GRADE 10 LEARNERS' UNDERSTANDING OF KEY MATHEMATICS CONCEPTS IN SELECTED SECONDARY SCHOOLS IN CHIBOMBO DISTRICT OF ZAMBIA

by

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DECLARATION

I **Choonya Caesar**, declare that **Grade 10 learners' understanding of key Mathematics concepts in selected Secondary Schools in Chibombo District of Zambia** is my original work and that I never plagiarised from any other source. This work is submitted in fulfilment of the requirements for the degree of Masters of Education in Mathematics Education at the University of South Africa, Pretoria. I would also like to state that this work has never been submitted to this university or any other university before.

Signature: Date: 15 December, 2018.

Choonya Caesar

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ABSTRACT

This dissertation presents a study on Grade 10 learners' understanding of key Mathematics concepts in selected secondary schools in Chibombo District of Zambia. The performance of learners in Mathematics countrywide has not been gratifying. This study therefore sought to investigate the Grade 10 learners' understanding of key Mathematics concepts. This poor performance in Mathematics by learners in many schools at grade 12 final examination in Chibombo District prompted the researcher to conduct this study. The target population was 250 Grade 10 learners in five secondary schools in Chibombo District. The data were collected by means of questionnaire and focus group discussions. A concurrent triangulation design was adopted for the study in which the researcher simultaneously gathered both quantitative and qualitative data merged it using both quantitative and qualitative data analysis.

The study revealed that Grade 10 learners' understanding of the key Mathematics concepts of zero, equal, function, and variable is unsatisfactory. However, it is surprising how these learners can reach secondary school level with such a weak understanding of mathematical concepts, which are ubiquitous in the subject. It is anticipated that the findings of this study will provide curriculum developers with new insights into this problem so that they come up with a curriculum that focuses on learners' understanding of basic concepts that serve as building blocks to next levels. The Ministry of General Education should rigorously monitor this process and provide both technical and financial support required.

Finally, based on the findings of this study, the researcher gave some recommendations for improving learners' understanding of mathematical concepts in schools.

KEY WORDS

Mathematics, concepts, zero, equal, variable, function

DEDICATION

I dedicate this study to my mother Domily Mutinta Chiputa, who is my inspiration and to the almighty God, the source of my strength.

LIST OF APPENDICES

LIST OF ABBREVIATIONS

ANS - Approximate Number System

DEBS - District Education Board Secretary

ECZ - Examinations Council of Zambia

ELL - English Language Learners

JSSLE – Junior Secondary School Leaving Examination

MOGE – Ministry of General Education

SACMEQ – Southern and Eastern Africa Consortium for Monitoring Educational Quality

UNISA - University of South Africa

WMCF – Where Mathematics Comes From

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CHAPTER ONE

GENERAL ORIENTATION

1.1 INTRODUCTION

Mathematics is one of the compulsory subjects in the secondary school curriculum. Mathematics is also a key requirement for learners' admission to tertiary institutions such as colleges and universities. Notwithstanding this, there are still many learners at the secondary school level facing challenges in learning Mathematics. These challenges are reflected in the misconceptions that learners make in relation to key Mathematics concepts. This chapter presents the background to the study, purpose of the study, theoretical framework and statement of the problem. The aim and objectives of the study, research questions, significance of the study, and delimitation of the study and definition of concepts are also discussed in this chapter.

1.2 BACKGROUND TO THE STUDY

Mathematics is an important tool for the development and improvement of a person's intellectual competence in several aspects such as spatial visualisation, logical reasoning, analysis, and abstract thinking. It is therefore important that learners acquire enough knowledge and understanding in Mathematics so that they are able to function as required of them by the society in which they belong. However, despite the significant role Mathematics plays in life, most learners face challenges relating to the understanding of key mathematical concepts required in the learning of the subject. Chibombo District found in the Central Province of Zambia has not been spared from this challenge as the learners' first language is Lenje and Tonga. Language has posed a challenge to learners in the learning and understanding of Mathematics in schools owing to the fact that lessons are conducted in their second language, which is English. The interaction between the teacher and learners and between learners in the classroom is not effective. Hence understanding of mathematical concepts become hindered by lack of a mastery of the language that is intended to drive that understanding. According to Smith and Ennis (1961: 112), "language is both the instrument and the vehicle of teacher-student interaction". It means that language is the conveyer belt through which meaningful learning and understanding on the part of the learner and teaching on the other side of a teacher takes place.

Vygotsky (1978) in Woolfolk (2010) argues that human capacity for language enables children to provide auxiliary tools in the solution of difficult tasks, to overcome impulsive action, to plan a solution to a problem prior to its execution, and to master their own behaviour. When learners are challenged to think and reason about Mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing (National Council Teachers of Mathematics 2000). Listening to others argue about mathematical concepts and being able to contribute in every way possible fosters understanding of Mathematics among learners.

1.3 PURPOSE OF THE STUDY

A focus on investigating learners' understanding of mathematics is important because an adequate understanding of mathematics can promote learners' performance in the subject. However, learners' understanding of the subject largely depends on their understanding of specific concepts that are key in the subject. Also, understanding key mathematical concepts is critical for cognitive development because it provides a way to express ideas and ask relevant and meaningful questions related to mathematics, and to think mathematically (Woolfolk, 2010). Learners need to understand each other and the content of the subject during their interactions as they solve mathematics problems.

Problem solving is one of the pedagogical approaches to teaching, which has highly been recommended in Mathematics Education. In order for this approach to be of reality, learners should understand the technical words used in the mathematics task; otherwise, it will not be worthwhile to pursue such an approach. Vygotsky (1978) in Woolfolk (2010) argues that human capacity for language enables children to provide auxiliary tools in the solution of difficult tasks, to overcome impulsive action, to plan a solution to a problem prior to its execution, and to master their own behaviour. When learners are challenged to think and reason about Mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing (NCTM, 2000). As noted earlier listening to others argue mathematical concepts and being able to contribute in every way possible fosters learners' understanding of Mathematics. Grant and Cook (2011) argue that understanding and success in Mathematics is influenced by English language proficiency in both productive (writing and speaking) and receptive (listening and reading) skills. Therefore, helping learners with their technical and general language proficiency helps them understand key mathematical concepts. Most of the research studies relating to understanding of key concepts have focused on lower grades with only a few on the senior secondary school learners. This research involved learners at the senior secondary school level, to examine their competence in relation to key concepts in their Mathematics syllabus. As noted earlier, the performance of learners in the Mathematics final examinations is dependent on how well learners understand the key concepts used in the syllabus. Questions in the final examinations often involve these key concepts.

1.4 THEORETICAL FRAMEWORK OF THE STUDY

Mathematics is one of the core subjects in the Zambian education curriculum. The status quo of Mathematics education in Zambia is not very different from the neighbouring countries such as Malawi, Mozambique and Namibia. A study by Musonda and Kaba (2011) on Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ), reports that Zambian learners are ranked the worst in Mathematics and reading skills in Southern and East Africa. The report highlighted that one of the contributing factors to poor performance among the Zambian learners is medium of instruction, which is English. The challenge of low learner performance in Mathematics in particular is seen largely as emanating from the demands of Mathematics knowledge to be transmitted in English, which is a second language to many. This is because language and Mathematics are closely connected. Mathematics has in fact been described as a language and that it has a language of its own. A proficiency in language of instruction means proficiency in understanding key mathematical concepts. Learners' lack of proficiency in the language of mathematics makes them unable to discuss mathematical concepts among themselves and the teacher and problem solving becomes a myth. Windsor (2010) indicates that a classroom environment that values and promotes collaborative learning situations and provides learners with opportunities to communicate mathematical ideas and conjectures can facilitate understanding of mathematical concepts better. This means that there is a clear linguistic demand that is associated with learners' ability to learn, understand and express their understanding of mathematics concepts.

In trying to address the linguistic challenge hindering desired performance of learners in schools, the Zambian government has introduced local languages as medium of instruction in grades 1 to 4 before embarking on English as a language of instruction in upper grades. This emphasis to shift at lower grades from English to local language is expected to enable learners express themselves in a meaningful way and be able to participate in their own learning processes and present cognitive overload in pupils (Linehan, 2004). It is believed that this policy shift will result to improved understanding of key concepts in all subject areas and enhance learner performance in higher grades as it will reduce the load on the learner to learn English as a language and subject key concepts at the same time.

The investigation in this research involved the following mathematical concepts: zero, equal, variable, and function. These concepts were chosen because of their significance and relevance in the study and mastery of many topics of the mathematics curriculum, particularly those that are linked to algebra and algebraic thinking (Bardini, Radford & Sabena, 2014). Research findings related to these concepts have revealed important aspects that need to be considered in order to obtain insights into learners' learning and understanding in mathematics generally. These studies have also indicated knowledge gaps that are critical in relation to these specific concepts. For example, on the concept of zero, the findings from Russell and Chernoff (2011) indicate that zero is understood to mean 'nothing' which means 'nothing of something'. It is also understood as a starting point in all situations such as in integers and not a number itself. The research indicates that zero is important only as a place value holder. Research by Akgun and Ozdemir (2006) show that students hold the idea of a variable as a general number. As a result, learners experience trouble in understanding and using variable. Akgun and Ozdemir (2006) quote Ursini (1990) as having found out those learners encounter major problems in understanding variables especially in cases where they are subjected to many different variables. Elia and Spyrou (2006) found that learners understand functions as a bijective correspondence. If two different values of x correspond to the same value of y, then the expression is not a function. Clement (2001) and Elia and Spyrou (2006) found that the students in both research studies understand that a function should always be continuous and that functions should always include some algebraic formula. They do not understand the fact that functions can also be expressed in words. Therefore, it was important to reflect on how learners in the specific context of Zambian schools respond in relation to previous research findings involving these concepts.

1.5 STATEMENT OF THE PROBLEM

According to the Zambian Minister of General Education MOGE Phiri (2015) 'Although we acknowledge improved performance in some individual subjects compared to the 2013 national examination session, the performance in Mathematics is generally poor'. It is very clear from the Minister's statement that the real problem in the performance of learners in schools is in Mathematics. Examinations Council of Zambia Examiners' Report (2015) identified lack of understanding of key mathematical concepts as one of the challenges learners face in answering the word questions and grasping mathematical concepts. From a total of 124,743 candidates who sat for the Mathematics examination in 2015, 13 282 (10.1%) scored zero in paper 1 and 9 038 (7.24%) scored zero in paper 2. One of the attributes to this poor performance is the inability of learners to understand and be able to apply mathematical concepts correctly in the examination. For instance, where questions asked candidates to expand, they instead tried to solve. The Examination Council of Zambia Examiners' Report further indicated that the candidates demonstrated lack of understanding of the key mathematical concepts as they fail to use concepts such as equal sign correctly. Learners have not been able to understand the mathematical concepts as taught in schools largely because of language barrier. In this regard, Grant et al. (2011) argue that success in Mathematics is influenced by English language proficiency in both productive (writing and speaking) and receptive (listening and reading) skills. Mathematical concepts can best be learnt when both the teacher and learners are proficient in the language of instruction. This research therefore intended to investigate Grade 10 learners' understanding of mathematical concepts, to explore how learners understand the following key mathematical concepts: zero, equal, variable and function.

1.6 AIMS AND OBJECTIVES OF THE STUDY

The aim of this study was to explore Grade 10 learners' understanding of concepts zero, equal, variable and function. These were selected from their Mathematics curriculum. The objectives of the study were to:

- Describe Grade 10 learners' understanding of the following mathematical words: zero, equal, variable, and function.
- Examine the extent to which Grade 10 learners are able to apply these key mathematical concepts correctly.
- Describe the extent to which these learners' understanding of Mathematics concepts is related to quality learning of Mathematics.

1.7 RESEARCH QUESTIONS

The main research question of the study was:

1.7.1 Main question

- To what extent and in what ways do Grade 10 learners understand key Mathematics concepts?

1.7.2 Sub questions

In the study, the following sub-questions were answered:

- What are Grade 10 learners' understandings of the mathematical concepts: zero, equal, variable, and function?
- To what extent are Grade 10 learners able to apply these key mathematical concepts correctly?
- To what extent is these learners' understanding of Mathematics concepts promotes quality learning of Mathematics?

1.8 SIGNIFICANCE OF THE STUDY

This study explored how Grade 10 learners' understanding of key mathematical concepts, equal, function, variable, and zero in selected secondary schools in Chibombo District of Zambia. It was envisaged that the study would help subject teachers in schools be able to understand and appreciate the extent to which learners understand these key Mathematics concepts. This would enable the teachers to come up with specific measures aimed at addressing the challenges in order to improve learner performance in Mathematics. This study would help learners understand key Mathematics concepts in order for them to able to make and use generalisations of their world. Learners would also be able to utilise analogies and make correct connections. It would also ensure that learners begin to appreciate the dynamic tools of communication used in Mathematics in order to be precise in any mathematical argument put forward. Ultimately, this study would help to improve the quality of answers learners write in their final national examinations and this is likely to improve the results. The researcher believes the study would also give perceptiveness and cognition to teachers and learners about the importance of learners' understanding of key Mathematics concepts. The findings of this study would be made available to the schools so that appropriate measures can be put in place by school management to help alleviate the challenges identified.

1.9 DELIMITATION OF THE STUDY

This study has a limitation in the sense that only Grade 10 learners were targeted in trying to understand a phenomenon that cuts across grades in a secondary school. Therefore, the results cannot be generalised to all secondary school learners. This grade was picked to study learners' understanding of key Mathematics concepts because it is the middle grade in the secondary school education and more so that these learners have just passed the junior secondary school leaving examination (JSSLE). The size of the sample used in the study was small when compared to the number of all Grade 10 learners in the district. As such, the researcher could not make generalisations using the findings of this study. The study is limited to the learners only without bringing in their teachers into perspective to get a balanced view on the subject. The research design used may not be the best out of all designs as a result a suggestion is made that other studies on this same topic should try and explore other designs and be able to compare the findings. The focus in this research was on those concepts where other researchers have already done some researches so that results can be compared with some findings from earlier researches.

1.10 DEFINITION OF MATHEMATICAL CONCEPTS

1.10.1 Mathematics

Lakoff and Nunez (2000) define Mathematics as a subject that is precise, consistent, stable across time and human communities, symbolisable, calculable, generalisable, universally available, consistent within each of its subject matters and effect as a general tool for description, explanation, and prediction in a vast number of everyday activities ranging from sports, to building, business, technology, and science. On the other hand, Bernkopf (1975) defines Mathematics as the study of abstractions and their relationships in which the only technique of reasoning that may be used to confirm any relationship between one abstraction and another is deductive reasoning.

1.10.2 Key Concepts

Key concepts in Mathematics are those, which are significant in the field of Mathematics and have some abstract meaning to the subject. Concepts such as equal, function, variable, zero, and many others are significant in the learning of Mathematics.

1.10.3 Understanding

Understanding of these key concepts in this research is taken to mean the ability to comprehend the meaning and being able to apply such a concept correctly in Mathematics.

1.10.4 Zero

Zero is something that represents nothingness.

1.10.5 Equal

The equals sign or equality sign (=) is a mathematical symbol used to indicate equality between two or more expressions.

1.11.6 Variable

A variable is a symbol that represents a number.

1.11.7 Function

A function is a relation or an expression involving one or more variables.

1.11 RESEARCH DESIGN AND METHODOLOGY

1.11.1 Research design

As noted in the introduction to this chapter, the purpose of this research was to investigate the Grade 10 learner's understanding of selected Mathematics concepts from the senior secondary school curriculum. In doing so, the research will use mixed methods research design where both quantitative and qualitative research methods are to be used in providing complete answers that meet the purpose of the study. According to Creswell (2007:37), 'Qualitative research design begins with assumptions, a worldview, the possible use of a theoretical lens, and the study of research problems inquiring into the meaning of individuals or groups ascribe to a social or human problem'. The fact that the problem in this research is within the understanding of the behaviour of humans, then qualitative research design is suitable to address some questions in this study. Quantitative research design, which emphasises on objectivity in measuring and describing phenomena, was used in the research. Qualitative and quantitative research designs allow the researcher to collect data directly from the source thereby promoting validity on the data-set collected. It promotes detailed narrative, which ensures in-depth understanding of the behaviour under study. Qualitative methods are typically more flexible – that is, they allow greater spontaneity and adaptation of the interaction between the researcher and the study participant. For example, qualitative methods ask mostly "open-ended" questions that are not necessarily worded in exactly the same way with each participant. The fact that the research is on the understanding of Mathematics key concepts by the Grade 10 learners and that data collected by interacting with the learners, qualitative research design provides the best platform as its focus is on the participants understanding, descriptions and meanings. Through careful data preparation, coding and interpretation, the results of qualitative content analysis can support the development of new theories and models, as well as validating existing theories and providing thick descriptions of particular settings or phenomena (Zhang & Wildemuth, 2005). The fact that the research involves human beings means that the design will not be static and that certain aspects of the research will change as the research progress and qualitative research design is one that can accommodate such changes in the process. This research is a triangulation type of mixed method design where both quantitative and qualitative data collection is implemented at the same time.

1.11.2 Research methodology

Research methodology is concerned with the application of a variety of interchangeable methods and techniques in the quest of valid knowledge (Mouton, 2002:35). The procedures by which researchers go about their work of describing, explaining and predicting phenomena are what are called research methodology (Rajasekar et al. 2013:5). The research methodology is determined by the research approach followed in the study. In line with the aim of the study namely, to investigate grade 10 learners' understanding of key Mathematics concepts, a mixed method approach would be used in this study. A mixed method research is an enquiry process that provides the researcher with opportunities to compensate for inherent method weaknesses, on inherent method strengths, and offset inevitable method biasness (Greene, 2007:3). Two data collection instruments were used to collect data from the participants. These are open-ended questionnaire and structured interview schedule for focus group discussions.

1.12 POPULATION AND SAMPLE

The population of the study was 1400 Grade 10 learners from all the eight secondary schools in Chibombo District at the time when the study was conducted. However, 250 Grade 10 learners were chosen from five secondary schools using stratified purposeful, which is based on the judgment of the researcher. The 250 Grade 10 learners on average of 50 learners per school were given the questionnaire to supply the information required. From a total of 50 learners per secondary school who filled in a questionnaire, four learners were picked per each of the five secondary schools for focus group discussion. The selection was based on the responses in the questionnaire.

1.13 DATA COLLECTION

1.13.1 Questionnaire

In line with mixed method research design adopted for this research, a questionnaire was used as one of the data collection instruments. The purpose of the questionnaire was to collect data from the informants who are perceived to have the information needed. A questionnaire can easily reach out many participants in all the schools at once. Schools in Chibombo district are distributed across a wide geographical area and gathering information in such areas requires the use of a questionnaire. The research questionnaire will bring out the Mathematics concepts to the learners to show their understanding and will give an insight of how the understanding of these concepts can promote teaching and learning of Mathematics. After analysing the data from the questionnaire, special cases of interest arising from the responses (high performing and the low performing learners) went for focus group discussion in order to collect more data. Four learners from each of the five schools were selected for focus group discussion.

1.13.2 Focus group discussion

Focus group discussion is a rapid assessment, semi-structured data gathering method in which a purposively selected set of participants meet to discuss issues and concerns. These issues and concerns should be based on a list of key themes drawn up by a researcher or facilitator (Kumar, 1987). This qualitative research technique was developed in order to give researchers a better understanding of data. In addition, focus group discussion has become extremely popular because it provides a quick way to learn from the target audience (Debus, 1988; Kruger, 1988). Focus group discussions are relatively cheap and flexible. It allows participants to question each other and to elucidate their responses. However, the multiple voices of the participants result in researcher's ability to control the discussion. In this study, four learners per school were identified among those who responded to the questionnaire with help from their teacher for focus group discussion on the key mathematical concepts.

1.14 DATA ANALYSIS

This study is a mixed method design. The data will be analysed both quantitatively and qualitatively. The data collected from open-ended items in the questionnaire were analysed by the use of descriptive statistics in form of percentages and frequencies relating to commonalities in learners' meanings of the Mathematics concepts. Responses from learners were put into categories according to emerging themes (Creswell, 2007) linked to meanings of particular concepts and their applications in Mathematics. The data to be collected from open-ended items in the focus group discussions were transcribed, read, re-read and analysed thematically using thematic analysis, as themes and sub-themes emerge from the data. Thematic analysis is a method for identifying, analysing and reporting patterns (themes) within data (Braun & Clarke, 2006:79). It is also seen as a qualitative descriptive method that provides core skills to researchers for conducting all forms of qualitative research data analysis. According to Braun and Clarke (2006:87), the qualitative data analysis is as follows:

STAGE	ACTIVITY	DESCRIPTION
1	Familiarising with data	Transcribing data, reading and rereading the data, noting down initial ideas.
2	Generating initial codes	Coding interesting features of the data systematically across the entire data set, collating data relevant to each code.
3	Searching for themes	Collating codes into potential themes, gathering all data relevant to each potential theme.
4	Reviewing themes	Checking if themes work in relation to the coded extracts and the entire data set, generating a thematic map
5	Defining and naming themes	Ongoing analysis for refining the specifics of each theme and the overall story that the analysis tells, generating clear definitions and names of each theme.
6	Producing the report	The final opportunity for analysis. Selecting of vivid, compelling extract examples Final analysis of selected extracts, relating back to the research questions and literature Producing a report of the analysis

The responses from learners will be compared with findings involving participants from other studies linked with learners' understanding of Mathematics concepts (see for example, Russell & Chernoff, (2011); Akgun & Ozdemir, (2006); Mbewe, (2014)).

1.15 ETHICAL CONSIDERATIONS

After obtaining ethical clearance from University of South Africa (Unisa), the researcher wrote a letter to the Chibombo District Education Board Secretary (DEBS) requesting permission to carry out research in the District. The following subject areas were adhered to in accordance to the demands of ethical clearance for the research to collect and use data from the participants.

1.15.1 Permission

The researcher sought permission from the DEBS who is in charge of educational institutions in the district by submitting a letter to seek permission. In addition, the researcher sought permission from the head teacher of every institution involved by submitting a consent letter.

1.15.2 Informed consent and assent

The head teacher of the school gave permission representing the parent in case of participants in boarding schools. The participants to the research who are not in boarding will be asked to give the researcher phone numbers of their parents so that the researcher can phone them to determine whether they are willing to let their child participate in the study. The participants were asked to give assent before participating in a study.

1.15.3 Voluntary participation

The participants were informed of their right to participate voluntarily. More importantly, participants were free to terminate their participation at any stage when they feel like without any consequences.

1.15.4 Confidentiality

Participants were assured of their anonymity and in this regard, the researcher will assure participants that their names or identities and the name of the school were not being disclosed. The findings were published in a report to be written and since this research is purely academic, the report will be circulated for library use. The participants were given feedback through a meeting which were arranged by the researcher and the school administration.

1.15.5 Protection from harm

The researcher also assured the participants that their participation in this research was not cause them any physical soreness, abasement and effusive stress.

1.16 RESEARCH STRUCTURE

Chapter 1 focuses on the outline of the background to the problem, the problem statement, research questions, the significance of the investigation, aims and objectives, roadmap to data analysis, delimitation, theoretical framework, description of the methods of investigation and definition of concepts, and ethical considerations.

Chapter 2 focuses on a review of the literature and the theoretical background of the investigation. Primary and secondary sources in relation to the topic were used. The literature review will concentrate on a wide review of relevant writings such as the latest articles, journals, and major books on the subject, including dissertations published.

Chapter 3 focuses on research design and methodology. Here the methods of investigation including specific procedures, research population and sampling, data collection and treatment were outlined and described.

Chapter 4 concentrates on data analysis and interpretation of data. In this chapter, the responses of the respondents were presented and elucidated by means of charts and tables, which were accompanied by a detailed analysis and interpretation.

Chapter 5 serves as a synthesis and comprises summary of each chapter, summary of findings as well as the conclusions or results. Finally, the recommendations for the future are provided.

1.17 CONCLUSION

In this chapter, the researcher gave an overview to the study, introduction, and discussed the background of the study about secondary school learners' understanding of key Mathematics concepts. The objectives, problem statement, research questions, theoretical framework, significance of the study, summary of research design and literature, were also presented and highlighted. In the next chapter, the researcher presents literature review on learners' understanding of key Mathematics concepts.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

The previous chapter dealt with the contextual definition and other preambles of the study. In this chapter, various literature sources will be reviewed in order to explore the learners' understanding and application of key Mathematics concepts of equal, zero, function, and variable. Literature review is a brush up of the existing scholarship or available body of knowledge, which helps the researchers to see how other scholars have investigated the research will share problem in which they are interested (Mouton, 2002:87). Literature review discusses the results of other studies that are closely related to the study being reported (Creswell, 1994: 34). In addition, literature review enables me to learn how other researchers have set, mark, or draw the boundaries of similar problems, collected data, theorised and conceptualised issues related to learners' understanding of key Mathematics concept like zero, function, equal, and variable. The literature review includes the findings and suggestions yielded and the relevant literature will justify the framework of the current study.

2.2.1 The mathematical concept of equal sign

The equal sign is with no doubt the most prevalent symbol in school Mathematics and developing an understanding of it has been considered a straightforward issue in Mathematics education (Knuth et al., 2008:514). The understanding of the equal sign concept is essential to algebraic understanding as it is seen as the foundations of the transition from arithmetic to algebraic reasoning (Hunter, 2007:421). Algebra is one such an important branch of Mathematics where learners have to grasp the concepts involved in order for them to be able to operate and live a normal life both in classroom situation and outside situations. Algebraic thinking begins to develop in children in their early grades and their knowledge of properties of numbers and operations creates the basis upon which to learn algebra. The equality concept is one such fundamental concept to algebra understanding and considerable research attention has been given to it (Behr et al. 1980; Kieran, 1981; Falkner et al., 1999; McNeil & Alibali, 2005). The presence of the equal sign almost everywhere at all levels of Mathematics highlights its significance in the learning and teaching of Mathematics (Knuth et al., 2006:298). This is why it is important for all learners to understand this concept in their early levels of their education for them to be able to operate in their higher levels of education and after school effectively and efficiently. Carpenter, Falkner and Levi (1999: 234) argue that the equal concept is central to arithmetic because children need to think about relationships expressed by number sentences as children with this ability will be able to represent arithmetic ideas, communicate them and further reflect on the same ideas. Children as young as kindergarten age may have appropriate understanding of the equality relations involving collections of objects but have difficulty in relating this understanding to symbolic representations involving the equal sign (Falkner et al., 1999:233).

2.2.2 Learners understanding of the equal concept

The equal sign (=) is omnipresent in Mathematics as it is common in almost all the Mathematics done, and a convoluted concept of the symbol is indispensable for understanding many topics in Mathematics such as algebraic equations (Knuth, 2006). However, research studies in Mathematics Education have indicated that many elementary school students (ages 7 to 11) have an inadequate understanding of the equal sign (Baroody & Ginsburg, 1983; Behr, Erlwanger, & Nichols, 1980). Instead of interpreting it as a relational symbol of mathematical equivalence, most students interpret the equal sign as an operational symbol meaning “find the total” or “put the answer (Carpenter, Franke, & Levi, 2003; Kieran, 1981; Rittle-Johnson & Alibali, 1999). Students do not only provide operational interpretations when asked to define the equal sign, but also rate operational interpretations such as “the total” and “the answer” as smarter than relational interpretations such as “equal to” or “two amounts are the same” (McNeil & Alibali, 2005). Operational interpretation of the equal sign by students is one where it is viewed as a tool to produce the answer to a question. It is applied in instances where one wants to indicate the final stage of the solution. Relational interpretation of equal sign as a mathematical equivalence is where the equal sign is taken as a tool to indicate the relation that exists on either side of the equal sign. In view of this evidence, some researchers have contended that the operational interpretation of the equal sign is a by-product of students’ experiences with the symbol in elementary school Mathematics. In elementary school, students often encounter the equal sign in the context of equations that have operations on the left side of the equal sign and the answer blank on the right side (Baroody & Ginsburg, 1983; Carpenter et al., 2003; McNeil & Alibali, 2005; Seo & Ginsburg, 2003). It is clear that the concept of equivalence is an elusive one not only for elementary school students but for high school students as well. Early elementary school children view the equal sign as a symbol that separates a problem and its answer (Kieran, 1981:4).

It seems reasonable to suggest that the contexts in which teachers using the curriculum present the equal sign play a major role in the development of students 'understanding of the equal sign and researchers have argued that knowledge of concepts may be context dependent' (Barsalou, 1982; Thelen & Smith, 1994; Munakata, McClelland, Johnson, & Siegler, 1997). Individual students may exhibit knowledge of a concept in some contexts, but not in others. This may be especially true of newly emerging concepts, such as the relational understanding of the equal sign as opposed to operational interpretations.

The notion of 'equal' is complex and difficult for students to comprehend, and it is also a central mathematical idea within algebra (RAND Mathematics Study Panel, 2003: 53). In the quest to improving students' understanding of the equal sign, and thus their preparation for algebra, may require changes in teachers' instructional practices as well as changes in elementary and middle school Mathematics curricula. Stacey and MacGregor's (1997) recommend that teachers should present students with statements of equality in different ways to further develop students' notions of equivalence.

2.2.3 Learners' misconception of the equal concept

The operational and relational understanding of the equal sign has been quite disputatious among researchers, who bring forward variables that may hinder or promote the understanding of the operations. One school of thought contends that students` early exposure to arithmetic operations related to addition, subtraction, multiplication and division has a bearing on how they interpret, use and understand the equal sign (Kieran 1981; Baroody & Ginsburg, 1983). This school of thought gives rise to a relational view that proposes equivalence. The other philosophical system postulates that the way individual students learn may have an underlying obstruction of having a relational interpretation of the equal sign (Molina & Ambrose, (2008). In essence, it would not be wrong to assume that both cognition and instructional factors do contribute to students` perception of the equal sign as a symbol operator (Baroody & Ginsburg, 1983). Because of learners' perception of the equal sign as an operational symbol, they consider the equal sign as a signal for them to "do something" or to "find the total" or that "the answer.

Students viewing the equal sign as a signal to “do something”, look to the right side of the equal sign (Cobb, 1987) the same as those who view the equal sign as a prompt to find the total even when finding the total is not appropriate (McNeil & Alibali, 2005). When students face unfamiliar number sentences other than of the form 8 + 2 + 6 = _ + 10, they add all the four numbers and get 26 as their answer. Meanwhile for students who interpret the equal sign as meaning “the answer comes next” will put 16 as the answer in the blank space (Sherman & Bisanz, 2009). Furthermore, Falkner et al.’s (1999) study of students` attempt to solve the equation 8 + 4 = □ + 5 got answers of 12, 17 or both 12 and 17 as the missing addend. Evidence of operational understanding has been recorded even among secondary school students. Kieran (1981, p. 324), for example, found 12 and 13 year-olds had difficulty in assigning meaning to expressions like 3a or a + 3 because, as one student stated, “there is no equal sign with a number after it”. McNeil and Alibali (2005) found this poor understanding of the equal sign even among college students. These studies show that misinterpretation of the equal sign does not get better with students` progress through grade levels. Jones (2009) argues that “the do something” interpretation of the equal sign seems to hold as he agrees with Kieran (1981).

2.3 THE MATHEMATICAL CONCEPT OF ZERO

Zero is the integer denoted 0 that, when used as a counting number, means that no objects are present. It is the only integer that is neither negative nor positive. A number which is not zero is said to be non-zero. Although many details about the development of understanding natural numbers are already known, the development of understanding zero is hardly known, and it is not integrated into the numerical cognition models (Wynn, 1990). The development of numerical knowledge cannot specify how the understanding of zero is incorporated into a more holistically numerical knowledge. There is an agreement that numerical information is probably processed by two representations (Krajcsi et al., 2017). First, the approximate number system (ANS) can represent numbers in an imprecise way: infants are able to discriminate two values if the difference is larger than a specific ratio (Feigenson, Dehaene, & Spelke, 2004), which ratio (the Weber-fraction) improves with age. Secondly, there is an object file system or visual indexes that can represent objects up to a limit of 3 or 4 items, which system can implicitly store the number of those objects (Feigenson et al., 2004). However, it is not clear whether these systems can represent zero.

The next important step in the development of number understanding is the understanding of the exact large symbolic numbers in pre-schoolers. At around the age of three or four years, children start to understand the conceptual principles of number use which is termed as the understanding of cardinality principle (Lipton & Spelke, 2006; Sarnecka & Carey, 2008). Number conception in the age group three or four years of age is mostly measured with the “give a number” task in which children are asked to give a specific number of objects from a pile of objects (Wynn, 1990). With this task, one can determine what number understanding phase a child is in. Pre-schoolers become cardinality-principle knowers when they begin to give any amount of items that is in their known counting list. However, this last phase is believed to depict the real understanding of exact symbolic numbers (Lipton & Spelke, 2006; Sarnecka & Carey, 2008). There is no consensus what representational changes occur when a pre-schooler understands the cardinality principle.
Their finding was that the understanding and the use of zero were delayed compared to the use of positive numbers. In a detailed analysis, the researchers’ conclusion was that there are three typical behavioural phases. In the first phase, children could name the zero symbol, although they did not understand the meaning of it. In the second phase, the children could count backward to zero, understanding that zero means nothing. Finally, in the third phase, the children knew that zero was the smallest number, and they could compare numbers even if one of the numbers was zero. The progress of the development was slow: four-year-old children did not reach the first phase, and only six-year-old children were able to reach the final phase. However, a later study found evidence that understanding zero is easy for pre-schoolers. Bialystock and Codd (2000) investigated the understanding and spontaneous notation of positive integers, zero, and fractions in preschool children. They found that understanding zero is not harder than understanding positive integers, a conclusion that is not in line with the previously described study by Wellman and Miller (1986) and Schlin and Skosnik (2010). Pre-schoolers can handle zero on the same level as they handle positive integers. Although the linguistic form can cause difficulties for them, this is independent of their conceptual understanding. Children's achievement of this understanding occurred in three phases. At each phase, understanding of zero lagged behind comparable understanding of other small numbers (Wellman & Miller, 1986). The second study, with 51/2–10-year-olds, investigated children's developing conception of simple algebraic rules, such as a + 0 = *a*. Results showed that even the younger children had some understanding of several algebraic rules. The older children had acquired more such knowledge, but at all ages algebraic

understanding was advanced for rules pertaining to zero, in comparison to those pertaining to other small numbers. These results suggest that zero plays a special role in children's increasingly algebraic knowledge of number. In addition, Wellman and Miller (1986) conclude that since zero is difficult to conceive of and use originally, children develop special rules for its use, and this provides a first step towards their formulation of more general algebraic rules to use in later stages of development.

2.4 UNDERSTANDING MATHEMATICAL CONCEPT OF FUNCTION

The mathematical concept of function plays an important role throughout the Mathematics curriculum (Lisa, 2001:745). The function concept has received rather much attention within the Mathematics Education research community, with the production of outputs such as conference papers and books (Harel & Dubinsky, 1992; Romberg, Fennema, & Carpenter, 1993). A major reason for this is the role that the function concept is often seen to play as a unifying concept in Mathematics (Sierpinska, 1992: 32). Mathematical function thinking should permeate all of Mathematics and students are expected to be brought up to functional thinking. Developing a sense for functions in students should be one of the main goals of the school and collegiate curriculum (Eisenberg, 1992:153). The teaching and learning of the concept of mathematical function is a widely researched topic. Vinner and Dreyfuss (1989) argue that students' mental images of functions may be different from its mathematical definition. Other studies confirm that students' function conceptions in many cases are limited. Bloch (2003) argues that students cannot think of different functions that provide a graph through the same pair of points. Meel (1998) argues that many of the participants in his study held a rule-based interpretation of the function concept, which prevents them from an object-view on the concept, which is needed, for the understanding of the notion of composite function. In another study on the understanding of concept of a function, Oehrtman et al (2008) stress the relevance of students' reasoning abilities and on a conceptual orientation on the notion of function, which needs time.

Bardini et al. (2014) argue that in their research students exhibited the understanding of concept of function in the following:

- *A function is a rule, which relates the values of one variable quantity to the values of another variable quantity, and does so in such a way that the value of the second variable quantity is uniquely determined by the value of the first variable quantity.*

- *It is an equation that has one y value for every x value. If you draw a line through the plot of the graph, the line will only pass through the plot once.*
- *A mapping, usually in the form of a rule, from one set to another, with the property that each input has a unique output.*
- *A graph that passes the vertical line test.*
- *It is like a machine that has an input and an output. And the output is related somehow to the input.*
- *Anything that takes an input and spits out a single output. Example: an apple juicer.*
- *It is a mechanism that alters numbers put in to equal different numbers when they come out.*
- *A function is a one-to-many relation.*
- *Something with the ability to do something else.*
- *Something with x in it.*
- *Describe the real world in mathematical terms.*

It is clear that some students had a well-connected working understanding of the function concept, equally others had attempted to memorise definitions, and still others displayed naïf conceptual understanding.

2.4.1 Misconceptions about mathematical concept of function

A study carried by Oehrtman, et al. (2008) found that students often confused the visual attributes of a real-world situation with similar attributes of the graph of a function that models the situation. School Mathematics tends to focus on special features of graphs, for example, maximum turning point, minimum turning point, point of inflection, and gradient. Oehrtman, et al. (2008) maintain that the superficial similarity of these features of graphs and the real world setting often leads to confusion, even for students with a strong understanding of functions. Students often think of the graph of a function as a picture of a physical situation rather than as a mapping from a set of input values to a set of output values. Developing an understanding of function in such real-world situations that model dynamic change is an important bridge for success in advanced Mathematics. The definition of function brings confusion to both the teachers and their students. There have been many debates as to which definition would best be used for function. Most teachers could provide an informal description of function that would be useful for explaining to someone who did not understand the concept. However, the formal definition of function caused considerable confusion. Norman (1992:224) argues that teachers' response to the definition of a function in a study as a situation where each member of the domain does not match with a unique member of the range [referring to the set {(1, 3), (2, 3), (3, 3)}]. But it does vice versa, so each member of the range matches with a unique member of the domain. This means that this teacher had not built strong connections between their informal definitions of function and what they viewed as the formal mathematical definition. In addition, Oehrtman, et al. (2008), referring to Carlson's 1998 study, reports that many students believed that constant functions (e.g., $y = 4$) were not functions because they do not vary but constant. Many students believe that a function should have a variable (e.g., $y=4x$) where the value of x can be any integer. Carlson (1998) undertook a study that included high achieving students who had either just completed college algebra or, at least, some calculus subjects. The college algebra course had included an introduction to functions. Carlson (1998) found that many students did not understand function notation, had difficulty understanding the role of the independent and dependent variables in a given functional relationship, could not explain what is meant by expressing one quantity as a function of another, and were unable to speak the language of functions.

2.5 DEFINITION OF THE CONCEPT OF VARIABLE

Variables are of critical importance in Mathematics. The concept of variable is central to Mathematics teaching and learning in school (Schoenfeld & Arcavi, 1988:420). The understanding of the concept provides the sound basis for the transition from arithmetic to algebra. This transition is important for the meaningful application of all Mathematics at higher levels of education. Despite the importance of the concept, most teachers of Mathematics have not treated variables as an important concept where they need to engage the learners to establish a meaningful understanding. The variable concept is seldom discussed in most of the lessons where algebra is given. However, this concept is the basis of everything that students learn. However, most basic textbooks do not explain or even mention the concept of variable. Skemp (1987) argues that a variable is a key concept in algebra. The concept variable is one of the most essential concepts in taught Mathematics; beginning with primary school and continuing throughout high school (Sasman et al., 1997). Akg¨un &and Ozdemir (2006) mentions Rajaratnam who argued in the late fifties that this concept is so important that its discovery was a milestone in the history of Mathematics. Nunn (1919) argues that discovery of variables is probably the most important event in the history of humanity and the sovereignty of their use will remain as one of the most important successes of the history of humanity. Despite the importance of the concept, much of the Mathematics curricular continue discussing variables like simple terms (Kieran 1981). Furthermore, Akgun and Ozdemir (2006) defines a variable as a quantity that can take infinite numerical values. Many textbooks distinguished quantities that represent a unique value (constants) from quantities that can stand for several values (variables). For example, in the canonical equation of the circle,$x^2 + y^2 = b^2$, x and y are variables and b is constant. Tarski (1941) argues that as opposed to the constants, the variables do not possess any meaning by themselves. A variable number cannot have any specified property as the only properties that such a number can have would change from case to case.

2.5.1 Learners' understanding of concept of variable

Understanding the concept of variable is crucial in constituting a strong foundation for further algebraic concepts related to it (Balyta, 1999). This understanding is the basis of all advanced studies in Mathematics (Graham & Thomas, 2000). It is thus to a great depth important for all students to gain confidence in using variables. Because variable is a concept that requires a rich concept-image, students are not expected to understand that concept easily (Tall & Vinner, 1981). Many students think that all variables are letters that stand for numbers when in real sense the values a variable take are not always numbers. Variables often represent points as seen in the use of variables A, B and C in an isosceles triangle where AB=BC (Usiskin, 2017). In real sense, variables are often used to mean different phenomena. For instance, q and p often stand for propositions and the variable *f* stand for a function while in linear algebra the variable A may stand for a matrix (Usiskin, 2017:10). This shows that a variable is not only represent a number but many other issues such as propositions, points on the vertex of an object or indeed naming of sets and matrices. Students understand the concept of variable as a symbol that can be replaced by any element of some designated set of numbers called the domain of the variable (Schoenfeld & Arcavi, 1988). This is the input from the domain in the function that has a variable to give an output in the range. The variable takes different values from the domain set to give different values in the range. Variables are assumed to represent an empty space into which an arbitrarily element from a fixed set can be substituted. They are applied in laws and solutions of problems expressed in terms of variables. In a right angled triangle with vertices as A, B and C, variables a, b and c vertically opposed to the respective vertices are used as lengths. From this situation, the Pythagoras theorem can be stated as $a^2 + b^2 = c^2$ where *a, b* and *c* are variables used in Pythagoras law where a and b represent the two adjacent sides of a right-angled triangle and c is the hypotenuse side. Students rarely interpret algebraic letters standing for variables because they get used to studying numbers (Macgregor & Stacey, 1997). After taking on letters as variables, students are more likely to accept and use them as unknown than varying quantities and generalised numbers (Kieran, 1992). Students of the middle class interpret letters as standing for objects or words (Kuchemann, 1981; Macgregor & Stacey, 1997). Another critical understanding of variables by students is that they tend to associate letters with their positions in the alphabet. They tend to think that from the variables *a, b, c* and *d, a* is taken to be the smallest number representation and *d* is the biggest variable in size of the four (Watson, 1990).

2.6 CONCLUSION

Various factors have been identified concerning the learners' understanding and application of the mathematical concepts of zero, equal, function, and variable. The concept of zero has been identified as the smallest number and that the understanding of it is not harder than that of positive numbers. Zero is difficult to conceive and use at first by children and so they develop rules for its use. It is taken to mean 'nothing'. Most students interpret the concept of equal as an operational symbol instead of it being relational. It is used to put the answer or to find the total more often than to relational interpretations such as 'equal to' or 'two amounts are the same'. The concept of function is based on the rule: equation that has one y value for every x value. It is argued to be a machine that has input and output and the two are related by the function. On the concept of variable, the findings from other researchers are that variables are letters that stand for numbers. Variables are also seen to apply in representing the points on diagrams and in formulation of propositions.

Research methodology and design of the study will be presented in the subsequent chapter.

CHAPTER THREE

RESEARCH METHODOLOGY AND DESIGN

3.1 INTRODUCTION

Chapter 2 dealt with a review of the literature that expatiates on the learners' understanding of key Mathematics concepts. The literature review focused on learning how other scholars have conjectured and gestated issues related to learners' views on some key Mathematics concepts such as equal, function, variable, and zero. This study was aimed at systematic collection and interpretation of information, which will enable the researcher to suggest solutions for the waning performance in Mathematics in most schools in Chibombo District and nation as a whole. The current juncture is putting so much coerce on teachers and school administrators to improve their practice by adopting more responsive ways of teaching and managing schools for the purpose of improving learner performance in schools. This chapter provides an account of the research design and methodology used to conduct this research. This includes a description on the research paradigm, research questions, research design, sampling, the data collection methods, and the methods for data analysis. The researcher hopes that this study will provide valuable input to the improvement of learners' understanding of some key Mathematics concepts, which will assist in the improvement of learner performance in Mathematics in Chibombo District in particular and Zambia in general.

3.2 RESEARCH PHILOSOPHICAL FOUNDATIONS

Although philosophical ideas remain largely hidden in research, they still influence the practice of research and need to be identified (Creswell, 2014:35). Researchers preparing research project make it clear on the philosophical ideas to follow. This explains why they prefer the research designs of qualitative, quantitative or mixed research. This set of beliefs about how the world is viewed and how it is understood and studied is called a "paradigm" (Guba, 1990:17). According to Schwandt (2001:183), a research paradigm is a shared worldview that represents the beliefs and values in a discipline and that guides how problems are shared. Therefore, a paradigm leads us into soliciting answers in certain areas and use suitable approaches to systematic inquiry such as how do we study the world (methodology), what do we hold to be true (axiology) and more importantly, how do we know what we know (epistemology). A paradigm influences the researcher's conclusion on the research questions and methodology to employ (Morgan, 2007:49). Research should be based on a paradigm that elucidates the study and as such, all researchers must consider the fundamental interaction of such eye shots when conducting research (Creswell & Piano Clark, 2007:21). Particular paradigm may be associated with certain methodologies. A positivistic paradigm is associated with a quantitative methodology, while a constructivist or interpretative paradigm is typically a qualitative methodology (Chalisa & Kawulich, 2011:2). No single paradigm is correct as the onus remains on the researcher to determine the paradigm to use. However, factors such as assumptions about the nature of reality and knowledge, theoretical framework, literature and research practice, values, and ethical principles influence the choice of a certain paradigm (Chalisa & Kawulich, 2011:3). There are basically four worldviews that are widely used in research, namely, post positivism, constructivism, transformative, and pragmatism (Creswell, 2017:36). The post positivism is a worldview that represents the traditional form of research. The assumptions of this worldview hold true for quantitative than qualitative research. It is sometimes called the scientific research or doing science research (Creswell, 2014:36). The proponents of this worldview believe that causes are most likely determining effects and that knowledge is absolute truth and cannot be found. Social constructivism is another philosophical worldview that seeks understanding and theory generation. Unlike post positivism, social constructivism holds true for qualitative more than quantitative research. The purpose of constructivist research is to make some meaning of the world around, using generated data through research. Another group of researchers holds to the assumptions of the transformative approach (Denzin & Lincoln, 2008:29). A transformative worldview holds that research inquiry needs to be intertwined with politics and a political change agenda to confront social oppression at whatever level it occurs (Mertens, 2010 in Creswell, 2014:37). The research problem, questions and aim are of a multifarious nature. Arising from this, both qualitative and quantitative approaches were selected for this research. The combination of research approaches has led to the adoption by many scholars of a pragmatic research as a philosophical underpinning of conducting a mixed method research (Creswell & Garrett, 2008:327; Creswell, 2009:10). The reason for choosing a pragmatic research is because this particular paradigm provides a theoretical basis and is also regarded as the philosophical partner of mixed methods research. This study will be based on a pragmatic paradigm because the aim of the study is to describe Grade 10 learners' understanding of the following mathematical words: zero, equal, variable, and function.

3.3 RESEARCH QUESTIONS

The main research question of the study was:

To what extent and in what ways do grade 10 learners understand key Mathematics concepts?

In the study, the following sub-questions were answered:

- What are Grade 10 learners' understandings of the mathematical words: zero, equal, variable, and function?
- To what extent are Grade 10 learners able to apply these key mathematical concepts correctly?
- To what extent is these learners' understanding of Mathematics concepts promotes quality learning of Mathematics?

3.4 AIM OF THE STUDY

The aim of this study was to explore Grade 10 learners' understanding of concepts equal, function, variable, and zero selected from their Mathematics curriculum.

3.5 OBJECTIVES OF THE STUDY

The study sought to:

- describe Grade 10 learners' understanding of the following mathematical words: zero, equal, variable and function.
- examine the extent to which Grade 10 learners are able to apply these key mathematical concepts correctly.
- describe the extent to which these learners' understanding of Mathematics concepts is related to quality learning of Mathematics.

3.6 RESEARCH DESIGN

Cohen and Manion (1994:29) describe research as the process of arriving at dependable solutions to problems through planned and systematic collection, analysis and interpretation of data. Research design indicates the general plan: how the research is set up, what happens to the subjects, and what methods of data collection are used (McMillan & Schumacher, 2010:20). The main function of a research design is to enable the researcher to anticipate what the appropriate research decisions are likely to be, and to maximize the validity of the eventual results (Mouton, 1996:107). The main aim of research design is to specify a plan for generating empirical evidence that will be used to answer the research questions. The research problem will determine the methods and procedures, types of measurement, the sampling, the data collection, and the data analysis to be employed for the proposed research (Zikmund, Babin, Carr & Griffin 2010:66). A mixed methods research design was used for research to determine the Grade 10 learners' understanding of key Mathematics concepts. A mixed research design is a research in which the researcher collects and analyses data, integrates the findings, and draws inferences using both qualitative and quantitative approaches and methods in a single study or a programme of inquiry (Tashakkori & Creswell, 2007 in McMillan & Schumacher, 2010:396). In another attempt to define mixed method design, (Johnson, Onweugbuzie and Turner, 2007 in McMillan and Schumacher 2010:396) assert that it is a research in which the researcher combines elements of qualitative and quantitative research viewpoints, data collection, analysis, inference techniques for the broad purposes of breadth, and depth of understanding and collaboration. This is what makes mixed method research design is unique and able to provide insights that are not possible when either qualitative or quantitative approach is used in isolation. It is in this vein that this research follows the mixed method research approach.

A concurrent triangulation design was used in this study to determine Grade 10 learners' understanding of some key Mathematics concepts: equal, function, variable, and zero in selected secondary schools in Chibombo District of Central Province in Zambia. A concurrent triangulation design is a mixed method design in which both quantitative and qualitative data are collected at the same time, merges them using both qualitative and quantitative data analysis methods and then interprets the results together to provide a better understanding of a phenomenon of interest

(McMillan & Schumacher 2010:403). Triangulation is used when the strength of one method offset the weakness of the other and this makes the results more meaningful.

A concurrent triangulation design is preferred in this study because the researcher believes that it is be capable of yielding new insights and highly enlightening into the problem under investigation. The researcher also considers concurrent triangulation design as the best design for breaking new ground for a better understanding of the Grade 10 learners' understanding of some key mathematical concepts; equal, function, variable, and zero in selected secondary schools in Chibombo District in Zambia.

3.7 RESEARCH METHODOLOGY

Research methodology brings up the application of a variety of standardised methods or approaches and techniques in the pursuance of valid knowledge (Mouton, 2002:35). There are basically two research design approaches, namely, qualitative and quantitative.

3.7.1 Qualitative approach

Denzin and Lincoln (2008) define qualitative research as a research that focuses on interpretation of phenomena in their natural settings to make sense in terms of meanings people bring to the setting. Qualitative research has the advantage in the sense that issues can be examined in detail and depth and data collection is informal and fun in most cases thereby encouraging participants in the research. It also provides a comprehensive textual description of how the public experience a given research problem. It espouses the human aspect in research about the behaviour, emotions, opinions, and beliefs of individuals. However, data collected from a few cases or individuals are used to generalise the larger population.

3.7.2 Quantitative approach

Quantitative research emphasises objectivity in measuring and describing phenomena by using numbers, statistics, structure, and control (McMillan & Schumacher, 2010:20). Quantitative research is an approach for testing objective theories by examining the relationship among variables (Creswell, 2014:32). The data collection under quantitative research is conducted using structured research instruments such as questionnaire. The results are based on larger samples sizes as a result can be used to generalise concepts more widely and be able to make predictions on future results. The dominant aim of this research is to relegate features, enumerate them and construct models in order to explain what has been ascertained.

3.7.3 Mixed method approach

Mixed method research is a type of research where the researcher combines aspects of both qualitative and quantitative for the purpose of breadth and depth of understanding of the phenomena under study (Onwuegbuzie et al., 2007:114). Mixed methods research is an approach to inquiry involving collecting both quantitative and qualitative data, integrating the two forms of data, and using distinct designs that may involve philosophical assumptions and theoretical frameworks (Creswell, 2014:33). The mixed methods research provides strength that offset the weaknesses of both quantitative and qualitative research. Quantitative research has a weakness of understanding the context in which people talk. Conversely, qualitative research compensates for this weakness arising from the use of quantitative research in isolation. Qualitative research brings in the aspect of biasness in interpreting the findings through the researcher's subjectivity. The smaller number of participants in qualitative research used to generalise the findings is another weakness of qualitative research (Onwuegbuzie et al., 2007:123). The core assumption of this form of inquiry is that the combination of qualitative and quantitative approaches provides a more complete understanding of a research problem than either approach alone.

3.7.4 Population and sample

3.7.4.1 Population

A population is a group of elements or cases, whether individuals, objects, or events, that conform to specific criteria and to which we intend to generalise the results of the research (McMillan && Schumacher, 2010:129). Population is an aggregate or totality of all the objects, subjects or members that conform to a set of specifications (Polit & Hungler, 1993:37). In addition, a population of a study is the entire group of persons, set of objects or events the researcher is interested in gaining information and drawing conclusions about (Van Rensburg, 2010:150). Five secondary schools in Chibombo District were selected purposefully for this study in line with the research design adopted. The population for this study was all the Grade 10 learners in all the eight (8) secondary schools in the district. The table below shows the population of this study.

Table 3.1: Population of the study

s/n	**Secondary school**	**Number of grade 10 learners**
1	1	344
2	2	201
3	3	200
4	4	185
5	5	150
6	6	120
7	7	105
8	8	95
	TOTAL	1400

Source: field data (2018)

3.7.4.2 Sampling

Sampling is the process of selecting subjects or participants from a population of interest so that after studying the sample we can generalise results back to the population from which they were chosen (McMillan & Schumacher, 2010:129). It is clear therefore that sampling is selecting of participant to the study from the population. The selection of participants to be part of the sample was done by using probability sampling for mixed methods studies. Stratified purposive sampling strategy was used in the selection of Grade 10 class in the secondary schools in the district. In this kind of sampling, the quantitative approach of stratifying the population is followed by purposive selection of a small number of cases from each stratum that are studied intensely (Schumacher & McMillan, 2010:399). Using stratified purposive sampling, participants are selected on strong suspicion that they are holders of important information needed to solve the problem under study. Accordingly, the research targeted all five Senior Secondary Schools in the District. Two of the Senior Secondary Schools are along the line of rail with slightly better qualified staffing levels and many homes use the same language of communication as that of instruction in schools. The other three Senior Secondary schools are far away from the line of rail where language of instruction in schools is different from the language used in their homes and have had no access to Early Childhood Education.

Once the schools are selected, a class was targeted per school of which there were a total of fifty (50) participants per class. Gender equality and performance in class was being paramount in selecting participants. The researcher made sure that there was a 50% representation from both sexes. Teachers in each school were requested to indicate on the class list the names of learners who are above average, average or below average. For every class selected, the researcher requested a record of performance in Mathematics for all the learners in that class in order to compare learners' responses in the questionnaire against their (learner) performance in class administered tests. This is important in order to determine whether learners' responses in the research instrument bear any connection to learners' profiles in their class assessments or tests. Selection of learners for focus group discussions was done after a preliminary analysis of the responses from the questionnaire. On average, each class is expected to have a minimum of50 learners. This means that there were approximately 250 participants. Chibombo District has an approximate population of 1400 pupils at senior level in all the eight secondary schools. The table below show the schools and number of learners who participated in the study. These schools were selected on the basis that they are easily accessible.

Table 3.2: Showing total number of learners who participated in the study

S/N	SECONDARY SCHOOL	NUMBER OF LEARNERS
1	1	50
2	2	50
3	3	50
4	4	50
5	5	50
Total		250

Source: field data (2018)

3.7.5 Data analysis

This study is a mixed method design. Data are analysed both quantitatively and qualitatively. The data collected from open-ended items in the questionnaire was analysed by the use of descriptive statistics in form of percentages and frequencies relating to commonalities in learners' meanings of the Mathematics concepts. Responses from learners were put into categories according to emerging themes (Creswell, 2007) linked to meanings of particular concepts and their applications in Mathematics. The data collected from open-ended items in the focus group discussions were transcribed, read, re-read and analysed thematically using content analysis, as themes and sub-themes emerge from the data. Responses from learners were compared with findings involving participants from other studies linked with learners' understanding of Mathematics concepts (see for example, Russell & Chernoff, 2011; Akgun & Ozdemir, 2006).

3.7.6 Research validity and reliability

Validity is defined as the extent to which a test measures what it claims to measure and does so without accidentally including other factors (Gregory 1992:117). A study should be able to assure the public its validity in so far as results are concerned. In this study, the validity of my investigation was ensured by reviewing the data collection instruments so that they measure exactly what they intend to measure. Pre-testing of the instruments was done to ensure that all ambiguous items were identified and corrected. Validity was also upheld by assuring the respondents that their identities will not be revealed; and ensuring that the instruments that I use to collect the data are valid. A valid measuring instrument is the one that measures what it is supposed to measure, and yields scores whose differences reflect the true differences of the variable or concept being measured rather than random or constant errors (De Vos, 2005:166). In this vein, the researcher discussed the research problem with the participants before the instrument was distributed. The researcher also ensured that the study was reliable. Reliability is the extent to which the results are similar over different forms of the same instrument or occasion of data collection (Schumacher & McMillan, 2010:179). In this study, reliability was achieved by subjecting the instrument to a pilot study. The aim of the pilot study was to ensure participants understood the questions they were required to respond to, and to determine whether there were any items or questions that needed refining or restating. The pilot of the research questionnaire involved learners who were in one of the schools that did not form part of the main study. The results of the pilot study indicated that learners did not show any difficulty in understanding the items in the instrument.

3.8 DATA COLLECTION

3.8.1 Use of questionnaire

In line with mixed method research design adopted for this research, a questionnaire (see Appendix F) was used as one of the data collection instruments. The table below shows the questionnaire items that were given to learners to complete.

Concept	My understanding of the concept	Application of the concept in mathematics
zero		
Equal		

Function		
Variable		

In column 2, learners were requested to state the mathematical meaning of the concept. In column 3, they were requested to show the application of the specific concept in mathematics. For example, learners would demonstrate knowledge of the application of the concept by indicating a particular mathematics topic or area in which the concept is presented or learned in school. The purpose of the questionnaire was to collect data from the informants who are perceived to have the information needed. A questionnaire can easily reach out many participants in all the schools at once. Schools in Chibombo district are distributed across a wide geographical area and gathering information in such areas requires the use of a questionnaire. After analysing the data from the questionnaire, special cases of interest arising from the responses (high performing and the low performing learners) participated in a focus group discussion in order to collect triangulatory data. Four learners from each of the five schools were selected for focus group discussion.

3.8.2 Use of focus group discussion

Focus group discussion is a rapid assessment, semi-structured data gathering method in which a purposively selected set of participants meet to discuss issues and concerns. These issues and concerns should be based on a list of key themes drawn up by a researcher or facilitator (Kumar, 1987). This qualitative research technique was developed in order to give researchers a better understanding of data. In addition, focus group discussion has become extremely popular because it provides a quick way to learn from the target audience (Debus, 1988; Kruger, 1988). Focus group discussions are relatively cheap and flexible. It allows participants to question each other and to elucidate their responses. However, the multiple voices of the participants may pose a challenge in the researcher's ability to manage the discussion. In this study, four learners per school were identified among those who responded to the questionnaire with help from their teacher for focus group discussion, to solicit further insights into their understanding of the key mathematical concepts.

3.9 LIMITATIONS OF THE STUDY

When this study was designed, the researcher immediately became aware of several limitations in the design of the study. The following were identified as limitations:

- This study was delimited to one District of central province, namely, Chibombo District, while there are ten districts. It is the cyclorama of the researcher that this delimitation has to some extent decreased the generalisability of the research findings.
- Delimiting the research area to three only out of the seven educational zones further decreased the generality of the research findings. An educational zone is a group of both secondary and primary schools in a certain geographic area who meet on regular basis to share ideas and experiences on how best they can improve on the provision of quality education through what they have called school based continuing professional development. It would have been ideal if the study were extended to more zones.
- Eleven participants have not submitted the questionnaires to the researcher thereby reducing the number of participants who participated in this study from the initial 250 to 239.
- The study was limited to a sample from five schools only. A sample from five schools only may not be fully representative of the perceptual experience of all learners in central Province and Zambia as a whole. In spite of these limitations, the researcher believes that the findings of this study will contribute to solving the learners' problems in understanding some key mathematical concepts.

3.10 CONCLUSION

In this chapter, the researcher discussed the research design for the study. The research methodology used in the study, population, sample design and size and data collection procedures were discussed. In chapter 4, the researcher discusses data analysis and interpretation.

CHAPTER FOUR

FINDINGS

4.1 INTRODUCTION

In this study, Grade 10 learners in selected secondary schools in the Chibombo District of Central Province (Zambia) were requested to state their understanding of the following mathematical concepts: zero, equal, function, and variable. Participants were requested to state their own understanding of each concept and indicate the application of each of these concepts in Mathematics. A total of 250 learners took part in answering questions in the questionnaire. In addition, 239 out of 250 learners returned completed questionnaires, representing a 95.6% return rate. Twenty (20) learners out of 239 were selected for the focus group interview. This chapter provides an account of the research findings from the data analysis and interpretation.

4.1.1 Analysis of learners' responses linked to the concept of zero from the questionnaire

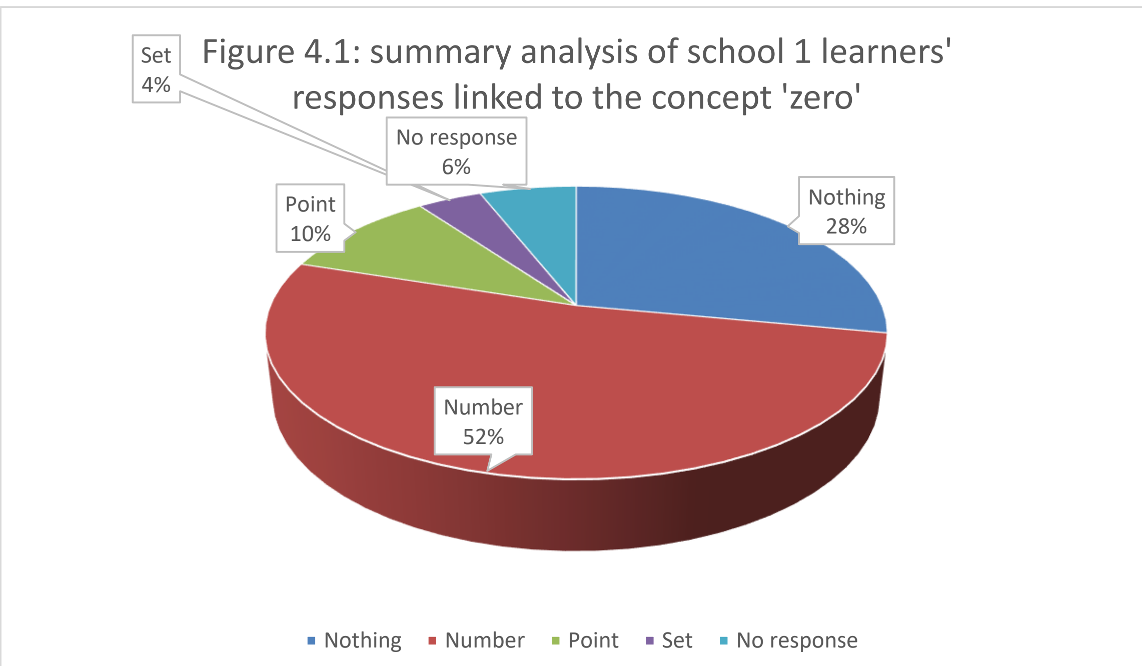
4.1.1.1 Responses from school 1

The responses from the learners on their understanding of the concept zero were widespread. This can be seen in Appendix J where learners defined 'zero' as a number that represents nothing or no number at all and it is used as starting point on the *x* and *y* axes. This means that zero is at the intersection of the two axes on the co-ordinate plane. Others defined the concept of zero as a number which is not significant or represents nothing and it is not found in the set of positive or negative numbers. Learner L45S1 from school 1 indicated, 'zero is a number which has no value and comes first when counting numbers'. This learner viewed zeros as a number that is "valueless" (no value) and is the first number on the list of counting numbers. The learner regarded zero to be the smallest counting number. A total of 14 learners (out of the 50) from school 1 (i.e. 28%) associated zero with nothing. This can be seen in Appendix H.

There were also learners who considered zero as a starting number and the lowest number in Mathematics. This meant that zero is taken to mean the lowest number used for counting and is the smallest number known in Mathematics. According to Learner L26S1, 'zero is a starting number in Mathematics and can be applied in addition and subtraction of numbers. This learner viewed zero as the first number in Mathematics implying that it is the lowest number of all the numbers in Mathematics. From a total of 50 learners in school 1, 26 (i.e. 52%) viewed zero as a number.

Another group of learners considered zero as a 'point'. They took zero as a point from where all measurements on scales should start. For such learners, zero seems to be the beginning or starting point of the mathematical work as one strives to find the answer when solving equations, by equating to zero. Five (5) out of 50 learners (i.e. 10%) from school 1 considered zero as a point. Some learners indicated that they understood zero as a set to mean an empty set and that zero is applied in denoting an empty set. This implies that an empty set is denoted using zero. Two out of 50 learners (i.e. 4%) associated the concept zero with the concept of a set. In addition, three learners (i.e. 6%) associated the concept to 'no response' from school 1.

Fig. 4.1 below shows a summary of the analysis of School 1 learners' responses related to understanding of the concept of zero.



Dominant categories linked to School 1 learners' responses to the concept of zero

From the information given in Fig. 4.1, it is clear that the dominant category related to learners' understanding of the concept zero is that it is a 'number'. That is, slightly more than half of the learners from school 1 considered the concept of zero as a number. In relation to this category of number, there were subcategories linked to it. For example, some learners viewed zero as a "starting number". Others viewed it as a just a number used in Mathematics, a point on a scale of measurement. Learner L50S1 viewed zero as not a number because when it is divided by 1, the result is 1 again. This is a misconception since zero divided by 1 is equal to zero and not 1. The second dominant category of responses concerned those learners who considered zero as "nothing". More than a quarter of the learners in School 1 indicated the concept of zero as nothing. Some learners viewed zero to mean an empty set. An empty set is associated with nothing since it does not have elements or members in it. Learner L37S1 viewed zero as a word that does not mean anything, and can be applied in situations where there is no solution to a given mathematical question.

4.1.1.2 Responses from school 2

An analysis of School 2 learners' responses on the understanding of the concept of zero indicates that 17 learners (i.e. 34%) associated the concept of zero with nothing. Another 21 (i.e. 42%) of the learners indicated that they associated zero as a number, while 10 learners (i.e. 20%) did not give any response to the task given to them. Detailed responses from each of the learners who associated zero to the dominant categories of 'number' and 'nothing' are given in Appendix L.
Fig. 4.2 below shows a summary of analysis of School 2 learners' responses related to understanding of the concept of zero.

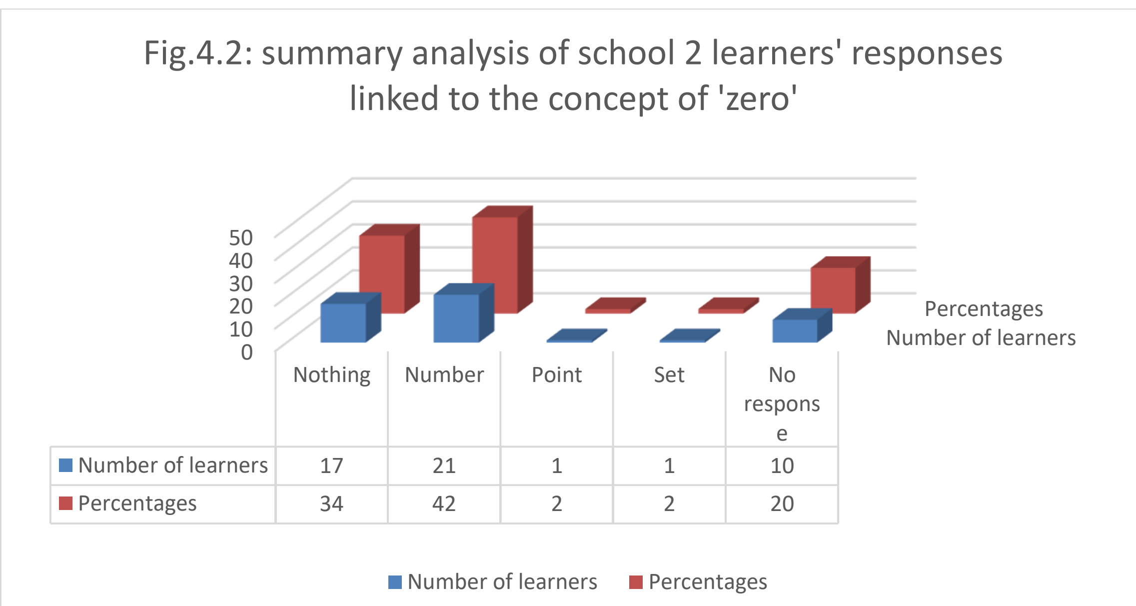


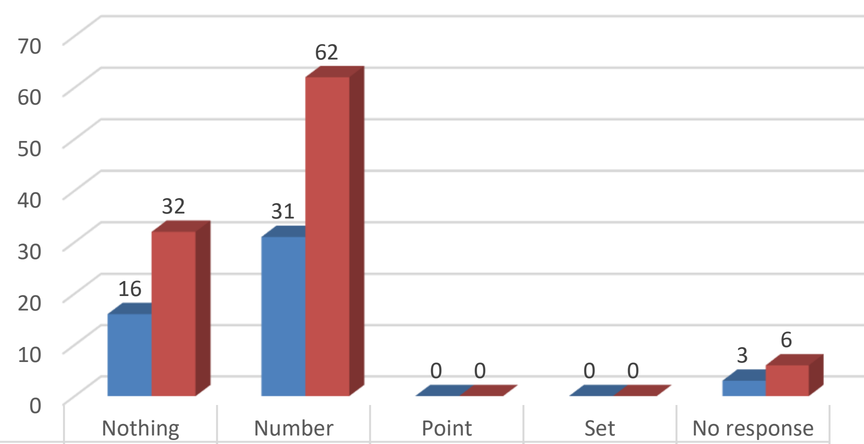
Fig.4.2: summary analysis of school 2 learners' responses linked to the concept of 'zero'

4.1.1.3 Responses from school 3

An analysis of School 3 learners' responses on the understanding of the concept of zero indicated that 16 (i.e. 32%) of the learners associate the concept of zero with nothing. Most of the learners in School 3, 31 (i.e. 62%) indicated that they associated zero as a number, while 3 (i.e. 6%) did not give any response to the task given to them. However, no learner in school 3 associated the concept of zero to either a 'point' or a 'set' category. School 3 had a larger number of learners associating zero to a number. Detailed responses from each of the learners who associated zero to the dominant categories of 'number' and 'nothing' are given in Appendix M.

Fig. 4.3 below shows a summary of analysis of School 3 learners' responses related to understanding of the concept of zero.

Fig 4.3: Summary analysis of school 3 learners' responses linked to the concept 'zero'

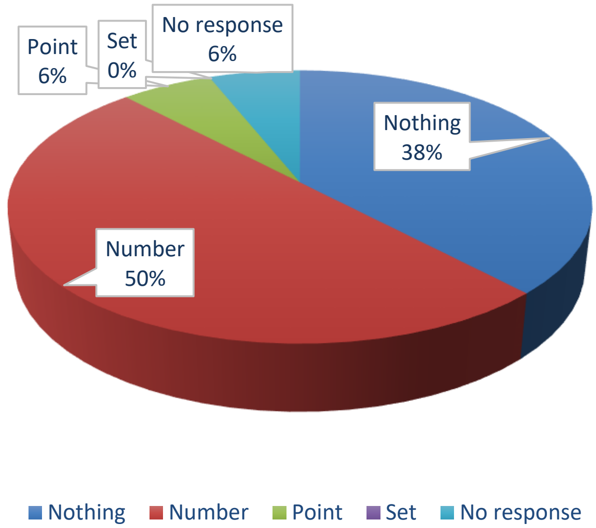


	Nothing	Number	Point	Set	No response
Number of learners	16	31	0	0	3
Percentage	32	62	0	0	6

4.1.1.4 Responses from school 4

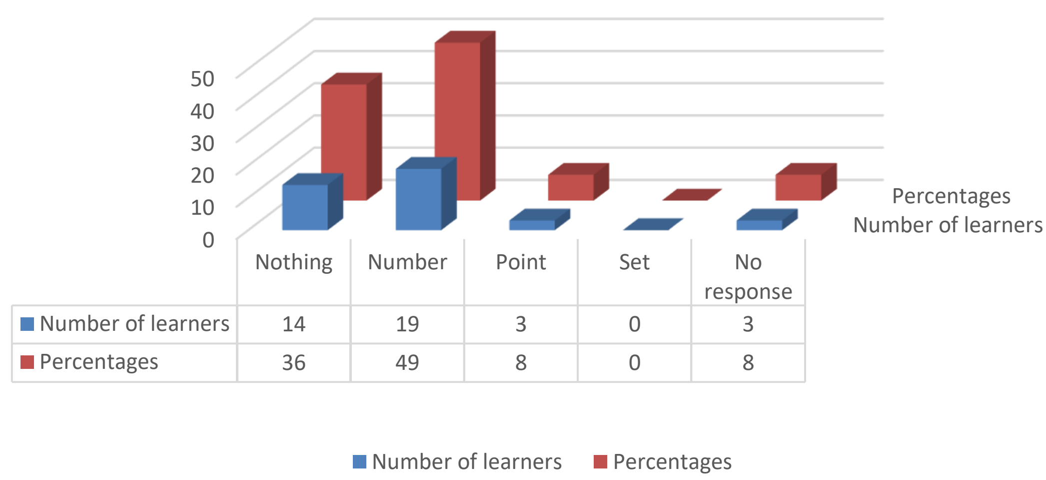
An analysis of School 4 learners' responses on their understanding of the concept of zero indicate that 19 (i.e. 38%) of the learners associate the concept of zero with 'nothing'. Most of the learners in School 4, 25 (i.e. 50%) indicated that they associated zero as a 'number' while three (i.e. 6%) did not give any response to the task given to them. However, no learner in School 4 associated the concept of zero to either a ‘set’. On the other hand, three learners (i.e. 6%) indicated that they understand the concept of zero to mean a ‘point’. Detailed responses from each of the learners who associated zero to the dominant categories of ‘number’ and ‘nothing’ are given in Appendix N. Fig 4.4 below shows a summary of analysis of School 4 learners’ responses related to understanding of the concept of zero.

Fig 4.4: Summary analysis of school 4 learners ' responses linked to the concept 'zero'



4.1.1.5 Responses from school 5

Only 39 out of 50 learners in School 5 completed the questionnaire. An analysis of these learners’ responses showed that 14 (i.e. 36%) of the learners associated the concept of zero to nothing. Like the other four schools (i.e. School 1, School 2, School 3, and School 4) more learners (19 out of 39 from School 5, representing 49%) associated the concept of zero to mean a number, while three learners out of 39 (i.e. 8%) did not give any response to the task given to them. No learner in School 4 associated the concept of zero to either a ‘set’. On the other hand, three learners (i.e. 8%) indicated that they understand the concept of zero to mean a ‘point’. Detailed responses from each of the learners who associated zero with the dominant categories of ‘number’ and ‘nothing’ are given in Appendix P. Figure 4.5 below shows a summary of analysis of School 5 learners’ responses related to understanding of the concept of zero.



	Nothing	Number	Point	Set	No response
Number of learners	14	19	3	0	3
Percentages	36	49	8	0	8

Fig 4.5: Summary analysis of school 5 learners responses linked to the concept 'zero'

Emerging themes linked to analysis of Schools 1-5 learners' responses to the concept of zero

From the information given in the tables above from the five secondary schools, it is clear from the analysis that two categories stand out. Dominated categories in learners' responses in relation to their understanding of the concept of zero were 'number' and 'nothing'. In the five schools, 126 (i.e. 52.7%) of the learners associate zero with a 'number' and 77 (i.e. 32.2%) associated zero with 'nothing'. A few learners, 12 (i.e. 5.0%) and four (i.e. 1.7%) associated the concept of zero to a 'point' and a 'set' respectively. A total of 22 learners (i.e. 9.2%) did not respond to the task. Figure 4.6 below shows a summary analysis of Schools 1-5 learners' associations of zero with other concepts.

Table 4.1: Summary of learners' associations of zero with other concepts

School ID	Association with nothing (n=)	Association with number (n=)	Association with point (n=)	Association with set (n=)	No response (n=)
S1	14	26	05	02	03
S2	17	21	01	01	10
S3	16	31	00	00	03
S4	19	25	03	00	03
S5	14	19	03	00	03
Total	80 out of 239 (33.5%)	126 out of 239 (51.0%)	12 out of 239 (5.0%)	03 out of 239 (1.3%)	22 out of 239 (9.2%)

Some learners see the concept of zero as being applied in situations where two numbers of the same value are subtracted. The result of the difference between two numbers of the same value is always zero. For example, if 5 oranges are taken away from 5 oranges, zero oranges remain. In solving algebraic equations like quadratic equations, zero is always applied by equating it to the algebraic expression on the other side of the equal sign. A participant also indicated that the concept of zero can be applied to denote an empty set and also to separate negative from the positive numbers. Zero is taken to mean an empty set once it is the only element in a particular set and a number to demarcate the two sets of numbers i.e. the negative and positive numbers on a number line.

The results of the analysis from the five schools indicated that the majority of learners associated zero with either a number or as nothing (see Table 4.6 above). Very few learners in all of the five schools associated zero with either a 'point' or a 'set'. In all the five schools, the number of learners who associated the concept of zero with a 'number' is more than other categories. School 3 come out different from the other schools as it had the biggest difference (over 50%) between the number of learners who associated the concept of zero to a 'number' than 'nothing'.

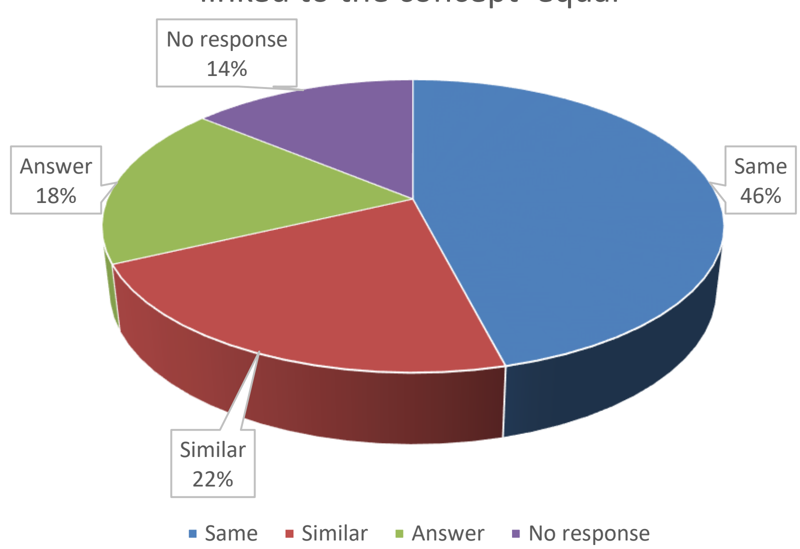
4.1.2 Analysis of learners' responses linked to the concept of equal

4.1.2.1 Responses from School 1

In this section, School 1 learners' responses on their understanding of the equal concept of equal are discussed. The dominant category in the analysis of learners' responses to the understanding of the concept of equal is 'same' (see Appendix Q). Some learners indicated that the concept of equal means 'similar' while others associated it to an 'answer'. However, other learners did not respond to the questions. Learners understand equal as a sign that is put to show that an answer to a problem being solved is about to be given and is applied in solving equations. Learner L25S1 indicated that the equal sign is used to show that two things are the same and is used before the final answer. The learners viewed equal sign as a symbol that is put before the final answer is given to any question. Other categories indicated by the learners from School 1 on the concept of equal are 'same' 23 (i.e. 46%), 'similar' 11 (i.e. 22%), 'no response' 7 (i.e. 14%) and 'answer' 9 (i.e. 18%).

Fig 4.6 below shows a summary analysis of School 1 learners' responses to concept of equal.

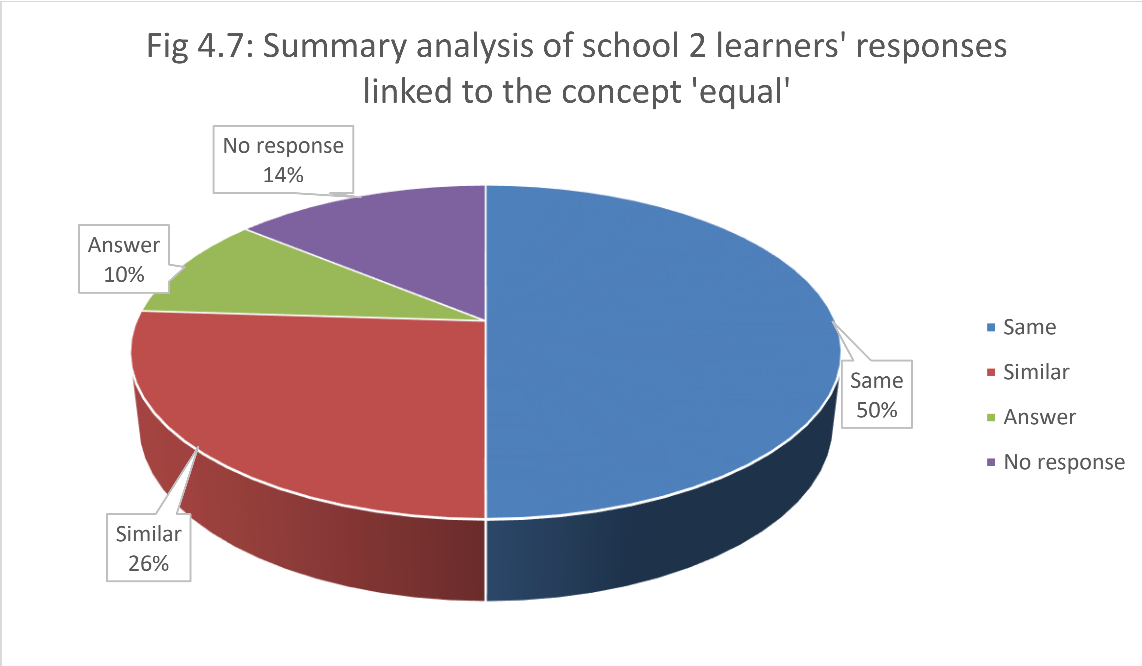
Fig 4.6: Summary analysis of school learners' responses linked to the concept 'equal'



4.1.2.2 Responses from School 2

The dominant category in the analysis of learners' responses to understanding of equal by the learners in school 2 is 'same' 25 (i.e. 50%) while the association of equal to 'answer' is 5 learners (i.e. 10%). Some learners 13 (i.e. 26%) associated equal to ‘similar’ representing 2%. Some learners did not respond to the task given 7 (i.e. 14%).

Fig 4.7 below shows a summary analysis of School 2 learners’ responses to concept of equal.

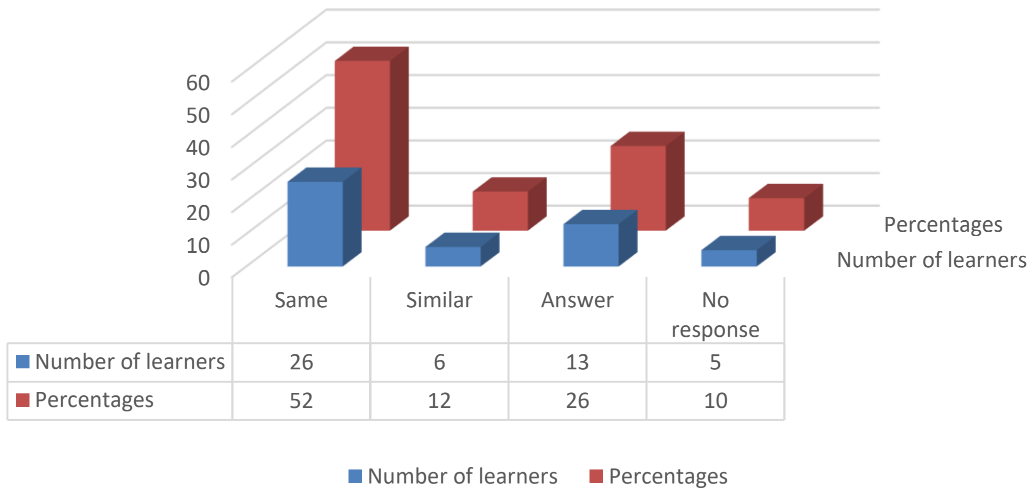


4.1.2.3 Responses from school 3

An analysis of School 3 learners’ responses to understanding of equal indicated that the dominant category given by 26 learners (i.e. 52%) was ‘same’ while those who associated it to ‘answer’ were 13 (i.e. 26%). Some learners, 6 (i.e. 12%), indicated that the concept of equal is associated to ‘similar’ and only five learners (i.e. 10%) did not respond to the task given.

Fig 4.8 below shows a summary analysis of School 3 learners’ responses to concept of equal.

Fig 4.8: Summary analysis of school 3 learners' responses linked to the concept 'equal'

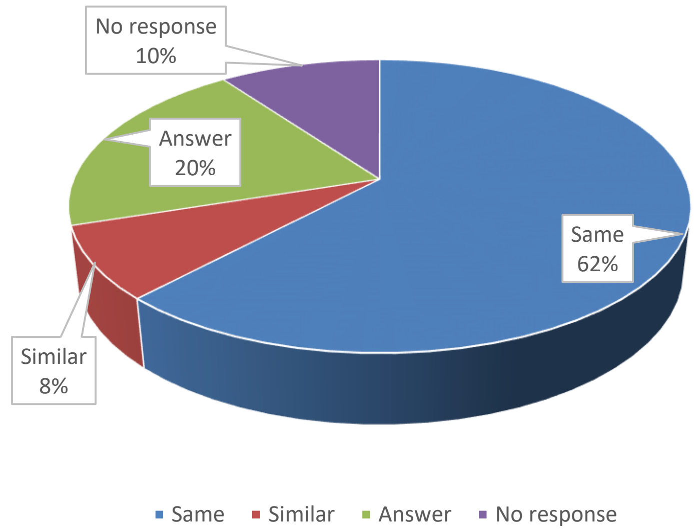


4.1.2.4 Responses from school 4

An analysis of School 4 learners' responses on their understanding of the concept "equal" showed that the dominant category was 'same'. This was given by 31 learners (i.e. 62%). The category 'answer' was given by 10 learners (i.e. 20%). Only four learners (i.e. 8%) associated the concept equal to 'similar' while five learners (i.e. 10%) did not respond to the given task. Learner L4S4 asserted that the concept of equal "is a sign that helps to identify the size of properties or goods that have weight or same number". This learner's response brings out three aspects of the concept equal in relation to the size of two or two items. The second aspect of equality is in terms of weight of items while the third aspect of equality is in terms of numbers in general.

Fig 4.9 below shows a summary analysis of School 4 learners' responses to concept of equal.

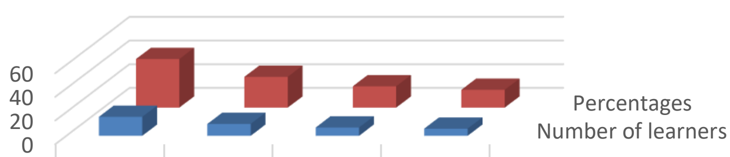
Fig 4.9: Summary analysis of school 4 learners' responses linked to the concept of 'equal'



4.1.2.5 Responses from school 5

About 16 learners out of 39 (41%) from school 5 indicated that they associate the concept of equal to ‘same’ and 10 (26%) associated equal to ‘answer’. The learners who associated equal to same asserted that equal is a mathematical symbol that demonstrates sameness on both sides. On the other hand, the learners who associated equal to ‘answer’ argue that equal is applied in situations where one is about to put the final answer to a mathematical problem. Other categories from School 5 were ‘similar’ 7 (i.e. 18%), and ‘no response’ 6 (i.e. 15%) as can be seen from Figure 4.10 below. The dominant category from School 5 was association of the concept of equal to ‘same’.

Fig 4.10: Summary analysis of school 5 learners responses linked to the concept of 'equal'



	Same	Similar	Answer	No response
Number of learners	16	10	7	6
Percentages	41	26	18	15

Number of learners   Percentages

Emerging themes linked to analysis of Schools 1-5 learners' responses to the concept of equal

The dominant category in learners' responses with regard to their general understanding of the concept of equal was 'same'. From the five schools, 121 learners (i.e. 50.6%) associated equal with 'same' while 47 (i.e. 19.7%) of the learners associate it to 'answer'. Other learners 41(i.e. 17.2%) associated the concept of equal to 'similar' and another 30 learners (i.e. 12.6%) did not give any response to the task.

Table 4.2 below shows a summary analysis of Schools 1-5 learners' association of equal with other concepts.

Table 4.1: Summary of schools 1-5 learners' associations of equal with other concepts

School ID	Association with same (n=)	Association with answer (n=)	Association with similar/identity(n=)	no response / I don't know (n=)
S1	23	09	11	07
S2	25	05	13	07
S3	26	13	06	05
S4	31	10	04	05
S5	16	10	07	06
Total	121 out of 239 (50.6%)	47 out of 239 (19.7%)	41 out of 239 (17.2%)	30 out of 239 (12.6%)

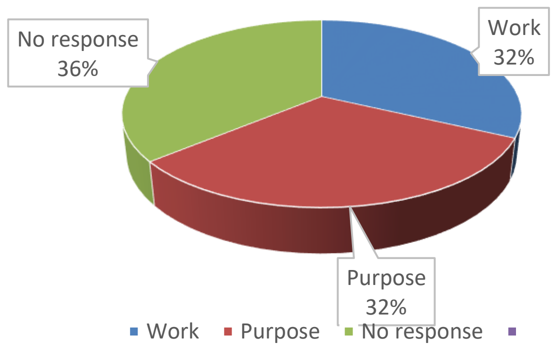
4.1.3 Analysis of learners' responses linked to the concept of function

4.1.3.1 Responses from School 1

The learners' responses on their understanding of the equal concept from School 1 are discussed in this section. The dominant category in the analysis of learners' responses to the concept of function was 'no response'. On the other hand, some learners indicated that the concept of function means 'work' while others associated it to 'purpose'. Learners considered the concept of function as related to the work that a certain part plays in a system. Moreover, they view the concept of function as an activity that some part within a certain system plays. The categories indicated by learners from School 1 on the concept of function are 'no response' 18 (i.e. 36%), 'purpose' 16 (i.e. 32%) and 'work' 16 (i.e. 32%).

Figure 4.11 below shows a summary of School 1 learners' responses to concept of function.

Fig 4.11: Summary analysis of school 1 learners' responses linked to the concept 'function'

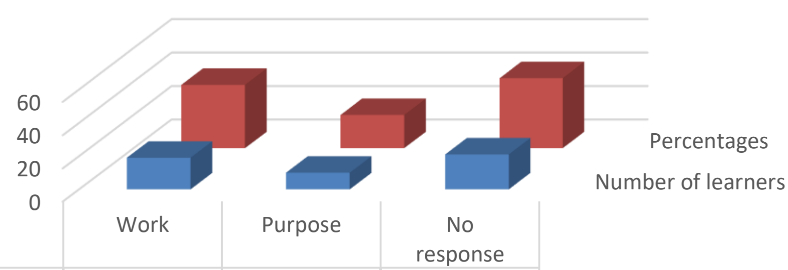


4.1.3.2 Responses from School 2

An analysis of School 2 learners' responses on their understanding of the equal concept is presented in Table 4.23 below. The dominant category in the responses to the concept of function from school 2 is 'no response'. On the other hand, some learners indicated that the concept of function means 'work' while others associated it to 'purpose'. The categories indicated by the learners from School 2 on the concept of function are 'no response' 21 (i.e. 42%), 'purpose' 10 (i.e. 20%) and 'work' 19 (i.e. 38%).

Figure 4.12 shows a summary of learners' responses to concept of function from School 2.

Fig 4.12: Summary analysis of school 2 learners' responses linked to the concept 'function'



	Work	Purpose	No response
Number of learners	19	10	21
Percentages	38	20	42

Similarly, the analysis showed that learners from Schools 3, 4 and 5 associated the concept of function with work and purpose according to the frequencies indicated in Table 4.3. In addition, many learners did not give responses to this task.

Table 4.3: Schools 1-5 learners' association of function with other concepts

School ID	Association with work (n=)	Association with purpose / use (n=)	No response (n =)
S1	16	16	18
S2	19	10	21
S3	06	11	33
S4	15	16	19
S5	11	06	23
Total	67 out of 239 (27.0%)	59 out of 239 (25.0%)	114 out of 239 (48.0%)

Emerging themes linked to analysis of Schools 1-5 learners' responses to the concept of function

From the information given in the Table 4.24, it is clear that the category "no response" stands out among the other two. Majority of the learners 114 (i.e. 48.0%) gave 'no response' to this task, indicating that they did not seem to understand what it means. Some learners 59 (i.e. 25.0%) associated function with 'purpose' and 67 (i.e. 27.0%) associated the concept to 'work'.

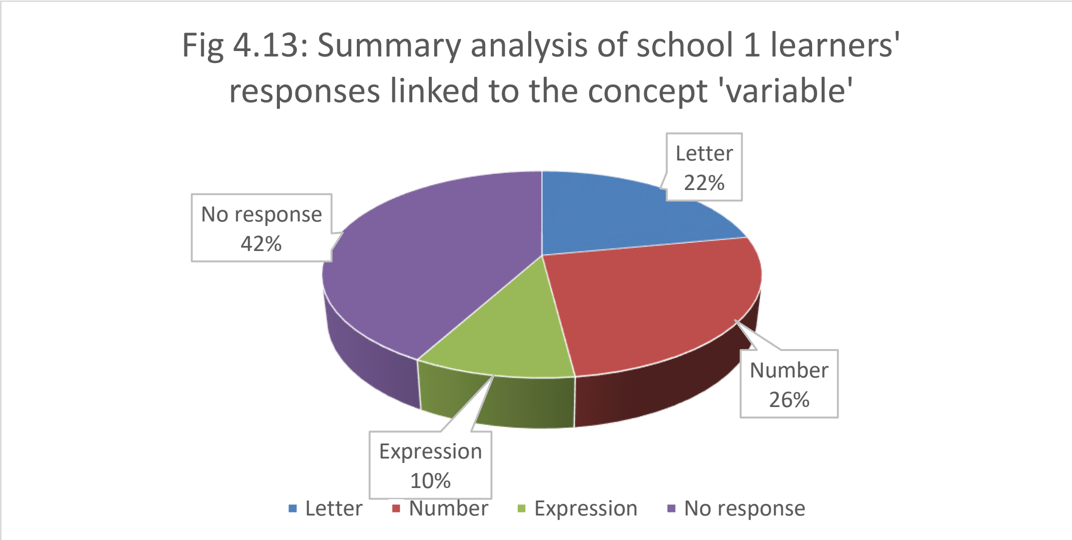
The way learners responded to this concept indicates that the majority of them viewed the concept function from an everyday life perspective. This is because the concept "work" and "purpose" are rarely associated with any formal area of mathematical knowledge.

4.1.4 Analysis of learners' responses linked to the concept of variable

4.1.4.1 Responses from school 1

The learners' responses on their understanding of the concept of variable from School 1 are analysed and presented in Figure 4.15 below. The dominant category in the responses to the concept of variable from School 1 was 'no response'. Furthermore, some learners in School 1 indicated that the concept of variable means 'letter' while others associated it with a 'number'. The categories of responses by the learners from School 1 on the concept of variable are 'no response' 13 (i.e. 26%), 'letter' 11 (i.e. 22%, 'number' 13 (i.e. 26%) and 'expression' 5 (i.e.10%).

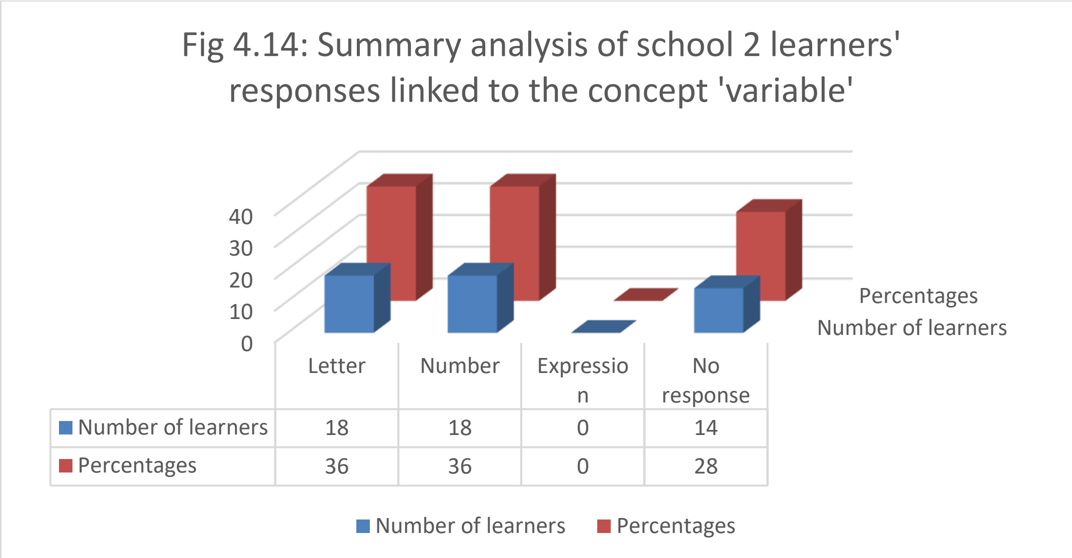
Figure 4.13 below shows a summary analysis of School 1 learners' responses to concept of variable.



4.1.4.2 Responses from School 2

An analysis of School 2 learners' responses on their understanding of the concept of variable is presented in Figure 4.16 below. Unlike in School 1 where the dominant category was 'no response', School 2 had 'number' and 'letter' as dominant categories. Some learners in School 2 indicated that the concept of variable means 'letter' while others did associate it to 'number'. The categories of responses by the learners from School 2 on the concept of variable are 'letter' 18 (. i.e. 36%), 'number' 18 (i.e. 36%), and 'no response' 14 (i.e. 28%). No learner in School 2 associated the concept of function to 'expression' as was the case is in School 1.

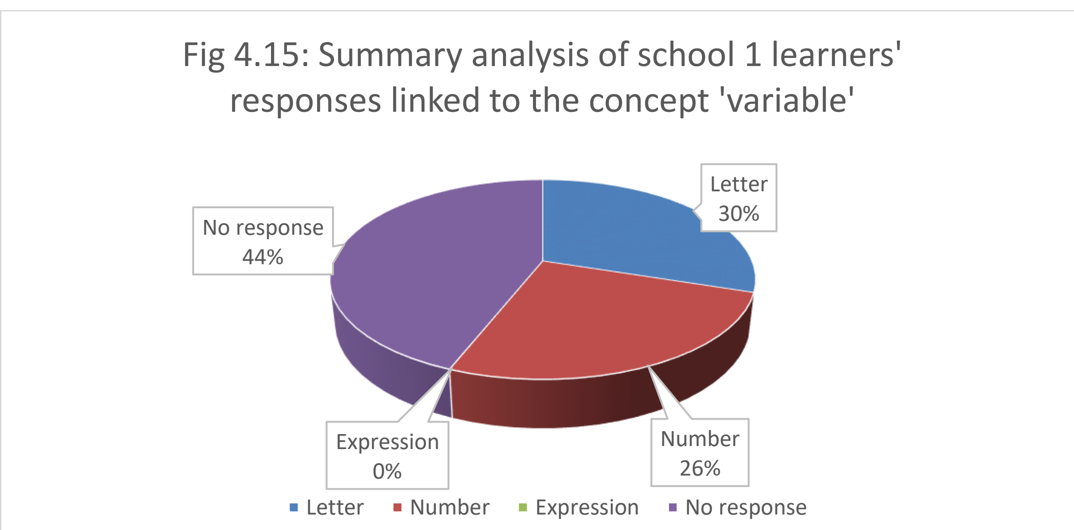
Figure 4.14 below shows a summary analysis of School 2 learners' responses to concept of variable.



	Letter	Number	Expression	No response
Number of learners	18	18	0	14
Percentages	36	36	0	28

4.1.4.3 Responses from School 3

Unlike in School 2 where there were two dominant categories of 'letter' and 'number', School 3 has 'no response' as the dominating category. Some learners in School 3 indicated that the concept of variable means 'number'. The categories of responses by the learners from School 3 on the concept of variable are 'letter' 15 (. i.e. 30%), 'number' 13 (i.e. 26%), and 'no response' 22 (i.e. 44%). Like in School 2, no learner in School 3 associated the concept of function with 'expression'. Figure 4.15 below shows a summary analysis of school 3 learners' responses to concept of function.



The summary analyses of Schools 4 to 5 learners' responses to the concept of variable are presented in Figures 4.16 and 4.17, below, respectively.

Fig 4.16: Summary analysis of school 4 learners' responses linked to the concept 'variable'

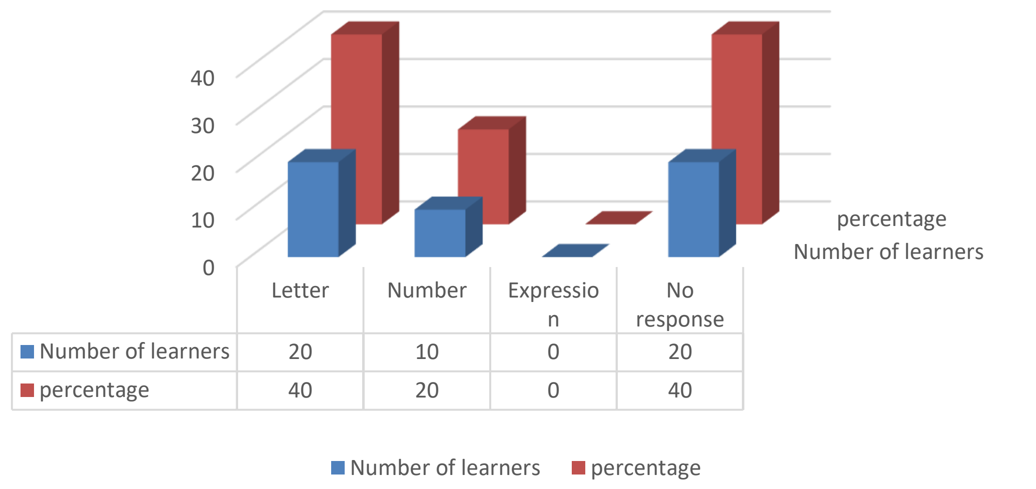
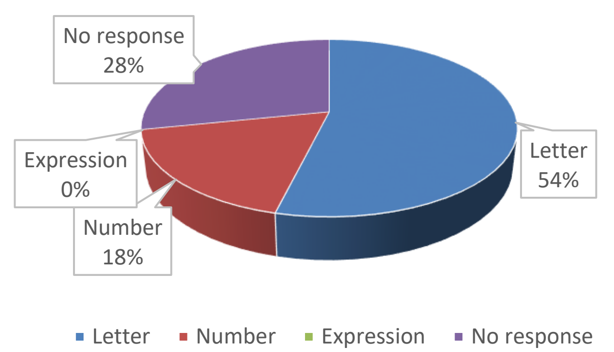


Fig 4.17: Summary analysis of school 5 learners' responses linked to the concept 'variable'



Emerging themes linked to analysis of Schools 1-5 learners' responses to the concept of variable

The dominant category in the responses given by the majority of learners 88 (i.e. 36.8%) with regard to their general understanding of the concept of variable was 'no response'. This suggests that they did not quite understand what it meant. About 85 (i.e. 35.6%) learners associated variable with 'letter', 61 (i.e. 25.5%) associated the concept to 'number' and five (i.e. 2.1%) indicated that they associated the concept to "an expression". Learners who associated variable to 'letter' held the view that a variable is a letter that represents a particular number in an algebraic expression.

Those who associated a variable with ‘number’ held the view that in an algebraic expression like $5a$, the number “5” represents a variable while a is a coefficient.
Table 4.4 shows a summary of schools 1-5 learners’ associations of function with other concepts.

Table 4.4: Schools 1-5 learner association of variable with other concepts

School ID	**Association with letter n=**	**Association with number n=**	**no response n=**	**Association with expression n=**
S1	11	13	21	05
S2	18	18	14	00
S3	15	13	22	00
S4	20	10	20	00
S5	21	07	11	00
Total	85 out of 239 (35.6%)	61 out of 239 (25.5%)	88 out of 239 (36.8%)	5 out of 239 (2.1%)

4.2 RESPONSES FROM FOCUS GROUP DISCUSSION

4.2.1 Analysis of learners’ responses to focus group discussions on the concept of zero

Four learners were selected from each of the five schools in order to carry out in-depth focus group discussions to establish the following two aspects:

- Learners’ general understanding of the concept of zero; and
- Learners’ thinking about an area or situation where the concept of zero can or cannot be applied in Mathematics.

4.2.1.1 Learners’ understanding of the concept of zero

A detailed transcript of all the interviews with learners is presented in Appendix H. Table 4.7 shows the responses from learners in all the five schools on their general understanding of the concept. This table is an extract from learners’ responses when they were asked specific questions in the interview, as can be seen in the following extract (with a group of learners in School 1 (S1)):
Interviewer*: What is your general understanding of the concept of zero in Mathematics?*
Learner L4S1: Means the eeee zero in Mathematics refers to something or number which does not exceed zero or in some other subjects, it can refer to being nullified or not valid.

Interviewer: *Which subject are you referring?*

Learner L4S1: *Like civic education under the topic elections where some votes are not counted because they were incorrectly marked.*

Learner L13S1: *Zero is nothing*

Interviewer: *what do you mean when you say "zero is nothing?"*

Learner L13S1: *No response*

Learner L19S1: *Means nothing because it cannot be used in anything. When we want to add up numbers and only have zeros, it means the answer is zero again. Therefore, we say zero is nothing.*

Learner L23S1: *No response.*

After reading the transcripts, I extracted key parts of the data (what learners said) that related closely to learners' understanding of the concept of zero, for each of the five schools. Proceeding in this manner, I compiled all the relevant responses into a table, as can be seen in Table 4.5 below.

Table 4.5: Learners' responses on their understanding of the concept of zero in five schools

LEARNER ID	**LEARNERS' RESPONSES**
Learner L4S1	Refers to number which does not exceed zero, being nullified or not counted.
Learner L13S1	Means nothing
Learner L19S1	Means it cannot be used in anything
Learner L23S1	No response
Learner L16S2	Any number …eeeeeh beginning zero to infinity
Learner L7S2	Whole number greater than negative numbers but less than 1
Learner L29S2	No idea
Learner L37S2	No answer
Learner L4S3	Means nothing
Learner L19S3	A natural number which is neither negative nor positive
Learner L11S3	Arbitrary nothing
LEARNER ID	**LEARNERS' RESPONSES**
Learner L28S3	It's a number with no value

Learner L13S4	It’s a non-integer number
Learner L6S4	A neutral number
Learner L10S4	It’s a number which forms part of the integers and separates positive numbers from negative numbers
Learner L16S4	It is eeeeeh pause for some time … nothing but 1 and 0 means 10 and 0 and 1 is 1.
Learner L1S5	Is one of the numbers
Learner L10S5	If you have been asked to choose or permute the numbers
Learner L22S5	Number between the ascending and descending of negative and positive
Learner L28S5	Number greater than positive numbers but less than negative numbers

In the quantitative analysis, and in relation to emerging themes linked to analysis of Schools 1-5 learners’ responses to the concept of zero, I noted that the dominant categories in learners’ responses in relation to their understanding of the concept of zero were ‘number’ and ‘nothing’. In the five schools, 52.7% of the learners associated zero to a ‘number’ and 32.2% associated zero to ‘nothing’. Only a few learners (i.e. 5.0% and 1.7%) associated the concept of zero to a ‘point’ and a ‘set’ respectively. Some learners (i.e. 9.2%) did not respond to the task. This information is summarised in Table 4.6 (summary analysis of Schools 1-5 learners’ associations of zero with other concepts).

In Table 4.7 above, the qualitative data from interviews with the 20 learners confirms key aspects of the analysis from the quantitative data. Key of these aspects is connected to learners associating zero with “number”.

The table below indicates response categories from the analysis of responses from interviews with 20 selected learners.

Response category: Associations of zero with "number" (n = 12)
• Refers to number which does not exceed zero, being nullified or not counted (L4S1) • Any number …eeeeh beginning zero to infinity (L16S2) • Whole number greater than negative numbers but less than 1 (L7S2) • It's a number with no value (L28S3) • It's a non-integer number (L13S4) • A neutral number (L6S4) • A natural number which is neither negative nor positive (L19S3) • It's a number which forms part of the integers and separates positive numbers from negative numbers (L10S4) • Is one of the numbers (L1S5) • If you have been asked to choose or permute the numbers (L10S5) • Number between the ascending and descending of negative and positive (L22S5) • Number greater than positive numbers but less than negative numbers (L28S5)
Response category: Association of zero with "nothing" (n = 4)
• Means nothing (L13S1) • Means nothing (L4S3) • Arbitrary nothing (L11S3) • It is eeeeeh [pause for some time …] nothing but 1 and 0 means 10 and 0 and 1 is 1 (L16S4)
Response category: No response / No idea (n = 3)
• No response (L23S1) • No idea (L29S2) • No answer (L37S2)
Response category: Other responses (n = 1)
• Means it cannot be used in anything (L19S1)

As can be seen from the table above, more than half (i.e. 12 out of 20) of the learners in the interview associated zero with number. This is consistent with the analysis of the quantitative data in which it was found that 52.7% (i.e. more than half) of the participants associated zero with number. In the qualitative data, we note that learners have made several associations in relation to what kind of number zero is. For example, five (5) of these learners associated zero with **location** (on an imagined number line):

- *Whole number greater than negative numbers but less than 1 (L7S2)*
- *A neutral number (L6S4);*
- *It's a number which forms part of the integers and separates positive numbers from negative numbers (L10S4)*
- *Number between the ascending and descending of negative and positive (L22S5)*
- *Number greater than positive numbers but less than negative numbers (L28S5)*

Learner L4S1 viewed zero as a "number which does not exceed zero". Although this description of zero appears to relate to location, this description seems tautological. It does appear trivial because it is tantamount to description zero as a number that does not exceed itself!

Other learners' comments were linked to the idea of value. For example, learner L28S3 noted that zero is "a number with no value". Learner L4S1 described zero as related to instances where something is "being nullified" or "not counted". There were other comments from learners that indicated the possibility of misconceptions related to understanding of zero. For example, learner L19S3 indicated that zero is "a natural number which is neither negative nor positive", while learner L13S4 observed that zero is "a non-integer number". These two responses of learners are contrary to what is known about the number zero. This is because although zero is a number, it is not a natural (or counting) number while it is an integer.

These above comments about zero in relation to number were more elaborate in the sense that they went beyond just noting that zero "is one of the numbers" (L1S5).

In contrast to the quantitative data analysis, none of the learners interviewed directly associated zero with "point" or "set".

4.2.2.2 The areas or situations where the concept of zero can or cannot applied in Mathematics

In the quantitative analysis, some learners saw the concept of zero as being applied in situations where two numbers of the same value are subtracted. The result of the difference between two numbers of the same value is always zero. In solving equations such as quadratic equations, zero is always applied by equating it to the algebraic expression on the other side of the equal sign. The concept of zero was noted to denote an empty set and to separate negative from the positive numbers. Zero was taken to mean an empty set. In learning whole numbers and other numbers, zero is applied. It is applied in matrix and in temperature reading. Zero is applied in situations where you cannot go further when your answer is zero. It is used to indicate that there is nothing and also in counting. It is used with other numbers as it cannot be used on its own.

In both quantitative and qualitative data, the learners indicate that the concept of zero is applied in number line and in writing big numbers like thousands. For example, in quantitative data, learner L2S4 indicated, that the 'concept of zero is applied in numbers such as 10, 100, 1000, 10000'. This is in line with learner L10S5 who remarked 'the concept of zero is applied in writing huge numbers in thousand or trillion'. Learner L4S4 indicated 'the concept of zero is used to show the starting point of the number line or numbers itself'.

In contrast, in the qualitative data, we note that learners have made association in relation to the application of zero as 'used in integers and separate the negative numbers from the positive numbers'. It is also reviewed that zero is applied in graphs such as the half-life and travel graphs. Some learners indicate that the concept of zero is applied in matrices, vectors and in money.

4.2.2 Analysis of learners' responses to the concept of equal

Four learners were selected from each of the five schools in order to carry out in depth focus group discussions to establish the following two aspects:

- Learners' general understanding of the concept of equal, and
- Learners' thinking about an area or situation where the concept of equal can or cannot be applied in Mathematics.

4.2.2.1 Learners' understanding of the concept of equal

A detailed transcript of all the interviews with learners is presented in Appendix H. Table 4.8 shows the responses from learners in all the five schools on their general understanding of the mathematical concept of equal. This table is an extract from learners' responses when they were asked specific questions in the interview, as can be seen in the following extract (with a group of learners in School 3):

Interviewer*: What is your general understanding of the concept of equal in Mathematics?*

L28S3: *Equal means .... paused.... an example may be in class the number of girls is equal to the number of boys.*

L11S3: *Equal means same number which means that same number of girls and boys is the same number.*

L19S3: *Like what my friends have said, it means same numbers.*

Interviewer*: What about the other, what can you say about equal concept in Mathematics?*

L4S3: *No idea sir*

After familiarizing myself with the transcripts, I extracted key parts of what learners said that related closely to their understanding of the concept of equal, for each of the five schools. Learners' responses were then compiled into a table, as can be seen in Table 4.6

Table 4.6: Learners' responses on their understanding of the concept of equal in five schools

LEARNER ID.	**LEARNERS' RESPONSES**
Learner L19S1:	It is a mathematical symbol used to show the answer.
Learner L23S1:	Paused for 2 minutes. No response
Learner L13S1:	Equal is to equate something on the right hand side to left hand side.
Learner L4S1:	It is like same, equivalent, parallel are almost the same as equal
LEARNER ID.	**LEARNERS' RESPONSES**
Learner L37S2:	shows that anything you are calculating is equal e.g 1 = 0 or y= 0
Learner L16S2:	Both sides are equilibrium and balanced or equal
Learner L7S2:	Silent for some minutes
Learner L29S2:	The same mmmmmmmm. No answer.
Learner L28S3:	Equal means .... paused.... an example may be in class the number of girls is equal to the number of boys.
Learner L11S3:	Equal means that same number of girls and boys is the same number.
Learner L4S3:	No idea sir
Learner L19S3	Like what my friends have said, it means same numbers
Learner L10S4:	Used to represent two or more objects which are the same.
Learner L13S4:	Means two or more things are equal like if you get a k10 and a book like this one we can say the book and the k10 are equal because a book was bought at k10.
Learner L16S4	Equal was derived from equality like when you are given something meaning both sides must be the same.
Learner L6S4	When things are equal it means they have to give the same output
Learner L22S5:	Equality of two things e.g. 5 chairs are equal to 5 desks. Silence for 3 minutes.
Learner L10S5:	No response
Learner L1S5	No response
Learner L28S5	No response

In the quantitative analysis, and in relation to emerging themes linked to analysis of Schools 1-5 learners' responses to the concept of equal, the researcher noted that the dominant categories in learners' responses in relation to their understanding of the concept of equal were 'same' and 'answer'. In the five schools, 49.0% of the learners associated zero to a 'same' and 14.2% associated zero to 'answer'. A considerable number of learners (i.e. 6.7%) associated the concept of equal to 'similar or identity'. Some learners (i.e. 6.7%) did not respond to the task. This information is summarised in Table 4.16 (summary analysis of Schools 1-5 learners' associations of equal with other concepts).

In Table 4.8 above, the qualitative data from interviews with the 20 learners confirms key aspects of the analysis from the quantitative data. Key of these aspects is connected to learners associating equal with "same". The table below indicates response categories from the analysis of responses from interviews with 20 selected learners.

Response category: Associations of equal with "same" (n = 11)
• Equal is a situation where you equate something which is on the right hand side and left hand side. (L23S1) • What is on the left hand side should same to what is on the other side. (L13S1) • Words like same, equivalent, parallel are almost the same as equal. (L13S1) • Equal is a symbol which shows that anything you are calculating is equal e.g I might equate 1 to 0 or y to 0 which means that y and 0 have the same value. (L37S2) • The same mmmmmmmm (L29S2) • Equal means same number which means that same number of girls and boys is the same number (L11S3) • like what my friends have said, it means same numbers (L19S3) • When things are equal it means they have to give the same output (L6S4) • Equal was derived from equality like when you are given something meaning both sides must be the same(L16S4) • Used to represent two or more objects which are the same. It can be used to indicate that two things or more if brought together are equal(L10S4) • Equal means .... paused.... an example may be in class the number of girls is equal to the number of boys(L28S3)

Response category: Association of equal with “answer” (n = 1)
• It is a mathematical symbol that is used when you multiply or add numbers to show the answer (L19S1)
Response category: Association of equal with “No response / No idea” (n = 7)
• No response (L1S5) • No response(L28S5) • No response(L10S5) • Paused for two minutes. No response (Learner L4S1) • No response(L23S1) • No idea sir(L4S3) • Silent for some minutes … no response(L7S2)
Response category: Association of equal with “balanced” (n = 1)
• Both sides are equilibrium and both sides are balanced, what is on the left hand side is equal to what is on the right hand side.

As can be seen from the table above, slightly more than half (i.e. 11 out of 20) of the learners in the interview associated the concept of equal with ‘same’. This is consistent with the analysis of the quantitative data in which it was found that 49.0% (i.e. slightly less than half) of the participants associated equal with ‘same’. Although it is less than half (in terms of per cent), the ‘same’ is still the dominant theme from the quantitative data obtained. In the qualitative data, we note that learners have made several associations in relation to what the concept of equal is. For example, three (3) of these learners associated the concept of equal to mean reaching a state of equilibrium where what is on one side be the same as what is on the other side:

- *Both sides are equilibrium and balanced or equal (L16S2)*
- *Equal means that same number of girls and boys is the same number(L11S3)*
- *Equal is a situation where you equate something which is on the right hand side and left hand side (L13S1)*

In contrast, there are more of learners in the qualitative data obtained through interview (i.e. 7 out of 20) who have indicated ‘no response or I don’t know’ representing 35.0% of the learners interviewed. However, only 6.7% of the learners (i.e. 16 out of 239) indicated ‘no response or I don’t know’ in the quantitative data obtained.

4.2.2.2 The areas or situations where the concept of equal can or cannot be applied in Mathematics

In the quantitative analysis, some learners saw the concept of equal as being applied in situations where one has to give an answer or make comparisons. It is a mathematical symbol used to show that things are the same. It is also used in simultaneous expressions like 2x+3=6 (L4S2). It is used when you are writing an equation and you find the answer we put equal or if we are dealing with greater than or less than (L14S2). The concept of equal is applied in the topic similarity (L21S4). The concept of equal is also used to separate two equations and to show that two or more things are balanced.

In qualitative data, learners indicated that the concept of equal is applied in putting the answer to a mathematical problem. The concept of equal is mostly used in equations for example, if you are given$2x + 3y = 5$, the equal sign has to be there. If it is missing, then it will not be a question but a statement. Equal concept is also used to show that both sides are equilibrium and both sides are balanced, what is on the left hand side is equal to what is on the right hand side. Sometimes we use equal sign when we are told to calculate an angle, you are told to calculate a certain side may be there are cases where we have sides of a triangle like AB = 12cm, BC = 13cm and AC = 13cm so we use equal sign to show that side BC and side AC are same. The concept is also used when we want to mean the variable is the subject of the formula by using the equal sign. Learners also highlighted areas or situations where the concept of equal is not applied as seen from this extract from what they brought out:

L37S2: *When calculating inequalities*

Interviewer: *Why don’t we use equal sign in inequalities?*

L37S2: *Because it deals with values which are less or greater than... silence.*

L7S2: *Also in ratio when you are expressing in ratios you cannot use equal sign. This is because ratios deal with comparison between two or more similar quantities so we cannot use equal symbols as they are written in fraction term.*

Silence for three minutes

L28S3: *Maybe when you are dealing with negative and positive numbers*

Interviewer: *Why don't we use equal symbol in such situations*

L28S3: *Because negative numbers are not equal to positive numbers as negative numbers are less than positive numbers.*

4.2.3 Analysis of learners' responses to the concept of function

4.2.3.1 Learners' understanding of the concept of function

A detailed transcript of all the interviews with learners is presented in Appendix J. Table 4.9 shows the responses from learners in all the five schools on their general understanding of the mathematical concept of function. This table is an extract from learners' responses when they were asked specific questions in the interview, as can be seen in the following extract (with a group of learners in School 2):

Interviewer: *What is your general understanding of the concept of function in Mathematics?*

L16S2: *A function is eeeeeh... a function ... it means that the first variable maps onto something.*

Interviewer: What are the views of others?

L7S2: *Mmmmmmm. Silence... I am still thinking.*

Interviewer: *how much time should I give you?*

L29S2: *Ummmm sir this concept is difficult to explain or define.*

Interviewer: *Have you ever used/heard of the concept of function in Mathematics?*

L37S2: *No answer sir, I do not understand the meaning of function but have heard of it.*

Interviewer: *Ok.*

After reading the transcripts, key parts of the data were extracted that related closely to learners' understanding of the concept of function, for each of the five schools. Proceeding in this manner, I compiled all the relevant responses into a table, as can be seen in Table 4.7 below.

Table 4.7: learners' responses on their understanding of the concept of function in five schools

LEARNER ID	LEARNERS' RESPONSES
Learner L23S1	It is defined as the mapping of an object to another object e.g. mapping of mangoes to apples. It must show the arrows to show the relationship
Learner L13S1	No response
Learner L19S1	No response
Learner L4S1	No response
Learner L16S2	A function is eeeeeh... a function ... it means that the first variable maps onto something.
Learner L7S2	mmmmmmm. Silence.... I am still thinking.
Learner L29S2	ummmm sir this concept is difficult to explain or define
Learner L37S2	No answer sir, I do not understand the meaning of function but have heard of it.
Learner L11S3	function ahhhh like in Mathematics we say the function of for example 2x+1, the whole thing is difficult to explain
Learner L4S3	No response
Learner L28S3	No response
Learner L19S3	No response
Learner L16S4	Mmmmmmmm no answer sir.
Learner L23S4	No response
Learner L6S4	No response
Learner L10S4	No response
Learner L10S5	It's a topic in Mathematics
Learner L22S5	No response
Learner L1S5	No response
Learner L28S5	No response

In the quantitative analysis, and in relation to emerging themes linked to analysis of Schools 1-5 learners' responses to the concept of function, the researcher noted that the dominant categories in learners' responses in relation to their understanding of the concept of function is 'no response or I don't know'. In the five schools, 33.4% of the learners associated function with a 'no response or I don't know' and 19.2% associated zero with 'purpose'. A considerable number of learners (i.e. 17.2%) associated the concept of function to 'work'. Some learners (i.e. 6.7%) did not respond to the task. This information is summarised in Table 4.26 (summary analysis of Schools 1-5 learners' associations of function with other concepts).

In Table 4.9 above, the qualitative data from interviews with the 20 learners confirms key aspects of the analysis from the quantitative data. Key of these aspects is connected to learners associating function with "no response". The table below indicates response categories from the analysis of responses from interviews with 20 selected learners.

Response category: Associations of function with "no response or I don't know" (n = 11)
• No response (L13S1) • No response (L19S1) • No response (L4S1) • No response (L4S3) • No response (L28S3) • No response (L19S3) • function ahhhh like in Mathematics we say the function of for example 2x+1, the whole thing is difficult to explain (L11S3) • Mmmmmmmm no answer sir. (L16S4) • No response (L23S4) • No response (L6S4) • No response (L10S4) • No response (L22S5) • No response. (L1S5) • No response (L28S5) • mmmmmmm. Silence.... I am still thinking. (L7S2) • ummmm sir this concept is difficult to explain or define (L29S2)

• No answer sir, I do not understand the meaning of function but have heard of it. (L37S2)
Response category: Associations of function with "others" (n = 11)
• It is defined as the mapping of an object to another object e.g. mapping of mangoes to apples. It must show the arrows to show the relationship (L23S1) • It's a topic in Mathematics (L10S5) • Is eeeeeh function it means that the first variable maps onto something. (L16S2)

As can be seen from the table above, almost all (i.e. 17 out of 20) learners in the interview associated the concept of function with 'no response or I don't know'. This is consistent with the analysis of the quantitative data in which it was found that 33.5% (i.e. slightly more than one-third) of the participants associated equal with 'no response or I don't know'. Although it is less than half (in terms of per cent), the 'no response or I don't know' is still the dominant theme from the quantitative data obtained. In the qualitative data, we note that learners have made some associations in relation to what the concept of function is. For example, three (3) of these learners associated the concept of function to mean the following:

- *It is defined as the mapping of an object to another object e.g. mapping of mangoes to apples. It must show the arrows to show the relationship* (L23S1)
- *It's a topic in Mathematics* (L10S5)
- *Is eeeeeh function it means that the first variable maps onto something.* (L16S2)

The dominant theme from the qualitative data is 'no response or I don't know', which is in conformity with what is in the quantitative data.

4.2.3.2 The areas or situations where the concept of function can or cannot be applied in Mathematics

In the quantitative analysis, some learners saw the concept of function as being applied in situations where it is used to identify what something does or work. The concept of function is mostly used to solve mathematical problems. It is used to get something to something else and also explain the work of something. The concept of function is also used to stand in for something that is happening. The concept of function was noted to denote how certain symbols or signs are used. A function was taken to mean the purpose of something or what it does. In learning how to solve problems in Mathematics, the concept of function is applied. It is applied to prove the answer given in a mathematical problem.

In contrast, in the qualitative data, we note that learners have made association in relation to the application of function as Learner L23S1 indicates that it is used in calculus to represent the function like f(x) = 2. It also applied in sets if they say f(x) = 5x and x is a prime number less than 10, and then asked to find the other set by replacing x with numbers. When we want to calculate the inverse, for example, you are given like f(x) =$\frac{2x-1}{5}$ if you want to find the inverse of that function. It can be applied in naming expressions like $f(x) = 2x^3 + 4 \ or \ g(x) = x^2 + 3x + 1$.

On the areas where the mathematical concept of function is not applied, some learners indicated

> '*Like in ratios... functions are not applicable… Pause for some time... Even in trigonometry because in trigonometry, we use sine, cosine and tangent. So, there is no need of using functions*'.

In both the qualitative and quantitative data of the concept of function, majority of the learners did not respond to the questions asked while others said 'I don't know sir'.

4.2.4 Analysis of learners' responses to the concept of variable

4.2.4.1 Learners' understanding of the concept of variable

A detailed transcript of all the interviews with learners is presented in Appendix J. Table 4.10 shows the responses from learners in all the five schools on their general understanding of the concept. This table is an extract from learners' responses when they were asked specific questions in the interview, as can be seen in the following extract (with a group of learners in School 5 (S5)):

Interviewer: *What is your general understanding of the concept of variable in Mathematics?*

L1S5: *is a letter which has a coefficient with it like 2x, 2 is a coefficient while x is a variable*

L22S5: *Variables are letters like x and y used in algebraic expressions.*

Interviewer: are x and y the only letters used in expressions?

L10S5: *There are many other variables used but the common ones are x and y.*

Interviewer: why is it like that?

L10S5: *Not sure.... may be its simple to write and recall.*

L28S5: *No response*

After reading the transcripts, I extracted key parts of the data (what learners said) that related closely to learners' understanding of the concept of zero, for each of the five schools. Proceeding in this manner, I compiled all the relevant responses into a table, as can be seen in Table 4.8 below.

Table 4.8: Learners' responses on their understanding of the concept of variable in five schools

LEARNER ID	LEARNERS' RESPONSES
Learner L23S1	no response
Learner L19S1	In Mathematics, we have terms which we use like coefficient and variable like in 5y. In 5y, 5 is a variable and y is a coefficient.
Interviewer	What is a variable then?
Learner L19S1	Sir, a variable is a number in simple terms
Learner L13S1	No response
Learner L4S1	No response
Learner L37S2	Silence...... any letter that is used to denote something like x e.g. if given equation $y = 2x + 1$ the equation has two variable which are y and x variable
Learner L16S2	No answer
Learner L7S2	No answer sir
Learner L29S2	No response
Learner L19S3	May be it's a number
Learner L28S3	A variable is anything that may carry a power
Interviewer	give an example
Learner L19S3	Like 2^3 and 2 is a variable.
Learner L4S3	No response
Learner L11S3	No response
Learner L13S4	: A letter which represents a number like in equation $x + y = 10, x\ and\ y$ are variables which represents members which can make the equation balance.
Learner L10S4	No response
Learner L16S4	No response
Learner L6S4	No response

Learner L1S5	is a letter which has a coefficient with it like 2x, 2 is a coefficient while x is a variable
Learner L22S5	Variables are letters like x and y used in algebraic expressions.
Interviewer	Are x and y the only letters used in expressions?
Learner L10S5	There are many other variables used but the common ones are x and y.
Interviewer	Why is it like that?
Learner L10S5	Not sure.... may be its simple to write and recall.
Learner L28S5	No response

In the quantitative analysis, and in relation to emerging themes linked to analysis of Schools 1-5 learners' responses to the concept of variable, the researcher noted that the dominant categories in learners' responses in relation to their understanding of the concept of variable is 'no response'. In the five schools, 31.8% of the learners associated the concept of variable with a 'no response' and 30.5% associated zero to 'letter'. A considerable number of learners (i.e. 16.7%) associated the concept with 'number'. Some learners (i.e. 2.1%) associated the concept to 'expression'. This information is summarised in Table 4.37 (summary analysis of Schools 1-5 learners' associations of variable with other concepts).

In Table 4.10 above, the qualitative data from interviews with the 20 learners confirms key aspects of the analysis from the quantitative data. Key of these aspects is connected to learners associating variable with 'no response' and 'letter'.

The table below indicates response categories from the analysis of responses from interviews with 20 selected learners.

Response category: Associations of variable with "no response or I don't know" (n = 12)
• No response (L23S1) • No response (L13S1) • No response (L4S1) • No answer (L16S2) • No answer sir (L7S2) • No response (L29S2) • No response (L4S3) • No response (L11S3) • No response (L10S4) • No response (L16S4) • No response (L6S4) • No response (L28S5)
Response category: Associations of variable with "letter" (n = 5)
• Silence... any letter that is used to denote something like x e.g. if given equation $y = 2x + 1$ the equation has two variable which are y and x variable(L37S2) • A letter which represents a number like in equation $x + y = 10, x \ and \ y$ are variables which represents members which can make the equation balance(L13S4) • is a letter which has a coefficient with it like 2x, 2 is a coefficient while x is a variable(L1S5) • Variables are letters like x and y used in algebraic expressions(L22S5) • A letter which represents a number like in equation $x + y = 10, x \ and \ y$ are variables which represents members which can make the equation balance. (L13S4)
Response category: Associations of variable with "number" (n = 3)
• May be it's a number(L19S3) • A variable is anything that may carry a power like 2^3 and 2 is a variable(L28S3) • In Mathematics, we have terms which we use like coefficient and variable like in 5y. in 5y, 5 is a variable and y is a coefficient. sir, a variable is a number in simple terms(L19S1)

As can be seen from the table above, more than half (i.e.12 out of 20) of the learners in the interview associated variable with ‘no response’. This is consistent with the analysis of the quantitative data in which it was found that 31.8% (i.e. highest number of learners) of the participants associated variable with ‘no response’. In the qualitative data, we note that learners have made several associations in relation to what variable is. For example, eight (8) of these learners associated variable with **letter or number:**

- *Silence...... any letter that is used to denote something like x e.g. if given equation* $y = 2x + 1$ *the equation has two variable which are y and x variable(L37S2)*
- *A letter which represents a number like in equation* $x + y = 10, x \; and \; y$ *are variables which represents members which can make the equation balance(L13S4)*
- *is a letter which has a coefficient with it like 2x, 2 is a coefficient while x is a variable(L1S5)*
- *Variables are letters like x and y used in algebraic expressions(L22S5)*
- *A letter which represents a number like in equation* $x + y = 10, x \; and \; y$ *are variables which represents members which can make the equation balance. (L13S4)*
- *May be it’s a number(L19S3)*
- *A variable is anything that may carry a power like* 2^3 *and 2 is a variable(L28S3)*
- *In Mathematics, we have terms which we use like coefficient and variable like in 5y. In 5y, 5 is a variable and y is a coefficient. sir, a variable is a number in simple terms(L19S1)*

Learner L1S5 viewed variable as “a letter which has a coefficient with it like 2x, 2 is a coefficient while x is a variable”. Other learners’ comments linked to the idea of letter were, for example, learner L13S4 noted that variable is “A letter which represents a number like in equation $\mathrm{x} + \mathrm{y} = 10,$ x and y are variables which represent members which can make the equation balance”. Learner L22S5 described variable as ‘letters like x and y used in algebraic expressions’. This view to some extent restricts variables to only x and y and not any other letter. There were other comments from learners that indicated the possibility of misconceptions related to understanding of variable. For example, learner L28S3 indicated that a variable is “anything that may carry a power like 2^3 and 2 is a variable”, while learner L19S1 indicated that “In Mathematics, we have terms which we use like coefficient and variable like in 5y. In 5y, 5 is a variable and y is a coefficient. Sir, a variable is a number in simple terms". These two responses of learners are contrary to what is known about the concept of variable. This is because although variable represents a number, it is not a number in itself.

In contrast to the quantitative data analysis, none of the learners interviewed directly associated variable with expression as in itself but that it is used in expressions.

4.2.4.2 The areas or situations where the concept of variable can or cannot be applied in Mathematics

In the quantitative analysis, some learners saw the concept of variable as being applied in situations when you want to compare things of the same variable. The concept of variable is mostly used in simultaneous equations and algebraic expressions where numbers are represented as letters. It is used in Mathematics to help the writer to know which one is variable. The concept of variable is also used to solve mathematical problems. The concept of function was noted to be used when we are doing algebra for example $7x + 3y - 24y^2$. Variables are also used to balance up the equation.

In the qualitative data, we note that learners have made association in relation to the application of variable as it is used in simultaneous equations like $2x+y=5$ where we are asked to find value of two variables at the same time (Learner L4S1). It is also used in inequalities like $250x+350y=1000$. It is used in calculus like where you are asked to differentiate $3x^2 + 2x$ with respect to x. Learner L16S2 indicated that variables are mostly used in simultaneous equations when we are told to find the values. Variables are even used in the topic variation for example; y varies inversely as a square of z. Learner L37S2 stated "Even in algebraic expressions, variables are used ... mostly x variables are the most used". Learner L19S3 observed that variables are used indices when raising a number to a power. Learner L6S4 indicated that every part of Mathematics uses variables in one way or the other.

On the areas where the mathematical concept of variable is not applied, some learners indicated that variables are not used in sets, since x and y are used and are not variables but members of the set. Of course zero. When dealing with zero, the concept of variables is not used because everything becomes zero. Learner L19S3 observed that it is not applicable when the power is negative.

In both the qualitative and quantitative data of the concept of function, majority of the learners did not respond to the questions asked while other gave 'I don't know sir'. In instances where the learners responded to the questions, the responses in both quantitative and qualitative are similar to a large extent.

4.3 CONCLUSION

The chapter presented the quantitative and qualitative data analysis emanating from the learners' responses in the questionnaire and focus group discussions respectively. It was noted during data analysis that some learners did not respond to the questions given in the questionnaire and among those selected for focus group discussions, some opted to remain silent. The dominant categories from findings of the data collected through questionnaire and also confirmed by findings from the focus group discussions on the four mathematical concepts of zero, equal, function, and variable indicate that learners associate the concept of zero to 'nothing' and 'number' and is applicable in writing decimal numbers. It is also applicable as a placeholder number. On the concept of equal, 'same' and 'answer' are the dominant categories, which learners have associated to concept. The findings also indicate that 'work', 'purpose' and 'no response' are the dominant categories of the concept of function. It is applied in calculus to represent the gradient of a curve. On the concept of variable, the dominant categories are 'letter' and 'no response' and that it is applied in algebra to represent the unknown. It is however not applicable to a constant.

The results and findings obtained during the study indicate that Grade 10 learners do not understand the key mathematical concepts as evidenced by the high numbers of learners in the 'no response' category. Some learners did not answer the questions in the questionnaire and opted not to submit the instrument to the researcher. This is an indication that the questions might have asked for concepts they do not understand. The learners' responses were scattered around at least two dominant categories on every mathematical concept asked. This is an indication that learners had different views and perspectives on their understanding and application of each of the concepts.
A detailed discussion of these ideas/key results in the light of the context of the study and also in light of the literature reviewed in Chapter 2 will follow in Chapter 5.

CHAPTER FIVE

DISCUSSION OF FINDINGS

5.1 INTRODUCTION

This chapter follows on from the analysis of the data presented in Chapter 4 and discusses four key issues identified in the analysis of the responses given by learners. The four key issues that emerged in the analysis are linked to:

- Learners' understanding of the concept of zero as a number or nothing and that "zero" is applied as a place holder and as an answer to arithmetic questions on the subtraction of two numbers of the same value;
- Learners' understanding of the concept of equal to mean same and that "equal" is applied in equations and to show the answer to a mathematical problem;
- Learners' understanding of the concept of function as purpose or work, and that "function" is applied to show the work of something; and
- Learners' understanding of the concept of variable to mean letter or number and that "variable" is applied to represent a number in algebra.

I highlight these four key issues and discuss them further in relation to the literature and to further evidence from the data from learners.

5.2 LEARNERS' UNDERSTANDING OF THE CONCEPT OF ZERO

The analysis in this research has revealed that learners have divergent views of what the concept of zero means. There are some differences as well as similarities observed in the quantitative and qualitative data presented on the concept of zero in the previous chapter. In the analysis of both the quantitative and qualitative data, most of the learners associated zero with a number. They view zero as a number just like any other number used in counting. A small number of learners view the concept as 'nothing', which means that it is a number that does not have value. They (learners) view zero as a symbolic number that is not used in counting and its presence is not significant as it does not add or subtract anything from what is already available. Very few learners in both the quantitative and qualitative data presented in Chapter 4 failed to express their understanding of the concept. This is an indication that learners indeed have ideas of what zero is about although some may not be able to give a precise definition.

In both data-sets, learners have indicated that the concept is applied in solving quadratic equations. A quadratic equation must be equated to zero so that the variable whose numerical value is supposed to be determined must be on one side of the mathematical expression while the other side should have a zero. Zero is viewed to be applied in relation to a number line, to separate negative numbers from positive ones.

On the other hand, there are some differences in the data-set presented in Chapter 4. A small number of learners associated the concept of zero to a point in the quantitative data while no learner did so in the qualitative data set. The concept has been understood by some learners to mean a point. Zero is assumed to be a point and as a symbol. In some cases, it is not taken to be a number like other numbers. The nature of the symbol for the number zero made some of the learners to regard zero as a point. A small number of learners who associated the concept to a set in the quantitative data-set did so to mean an empty set arising from the symbol that is used to indicate an empty set. The symbol $\emptyset$ is taken to represent zero by some learners when it actually means that a set is empty in that it has no elements or membership.

Learners' association of the concept of zero with nothing seems consistent with findings from various assertions in the literature made by different researchers. Some learners consider zero to mean nothing or not a number because of its "no item" property (Krajcsi et al., 2017; Sarnecka & Carey, 2008; Lipton & Spelke, 2006). This is in relation to its value in quantitative aspects, which seems to build in particular on the argument that zero can also be understood as denoting the concept of absence, void, nothingness, or the cardinality of the empty set (Wellman & Miller 1986; Schlin & Skosnik, 2010). On the contrary, some learners indicated that zero is a number like other numbers as it is applied in Mathematics. Learners' view that zero is a number seems to echo Krajcsi et al.'s (2017) argument that most adults know that zero is a number because it is only a step on the number line among the positive and negative numbers or because in writing multi-digit calculations; zero is handled in a similar way as the other digits. Learners asserts that zero is applied as a placeholder as it shows that there is no number at some place in the given number such as 503. The zero between 5 and 3 is to maintain the value of the number 503 hence; zero must be included between 5 and 3. The study also revealed that zero is applied in number line as it is a special number lying halfway between -1 and +1 on the number line. Some learners opted to remain silent during the focus group discussions while others did not answer the question related to this aspect in the interview. This demonstrates a lack of understanding of the concept of zero.

Further analysis in this research has revealed that there are some similarities and differences in the responses from the learners across the five schools on the concept of zero. The dominant category in all the five schools on the concept of zero is that it is associated with number. Most of the learners have indicated that they take zero to be a number though it has no numerical value. It is for this reason that the aspect of zero being nothing comes closer in terms of dominance in all the five schools than to it being a number. Moreover, a small number of learners in each of the five schools did not respond to the tasks given in both the quantitative or qualitative way. This is an indication of lack of understanding of the concept. The association of the concept of zero to a set or a point is only noted with very few learners in School 1 and 2.

Although most of the learners associated zero with number, there is a strong link of the concept with nothing. The concept of zero in the primary school curriculum is introduced at the lower level after other positive numbers 1 to 9 are introduced. The concept is introduced to signify emptiness of something in terms of possession and this is against the introduction of the other numbers with visible quantitative meanings. Very little or no effort at all is made by the teacher to make learners comprehend the concept. The rest remains an assumption that the learners will catch up with the theoretical understanding of the concept. Learners do appreciate the fact that zero is a number but it is also taken to signify emptiness. The concept of empty set is also introduced at middle primary school level and the symbol used to represent it is in form of the symbol for zero. This has generated the confusion in learner's mind on the meaning and application of zero.

5.3 LEARNERS' UNDERSTANDING OF THE CONCEPT OF EQUAL

The analysis in Chapter 4 has revealed that learners understand the concept of equal in different ways. There are some differences and similarities in the way learners responded to the concept of equal. The equal concept is understood to mean same or similar in both the qualitative and quantitative data. Very few learners did not associate the concept of equal with anything by giving no response to the task. This is an indication that they lack conceptual understanding of the equal sign. Very few learners associated the concept with 'answer' qualitatively than they did quantitatively.

The analysis has further revealed that learners understand the equal concept in relation to the equivalence that it creates between two sides of an algebraic equation. This seems consistent with Knuth et al.'s (2005) findings, which indicate that the concept of equal is understood by learners to mean that things on both sides of it are of the same value or simply the numbers on the right side are the same as numbers on the left hand side. Some learners understand equal as a mathematical symbol that show that two sides are equal, which means they are the same. What is on the left hand side is the same as what is on the right hand side. Few learners view the equal concept as a tool that is applied when giving an answer to a mathematical problem. More learners associated equal with same (relational) than with answer (operational). This is contrary to the assertion that the equal sign is usually introduced as an operational sign instead of relational sign of equivalence in the early stages of cognitive development (McNeil & Alibali, 2005). Learners responded this way on the concept of equal because they view the concept to play two major roles in Mathematics. These roles are the relational where the concept equal connects two mathematical statements of equivalence and the operational role where it is used to signify the occurrence of the answer to a mathematical problem. The learners view equal as a way to communicating the final answer to a problem. Learners' understanding of the concept is as relational as it is operational (Macgregor & Stacey, 1997; Carpenter et al., 2003; McNeil & Alibali, 2005). However, there is a shift in learners' understanding of the concept towards relational equivalence than operational equivalence. This shift towards relational equivalence of understanding the equal sign is critical in developing learners' algebraic thinking (Falkner, Levi & Carpenter, 1999; Knuth et al., 2006).

Further analysis of the learners' responses across the five schools show that there is a minimal difference in the way they responded to the tasks. In all the five schools, most of the learners understand the concept to mean 'same', which depicts its relational aspect and very few learners understand the concept to mean 'answer', which is the operational aspect of the concept. A very small number did not respond to the questions given to both the questionnaire and the focus group discussion. The silence exhibited by the learners is an indication that they do not understand the concept. Overall, Schools 4 and 5 had more learners who did not respond than any other school.

The Zambian primary education curriculum framework emphasises on the need for learners to understand and be able to apply the concept of equal sign correctly as early as the elementary stages of their primary education. This is in line with the conviction that learners' preparation and eventual progress in algebra is dependent on their strong background on their understanding of the equal sign. However, teachers make assumptions that once learners are introduced to the concept at elementary stages, they do not need to re-emphasise the concept to the learners to concretise their understanding. This leads to poor performance in Mathematics examinations at all levels as most of the questions in an examination l.

5.4 LEARNERS' UNDERSTANDING OF THE CONCEPT OF FUNCTION

The analysis has revealed that learners' understanding of the concept of function is in relation to two divergent aspects. Most of the learners understand the concept to mean 'work' of something. They view the concept to mean what something can do or are able to do. The notable similarity in the learners' responses to the concept of function is their association of the concept to work and that most of the learners did not respond to the task both in the quantitative and qualitative realms. This is an indication that these learners do not understand the concept. Very few learners associated the function with 'purpose' in the qualitative data-set than in the quantitative data-set. This viewpoint of the concept by the learners is not within the mathematical realm but rather in the everyday context. It is taken to mean how something works. The learners also understand function to mean the purpose of something or the reason for the existence of something. However, a substantial number of learners did not respond to the tasks in questionnaire and focus group discussions, which is an indication that they do not understand the concept.

The findings in this study confirm what other researchers have revealed earlier that learners' understanding of the concept of function is either operational or structural (Dubinsky, 1991; Sfard, 1991; Tall, 1996). A function may operationally be seen as a computational process, as a recipe to transform one number into another (or to link one number to another), whereas, structurally, a function can be thought of as a set of ordered number pairs (Sfard, 1991). The learners have indicated that a function is viewed as an input-output assignment of elements in one set to another connected by the arrow. There must be a number that must be substituted in the function to obtain another number which they have indicated as an output. In the same viewpoint a function is usually in a form of a rule, from one set to another with the property that each input has a unique output (Bardini et al., 2014). Learners view a function as a machine that works on the elements from one set and converts them into another element known as output for the other set. The study also reveals that function should connect two sets of numbers with each arrow pointing to exactly one element in the second set. This seems to confirm what Bardini et al. (2014) argued that a function is a mapping, usually in the form of a rule, from one set to another, with the property that each input has a unique output.

The responses from the learners across the five secondary schools indicate that there is little difference. Most of the learners in all the five secondary schools have demonstrated lack of understanding of the concept through their non-response to the tasks given to them on their understanding of the concept. A few learners have associated the concept to work or purpose in all the five schools. However, the difference lies in the numbers per school. For instance, Schools 4 and 5 had many learners who did not respond.

The concept of function is introduced in the upper primary Mathematics curriculum and runs through to tertiary level. There is however less emphasis on the proper understanding of the concept by the learners at this point. It is casually defined, followed by use in solutions to worked examples carefully selected by teachers to demonstrate its application. As a result, learners go to university without understanding this concept.

5.5 LEARNERS' UNDERSTANDING OF THE CONCEPT OF VARIABLE

The analysis in this research has revealed that learners' understanding of the mathematical concept of variable was two-fold. Some learners understand the concept as a letter that is used to represent a specific number in an algebraic expression. The learners' responses in both the quantitative and qualitative data-set are the same. Most of the learners in both data sets did not respond to the tasks given. A few of the learners associated the concept of variable to a number. One learner submitted that "*In Mathematics, we have terms which we use like coefficient and variable like in 5y. In 5y, 5 is a variable and y is a coefficient*". This statement is incorrectly grounded as it is clear that 5 is not a variable but a coefficient and to the contrary it is y which is a variable. Algebraic expressions use common alphabetical letters such as x and y commonly to represent the unknown number in order to make the mathematical statement correct. The learners responded this way because the equations they are asked to solve on a daily basis in Mathematics have letters (mostly x or y) and the answer they get is always a letter which is equated to a specific number. This is after reducing a complicated expression to a letter and a specific number like x=5 or y= -2. Therefore, this finding seems to follow closely on from Akgun and Ozdemir (2006) who argue that unknowns are variables that represent specific numbers and that in order to understand such a use of variable, students should be able to substitute variables in equations with numbers (one or several) to produce relations that are true numerical statements.

The concept of variable has been taught in its general form since the beginning and all letter symbols have been called variable (Kieran, 1989). One of the reasons that make the concept of variable difficult is the frequent use of "letters" and "verbal symbols". "Letters" and "verbal symbols" used in algebra are complex and have multiple representations of this concept (Schoenfeld & Arcavi, 1988). The learners' understanding of the concept of variable is restricted to letters representing a number in an algebraic expression. Most of the learners in the data analysis of this research indicate that variable is a letter that represents a specific number in the algebraic expression to make it true or correct. However, the concept of variable in Mathematics is much more than just a letter that is used to represent a number. Variables often represent points on the vertices of geometrical shapes such as A, B and C as vertices of an isosceles triangle where AB=BC (Usiskin, 2017). Variables can be used to represent the identity of someone who may not want to disclose their real identity for fear of victimization. For example, we might use Mr X or Mrs W and many others to refer to individuals who may want to hide their true identity. In real sense, variables are often used to mean different phenomena. For instance, *q* and *p* often stand for propositions and the symbol *f* stands for a function while in linear algebra the variable A may stand for a matrix (Usiskin, 2017). One learner argued that a variable is a number in simple terms like in 2^3 and 2 is a variable. This learner is too far from knowing what a variable is because 2 is a number and not a variable. This is a demonstration of lack of conceptual understanding by the learners of the concept of variable.

A few learners did not provide answers to the questionnaire and did not respond to the questions directed at them during the discussions. This is an indication that they (learners) do not understand the concepts in question and this impedes their development in the acquisition of mathematical knowledge and skills. Teachers must commit themselves by making sure learners are made to understand a concept before it is applied. There are several conceptual obstacles in proceeding in algebra (Stacey & MacGregory, 1997). One of the most important difficulties is the failure in understanding the concept of variable. This concept is rarely discussed in most of the classes where algebra courses are given (Sasman et al., 1997). If learners misunderstand the concept, they are likely unable to apply the concept correctly.

The learners' response across the five secondary schools on their understanding of the concept of variable is the same. Most of the learners have not responded to show their understanding of the concept and its application in Mathematics. Of few learners who responded, the majority linked the concept to a letter or a number. The only notable difference across the five schools in terms of the way they responded is that Schools 4 and 5 had more learners who did not respond to the task as compared to Schools 1, 2 and 3.

Any misconception in algebra has the capacity to cause poor performance in the final examination and eventually lead to constraints in learning Mathematics at tertiary level. This is largely because algebra is one of the most important branches of Mathematics and most of the questions in the final Mathematics examination in both Grade 9 and 12 are deeply rooted in algebra. We should thus fully understand the definitions used for the concept of variable and its application. This assertion seems consistent with Balyta's (1999) claim that understanding the concept of variable has a vital importance in constituting a strong basis for other algebraic concepts related to it. Graham and Thomas (2000) confirm that this understanding is the basis of all advanced studies. It is thus profoundly important for all students to understand the concept of variable as it is the cornerstone to learning algebra. This also seems to confirm Tall and Vinner (1981) who emphasises that students are expected to understand that concept to gain self-confidence in using variables. Since variable is a concept that requires a rich concept easily.

5.6 CONCLUSION

In this chapter, the key issues from the analysis of data on the four mathematical key concepts were discussed. Explanations were offered on why the learners responded the way they did during data collection. The link was established between the findings of this study and findings from other researchers. In Chapter 6, conclusion and recommendation are presented.

CHAPTER SIX

CONCLUSION AND RECOMMENDATIONS

6.1 INTRODUCTION

This chapter concludes the study by answering the research questions and providing recommendations for both further research and for teaching practice.

6.2 ANSWER TO RESEARCH QUESTION ONE

What are the Grade 10 learners understanding of the mathematical concepts of zero, equal, function, and variable?

To answer the first research question, I refer to the responses from the learners to the question on the meaning of each of the concepts.

The Grade 10 learners understand the mathematical concept of zero as a number that means nothing in terms of quantity meaning that it has no value. This is because when it is added to another number the result is the same number. It is a number before one and is neither positive nor negative. The learners' understanding of the concept of zero is restricted to what it is and what it represents (see Table 4.6).

The Grade 10 learners understand the mathematical concept of equal as a symbol that demonstrates sameness of two or more quantities. Most of the learners understand the equal sign to mean 'sameness', which means that what is on one side of the equal sign is same as what is on the other side. This is what many researchers have called the relational meaning of the equal sign. However, a few learners understand equal as a sign that is associated with the answer to any mathematical problem. This is what other researchers have called the operational meaning of the equal sign (see Table 4.6).

The Grade 10 learners understand the concept of function to mean the purpose or work of something. Most of the learners did not respond to this question, which is an indication that they do not understand the mathematical concept of function. The learners' understanding of functions is narrowly focused to substitution of some values to get corresponding values and include some wrong assumptions of the concept of function being 'purpose' and 'work' of something (Refer to Table 4.26).

Learners understand variable as a letter that represent the unknown number in a mathematical statement. Most of the learners though did not respond to this question, which is an indication that they do not have sufficient knowledge on the concept of variable (Refer to Table 4.37).

6.3 ANSWER TO RESEARCH QUESTION TWO

To what extent are Grade 10 learners able to apply these key mathematical concepts correctly?

The extent to which learners apply the mathematical concept of zero varies as the learners have identified so many areas of application of the concept of zero. Zero is applied as an answer to situations where two numbers of the same value are subtracted. It is also applied in solving of equations such as quadratic equations and also to denote the empty set. However, this is a misconception of zero as a concept by the learners as zero is not used to denote an empty set. A symbol, which looks like zero $\emptyset$ is used to denote an empty set. It is also used in matrices such as the identity matrix, transformational matrices. Zero is applied to separate the positive from the negative numbers on the number line.

The application of the equal sign in equations such as to separate the two parts of the mathematical statement, which must be equivalent. Learners have also indicated that equal sign is applied to show that what is on the left hand side is the same as what is on the right hand side, which is sometimes referred to as the symmetric and transitive or the left-right equivalence. The equal sign is also used to show the answer in a mathematical problem. A few learners interpret the equal sign as an operational symbol, which is more to do with a command to do something whereas most learners have demonstrated the relational application of the equal sign.

The application of the concept of function in Mathematics as viewed by the learners indicates that they (learners) lack the basic application principles of the function. Learners opted to remain silent on this concept in both quantitative and qualitative data set. A few learners gave answers such as ‘it is applied to show the work or purpose of something’. Most of the learners indicated that they do not know of any area in Mathematics where the concept can be applied.

The extent to which learners are able to apply the concept of variable is that they (learners) consider the concept of variable as applied naming of sets and angles such as set A = {a, e, i, o, u} and also angles such as angle B. Learners also indicated that the concept of variable is applied to create algebraic expressions where a mathematical statement representing the unknown is expressed in letters to represent the unknowns. This ascribed application of the concept of variable by the learners is attributed to variable as unknown. A variable is also applied in equations or sets where it is assumed to take a general number. Learners lack both the meaning and application of the concept correctly as seen from the responses they gave (Table 4.37).

6.4 ANSWER TO RESEARCH QUESTION THREE

To what extent are these learners' understanding of Mathematics concepts promotes quality learning of Mathematics?

The mathematical concepts of equal, zero, function, and variable are among the most essential concepts in taught Mathematics right from the primary school through the secondary school to the university level. These mathematical concepts are closely connected such that it is practically impossible to learn Mathematics with poorly co-ordinated understanding of these concepts. These concepts are more profoundly in Algebra though they are found in all other branches of Mathematics. One cannot talk of quality Mathematics learning with poor understanding of these key mathematical concepts. For instance, for learners to understand how to solve equations, they need to have sound knowledge and understanding of these key concept like the concept of zero to be able to solve equations by equating the expression to zero on the other side, concept of equal to demonstrate the relational aspect of the two sides as well as the concept of variable to represent the unknown or general number. The concept of function is applied to show the relation between two sets of numbers.

There is therefore a glowing need to have learners begin to understand these concepts to improve the quality teaching and learning of Mathematics at all levels. However, the scenario is gloomy in that the results from the study indicate that learners' understanding of the key mathematical concepts is not effective and yet these concepts are central to the attainment of quality teaching and learning of Mathematics at both primary and secondary school. They provide the basis for passage from pre-Mathematics to Mathematics and necessary for use at university level. The responses from the learners have given an indication that most of the learners have supposedly problem to understand these concepts. Ultimately, this is the main reason why the results at both junior secondary school leaving examination (JSSLE) and senior secondary in Mathematics are not impressive.

6.5 PURPOSE AND FINDINGS OF THE STUDY

The research was designed to investigate grade 10 learners' understanding of key Mathematics concepts of zero, equal, function, and variable in Chibombo District of Central Province in Zambia. The themes emerging from my analysis of data are that learners' associate the concept of zero to nothing and that it is applied in algebraic equations as well as being a place holder when writing numbers. On the concept of equal, learners associate it to same and that it is applied in solving quadratic equations and to show the answer to a mathematical question. The concept of function is associated to work or purpose of something and areas of its application have not been given. The learners have associated the concept of variable to letter and number. It is applied to represent a number in an algebraic expression. Though there are learners who did not respond to the tasks from both the questionnaire and the focus group discussions on all the concepts, more learners were recorded on the concept of function.

6.6 RELATIONSHIP WITH PREVIOUS RESEARCH

These findings are broadly in harmony with those of researchers such as learners' understanding of the concept of equal as an operational and relational interpretation was reviewed through the literature (Carpenter, Franke, & Levi, 2003; Rittle-Johnson & Alibali, 1999; McNeil & Alibali, 2005). Also reviewed was literature on learners' understanding of the concept of zero (Lipton & Spelke, 2006; Sarnecka & Carey, 2008; Bialystock & Codd, 2000). On the concept of function, various literatures were consulted (Oehrtman, et al. 2008; Bardini et al., 2014). On the concept of variable, learners' understanding of the concept was reviewed through the literature from many scholars (Akg¨un & Ozdemir, 2006; Graham & Thomas, 2000).

6.7 LIMITATIONS OF THE STUDY

This study involved learners in schools in one district of central province, namely, Chibombo District, while there are ten districts. This delimitation has limited the generalisability of the research findings. Delimiting the research area to three only out of the seven educational zones further decreased the generalisability of the research findings. An educational zone is a group of both secondary and primary schools in a certain geographic area who meet on regular basis to share ideas and experiences on how best they can improve on the provision of quality education through what they have called school based continuing professional development. It would have been ideal if the study was extended to more zones. A sample from five schools only may not be fully representative of the perceptual experience of all learners in central Province and Zambia as a whole. In spite of these limitations, the researcher believes that the findings of this study will contribute to solving the learners' problems in understanding some key mathematical concepts.

6.8 RECOMMENDATIONS

The researcher suggests the following recommendations:

6.8.1 Recommendations for further research

- A comprehensive research study using both quantitative and qualitative approaches should be conducted in another district in order to compare the findings and see if there is any correlation.
- Future research to target another category of senior secondary school learners' understanding of these concepts.

6.8.2 Recommendations to MOGE

- Mathematics as a subject should be promoted in schools so that learners can be motivated to understand all the mathematical concepts.
- Teachers should emphasise during Mathematics discourse on the application of the learnt concepts to solve problems as a way of inculcating the spirit of innovativeness in learners.
- Teachers should prepare tests items, which help to determine what learners understand about the underlying concept.

- Teachers should make sure learners understand the meanings and correct applications of the concepts and not just assumptions.

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8. APPENDICES

APPENDIX A: ETHICS APPROVAL

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2018/02/14

Ref: 2018/02/14/50881892/42/MC
Name: Mr C Choonya
Student: 50881892

Dear Mr Choonya

Decision: Ethics Approval from 2018/21/14 to 2021/02/14

Researcher(s): Name: Mr C Choonya
E-mail address: choonyac@yahoo.com
Telephone: +264 61 435 4099

Supervisor(s): Name: Prof WWJ Mwakapenda
E-mail address: mwakapendawwj@tut.ac.za
Telephone: +27 82 443 9243

Title of research:

Grade 10 learners' understanding of key mathematical concepts in selected secondary schools in Chibombo District of Zambia

Qualification: M Ed in Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2018/02/14 to 2021/02/14.

*The **Medium risk** application was reviewed by the Ethics Review Committee on 2018/02/14 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.
2. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

3. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
4. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
5. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
6. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
7. No field work activities may continue after the expiry date **2021/02/14**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2018/02/14/50881892/42/MC** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

M Claassens

Dr M Claassens
CHAIRPERSON: CEDU RERC
mcdtc@netactive.co.za

Prof V McKay
EXECUTIVE DEAN
Mckayvi@unisa.ac.za

Approved - decision template – updated 16 Feb 2017

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APPENDIX B: PARTICIPANT INFORMATION SHEET

KAFUSHI SECONDARY SCHOOL
P.O. BOX 80849
KABWE.
JANUARY 2017

Title:" **GRADE 10 LEARNERS' UNDERSTANDING OF KEY MATHEMATICS CONCEPTS IN SELECTED SECONDARY SCHOOLS IN CHIBOMBO DISTRICT OF ZAMBIA"**

Dear Prospective Participant

My name is Choonya Caesar and I am doing research with Professor Willy Mwakapenda, a professor in the Department of Mathematics Education, towards an M Ed. Degree at the University of South Africa. We invite you to participate in a study entitled 'Grade 10 learners' understanding of key Mathematics concepts in selected schools in Chibombo of Zambia'.

WHAT IS THE PURPOSE OF THE STUDY?

The purpose of this research is to find out Grade 10 learners' understanding of key Mathematics concepts in selected schools of Chibombo District.

WHY AM I BEING INVITED TO PARTICIPATE?

Your name was recommended by your teacher of Mathematics to be part of this group of other grade 10 learners in Chibombo District participating in this study.

WHAT IS THE NATURE OF MY PARTICIPATION IN THIS STUDY?

In this study, you will be invited to complete a short questionnaire on Mathematics concepts from your syllabus. The study also involves a face-to-face interview with the researcher who will be the interviewer and you as interviewee. In the interview, there will be a group focus discussion in a group of four grade 10 learners with the researcher. The information to be shared during focus group discussion will be treated as confidential by all the participants. No participant will be allowed to disclose any information to any unauthorized person. The face-to-face interview will be expected to last for not more than 20 minutes while the questionnaire will take no more than 30 minutes to complete.

CAN I WITHDRAW FROM THIS STUDY EVEN AFTER HAVING AGREED TO PARTICIPATE? Participation is voluntary and that there is no penalty or loss of benefit for non-participation. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written assent form.

WHAT ARE THE POTENTIAL BENEFITS OF TAKING PART IN THIS STUDY?

The benefit of participating in this study is that you will help assist the education authority to come up with ways to help improve the education system.

ARE THERE ANY NEGATIVE CONSEQUENCES FOR ME IF I PARTICIPATE IN THE RESEARCH PROJECT?

There are no foreseeable risks of harm or side-effects to you as a participant as the questions are not sensitive.

WILL THE INFORMATION THAT I CONVEY TO THE RESEARCHER AND MY IDENTITY BE KEPT CONFIDENTIAL?

The information the participant will convey to the researcher is going to be treated as confidential and will be stored in a computer with a strong password. However a report of the study may be submitted for publication, but individual participants will not be identifiable in such a report.

HOW WILL THE RESEARCHER PROTECT THE SECURITY OF DATA?

Hard copies of your answers will be stored by the researcher for a period of five years in a locked filing cabinet in the researcher’s house for future research or academic purposes; electronic information will be stored on a password protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. After five years, hard copies will be shredded and electronic copies will be permanently deleted from the hard drive of the computer.

WILL I RECEIVE PAYMENT OR ANY INCENTIVES FOR PARTICIPATING IN THIS STUDY?

There is no payment for participating in this study as it is purely academic.

HAS THE STUDY RECEIVED ETHICS APPROVAL

This study has received written approval from the Research Ethics Review Committee of the *CEDU*, Unisa. A copy of the approval letter can be obtained from the researcher if you so wish.

HOW WILL I BE INFORMED OF THE FINDINGS/RESULTS OF THE RESEARCH?

If you would like to be informed of the final research findings, please contact Mr. Choonya Caesar on cell number 0977 631438 or email choonyac@yahoo.com. The findings are accessible for a period of 5 years.

Should you have concerns about the way in which the research has been conducted, you may contact Professor Mwakapenda on cell number +27824439243 or email MwakapendaWWJ@tut.ac.za.

Thank you for taking time to read this information sheet and for participating in this study.

Thank you.

....

Choonya Caesar

APPENDIX C: REQUEST TO CONDUCT RESEARCH

THE DISTRICT EDUCATION BOARD SECRETARY

CHIBOMBO DISTRICT

P.O. BOX 80246

KABWE

JANUARY 2018

U.F.S

THE HEAD TEACHER

………………………..SECONDARY SCHOOL

KABWE

Dear Sir/Madam,

RE: **REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT SECONDARY SCHOOLS IN CHIBOMBO DISTRICT.**

I, Choonya Caesar am doing research with Professor Mwakapenda Willy, a Professor in the Department of Mathematics Education, towards a M Ed at the University of South Africa. We are inviting you to participate in a study entitled ‘’ **GRADE 10 LEARNERS' UNDERSTANDING OF KEY MATHEMATICS CONCEPTS IN SELECTED SECONDARY SCHOOLS IN CHIBOMBO DISTRICT OF ZAMBIA".** The aim of the study is to find out the grade 10 learners understanding of key Mathematics concepts. Chibombo has been selected because the nature of the diversity of the learners in the district. The study will entail asking the grade 10 learners on their understanding of selected Mathematics concepts. This will be done through a questionnaire and a face-to-face in-depth group interview discussion. A total of 250 learners are expected to take part in this study. Five secondary schools are targeted in this study. The group interview will involve four learners from each of the five schools.

The benefits of this study are to come up with some interventions in order to improve the performance in Mathematics. There actually no foreseeable potential risks in this exercise.

You will be welcome to request a summary of findings from the research. The feedback procedure will entail making request for those who would want to have the results.

Yours sincerely

....

Choonya Caesar

Researcher

APPENDIX D: PARENTAL CONSENT FOR MINOR

KAFUSHI SECONDARY SCHOOL

P.O. BOX 80849

KABWE

RE: A LETTER REQUESTING PARENTAL CONSENT FOR MINORS TO PARTICIPATE IN A RESEARCH PROJECT

Dear Parent

Your son/daughter/child is invited to participate in a study entitled ‘’ **GRADE 10 LEARNERS' UNDERSTANDING OF KEY MATHEMATICS CONCEPTS: STUDY OF SELECTED SECONDARY SCHOOLS IN CHIBOMBO DISTRICT OF ZAMBIA”**. I am undertaking this study as part of my Master's Research at the University of South Africa. The purpose of the study is to find out grade 10 learners understanding of key Mathematics concepts, and the possible benefits of the study is the improvement of performance in Mathematics. I am asking permission to include your child in this study because he/she has been recommended by his/her teacher. I expect to have 249 other children participating in the study. If you allow your child to participate, I shall request him/her to take part in a survey.

Any information that is obtained in connection with this study and can be identified with your child will remain confidential and will only be disclosed with your permission. His or her responses will not be linked to his or her name or your name or the school's name in any written or verbal report based on this study. Such a report will be used for research purposes only.

There are no foreseeable risks to your child by participating in the study. Your child will receive no direct benefit from participating in the study; however, the possible benefits to education are as indicated above. Neither your child nor you will receive any type of payment for participating in this study.

Your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusal to participate will not affect him/her in any way. Similarly you can agree to allow your child to be in the study now and change your mind later without any penalty.

The study will take place during regular classroom activities with the prior approval of the school and your child's teacher. In addition to your permission, your child must agree to participate in the study and you and your child will also be asked to sign the assent form which accompanies this letter. If your child does not wish to participate in the study, he or she will not be included and there will be no penalty. The information gathered from the study and your child's participation in the study will be stored securely on a password locked computer in my locked office for five years after the study. Thereafter, records will be erased.

If you have questions about this study please ask me or my study supervisor, Prof. Mwakapenda W.W.J, Department of Mathematics Education, College of Education, University of South Africa. My contact number is +260 977 631438 and my e-mail is choonyac@yahoo.com. The e-mail of my supervisor is MwakapendaWWJ@tut.ac.za. Permission for the study has already been given by REC and the Ethics Committee of the College of Education, UNISA.

You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. You may keep a copy of this letter.

Name of child:

Sincerely

______________________	______________________	______________
Parent/guardian's name (print)	Parent/guardian's signature:	Date:
______________________	______________________	______________
Researcher's name (print)	Researcher's signature	Date:

APPENDIX E: ASSENT FROM LEARNERS TO PARTICIPATE IN A RESEARCH PROJECT

KAFUSHI SECONDARY SCHOOL

P.O. BOX 80849

KABWE.

Dear Prospective Participant,

RE: REQUESTING ASSENT FROM LEARNERS IN A *SECONDARY SCHOOL* TO PARTICIPATE IN A RESEARCH PROJECT.

I am doing a study on Mathematics Education as part of my studies at the University of South Africa. The name of my supervisor is Professor Mwakapenda .W. and can be contacted on cell +27824439243 or email MwakapendaWWJ@tut.ac.za. The title of my study is "**GRADE 10 LEARNERS' UNDERSTANDING OF KEY MATHEMATICAL CONCEPTS IN SELECTED SECONDARY SCHOOLS IN CHIBOMBO DISTRICT OF ZAMBIA".**

Your Head teacher has given me permission to do this study in your school. I would like to invite you to be a very special part of my study. I am doing this study so that I can find ways that your teachers can use to make you understand Mathematics better. This will help you and many other learners of your age in different schools.

This letter is to explain to you what I would like you to do. There may be some words you do not know in this letter. You may ask me or any other adult to explain any of these words that you do not know or understand. You may take a copy of this letter home to think about my invitation and talk to your parents about this before you decide if you want to be in this study.

I would like to ask you to complete a questionnaire and attend interviews which will be face to face with the researcher in a group of three other learners to discuss your views on some Mathematics concepts. The whole exercise will take no longer than 1 hour.

I will write a report on the study but I will not use your name in the report or say anything that will let other people know who you are. You do not have to be part of this study if you don't want to take part. If you choose to be in the study, you may stop taking part at any time. You may tell me if you do not wish to answer any of my questions. No one will blame or criticise you. When I am finished with my study, I shall return to your school to give a short talk about some of the helpful and interesting things I found out in my study. I shall invite you to come and listen to my talk.

If you decide to be part of my study, you will be asked to sign the form on the next page. If you have any other questions about this study, you can talk to me or you can have your parent or another adult call me at: +260 977 631438. Do not sign the form until you have all your questions answered and understand what I would like you to do.

Researcher: Mr. Choonya Caesar Phone number: +260 977 631438.

Do not sign written assent form if you have any questions. Ask your questions first and ensure that someone answers those questions.

WRITTEN ASSENT

I have read this letter which asks me to be part of a study at my school. I have understood the information about my study and I know what I will be asked to do. I am willing to be in the study.

____________________	________________________	______________________
Learner's name (print):	Learner's signature:	Date:
____________________	________________________	______________________
Witness's name (print)	Witness's signature	Date:

(The witness is over 18 years old and present when signed.)

________________________	______________________	_________________
Parent/guardian's name (print)	Parent/guardian's signature:	Date:

____________________	________________________	______________________
Researcher's name (print)	Researcher's signature:	Date:

APPENDIX F: MATHEMATICS LEARNER'S QUESTIONNAIRE

Dear learner,

My name is Choonya Caesar, a student of Master of Education in Mathematics at the University of South Africa (UNISA) and my supervisor is Professor Mwakapenda .W.W.J. on +27824439243. Your teacher thought it would be a good idea for me to interact with you through the set of questions in the questionnaire so that we can assess your understanding of Mathematics concepts.

I would like to ask you some questions about your learning of Mathematics as a subject in school and your understanding of some Mathematics concepts. I hope to use this information to help both teachers and learners improve in teaching and learning of Mathematics in schools.

The exercise should take about 30 minutes.

MATHEMATICS LEARNER'S QUESTIONNAIRE

Name of School________________________Grade 10_____

Please put a tick (√) in the correct box against your gender.

Gender: [ ] Male [ ] Female

1. The table below has **three** columns;

Column 1 has a list of Mathematics concepts as they are used in your syllabus.

In column 2, you are requested to state the mathematical meaning of the concept.

In column 3, you are requested to show the application of the concept in Mathematics.

CONCEPT	**MY UNDERSTANDING OF THE CONCEPT**	**APPLICATION OF THE CONCEPT IN MATHEMATICS**
zero		
Equal		
Function		
Variable		

End

Thank you for your co-operation.

APPENDIX G: STRUCTURED FOCUS GROUP DISCUSSION QUESTIONS

Dear learner,

My name is Choonya Caesar, a student of Master of Education in Mathematics at the University of South Africa (UNISA) and my supervisor is Professor W.W.J. Mwakapenda, contact +27824439243. Your teacher thought it would be a good idea for me to interact with you through the set of questions in the questionnaire so that we can assess your understanding of Mathematics concepts.

I would like to ask you some questions about your understanding of the Mathematics concepts on the questionnaire you were given. The aim of this interview is to follow up on the responses you provided in the questionnaire. I hope to use this information to help me understand more about how you think about these concepts. It will also be useful to both teachers and learners towards improving the teaching and learning of Mathematics in schools.

This interview should take about 20 minutes and will involve your thinking and understanding of the concepts zero, equal, function, and variable.

Your responses to this interview will be kept confidential and will not be used against your performance in class. You are requested to respond as freely and honestly as possible.

1. Concerning the concept zero:

a) Comment on your general understanding of this concept. What is the meaning of the concept zero? Please give some examples to show what you mean.

b) In what area/situation of Mathematics is the concept of zero mostly used? Provide other examples in Mathematics in which this concept is applicable.

c) Are there any cases in Mathematics where this concept is not applicable? Explain why.

2. Concerning the concept equal:

a) Comment on your general understanding of this concept. What is the meaning of the concept equal? Please give some examples to show what you mean.

b) In what areas/situations of Mathematics is the concept of equal mostly used? Provide other examples in Mathematics in which this concept is applicable.

c) Are there any situations in Mathematics where this concept is not applicable? Explain why.

3. Concerning the concept function:

a) Comment on your general understanding of this concept. What is the meaning of the concept function? Please give some examples to show what you mean.

b) In what areas/situations of Mathematics is the concept of function mostly used? Provide other examples in Mathematics in which this concept is applicable.

c) Are there any situations in Mathematics where this concept is not applicable? Explain why.

4. Concerning the concept variable:

a) Comment on your general understanding of this concept. What is the meaning of the concept variable? Please give some examples to show what you mean.

b) In what areas/situations of Mathematics is the concept of variable mostly used? Provide other examples in Mathematics in which this concept is applicable.

c) Are there any situations in Mathematics where this concept is not applicable? Explain why.

5. Any other comments you would like to make regarding your understanding of each of these concepts?

End.

Thank you for your cooperation.

APPENDIX H: LEARNERS RESPONSES TO THE FOCUS GROUP DISCUSSION

GRADE 10 LEARNERS' RESPONSES TO THE GROUP INTERVIEW - SCHOOL 1

a) **mathematical concept of zero**

Question 1

What is your general understanding of the concept of zero in Mathematics?

Learner L4S1: Means the eeee zero in Mathematics refers to something or number which does not exceed zero or in some other subjects, it can refer to being nullified or not valid.

Interviewer: which subject are you referring?

Learner L4S1: like civic education under the topic elections where some votes are not counted because they were incorrectly marked.

Learner L13S1: zero is nothing

Interviewer: what do you mean when you say "zero is nothing?"

Learner L13S1: No response

Learner L19S1: means nothing because it cannot be used in anything. When we want to add up numbers and only have zeros, it means the answer is zero again. Therefore we say zero is nothing.

Learner L23S1: No response.

Question 2

In what areas/situation of Mathematics is the concept of zero mostly used?

Learner L4S1: we use the mathematical concept of zero in integers. Zero is used to separate the negative numbers from the positive numbers and is neither positive nor negative.

Learner L13S1: In Mathematics, we use zero especially in situations to do with money in commercial arithmetic. When we are dealing with money especially counting money if you are writing K700 and you leave out a zero then you will have K70. So zero must be recognized.

Interviewer: it must be recognized as what?

Learner L13S1: it must be recognized as a number which is important in that position because if we remove it, the meaning will change.

Learner L19S1: It is used in all topics in Mathematics because there is no topic which we can learn from the beginning up to the end without using the concept of zero.

Learner L13S1: it is used when given a question to identify the significant figures. For example 1002 the two zeros are recognized to be significant but when you are given 1 with zeros, the other zeros are not significant. in 0.100 we recognize the zeros after the point as significant.

Learner L23S1: zero is used when you want to plot a graph

Interviewer: what kind of graph are those?

Learner L23S1: like half life graphs in physics, travel graphs and XOY plane in Mathematics.

Question 3

In what areas/situations in Mathematics is zero not applicable?

Learner L13S1: zero is not used in the newly introduced topic in Mathematics called computer. This is because the topic is based on how to use computer and its programs.

Interviewer: what about the zero found on the keyboard? Why should it be there when it is not used?

Learner L13S1: no response

Learner L13S1: zero is used in computers like if you want to write a setcode, zero error will be displayed.

Interviewer: what is a set code and what is the meaning of zero on the set code?

Learner L13S1: a set code is a computer program

Learner L13S1: zero is not used in algebra, as we just use letters.

Interviewer: why don't we use zero in algebra?

Learner L13S1: because when we multiply zero with any letter the answer is zero like $0 \times x = 0$ but when we multiply 1 and x, the result is x. As a result, zero is not used in algebra.

b) mathematical concept of equal

What is your general understanding of the concept of equal in Mathematics?

Learner L19S1:It is a mathematical symbol that is used when you multiply or add numbers to show the answer.

Learner L23S1: no response

Learner L13S1: equal is a situation where you equate something which is on the right hand side and left hand side.

Interviewer: What do you mean with what you said?

Learner L13S1: What is on the left hand side should equal to what is on the other side.

Interviewer: Are there words you can use to mean equal?

Learner L13S1: Words like same, equivalent, parallel are almost the same as equal.

Learner L4S1: Paused for 2 minutes. No response

In what areas/situation of Mathematics is the concept of equal mostly used?

Learner L19S1: equal symbol is mostly used in equations. For example, if you are given $2x + 3y = 5$, the equal sign has to be there. If it is missing, then it will not be a question but a statement.

Interviewer: What do you mean to say it won't be a question but a statement?

Learner L19S1: when it has no equal sign, then we will not solve it to find an answer. but when it has equal symbol, then we are supposed to solve because it becomes a question.

Learner L4S1: it is also used in writing money like if you want to write K12, 230=00. So we use the equal sign in that.

Interviewer: why do we write money using the equal sign like that?

Learner L4S1: to show the ngwee and separate it from the kwacha.

Learner L13S1: no response

Learner L23S1: no response

In what areas/situation of Mathematics is the concept of equal not applicable?

Learner L4S1: it is not used in construction as we only deal with angles and sides.

Learner L13S1: it is also not used in inequations as we only use symbols like greater than and less than.

c) mathematical concept of function

What is your general understanding of the concept of function in Mathematics?

Learner L23S1: it is defined as the mapping of an object to another object e.g. mapping of mangoes to apples. It must show the arrows to show the relationship.

Learner L4S1: no response

Learner L13S1: no response

Learner L19S1: no response

In what areas/situation of Mathematics is the concept of function mostly used?

Learner L23S1: it is mostly used in calculus to represent the function like $f(x) = 2$.

Interviewer: give another example of a function used in calculus?

Learner L23S1: eeeeeh no response.

Learner L4S1: in sets if they say $f(x) = 5x$ and x is a prime number less than 10, and then asked to find the other set by replacing x with numbers

In what areas/situations of Mathematics is the concept of function not applicable?

Learner L4S1: in circle theorem because we just use angles, diameter and chords.

Learner L23S1: eeeeeh no response.

Learner L23S1: to show that something works or what it does or is able to do.

Learner L13S1: no response

Learner L19S1: no response

d) **mathematical concept of variable**

What is your general understanding of the concept of variable in Mathematics?

Learner L23S1: no response

Learner L19S1: in Mathematics, we have terms which we use like coefficient and variable like in 5y. in 5y, 5 is a variable and y is a coefficient.

Interviewer: what is a variable then?

Learner L19S1: sir, a variable is a number in simple terms

Learner L13S1: no response.

In what areas/situation of Mathematics is the concept of variable mostly used?

Learner L4S1: it is used in simultaneous equations like $2x+y =5$ where we are asked to find value of two variables at the same time.

Interviewer: how many equations are supposed to be solve together to find the values of two variables?

Learner L4S1: I have just stated one but they are supposed to be two.

Learner L19S1: it is also used in inequalities like 250x+350y=1000. It is used in calculus like where you are asked to differentiate $3x^2 + 2x$ with respect to x.

In what areas/situation of Mathematics is the concept of variable is not applicable?

Learner L23S1: no response

Learner L19S1: it is not applicable in fractions and percentages as only numbers are used.

Interviewer: any other comment as we end our discussion

Learner L13S1: sir zero is difficult to understand. We use it without understanding what it is.

Interviewer: what makes these concepts difficult to understand?

Learner L13S1: they are not defined and properly explained to us by our teachers when they are introduced to us.

Learner L19S1: teachers should try to make us learners understand these concepts you have asked us on today so that we are able to understand them properly. Otherwise we just use them because they are already in a question asked.

Interviewer: This is the reason you were asked to participate in the research so that we can come up with a way to help you understand.

Interviewer: do you then think your understanding of these concepts and many others can help you learn Mathematics and improve your performance?

Learner L13S1: yes.

Interviewer: explain how it can be that way.

Learner L13S1: hmmm

Learner L19S1: they help us understand Mathematics in general if you do not know how to differentiate a variable and coefficient you end up confusing yourself. It is important to understand these concepts.

Interviewer: Thank you boys and girls for your contributions during this discussion.

GRADE 10 LEARNERS' RESPONSES TO THE GROUP INTERVIEW - SCHOOL 2

a) mathematical concept of zero

Question 1

What is your general understanding of the concept of zero in Mathematics?

L16S2: Zero is a number.... it is a number or any number..... paused for a minute...begining zero to positive infinity.

L7S2: it is a whole number greater than negative numbers but less than 1.

L29S2: Zero is eeeeeh paused for a minute ..... I think zero is .... paused. I think that's what I can say.

L37S2: Zero is..... zero is..... paused. No answer sir.

In what areas/situations of Mathematics is the concept zero mostly used?

L7S2: mmmmmmmm. Concept zero.... laughs. It is mostly used in functions.

Interviewer: give an example of what you are trying to say.

L7S2: Like in cubic functions. There are so many functions where zero can be used except I cannot mention all of them.

Interviewer: Someone to give an example.

L29S2: Like in $x^2 + y = 0$ zero is used.

L16S2: In quadratic functions where the equation is equated to zero e.g. $x^2 + 5x - 2 = 0$ which is a quadratic equation.

Interviewer: Any other area where zero is mostly used

Silence for about 3 minutes

L37S2: In matrices such as $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Interviewer: What is the matrix supposed to mean?

Silence for some time with low sound consultations among the learners.

In what areas/situations of Mathematics is the concept zero not applicable?

L7S2: In trigonometry

Interviewer: Give an example

L7S2: there are equations where you are asked to equate zero to something like y=0 or to something like $sin60 = \frac{sinB}{18}$

Interviewer: are you sure you haven't used zero there?

L7S2: Yes

Interviewer: ok

L16S2: When you are asked to calculate quadratic expressions because when we are told to calculate an expression, we do not find the answer there we find the expression for example$(2x+1)(x+4)$ do not use zero

L37S2: Also in factorizing, when factorizing we do not use zero like when you are told to calculate e.g. $x^2+24x+1$. you will not use zero.

L29S2: Like what he said, when we are told to factorise we do not use zero because we are factoring out what is common and zero will not be common.

b) mathematical concept of equal

L37S2: Equal is a symbol which shows that anything you are calculating is equal e.g I might equate 1 to 0 or y to 0 which means that y and 0 have the same value.

L16S2: Both sides are equilibrium and both sides are balanced, what is on the left hand side is equal to what is on the right hand side.

Interviewer: what about others, what is your understanding of equal?

L7S2: Silent for some minutes

L29S2: the same mmmmmmmm. No answer.

In what areas/situations is the concept of equal mostly used?

L37S2: in most cases in Mathematics, we use equal sign to denote the final answer.

L7S2: Silence ...... sometimes we use equal sign when we are told to calculate an angle, you are told to calculate a certain side may be there are cases where we have sides of a triangle like AB = 12cm, BC = 13cm and AC = 13cm so we use equal sign to show that side BC and side AC are same.

L16S2: And we can also use equal when we want to mean the variable is the subject of the formula by using the equal sign.

Interviewer: Give a particular example of what you are talking about?

L16S2: For example if you are solving this calculation $x+2y \geq 90$ in this case if I want to make x the subject of the formula I say $2y \geq 90-x \; or \; 2y=90-x$.

In what areas/situations is the concept of equal not applicable?

L37S2: When calculating inequalities

Interviewer: why don’t we use equal sign in inequalities?

L37S2: because it deals with values which are less or greater than..... silence.

L7S2: Also in ratio when you are expressing in ratios you cannot use equal sign. This is because ratios deal with comparison between two or more similar quantities so we cannot use equal symbols as they are written in fraction term.

c) mathematical concept of function

What is your general understanding of the concept of function in Mathematics?

L16S2: A function is eeeeeh... a function ... it means that the first variable mapps onto something.

Interviewer: what are the views of others?

L7S2: mmmmmmm. Silence..... I am still thinking.

Interviewer: how much time should I give you?

L29S2: ummmm sir this concept is difficult to explain or define.

Interviewer: have you ever used/heard of the concept of function in Mathematics?

L37S2: no answer sir, I do not understand the meaning of function but have heard of it.

Interviewer: Ok.

In what areas/situations is the concept of functions used in Mathematics?

Silence for about 4 minutes

L16S2: When we want to calculate the inverse for example you are given like f(x) $=\frac{2x-1}{5}$ if you want to find the inverse of that function.

L7S2: No answer sir...

L37S2: No answer sir..

In what areas/situations is the concept of functions not applicable in Mathematics?

L7S2: Like in ratios.... functions are not applicable. Pause or a minute... Even in trigonometry because in trigonometry, we use sine, cosine and tangent. So there is no need of using functions.

Interviewer: Why are functions not applicable in trigonometry?

L7S2: mmm because it involves use of sides, angles and eeeeh

a) mathematical concept of variable

What is your general understanding of the concept of variable in Mathematics?

L37S2 Silence...... any letter that is used to denote something like x e.g. if given equation $y = 2x + 1$ the equation has two variable which are y and x variable

L16S2; no answer

L7S2: no answer sir

In what areas/situation of Mathematics is the concept variable mostly used?

L16S2: They are mostly used in simultaneous equations when we told to find the values.

L37S: Even in algebraic expressions, variables are used .... mostly x variables is the most used.

Interviewer: why is x the mostly used variable?

Silence for about 2 minutes

L7S2: variables are even used in the topic variation

Interviewer: Give an example

L37S2: y varies inversely as a square of z.

In what areas/situation of Mathematics is the concept variable not applicable?

L16S2: In computer programming and earth geometry

L7S2: No response

Interviewer: Has your understanding of these concepts and many others helped you in improving your performance in Mathematics?

L7S2: Yes, example the concept of equal has helped improve in Mathematics by knowing where to put the answer.

GRADE 10 LEARNERS' RESPONSES TO THE GROUP INTERVIEW - SCHOOL 3

a) **mathematical concept of zero**

Question 1

What is your general understanding of the concept of zero in Mathematics?

L4S3: Zero is nothing..... it means nothing.

L19S3: Zero is a neutral number which is neither negative nor positive because it is at the centre of negative and positive numbers.

L11S3: Zero is arbitrary nothing

L28S3: It means nothing or it's a number with no value.

In what areas/situations is the concept of zero mostly used?

L19S3: it is mostly used in drawing the Cartesian plane or number line.

L4S3: It is used in counting as we start from zero then 1....... . it is a natural number.

L11S3: silent for about 2 minutes.... no idea sir.

L28S3: The same sir I have no idea.

In what areas/situation in Mathematics is the concept of zero not applicable?

L28S3: Zero is not applicable in money. There is no amount written or used as zero.

Interviewer: why don't we use zero in money?

L28S3: Because zero means nothing so you cannot say I have zero kwacha.

Interviewer: what is the view of others on where the concept of zero is not applicable?

L11S3: Ahhh.. hmmmm... no idea sir.

L4S3: Even me I have no idea

L19S3: No idea sir

b) The meaning of the mathematical concept of equal

L28S3: Equal means .... paused.... an example may be in class the number of girls is equal to the number of boys.

L11S3: Equal means same number which means that same number of girls and boys is the same number.

L19S3: like what my friends have said, it means same numbers.

Interviewer: what about the other, what can you say about equal concept in Mathematics?

L4S3: No idea sir

In what areas/situation is the concept of equal mostly used?

L11S3: It is mostly used in sets like A = {a, b, c, d} and B = {1, 2, 3, 4} so set A is equal to set B because they have the same number of elements.

L28S3: In most equations like if you want to find the answer like $x = \frac{x_1}{x_2}$.

L4S3: No idea

L19S3: No idea as well

In what areas/situations is the concept of equal not applicable?

Silence for 3 minutes

L28S3: Maybe when you are dealing with negative and positive numbers

Interviewer: why don't we use equal symbol in such situations

L28S3: Because negative numbers are not equal to positive numbers as negative numbers are less than positive numbers.

c) The meaning of the mathematical concept of function

Silence for 3 minutes

L11S3: function ahhhh like in Mathematics we say the function of for example 2x+1, the whole thing is difficult to explain.

L28S3: mmmmm no idea sir

L19S3: Equally no idea

L4S3: I don't know sir

In what areas/situation is the concept of function mostly used?

Silence for about 5 minutes

In what areas/situation is the concept of function not applicable?

Silence for about 4 minutes.

d) The meaning of the mathematical concept of variable

L19S3: May be it's a number

L28S3: A variable is anything that may carry a power

Interviewer: give an example

L19S3: Like 2^3 and 2 is a variable.

In what areas/situation is the concept of variable mostly used?

L19S3: In indices

Interviewer: how is it used in indices?

L28S3: When raising a number to a power.

In what areas/situation is the concept of variable not applicable?

L19S3: it is not applicable when the power is negative.

Interviewer: why is it not applicable when power is negative?

No response for about 2 minutes

Interviewer: what do others have on the concept?

L19S3: On negative numbers

Interviewer: why is like that?

L28S3: Because if you have 2^{-1} the answer you will have will be $\frac{1}{2}$ then that cannot be a variable.

GRADE 10 LEARNERS' RESPONSES TO THE GROUP INTERVIEW - SCHOOL 4

a) mathematical concept of zero

Question 1

What is your general understanding of the concept of zero in Mathematics?

L13S4: Concept zero...... it's a non integer number used in Mathematics to solve problems.

Interviewer: give an example of a mathematical problem where zero is used.

L13S4: Silence for about 2 minutes... let me think give me time.

L6S4: it is neutral as it can either be positive or negative

L10S4: It forms part of integers. It separates positive numbers from the negative numbers.

L16S4: Zero is something like eeeeeeeeeeeh nothing if we can write 1 and 0 that means 10 but sometimes it can be 01 which means writing just 1.

In what areas/situations in Mathematics is the concept of zero mostly used?

L16S4: Almost everywhere as it forms large part of the problems we solve in Mathematics.

Interviewer: give specific examples of mathematical problems where zero is mostly used

L16S4: there are so many and cannot list all.

L6S4: Plays a major role like if you are given a function and asked to find that function you have to equate the function to zero in order to solve it hence zero plays a major role

L13S4: it is also used in equations, matrix like $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ etc.

L10S4: Yes like the use of number line, we use zero to separate the negative from the positive numbers. it is also used in vectors as position vectors like OP, OM etc.

In what areas/situations in Mathematics is the concept of zero not applicable?

L16S4: Solving questions that have to do with infinity, zero cannot be used.

Interviewer: why can't it be used?

L16S4: because infinity deals with large numbers. it is also not used in fractions because the denominator does not need to be zero.

Interviewer: what if denominator is zero?

L16S4: It means the fraction will be undefined.

b) Meaning of the mathematical concept of equal

Question 1

What is your general understanding of the concept of equal in Mathematics?

L6S4: When things are equal it means they have to give the same output.

Interviewer: what do you mean with same output?

L6S4: means the same value

L16S4: Equal was derived from equality like when you are given something meaning both sides must be the same.

L10S4: used to represent two or more objects which are the same. It can be used to indicate that two things or more if brought together are equal.

L13S4: Means two or more things are equal like if you get a k10 and a book like this one we can say the book and the k10 are equal because a book was bought at k10.

In what areas/situations in Mathematics is the concept of equal mostly used?

L13S4: When dealing with equations

Interviewer: Give an example

L13S4: When you are given an equation and you are told to find the value of y like 3+y=10

L10S4: In inequalities, linear programming

Interviewer: how is the concept of equal used in linear programming?

L10S4: When you want to shade the unwanted region or wanted region to demarcate the region to be shaded.

L16S4: used to satisfy an equation....... silent for 3 minutes.

In what areas/situations in Mathematics is the concept of equal not applicable?

L6S4: in integers. The negative numbers cannot be equal to positive numbers, which is impossible.

L10S4: No response

L13S4: mmmmmmmm no response

L16S4: I have no answer.

c) Meaning of the mathematical concept of function

L13S4: It is a relation between two objects or a linkage between two objects.

L10S4: Something given between two things

L6S4: A relation or mapping to connect object to images

L16S4: Mmmmmmmm no answer sir.

In what areas/situation in Mathematics is the concept of function applicable?

L10S4: In the topic functions itself.

L6S4: if given the objects and a function, we can use the function and object to find image.

L13S4: It can be applied in naming expressions like $f(x) = 2x^3 + 4 \;\; or \; g(x) = x^2 + 3x + 1$.

L16S4: No response.

In what areas/situation in Mathematics is the concept of function not applicable?

Silence for 2 minutes

L6S4: Topics like similar triangle and construction.

L10S4: No response

L16S4: No response

L13S4: No response

d) Meaning of the mathematical concept of variable

L13S4: A letter which represents a number like in equation $x + y = 10, x \; and \; y$ are variables which represents members which can make the equation balance.

L10S4: No response

L16S4: No response

L6S4: No response

In what areas/situations is the concept of variable mostly used?

L13S4: it is used in a topic called variations and systems of equations.

Interviewer: give an example of what you have said

L13S4: Like x varies directly as square of y or in simultaneous equations

$2x + 3y = 6$ and $x - 4y = 1$.

L6S4: Every part of Mathematics uses variables in one way or the other.

In what areas/situations is the concept of variable not applicable?

L10S4: Of course zero. When dealing with zero, variables concept is not used because everything becomes zero.

L16S4: No response

L6S4: No response

L13S4: No response

GRADE 10 LEARNERS' RESPONSES TO THE GROUP DISCUSSION - SCHOOL 5

a) **mathematical concept of zero**

Question 1

What is your general understanding of the concept of zero in Mathematics?

L1S5: What zero is ...... zero is I consider it to be one of the numbers....

L10S5: Aaaaa if you have been asked to chose or permute the number

L22S5: Zero is a number in between the ascending and descending of negative and positive.

L28S5: Zero is a number greater than positive numbers and less than negative numbers

L1S5: Zero is also a number at rest or origin.

In what areas/situation is the concept of zero mostly used?

L28S5: Zero is mostly used in drawing graphs like the travel graphs when you are told that a body moves from rest to some velocity. When it is at rest then it is at zero.

L22S5: on inequalities to express the range of numbers in a given set.

L10S5: it is also used on the number line to show the direction of negative and positive numbers.

In what areas/situations is the concept of zero not applicable?

Silence for 2 minutes

L1S5: In reducing fractions to lowest terms it should use zero especially in the denominator.

Interviewer: why shouldn't zero be used in the denominator?

L1S5: It will be undefined.

b) Meaning of the mathematical concept of equal

L22S5: Equality of two things .e.g 5 chairs are equal to 5 desks.

Silence for 3 minutes.

L10S5: No response.

In what areas/situations is the concept of equal mostly used?

L28S5: It is mostly used in equations like $5x + 2 = 9$ or simultaneous equations.

L1S5: Mostly used in situations where you are to give an answer to a mathematical problem.

L10S5: In simultaneous equations, equations, sets- mostly in all Mathematics topics.

L22S5: It shows that the steps you have made in your calculations are not independent but dependent on the previous step.

In what areas/situations is the concept of equal not applicable?

L10S5: In inequalities like where we write $\neq$

Interviewer: why is it not applicable?

L10S5: Like find x = + y ....silent for some time.

L22S5: When one side is greater than the other side, we cannot use equal concept.

c) Meaning of the mathematical concept of function

Silence for about 5 minutes

L10S5: It's a topic in Mathematics

L22S5: It's an equation with f(x)......silent for some time.

In what areas/situations is the concept of function mostly used?

L10S5: When learning the function itself. They are also used in graphs such as cubic graphs.

Interviewer: what is the view of others?

Silence for some time..

In what areas/situations is the concept of function not applicable?

No response from all the learners.

Meaning of the mathematical concept of variable

L1S5: is a letter which has a coefficient with it like 2x, 2 is a coefficient while x is a variable

L22S5: Variables are letters like x and y used in algebraic expressions.

Interviewer: are x and y the only letters used in expressions?

L10S5: There are many other variables used but the common ones are x and y.

Interviewer: why is it like that?

L10S5; Not sure..... may be its simple to write and recall.

Are there situations/ areas in Mathematics where the variables are mostly used?

L22S5: In equations, inequations.

Silence in almost 5 minutes.

Are there situations/ areas in Mathematics where the variables are not applicable?

L22S5: In sets, since x and y are used and are not variables but members of the set.

Silence in almost 5 minutes

APPENDIX I: LANGUAGE EDITING CERTIFICATE

EDITING AND PROOFREADING CERTIFICATE

7542 Galangal Street

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0008

21 February 2019

TO WHOM IT MAY CONCERN

This certificate serves to confirm that I have edited and proofread Mr C Choonya's dissertation entitled, **"GRADE 10 LEARNERS' UNDERSTANDING OF KEY MATHEMATICS CONCEPTS IN SELECTED SECONDARY SCHOOLS IN CHIBOMBO DISTRICT OF ZAMBIA".**

I found the work easy and intriguing to read. Much of my editing basically dealt with obstructionist technical aspects of language which could have otherwise compromised smooth reading as well as the sense of the information being conveyed. I hope that the work will be found to be of an acceptable standard. I am a member of Professional Editors' Guild.

Hereunder are my particulars:

____

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