THE IMPACT OF USING GEOGEBRA TO TEACH CIRCLE GEOMETRY ON GRADE 11 STUDENTS’ ACHIEVEMENT
Ugorji I. Ogbonnaya¹, Chimuka Alfred²
University of Pretoria¹, University of South Africa²
ugorji.ogbonnaya@up.ac.za¹, alfredchimuka7@gmail.com²

ABSTRACT- Circle geometry is among the new topics introduced into the school mathematics curriculum in South Africa. The teaching of the topic seems to be a challenge to most teachers and students find it difficult to grasp. Hence the need to find better ways of teaching the topic and thereby improve students’ learning of the topic led to this study that explored the impact of using GeoGebra to teach circle geometry on Grade 11 students’ achievement. Van Hiele theory of geometric understanding was used as the framework that guided the study. Using a non-equivalent group quasi-experimental research design, two secondary schools (experimental and control groups) located in a rural area in Limpopo Province of South Africa were used as sample. The experimental and control groups were taught using GeoGebra and the traditional teacher ‘talk-and-chalk’ method respectively. Data was collected by means of a pre-test of multiple choice questions and a post-test of open ended questions. The content validity ratio (CVR) for both the pre-test and post-test was +1 which attest to the validity of the tests. The Kuder-Richardson formula 20 (KR20) was used to calculate the reliability coefficient of the pre-test while the Spearman-Brown formula was used for the post-test. Reliability coefficients of 0.81 and 0.98 were obtained for the pre-test and post-test respectively. Descriptive and inferential statistics were used to analyse the data. The study found that there was a significant difference in achievement in favour of the students taught with GeoGebra at levels 1 and 2 of Van Hiele theory of geometric understanding. No significant differences were found at levels 3 to 5. The findings suggest that integrating GeoGebra with the teaching of circle Geometry may positively impact on students’ achievement on circle geometry.

Keywords: Circle geometry, GeoGebra, Mathematics, South Africa, student achievement, van Hiele

Introduction and background

The transformation of the South African school curriculum from the National Curriculum Statement (NCS) of 2002, revised in 2009, to the Curriculum and Assessment Policy Statement (CAPS) of 2012 brought many instructional challenges to teachers, in particular secondary school mathematics teachers. For example, in the NCS some topics, including circle geometry, were optional meaning that the topics were only offered by schools and candidates that wished to do so. These candidates were therefore required to write a separate examination paper (Paper 3) on the topics in addition to the compulsory Papers 1 and 2.

In the NCS, the majority of students that opted for Paper 3 between 2009 and 2013 did not do well in the examination (Department of Basic Education, 2013). As a result, most schools discouraged their students from opting for Paper 3. While the performance of students in mathematics across the country has been generally poor (Campbell & Prew, 2014; Howie & Plomp 2002), Limpopo Province has been one of the lowest performing provinces (National Senior Certificate Examination Diagnostic Report, 2014). Given the poor state of mathematics education in the country in general and Limpopo Province in particular, the inclusion of new topics in the CAPS curriculum was a curriculum change of great magnitude to teachers and students alike. The majority of mathematics teachers in the Further Education and Training (FET) band (Grades 10–12) are pedagogically ill-equipped to effectively teach the new topics as most of them were not taught the topic in their schooling. Most teachers find it extremely difficult to effectively present the topic so that students are enabled to grasp and construct the relevant mathematical knowledge. Our observations and discussions with many teachers show
that the circle is one topic that they find difficult to present in ways that the students easily understand. This could account for the students’ difficulties and poor achievement in circle geometry.

One possible way to improve students’ learning and achievement in circle geometry could be the integration of computer technology with the teaching and learning of the topic. Integrating computer technology software such as GeoGebra with mathematics teaching and learning is supported by many studies. Bester and Brand (2013) argue that computer technology assists students to make meaning of the learning material, and the interactive effects of sound, animation, narration and additional definitions provided by technology (computers) appeal to today’s learners, motivating them to concentrate better and to achieve higher average scores. Trifonas (2008) showed that achievement can be improved in the classroom with the active involvement of the students making optimal use of technological innovations. Similarly, Willougby and Wood (2008) argued that learning takes place on the computer without the learners realizing the amount of attention they are paying to the material. This could be because students seem to focus on their work longer when using technology (Bitter & Legacy, 2008). Segal and Stupel (2015) observed that computer technology stimulates investigative learning and improves the quality of teaching.

Ertekin (2014) investigated the effects of teaching analytical geometry using the software Cabri 3D on teacher trainees’ ability to write the equation of a given special plane, identify the normal vector of a plane and draw the graph of the plane. The result of the study indicated that students instructed with the software were significantly more successful than those who were not instructed with the software in terms of identifying the equations of special planes and their normal vectors and drawing their graphs.

Donevska-Todorova (2015) explored students’ conceptual understanding of the dot product of vectors in a dynamic geometry environment (DGE). The study revealed that DGE offers students multiple representations which are important by allowing them to acquire deeper knowledge about a specific mathematical concept rather than observing single static representations.

Perjesi-Hamori (2015) found that the use of the computer algebraic system (CAS) enabled students with limited mathematical skills to understand more complex tasks, such as solutions of multivariate interpolations and regressions, or those of partial differential equations. Vajda (2015) used computer algebra software to introduce the classical Chebyshev polynomials as extremal polynomials. The use of computer algebra in the study was reported to have made the exploration of extremal polynomials easy and enjoyable for students. Soon and Ang (2015) introduced queuing theory to students using computer simulations and found that they were able to understand basic probability theory and statistical concepts, such as the Poisson process and exponential distribution, without the need to know all about classical queuing theory.

Stols (2012) used the Van Hiele theory to investigate the geometric cognitive development of students in a technology-enriched environment (dynamic geometry software) compared with students in a learning environment without any technological enhancement. The results suggest that the technology-enriched environment helped to improve the conceptual geometric growth of students on Van Hiele Levels 1, 2 and 4. The study suggests that technology can help to create an active learning environment in which students can discover, explore, conjecture and visualise.

In light of the view that the integration of computer technology in the teaching of mathematics may enhance students learning of mathematical concepts, this study explored the impact of using GeoGebra to teach circle geometry on Grade 11 students’ achieving.
GeoGebra is dynamic mathematics software designed for teaching and learning mathematics in secondary school and at college level. It can be used in most mathematical disciplines or topics such as geometry, algebra, statistics and calculus (Hohenwarter & Preiner, 2007). It can also be used to visualise mathematical concepts as well as to create instructional materials. According to Prodromou (2015), GeoGebra can foster active student-centred problem-solving by allowing for mathematical experiments, interactive explorations, as well as discovery teaching. Prodromou (2015) also found that GeoGebra enhanced college students' understanding of specific statistical concepts as well as their performance of statistical tasks such as data analysis and inference and exploration of probability models.

In Slovakia, Guncuga, Majherova and Jancek (2012) found that GeoGebra can be a motivational tool for teaching and learning while in Malaysia, Noorbaizura and Leong (2013) studied the effect of using it to teach fractions to students. The study showed that the students in the experimental group performed better than those in the control group who were taught using the traditional learning method. The software also enhanced visualization and understanding of the concept of fractions for both the teacher and students.

Venkataraman (2012) in Singapore carried out a study on innovative activities to develop the geometrical reasoning skill in secondary mathematics with the help of GeoGebra and found that students taught with the software made progress towards mathematical explanations which provide a foundation for further deductive reasoning in mathematics. He found that the dynamic nature (drag feature) of the software influences the form of the explanation and that the students were able to generalize the solution and respond with an adequate statement. Venkataraman (2012) concluded that GeoGebra makes learning abstract concepts far more meaningful and helps students to visualize related concepts.

**Theoretical framework**

This study adopted the Van Hiele theory of geometric understanding which describes the development of geometrical reasoning as its theoretical framework. It is a pedagogical theory which describes geometrical understanding levels of students by focusing on problems students face when they learn geometry (Olkun & Toluk, 2003). According to Van Hiele theory, a student progresses through five stages/levels of development when learning geometry.

**Level 1: Visualization**

Students recognize the figures by appearance alone, often by comparing them to a known prototype. The properties of a figure are perceived. Students recognize triangles, squares, circles, parallelograms, and other shapes, but do not explicitly identify the properties of these figures. At this level, students make decisions based on perception, not reasoning (Ball, 1990). Also at this level, the properties of a figure cannot be understood; students make decisions based on perception, not reasoning, for example, a figure is a square, cube or rectangle because it looks like one.

**Level 2: Analysis**

Analysis is the process of identifying and examining each element of an object, or features of it, in detail in order to understand it. At this level, students see figures as collections of properties. They can recognize and name the properties of figures, but they do not see relationships between the properties. When describing an object, a student operating at this level might list all the properties the student knows, but does not make connections between figures (Clements & Battista, 1992).
properties are seen as discrete entities independent of one another. For example, an equilateral triangle can have three equal sides, three equal angles and three axes of symmetry but no property implies another.

**Level 3: Abstraction/Ordering**

An abstraction is an idea or principle considered or discussed in a purely theoretical way without reference to actual examples and instances. Students at this level are able to perceive relationships between properties and figures and can create meaningful definitions and give informal arguments to justify their reasoning.

**Level 4: Deduction**

Deduction is the reasoning process by which one concludes something from known facts or circumstances. At this level students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions.

**Level 5: Rigor**

Students at this level understand the formal aspects of deduction, such as establishing and comparing mathematical systems. The students can understand the use of indirect proof and proof by contrapositive methods as well as non-Euclidean systems (Simon, 1997).

**Methodology**

This quantitative study employed a quasi-experimental study, of non-equivalent comparison group design. The experimental group was taught using GeoGebra, while the control group was instructed using the traditional teacher ‘talk-and-chalk’ method. A quasi-experimental study was used because it enabled the researchers to estimate the causal impact of an intervention on its target population (Castillo, 2009).

Two schools from one circuit were used in this study, as the control and experimental groups. The two schools were similar in that both were public schools located in the same rural geographical area, having comparable teaching and learning resources and the students were of similar social economic backgrounds. The control group comprised 25 participants (10 girls and 15 boys), while the experimental group comprised 22 participants (9 girls and 13 boys). Each group was taught by a different teacher. The teachers were holders of university degrees in mathematics and had over twenty years of experience in teaching high school mathematics.

**Teaching in the experimental group**

The experimental group used the schools’ laptops with GeoGebra installed on them. The teacher instructed and demonstrated with a laptop connected to an overhead projector. Content development worksheets were used during lesson delivery. Each lesson was one-hour long and there were 10 in total. The worksheets had open-ended questions to allow students to explore different solution strategies and/or skills of answering circle geometry questions. The content development worksheets had the same content for both the control group and the experimental group. Each worksheet covered one or two circle theorems depending on the length of the procedures required to prove the theorem(s).

Teaching in the control group
The group was taught by their mathematics teacher using the traditional teacher ‘talk-and-chalk’ method. One day was used for the introduction of the topic since no computer introduction was needed. Four content development worksheets similar in content to those for the experimental groups were used. Each lesson was one hour long, and teaching was done for ten hours over a period of ten days.

**Data collection instruments**

Data was collected by means of a pre-test and post-test administered to both groups. The pre-test consisted of 15 multiple choice questions covering basic concepts on circles and geometry in general. It was used to determine if the groups were comparable at the outset of the study. The post-test was a comprehensive summative 30 question test that covered the levels of Van Hiele’s theory of geometric understanding.

**Validity and reliability of the tests**

The test items were evaluated by five experienced mathematics teachers and two mathematics subject advisors in the school district. They judged the tests’ coverage of the curriculum in terms of content and level of difficulty by rating each item as being essential or not. Using Lawshe’s (1975) formula, the CVR for each item in the test and the mean CVR across all items were calculated. The mean CVR for both the pre-test and post-test for this study was +1, an indication that all the panellists agreed that these tests were valid (Wynd, Schmidt & Schaefer, 2003).

The Kuder-Richardson formula 20 (KR20) was used to calculate the reliability coefficient (internal consistency) of the pre-test. A value of 0.81 was obtained, which showed that the pre-test had high reliability. The post-test's reliability was tested using the Spearman-Brown formula (Stanley, 1971) A reliability value of 0.98 was obtained which implied that that the post-test was very reliable.

**Data analysis**

Descriptive and inferential statistics were used to analyse the data in the Statistical Package for the Social Sciences (SPSS). The inferential statistics, namely independent samples t-test, was used to test if there were significant difference between the test scores of the groups in general and at the different levels of the Van Hiele’s theory of geometric understanding.

**Findings**

Table 1 shows the descriptive statistics for the pre-test and post-test scores for the two groups.

<table>
<thead>
<tr>
<th>Table 1: Groups’ pre-test and post-test scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Groups</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Control (25)</td>
</tr>
<tr>
<td>Experimental (22)</td>
</tr>
</tbody>
</table>
In the pre-test, the average score (M = 51.3; SD = 9.1) of the experimental group, was slightly lower than the control group average score (M = 51.8; SD = 7.8). To check if the difference between the achievements of the groups were statistically significant, independent samples t-test was computed. The results are shown in Table 2.

**Table 2: Independent samples t-test for pre-test**

<table>
<thead>
<tr>
<th>Levene’ s Test for Equality of Means</th>
<th>t-test for Equality of Means</th>
<th>F</th>
<th>Sig.</th>
<th>T</th>
<th>Df</th>
<th>Sig.</th>
<th>Mean Difference</th>
<th>Std. Error Difference</th>
<th>95% Confidence Interval of the Difference</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal variances assumed</td>
<td>.000</td>
<td>.985</td>
<td>.039</td>
<td>45</td>
<td>.970</td>
<td>-.46429</td>
<td>11.92605</td>
<td>-26.22895</td>
<td>25.30038</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal variances not assumed</td>
<td>-.039</td>
<td>12.337</td>
<td>.970</td>
<td>-.46429</td>
<td>12.00204</td>
<td>-26.53544</td>
<td>25.60686</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 shows that there was no statistically significant difference in the marks for the experimental group (M = 51.3; SD = 9.1) and control group (M = 51.8; SD = 7.8); t(45) = -0.039; p = 0.97). These results of the pre-test confirmed that the two groups (experimental and control) were of comparable/similar geometric ability before treatment; as such, any differences in geometric ability after treatment could be attributed to the treatment.

In the post-test, the experimental group post-test average score (M = 61; SD = 19.65), was higher than that of the control group (M = 44.76; SD = 21.21) as shown in Table 1.

Independent samples t-test for the post-test was carried out to check whether there was a significant difference between the two groups’ achievements in the post test. The result is shown in Table 3.

**Table 3: Independent samples t-test for post-test**

<table>
<thead>
<tr>
<th>t-test for Equality of Means</th>
<th>Levene's Test for Equality of Means</th>
<th>F</th>
<th>Sig.</th>
<th>T</th>
<th>Df</th>
<th>Sig.</th>
<th>Mean Difference</th>
<th>Std. Error Difference</th>
<th>95% Confidence Interval of the Difference</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal variances assumes</td>
<td>.578</td>
<td>.315</td>
<td>2.710</td>
<td>45</td>
<td>.009</td>
<td>16.24000</td>
<td>5.99258</td>
<td>4.17033</td>
<td>28.30967</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal variances not assumed</td>
<td>2.724</td>
<td>44.86</td>
<td>.009</td>
<td>16.2400</td>
<td>5.96289</td>
<td>4.22914</td>
<td>28.25086</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3 shows that there is a statistically significant difference in post-test marks of experimental group (M = 61; SD = 19.65) and control group (M = 44.76; SD = 21.21); t(45) = 2.71; p = 0.009). The groups’ post-test average scores at the Van Hiele levels of geometric understanding are shown in Table 4.

Table 4: Groups’ post-test average scores at Van Hiele levels.

<table>
<thead>
<tr>
<th>Group</th>
<th>Number of students</th>
<th>Level 1 (7 marks)</th>
<th>Level 2 (14 marks)</th>
<th>Level 3 (21 marks)</th>
<th>Level 4 (28 marks)</th>
<th>Level 5 (10 marks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>25</td>
<td>5.28</td>
<td>9.44</td>
<td>8.28</td>
<td>8.6</td>
<td>4.28</td>
</tr>
<tr>
<td>Experimental</td>
<td>22</td>
<td>7</td>
<td>13.45</td>
<td>10.36</td>
<td>12.41</td>
<td>5.59</td>
</tr>
</tbody>
</table>

The results show that the average of the experimental group was higher than the average of the control group at all the Van Hiele levels of geometric understanding. The independent samples t-tests for the groups at the Van Hiele Levels of geometric understanding were computed. The results are shown in Table 5.

Table 5: Independent samples t-test of the groups achievements at each of the Van Hiele Levels.

<table>
<thead>
<tr>
<th>Level</th>
<th>Levene's Test for Equality of Variances</th>
<th>t-test for Equality of Means</th>
<th>95% Confidence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
<td>Sig.</td>
<td>T</td>
</tr>
<tr>
<td>Level 1</td>
<td>Equal variances assumed</td>
<td>58.4</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Equal variances not assumed</td>
<td>5.03</td>
<td>24.00</td>
</tr>
<tr>
<td>Level 2</td>
<td>Equal variances assumed</td>
<td>20.740</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Equal variances not assumed</td>
<td>5.253</td>
<td>29.381</td>
</tr>
<tr>
<td>Level 3</td>
<td>Equal variances assumed</td>
<td>.053</td>
<td>.819</td>
</tr>
<tr>
<td></td>
<td>Equal variances not assumed</td>
<td>1.442</td>
<td>42.644</td>
</tr>
<tr>
<td>Level 4</td>
<td>Equal variances assumed</td>
<td>2.489</td>
<td>.122</td>
</tr>
<tr>
<td></td>
<td>Equal variances not assumed</td>
<td>1.730</td>
<td>37.399</td>
</tr>
</tbody>
</table>
The Table shows that at Van Hiele Level 1 there is a statistically significant difference between average marks of the experimental group (M = 7; SD = 0) and the control group (M = 5.28; SD = 1.95); t(45) = 4.71; p = 0) in favour of the experimental group at the first (visual) level of van Hiele levels of geometric understanding. Similarly, there is a statistically significant difference between the average post-test marks of the experimental group (M = 13.35; SD = 1.18) and control group (M = 9.32; SD = 3.73); t(45) = 4.98; p = 0) level 2 of the van Hiele levels of geometric understanding.

On the contrary, the table shows that there was no statistically significant difference between the average marks of the experimental group (M = 10.36; SD = 5.18) and the control group (M = 8.28; SD = 4.66); t(45) = 1.45; p = 0.15) at the third level of van Hiele levels of geometric understanding and also at the levels four of van Hiele levels of geometric understanding (experimental group, M = 12.23, SD = 7.55; control group, M = 8.88, SD = 5.37; t(45) = 1.77; p = 0.084) and five van Hiele levels of geometric understanding (experimental group, M = 5.64, SD = 3.49; control group, M = 4.28, SD = 3.32); t(45) = 1.36; p = 0.18).

**Discussion**

The study indicated a significant difference in achievement in favour of the students taught with GeoGebra. The students exposed to GeoGebra achieved a higher average score compared to the control group of students. The possible reasons for this finding could be that GeoGebra enabled students in the experimental group to check the correctness of their methods and the accuracy of their work. Being able to check one’s own work goes a long way in determining achievement levels. Because GeoGebra is dynamic, students in the experimental group had opportunities of re-examining their work, while those in the control group could not do the same easily.

The production of neat and accurate sketches which aids visualisation of geometric figures requires competence in technical drawing skills, which many teachers and students do not possess. However, GeoGebra-generated sketches are neat and accurate. GeoGebra allowed students in the experimental group real-time exploration geometric properties. Consequently, this improved the learning process in terms of speed and quality (Ljajko & Ibro, 2013). When students learn using GeoGebra they spend less time drawing diagrams (sketches) and making calculations; this allows them more time to explore the characteristics of different circle theorems. All these factors could have contributed to the superior achievement of the experimental group.

It is virtually impossible to have passive students when computer technology, such as GeoGebra, is used in the teaching and learning process. GeoGebra changes passive students to independent explorers and the role of the teacher is to direct and monitor students’ work. Mathematical concepts and procedures learnt using GeoGebra are long-lasting and better incorporated into students’ cognitive structure, which makes them easier to apply (Ljajko & Ibro, 2013).
The findings of this study agree with those of Okoro and Etukudo (2001) and Karper, Robinson and Casado-Kehoe (2005) that students taught with CAI packages in chemistry, mathematics and education in general, perform better than those taught with normal classroom instruction.

This study also concluded that the use of GeoGebra in the teaching and learning of circle geometry compared to conventional talk-and-chalk teaching and learning methods results in a significant difference in Levels 1 and 2 but not in the other three levels. Contrary to many studies on technology integration that have tended to wholesomely portray the notion that technology integration yields significant positive differences when compared to traditional teaching methods, this study has shown that significant change depends on the Van Hiele levels. GeoGebra could have enabled students to recognise and name different circle theorems and also state the angle properties of those theorems because it is highly interactive and offers countless opportunities to repeat tasks and view them several times, hence students were able to internalise concepts at only these two levels (1 and 2).

This finding is similar to Donevska-Todorova’s (2015) finding that analysis of mathematical concepts can be made easier by instructing students using technological devices and software because technology offers multiple representations rather than single and static representations that the teacher talk-and-chalk method offers. The results of Levels 1 and 2 are also similar to those obtained by Venkataraman (2012), who found that students taught with GeoGebra made progress towards mathematical explanations which provide a foundation for further deductive reasoning in mathematics (Levels 1 and 2).

The results of the other three levels (abstraction, deduction, and rigour) did not show any statistically significant difference of achievement between the experimental group and control group. This is possibly because at these levels students need to independently carry out sequences of logical analysis and presentation of a specific theorem. In this case, GeoGebra might not help much in attaining correct results.

**Conclusion**

The study found that the use of GeoGebra had positive impact on students’ achievement at levels 1 and 2 of Van Hiele levels of geometric understanding. Based on the findings of the study, the researcher recommends GeoGebra assisted instruction in the teaching and learning of circle geometry. As the outcomes from this study show, a mathematics curriculum enriched by GeoGebra may significantly help to improve students’ learning and achievement. Though this study was limited to circle geometry, it makes significant contribution to research on students’ learning of an aspect of geometry using technology in rural South African school context.

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