Using a cognitive load theory to account for the enhancement of high school learners' mathematical problem solving skills

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Abstract

This paper reports on a context-based problem solving instruction implemented to a group of high school mathematics learners to promote their problem solving skills. Participants consisted of a convenient sample (n = 57) of grade 10 learners who were of a disadvantaged socio-economic status and were low-performing in mathematics. Results from the achievement test suggested that the context-based problem solving instruction was effective in accelerating learners' problem solving performance ($p < 0.05$). Using a cognitive load theory, it is possible to explain aspects of learners' problem solving performance in terms of human cognitive architecture.

Keywords: problem solving; context-based problem solving instruction; cognitive load theory.

INTRODUTION

In this paper, a context-based problem solving instruction to promote high school learners' mathematics problem solving skills is presented. We describe a *context-based problem solving instruction* as a teaching approach. In this approach, problem solving knowledge of financial mathematics is uncovered when learners are exposed to tasks giving meaning to their everyday experience (Dhlamini, 2011, p. 135). The context-based problem solving instruction we propose is conceptualized within the context of a "worked examples approach" advanced through the cognitive load theory (Van Gog & Rummel, 2010, p. 156). A worked examples approach is an instructional device that provides a model for solving a particular type of problems by presenting the solution in a step-by-step fashion (Van Gog & Rummel, 2010). It is intended to provide the learner with an expert's solution, which the learner can use as a model for his or her own problem solving.

In Dhlamini (2011) the results of implementing a context-based problem solving instruction on grade 10 mathematics learners are documented. Participants in the study were of a disadvantaged socio-economic background and were low-performing in mathematics problem solving. Results of the study measured at pre- and post-stages suggested substantial improvement in learners' problem solving performance when dealing with contextualized tasks. The Dhlamini (2011) study applied the results of cognitive science research to design a treatment that teaches transfer explicitly, with positive effects. Hence in this paper assumptions of cognitive load theory (Sweller, 1988) are used to explain cognitive activities linked to observed learners' problem solving performance in terms of human cognitive architecture. In the next section we elaborate on these theoretical assumptions.

COGNITIVE LOAD THEORY ASSUMPTIONS

Cognitive load theory uses current knowledge about the human cognitive architecture to generate instructional techniques that promote learning and development of problem solving skills. Cognitive architecture can be defined as an underlying infrastructure that influences cognitive processes for an intelligent system, such as a human being (Langley, Laird & Rogers, 2009, p. 1). In that way all human mental life and behaviour involve the cognitive architecture. For instance, perceiving everything around us involves using our cognitive system so that we can recognize and categorize what we see, hear, taste, touch and smell.

The basic premise of cognitive load theory is that learners' cognitive architecture consists of a working memory with severely limited processing capacity and duration when dealing with novel information. Concerning its processing duration, almost all information stored in working memory and not rehearsed is lost within 30 seconds (Paas, Van Gog & Sweller, 2010, p. 117). Also, its capacity cannot deal with information more than about 7 elements of information simultaneously (Miller, 1956). Hence, if the working memory capacity is exceeded while processing information then some, if not all, of that information will be lost. However, the limitations of the working memory can only apply to new, yet to be learned information (Paas, et al., 2010). Well-learned material, held in long-term memory, suffers from neither of these limitations when brought into working memory (Ericsson & Kintsch, 1995). This means working memory should be occupied by task-relevant operations, especially when dealing with complex material. Hence cognitive load theory pleads for a proper use of working memory by means of efficient training.

Furthermore, according to cognitive load theory, human cognitive architecture also consists of an effectively unlimited long-term memory which interacts with a working memory to process information. Because long-term memory has unlimited capacity, it can permanently store chunks of domain-specific skills and information structures known as schemas or schemata. Schemata categorize elements of information according to how they will be used, thereby facilitating accessibility later when they are needed for related tasks (Sweller, Van Merriënboer & Paas, 1998). In terms of cognitive load theory, the presence of schemata in long-term memory is considered a prerequisite because schemas reduce the amount of mental effort in working memory that is needed to perform particular tasks (Van Gog & Rummel, 2010).

Mainly, cognitive load theory focuses on how constraints on our working memory help to determine what kinds of instruction are effective. According to cognitive load theory, teachers should design problem solving tasks that minimize the demand for processing in working memory. Hence learning activities should minimize the processing and storage of information that is not directly relevant for learning in order to avoid taxing the working memory processing capacity. To further illustrate the assumption of cognitive load theory, three types are distinguished.

Intrinsic cognitive load

This load refers to the complexity of the learning material that a learner intends to mentally learn (Van Gog & Rummel, 2010). However, the complexity is dependent on the intrinsic nature (difficulty level) of the learning material and also upon the learner's amount of prior knowledge. Learner's prior knowledge has been considered in this definition because the size of meaningful information chunks that a learner can handle without taxing his or her

working memory capacity is dependent upon it (Van Gog & Rummel, 2010). Hence a learning problem solving task that is complex for a beginner may indeed be simpler for an expert. Therefore, to compensate for the deficiency in learner's prior knowledge, learning material of high complexity is enhanced when the interacting elements are taught in isolation and the relevant interactions are instructed later, suggesting that intrinsic load can be manipulated by instruction (Moreno, 2006, p. 171).

Extraneous cognitive load
Extraneous cognitive load is defined as the cognitive load that is imposed by instructional designs that require learners to engage in activities "that are not directed at schema acquisition or automation" (Sweller, 1994, p. 299). This type of load is mainly dependant on the goal of instruction. For instance, when the goal of instruction is to construct problem solving schemas, extraneous cognitive load is imposed if instructional materials contain texts and graphics that are difficult to integrate with each other (Chong, 2005). In this case learners may be forced to use much of their working memory resources trying to establish coherence between the two sources of information. Consequently, little or no cognitive capacity will remain to foster learning and skill acquisition.

Germane cognitive load
Germane cognitive load is also known as effective cognitive load. This is because, unlike extraneous cognitive load, germane cognitive load is conceptualized as a load that contributes directly to learning. It is thereby influenced by the instructional designer. The manner in which information is presented to learners and the learning activities required of learners are relevant to what constitutes germane cognitive load (Chong, 2005). In the case of worked examples (to be discussed in next section), self-explanatory activities would be considered as a germane cognitive load.

The three types of cognitive load discussed above are additive (Gerjets, Scheiter & Cierniak, 2009). However, their sum cannot exceed the limits of the working memory capacity if learning is to occur. Hence cognitive overload results if the sum of the three cognitive load types requires more working memory resources than the learner has at his or her disposal during learning (Gerjets, et al., 2009, p. 45). In figure 1 we suggest techniques to manipulate cognitive load in order to foster learning. Here, we propose three stages to implement a context-based problem solving instruction.

Firstly, instruction must be designed in such a way that intrinsic load is optimized. This means a context-based problem solving task should be at an appropriate level of complexity for the learner's processing ability. As alluded earlier, this is achieved through sequential presentation of learning material, thus reducing the number of element interactivity that a novice memory has to simultaneously process at an instance. *Secondly*, extraneous load must be minimized. In terms of our study, this is achieved by presenting learning material located in learners' every day's experience. According to cognitive load theory, learning that takes place in familiar settings reduces the effects of cognitive load or the extraneous load. So for effective learning to happen extraneous load must be kept at a minimum (Van Gog & Rummel, 2010).

Figure 1: The relationship between cognitive load and learning*
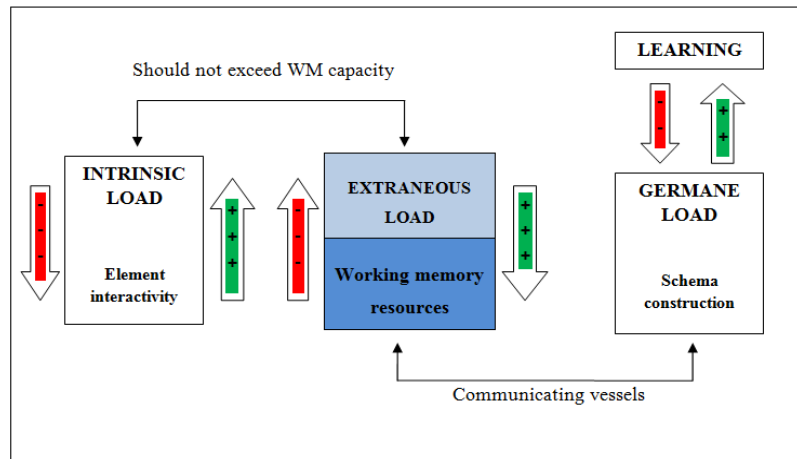
*green arrows (with positive signs) represent cognitive processes that support learning and problem skills development, and red arrows (with negative signs) represent cognitive processes that defeat learning and problem solving skills development

Thirdly, germane load should be optimized so that the working memory resources are optimally used. The germane load is optimized by keeping both intrinsic and extraneous loads at manageable levels. Once the extraneous load is effectively managed it can influence the levels of germane load. Hence the two loads are like communicating vessels. This 3-step instructional process that supports learning is represented by green arrows (with "+" insertions).

Having observed that the three components of cognitive load theory are manageable, it is reasonable to seek instructional techniques capable of substituting extraneous load with germane load. Employing the worked examples approach encourages self-explanation by learners. Self-explanation and lowering the split-attention effect are instructional techniques that have been used to substitute the extraneous cognitive load with germane cognitive load.

PRESENT STUDY

The study reported in this paper aimed to promote grade 10 learners' problem solving skills in mathematics. The study was located within *financial mathematics* in which the following aspects were treated: *simple and compound interests, higher purchase, inflation, and exchange rates*. Learners in this study were from a township background. A township is an area in South Africa segregated for occupation by persons of non-European descent, especially blacks (Macrae, 1994). Most of the learners from the township were from low socio-economic circumstances. Cognitive load theory (Sweller, 1988) was used to frame the study. Using cognitive load theory assumptions we conceived *learning* as the construction of learners' problem solving schemata.

In terms of this definition, learners' problem solving performance is a consequence of the ability to retrieve information in long-term memory. It seems various conditions affect the ability to retrieve information. According to Tulving and Thomson (1973, cited in Fulcher, 2003), the best conditions for retrieval are those that are most similar to those

during learning. According to cognitive load theory, learning that occurs in familiar settings ameliorates cognitive load associated with this process. So given this background, learners' real-world background was used as a tool to enhance their problem solving performance in this study. The aspects of this study are also documented in Dhlamini (2011). Hence the purpose of this paper is to provide plausible explanations for the observed learners' accelerated problem solving performance during the experiment.

Methods and design

The study employed a mixed-method approach, consisting of a quasi-experimental design and a descriptive survey design. Specifically this involved semi-structured interviews and classroom observations. Further, aspects of a descriptive survey design were included to account for the outcomes of the quasi-experimental study.

Study sample

A convenient sample (n = 57) of a grade 10 classroom located in a township setting in the Gauteng province participated in the study. The mean age of the participants was 18.44 (*SD* = 0.74). Given learners' township background and their link to a school with a low-performing profile in grade 12 mathematics, participants were considered to be of a disadvantaged socio-economic status (SES). "Disadvantage", in this study, is synonymous with "black", conflating race, class, language difference, cultural difference, educational difference and poverty. In South Africa it is standard to categorize someone by the color of their skin, as black, white and colored (Deaton, 1999). In this paper the term "black" is mainly used to refer to learners in disadvantaged township schools, and also as a representative term for township communities. To verify participants' socio-economic status and their suitability for participation, they completed a demographic questionnaire.

Table 1: Background characteristics of learners in the study

	Boys (n = 24)	Girls (n = 33)	Total (n = 57)
Median age			18.44 (*SD* = 0.74)
Learners' parentage status			
Living with both parents	3(5%)	6(11%)	16%
Living with single parent/live with guardian	7(12%)	12(21%)	33%
No parents	13(22%)	9(16%)	38%
No response on parent information	5(9%)	2(4%)	13%
			100%
Learners' parents' employment status			
Parent employed	5(9%)	7(12%)	21%
Parent self-employed	4(7%)	6(11%)	18%
Parent unemployed	17(30%)	11(19%)	49%
No response on employment status	4(7%)	3(5%)	12%
			100%
Learners' parents' education status			
Parent education: Poor	19(33%)	12(21%)	54%
Average	5(9%)	9(16%)	25%
Good	3(5%)	3(5%)	10%
No response on education status	2(4%)	4(7%)	11%
			100%
Learners' accessibility to computer at home			
Have a computer at home	7(12%)	4(7%)	19%
Do not have a computer at home	23(40%)	16(28%)	68%
No response about computer	2(4%)	5(9%)	13%
			100%

Table 1 shows that the sample was demographically comparable, and that most participants emerged from disadvantaged socio-economic backgrounds. Of the 57 participants 22 (38%) were without parents. Most participants' parents were not employed (48%), and 31 (54%) had parents with education level less than grade 12. A standard examination in South Africa is only written at the end of grade 12, which is also known as matriculation examination. The grade 12 examination results are used as an indicator of learners' performance at school level.

The educational status of a parent was considered poor when it was less than grade 12 (poor = less than grade 12). Also, average = grade 12, and good = more than grade 12. A majority of participants emerged from household backgrounds without a computer (68%). These results seem to suggest poor parental support experienced by most participants in the study. Hence the sample (n = 57) was suitable for participation in the study.

Information collected on school profile also corroborated Van der Berg's (2007) assertion that schools in disadvantaged townships are usually low-performing in mathematical terms. According to data elicited in this regard the school had achieved 31.3% in 2010 grade 12 end-of-the-year mathematics results. Although the school had both a computer and a science laboratory, both facilities were not optimally utilized by learners.

The school did not have a library for learners. Hence learning conditions in this school were considered not favourable and were disadvantaging to the learners. These observations are confirmed by participants' mean score in the pre-test (M = 18.52; SD = 6.827).

Instruments

We used an achievement test scores to measure learners' problem solving skills at pre- and post- stages. To construct the test we drew from the guidelines of the Department of Education (DoE) assessment documents. The test was constructed using cognitive levels suggested by the department (DoE, 2005, p. 26). To differentiate between learners on the basis of performance a taxonomical differentiation of questions suggested by the department of education was used (DoE, 2005, p. 12). To classify learners on the basis of performance we used the seven-point scale used in South African schools for reporting purposes. Since the test was out of 60 marks the following performance categories were established (see table 2):

Table 2: Classification of learners' achievement test scores

CRITERIA	CODE	DESCRIPTION
Low-performing	LO	Below 24 marks
Average-performing	AV	Between 24 and 42 marks
High-performing	HI	Above 42

Semi-structured interviews and classroom observations were also incorporated in the experiment. The aim of interviews was to understand the underlying thinking of participants, to enter participants' minds rather than taking only their written responses to context-based problem solving tasks. Interviews were constructed and conducted according to Cobb and Steffe's (1983) principles of clinical interviews. Face, content, construct and aspects of convergent validity were used to validate instruments. For the achievement test this was achieved through an expert panel in mathematics education and research. Spearman Brown's results confirmed the reliability of a test (r = 0.92) to measure learners' problem solving skills. Interviews took place at the end of the experiment and 4 learners were sampled for the interviews. Classroom observations took place during problem solving instruction.

Instructions and intervention

When designing and implementing a context-based problem solving instruction the flowing aspects of cognitive load theory (Sweller, 1988) were considered:

    Worked-examples: The potential of more robust learning was exploited with several worked-out context-based problem solving examples. All worked examples contained a problem with a modelled procedure for solving the problem (see appendix C). The modelled procedure is a step-by-step expert's solution given to learners to study. Sweller and his colleagues found that providing learners with many worked examples is more effective than providing them with a few worked examples followed by conventional instruction (Cooper & Sweller, 1987; Sweller & Cooper, 1985).

A worksheet with examples was given to learners at the beginning of instruction. Learners were allowed to work with peers in groups. Their task was to study examples and solve problems. At all times we were available for the learners and we provided assistance as needed. We used the example approach to demonstrate problems of the form:

> *An amount of R1 200 accumulates to R2 600 after 3 years. Find the interest rate if the investment earned simple interest.*

Problems of this type may result in a *'reversal error'* for learners where they write **P = 4C** instead of **C = 4P** (see Cooper, 1986; Wollman, 1983). In the context of our study, learners are prone to confuse the *principal value* (P) with the *future value* (A), or vice versa. They may write A = R1 200 and P = R2 600, instead of A = R2 600 and P = R1 200. The other possibility is that, due to their inadequate prior knowledge learners may spend time searching for cognitive mechanisms to match numbers with variables. According to cognitive load theory, this process is cognitively demanding and at the expense of mental resources that could otherwise be allocated to learning. However, if learners are exposed to worked examples they do not spend time searching or solving the problem, they rather devote all the available cognitive capacity to studying the worked-out solution procedure and constructing a cognitive schema for solving such problems (Van Gog & Rummel, 2010).

Split-attention effect: Another aspect of cognitive load theory that was considered for context-based problem solving instruction is the "split-attention effect" (Paas, Van Gog & Sweller, 2010). It is defined as the process of attending to two distinct sources of information (Paas, et al., 2010). The unnecessary visual search associated may heighten learners' cognitive load. An alternative instructional format to have all information physically located together may reduce the effect of split-attention. To test the influence of the split-attention effect on learners' context-based problem solving performance a '*split-attention detector*' was designed in this study (see appendix B).

The purpose of testing was to use the results to influence the design of instructional material for the experiment. In the activity given to the learners, one group of the class was given a context-based problem solving task in which both the problem and the subsequent questions were written on the same side of A4 page. The other group in the class was given the same task, but the problem was on one side of the A4 while the questions were on the flip side of the page. The purpose of this task was to observe the influence of a split-attention effect on learners' problem solving behaviour with an aim to maximize the efficiency of a context-based problem solving instruction.

It was observed that learners who were given context-based problem questions on the same page experienced minimal cognitive-related problems compared to learners subjected to split-attention inducing conditions. When the following questions were asked to learners in both groups some of the responses pointed to a group to which a respondent belonged and suggested whether or not the respondent experienced cognitive load. Some of the questions asked to the learners were:

> Q: Was it easy or difficult for you to do this task?

Most respondents subjected in the split-attention induced conditions responded with a "YES" answer. A follow-up question was advanced:

> Q: Why was it difficult for you to do this task?

One learner responded with a question:

> **L1:** *But meneer why did you write questions in another side*. [The word *meneer* is an Afrikaans word for Mister (Mr). In South Africa it is common for learners to refer to their male teachers as *meneer*, as a sign of respect]

Another learner: **L2:** *It was not fair for us because we were not working in one page.*

It was also observed that most learners who handled the split-attention inducing task took more time to complete the task. Results of the task demonstrated an advantage for the integrated versions of the task. From a cognitive load theory perspective, unnecessary visual search caused by the split-attention effect heightens learners' cognitive load, and working memory resources needed for learning are used to counter-act the effects of split-attention. So to reduce the working memory load we presented context-based tasks by physically integrating all aspects of the problems. For instance, there was no separate sheet for problem formulae. All formulae were integrated in problem sheets.

Given these guidelines, all activities were properly designed to optimize learning outcomes. For instance the *exchange rates* section normally includes an exchange currency rates table. In our case, the table, the problem and the questions were all integrated.

Results and analysis

Achievement test: The level of problem solving skill acquisition was measured by the performance in the achievement test. The pre-test (M = 18.54; SD = 6.827; n = 57) and post-test (M = 21.35; SD = 7.328; n = 57) were computed. Because the mean scores of the pre-test were low we assumed learners were in their early stages of problem solving skill acquisition. We also observed the increase of performance from the pre-test performance to post-test performance. To determine the effectiveness of the new instruction, the mean scores of the pre- and post-tests were compared using a t-test at the significance level of 0.05. The results of the t-test analysis are presented in table 3 below.

Table 3: Statistical results of the t- test analysis for the achievement test

Test	group	n	$\bar{x}$	SD	SEM	t	p-value
pre-test	grade 10 learners	57	18.54	6.827	0.90	2.116	0.0366
post-test	grade 10 learners	57	21.35	7.328	0.97		

Significant at 0.05 level

The results above suggest that the performance of the learners in a context-based problem solving achievement test improved significantly ($p < 0.05$). It is therefore possible to conclude that the context-based problem solving instruction designed to improve learners' problem solving skills is effective.

Several errors were committed by learners at the initial stage of the experiment. For instance, in question 2.2 of the achievement test the following errors were observed: 1) two types of reverse errors; and, 2) wrong choice of formula. The question was formulated as follows (see appendix A):

> **Q 2.2:** *R4 250 is invested for 6 years and grows to R14 740. Find the interest rate if interest is compounded annually.*

In this problem data can be arranged as follows: A = 14 740; P = R4 250; n = 6 years; i = ?, and the formula $A = P(1 + i)^n$ should be used. The following are samples of learners' script with "reverse errors":

Type 1 error	Type 2 error

In type 1 error the learner confused P and A values (Cooper, 1986). Another type of reverse error is reflected in type 2 error. Learners wrongly assigned the value of P to i. Most learners committed this type of error. Of the 57 learners 31 (54%) committed this type of error. When probed on this type of an error, one learner responded:

> **L3:** *If you take money to the bank you get interest. The money you get in the end is interest, bigger than your first money.*

Most learners conceded to this learners' response as they nodded in silence. To these learners a phrase such as "accumulated amount" referred to the interest which they associated with "i". Another type of error (type 3 error) that emerged in this question was learners' inability to select the correct formula for problem solving. The example is given below.

318

Type 3 error

This learner selected a wrong formula (simple interest formula instead of compound interest formula). Moreover, this learner committed a type 2 error by replacing "*A*" with "*i*". Of the 57 learners, 19 (33%) committed a type 3 error. These findings suggested that learners lacked problem solving skills at the beginning of the experiment.

Interviews and observations: Interviews and observations also corroborated the results of the achievement test. However, there was evidence that learners were progressing in problem solving skills acquisition. For instance, during classroom observations, whenever an extended period of silence was observed we asked the learner: "*What are you thinking?*". Learners' responses demonstrated their attempts in linking novel problems to previously encountered problems. For instance, this was one learner's response to the above questions.

L4: *I'm trying to think how did we do the same problem sir.*

According to Cobb and Steff (1983), these kinds of questions only cause minor interruptions of learners' actions and do not threaten the data's validity. Periods of self-reflection may indicate instances where learners are monitoring and assessing their actions to aid their understanding of the problem (Cobb & Steff, 1983).

We questioned learners as they worked through context-based problems. Learners' responses were coded in terms of whether they reflected problem solving schemata. For an example, the following learners' responses were coded as reflecting schema construction:

L5: *This problem reminds me of an earlier problem that we solved.*
L6: *I'm using the same step as in that problem.*
L7: *I'm solving this one like that one.*

Observations are that when faced with novel context-based problems, learners reported thinking about how an earlier problem (example) had been solved. The above responses demonstrated that schemata influenced their performance on problems that fell within the scope of those schemata. These results replicate Cooper and Sweller (1987) in which they questioned grade 8 learners as they worked novel algebra problem. Respondents demonstrated gains in schema constructions through their responses.

When one learner was asked about the context-based problem solving approach she responded: "*I think it was easy to solve problems after we did the examples*". In terms of cognitive load theory, the latter response suggested learners experienced reduced levels of cognitive load during a context-based problem solving instruction. According to Van Gog and Rummel (2010), example-based instruction, which was the pillar of context-based

problem solving instruction, should minimize learners' use of cognitive resources in activities that are not relevant to schema acquisition and automation (sources of intrinsic cognitive load and extraneous cognitive load) and maximize learners' use of cognitive resources in germane activities (sources of cognitive load) within the limits of working memory capacity.

SUMMARY AND CONCLUDING REMARKS

Using the assumption provided by the cognitive load theory in this paper, it is now possible to provide plausible explanation for the observed learners' accelerated problem solving performance. In our earlier discussions we demonstrated that complex tasks such as problem solving are high in element interactivity (extraneous load). Using this knowledge and the results of our experiment, we have argued that many of the elements involved in solving context-based problem solving tasks in financial mathematics interact with each other and so cannot be considered in isolation. Given that most problems in financial mathematics are presented in real-world world contexts, element interactivity may appear to be very high if the context of the problem is not familiar thus heightening the extraneous load that may hamper the desired learning (see figure 1).

In financial mathematics problems, learners not only have to identify relevant information, but they have to (a) simultaneously match specific key amounts with their corresponding symbols, and, (b) construct relationships between them. This process may pose challenges for a novice problem solver. According to cognitive load theory and figure 1, a rise in extraneous cognitive load reduces working memory resources needed for schema construction and automation. So to alleviate these cognitive challenges the following techniques were incorporated in the design of context-based instruction: 1) we minimized the effect of the split-attention phenomenon; 2) we used learners' real-world context during problem solving; 3) we employed example approach; and, 4) we encouraged self-explanations by learners. In terms of cognitive load theory, all of these instructional techniques contribute positively to learning (Van Gog & Rummel, 2010). Aspects of a 4-stage context-based problem solving model are explained (see figure 2).

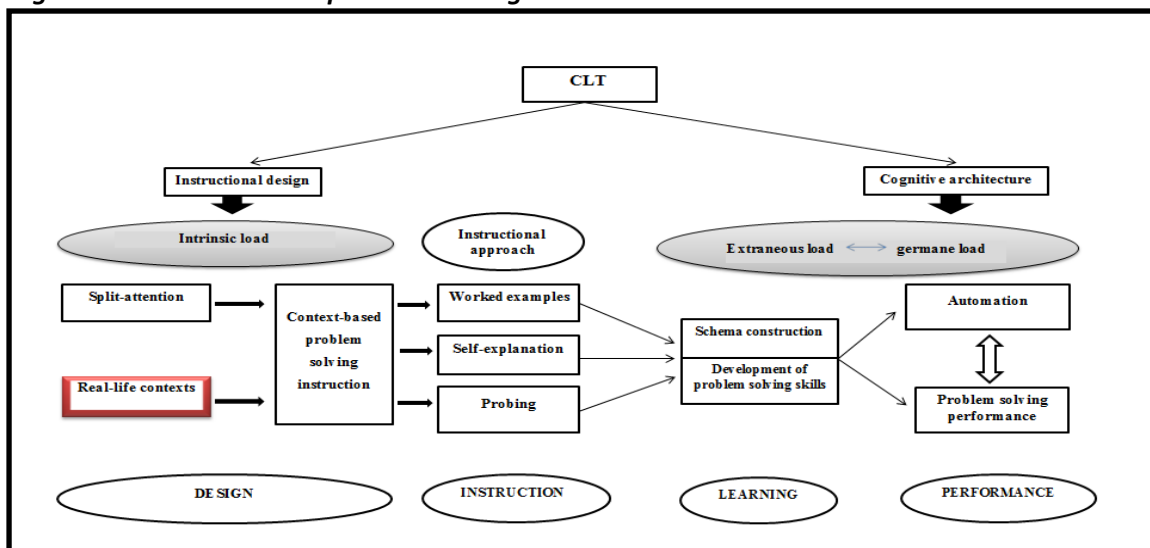*Figure 2: A context-based problem solving instructional model*

Figure 2 shows that four stages constituted the context-based problem solving approach. *Firstly*, in our design of instruction we eliminated the split-attention effect by physically integrating all aspects of a context-based task. Furthermore, to minimize the effects of extraneous load we located problem solving tasks within learners' real-world experiences. *Secondly*, during instruction we presented learners with various worked-out problems which were followed by problem solving. We probed learners to evaluate their problem solving progress. Probing was mostly prompted by learners' observed problem solving actions.

The *third* stage of our model represents the phase at which learners demonstrate their level of problem solving skills acquisition. This is observed through learners' actions and probing feedback. The *fourth* stage in our model is when learners demonstrate their performance and skill this is tested through a formal test. The findings of this position a context-based problem solving instruction as a robust instructional technique to bolster learners' problem solving skills.

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APPENDIX A: ACHIEVEMENT TEST

<table>
<tr><td colspan="2" align="center">ACHIEVEMENT TEST</td></tr>
<tr><td><u>SUBJECT</u></td><td>: Mathematics</td></tr>
<tr><td><u>LEVEL</u></td><td>: Grade 10</td></tr>
<tr><td><u>TOPICS COVERED</u></td><td>: Simple and compound interests, higher purchase and inflation.</td></tr>
<tr><td><u>QUESTION TYPE</u></td><td>: Long questions</td></tr>
</table>

QUESTION 1: Simple Interest

1.1 Thembi invests R50 000 for 10 years at an interest rate of 15% per annum (p.a.) Simple Interest. Find:

    1.1.1 The future value of the investment; (4)

    1.1.2 The simple interest received at the end of 10ᵗʰ year; (3)

    1.1.3 The simple interest received each year. (2)

1.2 Mapule's parents are unemployed, and she borrows money from the First National Bank in order to buy her school books. How much did Mapule borrow from the bank at 12% p.a. Simple Interest (SI) if she had to pay R510 interest after 5years? (5)

[14]

QUESTION 2: Compound Interest

2.1 Mrs Mokoena wants to start a 'spanza' shop (tuck shop) in order to make a living, but she does not have money. She then borrows R4000 from Capitec Bank at 5% p.a. compounded annually. How much will she need to pay after 6 years? (5)

2.2 R4 250 is invested for 6 years and grows to R14 740. Find the interest rate if interest is compounded annually. (4)

2.3 Calculate the compound interest on a loan of R800 at 7% p.a. if the interest is compounded half yearly. (3)

[12]

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QUESTION 3: Higher Purchase

3.1 A car radio costs R960. Uncle Tsepo buys a radio on Hire Purchase (HP) and agrees to pay a deposit of R100 and 24 monthly payments of R45. Calculate:

3.1. 1 The total amount paid, (4)

3. 1.2 The total simple interest paid. (4)

3.2 Nomsa buys a DVD player for R9 800. She takes out a higher purchase loan involving equal monthly payments over three years. The interest rate charged is 14% per annum simple interest. Nomsa also takes out an insurance premium of R10, 35 per month to cover the cost of damage or theft. Calculate:

3.2.1 The actual amount paid for the DVD player; (5)

3.2.2 The interest paid; (4)

3.2.3 How much must be paid each month. (2)

[19]

QUESTION 4: Inflation

Your brother wins a LOTO competition and decides to invest R50 000 now. He secures an interest rate of 9% p.a. compounded annually. The inflation rate is currently running at 12% p.a.

4.1 What will the future value of your brother's money be in 15 years from now? (4)

4.2 Due to inflation, what money will have the same buying power as R50 000 in 15 years' time? (3)

4.3 By how much will the buying power of your brother's money have declined after 15 years? (3)

[10]

QUESTION 5: General

Thembi spent R475 on two skirts and a pair of shoes. How much did she pay for the skirts and the shoes? [5]

Total score: 60

324

APPENDIX B: SPLIT-ATTENTION CONTEXT-BASED PROBLEM SOLVING TASK

Task:

Uncle Thabo wins R500 000 from a LOTTO and decides to invest 10% of his winnings. He goes to Standard Bank and invest his money for 10 years at an interest rate of 15% per annum simple interest. Find:

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1. the amount of money uncle Thabo invested with Standard Bank;

2. the accumulated amount of the investment after 10 years;

3. the simple interest received at the end of the 10th year;

4. the simple interest received each year.

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APPENDIX C: A SAMPLE OF AN EXAMPLE-APPROACH WORKSHEET AND A CORRESPONDING CONTEXT-BASED PROBLEM SOLVING TASK

Problem: How much money was invested five years ago if the value of the investment is currently R7000? The interest rate was 8% per annum simple interest.

Tools: **Formulae:** Simple interest formula $\longrightarrow$ $A = P(1 + in)$

Compound interest formula $\longrightarrow$ $A = P(1 + i)^n$

Notations:

P = present value of the investment (original amount at the beginning);

A = accumulated amount (future value) of the investment after n period;

n = time period;

$i = \dfrac{r}{100}$ for the simple interest rate r%.

Steps	Step-by-step explanation
$A = P(1 + in)$	**Step 1:** Choose correct formulae by using key words "simple" and "compound" in problem.
$A = 700; P=?; i=0,08; n = 5.$	**Step 2:** Arrange data by attaching each value in problem to the correct symbol.
$7000 = P(1 + 0,08 \times 5)$	**Step 3:** Substitute data in formula without changing the arrangement of formula.
$7000 = P(1 + 0,4)$	**Step 4:** Work on more complicated side and apply BODMAS. Start by multiplication inside bracket.
$7000 = P(1,4)$	**Step 5:** Add inside bracket.
$\dfrac{700}{(1,4)} = P$	**Step 6:** Divide by (1,4) both side to make P the subject of formula.
$P = R5000$	**Step 7:** Simplify and solve for P.

Problem task[9]

In 4 years Sipho wants to have saved R30 000 to open a tuck shop in township. He manages to receive an interest rate of 12% per annum simple interest. How much must he invest `now in order to achieve his goal?

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[9] The context in which this problem solving task is presented was manipulated to that of learners. The purpose of context manipulation was to reduce learners' extraneous cognitive load.