ANALYSING GRADE 2 LEARNERS' DIFFICULTIES IN SOLVING WORD PROBLEMS

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ABSTRACT–This article repots a study of the difficulties that primary school children experience whilst tackling school mathematical word problems. A case study of fifteen second grade children who were part of a broader doctoral study was conducted; this involved interviews which probed the children's views of their own difficulties and discussions with children as they tackled word problems. The data were qualitatively analyzed using a thematic analysis approach based on categories of difficulty identified from existing literature. Exemplars of transcripts and responses which show the children experiencing difficulties are included, as well as the children's opinions on their difficulties. Our interpretation of these findings, including proposed subcategories of difficulty, is also given. The report concluded with suggestions of methods-subject to further research- that teachers may use to help children overcome their difficulties with school mathematical word problems.

Keywords: word problems, mathematics

1. INTRODUCTION

Word problems are an integral part of the mathematics curricula, yet, on the other hand, they present one of the most common mathematics difficulties to children. Word problems not only have the potential to motivate students, developing new mathematical concepts and skills meaningfully but also developing the skills to apply mathematics effectively in our day-to-day activities (Verschaffel, Greer, & De Corte, 2000; Boaler, 1993; Hiebert et al., 1996). Word problems are defined as "textual descriptions of situations within which mathematical questions can be contextualized (Verschaffel et al., p.ix)" A word problem is a situation where "mathematical model can be applied to represent the quantities and relations present in the text and to find a solution to the given question" (Nortvedt, 2011, p.256). A corpus of research focusing on the assessment and understanding of young children's problem solving with respect to addition and subtraction on word problems can be categorized into (a) the difficulty level of different types of word problems; (b) "the strategies children use to solve those problems, and the nature of their errors"; and (c) "building explicit models of the knowledge structures and solution processes underlying children's performances on those problems".

Various variables affect word problems difficulty. They include context familiarity, number of words, sentence length, readability, vocabulary and verbal cues, magnitudes of numbers and sequences of operations (Caldwell & Goldin, 1979). The difficulty level of word problems also depends on mechanics, (e.g. problem length, readability, order of data, etc.), format (e.g. written or oral format) and context (Hembree & Marsh, 1993).

2. THEORETICAL UNDERPINNINGS

Several factors affect students' ability to solve word problems. These categories are presented and articulated below. Reading and understanding the language used within a word problem. It has been generally observed that students spend a considerable amount of time trying to understand the problem because they experience difficulty in making sense of WPs. Krick-Morales (2006) notes that "word problems in mathematics often pose a challenge because they require that students read and comprehend the text of the problem, identify the question that needs to be answered, and finally create a numerical equation" (p.1). Hence it is challenging to construct meaning by reading a problem statement superficially. According to Orton:

It is possible to read a story or novel in English in a fairly superficial way, and yet still derive meaning, message and moral. It is even possible to use rapid reading techniques, perhaps skipping sentences or descriptive paragraphs which are clearly not crucial. Non-fiction cannot generally be read in a superficial way without losing details that might be essential and mathematical text comes into this category. (p.133).

Consequently, the role of comprehending the text of the word problem is crucial because it is not only a means of conveying information; rather it is used to interpret the events and phenomenon in a way that provokes students' thinking (Panah, 2000).

Furthermore, WPs are interesting with respect to their role in teaching and learning of Mathematics because they require the integration of several competences: language understanding is one of them (Anderson, 2007; Thevenot & Oakhill, 2008). Therefore without understanding the language of WPs it is difficult to initiate the process of solving the problem. As a result, students' opportunities for success in solving the problem decrease. Garderen (2004) endorses this notion when he notes that "solving mathematical word problems is often hindered by the students' failure to comprehend the problem" (p.225). Further comprehension becomes even more problematic when the WP is expressed in the leaners' second or third language as is the case for many students in South African primary schools. Further, comprehension becomes even more problematic for second or third English language learners due to a lack of proficiency in the English language. Bautista, Mitchelmore and Mulligan (2009) assert that "learning mathematics in general, and solving word problems in particular, poses difficulties, given that large-scale assessments show that many students are not proficient in the language" (p.729). Likewise, research (e.g. Bernardo, 2002; Oviedo, 2005) has shown similar findings in that students' difficulties in comprehending WPs is due to a lack of understanding the language of the problem.

Another category that poses difficulties for children in solving word problems relates to recognizing and imagining the context in which a word problem is set. These difficulties arise when children cannot imagine the context in which a word is set or their approach is altered by the context in which the WP is given (Caldwell & Goldin, 1979; Nunes, 1993). Caldwell & Goldin (1987, 1979) carried out a study at junior school level (1979) and secondary school level (1987). The problems that they presented to schoolchildren were all word problems categorized as concrete or abstract, and hypothetical or factual. Concrete and abstract problems are defined in terms of the realism of their context, that is concrete problems are set in a realistic context and abstract problems have no immediate real world analogy. Hypothetical and factual problems differ in that factual problems simply describe a situation, while hypothetical problems suggest a possible change in the situation. In the Caldwell & Goldin (1979 & 1987) studies, the difficulty level of a problem was measured by the number of students who successfully solved the problem. Caldwell & Goldin (ibid.) found that abstract problems were significantly more difficult than concrete problems, a finding which is reflected in this study.

Difficulties can occur here with children's selection of, and aptitude with calculation strategies (for example formal algorithms, pencil and paper methods and calculators). The context in which a word problem is given and the size of numbers involved can impact on children's choice of a calculation strategy. (Verschaffel, De Corte and Vierstraete 1999: Nunes 1993: Anghileri 2001). Students tend to solve problems easily if presented with numerical version rather than as words; however, they may fail to solve WPs even though they can solve corresponding problems given in purely numerical format. This phenomenon has also been confirmed by studies on Filipino students (Bernardo, 2002; Bautista et al., 2009). Similarly, Dickson, Brown and Gibson (1984) argue that "a major source of difficulty experienced by children in the problem solving process is transforming the written word into mathematical operations and symbolization of these" (p.358). Therefore, Laborde (1990) argues that "understanding what is to be solved requires understanding the problem statement given in an oral or written form" (p.62). In this regard, a plethora of research has documented the difficulties that students encounter when solving WPs (e.g.Adetula, 1990; Badia & Armengol, 1998; Boggs, 2005; Jiang & Chua, 2010; Reed, 2001; Zhu, 2003).

Children appear to find it harder to from a number sentence for some word problem structures than others. These difficulties can result in children not being able to select a calculation to perform or selecting an incorrect calculation. (Carey 1991: English 1998). Personal experience indicates that while working on a WP, students mainly engage in calculations regardless of understanding the problem statement. Likewise, Tuohimaa, Aunola and Nurmi (2008) state that "children are usually asked to read (or listen to) the maths story or the problem presented, write down the mathematical operation necessary for completing the task, and then solve the problem and come up with an answer" (p.410). Learners are frequently limited to computation exercises and little or no time is spent in problem solving (Secada, 1991). Similar practices are carried out in the context of South Africa: teachers write down the problem statement numerically for students and students arrive at the answers without focusing on how and in what way the problem is stated. When students are asked to transform the problem statement into a numeric form they merely rely on key words to do so. Contrary to this, several researchers including Leader and Middleton (2004), Montague and Applegate (1993), Polya (1957), Rickard (2005), and Ridlon (2004) suggest that students should be taught to read and understand the problem, come up with a plan, solve the problem, and then check their answer against the fact in the story problem. Researchers relate WPs to problem solving and application while students and teachers in general see WPs as nothing more than exercises in the four basic operations (Blum & Niss, 1991). Therefore, consistent with the findings of Rickard's study (2005) a teacher needs to understand the difference between problem solving and merely engaging students in routine exercises. Unless teachers realize the importance of problem solving together with language, they may continue to reinforce traditional practices such as writing down data on the board and solving WPs for students for the purpose of exercise completion.

Children have been shown not to consider real-life factors and constrains when giving an answer to word problems which can result in in giving an answer that is impossible in the context and therefore incorrect (Verschaffel, De Corte and Lasure 1994; Wyndham and Säljö 1997; Cooper and Dunne 2000). That is, students did not apply common sense knowledge when solving WPs. The lack of ability to integrate commonsense knowledge with school-learned mathematics was reported in several studies (Greer, 1993; Reusser & Stebler, 1997; Verschaffel, De Corte, & Lasure, 1994; Yoshida, Verschaffel, & DeCorte, 1997). Some researchers reported that this difficulty in applying commonsense knowledge when solving WPs is a result of the instructional practices (Greer, 1993; Verschaffel, et al 1994). For example, school instruction may not require students to provide final solutions that are reasonable and based on the context of the problem. Instructional practices and experiences may also result in the belief that routine WPs can be solved using straightforward calculations without making reference to the context of the problems (Greer, 1993; Verschaffel & DeCorte, 1997). Perhaps, students will be able to apply common sense knowledge in solving WPs when the mathematics they learn is sensible to them. That is, they are able to "view that mathematics is a connected, coherent system in which there are reasons for such things as rules, procedures, and formulas, whether or not the individual yet knows or understands the reasons." (Grady, 2013, p.5).

3. METHOD

The aims of the study were to establish whether difficulties within the identified categories occur in South African Primary Schools and if they do, to find examples of children experiencing difficulties within these categories. I hoped that examples of children experiencing the range of difficulties may provide useful resource for increasing teacher's awareness of the difficulties.

Fifteen children were selected to take part in the study. These children were from a Grade 2 class from one township school which is part of three schools in my broader doctoral study. They were selected on the criteria that they were willing to and able to discuss the eleven word problems that I had given them as a paper and pencil test the previous week as a class. The results of the test had shown that they were working at a range of attainment levels in solving word problems. The second element involved the children working individually through sets of equivalent word problems and discussing their processes and difficulties with me. There were eleven sets of word problems that each child attempted. Each set was given in a different condition, with a different form of help given in in each. Each form of help corresponded to one of the previously identified category of difficulty. For example, I read the word problem to the child, explained any vocabulary and simplified sentences in condition one to correspond to the first category. I offered forms of help in the belief that if I gave a specific type of help and children then solved the problem, I could identify where the original difficulty lay and be aware of which kinds of help allow children to overcome certain types of difficulty.

4. DATA ANALYSIS

The first stage of data analysis involved analyzing and coding interview transcripts and recordings. Excerpts were coded under a category of difficulty if they showed opinions on that difficulty, a child experiencing that difficulty, or a child competently completing a process, therefore not having that difficulty. Any un-coded data were then checked for a need for new categories or reported as 'Other Findings'. The second stage of data analysis involved analyzing and coding all incorrect or no responses to word problems. Transcripts and children's jottings or workings were used to code as to which difficulty prevented a correct answer being given. Using the coded data, I was able to create subcategories within some of the five main previously identified categories of difficulty. Finally, we selected illustrative examples of children experiencing difficulties or giving opinions on difficulties from each category and subcategory. Examples were picked using the criteria of being typical and not extreme.

5. THE RESULTS AND DISCUSSION

Reading and understanding the language used within a word problem

The following example shows Thabo giving an incorrect response to a Join Start Unknown (JSU), problem because he has not read or comprehended all of the text in the question.

Betty had some toy cars. Philip gave her 4 more toy cars. Now she has 12. How many did Betty start with?

R: How many toy cars did Betty start with?

L: 16

R: how did you get 16?

L: I took 4 and 12[showing 4 fingers on one hand] combined it and then I counted 4 and 12 together then I get the answer.

R: Ok, so how many did toy cars did Thabo start with?

L: 16

R: Ok, but now listen, Betty had some toy cars, Betty had [pause] some toy cars, do you know the number of toy cars that Betty had?

L: Some

R: How many are 'some'? If you say Betty had some, do you know the number?

L: 4

R: Philip gave her 4, now she has, 12, so how many did Philip give her?

L: Betty had 3 toy cars then Philip gave her 4 more toy cars

R: Where are you getting 3?

L: I thought Betty had 3

R: Ok

The dimensions of this problem are problems that have the start missing. We acknowledge the possibility that Thabo may not have understood the question but his confidence in his final answer led us to believe that he has not only understood the question, but on further exploration of his misconceptions we realized that words with 'double meanings' were to blame. The word 'more' is naturally associated with addition, acted as a miscue, and prompted the pupil to add the numbers in the problem. While clues and key words can be of help in solving word problems, these words alone do not make sufficient sense and are not concrete enough to help Thabo conquer the entire word problem. Evidently the word 'more' was being associated with addition. The word 'more' does not always imply 'addition'; it means that its meaning depends on the way it is used. Hence giving more weightage to key words leads to partial success as it falls short of solving more complex problems where the meaning of words differ. Thabo had an instrumental understanding of the word 'more' relating to addition, when in this case a subtraction was needed. Thabo seemed to have approached the problem in the order in which he heard the problem. It also appears Thabo had not fully developed the ability to use inverse thinking. Thabo had not yet developed the ability to relate a part of a quantity to the total quantity and responded to the word problem by mapping what we would refer to as 'simple key' onto a procedure of addition. Consequently, the findings of this research reveal that the emphasis on key words to arrive at the correct representation and solution of problems can be counterproductive and may reside with students as incorrect strategy that may backfire in higher grades (Marshall, 1995).

Carrying out the mathematical calculation

We asked Ben to solve the following separate result unknown problem:

Mitchell had 12 toy cars. She gave 4 to Lynn. How many does she have left?

Below is the episode that followed in our discussion with him on solving this problem solution.

Researcher: how many toy cars does Mitchel have left?

L: Sir, is this minus?

R: That's what I want you to tell us

L: We say 12 plus 4, sir

R: If you say 12 plus 4, how many toy cars does Mitchell have left now?

L: 16

Ben applied the "wrong operation' in solving the Separate result unknown problem and set an incorrect equation- 12 + 4 = 16, resulting in in an incorrect solution. Ben does not appear to realize, or consider that it is impossible to for Mitchell to have 'more' cars since he gave some away. Ben's difficulties could also be attributed to miscomprehension of the word problem, which seems to be the main finding in this study. Ben's lack of linguistic knowledge resulted in lack of comprehension, translation and processing skills. It appears Ben could not examine the correctness or incorrectness of the given answer along with using his intuition knowledge and common sense. Clearly Ben did not have such knowledge and only dealt with calculation procedures, albeit, wrongly. While Ben exhibited calculation knowledge which relates to calculations in problem-solving-doing math operations, procedural skills and numerical computations the knowledge appeared to be fragmented.

6. RECOMMENDATIONS

As a result of examining my findings alongside existing literature, we have compiled a list of strategies that teachers and researchers could trial to help children to overcome difficulties with mathematical word-problems:

• Encourage children to read the word-problems thoroughly;

• Teach children which kinds of information may be important;

• Ensure that children practice solving word-problems to allow them to be able to recognize the structure of word-problems and therefore know when to use each calculation; Consider giving children manipulatives to support the solving of word-problems currently beyond the scope of their ability;

• Encourage children to write down their workings so that they do not become unnecessarily confused;

• Encourage children to check if their answer satisfies the criteria of a question. For example if it is in the correct format;

• Teach children to calculate with monetary values;

• Encourage children to check if an answer is possible in the context of the question.

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