Abstract: This paper presents the literature reviewed to reflect on the advances in multimedia in the teaching of mathematical problem solving. The use of multimedia in teaching mathematical problem solving has become a need in the 21st century. Multiple resources in addition to text material are necessary to stimulate prior learning. The stimulated prior knowledge can be encoded in the students’ memory in the context of the new knowledge; and then exploration on the problem posed can be advanced. In exploring the problem students can formulate conjectures, and then confirm and prove the conjectures to ultimately provide the solution. The purpose of this paper is to highlight the 21st century innovations in multimedia to enhance the teaching of school mathematics. The paper prompts a need in mathematics teachers to keep pace with developments in teaching resources. The paper also contributes in providing knowledge on the kinds of 21st century resources to be considered in mathematics classrooms, and guidelines on how to plan and co-ordinate multimedia for mathematical problem solving purposes.

Keywords: multimedia, mathematical problem solving, digital resources, text material, virtual manipulatives

1. INTRODUCTION

Technology and multimedia play a vital role in the contemporary society, and students are exposed to different media on daily basis out of the classroom context. Media (singular medium) are a means of transferring or delivering messages (Li-Ling Kuo, 1991). The author further motivated that resources are called teaching media when they transfer the message for the purpose of teaching. Multimedia refers to the use of a variety of media formats, for example, text, visuals, audio video, digital resources etc., (Newby, Stepich, Lehman, Russel & Ottenbreich-Leftwhich, 2011). In light of the given explanation about multimedia, different types of teaching and learning media and resources are referred to as multimedia in this paper. Regardless of the strong impact of technology and multimedia on societies, individuals and different organisations, the teaching and learning of mathematics in most of the South African classrooms is still centred on traditional talk, chalkboard and text teaching method. The traditional way of teaching mathematics therefore, presents mathematics as an abstract subject in classrooms. In this paper, keeping pace with advances in multimedia means the application of multiple advanced teaching resources to augment instruction. Multiple teaching resources are necessary in order to concretise mathematics and to stimulate the learning of mathematical problem solving. Mathematical problem solving means engaging in a task for which the solution method is not known in advance (Billstein, Libeskind & Lott, 2014). The mathematical problem solving skill that can be applied to develop the unknown solution method is outlined by Polya (2004) as a four process step. The four steps are: understanding the problem, devising a plan, carrying out a plan and looking back (ibid.). In applying the problem solving skill (four
process steps) multimedia and technology are necessary because multimedia and technology can support problem solving. Indiana university report (2012) outlined that multimedia and technology supported problem solving means that problem solving can be approached from diverse perspectives, content learning develops naturally as learners work towards finding a solution, learners assume an active role in the problem solving process, and teachers provide learning support as well as rich multiple media sources of information to assist students in successfully completing the mathematical problem solving process.

Active learning in mathematical problem solving can be prompted through the application of multimedia in teaching. This paper addresses the notion that technology encompasses the usage of digital resources as well as the application of creative skills to improvise in order to access multimedia. Therefore, teachers are advised to use digital resources at their exposure or available resources to be creative and improvise in order to advance multiple resources. The needs of students include learning through exposure to different media, learning through concrete mathematics and stimulated active participation in classrooms. Therefore, it is from this perspective that the paper incites a discussion about how to keep pace with advances in multimedia by bringing the concrete world of the technologically inclined student into the mathematics classroom. Furthermore, mathematics teachers are bound to know the essence of keeping pace with advances in multimedia; types of learning and teaching support material that can best serve the needs of students in the 21st century, how to plan and organise well-resourced classrooms; and strategies that can be employed to select advanced multimedia in Mathematics classrooms. The paper addresses the need for both resourced and under resourced schools to improve mathematics instruction by using diverse resources to include all students’ learning methods. Other students learn better through text, others through visual stimuli while the other group learn better through listening. In essence, multimedia must be used by teachers to deliver teaching information during mathematical problem solving so that students can get a wide range of learning experience through a wider range of channels (Ruiji, 2012).

2. THE ESSENCE OF KEEPING PACE WITH ADVANCES IN MULTIMEDIA IN MATHEMATICS

The Research Centre (2015) has highlighted that abstract concepts are essential to understanding and performing mathematics. In addition, Research Centre motivated that abstract concepts are also a source of difficulty for many students who struggle with mathematics, many of whom find even basic mathematics concepts difficult to understand. It is argued that a popular approach to help students understand abstract concepts is the use of manipulatives; and further manipulatives enable students and teachers to represent concretely the abstract concepts that students are learning in mathematics classrooms (Research Centre, 2015). Young (2006) defined manipulatives as physical objects that students can interact with to promote learning. In addition, Young (2006) argued that the use of physical manipulatives in Mathematics Education has been regarded as an effective teaching and learning strategy. In concurrence, (Newby, Stepich, Lehman, Russel & Ottenbreich-Leftwhich, 2011) averred that, teachers are supposed to organise different kinds of material (text, audio, visual) when planning the lessons and apply material in teaching. Further, the authors asserted that the application of different media in teaching can help students to organise new information, link it to the existing knowledge and encode it into short and long term memory. The encoded information can be retrieved when needed to formulate conjectures, confirm formulated conjectures, prove conjectures and ultimately provide solutions to problems (Newby et al., 2011; Polya, 2004). Multimedia, that is, diverse mathematical manipulatives are essential in helping
students to achieve higher order thinking skills and technical competence for mathematical problem solving purposes (Carbonara, 2005).

Further, actual objects have an appeal for the student, therefore, the student must be provided with things to see, touch and handle (Reddy & Nagaraju, 2007). In addition, Reddy & Nagaraju, (2007) highlighted that the provision of concrete material in the mathematics classroom should be adequate; without sufficient provision, the subject becomes abstract. It is consequently essential for teachers to apply multimedia in instructional programmes in order to avoid the common practice of using text only in teaching mathematical problem solving. Although the manipulatives or concrete materials are used primarily in the lower grades, they offer a useful means to introduce new concepts to students in all grades (Research Centre, 2015). Trends in International Mathematics and Science Study (TIMSS) score indicated that the dual nature of South African society is that learners from well-resourced schools produce better results than learners from under-resourced schools (TIMSS, 2013). In essence a well-resourced learning environment that can impact on active learning and better results is necessary.

A well-resourced learning environment influences active learning by allowing students to actively interact with multiple resources in order to learn by acquiring meaning; and consequently to solve problems by applying acquired knowledge, skills, attitudes and values (University of South Africa [UNISA], 2008). Students can become mentally active, literate, independent, confident and critical if they have access to a wide range of suitable learning support material (ibid.). The usage of multiple resources in teaching and learning mathematical problem solving can contribute towards developing students’ critical thinking, confidence and independence (Carbonara, 2005; UNISA, 2008). When tools are incorporated into students’ activities, they are not merely auxiliary components in the teaching of mathematics; they shape students’ actions and therefore their learning (Trigueros & Lozano, 2007). In order to meet the students’ individual needs, teachers must keep pace and get advanced in multimedia usage in this digital era. However, teachers in mathematics classrooms still continue to use the same teaching strategies (lectures) and techniques (writing on a large board) as it happened in the 1920s” (Barron, Orwig, Ivers & Lilavois, 2002). Barron et al. (2002) further argued that it is still difficult for education systems to keep pace with advances in multimedia and information access, and mathematics classrooms have not changed in terms of integrating technology or multimedia into teaching and learning. Learning and teaching support material (LTSM) as known at schools must shift from text book only practice to multiple resource usage and to incorporating technology as additional support in teaching and learning.

3. LEARNING AND TEACHING SUPPORT MATERIAL (LTSM) IN THE 21ST CENTURY

Learning and Teaching Support Material (LTSM) include: textbooks, magazines and newspapers, pre-recorded videos, audio cassettes, slides, multimedia packages, CD-ROMs, computer software and internet or World Wide Web, blocks, coins, tangrams, spinners, rulers, fraction bars, algebra tiles, geoboards, geometric planes and solid figures (UNISA, 2008; Newby, Stepich, Lehman, Russel & Ottenbreich-Leftwich, 2011; Reddy & Nagaraju, 2007). There is a need to incorporate diverse media during teaching in order to promote learning and further to help students to represent the abstract concepts concretely (Research centre, 2015; Young, 2006). Moving from the concrete to the abstract by using learning and teaching support material can help students to approach mathematical problem solving better. A better approach can be advanced because concrete materials can help students to organise new information in the context of the existing or pre-
knowledge during problem solving. (Newby, et al., 2011) emphasised that multimedia programs that incorporate audio, textual as well as pictorial information (visual) are programs that can help students to recognise meaningful prior learning and how the new information relates. Usage of physical objects or manipulatives in Mathematics classroom is essential and different classes of learning and teaching support material must be considered in each mathematical problem solving activity for active learning to take place.

Resources that can be used in Mathematics classrooms, that is, the LTSM can be classified into the following groups: (1) Audio and Audio visual material; (2) Visual material; (3) Text material; (4) Digital resources; and, (5) Mathematical Virtual Manipulatives (Newby, Stepich, Lehman, Russel & Ottenbreich-Leftwhich, 2011). Essentially, media should be used to supplement each other in every lesson. For a lesson to be more informative and to address the needs of all learners, more than one media should be applied in teaching mathematical problem solving in the classroom. Different media favours diverse situations and economic constraints; therefore, teachers should know each type and the relevance of each material in addressing objectives of a particular lesson and the needs of students in a particular environment. Teachers are also challenged to improvise in adverse classroom conditions by selecting accessible material to avoid applying text only in all lessons and topics. In addition teachers can supplement text with other material in order to cater for all learning objectives and individual students’ learning styles. The knowledge of classes of media can prevent the classification of other schools as under-resourced and hence the compromise of active teaching and learning in mathematics. At least one classification of media can be accessible in a particular environment and can be applied to supplement the textbook. The classifications of media are elaborated in the following four paragraphs to explicate the importance, types and usage of each media group.

The need for the use of audio-visual aids (sight and sound features) in mathematics classrooms springs from a realisation that teaching that is supplemented by sight and sound features facilitates the learning process (Reddy & Nagaraju, 2007). Further, Reddy & Nagaraju (2007) perceived that there is a serious lack of mathematical resources in schools, and the lack of resources contribute to challenges of teaching and learning in mathematics classrooms. Therefore, the authors motivated that audio visual materials may have an important role to play in mathematics classrooms. Audio Visual material includes the radio, recorders, CD players, video tape and television. In mathematics audio visual aids can be used as a means to extract scientific terminology and study the origin of concepts (Esteban, Gonzalez & Tejero, n.d.). When properly used and controlled, Audio Visual aids can accomplish bringing about significant changes in student behaviour and a variety of learning experiences, can make learning meaningful over a wide range of student abilities, can furnish rich experiences from which meaningful concepts will develop (Santos & Nishida, 1994). Presentations by experts in mathematics can also be accessed by teachers and learners through audio visual material. One such an example in South Africa is tutoring through the medium of television through Mindset learning programme where educational programmes are offered via satellite television with additional multimedia support (Matthee & Liebenberg, 2007). It is important to include audio-visual aids in mathematics lesson presentation in order to help students develop mathematical vocabulary, terminology or all essential concepts. Students can easily advance mathematical problem solving if mathematical vocabulary, terminology and concepts are well established.

By using visual materials (objects that students can see and touch) teachers are more likely to maintain students’ attention and encourage active participation and can also cater for different
learning styles (Akers, Cheung, Chen & Wong, 2010; Pragacha, 2014). Types of Visual resources are projectors, display boards, charts, posters, pictures magazines, newspapers, measuring instruments, drawings and cartoons. In Mathematics real objects are the best visual aids (Pragacha, 2014). Pragacha (2014) further emphasised that there are so many skills that correlate with real objects, for example, slicing cakes, fruits and others to help learners relate with fractions; using balls, containers and other real objects to recognise shapes and sizes. Mathematics becomes fun and interesting when learners see real objects, pictures and other forms of visual aids. They can relate easily to the lesson with the use of visual aids. Visual aids are easily accessible and teachers should consider applying some of them to supplement the text material in mathematics classrooms.

Text materials play a prominent role in schools and in mathematics classrooms in particular (Nicol & Crespo, 2006). In addition, Nicol & Crespo (2006) argued that in mathematics classrooms, text material provide frameworks of what is taught, how it might be taught, and the sequence for how it could be taught. In emphasis Mathmaine report (2011) stated that teachers prefer a textbook because teaching from a text can require less work than other approaches; many texts provide extra resources such as chapter tests, worksheets of extra problems as well as project support materials that save time for the teacher. Furthermore, the Mathmaine report (2011) outlined that the teacher’s edition of the text can also remind teachers of alternative approaches to a topic, give guidance on sequence and timing, and make it easier to coordinate with other teachers who are teaching the same subject. However, in contrast, the Mathmaine report (2011) stated that teachers can be constrained by textbooks if they seek to cover material at a faster pace, in greater depth, or with a more “constructivist” approach than the textbook author(s) used. Further, motivation was made that the explanations in texts can be simplistic or procedural; the problems in textbooks can be too few in number, too easy or not relevant enough for the students; and textbooks often seem to facilitate or encourage memorization instead of understanding (ibid.). Teachers might refrain from using text only if they see it necessary to develop their own problems and tests, and teach their own way. Text is good, and easily accessible in Mathematics, can be used everywhere at any time, can provide reading and writing skills, however, integration with other resources for active learning is necessary.

Digital resources or material that can be used in teaching and learning comprise computers, computer laboratory, computer software, internet/worldwide web, Geogebra programmes, digital textbooks, online exercises, E-mail Interactive Whiteboards (Ehmann, Gerhauser, Miller, Voggel & Wassermann, 2014; Gueudet, 2010). However, the necessity for the computer laboratory is the greatest obstacle for the integration of computer technology in mathematics education in schools (Ehmann et al., 2012). In addition Ehmann et al. (2012) emphasized that mobile technology is becoming more and more relevant in mathematics education. In the same note the South African educational landscape like many other third world environments advances itself to a unique solution for learner support (Matthee & Liebenberg, 2007). The authors explained the unique solution which is affordable as access to the Internet via technology enhanced learning, specifically mobile learning in a South African context. Digital resources can further assist teachers to select, modify and create teaching media. For example pictures, sketches, formulae and other information can be retrieved from the teachers’ computers, and other material like posters can be created for the classroom.

Mathematical virtual manipulatives are virtual images that do not physically exist as such but made by software to appear to do so (Research Centre, 2015). In concurrence Moyer, Bolyard, & Spikell, (2002) argued that virtual manipulatives are basically digital objects that resemble physical objects
and can be manipulated, usually with a mouse, in the same ways as their authentic counterparts. Further, the examples of mathematical virtual manipulatives are listed as blocks, base 10 blocks, tangrams, geometric planes and solid figures (Moyer, Bolyard, & Spikell, 2002; Research Centre, 2015). The Research Centre (2015) further highlighted that for the most part, classroom use of manipulatives has involved concrete or physical manipulatives; however, with the assistance of the World Wide Web virtual versions of concrete manipulatives (typically used in mathematics education such as Base 10 Blocks, Cuisenaire Rods, and Tangrams) are available at no cost online. Research Centre (2015) further hinted that if the virtual manipulatives are used appropriately (not used as an extra student activity), then they can provide students with opportunities for guided discovery which can help them to build a better understanding of mathematical concepts. Further, visual manipulatives “increase motivation and attention in students as well as teachers” (Young, 2006: 3). Application of virtual manipulatives with clear instruction and assistance from the teacher can equip learners with clear problem solving skills, and all learners with different learning styles, including learners with disabilities can benefit from problem solving activities in the classroom (Research Centre, 2015; Young, 2006). Virtual manipulatives need to be supplemented by other material in order to function effectively in the mathematics classroom.

Learning in this digital era, that is, in the twenty first century, has attained a gradual improvement and learners are best motivated through incorporating multi-media and technology into the text material. Ronis (2008) outlined technology as the adhesive that enables learners to approach mathematics from a more global perspective because “it offers learners the tools and information they need to explore mathematics in the real world” (p. 10). Introducing multiple resources in addition to text in mathematics classrooms is essential in the 21st century. Text is very essential in equipping students with reading and writing skills, however complementary resources are necessary for catering the needs of all students and for adherence in the digital orientation of diverse societies. The Gauteng Provincial Governance in South Africa has realised a need to develop smart schools within the province in order to “modernise the delivery of education in the province” (Premier’s address, Mail & Guardian, 2015: 1). The Premier in his address (Mail & Guardian, 2015) indicated that modernising the delivery of education will include launching paperless, technologically-enabled smart schools. However, the eradication of paper and text in learning will still remain deficient. The application of one type of material is insufficient to cater for diverse student needs, but the integration of diverse material can ensure individual and group learning. Using manipulatives (learning materials students can work with physically) in mathematics can afford the students the opportunity to comprehend the abstract (Ronis, 2008). In this regard text materials, audio visual, visual and virtual material remain a good enhancement to digital technology.

Reddy & Nagaraju (2007) argued that in advanced countries, for example western countries, the Educational Technology (that is, teaching using diverse material or multimedia) and its efficacy in different dimensions have been well explored. Educational Technology has been well explored means that Educational Technology is effectively being put into practice. Therefore, Reddy & Nagaraju (2007) motivated that there is an important need to create efficient learning conditions and optimise the learning outcomes by applying the concept of Educational Technology especially in mathematics. Mathematics teaching and learning in South African classroom is challenged to transform from textbook or text only to teaching and learning through application of multiple

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3 Gauteng is one of the nine provinces in South Africa.
resources. The application of multiple resources in teaching or educational technology should be done to activate students’ interest and knowledge acquisition in order to advance better mathematical problem solving skills in classrooms. In planning the lessons, teachers need to consider planning and organising resources accordingly in classrooms. Well planned and organised classrooms in terms of multimedia can “support teacher-student relationship as co-explorers in problem solving” (Kenderov, Sendova & Checlarova, 2012: 47).

4. THE IMPORTANCE OF PLANNING AND ORGANISING WELL-RESOURCED CLASSROOM FOR SUCCESSFUL MATHEMATICAL PROBLEM SOLVING ACTIVITIES

The role of the teachers is to facilitate knowledge acquisition by establishing environments in which learners are guided and encouraged to develop their problem solving skills (Taplin, 2014). In such environments diversity of the teaching and learning material is a need. Multimedia should play a major role to influence learner active participation and interest in mathematics classrooms. Organised classrooms, that is, classrooms with relevant and enough resources remain a key to successful teaching and learning (Masilo & Ramorola, 2013). Diverse learning environments require planning and organisation of resources that will address educational needs of such different environments. Proper planning and organisation of resources and activities in the educational environment can enhance learner interaction, interest and acquisition of more knowledge for problem solving. Problem solving has long been recognised as one of the hallmarks of mathematics (Billstein, Libeskind & Lott, 2014). Therefore, well planned classroom environments, that is, classrooms with relevant and enough resources, are needed for the improvement of mathematical problem solving skills in most of the mathematics classrooms in South Africa. A classroom that is well planned or organised for mathematical problem solving possesses a group learning situation, available resources students can use to find answers, and flexible time allocation (Ronis, 2008). Further, Ronis (2008) averred that availability of advanced multiple resources and sufficient time are key to inviting and attractive classrooms, and such classrooms in turn activate action, critical thinking and participation. Orderly learning environment, teacher and students’ commitment, as well as adequate supply and material support for teaching and learning were emphasised as school climate variables influencing student achievement (Zullig, Koopman, Patton & Ubbes, 2010). Therefore, an alternative way to keep the digitally orientated students interested, engaged and committed to problem solving is to introduce multiple resources as well as technology based multimedia in orderly mathematics classrooms. Most of the young minds in this era are technologically focused, therefore problem solving activities in diverse mathematics classrooms should be planned in a way that they will to some extent be supported by technology. Teacher commitment in delivering successful mathematical problem solving skills must be guided by relevant strategies in selecting learning and teaching support material.

5. STRATEGIES THAT CAN BE EMPLOYED TO SELECT ADVANCED MULTIMEDIA

The most important decision faced by teachers is how to select instructional or teaching media that can enhance problem solving skills. Newby et al., (2011) cautioned that: (1) there are always techniques and guidelines to follow in selecting teaching media resources; (2) teaching media can be combined during instruction; and (3) technology can be used to select, modify and create teaching media. Looking at the current trends in media usage, whereby a textbook is the main, and in some mathematics classrooms the only resource; it shows that teachers are lacking in terms of techniques to adopt relevant multimedia for instructional purposes. In acquiring teaching media resources the teacher must select teaching media resources from the available (Newby, et al., 2011), for example
using radio and videos recorders. Furthermore, the teacher can try to modify existing material to meet the students’ needs and create new material only when there is a need (Newby, et al., 2011), for example, using boxes and papers to make geometric figures. The ability to modify existing material and create new material based on the accessible sources is a trait that is needed by teachers in order to ensure that text is supplemented by relevant media in all lessons and in all classrooms’ situations.

In selecting resources and modifying existing resources several factors must be considered by teachers. Ramiszowski (1992), Reiser & Gagné (1983) cautioned that factors that influence the media choices are the instructional method, type of learning task and the special characteristics of some students. Some students learn best through text, some are motivated and empowered by visuals and practical, while some will learn best through handling digital equipment (Ramiszowski, 1992; Reiser & Gagné, 1983). In consensus, Santos and Nishida (1994) highlighted that lectures alone can be frustrating to students if students are having different intellectual development. Teacher skills and attitudes were also outlined as a prominent factor that can influence the choice of teaching and learning media (Ramiszowski, 1992). For example, if teachers are not skilled and interested in educational technology, then their choice of teaching and learning media will revolve around the traditional resources mostly text resources. It is important that teachers must get orientated to technology, and develop interest in multimedia usage. This can be done through teacher training on multimedia planning and usage.

Finding, selecting, modifying and creating teaching aids require time and effort; therefore there are a number of ways in which technology can be used to advance the processes of acquiring, selecting, modifying and creating relevant teaching media resources (Newby et al., 2011). The ways were outlined by Newby et al., (2011) as: (1) Using internet search engines to access relevant information based on subject and grade; combining the use of electronic storage devices (CDs, DVDs) after accessing information; (2) ability to cut and paste within word processing and desktop programmes to quickly add visuals, change text, and alter needed sequences (3) Using technology to evaluate materials to gain insight into when, why and how the materials were developed. In situations where learners do not have access to digital resources, while the teacher has access, the teacher can use accessible technology to develop appropriate materials, for example visual aids like posters and pictures.

Teacher familiarity with advanced multiple teaching media resources, as well as skills to use technology to plan and create multiple resources is of utmost importance to ensure that students’ learning needs are fulfilled. In addition, teacher knowledge of diverse teaching media can assist the teacher to ensure that the learning tasks are adequately addressed, and that diverse students’ characteristics and learning styles are catered for. The knowledge of planning and co-ordinating multiple resources is also a need to enable teachers to organise and keep well-resourced classrooms to ensure effective learning in mathematical problem solving.

6. CONCLUSION
This paper has presented the importance of advanced multimedia in mathematics classrooms. It also elucidated on types of learning and support material to assert that the teaching and learning resources supplement each other in order to cater for diverse students’ needs in acquiring mathematical problem solving skills. Further, elaboration has been done on the importance of planning and organising teaching and learning aids to maintain well-resourced mathematics classrooms for successful mathematical problem solving activities. Strategies that can be employed to select advanced multimedia were also explicated. The planning calls for teachers to creatively
improvise and create material according to the environment and students’ needs. A well planned and organised mathematics classroom must contain more than one material or LTSM. For example, at an under resourced school a teacher can still opt for text supplemented by visual or audio-visual aids as they are cheap and accessible. Teaching from the concrete using manipulatives to help students understand the abstract mathematics has been emphasised. Teaching that applies only one material regularly contributes to learning impairments to some of the students who cannot cope with one chosen and applied material. Emphasis is laid on the paper that mathematics teaching should keep pace with advanced media in this digital era and should focus on applying diverse material other than a single resource in order to activate students’ creative critical mathematical problem solving skills.

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