EXPLORING GRADE 9 LEARNERS' KNOWLEDGE OF PROPERTIES OF QUADRILATERALS

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ABSTRACT – A qualitative case-study approach which used an inductive inquiry strategy was undertaken to identify the properties of quadrilaterals that are (un)known to Mathematics learners exiting Grade 9. Data were collected in one public rural secondary school in Malamulele West Circuit. A 25-multiple choice test was administered to a group of 84 Grade 10 Mathematics learners because they had just exited Grade 9. They would have learnt all the properties of quadrilaterals and are still pursuing Mathematics. The collected data were analysed using the Statistical Package for the Social Sciences to calculate percentages and indicate the frequency of getting the properties right or wrong as expressed by the participants. The research revealed that the Grade 10 Mathematics learners did not know all the properties of all the quadrilaterals. The recommendation is that secondary school learners should be taught properties of quadrilaterals through a discovery approach.

Keywords: Properties of quadrilaterals, levels of geometric thinking

1. INTRODUCTION

Understanding geometry of shapes has many practical applications in professional and real-life situations. Many professionals like: engineers, architects, artists, real-estate agents, farmers and construction workers need to understand shapes (Jones, 2012). Individuals may as well need to understand shapes when doing home improvements, when gardening and even when planning a party (Jones, 2012). Despite the importance of understanding geometry of shapes, most learners struggle with it and do not perceive its value (Zilkova, 2015).

Geometry, concerned with shapes, size, relative position of figures, and the properties of space, has an important place in school Mathematics curricula as it develops learners' spatial ability and logical reasoning skills (French, 2004). Learners need to learn Geometry from primary school to secondary school. The sequence to be followed is: firstly, learn to identify shapes by appearance; secondly, know the properties of these shapes; thirdly, understand the relationship between the properties of shapes and; lastly, construct deductive proofs (Van Hiele, 1986). This means that learners who exit secondary schools must be able to construct deductive proofs. However, for the mastery of deductive proofs the knowledge of quadrilaterals and triangles is necessary (Van Hiele, 1986).

In South Africa, the knowledge of quadrilaterals was reported mostly on pre-service teachers. Luneta (2014) conducted a study with students who had just exited secondary school and were studying to become Mathematics foundation phase teachers. This study was about students' conceptual understanding of quadrilaterals. It was found that the majority of the students could not define the term quadrilateral and most of them could not tell the difference between a triangle and a quadrilateral. It was also found that, while these pre-service teachers had just passed Grade 12 mathematics and were expected to do deductive proofs, the majority of them operated at the level of recognising quadrilaterals by appearance alone. The specific quadrilaterals that were investigated and found to be unknown to the participants were the rectangle and rhombus.

Alex and Mammen (2014) however investigated Grade 10 learners' knowledge of quadrilaterals and triangles. They found that most learners were at the level of identifying quadrilaterals by appearance and not by its properties. They further found that many learners had difficulty with the properties of quadrilaterals although they did not mention the properties.

The studies carried out in South Africa just mention that learners identify quadrilaterals by appearance and that they do not know the properties of quadrilaterals, without specifying the properties that are unknown. In reality, though Grade 9 learners are expected to know properties of quadrilaterals and triangles, they exit the grade with limited knowledge. This explains why learners exit secondary school unable to construct deductive proofs as expected of them (Luneta, 2014). Limited knowledge of properties of quadrilaterals and triangles creates a gap in the construction of deductive proofs. Hence there are calls for interventions in learning the properties of shapes in Grade 9 to bring learners to expected levels of understanding geometry of shapes when they exit secondary school (Feza & Webb, 2005). However, literature that specifies the properties of quadrilaterals and triangles that learners have difficulties with as they exist Grade 9 is scarce in South Africa. Hence this study identified learners' knowledge of properties of six quadrilaterals (rectangle, square, parallelogram, rhombus, kite and trapezium) that are based on sides, angles and diagonals.

2. RESEARCH QUESTION

The study responded to the research question: What are the properties of quadrilaterals that are (un)known to learners exiting Grade 9?

3. THEORETICAL FRAMEWORK

The Van Hiele Model of the Development of Geometric Thought (1986) was considered to decide on the questionnaire used for collecting data and analysing it. The model has five different levels, namely, recognition or visualisation, analysis, ordering or informal deductive, deduction or formal deductive, and rigour. Because of the grade level of the learners that participated in this study, only visualisation and analysis are reported on in this paper. At the level of visualisation, it is expected that an individual has the geometric vocabulary, can identify specified shapes and can reproduce a given figure (Van Hiele, 1896). Recognition of properties of shapes is done at the level of analysis; learners are able to list properties of geometric figures. These properties of shapes can be generalised into a class of shapes. Therefore, one rectangle belongs to a class of rectangles because it has properties that correspond to the class (Crowley, 1987).

4. METHODOLOGY

A case study methodology was followed to identify properties of quadrilaterals that are (un)known to grade 10 learners. The choice of case studies was informed by Giorgi's (2009) view of case studies as effective for exploring understandings of people who have experienced that phenomenon. As a result, sampling by case type was used to decide on a sample of Grade 10 learners as they had been taught properties of quadrilaterals by the first author of this paper. The sample was the Grade 10 mathematics class at a secondary school in Malamulele West Circuit. In size, the sample had a total of 84 learners who had just exited Grade 9 and were still pursuing studies in Mathematics. Since the Van Hiele levels of geometric thinking are not age and gender related, it was not deemed necessary to consider biographical information to decide on a sample of participants.

Furthermore, Yin's (2002) view that case studies are empirical inquiries that investigate a case by addressing questions concerning the phenomenon of interest was used in collecting data. A questionnaire adapted from Van Hiele's (1986) test on properties of shapes, consisting of 25 multiple choice questions that learners responded to in a maximum time of an hour, was used for collecting data. The content coverage was framed on Van Hiele's (1986) second level (analysis) of geometric thinking where learners are able to identify particular properties of shapes, but not in a logical order. For content validity, the questions in the test were assessed by four experts: a mathematics head of

department; and three mathematics teachers from the study site (Carter & Porter, 2000). In order to establish the construct validity of the instrument, a pilot study was conducted with five Grade 10 learners from a different rural secondary school within Malamulele West Circuit (Polit & Beck, 2012). Cronbach's alpha coefficient value ranges from 0 to 1 as an overall assessment of internal consistency (reliability) of an instrument (Tavakol & Dennick, 2011). If its value is ≥ 0.70 then the instrument is reliable. Therefore, the overall Cronbach alpha coefficient computed using SPSS was 0.951, which meant the questions measured the underlying properties of six quadrilaterals.

Data were analysed by grouping them according to frequencies of correct and wrong answers using version 22 of SPSS. These frequencies were also represented as percentages of the total number of learners that participated in the study. A summary of the results of the analysis is provided in Table 1.

This is followed by a narrative of what the table captured.

5. ETHICAL ISSUES

Permission to carry out the study at the selected site was obtained from district education manager and principal of the school where data were collected. During the process of data collection and processing, anonymity and confidentiality of participants were assured. The participants were asked not to write their names on the answer sheets and told that the collected information was solely for the purposes of the study.

6. RESULTS OF THE ANALYSIS

The table below represents the results of the analysis. It is narrated to provide more clarity.

Table 1: Properties of quadrilateral and frequencies of getting correct and wrong answers.

Item No	Property of a Quadrilateral	Frequency of Correct Answer	Frequency of Wrong Answer
Q1	Both pairs of opposite sides of a rectangle are equal	62 (73.8%)	22 (26.2%)
Q2	The diagonals of a kite are perpendicular	27 (32.1%)	57 (67.9%)
Q3	Both pairs of opposite sides of a square are equal and parallel	17 (20.2%)	67 (78.8%)
Q4	Both pairs of opposite sides of a rhombus are equal	31 (36.9%)	53 (63.1%)
Q5	One pair of the sides of a trapezium is parallel	23 (27.4%)	61 (72.6%)
Q6	The diagonals of a parallelogram are not equal	34 (40.5%)	50 (59.5%)
Q7	The diagonals of a kite are not equal	17 (20.2%)	67 (79.8%)
Q8	The diagonals of a rhombus are perpendicular.	31 (36.9%)	53 (63.1%)
Q9	The diagonals of a trapezium do not bisect each other	24 (28.6%)	60 (71.4%)
Q10	Diagonals of a rectangle are equal	13 (15.5%)	71 (84.5%)
Q11	The shorter diagonal of a kite is bisected by its longer one	30 (35.7%)	54 (64.3%)
Q12	Diagonals of a square bisect each other at right angles	08 (9.5%)	76 (90.5%)
Q13	The longer diagonal of a kite bisects the shorter one	08 (9.5%)	76 (90.5%)
Q14	All the angles of a rectangle are equal	27 (32.1%)	57 (67.9%)
Q15	All sides of a square are equal	27 (32.1%)	57 (67.9%)
Q16	Both pairs of opposite angles of a kite are not equal	17 (20.2)	67 (79.8%)
Q17	Diagonals of a trapezium are not equal	27 (32.1%)	57 (67.9%)
Q18	Diagonals of a rectangle bisect each other	35 (41.6%)	49 (58.3%)
Q19	Diagonals of a parallelogram bisect each other	34 (40.5%)	50 (59.5%)
Q20	The sides of a rhombus are all equal	26 (30.9%)	58 (69.1%)

Q21	All the angles of a square are equal	27 (32.1%)	57 (67.9%)
Q22	A trapezium has one pair of opposite sides parallel	15 (17.9%)	69 (82.1%)
Q23	Rhombus has both pairs of opposite sides parallel	22 (26.2%)	62 (73.8%)
Q24	Rhombus has at least one interior obtuse angle	14 (16.7%)	70 (83.3%)
Q25	Consecutive angles of a rhombus add up to 180˚	22 (26.2%)	62 (73.8%)

Table 1 represents frequencies of students that got an answer correct or wrong for each item. For example, responses to item 1 show that 62 (73.8%) learners know that both pairs of opposite sides of a rectangle are equal. It can be observed that, on average, the majority of the learners provided wrong answers in all the 25 multiple choice questions with the exception of Question 1. In all the properties of quadrilaterals asked, more than 60% of the learners got the answers to items of the questionnaire wrong. The analysis of the answers provided by learners is silent on identifying learners who got answers correct. As a result nothing can be said about mutual inclusivity and exclusivity of the answers that were provided by the students.

Four items of the questionnaire tested learners' knowledge of the following properties of rectangles (Q1, Q10, Q14 and Q18). Table 1 shows that the property that is known by most of the learners is the first one with a frequency of 62 correct answers. The property that diagonals of a rectangle are equal' is less known by almost all learners. Seventy five percent of the learners did not identify it as corresponding with rectangles.

Four of the questionnaire items tested learners' knowledge of properties of squares (Q3, Q12, Q15, and Q21). Answers to these questions show that most of the learners did not know that diagonals of a square bisect each other at right angles with the exception of eight learners. That all angles and all sides of a square are equal was known by less than a third of the learners (27%). Lastly, only 20% of the learners knew that opposite sides of squares are both equal and parallel.

In the questionnaire, there were two items that tested knowledge on properties of parallelograms. The properties were: The diagonals of a parallelogram are not equal, and diagonals of a parallelogram bisect each other. In both cases about 41% of the learners got the answers correct. The percentage of learners who got these questions correct is more that the percentage of learners who got a correct answer with any of the properties of squares. This is remarkable because it is expected that squares are easier to visualise and that their properties are easier to notice.

Six of the items of the questionnaire were on properties of a rhombus (Q4, Q8, Q20, Q23, Q24, and Q25). About a third of the learners know that both pairs of opposite sides of a rhombus are equal and that its diagonals are perpendicular. The least known property was that a rhombus has at least one interior obtuse angle. The number of learners who knew that the sides of a rhombus are equal is 26 out of 84. This number is almost the same as the number of learners who knew that all sides of a square are equal – 27 out of 84 learners is almost the same number of learners.

Of the properties of parallelograms that were tested in the questionnaire, five of them were on properties of a kite (Q2, Q7, Q11, Q13, and Q16). Almost all of the properties tested were on diagonals of a kite. The frequency counts show that eight of the learners knew that the longer diagonal of a kite bisects the shorter one. Strangely, the 30 learners of the 84 learners thought that the shorter diagonal of a kite bisected the longer one. This is more than twice the number of learners who said the longer side of a diagonal bisects the shorter one. Data further showed that a fifth of the learners knew that diagonals of a kite are not equal. Also, almost a third of the learners know that diagonals of a kite are perpendicular. Lastly, a fifth of the learners knew that both pairs of opposite angles of a kite are not equal.

7. DISCUSSION AND CONCLUSIONS

7.1. THE RESEARCH QUESTION

The study reported in this paper responded to the question: What are the properties of quadrilaterals that are (un)known to learners exiting Grade 9? The analysis of data using frequency counts of correct and wrong responses of learners to a questionnaire on properties of quadrilaterals show that the only property of quadrilaterals that is known to learners exiting grade 9 states that both pairs of opposite sides of a rectangle are equal. This property had the highest score (73.8%) of frequencies representing correct scores. None of the correct scores had a frequency count of correct answers that is more than 42%.

In the sections that follow, the discussion focuses on diagonals, sides and angles of diagonals as they form the basis for properties of quadrilaterals.

7.2. DIAGONALS OF QUADRILATERALS

Items Q2, Q8, and Q12 of the questionnaire tested whether learners knew that a kite, a rhombus and a square are the quadrilaterals with perpendicular diagonals respectively. Table 1 shows that 63.1%, 67.9%, 90.5% got the answers to Q2, Q8, and Q12 wrong respectively. It can be concluded that more than 63% of the learners do not know properties of quadrilaterals that relate to perpendicular diagonals. Almost the whole class (90.5%) was clueless about the property that squares have perpendicular diagonals. This is inexplicable since in most instances, squares are the first to be introduced when learning about quadrilaterals and their properties.
Items Q9, Q11 and Q13 of the questionnaire were used to tap on learners the knowledge of quadrilaterals whose diagonals bisect. It was expected that learners knew that a kite and trapezium are the only quadrilaterals whose diagonals do not bisect each other. Table 1 shows that 71.4% of learners did not know that diagonals of a trapezium do not bisect each other (Q9). Furthermore, 90.5% of learners did not know that the shorter diagonal of a kite is bisected by the longer one (Q13). Consequently, it can be concluded that learners do not have the correct knowledge on the properties that relate to diagonals of the trapezium and kite.

It is expected that by the end of Grade 9, learners know that the only diagonals of quadrilaterals that are equal are those of squares and rectangles. Items Q6, Q7 and Q10 were used to probe learners' knowledge of the aforesaid property. The answers provided show that 84.5% of learners did not know that the diagonals of a rectangle are equal (Q10). Furthermore, 79.8% of learners wrote that the diagonals of a kite are equal (Q7), while 59.5% said that a parallelogram has equal diagonals (Q6). In essence, learners did not kn7ow the quadrilaterals whose diagonals are equal.

In summary, it is clear that the learners do not know the properties of diagonals of quadrilaterals. They have not yet attained Van Hiele's level 2 that relate to identifying shapes by properties relating to diagonals. This findings are not unique to learners. In a study carried out by Turnuklu, Akkas and Alayli (2013), teachers that participated were found to have problems with properties of quadrilaterals that related to diagonals. The reported study is specifically on which properties of quadrilaterals that are related to diagonals that are not known by learners; they are those that have to do with diagonals that are perpendicular, that bisect and that are not equal.

7.3. INTERIOR ANGLES OF QUADRILATERALS

The study assessed learners' knowledge on interior angles of quadrilaterals in relation to their individual angles, opposite angles and consecutive angles. Items Q14 and Q21 of the questionnaire sought to find out if learners knew quadrilaterals whose interior angles are all equal. Table 1 reveals that 67.9% of learners did not know that both a rectangle and a square have interior equal angles. While this may seem to be an obvious property of squares and rectangles, learners in this study showed that this is not necessarily the case.

While it is expected that learners should know that all quadrilaterals except the trapezium and kite have both pairs of opposite angles equal, this study showed that the majority, 79.8% of learners did not know (Q16). On the other hand, it is expected of learners to know that only a kite and a trapezium do not have consecutive angles that add up to $180°$ while the rest of the quadrilaterals do. Responses to item Q25 show that 73.8% of the learners did not know that the consecutive angles of a rhombus add up to 180°.

The responses to the test show that most learners were not at Van Hiele's level 2 on properties that relate to angles of the six quadrilaterals. This is however different from findings of a study conducted by Turnuklu, Akkas and Alayli (2013) that the teachers who participated in the study correctly defined properties related to angles of quadrilaterals.

7.4. SIDES OF THE QUADRILATERALS

The knowledge of properties on sides of quadrilaterals that the researcher found from learners is about the relationship between opposite sides and among all four sides. While it is expected that Grade 9 learners should know that squares and rhombi are the only quadrilaterals with all sides equal, 69.1% of learners did not know this (Q20) while 67.9% of them did not know that a square has all its sides equal (Q15). This is an unexpected finding since properties of squares that relate to sides are noticeable by appearance.

Another property of quadrilaterals, with the exception of kites and trapeziums, that relates to both pairs of opposite sides is that they are equal and parallel. Again, most of the learners (73.8%) did not know that a rhombus has both opposite sides parallel (Q23). Instead they claimed the property to be true with either kites or trapeziums. Also, 82.1% of learners did not know that a trapezium has one pair of opposite sides parallel (Q22). It can be concluded that learners did not have knowledge of properties of quadrilaterals that relate to their sides.

8. CONCLUSION

This study shows that Grade 9 learners who participated in this study were not at the analysis level of the Van Hiele Model of the Development of Geometric Thought (1986). They could not recognise the properties of these shapes. Unfortunately it cannot be claimed that the same finding could result if the shapes were given diagrammatically. As a result it cannot be claimed, as in the study by Alex and Mammen (2014), that they were at the visualisation level; the reported study did not tap on that knowledge.

The finding of this study reveals that generally all the properties of all the six quadrilaterals are unknown to learners. This may suggest that the properties of all the six quadrilaterals should be taken into consideration and no property should be assumed as easy or well known to learners. The suggestion is that this finding should go a long way in alerting the mathematics teachers on the learners' knowledge deficiency of properties of the six quadrilaterals. Hence, this should encourage them to redirect their efforts in making sure that they change their approach to teaching these

properties of quadrilaterals in the classroom. As such, the phases of Van Hiele's levels of geometric thinking have to be considered when designing instructional activities.

8.1. LIMITATION OF THE STUDY

The reported study was conducted on a small scale. The study may need to be widened to be more representative of the majority of Grade 9 learners. On the other hand, the findings of this study can be given more depth if qualitative data research could have been conducted. It should be interesting to find out if the findings will be consistent if data were collected in a less formal setting than a test.

Furthermore, a rigorous quantitative analysis of the data is recommendable for a study like this. It should be possible to track learners and the responses they choose in multiple-choice questions.

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