EFFECTS OF A PROBLEM-SOLVING HEURISTIC INSTRUCTIONAL APPROACH IN THE LEARNING OF ALGEBRA IN GRADE 6

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ABSTRACT-The aim of this study was to explore the effects of a problem-solving heuristic instructional design on learners' achievements in algebra. The impact of the heuristic teaching treatment was investigated with 198 learners from four different primary schools in the Zululand district of KwaZulu-Natal that were purposively sampled. A mixed method approach was employed that was broken down into three phases. Phase 1 was a qualitative study that involved a classroom observation of all four schools used in this study. The findings from phase 1 indicated all four the schools made use of the similar traditional methods of instruction. Phase 2 was a quantitative study that employed a pre-test-post-test non-equivalent quasi-experimental design. A hypothesis was formulated to investigate the effects of the heuristic teaching method on the learners' achievements in algebra. The findings from Phase 2 of the study supported the initial hypothesis of the improved scores in algebra through participation in the heuristic teaching treatment. Phase 3 explained how the heuristic teaching treatment was developed and implemented. The findings from phase 3 provided information on how a heuristic teaching treatment can be developed and used in Grade 6 algebra lessons.

Key words: Problem-solving heuristic instruction; Modelling-eliciting activity; Algebra.

1. INTRODUCTION

The South African Department of Basic Education (DBE, 2011a) has prioritised the improvement of the quality and levels of educational outcomes in the school system with a view to, among others, improving learners' performance in mathematics. The extent to which these outcomes are achieved is determined and monitored through the administration of the Annual National Assessments (ANA), which involves standardised literacy and numeracy skills tests written by learners in Grades 1 to 6 and 9. The Human Sciences Research Council (HSRC), commissioned to verify the 2014 ANA, found consistency of standards with few exceptions. Among others, the results showed that the percentage score of 41.8% was obtained at the national level, with 32.4% of the learners scoring above 50% (HSRC, 2014). This fell short of the target set by the DBE as per injunction of the president of the Republic of South Africa to have at least 60% of learners achieving above 50% by 2014. The results indicate that the general performance was still falling below this target (DBE, 2014). One of the challenging topics identified in the ANA diagnostic report of 2014 for the intermediate and senior phases (i.e., Grades 6 and 9) in mathematics was algebra: "A high percentage of learners were unable to see the relationship between the input and output values given in a table" (DBE, 2014, p. 43). It is argued that the mathematics curriculum has to provide differentiated pathways and choices to support every learner in order to maximise their potential (Curriculum Planning and Development Division of the Ministry of Education, 2013). Research supports the importance of developing conceptual understanding for future success in mathematics (National Council of Teachers of Mathematics, 2000). There is widespread agreement that teaching through the problem-solving approach holds the promise of fostering the learners' conceptual understanding of mathematics (English & Sriraman, 2010; Schroeder & Lester, 1989). The greater focus on the teaching of mathematics should be on a problem-solving conceptual development (English & Sriraman, 2010). By solving problems, learners develop a sound understanding of the relationship between the elements in the problem and number facts (Baroody, 1998; Carpenter, Fennema, Peterson, Chiang & Loef, 1989; Kilpatrick, Swafford & Findell, 2001; Reys, Lindquist, Lambdin, Suydam & Smith, 2001). Cai (2003) mentions, "While teaching through problem-solving starts with problems, only worthwhile problems give learners the chance both to solidify and extend what they know and to stimulate their learning" (p. 247). Kaur, Yeap and Kapur (2009) explain that an important component of improving mathematics learning through the problem-solving approach is for the educators concerned to be able to identify the types of mathematical problems that prompt learners' engagement, thinking, and the making of cognitive connections, and that the associated educator actions that support the use of these problems includes addressing the needs of individual learners. Lester (2013), sharing the same sentiments expressed by Kaur et al. (2009), proposes that the success of a problem-solving instructional approach depends on the consideration of a wide range of factors by the educator involved, which includes the selection of problems, the type of problem-solving experiences to use, the stage at which problem-solving instruction should be employed, the level of guidance to give to the learners and how to assess learners' progress. Educators must challenge learners with problems and at the same time offer support to those experiencing difficulty (Kaur et al., 2009).

Even though some research has been conducted on problem-solving driven instruction (see English, 2009; Maclean 2001; Tan, 2002;), a host of researchers has cited a lack of studies that address problem-solving driven conceptual development as it occurs with problem-solving competencies (Cai, 2003; Lester & Charles, 2003; Schoen & Charles, 2003). Research has not clearly explained how the development of mathematical concepts can interact with the development of problem-solving tools. Hence there is a need for a better integration of problem-solving approaches within all topic areas across the mathematics curriculum (English & Sriraman, 2010). Lester (2013) has also emphasised that further research is needed in this regard as the accumulation of knowledge in problem-solving instruction has been slow. In the light of this the study aims to investigate the effects of heuristic teaching instruction on learners' conceptions and achievements in algebra. The term *heuristic* is derived from the Greek meaning to 'find' or to 'discover'. It is an adjective for experience-based techniques that help in problem-solving, learning and discovery (Jaszczolt, 2006).

A problem-solving heuristic teaching method is an innovative approach to teaching and learning which is self-inviting and aids the self-directed development of the learner (Blinkston, 2000). The technique used in this teaching approach is based on the learners' experience that aids problem-solving and discovery learning, and has its roots in the correct signal learners receive from their immediate environment (Abonyi & Umeh, 2014). Higgins (1971) highlights four characteristics of the heuristic teaching method, as follows: (i) it approaches content through problems; (ii) it reflects on problem-solving techniques in the logical construction of instructional procedures; (iii) it demands flexibility for uncertainty and alternative approaches; and, (iv) it seeks to maximize learners' actions and participation in the educator-learning process. Authentic real-life problems, known as modelling eliciting activities (MEAs), are used to explore learner development in algebra in this study. These activities can be designed to lead to significant forms of learning because they involve mathematizing – by quantifying, dimensioning, coordinating, categorizing, algebraizing, and systematizing relevant objects, relationships, actions, patterns, and regularities (English, 2006; Lesh, Cramer, Doerr, Post & Zawojewski, 2003; Lesh & Zawojewski, 2007).

The Curriculum Assessment Policy Statement (see DBE, 2011b) categorizes algebra in grade 6 into three main components, namely number sentences where learners are expected to write number sentences with other representations such as mathematical problems; number patterns where learners are expected to look for relationships or rules in a number pattern using flow diagrams and determine input and output values of a number pattern, and geometric patterns where learners are expected to look for relationships or rules in a geometric pattern using flow diagrams and determine input and output values of a geometric pattern.

2. THEORETICAL FRAMEWORK

The problem-solving heuristic instructional approach explored two main theories, namely the Modelling and Modelling Perspective and the Action Process Object Schema (APOS) Theory. The Modelling and Modelling Perspective aided in the design of MEAs used in this study, whereas the APOS Theory was used as a framework in the heuristic teaching sequence to develop the learners' conceptual understanding in algebra at the level of grade 6. The Modelling and Modelling Perspective are based on six principles that arose out of the work of a number of researchers and educators, but was subsequently refined by Lesh, Hoover, Hole, Kelly & Post (2000). These six principles are as follows, namely the *reality principle,* which requires the activity to aid the learners to be able to interpret the problem given; *the model construction principle,* which requires the activity to be able to push the learners to explicitly describe and explain a given situation mathematically; the *self-evaluation principle,* which requires the activity to contain a criterion the learners themselves can use to revise and test their current way of thinking mathematically; the *model documentation principle,* which ensures that while working on the activity the learners create some form of documentation to reveal their thinking of the problem situation; the *model generalisation principle,* which requires the learners to be able to produce sharable and re-usable solutions. The *simple prototype principle* requires the activity to be as simple as possible, whilst at the same time being mathematically significant.

The APOS framework is a constructivist theory developed by Dubinsky (1991) that arose out of an attempt to understand the mechanism of *reflective abstraction as* introduced by Piaget, to describe the development of logical thinking in young learners. The theory has four main conceptions, namely *action conception*; it is a reaction to stimuli which the learner perceives as external. The learner at this level is able to conceptualize a rule to explain the goals of an MEA and is able to substitute input values to give output values in accordance with the goals of the MEA; *process conception;* as the learner repeatedly performs and reflects on the action, the action is perceived as part of the learner, and the learner is able to establish control over it. The learner is able to conceptualize a rule as an expression that transforms input values into output values without substituting in any values. We say the learner has interiorized the actions into mental processes; *object conception*; the learner reflects on the operations applied in a particular process; he or she becomes aware of the process as a totality, realises that transformations can act on that totality and can actually construct such transformations. When the learner is able to transform a rule by adding, subtracting, dividing or multiplying a constant to describe similar problem-situations, we say the learner has encapsulated the process conception into an object conception s*chema;* once constructed, objects and processes can be interconnected in various ways. A collection of actions, processes and objects can be organized in a structured manner to form a schema, which may also include previously-constructed schemas. It must be noted that actions, processes, and objects are hierarchical; they are not linear. Learners may need to alternate many actions and processes before we can say they have developed the process conception of a particular mathematical concept.

The APOS framework develops possible pedagogical strategies for learning a particular concept, known as the 'genetic decomposition'. Data are gathered in the process to either validate the teaching pedagogy, or to call for amendments to the teaching pedagogy (Dubinsky, 1991). A genetic decomposition is not unique to a particular mathematical concept but most importantly, it must predict what the learners must do to learn a particular mathematical concept.

3. OBJECTIVES OF THE STUDY

The following objectives were set out for the study:

To develop a heuristic teaching sequence in the learning of algebra in Grade 6 in line with the modelling and modelling perspectives and the APOS theory; and, To determine whether the heuristic teaching method has any effect on the learning of algebra in Grade 6.

4. THE RESEARCH QUESTIONS

The main research question is formulated, as follows:
Will there be any statistically significant improvement in the learners' achievements in algebra after being exposed to the heuristic teaching instruction?

A hypothesis was drawn from the main research question, as follows:
There is a statistically significant improvement in the algebra test scores of learners who participated in the heuristic teaching treatment.

The research question was then further expanded, as follows:
How can a heuristic teaching instruction be developed and used in Grade 6 algebra lessons to facilitate learning?

5. METHODOLOGY

5.1 The population, the sample and sampling technique

The population of the study was Grade 6 learners in quintile one schools (poorly resourced schools) in the Zululand district of Kwazulu-Natal and the sample for the study consisted of intact Grade 6 classes from four of the category of schools mentioned; one school chosen from each of the four circuits in the district was purposively sampled. Two of the schools represented the experimental group and the remaining two schools represented the control group.

5.2 Research design

The study followed a mixed-method approach that was broken down into three phases. A mixed-method approach combines both qualitative and quantitative methods of data collection and analysis that enable the study to answer the proposed research questions (Cresswell, Klassen, Plano Clark & Smith, 2011). Phase 1 was a qualitative study that involved Grade 6 class observation by the researcher. The aim was to investigate the teaching approach used by the mathematics educators in the four schools used for this study and to determine whether any of these schools had an advantage over the others with regard to teaching and learning. This enabled the study to control some of the variables other than the heuristic teaching treatment that could influence the outcome of this treatment. Phase 2 was a quantitative study that involved a pre-test-post-test non-equivalent quasi-experimental design. This approach enabled the researcher to draw comparisons of the pre-test and post-test between the experimental group and the control group after exposing the experimental group to the heuristic teaching treatment to assess its relative effectiveness when used with the experimental group. Phase 3 was a qualitative study that explained how the heuristic teaching treatment could be developed and implemented in Grade 6 algebra lessons.

5.2.1 Heuristic teaching treatment

The heuristic teaching approach explored two theories, namely the Action Process Object Schema (APOS) and the Modelling and Modelling Perspective (see section 2.0). The effects of the APOS Theory and the Modelling and Modelling Perspective enabled the researcher to develop a preliminary genetic decomposition, which is a specific mental construction the learners may make as they develop a conception in algebra. It was implemented flexibly. Problem-solving was used as a pedagogical approach using MEAs as a class of problems. The APOS Theory was used as a framework to develop the learners' understanding of algebra. The learners worked collaboratively in groups sharing ideas, commenting on each other's ideas, and writing their perceived answers on a common-group sheet as they were guided through the various stages of the preliminary genetic decomposition. The learners were given activities to do at home individually. This was to enable them to reflect on the various stages of the preliminary genetic decomposition. In the next teaching session it was expected from the learners in each group to discuss, agree on and present a common answer which was presented in a whole-class format.

5.2.1.1 Preliminary genetic decomposition

i). *Action stage:* The learners were guided to reflect on their understanding of the MEA to formulate a rule or relationship among the elements in the MEA in line with their understanding of the MEA. The learners were guided to test their rule by substituting input values to give output values in line with the objectives of the MEA.

ii). *Process stage*: As the learners repeated and reflected on these actions several times, they gradually began to interiorise the actions into a mental process, where they could conceptualise a rule as an expression that dynamically transforms an input value into an output value without performing any extensive calculation. Once this has been established, the learners are guided to manipulate the rule in order to be able to substitute output values to predict input values in accordance with the objectives of the MEA.

iii). *Object stage:* As the learners reflect on the operations applied in this process they gradually encapsulate the process-conception into an object-conception where they conceptualize a rule or relationship as a static object that can itself be transformed. At this stage the learners are guided in thinking of parallel problems similar to the MEA, and modify and manipulate the original rule to explain the goals of that activity.

iv). *Schema*: The learners are given activities that will enable them to reflect and inter-connect the various stages of actions, processes and object conceptions in a structured manner.

5.3 Data collection instruments

5.3.1 Class room observation instrument

The classroom observation tool adopted from Kotoka (2012, p.49) was used by the researcher to investigate and compare teaching approaches used by the mathematics educators in all four schools used for this study.

5.3.2 Pre-test and post-test

The same test was used in both pre-test and post-test and was made up of selected questions in algebra from the Standard Grade 6 CAPs-approved mathematics textbooks and the ANA examination for Grade 6 set from 2010 to 2013. Using the same test before and after intervention ensured that the cognitive demands of the algebra questions were maintained. There were 20 questions in all, 10 of them being multiple choice questions and the remaining 10 written questions. A total of 40 marks was allocated and afterwards converted to 100 percentage points.

The researcher content validated the test by consulting with mathematics subject advisors to check whether it was suitable for Grade 6 learners. The test was piloted among 30 learners in another class of Grade 6 learners that did not form part of the main study. The same test was written twice over a period of one month and the Cronbach alpha coefficient was calculated which stood at 0.74, hence confirming the reliability of the test.

5.3.3 Modelling eliciting activities (MEAs)

The MEAs used in this study were based on learners' background and interests in their school. It was developed based on the six principles of the modelling and modelling perspective. The proposed modelling task for this study contained features which included real-world relevance, accessibility, feasibility, sustainability and alignment with the learning of algebra in Grade 6 of the South African education system. The learners were not expected to have readily accessible procedure that guarantees or find the solution MEA, rather the MEA was supposed to create a blockage for the learners which is then used as a medium of interaction between the educator and the learner to develop the learners' conceptual understanding of algebra(see Kaur et al, 2009, p.5). The MEAs were validated by an independent researcher to verify whether the six principles of the modelling and modelling perspective were featured in the task and to check whether the task was a meaningful medium to develop learners' conception of algebra at the Grade 6 level. The MEAs were piloted by the researcher in a fifth school that did not form part of the main study to assess its feasibility and suitability for Grade 6 learners. These sessions were video and audio-recorded. Triangulation was used during the pilot study to assess the MEAs' reliability. The researcher compared data from transcripts and audio recordings, his own field notes and learners' worksheets to confirm the consistency of data gathered.

5.4 Data Collection Procedures and Method of Data Analysis

5.4.1 Class observation

The researcher twice attended regular Grade 6 mathematics classes at each of the four schools to investigate the nature of teaching approaches adopted by their mathematics educators between July 2014 and October 2014. The data was gathered by means of personal observation and recorded on the classroom monitoring instrument. The researcher documented and summarized the classroom activities with the adapted tool in order to understand the nature and quality of the teaching approaches adopted by the Grade 6 mathematics educators of the respective schools.

5.4.2 Administration of pre-test and post-test

All 198 learners in both the experimental and the control group participated in the pre-test and post-test. The pre-test was administered in July 2014 before the heuristic teaching treatment was administered to the experimental group while the post-test was administered in October 2014 after the heuristic teaching treatment had been administered to the experimental group. The researcher was not directly involved in conducting and marking the pre-test and post-test but was assisted by the Grade 6 mathematics educators of the schools used. After the pre-test and post-test had been marked, three scripts from each were sampled, using a simple random sampling technique, for moderation by the respective HODs of the four schools and the necessary corrections and adjustments made. Data was analysed using descriptive and inferential statistics. The descriptive statistics used were mean, and standard deviation and the inferential statistics used were analysis of covariance (ANCOVA), the test of homogeneity of regression slopes and the Johnson-Neyman technique. The quantitative analysis compared the pre-test and the post-test of the control and experimental group to check whether there were any changes in the post-test scores of the experimental group after the heuristic teaching treatment was implemented and whether the changes observed were statistically significant.

5.4.3 Heuristic teaching treatment

The teaching treatment was conducted between July and October 2014 by the researcher. The researcher negotiated with the school authorities of the two experimental schools to allocate one hour per week for this research as it was, for all practical purposes, impossible to conduct the study after school hours. In total 9 hours were used in teaching each of the Grade 6 classes in the two experimental schools. Ninety-two learners in the two experimental group schools participated in the teaching treatment. Experimental group 1 was divided into 9 groups of five members each and one group of four members. Experimental group 2 consisted of 8 groups of five learners each and one group of three learners. The researcher took on the role of researcher-educator, implementing the intervention himself and guiding learners through the various stage of the preliminary genetic decomposition. The researcher also offered further explanations and slightly lowered the cognitive demands of some aspects of the MEA at different stages of the preliminary genetic decomposition for learners experiencing difficulty. This enabled such learners to actively engage in the activity and to start or restart the activity at their own level of understanding. Data from the heuristic teaching instruction was gathered through transcripts from video and audio recordings, field notes made by the researcher and trained research assistants, as well as from group worksheets. Data gathered was used to explain how problem-solving heuristic instruction can be developed and used in the classroom.

5.5 Findings

5.5.1 Classroom observation

Processes and procedures used in all the classes with regards to teaching and learning in all four schools were similar. The mathematics educators provided a step-by-step explanation of each procedure; learners then practised the procedure through class exercises and homework and the remedial action given to point out and correct errors learners make. Learners in the four observed classrooms were passive rather than active receivers of these procedures. This placed all four schools at the same level with regard to resources used in teaching and learning and in effect controlled some of the extraneous variables that could contaminate the effects of the treatment on the experimental group. Through this the true effects of the heuristic teaching treatment could be measured.

5.5.2 Quantitative findings

Table 1 indicates a comparison between the control group and the experimental group in terms of the mean and standard deviation scores in the pre-test (before the intervention) and the post-test (after the intervention). Since the research could not achieve a complete randomization of the sample, the pre-test scores for the control group were compared with the scores of the experimental group to evaluate the pre-intervention equivalence of the two groups. Findings from the descriptive statistics also highlight the effects (improved score) of the experimental groups' participation in the teaching treatment.

Table 1: Changes in the scores in the pre-test and post-test for the control and experimental groups

Control	Mean	SD	n	Experimental	Mean	SD	n
Pre-test	13.54%	9.98	106	Pre - Test	15.60%	9.52	92
Post-test	13.92%	9.36	106	Post - Test	43.73%	16.8	92

From Table 1 it can be seen that the mean score of the control group is 13.6%, whereas that of the experimental group is 15.6%. This score indicates the learners' academic strength in algebra before the intervention. Even though the pre-test mean score for the control group is higher than that of the experimental group, the researcher could not draw any definite conclusions regarding the pre-intervention equivalence of the control and experimental groups, as there were a myriad of factors, such as the natural variation in their respective schools that could have contributed to this. All learners in both the experimental and the control groups achieved 35% or less in the pre-test.

The difference between the post-test and the pre-test mean scores of the control group indicate a 0.38% (approximately 0%) improvement, whereas those of the experimental group show a 28.18% improvement. The significant improvement in the experimental group's scores seems to suggest the positive effect of the heuristic teaching method and supports the hypothesis that the heuristic teaching method improved the learners' achievements in algebra. To support this claim, further testing was carried out with ANCOVA to verify the hypothesis. This was done by testing whether the difference between the pre-test and post-test scores of the learners in the experimental group was statistically significant. An assumption for using ANCOVA was to test whether the regression slopes associated with the treatment effects were equal. The regression slopes were found to be heterogeneous, meaning that the treatment effects are not the same at all levels of the covariate (pre-test). Further testing was done, using the Johnson-Neyman technique, to locate intervals of the covariate where the treatment was effective and intervals of the covariate where the treatment was not effective.

5.5.2.1 Findings from ANCOVA

At 95% confidence interval the post-test scores were compared with the pre-test scores where the pre-test scores were the covariate (see Table 2).

Table 2: ANCOVA summary table

Source	SS	df	MS	F	P - Value
Adjusted treatment	39016.16	1	39016.16	298.85	0.001
Error (Res_{within})	25458.48	195	130.56		
Total Residuals(Res_{total})	64474.64	196			
CRITICAL VALUES	$F_{0.05,\ 1,\ 195)}$	3.88958864	**Adjusted Means**		
CRITICAL VALUES	$F_{0.01,\ 1,\ 195)}$	6.76663859	**Pooled Regression Coefficient**		0.676
			Control	14.62	
			Experimental	42.92	
Hypothesis (H_A): There is a statistically significant improvement in the algebra test scores of the learners who participated in the heuristic teaching treatment.					
Null hypothesis (H_0): There is no statistically significant improvement in the algebra test scores of the learners who participated in the teaching treatment.					

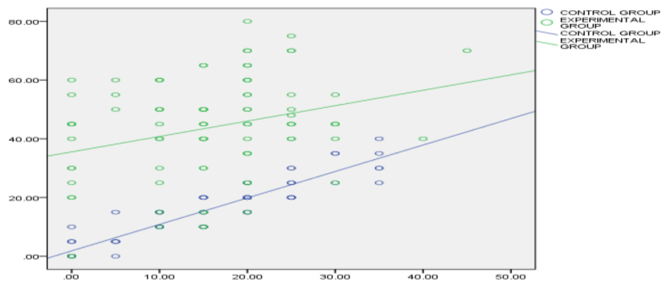
The F value obtained was then compared with the critical F value with 1 and 195 degrees of freedom; $F(_{0.05,\ 1,195)}$= 3.88958864. At 95% confidence interval the research rejected the null hypothesis since the F value (F= 298.85) was greater than the critical F value ($F(_{0.05,\ 1,195)}$= 3.88958864). Furthermore, the p-value (0.001) was far less than α (0.05). This implies that the heuristic treatment influenced the post-test scores of the experimental group and that the improved post-test scores obtained by the learners in the experimental group are statistically significant. The researcher performed a test for the homogeneity of the regression slopes since it was a condition for using ANCOVA. The regression slopes between the control group and the experimental group were found to be non-homogenous (see Table 3).

Table 3: Homogeneity of regression slope test

Source	SS	DF	MS	F	p-value
Heterogeneity of Slopes	641.69	1	641.69	5.01	0.026
Individual Residuals(res_i)	24816.79	194	127.92		
Within Residual(res_w)	25458.48	195			
H_o: $\beta_1^{Control} = \beta_1^{Experimental}$	**CRITICAL VALUES**		$F_{0.05,\ 1,\ 194)}$	3.88983904	
H_1: $\beta_1^{Control} \neq \beta_1^{Experimental}$	**CRITICAL VALUES**		$F_{0.01,\ 1,\ 194)}$	6.76732738	

At 0.05 significance level (95% confidence interval) the p-value (0.026) was less than α=0.05 and the F value was 5.01, which was larger than the critical F value 1 and 194 degrees of freedom ($F_{0.05,\ 1,194)}$ = 3.88983904). Figure 1 illustrates a scatter plot for the pre-test scores against the post-test scores for the control and the experimental groups and further indicates the heterogeneity of the regression slopes. The control group is represented by blue lines and the experimental group by green lines.

Y



$Y = \beta_1 x + \beta_0$
$Y = 0.90x + 1.76$ (Control)
$R^2_{control} = 0.930$
$Y = 0.53x+35.54$ (Experimental)
$R^2_{experimental} = 0.297$

X = Covariate

Figure 1: Scatter plot for pre-test against post-test and their corresponding regression equation for the experimental and control groups

Figure 1 shows the heterogeneous nature or convergence of the two regression lines of the respective control and experimental groups in relation to Table 3. This provides sufficient evidence to reject the null hypothesis (no difference between the two regression slopes). When there is heterogeneity between the regression slopes it implies that the magnitude of the treatment was not the same at all levels of the covariate (pre-test) which means an alternative to ANCOVA should be considered. The Johnson-Neyman technique was used to analyse the intervals of the pre-test where the treatment was effective and the intervals where the treatment was not effective. The results from the Johnson-Neyman technique indicate that the intervention was effective for the learners in the experimental group who had obtained marks of 55.4 or less ($X<55.4$) in the pre-test.

5.5.3 How a problem-solving heuristic approach can be developed and used in algebra lessons to facilitate learning

Integrating the two theories, namely the Modelling and Modelling Perspective and the APOS Theory proved useful in designing algebra concept-construction activities that helped the learners in grade 6 to develop a sound conceptual understanding of algebra. The Modelling and Modelling Principle can guide the design of an MEA that can provide a way for the educator to develop the learners conceptual understanding in algebra as this gives the educator the opportunity to explore how the learners think in terms of algebra through their engagement in the modelling task. MEA provides an effective platform for cooperative learning in algebra. The learners' engagement in MEAs can result in the development of sound algebra concepts, as an MEA requires the learner to develop powerful mathematical ideas to be able to solve the problem at hand. MEAs provide the platform for learners to document their own thinking and learning development in algebra. Teaching through MEAs gives the educator the tools to understanding how the learners are thinking about a particular mathematical procedure. This can guide the educator to manage a multiplicity of ideas that can be used to support multiple developments of the learners' ideas. The APOS theory is a viable learning framework that can guide the educator through the process of developing the learners' conceptual understanding in algebra as it occurs though the learners mathematizing an MEA.

6. CONCLUSION

The results of this study shows a statistically significant improvement in the post-test scores of the learners in the experimental group who participated in the heuristic intervention, and almost a zero improvement in the learners in the control group who did not participate in the intervention. It can therefore be concluded that the usual teaching the learners received was not effective enough to develop their conceptions in algebra. The experimental group's ability to improve their post-test scores highlighted the effectiveness of the teaching treatment. The heuristic teaching treatment can further be developed in respect of other mathematical concepts on the basis of the Modelling and Modelling Perspective and the APOS Theory as they have proved to be effective.

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