# Mathematical modeling abilities among undergraduate learners: a challenge to assess through ODL?

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**Abstract**
Teaching of mathematical modelling has been considered essential to the development of student mathematical competencies and various curricula around the world have included modelling and applications into their curricular designs. This paper presents a pilot study on the evaluation of mathematical modelling abilities of learners and what would be some challenges in assessment of modelling activities in the distance learning mode of instruction. Forty (40) modelling activities were analysed and the assessment of these was based on Dossey's (2002) modeling structure that was exposed to the learners throughout the year course. The findings indicated an average performance of students in modelling activities, and the challenge was mainly in the assessment of modelling stages and transition process without a face to face interaction. It was possible that with the use of discussion classes, video conferences and Satellite Broadcasts students could be more engaged in modelling activities with assistance of the lecturers for more understanding of assessing student abilities of modelling in the ODL context.

***Key words:*** mathematical modelling, assessment of modelling skills, assessment of modelling in ODL context

**Introduction**
The relationship between mathematics and the real world has been at the centre of discussion for the past many years and in particular after 'modern mathematics' era came under criticisms. The 'real world' in mathematics is described to mean everything in everyday life such as the nature, culture, society, scientific and scholarly disciplines different from mathematics, etc. In the 'real world' the learner would encounter a problem outside the mathematics domain but use the mathematical knowledge to try to find the solution. "Modelling and Applications" has been used as an umbrella term to denote all activities that link mathematics to everyday and/or other sciences (Blum & Niss, 1991). Teaching of mathematical modeling and applications gear to reflect the role of mathematics in society by drawing about socio-cultural dimensions of mathematics (Atweh et al 2001; D'Amboise 1986, 1999). Activities engaged in modeling should be extracted from everyday life where students investigate the problem with reference to reality via mathematics (Barbosa, 2008). Mathematical modeling has been considered fundamental to development of student

competencies and the integration into mathematics curricula has been emphasized (Blum, 2006; Blum, Galbraith & Niss,, 2007). While modelling and applications have indicated to play an important role in many countries there still exists a substantial gap between the ideals of educational debate and innovative curricula, on the one hand, and everyday teaching practice on the other hand (Blum, 2006). Different perspectives in mathematical modeling have been debated and categorized into six perspectives, i.e. realistic, contextual, educational, epistemological, cognitive and socio-critical perspectives (Kaiser & Sriraman, 2006).

Despite many debates on modeling and applications, not many studies have focused on students' activities in modeling and as argued by Borromeo Ferri (2006), aspects of students' cognition at the micro level have received little attention. This paper provides a pilot analysis as contribution to the debate on modeling by analyzing students' abilities in the modeling activities and the nature of modeling activities that the students were engaged in during their year course of study.This course is for the BEd (Senior & Further Education Training-FET) degree and the Postgraduate Certificate in Education (PGCE).

**Objectives of the study**

i) to evaluate mathematical modelling abilities of the learners in the year programme
ii) to find out the nature of modelling activities done by the learners
iii) to create a dialogue among mathematics educators on assessment of modelling activities and in particular through a distance learning mode

**Conceptual framework**

This study adopted Dossey's (2002) modelling structure that was described in the students' study guide given to each registered candidate. Dossey's (2002) book is also a prescribed text for this course. The modelling structure describes the following components that are represented in figure 1 below:

Step 1: Identify the problem.
Step 2: Make assumptions
(a) Identify and classify the variables that possibly explain
the patterns that are observed.
(b) Determine the interrelationships between the variables.
Step 3: Solve or interpret the model.
Step 4: Verify the model
(a) Does it address the problem?
(b) Does it make sense?
(c) Do the model's predictions fit in with real-world data?

Step 5: Implement the model
Step 6: Retain the model

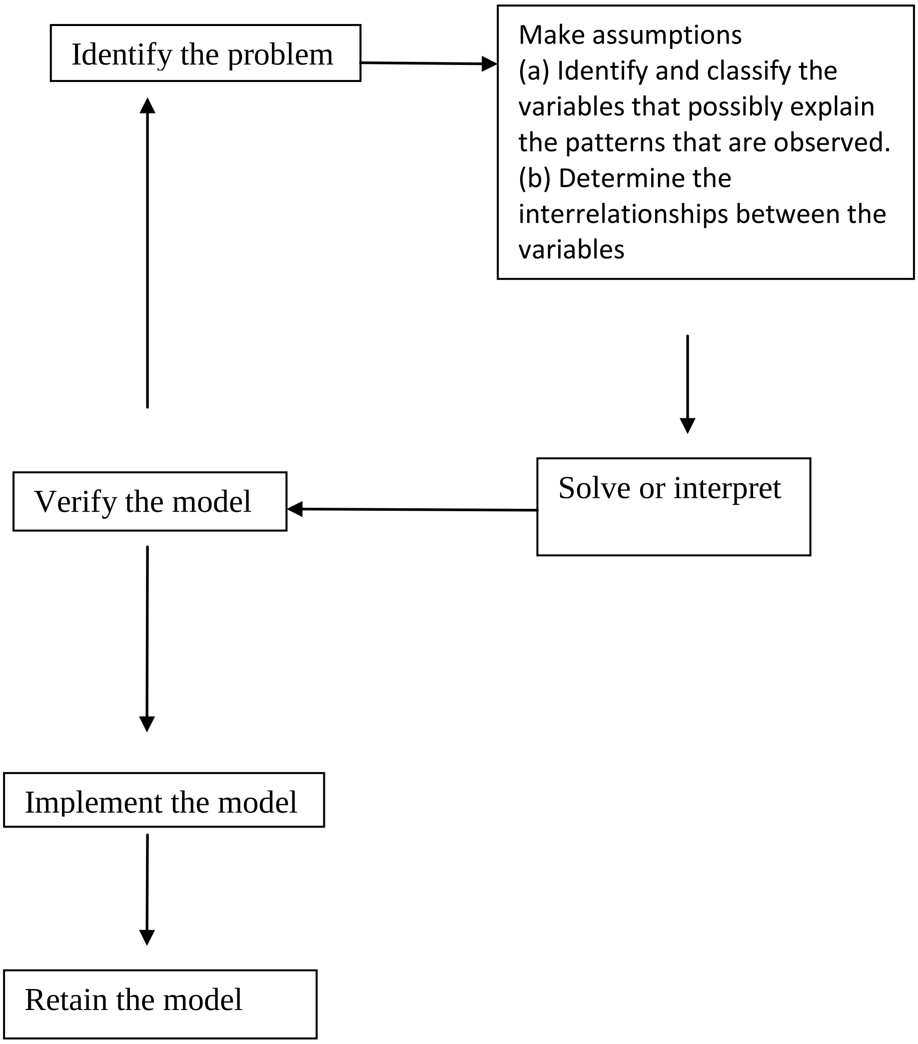



Fig. 1: Dossey's (2002) modelling structure

The students in this study were given the fish problem below with its solution and asked to indicate the steps as provided by Dossey's (2002) modelling structure. The researcher assessed students' abilities by categorizing the modelling process into three areas: *Objectives identification, Modeling stages and Explanation of modeling process.* The premise put forward by the researcher was that good performance in these areas was an indicator of students' knowledge in modelling. Dossey's (2002) modelling process was also considered by the researcher as adequate for evaluation of student modelling abilities and is one of the structures used to assess modelling abilities.

***The fish problem***

Determine the weight of the fish caught from the pond by establishing a general rule to predict the catch. The weight of fish is assumed to be the function of the size (volume) of the fish caught and the average weight density assumed constant. Also it is assumed that the fish are geometrically similar and the volume of any one of them was proportional to the cube of the characteristic dimension.

Solution process:

Assumptions made

(i) The fish were geometrically similar

(ii) The volume of any fish was proportional to the cube of the characteristic dimension, v being the volume of the fish and l the length of the fish, i.e. $v \alpha l^3$

(iii) A constant average density was assumed and weight of the fish was determined by multiplying the volume and the weight density, $w \alpha l^3$

(iv) The lengths and weights of fish are provided and graphed to obtain an expected approx. straight line. The approximations give a model represented as $w = 0.0085l^3$

**Study Design & Methodology**

The study involved students taking the module Subject Didactics (mathematics) as a year course in a distance learning mode. The module has a number of mathematics topics and the topic on modelling that was considered by this study, was one of them. The study involved assignments on modelling given to learners as part of their portfolio activities. In the tutorial letter, students were provided with a number of modelling activity examples and were then asked to work on a number of modelling problems. This study is quantitative in design assessing the learners' mathematical modelling abilities in a given problem and the nature of modelling activities the students used. The activity given to learners involved a real life problem on fishing and students were required to do the following: To (i) Identify the objectives/goals of the fishing problem, (ii) indicate the modelling stages of this example as Dossey (2002) applies them in the solution process, and (iii) write down the information explaining the modelling process in indicated stages in (ii) above. The nature of the modelling activities was identified from student portfolios presented. The assessment of student mathematical modeling abilities was based on Dossey's (2002) modeling structure (outlined in conceptual framework above) that was exposed to the learners throughout the year course. A scoring scale of 0 up to 4 was used in modeling knowledge abilities. From a total of 89 portfolios that were submitted by students, portfolios that had worked on modelling activities were selected and analysed. Forty (40) out of 89 were found to have modelling activities and these were analysed for this study.

**Findings**

The modeling activity was assessed using the following scale:

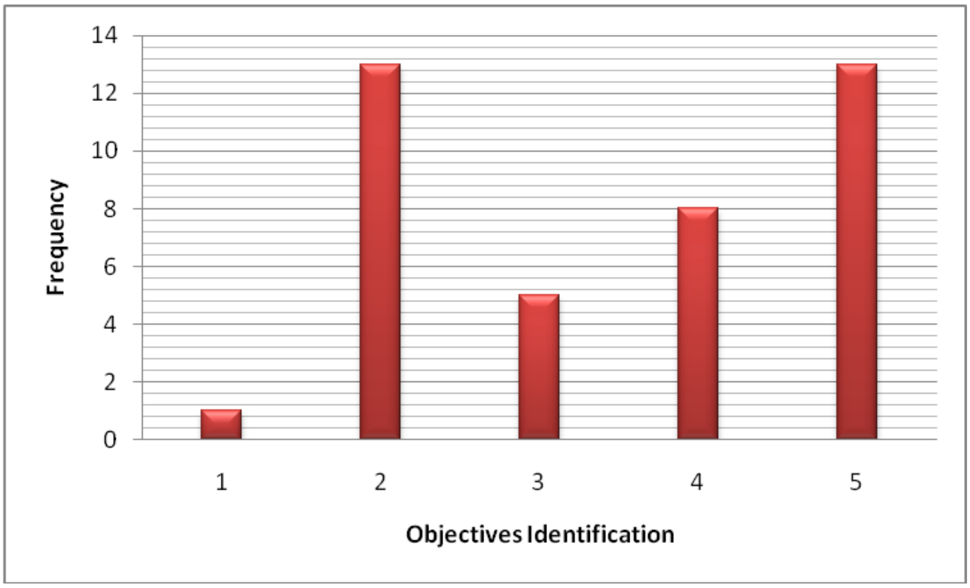
1- Poor
2- Average
3- Good
4 -Very Good
5=Excellent

Table 1: Student scores in modelling activities

| Student | Objectives identification | Modelling stages | Explanation of modelling process | Averages |
|---|---|---|---|---|
| 1 | 5 | 5 | 1 | 3.66 |
| 2 | 3 | 2 | 2 | 2.33 |
| 3 | 4 | 1 | 3 | 2.66 |
| 4 | 5 | 4 | 5 | 4.66 |
| 5 | 3 | 2 | 2 | 2.33 |
| 6 | 2 | 2 | 2 | 2.00 |
| 7 | 2 | 2 | 2 | 2.00 |
| 8 | 2 | 2 | 4 | 2.66 |
| 9 | 2 | 2 | 3 | 2.33 |
| 10 | 4 | 2 | 2 | 2.66 |
| 11 | 2 | 2 | 4 | 2.66 |
| 12 | 2 | 2 | 2 | 2.00 |
| 13 | 3 | 2 | 2 | 2.33 |
| 14 | 3 | 5 | 4 | 4.00 |
| 15 | 4 | 4 | 4 | 4.00 |
| 16 | 3 | 3 | 4 | 3.33 |
| 17 | 5 | 5 | 5 | 5.00 |
| 18 | 2 | 2 | 2 | 2.00 |
| 19 | 5 | 5 | 5 | 5.00 |
| 20 | 2 | 2 | 2 | 2.00 |
| 21 | 5 | 5 | 5 | 5.00 |
| 22 | 5 | 4 | 4 | 4.33 |
| 23 | 4 | 3 | 1 | 2.66 |
| 24 | 4 | 4 | 4 | 4.00 |
| 25 | 4 | 5 | 2 | 3.66 |
| 26 | 5 | 2 | 2 | 3.00 |
| 27 | 2 | 5 | 5 | 4.00 |
| 28 | 5 | 4 | 4 | 4.33 |
| 29 | 2 | 3 | 2 | 2.33 |
| 30 | 1 | 5 | 2 | 2.66 |
| 31 | 5 | 3 | 2 | 3.33 |
| 32 | 4 | 3 | 1 | 2.66 |

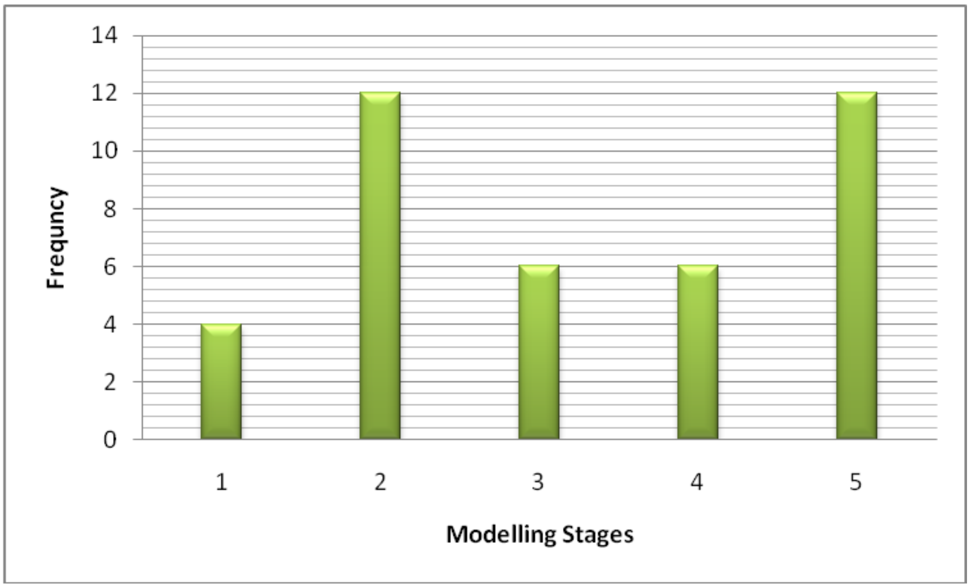
| | | | | |
|---|---|---|---|---|
| 33 | 5 | 4 | 3 | 4.00 |
| 34 | 2 | 3 | 1 | 2.00 |
| 35 | 5 | 5 | 4 | 4.66 |
| 36 | 5 | 5 | 5 | 5.00 |
| 37 | 2 | 1 | 2 | 1.66 |
| 38 | 4 | 5 | 2 | 3.66 |
| 39 | 3 | 1 | 1 | 1.66 |
| 40 | 5 | 5 | 1 | 3.66 |
| Averages | 3.50 | 3.28 | 2.83 | 3.20 |

Fig. 2: Objectives identification



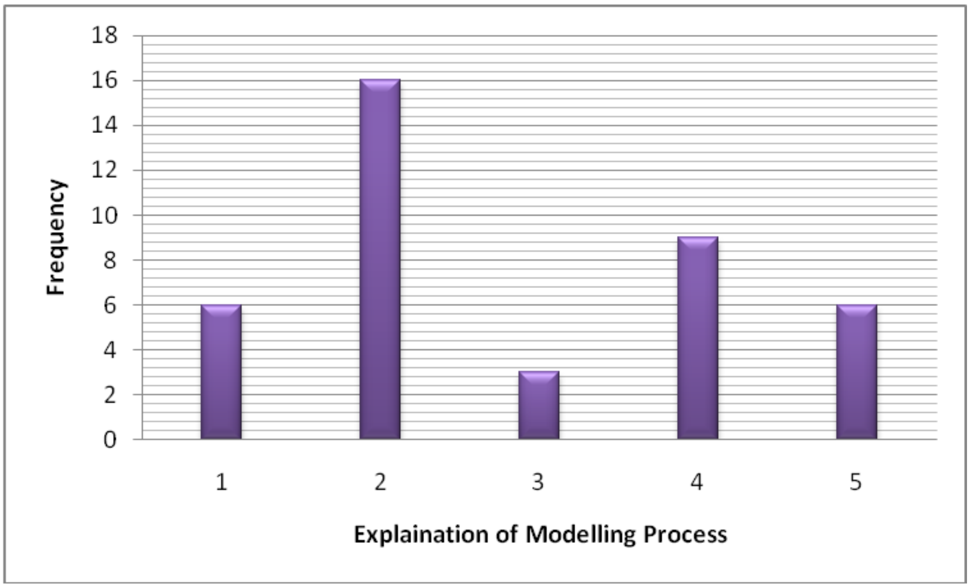

The students' average score of 3.5 in *Objectives identification* was above average and showed students were capable of identifying modeling objectives in the process. However, as indicated in above figure, there were many students who scored at the averagely of 2 and also almost the same number who scored very highly at average scale of 5.

Fig.3: Modeling stages




The students' average score of 3.28 in *Modeling stages* was above average and also showed that students were knowledgeable in modeling stages. However, being similar to previous findings in fig. 1 above, there were also many students in fig. 2 above who scored at the averagely of 2 and almost the same number who scored very highly at average scale of 5. This similarity was an indication that it was almost the same students who scored better in both assessed areas of *Objectives identification* and *Modeling stages.*

Fig. 4: Modeling process explanations




The students scored below the average (at 2.83) in explanation of the modeling process (fig. 4 above). The findings showed that many students performed poorly in this area with an average of 2. The findings indicated that while many students performed well in identification of *modeling objectives* and *modeling stages*, many of them were unable to explain the *modeling process*. The overall score in this activity was 3.20 (slightly above average) and indicated that students were knowledgeable in the modeling process.

## Discussion

The findings indicated that students were able to describe the objectives of modeling and the description of the modeling stages with a 'good' score. However, students averagely explained the modeling process to draw the conclusions. The latter, involved evaluation of students' understanding and interpretation of the modeling activity, an important aspect in modelling. Performance in *Objectives identification* can be considered to be at the lower level of modeling abilities than *Modeling stages* and *Modeling process explanations*. A good performance in *Modeling stages* was expected to be reflected in *Modeling process explanations* stage; a good performance in the former without the latter defeats the main goal of the modeling process where the student had to explain in his/her own words and apply the model. Most of the students certified the model because it provided an approximate answer but they failed to explain how the model could be validated and improved. Student explanations indicated that the modeling process was a one-loop exercise to obtain the answer. The issue they did not tackle was what was to be done if an approximate and seemingly realist answer was not obtained or to obtain a more accurate answer. Failure to explain the modeling loops of the model were indications of low abilities in modeling. While these findings provided a general overview of the modeling abilities of learners in a piloted module, it was also important to evaluate students' competencies at each stage of the modeling process and the transition from one stage to the next, the process that this study was not able to perform. Some studies like those of Houston and Neill (2003) and Maaβ (2006) investigated students' skills and competencies and compared them to some normative descriptions of modelling. Borromeo Ferri (2006) and Galbraith and Stillman (2006) also investigated the transition among the modelling stages by the students. Other studies have indicated that normative models may be inappropriate for describing students' actions, such as those expressed by diagrammatic representations (Barbosa, 2008). Borromeo Ferri (2006) was of the view that differentiation among modelling stages was only theoretical, and that it was difficult to distinguish between them empirically. These studies imply that there are still many ways yet to explore in order to understand the aspects of the students' cognition in modelling. In efforts to understand these, researchers have applied various ways such as analyzing modelling routes (Borromeo Ferri, 2006), blockages (Galbraith & Stillman, 2006), styles and ideal types (Busse, 2005; Maull & Berry, 2001; Maaβ, 2006), reasoning (English, 2003), metaphor (Carreira, 2001), etc. and the need is still required.

## Conclusion

The failure of students to explain well the modelling process was an indication that students had low knowledge in mathematical modelling. The activity provided in this paper cannot claim to be the best way of assessing modelling abilities among students though it is one of the ways to identify students' abilities through given assignments. The assessment of modelling abilities through distance learning tends to give a challenge in the assessment of modelling activities. The challenge in the distance learning instruction is how to evaluate adequately student stages in the modelling process without a face to face touch. It is probably possible that with the use of more discussion classes that can be provided through video conferences and Satellite Broadcasts facilities students can be more engaged in modelling activities with assistance of the lecturers.

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