IDENTIFYING SUBSCALES OF VISUALISATION IN A LITERACY TEST FOR ENGINEERING STUDENTS

Hanti Kotze
University of Johannesburg
South Africa
hantik@uj.ac.za

Gerrie J Jacobs
University of Johannesburg
South Africa
gjacobs@uj.ac.za

Erica Spangenberg
University of Johannesburg
South Africa
ericas@uj.ac.za

ABSTRACT– This paper aims to develop and refine a literacy test to measure the construct of visualisation. The ability to reason visually and to fluently interpret multiple representations is a cognitive process referred to as visualisation. Most engineering subjects make use of symbolic, numeric and graphic representations. Unlike symbolic approaches and routines, visualisation with computer algebra system (CAS) technology demands cognitive versatility since visual processes tend to be mostly non-procedural and unpredictable. Engineering educators are faced with a dual concern: incoming students' poor visualisation and the swelling demand for CAS-competent recruits in the engineering workplace. A cohort of second year engineering students participated in a quantitative visualisation literacy test measuring various theory-related components of visualisation. Items were either sourced from literature or custom-designed, both curricular topics and unrelated contexts were selected to gauge students' visual reasoning in novel settings. Using exploratory factor analysis, three subscales of visualisation were identified – translation, new insights and visual reasoning which explained 45% of the total variance. Students underperformed in items relating to unconventional contexts and also struggled to reason visually, even within curricular contexts. A visualisation literacy test can help to identify students' visualisation strengths and weaknesses and facilitate the transition between visually-deprived and visually-demanding learning environments.

Keywords: Visualisation; computer algebra system; factor analysis, engineering diploma students.

1. INTRODUCTION AND BACKGROUND

Engineering graduates of the 21st century face a technologically-driven world of work that are different from that of a few decades ago. In their critical analysis on "being educated in the 21st century", Prinsloo and Louw (2006:288) highlight the obligation of universities to create new contexts with new technologies that can lead to a new type of professionalism in global citizenship. Higher Education Institutions (HEIs) therefore need to continuously revise course content and align teaching and learning strategies with the practices endorsed by the engineering industry (The Royal Academy of Engineering, 2007). This inspired Mathematics educators to experiment with computer algebra systems (CAS) – such as Mathematica – which supports a symbolic, numeric and graphic interface (Schmidt, Rattleff & Hussmann, 2010).

While fostering such ideals, the reality facing South African learners in Basic Education is a world apart. School curricula do not promote the use of visualisation which can create visualisation deficits for many students who enter engineering programmes. Moreover, many South African schools experience a lack of educational and human resources and the effects thereof reverberates to Mathematics departments in HEIs (Howie, Scherman & Venter, 2008). When students are predominantly schooled in analytical procedures, a technology-rich environment implicates a shift from the traditional paper-and-pen-based techniques. The interpretation of computer graphs and tables demand conceptual skills which co-evolve with theoretical knowledge and technological skills (Kieran & Saldanha, 2008). With these different foci, the school-to-higher Mathematics transition is problematic. In a CAS environment, new obstacles come to the fore as students lack epistemological cogency to bridge the "cognitive discontinuity" (Yerushalmy, 2005:37) caused by a new computer language and new demands on their visualisation with CAS.

Striving to better understand incoming students' visualisation status, the following research questions are posed: *What are the subscales of visualisation required from engineering students studying in a CAS environment? How do students' visualisation strengths and weaknesses compare with their pre-entry performance?* This paper therefore aims to identify the subscales of visualisation through a literacy test.

2. THEORETICAL PERSPECTIVES

2.1. Theoretical framework

Long before the notion of CAS was coined, Janvier (1987) described the source-target framework. This framework encapsulates four different modes of representations encountered in most mathematical manipulations, whether that be with or without CAS. The process of "going from one mode of representation to another" is defined as a translation (Janvier, 1987, p. 27). The source-target framework as seen in Table 1 represents a 4x4 matrix. Crisscrossing the matrix from source to target, one can follow either a direct path or an indirect path. For instance, the third row of Table 1 suggests that the translation from table (as source) to formula (as target) has a direct path and an indirect path. The direct route would be table $\rightarrow$ formula as is custom in reading off a matching row-column pair. Alternatively, one can follow an indirect path, lingering at the stopover-points and execute the process as table $\rightarrow$ situation $\rightarrow$ graph $\rightarrow$ formula. The reverse equivalent of this translation would be from formula (as source) to graph (as target). For this purpose, row five of Table 1 proposes the direct path formula $\rightarrow$ graph and alternatively, the indirect route formula $\rightarrow$ situation $\rightarrow$ table $\rightarrow$ graph. According to Janvier, the cognitive processes involved in the indirect route are "substantially different" from those required in the direct route (p. 29). The thrust of the matter is that CAS exclusively utilise the direct version where cognitive processes are outsourced to the computer.

Table 3: Translation process within four representations (Janvier, 1987, p. 28).

To: / From:	Situations and verbal descriptions	Tables	Graphs	Formulas
Situations and verbal descriptions		Measuring	Sketching	Modelling
Tables	Reading		Plotting	Fitting
Graphs	Interpretation	Reading off		Curve fitting
Formulas	Parameter recognition	Computing	Sketching	

2.2. Visualisation in a CAS environment

When students are largely schooled in analytical approaches and routines, visualisation with CAS technology may demand cognitive versatility for which students are not prepared. Natsheh and Karsenty (2014) explored the visualisation of Palestinian students in grade 12 and concluded that teachers are oblivious to visualisation and its potentials in Mathematics Education. This resonates with Presmeg (2014) who voices concerns that "we cannot simply expect students to be visual upon arrival at university". In the South African higher educational landscape, little attention has been given to the role of visualisation in the teaching and learning of Engineering Mathematics. In her study involving engineering students at the University of Witwatersrand, Berger (2010) found that the skills required for the interpretation of CAS outputs are different from that in a paper-and-pen environment. Unlike symbolic approaches and routines, visualisation with CAS technology demands cognitive versatility that tends to be mostly non-procedural and unpredictable.

Various components of visualisation have been researched, especially by the Israeli savants including Arcavi, Hershkowitz, Karsenty, Natsheh and Yerushalmy. Of late, the focus has been on visual reasoning (Natsheh & Karsenty, 2014). In the Palestinian context, Natsheh and Karsenty found that the visualisation repertoire in school Mathematics merely involves the use of illustrations. Tasks where images are employed to solve problems, to make inferences and to interpret and reason meaningfully are absent. They concluded that such tasks could in fact enhance cognitive insights and argued for the inclusion of such opportunities in the school curriculum. Increasingly, researchers (Hershkowitz, 2014; Nardi, 2014) have related to visualisation as a specialised mode of thinking, a language wherein patterns, symmetry and dimensions are building blocks to problem-solving and visual thinking. These may be regarded as stepping stones to translations (Arcavi, 2003), which is at the heart of visualisation with technology. In a CAS environment, translations involve the activity of interacting with diagrams, symbols, numbers and/or written text, all of which are different but equivalent forms of the same concept. The translation of multiple representations is therefore a cognitive process that also require technological and procedural skills.

The implication is that unrehearsed students may be unprepared to master cognitive, non-algorithmic visualisation challenges posed in a CAS domain. Visualisation literacy translates into the mastery of visualisation. Mudaly (2010, p. 67) uses the parabola to illustrate the point:

Visual literacy would refer to many pictures of parabolas, each one conjuring images that show parabolas having minima and maxima, the different shapes of the parabola, the effect of changing the values of c and the coefficients of x^2 *and* x*. An entire series of possibilities flashes through the mind.*

However, Duval (2014, p. 167) has a more prudential view and refutes the nouns 'skill' or 'competence' which should only be "applied afterwards in an assessment perspective".

3. RESEARCH DESIGN AND METHODOLOGY

3.1. Paradigm and research approach

This study espouses a constructivist paradigm. Championed by Piaget, constructivism proposes a balance between assimilation and accommodation (Riegler, 2012). Assimilation acknowledges that new experiences are dependent and governed by previous experiences. Methodologically, students' experiences need to be integrated through the discovery of new insights that correspond with their reality. From this perspective, accommodation involves the facilitation of assimilation. A "modification of the cognitive apparatus" strives to fit the new experiences through empirical activities so as to progressively advance knowledge (Riegler, 2012, p. 241). Lecturers are thus tasked to modify existing structures in innovative ways to accommodate and assimilate students' reality.

This study follows a quantitative design. Exploratory factor analysis and in particular, principle axis analysis, was used to extract different subscales of visualisation. Factor analysis is used in educational studies to develop and refine assessment instruments such as tests and questionnaires where underlying dimensions of a latent variable are measured (Williams, Brown & Onsman, 2012). A salient use of factor analysis is the reduction of variables with a common structure into a composite factor; hence assisting in the refinement of theory. The IBM SPSS Statistics version 23 was used to conduct all statistical analyses.

3.2. Participants

A 2015 cohort of 113 second year students registered for Engineering Mathematics 3 (EM3) participated in a pre-test and post-test which formed part of a quasi-experiment. The pre-test coincided with a Mathematica semester test while the post-test was synchronised with an exam. Complementary to the theory lectures, CAS is used to study matrices and explore differential equations (DE) numerically. The latter topic is introduced in Engineering Mathematics 2 (EM2), a first year module which is a prerequisite to EM3.

3.3. Data collection

3.3.1 Biographical data

Biographical data was collected from 110 students one week after the pre-test was written; three students (2.7%) were absent. Table 2 reflects the participant profile. The majority of students were male (87%), were between 23 and 24 years old (37.2%), came from the Limpopo province (35.4%), passed EM2 with a final mark of between 50% and 59% (50.4%), had a personal computer or laptop (78.8%) and owned a prescribed textbook (89.4%). Data was used from all participants who wrote the pre-test and post-test, regardless of missing biographical data.

Table 4: Participant profile according to biographical data.

	Frequency	Percentage
Gender		
Male	87	76.9
Female	23	20.4
Age group		
19-20	11	9.7
21-22	41	36.3
23-24	42	37.2
>24	16	14.1
If South African – home province		
Limpopo	40	35.4
Gauteng	27	23.9
Mpumalanga	14	12.4
North West	11	9.7
KZN	10	8.8
Eastern Cape	4	3.5
Free State	1	.9
If non-South African, home country		
Burundi	1	.9
DRC	1	.9
Mozambique	1	.9
Final Engineering Mathematics 2 mark (%)		
50-59	59	52.2
60-69	34	30.1
70-79	14	12.4
80-100	6	5.3
Available learning materials		
Laptop/PC	89	78.8
Textbook	101	89.4

3.3.2. The literacy test

The pre-test and post-test involved 13 items each; all test items were assessed on a dichotomous scale that awarded either 1 for a correct answer or 0 for an incorrect answer. Each item carried an equal weight to the test total. For the purposes of this study, only the pre-test is analysed and henceforth referred to as the literacy test. A variety of visualisation characteristics were incorporated into 13 items as seen in Table 3. This included four types of representations namely textual, symbolic, numeric and graphic. The use of Mathematica software and pocket calculators was allowed during the test. Students had 25 minutes to complete the test and were requested not to guess but rather leave a question unanswered if they didn't know the answer. This way, scores would not be affected while guessing would not impact on reliability (Yu, 2001).

Table 5: Visualisation characteristics incorporated in literacy test according to literature.

Visualisation characteristic	Test item	Literature source
Connect and translate different representations	A1.1 -1.4, A2.1-2.4, A3, A5, A4.1-4.3	Arcavi (2003)
Develop new understandings	A2.1, 2.2	Arcavi (2003)
Coordinate different tools	A2.2	Arcavi (2003)
Discover unknown ideas	A2.3	Arcavi (2003)
Represent new concepts	A2.3	Arcavi (2003)
Interpret new diagrams, images and graphs	A2.4	Arcavi (2003)

Communicate information	A2.4	Arcavi (2003)
Enact visualisation as proof for new insights	A2.1-2.4	Natsheh & Karsenty (2014)
Integrate procedural and conceptual knowledge	A4.1-4.3	Arcavi (2003)
Visual reasoning	A4.1-4.3,2.3, A3, A5	Nardi (2014)

Most of the test items were sourced from literature but three questions were specially created; these are exhibited in Figure 1. Other test items appear in Appendix A. The three custom-designed items are typical translations encountered in solving DE with CAS; conceptual knowledge of differentiation and integration was required as well as visual reasoning.

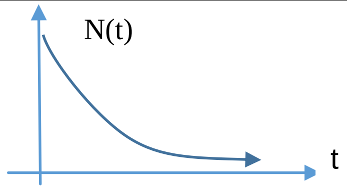
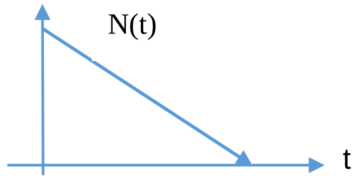
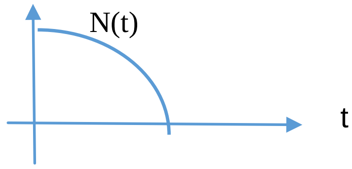
For each of the following differential equations on the left, find a matching solution curve on the right if $c > 0$:	
4.1 $\frac{dN}{dt} = -c$	A 
4.2 $\frac{dN}{dt} = -cN$	B 
4.3 $\frac{dN}{dt} = -c\,t$	C 

Figure 10: Three custom-designed items of the literacy test.

3.4. Ethical considerations

The study was set in a natural educational setting and formed part of the normal curricular schedule of EM3. By scheduling the literacy test to coincide with a Mathematica semester test, formal assessment conditions and regulations could be implemented. An additional advantage was that formal assessments usually attract the highest attendance since certain students (repeating students who work full-time) do not attend weekly lectures. Therefore, no one was excluded or negatively affected.

3.5. Reliability and validity

The internal consistency of the 13 items of the literacy test was calculated with Cronbach's alpha coefficient (α). Where an instrument has more than 12 items, the desired α should be around .7 (Field, 2009). Alpha was found to be .642 and ranged between -.213 and .791 in the inter-item correlation matrix. Subsequent to factor analysis, two items were removed (Q3 and Q5) due to poor factor loading (Field, 2009). This resulted in an improved alpha of .732 and the inter-correlation of the remaining 11 items improved to range between -.122 and .846. Field suggests that items of an instrument that are related be grouped together to form a subscale and alphas be applied separately to subscales. Once subscales of visualisation were extracted with factor analysis, alpha values were calculated and are recorded in Table 4.

Table 6: Reliability of extracted factors.

	Cronbach's α

Factor 1	.847
Factor 2	.688
Factor 3	.552
Overall	**.732**

The literacy test was moderated by another subject expert. Disagreements on test scores were settled by mutual consent. The inter-rater agreement was calculated by means of Pearson's product-moment correlation coefficient (r = 0.81). Data captured in SPSS was verified by the same subject expert. Missing data was recorded as such but a zero score was allocated in test totals (Yu, 2001). For face validity, two tutors were asked to inspect the test and judge whether the contents summoned visual activity. The mainstay of factor analysis is the measurement of a construct and the identification of items that should be discarded to enhance construct validity; therefore, this research design confirmed construct validity (Williams, Brown & Onsman, 2012). Content validity of the literacy test was secured by incorporating ten items (out of 13) from literature. The curricular context and CAS environment were deliberated and test items were aligned according to students' projected visualisation.

4. DATA ANALYSIS AND DISCUSSION

4.1. Factor analysis

4.1.1 Suitability of data

Several tests were performed to validate the suitability of data for factor analysis. Firstly, the sample size has to be greater than 100 and the sample to item ratio can range from 3:1 up to 20:1 (Williams, Brown & Onsman, 2012). With 113 students in the sample and 11 test items, the ratio in this study is 10:1 and therefore falls well within these restrictions.

Secondly, the factorability of the correlation matrix needs to be inspected. The correlation matrix is given in Table 5 and contains the Pearson correlation coefficients (r) of the literacy test items. The correlations between items are read off in cross-wise pairs. In particular, the off-diagonal coefficients can be scanned for clusters that may reveal test items that correlate well with certain items but not with items outside that cluster (Field, 2009). In Table 5, potential clusters are indicated with overlaying triangles. These three triangles help to identify correlation coefficients of items that correlate with at least one item in each triangle within the threshold of $r = \pm.30$ (Field, 2009). Essentially, acceptable correlation coefficients in-between items already suggest that certain items could be measuring subscales of the same underlying construct.

Table 7: The correlation matrix of the literacy test items.

	A1.1	A1.2	A1.3	A1.4	A2.1	A2.2	A2.3	A2.4	A4.1	A4.2	A4.3
A1.1	**1.000**	.695	.565	.565	.239	.289	.072	.042	.016	.115	.199
A1.2	.695	**1.000**	.677	.587	.253	.220	.180	.149	.014	.169	.177
A1.3	.565	.677	**1.000**	.427	.369	.069	.135	.045	.039	.125	.103
A1.4	.565	.587	.427	**1.000**	.283	.150	.264	.068	.040	.088	.193
A2.1	.239	.253	.369	.283	**1.000**	.272	.393	.222	.063	.163	.070
A2.2	.289	.220	.069	.150	.272	**1.000**	.437	.365	-.054	.137	.111
A2.3	.072	.180	.135	.264	.393	.437	**1.000**	.417	.043	.226	.133
A2.4	.042	.149	.045	.068	.222	.365	.417	**1.000**	.000	.092	.149
A4.1	.016	.014	.039	.040	.063	-.054	.043	.000	**1.000**	.319	.285
A4.2	.115	.169	.125	.088	.163	.137	.226	.092	.319	**1.000**	.276
A4.3	.199	.177	.103	.193	.070	.111	.133	.149	.285	.276	**1.000**

Thirdly, the Kaiser-Meyer-Olkin (KMO) statistic that measures the sampling adequacy and Bartlett's Test of Sphericity were performed. Like all other statistics, KMO values range between 0 and 1. A lower threshold of .50 indicates that data is suitable for factor analysis (Field, 2009). Since the KMO statistic for this study was .731, it can be concluded that factor analysis would be appropriate to explore the data. The null hypothesis of Bartlett's Test of Sphericity is that the correlation matrix should resemble an identity matrix. Under this assumption, the zeros off the main diagonal of the identity matrix would imply that there is no correlation between any of the test items. However, this would defeat the purpose of factor analysis namely, to establish the “underlying dimensions between measured variables and latent constructs” (Williams, Brown & Onsman, 2012, p. 2). Consequently, data would only be compliant with factor analysis if the null hypothesis is rejected and Bartlett’s Test of Sphericity is significant ($p < .05$). The data passes Bartlett’s Test of sphericity χ^2 (55) = 180.133, $p < .001$ which indicates that correlations between test items were sufficiently large to conduct factor analysis.

4.1.2 Data extraction and criteria

The most common extraction methods used in factor analysis are principle component analysis (PCA) and principle axis analysis (PA). Researchers are advised to decide whether they want to explore data or test a certain hypothesis. In case of the former, PCA or PA should be selected and in the latter case, confirmatory factor analysis would be more suitable (Field, 2009). Regarding exploratory factor analysis, there are different views on the method of choice. PCA is preferred when no *a priori* theory is available to the researcher (Williams, Brown & Onsman, 2012). On the other hand, Field (2009) is of the opinion that “principal component analysis and principal axis analysis are the preferred methods and usually result in similar solutions” (p. 636-637).

While various extraction criteria are available, Williams, Brown and Onsman (2012) report that it is advisable to use multiple extraction techniques. These include Kaiser’s criterion, the Scree test and verification of the cumulative percentage of variance extracted. Firstly, Kaiser’s criterion states that only eigenvalues with a magnitude greater than one (Field, 2009) need to be accommodated. Since the visualisation test had a small number of test items ($n = 11$), a slightly adjusted lower bound of 1.14 was set for the extraction of eigenvalues.

Secondly, a scree plot was generated to visualise the one-on-one relationship between all possible factors and their associated eigenvalues (Williams, Brown & Onsman, 2012). These authors regard the interpretation of scree plots as subjective and flexible. Typically, a scree plot displays a sudden vertical drop and then tapers off as the eigenvalues contract horizontally. A point of inflection (Field, 2009) can be obtained by constructing two dotted lines as indicated in Figure 2. Guided by the Kaiser criterion, all eigenvalues less than 1.14 are connected with an almost horizontal dotted line. Aiming for the steepest gradient, a second dotted line is constructed to connect the largest eigenvalue with the eigenvalue closest to the Kaiser threshold. Only factors above the point of inflection are retained (Field, 2009) so that the scree plot visually suggests that only three factors (of the possible 11) should be extracted.

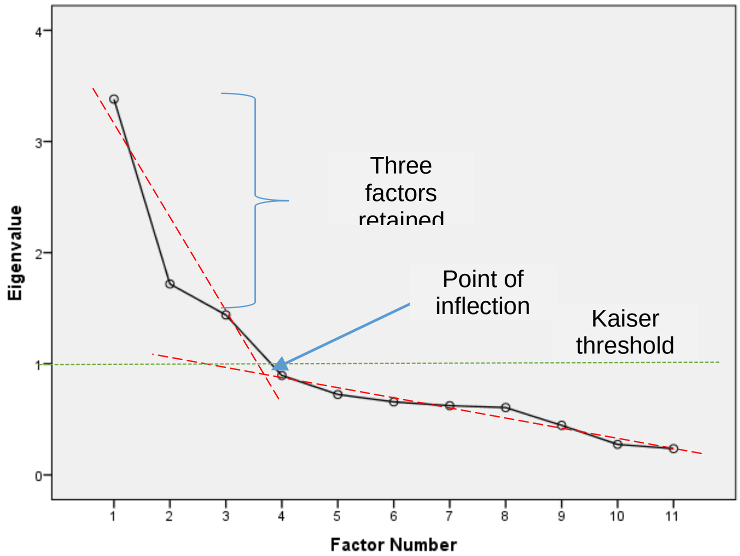


Figure 11: Scree plot with point of inflection as used for factor extraction.

Thirdly, the cumulative percentage of variance explained by the eigenvalues was considered. Table 6 projects the 11 eigenvalues (since there were 11 items in the test) associated with each factor before extraction, after extraction and after rotation (Field, 2009). The initial eigenvalues are listed in descending magnitudes and each eigenvalue represents the variance explained by its associated factor. Before extraction, the respective eigenvalues explained 30.735%, 15.616% and 13.086% of the total variance. Using principle axis analysis as the extraction method, all factors with eigenvalues greater than 1.14 were then extracted. After extraction, the cumulative percentage of the common variance was calculated which would be lower than the initial values for the total variance. Only three factors emerged which explain 44.781% of the cumulative percentage of variance. From here on, statistics for all redundant factors were suppressed by SPSS. The optimised factor structure after rotation is described next.

Table 8: Variance explained for each eigenvalue.

Factor	Initial eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of variance	Cumulative %	Total	% of variance	Cumulative %	Total	% of variance	Cumulative %
1	3.381	30.735	30.735	2.922	26.564	26.564	2.452	22.293	22.293
2	1.718	15.616	46.351	1.203	10.934	37.498	1.513	13.757	36.049
3	1.439	13.086	59.437	.801	7.283	**44.781**	.961	8.732	44.781
4	.895	8.134	67.571						
5	.724	6.581	74.152						
6	.657	5.972	80.124						
7	.623	5.662	85.787						
8	.606	5.509	91.296						
9	.446	4.055	95.351						
10	.275	2.497	97.848						
11	.237	2.152	100.00						

4.1.3 Rotation method

Rotational techniques are applied to maximise high item loadings and minimise low item loadings (Williams, Brown & Onsman, 2012). Effectively, rotation will equalise "the relative importance of the extracted factors" (Field, 2009). The other advantage of rotation is that it simplifies the interpretation of factor analysis. Two rotation methods are available, orthogonal and oblique rotation. Orthogonal rotation was deemed suitable for this study since the underlying factors are assumed to be independent (Field, 2009), this will result in factor structures that are uncorrelated (Williams, Brown & Onsman, 2012). In particular, the orthogonal varimax rotation method with Kaiser Normalisation was used for the data in an attempt to "maximize the dispersion of loadings within factors" (Field, 2009, p. 644). The rotation algorithm of choice produces a factor model with adjusted Pearson correlations that differs from the original correlation matrix. These differences are indicative of the closeness of the fit of the factor model; the smaller the differences, the better the fit. By default, SPSS generates a residual matrix (omitted). A summary revealed that 18% of residuals had an absolute value greater than 0.05 where the recommended (Field, 2009) percentage should be less than 50%. Principle axis analysis generated a rotated factor matrix that revealed factor loadings of all test items onto the three rotated factors (Field, 2009). The rotated factor matrix is displayed in Table 7 where factor loadings of .4 are considered to be important (Field, 2009).

Table 9: The rotated factor matrix.

	Factor 1	Factor 2	Factor 3
A1.2	.865		
A1.1	.800		
A1.3	.720		
A1.4	.632		
A2.3		.762	
A2.2		.584	
A2.4		.556	
A2.1		.415	
A4.1			.653
A4.2			.517
A4.3			.452

4.1.4 Labelling of factors

Factor analysis is an exploratory tool and does not depend solely on the interpretation of quantitative statistics (Field, 2009). Guided by *a priori* theory, substantiated by the scree plot and authenticated by the rotated factor matrix, three prominent factors (subscales) of visualisation were identified. Test items A1.1 – A1.4 loaded highly onto factor 1 which was labelled *translation*. The items were all related to the translation from numerical data to the equivalent graphical representation. Factor 2 was labelled enactment of insight in novel situations or in short, *new insights* (Natsheh & Karsenty, 2014). The four test items that loaded onto factor 2 tested various visualisation strands (refer Table 3): communicate and represent information, understanding a new concept, interpret new understandings, construct and interpret a diagram, coordinate different tools, connect different representations and visual reasoning. All these activities related to a novel situation where new insights into each test item were required, hence the label *new insights*. Factor 3 was labelled *visual reasoning*. To link the DE (source) to the graph (target), the last three test items required several stepping stones: analytical manipulations, functional identification, translations from symbolic to graphic representations and coordinated visual reasoning.

4.1.5 Graphical representation of factors

Figure 3 displays a factor plot in the 2-D rotated factor space. *Translation* is represented on the horizontal axis and *visual reasoning* on the vertical axis. The circles represent test items with large coordinates on the translation-axis since they correlate highly with *translation* but weakly with *visual reasoning*. Contrarily, the triangles represent test items with large coordinates on the *visual-reasoning*-axis; these items correlate highly with *visual reasoning* but weakly with *translation*. In general, variables with large coordinates on one axis but small coordinates on the other are related to a unique factor (Field, 2009). The cluster of circles (ellipse) load onto *translation* but the cluster of triangles is best explained by *visual reasoning*.

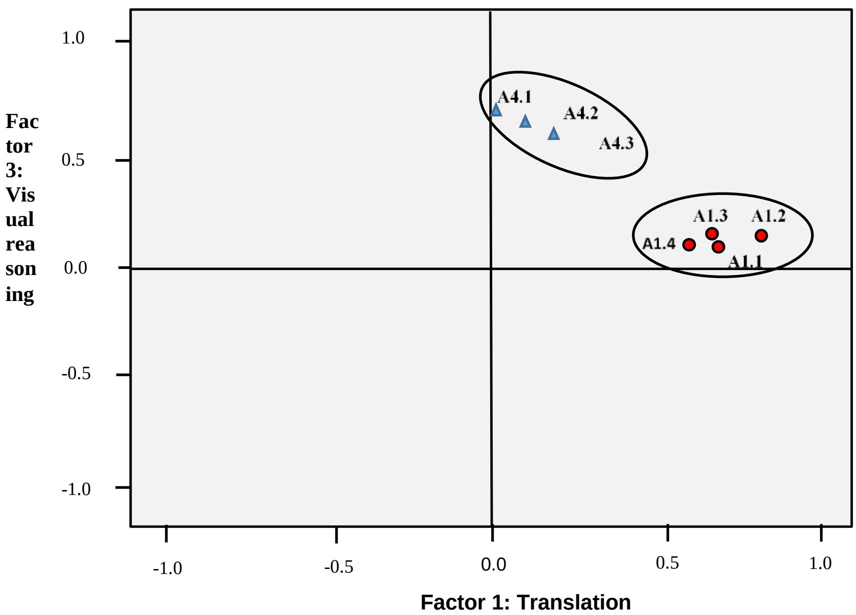


Figure 12: Factor plot projecting translation against visual reasoning.

4.1.6 Missing data

Table 8 lists missing data for each test item and accounts for the number of students who did not answer a particular item. The highest number of missing items in the test was a total of 27 students (24%) who did not answer item A2.2 and 34 students (30%) who left item A2.4 unanswered. Since these items loaded onto *new insights*, these statistics are evidence that students struggled to adapt to novel situations. Item A2.2 required students to numerically verify a symbolic model, hence a translation from one representation to another. In order to answer item A2.4, students had to interpret item A2.3 in their own words. Since only nine missing answers were recorded for item A2.3, it can be concluded that 25 more students were in a position to attempt item A2.4 but did not know how. The intricate connectedness of various visualisation processes possibly intimidated unskilled visualisers.

Table 10: Descriptive statistics of 11 test items.

	Mean	Std. Deviation	Analysis N	Missing N
A1.1	.84	.369	112	1
A1.2	.85	.362	111	2
A1.3	.90	.300	111	2
A1.4	.86	.346	109	4
A2.1	.89	.312	111	2
A2.2	.50	.503	86	**27**
A2.3	.56	.499	104	9
A2.4	.30	.463	79	**34**
A4.1	.35	.480	94	19
A4.2	.17	.375	96	17
A4.3	.28	.449	98	15

4.2 Discussion

Once the test items that loaded onto each factor were identified, composite scores could be calculated for each student in each of the three visualisation subscales (factors). These respective scores were converted to percentages and the means calculated. Figure 4 compares the subscale means with students' pre-entry (EM2) scores. Students performed best in *translation* where the mean scores ranged from 79.82% – 100%. The second best mean scores were recorded for *new insights* with scores between 42.42% and 66.67%. A minimum mean of 14.14% and a maximum mean of 27.78% revealed that students hugely struggled with *visual reasoning*. Regardless of their EM2 scores, students struggled to identify graphical solutions of the DE in items A4.1 – A4.3. The reader will note that the solution to item A4.1, $\frac{dN}{dt} = -c$ involves integration and results in $N = -ct + k$. Subsequently, this equation should be identified as a straight line with a negative slope, hence option B would have been the appropriate answer (refer Figure 1). Although DE were discussed in EM2, it cannot be assumed that students can solve *and* identify the correct solution curve. When analytical procedures are taught in isolation, it compromises effective utilisation of equivalent representations.

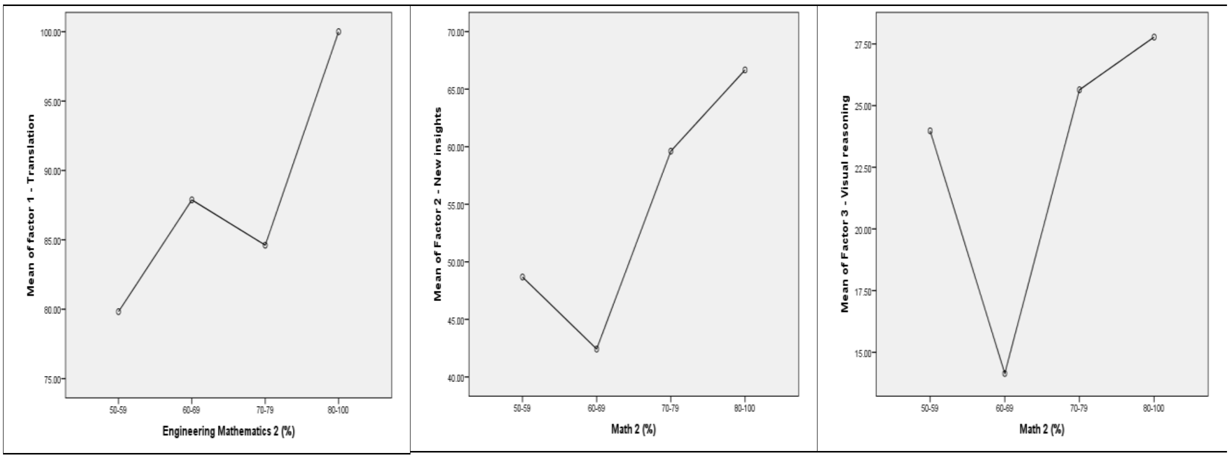


Figure 13: Engineering Mathematics 2 scores compared with three factor means.

Students' pre-entry performance in EM2 was also explored with a boxplot (Field, 2009) in Figure 5. Students with EM2 scores between 50%-59% (n = 57) had the same median (100%) for *translation* as all other students in the cohort. The box-whisker-dispersion indicates that 50% of this group obtained scores of between 50% and 100% (Field, 2009). With only a few outliers, the overall scores for *translation* were high, an indication that most students could translate from numerical to graphical representations. A drop in the *new insights* scores can be observed in all but the top performers (80%-100%). Equal dispersions were recorded for the lower-end performers (60%-69% and 70%-79%) whereas the two remaining top-end scores reflect medians of 50% and 75% respectively. This indicates that overall, students were less capable to enact proof of *new insights* than to translate. The lowest scores in the literacy test were recorded for *visual reasoning*. Considering the respective dispersions of the previous two factors, the sudden drop in dispersion and median scores (mostly zero) suggest that students across the board, struggled most with *visual reasoning*.

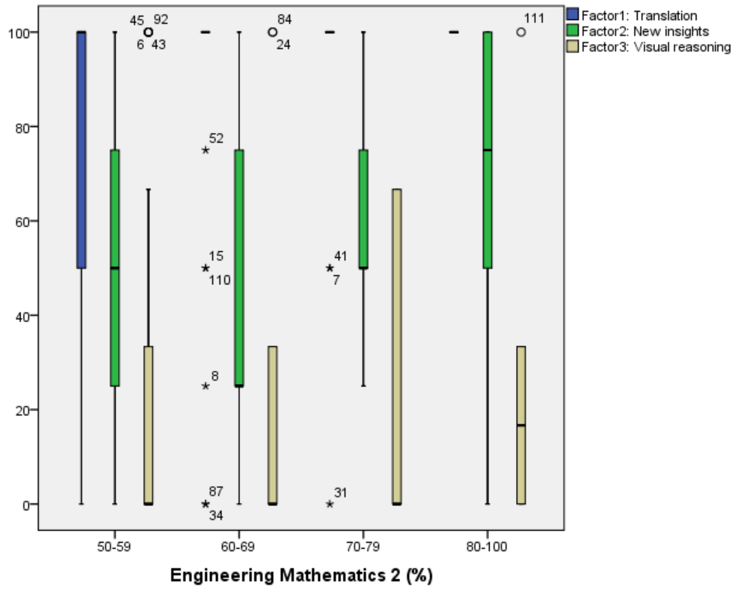


Figure 14: Boxplot with whiskers from minimum to maximum, comparing pre-entry Engineering Mathematics 2 scores with extracted factors.

The availability of a personal computer (PC) or laptop was note-worthy. Figure 6 reveals that students with recourse to a CAS device (which excludes smart phones and tablets) are more likely to adapt to novel situations and display *new insights*. Activities wherein CAS codes are repeated with different inputs, expose students every time to a new output, demanding decision-making and critical evaluation of each new output. In a CAS environment, experimentation can serve as an incubator for deeper understanding and interpretation of diagrams. The availability of a CAS device did not benefit students' *visual reasoning*.

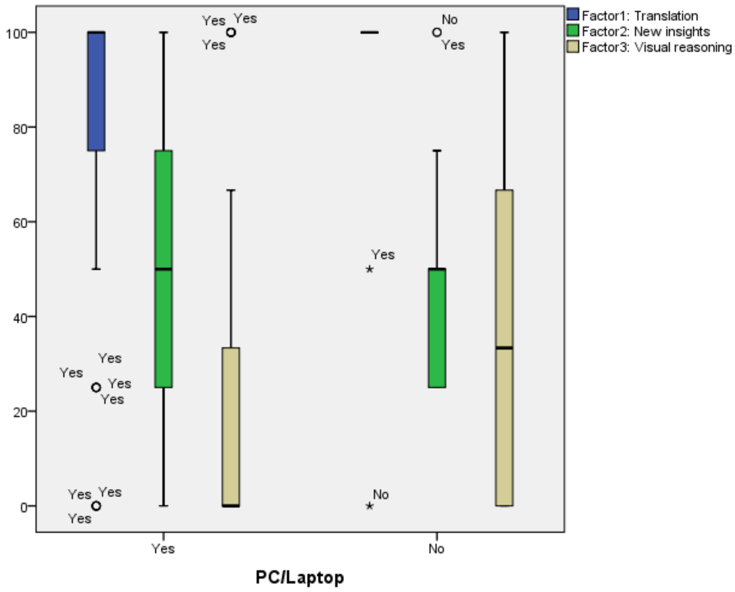


Figure 15: Comparison between extracted factors and the availability of a PC or laptop. Outliers indicate to textbook availability.

5. CODA

When new technologies are introduced into curricula, teaching and learning approaches should co-evolve to keep pace with different demands on engineering students' visualisation. A visualisation literacy test revealed three dominant subscales of visualisation. Students could *translate* in a direct path (in the sense of Janvier) but struggled with translations that required an indirect path. Where cumulative visualisation processes had to be navigated, *visual reasoning* proofed to be the most challenging subscale of visualisation, even for top performers. Many students avoided the test items relating to novel situations where the enactment of *new insights* challenged students on a non-procedural level. A visualisation test can firstly help identify the strengths and weaknesses of engineering students' visualisation. Secondly, it can guide suitable activities or interventions to help engineering students bridge the cognitive gap between visually-deprived and visually demanding environments. It is recommended that, regardless of entry-level scores, incoming cohorts be monitored with a visualisation literacy test in order to identify specific visualisation components in need of remedial activities.

ACKNOWLEDGEMENT

Our sincere thanks to Ms. Juliana van Staden from STATKON for her guidance in statistical data analysis.

REFERENCES

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics*, 52, 215–241.

Berger M. (2010). Using CAS to solve a mathematics task: a deconstruction. *Computers & Education*, 55(1), 320–332.

Duval, R. (2014). Commentary: linking epistemology and semio-cognitive modeling in visualization. *ZDM Mathematics Education*, 46(1), 159–170.

Field, A. (2009). *Discovering statistics using SPSS,* 3rd Edition. London: SAGE.

Hershkowitz R. (2014). Looking at visual thinking and visual communicating in mathematics learning through the lens of examples. In M.N. Fried & T. Dreyfus, *Mathematics & mathematics education: searching for common ground*, 198–204, Dordrecht: Springer

Howie, S., Scherman, V. & Venter, E. (2008). The gap between advantaged and disadvantaged students in science achievement in South African secondary schools. *Educational Research and Evaluation*, 14(1), 29–46.

Janvier, C. (1987). Translation processes in mathematics education. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 27–32). New Jersey: Hillsdale.

Kieran C., & Saldanha, L. (2008). Designing tasks for the co-development of conceptual and technical knowledge in CAS activity: an example from factoring. In G.W. Blume & M.K. Heid (Eds.), *Research on technology in the teaching and learning of mathematics: Cases and perspectives*, Vol. 2, 393–414. Charlotte, NC: Information Age.

Mudaly, V. (2011). Thinking with diagrams whilst writing with words, Pythagoras, 71, 65–75.

Nardi, E. (2014). Reflections on visualization in mathematics and mathematics education. In M.N. Fried & T. Dreyfus, *Mathematics & mathematics education: searching for common ground*, 3–24, Dordrecht: Springer.

Natsheh I. & Karsenty, R. (2014). Exploring the potential role of visual reasoning tasks among inexperienced solvers. *ZDM – International Journal on Mathematics Education*, 46(1), 109–122.

Presmeg, N. (2014). Visualization in the learning and teaching of mathematics. In M.N. Fried & T. Dreyfus (Eds.), *Mathematics & mathematics education: searching for common ground*, 210–220. Dordrecht: Springer.

Prinsloo, P. & Louw, H.A., (2006). Being educated in the twenty-first century: an exploration. *South African Journal of Higher Education*, 20(2), 288–298.

Riegler, A. (2012). Constructivism. In L. L'Abate (Ed.), *Paradigms in Theory Construction*, 235–255, New York: Springer.

Schmidt, K., Rattleff, P. & Hussmann, P.M. (2010). The Impact of CAS use in introductory engineering mathematics. In A.D. Fitt, J. Norbury, H. Ockendon & E. Wilson (Eds.), *Progress in industrial mathematics at ECMI 2008, Mathematics in Industry 15*, 653–659. Berlin: Springer Berlin.

The Royal Academy of Engineering (2007). *Educating engineers for the 21st century*. Available from http://www.raeng.org.uk/publications/reports/educating-engineers-21st-century

Williams, B., Brown, T. & Onsman, A. (2012). Exploratory factor analysis: a five-step guide for novices. *Australian Journal of Paramedicine*, 8(3), 1–13.

Yerushalmy, M. (2005). Functions of interactive visual representations in interactive mathematical textbooks. *International Journal of Computers for Mathematical Learning*, 10(3), 217–249.

Yu, C. (2001). An introduction to computing and interpreting Cronbach coefficient alpha in SAS. *Proceedings of 26th SAS user group international conference*, 1–6.

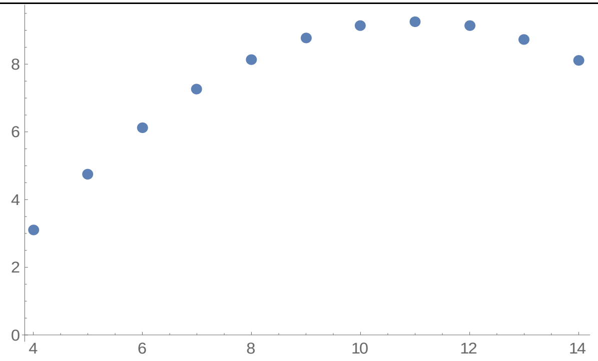
APPENDIX A: Literacy test

Match the **X** and **Y** data on the left with one of the graphs on the right (Eg. A1.1A, A1.2B, A1.3C, A1.4D):

A1.1

X	Y
10.0	8.04
8.0	6.95
13.0	7.58
9.0	8.81
11.0	8.33
14.0	9.96
6.0	7.24
4.0	4.26
12.0	10.84
7.0	4.82
5.0	5.68

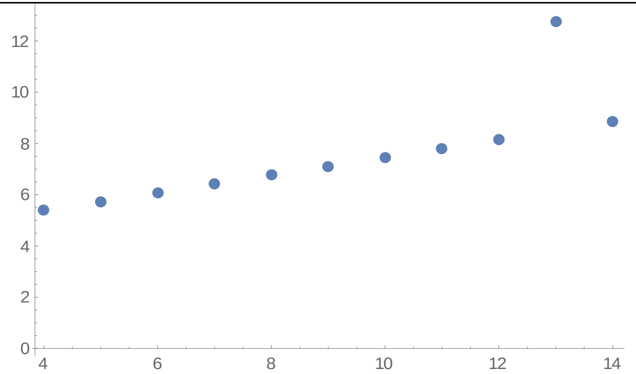
A



A1.2

X	Y
10.0	9.14
8.0	8.14
13.0	8.74
9.0	8.77
11.0	9.26
14.0	8.10
6.0	6.13
4.0	3.10
12.0	9.13
7.0	7.26
5.0	4.74

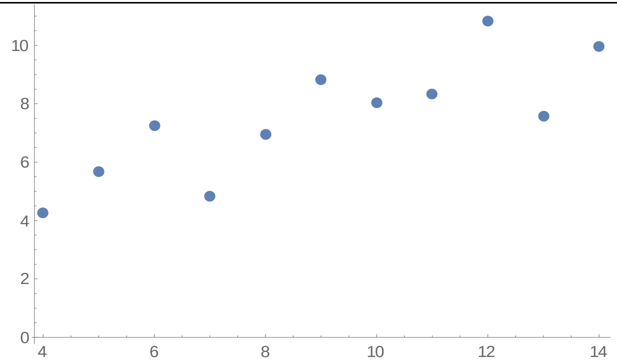
B



A1.3

X	Y
8.0	6.58
8.0	5.76
8.0	7.71
8.0	8.84
8.0	8.47
8.0	7.04
8.0	5.25
14.0	12.50
8.0	5.56
8.0	7.91
8.0	6.89

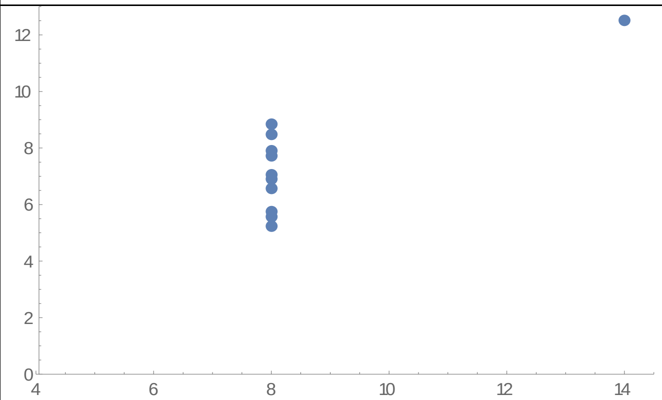
C



A1.4

X	Y
10.0	7.46
8.0	6.77
13.0	12.74
9.0	7.11
11.0	7.81
14.0	8.84
6.0	6.08
4.0	5.39
12.0	8.15
7.0	6.42
5.0	5.73

D



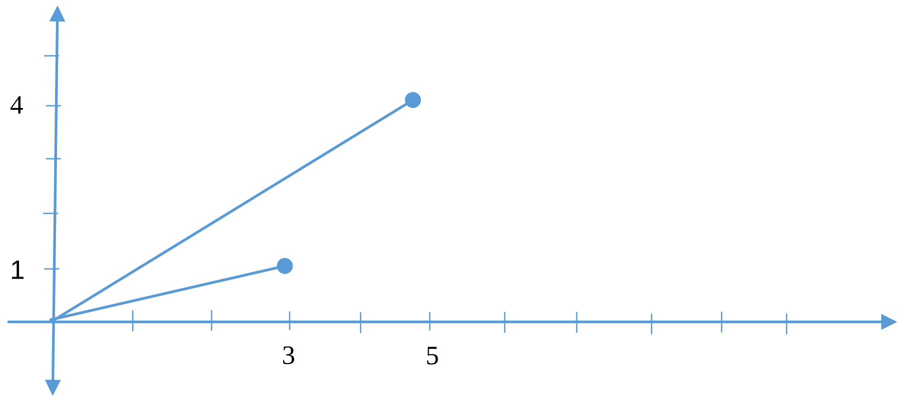
Source: Arcavi (2003, p. 218).

A2. For two positive fractions $\frac{a}{b} < \frac{c}{d}$, the mediant is defined as $\frac{a+c}{b+d}$ such that $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$, that is, the mediant always lies strictly in between the two fractions. Consider the fractions $\frac{1}{3}$ and $\frac{4}{5}$:

A2.1 Calculate the mediant.

A2.2 Show how you can check with your calculator that $\frac{1}{3} < \text{mediant} < \frac{4}{5}$.

A2.3 In the diagram below, $\frac{1}{3}$ is represented as the point (3,1) on the Cartesian plane. The gradient of the line from (0, 0) to (3,1) corresponds with $\frac{1}{3}$. Similarly, the gradient of the line from (0, 0) to (5, 4) corresponds with $\frac{4}{5}$ as shown on the diagram. Draw a line on the diagram that represents the mediant of $\frac{1}{3}$ and $\frac{4}{5}$.



A2.4 Describe how you can use this diagram with the three lines to explain that $\frac{1}{3} < \text{mediant} < \frac{4}{5}$?

Source: Arcavi (2003, p. 220)