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Preface

Philip Machanick, Overall Chair: SAICSIT'99

Running SAICSIT'99, the annual research conference of the South African Institute for Computer Scientists and Information Technologists, has been quite an experience.

SAICSIT represents Computer Science and Information Systems academics and professionals, mainly those with an interest in research. When I took over as SAICSIT president at the end of 1998, the conference had not previously been run as an international event. I decided that South African academics had enough international contacts to put together an international programme committee, and a South African conference would be of interest to the rest of the world.

I felt that we could make this transition at relatively low cost, given that we could advertise via mailing lists, and encourage electronic submission of papers (to reduce costs of redistributing papers for review).

The first prediction turned out to be correct, and we were able to put together a strong programme committee.

As a result, we had an unprecedented flood of papers: 100 submitted from 21 countries. As papers started to come in, it became apparent that we needed more reviewers. It was then that the value of the combination of old-fashioned networking (people who know people) and new-fashioned networking (the Internet) became apparent. While the Internet made it possible to convert SAICSIT into an international event at relatively low cost, the unexpected number of papers made it essential to find many additional reviewers on short notice. Without the speed of e-mail to track people down and to distribute papers for review, the review process would have taken weeks longer, and it would have been much more difficult to track down as many new reviewers in so little time.

Even so, the number of referees who were willing to help on short notice was a pleasant surprise.

The accepted papers cover an interesting range of subjects, from management-interest Information Systems, to theoretical Computer Science, with subjects including database, Java, temporal logic and implications of e-commerce for tax.

In addition, we were very fortunate in being able invite the president of the ACM, Barbara Simons as a keynote speaker. Consequently, the programme for SAICSIT'99 should be very interesting to a wide range of participants.

We were only able to find place in the proceedings for 36 papers out of the 100 submitted, of which only 24 are full research papers. While this number of papers is in line with our expectation of how many papers would be accepted in each category, we did not have a hard cut-off on the number of papers, but accepted all papers which were good enough, based on the reviews. Final selection was made by myself as Programme Chair, and Derrick Kourie, as editor of the South African Computer Journal. Additional papers are published via the conference web site.

We believe that we have put together a quality programme, and hope you will agree.

Acknowledgments

I would like to thank the South African Computer Journal production team, Andries Engelbrecht and Herna Viktor, respectively from the Department of Computer Science and Informatics, University of Pretoria, for their work on producing the proceedings.

The reviewers listed overleaf did an excellent job: many wrote very detailed reports, sometimes after being called in on very short notice. Inevitably, there were some glitches resulting from the unexpected workload, but the buck stops with the programme chair: I promise to do better next time.

I would also like to thank my own department for putting up with the extra work and expense that running a conference entails. I tried not to burden them with too much extra work, but our secretaries, Zahn Gowar and Leanne Reddy, inevitably had to take on some extra work. John Ostrowick provided valuable assistance with design of our web pages and call for papers poster. Carol Kemick, who handles our finances and membership records, did a fine job of keeping up with the demands of the conference.

Finally, I would like to thank our sponsors, whose contribution made this conference been possible:

- PricewaterhouseCoopers - sponsored generous prizes and the conference banquet
- National Research Foundation (NRF) - provided financial support
- University of the Witwatersrand - provided financial support
- Programme for Highly Dependable Systems, University of the Witwatersrand - provided financial support
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Editorial

- Apple Computer – provided equipment for the conference
- Qualica – provided technical support including helping with the conference web site

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For more information about SAICSIT, including a pointer to the conference site, see <http://www.saicsit.org.za>.

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Research Article

Some Automata-Theoretic Properties of \(\cap\)-NFA

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Abstract

We prove that the shortest word accepted by an \(n\)-state intersection selective nondeterministic automaton with nonempty language can be of length \(O(e^{\sqrt{n \log n}})\).

Keywords: Nondeterminism, automata theory
Computing Review Categories: F.1.1, F.4.3

1 Introduction

Traditional nondeterministic finite automata [6] allow for a nondeterministic choice of one particular path from the union of all possible sets of paths at a certain point. Selective nondeterministic automata, on the other hand, perform a given associative and commutative binary operation * on all possible sets of paths, and then make the nondeterministic choice from this resultant set. In this sense the traditional NFA is a selective NFA with the set operation taken as union. The succinctness properties of selective NFAs were investigated in detail in [8].

In this paper, we consider some well-known automata-theoretic properties of traditional NFAs, and investigate those properties in the case of selective nondeterministic automata, with the * operation taken as intersection (\(\cap\)-NFAs). In particular, if \(M\) is a traditional NFA with \(n\) states and \(L(M) \neq \emptyset\), then it is known that \(M\) accepts a word of length strictly less than \(n\) [3]. In this article we prove that this property does not hold for \(\cap\)-NFAs.

We give a formal definition of selective nondeterministic finite automata (or \(*\)-NFA) and \(\cap\)-NFA in section 2. In section 3 we show that it is possible to find an \(n\) state \(\cap\)-NFA which accepts a non-empty language \(L(M)\), in which the shortest word has length \(O(e^{\sqrt{n \log n}})\).

2 Definition of \(\cap\)-NFAs

The formal definition of the \(*\)-NFA is identical to the well-known definition for the traditional NFA except that the union operation is replaced by the \(*\) operation, where \(*\) is any associative commutative binary operation on sets.

Definition 1 A \(*\)-NFA \(M\) is a 6-tuple \(M = (Q, \Sigma, \delta, q_0, F, *)\), where \(Q\) is the finite non-empty set of states, \(\Sigma\) is the finite non-empty input alphabet, \(q_0 \in Q\) is the start state and \(F \subseteq Q\) is the set of final states. \(\delta\) is the transition function such that \(\delta : Q \times \Sigma \rightarrow 2^Q\), and \(*\) is any associative commutative binary operation on sets.

The transition function \(\delta\) can be extended to \(\delta : 2^Q \times \Sigma \rightarrow 2^Q\) by defining

\[\delta(A, a) = \bigcup_{q \in A} \delta(q, a)\]

for any \(a \in \Sigma\) and \(A \in 2^Q\).

The extension of \(\delta\) to \(\delta^* : 2^Q \times \Sigma^* \rightarrow 2^Q\) is straightforward.

A selective nondeterministic intersection NFA (\(\cap\)-NFA) is defined as in definition 1 above, but with \(*\) taken as intersection.

Acceptance for an \(\cap\)-NFA is defined as follows:

Definition 2 Let \(M\) be an \(\cap\)-NFA \(M = (Q, \Sigma, \delta, q_0, F, \cap)\), and let \(w\) be a word in \(\Sigma^*\). Then \(M\) accepts \(w\) iff the final state set \(F\) is contained in \(\delta^*(q_0, w)\); that is, if \(F \subseteq \delta^*(q_0, w)\).

Note that in the case where the final state \(F\) contains only one state (that is, \(F = \{q\}\) for some \(q \in Q\)), the definition of acceptance for the traditional NFA and the \(\cap\)-NFAs is equivalent.

Theorem 1 Let \(L(M)\) be a language accepted by a \(*\)-NFA \(M\). Then there exists a dfa \(M'\) that accepts \(L(M)\).

Proof: See [8].

Example 1 Let \(M\) be a traditional NFA defined by

\[M = (\{q_1, q_2, q_3\}, \{a\}, \delta, q_1, \{q_3\})\]

with \(\delta\) given by

\[
\begin{array}{c}
q_1 \quad a \\
q_1 \\
q_1, q_2, q_3
\end{array}
\]

Then the DFA equivalent to \(M\) is given by

\[
\begin{array}{c}
q_1 \\
q_1, q_2, q_3
\end{array}
\]
On the other hand, suppose that $M$ were an $\cap$-NFA. Then its equivalent DFA (with non-reachable states removed) would be given by

![Diagram](image.png)

It should be clear that the traditional NFA simply takes its nondeterministic choices from the union of its possibilities, while the $\cap$-NFA prunes its possibilities using the corresponding set operation.

The interested reader may note at this point an analogy between boolean automata [5] (or alternating automata [11]) and $\cap$-NFAs. Boolean automata allow any combination of boolean operators in their transition function, and in that sense is more general than the $\cap$-NFAs proposed above. Indeed, it can be shown that there are boolean automata with $n$ states for which the corresponding minimal DFA has $O(2^n)$ states [5]. We show in [8] that boolean automata correspond to $\cap$-NFAs with a transition function from the set of states to the powerset of the powerset of states (called $\cap$-SNFAs).

It can be shown (see [8]) that an $n$-state boolean automaton with no negation and only the OR operation in the boolean formulae of the transition function can be simulated by an $n$-state $\cup$-NFA, and vice versa. The $\cap$-NFA has no such direct translation into a one operation boolean automaton which preserves the number of states.

3 The Shortest Word Accepted by an $n$-state Unary $\cap$-NFA

It is trivial to construct an example of an $n$-state unary $\cap$-NFA which accepts a non-empty language such that the shortest word in this language has length greater or equal to $n$: the shortest word accepted by the $\cap$-NFA in example 1 has length three.

The interesting question is, how long could such a shortest word could be in the general case? We first prove that it is possible to construct a unary $\cap$-NFA $M$ with an equivalent minimal DFA with $O(e^{\sqrt{n\log n}})$ states. Using this construction, we show that it is possible to choose a final state set for $M$ such that the shortest word accepted by $M$ is of length $O(e^{\sqrt{n\log n}})$.

We use $n$-state $\cap$-NFAs with a special form of the transition function, namely, for every $q_i \in Q$ we assume that $\delta(q_i, a)$ is one of those $n$ subsets of $Q$ which has cardinality $n - 1$, and we assume that all the entries in the transition table are distinct (see example 2 below). Then the complement of each entry in the transition table defines a permutation of the states of $Q$ (the reader may consult [7] for more information on permutations). We set up a specific permutation for which we prove a bound on the first possible position where a repetition can occur. This result translates back directly to the $\cap$-NFA.

**Example 2 (n-NFA)** Let $n = 4$. An example of the special form of the transition function as described above is given by

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$a$</th>
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<tr>
<td>$q_1$</td>
<td>${q_1, q_3, q_4}$</td>
</tr>
<tr>
<td>$q_2$</td>
<td>${q_2, q_3, q_4}$</td>
</tr>
<tr>
<td>$q_3$</td>
<td>${q_1, q_2, q_4}$</td>
</tr>
<tr>
<td>$q_4$</td>
<td>${q_1, q_2, q_3}$</td>
</tr>
</tbody>
</table>

We first establish some notation; let $n > 1$ be arbitrary but fixed for the following discussion.

- Let $Q = \{1, 2, \ldots, n\}$, and let $A_i = Q \setminus \{i\}$ for $1 \leq i \leq n$.
- For $\pi$ a permutation on $Q$, we indicate the image of $j$ under the permutation by $\pi(j)$. Therefore,
  \[ A_{\pi(j)} = Q \setminus \{\pi(j)\}. \]
- A cycle of length $k$ of the permutation $\pi$ is given by
  \[ (j \pi(j) \pi^2(j) \ldots \pi^{k-1}(j)), \]
  where $\pi^k(j) = j$, and $\pi^m(j) \neq j$ for $1 \leq m < k$.
- The order of a permutation, indicated by $o(\pi)$, is the least common multiple of all its cycle lengths.
- For a subset $B = \{i_1, \ldots, i_k\}$ of $Q$ and permutation $\pi$ on $Q$, we define $\pi(B)$ as
  \[ \pi(B) = \{\pi(i_1), \pi(i_2), \ldots, \pi(i_k)\} = \bigcup_{i \in B} \{\pi(i)\}. \]

- We indicate the complement over $Q$ of a set $B \subseteq Q$ by $c(B)$.

Let $M_n = (Q, \{a\}, \delta, q_0, F, c)$ be a unary $\cap$-NFA with state set $Q$, alphabet $\{a\}$, initial state $q_0 \in Q$ and final state set $F \subseteq Q$. We assume that $\delta$ has the special form

\[ \delta(i, a) = A_{\pi(i)} \]

where $\pi$ is some permutation of $Q$.

We first establish the relationship between the transition function $\delta$ of $M_n$ and a permutation of a subset $B$ of the states of $M_n$.

**Lemma 1** In the unary $\cap$-NFA $M_n$ defined above, $\delta(B, a) = c(\pi(B))$ for any non-empty subset $B$ of $Q$. 

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Proof:
\[ \delta(B, a^k) = \bigcap_{j \in B} \delta(j, a) \quad (\text{since } M_n \text{ is an } \cap\text{-NFA}) \]
\[ = \bigcap_{j \in B} A \cup_c(j) \quad (\text{by (3)}) \]
\[ = c(\bigcup_{j \in B} A \cup_c(j)) \quad (\text{De Morgan}) \]
\[ = c(\bigcup_{j \in B} \{c(j)\}) \quad (\text{by (1)}) \]
\[ = c(\bigcup_{j \in B} \{c(j)\}) \quad (\text{by (2)}) \]
\[ \square \]

Lemma 2 \( c(\pi(B)) = c(\pi(c(B))) \) for any \( B \subseteq Q \).

Proof: \( \pi \) is a bijection of \( Q \); hence, since \( B \) and \( c(B) \) are disjoint, it follows that \( \pi(c(B)) \) and \( c(\pi(B)) \) are disjoint. Moreover, \( B \cup c(B) = Q \) and hence \( \pi(B) \cup \pi(c(B)) = Q \). It follows that \( c(\pi(B)) = c(\pi(c(B))) \).

We can now show that the behaviour of \( M_n \) on a word of length \( k \) can be described by \( k \) applications of the permutation on \( B \):

Theorem 2 For any non-empty subset \( B \) of \( Q \),
\[ \delta(B, a^k) = \begin{cases} \pi^k(B) & \text{if } k \text{ is even} \\ c(\pi^k(B)) & \text{if } k \text{ is odd} \end{cases} \]

Proof:
\[ \delta(B, a^k) = \delta(\ldots \delta(\delta(B, a), a), a) \quad (\text{by lemma 1}) \]
\[ = c(\bigcup_{j \in B} \{c(\pi(\pi(\ldots(\pi(c(B)))))))\}) \quad (\text{by reordering and lemma 2}) \]
\[ = c^k(\pi(B)) \]
\[ = \begin{cases} \pi^k(B) & \text{if } k \text{ is even} \\ c(\pi^k(B)) & \text{if } k \text{ is odd} \end{cases} \]
\[ \square \]

Corollary 1 Suppose \( o(\pi) = k \). If \( k \) is even, then \( \delta(B, a^k) = B \); in any event, \( \delta(B, a^k) = B \).

Given the relationship between the transition function of \( M_n \) and the permutation as described above, we want to choose a \( B \) from which one could construct a long sequence before the first repetition occurs (that would enable us to find a long sequence of states in the DFA equivalent to \( M_n \), before any cycle occurs). Such a \( B \) is described in theorem 3 below.

Theorem 3 Suppose that \( n = r_1 + r_2 + \ldots + r_k \), such that for each \( i \) the \( r_i \) are mutually relatively prime, \( r_i > 2 \) and odd. Let \( \pi \) be the permutation
\[ (1 2 \ldots r_i 1)(r_1 + 1)(r_1 + r_2)\ldots((r_1 + \ldots + r_{k-1} + 1)\ldots n). \]

Suppose \( B \subseteq Q \) contains at least one and at most \( r_j - 1 \) consecutive elements of the \( j \)-th cycle, with \( 1 \leq j \leq k \). Then the sequence
\[ \pi^0(B), c(\pi(B)), \pi^2(B), c(\pi^2(B)), \ldots, c(\pi^{2r_1r_2\ldots r_{k-1} - 1}(B)) \]
contains no repetition.

Proof: The order of the permutation \( \pi \) is the least common multiple of its cycle lengths. Since the \( r_i \) are odd and relatively prime, it follows that
\[ o(\pi) = \text{lcm}(r_1, r_2, \ldots, r_k) = r_1r_2\ldots r_k, \]
which is odd.
Since \( o(\pi) \) is odd, the sequence has the form
\[ \pi^0(B), c(\pi(B)), \pi^2(B), \ldots, \pi^{o(\pi) - 1}(B), c(\pi^{o(\pi)}(B)), \]
\[ \pi^{o(\pi) + 1}(B), \ldots, c(\pi^{2r_1r_2\ldots r_{k-1} - 1}(B)). \]

But \( \pi^{o(\pi) + j}(B) \) is simply \( \pi^j(B) \), and hence \( c(\pi^{o(\pi) + j}(B)) = c(\pi^j(B)) \). By rewriting and rearranging the sequence we get
\[ \pi^0(B), c(\pi(B)), \ldots, \pi^{o(\pi) - 1}(B), c(\pi^0(B)), \]
\[ c(\pi^1(B)), \ldots, c(\pi^{o(\pi) - 1}(B)). \]

From the way in which \( B \) was chosen, it follows that there can be no repetition of the form \( \pi^j(B) = \pi^k(B) \) for \( 0 \leq j < k \leq o(\pi) - 1 \). Similarly, there can be no repetition of the form \( c(\pi^j(B)) = c(\pi^k(B)) \) for \( 0 \leq j < k \leq o(\pi) - 1 \). The only other possibility is that there is a repetition of the form \( \pi^j(B) = c(\pi^k(B)) \) for \( 0 \leq j < k \leq o(\pi) - 1 \). But since \( \pi^j(B) \) and \( c(\pi^k(B)) \) contain the same number of elements from each cycle of \( \pi \), and all the cycles of \( \pi \) have odd length (by the construction of \( \pi \)), it follows that \( \pi^j(B) \) and \( c(\pi^k(B)) \) contain different numbers of elements from each cycle. Therefore, there can be no repetition of the form \( \pi^j(B) = c(\pi^k(B)) \) for \( 0 \leq j < k \leq o(\pi) - 1 \).

The result follows.

\[ \square \]

Theorem 4 Suppose that \( n = r_1 + r_2 + \ldots + r_k \), such that for each \( i \) the \( r_i \) are mutually relatively prime, \( r_i > 2 \) and odd. Let \( \pi \) be the permutation
\[ (1 2 \ldots r_i 1)(r_1 + 1)(r_1 + r_2)\ldots((r_1 + \ldots + r_{k-1} + 1)\ldots n). \]

Suppose \( B \subseteq Q \) contains at least one and at most \( r_j - 1 \) consecutive elements of the \( j \)-th cycle, with \( 1 \leq j \leq k \), while \( |B| \leq \lfloor n/2 \rfloor \).

Let \( M \) be an \( n \)-state unary \( \cap\text{-NFA} \) such that \( M = \{(0, 1, 2, \ldots, n), \{a\}, \delta, 0, F, \gamma\} \), where \( 0 \) is a dummy start state such that \( \delta(0, a) = B \) and \( \delta \) has the special form given in (3). Then it is possible to find a final state set \( F \) such that the minimal DFA \( M' \) equivalent to \( M \) has \( 2r_1r_2 \ldots r_k \) states.

Proof: We have shown in theorem 3 that the cycle length of \( M \) is \( 2r_1r_2 \ldots r_k \), and it simply remains to find a final state set \( F \) such that the DFA equivalent to \( M \) is minimal. Since the cycle length is independent of the choice of final states we are at liberty to make any choice of final states which will lead to a minimal DFA. Consider the last element in the sequence in theorem 3 above:
\[ c(\pi(2r_1r_2\ldots r_{k-1} - 1)(B)). \]

In the DFA \( M' \), this element corresponds to a compound state \( S = [s_1, s_2, \ldots, s_m] \). Choose the final state set
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$F$ of $M$ as $F = \{s_1, s_2, \ldots, s_m\}$. By the acceptance condition for an $\cap$-NFA, the only final states in the DFA are those which contain all of the $s_i$ in $F$. But if $B$ has cardinality $|B| \leq n/2$, then $|c(\pi^i(B))| \geq n/2$ for any $i$; hence the only state in the DFA which can contain all the $s_i$ in $F$ is $[S]$.

It follows that $M'$ has only one final state, and since there is only one final state in the cyclic unary DFA, it follows that $M'$ is minimal, and the theorem holds.

The length of the sequence in theorem 4 above is given by $\sum_{i=0}^{n-1} \mu_i$.

Finding a good approximation for $F(n)$ is known as Landau’s problem [4]; Chrobak [2] uses the approximation $F(n) = \frac{1}{6} n \log n + O(n)$.

We can now show that there is an $\cap$-NFA $M$ which is minimal, and hence the sequence contains no repetition.

Theorem 4 For any $n \geq 1$ there is an $\cap$-NFA $M$ with $n + 1$ states such that the minimal DFA equivalent to $M$ has $O(F(n))$ states.

Proof: Suppose $F(n) = \frac{1}{6} n \log n + O(n)$.

where $r_1 + r_2 + \ldots + r_k = n$, and let $\sigma$ be the permutation $(1 \ldots r_1)(r_1 + 1 \ldots r_1 + r_2) \ldots (r_1 + \ldots + r_k - 1 \ldots n)$. Then $\sigma(\pi) = \frac{1}{6} n \log n + O(n)$.

Suppose now that $B \subseteq Q$ contains exactly one element of each cycle. Then the sequence $B, c(\pi(B)), \pi^2(B), c(\pi^3(B)), \ldots, c(\pi^{F(n)-1}(B))$ if $F(n)$ is even, and

$B, c(\pi(B)), \pi^2(B), c(\pi^3(B)), \ldots, \pi^{F(n)-1}(B)$

if $F(n)$ is odd, contains no repetition. To see this, note that there can be no repetition of the form $\pi^i(B) = \pi^j(B)$ for $i < j < F(n)$, since $\sigma(\pi) = F(n)$ and $B$ contains exactly one element from each cycle. Similarly, there can be no repetition of the form $c(\pi^i(B)) = c(\pi^j(B))$. It remains to show that there can be no repetition of the form $\pi^i(B) = c(\pi^j(B))$ for $i < j < F(n)$. Note that $B$ (and hence $\pi^j(B)$) contains exactly one element from each cycle, and therefore $\pi^j(B)$ and $c(\pi^j(B))$ cannot contain the same number of elements unless all the cycles have length two. This is clearly impossible by the definition of $F(n)$, and hence the sequence contains no repetition.

Let $M$ be an $n + 1$-state unary $\cap$-NFA such that $M = \{(0, 1, 2, \ldots, n), \{\alpha\}, 0, F, \cap\}$, where 0 is a dummy start state such that $\delta(0, a) = B$ and $\delta$ has the special form given in (3). We claim that it is possible to find a final state set $F$ such that the minimal DFA $M'$ equivalent to $M$ has $O(F(n))$ states.

We know that the cycle length of $M$ is at least $F(n)$. Select the final state set $F$ such that the last and second last element of the sequence above. Then, by the same reasoning as in the proof of theorem 4, it follows that the DFA $M'$ is minimal.

The result holds.

We illustrate our result with an example.

Example 3 Construct an $\cap$-NFA $M$ such that

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>${1, 2, 5, 6}$</td>
</tr>
<tr>
<td>$1$</td>
<td>${2, 3, 4, 6, 7, 8}$</td>
</tr>
<tr>
<td>$2$</td>
<td>${1, 2, 4, 5, 6, 7, 8}$</td>
</tr>
<tr>
<td>$3$</td>
<td>${2, 3, 4, 5, 6, 7, 8}$</td>
</tr>
</tbody>
</table>

Note that we chose $\delta(0, a) = \{1, 2, 5, 6\}$. Then $|B| = 8/2 = 4$, and $B$ contains at least one and at most two elements from the first cycle, and at least one and at most four elements from the second cycle.

We refrain from listing the DFA equivalent to $M$ in full, giving only the first four and the last four states:

$\delta$ | $\alpha$ |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>${1, 2, 5, 6}$</td>
</tr>
<tr>
<td>$1, 2, 5, 6$</td>
<td>${1, 4, 5, 8}$</td>
</tr>
<tr>
<td>$1, 4, 5, 8$</td>
<td>${1, 3, 7, 8}$</td>
</tr>
<tr>
<td>$1, 3, 7, 8$</td>
<td>${3, 5, 6, 7}$</td>
</tr>
<tr>
<td>$3, 5, 6, 7$</td>
<td>${2, 3, 4, 5}$</td>
</tr>
</tbody>
</table>

Excluding the dummy start state 0, the DFA has exactly thirty states; that is, $2 \times (3 \times 5)$. Choose the final state set of $M$ as $F = \{2, 6, 7, 8\}$. Then the final states of the DFA are those which contain all of the elements of $F$; hence, the only final state in the DFA is the state $\{2, 6, 7, 8\}$.

The shortest word accepted by $M'$ has length 30.
Theorem 6 There is a unary \( \cap \)-NFA \( M \) which accepts a non-empty language \( L \) such that the shortest word in \( L \) has length \( O(e^\sqrt{\log n}) \).

Proof: Directly from theorem 5.

4 Conclusion

We illustrated a new automata-theoretic property of selective nondeterministic intersection automata (\( \cap \)-nfas), namely, we showed an \( \cap \)-nfa \( M \) with \( n \) states such that the length of the shortest word in \( L(M) \) is \( O(e^\sqrt{\log n}) \).

References


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