

CORPORATIVE LEARNING AS A TOOL TO DEVELOP CONCEPTUAL AND PROCEDURAL KNOWLEDGE IN SOLVING ALGEBRAIC EQUATIONS: A CASE STUDY OF GRADE 11 MATHEMATICS LEARNERS

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ABSTRACT

This study explored Grade 11 Mathematics learners' conceptual and procedural knowledge in solving algebraic equations using cooperative learning. This qualitative case study was carried out in a classroom setting in which 34 Grade 11 Mathematics learners participated. The data was collected using a round-table discussion and reflective interviews. The study shows that learners learn better when learning in groups than when learning as individuals. Thus, in this article, we argue that cooperative learning can develop conceptual understanding in solving algebraic equations.

Keywords: algebraic equations, conceptual knowledge, cooperative learning, procedural knowledge

INTRODUCTION

The ability of students to use their Mathematics for purposes beyond doing routine examples and passing formal tests and examinations is a matter of worldwide interest. The concern is at the centre of efforts to give impetus to this application in many national contexts as expressed, for example, in Blum and Niss (1991). Algebraic equations and inequalities play an important role in various mathematical topics, including algebra, trigonometry, linear programming and calculus (Bazzini & Tsamir 2004; Kieran 1989). Algebra is the branch of Mathematics that deals with symbolising and generalising numerical relationships and Mathematics structures and operating with those structures. Further, Van de Walle, Karp and Bay-Williams (2016) indicate that algebraic reasoning involves representing, generalising and formulating patterns and regularity in all aspects of Mathematics. According to the Department of Basic Education (DBE 2011), learners in Grade 10 are supposed to solve linear equations, quadratic equations and literal equations (changing the subject of a formula). The Curriculum Assessment and Policy Statement (CAPS) document makes it clear what must be done in algebraic equations. In our experience, we found that most of the Grade 11 learners are unable to find the subject of the formula, which was supposed to be dealt with in Grade 10. It was this reason that this study sought to explore Grade 11 Mathematics learners' conceptual and procedural knowledge in solving quadratic (algebraic) equations using cooperative learning. The study was guided by the following questions:

- What are learners' conceptual and procedural knowledge of algebraic equations?
- What are learners' experiences when solving equations through cooperative learning?
- What is the effect of cooperative learning on learners' conceptual and procedural knowledge of solving equations?

What is conceptual and procedural knowledge?

Kharatmal (2009) refers to conceptual understanding as an integrated and functional grasp of mathematical ideas. Kilpatrick and Swafford (2002) further give the definition of conceptual understanding as being able to comprehend Mathematics concepts, to perform operations and relate the concepts. In this study, conceptual knowledge was taken as an integrated grasp of mathematical ideas wherein comprehension, operations and relations of abstract or generic ideas are generalised. Furthermore, Van de Walle (2004) explains that conceptual knowledge of Mathematics consists of logical relationships constructed internally and existing in the mind as part of the network of ideas. This is the type of knowledge Piaget (1964) referred to as

logico-mathematical knowledge. This is the knowledge made up of relationships between objects, which are not inherent in the objects themselves but is introduced through mental activity. By its very nature, conceptual knowledge is knowledge that is understood. This is the knowledge, according to Skemp (1976), which produces relational understanding. This kind of knowledge is referred to as “flexible” knowledge (Boaler 1997), that is, knowledge which can be used in a new situation.

On the other hand, procedural knowledge is derived from procedure and knowledge. According to Rittle-Johnson and Schneider (2013), a procedure is a series of steps, or actions, done to accomplish a goal. Knowledge is only an explanation and an assumption but not the final answer for all questions; it will be discarded along with the human process and a new assumption will appear. The knowledge of the procedure is knowing how or the knowledge of the steps, or actions, done to accomplish the goal.

Furthermore, procedural knowledge of Mathematics, according to Van de Walle (2007), is knowledge of rules and procedures that one uses in carrying out routine mathematical tasks and includes the symbolism that is used to represent Mathematics. This is the knowledge produced by lack of connections of mathematical ideas. If mathematical ideas are seen as isolated from each other, the knowledge produced is referred to as “procedural”. Conceptual knowledge and procedural knowledge can be evidenced when learners solve mathematical problems.

Conceptual and procedural knowledge of algebraic equations

Panasuk and Beyranevand (2010) state that conceptual knowledge in algebra can be characterised as the ability to recognise the functional relationship between known and unknown, and independent and dependent variables, and to distinguish between and interpret different representations of the algebraic concept. On the other hand, Bulk, Hull and Miles (2013) define the conceptual knowledge of algebra as a comprehension of mathematical concepts, operations and relations. Students demonstrate conceptual knowledge in algebraic equations when they provide evidence that they can recognise, label and generate examples of concepts; as well as use and interrelate models, diagrams, manipulative and varied representations of concepts.

Further, learners demonstrate conceptual knowledge by being able to identify and apply principles; know and apply facts and definitions; compare, contrast and integrate related concepts and principles; and recognise, interpret and apply the signs, symbols and terms used to represent concepts (Bulk *et al.* 2013). The description given by Panasuk and Beyranevand (2010) and Bulk *et al.* (2013) were used in the paper to provide what should be considered when identifying whether or not a learner has a conceptual knowledge of algebra.

Cooperative learning

Mabrouk (2007) states that cooperative learning is when students are working in teams on an assignment or a project under conditions in which certain criteria are satisfied. Cooperative learning is used as the teaching method in which children work in small groups to help one another learn. Cooperative learning was used as it provided the exact meaning of the intention of the study. Hopkins and Salvin (2008) further indicate that in a cooperative learning classroom, students are expected to help each other, to discuss and argue with each other, to assess each other’s current knowledge, and fill in gaps for individual understanding.

In their work, Kagan (1994) and Al-Yaseen (2014) used different ideas to develop a way of implementing cooperative learning. In their study, Zakaria, Chung Chin and Daud (2010) argue that student-centred approaches, such as cooperative learning, improve Mathematics achievement and attitudes towards Mathematics among students. They therefore suggest that cooperative learning is an effective approach that Mathematics teachers need to incorporate in their teaching.

RESEARCH DESIGN AND METHODOLOGY

In this case study, 34 Grade 11 Mathematics learners from a rural school in the Malokela circuit in Limpopo were engaged in the round-table cooperative learning method. The round-table method was chosen because learners are given an opportunity to brainstorm, review

and practise throughout the sessions. There were 6 learners in groups 1 to group 5 and 7 learners in group 6. According to Kagan (1994), there are three steps in the round-table method. Firstly, the teacher asks a question, which has multiple answers, and then each student writes a response or a portion of a response. Secondly, after writing their response, they pass the paper to the next person. Thirdly, one group member may be asked to share with the whole class what their group has written. Learners were observed during the round-table method, and eventually a sample of learners were interviewed based on their participation on their respective groups. The round-table activity can be done with one piece of paper per group or with one piece of paper per group member.

To capture what exactly transpired in the study, the concepts procedural, conceptual knowledge and cooperative learning were used as the lens to explore learners' knowledge in solving algebraic equations.

Findings and discussion

We organised our analysis in terms of themes from literature – manipulative and varied representations; and recognising, interpreting and applying signs – and from the data (communication, confidence and motivation; and comparing, contrasting and integrating related concepts). The first author (JF) conveniently selected Learner 2 from group 1 (L2G1), Learner 18 from group 3 (L18G3) and Learner 37 from Group 6 (L37G6) for interviews. The pseudo-codes L for learner were used, G for group number and JF for the first author.

Manipulative and varied representation

During the round-table method, learners were busy organising the algebraic equations by using the basic skills of additive inverse and multiplicative inverse to come up with the letter that they were requested to make the subject of the formula. In this case, learners were engaged in the manipulation and organisation of the variables to come up with different representations of the given algebraic equations. Learners were able to explain the way in which they managed to make the particular letter the subject of the formula (find the value of x). For example, from Figure 1, $10x$ is made by 10 and x as a product, which means that the factors are 10 and x . To separate 10 from x , learners identified that there is a need to divide or use the multiplicative inverse of 10 which is $\frac{1}{10}$.

Solving the following equation

$$(1) \quad (2x-3) \times 4x = 4-6x$$

$$8x-36-4x = 4-6x$$

$$8x-40x+6x = 4+36$$

$$10x = 40$$

$$\frac{10x}{10} = \frac{40}{10}$$

$$x = 4$$

Figure 1: Example 1 from student work

Recognise, interpret and apply signs

Learners were able to recognise the operation signs as +, −, × and ÷, and they recognised the bracket as multiplication in the algebraic equation. The purpose of recognising the operations signs helped learners in applying the signs and how to remember the concepts such as multiplicative inverses and additive inverses. Other learners were able to use the signs differently for the same purpose. Some learners used the idea of the multiplicative inverse whereas some used the idea of division. Because learners were explaining to each

other in their respective groups and the whole class, the idea of multiplicative inverse and division were made clear to others who did not understand the relationship.

JF: What happened to the $4(2x - 9)$ on this step?

L2G1: I removed the bracket by multiplying 4 with $2x$ to get $8x$ and again 4 with 9 to get 36.

JF: What happened to the minus sign in between $2x$ and 9? Did it have any effect?

L2G1: Yeah, Meneer, (meaning Mr) 4 is positive and, eh when it multiplies the minus, the answer is minus.

So that is why I got $8x - 36$.

JF: Can you explain what happened on the next step?

L2G1: I grouped like terms. Those with x 's are on the left and those without x 's are on the right.

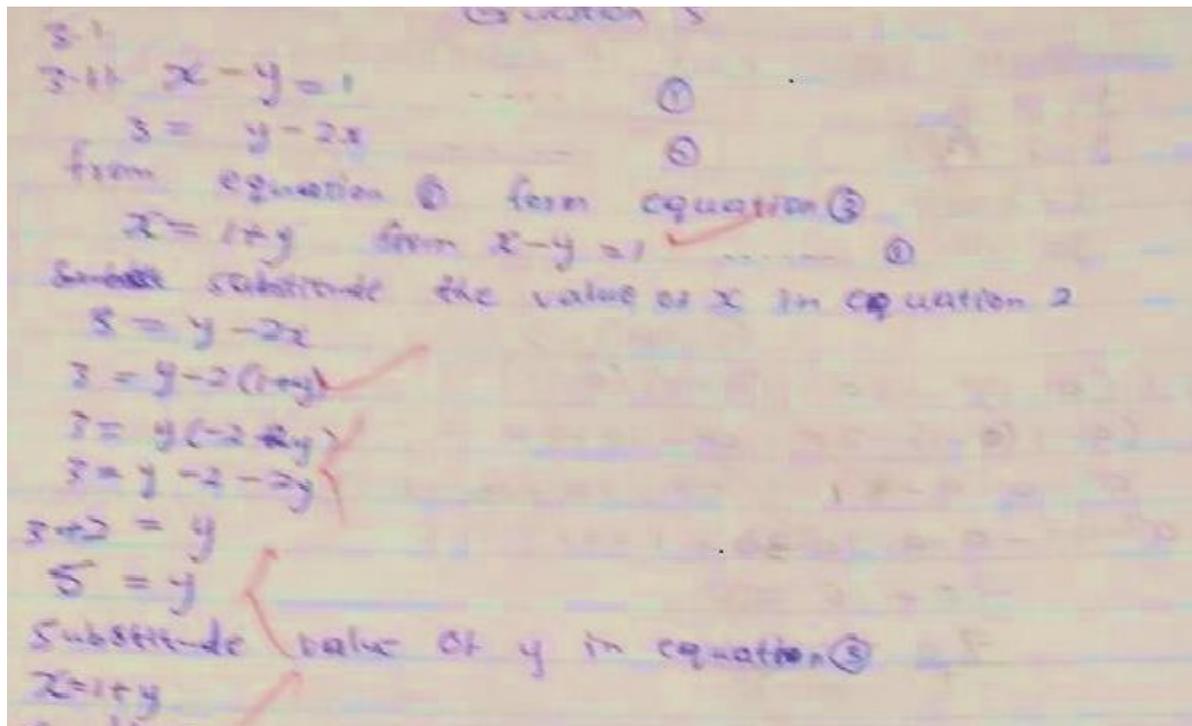


Figure 2: Example 2 from student work

From figure 2, it is found that some learners were unable to recognise how to multiply when brackets are having a coefficient number that has a negative sign. The interview helped learners to recognise and multiple the problem. Before the interview, some learners could not to see a relationship between the division and multiplication signs on the algebraic equations; especially when the variable has a coefficient; learners were finding it difficult to remove the constant from the variable. The following excerpt is from one learner from a group of learners who were able to identify the operations signs and the purpose of the bracket. However, the learner was not doing well like his or her peers. When asking L13G3 about the results and process, the responses were as follows:

JF: You were silent in your group at the first stages. Can you explain to me why or what happened?

L13G3: Hmm, I did not understand, Meneer (meaning Mr).

JF: Explain to me what your role in the group was.

L13G3: I was also writing and giving eh, the answers to others.

The learner removed the brackets by correctly multiplying the number 4 with $2x - 9$. In this case, the learner was able to use the bracket correctly and the multiplication was done correctly. When interviewed, the learner was able to explain the process and eventually that helped other group members to see the use of brackets. Again, the learner was able to identify the additive inverse. Though the learner used a language that could not be attributed to the Mathematics language, the process was actually the correct Mathematics process of using the additive inverse. From the second learner, it emerged that L13G3 was unable to

understand the equation itself and the basics skills of identifying the subject, the operations and manipulation. The learner was not going to understand the concept of solving algebraic equations.

Compare, contrast and integrate concepts

Learners compared the solutions to each problem in their respective groups and presented their agreed solution to the other groups. Within the groups, learners were contrasting their solutions until they decided on the best solution, which made sense to all. The purpose of each group member to present their group agreed solution was to compare, contrast and integrate concepts with other groups.

Handwritten algebraic solution for Figure 3(a):

$$2-x = 2(3x+1)$$

$$2-x = 6x+2$$

$$2-2 = 6x+x$$

$$0 = \frac{7x}{7}$$

$$0 = x$$

Figure 3(a)

Handwritten algebraic solution for Figure 3(b):

$$(2) \quad \frac{2-x}{3x+1} = 2$$

$$= (2-x) = 2(3x+1)$$

$$2-x = 6x+2$$

$$-x-6x = 0$$

$$\frac{-7x}{7} = \frac{0}{7}$$

$$x = 0$$

Figure 3(b)

The above figures (figure 3(a) and 3(b)) indicate different solutions given by different learners within a group. The learners were circulating their solutions, comparing and contrasting as mentioned earlier on.

Communication, confidence and motivation

It emerged that when the learners were interacting; they were communicating and motivating each other to further engage in solving the problems. Whilst they were communicating and motivated, learners gained the confidence to solve even other problems. In this case, the teacher showed an interest in learners' opinions. The learners felt that their thoughts or ideas were appreciated. This increased self-esteem and confidence. A confident student is less likely to second guess his answers in tests, and a self-assured student is more likely to speak up in class. Class participation leads to increased learning for the entire class. When asking L37G6 about the results and process, the *responses were as follows*:

JF: I see that your way of solving the algebraic equations are completely different. Explain how you were able to move towards the ability to solve.

L37G6: In the group, eh, we, were talking to each other. Other learners were showing us the way they are solving the problems and eh, mmm, yeah, I understood, and I was able to show mine and explain how I solved it.

Conclusion

In this article, we were guided by three questions: What are learners' conceptual and procedural knowledge of algebraic equations, what are learners' experiences when solving equations through cooperative learning? What is the effect of cooperative learning on learners' conceptual and procedural knowledge of solving equations?

It was found that cooperative learning provided learners with an opportunity to explore different ways and strategies of solving quadratic algebraic equations. Further, it emerged that learners' procedural knowledge of solving algebraic equations was developed. The learners who were unable to follow the procedure for solving algebraic equations were helped by the explanations they got from their peers. The reason could be that learners were able to communicate using their own level of communication or language. The study also found that learners' conceptual and procedural knowledge were stimulated when they were cooperatively engaged. Learners felt very comfortable with the process of cooperative learning to the extent that they were motivated to solve more algebraic equations. The communication amongst themselves, as peers, made them more confident and able to engage positively. Confidence helps learners to tackle even difficult problems with the hope that they could find a solution. Drawing on the findings in the study, we recommend that when cooperative learning is used, there should be enough time, especially with Mathematics concepts that seem difficult for learners, for discussing and exchanging ideas. Further, teachers should use arranged algebraic equations in terms of difficulty in order to explore the various ways in which learners could develop both conceptual and procedural knowledge.

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