STUDENT TEACHERS' AUTHENTIC INTRODUCTION TO MATHEMATICAL MODELLING: A DESIGN-BASED APPROACH

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ABSTRACT–In this paper; the authors report on the process of getting third year student teachers gradually acquainted with mathematical modelling through a design-based research (DBR) approach over two iterations. In accordance with the 'Learning to be' framework, the inclusion of real-life examples and applications worldwide is an essential component of mathematics curricula - also in South Africa. Such authentic examples might enhance the modelling competencies of students but it depends largely on educators who are well prepared to teach modelling. Local and international research reports confirm the challenges of mathematics educators when teaching modelling. Hence, this study was conducted to prepare student teachers for these challenges. The mixed methods-generated data reveal the development of their competencies and a pertinent improvement in their motivation.

Keywords: Mathematical Modelling; Student Teachers' Mathematical Modelling Competencies; Student Teachers' Motivation; Design-based Research.

1. INTRODUCTION

Traditional problem solving and mathematical modelling are connected as both contribute towards authentic learning experiences. Kang and Noh (2012) view problem solving as a special case of modelling. Although problem solving activities (containing carefully defined information and asking for specific procedures) were part of school mathematics curricula for many years, modelling-eliciting activities (conversing open-ended real-world scenarios with no explicit procedures) were only introduced over the last decade (Kang & Noh, 2012). The theme was also incorporated into the South African Grade 10 to 12 mathematics curriculum (CAPS, 2011), promptly expecting teachers and students to take part in modelling activities. The authentic environment created through these activities requires students to move through the stages of a modelling cycle, making sense of the real-world problem through collaboration, variable introduction, model formulation, procedural application, interpretation of results and model validation (Balakrishnan, Yen & Goh, 2010; Kang & Noh, 2012). Through such learning opportunities, teachers might gain access to their students' thinking and planning strategies.

Characteristics of the mathematical knowledge domain teachers require for teaching include the comprehension of mathematical procedures and results, as well as an understanding of student engagement, thinking and reasoning (Adler & Davis, 2006). Questions have been raised whether mathematics teacher education programmes provide opportunities to adequately prepare teachers for their task (Adler & Davis, 2006). Furthermore, the gap between teacher education and practice is a transferred problem over many years of research and Korthagen (2001) advises teacher education should start with student teachers' experiences. Teachers will only fully appreciate the benefits and importance of model-eliciting activities in the learning environment if they themselves were exposed to such tasks during their formal education. Moreover, research reports (Jacobs & Durandt, 2017; Stillman, Galbraith, Brown & Edwards, 2007) address some recentness challenges experienced by teachers in the teaching and learning of mathematical modelling and on fluctuations in their attitudes towards such activities. Even if student teachers enjoy and value modelling activities, they often lack confidence and motivation to solve and to teach it. The ideal is thus to close the perceived gap between teacher education programmes and practice by developing student teachers' required mathematical and modelling competencies and by inspiring a growth mind-set.

The purpose of this paper is to track and report on the mathematical and modelling competencies, as well as the motivation levels of a group of third year mathematics student teachers at a public university who has been taking part in authentic modelling activities, over a period of approximately six months, involving two iterations. The two research questions are: (1) what are the nature of the thinking and planning strategies of participants confronted with modelling activities over a period of two iterations, and (2) are there any changes in their level of motivation based on their exposure? This design-based inquiry forms part of a broader project, which intends to close the gap between the theory and practice of mathematical modelling, by designing a set of teaching and learning principles.

2. THEORETICAL PERSPECTIVES

The theoretical framework underlying this inquiry is rooted in the *Learning to be* philosophy of Bruner (1973), as cited in Amory, Gravett and Van der Westhuizen (2008), who distinguishes between two types of learning in becoming a practitioner of knowledge and a professional domain. To prepare students for a complex world and to enable them to act purposefully in situations they are going to encounter, learning in teacher preparation programmes should be significant. Bruner's first type of learning is learning *about*, which involves the learning of facts, concepts and procedures, and the second type, learning *to be*, refer to the practices of the knowledge domain. Learning, according to Bruner, requires the utilisation of the conceptual framework and theories of the domain, giving rise to practices of inquiry, interpretation and the eventual solving problems.

Teaching and learning about mathematical *models* is not the same as teaching and learning about mathematical *modelling*. With the former, the emphasis is the product and its efficiency, as it would play a role in decision-making. Kang and Noh (2012) refer to six common principles in evaluating a model, namely (1) accuracy, (2), realism, (3) precision, (4) robustness, (5) generalisability and (6) fruitfulness. In the teaching and learning of mathematical modelling the focus is on the process to arrive at a real-world solution. This process is cyclic in nature and although many examples exist, the four elementary stages of Balakrishnan et al. (2010) were considered in this inquiry. Their sequential stages start from representing a real-world problem mathematically, then using appropriate mathematics to solve the problem, followed by a sense making of the solution in terms of relevance and appropriateness and then a final reflection to examine assumptions and possible limitations. The inclusion of modelling activities in the curriculum exists either to teach modelling competencies or mathematical content (Stillman et al., 2007). Niss, Blum and Galbraith (2007, P. 12) define modelling competency as "The ability to identify relevant questions, variables, relations or assumptions in a given real world situation, to translate these into mathematics and to interpret and validate the solution of the resulting mathematical problem in relation to the given situation", as well as "the ability to analyse or compare given models by investigating the assumptions being made, checking properties and scope of a given model". Research reports (Stillman et al., 2007) point towards remarkable mathematical accomplishments by students moving through the modelling cycle, but also numerous challenges in changeover between the different stages of the cycle.

Kim and Cho (2016, p. 1572) describes motivation as "a kind of internal status that riggers certain actions, while presenting and maintaining the direction of the actions". An overview of the literature reveals motivation encourages students to become interested in specific mathematical topics and displays the curiosity needed to continue with studying these topics. They argue to support student development of intrinsic motivation, mathematics educators should emphasise not only the teaching and learning of procedures and problem solving strategies, but should also create opportunities to allow students to grow in confidence by doing mathematics. If teachers could pay more attention to the interaction of these factors, the engagement of students in more challenging mathematics could be improved. Put simply, motivation is the reason for engagement in any pursuit.

3. RESEARCH DESIGN AND METHOD

This inquiry was conducted from a *pragmatic* viewpoint (Creswell, 2013) and allowed the researchers a working path to answer the research questions. Data were collected through a structured process of design-based research (DBR) via mixed methods. Anderson and Shattuck (2012, P. 2) define DBR as "A methodology designed by and for educators that seeks to increase the impact, transfer, and translation of education research into improved practice. In addition, it stresses the need for theory building and development of design principles that guide, inform, and improve both practice and research in educational contexts". They further emphasise the importance of the reflection step in all stages of DBR. This inquiry forms part of a broader study conducted over three DBR phases (view table 1). The first phase focused on problem identification and needs analysis, the second on design development and implementation and the third on a reflective analysis and evaluation. In particular, this inquiry fits into the second DBR phase and data were collected over two iterations. The findings of this inquiry enabled the researchers to reflect on and to design the next DBR stage.

Forty-nine student teachers (mean age of 22.4 years) participated in the project in 2015. Except for a few cases, the same participants were involved in both iterations. Unexperienced in school practicum and as modellers, the participants were assigned to 10 groups via proportional stratified sampling, incorporating at least one high(er), one moderate and one low(er) achiever per group. The participants were exposed to a series of carefully planned authentic modelling activities over two iterations (view table 1), each grounded on design guidelines by Tan and Ang (2013) and findings stemming from data collected during the preceding pilot study (Jacobs & Durandt, 2017).

Table 1. Exposure of participants to modelling activities during the DBR phases of the overarching study

DBR Phase 1	DBR Phase 2		DBR Phase 3
	Iteration 1	Iteration 2	
Pilot Study	Rugby World Cup *2015* Modelling Activity (on level 3)	Session 1: Production of a Sweets Factory and Nutrient *Requirements* Modelling Activities (both on level 1)	
		Session 2: *Traffic Flow* Modelling Activity (on level 3)	
		Session 3:	
		Session 4:	

The session in iteration 1 kicked off with an introduction to the purpose of the project, gaining ethical consent from participants, presenting an overview of the modelling cycle, and introducing the modelling activity. During the activity the participants were expected to move through the stages of the modelling cycle (Balakrishnan et al., 2010), and to present their suggested solution to the authentic problem in groups. The activity, labelled *World Cup Rugby 2015*, a level 3 modelling problem (Kang & Noh, 2012), contained information that was open-ended, not suggesting any specific procedure. All groups documented their working on predesigned worksheets. The second iteration (informed by the findings from iteration 1) followed similar design guidelines, although it was implemented over a longer period (four weekly two hour sessions). During this iteration, student teachers were exposed to the role of modellers as well as facilitators of modelling activities. For the scope of this inquiry, only qualitative data collected from the first two sessions were analysed,

focusing on student teachers as they took on the role of modellers themselves and to what extent they demonstrated mathematical and modelling competencies. The activities (viewed as level 1 tasks) in the first session of iteration 2, entitled *Production of a Sweets Factory* and *Nutrient Requirements*, both contained already defined information and suggested a certain procedure (Kang & Noh, 2012). The modelling activity in the second session, labelled *Traffic Flow*, was a level 3 problem. In both sessions group work was documented and submitted for analyses afterwards. Following both iterations, participants (individually) completed a questionnaire based on their experiences. The Attitudes toward Mathematical Modelling Inventory (ATMMI) is a locally tested and adapted (from Schachow (2005)) instrument to gain information regarding student teachers attitudes towards mathematical modelling as a topic. The instrument consists of 40 Likert-scale items grouped in four dimensions; value (10 items), enjoyment (10 items), self-confidence (15 items) and motivation (5 items). For the scope of this inquiry, the researchers only considered data from the *motivation* dimension of the ATMMI and monitored the percentage change over the two iterations.

Qualitative data were analysed by following the direct content analysis method (Hsieh & Shannon, 2005), and the researchers purposefully addressed the trustworthiness of findings (Creswell, 2013). Quantitative data were analysed via the Statistical Package for the Social Sciences (SPSS, version 24) and internal consistency was confirmed by Cronbach's alpha coefficients (in all cases > than .8).

4. FINDINGS

4.1 Qualitative findings

The researchers identified three coding categories from key concepts in the literature (Kang & Noh, 2012; Stillman et al., 2007) to analyse the working documents in both iterations. These coding categories are modelling competence, mathematical competence and model competence. First, findings in the category *modelling competence* relate to how participants embarked on the modelling cycle. They experienced more challenges in iteration 1 than in iteration 2. During the first iteration, not all groups completed all stages of the modelling cycle, although most groups attempted to. One group chose appropriate methods to confirm calculations and eight groups linked their results to the real-world. Five groups indicated a need to acquire additional information and considered implications of decisions. In iteration 2 all groups revealed improved modelling competencies by successfully moving through the cycle. They all identified relevant information from the problem and simplified assumptions to enable mathematics to be applied. Most groups indicated a need to acquire additional data and considered the implications of their respective results. In both iterations, groups were lacking the skill to check their results in an authentic situation.

Second, findings in the category *mathematical competence* relate to the groups' accuracy in their representations of the real-world problem mathematically, their selection of applicable content knowledge and the efficiency of their performing procedures. Although the majority of groups recognised relevant variables from the data and accurately mathematised the problem through both iterations, more groups revealed an improved mathematical competence in the later iteration. This is suggested by the increased number of groups, which selected appropriate mathematical formula (consistent with the representation), while also correctly applying mathematical procedures. Generally, in the later iteration 90% of groups selected appropriate mathematical formula consistent with their representations and 70% of groups correctly performed mathematical calculations, likewise comparing to 60% of groups in iteration 1.

Findings in the third category, *model competence*, relate to the precision, validity and applicability of the distinct mathematical models. Throughout the analysis the researchers' focused on the efficiency of the mathematical models and the sensibility of solutions – not their mathematical abilities as such.

Document analyses indicate that 80%+ of group models in the later iteration provided practical solutions based on correct assumptions. They all then made sensible suggestions for improvement of their mathematical models and discussed its applicability. In contrast, during the early iteration less than half of the groups' models delivered neither practical output values nor useful conclusions, although 60% were realistic (based on original correct assumptions). It seems model competence is more closely related to mathematical competence, as expected, and it became more challenging for groups to provide practical solutions and sensible suggestions if their output values were inaccurate.

4.2   Quantitative findings

This paper only reports on possible changes in the motivation of participants in the modelling activities from the first to the second iteration. Only 43 participants completed both the pre and post dimensions on motivation and other data were therefore excluded. Figure 1 displays two parallel Boxplots comparing the distribution of average motivation pre-scores and post-scores. An average score for the *motivation* dimension (maximum 25) for each participant is calculated by dividing the total by 5. Motivation pre-scores indicate a lower mean value than post-scores (3.51 versus 3.73), with a smaller interquartile range (1.00 versus 1.20). Outliers were detected, two cases in pre-scores and only one case in post-scores. The maximum value (5.00) for post-scores was higher than the maximum value for pre-scores (4.80).
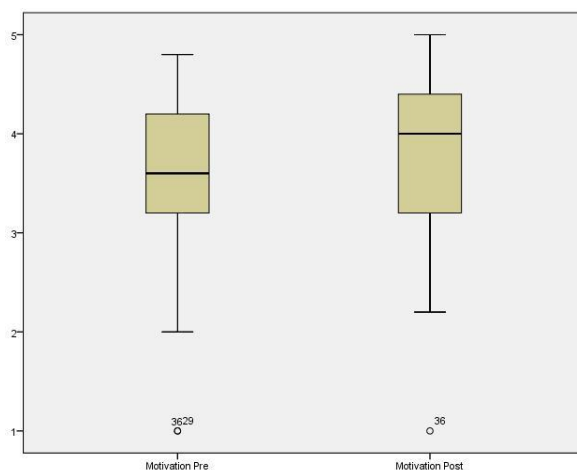

Figure 1: The distribution of average motivation pre-scores versus post-scores

Table 2 presents the Wilcoxon Signed Ranks indicating the change in motivation scores between the pre and post intervention. The motivational level of 25 participants increased while that of 14 participants decreased. Four participants sustained their motivational levels. The percentage change in motivation is calculated as the difference between the motivation post-score and pre-score totals, divided by motivation pre-score total, times 100. Table 3 illustrates the output values of a one-sample t-test. The t-test reveals that participants' percentage change in motivation scores between the two iterations are significant (M = 13.34, SD = 40.570, p < .05). It can thus be deduced (only in respect of this study at a 95% level of confidence) that student teachers' on average experience a positive change in motivation when participating in modelling activities more frequently.

Table 2. Wilcoxon Signed Ranks Test

		N	Mean Rank	Sum of Ranks
Post Motivation scores – Pre Motivation scores	Negative Ranks	14[a]	19.14	268.00
	Positive Ranks	25[b]	20.48	512.00
	Ties	4[c]		

		Total	43			

a. Post Motivation scores < Pre Motivation scores.
b. Post Motivation scores > Pre Motivation scores.
c. Post Motivation scores = Pre Motivation scores.

Table 3. One-Sample Test

	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval	
Change	2.156	42	0.037	13.34177	0.8560	25.8275

5. CONSLUSION

This inquiry monitors and reports on the mathematical and modelling competencies, as well as the motivation levels of a group of third year mathematics student teachers at a public university who participated in authentic modelling activities over two iterations. It was found that student teachers mathematical, modelling and model competencies improved as they are more often exposed to authentic problems, through a well-planned series of activities. These activities included level 1 and 3 model-eliciting tasks. Although student teachers still lack particular skills (to a greater or lesser extent) such as examining assumptions and checking results in the final stage of the modelling cycle, accurately performing mathematical procedures, and making sensible suggestions regarding their respective models, their desire to engage with modelling activities has grown significantly from the first iteration to the second. This paper is a precursor to further research, to close the gap between theory and practice of mathematical modelling through a well-structured set of guiding principles that progressively build student teachers' competencies and motivation towards such activities.

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