MISCONCEPTIONS ASSOCIATED WITH KINEMATIC GRAPHS IN PHYSICS

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ABSTRACT - Graphical knowledge, core component of the science curriculum, is an essential skill for any technological innovation. A lack of such a skill is a reflection of poor conceptual understanding. To assess students' understanding of kinematic graphs, we have explored the use of Beichner's (TUG-K, version 3.2) diagnostic test for eliciting graphical misconceptions amongst South African students. This data was disaggregated into gender groups to determine if there was any bias towards any particular group. Results were statistically analysed between the various gender groups. The results indicate that students experienced considerable difficulty in virtually all aspects of the test survey. Further, it is revealed that students experienced many difficulties based on misconceptions that are deeply rooted in their prior school knowledge. Interpretation of kinematic graphs using the variables position, velocity and acceleration appears to be the most problematic section in the science curriculum. These sections are the "building blocks" for the later sections to follow. It is suggested that educators modify their teaching instruction to remedy this paltry situation. This has a serious impact on students wishing to pursue higher education studies in science or other related fields of study.

Key words: Graphical; Misconception; Skill; Resolve and Kinematic.

1. INTRODUCTION

Graphs are commonly used in many gateway subjects such as mathematics and science to convey vital information, yet students have difficulties in interpreting graphs (Zucker et al., 2013), especially if it involves variables of a kinematic nature such as position, velocity and acceleration. The difficulties that students experience with graphs has been around for a few decades (Beichner, 1994; McDermott et al., 1987). It is a known fact that students of all ages have difficulties in interpreting graphs (Zucker et al., 1998), even students at honours level (McDermott et al., 1987). In this respect, it could be inferred that their knowledge is brittle and much compartmentalised to what was initially taught at school level (Zucker et al., 2013). For students to attain good skills at kinematic graphs, they will have to undergo some developmental process, which cannot be fully achieved from school textbooks or from the way this section is done at most schools. It can be said that for those that acquire success in this aspect must have drawn inspiration from other resources such as journals for which the information they got was more informative and more descriptive (Bowen et al, 1998). Thus in becoming a scientist, the skills that they have allows them to work comfortably with the section of graphs (Beichner, 1994), which is an integral part of any experimentation and the heart of science (Mckenzie et al., 1986).

Many students have difficulties in making connections between the various kinematic variables and its related concepts to real world applications. Instead some of them often perceive graphs as a literal picture of a determined motion, for example, a rising line with a steep gradient is interpreted as a hill (Leinhardt et al., 1990) or any straight line to represent constant velocity.

Many researchers are looking for alternate forms of instruction in order to improve the students understanding of kinematic graphs. Some have used the microcomputer-based labs (MBL) to show how kinematic variables (position, velocity and acceleration) are measured. This approach has proved to be useful, since according to Dykstra et al. (1992), students use the MBL activities to help them to arrive at a differential view of kinematic variables instead of an undifferential view of them (Hale, 2000). In this scenario, students observe graphs as a realistic representation of motion. Other researchers such as Kekule (2015) probed students' deeper level of thinking through eye-tracking research methods. A similar approach was also undertaken by Madsen et al. (2012) and Kozhevnikov et al. (2007) following eye-tracking methods. Kozhevnikov et al. (2007) considered a position vs time graph and implored students to describe the motion of a curve that remained constant for a while and then decreases to zero. Students were required to describe the graph in terms of spatial symmetry. Students of high spatial ability spent a lot of time studying the axes and the line segments in more detail as compared to students with low spatial ability that had only focussed a lot of time in describing the graph as a physical picture.

Other researchers, such as Merhaz et al. (2009) considered reversing the order of traditional teaching to include experimentation as a precursor to teaching of fundamental concepts. This method which they regarded as taking a minimum of teaching time proved to be very valuable and achieved desired success than the other way around. Some researchers have used SmartGraphs, a web-based software, as an addition to formal teaching with the objective of helping students overcome their misconceptions regarding kinematic graphs (Zucker et al., 2013). This method although used as a supplement to normal teaching is a way forward in remediating some of the problems teachers face with this section.

In terms of evaluation of teaching, there are many approaches followed by researchers. Some of them use the Multiple Choice Questions (MCQ) (Beichner, 1994) and other use the interviewing technique (McDermott et al., 1987). Whichever method that is used has its specific benefit. Whilst the interviewing technique is most fruitful, the MCQ method is also useful in that it provides a diagnostic tool to detect misconceptions. In this report we will only consider the latter method to detect misconception in graphs of a kinematic nature.

The aim of this study was to replicate a similar study undertaken by Beichner (1994) to find:

1. Similarity between the misconceptions (and additional ones) experienced by South African learners and those of the developed world,
2. If there was any gender bias in performance towards any particular group, and
3. Possible areas for pedagogical instructional change.

2. METHODS

2.1 Participants

A sample of first year university physics students comprised of 31 females and 52 males pursuing physics disciplined study participated in this study. Prior to the administration of the Test, students were informed that the study was voluntary and that they would not be disadvantaged in any way and that the marks they got from such an evaluation would not jeopardise their semester marks in any way. Permission was also sought from the learners and the teachers (responsible for teaching these classes) before administrating the Test. Learners were also informed about the confidentially of this test and the conditions stipulated by the designers of the test. This was due to the fact that this test questionnaire had taken many years to develop and was regularly used for assessments.

2.2 Instrument and Procedure

This study made use of a well-established Test assessment on graphical misconceptions, known as Test of Understanding of Graphs in Kinematics (TUG-K) developed by Beichner (1994). We obtained written permission from Prof Beichner to conduct a similar research amongst South African learners, but that the test itself should be carefully monitored. Copies of the unused tests should not be floating amongst learners and care was taken to collect every written test paper administered. This instrument is an ideal instrument to measure graphical misconceptions and has been used extensively at many international universities. The Test itself consists of 26 questions and 5 most commonly known mistakes or distractors made by learners. The results from this assessment will be benchmarked against the participants from Prof Buchner's cohort of students. This evaluation also required the students to indicate their gender as well as the grades they obtained from the National Senior Certificate (NSC) examinations. The duration of the Test was one hour. Also a t-test was done to show the statistical differences between the performances of South African students to the students of Beichner's group.

Note to the evaluator: Prof Beichner wants confidentially of these tests and requested that the examiners refer to the website for a copy of the test. The website address is: http://www.physPort.org/assessment, and once in this website select “assessments”. There are many assessments in this website, but the one we are concerned with is the: Test of Understanding of Graphs in Kinematics (TUG-K) and the version is 3.2. The website will require one to register and a code will be provided.

2.3 Conceptual framework

Literature study, in particular studies undertaken by Beichner (1994) and McDermott et al. (1987), have captured the relevant theories that could point towards meaningful comparisons between the competence of South African learners and those from the developed world. The main focus of this study is an interpretation of a questionnaire, eliciting misconceptions from students of a kinematic nature. In this respect, Beichner has highlighted the following student difficulties:

1. Misconception of graphs as a picture representation of an event.
2. A mix-up between slope and height in a graph.
3. Confusion in determining the slope of a curve that does not pass through the origin.
4. Inability to interpret the area under a given curve.

McDermott et al. (1987) also highlighted similar difficulties experienced by students. In this study, graphical misconceptions will be compared to these 4 points. Further, it will be gauged how students will make use of existing knowledge (from school experiences) to interpret new information presented to them. The important question is whether they have matured or their mind-set is still deeply rooted in the knowledge learnt at school.

3. RESULTS

Table 1 show the number of learners that have selected a particular item in the Test. Results are further aggregated along gender lines to determine if there was any significant bias towards any particular group. Each of the items (number 1 to 26) of the Test refers to TUG-K questionnaire. Results are expressed as a percentage (for 31 females and 52 males).

Item No	Distractors: Females					Distractors: Males				
	A	B	C	D	E	A	B	C	D	E
1	23	39	6	10	23	17	38	10	23	12
2	10	23	42	6	19	2	27	35	0	37
3	13	0	39	29	19	12	0	50	29	10
4	0	19	23	13	45	8	17	21	12	42
5	0	0	16	58	26	4	4	19	58	15
6	52	23	0	6	19	48	29	4	8	12
7	10	10	48	32	0	10	27	46	17	0
8	10	23	52	6	10	12	21	29	19	19
9	0	52	6	29	13	8	65	10	4	13
10	22	52	3	0	23	42	23	10	6	19
11	45	19	32	0	3	37	13	25	15	10
12	13	16	23	32	16	19	13	25	33	10
13	19	48	3	29	0	44	23	8	25	0
14	29	29	6	26	10	31	40	2	19	8
15	26	26	26	0	23	31	21	25	6	17
16	3	32	10	13	42	10	25	15	6	44
17	16	13	3	58	10	21	25	2	46	6

18	13	19	13	35	19	8	17	19	46	10
19	3	35	35	23	3	4	35	29	31	2
20	3	19	29	35	13	2	15	35	37	12
21	6	29	42	16	6	4	19	25	44	8
22	16	26	13	6	39	13	25	25	8	29
23	6	6	23	29	32	4	17	17	31	31
24	6	81	6	6	0	2	75	8	5	10
25	13	3	10	68	6	8	8	19	56	10
26	16	10	3	55	16	8	21	12	40	19

In Table 2, we have the grade 12 physical science results for both females and males.

Grade range (percentage)	2 (30-39)	3 (40-49)	4 (50-59)	5 (60-69)	6 (70-79)	7 (80-100)
Females	0	1	1	10	14	5
Males	0	0	5	15	22	10

Of the 26 questions that were answered by the students, only 3 questions (14, 15 and 19) and 6 questions (10, 13, 14, 15, 19 and 20) were correctly answered by the males and females, respectively. This amounts to a mere 12% (females) and 23% (males) of the candidates that were able to identify the correct options to the Test and a clear indication that both genders were struggling to cope with the interpretation of kinematic graphs from a South African point of view.

As far as the grades were concerned, in both cases the average grades between the males and females were 6 (70-79%). Potentially these students were above average and were expected to perform well for this kind of assessment.

3.1 Analysis of individual items in the Test

Questions that displayed the largest discrepancies between the correct answer and the largest incorrect answer between the males and females responses were the following questions: 5, 7, 8, 9, 11, 16, 17, 18, 24, 25 and 26. Only 3 of these questions (5, 24 and 25) showed commonality in conceptual misunderstanding between the 2 cohorts of learners. Of all questions from the Test, a question 5 was the easiest and question 1 was the hardest to comprehend. In question 5, 58% of both the males and females have chosen option D. Further, of the questions mentioned, in particular questions 5, 8 and 11 referred to the position vs time graph of motion. Students have had considerable difficulty in determining gradient of lines that does not pass through the origin for these graphs. In determining the gradient of the line in question 5, students could have performed the following calculation:

Gradient (at t = 2s) = $\frac{\Delta v}{\Delta t} = \frac{10-0}{2-0} = 5\ \mathrm{m/s}$.

A similar calculation could have been done for the gradient in question 18:

Gradient (at t = 3s) = $\frac{\Delta v}{\Delta t} = \frac{3-10}{3-0}$ = -2.3 m/s.

For this question, 46% of the males and 35% of the females choose option D, in obtaining a velocity of v = -2.3 m/s at t = 3s instead of the correct option A. A small percentage of the males (8%) and females (13%) obtained the correct answer for this question in the questionnaire.

A large majority of the female (52%) students and a smaller percentage of male (29%) students have chosen option C for question 8. In choosing this option, they have treated the graph as a picture representation of the real problem. Some male students (21%) and female students (23%) have described this graph as an object that rolled down a hill before coming to a rest. Students have physically translated the velocity vs time graph of question 11 exactly as a picture representation of its position vs time graph. This was done by 37% of the male and 45% of the female students, while only a small percentage of these students (15% for the males and 0% for the females) have chosen the correct option D for this question. This thinking is further replicated in question 9 in describing an object starting from rest and undergoing constant positive acceleration for 10 seconds before continuing at a constant velocity. For this question, the 65% of the male students and 52% of the female students choose this incorrect option B.

For question 16, a vast majority of students have chosen option D (44% for the males and 42% for the females) as opposed to the correct option C (6% for the males and 13% for the females). A common mistake made by these students was a mix-up between the gradient of a line vs area under a curve to determine the velocity of an object within a stipulated time interval. A possible explanation for the answers given by the students:

1. Gradient of the straight line: $\frac{\Delta v}{\Delta t} = \frac{2-0}{3-0} = 0.67\ m/s$ (25% of the males and 32% of females choose this option).

2. Area under the curve: Area = a x t = 2 x 3 = 6 m/s (44% of the males and 42% of the females choose this option).

3. Area under the a-t curve: Area = ½ b x h = ½ (3) (2) = 3 m/s (6% of the males and 13% of the females choose this option).

4. Change in velocity: $\Delta v = \frac{2m}{s} - 0\frac{m}{s} = 2\ m/s$ (15% of the males and 10% of the females choose this option).

5. Rate of change in velocity: $\frac{\Delta v}{\Delta t} = \frac{3-0}{2-0} = 1.5m/s$ (10% of the males and 3% of the females choose this option).

A possible misconception associated with this question was the use of the words "change in velocity" or the rate of change in velocity. In this respect, the possible answers as indicated in 4 and 5 above could have been a possible explanation.

The students' responses to question 17, which required them to give a description of a constant velocity versus time graph of a moving object. The physical picture representation of the moving object yet again appears for this question. Since the velocity is a constant for a certain time interval, the students were still stuck with the idea that the object has a constant, non-zero, acceleration (46% of the males and 58% of the females choose this option), unaware of the fact that the gradient of the velocity versus time graph gave the acceleration, and area under the velocity versus time of the same graph gave the displacement of the object.

Students have struggled with question 18, as they have struggled with question 5. They struggled to determine the gradient of the line in a position versus time graph. Most of the males (46%) and females (35%) have chosen option D instead of option A (8% for the males and 13% for the females).

Although questions 19 and 20 have been identified as questions for which both cohorts of students got correct, their responses to the rest of the distractors in these questions raised some alarming concerns. For example in questions 19 and 20, the percentage of students that got these questions correct against their responses to the other distractors, reveals a large degree of uncertainty, as could be seen below:

Question 19: Correct option B (35% for the males and 35% for the females), incorrect option C (29% for the males and 23% for the females).

Question 20: Correct option D (37% for the males and 37% for the females) and incorrect option C (35% for the males and 29% for the females).

Whilst a somewhat systematic approach is given of how to determine the displacement of an object from a velocity versus time graph, some students were confused of when to use area or slope to solve these kinds of problems (question 19). The same argument prevailed for a similar type of problem given in question 16. If they thought that finding the area under the curve was the correct way of solving this problem, then much must be said about their mathematical skills because they treated finding the area of a triangle as base x height (31% of the males and 23% of the females choose this option). For the answers given for question 20, reveals a lack of conceptual understanding since they looked at any graph that showed a positive gradient (diagrams II, IV and V) or any permutations of them. In choosing these options, care was not taken about the ordinate (dependent) components of the graph. Preference for positive gradient graphs can be seen from their choices: 35% for the males and 29% of the females choose option C (incorrect), 35% for the males and 35% for the females choose option D (correct), 15% for the males and 19% for the females choose option B (incorrect) and 12% for the males and 13% for the females choose option E (incorrect).

A translation of the velocity versus time graph in question 21 to a position versus time graph was reasonably well done by both cohorts of students since although 19% of the males and 29% of the females choose the correct option B, a further 44% of the males and 16% of the females choose option D which was a close replica to the correct answer B. The velocity of the object between 1 and 2 seconds in their calculation was:

$v = \frac{\Delta s}{\Delta t} = \frac{2-1}{2-1} = 1\mathrm{m/s}$ instead of $v = \frac{\Delta s}{\Delta t} = \frac{3-1}{2-1} = 2\ \mathrm{m/s}$.

This error has a compounding effect on their calculation of the value of velocity in the last 3 seconds of the object's motion.

In a similar scenario to question 20, where students were required to identify graphs that displayed a constant non-zero acceleration, students choose any combinations of the graphs that showed a constant acceleration (diagrams III and V) in question 22 without looking at the ordinate parameter to this problem. This is also true for question 26 where students were limited to the following diagrams II, III or IV to show that the velocity of an object was uniformly increasing.

3.2 Similarity in misconceptions between Beichner's cohort of students and South African students

In this section, we highlight the disparity between the correct and incorrect answers to the Test questionnaire.

Physical (visual) description of kinematic graphs

1. It appears that both the South African students and the international students have made the same mistake when describing a decreasing velocity versus time graph in question 24. Most of the international students (72%) have chosen option B (uniformly decreasing acceleration) and South African students have also chosen option B (81% of the females and 75% of the males) instead of the correct option A. In this question the object has to move with a constant but not uniformly decreasing acceleration.

2. In question 8, the international students were divided in their responses to the options given. They have chosen C (37%) and D (37%) as opposed to option C (52% of the females and 29% of the males) for South African students. It appears that both cohort of students have focussed on the visual description for this graph, and in the case of South African students, they have not fully conceptualised the meaning of gradients of kinematic variables.

3. For question 12, the international students were in overwhelming agreement that the best option for this question was B (67%) as opposed to South African learners who were split between 2 options C (23% for the females and 25% for the males) and D (32% for the females and 33% for the males). Only a small percentage of South African students have chosen option B, since they were more focussed on a physical description of a positive gradient or any permutations of the graphs that reflected that shape.

4. For question 9, the responses from South African students were almost identical to their international counterparts. Both the females (52%) and males (65%) students have chosen option B, while the international students have chosen option B (57%). For this question, the students found much difficulty in expressing a position versus time graph in terms of acceleration versus time and therefore choose option B as their best option.

5. For question 6 which required students to determine the velocity from an acceleration versus time graph that did not pass through the origin, the international students have chosen option A (45%). Likewise South African students have chosen option A (52% for the females and 48% for the males). Students generally have a problem of finding the differences in the y and x-values of the graph.

4. DISCUSSION

Judging from the number of questions that both the males (6 out of 26) and females (3 out of 26) obtained for the Test, it is clear that the test was beyond their level of comprehension, and a poor level of kinematic understanding. From a statistical point of view, it was also found that males have performed a little better than the females in the Test ($t = 2.24$, $df = 81$, $p < 0.02$). This value of t is much lower than the value that Beichner (1994) obtained from his research in respect of both genders ($t = 5.66$, $df = 491$, $p < 0.01$). In those results a vast majority of students got at least 13 out of 21 questions correct for a slightly reduced version of the same test. It is surprising that despite these students, whose APS scores were on average a score of 6 (70% to 79%) and who were exposed to a semester of physics (Newtonian Mechanics), including calculus based mathematics at a South African university were struggling to interpret kinematic graphs at a fundamental level. This concurs with Beichner's (1994) line of argument that students were doing no better after spending some time working on graphs of a kinematic nature. Students came to the university with many difficulties associated with graphs that were deeply rooted in their personal experience with the section (Hale, 2000). To remedy these misconceptions will be a challenge for them since it is difficult for them to abandon the concepts they learnt at school and to replace them with new concepts. They must confront these misconceptions head-on as they travel through a journey of self-revelation.

A close analysis of the student responses from the Test reveals many areas of concern. Students have much difficulty in determining slopes of lines that did not pass through the origin. As a common practice, students tended to take the upper value of the ordinate (y value) without subtracting the lower ordinate value, and divided this value by the maximum time for the object's motion. This was fine if it was a straight line but a bit problematic if the position versus time graph consisted of time intervals where the curve was a straight line and sections of the graph consisted of curves. Some of the students were erroneously finding the slope of a line from acceleration versus time graph instead of finding the area under the curve. This was evident from their responses to the following questions:

Question 1, question 10B (52% of the females and 23% of the males), question 20C (29% of the females and 35% of the males), 22B (26% of the females and 25% of the males), 25B (3% of the females and 8% of the males) and question 26D (including graph IV) (55% of the females and 40% of the males choose these options. Stemming from these misconceptions was a mix-up between the concepts of area and slope in kinematic graphs (Lea, 1993). This error was frequently observed when students attempted to determine the gradient of lines that did not pass through the origin, especially in question 1. If a graphical description of an acceleration versus time graph was required, they often linked it to a physical picture of a body having a constant velocity and did not see the implication of time in this problem ($v = at$).

Question 1, 10 and 16 which required students to determine the "change in velocity" of an object from acceleration versus time graph was most poorly done. They had literally calculated $\Delta v = v_1 - v_2$ for this problem. Judging from the responses, it seemed that a fair amount of students have done these problems in this format: Question 16C (10% of females, 23% of males) and question 10C (10% of females, 23% males). The determination of area from a velocity versus time graph was not clearly evident from their responses to find the distance (d = vt) travelled by the object.

The physical description of a curve as a picture (Beichner, 1994) was most evident from both cohorts of students. Their description of a graph of having a positive or negative gradient has been determined without taking into consideration the ordinate variables (position, velocity, acceleration) of the graph. It seemed that students with a lower spatial ability tended to focus on the physical picture as compared to students with a high spatial ability that tended to focus on the key features of the graph.

Further, a translation from one kinematic situation to another was another area of concern for these students. Their translation of the graphs in question 11 (position versus time), question 14 (velocity versus time), question 15 (acceleration versus time) and question 21 (velocity versus time) to other kinematic variables in most cases was almost a picture representation of them. Question 11 was a good illustration of what an exact translation of a position versus time graph to a velocity versus time graph looked like (45% of the females and 37% of the males choose this option), and clearly a thinking of lower spatial ability. A replica representation of the velocity versus time graph to acceleration versus time graph was also observed from their responses to question 15.

Their mathematical skills in their approach to kinematic graphs were average. Some students treated the area of a triangle to be base x height and some were unable to differentiate between a steep and shallow gradients or positive and negative gradients, even though they were exposed to a semester of engineering mathematics. This can be observed from their responses to questions 2 and 11.

5. CONCLUSION

There appears to be much similarity between the misconceptions on kinematic graphs experienced by students from developed world to students of South African origin. However, South African students looked at graphs from a superficial point of view. This has serious implication for pedagogical instruction. Much has been said about males doing better than females in mathematics and science in research and the same scenario is also applicable here. If the international students could achieve a higher performance in kinematic graphs then the same should be applicable to these students as these questions were not above their cognitive level. Even though these students have an average APS score of 6, coupled with the fact that they have been exposed to a semester of teachings in mathematics and science, this has had a least impact on their performance. It seems that their mind-set is deeply rooted in the knowledge learnt at school. Uprooting such knowledge would be a challenge for both the students and the lecturer in order to bring these students in par with the international students.

In terms of pedagogical instruction, there are many sections that the teachers need to look at:

1. Area and gradient mix-up: Teachers need to be more pragmatic in their approach in teaching each of these sections. Graphs of all shapes without reference to the kind of motion should be given as an exercise and students should be asked to depict (and describe) the type of motion that unfolds. However, other approach of doing an experiment first before conceptualisation appears to have worked successfully from international studies. Other methods, such as computer simulation will imprint the kind of knowledge that is required. The time evolution of any motion is of utmost importance and is a key for their understanding of kinematic graphs.

2. Shape interpretation: In many cases in this study, students have transformed graphs from one kinematic variable exactly into another without a deeper conceptual understanding of the situation. Such errors could have been easily remedied by explaining the meaning of each of these graphs. Students need to move forwards and backwards between different kinetic variables (in particular to

graphs) to ensure that if the y variables are changed, they implicate a new meaning and does not always have the same shape.

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