QUANTIFYING CONTENT KNOWLEDGE GAPS IN THE FOUNDATION MATHEMATICS FOR ENGINEERING STUDENTS

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ABSTRACT: This paper aims to provide a method that can help in identifying content knowledge gaps and highlights the importance of making early identification of the students’ content knowledge gaps in mathematics. A cohort of 203 first year students in the extended Engineering Diploma programme was used as the sample in this probe. Using learner responses from the pre and post assessments, conceptual knowledge gaps were identified and this provided us with insight into some of the underlying reasons for their underperformance in foundation mathematics at a South African university. The educational background and the entry level characteristics in terms of the achievement levels in Mathematics of the sample from the National Senior Certificate (NSC) were scrutinized. A test in the form of a survey, which comprises of similar sections to the NSC examinations were set with the objective to find discrepancies in conceptual understanding.

Key words: Subject Content Knowledge; Content Knowledge Gaps; Extended programme; pre and post-test; competency tests

1. INTRODUCTION

Foundation mathematics is mainly offered by universities to students who want to improve their skills and competence in mathematics for the purposes of main stream admission. Extended programmes at most universities have been established to address the deficiencies in content knowledge gaps that students bring with them for Higher Education studies.

Curriculum reform in South Africa has come with a lot of criticism from educational practitioners. Since the advent of Outcome Based Education (OBE) to the recently revamped but also revised National Senior Certificate (NSC) there was a lack of clarity on the curriculum content. This culminated in the Curriculum and Assessment Policy Statement (CAPS), an amendment of NSC. Recent studies in Kwa-Zulu Natal (KZN) revealed that when teachers were assessed using the same questions from NSC examination papers to determine their level of subject knowledge and understanding of Mathematics they scored an average of 57% according to (htxt.africa) (2014). Pedagogical content knowledge is an intersection of the teacher’s knowledge of teaching methods, educational theories and assessment strategies on the one hand and subject content knowledge on the other, as suggested by Chai et al. (2013). This implies that on average these teachers’ content knowledge gap is 43% which they exhibit either teaching wrongly or never teach certain sections of the syllabus at all or perhaps only teaching sections of the syllabus which they understand.

CAPS brought into the curriculum new concepts which were either optional or not part of NCS mathematics curriculum. In this respect topics such as Euclidean Geometry, Probability and reciprocal Trigonometric functions were recalled, while other topics such as Remainder and Factor Theorem formed part of the grade 12 algebra syllabus in 2011. Olivier (2013) found rudimentary evidence drawing from the knowledge of teachers that elucidates that many subject advisors, or those who were tasked to present CAPS training at district level on their behalf, were not knowledgeable or competent enough to do justice to the intended training programme. It is
distinctly clear that the roll out of CAPS in mathematics coupled with basic training in this area has not achieved the desired expectation of the Further Education and Training (FET) teachers at grass roots level. In this regard the subject content skills needed by teachers seems to be much dire need than to receive additional training on how to implement revised CAPS strategies.

A lot of studies have been done on misconceptions by learners on mathematical concepts and these misconceptions persist due to the fact that we fail to eradicate the problem at grass root level, which is the teacher subject content knowledge. Certain conceptual relations that are acquired may be inappropriate within a certain context; a misconception does not exist independently but is contingent upon a certain existing conceptual framework as indicated by Pines (1985). Smith et al. (1993) said that “misconceptions develop because students have the tendency to incorrectly generalise prior knowledge when dealing with new tasks. This is the kind of knowledge that needs early identification and remediation before it is embedded and then used in a wrongful construct”.

This paper involves a survey of measurable subject learning gaps in mathematics, i.e. the extent to which prospective engineering students have deficiencies in Mathematics per topic or concept; and thereby giving lecturers an opportunity of identifying them for remediation. It is our job to be effective in helping students to integrate old and new knowledge so that from that trajectory, they can use that knowledge to analyze and synthesize at a much higher cognitive level as they move forward in their careers paths (Ginsberg, 2010). The South African grade 12 classes of 2013 were subjected to a curriculum in mathematics that was pitched at an assessment level that was equivalent to the discontinued standard grade mathematics syllabus (Collier-Reed, 2010). The type of learners that were accepted into the foundation programme had the following range of grade 12 mathematics marks, as shown in the table below.

Table 1: Mark range of mathematics marks, based on the grade 12 examination

<table>
<thead>
<tr>
<th>Mark Level (%)</th>
<th>Grade 12 Exam</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 39</td>
<td>0</td>
</tr>
<tr>
<td>40 – 59</td>
<td>96</td>
</tr>
<tr>
<td>60 – 79</td>
<td>101</td>
</tr>
<tr>
<td>80 – 100</td>
<td>6</td>
</tr>
</tbody>
</table>

The criteria used to place students in the Extended Programme were as follows:

1. Either Maths or Physical Science is below minimum requirements.
2. APS score lower than minimum requirement of 30.
3. Mainstream classes were full; these are usually students with a High Potential and make up a very small cohort.

These learners were then subjected to a pre and post test to determine discrepancies in their conceptual understanding which is the focus of our research.

2. LITERATURE REVIEW

Engineering Mathematics can be classified as an applied science, since it is there to link the mathematical skills with real technical applications and hence a solid foundation should be in place. Factual knowledge is closely integrated with creativity, problem solving and analysis according to
Christodoulou (2014) and all of these are possible if the factual knowledge is exact, especially in this subject that has very specific laws, axioms, theorems and procedures. Christodoulou (2014) further mentioned that when the relevant knowledge base is not in place, learners struggle to develop the understanding of a topic.

Basic things like factorizing, exponents, differentiation and solving equations are usually assumed to be in place when students meet minimum requirements to study at tertiary institutions. All that there is to intelligence is the simple accrual and tuning of many small units of knowledge that in total produce complex cognition; the whole is no more than the sum of its parts, but it has a lot of parts Anderson (1996). And according to Heick (2014) teachers need to shift their focus from teaching and re-teaching and change into systemic-guided diagnoses of academic performance barriers.

In the US, according to Saderholm et al. (2010), the Diagnostic Teacher Assessment in Mathematics and Science (DTAMS) was developed to measure the content knowledge and pedagogical content knowledge of middle school teachers; the results of the study confirm trends in middle school mathematics teacher preparation and certification, and the help needed to explain student achievement levels.

Examination Markers in South Africa are not tested for their competency, their subject knowledge or for their ability to interpret answers which are phrased differently from the examination memorandum. According to DA (2014), the Western Cape is the only provincial government that conducts rigorous competency tests for its Matric markers. According to NSC Technical Report of Department of Basic Education (DBE) (2012) plans were afoot to finalise the policy in January 2013 so that the competency test could be implemented in seven subjects in March 2013. The subjects selected for this competency test in 2013 were: Mathematics, Physical Sciences, Life Sciences, Accounting, History, Geography and English First Additional Language. More subjects will be included incrementally and by 2015, all markers for all subjects will be subjected to the competency test before being appointed. Personnel Administration Measures (PAM) determined by the Minister of Education in terms of the Employment of Educators Act 1998 (the Act) is silent on the issue of competency tests for Examination Markers and hence the provinces are not obliged to test educators before they are appointed as markers. In fact this would have been in line with the normal practice among countries in sub-Saharan Africa. This is a tragedy of the highest order and shows the Department’s unwillingness to address key issues.

South Africa is producing too few qualified teachers, especially in gateway subjects such as mathematics and physical science. The teacher training system is producing about a third of the country’s requirement of about 25 000 new teachers a year. It needs to produce about 15 000 more teachers a year, particularly in scarce subjects (such as maths, science, commerce, and technology), according to the report by the Centre for Development and Enterprise (CDE, 2011). Furthermore, CDE is of the opinion that there is a shortage of mathematics teachers, yet many qualified maths teachers are not teaching mathematics despite being willing to do so. Another study cited by CDE researchers found that 16 581 teachers in the Eastern Cape were qualified to teach mathematics, with 5032 of this cohort practicing teachers, and a further 2058 teachers were teaching without the necessary qualification. CDE also recommended that teachers need to be well remunerated so that they could do justice to teaching based on the fact that in other countries in Africa, they are well paid and they constantly undergo competency tests.

According to the South African Council of Educators (SACE) (2009), 13 South African universities produced 713 (12,1%) Mathematics educators out of a total enrolment of 5899 but at that year the number of learners taking the subject needed about 2832 (48%) new educators. Whilst they are in the education system, very little or nothing is done to revive or test the educators’ content
knowledge and the only people at the receiving end are the learners who are always ranked towards
the bottom in every competition or ranking as shown by Mullis (2011). Continuous professional
development is virtually non-existent in South Africa.

The South African Council for Educators (SACE), as a statutory body for professional educators will
have overall responsibility for the implementation, management and quality assurance of the
Continuous Professional Teacher Development (CPTD) system as indicated in the Government
Gazette No. 29832 Section 53 (2007). In the same policy subsection 4a stipulates that teachers must
take the initiative to identify the areas in which they need further development and should approach
the department for assistance to access training opportunities. The initiator of reforms, in this case
the Department of Basic Education, should be in a better position to be able to identify sections of
each subject which teachers need to be trained on and at the same time teachers should be willing
to be trained.

According to the Integrated Strategic Planning Framework for Teacher Education and Development
in South Africa ‘The basket of education services that district offices, circuit offices and their staff are
required to deliver are described in Annexure 2 of the guidelines document DBE (2011). Specifically,
in relation to teacher development, the services include the following: Assist educators to identify,
assess and meet the needs of learners DBE (2011: 28).’ This seems to contradict the responsibility of
SACE but if properly managed and synchronized at national level, continuous teacher development
can benefit the educators and ultimately learners.

The largest teacher union in South Africa, South African Democratic Teachers Union (SADTU) was
responsible for launching the SADTU Curtis Nkondo Professional Development Institute, according to
The Politician Magazine and Newspaper of May 6 2013, a novel idea but according to the Business
Day of 28 August 2013 SADTU has effectively blocked the introduction of competency tests for
matric exam markers, despite the exam quality watchdog Umalusi having told Parliament it
’strongly’ recommended the introduction of such tests. One wonders how they will go about
diagnosing the areas of concern in the subjects they will be trained on at the institute.

According to DBE (2012), the current state of Mathematics, Science and Technology (MST) across the
country and the lack of a working national framework for improving MST warrant the establishment
of national MST office to develop, implement, coordinate and manage the roll out of a revised MST
Strategy. It is recommended that such an office be set up at DBE, with the appropriate level of
authority and adequate human and other resources, to coordinate and assist provinces to
implement the Strategy at provincial level. The proposed office should implement mechanisms for
schools and district offices to elevate concerns and issue to the DBE to allow a system wide
communication structure that accommodates all voices.

In Gauteng Province, the Sci-Bono Discovery Centre and in Limpopo Province, MASTEC INSTITUTE
are examples of government institutions created to help redress the gaps in Mathematics, Science
and Technology knowledge and skills. The Gauteng Education MEC in his 5 year plan presentation,
highlighted the fact that the grade 6 Mathematics provincial average in 2013 at 45% was the best in
three years while the grade 9 Mathematics 2013 average stood at 15% according to GDE (2014). The
2011 Annual National Assessment (ANA) results were released by Basic Education Minister Angie
Motshhekga in June 2011 and the figures indicated that on average students had not achieved the
competencies specified in the curriculum. For example in grade 6, the national average performance
in mathematics was 30 per cent. This just shows how deep the crisis of the lack in Mathematics
content knowledge is concerned.
In Limpopo 57 Mathematics educators selected from all circuits in the province undertook an extensive continuous professional development effort offered by the University of Pretoria in conjunction with MASTEC to prepare and equip them on new concepts in CAPS and consolidate those concepts which were retained from previous curricula LDoE (2011). The number of participants leaves a lot to be desired though they were expected to go back and cascade the information, knowledge and skills gained to their respective circuits. Lack of capacity in the institute hampered plans to assist districts’ and circuits’ subject specialists carry out similar interventions at local level.

The Western Cape Department of Education with the help of nonprofit organizations (NPO’s) have WCED Programmes for Learners and Educators during School Holidays in which educators are empowered to teach the challenging sections of the Mathematics curriculum to their grade 12 learners with exam preparations WCED (2013).

Educational experts argue that the approach to teacher training must be reviewed and strengthened. They further commented that training activities, methods and processes implemented by provinces needed strengthening. Also there are issues of accreditation; training incentives and recognition need review. The design and management of a teacher training system that has effective benchmarks, effective engagement with training providers such as HEIs and NGOs, and the effective identification of which teachers need what training and support, proper monitoring and evaluation, post-training support in schools, and many other important elements of the system need to be planned DBE (2012).

The then Department of Education (DoE) now DBE together with National Business Initiative started a national group of schools called Dinaledi Schools and gave these schools additional support to improve Maths, Science and Technology through the provision of equipment and educator training as well as learner support to boost the number of learners graduating from secondary schools with good Mathematics and Science grades. In one study cited by CDE (2011) researchers, 73 matric physical science teachers from South Africa’s Dinaledi schools (which are provided with extra maths and science resources) were tested on basic problem-solving skills. Only 60 per cent were able to solve the problems involved.

If Higher Education Institutions (HEI’s) are to remain relevant, and increasing access to higher education in an attempt to alleviate the critical shortage of science and engineering graduates in South Africa, we have to find ways to respond adequately to the problem of assisting under-prepared first-year students “bridge the gap” and succeed in their studies (Wolmarans et al, 2010).

3. METHODOLOGY

3.1 Research Design

A survey in the form of a pre-test was administered to 203 first year Foundation Mathematics students engaged in the Extended Engineering diploma at the university for the year 2014, during the month of February. Subsequently, a total of 171 students wrote a post-test during the month of October of the same year. The reason why fewer students wrote the post test is because of drop-outs.
# Test Foundation Mathematics

Date: …………………..

**Answer all the questions. Round off to 2 decimal places where applicable.**

## Question 1

1.1 Factorize the following completely:

1.1.1 \( 2x^2 + xy - 6y^2 \)  

1.1.2 \( \frac{1}{4}m^2 - 36p^6 \)

1.1.3 \( 27a^6 + 64b^9 \)

1.1.4 \( (b - 2)^2 - 4(b - 2) + 3 \)

1.2 Solve for x:

1.2.1 \( (x^2 - 9)(2x + 1) = 0 \)

1.2.2 \( x^2 + x - 13 = 0 \)

1.2.3 \( 2.3^x = 81 - 3^x \)

1.3 Given \( x + 2y = 5 \) and \( 2x + 3y = 8 \). Solve for \( x \) and \( y \) simultaneously.

1.4 Given \( x - y = 2 \) and \( x^2 + y^2 - 7 = xy \). Solve for \( x \) and \( y \) simultaneously.

## Question 2

The graph of \( f(x) = \left( \frac{1}{3} \right)^x \) is sketched below.

![Graph of f(x) = (1/3)^x]

2.1 Write down the domain of \( f \).

2.2 Write down the equation of the asymptote of \( f \).

2.3 Write down the equation of \( f^{-1} \) in the form \( y = \ldots \).

## Question 3


$S (1 ; 18)$ is the turning point of the graph of $f(x) = ax^2 + bx + c$. $P$ and $T$ are $x$ – intercepts of $f$.

The graph of $g(x) = -2x + 8$ has an $x$ – intercept at $T$. $R$ is a point of intersection of $f$ and $g$.

### Question 3

#### 3.1 Calculate the coordinates of $T$. (2)

#### 3.2 Determine the equation for $f$ in the form $f(x) = ax^2 + bx + c$. Show ALL your working. (4)

#### 3.3 If $f(x) = -2x^2 + 4x + 16$, calculate the coordinates of $R$. (4)

#### 3.4 Use your graph to solve for $x$ where:

##### 3.4.1 $f(x) \geq g(x)$ (2)

##### 3.4.2 $-2x^2 + 4x + 16 < 0$ (4)

### Question 4

Given: $f(x) = -x^3 - x^2 + x + 10$.

#### 4.1 Write down the coordinates of the $y$ – intercept of $f$. (1)

#### 4.2 Show that $(2 ; 0)$ is the only $x$ – intercept of $f$. (4)

#### 4.3 Calculate the coordinates of the turning points of $f$. (6)

### Question 5

#### 5.1 Use the definition of the derivative (first principle) to determine $f'(x)$ if $f(x) = 2x^3$. (5)

#### 5.2 Determine $D'(x)$ if $y = x^{-4} + 2x^3 - \frac{x}{5}$.
5.3 Determine $\frac{dy}{dx}$ if $y = \frac{2\sqrt{x} + 1}{x^2}$ \hfill (4)

TOTAL = 79

3.2 Data Collection

The sample students were registered in 2014 for either of the Extended Programmes in Industrial, Civil, Electrical or Mechanical engineering offered by the Academic Development Center (ADC) of the university. These students took part in the pre and post-test prepared by the department. The pre and post-test comprised of questions covering the following concepts: factorization, equations, functions (exponential, linear, quadratic and cubic) and differential calculus, which are some of the pre-requisite topics for Engineering Mathematics. Marks were recorded in terms of concepts.

3.3 Data Analysis

Statistical Package for the Social Sciences (SPSS) was used to analyze the data and the excel spreadsheet was used to group students into different categories, and finally a graph was drawn to show the emerging gaps per topic for both the pre and post-tests. For comparison purposes, marks were grouped into four categories, namely:

- At Risk – students who have the greatest risk of not doing well and need extra remedial support.
- Low Potential – students who need support to succeed.
- Potential – students who may progress with little or no support.
- High Potential – students who have the most probability to succeed.

Table 2: The percentage of students in each category level for the pre and post-tests.

<table>
<thead>
<tr>
<th>Mark Level (%)</th>
<th>Category</th>
<th>Pre-test (%)</th>
<th>Post-test (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 39</td>
<td>At Risk</td>
<td>44.3</td>
<td>13.5</td>
</tr>
<tr>
<td>40 – 59</td>
<td>Low Potential</td>
<td>40.4</td>
<td>36.3</td>
</tr>
<tr>
<td>60 – 79</td>
<td>Potential</td>
<td>13.3</td>
<td>41.4</td>
</tr>
<tr>
<td>80 – 100</td>
<td>High Potential</td>
<td>2.0</td>
<td>8.8</td>
</tr>
</tbody>
</table>

The gaps were identified by calculating the mean percentage and subtracting from 100% in each topic. All topics were then broken down into sub-topics to do a further item or sub-topic analysis to show the main contributor to the deficit gap is content knowledge on each topic.

4. RESULTS AND DISCUSSION

The pre and post test scripts were marked by lecturers who were offering the course and were moderated by a colleague in the department for consistency.

The diagnostic test paper consisted of the following topics with a total mark per topic included in brackets: factorization (12), equations (23), exponential function (5), quadratic function (16), cubic
function (11) and differential calculus (12). The learning deficit gap per topic is represented in Figure 1 below.

**Figure 1: Percentage content knowledge gap per topic**

![Content Knowledge Gaps](image)

The most glaring gap is in *factorization*, a lower grade concept which is usually assumed to be known to students. It is also peculiar that the learning deficit gap in *equations* is much smaller than in factorization since factorization is applied in solving equations. This analysis gave rise to a deep interrogation of the discrepancies in factorization as reported by Nel (2014).

The following table (highlighting 2 sections that have major gaps) indicates the various sub-topics and the associated subject content knowledge gaps per sub-topic for *factorization*, the *cubic function* and *differentiation* (with the mark allocation given in brackets):

**Table 3: Content knowledge gaps in the pre and post-test for 2 identified topics: Factorization and Cubic Functions.**

<table>
<thead>
<tr>
<th>Main Topic</th>
<th>Subtopics</th>
<th>Pre-test Gap (%)</th>
<th>Post-test Gap (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factorization</td>
<td>Quadratic trinomial (2)</td>
<td>63</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>Difference of squares (3)</td>
<td>86</td>
<td>78</td>
</tr>
<tr>
<td></td>
<td>Sum of cubes (3)</td>
<td>97</td>
<td>87</td>
</tr>
<tr>
<td></td>
<td>Mixed quadratic (4)</td>
<td>55</td>
<td>54</td>
</tr>
<tr>
<td>Cubic Function</td>
<td>$y$ -intercept (1)</td>
<td>8</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>$x$ -intercepts (4)</td>
<td>70</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>Turning points (6)</td>
<td>47</td>
<td>52</td>
</tr>
</tbody>
</table>

Factorization was further divided into subtopics of individual expressions that needed to be factorized and the most glaring is the *sum of cubes* at a gap deficit level of 97% (even in the post test, the gap was 87%). Expression and equations of power 3 are dealt with in grade 12 under application of differential calculus when drawing cubic functions and also need the factor theorem, with grade 11 concepts used to do the factorizations.
In cubic functions, the only aspect that students excelled in, was finding the \( y \) –intercept and the hardest to obtain was determining the \( x \) –intercepts. They could also differentiate an expression with exponents that are integers to some extent. Determination of the \( x \) – intercepts has a gap deficit of 70% (with a post-test gap of 65%) which ties in very well with the gap deficit in factorization of cubes as most students could not even attempt to apply the factor theorem. McNamara and Shaughnessy (2010) suggested that students struggle with fractions, because a fraction has many meanings (part-whole, measurement, division, operator, and ratio) and is written in a new way. In addition, most teachers introduce fractions without focusing on a conceptual understanding, and then students attempt to link their existing knowledge of whole numbers to fractions.

To analyze the data more meaningfully, marks were converted into percentages and statistically analyzed to find the mean, median, mode, and standard deviation:

Table 4: Statistical representation of percentages marks per topic for the pre-test and post test

<table>
<thead>
<tr>
<th>Topic</th>
<th>Mean Pre-test</th>
<th>Mean Post-test</th>
<th>Median Pre-test</th>
<th>Median Post-test</th>
<th>Mode Pre-test</th>
<th>Mode Post-test</th>
<th>Standard Deviation Pre-test</th>
<th>Standard Deviation Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factorization</td>
<td>19</td>
<td>33</td>
<td>17</td>
<td>42</td>
<td>0</td>
<td>50</td>
<td>20</td>
<td>26</td>
</tr>
<tr>
<td>Equations</td>
<td>55</td>
<td>62</td>
<td>57</td>
<td>76</td>
<td>4</td>
<td>81</td>
<td>23</td>
<td>20</td>
</tr>
<tr>
<td>Exponential Function</td>
<td>46</td>
<td>38</td>
<td>40</td>
<td>40</td>
<td>0</td>
<td>60</td>
<td>34</td>
<td>31</td>
</tr>
<tr>
<td>Quadratic Function</td>
<td>45</td>
<td>43</td>
<td>38</td>
<td>56</td>
<td>38</td>
<td>63</td>
<td>26</td>
<td>28</td>
</tr>
<tr>
<td>Cubic Function</td>
<td>34</td>
<td>46</td>
<td>18</td>
<td>64</td>
<td>9</td>
<td>9</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td>Differentiation</td>
<td>44</td>
<td>54</td>
<td>42</td>
<td>67</td>
<td>0</td>
<td>58</td>
<td>30</td>
<td>25</td>
</tr>
<tr>
<td>TOTAL</td>
<td>42</td>
<td>57</td>
<td>43</td>
<td>60</td>
<td>56</td>
<td>57</td>
<td>18</td>
<td>17</td>
</tr>
</tbody>
</table>

It is clear that after remediation the students have performed much better in most of the sections of the post-test, as reflected by the comparative scores in table 4. As far as the mode is concerned, students’ performance is encouraging.

CONCLUSION AND RECOMMENDATIONS

The paper outlined a method to be used to measure subject content knowledge gaps using responses of learners through concepts or topics. Focused attention at these specific areas and its network of linkages could be a feasible way of addressing the dire situation the students find themselves in at this late stage of their studies. This kind of analysis will not only be able to pinpoint the real causes of poor performance in each topic but also identifies and ranks the causes in a quantifiable manner to help the educational practitioners to prioritize and allocate meaningful time appropriately to different forms of interventions as a way forward. However, such strategic interventions should be a norm for all students pursuing science related qualification programmes at tertiary institutions.

One of the findings established in this research is the learners’ inability to factorize mathematical expressions to the power of three. For students to overcome this problem, theoretical knowledge and applications of factor theorem becomes a pre-requisite. Reinforcing these knowledge gaps and
its associated action plans will go a long way in empowering the learners to be more confident in tackling such problems. From this prognosis, it is imperative to ask the following questions: How much influence does the factor theorem have on the factorization of cubic functions? Is it justifiable to abandon and delineate learners whose grade 12 results show unacceptable levels of achievements? Who is responsible for the state of affairs that these learners find themselves in?

Similar research needs to be done with any subject and at every level of education where students write a formal assessment; so the DBE, provincial departments of education, education districts and circuits, NPO’s (Non-Profit Organizations) in education, schools, universities, technical-educational institutes etc. can quantifiably measure the content knowledge gaps which may help in measuring the impact of training sessions, measure educators teaching ability through learner scripts and use such data to identify areas where educators need focused development. Such a diagnosis with robust interventions is a way of tackling the problems head-on. Such a plan of action is necessary as South African learners are finding themselves at the bottom of the class in internationally bench marked tests in mathematics. The fact that other African countries are performing much better with lesser resources than South Africa makes a wake-up call for all educators and policy makers in this country to make bold steps to rectify the situation.

Further research needs to be done to determine the competency of the educators as well as those tasked to deliver subject materials to educators. The learner is highly dependent on the educator. It goes without saying that the training of educators should be done by competent trainers who should themselves prove their competency skills before being approved to offer training. This vicious cycle of remediation could prove to be instrumental in making noticeable changes in education.

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