

CONNECTION BETWEEN PRIOR KNOWLEDGE AND STUDENT ACHIEVEMENT IN ENGINEERING MATHEMATICS

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ABSTRACT—In the paper the author reports on connections between first-year engineering mathematics students' prior mathematical knowledge and their achievement in an engineering mathematics (year) course in an extended curriculum programme. The importance of links between prior knowledge in learning and performance is widely supported in educational studies, also in learning mathematics. Furthermore, the transition from school to university is a substantial hurdle in the learning trajectory of many students, and particularly in science and engineering courses. The author exposed students to a carefully designed diagnostic entrance test that consists of different content and knowledge components, and correlated these results, by means of regression analysis, with students' achievement scores at the end of term 1, term 2, and the first semester. The results show a moderate correlation. This research is meant as the beginning of exploring possibilities to predict study success in mathematics by means of an entrance test and of developing possibilities of remedying students' entrance deficiencies and thus increasing study success. More broadly, this inquiry was conducted to explore adequate ways of assessing prior knowledge that could be useful in supporting students' learning of engineering mathematics in a South African context.

Keywords: Engineering Mathematics; Extended Curriculum Programme; Knowledge components; Prior Mathematical Knowledge; Student Achievement.

INTRODUCTION AND AIMS

Success in studying a first year engineering mathematics course often relies largely on prior knowledge of the mathematics that precedes calculus: algebra, analytic geometry, functions, and trigonometry. Supporting this idea, many calculus textbooks include sections on prior knowledge to support students to diagnose weaknesses that they might have in these areas, to refresh their skills and to review key concepts (e.g. Stewart, 2016). There is a lively discussion in many countries and on the international level about the prior mathematical knowledge of university beginners and about consequences for their success in tertiary studies with a mathematical component (such as mathematics, science and engineering studies) (e.g. Greefrath, Koepf & Neugebauer, 2017; Hailikari, Nevgi & Lindblom-Ylänne, 2007; Rach & Heinze, 2017). According to empirical findings at the secondary school level, South African students have severe shortcomings in basic mathematical skills (e.g. Bernstein, 2013; TIMMS reports) and the transition from school to university appears to be another obstacle in their learning path. Du Plessis and Gerber (2012) explained a few key aspects that describe students' preparedness for university. They conducted an action research study on the academic achievement of two cohorts of first-year students, majoring in mathematics and accounting, at a public university in South Africa. They concluded that a combination of aspects is related to students' under-preparedness in the academic domain: English reading or writing ability, mathematical ability and effective study habits. Furthermore, for some time, in the public domain concern has been expressed about the South African national Grade 12 examination and results (e.g. Ramphele, 2009). As an attempt to bridge prior knowledge gaps and support under-prepared students, many South African universities introduced extended curriculum programmes. Commonly, students are placed in such programmes due to lower grades, and particularly in mathematics and science, achieved in their final year of school (Du Plessis & Gerber, 2012). From a study conducted in Germany, with 182 students majoring in mathematics, Rach and Heinze (2017) emphasised the challenges students face in learning mathematics at the beginning of university studies, related to the difference in character between school mathematics and scientific university mathematics and the different demand in learning cultures at the respective platforms.

In addition to mathematical skills and abilities, the professional development of engineering students requires problem-solving abilities in mathematics for real-life situations. Students

usually have difficulties to solve problems if well-defined procedures are not clear and this could even be worse if students lack basic prior knowledge in mathematics. In this study, the author considered the special situation of possible under-prepared students in a first-year engineering mathematics course in an extended curriculum programme and this led to the exploration of possibilities to predict study success in mathematics by means of an entrance test and of developing possibilities of remedying students' entrance deficiencies and thus increasing study success. The study from Greefrath et al. (2017), on a sample of degree programme electrical engineering and computer science students in Germany, highlighted the interesting possibility of making statements about future academic success by using a short test at the start of a course. Tests can serve a number of purposes: to select students, to provide student support, and for research purposes. The overarching goal is to use the results of this study to streamline the diagnostic test to improve the prediction of study success and to match adequate student support within the context.

The research questions were:

Which particular mathematical knowledge components preceding the study of calculus can be identified as particular strengths and weaknesses in students' prior knowledge?

How strong is the correlation between prior knowledge components and student achievement in a first-year engineering mathematics course at the end of term 1, term 2 and the first semester?

THEORETICAL FRAMEWORK

Mathematical knowledge comprises much more than operations with numbers or variables. It should, in particular, help students understanding the world better and finding solutions for real-life situations. The theoretical framework that guided the design of the diagnostic test lies with the notion how prior knowledge affects learning (Hailikari et al., 2007), the pragmatic approaches worldwide (compare Greefrath et al., 2017) where the aim is to measure students' knowledge of school mathematics in some important areas, as an important prerequisite for their academic success, the theoretical strands of mathematical knowledge from Kilpatrick, Swafford & Findell (2001), and for a small part of the test also the intention to measure students' modelling competencies (Blum & Borromeo Ferri, 2009).

Kilpatrick et al. (2001) described the five different strands of mathematical knowledge, which in combination indicate mathematical proficiency as: (i) *conceptual understanding*, (ii) *procedural fluency*, (iii) *strategic competence*, (iv) *adaptive reasoning*, and (v) *productive disposition*. The diagnostic entrance test in this study (view sub-section 3.2) is mostly related to procedural fluency (the skill of performing flexible procedures accurately, efficiently and appropriately), as well as partly to conceptual understanding (the ability to grasp mathematical concepts, operations and relationships) and productive disposition (the ability to view mathematics as sensible, useful and worthwhile). Hailikari et al. (2007) explained how domain-specific prior knowledge in the context of higher education has been explored in many studies (e.g. Dochy, De Ridjt & Dyck, 2002; Weinert, 1989) from different perspectives and how in general prior knowledge interacts with different phases of information processing. In most studies, it is argued that prior knowledge facilitates learning substantially. Thus, if educators have 'tools' to identify misconceptions at the beginning of the learning process, they can consider these in their teaching, because "if students possess inaccurate prior knowledge and misconceptions within a specific domain it can make it difficult to understand or learn new information" (Hailikari et al., 2007, p. 321).

According to Greefrath et al. (2017), tests at the start of studies can have distinct functions: (1) the aim of recording the current performance level of students, or (2) generating a prediction of how successful students will be. They explained these tests should be optimized to improve the prediction of study success. Thus, the question regarding the quality of prediction cannot yet be clearly answered. Nevertheless, Greefrath et al. (2017) showed significant correlations between the results of a mathematics test at the beginning of a course and the examination results at the end.

Calculus as a content area is introduced to students already at school, but at university it is the main content and focus of first-year mathematics. Calculus is concerned with change and motion and is fundamentally different from school mathematics – less static and more dynamic (Stewart, 2016), with a new and cognitively complex kind of thinking involving infinitesimal concepts. Furthermore, engineering students should be prepared for real-life problem situations and in this regard, Blum and Borromeo Ferri (2009) explained the importance and general objectives of mathematical modelling for students. Modelling ought to help students to understand the world better; to support mathematics learning (motivation, concept formation, comprehension, retaining); to contribute to developing various mathematical competencies and appropriate attitudes; and to contribute to forming an adequate picture of mathematics.

METHODOLOGY

Participants

The sample consisted of 41 engineering mathematics students at the University of Johannesburg. They were all first-year students registered in the engineering extended curriculum programme, due to their lower mathematics marks in the final school year (with an entrance minimum requirement of 50% in Grade 12 mathematics). As part of the engineering programme, all students were enrolled for a year course in engineering mathematics. Although the language of instruction is English, it is usually not students' home language (only for 16% of the group).

Research design and data collection

The study was quantitatively oriented and students' average scores from the diagnostic test as well as averages at the end of term 1, term 2 and the first semester, were collected. The **diagnostic test** was designed based on guidelines from Stewart (2016) on the required mathematical knowledge preceding calculus and the theoretical framework informing this inquiry (Hailikari et al., 2007; Kilpatrick et al., 2001). Stewart's guidelines included the content areas *algebra*, *analytical geometry*, *functions* and *trigonometry*. The author's experience on teaching first-year engineering mathematics and the input from other research specialists in the field of mathematics education (with a focus on the teaching and learning of modelling) led to the inclusion of another two content areas in the diagnostic test: *calculus* and *modelling*. The test consists of 25 tasks (with altogether 32 items, and 38 marks as maximum) of which the format and demand ought to be mostly familiar to high school students in South Africa. The only unfamiliar task (related to format and demand) was the second of two modelling tasks in the final section of the test (where the approximate volume of a hot air balloon had to be calculated based on a photo). All test items address mathematical concepts that occur in the South African school curricula and their solution displays mathematical proficiency. Table 1 provides an overview of the key aspects of the diagnostics test. The major knowledge component was procedural fluency in central areas of school mathematics, for instance simplifying algebraic expressions, solving equations or drawing graphs of elementary functions. Both the sections on *algebra* and *functions* were more emphasised since these sections are dominant in the term 1 and term 2 engineering mathematics course curricula in the extended curriculum programme (view Table 2).

Table 1. Features of the diagnostic test

Section	Total	Knowledge component	Strands of knowledge
A	12	Algebra	With a focus on conceptual understanding and procedural fluency
B	7	Analytical geometry	
C	8	Functions	
D	3	Trigonometry	
E	2	Calculus	
F	6	Modelling	With a focus on productive disposition

Table 2. Spreading of content in the engineering mathematics course in the extended curriculum programme

	Semester 1	Semester 2
Term 1	The binomial series The theory of matrices and determinants Solving simultaneous equations using Cramer's rule Algebraic functions and their graphs	
Term 2	Manipulation of formulae Exponential and logarithmic functions Trigonometry and sinusoidal graphs	
Term 3		Limits Differentiation and applications
Term 4		Integration and applications

The internal consistency of the diagnostic test (calculated on 30 items after 2 items were deleted) was determined using *McDonalds' Omega* (with $\omega = 0.79$ which is very satisfactory). The researcher oriented herself towards other such tests (such as Greefrath et al., 2017 and Rach & Heinze, 2017) and adapted them to the South African situation, and at the same time implemented modelling as a new element. Furthermore, a validation of the test items was conducted by an expert.

The diagnostic test was administered in the first week of the academic semester, before the introduction of any new university mathematics content areas, during an official lecture period (90 minutes). Participants were not informed of the test ahead of time, and were not allowed to use scientific calculators in sections A – E, which was unfamiliar for them, but in section F it was necessary and allowed. The academic semester consists of 14 weeks, divided in two terms of 7 weeks each. Students have six periods of mathematics teaching per week (4 sessions for lectures and 2 sessions for tutorial work). During each term, students were required to take part in continuous and formal assessment opportunities. Continuous assessment opportunities included homework tasks and class tests and covered only a small portion of content. A semester test was the only formal assessment opportunity per term and covered a large portion of content. There was an overlap between the diagnostic test content and the semester test content (in both cases), for example algebraic manipulations or drawing function graphs. In both semester tests, the primary mathematical knowledge components were again conceptual understanding and procedural fluency. Both these assessment opportunities contributed towards a final term mark at the end of term 1 and term 2 respectively, in a 1:4 ratio. The sum of the two terms generated the data for the first semester average.

DATA ANALYSIS AND DISCUSSION OF FINDINGS

Participants' average scores from the diagnostic test, averages at the end of term 1, term 2 and the first semester, were analysed via the software package Excel. First, diagnostic test results were analysed per prior knowledge section and presented as percentages (view Figure 1); as *known* (for correct responses) and as *gap* (for incorrect responses). This analysis was done to determine which particular mathematical knowledge components that precede the study of calculus can be identified as strengths and weaknesses in students' prior knowledge. In section A (algebra) and section B (analytical geometry), 45% of responses were known, in section C (functions) 28%, in section D (trigonometry) 22%, in section E (calculus) 39% and in section F (modelling) 13% of all responses. Total scores for the diagnostic test revealed that 34% of responses were known. These results (total scores and prior knowledge per content section) were much lower than expected by the researcher, although a very low *known* percentage was expected in section F (modelling). In all content areas a *gap* of more than 50% was identified. It should be mentioned that the diagnostic test results were not reflected against prior knowledge background variables, which could be considered in a follow-up study. These background variables could be related to the familiarity with calculators and computers, the cultural background, or final grades in school mathematics (compare Hailikari et al., 2007).

Further analysis revealed 18 from 41 participants' total test scores were in the interval from 0% – 29%), 8 from 41 in the interval from 30 % – 39%, 7 from 41 in the interval from 40% – 49%, 5 from 41 in the interval from 50% – 59%, and 3 from 41 in the interval from 60% – 69%. No participants achieved 70% or above.

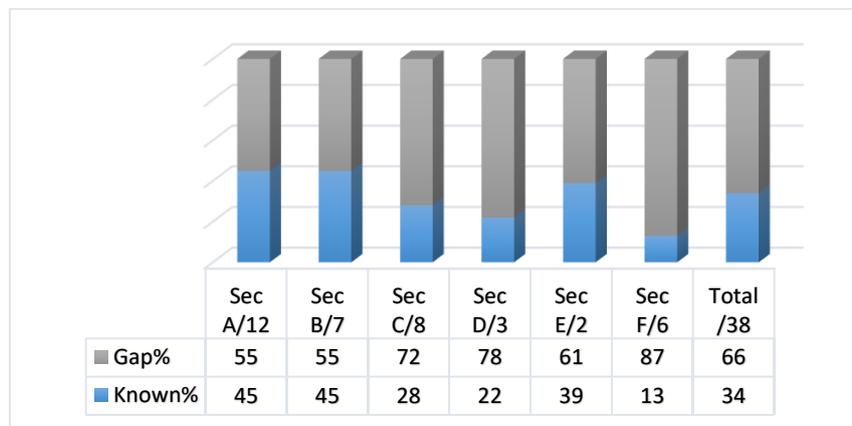


Figure 1: Knowledge gap per section of the diagnostic test

Second, regression analyses were carried out to analyse how far the diagnostic test results predict student achievement at three different time intervals, at the end of term 1, term 2, and the first semester. At the end of all three time intervals, a moderate positive correlation with the diagnostic test was found (term 1: $r = .40$, $R^2 = .157$; term 2: $r = .40$, $R^2 = .154$; semester 1: $r = .44$, $R^2 = .197$). Figure 2 shows the regression analysis and the line of best fit between the diagnostic test and term 1 average, term 2 average, and semester test 1 average, as well as a box plot for descriptive statistics for the diagnostic test and semester 1 averages.

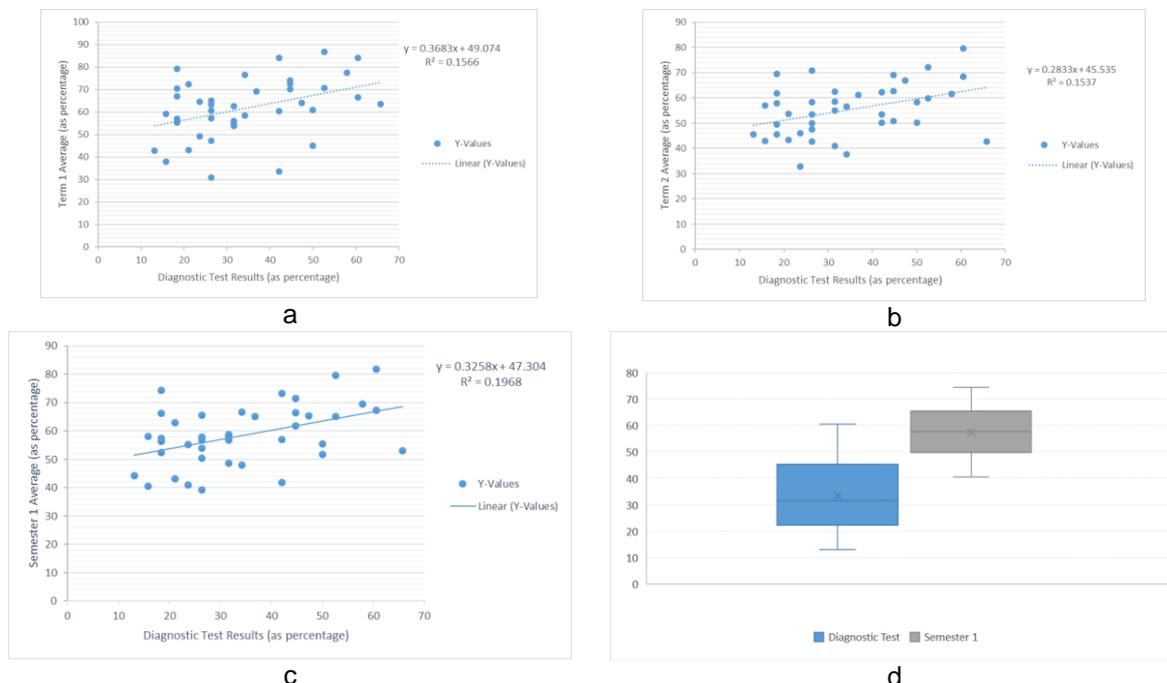


Figure 2: Regression analysis between the diagnostic test and (a) Term 1 average, (b) Term 2 average, and (c) Semester test 1 average; (d) Box plot for the diagnostic test and semester 1 averages

The strongest correlation was with the first semester averages, although the coefficient of determination (R^2) that measures the accuracy of our prediction tells us that only 19.7% of the variation in students' first semester marks is explained by their diagnostic test results. The

various correlations are not particularly strong and the scatter plots are widely spread. Thus, it is not quite clear what the results are telling, but it is just the beginning of exploring possibilities to predict study success in mathematics by means of an entrance test and of developing possibilities of remedying students' entrance deficiencies and thus increasing study success.

Table 3 displays descriptive statistics for diagnostic test results, term 1 data, term 2 data and the semester mark results. At all three time intervals (term 1, term 2 and semester 1) descriptive statistics surpasses the diagnostic test results. It should be interesting to repeat the regression analysis at the end of the second semester when all university mathematics content areas are introduced and to compare the results.

Table 3. Descriptive statistics (as percentages)

	Diagnostic test	Term 1 mark	Term 2 mark	Semester mark
Minimum	13	34	33	41
Maximum	61	79	71	74
Mean	33	60	55	57

Hence, the findings of this inquiry are consistent with previous research that indicated a correlation between prior knowledge components and student achievement at higher education level (compare Hailikari et al., 2007). Following the notion of Dochy et al. (2002) the rather substantial gap in prior knowledge seems to require particular attention in this context.

CONCLUSION

In this study, the author considered the special situation of possible underprepared students in a first-year engineering mathematics course in an extended curriculum programme and this led to the exploration of a way to assess their prior knowledge in mathematics with the intention to correlate the results with student's academic achievement. The theoretical framework that guided this research initiative is mainly connected to the different stands of mathematical knowledge from Kilpatrick, Swafford and Findell (2001) and the notion how prior knowledge affects learning (Hailikari, 2007). In answering the two research questions, *Which particular mathematical knowledge components preceding the study of calculus can be identified as strengths and weaknesses in students' prior knowledge and How strong is the correlation between prior knowledge components and student achievement in a first-year engineering mathematics course at the end of term 1, term 2 and the first semester*, quantitative data were collected and analysed.

Results revealed, partly unexpected, substantial gaps in the content areas preceding calculus, especially in *functions* and *trigonometry*, and in both the sections *calculus* and *modelling*. It seems, from other studies, that prior knowledge facilitates learning and content gaps should be considered in planning suitable activities for teaching. Furthermore, a moderate positive correlation was shown between prior knowledge according to the diagnostic test and student achievement at the end of three different stages. Additional data could support the effort to optimize the diagnostic text to improve the prediction of study success. With these results, the author has now a better basis for meeting the needs of first-year engineering students in an extended curriculum programme.

ACKNOWLEDGEMENT

The author gratefully acknowledges the valuable contribution of Werner Blum in the construction of the diagnostic test.

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