VALUES THAT PRIVATE SCHOOL LEARNERS ASSOCIATE WITH MATHEMATICS LEARNING

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ABSTRACT–The international *What I Find Important [WIFI] in mathematics learning* project involves 21 teams from 17 countries, with a University of Johannesburg team involved since 2016. The international project strives is to collect data (from various contexts) on values (fundamental convictions that guide behaviour) that students associate with mathematics learning. This paper reports on the piloting of sections of the international questionnaire among a convenience sample of 85 grade 8 and 9 students from a private school in Johannesburg, conducted in an ex-post facto manner from a post-positivist paradigm. Besides values that these students associate with mathematics learning, the paper also explored differences between values that males and females subscribe to. These private school students pertinently value *process* (how an answer is obtained) over *product* (what an answer is) and *effort* (hard, regular and diligent work) over *ability* (talent and potential). Furthermore, female students desire that mathematics be *explained* to them, while male students prefer *exploring* the subject on their own. This is a first pilot and the intention is to collect adequate data on values that private school students associate with mathematics learning, which will be compared with private schools' mathematics teachers' pedagogical strategies.

Keywords: Values in mathematics learning; Third wave project in mathematics education; Private school mathematics students.

1. BACKGROUND CONTEXT, RESEARCH PROBLEM AND PURPOSE

The South African (SA) schooling system has been poorly rated by international tests of educational achievement such as TIMSS, PIRLS and SACMEQ over the past two decades. A report by Spaull (2013, p. 3) signifies that SA performs worse than most middle- and low-income African countries. An analysis of the National Senior Certificate (NSC) examination results reveal that only 50% of the students that start school in Grade 1, will make it to Grade 12 and just 40% of the latter group will eventually attain a Grade 12 qualification. The indication is there that SA schools are "significantly below where they should be in terms of the curriculum, and more generally, have not reached a host of normal numeracy and literacy milestones" (Spaull, 2013, p. 4). A recent report by the Minister of Education (DBE, 2014) on the so-called Annual National Assessments, which are annually conducted in some of the lower grades, directly points a finger at the inappropriate quality of mathematics teaching and learning throughout the entire school system. A lack of understanding of mathematical concepts by the majority of students and unsatisfactory levels of mathematical content knowledge of the majority of mathematics teachers become evident. In a study conducted by Uusimaki and Nason (2004, p. 370), on the beliefs and anxieties of pre-service mathematics teachers, participants' negative beliefs and anxiousness are firstly attributed to personality factors, symbolised by an "unwillingness to ask questions due to shyness, low self-esteem and for females viewing mathematics as a male domain". A second major contributor to mathematics anxiety is prior school experiences and especially exposure to underprepared and non-supportive teachers.

Educators and researchers are pertinently attending to the abovementioned issues of poor and underachievement in mathematics at various levels, trying to find solutions, or to discover changed or new approaches or strategies that might just improve the status quo. Over the past decade, several international mathematics educators-cum-researchers of note (compare Clarkson, Bishop & Seah, 2010; Jerrim, 2014; Law, Wong & Lee, 2011; Österling & Andersson, 2013; Seah, 2008; 2010a, 2010b, 2011a, 2011b; and others), have put an emphasis on the *culturally-referenced* nature of mathematics in their research. In 2008, the international '*Third wave project: A values approach to optimising mathematics education*' was conceptualised by Seah (2011a). The project is conducted by an international consortium of research teams via parallel inquiries into the nature and harnessing of values aimed at the enhancement of mathematics learning in schools. This paper reports on the piloting of sections of the project questionnaire among grade 8 and 9 students of a private school in a Johannesburg suburb, attempting to collect data from a variety of South African teaching-learning contexts.

The role that gender fulfils in mathematics achievement at high school level has also been the focus of various studies over the last decade. Niederle and Vesterlund (2010, p. 130) report as follows: "Over the past 20 years the fraction of males to females who score in the top five percent in high school math has remained constant at two to one". Early childhood education researchers, Berenbaum, Martin, Hanish, Briggs and Fabes (2008) highlight two important gender differences, already visible at a young age. The first is that boys have and develop superior spatial orientation skills and they secondly tend to engage in play that is more movement oriented and spatially complex. This research therefore also aims to compare what values female and male students associate with mathematics learning. The inquiry is conducted at a private school, where there is abundant resources and teachers, probably better qualified to teach the curriculum than in most public schools. However, the shortage in properly trained and knowledgeable mathematics teachers is a growing concern and although private schools might be in a position to appoint more and 'better' teachers, the pool of quality mathematics teachers remains very limited. Answers will be sought to the following two research questions, by collecting the views from the sample of grade 8 and 9 students:

What are the most important values that private school students associate with mathematics learning?

Are there differences between the values that male and female mathematics students at this private school subscribe to?

2. THEORETICAL PERSPECTIVES

An overview of the international third wave project in mathematics education

There have been many attempts by mathematics educators and researchers in the international arena to find an appropriate answer(s) to the question: *How can mathematics teaching & learning be made more effective*? (Seah & Wong, 2012, p. 33). The initial research foci were on *cognitive* approaches of which mental problem solving schemes, for example higher order thinking and the eradication of mathematics misconceptions. This is approach was known as the first wave of research in mathematics education (Law, Wang & Lee, 2011, p. 72). By the turn of the 21st century the second wave of research came to the forefront and placed a lens on *affective* matters – the beliefs, attitudes, emotions and motivation of both educators and students. The past few years saw a movement towards investigations into the socio-cultural nature of mathematics teaching and learning (Seah, 2010). This approach was labelled as the discursive approach (Law, Wang & Lee, 2011, p. 72), with the intention of exploring discourses on important *values* in mathematics teaching and learning, across various cultures, countries and regions. This was the origin of the third wave of research in mathematics education. Lazaridou (2007) distinguishes between a belief or attitude as what somebody considers to be true and a value is regarded as the importance someone accords to this belief or attitude. Hence, values form the principles or fundamental convictions that guide behaviour and decision-making, which are closely connected to someone's integrity and identity (Halstead, 1996, p. 5).

Prof Seah (2008, 2010a, 2010b, 2011a) conceptualised the '*Third wave project: A values approach to optimising mathematics education*' in 2008. He involved a consortium of 21 research teams (from 17 countries) via parallel inquiries in this project of which this study in SA is one. In this third stage the emphasis will be to validate the new data collection instrument (questionnaire), entitled '*What I Find Important [WIFI] in mathematics learning*' (Seah, 2011a).

2.2. Theoretical framework that underpins this study

This investigation is framed by two related value theories. Geert Hofstede (1997) formulated the first value informing the theoretical framework as the cultural dimensions. The discovery that he made is that every culture (teaching and learning contexts are also regarded as unique cultures) can be defined in a five-dimensional space (more about it below) has particular relevance. The second is the conception of values in mathematics education and more specifically the distinction among *mathematical*, *mathematics educational* and *educational* values in the mathematics 'classroom' as stated by Alan Bishop's (1996). Bishop (1988) initially determined three pairs of complementary values for (western) mathematics, namely *rationalism* and *objectivism, control* and *progress*, and *mystery* and *openness* (compare Shinno, Kinone & Baba, 2014) and this will serve as a pertinent theoretical underpinning.

Values are characteristic of particular sociocultural contexts, drawing their sources from the discourses, practices and norms of participants and of the interactions amongst themselves and their teachers and thus regarded as *sociocultural*, rather than as affective by nature. Affective versus sociocultural values correspond with participants' internal/personal versus external/cultural origins as viewed by Seah, Atweh, Clarkson and Ellerton (2008). McLaren (1998), indicate that values are not just embedded in the teaching and learning elements which a culture extends to its members, but are also intrinsic to the curriculum (Apple, 2000). Therefore, values are thus seen as sociocultural and/or personal convictions that individual students consider important enough to integrate in his/her learning and behaviour.

Values in mathematics education

Bishop (1996) classified values in mathematics education as mathematical, mathematics educational or simply educational. Dede (2009, p. 1230) went further to connect *educational* values to societal or communal values, *mathematical* values to the discipline of mathematics and *mathematics educational* values to the pedagogy (teaching and learning practices and norms) of mathematics. The teacher's cognitive and pedagogical behaviours are (perhaps unconsciously) guided by a personally held system of beliefs, values and principles (Opdenakker & Van Damme, 2006, p. 16). However, the quality of teaching (her/his pedagogy) may just be a piece of puzzle in the overarching conceptualisation of effective mathematics learning and it has been found that many students perform well in mathematics regardless of the teachers they have, or the class they find themselves in. In a study by the Nuffield Foundation (Askew, Hodgen, Hossain & Bretscher, 2010, p. 12) a review of more than 500 studies finds that in respect of mathematics, "high attainment may be much more closely linked to cultural values than to specific mathematics teaching practices", and this is also linked to achievement in mathematics.

2.4. The categories of values in mathematics education

The three categories of values in mathematics education as identified by Bishop (1996) and discussed above have particular relevance in this study. These *Mathematical* values relate to the extent to which aspects of Western mathematics is valued. Bishop (1988) proposed three pairs of complementary mathematical values, namely rationalism and objectivism; control and progress; and mystery and openness. The extent to which aspects of lecturing venue norms and practices are expressed as *Mathematics educational* values that relate to the teaching and learning of mathematics are considered as important. Bishop (1999, p. 20) provides specific examples in this category and these values are when the mathematics teacher give instructions such as: "Make sure you show all your working in your answers" or "Don't just rely on your calculator when doing calculations". Finally, the *general educational* values are not subject-specific, and derived from the "general educational and socialising demands of society" (Bishop, 1999, p. 21).

Seah (2011a, p. 10) emphasises the theory of Hofstede's (1997) that postulates that every culture has a teaching and learning environment with a unique culture. This can be defined in a five-dimensional model along the cultural dimensions of (i) power–distance, (ii) collectivism–individualism, (iii) femininity–masculinity, (iv) uncertainty–avoidance, and (v) life orientation. In the first stage of the WIFI project it was found that Hofstede's cultural dimensions don't specifically relate to Bishop's mathematical or mathematics educational values, however, they relate to the third category of general educational values. There is a distinct difference in character between Hofstede's (1997) *cultural dimensions* and Bishop's (1996) *general educational values.* With the Hofstede's cultural dimension norms and ways of thinking are internalised by members of different cultures whereas, Bishop's general educational values capture the range of ideals and standards, which a culture has selected to inculcate through education. A culture often makes a conscious selection of values it wants to pass on to the next generation, as evidenced in curricula, assessments, websites and promotional documents and might not have control over the historical development of the values encapsulated in the cultural dimensions. Therefore, cultural dimensions are mostly implicit, general educational values manifest more explicit (Seah, 2011a).

There is a need for a fourth category of values in the mathematics classroom or lecture venue according to Seah (2005), to reveal the full account for the principles and convictions that are valued and co-valued amongst all stakeholders, for example parents, alumni, the government, assessors, etc. This fourth category represents *cultural values* (from Hofstede's ,1997) cultural dimensions) and bear particular relevance in respect of multicultural mathematics lecturing and classroom settings. The four categories are represented in data collected by the international project, and interpreted in respect of these four sets of mathematical, mathematics education, general educational, and cultural values.

Educational values in mathematics

Bishop (1996) illustrates mathematical and general educational values of mathematics educational values without any prototypes. With the variety of mathematical pedagogies used in and by different cultures it is a challenge to display or express any value by students in a mathematics teaching and learning setting and this might now be regarded as mathematics educational, which is not ideal. Dimensions similar to Bishop's (1988) pairs of complementary mathematical values and Hofstede's (1997) five cultural dimensions is necessary according to Seah (2011a). Seah (2010a; 2010b; 2011a, p. 12) explored values that learners/students associate with effective mathematics learning and eventually reports six (6) mathematics educational value dimensions in the initial stage of the Third Wave project namely:

pleasure – effort

process – product

application – computation

facts and theories – ideas and practice

explanation – exploration and

recalling – creating.

These six value dimensions (Seah, 1999) were compared with the five mathematics educational values Dede (2011) as projected on different continua:

formalistic – activist view of mathematics learning;

relational – instrumental view of mathematics learning;

relevance – theoretical view of mathematics learning;

accessibility – specialism view of mathematics learning; and,

process – tool and procedural view of mathematics learning.

Seah (2011a, p. 13) reported on the comparison of the five mathematical value dimensions, as follows:

The relevance view represents the *progress–control* dimension.

The accessibility view epitomises the *openness–mystery* dimension.

The formalistic view embodies the *explanation–exploration* dimension.

The relational view signifies the *recalling–creating* dimension.

The process (tool or procedural) view denotes the *process–product* dimension.

The data collection instrument had each of these five mathematical value dimensions presented as a continuum between two extreme values. The students were required to value the two extremes along any continuum.

3. RESEARCH METHODOLOGY

3.1. Research paradigm and method

The research *paradigm* is the researchers' worldview, as portrayed by the matrix of beliefs, perceptions and underlying assumptions, which guided them in approaching this inquiry. The main paradigm underlying this study relates to the quantification of female and male students' values, which they might associate with the learning of mathematics. The investigation was thus conducted from a post-positivist stance. Post-positivism is a milder form of positivism, allowing for measurement, comparison and scientific engagement with the participants (Phillips & Burbules, 2000).

The research *method* used is quasi-experimental and causal-comparative, conducted in an ex post facto manner (Baltimore County Public Schools, 2010; Gay, Mills & Airasian, 2006, p. 217). This implies that the views of two naturally formed groups of participants (female and male students) have been collected and compared in respect of the variable 'values in mathematics learning'. The authors agree that this study's 'reality' can never be known fully, but that measuring it in a scientific manner could disclose valid elements of its nature.

3.2. Participants

The participants were a convenient sample of 85 mathematics students, who were either in Gr 8 or in Gr 9 in a private school in a suburb to the northwest of Johannesburg, in June 2016. The majority are *female* (n=52 or 61.2%) and they very evenly divided between the two *grades*, with 42 in Gr 8 and 43 in Gr 9. Ethnically, more than one in four (43.5%) are *white* and almost a quarter (22.4%) are *black*. Nearly three quarters (74.1%) are *English* language speakers, 44.7% are *15 years* of age*,* about the same number (42.4%) are *14 years* old and the others are younger.

Twenty three (44.2%) of the 52 female students, agreed or strongly agreed with the Likert Scale item, "*I am doing well in mathematics*", while ten (30.3%) of the 33 male students responded likewise. Just more than a quarter (25.9%) of all the participants (14 females and 8 males) disagreed or strongly disagreed with this statement, and they are thus of the opinion that they are not doing well in mathematics.

3.3. Data collection, administration and capturing

Two of the four components of the international project questionnaire (Seah, 2011a) were piloted in this inquiry. *Section A* collects relevant demographical information via closed and open-ended questions. *Section B* contains ten bi-polar items, exploring participants' views on mathematics learning, presented as semantic differentials. The semantic differential, devised by Osgood, Suci and Tannenbaum (1957), is widely acknowledged (Fennell & Baddeley, 2013) for effectively exploring connotative (affective) meanings. Participants were expected to take a stance from one of five positions on each item, on a horizontal line as follows: [left] -2…-1…0…+1…+2 [right]. In semantic differentials (Sapsford, 2007), two opposing adjectives are usually used, for example Cold–Hot or Tall–

Short. The data collection instrument in this study presents two bipolar activities, which are not opposites in the same way as the opposing adjectives have been. The instrument was presented in a printed format to participants at the end of the second quarter (in June) of the year and their responses were hence captured in a Microsoft Excel worksheet.

3.4. Data analyses

The demographic data generated by Section A were analysed via the frequencies and cross-tabulations options of the Statistical Package for the Social Sciences (SPSS, version 23). The semantic differential data of Section B were analysed via select descriptive statistics, followed by testing for normality and hence for potential differences between the views (values) of female and male participants.

The Shapiro-Wilk W test was employed to determine whether data on the ten semantic differential items were normally distributed or not. The significance of the results indicates that normality cannot be assumed. A scrupulous analysis of the distributions' P-P and Q-Q plots, as well as 'common sense' (supported by the website: stats.stackexchange.com) led the authors to believe that a parametric test, like the independent samples t-test (via SPSS), would be sufficiently robust to handle this perceived 'non-normality'.

3.5. Ethical measures and participants' consent

The goal of the inquiry, the nature of the data collection instrument and participants' rights and responsibilities are outlined on the cover page of the data collection instrument and this information was conveyed to them at the start of the data collection session. Beforehand, individual written consent was also obtained from the school leaders to safeguard the confidentiality of collected data and the anonymity of each student.

3.6. Validity and reliability measures

This investigation piloted the international WIFI questionnaire for the first time at school level in South Africa. Validity and reliability measures of data generated via semantic differential items have been widely explored and reported on. Friborg, Martinussen and Rosenvinge (2005, p. 973) suggest that a semantic differential scale "may effectively reduce acquiescence bias without lowering psychometric quality", but that a drawback might be the cognitive demand placed on participants. Österling (2013, p. 39) advises that the extremes of this scale should be "opposites and preferably adjectives instead of nouns or verbs". The validity and reliability of the semantic differential section of the questionnaire will be explored when more data beyond this initial pilot have been collected. Principal Component Analysis as a validity and a test-retest technique as a reliability measure will also be considered then.

4. FINDINGS AND DISCUSSION

4.1. Participants' values in respect of the ten semantic differential pairs

Four descriptive statistical measures, the mean, standard deviation, median and mode are presented for all participants in respect of the ten semantic differential items in table 1 on the next page. Four noticeably *negative* means (items 1, 6, 7 and 9 in the table), supported by median scores of -1 and mode scores of -2, supplemented by two evidently positive means (items 3 and 8), reinforced by median scores of +1 and mode scores of +2, as well as standard deviations of not too an excessive magnitude, signify that the majority of the participants value:

process (how an answer is obtained) over *product* (what an answer is);

explanation (mathematics explained by someone else) over *exploration* (explore mathematics on their own);

recalling (to be able to remember ideas, concepts, rules and formulae) over *creating* (constructing ideas, rules, concepts and formulae);

openness (sharing mathematics with others) over *mystery* (maintaining mathematics' magical character);

effort (hard, regular and diligent work) over *ability* (talent and potential); and,

rationalism (attempting mathematics via logical thinking and reasoning) over *objectivism* (being more pragmatic and concrete).

Table 1: Descriptive statistics for the semantic differential items: All participants (n=84)

Semantic differential items	Values	Mean	Std dev	Mdn	Mode
HOW the answer to a problem is obtained versus WHAT the answer to a problem is.	Process vs Product	-0.43	1.301	-1	-2 & -1
Feeling relaxed or having FUN versus hard WORK when doing mathematics.	Fun vs Work	0.25	1.352	0	0
Leaving it to ABILITY versus putting in EFFORT when doing mathematics.	Ability vs Effort	0.99	1.135	1	2
APPLYING mathematical concepts versus using a RULE or formula to solve a problem.	Application vs Computation	0.31	1.202	0	0
Truth and FACTS that were discovered versus IDEAS and practices used in life.	Facts vs Ideas	0.04	1.103	0	0
Someone TEACHING or explaining to me versus EXPLORING mathematics myself.	Explanation vs Exploration	-1.06	1.255	-2	-2
REMEMBERING versus CREATING mathematics ideas, concepts, rules or formulae.	Recalling vs Creating	-0.75	1.269	-1	-2
TELLING what a triangle is versus letting me see concrete EXAMPLES first.	Rationalism vs Objectivism	0.62	1.447	1	2
Demonstrating and explaining mathematics to OTHERS versus keeping mathematics MAGICAL.	Openness vs Mystery	-0.63	1.240	-1	-2
Using mathematics to predict and explain (CONTROL) versus for development and PROGRESS	Control vs Progress	0.01	1.125	0	0

No clear value preference emerged in respect of item 2 (*having fun* versus *hard work*), although 40.5% of the participants valued hard work, 28.2% associated more with fun, while 30.6% were neutral on this matter. The same can be said in respect of item 4 (*applying mathematical concepts versus using rules and formulae* to solve problems), item 5 (*discovering truths and facts versus ideas and practices* in real life) and item 10 (using mathematics for *control* or for *progressio*n purposes). The relative immaturity of the participants –students in grades 8 and 9 are still very much teacher-driven in respect of their learning – could have contributed to these four more implicit value preferences.

4.2. Values projected by female versus male students

The independent samples t-test, interrogating equality of means on all ten semantic differential items, was utilised to detect differences between the values that female students associate with mathematics learning versus values that male students subscribe to. Table 2 below displays relevant descriptive statistics, as well as the t- and p-values. The respective F-scores and p-values generated via Levene's test confirmed that all variances can be assumed as equal.

Table 2: Group statistics and independent samples T-test findings

Semantic diff values	Groups	Mean	Std dev	Mean diff	t-value	Sig. (2-tailed)
Process vs Product	Female (n=52)	-0.60	1.257	-0.440	-1.516	.133
	Male (n=32)	-0.16	1.347			
Fun vs Work	Female	0.29	1.377	0.101	.331	.742

	Male	0.19	1.330			
Ability vs Effort	Female	1.12	1.060	0.334	1.316	.192
	Male	0.78	1.237			
Application vs Computation	Female	0.27	1.257	-0.106	-.390	.698
	Male	0.38	1.289			
Facts vs Ideas	Female	0.06	0.998	0.058	.232	.817
	Male	0.00	1.270			
Explanation vs Exploration	Female	-1.25	1.135	-0.500	-1.797	.076
	Male	-0.75	1.391			
Recalling vs Creating	Female	-0.79	1.177	-0.101	-.352	.726
	Male	-0.69	1.424			
Rationalism vs Objectivism	Female	0.71	1.419	0.243	.745	.459
	Male	0.47	1.502			
Openness vs Mystery	Female	-0.58	1.289	0.142	.507	.614
	Male	-0.72	1.170			
Control vs Progress	Female	0.12	1.149	0.272	1.076	.285
	Male	-0.16	1.081			

The independent-samples t-test unfortunately detected no statistically significant differences between female and male students regarding the values that they associate with the ten semantic differential pairs. The **most noteworthy** differences in the view of the authors, although they are not significant, between the values that the two genders subscribe to, are in respect of the pairs:

explanation–exploration, where female students to a greater extent than male students, wanted mathematics to be explained to them, instead of exploring it on their own and

process–product, where females associated more than males with the process how solutions to problems are found, than purely getting to the answer (the product).

In highlighting factors that were found to contribute to gender differences in mathematics achievement, Haroun, Ng, Abdelfattah and AlSalouli (2016, p. S384) cite four pertinent contributors, namely "stereotypes of mathematics as a male domain", teachers' expectations, teachers' gender and students' prior teaching and learning experiences in mathematics. These four contributors to potential differences in the mathematics learning values of the two genders, but also possible others, will be carefully monitored during follow-up studies at private (and other) schools, which form part of the bigger project.

5. IN CONCLUSION

The two research questions that this study attempted to answer are:

What are the most important values that the participants associate with mathematics learning?

Are there differences between the mathematics learning values that male and female participants at this private school associate with?

In respect of the first research question, it was found that majority of the 85 grade 8 and 9 mathematics students, enrolled in the selected private school, evidently value *process*, *explanation*, *recalling*, *openness*, *effort* and *rationalism* over *product*, *exploration, creation, mystery, ability and objectivism*. In a study involving Japanese mathematics students (Grade 5 and 9), Shinno, Kinone and Baba (2014) found that their participants explicitly value *process*, *effort*, *openness and recalling* over *product*, *ability*, *mystery* and *creating*. It was a disappointment to the Japanese researchers that their students valued the *recall* of mathematical ideas and concepts to a greater extent than their *discovery* or *creation*. The South African study generated rather similar findings. This raises questions, because of the enabling teaching-learning environment of private schools and especially the quality of their teachers. However, this is just the first pilot study, and while it comforting to know that the international project questionnaire does generate usable data, a further more penetrative exploration of mathematics students' values in private schools' educational environment is definitely necessary.

In respect of the second research question, male and female students in this study displayed just subtle differences between their respective mathematics learning values. It was found that the majority of female students places more value on the role and influence of 'someone' (a 'knowledgeable other'), who should *explain mathematics content to them*, while the majority of the male students seems to be more contend to explore mathematical concepts *on their own*. It was also insightful to note that most female students value the (sometimes step-by-step) *process* to get towards solutions more, while most male students seem more eager to find an accurate answer to a problem, thus attaching greater value to mathematical *product*s. Awareness of values in the teaching and learning of mathematics is imperative in striving for improved student/learner achievement. The discovery of student values, which might constantly be in a state of flux, and the alignment of detected values with the educator's pedagogy, should be a regular strategy within any teaching-learning setting of note.

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