Evidence from written work shows that learners depended on the calculator to calculate the points of the graphs. Van Putten (2014) contends that teachers should find out where their learners are, and begin at that level, otherwise success will remain elusive. Furthermore, constructivism states that to construct and understand a new idea, learners use prior knowledge. These last two views together, informed the researcher on the importance of administering the diagnostic test before the Grade 11 worksheet activities began. The learners' errors gave the researcher an insight into the learners' previous knowledge constructions (Murphy, 1997). In light of this, the researcher altered the tasks in the worksheets in order to accommodate the participants' operant levels by incorporating some Grade 10 content on trigonometric functions. This is supported by Stiff (2001) who insists that integrating new knowledge with existing knowledge creates a deeper understanding of mathematics concepts by learners and is aligned to constructivist learning practices.

A more detailed account of worksheet activities done by learners using GeoGebra is presented in the following section.

4.3 HOW GEOGEBRA ENVIRONMENTS HELP LEARNERS IN UNDERSTANDING TRIGONOMETRIC FUNCTIONS

This section sought to analyse (and triangulate) worksheet activities and one-on-one interviews on these activities in an attempt to answer the first sub-question:

How do GeoGebra environments help learners in understanding trigonometric functions?

A total of six worksheets were done by the learners. The learners worked in groups of their choices as they interacted with GeoGebra, tackling the various tasks in the worksheets. This is in accordance with the modern constructivist view that technology should be integrated into the day-to-day teaching and learning of the class and that learners should be viewed as collaborators (Mills, 2006). The learners were encouraged to discuss the various worksheet tasks with each other and to reflect on their own learning. This was done in order to improve their learning and reasoning through discussions and explanations amongst group members, which is line with the understanding theory, and supported by Mills (2006). Some of the learners reported (during focus-group interviews) how they benefited in working as groups in the use of GeoGebra (see Section 4.5).

4.3.1 Analysis of worksheets

On the first day of worksheet activities that were done in groups, the researcher trained learners on the operations of GeoGebra (as discussed in Chapter 3 Section 3.9.2), before they worked on Worksheets 1, 2 and 3. In the worksheet activities done in the study, learners used GeoGebra as a cognitive, mind or learning tool in exploring trigonometric functions and constructing of new knowledge which affirms constructivist learning tenets as reported by Sabzian et al (2013) and Pfeiffer (2017) (see Chapter 2 Section 2.9.4). In addition, the environment allowed learners to use GeoGebra as a tool to construct mind models in line with Chart's (2017) theory of understanding. During the cleaning and screening of data, the researcher targeted and tracked L7, L12, L17, L19, L24 and L26 for reasons stated in Chapter 3 Sections 3.6.2 and 3.8.2 (see also Tables 3.1 and 3.4). Written work on the worksheets was analysed qualitatively worksheet by worksheet in conjunction with smart recordings that could be replayed. Again, screenshots were captured from these recordings and pasted in the write-up during analysis.

Worksheet 1: The effects of the parameter *q on* the graph of
***y = sin x* for *x* = [-360°; 360°]**

The learners' task was to use GeoGebra to draw the sine graph and its six transformations on the same set of axes, describe each graph and then provide a generalisation (see Appendix I). It was easy for all the three groups [Group1 (that included L7 and L19); Group 2 (that included L24 and L26); Group 3 (that included L12 and L17)] to describe all the other graphs in relation to the mother sine graph. This result showed that the interaction with GeoGebra enhanced the learners' understanding because they (L7, L24 and L26) could not answer a similar problem involving *q* in Question 4 of the diagnostic Test. This finding could be explained in terms of collaborative visualisation a tenet of constructivist learning that Wu et al (2002) and Irawan et al (2019) see as aiding learners in understanding mathematical concepts. Add Chart (2017) However, two of the three groups attempted the generalisation and only one group (Group 3) was able to come up with a partially correct answer "The value of *q* shifts the graph up and down".

Worksheet 2: The effects of parameter *a* on the graph of
***y* = *tan x* for *x* = [-360°; 360°]**

Worksheet 2 required learners to draw the tangent graph and its families and use the graphs to generalise their observations (see Appendix J). In this worksheet, the researcher instructed the groups to draw the mother graph and any other three graphs of their choice. This was meant to reduce the time taken by each group in using the smartboard. One group (Group 1) provided two acceptable responses by describing the graphs in terms of steepness and closeness to the asymptotes. Figure 4.5 shows the graphs drawn by Group 1 using GeoGebra. They were extracted from the smart recording of the work done by the group. Figure 4.6 shows their attempted descriptions of the graphs.

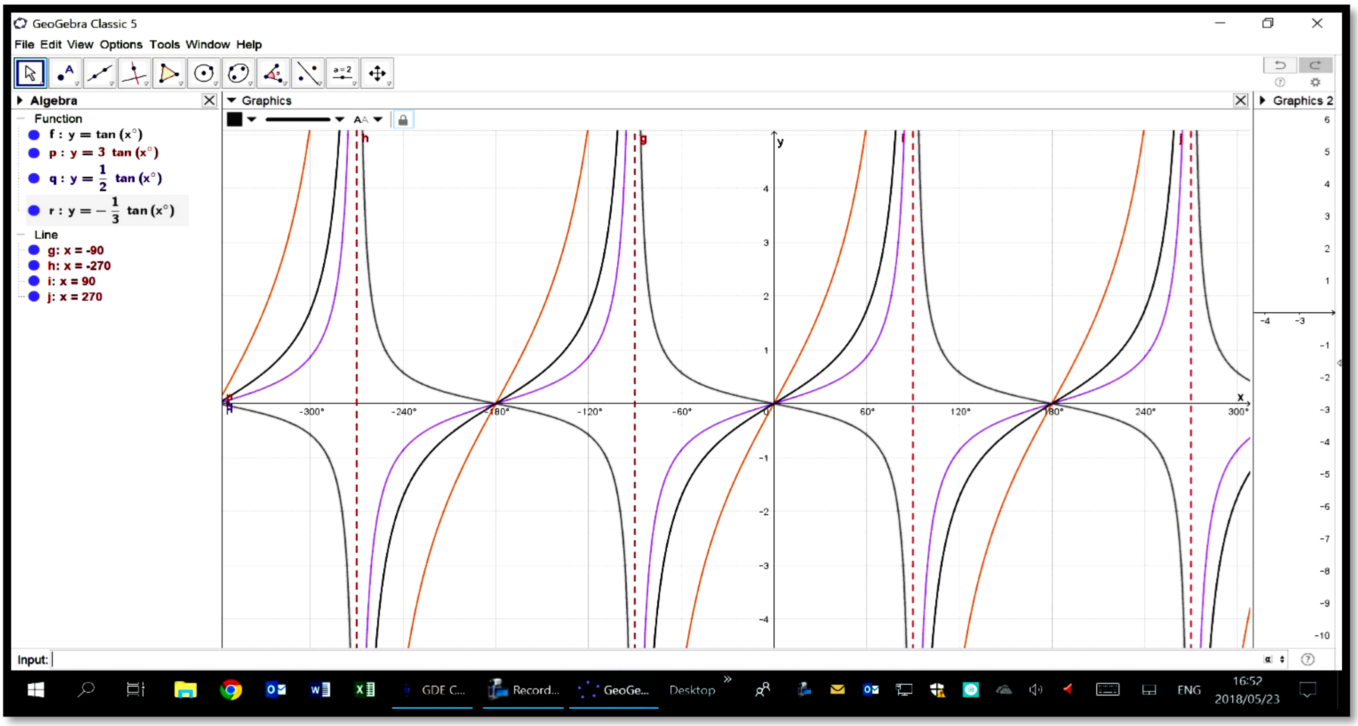
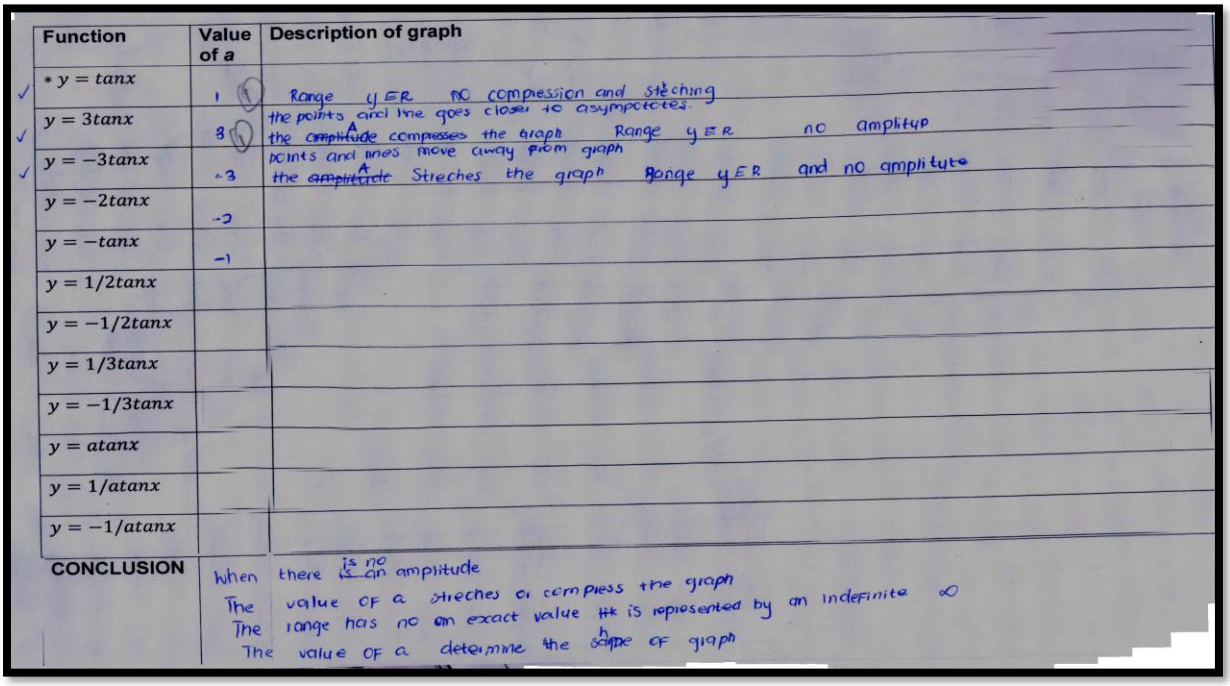


Figure 4.5 Graphs drawn by Group 1 for Worksheet 2

Function	Value of *a*	Description of graph
$*\,y = tanx$	1	
$y = 3tanx$	3 (1)	~~The~~ ~~as~~ From the mother graph the ~~gue~~ graph 3 tan x is shifting away from the asymptotes ~~and~~ it is more steeper than the mother graph. (1)
$y = -3tanx$	-3	
$y = -2tanx$	-2	
$y = -tanx$	-1	
$y = 1/2tanx$	½ (1)	The graph y = ½ tan x is moving closer to the asymptotes and it is less steeper than the mother graph. (1)
$y = -1/2tanx$	-½	
$y = 1/3tanx$	⅓	
$y = -1/3tanx$	-⅓ (1)	The graph y = -⅓ tan x it is the reflection of the mother graph about the x-axis. (1)
$y = atanx$	a	
$y = 1/atanx$	1/a	
$y = -1/atanx$	-1/a	
CONCLUSION	We noticed that when the value of "a" is greater than zero (0) the graph shifts away from the asymptote and vise versa. therefore when "a" is less than zero it is the reflection of the mother graph. therefore "a" affects the shape of the graph.	

Figure 4.6 Group 1 responses to Worksheet 2

In their conclusion, Group 1 found it difficult to express themselves, but their attempt had clues of what was seen in the graphs. The group used 'the value of *a* is greater than 0' instead of 'greater than 1'. Group 2 struggled to provide acceptable descriptions (see Figure 4.7). However, their descriptions had terms like 'points and lines go closer to asymptotes'; 'the *a* compresses the graph'.



Function	Value of a	Description of graph
$*\ y = \tan x$	1	Range $y \in R$ no compression and stetching the points and line goes closer to asymptotes.
$y = 3\tan x$	3	the ~~amplitude~~ A compresses the graph Range $y \in R$ no amplitup
$y = -3\tan x$	-3	points and lines move away from graph the ~~amplitude~~ A Streches the graph Range $y \in R$ and no amplitute
$y = -2\tan x$	-2	
$y = -\tan x$	-1	
$y = 1/2\tan x$		
$y = -1/2\tan x$		
$y = 1/3\tan x$		
$y = -1/3\tan x$		
$y = a\tan x$		
$y = 1/a\tan x$		
$y = -1/a\tan x$		
CONCLUSION	When there is no amplitude The value of a streches or compress the graph The range has no ~~an~~ exact value ~~it~~ it is represented by an indefinite ∞ The value of a determine the shape of graph	

Figure 4.7 Group 2 responses to Worksheet 2

Group 3 did not provide any correct descriptions but concluded by saying that the value of *a* stretches or compresses the graph (see Figure 4.8).

Function	Value of a	Description of graph
* $y = tanx$	1	
$y = 3tanx$	3	Range : $y \in \mathbb{R}$ the graph expanded 3 units. (b)
$y = -3tanx$	-3 (1)	
$y = -2tanx$		
$y = -tanx$		
$y = 1/2tanx$	$\frac{1}{2}$ (1)	range : $y \in \mathbb{R}$ the graph (b)
$y = -1/2tanx$		
$y = 1/3tanx$	1/3 (1)	range : $y \in \mathbb{R}$ (b)
$y = -1/3tanx$		
$y = atanx$		
$y = 1/atanx$		
$y = -1/atanx$		
CONCLUSION	The value of a Streches & or compresses the graph.	

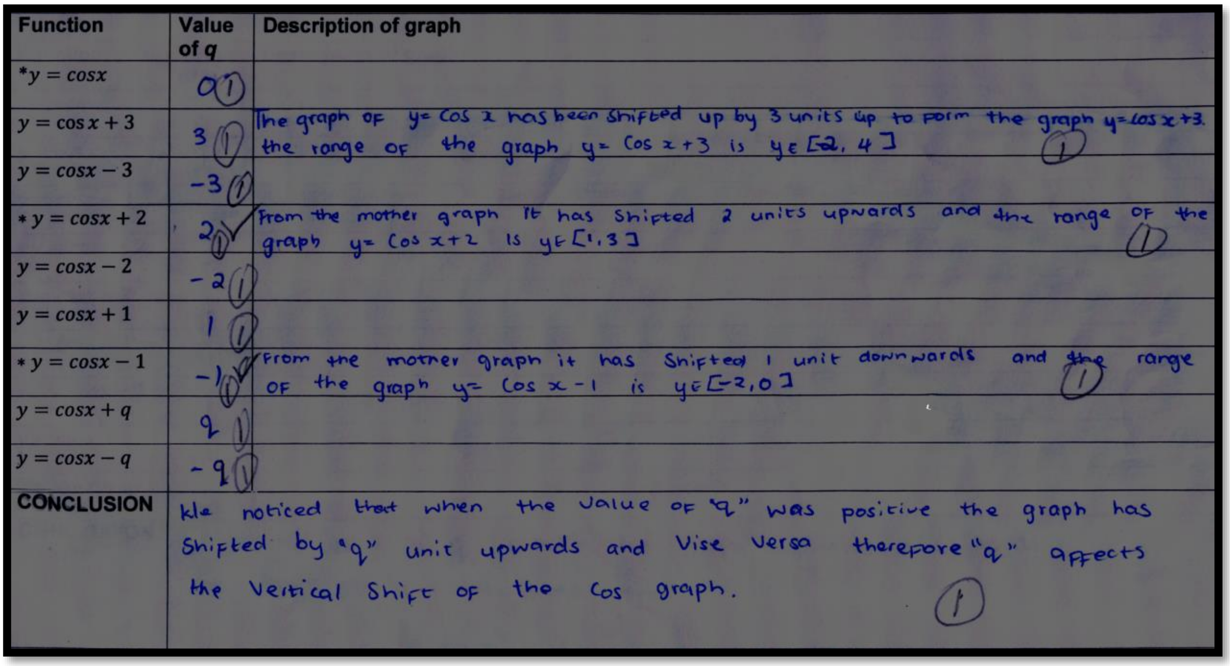
Figure 4.8 Group 3 responses to Worksheet 2

The learners' attempts show the power of GeoGebra in the learners' understanding of the effects of parameters on trigonometric functions. This is supported by Shelly et al (2008) who note that many learners are visual learners. The appearance of the projected graphs helped the learners comprehend the behaviour of the graphs (stretching; compressing; moving closer to the asymptote) as the value of the parameter *a* was varied. This result coincides with the research done by Naidoo and Govender (2014) that revealed that the nature of the GeoGebra program allows the mathematics learner the freedom to manipulate and visually notice instantaneous changes and behaviour of graphs. Again this finding could be explained in terms of constructivist learning principles in which a cognitive tool (in this case GeoGebra) enhances learners' learning and understanding of mathematical concepts by easing surrounding cognitive processes (Demir, 2012).

Worksheet 3: The effects of parameter *q* on the graph of *y = cos x* for *x* = [-360°; 360°]

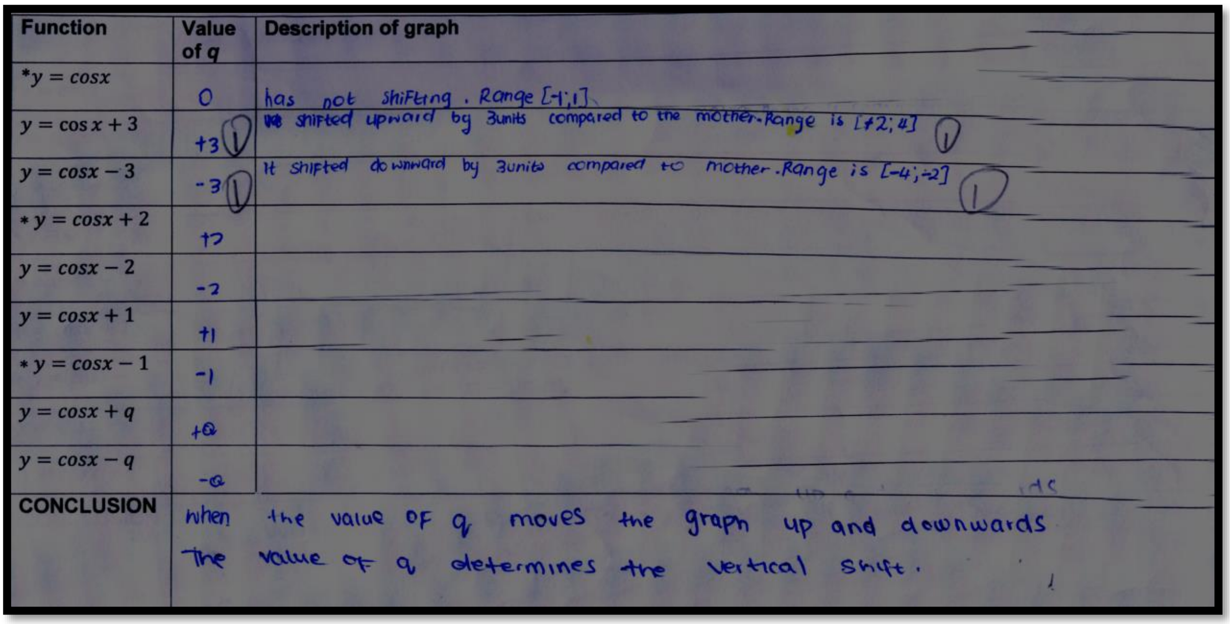
This worksheet required learners to use GeoGebra to draw the cosine graph and its families (or transformations) and make a generalisation from the graphs (see Appendix

K). All the groups were able to identify the values of *q* for the given graphical equations. The descriptions of the graphs were presented in varied, interesting and mathematically correct ways by each of the groups (see Figures 4.9, 4.10 and 4.11).



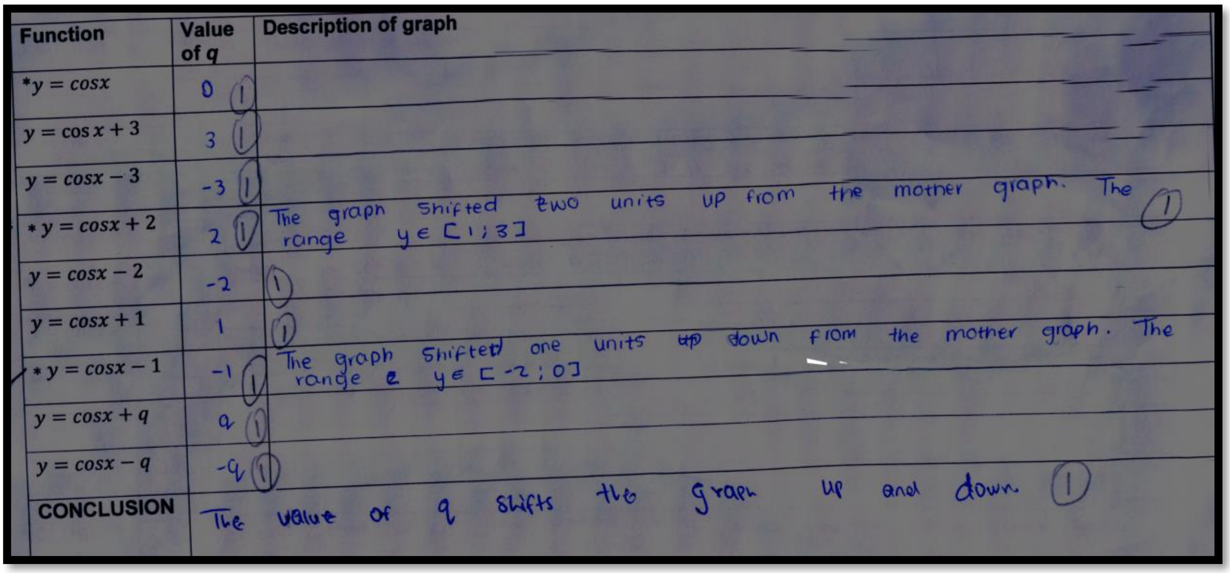
Function	Value of *q*	Description of graph
$^*y = cosx$	0 1	
$y = \cos x + 3$	3 1	The graph of y= Cos x has been shifted up by 3 units up to form the graph y=cos x+3. the range of the graph y= Cos x+3 is y∈[2, 4] 1
$y = cosx - 3$	-3 1	
$*y = cosx + 2$	2 1	From the mother graph It has Shifted 2 units upwards and the range of the graph y= Cos x+2 is y∈[1,3] 1
$y = cosx - 2$	-2 1	
$y = cosx + 1$	1 1	
$*y = cosx - 1$	-1 1	From the mother graph it has Shifted 1 unit downwards and the range of the graph y= Cos x-1 is y∈[-2,0] 1
$y = cosx + q$	q 1	
$y = cosx - q$	-q 1	
CONCLUSION	We noticed that when the value of "q" was positive the graph has Shifted by "q" unit upwards and Vise Versa therefore "q" affects the Vertical Shift of the Cos graph. 1	

Figure 4.9 Group 1 responses to Worksheet 3



Function	Value of *q*	Description of graph
$^*y = cosx$	0	has not shifting. Range [-1;1]
$y = \cos x + 3$	+3 1	shifted upward by 3units compared to the mother. Range is [+2; 4] 1
$y = cosx - 3$	-3 1	It shifted downward by 3units compared to mother. Range is [-4;-2] 1
$*y = cosx + 2$	+2	
$y = cosx - 2$	-2	
$y = cosx + 1$	+1	
$*y = cosx - 1$	-1	
$y = cosx + q$	+q	
$y = cosx - q$	-q	
CONCLUSION	when the value of q moves the graph up and downwards The value of q determines the vertical shift.	

Figure 4.10 Group 2 responses to Worksheet 3



Function	Value of q	Description of graph
$*y = cosx$	0 1	
$y = \cos x + 3$	3 1	
$y = cosx - 3$	-3 1	
$*y = cosx + 2$	2 1	The graph shifted two units up from the mother graph. The range $y \in [1;3]$ 1
$y = cosx - 2$	-2	1
$y = cosx + 1$	1	1
$*y = cosx - 1$	-1 1	The graph shifted one units ~~up~~ down from the mother graph. The range e $y \in [-2;0]$
$y = cosx + q$	q 1	
$y = cosx - q$	-q 1	
CONCLUSION	The value of q shifts the graph up and down. 1	

Figure 4.11 Group 3 responses to Worksheet 3

In this worksheet all the groups attempted the generalisations and the answers showed different levels of understanding and depth of mathematical language. This revealed some improvements in understanding *q* from Question 4 of the diagnostic test and Worksheet 1. The learners' success in generalisation of such mathematical concepts is in line with constructivist learning that is easily achieved by use of technology as posited by Sheehan and Nillas (2010). These results suggest that the learners' interaction with GeoGebra and discussions amongst themselves allowed them to reflect on and refine their responses to the worksheets. According to constructivist learning, discussions amongst group members engages their minds and sharpens their communication and argumentation skills (Pagan, 2005). This accords with Naidoo and Govender's (2014) view that the use of technology-based tools enriches learning, since visual data promotes and challenges explanation and justification, as opposed to the traditional methods of note taking, chalk and talk. Following along similar lines, is Trung's (2014) research that showed that learners wrote well, briefly and they also expressed mathematical language coherently and fluently after learning according to discovery learning with GeoGebra.

Worksheet 4: Sketching graphs with a combination of parameters *a* and *q*

On this day only one group attended the session. This was due to School Based Assessments (SBAs) deadlines that were looming as the term end approached and transport issues. The group (Group 1B) present was made up of five learners including L12 and L7. The other three learners were not considered as participants due to their erratic attendance. The worksheet is in Appendix L.

For Worksheet 4, learners were required to draw graphs that combined parameters *a* and *q*. The second part expected learners to draw a graph by hand. The group provided the correct response to 2(b). L7 had failed to describe a similar function in Question 4 in the diagnostic test. The hands-on experience with drawing the graphs in 2 (b) on the same set of axis in different colours helped L7 and L12 to understand the relationship between the sine graphs. Perhaps the display offered by GeoGebra afforded the learners the opportunity to easily compare the features of the two graphs. Although the learners had challenges in interpreting the effects of $a < 0$, they were able to state that it is a reflection but could not identify the line of reflection: “The graph of $y = -\tan x - 3$ is the reflection of the mother graph and it has shifted 3 units downwards from the mother graph.” In this instance the use of GeoGebra and collaboration amongst learners could have provided scaffolding which is an important characteristic of social constructivist learning (Murphy, 1997).

The last task required the group to sketch the graph $y = -3\sin x + 1$ on paper without the use of GeoGebra. The learners managed to draw the graph correctly as shown in Figure 4.12. This shows that the learners were able to use the experience they gained in interacting with GeoGebra and drew the required graph by hand. This is an important skill for Grade 11 learners. It is therefore evident that teamwork and interaction facilitated the achievement in this task. The GeoGebra software enabled learners to understand the effects of *a* and q on trigonometric functions by allowing them to project multiple graphs under the constraints of the two parameters (Ernest, 1995 and Jonassen, 1991, cited in Murphy, 1997).

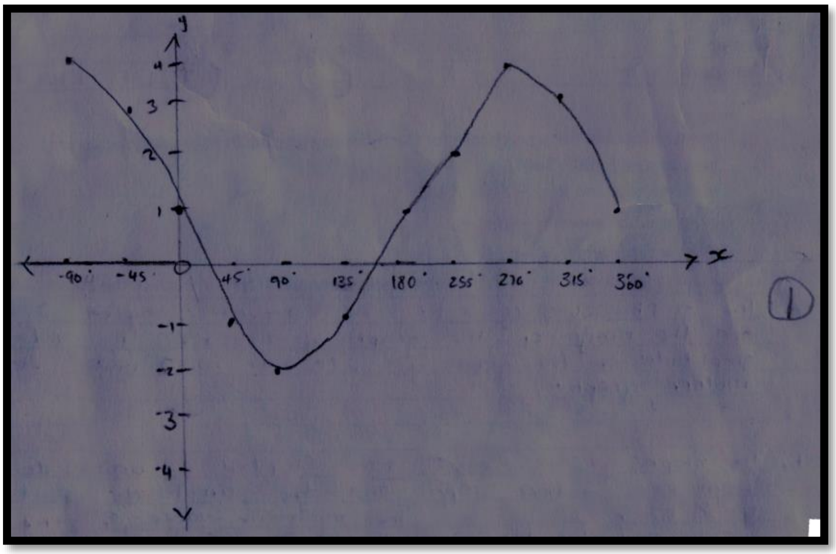
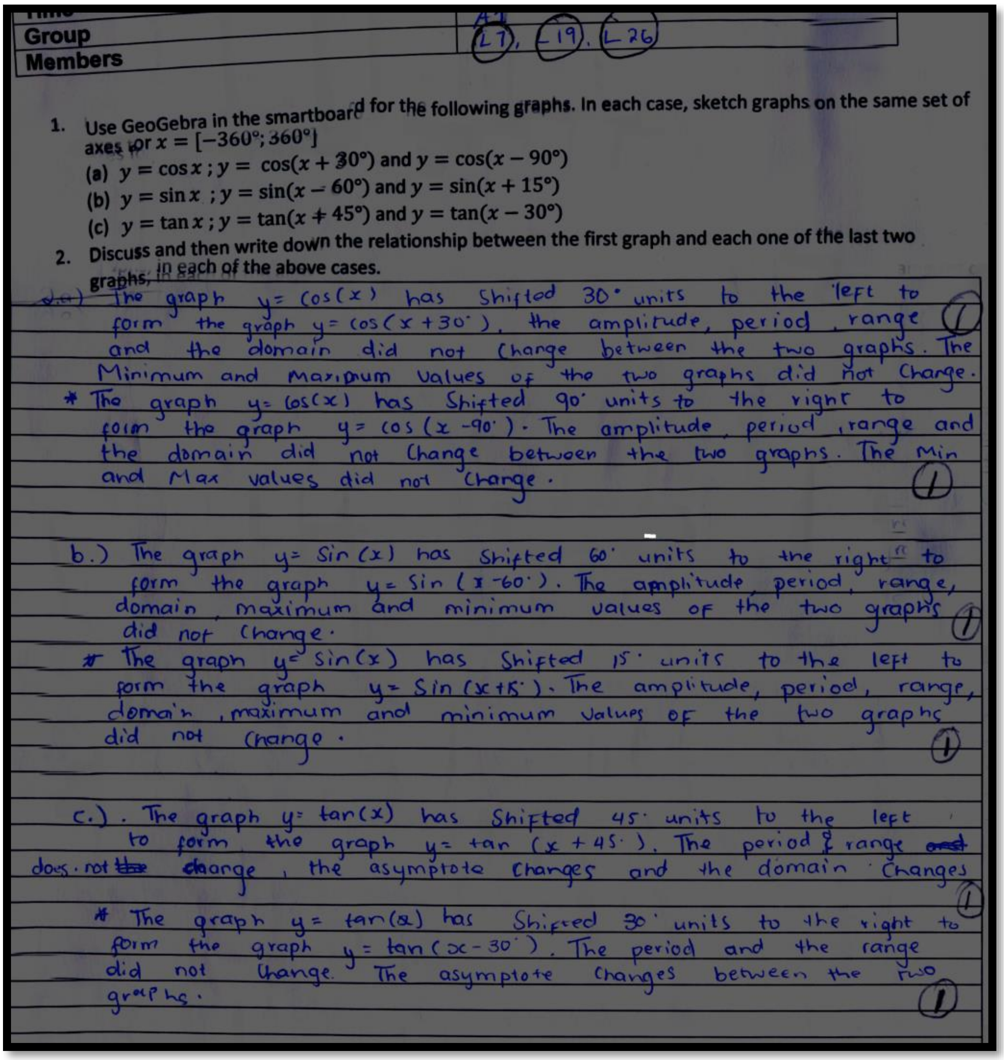


Figure 4.12 Group 1B's trigonometric graph drawn by hand

Worksheet 5: The effects of parameter *p* on sine, cosine and tangent graphs.

In this worksheet (see Appendix M) learners were required to use GeoGebra in the smart board to draw graphs in each of the cases on the same set of axes. The second part expected learners to describe the relationship between two graphs in each of the cases whilst the last part required learners to sketch a graph by hand. On the day, learners worked in two groups of three [Group A1 (which had L7, L19 and L26)] and four [Group A2 (which included L12, L17 and L24)]. Group A1 provided correct responses for all the tasks except the last part that required the learners to describe in their own words the graph they had sketched (see Figures 4.13a and 4.13b). However, the learners were able to realise that the graph had been shifted 45° to the left.



Group	A1
Members	L7, L19, L26

1. Use GeoGebra in the smartboard for the following graphs. In each case, sketch graphs on the same set of axes for $x = [-360°; 360°]$
   (a) $y = \cos x$; $y = \cos(x + 30°)$ and $y = \cos(x - 90°)$
   (b) $y = \sin x$; $y = \sin(x - 60°)$ and $y = \sin(x + 15°)$
   (c) $y = \tan x$; $y = \tan(x + 45°)$ and $y = \tan(x - 30°)$
2. Discuss and then write down the relationship between the first graph and each one of the last two graphs, in each of the above cases.

a.) The graph y= cos(x) has shifted 30° units to the left to form the graph y= cos(x+30°), the amplitude, period, range and the domain did not change between the two graphs. The Minimum and Maximum values of the two graphs did not change. 1
* The graph y= cos(x) has shifted 90° units to the right to form the graph y= cos(x-90°). The amplitude, period, range and the domain did not change between the two graphs. The Min and Max values did not change. 1

b.) The graph y= Sin(x) has shifted 60° units to the right to form the graph y= Sin(x-60°). The amplitude, period, range, domain, maximum and minimum values of the two graphs did not change. 1
* The graph y= sin(x) has shifted 15° units to the left to form the graph y= Sin(x+15°). The amplitude, period, range, domain, maximum and minimum values of the two graphs did not change. 1

c.). The graph y= tan(x) has shifted 45° units to the left to form the graph y= tan(x+45°). The period & range does not change, the asymptote changes and the domain changes 1
* The graph y= tan(x) has shifted 30° units to the right to form the graph y= tan(x-30°). The period and the range did not change. The asymptote changes between the two graphs. 1

Figure 4.13 (a) Group A1 responses to Worksheet 5 second part

The sketch in 4.14 (b) is an improvement from that of using a calculator or table of values in the diagnostic test (where many dots of coordinates appeared) to a smoother curve (where only intercepts and turning points appeared). This informed the researcher the use of GeoGebra by learners enabled them to sketch graphs from abstraction. Furthermore, the learners appear to be taking control of their learning using technology, constructing their own knowledge and understanding which is synonymous with constructivist tenets.

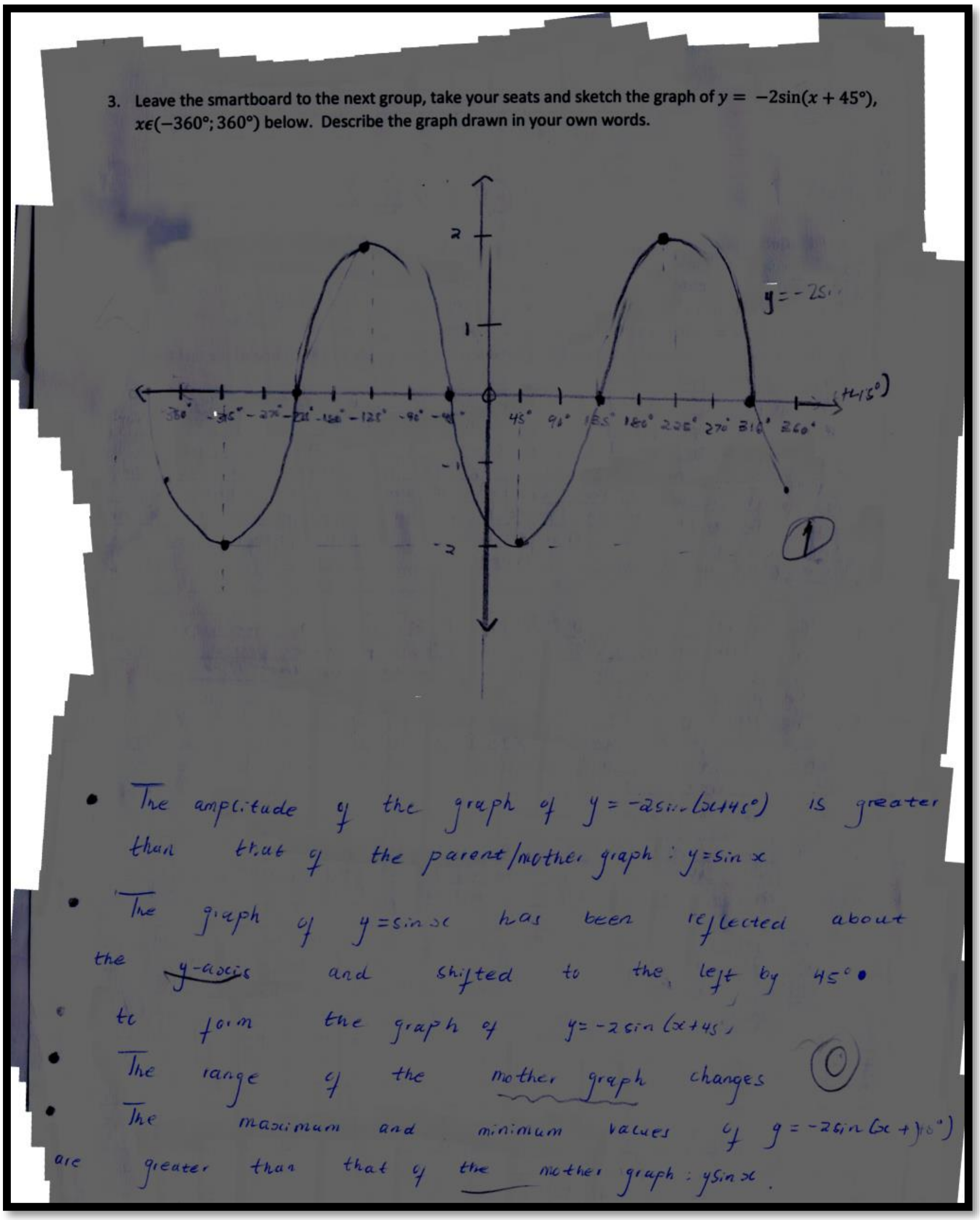
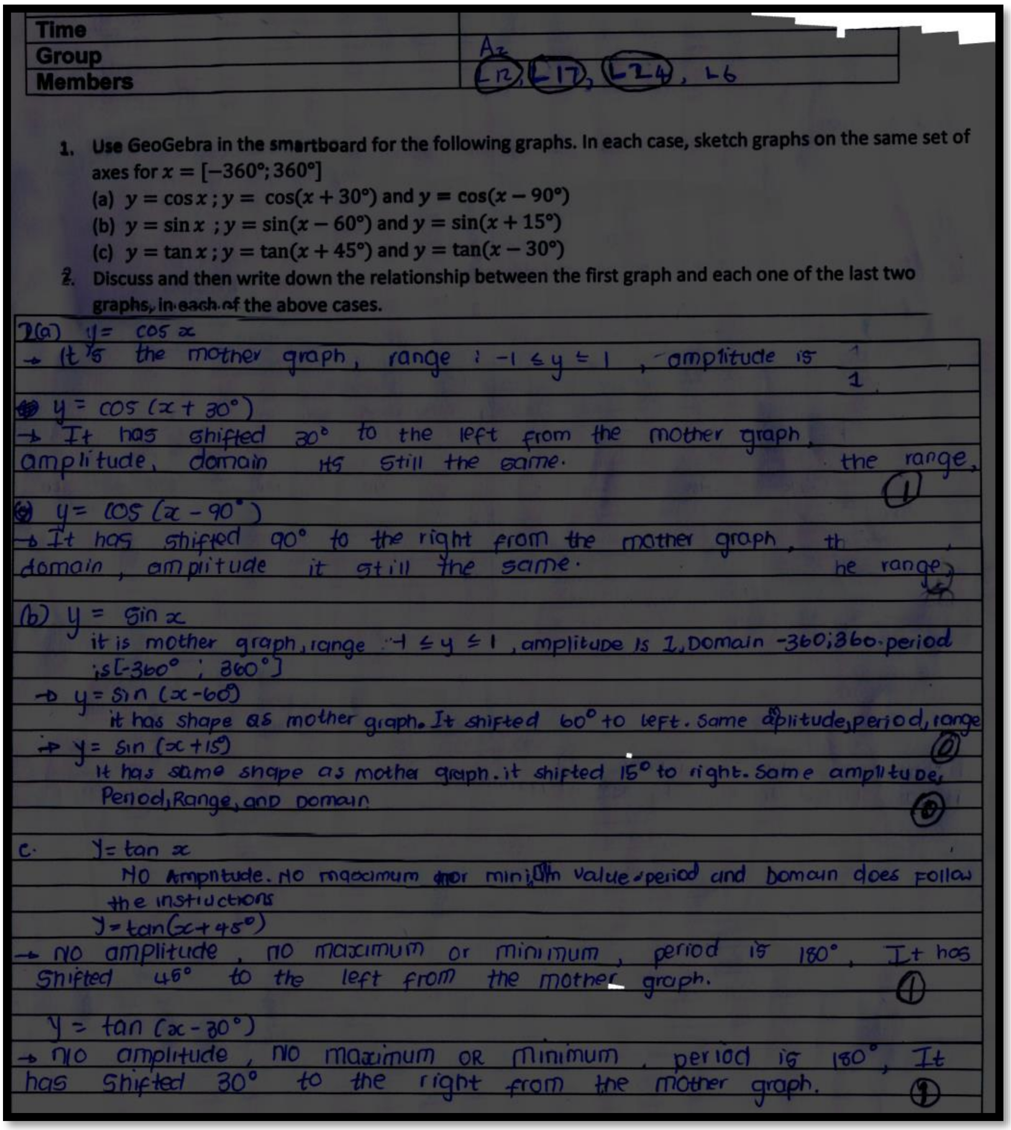


Figure 4.13 (b) Group A1responses to Worksheet 5 third part

Group A2 (L12, L17, L24 and L6) had difficulties in describing the second set of graphs after having managed the first and third sets (see Figure 4.14a). In addition, they were challenged by the sketching and hence the description of the last graph (see Figure 4.14b).



Time	
Group	A2
Members	L12, L17, L24, L6

1. Use GeoGebra in the smartboard for the following graphs. In each case, sketch graphs on the same set of axes for $x = [-360°; 360°]$
   (a) $y = \cos x$; $y = \cos(x + 30°)$ and $y = \cos(x - 90°)$
   (b) $y = \sin x$; $y = \sin(x - 60°)$ and $y = \sin(x + 15°)$
   (c) $y = \tan x$; $y = \tan(x + 45°)$ and $y = \tan(x - 30°)$
2. Discuss and then write down the relationship between the first graph and each one of the last two graphs, in each of the above cases.

2(a) $y = \cos x$
→ It is the mother graph, range : $-1 \leq y \leq 1$, amplitude is 1

$y = \cos(x + 30°)$
→ It has shifted 30° to the left from the mother graph, the range, amplitude, domain its still the same.

$y = \cos(x - 90°)$
→ It has shifted 90° to the right from the mother graph, the range, domain, amplitude it still the same.

(b) $y = \sin x$
it is mother graph, range $-1 \leq y \leq 1$, amplitude is 1, Domain -360;360. period is [-360° ; 360°]
→ $y = \sin(x - 60)$
it has shape as mother graph. It shifted 60° to left. Same amplitude, period, range
→ $y = \sin(x + 15)$
It has same shape as mother graph. it shifted 15° to right. Same amplitude, Period, Range, and Domain

c. $y = \tan x$
No Amplitude. No maximum nor minimum value period and Domain does follow the instructions
$y = \tan(x + 45°)$
→ no amplitude, no maximum or minimum, period is 180°. It has shifted 45° to the left from the mother graph.

$y = \tan(x - 30°)$
→ no amplitude, no maximum or minimum, period is 180°, It has shifted 30° to the right from the mother graph.

Figure 4.14(a) Group A2 responses to Worksheet 5 second part

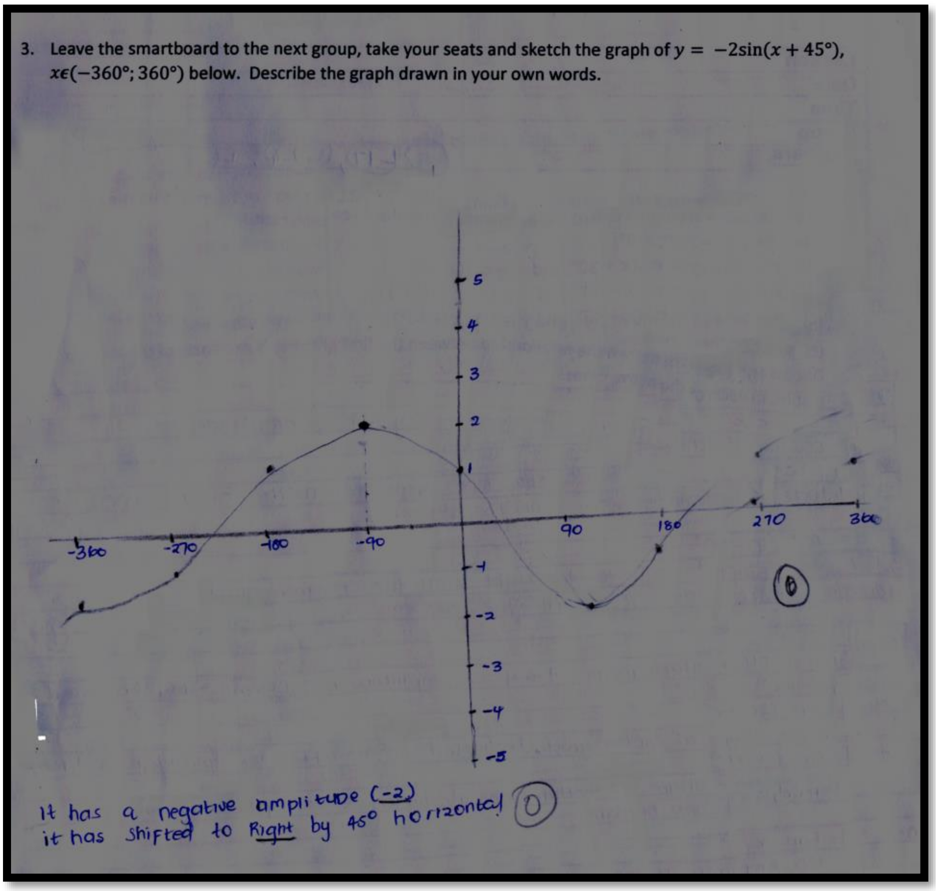


Figure 4.14(b) Group A2 responses to Worksheet 5 third part

The varied results of Group A2 clearly show that not all learners find using digital media easy and that it is important to develop lessons that enable learners to progress at their own pace (Shelly et al, 2008).

Worksheet 6: The effects of parameter *k* and *a*

Worksheet 6 (see Appendix N) required learners to use GeoGebra in the smartboard to draw graphs in each of the cases on the same set of axes. The second part expected learners to describe the relationship between two graphs in each of the cases and the last part assessed drawing graph by hand.

On the day there were two groups [Group A1 (including L7, L19 and L26), Group A2 (including L12, L17 and L24)]. The naming was similar to that of the previous day because the combination of targeted learners was the same. Both groups were able to state the relationship between the given graphs in each case. In some cases, the learners mentioned that the period is doubled or halved. It is important to state the actual period in degrees. This outcome compels educators to be involved and support learners in improving mathematical language associated with trigonometric functions. It is interesting that both groups were able to sketch the graph of $y = \cos\frac{1}{3}x + 2$. Group A1 were able to conclude that the period of the new graph is three times that of the mother graph, but just said that the graph had been 'shifted about 2 units' without indicating the direction. On the other hand, Group A2 were not able to describe the graph they sketched. This was most likely due to time factors.

4.3.2 Analysis of one-on-one interviews on worksheets

The purpose of this sub-section is to follow up on (probe) group work responses to worksheet activities and for triangulation purposes. The researcher sought for corroboration between written group work and individual oral responses to worksheets content aspects, an approach used by Ng and Hu (2006). L19, L17 and L24's interviews on Worksheets 3, 4 and 6 respectively were considered by the researcher after sieving the transcriptions.

Excerpt 1: L19

R: *May you describe the graph of $y = cos\, x + q$ (worksheet 3)*.

L19: *So in comparison to the parent graph, I would say $y = cosx + q$, the parent graph has then shifted by q upwards to form $y = cos\, x + q$, the period and the range, the minimum and the maximum values, all remain the same, while there was only vertical shift which is upwards by q units.*

This learner was able to give a clear answer addressing all the other features of the graph that are usually affected by parameters. In this worksheet learners had the chance to interact with the GeoGebra getting the chance to project on the smartboard, several cosine graphs as shown in Figure 4.15.

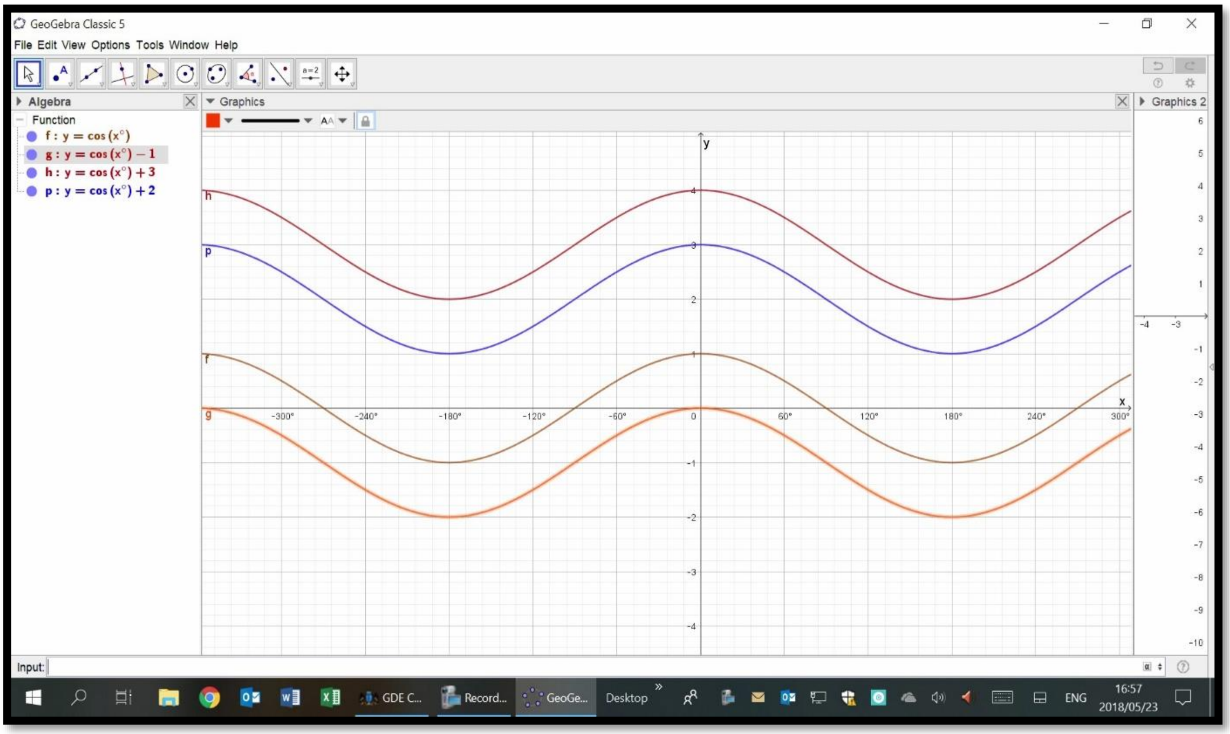
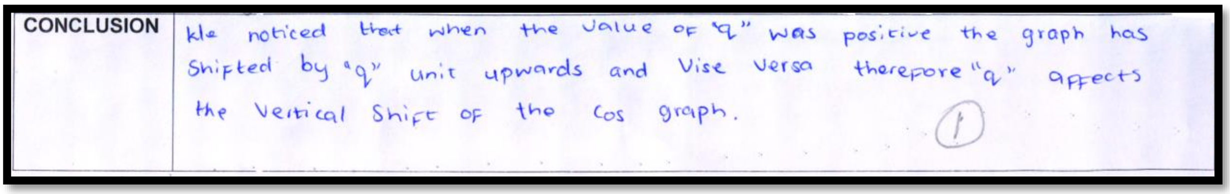


Figure 4.15: Screen shot (Video photo) for L19's group (Group 1 - Worksheet 3) from Smart recorder



CONCLUSION	We noticed that when the value of "q" was positive the graph has shifted by "q" unit upwards and Vise versa therefore "q" affects the vertical shift of the cos graph.

Figure 4.16: L19's group response to Worksheet 3

The use of GeoGebra in investigating these graphs made it easier for L19 and his group members to come up with such a generalisation (see Figure 4.16). This result agrees with Wiggins (2014) who maintains that understanding requires focused inferential work and being helped (in this case by GeoGebra software) to generalise from one's specific knowledge is key to genuine understanding. L19's response here and in the focus-group interviews resonates well with his group's response in Worksheet 3. The learner also confirmed during focus-group interviews (see Section 4.5 Excerpt 9) that he used the mother (basic) graph to understand the behaviour of other graphs (children). This reveals the power of GeoGebra in enhancing learners' understanding of trigonometric functions.

Excerpt 2: L17

R: *Can you describe the second graph in relation to the first in 1(c) that says* $y = tanx$ *and the graph of* $y = -tan(x) - 3$ *(Worksheet 4).*

L17: *The second graph in relation to this first graph, I think they all, they doesn't have like the maximum and the minimum point and also there is no amplitude. This negative sign which is between the y and the* $tanx$*, it just shows that it has been reflected up and down like it has been swopped and also the – 3 just emphasise the point there, the* y *–intercept only.*

This learner was absent on Day 3. The answer indicates that the learner did not explore the tangent graph enough using GeoGebra. An implication of this is the possibility that consistency in attendance when covering one concept to another may support such a learner in constructing knowledge without breakages. Again, the learner had challenges in mathematical language where 'reflected up and down like it has been swopped' meant reflection in the x-axis. However, L17's argument revealed that the learner could link the algebraic and graphical form of the function (since the learner mentioned reflection and the correct y-intercept), owing to the interaction with GeoGebra during worksheet activities.

GeoGebra afforded learners visualisation that resulted in learners constructing understanding by seeing the change in behaviour of graphs. This is supported by Excerpt 3.

Excerpt 3: L24

R: *Can you describe the two graphs for 1(b), the graph of* $y = sin2x$ *and the graph of* $y = sin\frac{1}{2}x$ *in your own words? (Worksheet 6)*

L24: *In the graph* $y = sin2x$*, I think it changes, it becomes smaller, the way it becomes smaller, it closes the space if you add the 2 in the graph of* $y = sin2x.$ *In the graph of* $y = sin\frac{1}{2}x$ *the graph distance in the* x*-axis I think it increases; it increases.*

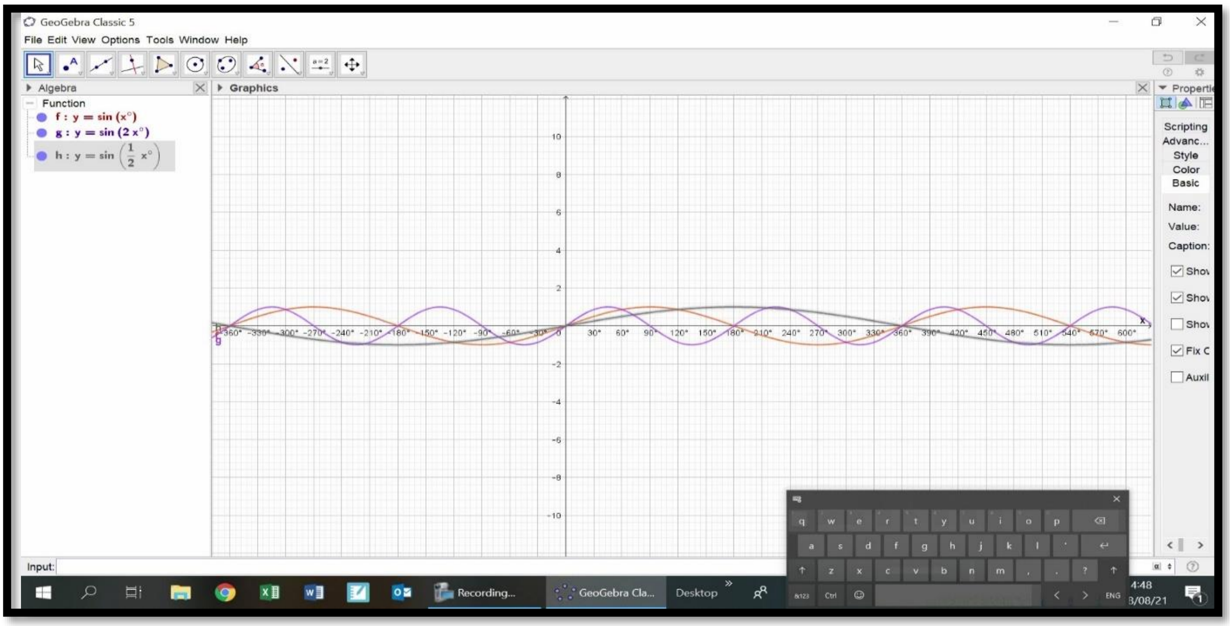


Figure 4.17: Screen shot (Video photo) for L24's group (Group A2 - Worksheet 6) from Smart recorder

The learner referred to the period when he talked about the 'increase and becoming smaller' of the distance in the *x*-axis. Here the learner referred to the crests and troughs (the periods of the graphs) that responded to the changes in the values of *k*. Group A2 (where L24 belonged) had obtained the correct answer by saying '$y = sin2x$ *- it is the same as the mother graph, the period is reduced by 2, the period is 180°, the range hasn't changed nor the amplitude'*; '$y = sin½x$ *- the graph is the same as the mother graph, the period increased 2 x than the mother graph, the period is 720°.'* This is evidence that the learner observed and understood how the sine graphs behaved as the group varied the values of the parameter *k* with GeoGebra. The software therefore afforded the learners to construct vividly their knowledge and understanding of this aspect of trigonometric functions in parameter *k*. From Chart's (2017) theory of understanding, mental models built by the learners during interaction with GeoGebra could have helped them to describe or explain the effects of the parameters.The respective graphs are shown in Figure 4.17. The software uses different colours for each graph thereby capturing and holding learners' attention (Shelly et al, 2008). Again, group and individual responses to this worksheet reveal that GeoGebra enhanced the learners' understanding of trigonometric functions by enabling them to relate and interpret algebraic, geometric and graphical forms an aspect that was reported as difficult in Naidoo and Govender's (2014) study.

4.3.3 Summary remarks

The individual learners' responses to worksheet content aspects during one-on-one interviews agreed with written group work in the worksheets. This result suggested that worksheet activities brought genuine learning gains to participants. The Grade 11 CAPS (2011:32) section on trigonometric functions requires learners to start the topic from "Point by point plotting of basic graphs defined by $y = \sin\theta$, $y = \cos\theta$ and $y = \tan\theta$ for $\theta \in [-360^0; 360^0]$." This is revision of Grade 10 work, the only difference is the interval where Grade 10 uses $\theta \in [0^0; 360^0]$. By the end of the topic in Grade 11, learners should be able to sketch and interpret the graphs of the functions involving at most two parameters at a time. Worksheets 1 to 4 allowed learners to use GeoGebra to explore Grade 10 trigonometric functions. A constructivist principle states: "To construct and understand a new idea, connections have to be made between the new one and old ones." (Van Putten 2014:5). The tasks in the worksheets linked Grade 10 (where calculator, paper and pencil were used) and Grade 11 content and simultaneously orientated learners to the use of GeoGebra. It is evident that worksheet activities tackled served as self-discovery tasks that allowed learners to investigate, notice and make generalisations, which is in agreement with constructivist tenets and the research by Naidoo and Govender (2014). It can be concluded from the learners' performance in the worksheet activities that GeoGebra environments (the use of GeoGebra) helped them to understand trigonometric functions.

4.4 LEARNERS' UNDERSTANDING OF TRIGONOMETRIC FUNCTIONS AFTER INTERACTING WITH GEOGEBRA

This section presents the analysis of the trigonometric functions test (see appendices Q and R for the test and its memorandum respectively) and the associated one-on-one interviews in quest of answering the second sub-question: *How is learners' understanding of trigonometric functions after interaction with GeoGebra?*

Six learners (L7, L12, L17, L19, L24 and L26) volunteered to sit for the test that sought to evaluate their understanding (knowledge and skills on) of trigonometric functions after interaction with GeoGebra during worksheet activities. The one-on-one interviews were meant to follow up on written work in the test so as to authenticate the learners' understanding, to check if their answers in the test were not through memorisation.

4.4.1 Analysis of the trigonometric functions test

The results obtained from the preliminary analysis of the trigonometric test are shown in Tables 4.1 to 4.4. A more detailed analysis of the test, question by question, is given in the following sections.

Question 1: sketching trigonometric functions with parameters *a*, *k*, *p* and q

In Question 1, the learners were required to sketch the graphs of $y = \cos 2x$, $f(x) = \sin \frac{1}{2}x$, $y = 2\sin 3x$, $f(x) = \tan(x + 45^0)$, and $y = \cos(x - 30^0)$. As a key element of this research, the learners used GeoGebra as a tool to build their understanding of the effects of the parameters *a*, *k*, *p* and *q* on trigonometric graphs. In addition, since constructivism suggests that learners should be active participants in the development of their own learning (Van Putten, 2014), the learners were expected to be able to sketch the graphs without much difficulty. However, the mean score for Question 1 was a disappointing 47%, with a minimum score of 0% and a maximum of 100%. In particular, most learners (five out of six) experienced problems in sketching the graph of $y = 2\sin 3x$. This result suggests that more research is needed in the area of trigonometric functions that combine parameters *a* and *k*.

Table 4.1 Summary of learners' responses to Question 1 of the trigonometric functions test

Question	Aim of task	Task	Number correct
1	Sketching trigonometric functions with parameters a, k, p and q	1.1 $y = \cos 2x \ , x\epsilon[0°; 360°]$	3 (L17,L19 and L26)
		1.2 $f(x) = sin½x \ , x \ \epsilon[0°; 360°]$	3 (L12, L19 and L26)
		1.3 $y = 2\sin 3x, x\epsilon[-360°; 360°]$	1 (L19)
		1.4 $f(x) = tan(x + 45°), x\epsilon[-180°; 360°]$	3 (L12, L19 and L24)
		1.5 $y = cos(x - 30°), x\epsilon[-180°; 360°]$	4 (L12,L17 L19 and L24)

Relating the sketching of graphs done in Worksheets 4,5 and 6 and diagnostic test to individual work in this Question 1, it is clear that the interaction with GeoGebra enhanced the learners' skills in sketching of trigonometric functions. This is seen when comparing the quality of sketches on Figures (4.1 to 4.3) with sketches on Figures 4.19, 4.20, 4.21, 4.22 and 4.24. L7 struggled throughout and the learner obtained an overall of 31% in the trigonometric functions test. The learner could not draw basic (mother graphs) graphs of sine, cosine and tangent in the diagnostic test (see Figures 4.4 a to c). Interestingly, the learner sketched the same graphs better in the Trigonometric functions test, although the sketches could not add up to the required solutions (see Figures 4.20 a to c). The same result came up when comparing other learners' sketches in the diagnostic test (see Section 4.2 Figures 4.1 to 4.3) and those in the worksheet activities (see Section 4.3.1 Figures 4.12, 4.13b and 4.14b). This finding accords with Fahrudin and Pramudya's (2019) observation that GeoGebra assists struggling learners to understand trigonometric concepts easily. This clearly shows that the use of GeoGebra during worksheet activities enhanced the learners' understanding of basic concepts of trigonometric functions through collaborative visualisation. It is therefore important for mathematics educators to create constructivist learning environments like the amalgamation of worksheets and GeoGebra in this research. The interaction with GeoGebra has tremendous benefits in mathematics classrooms towards the understanding of trigonometric functions.

Given enough time it proves that all learners of varying abilities could do better in this topic.

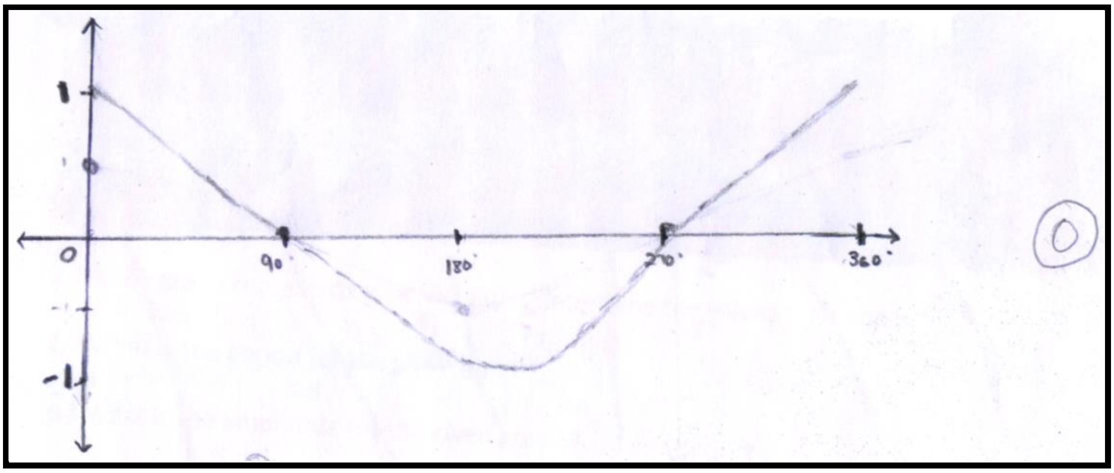


Figure 4.18a L7's answer to 1.1 in the trigonometric functions test

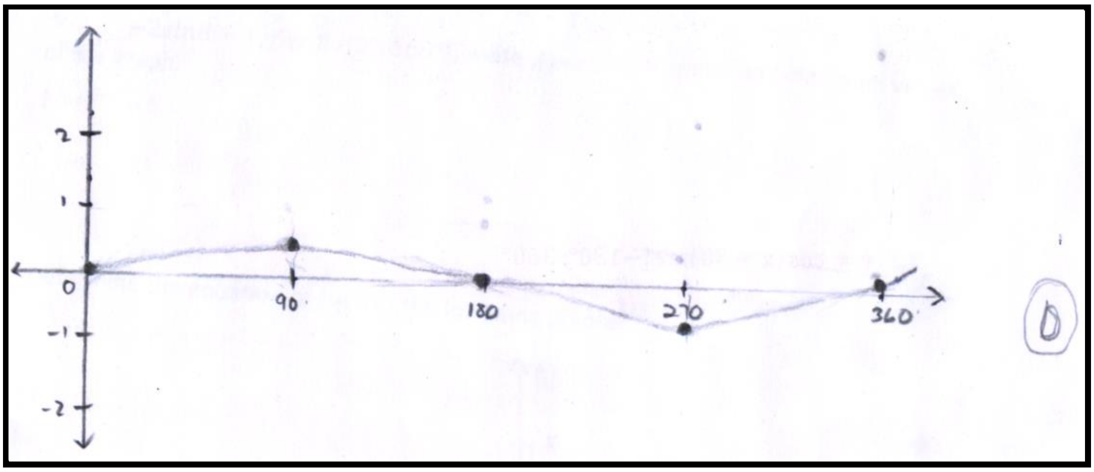


Figure 4.18b L7's answer to 1.2 in the trigonometric functions test

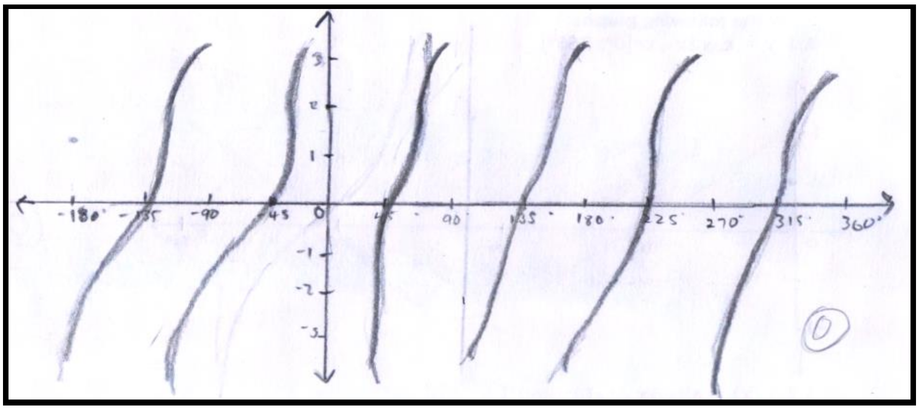
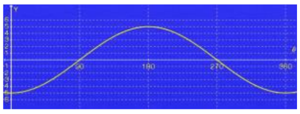


Figure 4.18c L7's answer to 1.4 in the trigonometric functions test

Question 2: Interpreting a trigonometric function graph

This question required learners to identify values of parameters in $y = b\cos a\theta$ associated with the drawn graph. Furthermore, learners were supposed to study the graph and determine its amplitude and period. The results, as shown in Table 4.2, indicate that learners did better in this area. The mean score for Question 2 was 63%, with a minimum score of 0% and a maximum of 100%. This outcome serves as a solution to Le Roux's (2014) observation that learners often struggle to interpret graphs. In this research, learners' interaction with GeoGebra enabled them to correctly interpret the graph.

Table 4.2 Summary of learners' responses to Question 2 of the trigonometric functions test

Question	Aim of task	Task	Number correct
2	Interpreting a trigonometric function graph $y = bcosa\theta$ 	2.1 value of a (equivalent to parameter k)	4 (L12,L17,L19 and L24)
		2.1 value of b (equivalent to parameter a)	2 (L7 and L19)
		2.2 period	5 (L7,L12,L17,L19and L24)
		2.3 amplitude	4 (L7,L17,L19 and L24)

In this question, the learners were able to identify more features of trigonometric functions (or graphs) compared to a similar question in the diagnostic test (Question 2). L7's responses to Question 2 in the diagnostic test were all incorrect but in this question (under the trigonometric functions test) learner obtained three out of four, being able to interpret or relate the algebraic, geometric and graphical forms of trigonometric functions. This function in question has combined parameters *a* (found in Grade 10) and *k* (found in Grade 11). L24 also did better in this question after having struggled in the diagnostic test. In contrary, L26's performance is odd in the sense that the learner obtained incorrect answers yet the learner had shown understanding of amplitudes in the diagnostic test.

Question 3: Interpreting the equation of the trigonometric function

In Question 3, learners were required to write down the minimum and maximum value of $y=-2\sin\theta-1$. Learners obtained a mean score of 67% on this question, with a minimum score of 0% and a maximum score of 100%, revealing better understanding enhanced by the use of GeoGebra. The diagnostic Report (2016: 172) states that: "…candidates confused maximum value with range and gave the answer as an interval instead of a single value. Candidates could not realise that the answer they were looking for could be obtained from the y-coordinate of the maximum turning point." A probable explanation to this report is that learners in our schools are being taught trigonometric functions using traditional methods. In accordance with the present results, previous studies have demonstrated that GeoGebra has the potential to help learners interpret trigonometric functions (Naidoo and Govender, 2014).

Table 4.3 Summary of learners' responses to Question 3 of the trigonometric functions test

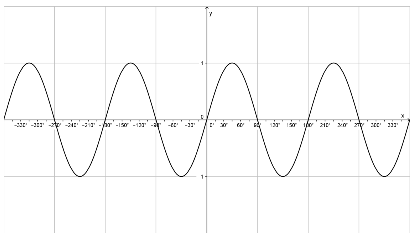
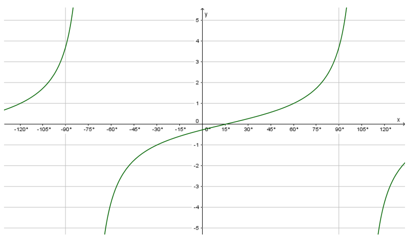
Question	Aim of task	Task	Number correct
3	Interpreting the equation of a trigonometric function	Minimum value of $y = -2sin\theta - 1$	5 (L7,L12, L17,L19 ,L26)
		Maximum value of $y = -2sin\theta - 1$	3 (L12,L1 7,L19)

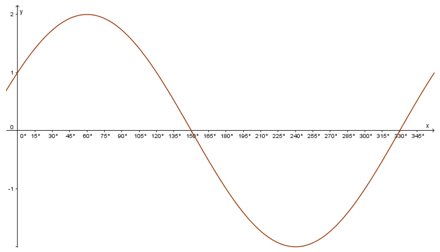
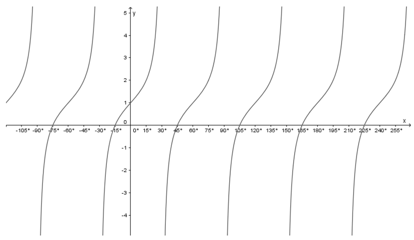
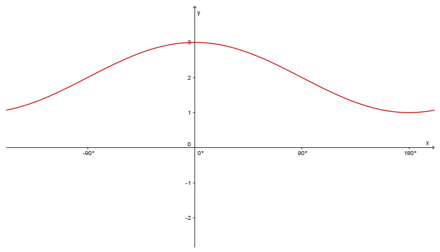
The requirements of this question are related to Question 3 of the diagnostic test which required learners to determine the range. The better performance by L7 and L26 (from 0% in the diagnostic test to 50% in the current question) indicated that the interaction with GeoGebra during worksheet activities enhanced the learners' understanding of range, maximum and minimum values of trigonometric functions.

Question 4: Determining the equation of given sketches

In this question, participants were expected to determine the equations of given trigonometric functions graphs (see the sketches in Table 4.4). Learners obtained a mean score of 13%, with a minimum score of 0% and a maximum score of 40%. Out of 30 responses 24 were incorrect (see Table 4.4). This is a discrepancy since learners were expected to obtain better marks after using GeoGebra in learning trigonometric functions. During one-on-one interviews learners also experienced problems in coming up with equations of graphs in Question 4 (see Section 4.4.2). A possible explanation for this might be that the learners were rushing against time. These results were unexpected, although they match those of Kepceoglu and Yavuz (2016). This suggests that this aspect needs to be given much more time and practice with GeoGebra.

Table 4.4 Summary of learners' responses to Question 4of the trigonometric functions test

Question	Aim of task	Task	Number correct
4	Determining equations of given sketches.	4.1 	2 (L12 and L19)
		4.2 	2 (L7 and L19)

		4.3 	2 (L12 and L19)
		4.4 	0
		4.5 	0

4.4.2 Analysis of one-on-one interviews on Trigonometric FunctionsTest

The participants were interviewed on Questions 1.3, 1.4, 1.5, 2.1, 4.1, 4.2 and 4.3 that were chosen at random by the researcher.

Question 1.3

The researcher asked L17 to describe the graph of $y = 2\sin 3x$ in Question 1.3 that they were asked to draw. The learner responded as follows: "The graph has an amplitude of 2 and the period of the graph has been changed from 360° to 720°. From 360° to 720° whereby, I took this 360° then multiplied it by$1/3$". This response agrees with the sketch drawn by L17 in the test in terms of the amplitude (see Figure 4.19). In the sketch the learner had a period of 360°. The learner remembered during the interview that the period had to be reduced three times, although the answer she gave is 720° not 120°. This response reveals that the learner understood the effects of the parameters *a* and *k* but had difficulties in multiplying with fractions without using a calculator.

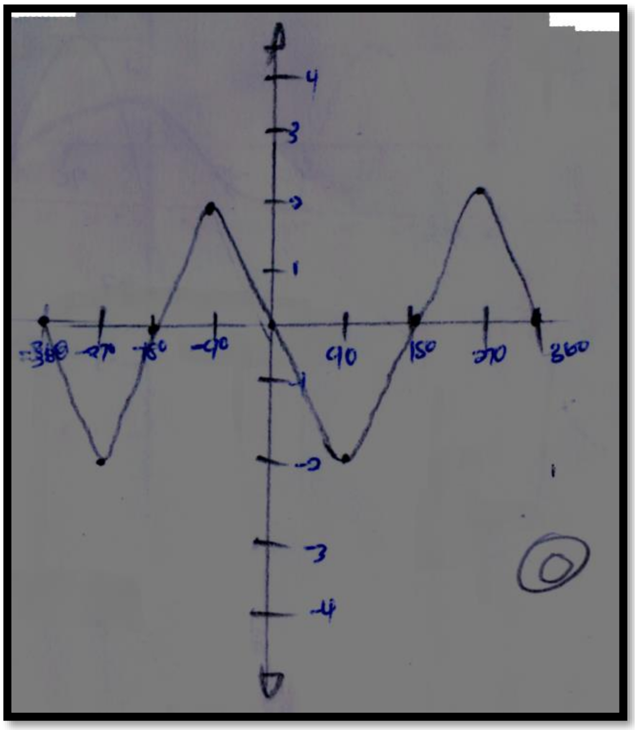


Figure 4.19: L17's answer to 1.3 in the trigonometric test

L17's sketch shows that the learner reflected the sine graph in the *x*-axis. The utterances given by the learner in the interview revealed better understanding of the trigonometric function than the drawn sketch. Barmby et al (2007) insist that we can use learners' errors to interpret how the learners understand mathematics concepts, which is what was done here by the researcher.

Question 1.4

Three learners L19, L24 and L12 were interviewed on Question 1.4 and their respective responses have been captured in Excerpt 4 below. The sketches drawn by L19 and L24 are in Figures 4.20 and 4.21 respectively.

Excerpt 4: L19 and L24

R: *Can you describe the graph that you drew in Question1.4 in the test.*

L19: *In comparison to the mother graph, the graph has shifted about 45º to the left. This change has also caused the asymptotes of the graph to change slightly by 45º to the left.*

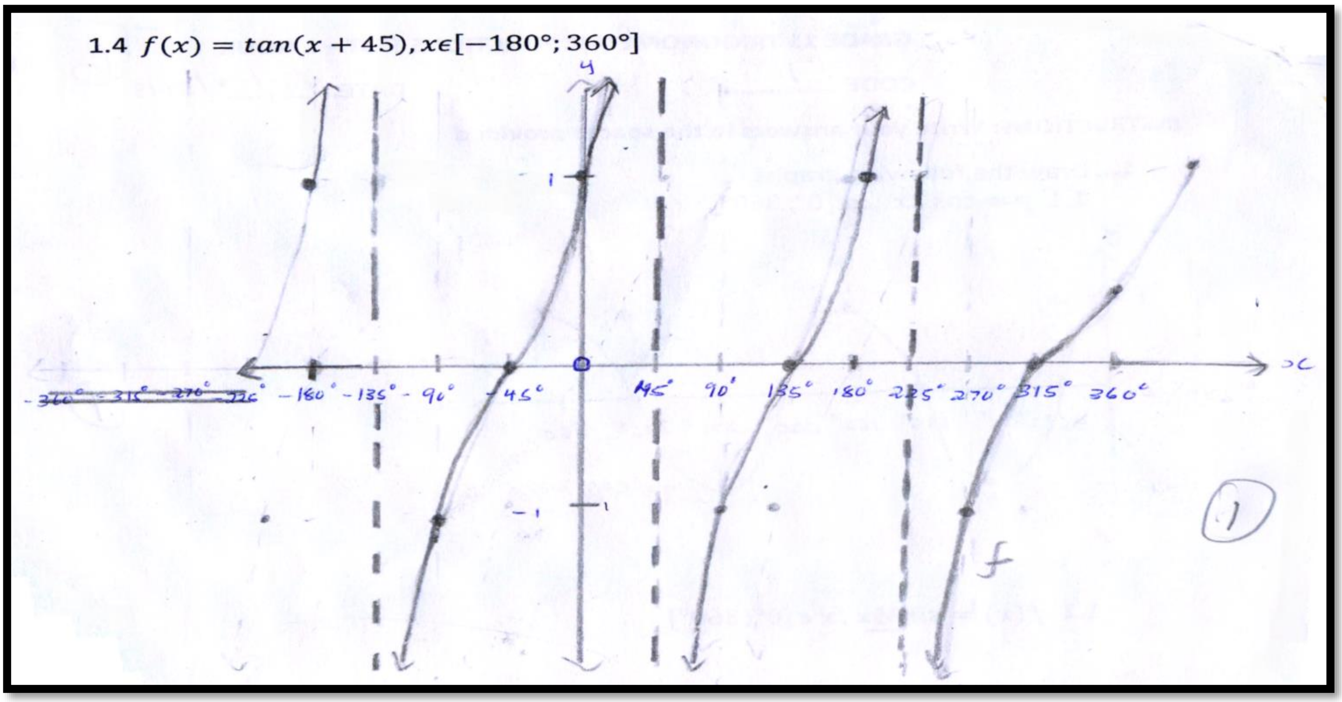


Figure 4.20: L19's answer to Question 1.4 in the trigonometric test

L24: *It adds 45º to the left.*

R: *Ok.*

L24: *And then it starts from negative 180° it means the x-axis it starts from negative 180° and ends in 360º. In terms of the amplitude, nothing changes, it is still the same and there is no negative sign, so the way it is supposed to be.*

R: *What does the negative sign do the graph?*

L24: *It changes the way it looks. It changes from, the shape changes, it becomes different from the original shape.*

R: *How does it become different?*

L24: *It becomes different, the way it looks from the original.*

R: *Be clear, please.*

L24: *It looks upside down.*

R: *Ok. I see.*

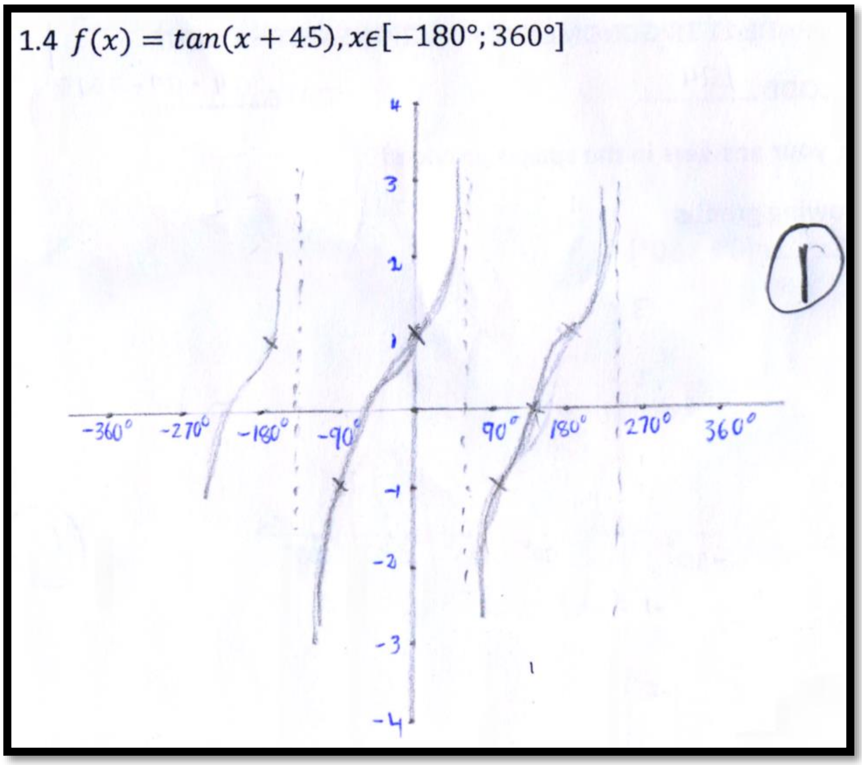


Figure 4.21 L24’s answer to Question 1.4 in the trigonometric test

Excerpt 5: L12

R: *How does the graph in 1.4 look like?*

L12: *The graph* $y = tan(x + 45°)$*. This graph has the same shape as the mother graph* $tanx$*, but this graph it has shifted 45º to the left.*

R: *Okay, what compels you, what informs you that it has shifted 45º to the left?*

L12: *Because it has a positive 45º, therefore it causes the graph to shift to the left.*

R: *Okay. Thank you very much.*

Learners understand differently according to constructivists and their justifications and refinement of answers are also different (Ndlovu, 2013). The learners’ remarks are consistent with earlier research findings. Weber (2005) recommended that learners should learn mathematics with understanding rather than to memorise procedures and acquire reliable methods for producing correct solutions on paper-and-pencil exercises. Based on L19, L24 and L12’s responses, it was clear that the learners understood very well the effects of the parameter *p* on the tangent graph. Again, L24 and L12 responded well to the follow up questions that sought clarifications on the initial responses to the main question. This in accordance with Skemp (1997, cited in Weber, 2005) who maintained that learners should be able to explain why the procedures they apply are mathematically appropriate and justify why mathematical concepts have the properties that they do.

Question 1.5

As a matter of follow-up on the understanding of the effects of parameter *p* on trigonometric functions, the researcher further requested L12 to describe $y = \cos(x - 30^0)$ in Question 1.5. The learner managed to respond in a single sentence as follows: "The graph has the same properties as the mother graph, that is the cosine graph, but the graph has moved 30° to the right." L12's responses to Questions 1.4 and 1.5 instils the learner's deep understanding of the effects of *p* on the functions. Furthermore, the learner provided the correct sketch in the trigonometric functions test (see Figure 4.22).

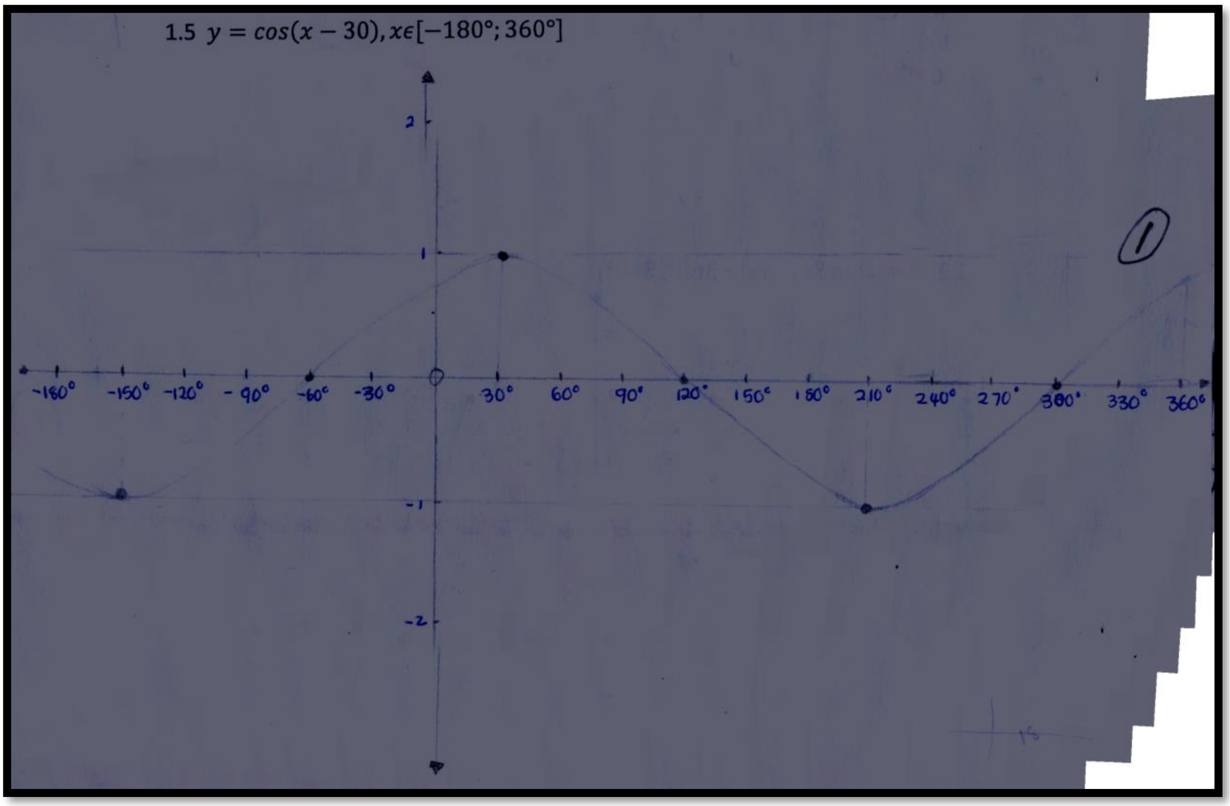


Figure 4.22: L12's answer for Question 1.5 in trigonometry functions test.

Question 2.1

L12, L24 and L19 were interviewed on this question that sought to test the learners' ability to link the algebraic and the graphical forms of trigonometric functions. They were also required to identify the values of the period (parameter *k*) and the amplitude (parameter *a*). L12 provided the correct period but confused the amplitude and gave an incorrect value of 10. The excerpts below present interesting responses from L24 and L19.

Excerpt 6: L24

R: *From the test that you wrote, let's look at this graph in 2.1. What are the values of* a *and* b*?*

L24: *I will start off with the value of* b*, the value of* b *it is 5.*

R: *Ok.*

L24: *And then the value of* a*, I think it is 1.*

R: *Ok. Can you say why the value of* a *is 1 and the value of* b *is 5?*

L24: *Because the amplitude is 5. That is why I say* b *is 5 because I try to look at the amplitude where it starts and where it ends*.

R: *Okay.*

L24: *Then the value of* a*, I see no change in the graph in terms of what the value of* a *does.*

R: *What does the value of* a *represent?*

L24: *The value of a represent the shape of the graph. It is length.*

R: *It is length?*

L24: *Yes.*

Excerpt 7: L19

R: *From the test that you wrote, 2.1, may you help me there by giving the values of* a *and* b*?*

L19: *I think the value of* b *is negative 5 and the value of* a *I think it is 1.*

R: *Okay, can you tell me the reason why you are saying the value of a is 1, what made you to say the value of a is 1?*

L19: *Because the period of the graph is 360º. The period of the graph is 360º which is the cycle for 1 cosine graph if you could look at the graph it forms the one cycle at 360º.*

R: *Okay. And then if I check here, you said the value of b is negative 5, why negative 5?*

L19: *In comparison with the mother graph, this graph has reflected so I think it is reflected about the x-axis. Because of that, then the value must be negative, and the amplitude is 5. So that is why I concluded that it is negative 5*.

Reflecting on L19 and L24's responses, L24 did not realise that the graph was a reflection in the *x*-axis hence obtaining the value of *b* as 5. L19 revealed during focus-group interviews that GeoGebra helped him understand trigonometric functions from the mother graph (see Section 4.5 Excerpt 9). In this case, his explanation developed from his knowledge of the mother graph.

Question 4

The whole of Question 4 required learners to determine the equations of given trigonometric functions graphs. L12 and L19 provided the correct equation for the graph found in Question 4.1 during the interviews. They both said that they were influenced by the period of the graph which is 180° and corresponding to $k = 2$. On the other hand, L24 took the equation of the graph to be $y = \sin x$. The results agree with the learners' responses in the trigonometric functions test. It is interesting that L17 obtained a wrong equation for Question 4.3 in the test but provided the correct answer in this interview. This shows the value of using varied ways to assess the learners' understanding of mathematical concepts, an approach supported by Hiebert et al (1992, cited in Barmby et al, 2007).

4.4.3 Summary remarks

The analysis of the learners' responses in the trigonometric functions test showed that learners had good understanding on Questions 2 and 3, moderate understanding on Question 1, and had difficulties with Question 4. The overall mean score for the test was 45%, with a minimum score of 19% and a maximum score of 88%. The results of the follow-up on the learners' written work through one-on-one interviews sealed the fact that the solutions provided by the learners in the trigonometric functions test reflected learners' understanding and not memorisation. The learners' overall scores in trigonometric functions test are as follows in ascending order: L26-19%; L7-31%; L24-31%; L17-44%; L12-56% and L19- 88%. Despite the moderate and low performance in Questions 1 and 4 respectively, the learners' responses revealed that the use of GeoGebra enhanced the learners' understanding of trigonometric functions.

4.5 LEARNERS' EXPERIENCES AND VIEWS ON THE USE OF GEOGEBRA IN EXPLORING TRIGONOMETRIC FUNCTIONS

This section focuses on analsying data collected from focus-group interviews in an attempt to answer the third sub-question: *What are learners' experiences and views on the use of GeoGebra in exploring trigonometric functions?*

The interviews sought to delve into the learners' learning benefits, challenges, suggestions, surrounding their use of GeoGebra in worksheet activities. The interviews were held after the one-on-one interviews were done and five (L12, L17, L19, L24 and L26) of the six targeted learners participated. The sixth learner, L7 was absent. The focus-group interviews were valuable in this research for the purposes of triangulation and supporting other sources of data (diagnostic test, worksheets, trigonometric test, one-on-one interviews, and smart recordings). Focus-group interviews have tremendous benefits as stated in Chapter 3 (see Section 3.8.5). The researcher used open-ended questions that resulted in rich qualitative data that assisted in providing a deeper interpretation of learners' experiences. The researcher formulated the Focus-group interviews questions around the research question with the aim of eliciting responses that could help to answer the research question.

4.5.1 Analysis of focus-group interviews

The discussions during the focus-group interviews resulted in the emergence of the following overlapping themes: GeoGebra as a learning tool; GeoGebra a better tool than a calculator; learners taking charge of their learning; collaborative work; and other findings. The researcher interpreted and understood these themes through the lenses of constructivist and understanding theories. It was difficult for the researcher to interpret any particular theme or finding using one clear cut tenet of these theories. The tenets seem to be networked in any constructivist learning environment.

GeoGebra as a learning tool

The researcher asked learners to report on their general experiences in learning trigonometric functions using GeoGebra. The intention was to explore what and how the learners think about the use of GeoGebra and why they think that way. From L19, L26 and L24's utterances, the use of GeoGebra enabled the learners to know more about trigonometric functions (see Excerpt 8). This suggests that learners engaged GeoGebra as a learning or cognitive tool in their mind processes to enhance their understanding of trigonometric functions during solution of tasks. The use of GeoGebra in this way is within the boundaries of the constructivist perspective as observed by Demir (2012).

Excerpt 8: L19, L26 and L24

L19: *The first thing I am glad to be part of this programme, it has helped me to know about trigonometric functions and also help me to use the software which is GeoGebra, although it was tough at the beginning. We managed to work it through, and we now know the basics of GeoGebra and how to use it and also the different properties of trigonometric functions. So, I am happy to be part, to have been part of this programme, thanks.*

R: *Okay. Can you please elaborate, where you say it was tough at the beginning, what was tough?*
L19: *The using of the GeoGebra software.*
R: *Okay. Thank you. Can we have another one, next person?*

L24: *I am L24 and my experience was good, it was an exciting. As I went on, I learned new stuff and I know the tricks of using GeoGebra. Now I know how to use it, and I also learned how to use the mother graph, it helps me to understand the whole thing much easier. Thank you.*

R: *Okay. Next person.*

L26: *Okay, I am L26. Okay, my experiences in GeoGebra, at first for me it was difficult because during school hours, in class, I quite didn't understand what was happening, until we started attending this session. It helped me a lot to understand what is going on with the functions and the properties*.

The utterances: '*we now know the basics of GeoGebra and … also the different properties of trigonometric functions*'; '*and I also learned how to use the mother graph, it helps me to understand the whole thing much easier.'; '. It helped me a lot to understand what is going on with the functions and the properties*.' by L19, L24 and L26 respectively indicate that the Grade 11 learners' experience with GeoGebra enhanced their understanding of trigonometric functions. Again, from Excerpt 8, it appears that being able to operate GeoGebra software opened doors for the learners to explore trigonometric functions using the software. This confirms Mills' (2006) view that constructivist learning environments are active and exploratory in nature. In addition, being able to operate GeoGebra by learners is a pre-requisite in the use of the software for learning purposes (Shelly et al, 2008). L17 elaborates this in her response " …at first it was quite hard due to settings, like the intervals and also the units of measuring the distances but as time goes on, I learned using the mother graph is like the basic and also using, knowing the properties to help me construct my graphs but what I can say is that GeoGebra like is quite fun because you learn as you go." What L17 said also indicates that the use of GeoGebra as a learning tool was motivating by being 'fun'. This agrees with Pfeiffer (2017) who insists that the learning of mathematics becomes fun and attractive when technology tools like GeoGebra are used to explore concepts. The researcher is also confident that GeoGebra enabled Grade 11 learners to master the properties of trigonometric functions that in turn helped them to sketch the graphs with ease. For example, what L17 sees as 'knowing the properties to help me construct my graphs '. It is therefore clear that the use of GeoGebra during worksheet activities was pivotal in providing a learner centred environment that supported understanding of trigonometric functions.

Reflecting on L26’s response, understanding trigonometric functions was difficult without GeoGebra. This is supported by L12 who says “…at the beginning I didn’t understand what was going on, but now I understand a lot.” This learner’s response is consistent with Naidoo and Govender’s (2014) findings that the use of GeoGebra allowed learners to develop a well-founded and enhanced mathematical understanding of trigonometric functions graphs. It follows that the learners used GeoGebra as a learning tool to construct their knowledge and understanding.

GeoGebra a better tool than a calculator approach

The learners revealed that their drawings improved from that of using dots, that is, from using a calculator (which is quite laborious) to free sketching using knowledge acquired from the use of GeoGebra. DBE examiners argue that drawing trigonometric graphs from the tabular method limits learners’ understanding of graphical features (DBE diagnostic report, 2013 and 2015). The interaction with GeoGebra in the current study enhanced the learners’ understanding of the graphical representation of trigonometric functions. This is detailed by three interviewees in Excerpt 9 below.

Excerpt 9: L12, L19 and L24

L12: *I am L12, I am glad to be part of this programme because I have learned a lot. I have learned to use GeoGebra, I have learned to draw graphs without the use of a calculator and in the beginning, I didn’t understand what was going on, but now I understand a lot.*

L19: *GeoGebra has allowed me to integrate the mother graphs or the parents graphs with other graphs that you can draw, we have been able to establish a relationship between the mother graph and also other graphs of the trig functions, I think it helped me that way and I can now use the mother graph without the use of the calculator, I am able to draw the other graphs at any time.*

R: *Okay, thank you very much.*

L24: *I agree with what L19 has said, GeoGebra has helped me to interpret the graphs but like as L19 said, like the relationship between the mother graphs and the other graphs, yes.*
L26: *The dots came because people used calculators, they use calculators to plot and if you use your knowledge, on what you know about the topic, you understand it and you draw it as you know it and this is how I see changed the appearance*.

This is exemplified by L12's work in the diagnostic test as compared to that in the Trigonometry test after interacting with GeoGebra (see Figures 4.23 and 4.24).

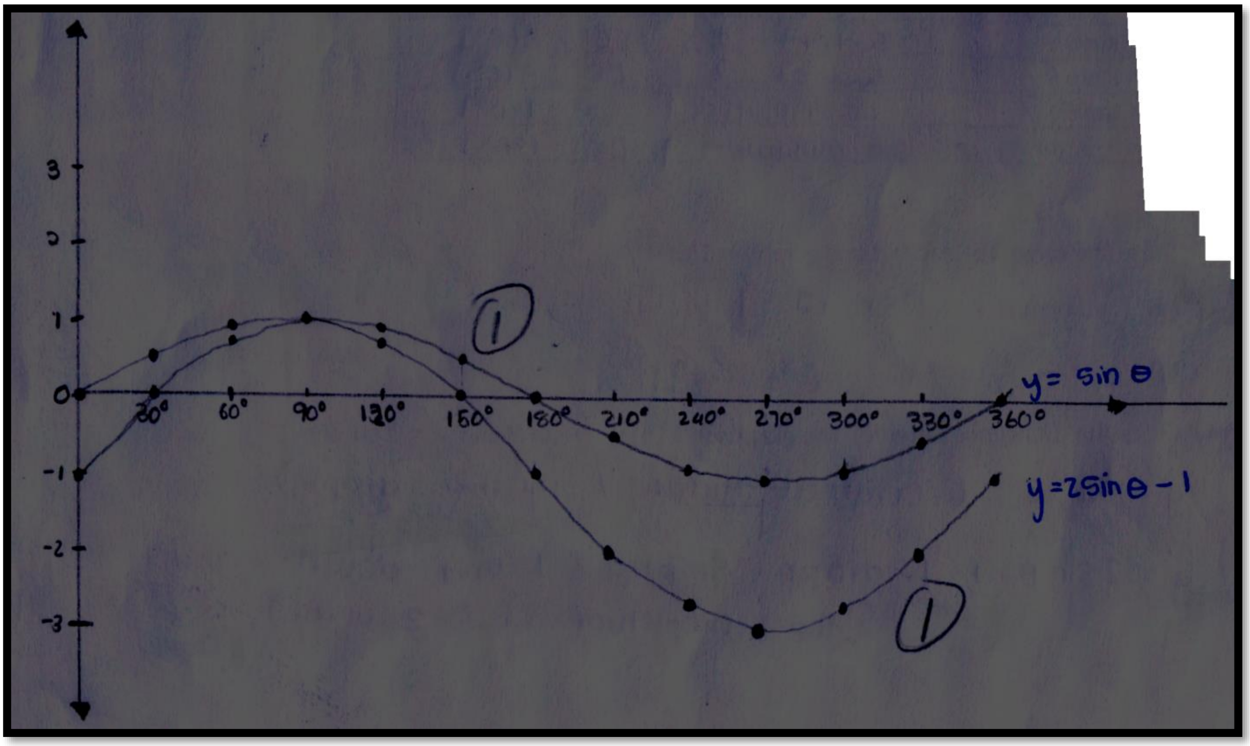


Figure 4.23: L12 using a calculator to draw trigonometric graphs in the diagnostic test.

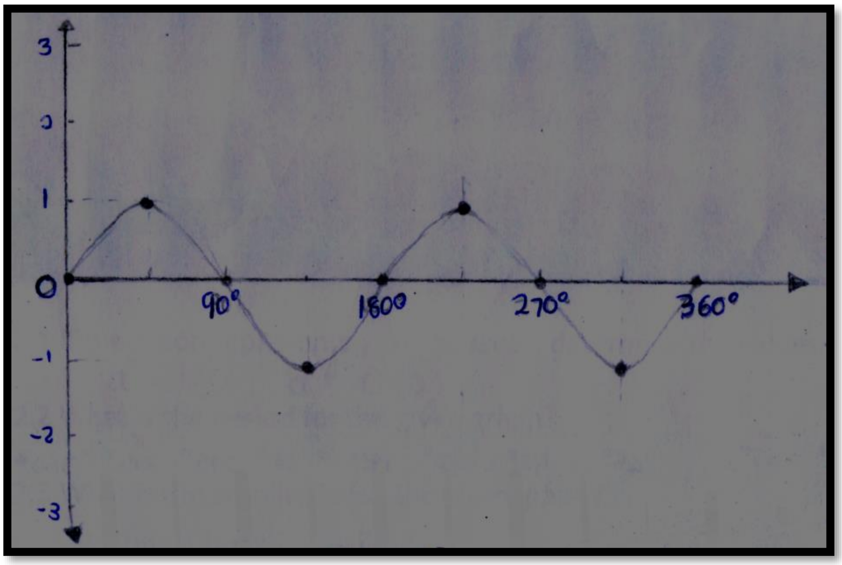


Figure 4.24: L12's sketch in the Trigonometry functions test (after interacting with GeoGebra).

The learners' responses coincide with Naidoo and Govender's (2014) findings that GeoGebra aids the element of visualisation that plays an important role in the learners' exploration of trigonometric functions graphs. Unlike the calculator, GeoGebra enabled learners to project several graphs (in varying colours) during worksheet activities, and relate them to the mother graph and this saw them being able to sketch without table values from a calculator. Comparing all the graphs drawn by participants before and after interacting with GeoGebra, shows that the tool enabled learners to draw graphs from abstraction. In fact, Van Woudenberg (2017) found that GeoGebra is a better tool to replace a calculator in the learning of some mathematics concept like those in trigonometric functions.

Stiff (2001) noted that learners in constructivist learning environments can deepen their understanding by constructing or building new knowledge on prior knowledge or experiences. During the focus-group discussions, learners revealed that GeoGebra enabled them to sketch other graphs (families) by relating to their mother (basic) graphs (see Excerpt 9). These findings corroborate those of Jenkin, van Zyl and Scheffler (2015) whose ideas encourage learners to always start with the basic graph (mother graph) when sketching trigonometric functions graphs followed by considering the effects of parameters. The participants in this research went further to draw trigonometric graphs without the mother graph and this showed the strength of

GeoGebra in enhancing learners' understanding of trigonometric functions. This is in line with Mills' (2006) observations that learners who are active in technology integration use technology to manipulate subject content and at the same time acquire advanced reasoning and understanding skills. In consistence with this is the constructivist view that technology (in this case GeoGebra) is used by learners as a tool in lessening cognitive burdens (Demir, 2012).

Some learners, like L24, reported that they had difficulties in differentiating between sine and cosine graphs before they interacted with GeoGebra. The same gap was reported by the DBE Diagnostic report (2015).

L24: *For me I think GeoGebra helped me in understanding it more better, and understand the difference between the sine and the cosine graphs because they are the same; the other one start from zero and the other one looks down and the signs and I can think and write my own equation and draw it and then check it, if it is correct in the GeoGebra. So GeoGebra I think it acts as a memo for any question you have on the question while GeoGebra that you are doing.*

In support of L24, L26 commented that "Well GeoGebra has helped me to understand the shifting of graphs. I understand the *p* and *q* the better now." Earlier research by Weber (2005) reports that many approaches to teaching trigonometry primarily stress procedural skills and such (traditional) approaches do not allow learners to understand sine and cosine functions. In the current research, L24 and L26's inputs during the Focus-group discussion revealed that GeoGebra enhanced the learners' understanding of basic trigonometric function graphs and the effects of parameters on these graphs. From a constructivist perspective, this finding suggests that GeoGebra was used by learners as a cognitive tool, which is defined as both a mental mechanism and digital device that supports, guides, and extend the thinking processes of users (Derry, 2000 cited in Mills, 2006). In addition, cognitive tools (in this case GeoGebra) function as intellectual partners to stimulate and facilitate critical thinking and higher order learning in learners and have the potential to augment teaching and learning in several ways (Jonassen, 2000 and Oliver, Omari, Herrington, and Herrington, 2000 cited in Mills, 2006). During the focus-group discussions, some learners also reported that GeoGebra helped them to relate the algebraic and the graphical part of trigonometric functions and this is an important skill in FET. However, others still had difficulties in this area as revealed by their performance in Question 4 of the trigonometric functions test and their utterances in the associated interview (see Sections 4.4.1 and 4.4.2).

Learners taking charge of their learning

Based on what they had experienced, learners were asked how the use of GeoGebra in the teaching and learning of trigonometric functions could be improved. L26 suggested that learners should be afforded individual access to GeoGebra software. The learner prefers to have his or her own software whilst the educator uses the smartboard for demonstration purposes. This was supported by L12:

L12: *I agree with L26 because accessing the GeoGebra individually allows you to access it every day, you can access it while, when we access it as a group, it is like we have a limited time. For example, with the teacher, if it is only the teacher, she is the one who is having the access to the GeoGebra, we can't have access to it every day so we can't understand. We end up forgetting the settings of the software, so I prefer it individually that we can therefore be able to use it every day*.

In the same vein, L17 was of the idea that the software should be installed in their tablets.

L17: *Sir like we as our school, we are so privileged that we have tablets, so and those tablets we can access the GeoGebra, so it doesn't have a limited time where it expires, so I think, yes using our tablets or cell phones, yes it is how we can access it almost every day and also in class, we also have the smart board.*

The school is a full ICT school where learning and teaching should be paperless. Unfortunately, the Gauteng Department of Education has not been able to supply all the learners with tablets. The researcher's findings during school visits in Term 1 2019 reveal that tablets had been supplied but there was a shortage of 38 in Grade 12 whilst Grades 8 to 11 had nothing at all up to the end of the term. The GDE had promised to deliver the supplies within two weeks of opening.

GeoGebra can be installed on each learner's tablet and hence afford access to the software anytime. During the focus-group discussions, learners also preferred to be assigned with homework on trigonometric functions. This is in line with the constructivist perspective where technology integration results in learners assuming responsibility for their own learning and being motivated to pursue deeper understanding of curriculum content better than a classroom focused solely on accomplishing curriculum objectives (Mills, 2006). In addition, Shelly et al (2008) encourage educators to assign learners with classwork and homework that is completed using technology because this allows learners to see how far their imagination can take them.

In the current research, the researcher could not assign participants with homework because of the inaccessibility of GeoGebra after school time. The access of GeoGebra through tablets could also allow learners to practice the operations (setting up) of the software and to do investigations that will enhance their understanding of trigonometric functions. The learners also wished to access GeoGebra the same way they do with the Siyavula App as discussed in Chapter 2 (see Section 2.2). In our CAPS curriculum, work in the form of SBAs, homework and classwork can be assigned to learners once GeoGebra is mediated to educators and District Subject Advisors and adopted as a learning and teaching tool in trigonometric functions and related topics.

Collaborative work

Whilst some learners opted for individual access to GeoGebra, others would like to work in pairs or groups as discussed in Excerpt 10 below:

Excerpt 10: L17, L19 and L24

L17: *I think as pairs, it is quite challenging, but we learn as we go, where we have like as individuals, we pair ourselves, like some they set up the GeoGebra, some they plot and some they capture the information. So, I think group wise it is fine because everyone has a like a routine on how to do it, how to use it. Some they use the settings, some they plot the graph and some they capture it and all we discuss it.*

R: *No problem, she prefers group work to individuals. Anyone else?*
L19: *I agree with L17, because when they are using GeoGebra in group work it will be easy because if for example I had got the information, incorrectly I could ask maybe someone to help me out.*
R: *Okay, other inputs?*
L24: *I prefer using it in pairs because in pairs it is much easier, you understand as it goes, you are able to ask the next person, if there is two of you, the concentration levels are also high so that is my take on how to learn it.*

The interviewees' responses revealed their preference for collaboration (or working in groups) in learning that is brought about by technology integration and characterises learner centred classrooms where learners manage their own learning to accomplish learning tasks (Mills, 2006). Furthermore, the use of GeoGebra during worksheet activities increased learner-to-learner and learner-to-educator interaction and this is in line with the fifth principle on 'Principles of Learner-Centred Teaching' that requires educators to do more to get learners learning from and with each other (Mills,2006). Most of the learner achievements in the worksheet activities were brought by robust discussions amongst learners during group work. From what L24 is saying '…you are able to ask the next person …' agrees with the constructivist perspective that technology integrated collaborative work results in scaffolding (ZPD) by group members and technology (in this case GeoGebra) (Lombardi, 2017; Maroske, 2015; Bakker et al, 2015; Wu et al 2002).

Other findings

One learner also preferred the theme or background of the screen to be improved (through settings) so that it becomes more attractive. This view is supported by Shelly et al (2008:23) who argue that "Effective learning techniques seek to capture students' attention to eliminate boredom and arouse natural curiosity…Effective techniques should stimulate the sense of wonder and maintain the interest." The researcher will explore the GeoGebra manual in search of theme settings that could be attractive to learners since mathematics learning also thrives in aesthetic values.

All the learners who participated in the focus-group interview reported that they were not taught trigonometric functions using GeoGebra before. The same response emerged during one-on-one interviews. This indicates the underutilisation of GeoGebra resources by mathematics educators. Research to establish why educators are not using GeoGebra in topics like trigonometric functions is needed. Perhaps TPCK is required for pre- and in-service educators. According to National Educational Technology Standards for Teachers, educators should identify, locate and evaluate technology resources in their environment for the purposes of integrating it in teaching and learning of subject matter (Mills, 2006).

4.5.2 Summary remarks

The focus-group interviews (discussions) enabled learners to squeeze out their experiences of learning trigonometric functions using GeoGebra and rich data were generated. The discussions validated the research and its findings. From the interviewees it is clear that their understanding was enhanced by the use of GeoGebra that helped them to master basic properties of sine, cosine and tangent graphs. This in turn resulted in interviewees being able to easily conceptualise the effects of parameters on the functions. There is also evidence that GeoGebra enabled learners to draw or sketch graphs without using table of values from calculators, as indicated in their sketches in the worksheets and Trigonometric test solutions. Working together during worksheet activities benefited the learners in understanding trigonometric functions from one another.

4.6 CONCLUSION

Data were analysed, and findings discussed and interpreted to answer the research primary question of the study:

How does the use of GeoGebra enhance Grade 11 learners' understanding of trigonometric functions?

This was facilitated by answering the three sub-questions:

(i) How do GeoGebra environments help learners in understanding trigonometric functions?

(ii) How is learners' understanding of trigonometric functions after interaction with GeoGebra?

(iii) What are learners' experiences and views on the use of GeoGebra in exploring trigonometric functions?

The chapter explored the power of GeoGebra in learners' understanding of sine, cosine and tangent functions. The researcher found that GeoGebra enhanced the learners' understanding of trigonometric functions by allowing the learners to vary the values of parameters instantaneously. Written work is not the only way to evaluate learners' understanding, instead more than a single way enriches the process. The researcher employed triangulation of worksheets, tests, and one-on-one interviews to strengthen the findings of the study. Focus-group interviews nourished the study by producing rich data for analysis from the learners' experiences. Implications and benefits of using GeoGebra were enriched by learners during focus-group discussions. Findings discussed in this chapter have shown how the use of GeoGebra enhanced Grade 11 learners' understanding of trigonometric functions. The following chapter deals with the summary, conclusions and recommendations of the study.

CHAPTER 5

SUMMARY, DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

This chapter provides a summary of preceding chapters, discussion of the major findings of the study and conclusions, the limitations and delimitations of the study, the implications of findings and recommendations for further research.

5.2 SUMMARY OF CHAPTERS

Chapter 1 gave the general introduction of the study. The purpose of this study was to explore Grade 11 learners' understanding of trigonometric functions using GeoGebra software. The chapter discussed the reasons that motivated the study. The study was a response to learners' struggles in trigonometric functions as reported by DBE Diagnostic reports 2011 to 2018, whilst GeoGebra software is available in smartboards supplied to Tshwane South District secondary schools. The scarcity of literature in the learning of trigonometry and trigonometric functions also provoked further the pursuance of this study. In addition, the main research question and three sub-questions, methodological considerations and significance of the study and assumptions were also provided in this chapter. The chapter ended by presenting definition of key terms which are functions, GeoGebra, parameter, technology/digital media/ICTs, technology integration.

Chapter 2 reviewed related literature that defined the strategic position the study occupied within the broad conceptual map of the body of knowledge on the use of GeoGebra to date and pointed out the frontiers or areas where the search of new knowledge concentrated. This was achieved through the following sub-headings: history of digital media in mathematics education; the importance of integrating technology in mathematics education; constraints in the integration of technology in mathematics education; the importance of trigonometry in the curriculum; the integration of technology in trigonometry and trigonometric functions; the smartboard; GeoGebra; and theoretical framework. Constructivist learning and understanding theories were compacted in this chapter to form the theoretical framework of the study. Constructivist learning insists that learners construct their own knowledge and the new knowledge is constructed on assumed or prior knowledge and experiences. Chart's (2017) theory of understanding is of the view that learners' understanding is possession of mental models which enable learners to simulate and explain subject matter. The two theories are in harmony and they were triangulated mainly to assist the researcher in planning and designing data collection, and in interpreting and understanding data and findings.

In Chapter 3 the methodology of the qualitative and exploratory case study was presented. The qualitative case study approach enabled the researcher to explore in depth Grade 11 learners' understanding of trigonometric functions using GeoGebra by making use of several sources of data. The philosophical underpinnings of the research methodology are interpretive (constructivist) in nature. The research setting was described, Grade 11 learners were sampled purposefully and six participated on voluntary basis. Piloting of research instruments was also discussed. The chapter went a step further to discuss and justify the data collection procedures, instruments and data analysis strategies employed in the study. Data was collected using a diagnostic test, six worksheets, a trigonometric functions test, one-on-one interviews and focus-group interviews. All instruments used in the study were piloted before the main study commenced. The content validity of the two tests was achieved through moderation and validation by two senior mathematics educators. All instruments were approved and authenticated by the researcher's supervisor and the UNISA College of Education Ethics review Committee. Trustworthiness of the study was discussed and the rigour was accomplished through techniques under credibility, dependability, confirmability and transferability. Ethical considerations were also discussed in the last section of the chapter.

Chapter 4 presented, analysed and discussed the results of data that were collected through worksheets, tests, one-on-one interviews and focus-group interviews in order to answer the main question through three sub-questions. Tests were analysed question by question and triangulated with one-on-one interviews. The analysis of worksheets was also done worksheet by worksheet and triangulated with one-on-one interviews. The transcribed data from the focus-group interviews were analysed by organising and categorising it in search of the emergence of patterns, critical themes and meanings.

5.3 MAJOR FINDINGS OF THE STUDY

The focus of this research was to explore learners' understanding of trigonometric functions using GeoGebra software. The main research question to be answered was:

How does the use of GeoGebra enhance Grade 11 learners' understanding of trigonometric functions?

The main question was answered by addressing the following three sub-questions:

(i) How do GeoGebra environments help learners in understanding trigonometric functions?
(ii) How is learners' understanding of trigonometric functions after interaction with GeoGebra?
(iii) What are learners' experiences and views on the use of GeoGebra in exploring trigonometric functions?

The results of the analysis of worksheets, tests, one-on-one interviews, and focus-group interviews done in Chapter 4, indicated major findings that answered the study's research question. In the following sub-sections, the researcher discussed and explained how the findings fitted with the research literature and theoretical framework that was discussed in Chapter 2.

5.3.1 Findings based on how GeoGebra environments help learners in understanding trigonometric functions

The findings emanating from the analysis of worksheet activities and one-on-one interviews on worksheets (see Chapter 4 Section 4.3) revealed that the use of GeoGebra enabled learners to: describe behaviour of and relationships between graphs; make generalisations in parameters *a* and *q*; interpret the algebraic and graphical forms of trigonometric functions; sketch graphs by hand without use of table of values. The learners were able to generalise the vertical translation of parameter *q* in Worksheets 1 and 3 (see Chapter 4 Section 4.3.1). A possible explanation for this is that GeoGebra allowed the participants to project as many graphs as possible within a short time, while investigating a single parameter's effects. This corroborates Ellis' (2011) research findings that exposing learners to physical or visual representations of mathematical relationships (in this case projected colourful graphs like those in Figures 4.5 and 4.15) promotes generalisation processes in learners. A study by Geraniou, Mavrikis, Hoyles and Noss (2010) agrees with that of Ellis (2011) in that collaboration through group work (a social constructivist tenet) and use of technology (GeoGebra) as a learning tool or use of technology in constructing knowledge (a cognitive constructivist tenet) create intelligent exploratory environments that support and shape learners' generalising activities. Answering the research question is the fact that GeoGebra enabled learners to relate numerical values of parameters, algebra and the drawn graphs which is a big step in understanding trigonometric functions. This coincides with Forster's (1999) study that found that technology fosters understanding in learners because it affords mathematics concepts to be pursued in visual, numeric and symbolic ways. The CAPS require learners to be able to flexibly switch between the algebraic and graphical representations of the functions.

The current study also found that understanding of trigonometric functions was a result of GeoGebra (a visual software) that afforded learners to vividly visualise the behaviour of graphs while the values of parameters were changed as shown in the learners' responses to Worksheet 2 (see Figures 4.5 to 4.8). This finding is portrayed by the phrases or words used by learners in their conclusion for parameter *a* in the tangent graph such as: 'the graph shifts away from the asymptote and is steeper', 'graph is moving closer to the asymptote and is steeper than the mother graph', '*a* compresses or stretches the graph'. It is evident that the attractive graphical displays on the smartboard engaged the learners' minds to build knowledge and understanding in this aspect of graphs, which accords with constructivist and understanding theories' tenets. In accordance with the present results, scholars like Malabar and Pountney (2002); Nghifimule and Schafer (2018); Summit and Rickards (2013); Caligaris et al (2015); McLoughlin and Loch (2013) have demonstrated that software environments, tools and objects (like that offered by GeoGebra) are visually stimulating, promote rich constructions, conceptual understanding and understanding of higher order concepts by learners. In addition, the outcome on worksheet activities done in small groups concur with Caligaris et al (2015) observations that learners discover concepts better when they are watching and doing, interacting and engaging in visual arguments. These findings are broadly in harmony with those of researchers such as Naidoo and Govender (2014) (see Chapter 2 Section 2.6).

Another important finding was that the learners' interactions with peers and GeoGebra enabled the learners to sketch, by hand, graphs involving two parameters *a* and *q*, *a* and *p*, *k* and *q* (see Figure 4.12, Figure 4.13(b) and Worksheet 6 in Chapter 4 Section 4.3.1). The ability to sketch trigonometric graphs on paper is a very important skill in the FET phase. It seems possible that this result was due to the scaffolding brought by the technology-rich and collaborative environments. In fact, the designing (by the researcher) of worksheet activities done in groups created a confluence of collaboration learning and scaffolding which are both aligned to the social constructivist perspective (Van de Pol, Mercer and Volman, 2019; Lombardi, 2017; Bakker et al, 2015). The mingling in such constructivist environments allows learners to develop skills to construct their knowledge and understanding from what *is* known to what is to *be* known (which is the zone of proximal development hand-in glove with scaffolding), through the help of the software and peers (Murphy, 1997). This outcome bears some resemblance to Dragon, McLaren, Mavrikis and Geraniou's (2011) research reports that collaboration (during group work activities) backed by technology fosters productive dialogue and arguments while exploring troublesome concepts, resulting in learners manipulating abstract ideas. Also, Maroske's (2015) study reported that learners scaffold one another's understanding during collaboration in group activities through listening, questioning, explaining, evaluating one another's responses and building on one another's ideas. Furthermore, separate studies by Wu et al (2002) and Bakker et al (2015) agree that technology integrated collaborative environments promote conceptual understanding through digital and peer scaffolds. During focus-group interviews learners confirmed this by revealing that they previously used table values to draw graphs but were now able to draw by free hand after interacting with GeoGebra (see Chapter 4 Section 4.5.1 Excerpt 9). The finding suggest that active dialogue, integrating experiences, sharing of knowledge and understanding, and interacting with GeoGebra during worksheet activities afforded the learners a new technique of freely drawing graph sketches (Pagan, 2006; Faulkenberry and

Faulkenberry, 2006). The current study therefore underlines the fact that environments characterised by collaboration, visualisation, scaffolding and GeoGebra enhance learners' understanding of trigonometric functions because it enabled learners to transfer what they had learnt using software to sketches on paper.

5.3.2 Findings based on learners' understanding of trigonometric functions after interacting with GeoGebra

This section discusses findings from the analysis of the trigonometric functions test and one-on-one interviews on the test. The test evaluated learners' individual understanding of trigonometric functions that was gained during worksheet activities that were informed by constructivist and understanding theories. The findings showed that the six learners who were of mixed ability, scored an average of 45% in the test with a minimum of 19% and a highest of 88% (see Chapter 4 Section 4.4.1 Tables 4.1 to 4.4). This indicates that the participants' interaction with GeoGebra during worksheet activities enhanced their understanding of trigonometric functions. The findings observed in this study mirror those of previous studies (by Naidoo and Govender, 2014; Kepceoglu and Yavuz, 2016; Bakar, Ayub, Luan and Tarmizi, 2010; Thambi and Eu, 2013; Trung, 2014; Arbain and Nurbiha, 2015; Yildir et al, 2017) that have examined the use of GeoGebra in the teaching and learning of trigonometric functions and other mathematics topics and yielded positive results.

The quality of sketches in the trigonometric functions test revealed some improvement from the diagnostic test and this is shown in the comparison of Figures 4.1 to 4.3 with Figures 4.19, 4.20, 4.21, 4.22, and 4.24. This is also exemplified by comparing L7's sketches in Figures 4.4 a to c with Figures 4.20 a to c. These findings are in line with research done by Ng and Hu (2006) in Singapore. Their research used web-based simulation as the technology and the results showed an improvement by learners in sketching trigonometric graphs. The CAPS examiners require learners to be skilled in sketching graphs showing relevant shapes, amplitudes and periods. Educators could achieve this by allowing learners to use GeoGebra that engages learners' hands and minds in learning trigonometric functions, as prescribed by the constructivist perspective. The participants in the current research have revealed that such an experience enabled them to understand the properties of trigonometric functions which in turn eases free sketching (see Chapter 4 Section 4.5.1 Excerpt 8 and 9).

The other finding was that of the learners' capability to interpret the algebraic and graphical forms of trigonometric functions as shown by the learners' responses to Questions 2 and 3 (see Chapter 4 Section 4.4.1) and one-on-one interviews (see Chapter 4 Section 4.4.2). In this aspect, learners were able to describe and state relationships between basic graphs and new graphs generated by parameters. This is an important area that learners should be versatile in as recommended by the DBE diagnostic report (2012 and 2014). The researcher believes that the use of GeoGebra in projecting the 'mother' graph with its multiple and different coloured 'children' instantaneously in the same Cartesian plane was the basis of learners' understanding of this aspect. Such a finding accords with social constructivist learning and understanding theories where collaborative visualisation occurred during worksheet activities. The learners worked together in explaining to one another what they had drawn and seeing on the smartboard. This is supported by Hoover (1996) who sees group interaction, from a constructivist perspective, as enhancing individuals' understanding by comparing it with that of other members. The set up afforded the Grade 11 learners to compare the characteristics of basic graphs and the corresponding transformations easily in a GeoGebra environment (through discussions as remarked by interviewees in Chapter 4 Section 4.5.1 Excerpt 10), which is difficult in a chalkboard situation. Again, from Chart (2017) learners built mind models (during interaction with GeoGebra) that helped them in explaining the relationship between the graphs.

Learners experienced difficulties in Question 4 where they were required to determine the equations of already drawn graphs (see Chapter 4 Sections 4.4.1 and 4.4.2). These findings are consistent with Kepceoglu and Yavuz's (2016) research findings. Kepceoglu and Yavuz (2016) argue that the dominance of algebraic representation in mathematics teaching cause the difficulties in this aspect. Educators are therefore encouraged to use GeoGebra that promotes visual form of learning trigonometric functions than the algebraic one. The researcher holds that this aspect is of a higher order and should be given more time in respect to researching the phenomenon as prescribed by constructivist works (Adom et al, 2016).

5.3.4 Findings based on learners' experiences and views on the use of GeoGebra in exploring trigonometric functions

The focus-group discussions revealed that the use of GeoGebra in learning trigonometric functions was a good experience and it excited and motivated learners (see Chapter 4 Section 4.5.1 and Excerpt 8). This finding confirms Kalender's (2007, cited in Adom et al, 2016) remark that the constructivist perspective values motivation as a necessary ingredient to learning. Pfeiffer's (2017) study also reports that the use of GeoGebra in learning trigonometric functions by a group of learners motivated them because there was articulation of ideas amongst the learners. It therefore follows that the motivated Grade 11 learners' understanding of trigonometric functions was enhanced because they enjoyed exploring concepts using GeoGebra.

Grade 11 learners reported that the use of GeoGebra helped them to understand properties of basic trigonometric functions and in turn the effects of parameters. One learner also reported the capability of identifying the difference between sine and cosine graphs. This finding agrees with Pfeiffer's (2017) argument that the use of GeoGebra in exploring the features of trigonometric functions enhances learners' understanding of the problematic topic. Scholars like Petre (2010, cited in Pfeifer, 2017) insist that this use of GeoGebra can only thrive when amalgamated with learner-centred environments (like the worksheet activities in the current study) that were informed by constructivist and understanding theories. From a social constructivist point of view Grade 11 learners used GeoGebra as a cognitive tool to visualise the effects of parameters (Demir, 2012). According to the theory of understanding, learners used GeoGebra to easily construct mind models.

Responses from learners during focus-group discussions revealed that the use of GeoGebra moved them from drawing graphs by plotting point-by-point to sketching smooth curves from abstraction (see Chapter 4 Section 4.5.1 and Excerpt 9). From the Theory of understanding's perspective the learners' interaction with GeoGebra enable them to simulate graphs. DBE examiners report that this level of sketching is a reflection that learners have a deeper understanding of trigonometric functions (Diagnostic report, 2011). This research has shown that the use of GeoGebra under the influence of constructivist learning approaches enhances learners' understanding of trigonometric functions. Learners will do better in this topic during examinations if educators use this approach.

Analysis of focus-group discussions also revealed that Grade 11 learners wanted to access GeoGebra conveniently both at school and at home. The learners suggested that the software be installed in their tablets. Unfortunately, the GDE is unable to supply enough tablets to all learners every year and the supplies are erratic most of times. Again, the policy of collecting all the tablets again at the end of each year may disadvantage learners who would have installed GeoGebra, since different batches are bought in the year that follows. Tablets are only supplied to non-fee paying schools while fee paying schools cannot afford them and do not see them as a priority. This situation in GDE schools leaves the understanding of trigonometric functions using GeoGebra at stake.

Interviewees indicated that they valued and benefited from collaborative or group work during interaction with GeoGebra. Learners elaborated how they shared duties and sought for assistance in trigonometric functions matters during worksheet activities (see Chapter 4 Section 4.5.1 and Excerpt 10). This finding resonates with findings in Section 5.3.1. Again, this finding corroborates Becta's (2003, cited in Pfeiffer 2017) notion that learner interaction in technology savvy environments promotes exchange of information and content knowledge. The GeoGebra environments used in this research exploited social constructivist learning opportunities where Grade 11 learners constructed their understanding and knowledge from interacting with their peers (or surrounding). Such a situation allows learners to learn from their peers, to be scaffolded by their peers in a zone of proximal development (Wu et al, 2002).

5.4 LIMITATIONS AND DELIMITATIONS OF THE STUDY

The findings of this qualitative case study are those of the school concerned and cannot be generalised to other schools in Tshwane District and the whole country. However, other academics may find benefit in applying and replicating this research in other schools with similar contexts. The Grade 11 class in the school had 39 learners but only 6 maintained participation in this study. The reasons were: participation was voluntary; most learners use scholar transport that left soon after the last lesson at 14:15 whilst the data collection session started at 14:30; there were learners who also cited business in other subjects' School Based Assessment tasks. Notwithstanding the limited sample, this research offers valuable insights into the power of using GeoGebra in learning of trigonometric functions.

The CAPS document prescribes the studying of real-life applications of trigonometric functions. However, the researcher could not cover this area due to the school's limited time schedules and the quest to narrow the scope of the research. The exploration concentrated only on the varying of parameter values.

The researcher was unable to capture data from learners' discussions during group work on worksheets. This was due to the lack of a microphone that could have been connected to the smart recorder. In spite of its limitations, the study certainly provides remedies to the challenges existing in the teaching and learning of trigonometric functions.

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5.5 IMPLICATIONS OF THE FINDINGS

The main aim in this research was to explore Grade 11 learners' understanding of trigonometric functions using GeoGebra. The researcher has done this through worksheets, trigonometric tests, one-on-one interviews and focus-group interviews. This section discusses how this research might benefit or be used by professionals in the field of mathematics education, as well as those in related disciplines.

The present study offers suggestive empirical evidence that may prompt mathematics educators and learners to exploit the potentialities of GeoGebra in the teaching and learning of trigonometric functions. The results of this study indicate that the use of GeoGebra, under the guidance of constructivist and understanding theories, enhances learners' understanding of trigonometric functions.

This research is timely after DBE's white paper on ICT integration and GDE's injection of smartboards in non-fee paying schools, while learners are not doing well in trigonometric functions (Diagnostic reports 2011, 2012, 2013, 2014, 2015, 2016, 2017 and 2018). On the face of this, perhaps the GDE may consider installing GeoGebra in laptops and tablets that they supply to educators and learners respectively. This will enable educators to plan and prepare trigonometric lessons, and learners to use GeoGebra to do their homework and investigations outside the classroom.

The findings of the study appear to support the need for all universities to incorporate the use of GeoGebra in the teaching and learning of trigonometric functions in their pre-service mathematics courses. The study also provides indications to mathematics District Subject Advisors on the need to develop in-service mathematics educators in the desirable and necessary skills of using GeoGebra in teaching and learning of trigonometric functions (see Section 2.4). Shelly et al (2008:16) note that “Marc Prensky stated in 2001 that digital kids are the digital natives and teachers are the digital immigrants.” Educators should be more conversant with technology than their learners.

5.6 RECOMMENDATIONS FOR FURTHER RESEARCH

- The current study considered the use of GeoGebra with only Grade 11 learners. A prolonged research that could track participants from Grade 10 to 12 in the learning of trigonometric functions using GeoGebra would be valuable in order to validate and support this and related research.
- A similar study could be carried out involving more than one school (more than one case) so that data can be analysed across cases.
- Future research could be done in different set ups, locations and contexts. The study should be repeated with all participant learners having tablets (installed with GeoGebra) so that homework could be assigned to learners.
- One avenue for further study would be research into learners’ determination of graphical equations of already drawn trigonometric graphs. The learners in the current study did not do well in Question 4 of the trigonometric test and Kepceoglu and Yavuz (2016) had similar findings. GeoGebra should be used to explore this area.
- The smart recorder is a very powerful tool that should be embraced in the field of research. The researcher attended the Mathematics strategy launch

Conference in 2018 where one of the delegates suggested that learners should be allowed to use technology to complete tasks and mathematics practitioners should come up with ways of marking such work. The smart recorder could be used by mathematics researchers to collect rich data. The current research only recorded group work activities without capturing participants' discussions during group work like that of Demir (2012) using a simple voice recorder. Demir (2012) analysed the data collected from group discussions. Further research in the use of GeoGebra for trigonometric functions could also be conducted with a microphone mounted to the smartboard to record the researcher and learners' voices.

- The current research is a qualitative case study. Research that is both qualitative and quantitative could establish more interesting findings in learning trigonometric functions using GeoGebra.
- Smartboards and tablets are already in schools. The current research found out that learners are not being taught trigonometric functions using GeoGebra that is available in these smartboards (see Section 4.5.1). Furthermore, the researcher observed, during school visits, that tablets are not always enough for Grade 10 to 12 learners. During some years, supplies are not even delivered to schools as promised by GDE. A further study could involve learners, mathematics educators, mathematics district subject advisors across the districts, teacher development units in districts, the ICT units in districts, and universities' mathematics education departments responsible for training mathematics educators and investigate: At what level are all these entities or units ready to integrate ICT resources supplied by GDE, in particular use GeoGebra in the teaching and learning of trigonometric functions? Considerably more work will need to be done to determine the status of resources (hardware, software, connectivity, tablets for learners, curriculum implementers with the know how in integration) in terms of quantity and compatibility in schools and the whole education sector to fully integrate mathematics topics like trigonometric functions.

5.7 CONCLUSION

The purpose of the current research was to explore Grade 11 learners' understanding of trigonometric functions using GeoGebra software. Data were collected through worksheets, a smart recorder, a trigonometric test, one-on-one interviews and focus-group interviews in order to answer the research question. The findings revealed that use of GeoGebra enhances learners' understanding of trigonometric functions and has profound implications and benefits to learners, educators, and all stakeholders in the mathematics education fraternity. The use of GeoGebra will foster in learners, the skill of thinking, investigation, understanding and explanation.

"We need to embrace technology to make learning more engaging. Because when learners are engaged and they are interested, that's where learning takes place" (Anonymous, 20 Popular Technology in Education quotes).

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LIST OF APPENDICES

APPENDIX A

ETHICS APPROVAL CERTIFICATE

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2018/04/18

Ref: 2018/04/18/36469262/12/MC
Name: Mr LS Makandidze
Student: 36469262

Dear Mr Makandidze

Decision: Ethics Approval from 2018/04/18 to 2021/04/18

Researcher(s): Name: Mr LS Makandidze
E-mail address: lancymakandidze@yahoo.co.uk
Telephone: +27 63 331 5506

Supervisor(s): Name: Prof MF Machaba
E-mail address: emachamf@unisa.ac.za
Telephone: +27 72 359 5509

Title of research:

Exploring learners' understanding of trigonometric functions using GeoGebra software: A case of grade 11 Mathematics learners at a school in Tshwane South District.

Qualification: M Ed in Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2018/04/18 to 2021/04/18.

*The **Medium risk** application was reviewed by the Ethics Review Committee on 2018/04/18 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane

1. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.
2. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
3. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
4. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
5. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
6. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
7. No field work activities may continue after the expiry date **2021/04/18**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2018/04/18/36469262/12/MC** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

M Claassens

Dr M Claassens
CHAIRPERSON: CEDU RERC
mcdtc@netactive.co.za

V McKay

Prof V McKay
EXECUTIVE DEAN
Mckayvi@unisa.ac.za

Approved - decision template – updated 16 Feb 2017

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APPENDIX B

REQUESTING PERMISSION OF THE GAUTENG DEPARTMENT OF EDUCATION

UNISA college of education

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

For admin. use only:
Ref. no.:

GDE RESEARCH REQUEST FORM

REQUEST TO CONDUCT RESEARCH IN INSTITUTIONS AND/OR OFFICES OF THE GAUTENG DEPARTMENT OF EDUCATION

1. PARTICULARS OF THE RESEARCHER

1.1	**Details of the Researcher**
Surname and Initials:	Makandidze L.S.
First Name/s:	Lancelot Sibanengi
Title (Prof / Dr / Mr / Mrs / Ms):	Mr
Student Number (if relevant):	36469262
SA ID Number:	7403186113188
Work permit no. (If not SA citizen)	

1.2	**Private Contact Details**

Home Address	***Postal Address (if different)***
32 Fonteinhoek	
533 Jasmyn Avenue	
Silverton	
Postal Code: 0184	***Postal Code:***
Tel:	***Cell: 072 3780 419***
Fax:	***E-mail: lancymakandidze@yahoo.co.uk***

2. PURPOSE & DETAILS OF THE PROPOSED RESEARCH

2.1	**Purpose of the Research (Place cross where appropriate)**
Undergraduate Study - Self	
Postgraduate Study - Self	x
Private Company/Agency – Commissioned by Provincial Government or Department	
Private Research by Independent Researcher	
Non-Governmental Organisation	
National Department of Education	
Commissions and Committees	
Independent Research Agencies	
Statutory Research Agencies	
Higher Education Institutions only	

2.2	**Full title of Thesis / Dissertation / Research Project**
Exploring learners' understanding of trigonometric functions using GeoGebra Software : A case of Grade 11 Mathematics learners at a school in Tshwane South District	
2.3	**Value of the Research to Education (Attach Research Proposal)**

The research seeks to explore ways in which technology integration can help
Improve the performance of learners in trigonometric functions.

2.4	**Date**
***Envisaged date of* completion of research in GDE Institutions**	**30/06/18**
***Envisaged date of* submission of Research Report and Research Summary to GDE:**	**30/01/19**

2.5	**Student and Postgraduate Enrolment Particulars**
Name of institution where enrolled:	UNISA
Degree / Qualification:	Master of Education
Faculty and Discipline / Area of Study:	Mathematics Education
Name of Supervisor / Promoter:	Prof M.F. Machaba

2.6	**Employer**
Name of Organisation:	Gauteng Department of Education – Tshwane South District D4
Position in Organisation:	District Subject Advisor [FET – Mathematics]
Head of Organisation:	Mrs H. Kekana
Street Address:	President Towers 265 Pretorius Street Pretoria
Postal Code:	0001
Telephone Number (Code + Ext):	(012) 401 6300
Fax Number:	(012) 4016318
E-mail:	Hilda.Kekana@gauteng.gov.za

2.7	**PERSAL Number (GDE employees only)**

2	2	8	2	3	2	4	7

3. PROPOSED RESEARCH METHOD/S

(Please indicate by placing a cross in the appropriate block whether the following modes would be adopted)

3.1 ***Questionnaire/s (If Yes, supply copies of each to be used)***

YES		**NO**	**x**

3.2 ***Interview/s (If Yes, provide copies of each schedule)***

YES	**x**	**NO**	

3.3 ***Use of official documents***

YES	**x**	**NO**	
If Yes, please specify the document/s: Policies and Circulars on ICT use in the Department of Basic Education; lesson plans			

3.4 ***Workshop/s / Group Discussions (If Yes, Supply details)***

YES		**NO**	**x**

3.5 ***Standardized Tests (e.g. Psychometric Tests)***

YES		**NO**	**x**
If Yes, please specify the test/s to be used and provide a copy/ies			

4. INSTITUTIONS TO BE INVOLVED IN THE RESEARCH

4.1 *Type and NUMBER of Institutions (Please indicate by placing a cross alongside all types of institutions to be researched)*

INSTITUTIONS	Write NUMBER here
Primary Schools	
Secondary Schools x	01
ABET Centres	
ECD Sites	
LSEN Schools	
Further Education & Training Institutions	
Districts and / or Head Office	

4.2 Name/s of institutions to be researched (Please complete on a separate sheet if space is found to be insufficient)

Name/s of Institution/s
Nellimapius Secondary School

4.3 **District/s where the study is to be conducted**. ***(Please indicate by placing a cross alongside the relevant district/s)***

District/s			
Ekurhuleni North		***Ekurhuleni South***	
Gauteng East		***Gauteng North***	
Gauteng West		***Johannesburg Central***	
Johannesburg East		***Johannesburg North***	
Johannesburg South		***Johannesburg West***	
Sedibeng East		***Sedibeng West***	
Tshwane North		***Tshwane South***	x
Tshwane West			

If Head Office/s (Please indicate Directorate/s)

4.4 Number of learners to be involved per school (Please indicate the number by gender)

Grade	***1***		***2***		***3***		***4***		***5***		***6***	
Gender	**B**	**G**	**B**	**G**	**B**	**G**	**B**	**G**	**B**	**G**	**B**	**G**
Number												

Grade	***7***		***8***		***9***		***10***		***11***		***12***	
Gender	**B**	**G**	**B**	**G**	**B**	**G**	**B**	**G**	**B**	**G**	**B**	**G**
Number									**16**	**14**		

4.5 Number of educators/officials involved in the study (Please indicate the

number in the relevant column)

Type of staff	***Educators***	***HODs***	***Deputy Principals***	***Principal***	***Lecturers***	***Office Based Officials***
Number						

4.6 Are the participants to be involved in groups or individually?

Groups		***Individually***	**x**

4.7 Average period of time each participant will be involved in the test or other research activities (Please indicate time in minutes)

Participant/s	**Activity**	**Time**
3 to 6 learners	Interview	5-10
Class of 30 learners	Learning	6 periods (6x45 = 270)

4.8 Time of day that you propose to conduct your research.

During school hours (for limited observation only)	**x**	***After School Hours***	***x***

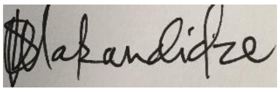
4.9 School term/s during which the research would be undertaken

First Term		***Second Term***	***x***	***Third Term***	

CONDITIONS FOR CONDUCTING RESEARCH IN GDE

Permission <u>may be granted</u> to proceed with the above study subject to the conditions listed below being met and permission may be withdrawn should any of these conditions be flouted:

1. *The District/Head Office Senior Manager/s concerned, the Principal/s and the chairperson/s of the School Governing Body (SGB.) must be presented with a copy of this letter.*
2. *The Researcher will make every effort to obtain the goodwill and co-operation of the GDE District officials, principals, SGBs, teachers, parents and learners involved. Participation <u>is voluntary</u> and additional remuneration will not be paid;*
3. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal and/or Director must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
4. *Research may only commence from the second week of February and must be concluded by the end of the THIRD quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
5. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
6. *It is the researcher's responsibility to obtain written consent from the SGB/s; principal/s, educator/s, parents and learners, as applicable, before commencing with research.*
7. *The researcher is responsible for supplying and utilizing his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institution/s, staff and/or the office/s visited for supplying such resources.*
8. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study <u>may not appear</u> in the research title, report or summary.*
9. *On completion of the study the researcher <u>must</u> supply the Director: Education Research and Knowledge Management, with electronic copies of the Research Report, Thesis, Dissertation as well as a Research Summary (on the GDE Summary template).*
10. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned;*
11. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director/s and school/s concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

<table>
<tr><th colspan="2">DECLARATION BY THE RESEARCHER</th></tr>
<tr><td colspan="2">1. I declare that all statements made by myself in this application are true and accurate.</td></tr>
<tr><td colspan="2">2. I accept the conditions associated with the granting of approval to conduct research and undertake to abide by them.</td></tr>
<tr><td>Signature:</td><td></td></tr>
<tr><td>Date:</td><td>14 March 2018</td></tr>
<tr><th colspan="2">DECLARATION BY SUPERVISOR / PROMOTER / LECTURER</th></tr>
<tr><td colspan="2">I declare that: (Name of <u>Researcher</u>) Machaba Masilo France</td></tr>
<tr><td colspan="2">1. is enrolled at the institution / employed by the organisation to which the undersigned is attached.</td></tr>
<tr><td colspan="2">2. The questionnaires / structured interviews / tests meet the criteria of:
• Educational Accountability;
• Proper Research Design;
• Sensitivity towards Participants;
• Correct Content and Terminology;
• Acceptable Grammar;
• Absence of Non-essential / Superfluous items;
• Ethical clearance</td></tr>
<tr><td colspan="2">3. I will ensure that after successful completion of the degree / project an electronic copy of the Research Report / Thesis / Dissertation and a Research Summary (on the GDE template) will be sent by the researcher to the GDE.</td></tr>
<tr><td>Surname:</td><td>Machaba</td></tr>
<tr><td>First Name/s:</td><td>Masilo France</td></tr>
<tr><td>Institution / Organisation:</td><td>UNISA</td></tr>
<tr><td>Faculty / Department (where relevant):</td><td>Mathematics Education</td></tr>
</table>

Telephone:	**012 429 8582**
E-mail:	**emachamf@unisa.ac.za**
Signature:	
Date:	**14 March 2018**

ANNEXURE A: **ADDITIONAL INFORMATION FOR GROUP RESEARCH**

This information must be completed by **every** researcher/ student who will be visiting GDE Institutions for research purposes.

By signing this declaration, the researcher / students accepts the conditions associated with the granting of approval to conduct research in GDE Institutions and undertakes to abide by them.

Supervisor/ Promoter / Lecturer's Surname and Name Machaba Masilo France

DECLARATION BY RESEARCHERS / STUDENTS:

Surname & Initials	Name	Tel	Cell	Email address	Signature

N.B. This form (and all other relevant documentation where available) may be completed and forwarded electronically to Gumani.mukatuni@gauteng.gov.za and please copy (cc) ResearchInfo@gauteng.gov.za. The last 2 pages of this document must however have the original signatures of both the researcher and his/her supervisor or promoter. It should be scanned and emailed, posted or hand delivered (in a sealed envelope) to Gumani Mukatuni, 7th Floor, 6 Hollard Building, Main and Simmonds Streets, Johannesburg. All enquiries pertaining to the status of research requests can be directed to Gumani Mukatuni on tel. no. 011 355 0775.

APPENDIX C

REQUESTING PERMISSION OF THE DISTRICT DIRECTOR

32 Fonteinhoek

533 Jasmyn Avenue

Silverton 0184

CELL 0723780419

23 March 2018

THE DISTRICT DIRECTOR

GAUTENG DEPARTMENT OF EDUCATION

TSHWANE SOUTH DISTRICT OFFICE

265 Pretorious Street

President Towers

Pretoria 0002

TELL:(012)4016317

Dear Sir/ Madam

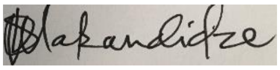
REF: PERMISSION TO CONDUCT RESEARCH IN TSHWANE SOUTH DISTRICT

I, Lancelot Sibanengi Makandidze am doing a research under the supervision of Dr M.F. Machaba a senior lecturer in the Department of Mathematics Education towards an M.E.D at the University of South Africa. We (my supervisor and I) are cordially inviting grade 11 Mathematics learners in your district to participate in a study entitled, "Explaining learners' understanding of trigonometric functions using GeoGebra software: A case of grade 11 Mathematics Learners in Tshwane South District.

The main aim of the study is to explore grade 11 learners' understanding of trigonometric functions when taught using Geogebra software in the smartboard. This study will employ qualitative research method. It will use case study design in particular to explore the main research question: 'How does the integration of teaching specifically Geogebra enhance learners' understanding of trigonometric functions? We will be studying a class of grade 11 Mathematics learners through an investigative task, direct observation and face-to-face unstructured interviewing in the comfort of their school environment. This study seeks to facilitate an improvement in grade 11 learners' understanding of trigonometric functions and thereby improve their performance in Mathematics. The study will motivate other teachers to use smartboards in teaching trigonometric and other topics. There are no potential risks involved. There will be no re-imbursement or any incentives for participating in the research.

If you would like to be informed of the final findings, kindly contact Makandidze L.S. on 0723780419 or email at lancymakandidze@yahoo.co.uk. Should you have any concerns about the way in which the research has been conducted, you might contact Dr. M.F. Machaba on 0124298582 or email emachamf@unisa.ac.za.

Yours faithfully



Makandidze L.S (Mr)

APPENDIX D

REQUESTING PERMISSION OF THE PRINCIPAL

UNISA | college of education

Title of the research: 'Exploring learner's understanding of trigonometric functions using GeoGebra software: a case of grade 11 Mathematics learners at a school in Tshwane South District.'

Date ____________________

The Principal

____

____

Dear Sir/Madam,

REF: REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT YOUR SCHOOL

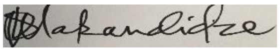
I, Lancelot Sibanengi Makandidze am doing a research under the supervision of Dr M.F. Machaba a senior lecturer in the Department of Mathematics Education towards an M.E.D at the University of South Africa. We (my supervisor and I) are cordially inviting grade 11 Mathematics learners in your school to participate in a study entitled, "Explaining learners' understanding of trigonometric functions using Geogebra software: A case of grade 11 Mathematics Learners at a school in Tshwane South District.

The main aim of the study is to explore grade 11 learners' understanding of trigonometric functions when taught using Geogebra software in the smartboard. This study will mixed research method. It will use case study design in particular to explore the main research question: 'How does the integration of teaching specifically Geogebra enhance learners' understanding of trigonometric functions? We will be studying a class of grade 11 Mathematics learners through pre and post-test and face-to-face unstructured interviewing in the comfort of their school environment. This study seeks to facilitate an improvement in grade 11 learners' understanding of trigonometric functions and thereby improve their performance in Mathematics. The study will motivate other teachers to use smartboards in teaching trigonometric and other topics. There are no potential risks involved. There will be no re-imbursement or any incentives for participating in the research.

The study will be conducted during school hours for not more than 8 days. The interviews will be 5-10 minutes for each of the 3-6 learners who will be interviewed. The learners to be interviewed will be selected according to the answers on their scripts.

If you would like to be informed of the final findings, kindly contact Makandidze L.S. on 0723780419 or email at lancymakandidze@yahoo.co.uk. Should you have any concerns about the way in which the research has been conducted, you might contact Dr. M.F. Machaba on 0124298582 or email emachamf@unisa.ac.za.

Yours faithfully



Makandidze L.S (Mr)

APPENDIX E

INFORMATION LETTER TO REQUEST ASSENT FROM THE LEARNERS

Date ___________________

Dear ____________________,

I, Lancelot Sibanengi Makandidze (the researcher) am doing research under the supervision of Dr M. F. Machaba, senior lecturer in the Department of Mathematics Education towards an MED at the University of South Africa. You are cordially invited to participate in a research study entitled 'Exploring learners' understanding of trigonometric functions using Geogrbra software: A case of Grade 11 Mathematics learners in Tshwane South District.' It is important for you to fully understand what is entailed on the research to enable you to make an informed decision whether to participate or not. If you have any queries regarding the research study after reading this form please do not hesitate to consult me or my supervisor on the contact details given in paragraph six.

The aim of the study is to explore grade 11's understanding of trigonometric functions when taught using Geogebra software in the smartboard. This study will mixed research method. It will use the case study design in particular to explore the main research question: 'How does the integration of technology specifically Geogebra, enhance learners' understanding of trigonometric functions.

The researcher would like to collect data from you through pre and post test and face-to-face unstructured interviewing at your school. Your participation in this study is voluntary. The interview will take approximately 5-10 minutes in length. You may decline to answer any of the interview questions if you so wish. Furthermore, you may decide to withdraw from this study at any time without any negative consequences.

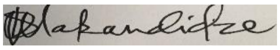
Audio-recording will enable the researcher to capture every bit of information that you will have volunteered to give for the purpose of analysis and verification. Please be advised that this exercise is voluntary, you may decline to answer any of the interview questions if you so wish. Shortly after the transcription has been completed, I will send you a copy of the transcript to give you an opportunity to confirm the accuracy of our conversation to add on to clarify any points.

The study will be conducted during school hours for not more than 8 days. The interviews will be 5-10 minutes for each of the 3-6 learners who will be interviewed. The learners to be interviewed will be selected according to the answers on their scripts.

All information you provide is considered completely confidential. Your name will not appear in any publication resulting from this study and any identifying information will be omitted from the report. However, with your permission, anonymous codes may be used. Data collected during this study will be kept in a filing cabinet under lock and key for 5 years. There are no known or anticipated risks to you as a participant in this study. You will not be reimbursed or receive any incentives for your participation in the research. If you would like to be informed of the final research findings, please contact Makandidze L. S. on 0723780419 or email lancymakandidze@yahoo.co.uk. Should you have concerns about the way in which the research has been conducted, you may contact Dr. M. F. Machaba on 0124298582 or email emachamf@unisa.ac.za.

Thank you for taking time to read this information sheet and for participating in this study.

Thank you



Makandidze L.S. (Mr)

APPENDIX F

ASSENT TO PARTICIPATE IN THIS STUDY (Return slip)

I, ___________________ (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty.

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I agree to the recording of the interview.

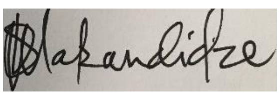
I have received a signed copy of the informed consent agreement.

Participant Name & Surname (please print) ______________________________________

_____________________________ ____________________________________

Participant Signature Date

Researcher's Name & Surname (please print) ______________________________________



Researcher's signature Date

APPENDIX G

INFORMATION LETTER TO REQUEST CONSENT FROM PARENTS (Return slip)

Dear Parent

Your …………………<son/daughter/child> is invited to participate in a study entitled "Exploring learners' understanding of trigonometric functions using GeoGebra software: A case of grade 11 Mathematics Learners in Tshwane South District" I am undertaking this study as part of my master's research at the University of South Africa. The purpose of the study is to explore the understanding by learners of trigonometric functions when taught using GeoGebra software. And the possible benefits of the study are the improvement of performance by learners in the topic. I am asking permission to include your child in this study because s/he is doing grade 11 mathematics. I expect to have 29 other children participating on the study.

If you allow your child to participate, I shall request him/her

- Take part in an interview at school after completing a pre-and post-test
- Complete a pre-and post-test in class before and after being taught trigonometry respectively

I am asking for permission to tape record your child during the interview. Any information that is obtained in connection with this study and can be identified with your child will remain confidential and will only be disclosed with your permission. His/her responses will not be linked to his/her name or your name or the school's name in any written or verbal report based on this study. Such a report will be used for research purposes only.

The study will be conducted during school hours for not more than 8 days. The interviews will be 5-10 minutes for each of the 3-6 learners who will be interviewed. The learners to be interviewed will be selected according to the answers on their scripts.

There are no foreseeable risks to your child by participating in the study. Your child will receive no direct benefit from participating in the study; however, the possible benefits to education are that the understanding of trigonometry will be made easier. Neither your child nor you will receive any type of payment for participating in this study.

Your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusal to participate will not affect him/her in any way. Similarly, you can agree to allow your child to be part of the study now and change your mind later without any penalty.

The study will take place during regular classroom activities with the prior approval of the school and your child's teacher. However, if you do not want your child to participate, an alternative activity will be available in the form of a revision exercise.

In addition to your permission, your child must agree to participate in the study and you and your child will also be asked to sign the assent form which accompanies this letter. If your child does not wish to participate in the study, he or she will not be included and there will be no penalty. The information gathered from the study and your child's participation in the study will be stored securely on a password locked computer in my office for five years after the study. Thereafter, records will be erased.

The benefits of this study are improved performance in trigonometric functions by learners. There are no potential risks in this study. There will be no reimbursed or any incentives for participation in the research

If you have questions about this study please ask me or my supervisor, Dr M.F. Machaba, Department of Mathematics Education, University of South Africa. My contact number is 0723780419 and my email is lancymakandidze@yahoo.co.uk. The email of my supervisor is emachamf@unisa.ac.za . Permission for the study has already been given by the Tshwane South Department of Education District Director, the Principal and SGB of the school and the Ethics committee of the College of Education, UNISA.

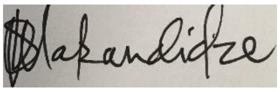
You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. You may keep a copy of this letter.

Name of child:

Sincerely

……………………………………………………….. ……………………………………….. ……………………..

Parent/Guardian's name (print) Parent/Guardian's signature Date



………………………………………………………… ……………………………………….. ………………………

Researcher's name (print) Researcher's signature Date

APPENDIX H

APPROVAL LETTER FROM GDE

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	27 March 2018
Validity of Research Approval:	05 February 2018 – 28 September 2018 2017/407
Name of Researcher:	Makandidze L.S.
Address of Researcher:	32 Fonteinhoek
	533 Jasmyn Avenue
	Silverton 0184
Telephone Number:	072 3780 419
Email address:	lancymakandidze@yahoo.co.uk
Research Topic:	Exploring learners' understanding of trigonometric functions using GeoGebra Software : A case of Grade 11 Mathematics learners at a school in Tshwane South District
Type of qualification	Master of Education
Number and type of schools:	One Secondary Schools
District/s/HO	Tshwane South

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

28/03/2018

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

1. *The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*
4. *A letter / document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

Ms Faith Tshabalala
CES: Education Research and Knowledge Management

DATE: 28/03/2018

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

APPENDIX I

WORKSHEET 1

The effects of parameter a on the graph of $\boldsymbol{y = sinx}$ for $\boldsymbol{x = [-360; 360]}$

Use the GeoGebra software in the smartboard to draw the graphs below on the same set of axes and use them to complete the table.

Function	**Value of *q***	**Description of graph**
$* \, y = sinx$		
$y = \sin x + 3$		
$* \, y = sinx - 3$		
$* \, y = sinx + 2$		
$y = sinx - 2$		
$y = sinx + 1$		
$y = sinx - 1$		
$y = sinx + q$		
$y = sinx - q$		
CONCLUSION		

APPENDIX J
WORKSHEET 2

The effects of parameter a on the graph of $y = tanx$ for $x = [-360; 360]$

Use the GeoGebra software in the smartboard to draw the graphs below on the same set of axes and use them to complete the table.

Function	**Value of *a***	**Description of graph**
$*\, y = tanx$		
$y = 3tanx$		
$y = -3tanx$		
$y = -2tanx$		
$y = -tanx$		
$y = 1/2tanx$		
$y = -1/2tanx$		
$y = 1/3tanx$		
$y = -1/3tanx$		
$y = atanx$		
$y = 1/atanx$		
$y = -1/atanx$		

APPENDIX K
WORKSHEET 3

The effects of parameter q on the graph of $y = cosx$ for $x = [-360; 360]$

Use the GeoGebra software in the smartboard to draw the graphs below on the same set of axes and use them to complete the table.

Function	**Value of *q***	**Description of graph**
$*y = cosx$		
$y = \cos x + 3$		
$y = cosx - 3$		
$* y = cosx + 2$		
$y = cosx - 2$		
$y = cosx + 1$		
$* y = cosx - 1$		
$y = cosx + q$		
$y = cosx - q$		
CONCLUSION		

APPENDIX L

WORKSHEET 4

1. Use GeoGebra in the smartboard for the following graphs. In each case, sketch graphs on the same set of axes for $x = [-90°; 360°]$

   (a) $y = \cos x$ and $y = \ -2\cos x + 1$

   (b) $y = \sin x$ and $y = \frac{1}{2} sinx - 2$

   (c) $y = \tan x$ and $y = -tanx - 3$

2. Discuss and then write down the relationship between graphs in each of the above cases.

3. Leave the smartboard to the next group, take your seats and sketch the graph of $y = \ -3sinx + 1$ below

APPENDIX M

WORKSHEET 5

1. Use GeoGebra in the smartboard for the following graphs. In each case, sketch graphs on the same set of axes for $x = [-360°; 360°]$

   (a) $y = \cos x$; $y = \cos(x + 30°)$ and $y = \cos(x - 90°)$

   (b) $y = \sin x$; $y = \sin(x - 60°)$ and $y = \sin(x + 15°)$

   (c) $y = \tan x$; $y = \tan(x + 45°)$ and $y = \tan(x - 30°)$

2. Discuss and then write down the relationship between the first graph and each one of the last two graphs, in each of the above cases.

3. Leave the smartboard to the next group, take your seats and sketch the graph of $y = -2\sin(x + 45°)$, $x\epsilon(-360°; 360°)$ below. Describe the graph drawn in your own words.

APPENDIX N

WORKSHEET 6

1. Use GeoGebra in the smartboard for the following graphs. In each case, sketch graphs on the same set of axes for $x = [-360°; 180°]$

   (a) $y = tanx$; $y = \ tan2x$ and $y = tan\frac{1}{3}$

   (b) $y = \sin x$; $y = sin2x$ and $y = \sin\frac{1}{2}x$

   (c) $y = \cos x$; $y = \cos\frac{1}{2}x$ and $y = \cos 3x$

2. Discuss and then write down the relationship between the first graph and each one of the last two graphs, in each of the above cases.

3. Leave the smartboard to the next group, take your seats and sketch the graph of $y = \ \cos\frac{1}{3}x + 2,\ \ x\epsilon(-360°; 360°)$ below. Describe the graph drawn in your own words.

APPENDIX O

GRADE 11 DIAGNOSTIC TEST

INSTRUCTIONS: Write your answers in the spaces provided

1. Plot the following graphs for $0° \leq \theta \leq 360°$

   (a) $y = \cos\theta$ and $y = -½\cos\theta + 2$ on the same set of axes

   (b) $y = \sin\theta$ and $y = 2\sin\theta - 1$ on the same set of axes

   (c) $y = \tan\theta$ and $y = -3\tan\theta$ on the same set of axes

2. Use the graphs from 1 (a), (b) and (c) to complete the table below

Graph	**Amplitude**	**Period**
$y = \cos\theta$		
$y = -½\cos\theta + 2$		
$y = \sin\theta$		
$y = 2\sin\theta - 1$		
$y = \tan\theta$		
$y = -3\tan\theta$		

3. State the range for each of the graphs in 1(a)

   (a) $y = \cos\theta$

   (b) $y = -{}^{1}/_{2}\cos\theta + 2$

4. State the difference between the graphs in 1 (b) $y = \sin\theta$ and $y = 2\sin\theta - 1$

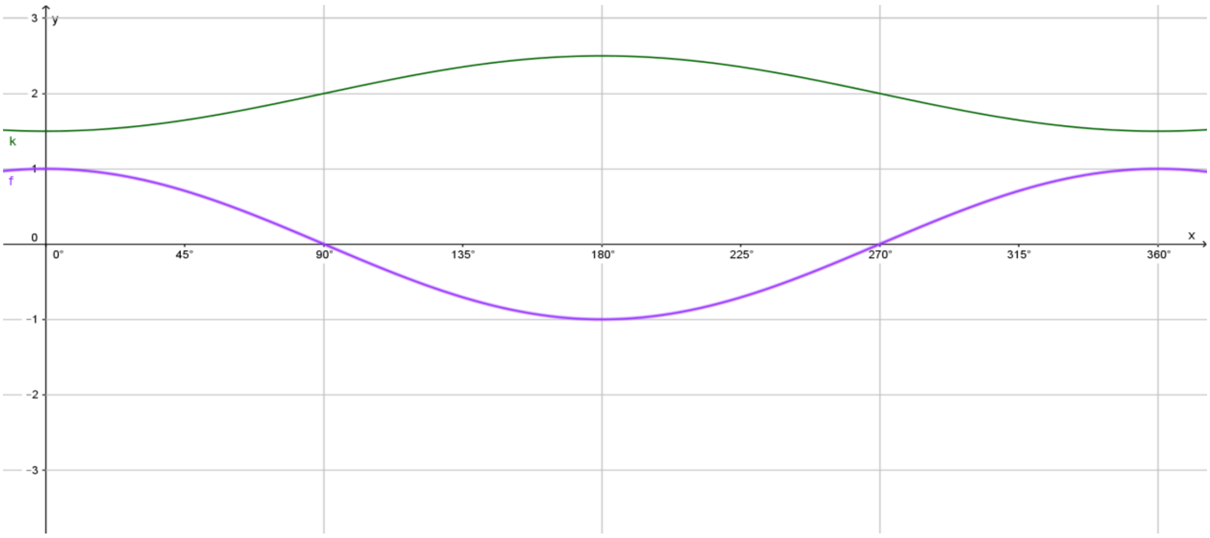
APPENDIX P

GRADE 11 DIAGNOSTIC TEST MEMORANDUM

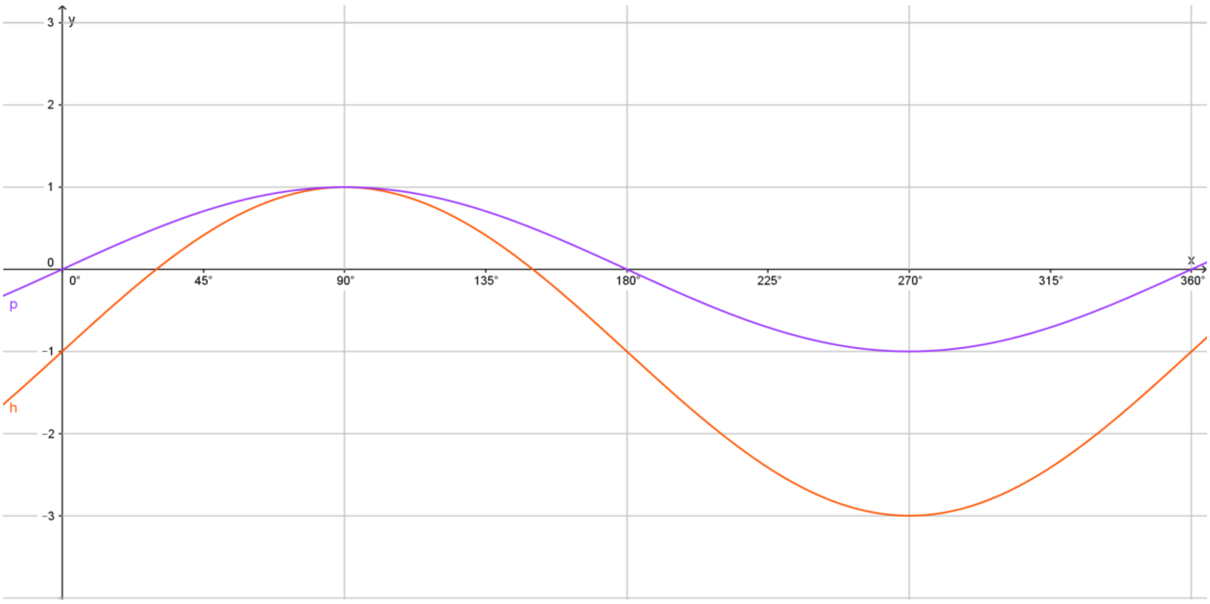
INSTRUCTIONS: Write your answers in the spaces provided

4. Plot the following graphs for $0° \leq \theta \leq 360°$

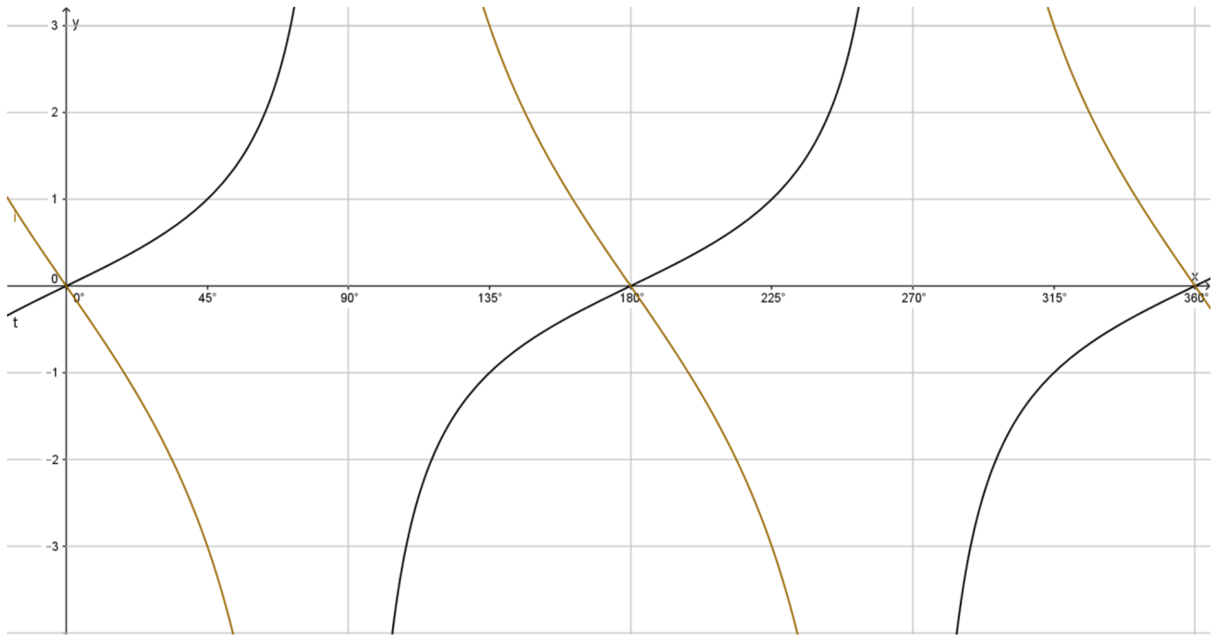
(d) $y = \cos\theta$ and $y = -½\cos\theta + 2$ on the same set of axes



(e) $y = \sin\theta$ and $y = 2\sin\theta - 1$ on the same set of axes



(f) $y = \tan\theta$ and $y = -3\tan\theta$ on the same set of axes



5. Use the graphs from 1 (a), (b) and (c) to complete the table below

Graph	Amplitude	Period
$y = \cos\theta$	**1**	**360°**
$y = -½\cos\theta + 2$	**1/2**	**360°**
$y = \sin\theta$	**1**	**360°**
$y = 2\sin\theta - 1$	**2**	**360°**
$y = \tan\theta$		**180°**
$y = -3\tan\theta$		**180°**

6. State the range for each of the graphs in 1(a)

(a) $y = \cos\theta$ **The range is $-1 \le y \le 1$**

(b) $y = -{}^{1}/_{2}\cos\theta + 2$ **The range is $1½ \le y \le 2½$**

4. State the difference between the graphs in 1 (b) $y = \sin\theta$ and $y = 2\sin\theta - 1$

-The graph of $y = 2\sin\theta - 1$ has double the amplitude of the graph of $y = \sin\theta$

-The graph of $y = 2\sin\theta - 1$ has been shifted 1 unit vertically downwards from the position of $y = \sin\theta$

[accept any answer(s) that enshrine the two difference]

APPENDIX Q

GRADE 11 TRIGONOMETRIC FUNCTIONS TEST

CODE …………….. **DATE…………….**

INSTRUCTIONS: Write your answers in the spaces provided

1. Draw the following graphs

   1.1 $y = \cos 2x \ , x\epsilon[0°; 360°]$

   $f(x) = sin½x \, , x \ \epsilon[0°; 360°]$

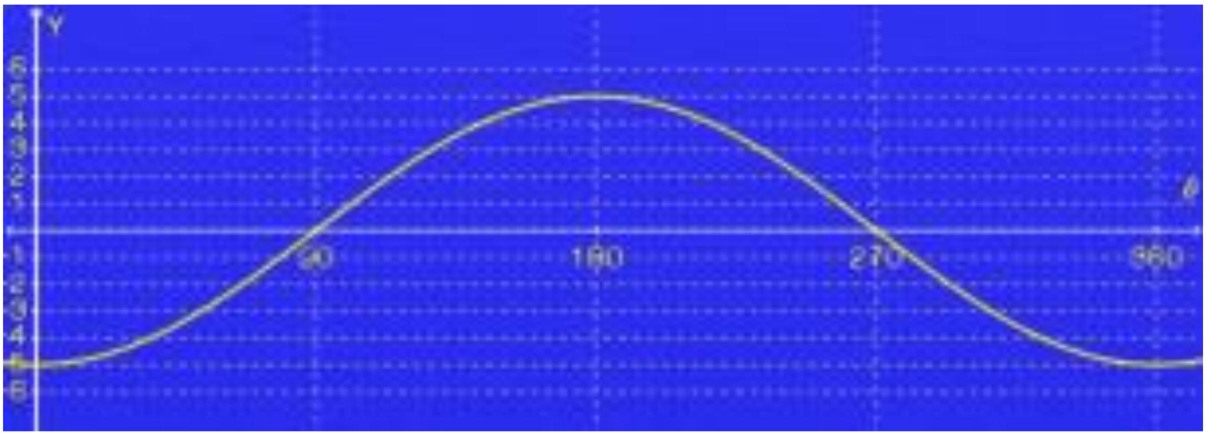
   1.2 $y = 2\sin 3x \, , \ \ x\epsilon[-360°; 360°]$

   1.3 $f(x) = tan(x + 45), x\epsilon[-180°; 360°]$

   1.4 $y = cos(x - 30), x\epsilon[-180°; 360°]$

2. Use the graph to answer the questions

*



2.1 If the graph represents $= bcosa\theta$, determine the values for a and b.

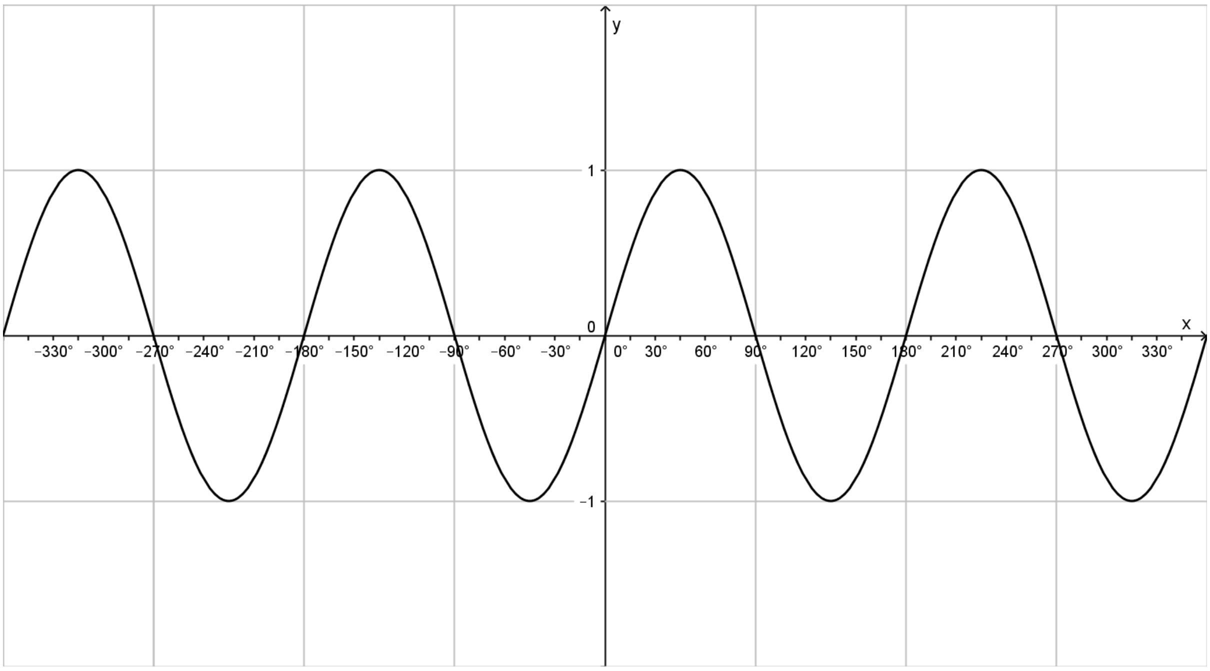
2.2 What is the period for the given graph?

2.3 What is the amplitude for the given graph?

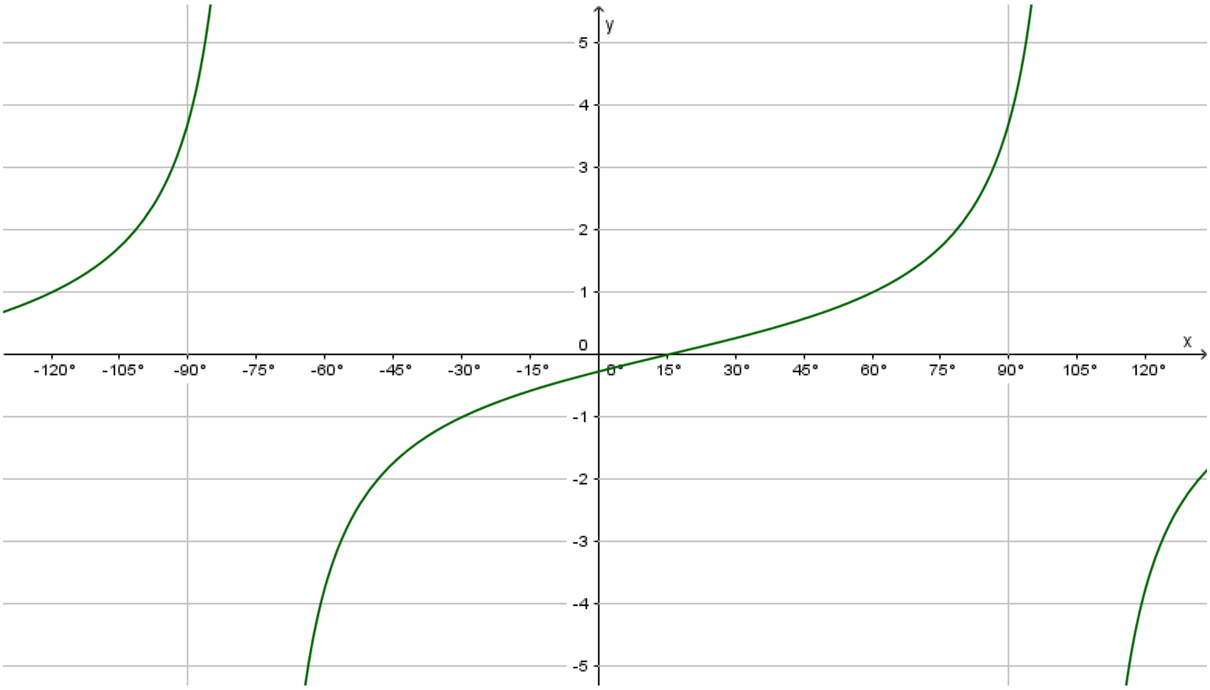
3. If $y = -2sin\theta - 1, for\ \theta\epsilon[0°; 360°]$, write down the minimum and maximum values of the graph.

4. Determine the equation of each of the following graphs:

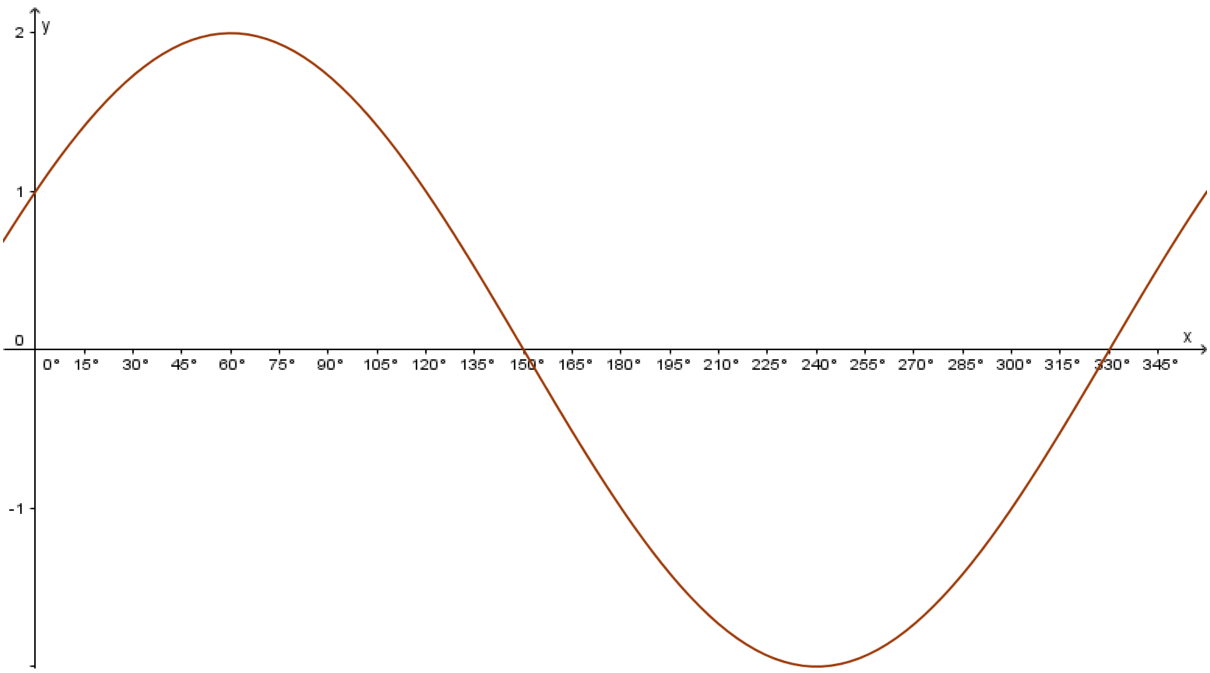
4.1



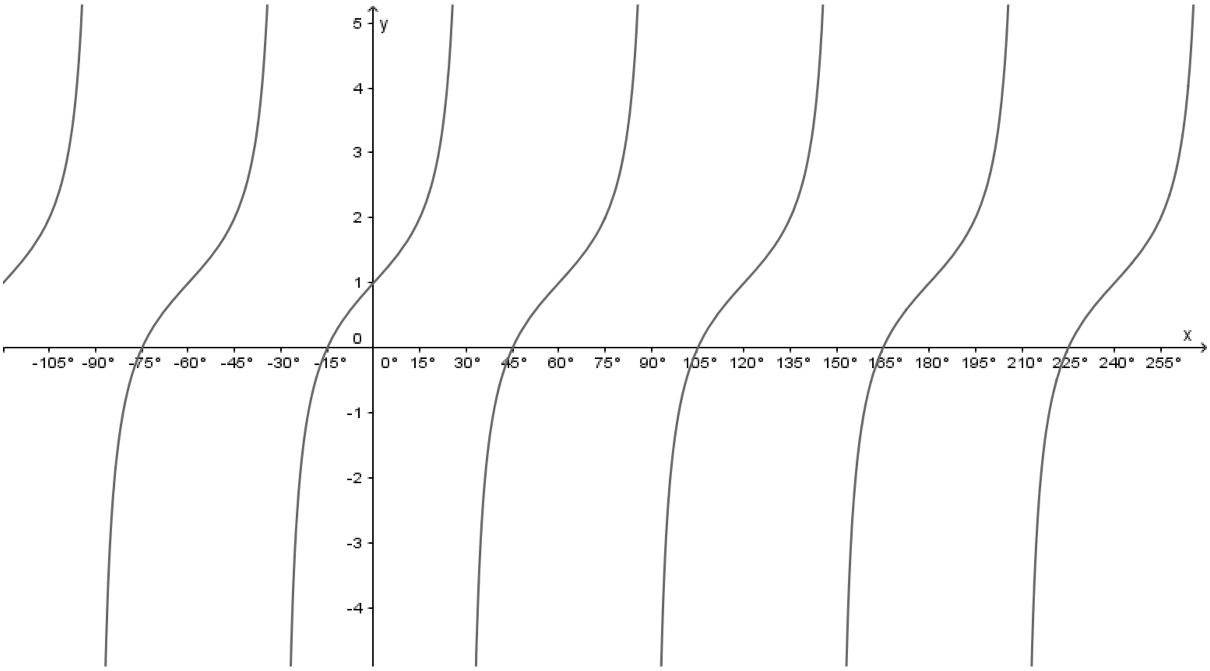
4.2


y
x
5
4
3
2
1
0
-1
-2
-3
-4
-5
-120°
-105°
-90°
-75°
-60°
-45°
-30°
-15°
0°
15°
30°
45°
60°
75°
90°
105°
120°

4.3

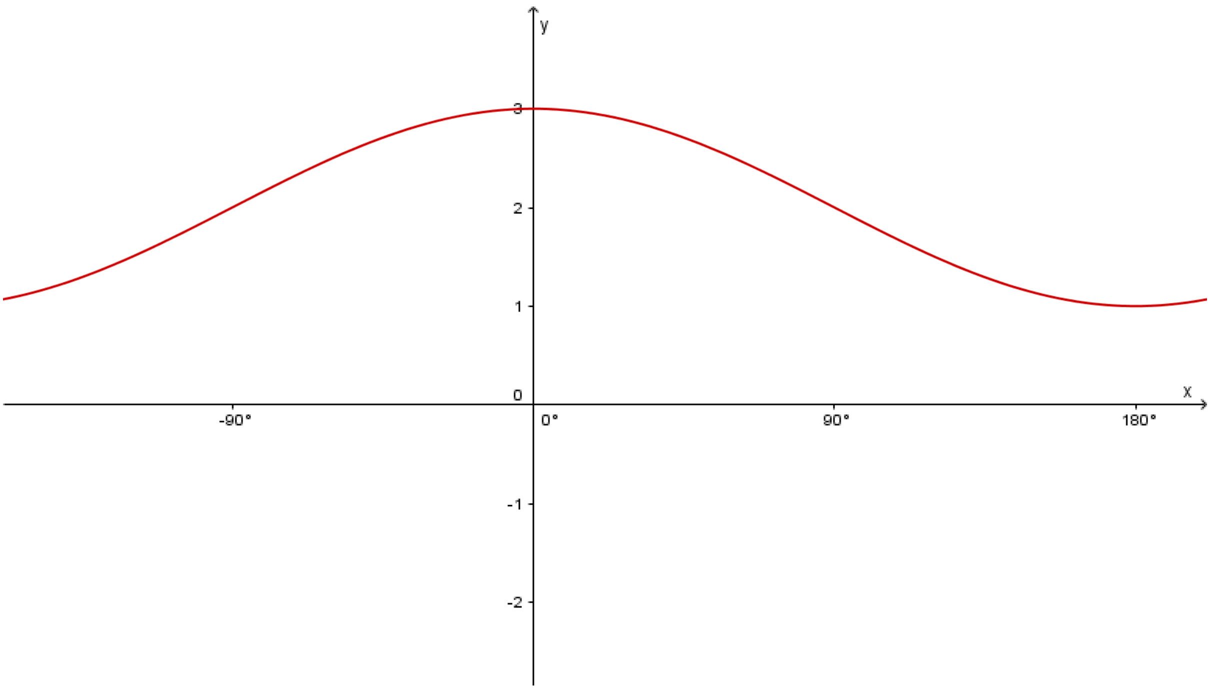

y
x
2
1
0
-1
0°
15°
30°
45°
60°
75°
90°
105°
120°
135°
150°
165°
180°
195°
210°
225°
240°
255°
270°
285°
300°
315°
330°
345°

4.4



y
x
5
4
3
2
1
0
-1
-2
-3
-4
-105° -90° -75° -60° -45° -30° -15° 0° 15° 30° 45° 60° 75° 90° 105° 120° 135° 150° 165° 180° 195° 210° 225° 240° 255°

4.5



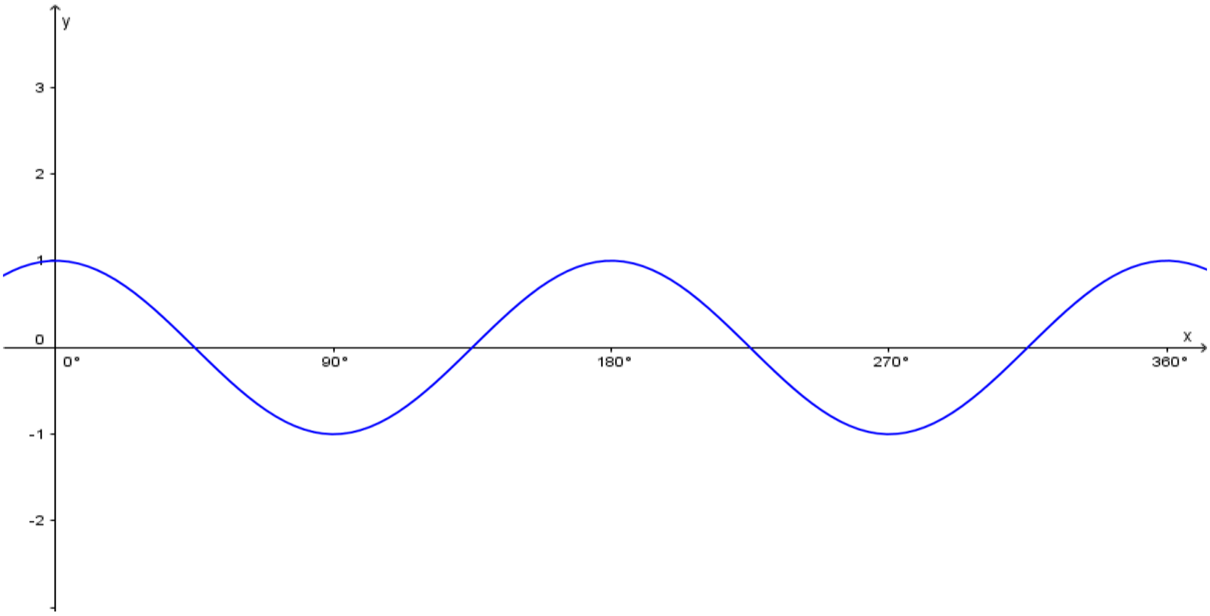
y
x
3
2
1
0
-1
-2
-90° 0° 90° 180°

APPENDIX R

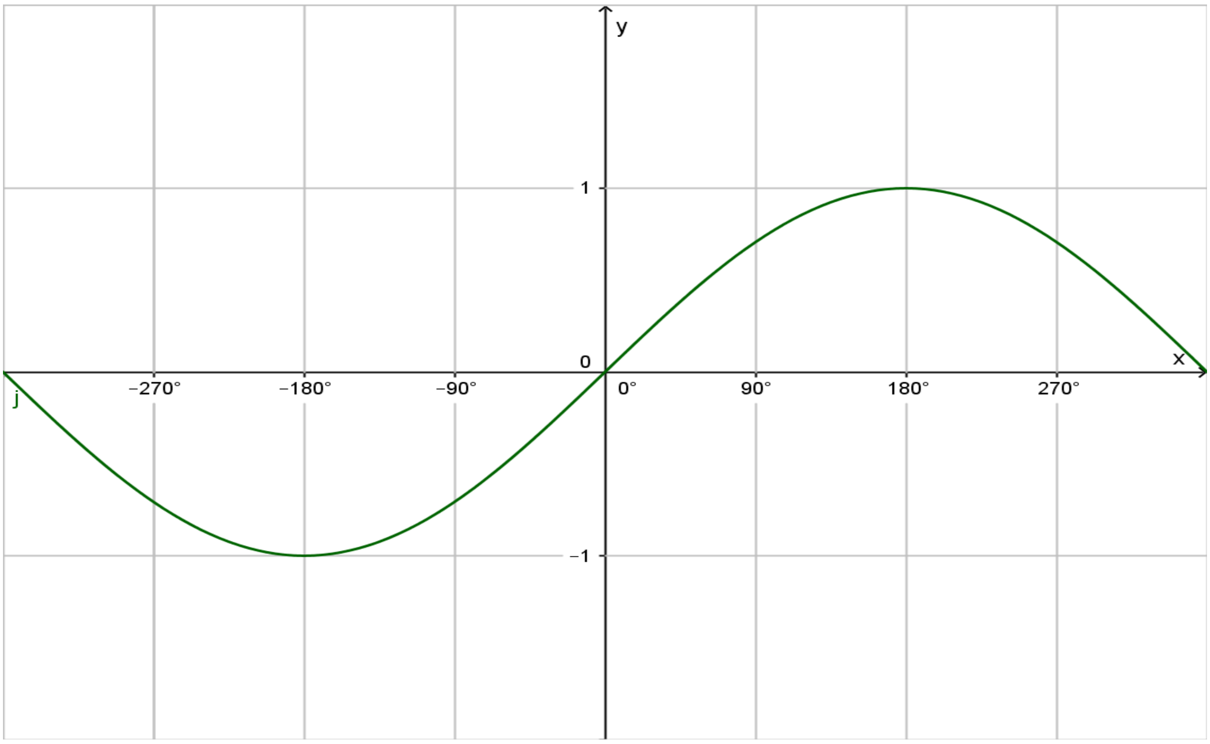
GRADE 11 TRIGONOMETRIC FUNCTIONS TEST MEMORANDUM

1. Draw the following graphs

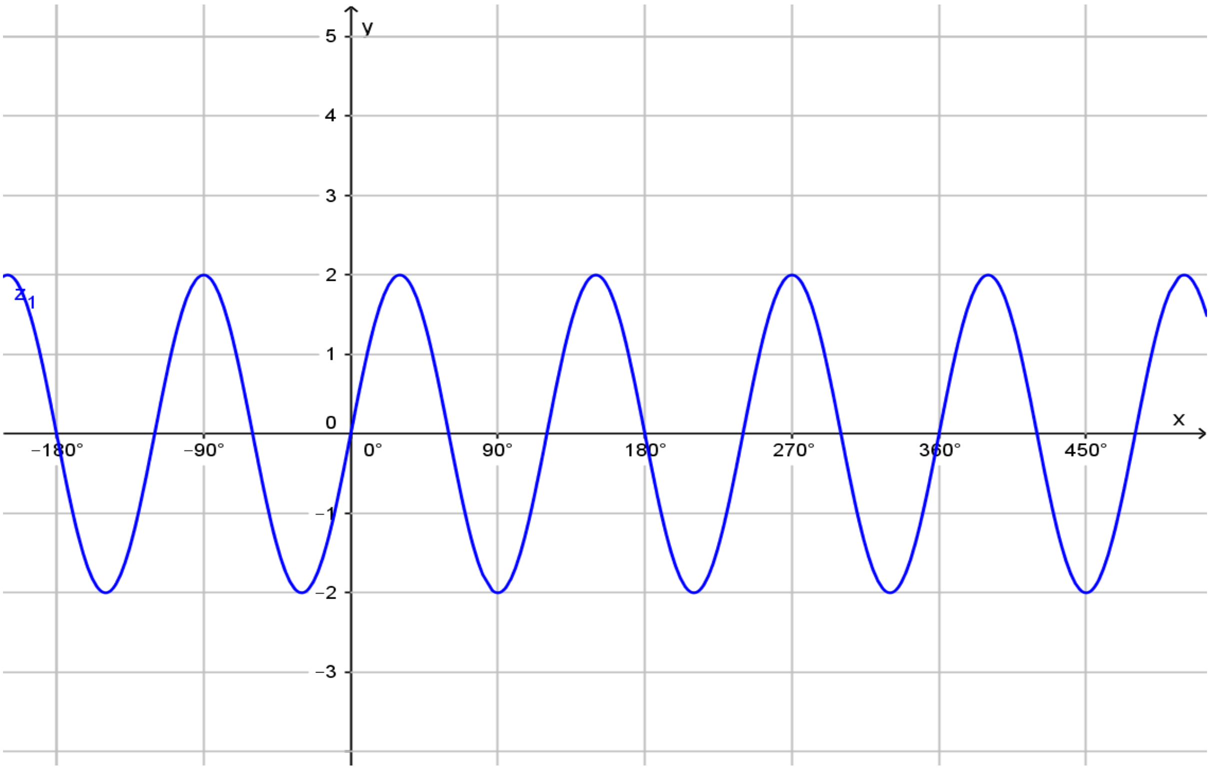
1.1 $y = \cos 2x \ , x\epsilon[0°; 360°]$



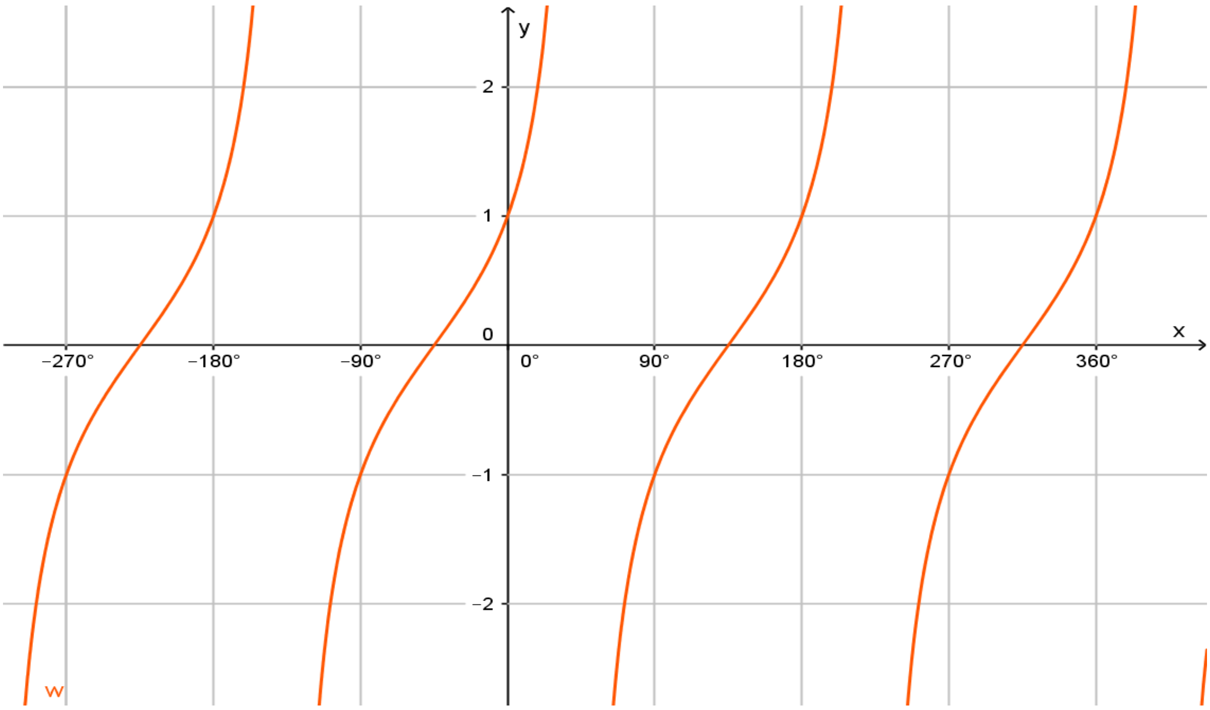
1.2 $f(x) = sin½x \ , x \ \epsilon[-360°; 360°]$



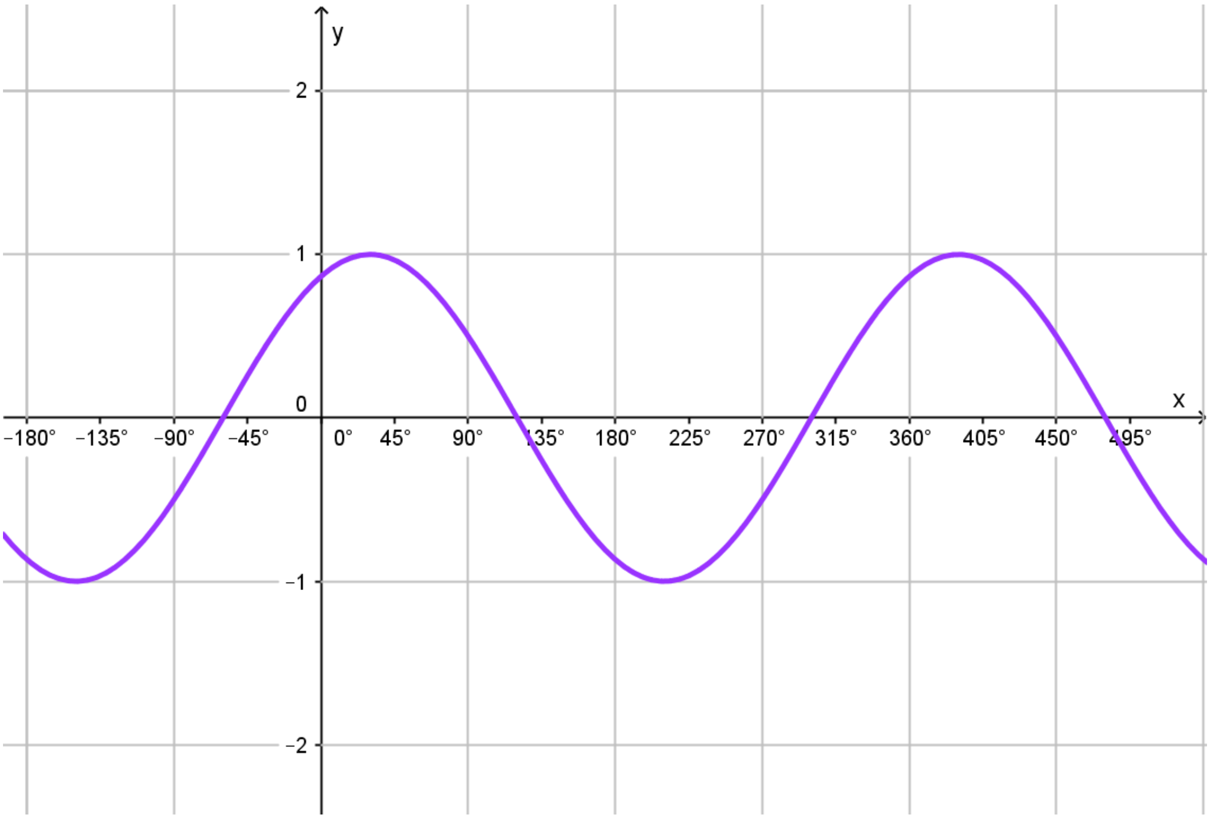
1.3 $y = 2\sin 3x \ , \ \ x\epsilon[-180°; 360°]$



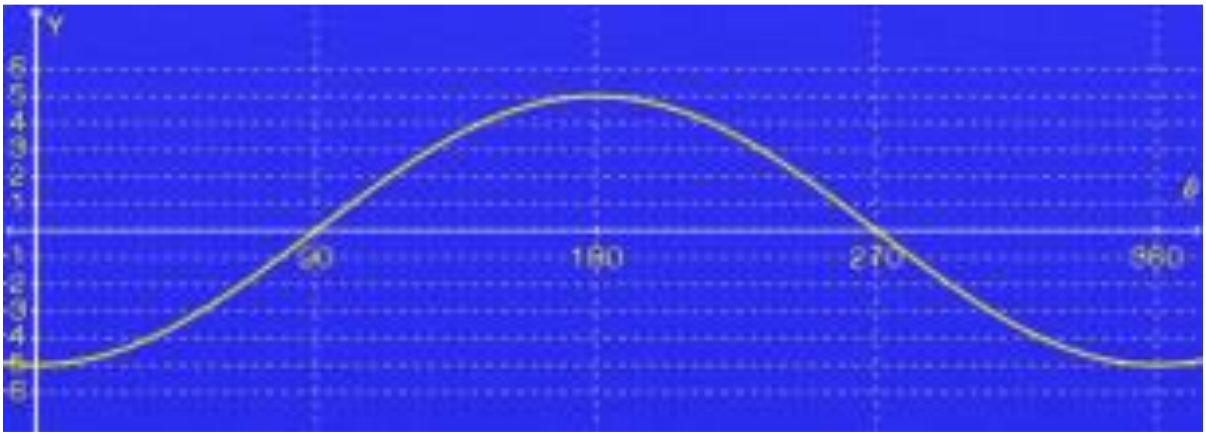
1.4 $(x) = tan(x + 45), x\epsilon[-180°; 360°]$



1.5 $y = cos(x - 30), x\epsilon[-180°; 360°]$



2. Use the graph to answer the questions



2.1 If the graph represents $= bcosa\theta$, determine the values for a and b.

a =1 and b= -5

2.2 What is the period for the given graph?

360°

2.3 What is the amplitude for the given graph?

5

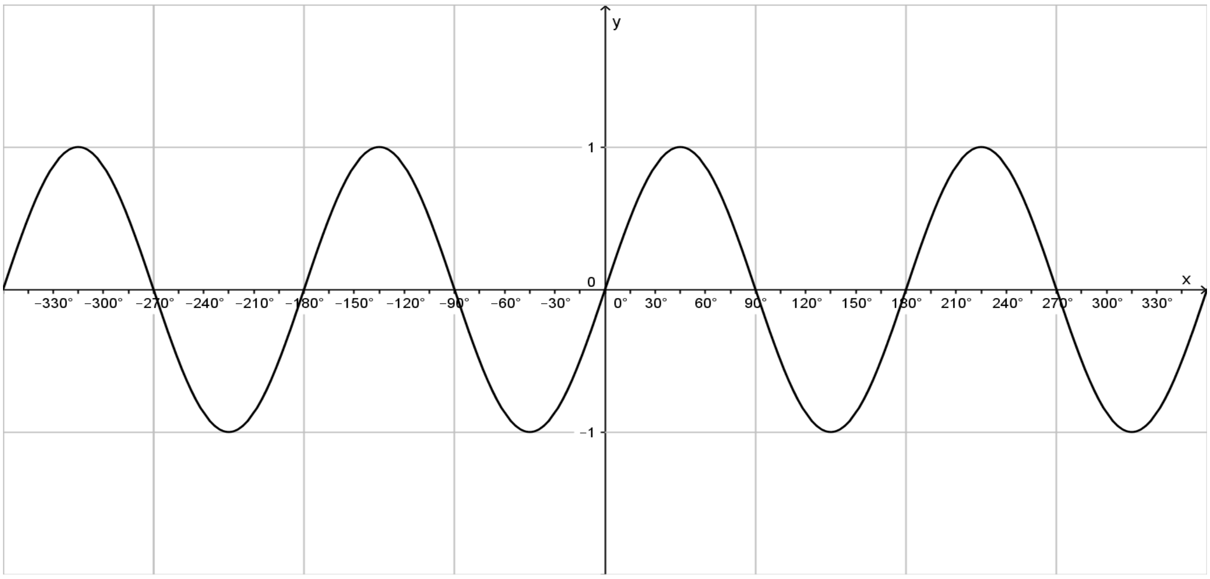
3. If $y = -2sin\theta - 1, for\ \theta\epsilon[0°; 360°]$**,** write down the minimum and maximum

values of the graph.

Minimum = -3 and Maximum = 1

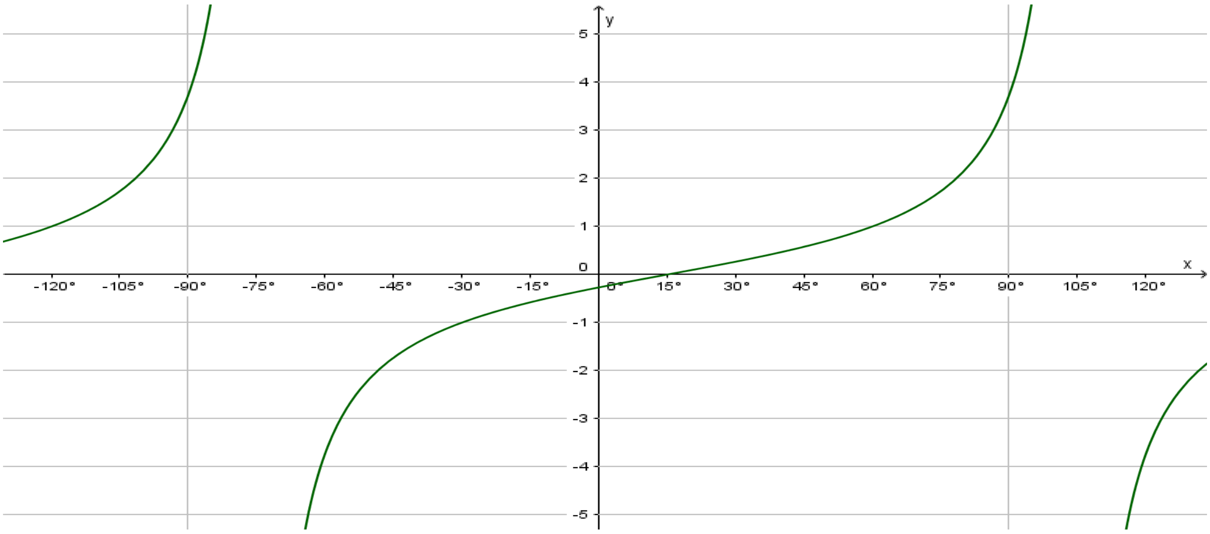
4. Determine the equation of each of the following graphs:

4.1



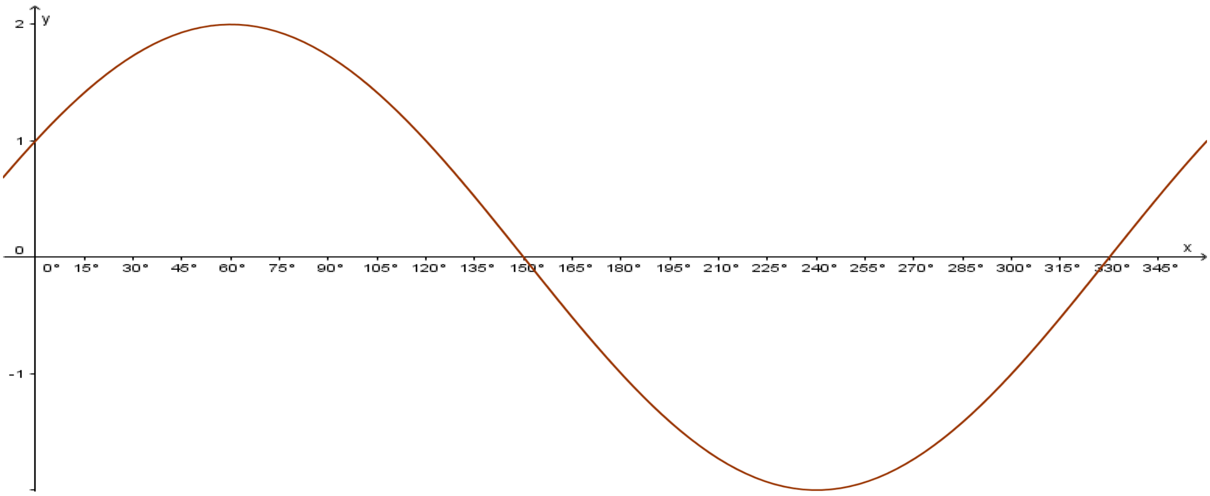
$y = sin2x$

4.2



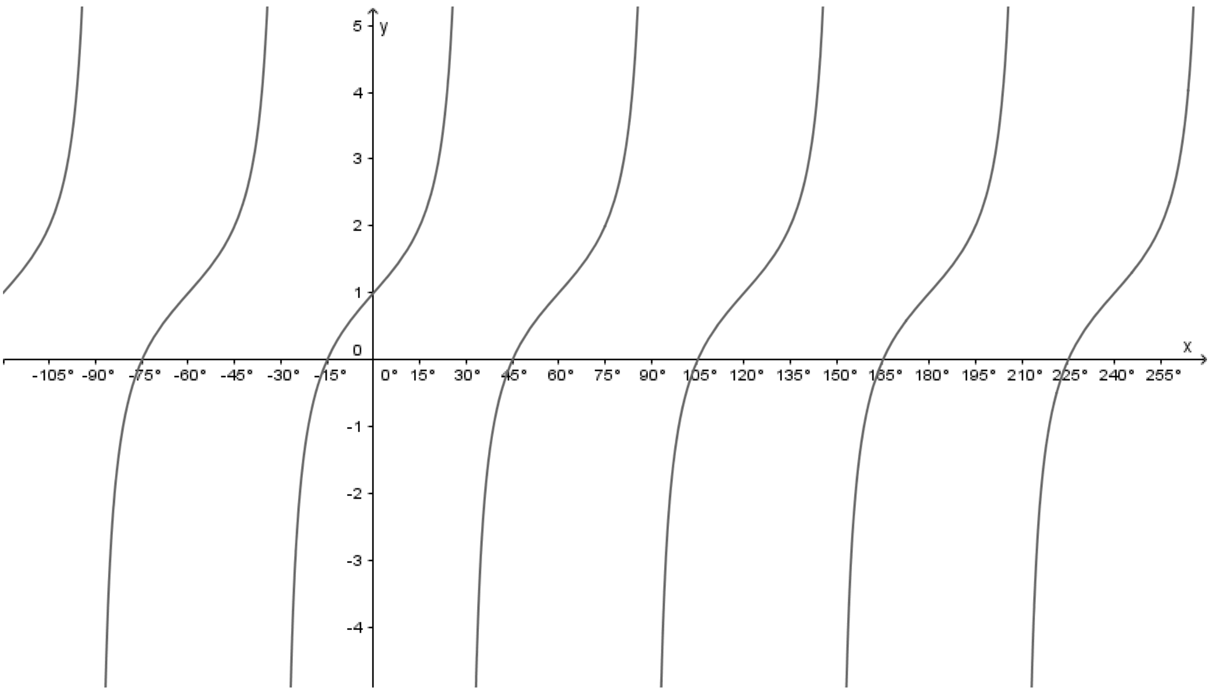
$y = \tan(x - 15)$

4.3



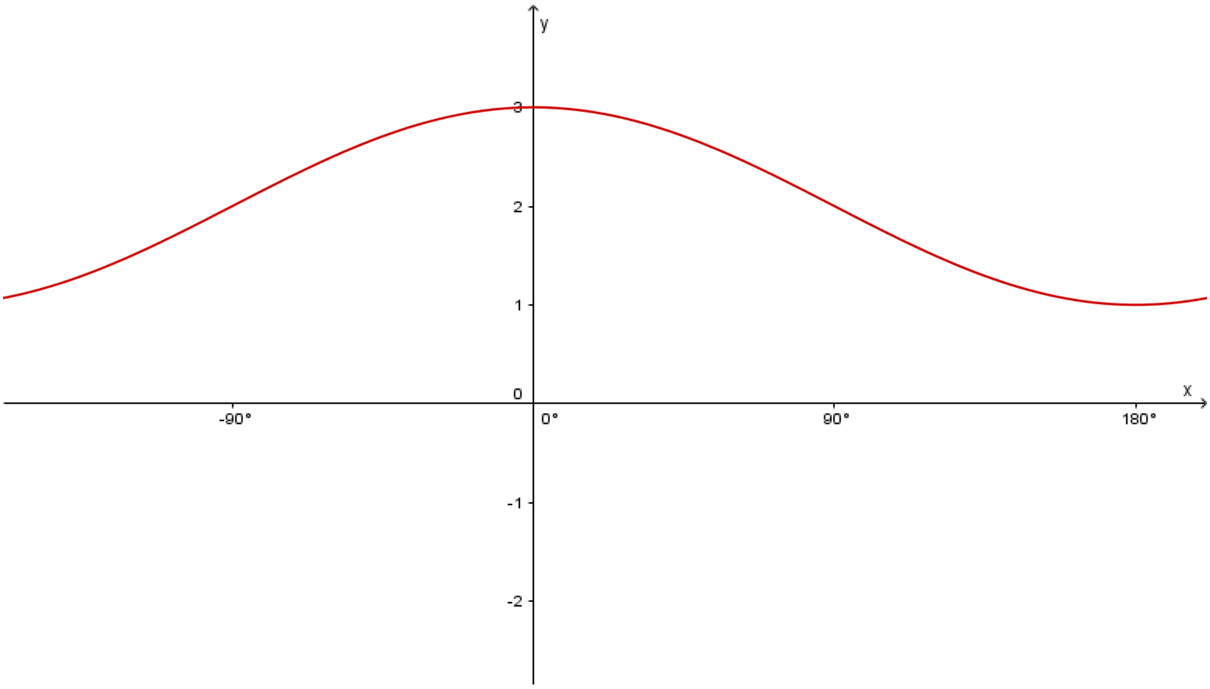
$y = 2\cos(x - 60) OR\ y = 2\sin(x + 30)$

4.4



$f(x) = tan3x + 1$

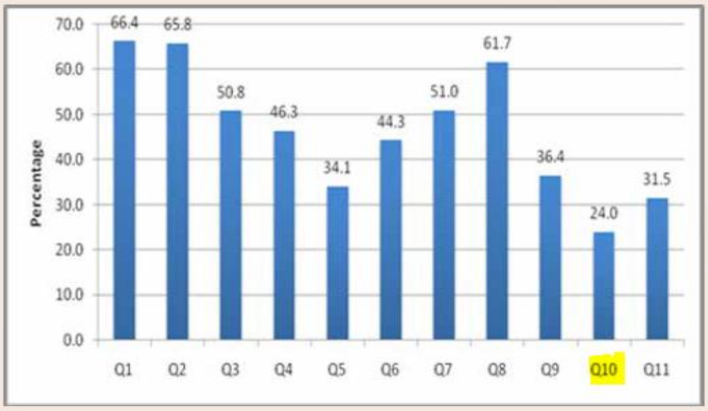
4.5



$f(x) = \cos x + 2$

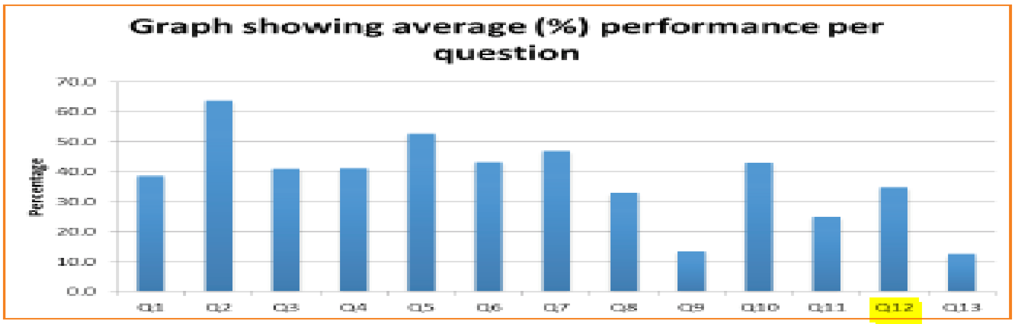
APPENDIX S

DEPARTMENT OF BASIC EDUCATION DIAGNOSTIC REPORTS AVERAGE PERCENTAGE PERFORMANCE ON TRIGONOMETRIC FUNCTIONS QUESTIONS



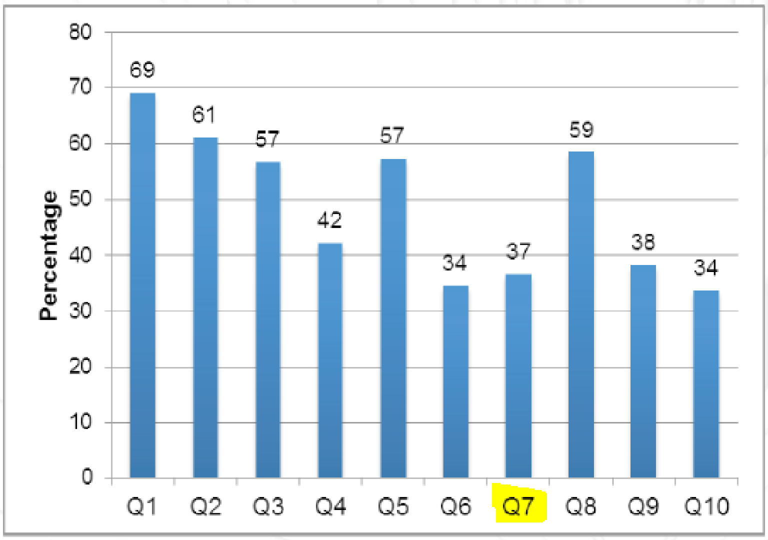
Q1	Data handling
Q2	Data handling
Q3	Data handling
Q4	Data handling
Q5	Analytical geometry
Q6	Analytical geometry
Q7	Transformation geometry
Q8	Trigonometry
Q9	Trigonometry
Q10	Trigonometry functions
Q11	Trigonometry
Q12	Trigonometry
Q13	Transformation geometry

Average percentage performance per question for 2012 Paper 2 (Trigonometric functions – Question10) (DBE Diagnostic Report, 2012:132)



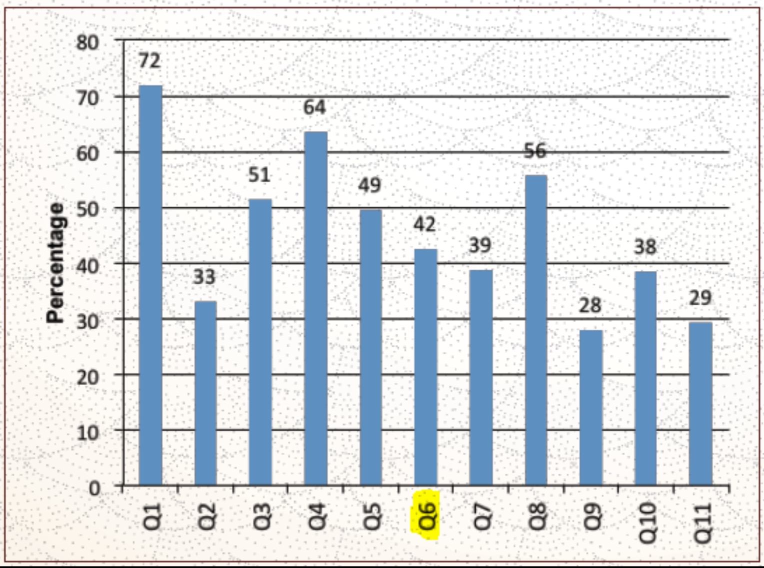
No.	Type of Question	Maximum	Average %
1	Data handling	6	38.7
2	Data handling	7	63.7
3	Data handling	5	40.9
4	Data handling	7	41.2
5	Analytical Geometry	15	52.7
6	Analytical Geometry	24	43.1
7	Transformation Geometry	6	46.9
8	Transformation Geometry	10	32.8
9	Transformation Geometry	10	13.5
10	Trigonometry	8	42.9
11	Trigonometry	30	24.8
12	Trigonometric functions	13	34.8
13	Trigonometry	9	12.5
	Av. Total	150	

Average percentage performance per question for 2013 Paper 2 (Trigonometric functions – Question 12) (DBE Diagnostic Report, 2013:137)



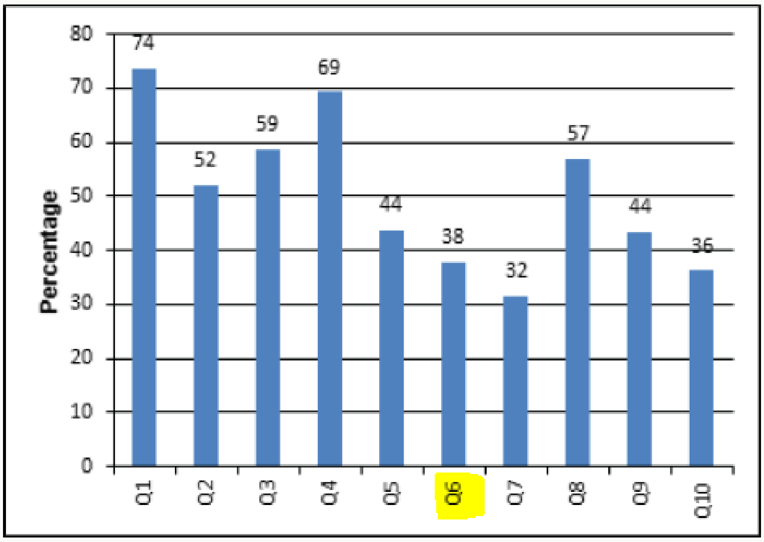
Q1	Data Handling
Q2	Data Handling
Q3	Analytical Geometry
Q4	Analytical Geometry
Q5	Trigonometry
Q6	Trigonometry
Q7	Trigonometry
Q8	Euclidean Geometry
Q9	Euclidean Geometry
Q10	Euclidean Geometry

Average percentage performance per question for 2014 Paper 2 (Trigonometric functions – Question 7) (DBE Diagnostic Report, 2014:122)



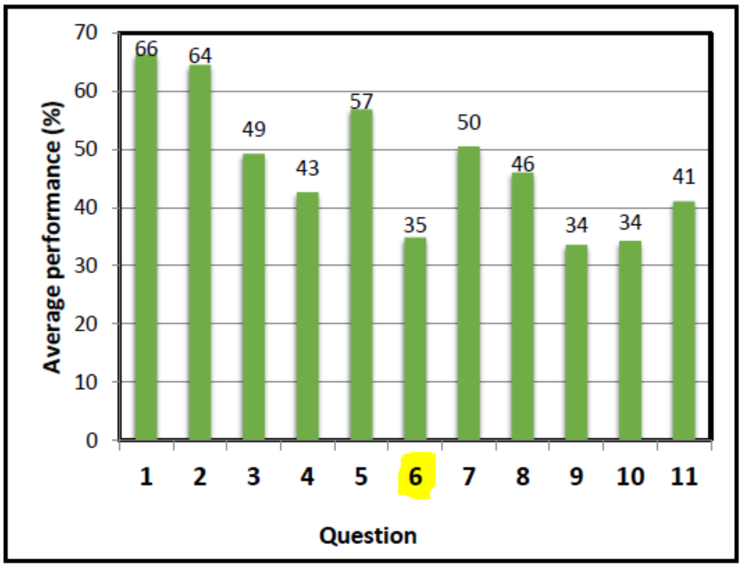
Q1	Data Handling
Q2	Data Handling
Q3	Analytical Geometry
Q4	Analytical Geometry
Q5	Trigonometry
Q6	Trigonometry
Q7	Trigonometry
Q8	Euclidean Geometry
Q9	Euclidean Geometry
Q10	Euclidean Geometry
Q11	Euclidean Geometry

Average percentage performance per question for 2015 Paper 2 (Trigonometric functions – Question 6) (DBE Diagnostic Report, 2015:163)



Q1	Data Handling
Q2	Data Handling
Q3	Analytical Geometry
Q4	Analytical Geometry
Q5	Trigonometry
Q6	Trigonometry
Q7	Trigonometry
Q8	Euclidean Geometry
Q9	Euclidean Geometry
Q10	Euclidean Geometry

Average percentage performance per question for 2016 Paper 2 (Trigonometric functions – Question 6) (DBE Diagnostic Report, 2016:165)



Q1	Data Handling
Q2	Data Handling
Q3	Analytical Geometry
Q4	Analytical Geometry
Q5	Trigonometry
Q6	Trigonometry
Q7	Trigonometry
Q8	Euclidean Geometry
Q9	Euclidean Geometry
Q10	Euclidean Geometry
Q11	Euclidean Geometry

Average percentage performance per question for 2017 Paper 2 (Trigonometric functions – Question 6) (DBE Diagnostic Report, 2017:164)

APPENDIX T

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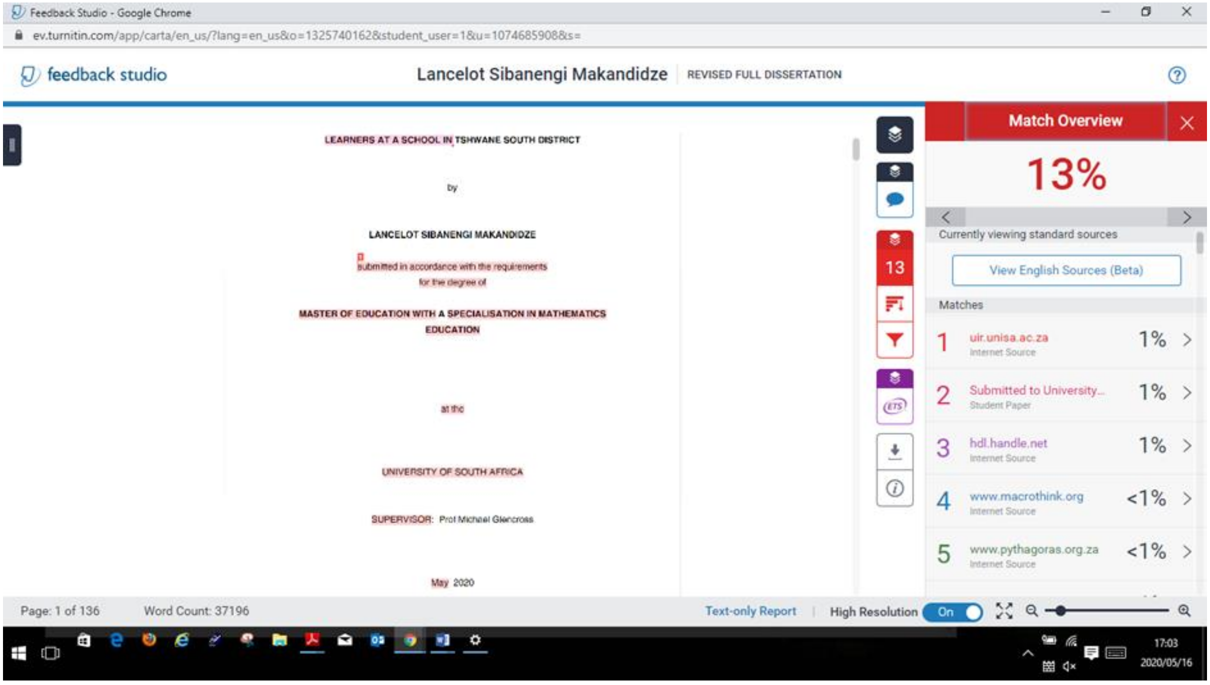

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EXPLORING LEARNERS' UNDERSTANDING OF TRIGONOMETRIC FUNCTIONS USING GEOGEBRA SOFTWARE: A CASE OF GRADE 11 MATHEMATICS LEARNERS AT A SCHOOL IN TSHWANE SOUTH DISTRICT

by

LANCELOT SIBANENGI MAKANDIDZE

submitted in accordance with the requirements for the degree of

MASTER OF EDUCATION WITH A SPECIALISATION IN MATHEMATICS EDUCATION

APPENDIX U

EDITING CERTIFICATE

CONFIRMATION OF EDITING **11 May 2020**

This is to confirm that the M Ed dissertation entitled: **EXPLORING LEARNERS' UNDERSTANDING OF TRIGONOMETRIC FUNCTIONS USING GEOGEBRA SOFTWARE: A CASE OF GRADE 11 MATHEMATICS LEARNERS AT A SCHOOL IN TSHWANE SOUTH DISTRICT** by **LANCELOT SIBANENGI MAKANDIDZE** has been re-edited for language usage. The examiners' language corrections have been implemented.The correlation of in-text references and the reference list remains the responsibility of the student.

Signed:

Eleanor M Lemmer
864 Justice Mohamet Street
Brooklyn
Pretoria
ID 5107110118088

EM Lemmer.