<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>T McDonald</td>
<td>A Proposed Computer Network for Researchers</td>
<td>95</td>
</tr>
<tr>
<td>T H C Smith</td>
<td>Finding a Cheap Matching</td>
<td>100</td>
</tr>
<tr>
<td>P J S Bruwer</td>
<td>Ranking Information System Problems in a User Environment</td>
<td>104</td>
</tr>
<tr>
<td>S W Postma</td>
<td>The Parallel Conditional</td>
<td>109</td>
</tr>
<tr>
<td>N C K Phillips</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D G Kourie</td>
<td>Experiences in CSP Trace Generation</td>
<td>113</td>
</tr>
<tr>
<td>R J van den Heever</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G de V de Kock</td>
<td>Die Meting van Sukses van Naampassingsalgoritmes in 'n Genealogiese Databasis</td>
<td>119</td>
</tr>
<tr>
<td>R Short</td>
<td>Learning the First Step in Requirements Specification</td>
<td>123</td>
</tr>
<tr>
<td>E C Anderssen</td>
<td>Frame Clipping of Polygons</td>
<td>129</td>
</tr>
<tr>
<td>S H von Solms</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
QUÆSTIONES INFORMATICÆ

The official journal of the Computer Society of South Africa and of the South African Institute of Computer Scientists

Die amptelike vaktydskrif van die Rekenaarvereniging van Suid-Afrika en van die Suid-Afrikaanse Instituut van Rekenaarwetenskaplikes

Editor

Professor J M Bishop
Department of Computer Science
University of the Witwatersrand
Johannesburg
Wits
2050

Editorial Advisory Board

Professor D W Barron
Department of Mathematics
The University
Southampton S09 5NH
UNITED KINGDOM

Professor G Wiechers
77 Christine Road
Lynwood Glen
Pretoria
0081

Professor K MacGregor
Department of Computer Science
University of Cape Town
Private Bag
Rondebosch
7700

Professor H J Messerschmidt
Die Universiteit van die Oranje-Vrystaat
Bloemfontein
9301

Dr P C Pirow
Graduate School of Business Admin.
University of the Witwatersrand
P O Box 31170
Braamfontein
2017

Professor S H van Solms
Departement van Rekenaarwetenskap
Randse Afrikaanse Universiteit
Auckland Park
Johannesburg
2001

Professor M H Williams
Department of Computer Science
Herriot-Watt University
Edinburgh
Scotland

Production

Mr Q H Gee
Department of Computer Science
University of the Witwatersrand
Johannesburg
Wits
2050

Subscriptions

The annual subscription is

<table>
<thead>
<tr>
<th></th>
<th>SA</th>
<th>US</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individuals</td>
<td>R20</td>
<td>$7</td>
<td>£5</td>
</tr>
<tr>
<td>Institutions</td>
<td>R30</td>
<td>$14</td>
<td>£10</td>
</tr>
</tbody>
</table>

to be sent to:
Computer Society of South Africa
Box 1714 Halfway House 1685

Quæstiones Informaticæ is prepared by the Computer Science Department of the University of the Witwatersrand and printed by Printed Matter, for the Computer Society of South Africa and the South African Institute of Computer Scientists.
The Parallel Conditional

S W Postma and N C K Phillips

Department of Computer Science, University of Natal, P O Box 375, Pietermaritzburg, 3200

Abstract

The parallel conditional is a new but natural programming language construct. It is particularly suited to evaluation on parallel machines and generalizes other well known conditionals.

Received November 1987, Accepted August 1988

1. Introduction

The programming languages in common use at any time tend to reflect the architecture of the computers that are in use at that time. Thus the long use of the inherently sequential and deterministic von Neumann architecture has led to the widespread use of languages that are inherently sequential and deterministic. However, the advent of relatively affordable parallel machines has greatly stimulated interest in parallel programming languages, and in individual language constructs that are intended for parallel evaluation. Our present purpose is to explore one such new construct, the parallel conditional.

The parallel conditional is a natural generalisation of both LISP's COND and Dijkstra's guarded conditional that is suited to parallel processing. It arose while designing the new language QUADLISP and experience with it in this context suggested that it would be worthwhile to make an independent study of it, and of its relationship with other conditionals.

Given n predicate-expression pairs (P1, X1), (P2, X2), ... , (Pn, Xn) a conditional selects an Xi according to some condition formulated in terms of the Pi. We investigate the guarded conditional of Dijkstra [1], the COND of LISP, and a new construct, the parallel conditional. In each case, following Dijkstra, the Pi are called guards.

2. Conditionals and Evaluation Strategies

Dijkstra's conditional selects non-deterministically some Xi where Pi is true, provided that all Pi are defined. If there is no such Pi or if some Pi is undefined, his conditional "aborts" - which we shall take to mean "becomes undefined". Dijkstra's notation for his conditional is

\[ \text{if } P1 \rightarrow X1 \text{ ... } \text{ and } \ldots \text{ and } Pn \rightarrow Xn \text{ fi} \]

and we call this expression DC (for Dijkstra's Conditional).

LISP's COND conditional selects the Xi such that Pi is true and Pj is false for 1 \leq j < i. If there is no such Pi then the value of the COND is undefined. Note that if COND selects X1 then Pj may be undefined for j \geq 1. The LISP notation for the COND conditional is \( (\text{COND } (P_1 X_1) \ldots (P_n X_n)) \), and we call this expression LC (for LISP's Conditional).

The parallel conditional is the simplest of the three conditionals to describe: it selects non-deterministically an Xi such that Pi is true, although all Pj need not be defined, and is undefined if there is no such true Pi. Our notation for the parallel conditional is \( (P_1 \rightarrow X_1, \ldots, P_n \rightarrow X_n) \), and we shall call this expression PC (for parallel Conditional). An essential difference between these conditionals lies in the evaluation strategies that are appropriate for the guards P1, ..., Pn in each case.

For PC it is appropriate to evaluate the Pi in parallel, and to return Xi, where Pi is a guard which evaluates to true. We intend non-determinism to be the "don't care" variety, so that it would be legal to return Xj where Pi is the first guard to evaluate to true. Note that the remaining guards may be true or false or even undefined: their attempted evaluation would be terminated on selecting Xi.

For LC the sequence P1, ..., Pn should be evaluated sequentially from the left until a true Pi is reached. Xi is then returned, and the remaining Pj are not evaluated (and could even be undefined).

For DC the guards can be evaluated either in parallel or sequentially, but since it is required that they all be defined, their complete evaluation should be attempted before selecting an Xi.

PC is defined if some Pi is true; LC is defined if some Pi is true and where Pi are defined for j < i; DC is defined if some Pi is true and all Pi are defined.

3. Correspondence with Three Valued Regular Logics

\( (P_1 \rightarrow X_1, P_2 \rightarrow X_2) \) is defined precisely when P1 \text{ por } P2 is true, where \text{ por } is the three-valued disjunction defined by the following truth table (# denoting undefined):

<table>
<thead>
<tr>
<th>( P_1 \rightarrow X_1 )</th>
<th>( P_2 \rightarrow X_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>true</td>
<td>true</td>
</tr>
<tr>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>undefined</td>
<td>#</td>
</tr>
</tbody>
</table>

Questiones Informaticae 6 (3) 109-112, November 1988 109
por, cor and wor are commonly called the parallel, conditional and weak three-valued disjunctions. Each of these, together with three-valued negation, generates one of the four regular three-valued logics of Kleene [2]. (The fourth regular three-valued logic is a trivial symmetric variation of the one that corresponds to cor.) The regular logics are the only three-valued extensions of two-valued logic that have the property that the propositional connectives are partial recursive predicates when applied to partial recursive arguments.

A more striking relationship between conditionals and the regular three-valued logics will appear in the next section, but firstly we clarify what we mean by "undefined".

The various meanings of "undefined" are manifest, and trying to be quite precise about them leads to sticky philosophical problems that we wish to avoid. In this article we have three situations in mind. Firstly, an expression may be defined for some values of sub-expressions, and undefined for others. For example, $4/c$ is undefined when $c$ has value zero. Secondly, an expression is undefined if attempting to evaluate it results in a non-terminating process. A famous example of this from the lambda calculus is the expression $(\lambda x.((x)x)(\lambda x.(xx)))$. Lastly, if we are working in the context of three-valued logic, a partial predicate $P$ is said to be undefined at $(x_1,\ldots,x_n)$ if $(x_1,\ldots,x_n)$ is not in the domain of $P$, and in this case the truth-value of $P(x_1,\ldots,x_n)$ is #.

**4. Weakest Precondition Semantics for Conditionals**

Weakest precondition semantics, invented by Dijkstra and described in [1], can be used to illuminate both the differences between the conditionals and their correspondence with regular logics.

Let $Q,R$ be predicates, $X$ be a computer process. Then $Q(X) \Rightarrow R$ is the assertion: if $X$ is started with $Q$ true, then $X$ terminates, and on termination $R$ is true. The weakest precondition for $R$ to be true after executing $X$ is defined to be the weakest $Q$ such that $Q(X) \Rightarrow R$, and is denoted $wp(X,R)$. Weakest preconditions are total predicates. Thus for any $X$, $wp(X,false)$ is false (here false is the constant predicate whose value is $f$), while if $X$ is a non-terminating process then, for any $R$, $wp(X,R)$ is false. The semantics of a process $X$ can be determined by specifying how $wp(X,R)$ is constructed for any $R$.

Assuming that all the guards, the $P_i$, are total, Dijkstra defined $wp(DC,R)$ to be

$$(P_1 \text{ or } \ldots \text{ or } P_n) \Rightarrow (P_1 \Rightarrow wp(X_1,R)) \Rightarrow \ldots \Rightarrow (P_n \Rightarrow wp(X_n,R))$$

where the logical connectives are as usual in two-valued logic.

We are interested in the case that the guards are partial predicates, especially since this is particularly appropriate to the conditionals LC and PC. In this case Dijkstra suggests that the above $wp(DC,R)$ be prefixed, with a cand, by the requirement that the initial state lies in the domain of all the guards. (cand is the and of the regular logic determined by negation and cor). Since the use of cand already involves three-valued logic, we look for an alternative to Dijkstra's suggestion that will allow firstly, expressing "the initial state lies in the domain of all the guards" by using logical connectives, and secondly can be modified easily to produce wp's for the other conditionals. Our solution is to define $wp(DC,R)$ to be

$$(P_1 \text{ or } \ldots \text{ or } P_n) \Rightarrow (P_1 \Rightarrow wp(X_1,R)) \Rightarrow \ldots \Rightarrow (P_n \Rightarrow wp(X_n,R))$$

where wor is weak disjunction and $\lambda$ and $\Rightarrow$ have the tables below.
in Bochvar's exterior logic which is discussed in Rescher [3]. \( \lambda \) yields \( f \) if one of its arguments is \( \# \), so our expression for \( \text{wp}(\text{DC}, R) \) is a total predicate and has value \( f \) when one or more of the guards is undefined. In the case that all guards are total, our \( \text{wp}(\text{DC}, R) \) is equivalent to Dijkstra's.

Some reflection on the informal semantics for LISP's \( \text{COND} \) reveals that \( \text{wp}(\text{LC}, R) \) can be defined in Bochvar's exterior logic which is discussed in Dijkstra's. Some constructs, including the other conditionals, can be defined in terms of the parallel conditional and negation, so the parallel conditional is relatively strong. We list some results.

1. The parallel conditional and negation can define the regular logics. To show this we need only define \( \text{por} \), \( \text{cor} \) and \( \text{wor} \).

\[
\begin{align*}
\text{por } Q &= (P \rightarrow \pi_1, Q \rightarrow \pi_2, \cdots, P \rightarrow (Q \rightarrow \text{if} Q \rightarrow \text{if})), \\
\text{cor } Q &= (P \rightarrow \pi_1, \neg P \rightarrow (Q \rightarrow \text{if})), \\
\text{wor } Q &= (P \rightarrow (Q \rightarrow \text{if}) \rightarrow (P \rightarrow \pi_1, \neg P \rightarrow (Q \rightarrow \text{if}), \neg Q \rightarrow (P \rightarrow \pi_2, \neg P \rightarrow \text{if})).
\end{align*}
\]

2. The parallel conditional and negation can define \( \text{if-then-else} \). For, define if \( P \) then \( X \) else \( Y \) to be \( (P \rightarrow X, \neg P \rightarrow Y) \).

3. The parallel conditional and negation can define LISP's \( \text{COND} \). For, define

\[
\begin{align*}
\text{COND } (P_1, X_1) \text{ to be: } & (P_1 \rightarrow X_1) \text{ and } \\
\text{COND } (P_1, X_1) \cdots (P_n, X_n) \text{ to be } & \{ P_1 \rightarrow X_1, \neg P_1 \rightarrow (\text{COND } (P_2, X_2) \ldots (P_n, X_n)) \}
\end{align*}
\]

if \( n > 1 \).

4. The parallel conditional and negation can define where, for \( 1 \leq i \leq n \), \( Y_i \) is the expression which results on removing \( P_i \rightarrow X_i \) and \( \neg P_i \rightarrow X_i \) from

\[
\begin{align*}
& \text{if } P_1 \rightarrow X_1[1 \ldots] \rightarrow P_n \rightarrow X_n \text{ fi. For, define } \\
& \text{if } P_1 \rightarrow X_1 \text{ fi to be } (P_1 \rightarrow X_1), \\
& \text{if } P_1 \rightarrow X_1[2] \rightarrow X_2 \text{ fi to be } (P_1 \rightarrow [P_2 \rightarrow X_1, \\
& \quad \neg P_2 \rightarrow X_1], P_2 \rightarrow (P_1 \rightarrow X_2, \neg P_1 \rightarrow X_2)), \\
& \text{if } P_1 \rightarrow X_1 \ldots [P_n \rightarrow X_n \text{ fi to be } (P_1 \rightarrow Y_1[, \\
& \quad P_n \rightarrow Y_n)]
\end{align*}
\]

5. The parallel conditional can select one of \( X_1, \ldots, X_n \) non-deterministically. For, \( \{\pi \rightarrow X_1, \ldots, \pi \rightarrow X_n\} \) does this selection.

Finally, it must be mentioned that several other parallel non-deterministic operators have been proposed in the literature. It is difficult to quantify the relative strength of these operators, since rarely is one obtainable from another without using considerable additional machinery. For example, consider the ambiguous function \( \text{amb} \) of McCarthy [4], \( \text{amb}(x, y) \) selects non-deterministically one of \( x, y \) if both are defined, otherwise whichever is defined, but is itself undefined if neither is defined. In the case that all of \( X_1, \ldots, X_n \) are defined, \( \text{amb}(X_1, \ldots, X_n) \) is \( \{\pi \rightarrow X_1, \ldots, \pi \rightarrow X_n\} \). But if the \( X_i \) are possibly all undefined, \( \text{amb}(X_1, \ldots, X_n) \) is \( \{\text{is-defined}(X_1) \rightarrow X_1, \ldots, \text{is-defined}(X_n) \rightarrow X_n\} \), where \( \text{is-defined}(X) = t \) if \( X \) is defined, \( f \) otherwise. Since the is-defined predicate is in general not computable, this relationship between \( \text{amb} \) and the strong conditional is not very illuminating. On the other hand, if \( \text{amb} \) and local scoping were available in LISP, then consider

\[
\text{(let} ([\text{amb}(\text{if } P_1 \text{ then } 1 \text{ else LOOP}) \ldots (\text{if } P_n \text{ then } n \text{ else LOOP}))]) \text{ (COND } (\text{equal } j 1) X_1 \ldots (\text{equal } n j) X_n))
\]

where \( \text{LOOP} \) is a non-terminating LISP process. This would have the effect of \( (P_1 \rightarrow X_1 \ldots P_n \rightarrow X_n) \), but again this is not very illuminating, especially since we have used \( \text{COND} \), which we have shown can itself be considered to be a special case of the parallel conditional.

6. Implementation

By distributing processes to parallel processors, a useful result can be obtained even if some of the individual processes might abort in error or loop forever. The straightforward way to implement the parallel conditional is to distribute the evaluation of the guards to parallel processors. From the implementation point of view the only interesting problem is how to ensure clean termination of the evaluation of remaining guards after a \( P_i \) has been found to be true. Such problems have been extensively studied and a variety of solutions can be found in Brinch Hansen [5], Hoare [6] and elsewhere.

We are implementing the parallel conditional in an extension of LISP called QUADLISP [7].
7. Conclusion

The parallel conditional proves to be a powerful unifying concept. In programming languages it has the same role as the (unbounded) minimisation operator in recursive function theory – it is used to specify partial functions. The concept can thus be used to simplify the definition of the semantics of programming languages in readily understood concepts, and also used to simplify the implementation of language processors by requiring fewer basic implementation routines.

References

NOTES FOR CONTRIBUTORS

The purpose of the journal will be to publish original papers in any field of computing. Papers submitted may be research articles, review articles and exploratory articles of general interest to readers of the journal. The preferred languages of the journal will be the congress languages of IFIP although papers in other languages will not be precluded.

Manuscripts should be submitted in triplicate to:

Professor J. M. Bishop
Department of Computer Science
University of the Witwatersrand
Johannesburg
Wits
2050

Form of manuscript
Manuscripts should be in double-space typing on one side only of sheets of A4 size with wide margins.

The first page should include the article title (which should be brief), the author’s name and affiliation and address. Each paper must be accompanied by an abstract less than 200 words which will be printed at the beginning of the paper, together with an appropriate key word list and a list of relevant Computing Review Categories.

Manuscripts may be provided on disk in any Apple Macintosh package or in ASCII format.

For authors wishing to provide camera-ready copy, a page specification is freely available on request from the Editor.

Tables and figures
Tables and figures should not be included in the text, although tables and figures should be referred to in the printed text. Tables should be typed on separate sheets and should be numbered consecutively and titled.

Figures should also be supplied on separate sheets, and each should be clearly identified on the back in pencil with the authors name and figure number. Original line drawings (not photocopies) should be submitted and should include all the relevant details. Photographs as illustrations should be avoided if possible. If this cannot be avoided, glossy bromide prints are required.

Symbols
Mathematical and other symbols may be either handwritten or typewritten. Greek letters and unusual symbols should be identified in the margin. Distinction should be made between capital and lower case letters; between the letter O and zero; between the letter I, the number one and prime; between K and kappa.

References
References should be listed at the end of the manuscript in alphabetic order of the author’s name, and cited in the text in square brackets.

Journal references should be arranged thus:


Proofs
Proofs will be sent to the author to ensure that the papers have been correctly typeset and not for the addition of new material or major amendment to the texts. Excessive alterations may be disallowed. Corrected proofs must be returned to the production manager within three days to minimise the risk of the author’s contribution having to be held over to a later issue.

Only original papers will be accepted, and copyright in published papers will be vested in the publisher.

Letters
A section of “Letters to the Editor” (each limited to about 500 words) will provide a forum for discussion of recent problems.