Investigating grade 11 learners' misconceptions in understanding of quadratic functions in some South African's schools

Emmanuel Ibeawuchi & Ngoepe Mapula G
College of Education, University of South Africa
Email: emmaibeawuchi@gmail.com; ngoepmg@unisa.ac.za

Abstract
In this paper, we report an investigation of the misconceptions which impede grade 11 learners' understanding of quadratic functions. The sample consists of one hundred and seventy grade 11 learners selected from seventeen mathematics teachers' classrooms. A qualitative descriptive design was used.The results show that some of the misconceptions which impede learners' understanding of quadratic functions were: treating two different functions as equivalent; limiting the graph of the quadratic functions only to the visible region; ascribing linearity to the quadratic function; and describing a special point by only one coordinate. Learners' misconceptions showcase the level of learners' understanding. The findings of this study suggest that the learners should be given appropriate learning activities that deal with the perceived misconceptions and enhance learners' learning.
***Keywords**: misconceptions, quadratic functions*

Introduction

In mathematics education the knowledge of the cognitive obstacles which may impede learners' understanding in specific topics is vital to all stakeholders including the curriculum designers and implementers. The above assertion remains valid because in the process of making meaning out of the mathematics taught in the classroom, it is reasonablefor learners to make errors and develop misconceptions.Learners invent rules to explain the patterns around them. While many of these rules are correct in the situation they are used, their domain of application may be limited. The applicationsof those rules in extended domains may lead to misconceptions. However, owing to thecomplex nature of concept formation, a single misconception can impede learners' attainment in the concept. Therefore, conscious efforts must be made to identify the learners 'misconceptions so that appropriate corrective measures can be taken. Matz, (1982) has indicated thatlearners' misconceptions show the level of learners' understanding and can shed light into the implicit knowledge of the problem solver; however, misconceptions may remain hidden unless efforts are made to uncover them. The persistent poor performance of learners in mathematics in both national and international examinations in South Africa (Howie, 2003; Reddy, 2006) may necessitate a study of learners' misconceptions in specific topics in mathematics. Perhaps rich subject matter knowledge, together with the knowledge of learners' conceptionspreconceptions and misconceptions may be the knowledge that teachers need to try and prevent the continued underachievement in performance of learners in mathematics.

The present study is focused on the concept of functions, specifically the quadratic functions. However, studies on the learning of functions have focused principally on the formal definitions of what a function is (Vinner & Dreyfus, 1989; Markovits, Eylon, & Bruckheimer, 1986) and on the linear functions (Markovits, Evylon & Bruckheimer, 1983). When learning what a function is, learners readily accepted a one-to-one correspondence as a function but rejected a many-to-one correspondence as function (Markovits et al., 1986; Marnyanskii, 1975). Leinhardt et al (1990) suggested that this could be as the result of the learning sequence in which the first family of functions that the students were exposed to was the linear functions, which (apart from the constant function) is a one-to-one correspondence. Students also had problems in differentiating between a many-to-one correspondence and a one-to-many correspondence (Lovell, 1971; Markovits, et al., 1986; Thomas, 1975).

Students' idea of the forms the graphs of functions can take is very limited.In Lovell (1971), students accepted a graph to be that of a function if it has a linear pattern. Other studies (Vinner, 1983; Vinner & Dreyfus, 1989) have shown that although students recognised other patterns which are not linear as functions, but they required that such pattern should possess certain regular characteristics such as symmetry, constantly increasing or constantly decreasing, and so on to qualify as a function.

The quadratic functions have not received much attention from researchers particularly on the issue of students' misconceptions in the learning of quadratic functions. Students' difficulties and misconceptions about the abstraction notion of function and their difficulties and misconceptions with the linear functions is indicative that students will encounter misconceptions as they learn quadratic functions. Our purpose therefore is to investigate the misconceptions which impede learners' understanding of quadratic functions in grade 11. The research questions that directed the research is: What is the nature of the misconceptions that exists amongst grade 11 learners as quadratic functions is taught to them? The results have the possibility of enhancing effective teaching and learning of the concept of quadratic functions.

The teachers' knowledge of the learners' misconceptions will help the teacher to plan effective instruction, assist the learners to develop conceptual understanding, and be better equipped to assess learners' learning. Researchers such as (Hill, Schilling & Ball, 2004; Hill, Blunk, Charalambous, Lewis, Phelps, Sleep & Ball, 2008; Ball, Bass, & Hill, 2004; Adler, & Davis, 2006; Shulman, 1986; Marks, 1990) have identified teachers' knowledge of their learners' thinking (conceptions and misconceptions) about the concept as an integral component of the teachers' mathematics knowledge for teaching. Other components include; subject matter knowledge, and knowledge of representations and instructional strategies (Shulman, 1986). According to (Viri, 2003), if learning and teaching are viewed from the constructivist perspective, the teachers' knowledge of the learners' cognition is the most important part of the teachers' PCK. This assertion agrees with the findings of Hope and Townseed (1983) that experienced teachers who exhibit excellent knowledge of the subject matter but fail to take into consideration their learners' thinking about the subject matter often have difficulties in teaching the content.These findings support the assertion generally found in the literature that good knowledge of subject matter alone is not enough for effective teaching of mathematics, although a lack of knowledge of learners' misconceptions could be the result of poor subject matter knowledge of the educator.By getting acquainted with the learners' specific conceptions and ways learners' think about the subject matter,teachers may restructure their content knowledge into a form that enables meaningful communication with their students. For instance, for a representation to be powerful or more comprehensible to the learners the educator must know the learners' conceptions about a particular topic and also the possible difficulties the learners will come across with the topic. In line with this, Fennema, Frank, Carpenter, & Carley (1993) suggested a general instructional model in which research based knowledge is used to inform classroom instruction.

Conceptual framework

Various reasons why learners have misconceptions in specific topics in mathematics and how teachers can use this knowledge to assist learners to attain conceptual understanding in the subject matter have been provided by researchers. According to Dickson, Brown & Gibson (1999), the many misconceptions that learners develop seem to be primarily due to inadequate teaching. In their view, misconceptions occur when learners focus on the wrong criteria of solving problems and hence develop limited or false concepts. In this study, we have used the definition of misconception put forward by Leinhardt, Zaslavsky, & Stein (1990) and the idea of Alwyn, (1989) to conceptualisethe issue of misconceptions in learning. Misconception will therefore be considered as incorrect features in learners' mathematical knowledge that are repeatable and explicit. Some of these features may or may not be as a result of early learning (Leinhardt, Zaslavsky, & Stein, 1990). Some misconceptions stem from intuitions; features of learners' knowledge that arise most commonly from everyday experience and may exist prior to specific formal instruction (Leinhardt, Zaslavsky, & Stein, 1990). Learners'misconceptions can also be as a result of the tendency of learners to over-generalise pervious knowledge that was essentially correct in an earlier domain, to an extended domain where it is no longer correct (Alwyn, 1989). It is important to note at this juncture that when there is a difficulty in an aspect of Mathematics, it does not necessarily mean that a misconception is the reason for the difficulty. Rather, difficulties imply that there is something about the task that makes it especially difficult (Leinhardt, et al., 1990).

Methodology

The qualitative descriptive design will be used in this study. This design was employed to enable the researchersdescribe the learners' misconceptions as they learned quadratic functions. The sample comprised of 170 grade 11 learners from eight secondary schools. The schools and learners involved were a convenient sample. All the learners had completed the study of quadratic functions at most three months before the investigation was carried out. In terms of resources for mathematics teaching and learning, the schools were resources constraint schools. There were no graph boards, neither projectors nor computers for teaching and learning mathematics. However, each student had a calculator, and at least one mathematics textbook.

The research instrument

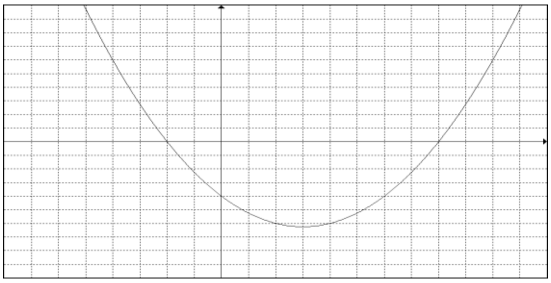
A questionnaire was used to gather data about learners' conceptions and misconceptionsof quadratic functions. The questionnaire consisted of thirteen items.Learners were required to give the right answer in each question and to give reason(s) for their answers.Learners 'choice of answer and reasons they provided for their choice were used to explore learners' conceptions and misconceptions about quadratic functions. Three of the questions were adopted from Zaslavsky (1997) while the researchers constructed ten questions in accordance withthe South African Curriculum Statements and the literature on quadratic functions.The items in the questionnaire were non-standard problems, most of which provided an opportunity for qualitative considerations and reasoning. The items can be classified as translation tasks (between graphical and algebraic representations).The questionnaire was validated by three experienced heads of department (HOD) of mathematics in secondary schools and two mathematics subject specialists. There was no time limit for the learners to complete the questionnaire since the researchers' interest was not on how fast the learners could answer the questions; rather the researchers' intent was to tap into the learners' conceptions and misconceptions in quadratic functions. The questionnaire took the learners about one to two hours to complete.

Sample questions used in the investigation

Three out of the thirteen questions used in conducting the research are provided below followed by the analysis of the knowledge that was deemed necessary for answering each of the questions.

Question 1

The parabola below is the graphical representation of the function $f(x) = x^2 - 3x - 4$



Which of the following has the same graph as $f(x) = x^2 - 3x - 4$?

a. $f(x) = 2x^2 - 6x - 8$ I chose this answer because………………………………………………….

b. $f(x) = 3x^2 - 9x - 12$ I chose this answer because……………………………………………...

c. $f(x) = 4x^2 - 12x - 16$. I chose this answer because………………………………………………..

d. All of the above. I chose this answer because…………………………………………………….

e. None of the above. I chose this answer because……………………………………………

An analysis of the graphical characteristics necessary for solving questions of the type in question 1

In order for learners to choose the correct option, it is necessary to consider the properties of each parabola in relation to that of $f(x) = x^2 - 3x - 4$ to ascertain if any of the parabola is same as $f(x) = x^2 - 3x - 4$. The characteristics of the three parabolas represented in the options and that of $f(x) = x^2 - 3x - 4$are shown in the table below.

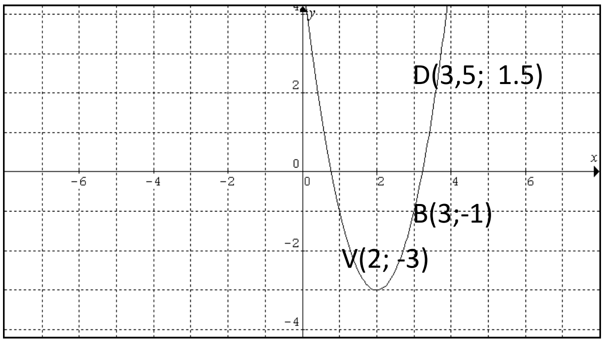
Table 1: Analysis of the graphical characteristics necessary for solving questions of the type in question 1

function	**symmetry**	**x-intercepts**	**y-intercept**	**Vertex**
$f(x) = x^2 - 3x - 4$	$x = \frac{3}{2}$	x = -1 or 4	y = - 4	$(\frac{3}{2}; -6.25)$
$f(x) = 2x^2 - 6x - 8$	$x = \frac{3}{2}$	x = -1 or 4	y = - 8	$(\frac{3}{2}; -12.5)$
$f(x) = 3x^2 - 9x - 12$	$x = \frac{3}{2}$	x = -1 or 4	y = - 12	$(\frac{3}{2}; -18.75)$

$f(x) = 4x^2 - 12x - 16.$	$x = \frac{3}{2}$	x = -1 or 4	y = -16	$(\frac{3}{2}; -25)$

Question 2

In the figure below, a parabola is graphed. V (2; -3) the turning point and two other points B (3; -1) and D (3.5; 1.5) on the parabola are given. C (3.25; 0.25) is the midpoint of line BD. Is point C on the parabola? Give reason(s) to justify your answer.

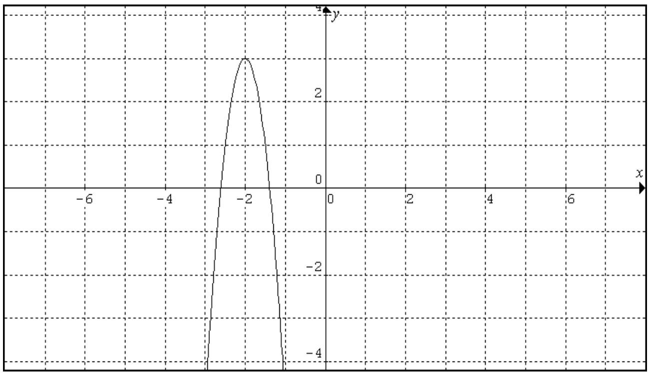


An analysis of the knowledge necessary for answering questions of the form in item 2 above

The learners can approach the task above by using the information provided on the graph to form the symbolic representation of the parabola (translating from the graphical representation to the algebraic representation) and then use the equation formed to verify if the given point satisfies the equation or not. Since the learners know the vertex (2;-3) and at least one other point on the parabola, they could form an equation using the canonical form of the quadratic function; $f(x) = a(x-2)^2 - 3$and substitute the other point (not the vertex) to calculate the value of 'a' and hence they could have obtained the algebraic representation as $f(x) = 2(x-2)^2 - 3.$ With the algebraic representation, they can verify ifpoint C (3.25; 0.25) is on the parabola or not. Alternatively, thelearners may choose to use the knowledge that since a parabola is a curve, and given that B and D aretwo points on the parabola, C cannot be on the parabola since B, C, and D arethree collinear points.

Question 3

If the parabola below is extended indefinitely, do you think that it will ever cut the y-axis?



An analysis of the knowledge necessary for answering questions of the form in item 3 above

In this question, quantitative information about the function was supressed so that the learners can use their conceptual understanding of the general characteristics of the quadratic functions, especially the infinite domain of the quadratic function to answer the question. For every x-coordinate in a parabola, there exists a y-coordinate, to extend the parabola indefinitely implied that its domain includes when *x* is equal to zero. Therefore, the parabola must cut the *y*-axis since every parabola has a *y*-intercept.

Data analysis

Content analyses of the reasons given by the learners for their choice of answers were done from which the researchers obtained thelearners' misconceptionsthat are reported in this study. To aid in identifying the learners' misconceptions, the responses given by learners in each question is organised using descriptive statistics. Zaslavsky's categories of conceptual obstacles in the learning of quadratic functions are used to provide structure for the findings.

Research Findings and Discussion

Table 2 gives the results of the responses of the 170learners to the 13 questions on quadratic functions. The table shows 1) the percentage of learnersthat chose the most popular wrong answer; 2) the percentage of learners that chose other wrong answers and 3) the percentage of learners that chose the correct answer.

Table 2 Learnersresponses (N = 170)

Question number	Percentage of learners that chose the most popular wrong answer	Percentage of learners that chose other wrong answers	Percentage of learners that chose the correct answer
1	08	11	81
2	64	Nil	36
3	66	Nil	34
4	13	15	72
5	66	Nil	34
6	12	Nil	88
7	Nil	17	83
8	68	10	22
9	Nil	90	10
10	87	Nil	13
11	29	38	33
12	28	46	26
13a	10	18	72
13b	11	15	74
13c	28	43	29

In Table 2 above, the "most popular wrong answer" represents the wrong option chosen by more learners. For instance in question 1, 08% of the learners chose option "e" which is the most popular wrong answer; 11% of the learners chose options "a", "b", and "d" i.e., the remaining wrong options; while 81% of the learners chose option "c", the answer right. From Table2, greater number of learners chose the wrong options in questions 2, 3, 5, 8, and 10. In each of questions 2, 3, 5, 8, and 10, the researchers analysed the reason(s) given by the learners to justify their answer. From the analyses of the learners' reasons for the choice of their answers the researchers identified the learners' misconceptions.

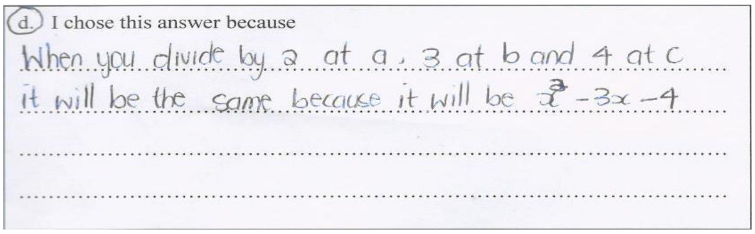
The Main Learners' Misconceptions

Four main learners' misconceptions in quadratic functions are reported are: Treating two different functions as equivalent; Limiting the graph of the quadratic functions only to the visible region; Ascribing linearity to the quadratic functions; Determining a special point by only one coordinate.

Treating two different functions as equivalent

Treating two different quadratic functions as equivalent is one of the misconceptions that impeded learners' understanding of quadratic functions. This was why 68 percent of the learners as can be seen in Table 2, decided that f(x) = $2x^2 - 6x - 8$; f(x) = $3x^2 - 9x - 12$; and f(x) = $4x^2 - 12x - 16$ are the same as f(x) = $x^2 - 3x - 4$. The learners' reasons was that after dividing by the common factor in each case we will get f(x) = $x^2 - 3x - 4$. However, two quadratic functions which have different values in their leading coefficients are totally two different functions.The reasons given by two students,which is a reflection of the reasons given by the sixty-eight of the learners is provided below

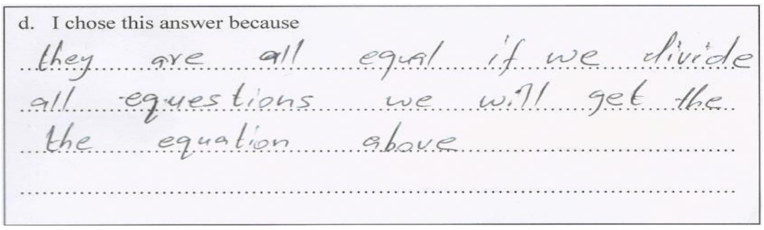
Learner A



d. I chose this answer because

When you divide by 2 at a, 3 at b and 4 at c it will be the same because it will be x^2 -3x -4

Learner B



d. I chose this answer because

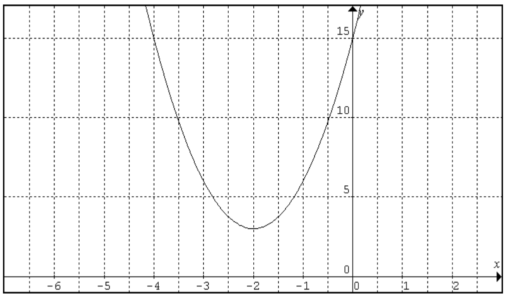
they are all equal if we divide all equestions we will get the the equation above

The learners' reasons show that they treated symbolic representations of quadratic functions as if they were treating quadratic equations. This misconception emanated because thelearners had already completed solving quadratic equations in which they were taught equivalent equations. In this concept, they were taught that equivalent equations are the same hence they could operate on the simpler equation. In this sense, when solving the equations; $2x^2 - 6x - 8 = 0$; $3x^2 - 9x - 12 = 0$; and $4x^2 - 12x - 16 = 0$, the learners can divide through by 2, 3, and 4 respectively to obtain $x^2 - 3x - 4 = 0$. However, a similar operation of dividing for instance a quadratic function f(x) = $2x^2 - 6x - 8$ by 2 to obtain y = $x^2 - 3x - 4$ results in two different functions which differ in all values except their$x$-intercepts and the x-coordinate of the vertex (see Table 1). A similar finding was made by Zaslavsky, (1997). The above misconception may have been developed as a result of the learning sequence in which the learners were only exposed to quadratic functions after they had completed extensive work on the quadratic equations earlier. Such misconceptions according to Alwyn, (1989) are a result of learners' "over generalisation of previous knowledge (that was correct in an earlier domain), to an extended domain (where it is not valid)".

Limiting the graph of the quadratic functions only to the visible region

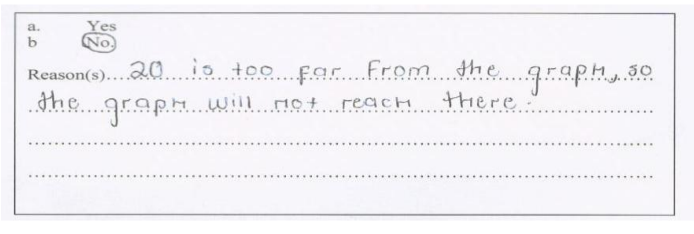
There was a tendency of learnersreading graphs like a picture.Learners interpreted the graph of quadratic functions based on only what is visible to them thereby ignoring the general characteristics of the functions. Although the graph of the quadratic functions may appear to be only that which is seen on paper or board, however, the graph goes to infinity as the domain of the quadratic functions is infinite. The pictorial interpretation of the graph of the quadratic functions is the reason why the learners inferred that the graph of the quadratic functions in Sample question 3 is asymptotic to the *y*-axis. They seem to have rejected that every quadratic function has a *y*-intercept regardless of whether it is shown on the graph or not. The picture reading of the graph was the reason why the 66percentanswered that the graph cannot cut the *y*-axis, although for every quadratic function there is a *y*-intercept.

The iconic interpretation of the graph of the quadratic functions was the reason why learners responded that it was not possible to have a point on the parabola below when *x* is 20.

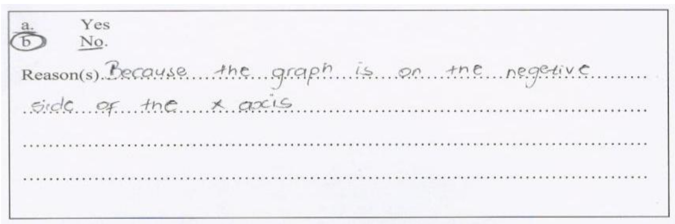


The reasons given by the learners were based on the reading of the parabola like a picture, while ignoring the general characteristics of the parabola. Two of the reasons given by the learners, which reflected the reasons given by 64percent of the sample, are presented below:

Learner A


a. Yes
b No
Reason(s) 20 is too far from the graph, so the graph will not reach there.

Learner B


a. Yes
b No.
Reason(s) Because the graph is on the negetive side of the x axis

The learners seem to have rejected the fact that for every parabola, any value of x has a corresponding value of y. The learners seem also to have rejected that the parabola has an infinite domain.The iconic interpretation of the graph which could be traced to the learners'intuition regarding picture reading has been reported in numerous studies including Leinhardt, et al (1990); and Zaslavsky (1997).

Ascribing linearity to the quadratic functions

There was the tendency of the learners ascribing linearity to the parabola. Learners were given two points, B (3; -1) and D (3.5; 1.5) on the parabola and were asked if another point C (3.25; 0.25) the midpoint of straight joining line C and D also lie on the parabola. 87 percent of the learners (see Table 1) responded that point C is also on the parabola. The learners' reason was that since the two points C and D lie on the parabola, therefor their midpoint also must lie on the parabola. For the students, the distance between points C and D on the parabola is a straight line. However, the distance between any two points on the parabola is a curve, hence; the parabola cannot pass through three collinear points no matter how close they may seem to be. The way in which the standard form of both the quadratic functions and the linear functions are written may have reinforced the learners' thinking that part of the parabola is linear. In most cases, quadratic functions are written as $f(x) = ax^2 + bx + c$ and the linear function as $f(x) = ax +c$. Learners seems to take $bx + c$ in the quadratic function as an equivalent form of $ax + c$ of the linear function. Ascribing linearity to the quadratic functions manifested itself in other formsincludinglearners' tendency of using straight lines instead of curve to join the points they plotted when they draw/sketch the parabola.
Reasons for learners' over-attachment to linearity may be that learners are only introduced to quadratic function in Grade 10 while they are exposed to the linear graphs as early as their preschool days when they "connect a dot"(Leinhardt, et al., 1990). In addition, since the linear function is the first family of functions which the learners are exposed to, they tend to over-generalise the conjectures which they make when they learn the linear function. This result agrees also with the articulation generally found in the literature that each curriculum sequence has its own attendant misconceptions.Over-attachment to linearity has been reported in many studies including Zaslavsky (1997) and Dreyfus & Eisenberg (1983).

Determining a special point by only one coordinate

Learners described the vertex of a parabola using only one of its coordinates while in reality the vertex is a point and must be described using two points. 66 percent of the learners stated that the parabolas $y = ax^2 + bx + 3$ *and* $y = ax^2 + bx + 7$ have the same vertex. The learners based their argument on the fact that the two parabolas have the same line of symmetry. They did not boarder to consider the y-coordinate of each of the parabolas. This impediment may have resulted from the way in which learners calculate the missing coordinate in the other special points; the y-intercepts, and the two x-intercepts (when they exist). In each of these special points, the other coordinate is always zero; hence, the learners need to determine only one coordinate.The learners extended their practice of finding only one coordinate when finding the x and y intercepts to the vertex.A similar finding was reported by Zaslavsky, (1997).

Implications and Conclusion

Our findings in this study show students' lack of understanding in moving from the symbolic representations to the graphical representations of the quadratic functions and students' misunderstanding concerning the infinite nature of the parabola. This has implication to the use of graphing technologies in the classroom to help learners overcome these misconceptions.The findings that we reported in this paper, is indicative of learners' possible lack of conceptual understanding in quadratic functions, which may be as a result of the traditional teaching and learning approach that is common in most of our mathematics classrooms. The methodology we used in this study whereby learners were required to give reason(s) for their choice of answer will be useful to researchers and teachers in teaching and assessing students' conceptual understanding in mathematics.

In conclusion, we need to appreciate that learners pass a lot of stages as they try to learn of which having misconception is one. Learners' misconceptions should not be viewed negatively; rather, they show the level of learners' development in the concept. The learners' misconceptions should be viewed from the learners' perspective andneed to be helped to overcome their misconceptions. This demands that teachers and curriculum developers should be abreast with learners' misconceptions in quadratic functions (and by extension in mathematics) and come up with appropriate learning activities that can deal with the perceived misconceptions and enhance learners' learning of quadratic functions.

References

Adler, J. & Davis, Z. (2006a).Opening another black box: Researching mathematics for teaching in mathematics teacher education.*Journal for Research in Mathematics Education*, *37*(4), 270-296

Adler, J. & Davis, Z. (2006b).*Studying mathematics for teaching inside teacher education*. Paper presented to at the 14th Annual Meeting of the Southern Africa Association for Research in Mathematics, Science, and Technology Education, University of Pretoria: South Africa.

Alwyn, O. (1989). Presidential address delivered at the thirteenth National Convention on Mathematics, Physical Science and Biology Education, Pretoria, 3 – 7 July 1989.

Ball, D. L., Bass, H., & Hill, H. (2004). *Knowing and using mathematical knowledge in teaching: Learning what matters*. Paper presented at the 12th Annual Conference of the Southern African Association for Research in Mathematics, Science, and Technology Education, Durban: South Africa.

Creswell, J.W. (2008). *Research design: Qualitative, quantitative, and mixed methods approaches* (2nd ed.). Thousand Oaks: Sage.

Dickson, L. Brown, M. & Gibson, O. (1999). *Children learning mathematics*, Cassel for The

Schools council: London.
Dreyfus, T., & Eisenberg, T. (1983).*The function concept in college students': Linearity, smoothness, and periodicity. Focus of Learning Problems in Mathematics, 5. 119 - 132*
Fennema, E., Frank, M.L., Carpenter, T.P., &Carley, D.A. (1993).Using children's mathematical knowledge in instruction.*American Educational Research Journal, 30(3), 403-434.*
Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction,* 26, 430-511.
Hill, H. C., Schilling, S. G., & Ball, D. L. (2004).Developing measures for mathematical knowledge for teaching.*The Elementary School Journal, 105*(1), 11 – 30.
Hope, J, & Townsend, M. (1983). Student teacher understanding of scienc concepts. *Research in science education*, 13, 177-183.
Howie, S.J. (2003). Conditions of schooling in South Africa and the effects on mathematics achievement.*Studies in Educational Evaluation 29, 22- 241*
Leinhardt, G., Zaslavsky, O., & Stein, M.K. (1990). Functions, graphs, and graphing: Task, learning, and teaching. *Review of Educational Research,* 60(1), 1-64
Marks, R. (1990). Pedagogical content knowledge: From a mathematical case to a modified conception. *Journal of Teacher Education ,41* (3), 3-11.
Markovits, Z., Evylon, B., & Bruckheimer, M. (1983).Functions: Linearity unconstrained. In R. Hershkowitz (Ed.).*Proceedings of the seventh international conference of the International Group for the Psychology of Mathematics Education* (271 – 277).Rehovot, Isreal: Weizmann Institute of Science.
Markovits, Z., Evylon, B., & Bruckheimer, M. (1986). Function today and yesterday. *For the Learning of Mathematocs,* 6(2), 18-28.
Marnyanskii, A. (1975). Psychological characteristics of pupils' assimilation of the concept of a function.In J. Kilpatrick, Wirszup, E., Begle, E., & Wilson, J. (Eds.).*Soviet Studies in the Psychology of Learning and Teaching Mathematics,* 13,163-172. Chicago: University of Chicago Press.
Matz, M. (1982).Towards a process model for high school algebra errors.In D. Sleeman& J. S. Brown (Eds.), *Intelligence tutoring system* (25 – 50).London: Academic Press.
Reddy, V. (2006).Mathematics and science achievement at South African schools in TIMSS2003. Pretoria: Human Sciences Research Council.
Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.
Thomas, H. L. (1975). The concept of functions. In Rosskopf, M (Ed.).Children's mathematical concepts (145 - 172). New York: Columbia University. Teachers' College.
Vinner, S. (1983).Concept definition, concept image and the notion of functions.*International Journal of Research in Mathematics Education in Science and Technology*, 14(3), 293 – 305.
Vinner, S., & Dreyfus, T. (1989).Images and definitions for the concept of function.*Journal for Research in Mathematics Education*, 20(4), 356-366.
Viri, J. (2003). Engineering teachers pedagogical content knowledge. *European Journal of Engineering Education, 28*(3), 353-359.
Zaslavsky, O. (1997). Conceptual obstacles in the learning of quadratic function.*Focus on Learning Problems in Mathematics*, *19* (1), 20-25.