EXPLORING PROSPECTIVE FOUNDATION PHASE TEACHERS' UNDERSTANDING OF WORD PROBLEMS: A CASE STUDY

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ABSTRACT– The purpose of this study was to investigate second year prospective teachers' understanding of the distinctions between addition and subtraction word problems. We share data on the responses we got when we asked 87 second year prospective teachers to write six number sentences into word problems. The article draws from a descriptive inquiry and highlights that most prospective teachers could not mathematical number sentences into word problems. Our findings seem to suggest that prospective teachers had difficulty articulating the distinctions between join and separate word problems, and could not to recognize the different roles of the unknown variables in join and separate word problems. Most teachers appeared not to have a coherent framework for distinguishing word problems and appeared to work on an ad-hoc basis. Lack of such knowledge points to the fact that teachers do not focus on problem types when teaching word problems and this may result in learners having a fragmented understanding of solving word problems. We suggest that prospective teachers need to be acquainted with the different word problems that they are expected to teach at the foundation phase.

Keywords: Addition and subtraction; Word problems; Prospective teachers

1. Introduction

From as early as the foundation phase (Grade R-3) learners' are exposed to solve word problems in South African primary schools (DBE, 2012). Yet, learners appear to have difficulties in solving word problems, do not understand words used in a word problems, have difficulties comprehending a word problem, do not understand specific vocabulary and do not have confidence or the ability to concentrate when reading word problems (Nunes et al., 1993). While, globally many studies have investigated how learners solve word problems, research on prospective teachers' abilities to distinguish between the different types of word problems has been generally sparse. In this article we explore 2nd year prospective foundation phase teachers' understanding of the distinctions between two of the most common word problems in the foundation phase viz: join and separate problems. Given the centrality of word problems in primary school curricula prospective teachers need to know the different kinds of word problems that they are expected to teach. While teachers need to know the distinctions between the different kinds of word problems research evidence points to the fact that some primary school teachers lack conceptual understanding of the distinctions between word problem types (Ma, 1999). Teaching is not an easy task, it is complex, is cognitively demanding, and involves problem solving and decision making and requires teachers who are reflective and thoughtful individuals (Carpenter et al., 1999).

2. Conceptualizing the study

We used Shulman's (1986) notion of pedagogical content knowledge (PCK) to get some insights into what prospective teachers should know about word problems because learners in classrooms of teachers with more knowledge about children's thinking are more successful than does children in classrooms of teachers with less knowledge about their children's thinking (Carpenter et al., 1999).Mathematical knowledge has been linked to student performance in standardized testing (Hill, Rowan, & Ball, 2005). "Effectiveness in teaching resides not simply in the knowledge a teacher has accrued, but how this knowledge is used in the classroom" (ibid).Children in classrooms of teachers with more knowledge about their children's thinking are more successful than does children in classrooms of teachers with less knowledge about their children' thinking (Carpenter et al,.1984). Thus, specific knowledge about children's thinking could be used by teachers in teaching word problems. In fact, (Carpenter et al., 1999), suggest that knowledge of children's thinking makes a difference in how children learn solving word problems. Shulman (1986) distinguishes between different categories of knowledge. Subject content knowledge relates to teachers' conceptual knowledge of the discipline of their teaching subjects, for example mathematics, science, life sciences or geography. Pedagogical content knowledge has three components according to Shulman (1986a): knowledge of a subject, knowledge of children's existing knowledge and beliefs about the subject, and knowledge of effective ways to represent this to children.

Knowledge of mathematics and knowledge of mathematical representations are related to content knowledge, while knowledge of students and knowledge of teaching are related to pedagogical content knowledge. Shulman (1986) defines content knowledge as the knowledge about the subject, for example mathematics and its structure. According to Shulman (1986) pedagogical content knowledge includes, 'the ways of representing and formulating the subject that make it comprehensible to others'… 'an understanding of what makes the learning of specific topics easy or difficult; the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons' (p.130). Therefore, based on Shulman's (1987) notion teachers need to have a 'thicker, and holistic understanding of representing subject matter to learners (Parker& Heywood, 2000). Prospective teachers need to be able to guide their learners to understand choosing and using useful representations, of word problems, assist learners how to understand the different roles, actions, and relationships of the unknown quantities in word problems. According to Grouws & Schultz (1996) PCK involves providing learners with useful representations in solving word problems, assisting students to 'see' unifying ideas in word problems, clarifying examples of the different types of word problems, and counter examples, making helpful analogies in explaining word problems, fleshing out important relationships in addition and subtraction of word problems, and making connections among addition and subtraction. Teachers of mathematics need to be able to transform content into forms that are accessible to learners, pedagogically powerful and yet adaptive to the variations in ability and background presented by the students (Shulman, 1987, cited in An, Kulm and Wu, 2004).

In order to construct mathematical concepts in students' mind, pedagogical content knowledge and mathematical content knowledge are desirable. The manner in which teachers relate their subject matter (what they know about what they teach) to their pedagogical knowledge (what they know about teaching) and how subject matter knowledge is a part of the process of pedagogical reasoning and are central to pedagogical content knowledge (Cochran, DeRuiter & King, 1993).

3.1. Understanding word problems types

Word problems presented in numeric format are easy to solve (Cummins *et al,* 1988) than the same presented as a *word problem*. When looking specifically at understanding word problems and how it affects students' ability to solve word problems Franke & Kazemi (2001) and Christou & Philippou (1998) both indicate that when solving word problems , students must understands what they are reading in order to successfully solve it. If children do not learn with understanding, or comprehend the task, each new problem is unfamiliar and isolated from all others. They will only be successful if given direction only that very type of problem for each problem they attempt. As children advance to more sophisticated domains, they continue to find word problems in those domains more difficult to solve than problems presented in symbolic format. Unfortunately, children perform particularly poorly on arithmetic word problems even when they perform well on corresponding arithmetic computation (Cummins et al,. 1991) suggesting that problem comprehension is a source of students' difficulties.

This discrepancy in performance on verbal and numeric format problems suggests that factors other than mathematical skill contribute to problem solving success because not all word problems are alike. Some problems are much easier to solve than others. For example, even very young children rarely make errors on combine problems (See Table, l) but frequently make errors on compare problems. This differential performance changes with age, with performance on these problems becoming nearly equivalent overtime. Maybe, because problem difficulty patterns change with age, some researchers have adopted a Piagetian view of solution performance characteristics on word problems. This view suggests that a problem proves troublesome for a child only in so far as the capacities required to process the problem are not yet possessed by the child (Cummins, et al., 1988) While this general view is fairly uncontroversial, researchers disagree as to which capacities develop over time to improve solution performance. Nonetheless, what is in agreement is that there are generally two perspectives: those that attribute improved solution performance to the development of logico-mathematical knowledge and those that attribute such improvement to the development of language comprehension skills.

3.2. The logico-mathematical perspective

This perspective holds that children fail to solve certain problems because they do not possess the conceptual knowledge required to solve them correctly (Cummins, et al., 1988) and attributes developmental trends in problem solving skill to the acquisition of knowledge concerning logical set relations such as part-whole, or subset-superset relations. The strongest assertion of this view argues that poor performance is a result of impoverished schemata that are capable of representing the integrity of individual sets but not their part-whole relations (Riley et al., 1999). Hence*,* children who are competent problem solvers understand and represent word problems as relations among parts and wholes as part-whole schemata that are contextualized to map smoothly onto standard arithmetic story problems structures (Cummins et al., 1988).

Skill and schema developmen*t* encompass join and compare problems (Riley & Greeno, 1988). For example, understanding phrases such as "*X* has n more marbles than *Y*" requires understanding that numbers can be values for operators and not just set cardinalities. The perspective proposes models of good, medium, and poor problem solving using schema type formalism (Riley, Greeno, & Heller, 1983; Briars & Larkin, 1984) and set knowledge is represented in these models as schemata that specify relations among sets of objects and good problem solving embraces elaborate schemata that specify high level set relations, such as part-whole or subset-superset relations.

3.3. The linguistic development view

The linguistic development view largely derived from cognitive psychology (Briars & Larkin, 1984; Carpenter & Moser, 1984) argues that that semantic structure is much more relevant than syntax in studying children's solutions of addition and subtraction problems (Carpenter & Moser, 1982). There are systematic differences in children's word problem–solving performance levels and comprehension is the most important source of problem difficulty because children do not yet have a repertoire of highly automatized schemata for representing the different problem types (Moreno & Mayer, 1999).

Certain word problems are difficult to solve because they employ linguistic forms that do not readily map onto children's existing conceptual knowledge structures and certain terms are difficult for children in understanding word problems (Cummins, 1991). Children often have difficulty in interpreting the term *compare* especially on class inclusion terms because children often transform them into simple possession terms when retelling word problems (Cummins et al., (1988) and skip over them when reading word problems containing such terms. According to (De Corte & Verschaffel, 1986) children perform better when problems containing *compare* terms are reworded to exclude the terms. Mayer (1982) suggests that faulty problem representations result from children's interpreting *comparative* terms as statements of simple possession or simple assignment. For example, "Mary has 5 more marbles than John: is interpreted as "Mary has 5 marbles". A child may understand part-whole set relations and yet be uncertain as to how the comparative verbal form (e.g., How many more X's than Y's?) maps onto them. If this were the case, we would say that the child had not yet acquired an interpretation for such verbal forms (Cummins, et al., 1988).

Another term that proves difficult for children is *altogether especially* when the term is used with conjunctives to indicate joint ownership, as in "Mary and John have 5 altogether". Children interpret the conjunction in this context to mean each child. For example, in retelling protocols such as the following (I = Interviewer, C = child):

> I: Pete has 3 apples; Ann also has some apples; Pete and Ann have 9 apples altogether; how many does Ann have?
> C: 9
> I: Why?
> C: because you just said it.
> I: Can you retell the story?
> C: Pete had 3 apples; Ann had also some apples; Ann had 9 apples; Pete also has 9 apples.
> (De Corte & Verschaffel, 1985, p.19)

An underlying reason impacting on solution errors on these problems could perhaps, reflect deficiencies in semantic knowledge, logico-mathematical knowledge, or both. To test the contributions of each perspective (Hudson, 1993) manipulated problem wording and observed its effects on solution performance. For example, considering the following problem:

> There are 5 birds and 3 worms.
> How many more birds are there than worms?

This is a relatively difficult problem for children, with correct performance ranging from 17% for nursery school children to 64% for first graders (Hudson, 1993). The logico-mathematical view holds that this problem is difficult because it requires sophisticated understanding of part-whole relations, which primary school children presumably do not yet possess. However, reported dramatic improvements in solution performance on this type of problem were observed when the final line was changed to the following: How many birds won't get a worm? (Hudson, 1993)

An unanswered question in this work, however, is just how children interpret the problems they are asked to solve, particularly those that employ troublesome language. This is of some importance because the errors that children make are often counter-intuitive. For example, the most commonly committed error on the birds/worms problem is to return the large number "5" as the answer to the problem. In fact, these "given number errors" constitute a significant proportion of errors committed on word problems (Riley et al., 1983; De Corte et al., 1985). It is not clear why children believe that the solution to a problem could consist of a number already given in the problem. Another term that is difficulty to children in understanding word problems is the term *some* as in "John had some marbles," is often improperly understood by children. The finest of reproduction to observe solution performance occurred when the simulation's vocabulary was altered to allow interpretation of this term as a quantity term.

3.4. Types of word problems

The Curriculum and Assessment Policy Statement (CAPS) document, Grades R - 3 promotes the teaching of word problems (DBE (2012) and mandates three basic type of addition and subtraction word problems, "change, combine, and compare problems". CAPS classification of word problems is similar to Riley et al., (1983)'s classification scheme who point that most standard problems can be classified into just three semantic types-: 1. Combine, 2. Change, and 3. Compare problems. Compare and combine problems describe static relations between quantities, whereas change problems describe *actions* that cause or decrease quantity. Examples of these problems are provided in table 1. We adopted (Carpenter et al., 1999)'s classification scheme as it appeared clearer and more specifically related to the content domain of addition and subtraction that was the focus of this study.

Table 1: Classification of word problem with number sentences
(Adapted from Carpenter et al., 1999).

Problem Type	Problem		
Join	*Result Unknown* Mike had 4 toy cars. Lynn gave him 8 more toy cars. How many cars does he have altogether? 4 + 8 = ?	*Change Unknown* Sue has 4 toys cars. How many more does she need to have 12 toy cars? 4 + ? = 12 12 – 4= ?	*Start Unknown* Betty had some toy cars. Philip gave her more toy cars. Now she has 12. How many did Betty start with? ? + 4 = 12 12– 4= ?
Separate	*Result Unknown* Michelle had 12 toy cars. She gave 4 to Lynn. How many does she have left? 12–4=? 12 – 4 =?	*Change Unknown* Sue had 12 toy cars. She gave some to Gary. Now she has 4 left? How many did she gave to Gary? 12 - ? = 4 12 – 4 =?	*Start Unknown* Betty had some toy cars? She 4 to Philip. Now she has 8 left. How many cars did Betty start with? ? – 4 = 8 4 + 8 =?
Part-Part-Whole	*Whole Unknown* Michelle has 4 red toy cars and 8 yellow toy cars. How many toy cars does she have? 4 + 8 =? 4+8=?	*Part Unknown* Sue has 12 toy cars, 4 red and the rest yellow. How many yellow cars does Sue have? 12 = 4 +? 12 - 4 =?	
Compare	*Difference Unknown* Michelle has 4 toy cars and Lynn has 12 cars. How many more cars does Lynn have than Michelle? 4 +? = 12 12 – 4 =?	*Compare Quantity Unknown* Sue has 4 toys. Gary has 8 more than does Gary have? 4 + 8 =? 4 + 8 = ?	*Referent Unknown* Betty has 12 toy cars. She has 4 more than Philip. How many cars does Philip have? 12 = 4 +? 12 – 4 =?

There are two basic types of change problems which involve action (Carpenter and Moser, 1984; Carpenter et al., 1999). The join-change-unknown problem describes a joining action rather than a separating action. In Join problems, there is an initial quantity and a direct or implied action that causes an increase in the quantity (Carpenter & Moser, 1984; Carpenter et al., 1999; Garcia, Jimenez & Hess, 2006). For separate problems, a subset is removed from a given set. In both classes of problems, the change occurs over time. Within both the join and separate classes, there are three distinct types of problems, depending on which quantity is the unknown.

For Join Result Unknown type problems, the initial quantity and the magnitude are given and the resultant quantity is the unknown. For Join Change Unknown problems, the initial quantity and the result of the change are given and the object is to find the magnitude of the change. In Join Start Unknown, the quantity is the unknown. Both combine and compare problems involve static relationships (Carpenter et al., 1999) for which there is no direct or implied action. Combine problems involve the relationship existing among a particular set and its two disjoint subsets. Two problem types exist: two subsets are given and one is asked to find the size of their union, or one of the subsets and the union are given and one is asked to find the size of their union, or one of the subsets and the union are given and the learner is asked to find the size of the other subset.

Compare problems involve the comparison of two distinct, disjoint sets (Carpenter et al., (1999). Since one set is compared to the other, it is possible to label one set the referent and the other the compared set. The third entity in these word problems is the difference, or amount by which the larger set exceeds the other. In this class of problems any of the three entities could the unknown, the difference, the referent set, or the compared set. There is also a possibility of having the larger set being either the referent set or the compared set. The difficulties that children experience in writing number sentences to represent certain types of problems may occur because the representations that they have been taught (a + b and a - b) do not correspond to their interpretation of the problems (Carpenter and Moser, 1984; Carpenter et al., 1999). Thus, a number of semantically distinct problems can be generated by varying the structure of the problem, even though most of the same words appear in each problem.

4.1. Method

We adopted a descriptive inquiry (Schumacher & McMillan, 2010) in order to get some insights into prospective teachers' knowledge on word problem types. A descriptive inquiry according to (Kumar, 2011) "attempts to describe systematically a situation, problem, phenomena or service, or provides information about, say the living conditions of a community, or describe attitudes towards an issue" (p.10). In this study the participants were a group 87 second- year foundation phase prospective teachers enrolled in the bachelor of education programme in a University of South Africa tested on their knowledge on distinguishing the different types of join and compare problems.

Data was collected from the cohort of 89 foundation phase teachers by means of a pencil and paper test. The prospective teachers were given six number sentences (9+5= x; 7+ x = 13; x + 6= 11; 7 - 5= x; 12 - x =7; and x - 5 = 17) and asked to write word problems that would best be represented by the number sentences. The number sentences consisted of 3 join and 3 separate problems. These number sentences correspond to the six join and separate problems presented in table 1. The expectation was that having completed successfully their first year at university and specializing in teaching mathematics at the foundation phase the prospective teachers would be able to write the number sentences as word problems. Themes that emerged from the data scripts showed word problems that correspond to the given number sentence, problems that did not directly match the given number sentence, incomplete word problems, word problems that did not make sense, word problems that did not directly match the given number sentence but had the same answer, and number sentences that were not answered, word problems that had different answers.

Table 2: Summary of teachers' responses on types of join and separate word problems

See explanation of codes below table.

Problem type	Join			Separate		
	Join Result Unknown 9 +5 = x	Join Change Unknown 7+x = 13	Join Start Unknown x + 6 = 11	Separate Result Unknown 7- 5 = x	Separate Change Unknown 12 - x = 7 Check the colour coding here	Separate Start Unknown x – 5 = 17
APWP	77.01%	63.21%	21.83%	75.86%	27.5%	10.34%
ICWP	10.34%	2.29%	12.64%	4.59%	13.79%	4.59%
NSWP	6.9%	17.24%	9.19%	2.29%	5.74%	2.29%
SAWP	5.79%	17.24%	57.47%	16.09%	51.72%	37.93%
DAWP	-------	-------	2.29%	2.29%	---------	20.68%
DANS						24.13%

Codes[4]: APWP-Appropriate word problem; **ICWP**- Incomplete word problems; **NSWP**-No Sense Word problems; **SAWP**- Word problems with the same answer, **DAWP**- word problems with different answers; **DANS-**Did not answer

4.2. Results and Analysis

The analysis focuses mainly on qualitative features of the data.

Task 1: 9+ 5 = x

A result unknown task that required prospective teachers to write the number sentence as a word problem. Seventy-seven percent of the student teachers were able to write appropriate word problems. Ten percent of the prospective teachers wrote incomplete word problems. Six point nine percent of the prospective teachers wrote answers that did not make sense. Five percent of the prospective teachers provided the correct answers although the word problems did not directly match with the word problem. We did not interrogate how the student teachers arrived at the correct answers. We can only speculate that most prospective teachers do not understand the relationship between the unknowns in word problems. Forty-three percent of the prospective teachers provided answers that we classified as making no sense, across both the join and separate problem types.

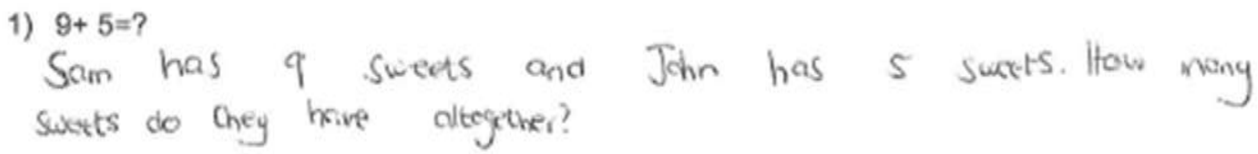

Figure 1: Appropriate word problem for the number sentence 9+ 5=?

Task 2: 7 + x = 13

Focus of this task was to identify the *change unknown quantity* and write the correct word problem. Sixty-three percent of prospective teachers were able to write appropriate word problems while seventeen percent provided answers that either made no sense or word problems that did not directly match the given number sentence. While most of the teachers were able to provide the correct answers their understanding of the join word problems types was not systematic and coherent. The number of prospective teachers who could write the appropriate word problems fell significantly from sixty- three percent to twenty –one percent in join start unknown problems. Twelve percent of the prospective teachers did not complete the task. The declining trend in correct word problems in writing appropriate word problems is evident across and within the entire join and separate problems in this study. Coincidentally, these are the same word problem types that learners find most difficult to solve.

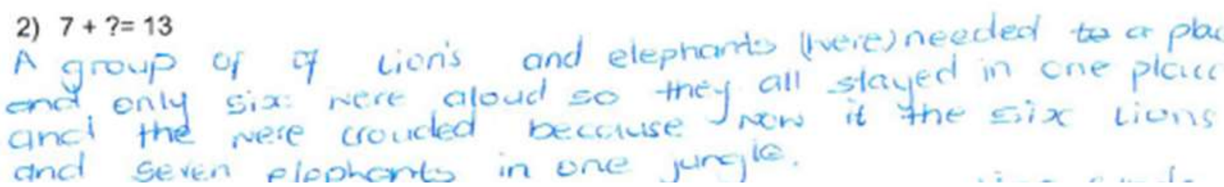

Figure 2: Erroneous writing of a join change unknown number sentence

Task 3: x + 6 = 13

This was a join start unknown word problem. The prospective teachers performed poorly in this type of word problem as only. Twenty one percent of the prospective teachers were able to write appropriate word problems, while nine percent could not provide complete answers and almost ten percent of the answers provided did not make sense. Fifty seven percent of the prospective teachers were able to only provide the same answer to the task notwithstanding the word problem not matching the given number sentence. It is important to note that in this task we begin to see a new trend emerging, a small number of teachers though significant failing to write the appropriate word problem and providing different answers to the task, again the reason for this are only speculative.

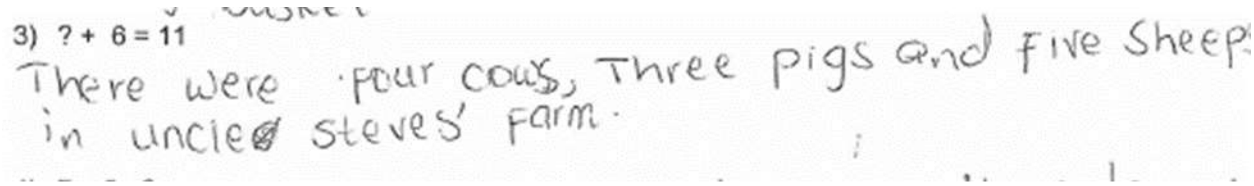

Figure 3: A word problem that does not make sense

Task 4: 7- 5 = x

A separate result unknown number sentence. Seventy five percent of the prospective teachers were able to write appropriate word problems for the task. Subtraction problems appear to be difficult to most prospective teachers. there appears to be a decline in appropriate word problems in comparison to change unknown problem for both join and separate problems Five percent of the students provided word problems that were incomplete and, two percent of the students' answers were incomplete, again it would only be a conjecture if we provide explanations as to why this was so, as this was not the primary objective of this study. But what is noteworthy is that two percent of the prospective teachers provided word problems that did not make sense and the same provided answers that were completely incorrect. The number of prospective teachers who provided answers that were appropriate but the word problems not directly matching the number sentence increased significantly when compared across the same variable. The increase was from five percent to 16%. The cumulative percentage points across both join and separate problems for problems that did not make sense declined from six percent to two percent on separate change unknown problem type. This decline is interesting as most of the results have been showing an increasing level of inappropriate word problems for separate word problems.

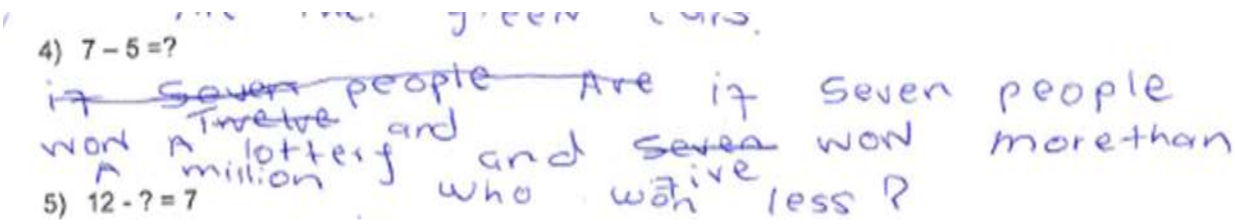

Figure 4: A word problem that does not make sense for the number sentence 7- 5 = ?

Task 5: 12 - x = 7

The focus of this task as stated in the instructions given to the prospective teachers was to write a word problem that resembled or represented the number sentence in words. Twenty seven percent of the prospective teachers were able to write the number sentence into appropriate word problems in comparison with join change unknown word problems. This further depreciates to 51% for separate change unknown problems on the number of prospective teachers who provided word problems that didn't directly match the number sentence but still had the same answer. An interesting occurrence is that while less than 20% of the prospective teachers performed poorly on join change unknown problems this number dramatically increases for separate change unknown problems to more than fifty of all the prospective teachers for the same attribute.

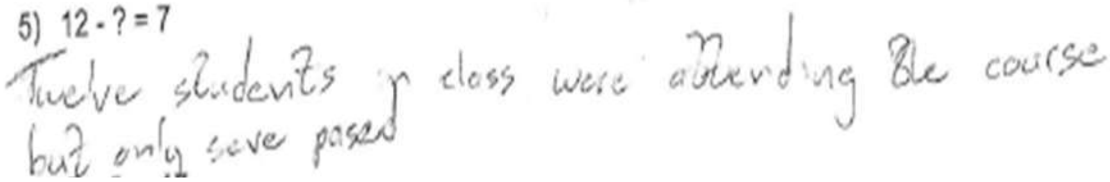
5) 12 - ? = 7
Twelve students in class were attending the course but only seve pased

Figure 5: Erroneous separate change unknown problem

Task 6: x – 5 = 17

The unknown quantity in this task was the start variable. A comparison across join and separate problem types revealed a decline from 21% for join problems to 10% in separate problems for appropriate prospective teachers who wrote appropriate word problems. Within the start unknown problem four percent of the prospective teachers wrote incomplete answers and two percent provided answers that did not make sense. It appears separate problem types are difficult for prospective teachers. There is a decline in performance both within and across categories of join and separate problems. It is in the start unknown category that 20% of prospective teachers did not attempt to answer the question. Twenty five percent of the prospective teachers not attempt to write the word problem.

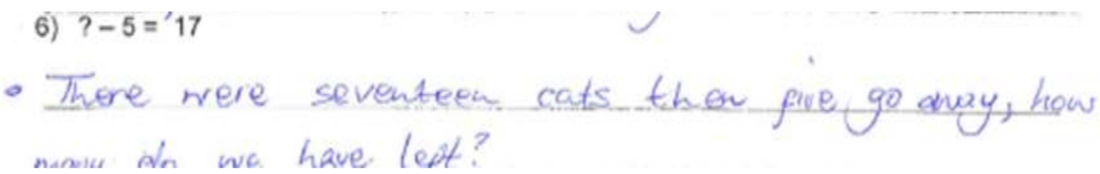
6) ? – 5 = '17
There were seventeen cats then five go away, how many do we have left?

Figure 6: Erroneous writing of the number sentence into a word problem

Most prospective teachers were able to distinguish between join result unknown problems types. The prospective teachers were able to identify the initial quantity, magnitude, and the result of the change. This backs earlier claims that join word problems presented in numeric format are the easiest to solve and that even young children rarely make errors (Cummins et al., (1988) and that children perform worse on arithmetic word problems than on comparable problems presented in numeric format (Carpenter et al.,(1980). This could be attributable to the fact that some teachers do not have sufficient knowledge on the distinctions between word problems. Our characterization of the prospective teachers knowledge on join result unknown word problems points to a high level of understanding of this problem type. However, some of the teachers only provided the correct answers for the task, suggesting that perhaps prospective teachers had comprehension problems (Cummins et al., 1998). Children are not only challenged to comprehend relationships between language and mathematical processes, but also to experience sense making (Greer, 1997).

The data also reveals that prospective teachers' knowledge on word problems was strongly limited practically, unstructured, and sometimes meaningless. Prospective teachers could be not thinking about 'what they should be teaching'. For example in problem 2, the prospective teacher repeats the number 6 twice. This might confuse learners into thinking that they need to be adding 6 and 6.This finding is consistent with research suggesting that inexperienced teachers have incomplete and superficial levels of pedagogical content knowledge (Carpenter, Fennema, Petersen & Carey, 1998). In addition, prospective teachers often rely on unmodified subject matter knowledge most often directly extracted from the text or curriculum materials and may not have coherent framework from which to present information (Cochran, DeRuiter & King, 1993). Prospective teachers are sometimes not developmentally ready to assume the roles required of them as good mathematics teachers (Brown & Borko, 1992).

In a study by (Brown & Borko, 1992) prospective teachers struggled, and sometimes failed, to come up with powerful means of representing subject areas to students. Further, their efforts are often time-consuming and inefficient (Brown & Borko, 1992). According to (Langrall et al., 1996) course instructors can influence prospective teachers' beliefs in a positive way if they consistently encourage individuals and collaborative groups to reflect on a limited but powerful set of pedagogical principles. The import of these two competences can be explained to lecturers to make them emphasize these in their courses.

5. Concluding remarks

Our analysis led us to characterize levels of prospective teachers' competence in identifying join and separate word problems into low, medium, and high levels. We defined levels of achievement in this study as competence-relevant levels that teachers need to possess to understand the two types of addition and subtraction word problems for effective instruction. The table summarizes the levels of prospective teachers' knowledge on distinguishing levels of teacher competences on distinguishing the two types of word problems addressed in this study.

Table 3: Levels of teacher competence on identifying word problem types

Problem type	Join problems	Level of teacher understanding	Separate problems
	Result unknown	High	Result unknown
	Star unknown	low	Star unknown
	Change unknown	low	Change unknown

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