THE DEVELOPMENT OF MATHEMATICAL PROBLEM-SOLVING SKILLS OF GRADE 8 LEARNERS AT A SECONDARY SCHOOL IN GAUTENG

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SUMMARY

This mixed methods research design, which was modelled on the constructivist view of schooling, sets out to investigate the effect of developing mathematical problem-solving skills of grade 8 learners on their performance and achievement in mathematics. To develop the mathematical problem-solving skills of the experimental group, a problem-centred teaching and learning environment was created in which problem posing and solving were the key didactic mathematical activity. The effect of the intervention programme on the experimental group was compared with the control group by assessing learners' problem-solving processes, mathematical problem-solving skills, reasoning and cognitive processes, performance and achievement in mathematics. Data were obtained through a mathematical problem-solving skills inventory, direct participant observation and questioning, semi-structured interviews and mathematical tasks. Data analysis was largely done through descriptive analysis and the findings assisted the researchers to make recommendations and suggest areas that could require possible further research.

Key concepts: mathematical achievement and performance; mathematical problem-solving skills; problem-centred teaching and learning approach; problem-centred teaching and learning environment; skills development.

1 INTRODUCTION AND BACKGROUND TO THE STUDY

South Africa participated in TIMSS 1995, TIMSS 1999 and TIMSS 2003 and was rated last in all three studies, even coming in behind African countries such as Ghana and other developing nations that spent far less of their budgets on education than South Africa does. TIMSS (Dossey, Giordano, McCrone & Weir 2006), which is formally known as the Trends in International Maths and Science Survey, is an international study whose aim is to assess the national curricular, school and social environment, and learners' achievements in mathematics and science in participating countries across the world. The poor performance can be attributed to a content-based curricular, rote learning, the teacher-centred teaching approach, the lack of a problem-centred teaching and learning approach and a lack of development of mathematical problem-solving skills in learners. In mathematics, learners develop these problem-solving skills only if genuine mathematical problem-solving takes place. Genuine mathematical problem-solving is believed to take place in a problem-centred teaching and learning environment. In order to develop the mathematical problem-solving skills of grade 8 learners, a problem-centred teaching and learning environment was created. The problem-centred approach is a teaching approach that aims to motivate learners to participate in the learning process and helps to enhance and foster mathematical problem-solving skills.

2 PURPOSE OF THE STUDY

The Curriculum and Assessment Policy Statement (2011) aims to produce learners that are able to "identify and solve problems and make decisions using critical and creative thinking". South Africa's poor performance indicate that traditional methods of teaching mathematics are failing to provide most learners with the skills to solve problems even though they will need these to function effectively in society. However the problem-centred teaching and learning approach takes focuses on teaching and learning through problem solving, promotes high-level engagement of learners, involves recall of facts; and uses a variety of mathematical problem-solving skills and procedures; the ability to evaluate one's own thinking and the coordination of knowledge; previous experience and intuition. Kadel (1992) advocates that learners in a problem-centred teaching and learning environment are learning to develop and apply mathematics problem-solving skills since they are expected to utilise their own resources and experiences when approaching new situations.

The importance of developing the mathematical problem-solving skills of learners has been emphasised for almost a century (Bruner 1961; Dewey 1910; 1916). The purpose of this study was therefore to explore how these mathematical problem-solving skills develop in grade 8 learners and to investigate their effect these learners' performance and achievement in mathematics.

3 LITERATURE REVIEW

3.1 The problem-centred teaching and learning approach (PCTLA)

The PCTLA involves the learning of mathematics through real contexts, problem situations and models (Van de Walle, Karp & Bay-William 2013). Problem-solving is used as a "vehicle for learning" (Human 1992). The problem-centred approach is perceived by Murray et al (1998) as teaching mathematics through problem solving: using problem-solving as a technique for helping learners to learn other concepts. Davis (1992) cited in Murray et al (1998) states the following:

Learning through problem-solving means that instead of starting with "mathematical" ideas and then applying them, we would start with problems or tasks, and as a result of working on these problems, the children would be left with a residue of “mathematics” and we would argue that mathematics is what you have left over after you have worked on problems. We reject the notion of “applying” mathematics because of the suggestion that you start with the mathematics and then look around for ways to use it.

The problem-centred teaching and learning approach model builds on the work of Human (1992), Olivier (1999), Van de Walle (1998; 2004), Murray, Olivier and Human (1992; 1993; 1998), Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier & Wearne (1996) and integrates the aspect of the zone of proximal development (Vygotsky 1978) and scaffolding (Wood, Bruner & Ross in Woolfolk 2007). It incorporates key aspects of metacognition apprenticeship (Flavell 1979), reflective thinking (Van de Walle 2004) and social interaction (Schoenfeld 2002; Nathan & Knuth 2003; Kilpatrick 1985; 1987). In this problem-centred teaching and learning approach, the aim was to assist learners to become effective thinkers and to develop good mathematics habits of mind (Cuoco, Goldeberg & Mark 1996), where the researchers guided them to work independently and at the same time acquiring important mathematical problem-solving skills.

3.2 Mathematical problem-solving skills found in literature

In the literature (Lenchner 1983; Bransford & Stein 1984; Gick 1986; Polya 1957; Kadel 1992; Hiebert & Wearne 1993; Van de Walle 1998; Adamovic & Hedden 1997; Dendane 2009), there are seven mathematical problem-solving skills that were of particular importance for this study.

3.2.1 Understanding or formulating the question in the problem

To formulate the question in the problem is the first basic skill that learners have to acquire. Formulating a problem can be extremely demanding (Dendane 2009). However, this is an important stage, because a correct solution cannot be generated without an adequate understanding of the problem. Learners who cannot understand or formulate the question in a given problem usually have difficulty solving the problem.

3.2.2 Understanding the conditions and variables in the problem

The learner has to develop a sense of how the conditions and variables relate to each other and clarifies the meaning of the information explicitly stated or implied in the problem. Kadel (1992) stresses that learners must be able to identify the known and the unknown variables in a given problem. Making a model, a diagram, a picture or a list of key ideas can help the process of understanding the conditions and variables in the problem.

3.2.3 Selecting or finding the data needed to solve the problem

Learners must be able to identify needed data and eliminate data not needed. They should be able to extract the relevant information from the given problem.

3.2.4 Formulating sub-problems and selecting appropriate solution strategies to pursue

This skill addresses the ability to determine if there are sub-problems or sub-goals to be solved and decide which strategy or strategies to use and when. For example, a learner may know how to multiply but not know when to multiply or might know how to find a pattern but not know when to look for a pattern in solving problems. During this stage, learners must develop the relationship between the known information and the unknown information by writing an equation with appropriate variables. As progress is made towards the solution of problems, the problem solver is often required to identify sub-goals to be reached. This mathematical problem-solving skill involves the learner making decisions on what problem-solving strategy or strategies to try. The learner searches for or generates possible solution strategies to the problem, which are then implemented and tested (Gick 1986).

3.2.5 Correctly implementing the solution strategy or strategies and solving sub-problems

Learners must know how to correctly implement solution strategies, ranging from being able to perform computations to solving equations by using mathematical operations. In effect, after identifying and ordering sub-goals, the learner must be able to attain them.

3.2.6 Giving an answer in terms of the data in the problem

The learner must be able to give an answer in terms of the relevant features of the problem. Learners must be able to include the right units in their final answer or solution.

3.2.7 Evaluating the reasonableness of the answer

The problem-solving process does not end after obtaining an answer; learners should be able to check the validity of their solution. If the solution is incorrect, they need to refer back to the previous steps to check for any errors in mathematical calculations, translations and the overall understanding of the problem. During this process the learner may reread the problem and check the answer against the conditions, variables and the question. Learners may also use various estimation techniques to determine if an answer is reasonable. Lenchner (1983) points out that the reasonableness of the answer can be achieved when learners write their own answers in complete sentences. If they write answers in complete sentences, they can easily review the statement of the problem and detect a possible error.

3.3 Strategies for developing learners` mathematical problem-solving skills

Learners’ mathematical problem-solving skills can hardly be developed by merely solving problems (Killen 1996). Approaches and learning situations, in which mathematical problem-solving skills can be developed, are discussed in this section.

3.3.1 Developing learners’ thinking skills

Before learners can develop mathematical problem-solving skills, they must first become effective thinkers. For learners to become effective thinkers, they must develop useful mathematics “habits of mind” (Cuoco et al 1996). Cuoco et al (1996) explain that habits of mind are dispositions or tendencies by learners to employ appropriate critical-thinking behaviours often and are what learners need to develop in order to think mathematically and to be successful problem solvers. Habits of mind are regarded as something students need to develop in order to think mathematically and to be successful problem solvers (Lesh & Doerr 2003). Learners should be taught to think about mathematics, the way mathematicians do. For them to think like mathematicians, Cuoco et al (1996) point out that they should be pattern-sniffers, experimenters, describers, tinkerers, inventors, visualisers, conjecturers and guessers.

3.3.2 Structuring learning situations to develop mathematical problem-solving skills

Comparing problems- By looking for similarities learners can consider what strategies were effective in solving other problems that have these characteristics (Killen 1996). The teacher can help learners to develop their ability to recognise similarities in problems by deliberately structuring a series of problems around a common theme.

Comparing strategies- Thinking about more than one strategy should be an important part of each learner’s plan for solving a problem. Teachers should teach learners to understand that when one problem-solving strategy does not work, there is a need to establish why it was not successful and then modify their approach or look for a new approach (Killen 1996).

Valuing the process- Learners that are used to traditional learning are likely to be more interested in producing correct answers to problems than using the problems as a vehicle for learning mathematics. Day in Killen (1996) suggests that the teacher can change the disposition of learners by placing most emphasis on the process of solving problems and placing least emphasis on obtaining the correct answer. This can be achieved by having learners express and record their ideas, thoughts, feelings and questions as they work through the problem-solving process. By exposing learners to frequent practice at thinking through a strategy they would use without actually proceeding to a solution and encouraging learners to realise that a particular approach is not the best option because it is the one they thought of or the one they thought of first.

Commitment and perseverance- Learners are unlikely to solve problems successfully without willingness and perseverance. No one has ever solved a problem without being involved in the process, teachers should emphasise to learners that they can become better problem solvers by showing an interest in the problem solving process (Schmalz 1989).

Encouraging learners to “think out loud” when solving problems- Thinking out loud forces learners to pay attention to their thinking and problem-solving because they become aware of the information they are using to solve problems and therefore more aware of how they are solving them (Posamentier & Jaye 2006).

3.3.3 Encouraging learners to pose their own problems

The CAPS (2011) advocates that for learners to develop essential mathematical skills they should “learn to pose and solve problems”. Learners’ ability to generate their own problems for other learners to solve is a good indicator of their mastery of content, concepts and principles that the teacher has been teaching and their ability to analyse problems. After learners generate their own problems, the teacher can discuss with them the way in which their problems can be made easier or harder and this should encourage them to look for factors to consider when they come across problems by themselves.

3.3.4 Enhancing learners’ thinking

Learners’ thinking can be enhanced by developing their skills to focus, gather, organise, evaluate and analyse.

4 THE RESEARCH DESIGN

This study, which employed one of the most popular mixed methods research design in educational research, the **convergent research design,** was guided by the research question: “Does the development of mathematical problem-solving skills of grade 8 learners in a problem-centred teaching and learning environment have an effect on their performance and achievement in mathematics?”

In convergent research design, the researcher simultaneously gathers both quantitative and qualitative data, merges them using both quantitative and qualitative data methods and then interprets the results together to provide a better understanding of the phenomena of interest (McMillan & Schumacher 2006). The aim of the convergent design is “to obtain different but complementary data on the same topic” (Morse 1991) in order to best understand the research problem. The convergent design also helps “to directly compare and contrast quantitative statistical results with qualitative findings” (Creswell & Plano Clark 2011) in order to elaborate well-substantiated conclusions.

According to Creswell and Plano Clark (2011), there are challenges in employing the convergent design, because much effort and expertise are required since equal emphasis is given to each data type. To overcome this challenge, the researchers familiarised themselves with the mixed methods design and the convergent design before conducting the research study. Creswell and Plano Clark (2011) further point out that it can be difficult to merge very different data sets and their results in a meaningful way. To overcome this challenge and to facilitate merging of the two data sets, this study was designed in such a way that the quantitative and qualitative data sets addressed the same concepts.

The quantitative research strand used the mathematical problem-solving skills inventory (see appendix B) and mathematical tasks to gather data. The qualitative strand employed direct participant observations and questioning and in-depth semi-structured interviews (McMillan & Schumacher 2006) to collect data from the learners. Semi-structured interviews with learners during and after problem-solving sessions were conducted in their classrooms. A few learners were selected at a time to interview, observe and question during the intervention. Data were transcribed immediately after the interviews.

4.1 Participants

Participants are individuals who participate in a study and from whom data are collected (McMillian & Schumacher 2006). The research was conducted at a secondary school in Gauteng, South Africa. This secondary school was chosen because of its "convenient location and accessibility", a valid and useful approach pointed out by McMillian and Schumacher (2006).

The experiment involved 57 grade 8 mathematics learners in the 2012 academic year. The grade 8 learners were chosen because the researchers would not interfere a lot with the teacher's schedule, which would have been the case for higher grades. The grade 8 mathematics learners were assigned randomly to either the control group or the experimental group by using a table of random numbers. A total of 28 learners were assigned to the experimental group and another 29 learners to the control group. The group of learners taught by the researchers using the PCTLA was called the experimental group, while the group that was taught by the current grade 8 mathematics educator using the traditional teaching method was called the control group. Both the control and experimental groups were involved in the study during their usual mathematics lessons, that is, 4.5 hours a week for ten weeks during the third term of the 2012 academic year. In total, during these ten weeks the respondents were expected to attend the intervention programme for a minimum of $10 \times 4.5 = 45$ hours.

4.2 Reliability and validity

Reliability refers to "consistency of measurement" (McMillan & Schumacher 2006), that is, the extent to which independent administration of the same instrument (or highly similar instruments) consistently yields similar results under comparable conditions (De Vos 2002). To address this issue, Cronbach alphas for the various constructs of the mathematical problem-solving skills inventory were calculated to determine their reliability. The reliability coefficients were generally above 0.8 which is excellent for this kind of instrument (McMillan & Schumacher 2006). The Spearman-Brown formula (McMillan & Schumacher 2006) was used to calculate the reliability coefficients of the mathematical tasks. The Spearman-Brown coefficients for this instrument were generally above 0.70, which is acceptable for this kind of instrument.

The consistency of the qualitative data was ensured by prolonging the data collection period, conducting the semi-structured interviews in the same classrooms that learners' mathematics lessons took place (natural setting) and by audio recording the semi-structured interviews and transcribing them verbatim on the same day.

Validity is the extent to which inferences and uses made on the basis of numerical scores are appropriate, meaningful and useful (McMillan & Schumacher 2006). Its purpose is "to check the quality of the data, the results and the interpretations" (Creswell & Plano Clark 2011). A research tool is said to be valid if it measures what it is supposed to measure. Pilot testing of the mathematical problem-solving skills inventory was conducted to ensure its reliability and validity. Content validity was ensured by making sure that all the problem-solving skills were represented by items and questions in the mathematical problem-solving skills inventory and tasks.

4.3 Ethical procedures

For this research, in compliance with the Unisa research ethics policy, all precautions were taken before the data collection process in order to adhere to the ethical measures to respect the integrity, confidentiality, anonymity, privacy, caring, consent and humanity of the participants.

5 RESULTS

5.1 The mathematical problem-solving skills inventory (MPSSI)

The MPSSI (see appendix B) was administered to both the experimental group and control group at the beginning and end of the intervention. Table 1 summarises the control group learners' mathematical problem-solving skills at the beginning and end of the intervention.

Table 1 T-test for the mathematical problem-solving skills inventory for the control group before and after the intervention

Control group	Pre-intervention		Post-intervention		
Problem-solving skills *n = 29*	Mean	Std. deviation	Mean	Std. deviation	*t*
Understanding or formulating the question in the problem	2.21	1.19	2.86	1.35	-2.86

Understanding the conditions and variables in the problem	3.29	0.91	3.29	0.61	0.00
Selecting or finding the data needed to solve the problem	2.86	1.41	3.14	1.03	-0.72
Formulating sub-problems and selecting appropriate solution strategies to pursue	2.7	1.33	2.93	1.33	-0.72
Correctly implementing the solution strategy or strategies and solving sub-problems	3.29	1.33	3.14	1.23	0.69
Giving an answer in terms of the data in the problem	2.93	0.92	3.00	0.88	-0.29
Evaluating the reasonableness of an answer	3.00	1.24	3.07	1.07	-0.25

$p < 0.05$

By merely looking at the above mean scores one may think there are differences between the means of the mathematical problem-solving skills of the control group learners at the beginning and at the end of the intervention. Therefore a paired t-test was run to test the null hypothesis:

H_0: There is no significant difference between the mean scores of the control group learners' mathematical problem-solving skills at the beginning and the end of the intervention.

The null hypothesis is rejected if the calculated t-value > critical t-value, and it is accepted if the t calculated < t critical at 56 degrees of freedom. The calculated t-values are low and less than the t-critical value = 2.0032 at the level of significance $p < 0.05$. Hence the null hypothesis was accepted it was concluded that learners in the control group did not perceive any change in their mathematical problem-solving skills.

Table 2 summarises the experimental group learners' mathematical problem-solving skills at the beginning and at the end of the invention.

Table 2 T-test for the mathematical problem-solving skills inventory for the experimental group before and after the intervention

Experimental group	Pre-intervention		Post-intervention		
Problem-solving skill $n = 28$	Mean	Std. deviation	Mean	Std. deviation	*t*
Understanding or formulating the question in the problem	3.13	0.99	7.20	1.15	11.48
Understanding the conditions and variables in the problem	2.60	1.06	5.00	1.31	11.59
Selecting or finding the data needed to solve the problem	2.87	0.92	6.40	1.06	19.04
Formulating sub-problems and selecting appropriate solution strategies to pursue	2.67	1.11	5.87	1.41	11.57
Correctly implementing the solution strategy or strategies and solving sub-problems	2.73	1.10	6.27	1.16	10.74
Giving an answer in terms of the data in the problem	2.40	0.99	8.93	1.33	13.81
Evaluating the reasonableness of an answer	2.87	0.83	7.13	1.36	11.93

$p < 0.05$

The results in the table above indicate that learners in the experimental group perceived a significant improvement in their mathematical problem-solving skills at the end of the invention. This is reflected by the higher means, t-scores and significance levels. The results reflect that learners in the experimental group regarded themselves as having developed mathematical problems-solving skills.

A null hypothesis H_0 was also tested:
H_0: There is no significant difference between the mean scores of the experimental group learners' mathematical problem-solving skills at the beginning and the end of the intervention.

The calculated t-values for the mathematical problem-solving skills were all high and more than the t-critical value = 2.0032 at the level of significance $p < 0.05$. Hence the null hypothesis was rejected and it was concluded that learners in the experimental group perceived an increase in their mathematical problem-solving skills.

5.2 Participant observation and questioning

It was observed that at the beginning of the intervention learners in the experimental group were hesitant to solve unfamiliar problems without the teacher's help and generally used impulse approaches to solving the given problems. When learners impulsively decided on a wrong problem-solving strategy they usually obtained incorrect solutions, unless they assessed their actions early enough to check whether the strategy lead to the correct solution. However, all learners in the experimental group were enthusiastic about this new method of learning even though they sometimes felt insecure about their attempts. Although learners were initially worried about being responsible for their own learning and finding their own ways of solving unfamiliar problems; they rose to the occasion, in their small groups interactions were lively and constructive.

Participant observation and questioning revealed that mathematical problem-solving skills development was gradual and new problem-solving skills were time consuming and gradual. As the intervention progressed, learners were able to make conscious decisions about choosing a problem-solving strategy. The various problem-solving strategies that learners chose showed signs of independent thinking and the development of their mathematical problem-solving skills. Learners were encouraged to "think out aloud" when solving mathematical problems. As they thought out aloud when solving problems, it became evident that they became more aware of the information they used to solve problems and they became more conscious of how they were solving the problems. Learners showed a particular enjoyment to working in small groups. Working in groups afforded them the opportunity to test out their ideas in a relaxed atmosphere. They discussed and modified ideas as they progressed with each other's help. With time, learners who were initially nervous and withdrawn began to comfortably contribute and enjoy group work and could easily explain their solutions to the whole class.

5.3 Semi-structured interviews

The data on the learners' subjective learning experiences were gathered in the semi-structured interviews that were conducted during the problem-solving sessions. As indicated earlier, the audio recordings were transcribed verbatim. The researchers read the transcribed data line by line to make sure it made sense and read through all the interview transcripts several times in order to immerse themselves in the data. During this process, the researchers wrote key concepts in the margins of each transcript and recorded notes in the form of short ideas.

As learners became familiar with the PCTLA and began to develop mathematical problem-solving skills, a culture of enquiry was established in the classroom. Learners were now able to take control of the given problems, problematise them, focus their attention on developing appropriate strategies and check on the correctness and reasonableness of solutions to the given problems. During semi-structured interviews, it was noticed that as the intervention progressed, learners began to verbalise given problems, understand the question in a problem and evaluate the reasonableness of a solution. They also began to take ownership of ideas and developed a sense of power in making sense of mathematics. Learners started to understand that most problems can be solved in more than one way and that some problems have more than one correct answer. They started to use a variety of strategies like working backwards, looking for patterns or making a list when solving problems.

At the end of the intervention learners demonstrated that they had developed mathematical problem-solving skills. This was evidenced by their improved ability in solving unfamiliar problems, by their ability to identify needed information, by their efficiency in selecting the data needed to solve a problem and at the same time ignoring non-essential information. Learners could give solutions to a problem and could clearly state the goal of the problem or task. Below are two solutions for two different tasks that were done by learner 24. The first solution was done by the learner at the beginning of the intervention on 23 July 2012. The second solution was completed by the learner at the end of the intervention on 20 September 2012.

First question

A hummingbird lives in a nest that is 8 metres high in a tree. The hummingbird flies 10 metres to get from its nest to a flower on the ground. How far is the flower from the base of the tree?

Solution to first question

$$a^2 = b^2 + c^2$$
$$10^2 = 8^2 + c^2$$
$$100 - 64 = c^2$$
$$36 = c^2$$
$$c = 6$$

The learner used an algorithm to solve the problem, did not attempt to use a diagram to present the solution and did not give the answer in terms of the data in the problem. In the semi-structured interview it was revealed that this learner was unable to verbalise the problem and could neither explain nor check the solution.

Second question

Peter has R100 pocket money and James has R40. They are both offered part-time jobs at different supermarkets. Peter earns R10 a day and James R25 a day. If they do not spend their pocket money or their daily wages, after how many days will they have the same amount of money?

This solution was given by learner 24 towards the end of the intervention:

peter starts with R100, James starts with R40.
peter gets R10 a day, James gets R25 a day.
I can make a list

peter		James
day 1	R100+R10=R110	R40+R25=R65
day 2	R110+R10=R120	R65+R25=R90
day 3	R120+R10=R130	R90+R25=R115
day 4	R130+R10= R140	R115+R25=R140

after 4 days they will have the amount of money.

There is clear evidence in the above solution that learner 24 had developed mathematical problem-solving skills at the end of the intervention by being exposed to the PCTL environment. The learner could formulate the question in the problem, could select data needed to solve the problem, was able to correctly implement a solution strategy and solve the sub-problems. At the beginning of the intervention learner 24 could not give the solution in terms of the data in the problem, but from the above solution it has now become evident that he had developed this skill by the end of the intervention.

5.4 Mathematical tasks

Mathematical tasks and written work were administered to the experimental group only. During the intervention, learners completed three tasks. All tasks were marked using the analytic scoring scale, see appendix A.

Table 3 Descriptive statistics for mathematical tasks

Task	N	Minimum	Maximum	Mean	Std. deviation
Task 1	28	44.00	69.00	57.8000	7.22298
Task 2	28	51.00	75.00	63.8000	8.09056
Task 3	28	60.00	88.00	72.6000	9.41731

Table 3 shows that the mean scores for each task improved as the intervention progressed. Task 3 showed the highest mean and the maximum score. From the results of the participant observation and questioning, semi-structured interviews and the MPSSI it was concluded that as the intervention progressed, learners showed a definite development in mathematical problem-solving skills. It was evident that learners' performance in the tasks improved as the intervention progressed.

6 DISCUSSION

The results from semi-structured interviews and learner observation and questioning evidenced that learners were developing mathematical problem-solving skills as the intervention progressed. Learners could formulate the question in a given problem, understand the conditions and variables in the problem, select or find the data needed to solve the problem, formulate sub-problems and select appropriate solution strategies to pursue, correctly implement the solution strategy or strategies and solve sub-problems, give an answer in terms of the data in the problem and evaluate the reasonableness of their solutions. It was observed that learners frequently checked and monitored their understanding during the problem-solving sessions. The transcribed data from the semi-structured interview audio-recordings revealed that learners had become conscious of how and why they were solving a given problem.

The data from the MPSSI indicated that learners in the experimental group perceived themselves as having developed mathematical problem-solving skills by the end of the intervention. However, the MPSSI indicates that learners in the control group did not show any change in their mathematical problem-solving skills at the end of the intervention. Learners in the experimental group achieved better results in their mathematical tasks as the intervention progressed. It seems reasonable to conclude that learners in the experimental group had developed mathematical problem-solving skills and this had a positive impact on their performance and achievement in mathematics.

From the above discussion it can be concluded that the quantitative results and qualitative findings converged. The learners had indeed developed mathematical problem-solving skills by being exposed to the PCTLA and this had a major impact on their performance and achievements in mathematics.

7 CONCLUSION

The empirical study was in the form of a mixed methods research design. The purpose of the qualitative research was to explore the development of mathematical problem-solving skills of grade 8 learners in a problem-centred teaching and learning environment. The quantitative strand's purpose was to test the effect of the development of mathematical problem-solving skills on grade 8 learners' performance and achievement in mathematics. From the results and findings of the study it can be concluded that the grade 8 learners had indeed developed mathematical problem-solving skills at the end of the intervention and this had a positive impact on their performance and achievement in mathematics.

7.1 Limitations of the study

The same equal sample was used to compare and merge the quantitative and qualitative data sets in a meaningful way. Since this was a small-scale study, a small sample was utilised in order to enhance the richness of the qualitative data. A small sample could result in low statistical power for the quantitative data and this could limit the ability to find individual participant differences (Creswell & Plano Clark 2011). This therefore resulted in the findings of the quantitative strand being limited in terms of their wider application.

7.2 Areas for possible further research and recommendations

Although this study clarified the problem under investigation, the results can be regarded as tentative. Further research may be conducted with a larger group. Deeper research is also needed concerning what happens in learners' minds during the process of developing mathematical problem-solving skills. Further research could investigate how learners move from basic mathematical skills to advanced mathematical problem-solving skills.

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9 Appendices

Appendix A: The analytic scoring scale

Understanding the problem	0	Complete misunderstanding of the problem
	1	Part of the problem misunderstood or misinterpreted
	2	Complete understanding of the problem
Planning a solution	0	No attempt or totally inappropriate plan
	1	Partially correct plan based on part of the problem being interpreted correctly
	2	Plan could have led to a correct solution if implemented properly
Getting a solution	0	No answer or wrong answer based on an inappropriate plan
	1	Copying error, computational error, partial answer for a problem with multiple answers
	2	Correct answer and correct label for the answer

Appendix B: The mathematical problem-solving skills inventory

Where 1=strongly disagree and 10=strongly agree

Problem-solving skill	Rating of skill
1. Understanding or formulating the question in a problem	1 2 3 4 5 6 7 8 9 10
2. Understanding the conditions and variables in the problem	1 2 3 4 5 6 7 8 9 10
3. Selecting or finding the data needed to solve the problem	1 2 3 4 5 6 7 8 9 10
4. Formulating sub-problems and selecting appropriate solution strategies to pursue	1 2 3 4 5 6 7 8 9 10
5. Correctly implementing the solution strategy or strategies and solve sub- problems	1 2 3 4 5 6 7 8 9 10
6. Giving an answer in terms of the data in the problem	1 2 3 4 5 6 7 8 9 10
7. Evaluating the reasonableness of the answer	1 2 3 4 5 6 7 8 9 10