FIRST PRINCIPLES STUDY OF STRUCTURAL, ELECTRONIC AND MECHANICAL PROPERTIES OF TERNARY Ru-Cr-X ALLOYS

By

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DECLARATION

( Student number 5579-223-5 )

I declare that the thesis hereby submitted to the University of South Africa, for the degree of Master of Science in Physics is my own original work and has not been submitted before to any institution. Further, I have acknowledged all the resources that I have used.

............................................................... 19-02-2018..............................

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ABSTRACT

We have investigated the effect of doping on the structural, electronic and mechanical properties of A15 RuCr$_3$ and A15 Ru$_3$Cr alloys using density functional theory. Various transition metal dopants, namely V, Mn, Fe, Co, Zr, Mo, Pd, Pt and Re were introduced in the 2x2x2 supercells constructed from the optimized unit cells of RuCr$_3$ and Ru$_3$Cr binary alloys. The calculated lattice constants for the pure metals, doped and undoped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ alloys were determined using the generalized gradient approximation. The results are in overall good agreement with the experimental and previous theoretical data. The relaxed lattice constants for the X doped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ structures (X = V, Mn, Fe, Co, Zr, Mo, Pd, Pt and Re) remained the same as the unrelaxed structures because the supercells were produced from the optimized unit cells. All the studied undoped systems were predicted to be unstable, in agreement with the previous theoretical study. Stability of doped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ compounds were investigated based on the heats of formation. In general, the Mn, Fe and Mo-doped compositions exhibited negative heats of formation whereas all the remaining dopants displayed positive heats of formation. The addition of Mn, Fe and Mo dopants to the unstable Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ tuned the heats of formation and enhanced the stability of the systems. The magnetic properties of the pure metals and their ternaries were studied, the metallic Mn was found to have a high-optimized magnetic moment of 5.0 $\mu$B while the optimized magnetic moment for Zr was found to be -0.00059 $\mu$B. Mn and Zr influence Ru-Cr systems to obtain high magnetization and low magnetization respectively. Densities of states were investigated to analyze the nature and stability of the alloys. It was found that all the Mn, Fe and Mo-doped Cr$_{48}$Ru$_{16}$ and Ru$_{48}$Cr$_{16}$ systems
show full-metallic behavior. Furthermore, the X-doped Cr$_{48}$Ru$_{16}$ compounds (X = Mn, Fe and Mo) were found to be more stable than the X-doped Ru$_{48}$Cr$_{16}$ compounds. The observed results are due to Cr rich alloys having low density of states around Fermi energy than Ru rich alloys. This finding is consistent with the heats of formation results. Elastic properties were studied to determine the strength and further analyze the stability of the systems. The ratio of bulk to shear modulus, and Poisson’s ratio are greater than 1.75 and 0.3 respectively indicating that the studied X-doped systems are ductile. In addition, the elastic constants revealed that the systems are mechanically stable. The Mn, Fe and Mo-doped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ could be considered as corrosion resistant materials in high temperature applications.
DEDICATION

This work is dedicated to my family, my mom Mokgadi Mnisi, my brothers (Samora, Lamburg, Aubrey and Nimrod) and my sisters (Angie, Peggy and Rose) who played a huge role in supporting me. I also dedicate this work to my late father Abram Mnisi.

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CHAPTER I

Introduction

In this chapter, we give a brief overview of the fundamental theory and experimental work done on transition metals X, to name few Cr, Ru Pt, Zr and Mn, Cr and their alloys. We further give a brief information about the gas turbine engine. The background will provide broader understanding of this research project. The importance of the transition metals and alloys are briefly summarized and lastly we state the objectives and outline of the study.

1.1 Overview

Substantial research based on alloy development has been encouraged due to the huge demand of material capable of withstanding high temperatures in engine industries. The need for high efficiency in gas turbine engines become more important as fuel becomes more costly. The engines are affected by many factors such as high temperature oxidation, hot corrosion [1] and the individual component efficiencies that influence the performance and the overall efficiency of the engine. The development of the new alloys having both the high temperature strength and high temperature corrosion resistant is another area of research that has drawn considerable interest. Transitional metals (TMs) alloys such as platinum, chromium, ruthenium and nickel alloys show a unique number of diverse applications globally and aid as a noble source for the growth in modern theory
of solids [2]. The current materials at solid state displaying high melting points are requested for high temperature structural usages [3].

Gas turbine engines are affected by the environmental threats that decrease the speed of the engines. High-pressure turbine sections obtained in modern gas turbine engine is outlined as one of the most aggressive environments [4-5]. However, when the engine starts to operate just after combustion had took place, and then highly oxidizing gas enters the turbine. Nevertheless, this only happen when the melting point of the super-alloy turbine blade is exceeded by 200°C. This result in the failure of thermal barrier coating in sustaining the temperature and thus cause the blades of the engines to melt [6-7].

Various researchers have studied the properties of binary metal alloys such as hardness, strength and ductility of the materials. The information serves as a guide for selection of suitable high temperature alloys for gas turbine engine applications. Previous study by Tibane M.M et.al [8] computed the heats of formation in the composition of AB, AB₃ and A₃B where A is Ru and B is Cr. The stability study was based on five phases (L₁₂, A₁₅, tP₁₆, DO₆ and DO₆') of Ru₃Cr and RuCr₃. In addition, the AB composition with two phases B₂ and L₁₀ were explored. The study predicted positive heats of formation for all the studied Ru-Cr structures, indicating the instability of the studied Ru-Cr binary alloys. The components coated with RuCr can be hazardous for engine industrial usage. However, the computed heats of formation are positive but closer to zero, suggesting the possibility of stability which may be achieved by adding materials to Ru-Cr alloys.
Addition of transition metals (TM) to alloys could enhance the stability of the alloys and improve its strength even at elevated temperatures [9].

1.2 Overview of Ni-Al, Pt-Al and Cr-Ru alloys

This section gives an overview of the binary and ternary alloys consisting of Ni, Pt, Al, Ru and Cr materials.

Nickel based super alloys can have cobalt, chromium, rhenium, aluminium and other elements as alloy additions. These alloys serve as an important source to gas turbine engines. The efficiency of the gas turbine engines are influenced by the increase in temperature of the working material where the material’s temperature used is limited by its high melting point. The NBSAs have been utilized in the engine industry as blades and vanes since the late 1940’s. The good accomplishment of the NBSAs lies to the fact that they possess exceptionally well characteristics of high melting points and resistant in oxidation [10].

The Ni$_3$Al structure based on the NBSAs is the one that has been studied the most. Furthermore, this alloy has a good ductility in both single and polycrystalline form when their stoichiometry is regulated appropriately. Single crystalline is the material’s crystal lattices in the whole samples that is continuous with no grain boundaries and are formed mostly by solidification process, whilst polycrystalline has a columnar grain structure with many grain boundaries that cause defects. Within Ni$_3$Al, Al is located at the corners and Ni situated on the faces of the cube. Two reasons for choosing to use single
crystalline NBSAs instead polycrystalline NBSAs is that they have a very good creep and fatigue properties [11].

More-over NBSAs are known to have remarkable properties; nowadays 85% of its melting temperature is utilized [10]. In search of possible materials, ceramics were the first candidates to be considered based on niobium, molybdenum and tungsten. In addition, ceramics are suffering from oxidation, creep at low temperatures and lower toughness [12]. In addition, due to the way they are processed it is much expensive than NBSAs.

These NBSAs have superb mechanical properties this is because they have a microstructure that consist of so many small precipitates in a softer matrix. However, NBSAs has a restricted temperature operation to the turbine engines, owing to the melting point of nickel (1543°C) and strengthening dissolution at ~1150°C. This set the current temperature of ~1100°C a limit to NBSAs [13].

This led to the development and study of Pt-based alloys due to their noble corrosion resistance and high melting points. Platinum-based alloys are high in expenditure and display low mechanical resistance at elevated temperatures. In addition, these alloys go hand in hand with the Platinum group metals (PGMs) such as (Platinum, ruthenium, rhodium, osmium, palladium and iridium). These metals are employed in automotive industries as catalysts, in glass manufacturing environment, in spacecraft for increasing the resistance of the rocket engine nozzles.

Platinum based-alloys with their properties can be applied in the chemical industry and can be used as a replacement to Ni super-alloys for high temperature components in the
turbine engines due to having an analogous FCC structure as Nickel. In addition, platinum (1796°C) has high melting point than nickel (1543°C) thus mean platinum has the potential to work better than nickel. Furthermore, during the study of corrosion and oxidation in Pt-based alloys, it was obtained that Pt-based alloys properties are superior to NBSAs [14] however, it is cost effective and high in density but platinum can be recycled as to compensate its expensiveness [15].

In platinum-based alloys, the most studied structure is Pt₃Al. Many researchers had a breakthrough in comparing the Pt-based alloys having high temperature mechanical strength compared with NBSAs. Compounds of L1₂ Pt-based alloys, which are Pt₃Cr, Pt₃Ti, and Pt₃Sn shows a similar temperature dependence of strength whilst the Pt₃Al indicates a normal dependence [15]. The Pt-Al system has been shown to have FCC structure analogous to the NBSAs [16]. This according to Wolff, will aid as the most important candidate for high temperature applications [17].

Furthermore, in checking the oxidation resistance characteristic, researchers obtained that Pt-Al-X ternary system is the best with that property more than the other systems [16, 18]. For Pt-Cr binary, work has been carried out showing that the phase diagrams of the structures were fruitfully analysed. They found that Pt₃Cr is similar to Li₂ AuCu₃ FCC structure and has an ordered cubic A15-type structure occurring at a slightly off-stoichiometric composition of 20 at.%, existing below ~1130°C using the energy dispersive X-ray spectroscopy [19].

Later on chromium (Cr) and ruthenium (Ru) had attracted interest to researchers due to their properties of high melting point, oxidation and corrosion resistance. When the Cr
reacts with other conventional materials they form strong alloys making the Cr to have good ductility. In addition, Cr (1863°C) is high in melting point than Pt (1796°C). However, Cr is easily oxidised and it exhibit a BCC crystal structure that is known to forbid close packing [20]. For (Ru), its interest lies on being an alloying material [21]. The Ru material increases the liquidus and solidus temperatures NBSAs. This property of Ru can be able to recover the temperature capabilities of these materials.

The Cr-Ru system was very deprived owing to being slow in diffusion rate and easy oxidation occurring at Cr when annealed at a particular temperature. This makes it problematic for experimentalists to study Cr-Ru system owing to lack of thermo-dynamical data of this system [22]. The enthalpy of formation was calculated for Cr2Ru by Mediema’s model and found to be -15kJ/mol [23]. The investigation carried on where Ru-Cr binary alloys been studied using the first principle calculations to predict the phase stability based on the structural, thermo-dynamical, elastic, electronic and dynamic properties.
1.3 Gas turbine engine

A gas turbine as shown in Figure 1.1 is a type of internal combustion engine that uses air instead of water for cooling. It consists of a shaft, turbine blade, combustor air inlet. The turbine blade is found just after the combustor in the turbine stage. The main role of the combustor is to add energy to the system to power the turbine [24]. In coming to the turbine blade that are responsible for energy extraction from the high temperature and high pressure gas produced by the combustor, normally it consist of three to four rows shown in Figure 1.2 below:-
When the hot gas expands from the combustor through the turbine stage it knocks out the first row of the blade since it is the one exposed to severe high temperatures coming from the hot gases. However, this first row is the one in need of coating in order for the turbine engine to be efficient. [11].

In designing the turbine engine one has to consider the turbine entry temperature (TET) [25]. This is where the hot gases are entering the turbine arrangement. In addition inside the turbine, the gas stream extracts the temperature that falls as the mechanical work [7]. When offering ways to shield the turbine blades from high temperature gases, thermal barrier coating (TBC) is considered. This coating consists of 4 layers namely: the bond coat, substrate, ceramic top coat and thermally grown oxide (TGO).
The ceramic top coat layer is good for insulating the blade and has low thermal conductivity. This layer is specifically designed to bear thermal cycles. The (TGO) is a type of an oxide that is formed when the bond coat reacts with the ceramic top coat in very high temperatures and it is 1-10µm in thickness. The bond coat is well known as a metallic layer containing oxidation resistance characteristic and it is 75-150µm. Moreover it is able to grip the ceramic top coat to the substrate. Mostly this layer is built upon nickel and platinum materials.

More advancement in engine materials serves as the important aspects in a progress of a flight or an aircraft. Major interest is on the super-alloys for cooling turbine blade systems and by so doing, the service temperature will meet a 20°C increase and even more. This is because the turbine blades are affected by oxidation and hot corrosion as well [26]. The super-alloys identified were the Nickel based super-alloys owing to its high melting points. However, they are well known as high temperature materials that exhibit unique resistance to mechanical and chemical degradation at temperatures near their melting points [7].

The backgrounds of the jet engine together with the based-alloys are more connected in terms of their developments since the more the based-alloys are improved the more the efficiency of the engines. In the modern generation of aero-aircrafts when they take off, their turbine entry temperature (TET) will exceed 1800K [27]. However, components in the turbine engines are able to withstand this temperature and this is due to the air-cooling them from time to time. More progress has been made in increasing the TETs for the last 40 years [28-29] due to a high demand of power and improved fuel consumption in turbines. In addition, turbine components can be made to operate at high TET by
insulating the metal surface for the metal temperature to be minimized [30]. Furthermore, this formed a basis of the initialization of the thermal barrier coatings [31].

1.4 Phase diagrams of Pt-Cr, Pt-Ru and Cr-Ru

This section gives a brief overview of the Pt-Cr, Pt-Ru and Cr-Ru phase diagrams, and firstly the structural characteristics of the metallic Pt, Cr, Ru, Fe, Mo, V, Mn and Co metals and their ternary alloys.

In the previous years, experimental work was a key for improvement and research. However, ways to study the phase diagrams led to the development of alloys [32]. When computational methods became accessible, made the study of phase diagrams to be enhanced employing the method of calculation of phase diagrams (CALPHAD) which had developed by Larry Kaufman [33]. This method is accountable for the formation of databases of many systems like nickel based super-alloys (NBSAs).

1.4.1 The Pt-Cr phase diagram

The Cr-Pt phase diagram in Figure 1.3 displays a broad homogeneity range from ~29 to 100 at.% of Pt solid solution at 1530°C. Waterstrat [35] has found the temperature and composition of the two eutectics. In addition, it has been obtained that Cr₃Pt melts congruently [36]. Recently thermo-calculation method was used to calculate the Cr-Pt phase diagram [37], which was observed to agree very well with the one of Müller [38] and Waterstrat [35] phase diagrams. Moreover, the whole composition range from 50-85 at.% CrPt₃ still remain cubic [35,39-42].
At 1500°C in Massalski’s phase diagram shows a Cr-rich side specified on the temperature of the eutectic and Pt-rich side is at 1530°C. However Oikawa’s diagram showed that the Cr-rich is probably higher [43]. When Pt is added in small amounts, it increases the Neel temperatures of pure Cr from 311.5 to 440K at 0.6 at.% Pt [44] and 583K at 2 at.% Pt [45] and 460K at 5 at.% Pt [46].

Figure 1.3 : Phase diagram of Pt-Cr by Oikawa [34]
1.4.2 The Pt-Ru phase diagram

![Phase diagram of Ru-Pt](image)

Figure 1.4: Phase diagram of Ru-Pt

The Pt-Ru phase diagram in Figure 1.4 shows the absence of phases but only the end-member solid solution (Pt) and (Ru). At 1000°C, 62% Ru dissolves in (Pt). In addition a region between ~62 and ~80 at.% Pt shows the existence of two-phase region of (Pt) and (Ru). Moreover at 2120°C a peritectic (a reaction in a phase diagram where the previous precipitates phase react with the liquid to produce new solid phase) reaction occurs and forms Pt while a Ruthenium solid solution at ~80 at.% Pt was observed. However, there is still a limited data on the solidus and liquidus curves. The calculated diagram of Spencer [47] was in good agreement with the ASM diagram [48].
1.4.3. The Ru-Cr phase diagram

Figure 1.5: Phase diagram of Ru-Cr [49]

Figure 1.5 present the phase diagram of Ru-Cr. The two intermediate phases of Cr$_2$Ru from 32-36 at.\%Ru and Cr$_3$Ru from 25±2 at.\% Ru are shown. The metallographic and x-ray diffraction techniques were used to establish this structure. At about 31.5 at.\% Ru the Cr$_3$Ru phase exists due to long annealing times that are required to reach equilibrium and a slow kinetics has been shown below 1000°C. Moreover, the phase of Cr$_3$Ru was found to be stable from 750 to 1000°C [50-55]. A peritectic point of Cr solution is visible at 37\% Ru around 1600°C, were the reaction between the previous precipitate and the liquid takes place to form the new solid phase. According to the diagram, at room temperature two or more structures occur from 18 to 58\% of Ru concentration. However, the liquidus and solidus curves of Cr-Ru are based on the work of Shurin AK [56-57]. The Cr-Ru
system is very challenging to investigate due to slow diffusion that grade from its large atoms and high melting point.

The Massalski phase diagram [49] was found to be similar to the one of Shunk [58] and it is built on the microstructural development of Ru-Cr alloys. The existence of $\text{Cr}_3\text{Ru}$ into the Cr-Ru phase diagram can be explained by the inhomogeneity cause when heat treatment and solidification occur and solid solution data [49] were the $\alpha$-chromium and $\text{RuCr}_3$ occur. This is due to the formation of grain boundaries from the precipitates. All of these happen when the volume fraction of the $\text{Cr}_3\text{Ru}$ increase with increasing Ru content [59].

1.5 Structural characteristics of pure Pt, Cr, Ru, Fe, Mo, V, Mn and Co metals and their ternary alloys

1.5.1 Platinum and its alloys

![Pure platinum metal](image)

Figure 1.6 : Pure platinum metal

Platinum is a metal that belongs to the platinum group minerals (PGMs) and it is the third highest in density within that group. It is silvery-white in colour, ductile and malleable
This metal is highly resistant to corrosion and oxidation when subjected to high temperatures [61]. However, platinum is a rare metal in the earth's crust and occur with copper and nickel ores in the same chemical deposit. In addition, this metal possesses a face centred cubic crystal structure [62]. In addition Pt has elastic properties as follows Young’s modulus =168 Gpa, Shear modulus=61 GPa, Bulk modulus =230 GPa, Poisson ratio = 0.38, Mohs hardness =4-4.5 [62-63].

This material is applied in various ways in the industry, like making jewellery due to its high resistant to tarnish, in the automotive industries being used as catalysts, in glass manufacturing environment and in spacecraft for increasing the resistance of the rocket engine nozzles. Platinum alloys are usually applied to gas turbine engines [64, 19].

1.5.2 Palladium and its alloys

![Image of Pure palladium metal]

Figure 1.7: Pure palladium metal

Palladium (Pd) is a chemical element forming part of the platinum group minerals (PGMs) which are Ruthenium, Osmium, Iridium, Platinum and Rhodium. However Pd is
a very scarce metal with low melting point as compared with the other PGMs. Palladium tend to occur not mixed with the other metal. In addition, it has an FCC structure just like platinum [64]. This element is applied extensively in the automotive industry as the catalytic converters in vehicles [65], furthermore, it is also used as medicine especially in dentistry and plays a major role in manufacturing of LCD television screens, computer, ceramic capacitors and cell phones [66]. Palladium is normally alloyed with Gold, Copper and Silver. These alloys can be used as ball bearing, springs, surgical instruments and electrical contacts. Meanwhile ruthenium when added to palladium it acts as a solid strengthener [67].

1.5.3 Vanadium and its alloys

![Pure vanadium metal](image)

Vanadium is a transitional metal that is hard, silvery grey, malleable and ductile. It is found in combined form of chemicals naturally. It occurs in 65 different minerals and fossil fuel deposits [68-69]. This metal possesses a base centred cubic structure similarly
to chromium. In addition, vanadium in steel can be applied in axles, bicycle frames and gears. The Vanadium metal has advantage being recycled and reused. Major uses of vanadium are when it acts as an alloy. Vanadium has high melting point thus mean when added to other material can form relatively high strength at elevated temperatures, good fabric-ability and lastly a good corrosion resistant alloy. Furthermore this metal forms a solid solution with Cr more than other materials therefore it has the most potential to ductilize Cr when used as an alloying agent [70-71]. The vanadium metal can act as materials for fusion blanket systems like coating and welding [72].

1.5.4 Zirconium and its alloys

![Zirconium metal](image)

Figure 1.9 : Pure zirconium metal

Zirconium is a chemical element that is grey-white, lustrous and robust transitional metal. This metal is highly resistant to corrosion and it has a high melting point. In addition it is found in a silicate mineral iron (ZrSiO₄) and has a 130kg concentration within the earth’s crust. Zirconium alloys shows extra-ordinary properties like high melting temperature,
high thermal, electrical conductivity and hardness. Furthermore, because of zirconium alloys properties they become very essential in the high temperature applications [73-75]. Zirconium material possess two phases of different crystal structure: when the temperature is below 860°C it has an α phase characterized by hexagonal close packed (HCP) and when the temperature is above 860°C it has a β phase of a base centred cubic (BCC) [76]. This material is compound forming with Rhenium alloys [77].

1.5.5 Manganese and its alloys

![Figure 1.10: Pure manganese metal](image)

Manganese is a chemical element that is pinkish-grey transitional metal. When manganese is exposed to heat and air it tarnishes. This material is found in substantial ores were one of it is manganese dioxide in the form of pyrolusite. In addition manganese is applied in dry cell batteries as a filler and brown/black pigment in paints. In the earth’s crust manganese is the 12th most abundant and it has the low melting point compared to the material discussed above. Furthermore, it possesses a BCC crystal structure and has
the potential to produce many alloys. When manganese is added to ferrite compound it raises its strength and the hardness of steel [78-79].

### 1.5.6 Rhenium and its alloys

![Pure rhenium metal](image)

Figure 1.11: Pure rhenium metal

Rhenium is a chemical element having the highest melting point. This material however is applied as catalysts due to its high resistance to chemical poisoning and mostly in turbine blades as nickel-based super-alloys for high temperature applications and occur as a by-product of copper and molybdenum ore’s extraction. In addition, Rhenium possesses a hexagonal close packed (hcp) crystal structure and it has a paramagnetic ordering [80-82]. In adding Rhenium to alloys, has played an important role as to improve the high temperature properties. [83]. Furthermore, when Rhenium is added to NI-Cr-Al alloy is has stabilizing effect to spheroidal morphology of the γ’ precipitates [84]. Few percentage of Rhenium to nickel-based super-alloys recovers the strength properties of
the alloy and has a Rhenium effect, meaning there is a simultaneous increase in plasticity and strength of the metal [77].

1.5.7 Molybdenum and its alloys

Molybdenum (Mo) is a chemical element that has the sixth-highest melting point of any element. In addition, Mo readily forms stable and hard carbides in alloys and appears as a by-product of tungsten and copper mining. Moreover, it is used as alloys for stainless steel and has the ability to withstand the extreme temperatures without expanding or softening. Mo is used in steel alloys due to its high corrosion resistance and weldability. When Mo is added to Rhenium system, a specific configuration phase was obtained to be stable at low temperatures [77]. This material has a BCC crystal structure and possesses a paramagnetic ordering [85-87].
1.5.8 Cobalt and its alloys

Cobalt (Co) is a chemical element that has an oxidation resistant characteristic because of passivating oxide film it is having. This material is produced as a by-product of copper and nickel mining [88]. In alloy application, Co is used as a metal in preparing magnetic, wear-resistant and high strength alloys. Moreover Co possesses a ferromagnetic ordering that occurs at two crystallographic structures: which are HCP and FCC and the difference between the two states is the transition temperature structure is 450 °C [89]. Furthermore, Cobalt as alloy is predicted to improve the workability of super-alloys [90].

1.5.9 Iron and its alloys

Iron (Fe) is a chemical element that is soft, low cost and high strength. In the earth’s crust Fe is the fourth common element. Furthermore, it is very important when mixed with other metals forming alloys used in steel formation for cutlery. In addition, it is applicable
in the construction of hulls of ships.

Figure 1.14: Pure iron metal

This material undergoes a phase change from bcc to FCC in the temperature from 912°C to 1400°C and it is a ferromagnetic material in nature [91-92]. The magnetic properties of iron being added to rhenium were studied and found that the predicted ground state shown reduced magnetic moments as compared with the pure elements [77].

1.5.10 Chromium and its alloys

Figure 1.15: Pure chromium metal
Chromium is a chemical element which is hard, lustrous and brittle. It has an average concentration of 100ppm and it is the 22nd most abundant element in the earth’s crust. This element has a high melting point of 1863°C. In addition this characteristic helps the element to resist corrosion and hardness. Chromium metals possess antiferromagnetic ordering at room temperature and below. Furthermore, it has a BCC crystal structure and does not allow atoms to pack together closely. The BCC unit cell has a close packing of 0.68 [93]. This material when added to manganese it enhances the hardness of steel. In addition, when 5% of chromium or more is mixed with manganese makes the steel to be air hardening by decreasing the quenching speed. In heat application, steel with chromium possess high critical temperatures [94-97].

1.5.11 Ruthenium and its alloys

Figure 1.16 : Pure ruthenium metal
Ruthenium is a chemical element belonging to the platinum group minerals in the periodic table. It is a rare metal and it is mostly abundant element in the earth’s crust. This element is inert to most other elements. Ruthenium is used mostly for wear resistant electrical contacts and as an alloying agent to Pd and Pt in metallurgical industry for high temperature application. Ruthenium is also used in the production of dental alloys as coating for oxidation resistance. In addition, this element has HCP crystal structure with a melting point between 2300°C and 2450°C. The magnetic ordering of Ruthenium is paramagnetic [98].

### 1.6 Objectives

In this thesis, we investigate the effect of transition metal dopants on the stability of previously studied RuCr$_3$ and Cr$_3$Ru alloys. The primary objective of this work is to search for beneficial dopants that could enhance the stability of the alloy coatings that are used to cover the gas turbine engines that are operating at high temperatures. The fundamental physical properties of transition metal ternary alloys are not fully explored. The discovered ternaries may improve the environmental threats that are encountered by the engines. A stable coating for a gas turbine engine is required to reduce hot corrosion and oxidation. To meet the objectives of this work we investigate the effect of dopants on the structural, electronic and mechanical properties of the A15 RuCr$_3$ and A15 Cr$_3$Ru alloys. We will consider the following dopants: Zr, Mn, V, Mo, Pt, Pd, Co, Fe and Re. We will predict the stability based on the heats of formation of Ru-Cr-X (where X
represent the dopant). In addition, we will also predict stability based on the electronic properties and elastic properties. We will compute the elastic constants ($C_{11}$, $C_{12}$ and $C_{44}$), elastic moduli, ratio of the bulk/shear modulus ($B/G$), as a proxy for ductility. All the calculation will be carried out using the density functional theory within the generalized gradient approximation and plane wave pseudo-potentials. We will employ the Cambridge serial total energy package (CASTEP) code for calculations and compare with the previous results.

### 1.7 Outline

This work consists of six chapters: In chapter one, we outline a brief introduction of Ni and Pt and their application. Moreover gives the theory of Ru-Cr, Pt-Cr, Pt-Ru alloys and their phase diagrams. Furthermore, the dopants used are described in this work individually. Chapter 2 outlines the methodology used in the current study. The density functional theory is used, were the plane wave pseudopotentials, generalized gradient approximation (GGA), local density approximation (LDA) and CASTEP software are analysed in order for the prediction of electronic, structural, elastic and mechanical properties. In Chapter 3 we present the results from the CASTEP software in the form of tables and graphs of lattice constants, heats of formation and magnetic moments. In chapter 4 the total density of states (DOS) and partial density of states (DOS) are analysed. More-over chapter 5 discuss the calculations of elastic constants. The results obtained will be compared with previous experimental and theoretical work. Chapter 6 summarize the whole work; give a conclusion and future work for the study. And lastly, the references used in the study.
CHAPTER 2
Theoretical methods

2.1 Introduction

In this chapter, we outline the ab-initio methods used in this study. The investigation in this thesis is based on the density functional theory (DFT) in a plane-wave pseudopotential framework. The Cambridge Serial Total Energy Package (CASTEP) [99] computational scheme was used to perform the calculations. CASTEP employs the DFT plane-wave pseudopotential methods, which perform first-principle quantum mechanics calculations that explore the properties of crystals and surfaces in various materials such as metals, minerals, semiconductors and ceramics. We will briefly outline the computational simulation using the ab-initio methods. We will further discuss the DFT, which predicts the ground state energy that is essential to predict the structural, magnetic and electronic properties in a vast class of materials.

2.2 Overview

Computer simulation techniques will be used to model our structures of interest and this will help to investigate properties of materials. The simulator inside the computer will model the real system and study the system’s behaviour and all this is done on the computer. This theoretical study is accurate and it is comparable with the experimental work. Moreover the wide most used technique is the ab initio method.
Ab initio on its own has a group of methods that tackle lots of properties like lattice parameter and band structures. The methods are Density Functional Theory (DFT), Car-Parinello ab initio Molecular Dynamics, Semi-Empirical methods and Hatree-Fock method. However, we will focus more on Density Functional theory because it is cheap and easy to use.

### 2.3 Density functional theory

Density functional theory (DFT) was born in 1960 under the invention of Kohn and Hohenburg and Kohn and Sham [100] in freezing the electron-electron interaction in a system. This is a theory well known to connect the ground state properties to the charge density. In addition, density functional theory reduces the strong interactions like complex electron-electron system and when the atomic nuclei are present, it maps it onto a single particle moving in an effective non-local potential.

Furthermore in DFT Hohenberg-Kohn verified that the total energy of many-electron system in an external potential is a distinctive functional of the electron density for a given location of an atomic nuclei. The ground state energy is outlined as the minimum value of the total energy functional on the system; additionally the density that produces this minimum value is the precise ground state density. Moreover the total energy is expressed as

$$E = E[\rho_\alpha(r), R_\alpha(r)]$$  \hspace{1cm} (2.1)
Where $E$ is the total energy depending on the type of organisation of the atomic nuclei, $R_\alpha$ denotes the positions of the nuclei in the system and $\rho_\alpha(r)$ is the electron density. This equation forms a wide array of essential tools for structural, electronic and magnetic properties at an atomic-scale. The Hohenberg-Kohn theorem outlines the possibility in applying the ground state density for the calculation of properties for a system; however, it does not grant a way to find the ground state energy. Then Kohn-Sham equations saw a path that can address the situation [101] by introducing different type of wave functions of single particle orbitals for the treatment of interaction and kinetic energy. The total energy equation (2.1), can be written by expressing the functional in terms of three terms as follows:

$$E[\rho] = T_\rho[\rho] + U[\rho] + E_{xc}[\rho]$$  \hspace{1cm} (2.2)$$

The symbol $T_\rho$ displays the sum of the kinetic energies of all independent particles, $U[\rho]$ is the classical coulomb energy having the electrostatic energy from the columbic attraction and $E_{xc}[\rho]$ is the exchange correlation energy responsible of the remaining complicated electronic contribution to the total energy. Density functional theory substitutes the real electrons by the effective electrons having similar mass, density distribution and charge. The effective electrons are independent when moving inside the effective potential however there is a correlation between the real and the effective electrons. More over when the effective electron is described by the single particle wave function $\psi_i$ its kinetic energy will be as follows
The number of electrons in state $i$ is denoted by $n_i$ and $\nabla^2$ is the gradient operator, $\psi_i(r)$ is the wave function describing the independent particles and. The coulomb energy that is classical having the electrostatic energy coming from the coulombic attraction can be show below as.

$$U[\rho] = U_{en}[\rho] + U_{ee}[\rho] + U[\rho]_{ion-ion}$$  \hfill (2.4)

Where $U_{en}[\rho]$ are the coulombic attraction nuclei and electrons, $U[\rho]_{ion-ion}$ the repulsion between the nuclei and classical repulsion between the electrons and $U_{ee}[\rho]$ which is the coulombic electron-electron repulsion. However, Kohn-Sham energy functional can be minimized by a set of wave functions given by an equation below

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 + V_{ion}(r) + V_H(r) + V_{XC}(r) \right] \psi_i(r) = \epsilon_i \psi_i(r)$$  \hfill (2.5)

Where $\epsilon_i$ shows the Kohn-Sham’s eigenvalue, $V_{ion}$ is the total spatial electron-ion potential, $\psi_i$ is the wave function of a state $i$ and $V_H$ as the Hatree-fock potential which can be given as

$$V_H(r) = e^2 \int \frac{\rho(r')}{|r - r'|} dr'$$  \hfill (2.6)

Whilst the exchange-correlation potential, $V_{XC}$ is given by the functional derivative shown below
\[ V_{xc}(r) = \frac{\delta E_{xc}[\rho(r)]}{\delta \rho(r)} \]  \hspace{1cm} (2.7)

Where \( \rho(r) \) is the electron density and can be given by

\[ \rho(r) = 2 \sum |\psi_i(r)|^2 \]  \hspace{1cm} (2.8)

Hence, the total Kohn-Sham total energy functional is given by

\[
E = 2 \sum_{\text{occ}} \varepsilon_i + U_{\text{ion-ion}} - \frac{e^2}{2} \int \int \frac{\rho(r) \rho(r')}{|r-r'|} drdr' + E_{xc}[\rho(r)] - \int \rho(r)V_{xc} dr
\]  \hspace{1cm} (2.9)

However still the exchange correlation remains to be unknown but it can be solved in applying the approximation method that follows below.

### 2.3.1 Local density approximation (LDA)

Many schemes have been established in order to approximate the exchange-correlation energy. Local density approximation is the easiest approach in explaining the exchange correlation energy of an electronic system. This approach can be applied in pseudo-potential calculations for total-energy of the system [102-103]. LDA assumes that the exchange-correlation energy rely only on the local electron density around each volume element \( dr \) given and can be expressed as

\[ E_{xc}[\rho] \approx \int \rho(r)e_{xc}[\rho(r)]dr \]  \hspace{1cm} (2.10)

Main emphases of LDA is that in any crystal, a surface or a molecule having an atomic arrangement consist of an electron density \( \rho(r) \) at each point \( r \) in space. The Local
Density Approximation (LDA) when it comes to the results for slow varying density systems like; nearly-free-electron metals it is more accurate and in addition it communicates nicely with semiconductors and insulators. However, LDA in strong correlated systems were independent particle picture is not considered it produces inaccurate results and more over the LDA with regard to the ground state energy and bulk modulus it obtains wrong values due to the bond lengths and lattice constants being underestimated [104]. Moreover, as for band gap values for these systems is less estimated as compared to the experimental work available [105]. To increase the accuracy of the result in LDA the GGA approximation comes into play and it is discussed below.

2.3.2 Generalized gradient approximation (GGA)

Generalized gradient approximation for Exchange functional having accurate expressions in DFT involves many applications globally [106-109]. This approximation advances the lacking in Local Density Approximation (LDA). The Generalized Gradient Approximation (GGA) takes care of the exchange-correlation energy density, consisting of density $\rho(r)$ and its gradient $\nabla \rho(r)$, whilst the local density approximation depends on the electron density. The GGA-functional uses Perdew and Wang (PW91) [110]. In this work we have used the PBE form of the GGA. Moreover GGA outlines more accuracy of the ground state energies, bulk modulus and band gap values than the Local Density Approximation (LDA), however it has some drawbacks as well which needs to be looked at for more improvement. The GGA exchange correlation energy is given as follows:
\[ E^{\text{GGA}}_s C(n) = \int dr \rho(r) \varepsilon^{\text{GGA}}_{\text{xc}} \left[ \rho(r), \| \nabla \rho(r) \| \right] \]  

(2.11)

The symbol \( \varepsilon_{\text{xc}} \) is the exchange correlation energy and \( \nabla \rho(r) \) is the gradient term.

### 2.4 Plane wave pseudo-potential (PWPP) method

Plane-wave pseudo-potential method has attained a lot of attention in studying many properties of materials globally. The method tends to make DFT simple by working only with the valence electrons under the assumption that there is no effect in chemical changes to their charge density.

#### 2.4.1 Pseudo-potential (PP) method

Electrons in matter are differentiated into two ways: those further away from the nucleus are the valence electrons and those that are closer to the nucleus are core electrons. The localization of the core electrons in closed inner atomic shells are very strong [111], more-over DFT wave functions best describes the features of core electrons showing more oscillation in the core state than the valence electrons in the valence state. This result is because of the orthogonal requirement. Plane wave set are normally needed for the description of the core regions, however this might be costly and computationally challenging.

The Kohn-Sham equations are used in the pseudo-potential approach for solving only valence electrons whilst the scattering potential from the core are described at a particular radius by pseudo-potential. In addition, this minimizes the total number of wave functions to be calculated, and will put more emphases on valence electrons calculations.
The full wave function close to the nuclei tends to lose data when the core electrons are treated by pseudo-potential. This might influence negatively other properties, i.e. the electric field gradient [112].

In solid state physics, electronic state near the Fermi level is related with the properties in question. However, most of the properties are mostly dominated by valence electrons. This brings to our attention that the core states and the core orbitals are generally irrelevant for the properties of interest and thus means that they can be disregarded or substituted. So the main task of the PP approach is to substitute the strong interaction potential that has core electrons that are bounded tightly with much weak effective potential [113]. Moreover, the PP must have similar scattering properties to the original potential in a core region. Furthermore, this PP does not have nodes since it is a smooth pseudo-wave function. Figure 2.1 below of the pseudo-potential and the true potential is shown.
Figure 2.1 : Schematic representation of all electron potential (solid line) and pseudo-potential (dotted line) with the corresponding pseudo wave function ($\psi_{\text{pseudo}}$) and the valence wave function ($\psi_V$) [114]

The PP is drawn presenting the similar phase shifts between the pseudo wave functions and the ion and core electrons for the valence wave function. Furthermore, the total phase shift produced by the core electrons and the ion is $\pi$ greater for each node contained in the core region by valence functions, than the production of phase shift by valence and ion electrons. Whilst on the outer side of the core region there is a similarity of two potentials and the scattering from the two potential cannot be distinguished.

2.4.2 Plane wave (PW) basis

Orbitals can be defined by the plane waves (PW) or linear combination of atomic orbitals (LCAO). The PW method has an exponential function as
\[ \eta^{PW} = e^{ikr} \]  \hspace{1cm} (2.12)

Where \( \vec{k} \) represents the wave vector and is related to the electron 's momentum. However, these functions are scattered across space and are not based around the nucleus. These functions need the periodic boundary conditions in order to calculate the infinite periodic systems well and cheap. The PW basis set uses an infinite, continuous basis set in expanding the electronic wave functions of the system. In addition, this method is best described by Bloch 's theorem stating that: the discrete PW basis set can expand the electronic wave function at each k-point by the following expression:

\[ \psi_{ki}(r) = e^{ikr} f_i(r) \]  \hspace{1cm} (2.13)

Where \( f_i(r) \) is a function that describes how periodic a solid is. However, the equation (3.8) contains two parts namely: the wavelike and the cell-periodic part. Moreover \( f_i(r) \) can be written as

\[ f_i(r) = \sum_G C_{i,G} e^{iGr} \]  \hspace{1cm} (2.14)

The symbol \( G \) displays the reciprocal lattice vectors of the periodic cell. In addition, each electronic wave function can be written as the summation of PW as follows below

\[ \psi_{ki}(r) = \sum_G C_{i,k+G} e^{i(k+G)r} \]  \hspace{1cm} (2.15)

\( C_{i,k+G} \) represents the coefficients of the PW which are set to be unknowns. This rely more to kinetic energy that is definite and can be outlined as \[ \left[ \frac{\hbar^2}{2m} \right] |K + G|^2 \]
For the expansion of the PW to converge it relies on the energy cut-off. In addition, the energy cut-off produces a finite set when introduced by the discrete PW basis set. In this work, we use plane-wave pseudopotential code that will be discussed in the next section.

2.4.3 Cambridge serial total energy package (CASTEP)

CASTEP code [99] is a package used in performing the DFT calculations, and has the ability to simulate interfaces, solids and surfaces for a wide range of materials such as semiconductors, ceramics, metals etc. This code deals with the weak pseudo-potential for materials by using the plane wave (PW) method. First principle calculation can simulate lot of properties like optical, electronic and mechanical that have assembly of electrons and nuclei within the DFT formalism; in addition, it does not need experimental input. CASTEP is well known to tackle problems in engineering, chemistry, material science and solid state. It is able to calculate the stresses, the forces of an atom in a unit cell taking place and heats of formation that is to be discussed below. However, CASTEP also has limitations that are associated with LDA i.e the underestimation of band gaps in insulators.

2.4.4 Heats of formation

Heat of formation is the change in enthalpy from the formation of 1mole of the compound from its constituent elements with all substances in their standard states at 1 atmosphere. The heats of formation can be illustrated by the following equation:

\[ \Delta H_f = E_{AxByCz} - xE_A - yE_B - zE_C \]  

(2.16)
Where for a given lattice (bcc, fcc and hcp), $E_A$, $E_B$, $E_C$ and $E_{AxByCz}$ are the equilibrium total energy of constituent elements $A$, $B$, $C$ and compound $(A_xB_yC_z)$. Atomic concentration $A$, $B$ and $C$ are highlighted by $x$, $y$ and $z$. The lower the heat of formation, the more stable the structure becomes. Heats of formation are able to approximate stability of phases which are not obtainable experimentally and offer ways to envisage phase diagrams. Based on the ab-initio calculations, heats of formation have been an accomplishment in predicting the stability of alloys and paving a good way in the construction of phase diagrams [115-118]. Hence investigating the relative stability of binary, ternary or quaternary alloys, considering the formation energy for individual structures may be the best way to tackle alloys. The results of the heats of formation for the studied crystal structures will be discussed in detail in the subsequent chapter.

### 2.4.5 Brillouin zone sampling

The average or the mean of the brillouin zone [119] in a particular wave-vector's periodic function involves many crystal calculation known to be complex and difficult. The brillouin zone sampling approach uses the finite number of k-points that contains small errors in the wave-function and it applies the Hamiltonian's eigenvalue for small changes of $k$ [120]. The first brillioun zone contains infinite k-points and for an infinite system there consist a finite number of occupied eigenstates of the Hamiltonian for each $k$-point. This scheme was initialized by Monkhorst Pack [121] based on a principle of metal
system needing a very fine k-point sampling unlike the semiconductors which require few k-points. The primitive reciprocal lattice vector \( k \) can be written with respect to the set of vectors \( q \) as

\[
K = n_1b_1 + n_2b_2 + n_3b_3
\]  

(2.17)

Where \( b_1, b_2, b_3 \) are the reciprocal lattice vectors and \( n_i \) are integers defined by the following expression:

\[
n_i = \frac{(2r-q-1)}{2q} \quad r = 1, 2, 3, \ldots, q
\]  

(2.18)

These k-points play a major role in the electronic potential on the bulk solids. To obtain accurate results, the wave functions are supposed to be calculated in all the k-points in the brilliouin zone. In principle, the electron wave-functions remain constant when a small change in the k-point space distance occur [111]. When calculation are performed with the finite, mesh of the k-points gives inaccurate results [122] that is why firstly the set of k-points must be determined by the position of the k-points and the shape of the brillioun zone in order to calculate only the electronic states of a material in question [123-124].

**2.4.6 Periodic boundary condition (PBC)**

The PBC are mainly used in molecular dynamics simulation to avoid problems with boundary effects caused by finite size. Surface effects are being tackled by the PBC, however in this investigation we are not interested with the surface effects. The applied simulation being used in this investigation will be for small number of particles. In the PBC approach the particles are enclosed in a box and there is a replication to infinity by rigid translation in all three Cartesian directions which fills the space completely. When
one of the particles is located at a point $r$ in the box, this can be assumed that this particle really represents an infinite set of particles located at

$$ r = la + mb + nc $$

(2.19)

Where $l,m,n$ are integers and $a,b,c$ are vectors that correspond to the edges of the box. The key point is that in the box each particle $i$ are thought as interacting with their images in nearby boxes together with other particles $j$ in the box and thus means interactions can go through box boundaries. In fact now the surface effects from the system are eliminated.
CHAPTER 3

Structural and thermodynamic properties

3.1. Introduction
In this chapter we present the structural properties of the Ru, Cr, undoped and transition metal doped A15 Ru₃Cr and Cr₃Ru alloys obtained by using computational methods that are outlined in chapter 2. The computed lattice constants of the Ru, Cr, their binary and ternary alloys Ru-Cr-X are compared with the available experimental and theoretical data. In addition, we present the lattice constants of the studied dopants. We also discuss the heats of formation and the magnetic moments of the binary and ternary alloys. We analyse the effect of dopants on lattice constants and heats of formation of Ru₄₈Cr₁₆ and Cr₄₈Ru₁₆ alloys.

3.2 Structural properties of metallic Ru, Cr and X

Density functional theory is well known to define the ground state properties of metals accurately. The ground state structures of Ru, Cr and the dopants X (X = V, Mn, Fe, Co, Zr, Mo, Pd, Re and Pt) were computed by using the self-consistent DFT calculations. The energy cut-off convergence for the plane-wave expansion of the wave function; and k-points convergence for the number of k-points to sample the k-space in the plane wave expansion are considered to make DFT calculations more accurate.
### Table 3.1: Computed lattice constants, cut-off and total energies for metallic Cr, Ru and X (X = V, Mn, Fe, Co, Zr, Mo, Pd, Re and Pt)

<table>
<thead>
<tr>
<th>Metal</th>
<th>Lattice constants a (Å)</th>
<th>E\text{cut-off} (eV)</th>
<th>E\text{Total} (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cr</td>
<td>2.791 2.850</td>
<td>600</td>
<td>-2465.381</td>
</tr>
<tr>
<td>Ru</td>
<td>2.681 2.726 2.706[128]</td>
<td>600</td>
<td>-2604.726</td>
</tr>
<tr>
<td>V</td>
<td>2.544 2.600</td>
<td>600</td>
<td>-1976.151</td>
</tr>
<tr>
<td>Mn</td>
<td>7.718 7.718</td>
<td>1100</td>
<td>-639.285</td>
</tr>
<tr>
<td>Fe</td>
<td>2.328 2.379 2.866[129]</td>
<td>1100</td>
<td>-859.958</td>
</tr>
<tr>
<td>Co</td>
<td>2.454 2.477 2.507[130]</td>
<td>1000</td>
<td>-1038.615</td>
</tr>
<tr>
<td>Zr</td>
<td>3.143 3.229</td>
<td>1600</td>
<td>-1282.935</td>
</tr>
<tr>
<td>Mo</td>
<td>2.701 2.734</td>
<td>600</td>
<td>-1935.303</td>
</tr>
<tr>
<td>Pd</td>
<td>2.714 2.781</td>
<td>600</td>
<td>-800.640</td>
</tr>
<tr>
<td>Re</td>
<td>2.735 2.774</td>
<td>1500</td>
<td>-2464.955</td>
</tr>
<tr>
<td>Pt</td>
<td>2.771 2.826 3.924[131]</td>
<td>600</td>
<td>-721.127</td>
</tr>
</tbody>
</table>

In order to determine the appropriate energy cut-off for Ru, Cr and X, we performed single energy calculations where we kept the k-points fixed while varying the plane-wave energy cut-off. We used the ultrasoft pseudopotential of Vanderbilt [125] in order to substitute the inactive core electrons close to the nucleus. These pseudopotential requires less computational resources than the norm-conserving potentials [126]. For the exchange correlation functional we used both the local density (LDA) and the generalized gradient (GGA) approximations. The cut-off energies of Ru, Cr and X are shown in Table 3.1.
From the single point calculation, the cutoff energy of 600 eV was chosen for metallic Ru, Cr, V, Mo, Pd and Pt. The cutoff energy of Mn and Fe was set to be 1100 eV. Lastly, the energy cutoffs of 1000 eV, 1500 eV and 1600 eV were selected for metallic Co, Re and Zr respectively. The kinetic energy cut-off is influenced by the atomic species of the material. The results of calculated lattice constants of Ru, Cr and X dopants using the LDA and GGA approximations are listed in Table 3.1, along with the available theoretical and experimental data. For all the studied metals except for Mn, we find that the LDA yield slightly smaller lattice constants than the GGA results. These findings show the well-known tendency of GGA and LDA to respectively overestimate and underestimate lattice constants. The experimental lattice constants are closer to the GGA results than LDA.

The k-points are the number of points used to sample the Brillouin zone in the k-space for the plane wave expansion. We performed the k-point convergence tests. We have carried out total energy calculation at fixed cutoff energy for all the studied metals while varying the number of k-points. We increased the number of k-points until the total energy converged to within 1 meV per atom. The Brillouin zone integrations were performed using the Monkhorst-Pack scheme [132]. The k-points for Mn, Ru, and Cr were chosen to be $3 \times 3 \times 3$, $28 \times 28 \times 2$ and $4 \times 4 \times 4$ respectively.
3.3. Binary and ternary alloys

3.3.1 Introduction

In this section, we explore the two binary alloys composed of the metallic Ru and Cr; which forms the basis of this study. We will also present the transition metal doped alloys. In this section, we outline the computational models and briefly describe the computational technique used in this study. The most relevant results obtained for the structural, thermodynamic and magnetic properties will be presented and discussed.

3.3.2 Computation model

a) Undoped binary alloys: A15 Ru₃Cr and Cr₃Ru

The Ru₃Cr and Cr₃Ru are the A15 type structures of space group Pm-3N (number 223). The A15 phases are described as a series of intermetallic compounds with a formula A₃B; where A is a transitional metal and B can be any element. In the case of Ru₃Cr, Ru and Cr represent the A and B respectively. The A15 unit cells of Ru₃Cr and Cr₃Ru are shown in Figure 3.1. A special characteristic of the A15 crystal structure is that the A atoms form mutually orthogonal linear chains that run throughout the crystal lattice. Furthermore, the A15 structures are close-packed structures in which the size directs their stability [133].
Figure 3.1: (a) Cr$_3$Ru structure: atoms are represented by balls of different colours, green (Ru) and grey (Cr), (b) Ru$_3$Cr structure: green balls (Cr) and grey balls (Ru).

Table 3.2: The atomic positions (Wyckoff notation) in the A15 Ru$_3$Cr and Cr$_3$Ru crystal structures.

<table>
<thead>
<tr>
<th>System</th>
<th>Atoms</th>
<th>Positions</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ru$_3$Cr</td>
<td>Cr</td>
<td>3a</td>
<td>0.25</td>
<td>0.0</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>Ru</td>
<td>1a</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cr$_3$Ru</td>
<td>Cr</td>
<td>1a</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Ru</td>
<td>3a</td>
<td>0.25</td>
<td>0.0</td>
<td>0.50</td>
</tr>
</tbody>
</table>

In Cr$_3$Ru (Fig.3.1 (a)), the Ru atoms are located at the corners and in the centre of the cube, while the Cr atoms are arranged in pairs on the cube faces. The spatial positions of Cr and Ru atoms are respectively 3a (0.25, 0.0, 0.50) and 1a (0, 0, 0) as listed in Table
3.2. The Ru₃Cr in Figure 3.1(b) has the same structure as the Cr₃Ru, the difference being in the position of the Cr and Ru atoms. In Ru₃Cr the Ru atom occupies the 3a site, while the Cr atom is on the 1a site as shown in Table 3.2.

**b) Supercells and Transition metal doped alloys**

In this section, we show the unit cell, supercells of the transition metal doped Ru₄₈Cr₁₆

and Cr₄₈Ru₁₆. The 2x2x2 supercells where constructed from the optimized structures of A15 Ru₃Cr and Cr₃Ru phases as shown in Figure 3.2. A supercell is made of the repetition of unit cells in a crystal lattice in order to avoid interactions between the neighboring atoms since the atoms are arranged periodically.

Our simulated supercells contain 64 atoms. We doped the supercell with one to six transition metal dopant atoms X (where X = V, Mn, Fe, Co, Zr, Mo, Pd, Re and Pt). We will introduce the Mn doped alloys for illustration purposes. Figure 3.3 illustrates the Ru rich alloys; Ru₄₇MnCr₁₆, Ru₄₆Mn₂Cr₁₆, Ru₄₅Mn₃Cr₁₆, Ru₄₄Mn₄Cr₁₆, Ru₄₃Mn₅Cr₁₆ and Ru₄₂Mn₆Cr₁₆ generated by CASTEP package. The Ru atoms are substituted by one to six Mn atoms as shown in figure 3.3 (a-f).

The crystal structures of Cr rich ternary alloys are shown in Figure 3.4. The following structures were modeled: Cr₄₇MnRu₁₆, Cr₄₆Mn₂Ru₁₆, Cr₄₅Mn₃Ru₁₆, Cr₄₄Mn₄Ru₁₆, Cr₄₃Mn₅Ru₁₆ and Cr₄₂Mn₆Ru₁₆ structures. The structures in Figures 3.4 (a –f) indicate the Cr rich alloys doped with different concentration of Mn atoms.
At first, the CASTEP simulator allows one to model a structure of interest. In this case, A15Ru₃Cr was modelled and the single point energy calculation was computed in order to optimize the k-points and kinetic energy cut-off. After that, we performed a geometry optimization to relax the equilibrium lattice parameter and to find the ground state energy. Furthermore, we built a 2x2x2 supercell on the relaxed structure and substitute Ru by Mn atom.
Figure 3.3: The schematic diagram of the Ru rich supercells used in the calculations. Each supercell contains 64 atoms (a) Ru$_{47}$MnCr$_{16}$, (b) Ru$_{46}$Mn$_2$Cr$_{16}$, (c) Ru$_{45}$Mn$_3$Cr$_{16}$, (d) Ru$_{44}$Mn$_4$Cr$_{16}$, (e) Ru$_{43}$Mn$_5$Cr$_{16}$ and (f) Ru$_{42}$Mn$_6$Cr$_{16}$ ternary alloys: Ru, Cr and Mn atoms are represented by green, grey and red balls respectively.
Figure 3.4: The schematic diagram of the Cr rich supercells used in the calculations. Each supercell contain 64 atoms (a) Cr$_{47}$MnRu$_{16}$, (b) Cr$_{46}$Mn$_2$Ru$_{16}$, (c) Cr$_{45}$Mn$_3$Ru$_{16}$, (d) Cr$_{44}$Mn$_4$Ru$_{16}$, (e) Cr$_{43}$Mn$_5$Ru$_{16}$ and (f) Cr$_{42}$Mn$_6$Ru$_{16}$ ternary alloys: Ru, Cr and Mn atoms are represented by green, grey and red balls respectively.
3.3.3 Computational method

The use of computer simulation techniques is becoming more important in the understanding of the microscopic behaviour of materials. The First principles calculations were carried out to determine the cutoff energy and k-points for the A15 phases (Ru$_3$Cr and Cr$_3$Ru) and the supercells of the doped, transition metal undoped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ alloys. The calculations were performed using theoretical lattice constants in the framework of DFT by CASTEP code. All the density-functional theory (DFT) calculations are executed using a plane-wave basis set with the projector augmented plane wave (PAW) method as implemented in the CASTEP code. The generalized gradient approximation is employed for the exchange-correlation functional. Structural, thermodynamical and magnetic calculations based on density functional theory (DFT) implemented in CASTEP code have been investigated for A15 Ru$_3$Cr and Cr$_3$Ru, undoped binary and RuCrX ternary systems.

3.3.4 Cutoff energy and k-points convergence

In order to determine the appropriate cutoff energy for the studied crystal structures, we carried out single point energy calculations where we increased cutoff energy and monitored the convergence of energy of approximately 1 meV/atom. The cutoff energy of 500 eV was chosen for both the binary and ternary alloys. As an illustration, Figure 3.5 (a) show the curves of total energy against cutoff energy of Ru$_{48}$Cr$_{16}$ supercell doped
We further performed the k-points convergence tests, where we carried out total energy calculations at fixed cutoff energy of 500 eV for each structure the number of k-points was varied. 20 x 20 x 20 mesh points in k-space based on Monkhorst-Pack scheme for Ru3Cr and Cr3Ru A15 phases sampled the Brillouin zone. In Figure 3.5 (b) we show the convergence of the total energies with respect to the number of k-points for Ru$_{48}$Cr$_{16}$...
supercell doped with different concentrations of Mn. The total energy with respect to the number of k-points was considered converged when the energy change per atom was within 1 meV per atom. The k-points were chosen to be $4 \times 4 \times 4$ for doped and undoped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ systems. The selected parameters are sufficient in leading to well converged total energy and geometrical configurations.

3.3.5 Geometry optimization

The first step in any ab-initio calculation is to find the optimised geometry of the crystalline structure. Geometry optimization is the way of finding the minimum energy of a system. Optimization plays a role in producing a relaxed structure, to ensure accurate examination of the system’s properties. In order to check the accuracy of the calculations, we first optimized the structures of A15 Ru$_3$Cr and Cr$_3$Ru, and the doped and undoped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ supercells at a fixed cutoff energy of 500 eV. In addition, the optimization was done using the GGA-PBE functional within the CASTEP code. The lattice constants of both the binary and ternary alloys were allowed to vary. The calculations were carried out until a good convergence was obtained. The minimized total energies of the structures were used to calculate the heats of formation and magnetic moments that we will discuss in the next section.
3.3.6 Lattice constants and heats of formation for the A15 RuCr\textsubscript{3} and Cr\textsubscript{3}Ru Alloys

Table 3.3: Calculated lattice constants and heats of formation of Ru\textsubscript{3}Cr and Cr\textsubscript{3}Ru in the A15 phase.

<table>
<thead>
<tr>
<th>System</th>
<th>Prototype notation</th>
<th>LDA</th>
<th>GGA</th>
<th>previous work</th>
<th>this work</th>
<th>previous work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ru\textsubscript{3}Cr</td>
<td>Cr\textsubscript{3}Si</td>
<td>4.694</td>
<td>4.777</td>
<td>4.788[8]</td>
<td>0.39</td>
<td>0.3225[8]</td>
</tr>
<tr>
<td>Cr\textsubscript{3}Ru</td>
<td>Cr\textsubscript{3}Si</td>
<td>4.537</td>
<td>4.623</td>
<td>4.623[8]</td>
<td>0.085</td>
<td>0.0796[8]</td>
</tr>
</tbody>
</table>

Firstly, we computed the equilibrium structural properties lattice constants and heats of formation of the binary. The calculated lattice constants, heats of formation are shown in Table 3.3. For comparison, we have included the results from a theoretical investigation of heats of formation for Ru\textsubscript{3}Cr and Cr\textsubscript{3}Ru using first principles calculations \[8\]. Our computed LDA lattice constants for both systems are slightly lower than the GGA results. It is owing to the GGA approximation, which always overestimates the results. It is evident that the lattice constant is lower in Cr rich than Ru rich alloy. We note here that the values of the GGA lattice constants are in good agreement with the theoretical results than the LDA. The heats of formation for Ru\textsubscript{3}Cr and Cr\textsubscript{3}Ru are predicted as 0.39 eV/atom and 0.085 eV/atom respectively. As seen, our results agree with the previous theoretical values of 0.3225 eV/atom and 0.0796 eV/atom for Ru\textsubscript{3}Cr and Cr\textsubscript{3}Ru respectively. The obtained positive values indicate that the Ru\textsubscript{3}Cr and Cr\textsubscript{3}Ru in the A15 phase are unstable. However, the values are closer to zero, especially for Cr\textsubscript{3}Ru, in complete agreement with
the theoretical work by M. M Tibane et.al [8]. Since the values are closer to zero, it may be possible to reduce heats of formation of the Ru$_3$Cr and Cr$_3$Ru by doping the systems with metallic dopants. We will discuss deeply the effect of doping in the next section.

3.3.7 Thermodynamic properties for the doped and undoped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ supercells

Researchers are devoted to improve the stability of high temperature materials by alloying with other elements and mechanical alloying. With the motivation of finding promising ternary alloys from A15 Ru$_3$Cr and Cr$_3$Ru binary alloys with improved physical properties in comparison with other ternary alloys, the present section focuses on the composition dependence of the thermodynamic properties of RuCrX ternary alloys by performing ab initio calculations, based of augmented plane-wave within DFT implemented in CASTEP code. These DFT simulations are beneficial to improve our understanding of the effects of substitution on the properties of binary alloys, and of the design of advance materials. We investigate the thermodynamic properties that are usually the basis of solid-state science and industrial applications. The study of the thermodynamic properties of materials is of importance to extend our knowledge on their specific behaviours when undergoing severe constraints of high pressure and high temperature environment [134].
In this section we research the effects of transition metals $X$ ($X = \text{Zr, Mn, V, Mo, Pt, Pd, Co, Fe and Re}$) on the heats of formation of the undoped $2 \times 2 \times 2 \text{Ru}_{48}\text{Cr}_{16}$ and $\text{Cr}_{48}\text{Ru}_{16}$ supercells (constructed from the unstable A15 Ru$_3$Cr and Cr$_3$Ru respectively). The Ru or Cr atom/s in the 2x2X2 $\text{Ru}_{48}\text{Cr}_{16}$ or $\text{Cr}_{48}\text{Ru}_{16}$ supercell structure were replaced by different doping concentrations of $X$. The heats of formation of the transition metal doped alloys were calculated using the First-principles calculation, and discussed in more detail in the following sections.

Table 3.4: calculated heats of formation of V, Mn, Fe and Co doped $\text{Ru}_{48}\text{Cr}_{16}$ and $\text{Cr}_{48}\text{Ru}_{16}$ alloys. 

<table>
<thead>
<tr>
<th>System</th>
<th>V</th>
<th>Mn</th>
<th>Fe</th>
<th>Co</th>
<th>Atomic percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Ru}<em>{47}\text{Cr}</em>{16}X$</td>
<td>0.272</td>
<td>0.138</td>
<td>-6.491</td>
<td>0.259</td>
<td>1.56</td>
</tr>
<tr>
<td>$\text{Ru}<em>{46}\text{Cr}</em>{16}X_2$</td>
<td>0.227</td>
<td>-0.022</td>
<td>-13.278</td>
<td>0.245</td>
<td>3.13</td>
</tr>
<tr>
<td>$\text{Ru}<em>{45}\text{Cr}</em>{16}X_3$</td>
<td>0.176</td>
<td>-0.193</td>
<td>-20.079</td>
<td>0.206</td>
<td>4.69</td>
</tr>
<tr>
<td>$\text{Ru}<em>{44}\text{Cr}</em>{16}X_4$</td>
<td>0.163</td>
<td>-0.329</td>
<td>-26.842</td>
<td>0.198</td>
<td>6.25</td>
</tr>
<tr>
<td>$\text{Ru}<em>{43}\text{Cr}</em>{16}X_5$</td>
<td>0.160</td>
<td>-0.468</td>
<td>-33.610</td>
<td>0.195</td>
<td>7.81</td>
</tr>
<tr>
<td>$\text{Ru}<em>{42}\text{Cr}</em>{16}X_6$</td>
<td>0.169</td>
<td>-0.593</td>
<td>-40.370</td>
<td>0.191</td>
<td>9.38</td>
</tr>
<tr>
<td>$\text{Cr}<em>{47}\text{Ru}</em>{16}X$</td>
<td>0.047</td>
<td>-0.079</td>
<td>-6.707</td>
<td>0.050</td>
<td>1.56</td>
</tr>
<tr>
<td>$\text{Cr}<em>{46}\text{Ru}</em>{16}X_2$</td>
<td>0.040</td>
<td>-0.212</td>
<td>-13.468</td>
<td>0.053</td>
<td>3.13</td>
</tr>
<tr>
<td>$\text{Cr}<em>{45}\text{Ru}</em>{16}X_3$</td>
<td>0.032</td>
<td>-0.345</td>
<td>-20.986</td>
<td>0.053</td>
<td>4.69</td>
</tr>
<tr>
<td>$\text{Cr}<em>{44}\text{Ru}</em>{16}X_4$</td>
<td>0.025</td>
<td>-0.478</td>
<td>-26.991</td>
<td>0.053</td>
<td>6.25</td>
</tr>
<tr>
<td>$\text{Cr}<em>{43}\text{Ru}</em>{16}X_5$</td>
<td>0.018</td>
<td>-0.612</td>
<td>-33.752</td>
<td>0.055</td>
<td>7.81</td>
</tr>
<tr>
<td>$\text{Cr}<em>{42}\text{Ru}</em>{16}X_6$</td>
<td>0.011</td>
<td>-0.745</td>
<td>-40.513</td>
<td>36.549</td>
<td>9.38</td>
</tr>
</tbody>
</table>
3.3.7.1 First-row transition metal doping

a) Ru-Rich Ternary Alloys

Firstly, we carried out structural optimization of 2x2x2 Ru$_{48}$Cr$_{16}$ supercell and then we introduced dopants into the optimized material. The heats of formation for the studied bulk A15 Ru$_3$Cr and the undoped Ru$_{48}$Cr$_{16}$ supercell were calculated to be 0.39 eV/atom and 0.294 eV/atom respectively. When Ru atom in the Ru$_{48}$Cr$_{16}$ supercell structure was replaced by one X atom (X = V, Mn, Fe and Co), it was observed that heats of formation decreased from 0.294 eV/atom to 0.272 eV/atom, 0.138 eV/atom, -6.491 eV/atom and 0.259 eV/atom respectively, as presented in Table 3.4. Based on these findings, it can be clearly seen that the introduction of transition metal atom has an influence in decreasing the heat of formation of the undoped Ru$_{48}$Cr$_{16}$. We further investigated the effect of dopants by substituting two, three, four, five and six Ru atoms in the supercell by V, Mn, Fe and Co dopants, which corresponds to the atomic percentage of 3.13%, 4.69%, 6.25%, 7.81% and 9.38% respectively. The results are presented in Table 3.4, the last column represent the atomic percentage of the dopants. We obtained positive heats of formation for Ru$_{48}$Cr$_{16}$ supercell doped with one to six Co or V atoms; we observed a decrease in heats of formation with increasing Co doping concentration. Similar trend was observed when the supercell was doped with one to five V atoms, however, when the system was doped with six V atoms we noted an increased heat of formation.

The calculated heats of formation for all the studied Mn doped system are lower than the original undoped Ru$_{48}$Cr$_{16}$ with $\Delta H_f$ of 0.294 eV/atom. We predicted the positive heat of formation when one Mn atom replaced Ru atom in Ru$_{48}$Cr$_{16}$, however the negative heats
of formation were obtained for the supercell doped with two to six Mn atoms. For two, four and six Mn doped systems, the $\Delta H_f$ were computed as -0.022 eV/atom, -0.329 eV/atom and -0.593 eV/atom respectively. All studied Mn doped supercells were predicted to be stable except for one Mn doped system. The stability of ternary Ru rich alloy has improved with increasing Mn concentration.

Heats of formation for Fe doped supercell at all concentrations are negative, indicating that the Fe doped systems are stable. As in the case of Mn doped system, the level of stability was enhanced as we increase the number of Fe atoms as shown in Table 3.4. The heats of formation for the two, four and six Fe doped systems are -13.278 eV/atom, -26.842 eV/atom and -40.370 eV/atom respectively. Fe has a significant decrease in $\Delta H_f$ as compared to other studied first-row transition metal dopants. The six Fe doped system has relatively lower heat of formation when compared to all studied Ru rich doped systems as shown in Table 3.4. The lower the heat of formation of the system, the more stable it becomes. From the previous statement, we conclude that the six Fe doped Ru$_{48}$Cr$_{16}$ atom is the most stable Ru rich structure as compared to the studied 1$^{st}$ row transition metal dopants. The decrease in the heats of formation for two to six X doped Ru$_{48}$Cr$_{16}$ supercell follow the order of Fe < Mn < V < Co, whereas for one X doped system we noted a decreasing order of Fe < Mn < Co < V.

b) Cr- Rich Ternary Alloys

In addition we also calculated the heats of formation of Cr$_{48}$Ru$_{16}$ doped with one to six atoms of Co, V, Mn and Fe, in this case Cr atoms where substituted by the dopants. The results are presented in Table 3.4. It is important to mention that the studied heats of
formation for the bulk A15 Cr$_3$Ru and the undoped Cr$_{48}$Ru$_{16}$ supercell were obtained to be 0.085 eV/atom and 0.055 eV/atom respectively. For Cr rich ternaries, the heats of formation for V, Mn or Fe-doped systems decrease with increasing dopant concentration. However, we observed that the values of Δ$H_f$ for Cr rich ternaries are lower as compared to the Ru rich ternaries. For example, Δ$H_f$ for two, four and six Mn-doped Cr$_{48}$Ru$_{16}$ we obtained -0.212 eV/atom, -0.478 and -0.745 respectively, whereas for two, four and six Mn-doped Ru$_{48}$Cr$_{16}$ we obtained -0.022 eV/atom, -0.329 eV/atom and -0.593 eV/atom respectively.

Co doped Cr$_{48}$Ru$_{16}$ system exhibits a different trend compared to the V, Mn or Fe-doped systems. Heats of formation for V, Mn or Fe-doped systems are lower as compared to the undoped Cr$_{48}$Ru$_{16}$. In Table 3.4 we noted that the heats of formation for one to four Co doped are lower than the undoped system, in addition the two to four Co doped system has yielded the same value of Δ$H_f$ = 0.053 eV/atom. For five Co-doped, Δ$H_f$ increased to 0.055 eV/atom, the heats of formation for the undoped Cr$_{48}$Ru$_{16}$ and five Co-doped system are equal. The value later increased further to Δ$H_f$ = 36.549 eV/atom for six Co doped system. For Cr rich ternaries, we obtained negative heats of formation for all studied Mn and Fe-doped systems, indicating that the structures are stable. Fe and Mn enhance the stability of the systems. The decrease in the heats of formation for all the studied X doped Cr$_{48}$Ru$_{16}$ supercell follow the order of Fe < Mn < V < Co.

From Table 3.4 it is apparent that the Fe doped Cr rich alloys are more stable than the Ru rich alloys. The results indicate that the six Fe doped Cr$_{48}$Ru$_{16}$ is the most stable structure.
compared to all the studied first-row transition metal doped systems. There are no available theoretical reports about the properties of our studied transition doped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ alloys to our knowledge.

### 3.3.7.2 Second-row transition metal doping

<table>
<thead>
<tr>
<th>System</th>
<th>Heats of Formation (ΔH$_f$)</th>
<th>Atomic percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ru$<em>{47}$Cr$</em>{16}$X</td>
<td>0.263 0.266 -14.860</td>
<td>1.56</td>
</tr>
<tr>
<td>Ru$<em>{46}$Cr$</em>{16}$X$_2$</td>
<td>0.264 0.245 -30.014</td>
<td>3.13</td>
</tr>
<tr>
<td>Ru$<em>{45}$Cr$</em>{16}$X$_3$</td>
<td>0.222 0.194 -44.877</td>
<td>4.69</td>
</tr>
<tr>
<td>Ru$<em>{44}$Cr$</em>{16}$X$_4$</td>
<td>0.227 0.198 -60.320</td>
<td>6.25</td>
</tr>
<tr>
<td>Ru$<em>{43}$Cr$</em>{16}$X$_5$</td>
<td>0.241 0.192 -75.453</td>
<td>7.81</td>
</tr>
<tr>
<td>Ru$<em>{42}$Cr$</em>{16}$X$_6$</td>
<td>0.267 0.195 -90.587</td>
<td>9.38</td>
</tr>
<tr>
<td>Cr$<em>{47}$Ru$</em>{16}$X</td>
<td>0.065 0.065 -15.074</td>
<td>1.56</td>
</tr>
<tr>
<td>Cr$<em>{46}$Ru$</em>{16}$X$_2$</td>
<td>0.073 0.081 -30.201</td>
<td>3.13</td>
</tr>
<tr>
<td>Cr$<em>{45}$Ru$</em>{16}$X$_3$</td>
<td>0.081 0.093 -45.329</td>
<td>4.69</td>
</tr>
<tr>
<td>Cr$<em>{44}$Ru$</em>{16}$X$_4$</td>
<td>0.092 0.100 -40.410</td>
<td>6.25</td>
</tr>
<tr>
<td>Cr$<em>{43}$Ru$</em>{16}$X$_5$</td>
<td>0.097 0.129 -75.585</td>
<td>7.81</td>
</tr>
<tr>
<td>Cr$<em>{42}$Ru$</em>{16}$X$_6$</td>
<td>0.065 0.146 -90.710</td>
<td>9.38</td>
</tr>
</tbody>
</table>

Similar studies were conducted on Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ alloys doped with Pd, Zr or Mo. The calculated heats of formation and atomic percentages are presented in Table 3.5. The heats of formation for all studied Pd and Zr doped systems were positive which are
expected to be unstable. In Ru rich ternaries, the heats of formation increased from one Pd doped to two Pd doped system, then decrease in three Pd doped system, and later increase from four to six Pd doped system. Whereas in Cr rich ternaries, the heats of formation increased from one Pd doped to five Pd doped system, then reduced in six Pd doped alloy as shown in Table 3.5. Heats of formation for Zr doped Ru rich system decreased from one Zr doped to three Zr doped system, then increased in four Zr doped system, then reduced in five Zr doped and lastly increased in six Zr doped system. For Cr rich ternaries, the heats of formation for Zr doped system increase with the increasing Zr concentration.

The calculations predicted the negative heats of formation for all studied Mo-doped systems. However, we obtained lower heats of formation for Cr rich Mo-doped as compared to Ru rich Mo-doped systems, except for the four Mo doped structure. For example $\Delta H_f$ of two, four and six Mo doped $\text{Ru}_{48}\text{Cr}_{16}$ are -30.014 eV/atom, 60.320 eV/atom and -90.587 eV/atom respectively. Whereas for two, four and six Mo doped $\text{Cr}_{48}\text{Ru}_{16}$ the calculated heats of formation were -30.201 eV/atom, -40.410 eV/atom and -90.710 eV/atom respectively, as listed in Table 3.5. When the content of Mo is increased, the heats of formation decreased compared with the original structure. Mo doped structures are the only stable structures as compared to all studied second row transition metal doped structures. We predicted the six Mo doped $\text{Cr}_{48}\text{Ru}_{16}$ as the most stable structure amongst all the studied Mo, Zr and Pd doped systems. The decrease in the heats of formation for all the studied X (second row) doped $\text{Ru}_{48}\text{Cr}_{16}$ and $\text{Cr}_{48}\text{Ru}_{16}$ supercell
follow the order of Mo < Zr < Pd, except for one X doped Cr_{48}Ru_{16} system which follow the order of Mo < Zr = Pd.

### 3.3.7.3 Third-row transition metal doping

Table 3.6: calculated heats of formation of Pt and Re doped Ru_{48}Cr_{16} and Cr_{48}Ru_{16} alloys.

<table>
<thead>
<tr>
<th>System</th>
<th>ΔH_f (eV/atom)</th>
<th>System</th>
<th>ΔH_f (eV/atom)</th>
<th>at.%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pt</td>
<td>Re</td>
<td>Pt</td>
<td>Re</td>
</tr>
<tr>
<td>Ru_{47}Cr_{16}X</td>
<td>0.280</td>
<td>0.276</td>
<td>Cr_{47}Ru_{16}X</td>
<td>0.058</td>
</tr>
<tr>
<td>Ru_{46}Cr_{16}X_2</td>
<td>0.265</td>
<td>0.252</td>
<td>Cr_{46}Ru_{16}X_2</td>
<td>0.067</td>
</tr>
<tr>
<td>Ru_{45}Cr_{16}X_3</td>
<td>0.209</td>
<td>0.217</td>
<td>Cr_{45}Ru_{16}X_3</td>
<td>0.071</td>
</tr>
<tr>
<td>Ru_{44}Cr_{16}X_4</td>
<td>0.217</td>
<td>0.214</td>
<td>Cr_{44}Ru_{16}X_4</td>
<td>0.078</td>
</tr>
<tr>
<td>Ru_{43}Cr_{16}X_5</td>
<td>0.224</td>
<td>0.214</td>
<td>Cr_{43}Ru_{16}X_5</td>
<td>0.084</td>
</tr>
<tr>
<td>Ru_{42}Cr_{16}X_6</td>
<td>0.263</td>
<td>0.218</td>
<td>Cr_{42}Ru_{16}X_6</td>
<td>0.094</td>
</tr>
</tbody>
</table>

Lastly, we investigated the effect of Pt and Re on the heats of formation of Ru_{48}Cr_{16} and Cr_{48}Ru_{16} alloys. The results are listed in Table 3.6. Our calculations predicted positive heats of formation for the entire Pt and Re doped systems indicating the instability of the alloys. The calculated heats of formation of two, four and six Pt doped Ru_{48}Cr_{16} are 0.265 eV/atom, 0.217 eV/atom and 0.263 eV/atom respectively. For comparison purposes, we obtained 0.067 eV/atom, 0.078 eV/atom and 0.094 eV/atom for Pt doped Cr_{48}Ru_{16} alloys. It is apparent that in all cases the values for Ru rich ternary alloys were higher as compared to the Cr rich ternary alloys. The heats of formation for both the Pt and Re doped Cr_{48}Ru_{16} present a regular increase with increasing dopant concentration, in addition, the values for Re doped Cr_{48}Ru_{16} are positive but closer to zero. Attempts to improve the stability of Cr_{48}Ru_{16}. However, by introducing Pt or Re dopants to Cr_{48}Ru_{16} system have been less successful.
Figure 3.6: The heats of formation (in eV/atom) against the number of dopant atoms for (a) X-doped Ru$_{48}$Cr$_{16}$ and (b) X-doped Cr$_{48}$Ru$_{16}$.

In Figures 3.6(a) and (b) we show the overall comparison of the heats of formation against the number of X atoms for all the X doped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ respectively. The data is presented using two graphs due to scale effects. In both Figures, the upper graphs represent the Pd, Zr, V, Pt, Mn and Re-doped systems. It can be clearly seen from the upper graphs that the heats of formation for X doped Ru rich systems are higher than those of Cr rich systems are. In addition we notice negative heats of formation on the graph of Mn doped structures only (in Fig 3.6 (a) and (b)) indicating that the structures are stable. The six Mn doped Cr rich structure was found to be more stable (with the heat of formation of -0.745 eV/atom) compared to all studied Pd, Zr, V, Pt, Mn and Re-doped
systems. The main graphs present data for the heats of formation for Pd, Zr, V, Pt, Mn, Re, Mo, Co and Fe doped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ systems respectively. The plots clearly show negative heats of formation for the Fe and Mo doped structures. However, the Mo doped Cr rich structures generally have the lowest heats of formation compared to all studied X doped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ systems. Comparing the heats of formation of the more stable six Mn, six Fe or six Mo doped Cr$_{48}$Ru$_{16}$, which were calculated as -0.745 eV/atom, -40.513 eV/atom and -90.710 eV/atom, it has turned out that the six Mo doped Cr$_{48}$Ru$_{16}$ is the most stable structure compared to all the X-doped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ systems. Despite few fluctuations, which require further research, our results seem to confirm more general conclusion that an increasing amount of Fe, Mn and Mo in the ternary alloys have stabilising effect on the properties.

### 3.3.8. Magnetic Moments

#### Overview

The magnetic moment of a material consists of contributions from spin and orbit polarization. The total spin magnetic moment in $\mu_B$ is the difference between the number of spin-up occupied states and the spin-down occupied states. The symbol $\mu_B$ is the Bohr magneton, the units for expressing magnetic moment of an electron caused by either its orbital or spin angular momentum [135-136].
In the current study, we used CASTEP code to calculate the magnetic moments of the systems. Magnetism originates from the spin and orbital magnetic moment of an electron. There are different types of magnetism in materials; however, for this section we define the ferromagnetism, anti-ferromagnetism and ferrimagnetism.

<table>
<thead>
<tr>
<th>Smearing width (eV)</th>
<th>Magnetic moment (µB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.02</td>
</tr>
<tr>
<td>0.2</td>
<td>0.00</td>
</tr>
<tr>
<td>0.1</td>
<td>0.12</td>
</tr>
<tr>
<td>0.01</td>
<td>1.78</td>
</tr>
<tr>
<td>0.001</td>
<td>2.00</td>
</tr>
<tr>
<td>0.0005</td>
<td>2.00</td>
</tr>
<tr>
<td>0.00025</td>
<td>2.33</td>
</tr>
</tbody>
</table>

The coupling between spins [137] can form ordered magnetic systems such as ferromagnetic, anti-ferromagnetic and ferrimagnetic. In ferrimagnets the magnetic moments of two sublattice in a system are not equal and result in a net magnetic moment, whilst in antiferromagnets the two sublattice moments are exactly equal but opposite, resulting in a net moment of zero. Lastly, in ferromagnets, the atomic moments exhibit very strong interaction produced by the electronic exchange forces and result in the parallel or anti-parallel alignment of atomic moments. Ferromagnets have a parallel alignment of moments that gives rise to a net magnetisation.
Table 3.8: Calculated magnetic moments (per atom) of individual atoms in ternary structures

<table>
<thead>
<tr>
<th>Metals</th>
<th>Magnetic moments (µB) this work</th>
<th>Magnetic moments (µB) previous work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ru</td>
<td>-0.000015</td>
<td></td>
</tr>
<tr>
<td>Cr</td>
<td>0.000050</td>
<td>±1.21[139]</td>
</tr>
<tr>
<td>Re</td>
<td>0.0005</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>-0.032</td>
<td></td>
</tr>
<tr>
<td>Fe</td>
<td>2.299</td>
<td>4.00[115]</td>
</tr>
<tr>
<td>Co</td>
<td>1.627</td>
<td>3.00[140]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.55[141]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.58[142]</td>
</tr>
<tr>
<td>Zr</td>
<td>-0.000059</td>
<td></td>
</tr>
<tr>
<td>Mn</td>
<td>5.0</td>
<td>5.0[140]</td>
</tr>
<tr>
<td>Mo</td>
<td>0.00018</td>
<td></td>
</tr>
</tbody>
</table>

3.3.8.1 Magnetic moments of the X-doped Ru<sub>48</sub>Cr<sub>16</sub> system

The magnetic moments of X (X = Zr, Mo, V, Mn, Fe, Co, Pt, Pd, Re) doped Ru<sub>48</sub>Cr<sub>16</sub> have been investigated by first-principles calculations using the pseudopotentials plane wave method. CASTEP code has been used to calculate the magnetic moments, which is the total integrated spin density calculated with a smearing width of 0.001 eV. It is important to mention that the magnetic moment is sensitive to the smearing width.

We performed smearing width tests on one Mn doped Cr<sub>48</sub>Ru<sub>16</sub> alloy, where we varied the width from 0.3 µB to 0.000025 µB. For each value, we then determined the magnetic moment, the results are provided in Table 3.7. The tests were performed in order to analyse the effect of smearing width on the magnetic moment of the system.
Table 3.9: Calculated magnetic moments (per unit cell) of X doped Ru$_{48}$Cr$_{16}$ alloys.

<table>
<thead>
<tr>
<th>System</th>
<th>Magnetic moments (µB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Zr</td>
</tr>
<tr>
<td>Ru$<em>{47}$Cr$</em>{16}$X</td>
<td>-0.041</td>
</tr>
<tr>
<td>Ru$<em>{46}$Cr$</em>{16}$X$_2$</td>
<td>-0.341</td>
</tr>
<tr>
<td>Ru$<em>{45}$Cr$</em>{16}$X$_3$</td>
<td>-0.007</td>
</tr>
<tr>
<td>Ru$<em>{44}$Cr$</em>{16}$X$_4$</td>
<td>-0.125</td>
</tr>
<tr>
<td>Ru$<em>{43}$Cr$</em>{16}$X$_5$</td>
<td>-0.05</td>
</tr>
<tr>
<td>Ru$<em>{42}$Cr$</em>{16}$X$_6$</td>
<td>-0.145</td>
</tr>
</tbody>
</table>

From the Table 3.8, it can be noted that a decrease in the smearing width leads to an increased accuracy of the magnetic moment. We obtained the same value when the smearing widths of 0.001 eV and 0.0005 eV were considered. The magnetic moment does not change significantly beyond 0.01 µB smearing width [138].

The magnetic moments for X doped Ru$_{48}$Cr$_{16}$ systems were calculated using a smearing width of 0.001 eV. We also computed the magnetic moments of the individual atoms in ternary structures in order to have a better understanding of magnetism; Table 3.8 presents the atomic moments of studied alloys constituencies. Table 3.9 provides a detail comparison of the calculated magnetic moment for different doping concentration. For each dopant, six magnetic moments corresponding to one, two, three, four, five and six X dopants were calculated.
We obtained negative magnetic moments for all the studied Zr and Re doped systems as seen in Table 3.8. The values reveal a striking variation of magnetic moment with various dopants and doping concentration.

For low content of Mo, magnetic moment decreases from 0.012 µB to 0.010 µB and -0.0126 µB for one, two and three Mo atoms doped systems respectively, then increases from four Mo doped (-0.058 µB) to five Mo doped (-0.012 µB) structures, then finally decrease for six Mo doped alloy (-0.098 µB). It is predicted that the 4d Mo electron substitution in 4d Ru will lead to anti-ferromagnetic coupling between Cr-Mo because of their half-filled 4d orbitals. A study was carried out by G.L. High et al using electrical resistivity measurements to show that (Cr$_{1-x}$Ru$_x$)$_{78}$Mo$_{22}$ is always paramagnetic at all temperatures in a range of 0 ≤ x ≤ 1 concentration of at % Ru and becomes antiferromagnetic in an array of 1 ≤ x ≤ 10 concentration at % Ru [143].

In Mn doped systems, we detected high and low magnetization in one Mn doped (0.012 µB) and three Mn doped (-0.126 µB) systems respectively. The positive magnetic moments were obtained for one to five Mn doped systems, however a negative value was obtained for six Mn doped alloy. Positive values indicate that the most significant contribution comes from Mn with a moment of 5.0 µB that is higher than 4.0 µB, indicating that Mn is highly magnetized [144]. Whereas, a negative magnetic moment is due to Ru major contribution of -0.000015 µB as compared to Mn (5.0 µB) and Cr (0.000050 µB) as listed in Table 3.9.
In comparison to the magnetic moments of the other studied dopants in Table 3.9, it is noted that the Mn doped systems have high magnetic point at one Mn doped (0.468 µB), two Mn doped (0.359 µB) and five Mn doped (0.569 µB) systems.

In addition, we computed high magnetic moments for three Fe doped (0.826 µB) and six Fe doped (0.285 µB) systems. The calculated values for all the studied Fe doping concentrations are positive except for the five Fe doped alloy (-0.645 µB). The positive values are due to Fe spin contribution of 2.299 µB (listed in Table 3.9), this value does not compare well with the experimental value of 4 µB. Negative value is an indication that Ru contributed more than Cr and Fe; which caused an antiferromagnetic coupling between Fe-Cr due to their half-filled d-orbital [145]. Recent investigation highlighted that negative iron as antiferromagnetic system; based on the assumption that Fe has negative neighboring ions in a face centered cubic lattice.

Five Co doped Ru_{48}Cr_{16} is highly magnetized (0.145 µB) whereas four Co doped system has low magnetization character of -0.254 µB. Lastly, an overall comparison of X doped Ru_{48}Cr_{16} shows that four Pt doped system has the highest magnetic moment (0.980 µB) as compared to all dopants and for all concentrations. Hence, we assume that the coupling between Pt-Cr in Pt doped Ru_{48}Cr_{16} structure can influence the system to behave like a ferrimagnetic material.

**3.3.8.2. Magnetic moments of the X-doped Cr_{48}Ru_{16} system**
We further performed a number of calculations for the magnetic moments of the X-doped Cr\textsubscript{48}Ru\textsubscript{16}. The results are shown in Table 3.10, our computed magnetic moments for X (Zr, Mo, V, Pd, and Re) doped Cr\textsubscript{48}Ru\textsubscript{16} are higher than the corresponding X doped Ru\textsubscript{48}Cr\textsubscript{16}. The values for five and six Mn doped, four and six Pt doped, two and six Co doped and one to three Fe doped Cr\textsubscript{48}Ru\textsubscript{16} are lower than the Ru rich doped systems.

From the two statements above, one can conclude that the studied Cr rich ternary alloys are generally more magnetic than the Ru rich ternaries. Our calculations predicted the positive magnetic moments for all the studied concentration of Zr, Mo, V, Fe, Co and Re doped Cr\textsubscript{48}Ru\textsubscript{16} systems. The results for Zr and Re doped Cr\textsubscript{48}Ru\textsubscript{16} are in contrast to the Ru rich system in Table 3.9 that indicated negative values for Zr and Re doped materials. Magnetic moments of Mo doped system increased from one to two Mo doped, and then decreased for three and four Mo doped, increased again for five Mo doped then finally
decrease for six Mo doped system. The two Mo doped is high in magnetization (2.511 μB) while four Mo doped system has low magnetization (0.068 μB). The positive values for all Mo doped systems are positive due to more contributions from Mo and Cr, we predict that Mo-Ru can couple ferromagnetically [146].

A zero magnetization was observed for four V doped Cr rich system. For low content of Mn (one to three Mn doped), it is evident that the addition of Mn content decreases the magnetic moment. The six Mn doped system is highly magnetized with a moment of 2.001 μB, while the five Mn doped system has a low magnetization character at -0.091 μB. The positive values for all Fe doped systems are influenced by high contribution of Fe as compared to Cr and Ru. Fe-Ru was predicted to couple Ferro-magnetically.

The results in Table 3.10 gives the magnetic moment of 1.444 μB and 0.006 μB for four Co doped and two Co doped systems respectively. We previously determined magnetic moment of atomic Co as 1.627 μB, this value does not compare well with the experimental value of 3 μB, however, there is a good agreement with theoretical values of 1.55 μB and 1.58 μB as indicated in Table 3.8. One, two, three and five Pt doped Cr rich systems have higher magnetic moments than the Pt doped Ru rich systems. These findings are influenced by highly magnetized antiferromagnetic Cr with a partial moment of 0.00005 μB.

One Pt doped system has high magnetization (1.53 μB) whereas six Pt doped show a sign of low magnetization character with -0.129 μB. When Pt bond with Cr they can form
alloy composition in a continuous mode between 100% Pt and 100% Cr. It was observed that Pt₃Cr alloy possess ferrimagnetism at a curie temperature of 170 °C [147].

The overall results of the comparison of the magnetic moments of all X-doped Cr₄₈Ru₁₆ indicate the high obtained magnetic moments as we increase the doping concentration as follows: One Re doped (2.980 µB), two Mo doped (2.511 µB), three Mo doped (2.023 µB), four Co doped (1.444 µB) five Mo doped (2.025 µB) and six Re doped (1.128 µB). One Re doped system is highly magnetized as compared to all X doped Cr₄₈Ru₁₆ systems. In the next chapter, we discuss the density of states in order to observe the d orbitals contribution of each metal around the Fermi energy.
CHAPTER 4

Electronic properties

4.1 Introduction

Density of states (DOS) of a system is defined as the number of different states at a particular energy level that electrons are allowed to occupy. A high DOS at a specific energy level means that there are many states available for occupation. A DOS of zero means that no states can be occupied at that energy level. The Fermi level is one of the important concepts in the solid-state physics of metals, its position of the Fermi level with the relation to the band energy levels is a crucial factor in determining the stability or the instability of the metal alloys. In this chapter, we discuss in more detail the DOS of metallic Ru, Cr, Fe, Pt, V and Mo. In addition, we will also discuss the DOS of six Fe, V, Mn and Mo doped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ alloys to provide more information about the investigated systems.

4.1.1 Methodology

Calculations have been performed within the DFT and the pseudo-potential approach, using the CASTEP package. The smearing width of 0.1 eV was used to ensure the convergence of the self-consistence procedure and eradicate discontinuity in the energy variation as the electron band passes the Fermi level. The ultra-soft pseudopotential was used to describe the electron-core interaction. The DOS calculations
Figure 4.1 : Periodic table for transition metals

were performed for Ru = [Kr] 4d^6 5s^2, Cr = [Ar] 3d^5 4s^1, V = [Ar] 3d^3 4s^2, Mn = [Ar] 3d^5 4s^2, Fe = [Ar] 3d^6 4s^2 and Mo = [Kr] 4d^5 5s^1. The studied metals belong to the 1st and 2nd rows of the transition metals in the periodic table as shown in Figure 4.1. V, Cr, Mn and Fe belong to the 1st row, while Mo and Ru belong to the 2nd row. We will also present the DOS of the predicted thermodynamically stable structures; the six Mn, Fe and Mo doped Ru Rich and Cr Rich alloys. Mo and Cr possess half- filled d-orbital that have high chance of attaining stability when alloyed with other materials. The six V doped system will be presented for a comparison purpose.
4.1.2 Density of states for pure metals

The density of state for metallic Cr is displayed in Figure 4.2. From the DOS we can clearly depict the continuity of the density of states, there is no band gap found at the Fermi level as the valence and conduction bands overlap significantly near the Fermi level. These observations indicate that Cr exhibit the metallic character. The d states contribute more to the DOS of Cr than the s and p states. The high peak of the valence band DOS is located at \( \approx 2.0 \) eV. The main peak of the valence band DOS is higher than the peaks of the conduction band DOS.

Figure 4.3 shows the calculated density of states of Ru. The main contribution in both the valence and conduction bands emanate from the d-states. The total DOS of metallic Ru consist of a Fermi level that falls into a pseudo-gap hence this metal may have stability characteristics [148,149,150].

From the DOS of metallic Mn is Figure 4.4 we note fewer bands below the Fermi as compared to a visible large number of bands in the conduction region. There is an overlap between the valence and conduction bands. The main peak of the total and partial DOS of Mn is located at the Fermi level; this is due to the 3d-Mn electrons. However, the bands at the Fermi level may be broken by doping. The states around 2.3 eV and 4.1 eV are the p electrons of the Mn atoms in the conduction band. The total DOS indicate two broad peaks at \( \approx 3.5 \) eV and 7.6 eV in the conduction region. In conclusion, we observed a semi-metal character for Mn material.
Figure 4.2: The calculated partial and total density of states for metallic Cr. Fermi energy corresponds to the solid line ($E_f$).
Figure 4.3: Partial and total density of states (DOS) of pure Ru metal. A solid line ($E_f$) shows the Fermi energy.
Figure 4.4 : Partial and total DOS of pure metallic Mn.
Figure 4.5 presents the DOS of metallic Fe. The Fermi energy level ($E_f$) falls on a largest peak located in the conduction side as shown by the DOS of metallic Fe. This can imply that the Fe metal exhibit the ductility characters [151-152]. We note a very small contribution of the Fe s and p states as compared to the d states. High DOS near the Fermi energy level is observed; this is due to the d-state contribution.

Figure 4.6 shows the calculated DOS of metallic Mo. Both the valence and conduction bands are mainly dominated by the Mo d-states. The main peak in the conduction band is slightly higher than the main peak in the valence band. We observed interesting features from the density of states of Mo; the narrow dip and broad dip were observed in the DOS at valence band and conduction band respectively. The Fermi level falls on a smaller peak together with the broad dip indicating the material’s stable behavior [151-152].

From the DOS of metallic V in Figure 4.7 we note that not all the bonding states are filled as the $E_f$ lies on the left of the narrow pseudo-gap [151-157]. The V d-states stretch from -3 eV to 5 eV. The contribution from the V s and p states is low. We observed a smaller main peak in the valence band compared to the conduction band. There is a visible separation of two d-state peaks by the narrow dip (pseudo-gap). We will discuss the density of states of the binary and ternary alloys in the next sections.
Figure 4.5: Partial and total density of states (DOS) for metallic Fe. A solid line presents the Fermi energy ($E_f$).
Figure 4.6: Total and partial densities of states for pure Mo metal. Zero energy corresponds to Fermi energy.
Figure 4.7: Total and partial DOS of transition metal V. The Fermi energy ($E_f$) indicates zero energy.
4.1.3 Density of states for the undoped Cr\textsubscript{48}Ru\textsubscript{16} and Ru\textsubscript{48}Cr\textsubscript{16} Alloys

Figure 4.8: Total and partial DOS for undoped (a) Ru\textsubscript{48}Cr\textsubscript{16} and (b) Cr\textsubscript{48}Ru\textsubscript{16} structures. The Fermi energy indicates zero energy.
In Figures 4.8 (a) and (b) a comparison of the calculated density of states for the undoped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ compounds are shown respectively. The total density of states (TDOS) of the Ru rich and Cr rich binary alloys, and the partial density of states (PDOS) of Ru and Cr are displayed. The solid lines in all the TDOS and PDOS represent the Fermi level. It can be seen from both DOS that there is no band gap at the Fermi level as valence band and conduction bands overlap near the Fermi level, as a result the Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ exhibits the metallic character. All the DOS curves in Figure 4.8 mainly originates from the Cr (3d) and Ru (4d) states, with small contribution from their s and p states.

It is clear from Figure 4.8 (b) that the total DOS of Cr$_{48}$Ru$_{16}$ alloy consist more states in the conduction band than those in TDOS of Ru$_{48}$Cr$_{16}$. We observed low electron density of states at the Fermi level from both the TDOS and PDOS of Cr$_{48}$Ru$_{16}$ and Cr and Ru respectively as shown in Figure 4.8 (b). Meanwhile a higher electron density of states was observed near the Fermi level in the TDOS of Ru$_{48}$Cr$_{16}$ and PDOS of Cr and Ru of Ru rich alloy as indicated in Figure 4.8(a). In both cases, we noted that the PDOS of Ru have fewer states at Fermi level as compared to the PDOS of Cr.

In the PDOS curve of Ru and Cr in Ru$_{48}$Cr$_{16}$ in Figure 4.8(a), the Fermi energy level (E$_f$) falls on top of a dip due to the presence of their d states. Position of the E$_f$ in relation to the band energy levels is crucial in determining the electrical properties of solids. When we compare the position of E$_f$ in DOS curves of Cr$_{48}$Ru$_{16}$ to Ru$_{48}$Cr$_{16}$, we observed a small shift of E$_f$ to the left in the TDOS and PDOS curves of Cr$_{48}$Ru$_{16}$. This shift gives a large variation in the number of states at the Fermi level that can be used to determine the
stability of alloys. In the next section, we investigate the effect of doping on the
electronic stability of Cr$_{48}$Ru$_{16}$ and Ru$_{48}$Cr$_{16}$ alloys.

4.1.4 Density of states for the X- doped Cr$_{48}$Ru$_{16}$ and Ru$_{48}$Cr$_{16}$ Alloys:

Ternary alloying may strengthen the electronic properties of the systems. In this section, we investigate the effect of Mn, Fe, Mo and V dopants on the electronic structure of the Cr$_{48}$Ru$_{16}$ and Ru$_{48}$Cr$_{16}$ alloys. We calculated the density of states for the most stable configurations of the X-(Mn, Fe, V and Mo) doped Cr$_{48}$Ru$_{16}$ and Ru$_{48}$Cr$_{16}$ systems. In both cases, the six X atom-doped systems were predicted as the most thermodynamically stable ternaries. We will also consider the systems doped with six V atoms to develop a better understanding of the electronic properties.

Figure 4.9 (a) and (b) show the calculated TDOS and PDOS of Ru$_{42}$Cr$_{16}$Fe$_6$ and Cr$_{42}$Ru$_{16}$Fe$_6$ respectively. From the PDOS of Ru and Cr in Ru$_{42}$Cr$_{16}$Fe$_6$ we noted that $E_f$ lies in a pseudo-gap. The TDOS of the Ru rich ternary exhibits the same behavior. The $E_f$ in the PDOS and TDOS of Cr rich alloys shifts to the right of the pseudo-gap. As a result, the value of $N(E_f)$ in TDOS of Cr rich alloy is slightly higher than in Ru rich system. The PDOS and TDOS of Ru$_{42}$Cr$_{16}$Mo$_6$ and Cr$_{42}$Ru$_{16}$Mo$_6$ structures are shown in Figure 4.10 (a) and Figure 4.10 (b) respectively. The main contribution for both configurations comes from the d states of Ru, Cr and Mo atoms. The DOS and PDOS curves of the Mo-doped Ru rich alloy reveal no peaks in the conduction band. The previous statement implies that there is no energy difference between the main peaks of the valence and conduction band of the Ru$_{42}$Cr$_{16}$Mo$_6$ ternary alloy. However, the energy
difference between the main peak of valence band and that of conduction band in the TDOS of Cr$_{42}$Ru$_{16}$Mo$_6$ was found to be 3.6 eV as shown in Figure 4.10 (b). The DOS of Ru$_{42}$Cr$_{16}$Mn$_6$ and Cr$_{42}$Ru$_{16}$Mn$_6$ structures are presented in Figure 4.11 (a) and Figure 4.11 (b) respectively. Most of the DOS lie in the valence band for both configurations; the DOS are mainly attributed to the d states of Cr and Ru. It is clear from the PDOS in Cr$_{42}$Ru$_{16}$Mn$_6$ that the s, p and d states are moved to lower energy region as compared to the Ru$_{42}$Cr$_{16}$Mn$_6$. In addition, the conducted band has shifted to low energy range. From the results of PDOS of Mn in Figure 4.11 (b), we note a narrow peak in the conduction band at 0.5 eV.

The TDOS and PDOS plots for Ru$_{42}$Cr$_{16}$V$_6$ and Cr$_{42}$Ru$_{16}$V$_6$ are displayed in Figure 4.12 (a) and (b) respectively. The valence band of both V-doped alloys mainly consists of the d states of V, Cr and Ru. The main peak in the PDOS curve of V in Cr rich alloy is higher compared to the PDOS of V in Ru rich alloys.

In general, the conduction band from all the DOS of Ru$_{42}$Cr$_{16}$Mo$_6$ is narrower than the DOS of the studied X-doped materials. In both configurations of Mo-doped systems, most of electrons are accumulated in the lower energy region as compared to Mn and Fe-doped systems in Figures 4.11 and 4.9 respectively, indicating higher level of stability for Mo-doped systems. To the best of our knowledge, there are no previous findings to compare with our computed results for the density of states of the six Mn, Mo, V and Fe–doped systems. In the next chapter, we present the results of the calculated elastic properties of the studied alloys.
Figure 4.9: Total and partial density of states for (a) $\text{Ru}_{42}\text{Cr}_{16}\text{Fe}_6$ and (b) and $\text{Cr}_{42}\text{Ru}_{16}\text{Fe}_6$ alloys.
Figure 4.10: Total and partial DOS for (a) Ru$_{42}$Cr$_{16}$Mo$_6$ and (b) Cr$_{42}$Ru$_{16}$Mo$_6$ alloys. A solid line represents the Fermi energy.
Figure 4.11: The partial and total density of states for (a) Ru$_{42}$Cr$_{16}$Mn$_6$ and (b) Cr$_{42}$Ru$_{16}$Mn$_6$ alloys. A solid line indicates the Fermi energy.
Figure 4.12: Partial and total density of states for (a) Ru$_{42}$Cr$_{16}$V$_6$ and (b) Cr$_{42}$Ru$_{16}$V$_6$ alloys. The position of the Fermi level is at 0 eV.
Chapter 5

Elastic constants

5.1 Introduction

Elastic properties are of importance to the solids because they relate to different solid-state properties such as interatomic potential, equation of state and phonon spectra. In addition, these properties are linked thermodynamically to the specific heat, melting points and thermal expansion. Furthermore, these elastic properties of a material are well known to be used in characterizing the structural stability and mechanical deformation under external load [158].

Within these properties we are more interested in calculating the, bulk, shear, Young’s modulus and Poisson’s ratio of pure metals, Ru₃Cr, Cr₃Ru, Cr₄₂Ru₁₆X₆ and Ru₄₂Cr₁₆X₆ (where X = Fe, Mn and Mo) systems. The bulk modulus represent the average bond strength of a material, shear modulus shows the material’s response to shear stress, while Young's modulus is used to determine the stiffness of a material. In addition, the ratio of bulk to shear will assist in order to tell if the material is ductile or brittle.

Important information will be extracted from the properties above (bulk, shear and Young’s modulus) of how stable the structures are, furthermore the Stress-Strain relation [159] will be used to compute the elastic constants. In addition, these elastic constants have a stability criterion obtained from Born mechanical stability for cubic, hexagonal and orthorhombic. However, in this study we will concentrate on cubic systems as this are what our structures possess. Moreover, for cubic we have (C₁₁, C₁₂ and C₄₄) and the
conditions are stated below as follows: \( C_{11} > 0, C_{12} > 0, C_{11} - C_{12} > 0, C_{44} > 0 \) \([160]\). The \( C_{44} \) and \( C_{66} \) are elastic constants which are more important in giving data for stability of other systems with regard to the hardness of the material. The Poisson’s ratio will be calculated using the formula below.

\[
\nu = \frac{3B_{VRH} - 2G_{VRH}}{2(3B_{VRH} + G_{VRH})}
\]

Where \( \nu \) is the Poisson’s ratio, \( B_{VRH} \) is the average bulk for Voigt, Reuss and Hill, and \( G_{VRH} \) is the average shear modulus of Voigt, Reuss and Hill. Furthermore, the bulk, shear and Young modulus will be calculated by (CASTEP) code based on density functional theory. From the bulk modulus we will detect the resistance to the change in volume, shear modulus will detect the stiffness on the material and the young modulus will information about the elasticity of the material. In this section the elastic constants, bulk, shear, young moduli, ratio of Bulk to shear and Poisson ratio of pure metals, A15Cr3Ru, A15Ru3Cr, A15Cr42X6Ru16 and A15Ru42X6Cr16 (X=Mo, Mn and Fe) systems are analysed. Table 5.1 show the computed elastic constants for the systems mentioned above.
Table 5.1: Calculated elastic constant $C_{ij}$ in GPa of Pure metals, undoped and X6 doped A15Ru$_{48}$Cr$_{16}$ and A15Cr$_{48}$Ru$_{16}$ where (X = Mo, Mn and Fe)

<table>
<thead>
<tr>
<th>System</th>
<th>$C_{11}$</th>
<th>$C_{12}$</th>
<th>$C_{13}$</th>
<th>$C_{16}$</th>
<th>$C_{33}$</th>
<th>$C_{44}$</th>
<th>$C_{66}$</th>
<th>$C_{12}-C_{44}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mo</td>
<td>457.75</td>
<td>163.59</td>
<td>93.75</td>
<td>5.22</td>
<td>5.21</td>
<td>-0.05</td>
<td>164.20</td>
<td>14.98</td>
</tr>
<tr>
<td>Mn</td>
<td>5.22</td>
<td>5.21</td>
<td>-0.05</td>
<td>5.26</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ru</td>
<td>569.74</td>
<td>179.18</td>
<td>164.20</td>
<td>14.98</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cr</td>
<td>639.39</td>
<td>76.79</td>
<td>63.09</td>
<td>13.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ru$<em>{48}$Cr$</em>{16}$</td>
<td>439.75</td>
<td>195.89</td>
<td>44.71</td>
<td>151.18</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cr$<em>{48}$Ru$</em>{16}$</td>
<td>581.66</td>
<td>209.49</td>
<td>128.23</td>
<td>81.26</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ru$<em>{42}$Cr$</em>{16}$Mo$_{6}$</td>
<td>426.70</td>
<td>210.38</td>
<td>224.87</td>
<td>-0.141</td>
<td>414.67</td>
<td>91.71</td>
<td>97.71</td>
<td>118.67</td>
</tr>
<tr>
<td>Cr$<em>{42}$Ru$</em>{16}$Mo$_{6}$</td>
<td>518.06</td>
<td>197.63</td>
<td>194.25</td>
<td>0.158</td>
<td>516.39</td>
<td>103.14</td>
<td>103.66</td>
<td>94.49</td>
</tr>
<tr>
<td>Ru$<em>{42}$Cr$</em>{16}$Mn$_{6}$</td>
<td>348.33</td>
<td>216.73</td>
<td>180.89</td>
<td>2.655</td>
<td>377.89</td>
<td>73.08</td>
<td>78.71</td>
<td>143.65</td>
</tr>
<tr>
<td>Cr$<em>{42}$Ru$</em>{16}$Mn$_{6}$</td>
<td>417.44</td>
<td>131.48</td>
<td>146.76</td>
<td>-0.260</td>
<td>454.75</td>
<td>99.07</td>
<td>99.28</td>
<td>32.41</td>
</tr>
<tr>
<td>Ru$<em>{42}$Cr$</em>{16}$Fe$_{6}$</td>
<td>343.22</td>
<td>190.93</td>
<td>194.87</td>
<td>-1.944</td>
<td>355.78</td>
<td>96.57</td>
<td>98.16</td>
<td>94.36</td>
</tr>
<tr>
<td>Cr$<em>{42}$Ru$</em>{16}$Fe$_{6}$</td>
<td>424.72</td>
<td>169.70</td>
<td>163.78</td>
<td>-0.545</td>
<td>424.82</td>
<td>87.49</td>
<td>87.17</td>
<td>82.21</td>
</tr>
</tbody>
</table>

5.1.1 Elastic constants of Pure metals and undoped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ systems

The elastic constants of the pure metals were computed and presented in Table 5.1; we noted that all the elastic constants satisfy the Born mechanical stability criterion. Ruthenium has high $C_{44}$ elastic constant value of 164.20 GPa that compare very well with Popoola et.al [15] results, while Manganese has the lowest value of -0.05 GPa.
In comparing the results in Table 5.1 for the Ru_{48}Cr_{16} and Cr_{48}Ru_{16} systems, it was found that C_{11}, C_{12} and C_{44} are increasing from A15 Ru_{48}Cr_{16} to A15 Cr_{48}Ru_{16}. However, the C_{44} in both configurations was observed to have the smaller values than the C_{11} and C_{12}. The C_{44} elastic constant is very essential in determining the hardness of a material. The Cr_{48}Ru_{16} has a large value of C_{44}, which implies that it has strong ability in resisting shear distortion in materials.

More over the C_{11} and C_{12} constants of Cr_{48}Ru_{16} are higher than Ru_{48}Cr_{16} meaning these materials are strong against compression in a, b and c axis. The Cauchy pressure calculated as (C_{12}-C_{44}) helps to describe the angular character of atomic bonding in metals [161] and show ductility or brittleness of a material. In Table 5.1, the Cauchy pressure values of Ru_{48}Cr_{16} and Cr_{48}Ru_{16} systems are positive meaning that there is a sign of metallic bonding and ductility. Furthermore, they follow a Cauchy pressure trend of

A15 Cr_{48}Ru_{16} < A15 Ru_{48}Cr_{16}

5.1.2 Elastic constants of X-doped Ru_{48}Cr_{16} and Cr_{48}Ru_{16} systems

The elastic constants for the six X atoms doped Ru_{48}Cr_{16} and Cr_{48}Ru_{16} structures (X = Mo, Mn and Fe) are discussed in details. The C_{44} elastic constant play a vital role in describing the hardness of a material. We noted a decreasing trend of C_{44} from six Mo-doped to six Mn-doped then later, to six Fe-doped Cr_{48}Ru_{16}. It can be seen that the six Cr_{42}Ru_{16}Mo_{6} has a high C_{44} constant of 103.14 GPa, whereas the Cr_{42}Ru_{16}Fe_{6} has the lowest value of 87.49 GPa. Further, The Cauchy pressure values are positive predicting
the possibility of metallic bonding. The six Mo-doped Cr$_{48}$Ru$_{16}$ has the highest C$_{44}$ and Cauchy pressure. The trend of decreasing Cauchy pressure for Cr rich alloys is as follows:

six Mo-doped > six Fe-doped > six Mn-doped

For the six X-doped Ru$_{48}$Cr$_{16}$ systems shown in Table 5.1, it is clearly seen that six Fe-doped Ru$_{48}$Cr$_{16}$ has the highest C$_{44}$ elastic constant of 96.57 GPa whilst the six Mn-doped Ru$_{48}$Cr$_{16}$ has the lowest C$_{44}$ of 73.08 GPa. The Cauchy pressure of six Mn-doped Ru$_{48}$Cr$_{16}$ is higher (143.65 GPa) whereas the six Fe-doped Ru$_{48}$Cr$_{16}$ has a lower value of (94.36 GPa). In addition, the Cauchy pressure trend for Ru rich alloys shown below:

six Mn-doped > six Mo-doped > six Fe-doped

Comparing the six X-doped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ systems, we observed that the elastic constants C$_{11}$ and C$_{44}$ and C$_{66}$ for six Mo and six Mn-doped Ru$_{48}$Cr$_{16}$ are lower compared to those of the six Mo and six Mn-doped Cr$_{48}$Ru$_{16}$ systems; however, the C$_{12}$ values are decreasing from Ru rich to Cr rich alloys in all configurations. From the above statements it is clear that Mo, Mn doped Cr$_{48}$Ru$_{16}$ can resist shear distortion easily than Mo, Mn, Fe doped Ru$_{48}$Cr$_{16}$ [162-163]. For Fe doped Ru$_{48}$Cr$_{16}$ system tend to have the same trend for C$_{12}$ values decreasing with increasing C$_{11}$ but opposite trend have been noticed for C$_{44}$ and C$_{66}$. 
Table 5.2: The calculated average bulk (B), shear (G) and young (E) modulus in GPa and Poisson's ratio (ʋ) of Pure metals, undoped and X₆ doped Ru₄₈Cr₁₆, Cr₄₈Ru₁₆ systems where (X = Fe, Mo and Mn)

<table>
<thead>
<tr>
<th>System</th>
<th>B</th>
<th>G</th>
<th>E</th>
<th>B/G</th>
<th>ʋ</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>189.05</td>
<td>91.82</td>
<td>127.84</td>
<td>2.06</td>
<td>0.29</td>
</tr>
<tr>
<td>Mo</td>
<td>261.65</td>
<td>112.37</td>
<td>186.73</td>
<td>2.33</td>
<td>0.31</td>
</tr>
<tr>
<td>Mn</td>
<td>5.21</td>
<td>-0.01</td>
<td>5.21</td>
<td>-521</td>
<td>0.50</td>
</tr>
<tr>
<td>Ru</td>
<td>308.71</td>
<td>187.42</td>
<td>183.56</td>
<td>2.06</td>
<td>0.25</td>
</tr>
<tr>
<td>Cr</td>
<td>264.33</td>
<td>120.92</td>
<td>183.71</td>
<td>2.19</td>
<td>0.30</td>
</tr>
<tr>
<td>Ru₄₈Cr₁₆</td>
<td>277.18</td>
<td>67.74</td>
<td>232.02</td>
<td>4.09</td>
<td>0.39</td>
</tr>
<tr>
<td>Cr₄₈Ru₁₆</td>
<td>333.55</td>
<td>148.91</td>
<td>233.61</td>
<td>2.23</td>
<td>0.35</td>
</tr>
<tr>
<td>Ru₄₂Cr₁₆Mo₆</td>
<td>293.49</td>
<td>94.52</td>
<td>230.48</td>
<td>3.10</td>
<td>0.35</td>
</tr>
<tr>
<td>Cr₄₂Ru₁₆Mo₆</td>
<td>303.02</td>
<td>123.07</td>
<td>220.98</td>
<td>2.46</td>
<td>0.32</td>
</tr>
<tr>
<td>Ru₄₂Cr₁₆Mn₆</td>
<td>256.44</td>
<td>82.96</td>
<td>201.13</td>
<td>3.09</td>
<td>0.35</td>
</tr>
<tr>
<td>Cr₄₂Ru₁₆Mn₆</td>
<td>249.30</td>
<td>114.87</td>
<td>160.73</td>
<td>2.17</td>
<td>0.35</td>
</tr>
<tr>
<td>Ru₄₂Cr₁₆Fe₆</td>
<td>245.39</td>
<td>88.06</td>
<td>186.68</td>
<td>2.79</td>
<td>0.32</td>
</tr>
<tr>
<td>Cr₄₂Ru₁₆Fe₆</td>
<td>253.19</td>
<td>101.91</td>
<td>185.25</td>
<td>2.48</td>
<td>0.32</td>
</tr>
</tbody>
</table>

5.1.3 Elastic moduli of Pure metals, undoped and six X doped Cr₄₈Ru₁₆, Ru₄₈Cr₁₆ structures, (X =Fe,Mn and Mo)

In Table 5.2 we present the calculated bulk, shear, Young's modulus, the ratio of bulk to shear modulus and Poisson’s ratio for the pure metals, the six X (X = Mn, Fe and Mo) doped and undoped Ru₄₈Cr₁₆ and Cr₄₈Ru₁₆. The trend in bulk modulus of pure metals
increases from Mn to Ru as shown in Table.5.2. The compressibility of the pure material increases in a trend as follows:

\[ \text{Mn} < \text{V} < \text{Mo} < \text{Cr} < \text{Ru}. \]

The shear modulus has a robust correlation with the hardness of the material. Ruthenium strikes us as having the highest shear modulus of 187.42 GPa while manganese has the lowest value of -0.01 GPa. However, the pure metals are ductile except manganese. From Table 5.2, it can be seen that the average Young’s modulus of Cr$_{48}$Ru$_{16}$ is higher than the Ru$_{48}$Cr$_{16}$ hence Cr$_{48}$Ru$_{16}$ system is stiffer.

The bulk modulus will assist in knowing which materials are more resistant to fracture. The Cr$_{48}$Ru$_{16}$ system has high ability to resist fracture due to its high bulk modulus (333.55 GPa) than Ru$_{48}$Cr$_{16}$ (277.18 GPa) system. In addition the shear modulus helps to determine the material’s response to shear stress and it is again found that Cr$_{48}$Ru$_{16}$ (148.91 GPa) system has more ability to resist shear stress than Ru$_{48}$Cr$_{16}$ (67.74 GPa) system.

To determine ductility or brittleness we used the ratio of the bulk to shear modulus under the criterion of Pugh [164] that says a material is ductile when B/G ratio is (greater than 1.75) whilst for low B/G ratio (less than 1.75) is brittle. It is clear in Table 5.2 that Ru$_{48}$Cr$_{16}$ has high B/G ratio of 4.09 than Cr$_{48}$Ru$_{16}$ (2.23), the Ru rich ternary alloy is more ductile and can be able to deform under tensile stress. The Poisson's ratio of Frantsevich et.al [165] suggested that the Poisson’s ratio (\(\nu\)) can be used to indicate ductility or brittleness. If the (\(\nu\)) > 0.3 it is ductile and otherwise it is brittle. The A15Ru$_3$Cr and
A15Cr₃Ru systems are all ductile; however Ru₄₈Cr₁₆ has higher value of 0.39 than Cr₄₈Ru₁₆ (0.35).

The Young’s, shear and bulk modulus of the six X (X=Mo, Mn and Fe) atoms-doped Cr₄₂X₆Ru₁₆ and Ru₄₂X₆Cr₁₆ structures are calculated and analysed. The six Mo atoms-doped Cr₄₈X₆Ru₁₆ have high Bulk of 302.03 GPa whilst Mn has a low bulk modulus of 249.30. A high and low Young’s modulus of 220.98 GPa and 160.73 GPa were obtained for six Mo-doped Cr rich and six Mn-doped Cr-rich alloys respectively. Lastly, the six Mo-doped Cr rich have a higher shear modulus of 123.07 GPa whereas the six Fe-atom doped-alloy possess a lowest shear modulus of 101.9 GPa. The trend in decreasing ductility based on B/G for the six X-doped Cr₄₈Ru₁₆ is as follows:

Six Fe-doped > six Mo-doped > six Mn-doped

It is apparent that six Fe-doped Cr rich alloy is the most ductile than those of six Mo-doped and six Mn-doped Cr rich alloys.

The results for the six X atoms-doped Ru₄₂Cr₁₆ in Table 5.2 indicate a decreasing trend in bulk and young’s modulus from 6Mo-doped to 6Mn-doped then to 6Fe-doped Ru rich alloys. In addition, the Ru₄₂Cr₁₆Mo₆ has the highest bulk and young modulus of 293.49 GPa and 230.48 GPa whereas the Ru₄₂Cr₁₆Fe₆ has the lowest values (245.39 GPa and 186.68 GPa). The ratio of bulk to shear (B/G) decreasing trend is presented below for six X-doped Ru₄₂Cr₁₆:

six Mo-doped > six Mn-doped > six Fe-doped
Moreover, Ru$_{42}$Cr$_{16}$Mo$_6$ is more ductile with a value of 3.10 whereas the six Fe atoms-doped Ru rich alloy possess a low value of 2.79. When comparing the ductility of all the six X-atoms doped Cr$_{48}$Ru$_{16}$ and Ru$_{48}$Cr$_{16}$ systems, we observed that the Ru$_{48}$Cr$_{16}$Mo$_6$ as the most ductile structure. Due to lack of experimental data, the elastic constant cannot be compared. It is highly suggested that experimental work should be done in order to verify the theoretical data.

In conclusion, the elastic constants for pure metals were computed and found that ruthenium is very hard with a $C_{44}$ value of 164.20 GPa that compare well with the theoretical data of Popoola et.al [15]. As experimental data is limited, we could not compare the other values from the pure metals. The Ru$_{48}$Cr$_{16}$ structure is more ductile with a B/G value of 4.09 than the Cr$_{48}$Ru$_{16}$ structure with 2.23. All the investigated structures were found to be mechanically stable. The elastic moduli for the doped and undoped structures were computed, and the Ru$_{42}$Mo$_6$Cr$_{16}$ was predicted to be the most ductile system with a value of 3.10 whereas Cr$_{42}$Ru$_{16}$Mn$_6$ has the lowest value of 2.17. The studied six X atoms-doped structures are predicted to be applicable in high temperature structural applications.
Chapter 6

Conclusion and Recommendations

6.1. Conclusion

By means of First-principles calculations, we have investigated the ground state properties of metallic Cr and Ru, their undoped and transition metal doped A15 Cr$_3$Ru and A15 Ru$_3$Cr alloys. Convergence tests for the k-points and kinetic energy cut-off were conducted, and the sufficient parameters were selected. The lattice constants for Cr, Ru, undoped and X-doped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ ($X =$ Pt, Mn, Fe, Co, Mo, Re, Pd and Zr) were determined based on the plane-wave pseudopotential DFT method. The equilibrium lattice constants for pure metals, Ru$_3$Cr and Cr$_3$Ru were calculated using LDA and GGA functionals, the GGA results are in good agreement with the experimental and theoretical results than the LDA. The unrelaxed and relaxed lattice constants of the X-doped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ remained the same because the 2x2x2 supercells were constructed from the optimized A15 Ru$_3$Cr and A15 Cr$_3$Ru respectively.

The stability of the studied alloys was investigated based on the heats of formation, density of states and elastic properties. The heats of formation of the Cr$_3$Ru and Ru$_3$Cr were found to be positive but very close to zero. These results clearly indicate that the studied binary alloys are unstable. We introduced one to six transition metals X dopant atoms to the 2x2x2 supercells of the A15 Cr$_3$Ru and Ru$_3$Cr alloys. The doping effect on the heats of formation of the undoped 2 × 2 × 2 Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ supercells was
investigated. We considered dopants from the first, second and third row transition metals in the periodic table.

Introduction of 1st row transition metal atom (V, Mn, Fe and Co) has an influence in decreasing the heat of formation of the undoped $\text{Ru}_{48}\text{Cr}_{16}$. The Mn and Fe doped alloys yielded negative heats of formation. We observed that values for Cr rich ternaries are lower as compared to the Ru rich ternaries. The $X$-doped $\text{Cr}_{48}\text{Ru}_{16}$ compounds (for all doping concentrations) indicated a decreasing trend in the heats of formation as follows: Fe < Mn < V < Co. Furthermore, Fe doped systems were predicted to be more energetically favorable in all studied configurations. The results predicted the six Fe doped $\text{Cr}_{48}\text{Ru}_{16}$ as the most stable structure compared to all the studied first-row transition metal Cr-rich and Ru-rich doped systems.

Similar analysis were conducted on the second row transition metal $X$ (Mo < Zr < Pd) doped $\text{Ru}_{48}\text{Cr}_{16}$ and $\text{Cr}_{48}\text{Ru}_{16}$ systems. We observed a decrease in the heats of formation for all the studied $X$ (second row) doped $\text{Ru}_{48}\text{Cr}_{16}$ and $\text{Cr}_{48}\text{Ru}_{16}$ alloys. However, the values for Zr and Pd doped systems remained positive. The study predicted negative heats of formation for Mo-doped Cr-rich and Ru-rich alloys, Mo doped structures are predicted as the only stable structures as compared to all studied second-row transition metal doped structures. The six Mo doped $\text{Cr}_{48}\text{Ru}_{16}$ was predicted as the most stable structure among all the studied Mo, Zr and Pd doped systems.

We observed positive heats of formation for the studied third-row transition metal (Pt and Re)-doped $\text{Ru}_{48}\text{Cr}_{16}$ and $\text{Cr}_{48}\text{Ru}_{16}$ systems, indicating the instability of the alloys. The heats of formation for both the Pt and Re doped $\text{Cr}_{48}\text{Ru}_{16}$ showed a regular increase with
increasing dopant concentration. Pt and Re dopants do not enhance the stability of Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ compounds.

Magnetic properties of pure metals, the doped and undoped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ alloys were investigated. We also looked at the effect of smearing width on the magnetic moment (per unit cell) of one Mn doped Cr$_{48}$Ru$_{16}$ alloy. A decrease in the smearing width leads to an increased accuracy of the magnetic moment. We obtained negative magnetic moments for all the studied concentrations of Zr and Re doped Ru-rich systems whereas the positive values were obtained for Zr, Mo, V, Fe, Co and Re doped Cr$_{48}$Ru$_{16}$ systems. These Zr and Re-doped Cr rich alloys are in total contrast with the Ru-rich results. The computed magnetic moments for X (Zr, Mo, V, Pd, and Re) doped Cr$_{48}$Ru$_{16}$ are higher than the corresponding X doped Ru$_{48}$Cr$_{16}$. The results predicted that Mo doped Ru$_{48}$Cr$_{16}$ would couple anti-ferromagnetically with Cr atom. For Ru rich Mn-doped system, we detected high and low magnetization in one Mn- doped and three Mn-doped systems respectively.

The Mo and Fe doped Cr$_{48}$Ru$_{16}$ compounds substituting Cr atom were predicted to couple ferromagnetically with Ru. The Mn doped Cr$_{48}$Ru$_{16}$ was found to be highly magnetized a moment of 2.001 $\mu$B, which is due to high magnetic value of Mn (5.0 $\mu$B). Some values of magnetic moments of the pure metals compares well with the experimental values. The Mo and Fe in Mo-doped and Fe-doped Cr$_{48}$Ru$_{16}$ were predicted to couple ferromagnetically with Ru. However, a negative moment of -0.645 $\mu$B was calculated for five Fe-doped Ru$_{48}$Cr$_{16}$, an indication that Ru contributed more than Cr and Fe. These results caused an antiferromagnetic coupling between Fe-Cr.
The total and partial densities of states for the Cr, Ru, dopants, doped and undoped Cr_{48}Ru_{16} and Ru_{48}Cr_{16} were calculated. The bands in all TDOS and PDOS curves are found to overlap at Fermi level, indicating metallic character of all the studied systems. The DOS of metallic Fe and Mn indicated the Fermi energy falling on top of a peak, a clear implication of ductility characteristics. Whereas the Ru DOS curve showed a structural stability features arising from the E_f falling on a pseudogap. The high peak of the metallic Cr is located at ≈ 2.0 eV. The DOS of Cr indicated a higher peak of the valence band as compared to the peaks of the conduction band DOS. From the TDOS and PDOS of metallic Mn, we noted that main peak located at the Fermi level.

TDOS and PDOS curves of the undoped Cr_{48}Ru_{16} and Ru_{48}Cr_{16} indicated that bands mainly originates from the Cr (3d) and Ru (4d) states, with small contribution from their s and p states. Comparing the DOS curves of the undoped binary alloys, we noted more states in the conduction band TDOS of Cr_{48}Ru_{16} than those in TDOS of Ru_{48}Cr_{16}. The stability is associated with low DOS at the Fermi level. We noted low electron density of states at the Fermi level from the DOS of Cr_{48}Ru_{16}, however, higher electron density of states were observed near the Fermi level in the DOS of Ru_{48}Cr_{16}.

We investigated the effect of Mn, Fe, Mo and V dopants on the electronic stability of Cr_{48}Ru_{16} and Ru_{48}Cr_{16} alloys. The s, p and d states in the PDOS of Cr_{42}Ru_{16}Mn_6 shifted to lower energy region as compared to the Ru_{42}Cr_{16}Mn_6. All the DOS of X doped Ru_{48}Cr_{16} compounds displayed high DOS around the E_f whilst X doped Cr_{48}Ru_{16} has low DOS. There was no peak in the conduction band DOS and PDOS curves of the Mo-doped Ru rich alloy. On the other hand, the energy difference between the main peak of valence band and that of conduction band in the TDOS of Cr_{42}Ru_{16}Mo_6 was found to be 3.6 eV.
The conduction band from all the DOS of Ru$_{42}$Cr$_{16}$Mo$_6$ was found to be narrower than the DOS of the studied X-doped materials. In both configurations of Mo-doped systems, most of electrons are accumulated in the lower energy region as compared to Mn and Fe-doped systems. These findings indicated a higher level of electronic stability for Mo-doped systems.

The elastic constants, bulk, shear, Young’s moduli for the pure metals, the doped and undoped Ru$_{48}$Cr$_{16}$ and Cr$_{48}$Ru$_{16}$ were calculated. The elastic constant $C_{44}$ is smaller than the $C_{11}$ and $C_{12}$ in all the studied structures. The values of $C_{11}$, $C_{12}$ and $C_{44}$ for the undoped Ru-rich alloys are higher as compared to Cr-rich binary alloys. The hardness of the material can be determined from the value of $C_{44}$ elastic constant. The higher value of $C_{44}$ for Cr$_{48}$Ru$_{16}$ alloy indicated that the material is harder than the Ru$_{48}$Cr$_{16}$ and it has strong ability in resisting shear distortion. The Cr$_{48}$Ru$_{16}$ system has high ability to resist fracture due to its high bulk modulus than Ru$_{48}$Cr$_{16}$ system.

From the elastic constants of the Cr$_{48}$Ru$_{16}$ and Ru$_{48}$Cr$_{16}$ alloys doped with six X-atoms (X = Fe, Mn and Mo) alloys, we noted that the $C_{11}$ and $C_{44}$ and $C_{66}$ increased from X (X=Mn, Mo) doped Ru$_{48}$Cr$_{16}$ to Cr$_{48}$Ru$_{16}$ system. The overall comparison predicted that six Mo-doped Cr$_{48}$Ru$_{16}$ can resist shear distortion more than all studied six X atoms-doped Cr rich and Ru-rich alloys. All the structures are mechanically stable according to Bohr mechanical stability criterion for cubic compounds. Among the X-doped Cr-rich systems, the Cr$_{42}$Ru$_{16}$Mo$_6$ exhibit the highest shear modulus (303 GPa) and Young’s modulus (123 GPa), while Ru$_{42}$Cr$_{16}$Mo$_6$ possess the largest shear modulus (230 GPa).
The Cauchy (C_{12}-C_{44}) was used to describe the angular character of atomic bonding in metals. To explore the ductility or brittleness of the material, we referred to the Cauchy pressure and the ratio of bulk to shear modulus. By analyzing the Cauchy pressure, the ratio of bulk modulus to shear modulus, we found that the ductility of X-doped follow the orders of Cr_{48}Ru_{16} < Ru_{48}Cr_{16}, Cr_{42}Ru_{16}Mn_{6} < Cr_{42}Ru_{16}Fe_{6} < Cr_{42}Ru_{16}Mo_{6}, and Ru_{42}Cr_{16}Fe_{6} < Ru_{42}Cr_{16}Mo_{6} < Ru_{42}Cr_{16}Mn_{6}.

In summary, we have investigated the effect of transition metal dopants on the structural, thermodynamic, electronic and elastic properties of A15 Cr_3Ru and A15 Ru_3Cr alloys. At first, we successfully used DFT methods to predict the stability based on the heats of formation. The results predicted the Cr_{42}Ru_{16}Mo_{6}, Cr_{42}Ru_{16}Fe_{6} and Cr_{42}Ru_{16}Mn_{6} as the more stable structures. Moreover, the results predicted the Cr_{42}Ru_{16}Mo_{6} as the most stable structure among all the studied structures. In general, the calculated density of states reported metallic behaviour for pure metals and their alloys. The TDOS and PDOS confirmed the stability of Fe-doped, Mo-doped and Mn-doped Cr-rich and Ru-rich alloys. Finally, the elastic properties calculations revealed both the studied Ru-rich and Cr-rich ternaries as mechanically stable structures. These study show that Mn, Fe and Mo dopants play an important role in improving the properties of metal alloys. We conclude that the Cr_{42}Ru_{16}Mo_{6}, Cr_{42}Ru_{16}Fe_{6} and Cr_{42}Ru_{16}Mn be considered for high temperature applications.
6.2. Recommendation

The heats of formation for Cr_{48}Ru_{16} doped with one to six atoms of Re, Pd, Zr and V were predicted as unstable, however the values were positive but very close to zero. We recommend that additional stability tests be conducted on Cr_{48}Ru_{16} with higher concentration of dopants (Re, Pd, Zr and V). Different alloy compositions may be beneficial in tuning the performance and the stability of the materials earmarked for high temperature structural applications. Molybdenum (Mo)-doped Cr rich alloys were found to be stable among all the studies configurations. We suggest complete theoretical analysis of the doping effect Niobium (Nb), Technetium (Tc) and Tungsten (W) on the ground state properties of Cr_{48}Ru_{16} and Ru_{48}Cr_{16}. The Nb, Tc and W metals are located on the immediate left, right and below the Mo in the periodic table respectively. The results can have a huge impact on our scientific understanding of the effect of metal dopants (from the same row/column) on the stability of the materials. Exploration of non-metallic dopants is a topic for future work. Due to lack of availability of experimental results, we recommend that stability experiments be conducted for comparison and to develop a deeper understanding.
REFERENCES


