DYNAMICS OF THE BREAKUP OF TWO–BODY HALO NUCLEI

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Dedication

I dedicate this thesis to

- My parents Mr Kajama Mueru Ferdinand and Mrs M’ Rutega Cirezi Concilie,
- Mr Birhashuma Nderhe Paul and Mrs M’ Rutega Kayange Maria,
- Mr Simba Kalolo Zachée and Mrs M’ Chimbeshe Njerina.
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- My brothers and sisters in the Lord in Living Hope Church.

The Lord has been merciful to me, making the end of this journey a reality. Words are not enough to thank Him!
I declare that “DYNAMICS OF THE BREAKUP OF TWO-BODY HALO NUCLEI” is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

Signature

Mr B Mukeru (Student Number: 44943725)

Date: 30 June 2015
Summary

In this thesis, the first-order and higher-order interferences on the total (Coulomb+nuclear), Coulomb and nuclear breakup cross sections in the $^{15}$C+$^{208}$Pb, $^{11}$Be+$^{208}$Pb breakup reactions are first studied at 68 MeV/u incident energy. It is shown that the first-order interference reduces by more than 60% the total breakup cross sections, by less than 3% the Coulomb breakup cross sections and by more than 85% the nuclear breakup cross sections, for both reactions. On the other hand, the high-order interference is found to reduce by less than 9% the total breakup cross section, less than 1% the Coulomb breakup cross section and less than 7% the nuclear breakup cross section for the $^{15}$C+$^{208}$Pb reaction. For the $^{11}$Be+$^{208}$Pb reaction however, the high-order interference reduces by less than 7% the total breakup cross section, by less than 1% the Coulomb breakup cross section and by less than 4% the nuclear breakup cross section. It is finally shown that even at first-order, the incoherent sum of the nuclear breakup cross sections is more important than the incoherent sum of the Coulomb breakup cross sections for the two reactions. The role of the diagonal and off-diagonal continuum-continuum couplings on total, Coulomb and nuclear breakup cross sections is also investigated for the $^{8}$B+$^{58}$Ni, $^{8}$B+$^{208}$Pb and $^{19}$C+$^{208}$Pb at 29.3, 170.3 MeV and 1273 MeV incident energies respectively. Qualitatively, we found that, the diagonal continuum-continuum couplings are responsible for the large reduction of the differential total and nuclear breakup cross sections at backward angles. At forward angles, this reduction is due to the off-diagonal continuum-continuum couplings. In the absence of these couplings, the nuclear breakup is the more dominant process, while when they are included, the Coulomb breakup becomes dominant. This shows that, the nuclear breakup is more affected by the continuum-continuum couplings than its Coulomb counterpart. Quantitatively, we found that, the off-diagonal continuum-continuum couplings reduce by 13.39%, 12.71% and 11.11% the total breakup cross sections for the $^{8}$B+$^{58}$Ni, $^{8}$B+$^{208}$Pb and $^{19}$C+$^{208}$Pb reactions, respectively.

Key words: Interferences; Integrated breakup cross sections; Continuum-continuum couplings; Differential breakup cross sections.
This thesis is based on the following publications:


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Chapter 1

Introduction

Since the discovery of the halo phenomenon \[1,2\], the exotic properties of halo and other loosely bound nuclei have been intensively investigated both experimentally and theoretically \[3-18\]. Quantum halos are regarded as systems with dominating few-body structure, and radii larger compared to the size of the classically allowed regions \[3-5\]. A halo nucleus is well described as a core nucleus to which one or two valence nucleons are loosely bound \[4, 6, 7\]. The consequence of the low binding energy is that halo nuclei exhibit bound states close to the continuum \[50\]. Among the well known one neutron halo nuclei, we can mention \(^{11}\)Be, \(^{15}\)C, \(^{19}\)C \[8,9,12,13,15-18\]. For the one proton halo nuclei, one mentions \(^{8}\)B, \(^{31}\)Ne \[10,11,17,18\].

Another consequence of the low binding energy is that a halo projectile breaks easily after its interaction with the target, making the breakup reactions to play a useful role in probing the structure of these nuclei \[4,8,9,13,14,51,59\]. Among the main focuses within this field, there are studies of the breakup cross sections and the influence of the breakup process on other reaction channels like fusion, elastic scattering, among others. Although the breakup of halo and other loosely bound nuclei has attracted intense attention in recent time, the full understanding of the dynamics of these reactions is far from being established, due to the complexity of the breakup process. In fact, one important question when investigating the breakup process of halo or weakly bound nuclei is, what is the main interaction that produces this breakup, the Coulomb or the nuclear interaction? Or what is the nature of their interferences and how important are these? If the answer to the first question cannot be predict to some extend, it is rather difficult to anticipate any answer to the second. Another important question is the role of the nuclear interaction in a Coulomb-dominated reaction. For example, how accurate the nuclear breakup
contribution can be eliminated to obtain a pure Coulomb breakup? Multipole transitions also play a significant role in the breakup process, in the sense that Coulomb and nuclear interactions are commonly expanded into multipoles for practical purposes, which may interfere. It is not well known how different multipole interferences affect the breakup cross sections, and eventually their effects on the Coulomb-nuclear interference.

Coulomb dissociation (CD) method based on the first-order approximation restricted to the $E1$ multipole $^{[52,60,61]}$, has been one of the methods intensively used to study the structure of halo nuclei $^{[8,13,14,52–56]}$. The $E1$ multipole restriction is mostly justified by the assumption that higher-order or non-first-order effects are negligible in the Coulomb breakup induced by a neutron-halo projectile $^{[13,14]}$. In particular, it was shown in Ref. $^{[52]}$, that the higher-order multipole transitions reduce by less than 4% the overall Coulomb breakup cross section. However, this method has received criticisms regarding the elimination of the nuclear breakup contribution to keep only the pure Coulomb breakup cross section $^{[51,62]}$, in the sense that the scaling method mostly used to eliminate the nuclear breakup contribution $^{[8,63]}$, was found to not be always reliable due to the significance of the Coulomb-nuclear interference $^{[62]}$. Various studies have also shown that small nuclear contribution does not necessarily mean negligible Coulomb-nuclear interference $^{[64–68]}$, thus raising more issues in the exclusion of the nuclear breakup cross section.

Another argument put forward for the $E1$ transition restriction in the analysis of the $^{15}C+^{208}Pb$ reaction was that, all the outgoing neutrons are in the $p$–waves and the breakup occurs in one-step $^{[58]}$. However, analyzing the same reaction in Ref. $^{[57]}$, using the CDCC (continuum discretized coupled channel) method $^{[69,70]}$, it was shown that all the outgoing neutrons are not in the $p$–waves and that the multi-step process plays an important role. While one could expect the Coulomb breakup cross section to fit the experimental data, the authors showed that the data are rather well fitted by the total breakup cross section. However, it is not clear whether this is an exclusive effect of the Coulomb-nuclear interference. On the other hand, to show that all the outgoing neutrons are not in the p-waves, a detailed partial wave analysis is required. On the light of these
contradicting conclusions, one may wonder as to whether this is a particularity of the $^{15}$C, or it can be generalized to other one neutron halo nuclei. An anticipated conclusion is not guaranteed given the different nuclear properties of these nuclei.

Although the first-and higher-order interference effects have received considerable attention, these effects are not yet fully understood for both total and nuclear breakup cross sections, while this could shed more light in the understanding of the role of the nuclear breakup in a Coulomb dominated reaction, induced by a neutron-halo projectile. For reactions induced by the proton-halo nucleus $^8$B, it was shown in Refs. [71,72], that for the total breakup cross section, the multipole interference plays a rather important role, and is strongly destructive. In Ref. [73,74], for example, where both Coulomb and nuclear breakups were considered separately, the authors obtained more pronounced effects of the different multipole transitions on the nuclear breakup cross section than on its Coulomb counterpart, for the $^{17}$F+$^{208}$Pb reaction. It is interesting to investigate whether similar conclusions can be drawn as well for reactions induced by neutron-halo projectiles.

The Coulomb breakup of $^{19}$C projectile impinging on $^{208}$Pb target at and 67 MeV/u have been measured and analyzed by different groups, using different approaches [8,58,59]. In [8], the Coulomb dissociation method was also employed, while in [59], the time-dependent Schrödinger equation was solved to investigate the nuclear and Coulomb breakups. In Ref. [8], it was concluded that the shape of the angular distribution is not affected by the nuclear breakup effects below the grazing angle ($\sim 2.7^\circ$). Later in Ref. [59], it was shown that even at 1.5° the nuclear effects are already important. But using a $^{19}$C binding energy of 0.53 MeV as in Ref. [8], the results overestimated the data for low excitation energies (see Fig.1(a) of Ref. [59]). On the light of these differences, a further investigation of these reaction is needed, for a better understanding of the nuclear breakup contribution and the effect of the Coulomb-nuclear interference.

Given the low binding energy, breakup reactions induced by halo projectiles exhibit, in general, strong continuum-continuum couplings (ccc), which have been intensively ana-
lyzed recently \cite{71,75,81}. In Refs. \cite{57,71,76,80}, the role of these couplings on the elastic scattering and breakup cross sections has been investigated for different reactions also by means of the CDCC method. The results obtained showed that, the inclusion of the ccc in the potential matrix elements, results in the reduction of the breakup cross sections. Similar studies were undertaken in Refs. \cite{77,79}, for the fusion cross sections, where it was also concluded that these couplings are responsible of the reduction of the fusion cross sections by increasing the Coulomb barrier. Particularly in Ref. \cite{77}, the authors showed that the ccc have a significant influence on the complete fusion cross section and it is also important to the total fusion cross section. However, most of these results were obtained by including in the potential matrix elements, either couplings to and from the ground state plus only diagonal couplings, or couplings to and from the ground state plus the ccc(both diagonal and off-diagonal couplings). It is therefore not clear how off-diagonal couplings affect the different reaction channels qualitatively and quantitatively, although in Ref. \cite{79}, it was mentioned that the role of off-diagonal couplings could be negligible on the fusion cross section. One could wonder as to how insignificant they are? It could be also important to know how they affect other reaction channels, like elastic breakup.

The use of the CDCC method for such studies, considered in the aforementioned works and this thesis, is mostly justified by the fact that it provides a nonperturbative approach in which to describe a breakup process, both Coulomb and nuclear breakups are treated at the same footing. Multipole excitations are fully taken into account as well as the final state interaction effects \cite{51,76}. On the other hand, the method includes accurately the ccc in the potential matrix elements. However, due to the inclusion of the ccc (which may be strong) in the potential matrix elements, the method is computationally expensive. As a result, this method, although promising, has been limited to low and medium energy reactions \cite{51}. From our experience, CDCC calculations converge faster when off-diagonal couplings are excluded than when they are included. It is therefore advantageous to have a clear idea on the role of off-diagonal couplings, as this could lead to an important simplification of the computational load.

In this thesis, we study the dynamics of the $^{11}\text{Be}+^{208}\text{Pb}$, $^{15}\text{C}+^{208}\text{Pb}$, $^{19}\text{C}+^{208}\text{Pb}$, $^{8}\text{B}+^{208}\text{Pb}$
and $^8\text{B} + ^{58}\text{Ni}$ breakup reactions. For the $^{11}\text{Be} + ^{208}\text{Pb}$ and $^{15}\text{C} + ^{208}\text{Pb}$ breakup reactions, where we consider the incident energy of 68 MeV/u, we mostly study the first-and higher-order interferences on the total, Coulomb and nuclear breakup cross sections. We aim especially to investigate how important these interferences are (meaning their magnitudes and nature) on the energy and angular distributions total, Coulomb and nuclear breakup cross sections, as well as on the Coulomb-nuclear interference, for a better understanding of the nuclear breakup contribution for these two reactions and testing the accuracy of excluding the nuclear breakup contribution when the different multipole breakup cross sections are summed incoherently, as it is the case in the CD method. The effects of these interferences on the angular momentum and impact parameter distributions are to be also investigated, in order to analyze their role on the nuclear absorption effect. The choice of these two reactions for this particular study is firstly justified by the availability of the experimental data [9, 14], making the comparison easy. Secondly, the two projectiles exhibit similar ground state configurations [59, 82]. Furthermore, the $^{11}\text{Be}$ ground state binding energy is lesser than the one of the $^{15}\text{C}$ nucleus, thus providing an opportunity to assess the role of the ground state binding energy on our findings. The methodology adopted here consists in the following steps: first we perform first-order (FO) CDCC calculations, and estimate the first-order interference. Second, we estimate the all-order (AO) interference, where all the different multipoles retained in the CDCC model space are included coherently and incoherently. Finally, the higher-order interference is then estimated by considering the difference between the first-and all-order interferences.

For the $^{19}\text{C} + ^{208}\text{Pb}$, $^8\text{B} + ^{208}\text{Pb}$ and $^8\text{B} + ^{58}\text{Ni}$ breakup reactions, which are analyzed at 1273 MeV, 170.3 MeV and 29.3 MeV, respectively, the prime objective is to investigate the effects of the ccc (diagonal and off-diagonal) also on the total, Coulomb and nuclear breakup cross sections, on the Coulomb-nuclear interference, as well as on the Coulomb barrier penetration. The effects of the couplings on the Coulomb barrier penetration, will shed more light on the dependence of the fusion cross sections on the breakup process. On the other hand, this will provide for the first time a clear understanding of the role of the off-diagonal ccc on the angular distribution breakup cross sections, both quantitatively and qualitatively. The choice of these reactions is motivated also on one hand, by the
fact that there are elastic scattering experimental data for the $^8B + ^{208}Pb$ and $^8B + ^{58}Ni$ reactions [83,84], which would guide our insight. On the other hand, there is an amount of theoretical works on these reactions, thus making the comparison easy. Moreover, the contradicting results for the $^{19}C + ^{208}Pb$ reaction as mentioned already [8,59], reveal that the analysis of this reaction is not fully complete regarding the nuclear breakup contribution. Furthermore, this study provides an opportunity of analyzing the effect of the valence nucleon charge on the results obtained. Also considering different target masses and incident energy regime, allows one to check the dependence of the results on the target masses and incident energies.

The thesis is organized as follows: In chapter 2, we present a theoretical description of the two- and three-body systems, where a brief description of the CDCC method is discussed. In chapter 3, the results obtained for the $^{11}Be + ^{208}Pb$ breakup reaction are presented and analyzed, while chapter 4 concerns the $^{15}C + ^{208}Pb$ breakup reaction. In chapter 5 on the other hand, we analyze the $^{19}C + ^{208}Pb$ breakup reaction, whereas the $^8B + ^{208}Pb$ and $^8B + ^{58}Ni$ reactions are analyzed in chapter 6. Each chapter starts with a description of the projectile, where some important properties of the halo nuclei are presented. The concluding remarks are reported in chapter 7.
Chapter 2

Description of few-body systems

In this chapter, we present the method employed to analyze the breakup of two-body halo nuclei. Starting from the simple case of two-body systems, we describe the elaborated CDCC method, suitable for the treatment of the three-body breakup process.

2.1 Description of two-body systems

When two particles collide, several phenomena may take place. They can combine and form a bound state, where its microscopic time is longer compared to the time during which the reaction occurs. Another phenomenon that may happen is the scattering, which is defined as a physical process during which a particle approaches the interaction region from afar and after passing through a potential, moves away again but sometimes in different direction, with different energy and quantum numbers. The energy of the scattering states is real and positive since the particle can move with real velocity in the asymptotically far regions. In this section, we briefly present a description of the two-body bound and continuum states. For completeness, we refer the reader to Refs. [85–88].

Let us consider two particles, interacting via a potential which does not depend on the orientation of the vector between them, meaning a spherical potential. The corresponding two-body Hamiltonian reads

\[
H_0 = \sum_{i=1}^{2} \frac{p_i^2}{2m_i} + V_{12}(r),
\]  

(2.1)

where \( p_i \) and \( m_i \) are the particle momenta and masses, respectively, \( r = |r_2 - r_1| \) is the rel-
ative distance between the two particles, and $V_{12}(r)$ the real effective two-body potential. Removing the center-of-mass motion, one writes the Hamiltonian (2.1) as follows

$$H_0 = -\frac{\hbar^2}{2\mu_{12}} \frac{d^2}{dr^2} + V_{12}(r), \quad (2.2)$$

where $\mu_{12} = \frac{m_1 m_2}{m_1 + m_2}$ is the two-body reduced mass and $r = |r|$. The two-body relative motion is described by the wave function $\phi_{k\ell}^{jm}(r)$, satisfying the following time-independent Schrödinger equation

$$H_0 \phi_{k\ell}^{jm}(r) = \varepsilon \phi_{k\ell}^{jm}(r), \quad (2.3)$$

where $\varepsilon$ is the energy in the center-of-mass system, and $k$ the wave number defined by

$$k = \sqrt{\frac{2\mu_{12}\varepsilon}{\hbar^2}}. \quad (2.4)$$

Assuming that only one particle has a spin $s$ with $m_s$ its $z$-projection, the total angular momentum $j$ is written as $j = \ell + s$, with $m = \nu + m_s$ its $z$-projection.

Given the spherical symmetry of the potential $V_{12}(r)$, it is convenient to expand the wave function $\phi_{k\ell}^{jm}(r)$ in terms of spherical harmonics to obtain

$$\phi_{k\ell}^{jm}(r) = \frac{1}{r} e^{i\ell} \left[ Y_{\ell}^{\nu}(\Omega_r) \otimes X_{s}^{m_s} \right]_{jm} \phi_{k\ell}^{jm}(r)$$

$$= \frac{1}{r} \sum_{\nu m_s} \langle \ell \nu s m_s | j m \rangle Y_{\ell}^{\nu}(\Omega_r) X_{s}^{m_s} \phi_{k\ell}^{jm}(r), \quad (2.5)$$

where $Y_{\ell}^{\nu}(\Omega_r)$ is the spherical harmonics associated with the angular momentum $\ell$, with $\Omega_r$ the polar angle of the vector $r$ with respect to some coordinate axis, $X_{s}^{m_s}$ the spinor associated with the spin of the particle, and $\phi_{k\ell}^{jm}(r)$ is the radial part of the wave function $\phi_{k\ell}^{jm}(r)$. For pure nuclear potentials [$V_{12}(r) = V_{12}^N(r)$], the radial wave function $\phi_{k\ell}^{j}(r)$ satisfies the following radial Schrödinger equation

$$\left[ -\frac{\hbar^2}{2\mu_{12}} \left( \frac{d^2}{dr^2} - \frac{\ell(\ell + 1)}{r^2} \right) + V_{12}^N(r) \right] \phi_{k\ell}^{j}(r) = \varepsilon \phi_{k\ell}^{j}(r), \quad (2.6)$$

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where the short-range nuclear potential is, in general given by

\[ V_{12}^N(r) = V_0(r) + l s V_S(r), \]  

(2.7)

with \( V_0(r) \) and \( V_S(r) \) the nuclear central and spin-orbit coupling terms, which are parametrized in this thesis using the Woods-Saxon parametrization, which is

\[ V_0(r) = V_0 f_0(r), \]
\[ V_S(r) = V_{so} \left( \frac{\hbar}{m \pi c} \right)^2 \frac{1}{r} \frac{d}{dr} f_S(r), \]

(2.8)

where \( \left( \frac{\hbar}{m \pi c} \right)^2 = 2 \text{ fm}^2 \) \[51\], \( V_0 \) and \( V_{so} \) are the depths of the central and spin-orbit coupling terms, respectively and

\[ f_x(r) = \left[ 1 + \exp \left( \frac{r - R_x}{a_x} \right) \right]^{-1}, \]

(2.9)

with the radius \( R_x \) being proportional to the nuclear size, \( R_x = r_x(A_1^{1/3} + A_2^{1/3}) \), where \( A_1 \) and \( A_2 \) are the mass numbers of the interacting nuclei, and \( r_x \) is a parameter whose typical value is 1.2 fm, and \( a_x \) the diffuseness parameter. If the particle 1 is a nucleon say, then \( R_x = r_x A_2^{1/3} \).

For charged particles, the potential \( V_{12}(r) \) contains an additional Coulomb term, such that

\[ V_{12}(r) = V_{12}^N(r) + V_{12}^C(r), \]

(2.10)

where \( V_{12}^C(r) \) is the Coulomb potential, which we consider to be a point-sphere Coulomb
potential, given by

\[
V_{12}^{C}(r) = \begin{cases} 
\frac{Z_1 Z_2 e^2}{R_C} \left( \frac{3}{2} - \frac{r^2}{2R_C^2} \right) & r < R_C \\
\frac{Z_1 Z_2 e^2}{r} & r \geq R_C,
\end{cases}
\]  

(2.11)

where \(Z_1\) and \(Z_2\) are the particle charges, \(R_C\) is the Coulomb radius, \(R_C = r_c \left( A_1^{1/3} + A_2^{1/3} \right)\), with \(r_c\) relating to \(r_x\). With the potential (2.10), the radial Schrödinger equation (2.6) can be rewritten as

\[
\left[ \frac{d^2}{dr^2} - \frac{\ell(\ell + 1)}{r^2} - \frac{2k\eta}{\hbar^2} V_{12}^{N}(r) \right] \phi_{k\ell}^{j}(r) = \varepsilon \phi_{k\ell}^{j}(r),
\]

(2.12)

where

\[
\eta = \frac{Z_1 Z_2 e^2}{\hbar v} = \frac{Z_1 Z_2 e^2 \mu_{12}}{\hbar^2 k} = \frac{Z_1 Z_2 e^2}{\hbar^2} \left( \frac{\mu_{12}}{2e} \right)^{1/2},
\]

(2.13)

is the Sommerfeld parameter.

### 2.1.1 Bound states

A bound state described by a wave function \(\phi_{n\ell_0}^{j_0}(r)\) \((n = 0, 1, 2, \ldots, n\) the number of nodes of the wave function\), satisfying the Schrödinger equation (2.3), is such that for a given angular momentum \(\ell_0\), the binding energy \(\varepsilon_0\) is negative \((\varepsilon_0 < 0)\). The wave function \(\phi_{n\ell_0}^{j_0}(r)\) is regular at the origin and has the following asymptotic boundary conditions at large distance \((r \to \infty)\) [51]

\[
\phi_{n\ell_0}^{j_0}(r) \to C_0 W_{-n, \ell_0 + \frac{1}{2}}(-2k_ir) \to C_0 e^{-kIr} \to 0,
\]

(2.14)

where \(C_0\) is the normalization coefficient and \(W_{k,b}(z)\) are Whittaker functions [89], which
behave asymptotically as
\[
W_{-\eta,0^+} \left(-2k_I r\right) \to e^{-k_I r + \eta_I \ln(2k_I r)}, \tag{2.15}
\]
with
\[
k = ik_I = i \sqrt{\frac{2\mu_{12}|\varepsilon_0|}{\hbar^2}}
\]
\[
\eta = -i\eta_I = -i\frac{Z_1 Z_2 e^2 \mu_{12}}{\hbar^2 k_I}. \tag{2.16}
\]
The bound states are normalized according to
\[
\int |\phi_{j0m_0}^j(r)|^2 dr = \int_0^\infty |\phi_{j0m_0}^j(r)|^2 dr = 1, \tag{2.17}
\]
and fulfill the following orthogonality property
\[
\langle \phi_{n_0m_0}^{j_0} | \phi_{n_0'\ell_0}^{j_0'} \rangle = \delta_{nn'} \delta_{\ell \ell_0} \delta_{j_0j_0'}. \tag{2.18}
\]

### 2.1.2 Continuum and Resonant states

As mentioned already, continuum states are associated with positive energies ($\varepsilon > 0$), and hence the wave number $k$ is continuous. In this case, the continuum wave function $\phi_{j\ell}^j(r)$, which is regular at the origin, has the following asymptotic boundary conditions at large distance ($r \to \infty$)
\[
\phi_{j\ell}^j(r) \to F_{\ell}(kr) \cos \delta_{\ell j} + G_{\ell}(kr) \sin \delta_{\ell j}
\to \sin[kr - \frac{1}{2}\ell \pi - \eta \ln(2kr) + \sigma_{\ell} + \delta_{\ell j}], \tag{2.19}
\]
where $F_{\ell}(x)$ and $G_{\ell}(x)$ are Coulomb functions [89], $\delta_{\ell j}$ the nuclear phase shifts, and
\[
\sigma_{\ell} = \arg \Gamma(1 + \ell + i\eta), \tag{2.20}
\]
the Coulomb phase shifts which, together with the phase \(-\eta \ln(2kr)\) distort the oscillatory behavior of the wave function in the asymptotic region. The boundary conditions \(2.19\), imply the following normalization of the continuum wave functions

\[
\int \phi^*_{kj\ell}(r)\phi_{k'j'\ell}(r) = \delta(k - k'),
\]

where \(\delta(k - k')\) is the delta function [89]. The continuum wave functions are not square-integrable functions. However, they satisfy the following orthogonality relation

\[
\langle \phi_{kj\ell}|\phi_{k'j'\ell'} \rangle = \delta(k - k')\delta_{\ell\ell'}\delta_{jj'}.
\]

They are also orthogonal to the bound states, meaning that

\[
\langle \phi_{kj\ell}|\phi_{n0\ell0} \rangle = 0.
\]

When the binding potential is not strong enough to form a bound state, the particle can get trapped inside the potential barrier for a time lesser than the time during which the reaction takes place. This short living state can be regarded as a resonant state, which is characterized, among others, by the fact that the corresponding phase shift approaches \(\pi/2\) as the incident energy approaches the resonance energy \((\varepsilon_r)\). For a better understanding of a resonant state, we follow [51], and first write the elastic scattering angular-integrated total cross section in terms of the phase shifts as follows

\[
\sigma = \sum_{\ell=0}^{\infty} \sigma_{\ell},
\]

where

\[
\sigma_{\ell} = \frac{4\pi}{k^2} (2\ell + 1) \sin^2 \delta_{ij} = \frac{4\pi}{k^2} (2\ell + 1) \frac{1}{1 + \cot^2 \delta_{ij}}
\]

is the partial angular-integrated cross section. The expansion of \(\cot \delta_{ij}\) around \(\varepsilon_r\), which
retains only the first term, gives
\[
\cot \delta_{lj} \simeq \frac{\partial \cot \delta_{lj}}{\partial \varepsilon} \bigg|_{\varepsilon=\varepsilon_r} (\varepsilon - \varepsilon_r).
\] (2.26)

Defining the width of the resonance \( \Gamma \) by
\[
\Gamma = 2 \left[ \left. \frac{\partial \cot \delta_{lj}}{\partial \varepsilon} \right|_{\varepsilon=\varepsilon_r} \right]^{-1}
\] (2.27)

and introduce equation (2.26) into equation (2.25), one obtains
\[
\sigma_\ell = \frac{4\pi}{k^2} (2\ell + 1) \frac{r^2}{4 + (\varepsilon - \varepsilon_r)^2},
\] (2.28)

which shows a clear peak at \( \varepsilon \simeq \varepsilon_r \). It is believed that many of the narrower and wider peaks observed in the cross sections plotted as functions of the energy are caused by the resonances. Consequently, the nuclear phase shifts will also peak at the resonance energy.

The radial bound and continuum state wave functions presented above are useful in describing the internal state of the projectile and/or target nuclei. However, they don’t provide enough information regarding the reaction dynamics between the two interacting partners. Given the definition of the scattering process, wave functions that describe the this process before and after collision and being eigenfunctions of the same Hamiltonian (2.6) are needed. They are known as stationary states \( \psi_{k\ell j}^{(\pm)}(r) \) [90, 91], and can be expanded in partial waves as follows [92]
\[
\psi_{k\ell j}^{(\pm)}(r) = \frac{4\pi}{k} \sum_{\ell jm} \sum_{\nu sm} \langle \ell sv m_s | jm \rangle Y_{\ell}^{\nu s*}(\Omega_k) \langle \ell \nu sm | jm \rangle Y_{\ell}^{\nu s}(\Omega_r) X_{m_s}^{\nu s} \psi_{k\ell j}^{(\pm)}(r),
\] (2.29)

where the radial part \( \psi_{k\ell j}^{(\pm)}(r) \) is given by
\[
\psi_{k\ell j}^{(\pm)}(r) = \frac{1}{r} e^{\pm i(\hat{\sigma}_\ell + \delta_{lj})} \phi_{k\ell j}^j(r),
\] (2.30)

where \( \hat{\sigma}_\ell \) is the Coulomb phase shift. The next section we describe the three-body systems,
where the bound, continuum and stationary states as presented are useful in describing the projectile, and therefore are important in the construction of the three-body wave function.

2.2 Description of three-body systems

We consider, in general a projectile (p) formed by a core c, to which a structureless valence nucleon v is loosely bound (p = c + v), such that the interaction of the projectile with the target t is treated as a three-body problem (p + t → c + v + t), as shown in Fig. 2.1.

![Figure 2.1: Three-body coordinate system for the collision between a two-body projectile and the target.](image)

2.2.1 Three-body Schrödinger Equation

Before writing the three-body Schrödinger equation, we start by describing the relevant ingredients. The projectile internal coordinate \( r \) and the projectile-target relative coordinate \( R \) are expressed in terms of Jacobi coordinates (whose advantage, among others, is to remove the center-of-mass motion when one is interested in the individual motion of each particle) as

\[
\begin{align*}
\mathbf{r} &= \mathbf{r}_c - \mathbf{r}_v \\
\mathbf{R} &= \mathbf{r}_t - \frac{m_c \mathbf{r}_c + m_v \mathbf{r}_v}{m_p},
\end{align*}
\]

(2.31)
where $r_i$ is the internal coordinate of the particle $i$, $m_c$ and $m_v$ are the core nucleus and valence nucleon masses, respectively, and $m_p = m_c + m_v$ the projectile mass. The core-target and nucleon-target relative coordinates $R_{ct}$ and $R_{vt}$, are obtained from the coordinates $r$ and $R$ as follows

$$
R_{ct} = R - \frac{m_v}{m_p}r
$$

$$
R_{vt} = R + \frac{m_c}{m_p}r. \hspace{1cm} (2.32)
$$

Similarly, the projectile momentum ($p$) and projectile-target relative momentum ($P$) are defined in terms of the particle internal momenta according to

$$
p = \frac{m_v p_c - m_c p_v}{m_p}
$$

$$
P = \frac{m_t(p_c + p_v) - m_p p_t}{m_t + m_p}, \hspace{1cm} (2.33)
$$

where $m_t$ is the target mass. The three-body Hamiltonian is written as

$$
H = H_0 + T_R + U(r, R), \hspace{1cm} (2.34)
$$

where $H_0$ is the projectile Hamiltonian similar to equation (2.1), $T_R$ the kinetic energy term associated with the coordinate $R$, defined as

$$
T_R = \frac{\hbar^2}{2\mu_{pt}} \frac{\partial^2}{\partial R^2}, \hspace{1cm} (2.35)
$$

with $\mu_{pt}$ the projectile-target reduced mass, and $U(r, R)$ is the phenomenological projectile-target potential, which is a sum of the core-target and nucleon-target potentials,

$$
U(r, R) = U_{ct}(R_{ct}) + U_{vt}(R_{vt})
$$

$$
= U^N_{ct}(R_{ct}) + V^C_{ct}(R_{ct}) + U^N_{vt}(R_{vt}) + V^C_{vt}(R_{vt}), \hspace{1cm} (2.36)
$$

with $U^N_x(R_x)$ and $V^C_x(R_x)$ the short-range nuclear and long-range Coulomb potentials,
respectively.

We then write the three-body Schrödinger equation as

\[(H - E)\Psi_{JM}(r, R) = 0, \quad (2.37)\]

where \(E\) is the three-body center-of-mass energy, related to the projectile-target momentum as

\[E = \frac{P^2}{2\mu_{pt}} + \varepsilon_k = \frac{\hbar^2 K^2}{2\mu_{pt}} + \varepsilon_k, \quad (2.38)\]

with \(K\) the three-body wave number, \(\varepsilon_k\) the projectile excitation energy, and \(\Psi_{JM}(r, R)\) is the three-body wave function. The subscript \(J\) represents the total angular momentum and is the only constant of motion, in the sense that it commutes with the Hamiltonian \(H\). It is given by \(J = j + L\), where \(j\) is the projectile total angular momentum already defined in section 2.1 and \(L\) the orbital angular momentum of the projectile-target relative motion. The subscript \(M = m + \Lambda\) is the \(z\)-projection of \(J\), where \(m\) and \(\Lambda\) are the \(z\)-projections of \(j\) and \(L\), respectively.

The numerical solution of the three-body Schrödinger equation \((2.37)\), especially in the case of projectile breakup is a formidable task. Among other challenges, one mentions the fact that scattering states extend to infinity in the coordinate \(r\). Moreover, the different couplings between continuum states in addition to the couplings from and to the ground state, which may be strong particularly for loosely bound projectiles. These challenges can be circumvented by using the CDCC method, which allows the inclusion of all the physically relevant couplings, and is discussed in the next section.
2.3 Continuum discretized coupled channels (CDCC) method

More details of the formalism of this method can be found in [69, 70, 93, 98]. Here we only present the important steps. To render the three-body Schrödinger equation more tractable numerically, this method assumes an inert core, which means that the core internal structure is not taken into account. In [99], where the core excitations were considered, it was concluded that these excitations do not change dramatically the breakup cross sections. We will also look closely into this assumption in the result chapters. On the other hand, the methods excludes explicit target excitations other than those due to the projectile-target effective complex potentials.

The CDCC method finds its origin in the expansion of the three-body wave function on a basis formed by the bound and continuum wave functions of the projectile. The infinity of the continuum states requires that this basis to be truncated with the requirement that the expansion on the truncated basis retains the most important part of the wave function. Therefore, the truncation procedure, followed by the discretization technique reduce the continuum wave functions to discretized ones, which are square-integrable, thus leading to the convergence of the potential matrix elements.

2.3.1 Discretization of two-body continuum wave functions

Discretized wave functions are obtained using commonly two different techniques. The pseudostate [70, 90, 97] technique and the average [69, 93, 94, 96, 100] one. In Ref. [94, 100], it was shown that these two techniques amount to similar results. For both techniques, the angular momentum $\ell$ is truncated by $\ell_{\text{max}}$ and the relative momentum $k$, by $k_{\text{max}}$, based on the convergence requirements. For the pseudostate technique, the internal Hamiltonian $H_0$ of the projectile is diagonalized using some basic functions. For example in [101], the cubic splines were used, while in [91, 102], the authors used the Lagrange-Legendre mesh functions. For the average technique on the other hand, which is used in this thesis, the continuum wave functions of the projectile are sliced into
$N_b$ bins $[0, k_1], [k_1, k_2], [k_2, k_3], \ldots, [k_{i-1}, k_i], [k_{N_b-1}, k_{N_b}]$ of widths $\Delta k_i = k_i - k_{i-1}, (i = 1, 2, 3, \ldots, N_b)$, and averaged over the relative momentum to obtain

$$\varphi_\alpha(r) = \sqrt{\frac{2}{\pi W_\alpha}} \int_{k_{i-1}}^{k_i} g_\alpha(k) \phi_{k\ell}^j(r),$$

(2.39)

where the normalization coefficient $W_\alpha$ is given by

$$W_\alpha^2 = \int_{k_{i-1}}^{k_i} |g_\alpha(k)|^2 dk,$$

(2.40)

with $g_\alpha(k)$ some weight function, and $\phi_{k\ell}^j(r)$ are the continuum wave functions of the projectile, being radial parts of eigenstates of $H_0$, normalized according to equation (2.19).

The subscript $\alpha = (i, \ell, s, j)$, represents the relevant quantum numbers describing the state of the bin, where the projectile ground state corresponds to $i = 0$. The bin energies are given by [90]

$$\varepsilon_\alpha = \frac{\hbar^2}{2\mu cvW_\alpha^2} \int_{k_{i-1}}^{k_i} k^2 |g_\alpha(k)|^2 dk.$$  

(2.41)

The weight function $g_\alpha(k)$ depends on the state of the bin. For instance, it is common to use $g_\alpha(k) = 1$, for non-S-wave nonresonant bins, which corresponds to

$$W_i = \sqrt{k_i - k_{i-1}} = (\Delta k_i)^{1/2},$$

(2.42)

so that the bin energies are

$$\varepsilon_i = \frac{\hbar^2 \hat{k}_i^2}{2\mu cv}, \quad \hat{k}_i^2 = \frac{1}{3}(k_i^2 + k_{i-1}^2 + k_i k_{i-1}).$$

(2.43)

For S-wave bins, it is convenient to use $g_\alpha(k) = k$ as this stabilizes the extraction of the three-body transition amplitude [103], in which case the normalization coefficient
becomes

\[ W^2_\alpha = \int_{k_{i-1}}^{k_i} k^2 dk = \Delta k_i k_i^2, \]  

(2.44)

and the corresponding bin energies read

\[ \varepsilon_\alpha = \frac{\hbar^2}{2\mu_{cv}} \frac{1}{5W^2_\alpha} (k_i^5 - k_{i-1}^5). \]

(2.45)

For resonant bins, we follow [75, 90, 104, 105], and write

\[ g_\alpha(k) = \left| \frac{i\Gamma}{2\varepsilon_\alpha - \varepsilon_r + \frac{i}{2}\Gamma} \right|, \]

(2.46)

where \( \Gamma \) is the resonance width given by equation (2.27). The advantage of bin wave functions \( \varphi_\alpha(r) \) over the pure continuum wave functions \( \phi_{k\ell}(r) \) is that they are square-integrable, and hence a radial integral involving bin wave functions would converge better. On the other hand, the bin wave functions are normalized to

\[ \langle \varphi_\alpha(r)|\varphi_\alpha'(r) \rangle = 1 \quad \text{if} \quad \alpha = \alpha', \]

(2.47)

once a large maximum radius \( r_{\text{max}} \) is taken.

### 2.3.2 Coupled Equations

In the CDCC method, the three-body Schrödinger equation is approximated as

\[ (H - E)\Psi_{J\ell M}^{\text{CDCC}}(\mathbf{r}, \mathbf{R}) = 0, \]

(2.48)

where \( \Psi_{J\ell M}^{\text{CDCC}}(\mathbf{r}, \mathbf{R}) \) is the approximated three-body CDCC wave function. Having constructed the bin wave functions, the rotational invariance of the three-body Hamiltonian
allows the three-body CDCC wave function to be expanded following [51,102]

\[ \Psi_{JM}^{CDCC}(R, r) = \frac{1}{rR} \sum_{\alpha} \sum_{L} \chi_{\alpha L}^{IJ}(R) \mathcal{Y}_{\alpha L}^{IJ}(r, \Omega_R), \tag{2.49} \]

where the expansion coefficient \( \chi_{\alpha L}^{IJ}(R) \) is the radial part of the wave function, and the basis functions \( \mathcal{Y}_{\alpha L}^{IJ}(r, \Omega_R) \), result from the coupling of the spherical harmonics and the discretized wave function of the projectile, that is

\[ \mathcal{Y}_{\alpha L}^{IJ}(r, \Omega_R) = [\Phi_{\alpha}^{m}(r) \otimes Y_{L}^{\Lambda}(\Omega_R)]_{JM} \]
\[ = \sum_{m\Lambda} i^{L}(j m L | J M) \Phi_{\alpha}^{m}(r) Y_{L}^{\Lambda}(\Omega_R), \tag{2.50} \]

where \( Y_{L}^{\Lambda}(\Omega_R) \) is the spherical harmonics associated with the angular momentum \( L \), with \( \Omega_R \) the angular part of the vector \( R \). The discretized projectile wave functions \( \Phi_{\alpha}^{m}(r) \) are defined as

\[ \Phi_{\alpha}^{m}(r) = \varphi_{\alpha}(r)[Y_{L}^{\nu}(\Omega_r) \otimes X_{m s}^{\nu}]_{jm}, \tag{2.51} \]

where the bin wave functions \( \varphi_{\alpha}(r) \) are given by equation (2.39). Introducing expansion (2.49) into the Schrödinger equation (2.48), followed by a projection onto the basis functions (2.50), one obtains the following coupled equations for the coefficient \( \chi_{\alpha L}^{IJ}(R) \)

\[ \left[ -\frac{\hbar^2}{2\mu_{pt}} \left( \frac{d^2}{dR^2} - \frac{L(L+1)}{R^2} \right) + V_{\alpha\alpha}^{IJ}(R) + \varepsilon_{\alpha} - E \right] \chi_{\alpha L}^{IJ}(R) \]
\[ - \sum_{\alpha \neq \alpha'} i^{L-L'} V_{\alpha\alpha'}^{IJ}(R) \chi_{\alpha'}^{LJ}(R) = 0, \tag{2.52} \]

where \( V_{\alpha\alpha'}^{IJ}(R) \) is the potential matrix element, describing the coupling between different states of the projectile and has the following form

\[ V_{\alpha\alpha'}^{IJ}(R) = \langle \mathcal{Y}_{\alpha L}^{IJ}(r, \Omega_R) | U(r, R) | \mathcal{Y}_{\alpha'}^{IJ}(r, \Omega_R) \rangle, \tag{2.53} \]

with \( U(r, R) \) given by equation (2.36).
2.3.3 Expansion of the potential matrix elements

The potential matrix element (2.53) represents a 5-dimensional integral over \((\Omega_r, \Omega_R, r)\). It is therefore convenient to separate its angular part from the radial part for a proper integration. This is done by first expanding the potential \(U(r, R)\) into multipoles following

\[
U(r, R) = \sum_{\lambda=0}^{\infty} U_\lambda(r, R) P_\lambda(\cos \theta),
\]

where \(U_\lambda(r, R)\) are the potential multipoles, \(P_\lambda(\cos \theta)\) Legendre polynomials, with \(\theta\) the angle between the vectors \(\mathbf{R}\) and \(\mathbf{r}\), and \(\lambda\) the multipole order. The potential multipoles are numerically evaluated as

\[
U_\lambda(r, R) = \frac{2\lambda + 1}{2} \int_{-1}^{1} U(r, R) P_\lambda(z) dz,
\]

after a change of variables \(z = \cos \theta\). For example, the three first potential multipoles are given by

\[
U_0(r, R) = \frac{1}{2} \int_{-1}^{1} \left[ U_{ct}(R_{ct}) + U_{ct}(R_{ct}) \right] P_0(z) dz
\]

\[
U_1(r, R) = \frac{3}{2} \int_{-1}^{1} \left[ U_{ct}(R_{ct}) + U_{ct}(R_{ct}) \right] z dz
\]

\[
U_2(r, R) = \frac{5}{4} \int_{-1}^{1} \left[ U_{ct}(R_{ct}) + U_{ct}(R_{ct}) \right] (3z^2 - 1) dz,
\]

which are commonly called zero-order, first-order and second-order potential multipoles, with

\[
R_{ct} = \left[ R^2 + \left( \frac{m_e}{m_p} \right)^2 r^2 - \frac{m_e}{m_p} 2rRz \right]^{1/2}
\]

\[
R_{vt} = \left[ R^2 + \left( \frac{m_e}{m_p} \right)^2 r^2 + \frac{m_e}{m_p} 2rRz \right]^{1/2}.
\]

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In order to obtain the analytical expression of the potential matrix element, we first write the Legendre polynomials in term of the spherical harmonics as follows:

\[
P_\lambda(\cos \theta) = \frac{4\pi}{2\lambda + 1} \sum_{\mu=-\lambda}^{\lambda} Y_\mu^\lambda(\Omega_R) Y_{\mu^*}(\Omega_r)
\]

\[
= \frac{4\pi}{2\lambda + 1} Y_\lambda(\Omega_R) Y_\lambda(\Omega_r).
\] (2.58)

The substitution of equations (2.58) and (2.54) into (2.53), results in the following expression:

\[
V_{LJ_{\alpha \alpha'}} L' (R) = \sum_{\lambda=0}^{\lambda_{\text{max}}} \frac{4\pi}{2\lambda + 1} \langle Y_{LJ_{\alpha}}(r, \Omega_R) | Y_\lambda(\Omega_R) U_\lambda(r, R) | Y_{LJ_{\alpha'}}(r, \Omega_R) \rangle. \tag{2.59}
\]

The use of the Wigner-Eckart theorem \[86,87\], allows one to separate the angular part of the potential matrix elements from its radial part, to have

\[
V_{LJ_{\alpha \alpha'}} L' (R) = \sum_{\lambda=0}^{\lambda_{\text{max}}} \frac{4\pi}{2\lambda + 1} \langle L || Y_\lambda(\Omega_R) \rangle \langle L || L' \rangle \left\{ \begin{array}{ccc} J & L & j \\ \lambda & j' & L' \end{array} \right\} 
\]

\[
\times \langle \Phi^m_{\alpha}(r) || Y_\lambda(\Omega_r) V_\lambda(r, R) || \Phi^m_{\alpha'}(r) \rangle \tag{2.60}
\]

where \( \lambda \) is truncated by \( \lambda_{\text{max}} \), \( \hat{\lambda} = 2\lambda + 1 \), and

\[
\langle L || Y_\lambda(\Omega_R) || L' \rangle = (-1)^L \sqrt{\frac{\hat{\lambda} \hat{\lambda'}}{4\pi}} \begin{pmatrix} L & \lambda & L' \\ 0 & 0 & 0 \end{pmatrix}. \tag{2.61}
\]

Similarly we have that

\[
\langle \Phi^m_{\alpha}(r) || Y_\lambda(\Omega_r) V_\lambda(r, R) || \Phi^m_{\alpha'}(r) \rangle = (-1)^{\ell + s + j} \langle \ell || Y_\lambda(\Omega_r) || \ell' \rangle \langle \varphi_\alpha || U_\lambda(r, R) || \varphi_{\alpha'} \rangle \times \left\{ \begin{array}{ccc} s & \ell & j \\ \lambda & j' & \ell' \end{array} \right\}, \tag{2.62}
\]
where also
\[ \langle \ell | Y_\lambda(\Omega_r) | \ell' \rangle = (-1)^\ell \sqrt{\frac{\ell! \ell'!}{4\pi}} \left( \begin{array}{c} \ell & \lambda & \ell' \\ 0 & 0 & 0 \end{array} \right), \] (2.63)

and the radial part is given by
\[ \langle \varphi_\alpha | U_\lambda(r, R) | \varphi_{\alpha'} \rangle = \int_0^{r_{\text{max}}} \varphi^*_\alpha(r) U_\lambda(r, R) \varphi_{\alpha'}(r) dr. \] (2.64)

Since there is no further angular decomposition, this radial integral can only be calculated numerically. It contains the whole information regarding the different couplings among the states of the projectile, depending on the strength of the potential multipoles. As this integral appears, it represents the continuum-continuum couplings, which can be further separated into diagonal continuum-continuum couplings, if \( \alpha = \alpha' \), and off-diagonal continuum-continuum couplings, if \( \alpha \neq \alpha' \). Another type of couplings are the couplings to and from the bound state, obtained by replacing the bin wave function \( \varphi^*_\alpha(r) \) by the bound state wave function \( \phi_{n\ell_0}^j(r) \). The integral (2.64) is integrated over \( r \), such that the potential matrix element (2.60) depends only on the relative coordinate \( R \).

The potential matrix elements (2.60), contains coherent effects of both the nuclear and Coulomb interactions. However, when the projectile is far from the target \( (r \ll R) \), the nuclear interaction effects are negligible, such that
\[
U(r, R) \approx U_{\text{Coul}}(R),
\]
\[
U_{\text{Coul}}(R) = \frac{Z_t Z_e e^2}{R_{ct}} + \frac{Z_v Z_e e^2}{R_{vt}},
\] (2.65)

where \( Z_t, Z_c \) and \( Z_v \) are the target, core and valence nucleon charges, respectively. To obtain the potential matrix element involving pure Coulomb potentials, we start by the
following multipole expansion \[119\]

\[
U_{\text{Coul}}(\mathbf{R}) = Z_t e \sum_{\lambda=0}^{\infty} (-1)^\lambda Z_{\text{eff}} \frac{4\pi}{\lambda R^{\lambda+1}} Y_\lambda(\Omega_R).Y_\lambda(\Omega_r),
\]  

(2.66)

where the effective charge \(Z_{\text{eff}}\) is given by

\[
Z_{\text{eff}}^\lambda = \left(\frac{m_e}{m_p}\right)^\lambda Z_e + \left(\frac{m_e}{m_p}\right)^\lambda Z_c.
\]  

(2.67)

Following similar steps as before, we end up with expression of the potential matrix element involving pure Coulomb interactions, reading

\[
V_{\alpha\alpha'}^{\text{Coul}}(R) = Z_t e (-1)^{\ell + \lambda} \sum_{\lambda=0}^{\lambda_{\text{max}}} \sqrt{\ell' \ell L \ell' \lambda \lambda} \frac{(-1)^{\lambda + s + j + j' + L + J}}{R^{\lambda+1}} \frac{1}{4\pi}
\]

\[
\times \left( \begin{array}{ccc} \ell & \lambda & \ell' \\
0 & 0 & 0 \end{array} \right) \left( \begin{array}{ccc} L & \lambda & L' \\
0 & 0 & 0 \end{array} \right) \left\{ \begin{array}{ccc} s & \ell & j \\
\lambda & j' & \ell' \end{array} \right\} \left\{ \begin{array}{ccc} J & L & j \\
\lambda & j' & L' \end{array} \right\}
\]

\[
\times \langle \varphi_\alpha(r) | \mathcal{M}_\mu^{E\lambda}(\Omega_r) | \varphi_{\alpha'}(r) \rangle,
\]  

(2.68)

where the electric multipole operator \(\mathcal{M}_\mu^{E\lambda}(\Omega_r) = e Z_{\text{eff}}^\lambda Y_\lambda^\mu(\Omega_r)\), such that the radial Coulomb integral is equal to

\[
\langle \varphi_\alpha(r) | \mathcal{M}_\mu^{E\lambda}(\Omega_r) | \varphi_{\alpha'}(r) \rangle = e Z_{\text{eff}} \int_0^{r_{\text{max}}} \varphi_\alpha^*(r) r^\lambda \varphi_{\alpha'}(r) dr.
\]  

(2.69)

Once the potential matrix elements are constructed, whether for the pure Coulomb interactions or in the presence of both the Coulomb and nuclear interactions, the coupled equations (2.52) are solved numerically with the usual boundary conditions at \(R \to \infty\), which are

\[
\chi_\alpha^{LJ}(R) \to \frac{i}{2} [H_\alpha^-(K_\alpha R) \delta_{\alpha\alpha'} - H_\alpha^+(K_\alpha R) S_{\alpha\alpha'}(K_\alpha)],
\]  

(2.70)
where $H^\pm_L$ are Coulomb-Hankel functions \cite{89}, defined as

$$H^\pm_\alpha(K_\alpha R) = G_\alpha(K_\alpha R) \pm iF_\alpha(K_\alpha R), \quad (2.71)$$

and which, asymptotically behave as

$$H^\pm_\alpha(K_\alpha R) \to e^{\pm i[K_\alpha R - \eta \ln(2K_\alpha R) - \frac{L^2}{2} + \sigma_L]}, \quad (2.72)$$

with $G_L(x)$ and $F_L(x)$ being Coulomb functions. In equation \[2.70\], $S_{\alpha\alpha'}(K_\alpha)$ are the partial S-matrices for exciting the bin state $\alpha$ to $\alpha'$, with the three-body wave number $K_\alpha$ related to the bin energies through

$$K_\alpha = \sqrt{\frac{2\mu_{pt}(E + \epsilon_\alpha)}{\hbar^2}}. \quad (2.73)$$

Different methods and techniques employed for the numerical solution of the coupled equations are discussed in \cite{51, 106}.

### 2.3.4 Differential breakup cross sections

The partial S-matrices obtained from the numerical solution of the coupled equations, are used to derive the inelastic scattering amplitudes for populating each bin state ($\alpha'$) from the initial state ($\alpha$), given by \cite{51, 88}

$$F_{mm'}(\Omega) = \sqrt{\frac{K_{\alpha'}}{K_0}} \sum_L \sum_{L'} \sum_J \sqrt{2L + 1} \langle L0jm|JM \rangle \langle L'j'm'|JM \rangle \times e^{i(\sigma_L + \sigma_{L'})} S_{\alpha\alpha'}(K_{\alpha'}) Y_{L'}^{\Lambda'}(\Omega), \quad (2.74)$$

where $\Omega = (\theta, \phi)$, $\sigma_L$ and $\sigma_{L'}$ are the initial and final Coulomb phase shifts, and $K_0$ is the initial three-body wave number, related to the projectile bound state energy through

$$K_0 = \sqrt{\frac{2\mu_{pt}(E - \epsilon_0)}{\hbar^2}}, \quad (2.75)$$
The wave number $K_0$ also relates to $K_\alpha$ through the energy conservation law

$$\frac{\hbar^2 K_0^2}{2\mu_{pt}} - \varepsilon_0 = \frac{\hbar^2 K_\alpha^2}{2\mu_{pt}} + \varepsilon_\alpha.$$  \hfill (2.76)

To obtain the angular and energy distributions of fragments after breakup, one needs to calculate the breakup transition matrix elements, which are related to the two-body amplitudes (2.74). The complexity of the numerical evaluation of the transition matrix elements evolves around the fact that they involve the original continuum wave functions that generated the bins. In order to circumvent this complexity, a smoothing procedure has been proposed \[94,107,108\]. For final state momenta $k$ and $K$, the transition matrix elements are given by \[107,108\]

$$T_{m^m}(k, K) = \langle \psi^{-}_{km_{m}}(r)e^{iK R}|U(r, R)|\Psi_{JM}(r, R)\rangle,$$  \hfill (2.77)

where $\psi^{-}_{km_{m}}(r)$ is given by equation (2.29), $e^{iK R}$ the plane wave describing the final state of the projectile-target motion and $\Psi_{JM}(r, R)$ the original three-body wave function. Using the orthogonality of the bin wave functions (2.47), and replacing the three-body wave function by its CDCC approximation, together with the smoothing procedure, the CDCC T-matrix elements is given by \[93,103\]

$$T_{m^m}(k, K) = \sum_{\alpha} \sum_{m} \langle \psi^{-}_{km_{m}}(r)|Phi_{m}(r)\rangle T_{m^m}(K_{\alpha})$$  \hfill (2.78)

where the factor $T_{m^m}(K_{\alpha})$ identifies to

$$T_{m^m}(K_{\alpha}) = \langle \Phi_{m}^{m^m}(r)e^{iK R}|U(r, R)|\Psi_{JM}^{CDCC}(r, R)\rangle,$$  \hfill (2.79)

with $\Psi_{JM}^{CDCC}(r, R)$ the CDCC wave function whose partial waves expansion is given by equation (2.49), and $\Phi_{m}^{m^m}(r)$ are the discretized wave functions of the projectile, given by equation (2.51). Using the expansion (2.29), and the orthogonality of the Clebsh-Gordon
coefficients \[86\], the factor \( \langle \psi_{km_s}^{-} (r) | \Phi_{\alpha}^m (r) \rangle \) is given by the following expansion

\[
\langle \psi_{km_s}^{-} (r) | \Phi_{\alpha}^m (r) \rangle = \frac{4\pi}{k} \sum_{\ell jm} \langle \nu s m_s j m \rangle \langle \psi_{k\ell j}^{-} (r) | \varphi_{\alpha} (r) \rangle Y_{\ell}^\nu (\Omega_k),
\]  

(2.80)

where using equations (2.30) and (2.39), we obtain

\[
\langle \psi_{k\ell j}^{-} (r) | \varphi_{\alpha}^m (r) \rangle = e^{i(\sigma_\ell + \delta_{j})} \sqrt{\frac{2}{\pi W_\alpha}} \int_{0}^{k_i} dr \phi_{kt}^j (r) \int_{k_{i-1}}^{k_i} g_\alpha (k) \phi_{kt}^j (r) dk
\]

\[
= \begin{cases} 
  e^{i(\sigma_\ell + \delta_{j})} \sqrt{\frac{2}{\pi W_\alpha}} g_\alpha (k) & \text{if } k \in [k_{i-1}, k_i] \\
  0 & \text{otherwise}.
\end{cases}
\]

(2.81)

Then equation (2.80), reduces to

\[
\langle \psi_{km_s}^{-} (r) | \Phi_{\alpha}^m (r) \rangle = \frac{4\pi}{k} \sum_{\ell jm} \langle \nu s m_s j m \rangle s(k) Y_{\ell}^\nu (\Omega_k),
\]  

(2.82)

where the factor \( s(k) \) reads

\[
s(k) = e^{i(\sigma_\ell + \delta_{j})} \sqrt{\frac{2}{\pi W_\alpha}} g_\alpha (k), \quad k \in [k_{i-1}, k_i].
\]

(2.83)

The dependence of the breakup process on the inelastic excitation, allows us to relate the transition matrix elements to the inelastic scattering amplitudes \( \mathcal{F}_{mm'} (\Omega) \), through

\[
\mathcal{T}_{mm'} (K_\alpha) = -\frac{2\pi \hbar^2}{\mu pt \sqrt{K_0 K_\alpha}} \mathcal{F}_{mm'} (\Omega)
\]

\[
= -\frac{2\pi^{3/2} \hbar^2}{\mu pt \sqrt{K_0 K_\alpha}} \sum_{L} \sum_{L'} \sum_{j} \sqrt{2L + 1} \langle L0jm | JM \rangle \langle L'j'jm' | JM \rangle
\]

\[
\times e^{i(\sigma_L + \sigma_{L'})} S_{\alpha \alpha'} (K_\alpha) Y_{L'}^\Lambda (\Omega).
\]

(2.84)
substituting equation (2.8.4) into (2.7.8), the transition matrix element reads

\[
T_{m_s}(k, K) = -\frac{8\pi^{5/2}h^2}{\mu pt} \frac{1}{i k K_0} \sum_{i f j m} s(k) Y_{\ell}^{\nu}(\Omega_k) \langle \ell \nu s m_s | j m \rangle \sqrt{K_{\alpha}'} \frac{K_{\alpha}}{K_0} \times \sum_{L} \sum_{L'} \sum_{J} \sqrt{2L + 1} \langle L0jm|JM \rangle \langle L'J'jm'|JM \rangle e^{i(\sigma_L + \sigma_{L'})} S_{\alpha a'}(K_{\alpha}) Y_{L'}^{\nu'}(\Omega).
\]

(2.85)

Once the transition matrix elements are constructed, the differential breakup cross sections can be obtained following [51, 92], as

\[
\frac{d\sigma}{d\epsilon d\Omega_k} = \frac{k_{\mu c} \mu^2 pt K_{\alpha}}{(2\pi)^5 h^6 K_0} \frac{1}{2j + 1} \sum_{m_s} \left| T_{m_s}(k, K) \right|^2 \frac{K_{\alpha}}{K_0} \times \sum_{L} \sum_{L'} \sum_{J} \sqrt{2L + 1} \langle L0jm|JM \rangle \langle L'J'jm'|JM \rangle e^{i(\sigma_L + \sigma_{L'})} S_{\alpha a'}(K_{\alpha}) Y_{L'}^{\nu'}(\Omega) \]

(2.86)

where \(\Omega_k = (\theta_k, \vartheta_k)\). An integration over \(\Omega_k\) leaves a double-differential breakup cross section, which reads

\[
\frac{d\sigma}{d\epsilon d\Omega} = \int d\Omega_k \frac{d\sigma}{d\epsilon d\Omega_k} \frac{d\Omega_k}{d\epsilon d\Omega_k} \frac{d\Omega_k}{d\epsilon d\Omega_k} = \frac{k_{\mu c} \mu^2 pt K_{\alpha}}{h^2 k 4\pi^{5/2} K_0} \frac{1}{2j + 1} \sum_{m_s} \sum_{i f j m} \sum_{\ell j m'} \langle \ell \nu s m_s | j m \rangle \langle \ell' \nu' s m_s | j' m' \rangle \times \int d\Omega_k Y_{\ell}^{\nu*}(\Omega) Y_{\ell'}^{\nu'}(\Omega) \frac{1}{\mu} \sum_{\ell' j m'} s(k) \sqrt{\frac{K_{\ell'}}{K_{\ell}}} \sum_{L} \sum_{L'} \sum_{J} \sqrt{2L + 1} \langle L0jm|JM \rangle \langle L'J'jm'|JM \rangle e^{i(\sigma_L + \sigma_{L'})} S_{\ell \ell'}(K_{\ell}) Y_{L'}^{\nu'}(\Omega) \]

(2.87)

Using the orthogonality of the Clebsh-Gordon coefficients and spherical harmonics, for example [56]

\[
\sum_{m_s} \langle \ell \nu s m_s | j m \rangle \langle \ell' \nu' s m_s | j' m' \rangle = \delta_{jj'} \delta_{mm'},
\]

(2.88)
equation (2.87), reduces to

\[
\frac{d\sigma}{d\varepsilon d\Omega} = \frac{\mu_{cv}}{\hbar^2 k} \frac{K_\alpha}{4\pi^{5/2} K_0^3 2j + 1} \sum_{\ell j} \left| \sum_{s(k)} \sqrt{\frac{K_{ij}}{K_i}} \sum_L \sum_{L'} \sum_J \sqrt{2L + 1} \right|^2 \times \langle L0jm|JM \rangle \langle L'N'j'm'|JM \rangle e^{i(\sigma_L + \sigma_{L'})} S_{ii'}(K_i) Y_{L'}^{N'}(\Omega) \right|^2. \tag{2.89}
\]

A further use of the orthogonality of the Clebsh-Gordon coefficients, gives the energy distributions breakup cross sections after an integrating over \(\Omega\), which is

\[
\frac{d\sigma}{d\varepsilon} = \int d\Omega \frac{d\sigma}{d\varepsilon d\Omega} = \frac{\mu_{cv}}{\hbar^2 k} \frac{K_\alpha}{4\pi^{5/2} K_0^3 2j + 1} \sum_J (2J + 1) \sum_{\ell j} \left| \sum_{i} \sqrt{\frac{K_{ij}}{K_i}} \sum_s(k) S_{ii'} \right|^2. \tag{2.90}
\]

The angular distributions breakup cross sections can be obtained by numerically integrating the double differential breakup cross section

\[
\frac{d\sigma}{d\Omega} = \int_{0}^{\varepsilon_{max}} d\varepsilon \frac{d\sigma}{d\varepsilon d\Omega}, \tag{2.91}
\]

or one can use for instance, equation (8.2.31) of [51].

In the following chapters, this formalism is used to analyze the breakups of one neutron-halo and one proton-halo nuclei on heavy and light targets. Energy and angular distributions breakup cross sections are considered.
Chapter 3

The $^{11}\text{Be}+^{208}\text{Pb}$ breakup reaction

In this chapter, we study the dynamics of the $^{11}\text{Be}+^{208}\text{Pb}$ breakup reaction. The bound and continuum states as well as the electric response functions of the projectile are first described in order to understand some important properties of the projectile. A partial wave analysis of the energy distributions differential total breakup cross sections is performed in order to analyze the contribution of each partial wave included in the CDCC model space. The effects of the first- and higher-order interferences on the total, Coulomb and nuclear breakup cross sections, as well as on the Coulomb-nuclear interference are also investigated in more detail. The major numerical calculations are performed using the FRESCO codes [116], not only for this reaction but also for all the other reactions described in this thesis.

3.1 Description of $^{11}\text{Be}$ projectile

The $^{11}\text{Be}$ nucleus is well known as a one-neutron halo nucleus [5, 6, 50], being successfully described as a core ($c \equiv ^{10}\text{Be}$) to with a valence neutron ($n$) is loosely bound ($^{11}\text{Be} \to ^{10}\text{Be} + n$). From structural point of view, we adopt the $^{11}\text{Be} \to |^{10}\text{Be}(0^+) \otimes n(2s_{1/2})\rangle$ ground state configuration as in [5, 13, 109], with a binding energy of $0.504 \pm 0.006$ MeV. Its first excited state is a $1p_{3/2}^-$ state, with an excitation energy of $0.184 \pm 0.007$ MeV, and a narrow resonance of $1.274 \pm 0.018$ MeV in the $d_{5/2}^+$ state [109].

The analysis of this reaction assumes an inert core $^{10}\text{Be}$, supposing that it is well distant from the neutron, such that its internal structure does not dramatically affect the reaction process, in such a way that the only $^{11}\text{Be}$ excitations considered are those of the
Table 3.1: Depths, diffuseness and radius parameters of the $^{10}$Be+$n$ potential, taken from Ref. [111].

<table>
<thead>
<tr>
<th></th>
<th>$V_{\ell=0}$</th>
<th>$V_{\ell&gt;0}$</th>
<th>$R_0$</th>
<th>$a_0$</th>
<th>$V_{SO}$</th>
<th>$R_{SO}$</th>
<th>$a_{SO}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{10}$Be + n</td>
<td>59.5</td>
<td>40.5</td>
<td>2.699</td>
<td>0.6</td>
<td>32.8</td>
<td>2.99</td>
<td>0.6</td>
</tr>
</tbody>
</table>

neutron. More precisely, we assume that the core excitations do not affect substantially the $^{11}$Be bound wave functions, and consequently, given the peripherality of the breakup process [110], these excitations could not result in a substantial effect on the breakup cross sections. For a better understanding of these assumptions, one can first show that the bound state wave functions are much more important at distances well beyond the core radius, or that these wave functions are significant at distances greater than the size of the halo. This amounts to saying that these bound state wave functions exhibit longer tails, extending beyond the size of the halo, which means that there is a considerable probability of finding the neutron outside the size of the halo. Yet another way to verify that the core excitations could not result in a substantial effect on the breakup cross sections, is to analyze the radial behavior of the electric response functions. This is due to the fact that the electric response function is directly related to the energy distributions breakup cross sections (this is much clear in the Coulomb dissociation method). Another advantage of such study is that it leads to the investigation the peripherality of the radioactive capture process, which is an important process from astrophysical point of view.

To obtain the bound and continuum wave functions useful in obtaining the ground and continuum state properties as well as the electric response functions, one solves numerically the radial Schrödinger equation (2.6), subject to the boundary conditions (2.14) and (2.19). The $^{10}$Be + $n$ interacting potential, $V_{ee}(r)$, given by equation (2.7), is a Woods–Saxon potential with a central plus a spin-orbit coupling components. The corresponding parameters, which are the depths of the central ($V_c$) and spin-orbit ($V_{SO}$) terms, and the corresponding radii and diffusenesses are listed in Table 3.1.
3.1.1 Bound state wave functions

The calculated bound and excited state wave functions are plotted in Fig. 3.1. Considering the $^{10}\text{Be}$ radius ($R_{\text{core}} = 2.36$ fm [5]), it is seen that these wave functions are much extended beyond the core radius, showing that small portions of these wave functions are located at $r \leq R_{\text{core}}$. For further insight into the extension of these wave functions beyond the core radius, we calculate the probabilities of finding the neutron outside $^{10}\text{Be}$, using the following equation

$$P = \int_{R_{\text{core}}}^{\infty} |\phi_{n\ell_0}^{j_0}(r)|^2 dr.$$  (3.1)

The obtained probabilities are 90.83% for the ground state and 81.29% for the excited state, which compare fairly well with the 91% and 87% of Ref. [5], showing that the neutron is well peripheral to $^{10}\text{Be}$. Let us now calculate the probabilities of finding the neutron inside and outside the size of the halo, for a better analysis of the extension behavior of these wave functions. We first calculate the size of the halo (which we define as the root-mean-square radius) given by

$$\sqrt{\langle r^2 \rangle} = \left[ \int_{0}^{\infty} |\phi_{n\ell_0}^{j_0}(r)|^2 r^2 dr \right]^{1/2},$$  (3.2)

where $\langle r^2 \rangle$ is the square of the $^{10}\text{Be} + n$ mean distance. The probabilities of finding the neutron inside and outside the halo size are then calculated using the following equations

$$P_{\text{in}} = \int_{0}^{\sqrt{\langle r^2 \rangle}} |\phi_{n\ell_0}^{j_0}(r)|^2 dr,$$

$$P_{\text{out}} = \int_{\sqrt{\langle r^2 \rangle}}^{\infty} |\phi_{n\ell_0}^{j_0}(r)|^2 dr.$$  (3.3)

The results are shown in Table 3.2 together with ground and excited state binding energies and the root-mean-square radii. The results show considerable probabilities of finding the neutron beyond the size of the halo, and thus reflecting the extension of the wave functions. On the light of these results, one can then argue that the core excitations are unlikely to
Table 3.2: Ground and first excited state separation energies ($S_n$), root-mean-square radii and the inside and outside probabilities for the $^{10}\text{Be} + n$ system.

<table>
<thead>
<tr>
<th>$n\ell j^\pi$</th>
<th>$S_n$(MeV)</th>
<th>$\sqrt{\langle r^2 \rangle}$(fm)</th>
<th>$P_{\text{in}}$(%)</th>
<th>$P_{\text{out}}$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2s_{1/2}^+$</td>
<td>0.504</td>
<td>7.036</td>
<td>70.67</td>
<td>29.47</td>
</tr>
<tr>
<td>$1p_{1/2}^-$</td>
<td>0.184</td>
<td>6.004</td>
<td>77.17</td>
<td>23.060</td>
</tr>
</tbody>
</table>

substantially affect the $^{11}\text{Be} + ^{208}\text{Pb}$ breakup cross sections.

![Figure 3.1: Bound and excited wave functions of the $^{10}\text{Be} + n$ system.](image)

3.1.2 Continuum states

Continuum states play an important role in the breakup process in the sense that for a loosely bound projectile, these states couple strongly with bound states. With this in mind, and given the fact that the electric response function requires the continuum wave functions, we briefly analyze these waves functions and the structure of the projectile in some partial waves. The calculated continuum wave functions in the $\frac{1}{2}^+, \frac{3}{2}^+$ and $\frac{5}{2}^+$ partial waves at incident the energies of 1.08 MeV, 6.52 MeV and 1.28 MeV are displayed in Fig.3.2. It is seen that these continuum states are resonant states, with a more pronounced resonance in the $\frac{5}{2}^+$ partial wave, as already indicated. These resonances are clearly observed in Fig.3.3 where the phase shifts are plotted as functions of the energy.
3.1.3 Electric response function

We analyze the radial behavior of the $E1$ electric response functions for the transitions from the ground state to the $p$ states, which are regarded as the most important transitions for the reaction under study. Following [112], one obtains the analytical expression of the electric response function, which reads

$$\frac{B(E\lambda)}{d\varepsilon} = \frac{\mu_{ee}}{\hbar^2 k} \sum_j (2j + 1)|\mathcal{H}_{\gamma\gamma}(k)|^2,$$

(3.4)
where

\[ H_{\gamma_0\gamma}(k) = eZ_{\text{eff}}^{\lambda}(-1)^{\ell_0+\ell+j+s+\lambda}\sqrt{\frac{\ell_0\ell}{\lambda}} \left( \begin{array}{c} \ell_0 & \lambda & \ell \\ 0 & 0 & 0 \end{array} \right) \left\{ \begin{array}{c} s & \ell_0 & j_0 \\ \lambda & j & \ell \end{array} \right\} \]

\times \int_0^\infty dr \phi_{n\ell_0}^j(r) r^\lambda \phi_{k\ell}^j(r), \tag{3.5}

with $Z_{\text{eff}}$ given by equation (2.67), $\gamma_0 = (\ell_0, s, j_0)$, $\gamma = (\ell, s, j)$, and the last factor is radial integral. Since the electric response function is generally expressed as a function of the energy, we analyze its radial behavior by considering the integrand of its radial integral. We then calculate this integrand for the transition $(0, \frac{1}{2}^+) \rightarrow (1, \frac{3}{2}^-)$, at three different arbitrary energies and present results in Fig. 3.4. One notices that for the three energies, the integrand builds its maximum around $r = 15 \text{ fm}$, and is negligible at $r \leq \sqrt{\langle r^2 \rangle}$, making it even more negligible in the range of the core radius. It follows that the neutron is captured well outside the core $^{10}\text{Be}$ to form the $^{11}\text{Be}$ nucleus, resulting in a peripheral neutron capture process. This is again a good indication of the fact that the core excitations could not result in a dramatic effect on the breakup cross sections.

Finally, we calculate the $E1$ electric response functions for the transitions from the ground state to the $p_{\frac{1}{2}^-}$, $p_{\frac{3}{2}^-}$ and $p$ (coherent sum of the transitions to $p_{\frac{1}{2}^-}$ and $p_{\frac{3}{2}^-}$) continuum states and display the results in Fig. 3.5. It is first shown that the transition to $p_{\frac{3}{2}^-}$ state is more important than the transition to $p_{\frac{1}{2}^-}$ state. Moreover, for each transition, the

Figure 3.4: Integrand of the radial integral (3.5), for the transition $(0, \frac{1}{2}^+) \rightarrow (1, \frac{3}{2}^-)$.
Figure 3.5: Electric response functions for the transitions from the ground state to the $p_{\frac{1}{2}^-}$ and $p_{\frac{3}{2}^-}$ and $p$ continuum states. The experimental data are from [118].

corresponding response function peaks in the vicinity of the ground state binding energy, which highlights the crucial role of the ground state binding energy in the breakup process.

After this brief description of the projectile, we now focus on the analysis of the dynamics of the $^{11}$Be+$^{208}$Pb breakup reaction. We first start by describing the relevant inputs.

### 3.2 The $^{10}$Be + $^{208}$Pb and $n + ^{208}$Pb potentials

To obtain the reaction breakup observables, one has to solve numerically the coupled equations (2.52), subject to the boundary conditions (2.70). This requires first of all the evaluation of the potential matrix element (2.53), which in turn requires the $^{10}$Be + $^{208}$Pb and $n + ^{208}$Pb potentials, which are discussed here. These are phenomenological complex potentials, taking care of the only target explicit excitations considered in the CDCC method as mentioned already. The $^{10}$Be + $^{208}$Pb and $n + ^{208}$Pb nuclear potentials are Woods–Saxon potentials, with a real and an imaginary terms, given by the following expression

$$U_{xt}(R_{xt}) = V_{xt}f^R_{xt}(R_{xt}, R^R_{xt}, a_{R}^{xt}) + iW_{xt}f^W_{xt}(R_{xt}, R^W_{xt}, a_{W}^{xt}),$$  \hspace{1cm} (3.6)$$

where $V_{xt}$ and $W_{xt}$ are the depths of the real and imaginary components, respectively,
\( R_{xt}^R, R_{xt}^W \) and \( a_{R}^{xt}, a_{W}^{xt} \) the corresponding radii and diffusenesses, and

\[
f_{xt}^R(R_{xt}, R_{xt}^R, a_{R}^{xt}) = \left[ 1 + \exp \left( \frac{R_{xt} - R_{xt}^R}{a_{R}^{xt}} \right) \right]^{-1},
\]

\[
f_{xt}^W(R_{xt}, R_{xt}^W, a_{W}^{xt}) = \left[ 1 + \exp \left( \frac{R_{xt} - R_{xt}^W}{a_{W}^{xt}} \right) \right]^{-1}, \quad (3.7)
\]

\( x = ^{10}\text{Be}, ^{1}\text{n} \) and \( R_{xt} \) is the core-target or neutron-target relative distance, given by equation (2.32). The different parameters of the nuclear potentials used in this thesis, taken from [111], are listed in Table 3.3.

Table 3.3: Parameters of the real and imaginary depths and the corresponding nuclear radii and diffusenesses for the \(^{10}\text{Be}+^{208}\text{Pb}\) and \(^{1}\text{n}+^{208}\text{Pb}\) nuclear potentials. These parameters are taken from Ref. [111].

<table>
<thead>
<tr>
<th>(^{10}\text{Be}/^{1}\text{n}+t)</th>
<th>( V )</th>
<th>( W )</th>
<th>( R_{R} )</th>
<th>( R_{I} )</th>
<th>( a_{R} )</th>
<th>( a_{I} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{10}\text{Be}+^{208}\text{Pb})</td>
<td>70.00</td>
<td>58.90</td>
<td>7.43</td>
<td>7.19</td>
<td>1.04</td>
<td>1.00</td>
</tr>
<tr>
<td>(^{1}\text{n}+^{208}\text{Pb})</td>
<td>29.46</td>
<td>13.40</td>
<td>6.93</td>
<td>7.47</td>
<td>0.75</td>
<td>0.58</td>
</tr>
</tbody>
</table>

For the \(^{10}\text{Be}+^{208}\text{Pb}\) Coulomb potential, we consider a point–sphere Coulomb potential, given by

\[
V_{ct}^{C}(R_{ct}) = \begin{cases}
\frac{Z_c Z_t e^2}{R_C} \left( \frac{3}{2} - \frac{R_{ct}^2}{2 R_C^2} \right) & R_{ct} < R_C \\
\frac{Z_c Z_t e^2}{R_{ct}} & R_{ct} \geq R_C,
\end{cases}
\]

(3.8)

where \( Z_c = 4, Z_t = 82 \) are the core, target charges, respectively, and \( R_C = 1.25(10^{1/3} + 208^{1/3}) \approx 10.1 \) fm is the Coulomb radius.

The \(^{10}\text{Be}+^{208}\text{Pb}\) and \(^{1}\text{n}+^{208}\text{Pb}\) potentials are not the only inputs required in the evaluation of the potential matrix element. The other crucial inputs are the parameters used to discretize the \(^{11}\text{Be}\) continuum, the bin integration parameter, as well as the multipoles into which the potentials are expanded. In addition to these, the solution of the coupled
equations requires the truncation of the angular momentum of the $^{11}\text{Be} + ^{208}\text{Pb}$ relative motion and the radial integration parameters. All these different parameters represent what is commonly known as the CDCC model space, which is described in the next section.

### 3.3 CDCC model space

The CDCC model space employed for this reaction includes, the $^{11}\text{Be}$ partial waves up to $\ell_{\text{max}} = 4$, and a maximum excitation energy $\varepsilon_{\text{max}} = 10$ MeV. The interval $[0, \varepsilon_{\text{max}}]$ is discretized into bins of widths $\Delta \varepsilon_i = 0.5$ MeV, for the $s$ and $p$ waves, $\Delta \varepsilon_i = 1$ MeV, for the $d$ and $f$ waves, and $\Delta \varepsilon_i = 2$ MeV, for the $g$ waves. We verified that the convergence of the results does not depend on a special discretization of the resonant states. The $^{10}\text{Be} + ^{208}\text{Pb}$ and $n + ^{208}\text{Pb}$ potentials are expanded into multipoles up to $\lambda_{\text{max}} = 4$, and the angular momentum of the projectile-target relative motion is truncated by $L_{\text{max}} = 9000\hbar$. Concerning the radial cutoffs, the energy bins are integrated out to $r_{\text{max}} = 60$ fm, where $\Delta_r = 0.1$ fm, while the coupled equations are solved up to $R_{\text{max}} = 1000$ fm, with $\Delta_R = 0.005$ fm. These parameters were selected based on the convergence requirements. The rest of the chapter is devoted to the discussion of the obtained results, starting with the energy distributions.

### 3.4 Energy distributions differential breakup cross sections

The energy distributions differential Coulomb, nuclear and total (coherent sum of the Coulomb and nuclear) breakup cross sections are investigated in this section. Different multipole transitions as well as the the effects of the first-and higher-order interferences on these breakup cross sections are also considered. However, we start with a brief discussion of the convergence of the results.
3.4.1 Convergence of the differential breakup cross sections and Partial waves analysis

Obtaining the convergence of the CDCC calculations is a tricky task. Therefore, in this subsection, before diving into the details of the reaction, we first address the convergence of the energy distributions breakup cross sections as functions of $\ell_{\text{max}}$, for example in order to ensure the sufficiency of the partial waves included in the CDCC model space. We keep all the other parameters fixed and increment $\ell_{\text{max}}$ from 0 to 4. The results are presented in Fig. 3.6, where it is noticed that partial waves up to f-waves are enough to ensure a good convergence of the energy distributions breakup cross sections, although the contribution of the g-waves is not negligible as we will see later, for the energy integrated breakup cross sections. On the other hand, one notices that higher partial waves ($\ell_{\text{max}} \geq 2$) result in a reduction of the breakup cross section. A further look at this figure reveals that the differential breakup cross sections are much concentrated at low excitation energies, with peaks around the ground state binding energy qualitatively similar to the results in Fig. 3.5. This further demonstrates the dependence of the breakup cross sections on the electric response function.

The results presented in Fig. 3.6 do not tell much about the contribution of each single partial wave. Therefore, one needs to plot separate differential breakup cross sections corresponding to each partial wave, for a better understanding of their importance. To this end, we present in Fig. 3.7 the differential breakup cross sections corresponding to each partial wave, together with their coherent sum (tot). One observes that the p-wave breakup cross section is much dominant (as already seen in Fig. 3.6), and dictates the shape of the total breakup cross section. However, it is seen that the contributions of the other partial waves are of great importance, especially at low excitation energies ($\varepsilon \leq 2$ MeV).
Figure 3.6: Convergence of the energy distributions differential breakup cross section as function of the partial waves.

Figure 3.7: Different partial waves differential breakup cross sections.

3.4.2 Total, Coulomb and nuclear breakup cross sections

As indicated elsewhere in this thesis, when a projectile breaks up as a result of its collision with a target, it is not straightforward known what interaction is responsible for this breakup, the Coulomb or the nuclear interactions, making the importance of the Coulomb and nuclear interactions together with the importance of their interference an open question in the analysis of the breakup process. Here, we briefly analyze the differential Coulomb and nuclear breakup cross sections as well their interference and compare the results with the experimental data, leaving a more detailed analysis to section 3.5.
Presented in Fig. 3.8 are the differential total, Coulomb and nuclear breakup cross sections. It is observed that the differential Coulomb breakup cross section dominates over both the differential total and nuclear breakup cross sections, and fairly fits the data at high excitation energies. The results also show that although the nuclear breakup is quite small compared to the Coulomb breakup cross section (and becomes even negligible at $\varepsilon \geq 1.5$ MeV), the Coulomb-nuclear interference is more significant and the total breakup cross section rather fits well the data at low excitation energies ($\varepsilon \leq 1$ MeV). A closer look at this figure reveals that, $\sigma_N < |\sigma_T - \sigma_C|$ (where $\sigma_T$, $\sigma_C$ and $\sigma_N$ are the total, Coulomb and nuclear breakup cross sections, respectively). This inequality indicates that a small nuclear breakup contribution does not automatically imply small Coulomb-nuclear interference, which is seen to be strongly destructive in this case. Our results disagree with the prediction of Ref. [13], where it was pointed out that the disagreement between the data and the Coulomb dissociation method (mostly at high excitation energies) could be due to the nuclear breakup and/or higher-order effects. It could be rather due to the description of the reaction. Later in this chapter, we will seek to understand the dominance of the Coulomb breakup cross section over the total breakup cross section and the smallness of the nuclear breakup cross section, observed in Fig. 3.8.
3.4.3 First-and higher-order interferences

Pure Coulomb breakup cross sections are commonly obtained after a restriction to the first-order multipole transition for a neutron-halo projectile, owing to the insignificant contributions of higher-order multipole transitions, and by assuming that the nuclear breakup cross sections are negligible to some extend. However, the exclusion of the nuclear breakup cross section for obtaining a pure Coulomb breakup cross section remains a subject of great debate, and the accuracy of the procedure involved is not yet fully established. Indeed, it is not verified whether the nuclear breakup cross section is as small as in Fig. 3.8, at the first-order multipole transition in order to be excluded without losing the accuracy. We address this question in more details by investigating the effects of the first-and higher-order interferences on the total, Coulomb and nuclear breakup cross sections as well as on the Coulomb-nuclear interference. Also we want to investigate whether the first-and higher-order interferences have anything to do with the importance of the Coulomb breakup cross section over the total breakup cross section and on the smallness of the nuclear breakup cross section, and the role of these interferences on the nature and magnitude of the Coulomb-nuclear interference.

The methodology adopted here is to first analyze the importance of each single multipole transition on the differential total, Coulomb and nuclear breakup cross sections. In what follows, \( \lambda = a \) denotes a single multipole transition of value \( a \), and \( \lambda_{\text{max}} = a \), stands for a coherent sum of \( \lambda = 0, \ldots, a \) and accounts for the interferences. The differential breakup cross sections are presented in Fig. 3.9. The total breakup cross sections presented in Fig. 3.9(a), show that the first-order (\( \lambda = 1 \)) breakup cross section is much dominant, followed by the second-order (\( \lambda = 2 \), which is negligible for \( \varepsilon \geq 1 \text{ MeV} \)), whereas the zero-order (\( \lambda = 0 \)) and the third-order (\( \lambda = 3 \)) cross sections appear to be negligible. Surprisingly, the \( \lambda_{\text{max}} = 1 \) breakup cross section is much lesser than the first-order breakup cross section, owing to the first-order (\( \lambda = 0, 1 \)) interference, which is seen to be strongly destructive. Including all the multipoles coherently, one finds that the all-order (\( \lambda_{\text{max}} = 4 \)) breakup cross section slightly differs from the \( \lambda_{\text{max}} = 1 \) breakup cross section, indicating a weak higher-order interference. We then conclude that the total breakup cross section is substantially reduced by the first-order interference.
Figure 3.9: Different multipole contributions on the differential Coulomb, nuclear and total breakup cross sections.

The Coulomb breakup cross sections presented in Fig. 3.9(b), indicate also a negligible
zero-order breakup cross section (it is multiplied by 20 for convenience). Moreover, it can
be seen that the first-order, the \( \lambda_{\text{max}} = 1 \), and the all-order breakup cross section curves
are hardly distinguishable, which shows more negligible first-and higher-order interfer-
ences. Therefore, the coherent and incoherent sums of the first-and all-order Coulomb
breakup cross sections are not expected to be that different. The higher-order breakup
cross sections were found to be insignificant and therefore not shown. Lastly, the nu-
clear breakup cross sections are presented in Fig. 3.9(c). We also observe as for the total
breakup cross sections, a much dominant first-order breakup cross section, peaking at
much lower energies and more extended to higher excitation energies. It is followed by
the second-order breakup cross section, which peaks around 0.5 MeV before dropping
systematically to become negligible for \( \varepsilon \geq 1 \) MeV. Unlike the total breakup, a non-
negligible zero-order breakup cross section is noticed, building its significant contribution
also around 0.5 MeV, while the third-order breakup cross section is seen to be negligible
for \( \varepsilon \leq 0.5 \) MeV. Looking carefully at this figure, one sees that \( \lambda_{\text{max}} = 1 \) and the all-order
breakup cross sections are hardly distinguishable, and are more less than the zero-order
breakup cross section, showing that the nuclear breakup cross section is substantially
reduced by these interferences much more than its total breakup counterpart. A simple
comparison between Figs. 3.9 (b) and 3.9 (c), shows that at the first-order transition, the
nuclear breakup cross section is more important than Coulomb breakup cross section. It
means that the nuclear breakup cross section cannot be simply disregarded to obtain a
pure Coulomb breakup cross section at the first-order multipole transition.

3.5 Energy integrated breakup cross sections

The results discussed in the previous section do not indicate clearly the effects of the
different multipole transitions on the Coulomb-nuclear interference. In this section, we
investigate the quantitative effects of the interferences on the total, Coulomb and nuclear
breakup cross sections and on the Coulomb-nuclear interference. A partial waves analysis
is also performed to assess how each multipole transition populates the different partial
waves. To this end, we first integrate the differential breakup cross sections, using the
following equation

\[
\sigma^\ell_x = \int_{\epsilon^\text{max}}^{\epsilon} \frac{d\sigma^\ell_x}{d\epsilon},
\]

(3.9)

where \( x \equiv T, C, N \). This integral is evaluated numerically and the obtained integrated total, Coulomb and nuclear breakup cross sections, which summarized in Table 3.5. The third, fourth and fifth columns contain the integrated total, Coulomb and nuclear breakup cross sections, respectively. Also in this table, \( \sigma^\ell_C + \sigma^\ell_N \) represents the incoherent sum of the integrated Coulomb and nuclear partial breakup cross sections (sixth column). The different Coulomb (\( \Delta^\ell_C \)) and nuclear (\( \Delta^\ell_N \)) contributions (in \%) to their incoherent sum are shown in the eighth and ninth column, respectively. The Coulomb-nuclear interference in each partial wave for the different multipole transitions is defined as

\[
\sigma^\ell_I = \sigma^\ell_T - (\sigma^\ell_C + \sigma^\ell_N).
\]

(3.10)

If \( \sigma^\ell_I < 0 \), then the Coulomb-nuclear interference is destructive or strongly destructive if \( \sigma^\ell_I \ll 0 \). This interference is said to be constructive if \( \sigma^\ell_I \geq 0 \). We define the incoherent sum of the partial breakup cross sections (\( S^\lambda_i, \lambda = i \)) for each single multipole transition as

\[
S^\lambda_i = \sum_{\ell=0}^{\ell_{\text{max}}} \sigma^\ell_x,
\]

(3.11)

and \( S^\lambda_{i_{\text{max}}}, \lambda_{\text{max}} = i \) stands for the incoherent sum of partial waves breakup cross sections, when the different multipoles are included coherently.

Starting with the different multipole transitions for the integrated total, Coulomb and nuclear breakups, the table shows that at zero-order, only transitions from the ground state to the s continuum states are accounted for. At first-order, transitions to all s, p, d, f and g continuum states are observed, where as already seen, the p-waves breakup cross sections are largely dominant. At second-order on the other hand, only transitions to s, d and g continuum states are noticed, where the d-waves breakup cross sections are more important. Finally at third-order, again transitions to all s, p, d, f and g continuum
states are possible, where f-waves breakup cross sections are more significant. It is known by exclusion laws that in the case where only couplings to and from the ground state are considered, at first-order for example, transitions to d-and g-waves are not allowed. However, due to continuum-continuum couplings among the bins, there are possibilities where \( \lambda + \ell + \ell' = \text{even}, (\lambda = 1) \), and therefore allowing the different transitions observed.

### 3.5.1 Quantitative effects of the first-and higher-order interferences on the integrated breakup cross sections

Let us now compare the integrated Coulomb and nuclear breakup cross sections for each partial wave and multipole transition. Considering first the zero-order transition, we notice that the nuclear breakup cross section is much larger than the integrated Coulomb breakup cross section, and contributes up to 92.14\% to the Coulomb+nuclear incoherent sum. Looking at the first-order transition, it is seen that the integrated nuclear breakup cross section is still dominant in all partial waves other than the p-waves, where the integrated Coulomb breakup cross section contributes up to 55.34\% to the incoherent sum. However, if we consider the sum \( S_1^\lambda \), one sees that the nuclear breakup cross section remains slightly dominant with a contribution up to 52.63\% to the incoherent sum. This analysis shows clearly that in any case, the nuclear breakup cross section cannot be excluded at this stage to obtain a pure Coulomb breakup cross section, even when only transitions to p-waves are considered. At second- and third-order transitions, we notice that the Coulomb breakup contributes only up to 3.59\% and 1.83\% to the incoherent sum, small contributions which were already seen for the differential breakup cross sections.

If one compares also the Coulomb breakup cross sections with the total breakup cross sections, it is observed that at zero-order, the total breakup cross section is largely more important than the Coulomb breakup cross section, and from the first-order to the third-order, the total breakup cross section is still much dominant. This indicates that single multipole transitions do not explain the importance of the Coulomb breakup cross section over the total breakup cross section observed in Fig. 3.8. We would like to emphasis that at the first-order for the Coulomb breakup cross section, although the p-waves breakup
cross section represent the more important contribution, but the contributions of the other partial waves represent 22.45% of $S_1^1$, which is not negligible. Therefore, for the pure Coulomb breakup, considering only the couplings to the p-waves, leaves out an important contribution from the other partial waves.

We then consider the effects of the first- and all-order interferences on the integrated total, Coulomb and nuclear breakup cross sections. The results indicate that the integrated total breakup cross sections are substantially reduced in each partial wave, resulting in a more lesser $S_\lambda^{\lambda_{\text{max}}}$ compared to $S_1^1$. Compared to the integrated Coulomb breakup cross sections, we see that in each partial wave, the Coulomb breakup cross section becomes more dominant (except in the s-waves), owing to the first-order interference. We can conclude in this case that the importance of the Coulomb breakup cross section over the total breakup cross section observed in Fig. 3.8 is mainly due to the first-order interference. Considering the integrated nuclear breakup cross sections on the other hand, we observe that these cross sections are dramatically reduced, which become much lesser than the Coulomb breakup cross sections in all the partial waves, and its overall contribution on the incoherent sum represents only 14.04%. Similarly it can be concluded that the much smaller nuclear contribution observed in Fig. 3.8 is mainly an effect of the first-order interference. Looking at the all-order transition, one observes that the integrated total and nuclear breakup cross sections are further significantly reduced, and the contribution of the nuclear breakup cross section to the incoherent sum drops to 09.15%, while the Coulomb breakup cross section is insignificantly affected in all the partial waves.

We compare our results with the experimental data of Ref. [13], which were obtained at an incident energy of 770 MeV. Although the incident energy considered in this thesis and in that reference differ by 22 MeV, the results are not expected to differ dramatically on the basis of this difference in incident energies. In that reference, a first-order pure Coulomb breakup cross section of $1510 \pm 92$ mb was obtained. Our results show a p-waves Coulomb breakup cross section of $1560.40$ mb. However, considering all the partial waves together, an overall sum of $2012.20$ mb is obtained. It is clear that if we were to use the same incident energy, our results would still disagree with the results from the reference,
owing mostly to the contributions of the partial waves, other than the p-waves.

We further look at the effects of the first-and higher-order interferences on the integrated total, Coulomb and nuclear breakup cross sections, by estimating the amounts reduced from these breakup cross sections, owing to these interference. To this end, we use the following equations

\[ \sigma_{I}^{\text{FO}}(\%) = 1 - \frac{S_1^{\lambda_{\max}}}{\hat{S}}, \]

\[ \sigma_{I}^{\text{AO}}(\%) = 1 - \frac{S_4^{\lambda_{\max}}}{\hat{S}}, \]  

(3.12)

where \( \sigma_{I}^{\text{FO}} \) and \( \sigma_{I}^{\text{AO}} \) are the first-and all-order interferences, respectively and

\[ \hat{S} = S_0^\lambda + S_1^\lambda + S_2^\lambda + S_3^\lambda. \]  

(3.13)

Therefore, the higher-order interference (\( \sigma_{I}^{\text{HO}} \)) is then estimated by

\[ \sigma_{I}^{\text{HO}}(\%) = \sigma_{I}^{\text{FO}} - \sigma_{I}^{\text{AO}}. \]  

(3.14)

The results obtained are presented in Table 3.4. It can be seen from this table that the all-order interference reduces the total breakup cross section by 78.50%, distributed as follows: 71.91% due to the first-order interference and 6.59% to the higher-order interference. For the Coulomb breakup cross section, a slight reduction of 3.07%, where 2.73% is due to the first-order interference and 0.34% to the higher-order interference is noticed. As for the nuclear breakup cross section, one sees that the all-order interference reduces 94.76%, where 91.47% is due to the first-order interference and 3.29% to the higher-order interference.
Table 3.4: Reduced amounts of the total, Coulomb and nuclear breakup cross sections due to the first-and higher-order interference.

<table>
<thead>
<tr>
<th></th>
<th>FO</th>
<th>AO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\sigma}$</td>
<td>$\sigma_{\lambda_{\text{max}}}^{\text{FO}}$</td>
</tr>
<tr>
<td><strong>Tot</strong></td>
<td>6490.95</td>
<td>1823.39</td>
</tr>
<tr>
<td><strong>Coul</strong></td>
<td>2073.07</td>
<td>2016.43</td>
</tr>
<tr>
<td><strong>Nucl</strong></td>
<td>3863.09</td>
<td>329.33</td>
</tr>
</tbody>
</table>

3.5.2 First-and higher-order interference effects on the Coulomb-nuclear interference

Let us now consider the effects of the different multipole transitions and the first-and higher-order interferences on the nature and magnitude of the Coulomb-nuclear interference. Starting with the zero-order transition, it is seen that at this transition, the Coulomb-nuclear interference is exclusively destructive, and is equal to the integrated Coulomb breakup cross section in magnitude. Considering the first-order transition, we see that this interference is strongly constructive in the s-waves, followed by the d-waves, whereas it is weakly constructive in the f-waves. It is rather strongly destructive in the p-waves and weakly destructive the g-waves. If one considers all the partial waves together, we notice that the overall Coulomb-nuclear interference is strongly constructive. At the second-order transition on the other hand, this interference is seen to be exclusively destructive wherever is nonzero, owing to the fact that in this case, the nuclear breakup cross section prevails over the total breakup cross section in each partial wave. At the third-order transition however, this interference is weakly constructive in the s-, p- and g-waves, and weakly destructive in the d- and f-waves. Concerning the effect of the first-order interference, one notices that the s-wave Coulomb-nuclear interference is dramatically reduced from 620.69 mb to 14.69 mb without affecting its nature. On the other hand, the p-wave Coulomb-nuclear interference is largely increased from -207.91 mb to -471.46 mb, keeping also its nature. The d-wave Coulomb-nuclear interference is substantially decreased from 250.50 mb to -33.86 mb and becomes destructive by nature. As for the f-waves, this interference, increases from 3.90 mb to -17.25 mb, becoming destructive, whereas this interference in the g-waves it is increased from -11.42 mb to -14.49 mb, with the same nature. Considering all the partial waves together, we find that the Coulomb-
nuclear interference is strongly destructive and more important than the nuclear breakup cross section in magnitude (329.33 mb against -522.37 mb). If we consider the all-order transition, we notice that the s-wave Coulomb-nuclear breakup cross section is largely increased (from 14.69 mb to -131.36 mb) and becomes strongly destructive. This interference in the other partial waves is increased without changing the nature, except in the g-waves where it is decreased from -14.49 mb to -10.25 mb. The overall Coulomb-nuclear interference is significantly increased from -522 mb to -816.16 mb. The results serve to conclude that the higher-order interference increases the Coulomb-nuclear interference. This increase can be, among other reasons attributed to the fact that the higher-order interference significantly reduce both the integrated total and nuclear breakup cross section, while it has an insignificant effect on the integrated Coulomb breakup cross section. This analysis demonstrates again that although the nuclear breakup cross section is rather small at the all-order transition, it cannot be simply excluded to obtain the pure Coulomb breakup cross section, given the importance of the Coulomb-nuclear interference.

These results show that the higher-order multipole transitions have small (< 10%) effects on the integrated total, Coulomb and nuclear breakup cross sections, while the total and nuclear breakup cross sections are substantially reduced at first-order. However, looking carefully at Fig.3.8 and Fig.3.9(a), we find that the first-order differential total breakup cross section alone overestimates the data mostly at low excitation energies. Therefore, higher-order multipole transition are also important in the analysis of this reaction, especially given the effect effect of the higher-order interference on the Coulomb-nuclear interference. Based on these results, it can be deduced that the dominance of the Coulomb differential breakup cross section over the total one observed in Fig.3.8 and in other works (for instance Ref. [57, 64]), can be attributed to the first-order interference effect. Moreover, it is clear that if the integrated breakup cross sections corresponding to the different multipoles were to be summed incoherently, the nuclear breakup cross section would largely prevails over the Coulomb breakup cross section, even at the first-order. In such case, the elimination of the nuclear breakup contribution for obtaining the pure Coulomb breakup cross section would rise concerns regarding the accuracy of
Table 3.5: Partial integrated breakup cross sections (in millibarns). The numerical integration is performed up to $\varepsilon_{\text{max}} = 5\text{MeV}$, $\sigma_{\text{CN}}^\ell = \sigma_{\text{C}}^\ell + \sigma_{\text{N}}^\ell$.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\sigma_0^\ell$</th>
<th>$\sigma_s^\ell$</th>
<th>$\sigma_p^\ell$</th>
<th>$\sigma_{\text{CN}}^\ell$</th>
<th>$\sigma_f^\ell$</th>
<th>$\Delta_C^\ell$</th>
<th>$\Delta_N^\ell$</th>
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</thead>
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<tr>
<td>0</td>
<td>145.80</td>
<td>12.43</td>
<td>145.80</td>
<td>158.22</td>
<td>-12.42</td>
<td>7.86%</td>
<td>92.14%</td>
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<tr>
<td></td>
<td>s</td>
<td>1263.02</td>
<td>226.82</td>
<td>415.51</td>
<td>642.33</td>
<td>620.69</td>
<td>35.31%</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>2611.95</td>
<td>1560.40</td>
<td>1259.46</td>
<td>2819.86</td>
<td>-207.91</td>
<td>55.34%</td>
</tr>
<tr>
<td>1</td>
<td>4903.45</td>
<td>2012.20</td>
<td>2235.48</td>
<td>4247.69</td>
<td>655.76</td>
<td>47.37%</td>
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</tr>
<tr>
<td></td>
<td>s</td>
<td>161.82</td>
<td>13.06</td>
<td>176.03</td>
<td>189.09</td>
<td>-27.27</td>
<td>6.91%</td>
</tr>
<tr>
<td></td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00%</td>
</tr>
<tr>
<td>2</td>
<td>1068.18</td>
<td>41.64</td>
<td>1117.08</td>
<td>1158.72</td>
<td>-90.54</td>
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<td>96.41%</td>
</tr>
<tr>
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<td>s</td>
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<td>1.36</td>
<td>42.21</td>
<td>42.57</td>
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</tr>
<tr>
<td></td>
<td>p</td>
<td>0.46</td>
<td>0.03</td>
<td>0.37</td>
<td>0.40</td>
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</tr>
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<td>3</td>
<td>373.52</td>
<td>6.80</td>
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<td>371.53</td>
<td>1.99</td>
<td>1.83%</td>
<td>98.17%</td>
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<tr>
<td>$\lambda_{\text{max}}$</td>
<td>1823.39</td>
<td>2016.43</td>
<td>329.33</td>
<td>2345.76</td>
<td>-522.37</td>
<td>85.96%</td>
<td>14.04%</td>
</tr>
<tr>
<td></td>
<td>s</td>
<td>337.43</td>
<td>225.90</td>
<td>96.84</td>
<td>322.74</td>
<td>14.69</td>
<td>70.00%</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>1297.31</td>
<td>1559.57</td>
<td>209.20</td>
<td>1768.77</td>
<td>-471.46</td>
<td>88.17%</td>
</tr>
<tr>
<td>4</td>
<td>1395.66</td>
<td>2009.36</td>
<td>202.46</td>
<td>2211.82</td>
<td>-816.16</td>
<td>90.85%</td>
<td>0.15%</td>
</tr>
<tr>
<td></td>
<td>s</td>
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<td>227.77</td>
<td>30.96</td>
<td>252.73</td>
<td>-131.36</td>
<td>87.77%</td>
</tr>
<tr>
<td></td>
<td>p</td>
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<td>1558.67</td>
<td>69.48</td>
<td>1628.15</td>
<td>-535.30</td>
<td>95.73%</td>
</tr>
</tbody>
</table>

the obtained results. Our conclusions are in line with the conclusions drawn in Ref. [74] for the $^{17}$F proton-halo projectile, where the angular distributions differential breakup cross sections were investigated, which we are also considering in the next section for the reaction under study.
3.6 Angular distributions breakup cross sections

We have studied in more details the effects on the first- and higher-order interferences on the energy distribution total, Coulomb and nuclear breakup cross sections as well as on the Coulomb-nuclear interference. However, to have a broad picture of the effects of these interferences, we also consider, as in [73, 74] for example, the angular distributions differential total, Coulomb and nuclear breakup cross sections. To this end, the angular distributions differential total, Coulomb and nuclear breakup cross sections are presented in Figs. 3.10(a), 3.10(b) and 3.10(c), respectively. For the total breakup, it is observed that the first-order, the \( \lambda_{\text{max}} = 1 \), and the all-order breakup cross sections oscillate around each other at \( \theta \leq 2^\circ \), and the first-order breakup cross section is more extended to larger angles. It is similarly seen that, although the zero-order breakup cross section is negligible, the first-order interference is more important and is responsible for the suppression of the first-order breakup cross section extension behavior. A further slight reduction of the \( \lambda_{\text{max}} = 1 \) breakup cross section due to the higher-order interference is also reported. It is obtained that the higher-order breakup cross sections are negligible at forward angles (\( \theta \leq 2^\circ \)), and slightly increased at backward angles (\( \theta \geq 2.5^\circ \)).

Looking at the Coulomb breakup, one notices that all the first-order, the \( \lambda_{\text{max}} = 1 \), and the all-order breakup cross sections are not distinguishable, showing again negligible effects of the first-and higher-order interferences. The higher-order breakup cross sections were again found to be negligible and consequently not plotted. Finally, for the nuclear breakup, we see that the breakup cross sections corresponding to the different multipole transitions are all significant. The first-order breakup cross section, which is less than the zero-order breakup cross section at \( \theta \leq 0.5^\circ \), is again seen to be much more extended to larger angles, followed by the second-order one, which is even more important around 3.5°. The third-order breakup cross section slightly extends to larger angles, with a maximum around 3°. Regarding the interferences, one sees that the first-order interference suppresses the extension of first-order breakup cross section to larger angles, where the \( \lambda_{\text{max}} = 1 \) breakup cross section becomes less than the higher-order breakup cross sections. However, it is noticed that although the higher-order breakup cross sections are important
Figure 3.10: Different multipole transition contributions on the angular distributions differential total (a), Coulomb (b) and nuclear (c) breakup cross sections.
at larger angles ($\theta \geq 2.5^\circ$), the higher-order interference does not dramatically reduce the $\lambda_{\text{max}} = 1$ breakup cross section. We then believe that if one were to perform an angular integration of the differential breakup cross sections, similar conclusions as for the energy distributions would be drawn.

In fact, we can show this by considering the angular distributions of the Coulomb-nuclear interference ($\frac{d\sigma}{d\Omega}$), given by equation (3.10). In particular we show that the same effects of the interferences on the Coulomb-nuclear interference observed in Table 3.5 can still be obtained by analyzing the angular distributions. The results for these angular distributions are presented in Fig. 3.11(a). This figure indicates a negligible Coulomb-nuclear interference at zero-order transition, similar to what is observed in Table 3.5 ($\sigma_I = -12.42$ mb). At the first-order transition, we observe that the Coulomb-nuclear interference is highly oscillatory at $\theta \leq 1^\circ$ and is exclusively constructive at $\theta > 1^\circ$, in line with the strongly constructive interference ($\sigma_I = 655.76$ mb) observed in the same table. The figure also shows that the effect of the first-order interference on the angular distributions differential Coulomb-nuclear interference is to lower the amplitude of the oscillations, also at $\theta \leq 1^\circ$, while this interference becomes exclusively destructive at backward angles ($\theta \geq 3^\circ$), in agreement with $\sigma_I = -522.37$ mb from the table. At all-order transition, a further reduction of the amplitude of the Coulomb-nuclear interference oscillations is observed, becoming more destructive at backward angles due to the higher-order interference, which agrees with $\sigma_I = -816.16$ mb. Still there is another way of analyzing the nature of the Coulomb-nuclear interference. That is to use the ratio

$$\hat{\delta} = \frac{1}{\sigma_C}(\sigma_T - \sigma_N)$$ (3.15)

If $\hat{\delta} \geq 1$, then the Coulomb-nuclear interference is constructive, otherwise it is destructive. The angular distributions of this ratio is shown in Fig. 3.11(b), where similar observations as in Fig. 3.11(a) are noticed.
$\lambda_{\text{max}} = 4$

$\lambda_{\text{max}} = 1$

$\lambda = 1$

$\lambda = 0$

(a)

$\theta$ (degrees)

$\delta d \Omega (\text{b/sr})$

(b)

$\theta$ (degrees)

$\delta d \Omega (\text{b/sr})$

$\theta_{\text{gr}}$ (degrees)

Figure 3.11: Angular distributions of the Coulomb-nuclear interference, (a) corresponds to equation (3.10), and (b) to equation (3.15)

3.7 Angular momentum and impact parameter distributions breakup cross sections

The extensions to larger angles (small impact parameters) of the differential total and nuclear breakup cross sections observed in Figs. 3.10 (a) and 3.10 (c), can be more analyzed using the concept of nuclear absorption. One may speculate around the conclusion that the extension to larger angles is a result of a weak nuclear absorption when the projectile approaches the target, allowing more projectile flux to penetrate the Coulomb barrier. On the other hand, the removal of this extension behavior owing to the first-order interference, is a reflection of the fact that this interference produces a stronger nuclear absorption at small impact parameters, such that more projectile flux is removed from the breakup channel to fuel probably other channels. Moreover, the non-negligible nuclear breakup cross sections even when the different multipoles are included coherently in such a Coulomb dominated breakup, shows that a considerable amount of nuclear flux survives the absorption and therefore is felt beyond the projectile-target relative distance. This is generally believed to be due to the the peripherality of the breakup process induced by loosely bound projectiles. For further insight into these conclusions, we consider in this section the the analysis of the angular momentum and impact parameter distributions breakup cross sections. To this end, we first write the classical relation relating the grazing impact parameter ($b_{\text{gr}}$) to the grazing angle ($\theta_{\text{gr}}$), and the grazing angular momentum
where we use $b_{gr} = 1.25(11^{1/3} + 208^{1/3}) \approx 10.186$ fm, corresponding to $\theta_{gr} = 3.74^\circ$, and $L_{gr} = 192.648\hbar$. The impact parameter distributions breakup cross sections can then be expressed in terms of the angular momentum distributions breakup cross sections, using

$$d\sigma_b = \sqrt{\frac{2\mu_{pl}(E + \varepsilon_0)}{\hbar^2}} \, d\sigma_L \Rightarrow \sigma_b = \sqrt{\frac{2\mu_{pl}(E + \varepsilon_0)}{\hbar^2}} \sigma_L. \quad (3.17)$$

The angular momentum and impact parameters distributions total breakup cross sections are shown in Figs. 3.12 (a) and (b), respectively. If we consider only the impact parameter distributions (although the same observations apply to both figures), we observe a narrow first-order breakup cross section, with a peak at almost the half of the grazing impact parameter ($b \approx 5$ fm), which shows negligible nuclear absorption. It is similarly seen also that the higher-order breakup cross sections are more significant below the grazing impact parameter and become negligible for $b \leq 15$ fm. This shows that as the projectile moves away from the target, the higher-order multipole effects vanish rapidly. Considering the effect of the first-order interference, one sees that the $\lambda_{max} = 1$ breakup cross section is negligible at $b \leq 10$ fm, after which it suddenly grows to peak at the grazing impact parameter, which shows that the first-order interference produces a strong nuclear absorption. It is observed that the effect of the higher-order interference is to lower the magnitude of the breakup cross section at the grazing impact parameter. This figure is a reflection of the results in Fig. 3.10 (a). It follows that the extension of the first-order breakup cross sections to larger angles is mainly due to the weak nuclear absorption, at the first-order multipole transition.

For the Coulomb breakup, the results are presented in Fig. 3.13, where one notices,
as expected that the different curves are hardly distinguishable, similar to what is observed in Fig. 3.10 (b). The extension to large impact parameter can be attributed to the natural long-range behavior of the Coulomb interactions. To analyze the effect of the nuclear interactions beyond the projectile-target relative distance, we plot in Fig. 3.14, the nuclear breakup cross sections. It is clearly seen that all the different breakup cross sections are extended beyond the grazing impact parameter, which reflects the fact that the nuclear breakup effects are felt beyond the projectile-target relative distance. Looking at the effects of the first-and higher-order interferences, it is seen that similar conclusions as in the case of the total breakup still apply. However, here we observe a rise of the zero-order breakup cross section, similar to what is observed in Fig. 3.10 (c).
In summary, in this chapter we have investigated in more details the contributions of the Coulomb and nuclear breakups, the role of the first-and higher-order interferences on the total, Coulomb and nuclear breakup cross sections, as well as on the Coulomb-nuclear interference for the $^{11}\text{Be} + ^{208}\text{Pb}$ breakup reaction. It is shown that both the total and nuclear breakup cross sections are more important than the Coulomb breakup cross section when considering single multipole transitions. However, the total and nuclear breakup cross sections are substantially reduced due to the first-order interference, becoming less than the Coulomb breakup cross section, which is insignificantly affected by the different interferences. It is seen that the first-and higher-order interferences affect the magnitude and nature of the Coulomb-nuclear interference. Given the importance of this interference, we concluded that the nuclear breakup contribution cannot be just excluded to obtain a pure Coulomb breakup cross section.

In the next chapter, we study similar dynamics for the same target but with a projectile with more binding energy, in order to understand, on one hand the effect of the ground state binding energy on these conclusions.
Chapter 4

The $^{15}$C+$^{208}$Pb breakup reaction

Motivated by the results obtained for the breakup of $^{11}$Be on the lead in chapter 3, we consider the same study for the breakup of $^{15}$C on the same target. The prime goal is to verify whether we can reach similar conclusions. These two projectiles are one-neutron halos, with similar ground state configurations, but differ in their ground binding energies, in their core nucleus masses. This provides an opportunity of analyzing the effects of the ground state binding energy on the conclusions drawn in chapter 3. This chapter also starts with a brief description of the projectile.

4.1 Description of $^{15}$C projectile

The $^{15}$C nucleus is a one-neutron halo nucleus, formed by the $^{14}$C core plus a neutron [9,12,113]. This neutron is loosely bound to the core, with a binding energy of 1.218 MeV. The ground state configuration adopted in this work is $^{15}$C $\rightarrow$ $^{14}$C($0^+$) $\otimes$ n($2s_{\frac{1}{2}^+}$), as in [9,12,113], with $j^\pi = \frac{1}{2}^+$. Its first excitation state is a $1d_{\frac{3}{2}^+}$ state, with an excitation energy of 0.478 MeV. The continuum exhibits a well known resonance in the $d_{\frac{3}{2}^+}$ state, with a resonance energy of $3.56 \pm 0.1$ MeV. We assume again an inert, and the direct implications are explained in section 3.1. The $^{14}$C + n potential used to obtain the bound and continuum wave functions, is the same as the $^{10}$Be + n potential, where we only adjust the depths of the central and spin-orbit coupling terms to fit the experimental binding energy. The values of these depths are $V_{l=0} = 52.18$ MeV, $V_{l>0} = 51.30$ MeV and $V_{so} = 20.77$ MeVfm$^2$. 
4.1.1 Ground and continuum state properties

The bound and excited wave functions are displayed in Fig. 4.1 (upper panel). Taking 

$$R_{\text{core}} = 2.30 \pm 0.07 \text{ fm} \quad [114],$$

we see that these wave functions are much extended beyond $R_{\text{core}}$. The probabilities of finding the neutron outside the core for the two states, are 87.07% for the ground state and 81.21% for the excited state, respectively. In Table 4.1 we present the calculated ground state and excited state binding energies, the corresponding root-mean-square radii and the inside and outside probabilities. The significant outside probabilities indicate also the extension of these bound state wave functions beyond the halo sizes.

Figure 4.1: Bound and excited state wave functions and continuum wave functions calculated at the resonance energies.
Table 4.1: Ground state and excited state binding energies ($S_n$), root-mean-square radii and inside and outside probabilities for the $^{14}$C + $n$ system.

<table>
<thead>
<tr>
<th>$nlj^\pi$</th>
<th>$S_n$(MeV)</th>
<th>$\sqrt{\langle r^2 \rangle}$(fm)</th>
<th>$P_{in}$ (%)</th>
<th>$P_{out}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2$s_{1+}$</td>
<td>1.218</td>
<td>5.268</td>
<td>67.94</td>
<td>32.06</td>
</tr>
<tr>
<td>1$d_{2+}$</td>
<td>0.478</td>
<td>3.817</td>
<td>70.56</td>
<td>29.44</td>
</tr>
</tbody>
</table>

Concerning the continuum states, the calculated continuum wave functions in the $s_{1+}$ and $d_{2+}$ partial waves, at 1.961 MeV and 3.561 MeV, respectively are displayed in Fig.4.1 (lower panel). This figure shows that these states are resonant states, with a pronounced resonance structure in the $d_{2+}$, as already mentioned. The resonance in the $s_{1+}$ is much less pronounced and the multiplication by two is for convenience. A good display of these resonance is obtained in Fig.4.2 where the phase shifts are plotted as functions of the energy.

Figure 4.2: Resonance structures in different partial waves.

4.1.2 Electric response functions

The integrand of the radial integral [3.5], which contains the radial information of the response function is calculated at the same three different arbitrary energies as in section 3.1.2 and the results are presented in Fig. 4.3 for the transition from the ground state to the $p_{\frac{3}{2}^-}$ continuum state. It is observed that for the three energies, the integrand builds
its maximum around 10 fm, and is more negligible for $r \leq R_{\text{core}}$. This shows again that the neutron capture process to form the $^{15}$C nucleus takes place well outside the core, making the capture process also peripheral in this case. Consequently, for the $^{15}$C+$^{208}$Pb breakup reaction, we can consider the core to be inert without loss of the accuracy. In Fig. 4.4 we present the $E1$ electric response functions for the transitions from the ground state to the $p_{\frac{1}{2}^-}$, $p_{\frac{3}{2}^-}$ and $p$ continuum states of the $^{14}$C($n,\gamma$)$^{15}$C capture reaction. We also observe that the transition to the $p_{\frac{3}{2}^-}$ continuum state is more important that the transition to the $p_{\frac{1}{2}^-}$ continuum state. Moreover, all these response functions peak as well in the vicinity of the ground state binding energy.
4.2 The $^{14}\text{C} + ^{208}\text{Pb}$ and $n + ^{208}\text{Pb}$ potentials

The nuclear $^{14}\text{C} + ^{208}\text{Pb}$ and $n + ^{208}\text{Pb}$ potentials, are also Woods–Saxon optical potentials given by the same equations (3.6) and (3.7). The parameters of the nuclear $^{14}\text{C} + ^{208}\text{Pb}$ potential, taken also from [111], are presented in Table 4.2. The corresponding parameters for the $n + ^{208}\text{Pb}$ potential are those given in Table 3.3. The $^{14}\text{C} + ^{208}\text{Pb}$ Coulomb potential remains a point-sphere Coulomb potential, given by equation (3.8), where the Coulomb radius is $R_C = 1.25(14^{1/3} + 208^{1/3}) \simeq 10.42$ fm, and $Z_c = 6$. Regarding the CDCC model space, we employ the same parameters as described in subsection 3.3, which were found to be sufficient in obtaining good convergence of the results for this reaction as well.

Table 4.2: Parameters of the real and imaginary depths, radii and diffusenesses of the $^{14}\text{C} + ^{208}\text{Pb}$ and $n + ^{208}\text{Pb}$ nuclear potentials. These parameters are taken from Ref. [111].

<table>
<thead>
<tr>
<th></th>
<th>$V$ (MeV)</th>
<th>$W$ (MeV)</th>
<th>$R_R$ (fm)</th>
<th>$R_I$ (fm)</th>
<th>$a_R$ (fm)</th>
<th>$a_I$ (fm)</th>
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</thead>
<tbody>
<tr>
<td>$^{14}\text{C} + ^{208}\text{Pb}$</td>
<td>70.00</td>
<td>58.90</td>
<td>7.67</td>
<td>7.42</td>
<td>1.04</td>
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4.3 Energy distributions differential breakup cross sections

In this section, we present the energy distributions total, Coulomb and nuclear breakup cross sections. A partial wave analysis is again performed for a good understanding of the contributions of the different partial waves retained in the CDCC model space for this reaction. The effects of the first-and higher-order interferences on the differential total, Coulomb and nuclear breakup cross sections and on the Coulomb-nuclear interference are investigated.

4.3.1 Partial waves analysis

Motivated by the contradicting results of Ref. [57][58], we first performed a partial waves analysis to assess the contributions of each partial wave. The partial differential breakup
cross sections, are shown in Fig. 4.5 along with their coherent sum. It is observed, as expected, a large dominance of the $p$-waves which also dictate the shape of the total breakup cross section. However, one sees that the contribution of the other partial waves is important, especially for $\varepsilon \leq 4$ MeV, which means that all the outgoing neutrons are not in the $p$-waves, in agreement with the findings of Ref. [57]. Again the $g$-wave breakup cross section was found to be insignificant, and is therefore not plotted. If we compare at this stage Figs. 4.5 and 3.7 we find that here the magnitudes of the breakup cross sections are largely reduced. Given the dependence of the breakup process on the binding energy, we can conclude that the reduction of the breakup cross section in this case is due to the large neutron ground state binding energy in $^{15}$C than in $^{11}$Be. However, qualitatively the results in both figures present similar features.

### 4.3.2 Differential total, Coulomb and nuclear breakup cross sections

Here we analyze the differential total, Coulomb and nuclear breakup cross sections, and compare the results with the data. To this end, we present in Fig. 4.6 the differential total, Coulomb and nuclear breakup cross sections. The results show that, the nuclear breakup cross section is much smaller than the Coulomb breakup cross section. It is also seen that the Coulomb breakup cross section is more important than the total breakup cross section.
cross section, and largely overestimates the data, which are rather well fitted by the total breakup cross section. Looking closely at this figure, we see again that $\sigma_N < |\sigma_T - \sigma_C|$, showing also the importance of the Coulomb-nuclear interference for this reaction. The importance of this interference is further revealed in the fitting of the experimental data by the total breakup cross section. It follows that although the nuclear breakup contribution is small, its exclusion significantly undermines that description of the data, due to the important of the Coulomb-nuclear interference.

The results in Fig. 4.6 are obtained when all the different multipoles are included coherently. We have seen in chapter 3 that in this case, the total and nuclear breakup cross sections are substantially reduced, owing to the first-order interference, which has negligible effect on the Coulomb breakup cross section. However, considering single multipole transitions, we found that both the total and nuclear breakup cross sections are much important than the Coulomb breakup cross section. We also perform similar calculations for this reaction, in order to check the dependence of these different breakups and the Coulomb-nuclear interference on the first-and higher-order interferences.
4.3.3 First-and higher-order interferences

We now consider the importance of the first-and higher-order interferences on the differential total, Coulomb and nuclear breakup cross sections, which will shed more light on the understanding of the dominance of the Coulomb breakup cross section of the total breakup cross section as already seen. Similar conclusions as in chapter 3 are not guaranteed in advance given the different ground state binding energies. In Figs. 4.7(a), 4.7(b) and 4.7(c), we present the respective differential total, Coulomb and nuclear breakup cross sections, corresponding to different multipole transitions. Looking at Fig. 4.7(a), it is observed that the first-order cross section is much dominant, followed by the second-order, whereas the third-order cross section is negligible for energies $\geq 0.5$ MeV. We see that the zero-order breakup cross section is negligible compared to the first-order breakup cross section. However, again the $\lambda_{\text{max}} = 1$ breakup cross section is much lesser than the first-order breakup cross section, indicating as well a strongly destructive first-order interference. It is similarly observed that the all-order breakup cross section curve only differs from the $\lambda_{\text{max}} = 1$ breakup cross section curve for energies between $0.5 \text{ MeV} \leq \varepsilon < 4 \text{ MeV}$ (in which interval the higher-order multipole effects are significant), owing to the higher-order interference. Once again the conclusion is that the differential total breakup cross section is substantially reduced by the first-order interference and also the higher-order interference is not negligible.

For the differential Coulomb breakup cross sections, one notices again a negligible zero-order breakup cross section (the multiplication by 10 is still for convenience). Moreover, it can be seen that the first-order, the $\lambda_{\text{max}} = 1$, and the all-order breakup cross section curves are hardly distinguishable, showing that first-and higher-order interferences are negligible. Comparing Figs. 4.7(a) and 4.7(b), it is noticed that the first-order total breakup cross section is more important than the first-order Coulomb breakup cross section. However, we also see that the substantial reduction of the total breakup cross section due to the first-order interference is the reason why in Fig. 4.6, the Coulomb breakup cross section is seen to be more important. Lastly, the results for the nuclear breakup cross sections show that similar conclusions as in Fig. 4.7(a) can be drawn, indicating that both the
Figure 4.7: Different multipole contributions on the differential Coulomb, nuclear and total breakup cross sections.

total and nuclear breakup cross sections are largely reduced by the first-order interference. However, a more stronger destructive higher-order interference is noticed in this case. Also
if one compares Figs. 4.7(b) and 4.7(c), it is clear that the first-order nuclear breakup cross section is more important than the first-order Coulomb breakup cross section. Therefore it amounts to saying that the small nuclear breakup cross section compared to the Coulomb breakup cross section observed in Fig. 4.6, is largely due to the first-order interference, which has an insignificant effect on the Coulomb breakup cross section, but substantially reduces the nuclear breakup cross section. These results lead us again to the conclusion that at the first-order transition, the nuclear breakup contribution cannot be excluded in order to obtain a pure Coulomb breakup cross section, as it is the case in the Coulomb dissociation method.

### 4.4 Energy-integrated breakup cross sections

In the previous section, we have discussed qualitatively the effects of the first-and higher-order interferences on the differential total, Coulomb and nuclear breakup cross sections. The quantitative effects of these interferences on the different breakup cross section are considered in this section. Such analysis is important as it provides an opportunity of investigating the effect of the first-and higher-order interferences on the magnitude and nature of the Coulomb-nuclear interference. To this end, we first obtain again the integrated total, Coulomb and nuclear breakup cross sections, using equation (3.9). The summary of the obtained results is presented in Table 4.3. In this table are presented, the integrated total, Coulomb and nuclear breakup cross sections (third, fourth and fifth columns), the Coulomb+nuclear incoherent sum (sixth column), the Coulomb-nuclear interferences (seventh column), and the Coulomb and nuclear contributions to their incoherent sum (eighth and ninth columns) as well as the different multipole transitions to continuum states and the sum of all the partial integrated breakup cross sections for each multipole transition.

Considering the transitions to the different continuum states, one observes that at zero-order, again only transitions to the s continuum states are accounted for. At the first-order, again possible transitions to all the s,p,d,f and g continuum states are noticed,
Table 4.3: Partial integrated breakup cross sections (in millibarns). The numerical integration is performed up to $\varepsilon_{\text{max}} = 5\text{MeV}$, $\sigma^{I}_{CN} = \sigma^{I}_{C} + \sigma^{I}_{N}$.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\alpha^{I}_{C}$</th>
<th>$\alpha^{I}_{N}$</th>
<th>$\Delta^{I}_{C}$</th>
<th>$\Delta^{I}_{N}$</th>
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<tbody>
<tr>
<td>0</td>
<td>$S^{I}_{0}$</td>
<td>80.84</td>
<td>14.03</td>
<td>94.87</td>
</tr>
<tr>
<td>s</td>
<td>429.05</td>
<td>57.14</td>
<td>174.12</td>
<td>231.26</td>
</tr>
<tr>
<td>p</td>
<td>1110.44</td>
<td>833.70</td>
<td>1076.54</td>
<td>1910.24</td>
</tr>
<tr>
<td>d</td>
<td>380.45</td>
<td>86.16</td>
<td>182.19</td>
<td>268.35</td>
</tr>
<tr>
<td>f</td>
<td>99.60</td>
<td>6.39</td>
<td>19.37</td>
<td>25.76</td>
</tr>
<tr>
<td>g</td>
<td>15.73</td>
<td>0.54</td>
<td>0.76</td>
<td>1.30</td>
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<tr>
<td>S^{I}_{1}</td>
<td>2035.26</td>
<td>983.93</td>
<td>1452.97</td>
<td>2436.90</td>
</tr>
<tr>
<td>s</td>
<td>105.79</td>
<td>0.00</td>
<td>112.47</td>
<td>112.47</td>
</tr>
<tr>
<td>p</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>d</td>
<td>323.93</td>
<td>0.45</td>
<td>329.52</td>
<td>329.96</td>
</tr>
<tr>
<td>f</td>
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<td>0.00</td>
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<tr>
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<td>S^{I}_{2}</td>
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<td>456.44</td>
<td>456.90</td>
</tr>
<tr>
<td>s</td>
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<td>0.00</td>
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<td>d</td>
<td>4.56</td>
<td>0.00</td>
<td>4.55</td>
<td>4.55</td>
</tr>
<tr>
<td>f</td>
<td>119.97</td>
<td>0.00</td>
<td>119.66</td>
<td>119.66</td>
</tr>
<tr>
<td>g</td>
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<td>0.00</td>
<td>2.05</td>
<td>2.05</td>
</tr>
<tr>
<td>S^{I}_{3}</td>
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<td>0.00</td>
<td>144.78</td>
<td>144.78</td>
</tr>
<tr>
<td>$\lambda_{\text{max}}$</td>
<td>$S^{I}_{1\text{max}}$</td>
<td>980.13</td>
<td>971.52</td>
<td>299.57</td>
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<td>s</td>
<td>184.05</td>
<td>61.85</td>
<td>58.12</td>
<td>119.97</td>
</tr>
<tr>
<td>p</td>
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<td>213.11</td>
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</tr>
<tr>
<td>d</td>
<td>106.91</td>
<td>83.52</td>
<td>28.17</td>
<td>111.69</td>
</tr>
<tr>
<td>f</td>
<td>19.25</td>
<td>6.41</td>
<td>0.8</td>
<td>6.49</td>
</tr>
<tr>
<td>g</td>
<td>2.60</td>
<td>0.58</td>
<td>0.08</td>
<td>0.66</td>
</tr>
<tr>
<td>$S^{I}_{4\text{max}}$</td>
<td>743.71</td>
<td>969.83</td>
<td>163.20</td>
<td>1133.03</td>
</tr>
</tbody>
</table>

where a large dominance of the $p$-wave breakup cross section is observed. At the second-order on the other hand, only transitions to the $s,d,$ and $g$ continuum states are seen, where the $d$-waves breakup cross section is more dominant. Finally, at the third-order, again possible transitions to all the $s,p,d,f$ and $g$ continuum states are noticed, where the $f$-wave breakup cross section is leading.
4.4.1 Quantitative effects of the first-and higher-order interferences on the integrated breakup cross sections

Let us now compare the integrated Coulomb and nuclear breakup cross sections for each partial wave and multipole transition. It is seen that at zero-order, the integrated nuclear breakup cross section is more important and contributes up to 85.20% to their incoherent sum. At first-order, we notice that the nuclear breakup cross section is still dominant in each partial wave, with an overall contribution of 59.62%. At higher-order, the Coulomb breakup cross section becomes much more negligible, such that $\sigma_C^0 + \sigma_N^I \simeq \sigma_N^I$. Comparing the integrated total breakup cross section to the integrated Coulomb one, we also observe that the total breakup cross sections are much important than their Coulomb breakup counterpart in each partial wave and single multipole transition.

The analysis of the first-order interference effect on the integrated total, Coulomb and breakup cross section shows that this interference substantially reduces the nuclear breakup cross sections, and the dominance of the nuclear breakup observed for the different multipole transitions is completely shifted to the Coulomb breakup, where the breakup cross sections become more important in all the partial waves with an overall contribution of 76.43% on their incoherent sum, much more that the overall nuclear contribution at the first-order transition. As for the total breakup, one sees that this interference amounts to a substantial reduction of the total breakup cross sections, and its $p$-wave breakup cross section becomes smaller than the $p$-wave Coulomb breakup cross section although its overall breakup cross section still slightly dominant. We conclude that the first-order interference alone, although it substantially reduces the total breakup cross section, does not explain the large dominance of the Coulomb breakup cross section, unlike in the case of the $^{11}$Be + $^{208}$Pb reaction.

When all the multipoles are included coherently, one notices that the total and nuclear breakup cross sections are further reduced and the nuclear breakup contribution to the Coulomb+nuclear incoherent sum drops to 14.40%. In this case, one notices that the all-order nuclear contribution is completely shifted to the zero-order Coulomb contribu-
tion, and vice versa, owing to the higher-order interference. Comparing the integrated total breakup cross section and the integrated Coulomb breakup cross section, we realize that the Coulomb breakup cross section becomes dominant in all the partial waves other than the \( f \)-and \( g \)-waves and its overall breakup cross section becomes dominant, given its large \( p \)-wave contribution. We can conclude in this case that the dominance of the Coulomb breakup cross section over the total and nuclear breakup cross sections observed in Figs. 4.5 and 4.6 can be attributed to the higher-order interference for the total breakup cross section and to the first-order interference for the nuclear breakup cross section. Our results for the \( S_{4}^{\lambda_{\text{max}}} \), and for the total breakup cross section, agree fairly with the 767 mb value of Ref. [113].

A further quantitative analysis of the effects of the first-and higher-order interferences on the total, Coulomb and nuclear breakup cross sections leads us to Table 4.4 where the amounts (in percentage) reduced from the integrated total, Coulomb and nuclear breakup cross sections are presented. The higher-order interference effect is again estimated using equations (3.12)–(3.14). From this table, it is observed that the all-order interference reduces by 72.50\% the total breakup cross section, distributed as follows: 63.75\% due to the first-order interference, and 8.74\% to the higher-order interference. It insignificantly reduces the Coulomb breakup cross section by 2.86\%, in the following distribution: 2.69\% due to the first-order interference and 0.17\% to the higher-order interference. For the nuclear breakup on the other hand, this interference dramatically reduces by 92.36\% the breakup cross section, where 85.97\% is due to the first-order interference and 6.39\% to the higher-order interference.

Table 4.4: Reduced amounts of integrated total, Coulomb and nuclear breakup cross sections due to the first-and higher-order interference.

<table>
<thead>
<tr>
<th></th>
<th>FO</th>
<th></th>
<th>AO</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>̂( S )</td>
<td>̂( S_{4}^{\lambda_{\text{max}}} )</td>
<td>( \sigma_{\text{FO}}^{\lambda_{\text{max}}} )</td>
<td>( \sigma_{\text{AO}}^{\lambda_{\text{max}}} )</td>
</tr>
<tr>
<td>Tot</td>
<td>2704.00</td>
<td>980.13</td>
<td>63.75%</td>
<td>743.71</td>
</tr>
<tr>
<td>Coul</td>
<td>998.41</td>
<td>971.52</td>
<td>2.69%</td>
<td>969.83</td>
</tr>
<tr>
<td>Nucl</td>
<td>2135.03</td>
<td>299.57</td>
<td>85.97%</td>
<td>163.20</td>
</tr>
</tbody>
</table>

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4.4.2 First-and higher-order interference effects on the Coulomb-nuclear interference

We first consider the effects of the different multipole transitions on the magnitude and nature of the Coulomb-nuclear interference. At zero-order, one notices that the Coulomb-nuclear interference is destructive and is equal in magnitude to the Coulomb breakup cross section. At first-order this interference is strongly destructive in the $p$-waves ($\sigma_I = -799.80$ mb), while is it constructive in all the other partial waves. This strong destructiveness in the $p$-waves is attenuated by the contribution of the other partial waves, such that the overall magnitude is reduced to $-401.64$ mb. At second-order however, one sees that this interference is exclusively destructive wherever it is nonzero, where its overall magnitude is roughly equal to its zero-order magnitude. At third-order on the other hand, the Coulomb-nuclear interference is much weakly constructive, owing to the fact that the Coulomb breakup cross section is just negligible and the nuclear breakup cross section is slightly larger than the total breakup cross section.

After discussing the effects of the different multipole transitions, let us now address the dependence of the Coulomb-nuclear interference on the first-and higher-order interferences. Looking at the first-order interference, we see that the Coulomb-nuclear interference is still destructive in the $p$-waves, but its magnitude is largely reduced by 54.37%, while it is dramatically reduced by 95.47% in the $d$-waves, where it becomes very weakly destructive, but still overall destructive by nature. One concludes that the first-order interference only affects the magnitude of the Coulomb-nuclear interference, not its nature in this case. At all-order, this interference becomes destructive in all the partial waves, except in the $g$-waves where it is weakly constructive and is increased by 33.81%, owing to the higher-order interference, showing a more important effect of the higher-order interference on the magnitude of the Coulomb-nuclear interference. This increase can be also generally explained by the fact that the higher-order interference further reduces the integrated total and nuclear breakup cross sections, while its effect on the integrated Coulomb breakup cross section is negligible.
4.5 Angular distributions differential breakup cross sections

The respective angular distributions differential total, Coulomb and nuclear breakup cross sections are shown in Fig. 4.8(a), Fig. 4.8(b) and Fig. 4.8(c), respectively. In Fig. 4.8(a), it can be seen that the non-first-order total breakup cross sections are all negligible around \( \theta \leq 2^\circ \). The zero-order breakup cross section, which corresponds to the smallest contribution, builds its maximum around 3\(^\circ\), the second-order at 4.5\(^\circ\), and the third-order breakup cross section, slightly important than the zero-order, builds its maximum around 3.5\(^\circ\). The first-order breakup cross section on the other hand, is much extended to larger angles, and amounts to the more important contribution as already observed for the energy distributions. It is observed that, although the zero-order breakup cross section is negligible compared to the first-order breakup cross section, the first-order interference is significantly important as it washes out the extension behavior to larger angles of the first-order breakup cross section. This interference is seen to be constructive at angles between 1\(^\circ\) and 3.5\(^\circ\), and beyond 3.5\(^\circ\), it is strongly destructive. Looking at the all-order breakup cross section, one concludes that the higher-order interference lowers the total breakup cross section at the whole range of angles, starting around 2\(^\circ\).

Considering the Coulomb breakup cross sections, one finds that the zero-order breakup cross section is rather negligible (the multiplication by 10 is again for convenience), while the first-order interference is seen to be responsible for the oscillatory behavior of the \( \lambda_{\max} = 1 \), and the all-order breakup cross sections, which are hardly distinguishable. Lastly, Fig. 4.8(c) shows that all the nuclear breakup cross sections are negligible at \( \theta \leq 1^\circ \), while the first-order breakup cross section is much extended at larger angles. Comparing the results in Figs. 4.8(a) and 4.8(c), it is seen that both the total and nuclear breakup cross sections exhibit some similarities regarding the effects of the first-and higher-order interferences. However, in this case, all the nuclear breakup cross sections are negligible at small angles (\( \theta < 2^\circ \)), which can be regarded as an effect of the short-range nature of the nuclear forces.
In the following section we address the effects of the first-and higher-order interferences on the nuclear absorption effect for a more clear understanding of the extension to larger angles of the first-order differential total and nuclear breakup cross sections, and of the suppression of this extension due to the first-order interference. To this end, we consider the angular momentum and impact parameter distributions total, Coulomb and nuclear breakup cross sections.

4.6 Angular momentum and impact parameter distributions breakup cross sections

We transform the angular momentum distributions breakup cross sections into impact parameter distributions breakup cross sections again using equations (3.16) and (3.17), where

\[ b_{\text{gr}} = 1.25(15^{1/3} + 208^{1/3}) \approx 10.50 \text{ fm}, \text{ which corresponds to } \theta_{\text{gr}} = 4.06^\circ, \text{ and } \]

\[ L_{\text{gr}} = 266.037\hbar. \]

The angular momentum and impact parameter distributions total breakup cross sections are respectively presented in Fig. 4.9(a) and 4.9(b). From both figures, the results show that the zero-order breakup cross section which amounts to the lowest contribution is negligible at small impact parameter \((b \leq b_{\text{gr}})\), due to the nuclear absorption. However, we observe weaker first-and higher-order nuclear absorptions since the first-order and higher-order breakup cross sections are all more significant at \(b \leq b_{\text{gr}}\).

Moreover, it is seen that the higher-order breakup cross sections rapidly vanish beyond the grazing impact parameter, as a result of negligible higher-order Coulomb breakup cross sections, while the first-order breakup cross section is much extended to larger impact parameters. Considering the first- and higher-order interferences, one sees that the \(\lambda_{\text{max}} = 1\) breakup cross section becomes negligible at \(b \leq 8 \text{ fm}\), showing a strong nuclear absorption due to the first-order interference. We also notice that beyond the grazing impact parameter, the first-order interference is negligible. This should be expected since beyond the grazing the Coulomb breakup cross section prevails, which insignificantly affected by the first-order interference. Furthermore, it is seen that the effect of the higher-order interference is to reduce the magnitude of the breakup cross section at the grazing impact parameter. It follows that the first-order interference produces a stronger nuclear absorp-
Figure 4.8: Different multipole transition contributions on the angular distributions differential total (a), Coulomb (b) and nuclear (c) breakup cross sections.
Figure 4.9: Angular momentum (a) and impact parameter (b) distributions breakup cross sections for the total breakup.

For the Coulomb breakup [Figs. 4.10(a) and 4.10(b)], where there are no nuclear interactions, we find that there are no absorption at small impact parameters. The different breakup cross sections present peaks at impact parameters lower than the grazing impact parameter, and as already seen, the first-order, $\lambda_{\text{max}} = 1$, and the all-order breakup cross sections are indistinguishable. A much extension of these breakup cross sections to larger impact parameters is observed, due to the Coulomb long-range behavior. The angular momentum and impact parameter distributions nuclear breakup cross sections are displayed in Figs. 4.11(a) and 4.11(b). We notice similarities with the total breakup cross sections, hence similar conclusions, although in this case, the long-range behavior is washed out. However, we find that all the different breakup cross sections are extended beyond the grazing impact parameter, i.e beyond the range of the nuclear forces, which is another good example of the peripherality of the $^{15}\text{C} + ^{208}\text{Pb}$ breakup reaction.
\[ \lambda = \lambda_{\text{max}} = 4 \]

Figure 4.10: Angular momentum (a) and impact parameter (b) distributions breakup cross sections for the Coulomb breakup

\[ \lambda = \lambda_{\text{max}} = 4 \]

Figure 4.11: Angular momentum (a) and impact parameter (b) distributions breakup cross sections for the nuclear breakup

### 4.7 Comparison between the $^{11}\text{Be}$ and $^{15}\text{C}$ breakups

We complete this chapter by comparing briefly the $^{11}\text{Be}$ and $^{15}\text{C}$ breakups. Quantitatively, we have seen that the breakup cross sections obtained for the $^{11}\text{Be} + ^{208}\text{Pb}$ reaction are much more larger than those obtained for the $^{15}\text{C} + ^{208}\text{Pb}$ reaction. We attributed this fact to the low $^{11}\text{Be}$ ground state binding energy. However, the results summarized in Tables 3.5 and 4.3 revealed that the way the first- and higher-order interferences affect the partial Coulomb-nuclear interferences for both reactions can significantly differ. This shows that the dependence of the breakup cross section on the ground state binding energy might not follow the same trend when it comes to the Coulomb-nuclear interference in each partial wave. Here, we briefly compare the effects of the first- and higher-order...
interference on the Coulomb-nuclear breakup cross section in each partial for the two reactions. To render the comparison easy, we plot in Figs. [4.12] the integrated partial Coulomb-nuclear interferences as functions of the partial waves for the two reactions. If we consider first the first-order transition, we find that in the s-waves, the Coulomb-nuclear interference is more strongly constructive for the $^{11}\text{Be} + ^{208}\text{Pb}$ reaction than for the $^{15}\text{C} + ^{208}\text{Pb}$ reaction. In the p-waves on the other hand, this interference is more strongly destructive for the $^{15}\text{C} + ^{208}\text{Pb}$ reaction. In the d-waves, we find that the interference is more destructive for the $^{11}\text{Be} + ^{208}\text{Pb}$ reaction. In the f- and g-waves however, the the Coulomb-nuclear interference is more constructive for the $^{15}\text{C} + ^{208}\text{Pb}$ reaction, while it becomes destructive in the g-waves for the $^{11}\text{Be} + ^{208}\text{Pb}$ reaction.

Considering the effects of the first-order interference, it is found that in the s-waves, the Coulomb-nuclear interference is substantially reduced but keeps its nature for the $^{15}\text{C} + ^{208}\text{Pb}$ reaction, while it is dramatically reduced and becomes weakly destructive for the $^{11}\text{Be} + ^{208}\text{Pb}$ reaction. In the p-waves, this interference still substantially reduced and remains destructive for the $^{15}\text{C} + ^{208}\text{Pb}$ reaction. However, for the $^{11}\text{Be} + ^{208}\text{Pb}$ reaction, one notices that this interference is rather substantially increased without affecting its nature. This interference is largely reduced and becomes weakly destructive for both reactions in the d-waves. The variation of this interference in the f- and g-waves is small at this stage. Looking at both figures, we observe that the higher-order interference has a pronounced effect on the Coulomb-nuclear interference for the $^{11}\text{Be} + ^{208}\text{Pb}$ reaction than for the $^{15}\text{C} + ^{208}\text{Pb}$ reaction.

The other major qualitative differences between the two reactions concern the angular distributions differential Coulomb breakup cross sections and the angular momentum and impact parameter distributions total breakup cross sections. It is observed in Fig. [4.8] (b) that the $\lambda_{\text{max}} = 1$ and the all-order breakup cross sections present oscillatory patterns as a result of the first-order interference. However, Fig. [3.10] (b) shows no difference between the first-order the $\lambda_{\text{max}} = 1$, and the all-order breakup cross sections. This shows that the effect of the first-order interference is much more pronounced for the Coulomb breakup for the $^{15}\text{C} + ^{208}\text{Pb}$ reaction than for $^{11}\text{Be} + ^{208}\text{Pb}$ reaction. Finally in Fig. [3.12]
we observe narrow first-order breakup cross sections than in Fig. 4.9. Narrow momentum distributions breakup cross sections are characteristic of breakup reactions induced by halo projectiles.

For these two reactions we left out another important aspect of the breakup reactions induced by loosely bound projectiles, which is the effect of the continuum-continuum couplings. These couplings have shown to play an important role in the breakup process of a loosely bound projectile. To this end, in the following chapters, we devote the major part of our investigations on the effects on the different continuum-continuum couplings on the breakup cross sections.
Chapter 5

The \(^{19}\text{C} + ^{208}\text{Pb}\) breakup reaction

In chapter 3 and chapter 4, we have presented a detailed analysis of the effects of the different multipole transitions and interferences on the energy and angular distributions total, Coulomb and nuclear breakup cross sections and on the Coulomb-nuclear interference. Another important aspect to consider when studying the breakup reactions are the effects of the ccc on the different breakup cross sections. As already pointed out once included in the potential matrix element, these couplings reduce significantly the breakup cross sections. This chapter and the next one are devoted to the study the effects of the ccc on the energy and angular distributions breakup cross sections. As in the previous chapters, we start with a brief description of the \(^{19}\text{C}\) projectile.

5.1 Description of \(^{19}\text{C}\) projectile

The \(^{19}\text{C}\) one-neutron halo nucleus is treated as a \(^{18}\text{C}\) core plus a loosely bound neutron \((^{19}\text{C} \rightarrow ^{18}\text{C} + n)\) \cite{8,12,15,18}. The widely adopted ground state configuration is \(^{19}\text{C} \rightarrow |^{18}\text{C}(0^+) \otimes n(2s_{1/2})\rangle\), to which corresponds a binding energy of 0.530\(\pm\)0.13 MeV, extracted from the data \cite{8}, and parity \(j^\pi = \frac{1}{2}^+\). We also adopt the same ground state configuration and binding energy. The \(^{18}\text{C} + n\) potential parameters used to obtain the bound and continuum wave functions are those of the \(^{14}\text{C} + n\) system, where again the depths of the central and spin-orbit couplings terms are adjusted to fit the experimental ground state binding energy. These two parameters are \(V_0 = 56.44\) MeV and \(V_{SO} = 23.761\) MeVfm\(^2\), respectively.
5.1.1 Ground and continuum state properties

In Fig. 5.1 are presented the ground state wave function (upper panel), and the continuum wave functions in different partial waves (lower panel) calculated at the three different energies. Considering $R_{\text{core}} = 2.82 \pm 0.19$ fm \[114\], one observes again that this wave function is largely extended beyond the size of the core, similar to the $^{10}\text{Be} + n$ ground state wave function plotted in Fig. 3.1. The two ground state wave functions compare rather well, despite the different core masses, owing largely to the slight difference in their ground binding energies. It is therefore clear that if we calculate the probabilities of finding the neutron inside and outside the size of the halo, we would end up with the same conclusions. For the continuum wave functions, one observes a more pronounced

Figure 5.1: Ground state wave function (upper panel) and continuum wave functions in different partial waves (lower panel), calculated at the resonance energies for the $^{18}\text{C} + n$ system.
resonance in the $d_{\frac{5}{2}^+}$ partial wave with a resonance energy of 0.801 MeV. Also $s_{\frac{1}{2}^+}$ and $d_{\frac{3}{2}^+}$ states are resonant states, with resonance energies of 0.961 MeV and 6.841 MeV, respectively. These resonances are clearly observed in Fig. 5.2, which represents the plot of the nuclear phase shifts as functions of the excitation energies.

5.1.2 Electric response functions

In Fig. 5.3, the integrand of the radial integral (3.5), is plotted for the same three arbitrary energies, where similar results as in Fig. 3.4 are observed, leading to similar conclusions. The electric response functions for the transitions from the ground state to $p_{\frac{1}{2}^-}$, $p_{\frac{3}{2}^-}$ and $p$ continuum states are shown in Fig. 5.4 where magnitudes lower than in Fig. 3.5 are noticed. This can be attributed, among other reasons to the slight difference in the ground state binding energies and the effective charges ($Z_{\text{eff}} = 0.364$ for $^{11}\text{Be}$ and $Z_{\text{eff}} = 0.316$ for $^{19}\text{C}$).
\[ C_1 + n = 0.5 \text{ MeV} \]
\[ \varepsilon = 0.3 \text{ MeV} \]
\[ \varepsilon = 0.1 \text{ MeV} \]

\[ \phi_{n_0}^{(r)} \phi_{k_0}^{(r)} (r) \text{[fm}^{-1/2}] \]

Figure 5.3: Integrand of the radial integral (3.5), for the transition \((0, \frac{1}{2}^+) \rightarrow (1, \frac{3}{2}^-)\).

Figure 5.4: Electric response functions for the transitions from the ground state to the \(p_{\frac{1}{2}^-}, p_{\frac{3}{2}^-}\) and \(p\) continuum states.

5.2 The \(^{18}\text{C} + ^{208}\text{Pb}\) and \(n + ^{208}\text{Pb}\) potentials

The parameters of the \(^{18}\text{C} + ^{208}\text{Pb}\) nuclear potential, taken from [59], are listed in Table 5.1, while for the \(n + ^{208}\text{Pb}\) nuclear potential, the same parameters presented in Table 4.2 are used. The \(^{18}\text{C} + ^{208}\text{Pb}\) Coulomb potential is still a point-sphere Coulomb potential, where \(Z_c = 6\), and \(R_C = 1.25(18^{1/3} + 208^{1/3}) \simeq 10.682\) fm. Regarding the solution of the coupled equations (2.52), the same CDCC model space already as described in section 3.3, is also employed for this reaction. This model space was found to be enough to guarantee the convergence of the results.
Table 5.1: Parameters of the real and imaginary depths, radii and diffusenesses of the $^{18}$C+$^{208}$Pb nuclear potential. The parameters are taken from Ref. [59].

<table>
<thead>
<tr>
<th></th>
<th>$V$ (MeV)</th>
<th>$W$ (MeV)</th>
<th>$R_R$ (fm)</th>
<th>$R_I$ (fm)</th>
<th>$a_R$ (fm)</th>
<th>$a_I$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{18}$C+$^{208}$Pb</td>
<td>200.00</td>
<td>76.20</td>
<td>5.39</td>
<td>6.58</td>
<td>0.9</td>
<td>0.38</td>
</tr>
</tbody>
</table>

5.3 Energy distributions breakup cross sections

The energy distributions total, Coulomb and nuclear breakup cross sections as well as the Coulomb-nuclear interference are considered in this section. For a clear investigation of the effects of the ccc on these breakup cross sections, we first consider the case where all the different couplings are included in the potential matrix element. After we will consider the case where the ccc are excluded, meaning that we retain only couplings to and from the ground state.

5.3.1 Partial waves analysis

The partial wave differential breakup cross sections, corresponding to each partial wave and where all the couplings are included, are displayed in Fig. 5.5. The results show similarities with the results presented in Figs. 3.7 and 4.5. That is to say that the $p$-wave differential breakup cross section is largely dominant, but the different contributions of the other partial waves are important, showing also that all the outgoing neutrons are not in the $p$-waves.

5.3.2 Differential total, Coulomb and nuclear breakup cross sections

The differential total, Coulomb and nuclear breakup cross sections are displayed in Fig. 5.6. It is not surprising to see that the Coulomb breakup cross section is more dominant over the total breakup cross section, given the results obtained for the other two reactions already investigated. One similarly notices that the nuclear breakup cross section is not
Figure 5.5: Differential partial waves and total breakup cross sections. All the different couplings are included.

insignificant, especially at low excitation energies ($\varepsilon \leq 2$ MeV). We also observe that $\sigma_T \ll \sigma_C + \sigma_N$, which reflects of a strongly destructive Coulomb-nuclear interference. In

Figure 5.6: Energy distributions differential total, Coulomb and nuclear breakup cross sections, obtained in the presence of all the couplings.

Fig. 5.7 we present the differential total breakup cross section and the incoherent difference of the total and nuclear breakup cross sections ($T - N$), a difference which is regarded as the Coulomb breakup cross section containing the effect of the nuclear breakup. Looking at the figure, one notices that the ($T - N$) breakup cross section fits well the data at low excitation energies ($\varepsilon \leq 1$ MeV), while for $\varepsilon \geq 1$ MeV, the total breakup cross section results in a good fit of the data. This fitness of the data by the ($T - N$) breakup cross
Figure 5.7: Differential total and incoherent the incoherent difference \((T - N)\) breakup cross sections. The experimental data are taken from [8].

section, highlights the importance of the nuclear breakup cross section for this reaction, especially at low excitation energies. If we were to apply the same technique, for example in Fig. 4.6 it is clear that the \((T - N)\) breakup cross section would underestimate the data, showing the in the \(^{15}\text{C} + ^{208}\text{Pb}\) reaction, the nuclear breakup cross section is more important than in the \(^{19}\text{C} + ^{208}\text{Pb}\) reaction.

### 5.3.3 Energy-integrated breakup cross sections and Coulomb-nuclear interference

A better understanding of the effects of the different partial waves on the Coulomb and nuclear breakup cross sections and hence on the nature and magnitude of their interference, requires the integrated breakup cross sections. Therefore, we integrate numerically the different differential breakup cross sections using again equation (3.9), and summarize the results in Table 5.2. The partial wave integrated total, Coulomb and nuclear breakup cross sections are shown in the second, third and fourth columns, respectively. The incoherent Coulomb+nuclear sum is shown in the fifth column, the integrated \((T - N)\) breakup cross sections in the sixth column, and magnitudes and nature of the Coulomb-nuclear interferences [which are obtained using equations (3.10) and (3.15)] in the seventh and eighth columns. The sixth row represents the sums of all the partial waves, defined
Table 5.2: Different partial waves contributions to the integrated breakup cross sections. The numerical integration is performed up to $\epsilon_{\text{max}}=8$ MeV. The experimental value for the total Coulomb dissociation cross section is $1.190\pm0.119b$ [8]. All the cross sections are expressed in barns.

<table>
<thead>
<tr>
<th>Part. waves</th>
<th>$\sigma^T_\ell$</th>
<th>$\sigma^C_\ell$</th>
<th>$\sigma^N_\ell$</th>
<th>$\sigma^T_\ell + \sigma^C_\ell$</th>
<th>$\sigma^T_\ell - \sigma^C_\ell$</th>
<th>$\delta_\ell$</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>0.1459</td>
<td>0.1696</td>
<td>0.0515</td>
<td>0.2211</td>
<td>0.0944</td>
<td>-0.0752</td>
</tr>
<tr>
<td>p</td>
<td>1.0503</td>
<td>1.3668</td>
<td>0.0514</td>
<td>1.4182</td>
<td>0.9989</td>
<td>-0.3679</td>
</tr>
<tr>
<td>d</td>
<td>0.1743</td>
<td>0.1586</td>
<td>0.0722</td>
<td>0.2308</td>
<td>0.1021</td>
<td>-0.0565</td>
</tr>
<tr>
<td>f</td>
<td>0.0377</td>
<td>0.0304</td>
<td>0.0290</td>
<td>0.0594</td>
<td>0.0087</td>
<td>-0.0217</td>
</tr>
<tr>
<td>g</td>
<td>0.0295</td>
<td>0.0123</td>
<td>0.0196</td>
<td>0.0319</td>
<td>0.0099</td>
<td>-0.0024</td>
</tr>
<tr>
<td>$\hat{\sigma}_x$</td>
<td>1.4377</td>
<td>1.7377</td>
<td>0.2237</td>
<td>1.9614</td>
<td>1.214</td>
<td>-0.5237</td>
</tr>
</tbody>
</table>

as

$$\hat{\sigma}_x = \sum_{\ell=0}^{\ell_{\text{max}}} \sigma^I_\ell, \quad x \equiv T, C, N. \quad (5.1)$$

Concerning the different partial waves contributions, the table shows that for the total and Coulomb breakup cross sections, the $p$-wave breakup cross section is more dominant, and represents 71.11% and 78.66% of $\hat{\sigma}_T$ and $\hat{\sigma}_C$, respectively, while it only represent 22.98% of $\hat{\sigma}_N$, where the $d$-waves breakup cross section is slightly more important, with a contribution of 32.28%. The table also shows that above the $d$-waves, both the Coulomb and nuclear breakups contribute almost equally. In fact, it can be observed that for the $g$-waves, although insignificant, the nuclear breakup cross section takes over the Coulomb breakup cross section. Comparing our results with the data, we that the integrated $(T-N)$ breakup cross section, agrees quite well with the $1.190\pm0.119b$ value of [8]. Looking at Table 4.3 one observes that, if we were to use the same technique for the $^{15}$C + $^{208}$Pb reaction, for example, the $(T-N)$ integrated breakup cross section value would underestimate the experimental value. We now turn to the Coulomb-nuclear interference. We observe that in each partial wave, $\sigma^I_\ell \leq 0$ and $\hat{\delta}_\ell \leq 1$, thus reflecting a destructive Coulomb-nuclear partial interference in each partial wave. One also notices that $\hat{\sigma}_N < |\hat{\sigma}_I|$, showing that the overall integrated nuclear breakup cross section is lesser than the overall Coulomb-nuclear interference.
5.3.4 Role of the continuum-continuum couplings

In Table 5.2, where all the couplings are present, one sees that the Coulomb-nuclear interference is exclusively destructive and the Coulomb breakup cross section is more important than the total and nuclear breakup cross sections. We now exclude the ccc, in order to verify their effects on these conclusions. The only couplings remaining are the couplings to and from the ground state. The differential total, Coulomb and nuclear breakup cross sections are presented in Fig. 5.8. The results show that the differential total breakup cross section becomes more important than the differential Coulomb breakup cross section for the whole energy spectrum shown. On the other hand, the Coulomb breakup cross section is more important that the nuclear breakup cross section for $\varepsilon \leq 1$ MeV, whereas the nuclear breakup cross section prevails over the Coulomb breakup cross section for rest of the energy spectrum, starting around $\varepsilon \simeq 1.2$ MeV. Looking at Figs. 5.6 and 5.8 it follows that the effects of the ccc is to substantially reduce the total and nuclear breakup cross sections more than the Coulomb breakup cross section.

To analyze the effects of these ccc on the Coulomb-nuclear interference, we present in Table 5.3 the different integrated breakup cross sections. The results first show that in each partial wave, the nuclear breakup cross section is more important that the Coulomb breakup cross section. In terms of partial wave contributions, one finds that for the three breakups, the $p$-wave breakup cross sections are still leading. In fact, they amount to 66.33% of $\hat{\sigma}_T$, 99.47% of $\hat{\sigma}_C$ and 39.92% of $\hat{\sigma}_N$. The results show that for this reaction, in the absence of the ccc, and for the Coulomb breakup, one can only consider transitions to the $p$ continuum states without loosing the accuracy. Comparing Tables 5.2 and 5.3 we can conclude that for the total and nuclear breakups, the ccc remove flux from the other partial waves to feed up the $p$-waves. In contrast, they remove flux from the $p$-waves to feed up the other partial waves for the Coulomb breakup. Quantitatively, it can be deduced that the inclusion of the ccc in the coupling matrix element, reduces by 75.67% the total breakup cross section, by 4.26% the Coulomb breakup cross section and by 95.60% the nuclear breakup cross section. This once again highlights the fact that the ccc affect dramatically the nuclear breakup cross section than its Coulomb counterpart. Considering
Figure 5.8: Differential total, Coulomb and nuclear breakup cross sections in the absence of the continuum-continuum couplings.

Table 5.3: Integrated partial breakup partial cross sections (in barns). The continuum—continuum couplings are excluded.

<table>
<thead>
<tr>
<th>Part. waves</th>
<th>$\sigma_T^\ell$</th>
<th>$\sigma_C^\ell$</th>
<th>$\sigma_N^\ell$</th>
<th>$\sigma_C^\ell + \sigma_N^\ell$</th>
<th>$\sigma_T^\ell - \sigma_C^\ell$</th>
<th>$\sigma_I^\ell$</th>
<th>$\delta_\ell$</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>0.9320</td>
<td>0.0029</td>
<td>1.6964</td>
<td>1.6993</td>
<td>0.9291</td>
<td>-0.7644</td>
<td>0.55</td>
</tr>
<tr>
<td>p</td>
<td>3.9195</td>
<td>1.8055</td>
<td>2.0217</td>
<td>3.8272</td>
<td>2.1140</td>
<td>0.0923</td>
<td>1.05</td>
</tr>
<tr>
<td>d</td>
<td>0.7358</td>
<td>0.0065</td>
<td>0.9639</td>
<td>0.9639</td>
<td>0.7293</td>
<td>-0.2346</td>
<td>0.76</td>
</tr>
<tr>
<td>f</td>
<td>0.2286</td>
<td>0.0002</td>
<td>0.2763</td>
<td>0.2763</td>
<td>0.2284</td>
<td>-0.0479</td>
<td>0.83</td>
</tr>
<tr>
<td>g</td>
<td>0.0936</td>
<td>0.0000</td>
<td>0.1058</td>
<td>0.1058</td>
<td>0.0936</td>
<td>-0.0122</td>
<td>0.89</td>
</tr>
<tr>
<td>$\hat{\sigma}_s$</td>
<td>5.9095</td>
<td>1.8151</td>
<td>5.0641</td>
<td>6.8792</td>
<td>4.0944</td>
<td>-0.9697</td>
<td>0.81</td>
</tr>
</tbody>
</table>

the magnitude and nature of the Coulomb-nuclear interference, the observation is that this interference is still destructive in all the partial waves other than the p-waves. One then concludes that the ccc do not affect the nature of the Coulomb-nuclear interference in partial waves other than the p-waves.
5.4 Angular distributions breakup cross sections and diagonal and off-diagonal continuum-continuum couplings

In section 5.3, we only considered the case where the diagonal and off-diagonal continuum continuum couplings are included simultaneously in the potential matrix element. Such procedure does not provide an opportunity to analyze, especially the role of the off-diagonal ccc, which are generally perceived as less important. In this section, we separate the two kinds of ccc, and study their role on the total, Coulomb and nuclear breakup cross sections, in order to investigate the role of the off-diagonal ccc. We use the following notations: dccc (diagonal continuum-continuum couplings), odccc (off-diagonal continuum-continuum couplings) and accc (all continuum-continuum couplings). For the sake of clarity, the symbol $\sigma_{nccc}$ represents the breakup cross section resulting from couplings to and from the ground state only, $\sigma_{dccc}$ cross section resulting from couplings to and from the ground state plus the dccc, and $\sigma_{odccc}$ from couplings to and from the ground state plus the odccc, while $\sigma_{accc}$ breakup cross section resulting from all the different couplings. To analyze the role of the odccc on the different breakup cross sections, we will compare the dccc and accc results.

5.4.1 Differential breakup cross sections

The differential total breakup cross sections are displayed in Fig. 5.9 (a). The results show that in the absence of the ccc, the breakup cross section is much extended at large angles, starting in the vicinity of the grazing angle ($\theta_{gr} = 2.8^\circ$). From the same angle, when the dccc are included, it is noticed that there is a substantial reduction of the breakup cross section which becomes negligible beyond $7^\circ$, although it is slightly increased at forward angles (between $1^\circ$ and $2^\circ$). The inclusion of the odccc, results in a reduction of the breakup cross section at $1^\circ \leq \theta \leq 7^\circ$. We find that the ccc have no effects on the Coulomb breakup cross section [see Fig. 5.9 (b)], other than removing its oscillatory behavior. As for the nuclear breakup [Fig. 5.9 (c)], similar conclusions as in the total
Figure 5.9: Differential total (upper panel), Coulomb (middle panel) and nuclear (lower panel) breakup cross sections for the different continuum-continuum couplings.

breakup case are reached.
5.4.2 Angular-Integrated breakup cross sections and Coulomb-nuclear interference

Qualitatively Fig. 5.9 shows that both the dccc and odccc play a rather significant role in reducing the extension of the breakup cross section at large angles for the total and nuclear breakup cross sections. Here we look at the effects of these different couplings on the integrated breakup cross sections. These breakup cross sections are obtained after a numerical integration of the following integral

\[
\sigma_x = \int_0^{\theta_{\text{max}}} d\sigma_x d\theta.
\]  

(5.2)

The results are shown in Table 5.4, where it is seen that in the case of nccc, the nuclear breakup is again the more dominant process and the corresponding breakup cross section amounts to 70.94\% for the incoherent Coulomb+nuclear sum. In the presence of all the ccc, it is observed that the Coulomb breakup prevails, and the resulting breakup cross section contributes up to 76.72\% on their incoherent sum. Another remarkable aspect is that the dominance of the integrated Coulomb breakup cross section over the integrated nuclear breakup cross section is an effect of the odccc.

To analyze the effects of the dccc and odccc on the different breakup cross sections, we use the following equations

\[
\beta_{x}^{\text{accc}}(\%) = 1 - \frac{\sigma_x^{\text{accc}}}{\sigma_x^{\text{nccc}}}
\]

\[
\beta_{x}^{\text{dccc}}(\%) = 1 - \frac{\sigma_x^{\text{dccc}}}{\sigma_x^{\text{nccc}}}
\]

\[
\beta_{x}^{\text{odccc}}(\%) = \beta_{x}^{\text{accc}} - \beta_{x}^{\text{dccc}}.
\]  

(5.3)

Considering the total breakup, we find that the ccc reduce by 70.39\% the breakup cross section, distributed as follows: 59.28\% due to the dccc and 11.11\% due to the odccc. For the Coulomb breakup, it is deduced that the breakup cross section is slightly reduced by 4.71\%, where 1.32\% is due to the dccc and 3.39\% to the odccc. Looking at the nuclear
Table 5.4: Integrated angular distributions cross sections in barns. The numerical integration is performed up to \( \theta_{\text{max}} = 8^\circ \).

<table>
<thead>
<tr>
<th>( \sigma_T^{\text{nccc}} )</th>
<th>( \sigma_T^{\text{dccc}} )</th>
<th>( \sigma_T^{\text{accc}} )</th>
<th>( \sigma_C^{\text{nccc}} )</th>
<th>( \sigma_C^{\text{dccc}} )</th>
<th>( \sigma_C^{\text{accc}} )</th>
<th>( \sigma_N^{\text{nccc}} )</th>
<th>( \sigma_N^{\text{dccc}} )</th>
<th>( \sigma_N^{\text{accc}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>378.211</td>
<td>224.216</td>
<td>111.971</td>
<td>113.660</td>
<td>112.162</td>
<td>108.308</td>
<td>277.429</td>
<td>123.381</td>
<td>32.859</td>
</tr>
</tbody>
</table>

breakup, we deduce that the breakup cross section is substantially reduced by 88.16\%, owing to the ccc, where 55.53\% is due to the dccc and 32.63\% to the odccc.

Let us now consider the effects of the ncce, dcc, and odccc on the Coulomb-nuclear interference. From the table we can deduce that, \( \sigma_I^{\text{nccc}} = 378.211 \text{ b} - (113.660 + 277.429 \text{ b}) = -12.218 \text{ b} < 0 \), \( \sigma_I^{\text{dccc}} = 224.216 \text{ b} - (112.162 + 123.381 \text{ b}) = -11.327 \text{ b} < 0 \) and \( \sigma_I^{\text{accc}} = 111.971 \text{ b} - (108.308 + 32.859 \text{ b}) = -29.196 \text{ b} < 0 \). These results show that the odccc increase the destructiveness of the Coulomb-nuclear interference. Moreover, it is found that these different ccc do not affect the nature of this interference, which remains destructive regardless whether they are included or not, as already seen for the energy distributions.

We also consider the qualitative effects of the dcc and odccc on the Coulomb-nuclear interference. To this end, we display in Fig. 5.10 the angular distributions of the ratio (5.15), for the ncce, dcc, and accc. The results indicate that in the absence of the ccc, this ratio is negative at large angles (\( \theta \geq 6^\circ \)), which implies a destructive Coulomb-nuclear interference. The inclusion of the dcc happens to strongly reduce the destructiveness of this interference at large angles, since it is observed that \( \frac{d\hat{\delta}}{d\hat{\theta}} \to 0 \), while these dcc increase this ratio at \( 2.5^\circ \leq \theta \leq 3.5^\circ \). This ratio is decreased at \( 2.5^\circ \leq \theta \leq 4.5^\circ \) due to the inclusion of the odccc.
5.5 Effects of the continuum-continuum couplings on the Coulomb barrier penetration

As already indicated elsewhere in this work, the reduction of the fusion cross sections due to the ccc, results from the fact that these couplings increase the Coulomb barrier, and thus, lowering of the tunneling. In this section, we show that using the breakup cross sections, we can reach the same conclusions regarding the effects of the ccc on the fusion cross sections, which in turn provides a good example of the dependence of the fusion cross sections on the breakup process. We first determine the breakup cross sections inside the Coulomb barrier due to the projectile flux that penetrates the barrier, and the breakup outside the barrier, due to the penetration hindrance. To this end, we calculate the integrated breakup cross sections inside ($\sigma_{IB}^X$) and outside ($\sigma_{OB}^X$) the barrier, using the following expressions [115]

$$\sigma_{IB}^X = \int_{\theta}^{\theta_{\text{max}}} \frac{d\sigma_x}{d\theta} d\theta$$

$$\sigma_{OB}^X = \int_{0}^{\theta_{\text{max}}} \frac{d\sigma_x}{d\theta} d\theta.$$  \hspace{1cm} (5.4)

Figure 5.10: Effects of the different continuum-continuum couplings on the Coulomb-nuclear interferences.
Table 5.5: Estimated Coulomb+nuclear, Coulomb and nuclear integrated breakup cross sections inside and outside the Coulomb barrier (in barns).

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Coulomb</th>
<th>Nuclear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>nccc</td>
<td>dccc</td>
<td>accc</td>
</tr>
<tr>
<td>$\sigma_{IB}$</td>
<td>261.071</td>
<td>92.626</td>
<td>21.657</td>
</tr>
<tr>
<td>$\sigma_{OB}$</td>
<td>123.287</td>
<td>138.629</td>
<td>92.569</td>
</tr>
</tbody>
</table>

The results are presented in Table 5.5 show that for the total breakup, and in the absence of the ccc, $\sigma_{IB}^{nccc}$ represents 67.92% of the $\sigma_{IB}^{nccc} + \sigma_{OB}^{nccc}$ sum. However, when the dccc are included, we deduce that the $\sigma_{IB}^{nccc}$ contribution on the same sum is reduced to 40.05%, which is further reduced to only 18.96%, when the odccc are included. This shows clearly the reduction of the flux that penetrates the Coulomb barrier as a result of the inclusion of the ccc in the potential matrix element.

Looking at the Coulomb breakup, as one could expect, it is seen that $\sigma_{OB} > \sigma_{IB}$ for the three different cases, where a slight decrease of the $\sigma_{IB}$ is observed as the dccc and odccc are included. A more clear effect of the ccc on the Coulomb barrier is seen for the nuclear breakup. Since there is no Coulomb barrier in this case, we see that $\sigma_{OB}^{nccc} \approx \frac{1}{6} \sigma_{IB}^{nccc}$, showing that almost all the projectile flux reaches the target. It is interesting to see that $\sigma_{OB} > \sigma_{IB}$, owing mostly to the odccc. This allows one to conclude that in the absence of the natural Coulomb barrier, the ccc create a kind of barrier that hinders the penetration of the projectile flux.

To summarize in this chapter, we have analyzed the effects of the ccc on the energy and angular distributions total, Coulomb and nuclear breakup cross sections. It is concluded that the ccc (both diagonal and off-diagonal), once included in the potential matrix element, the result in a substantial reduction of the total and nuclear breakup cross sections, while they insignificantly affect the Coulomb breakup cross section, compare to the total and nuclear nuclear breakup cross sections. For the angular distributions, we considered the dccc and odccc separately. We found that the dccc are responsible for the large reduction of the differential total and nuclear breakup cross sections at backward angles. At
forward angles however, this reduction is due largely to the odccc. Considering the effects of these different ccc on the Coulomb-nuclear interference, we noticed that the odccc have a pronounced quantitative effect on the interference than the dccc. However, we found that these couplings do not affect the nature of this interference, which is destructive regardless whether they are included are not. On the penetration of the Coulomb barrier, we found that the ccc reduce the flux that cross the Coulomb barrier, hence the lowering of the tunneling.

These results are obtained for a neutron-halo projectile and for a heavy target. It is interesting to verify whether these conclusions can be extended to a proton-halo projectile. To this end, in the next chapter, we consider the same analysis for a proton-halo projectile, for two different targets and different incident energies. This will provide an opportunity of assessing the effects of the target mass, energy regime on the above conclusions.
Chapter 6

The \( ^{8}\text{B} + ^{58}\text{Ni} \) and \( ^{8}\text{B} + ^{208}\text{Pb} \) reactions

Until now, we have limited our study to the breakup of neutron-halo projectiles. In this chapter, we now consider the breakup of the proton-halo projectile \( ^{8}\text{B} \). The main concern here is to verify whether the conclusions drawn for the neutron-halo breakup can be extended to these reactions. Particularly, we investigate the effects of the ccc on the energy and angular distributions total, Coulomb and nuclear breakup cross sections, on the Coulomb-nuclear interference as well as on the penetration of the Coulomb barrier. This will allow at least a qualitative comparison with the results obtained in chapter 5 and therefore assessing the role of the nucleon charge in the breakup process. On the other hand, considering different target masses and incident energies, allows the investigation of the effect of the target mass and the incident energy regime on the obtained results.

6.1 Description of \( ^{8}\text{B} \) projectile

The proton-halo nucleus \( ^{8}\text{B} \) is commonly described as \( ^{8}\text{B} \rightarrow ^{7}\text{Be} + p \). The spectrum of this nucleus is known to contain only one bound state, with \( j^\pi = 2^+ \), and a proton binding energy of 0.137 MeV. This proton is in a \( 1p_{3/2} \) orbit, coupled to the \( 3^2^- \) state of \( ^{7}\text{Be} \). The \( ^{7}\text{Be} + p \) potential used to calculate the bound and continuum wave functions, consists of both nuclear and Coulomb terms. The nuclear term contains a Woods-Saxon plus a spin-orbit coupling components, whose parameters, taken from Ref. [115] are, \( V_0 = 44.65 \) MeV, \( V_{SO} = 19.59 \) MeVfm\(^2\), \( R_0 = R_{SO} = 2.391 \) fm and \( a_0 = a_{SO} = 0.52 \) fm. The Coulomb term on the other hand, is a point-sphere Coulomb potential of radius \( R_C = 2.391 \) fm.
6.1.1 Ground and continuum state wave functions

Fig. 6.1 displays the bound (upper panel) and continuum (lower panel) wave functions. Looking at the bound state wave function, it is clear that we can draw similar conclusions, regarding the properties of the halo state as for the other projectiles. The continuum wave function show that the continuum of this nucleus in not structureless, as a clear resonance is observed. In Fig. 6.2, this resonance is seen to be in the \( j^\pi = \frac{3}{2}^- \) state, and other resonances are observed in the \( j^\pi = \frac{1}{2}^- \) and \( j^\pi = \frac{5}{2}^- \) states.

Figure 6.1: Ground state wave function (upper panel), and continuum state wave function, calculated at the resonance energy (lower panel).
6.2 Projectile-target interactions and CDCC model space

The different core-target and proton-target optical nuclear potentials are still Woods–Saxon potentials as for the previous reactions. However, here the proton-target imaginary potentials have additional surface terms. The proton-target potential parameters are given in Table 6.1 while the $^7$Be-target potential parameters are listed 6.2. The projectile-target Coulomb potentials are also point-sphere Coulomb potentials. The different parameters of the the CDCC model space employed for these reactions are given in Table 6.3.

Table 6.1: Optical model parameters for the nucleon–target used in the calculations. The parameters $V_0$, $r_0$, $a_0$ refer for to the depth, reduced radius and diffuseness of the real part, $W_V$, $r_V$, $a_V$ stand for the volume imaginary part, while $W_D$, $r_D$, $a_D$ correspond to the surface imaginary part. The reduced radii are converted to absolute radii as $R_x = r_x A_x^{1/3}$.

<table>
<thead>
<tr>
<th></th>
<th>$V_0$</th>
<th>$r_0$</th>
<th>$a_0$</th>
<th>$W_V$</th>
<th>$r_V$</th>
<th>$a_V$</th>
<th>$W_D$</th>
<th>$r_D$</th>
<th>$a_D$</th>
<th>$r_C$</th>
<th>Ref.</th>
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<td>$p + ^{58}$Ni</td>
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<td>1.17</td>
<td>0.75</td>
<td>7.24</td>
<td>1.26</td>
<td>0.58</td>
<td>2.59</td>
<td>1.26</td>
<td>0.58</td>
<td>1.25</td>
<td>[120]</td>
</tr>
<tr>
<td>$p + ^{208}$Pb</td>
<td>59.1</td>
<td>1.244</td>
<td>0.646</td>
<td>0.52</td>
<td>1.244</td>
<td>0.646</td>
<td>8.41</td>
<td>1.246</td>
<td>0.58</td>
<td>0.615</td>
<td>[74]</td>
</tr>
</tbody>
</table>

Figure 6.2: Resonance structure in different continuum states.
Table 6.2: Optical potential parameters for the core-target and CDCC input parameters. The different parameters are explained in Table 6.1. The reduced radii are converted to absolute radii as $R_x = r_x(A_x^{1/3} + A_y^{1/3})$

<table>
<thead>
<tr>
<th>Core−target potential parameters</th>
<th>$V_0$ (MeV)</th>
<th>$r_0$ (fm)</th>
<th>$a_0$ (fm)</th>
<th>$W_r$ (MeV)</th>
<th>$a_W$ (fm)</th>
<th>$r_C$ (fm)</th>
<th>Ref.</th>
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<tr>
<td>$^7\text{Be}+^{58}\text{Ni}$</td>
<td>150.0</td>
<td>1.190</td>
<td>0.50</td>
<td>60.0</td>
<td>1.150</td>
<td>0.62</td>
<td>[120]</td>
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<tr>
<td>$^7\text{Be}+^{208}\text{Pb}$</td>
<td>114.2</td>
<td>1.286</td>
<td>0.853</td>
<td>12.4</td>
<td>1.739</td>
<td>0.807</td>
<td>-</td>
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</tbody>
</table>

Table 6.3: CDCC model space input parameters.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$\ell_{\text{max}}$ ($\hbar$)</th>
<th>$\varepsilon_{\text{max}}$ (MeV)</th>
<th>$\lambda_{\text{max}}$</th>
<th>$r_{\text{max}}$ (fm)</th>
<th>$R_{\text{max}}$ (fm)</th>
<th>$\Delta_r$ (fm)</th>
<th>$\Delta_R$ (fm)</th>
<th>$L_{\text{max}}$ ($\hbar$)</th>
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</thead>
<tbody>
<tr>
<td>$^8\text{B}+^{58}\text{Ni}$</td>
<td>3</td>
<td>8</td>
<td>3</td>
<td>60</td>
<td>500</td>
<td>0.1</td>
<td>0.1</td>
<td>1000</td>
</tr>
<tr>
<td>$^8\text{B}+^{208}\text{Pb}$</td>
<td>4</td>
<td>9</td>
<td>4</td>
<td>60</td>
<td>1000</td>
<td>0.1</td>
<td>0.01</td>
<td>1000</td>
</tr>
</tbody>
</table>

6.3 Energy distributions breakup cross sections

The differential energy distributions and energy integrated total, Coulomb and nuclear breakup cross sections are discussed in this section. Cases where the ccc are included and excluded are considered, similar to the procedure adopted in chapter 5.

6.3.1 Partial waves analysis

The differential partial waves breakup cross sections for the $^8\text{B} + ^{58}\text{Ni}$ reaction are presented in Fig. 6.3 for the case where all the different couplings are included. This figure shows that the $p$-wave breakup cross section is dominant at much lower excitation energies, with a peak in the vicinity of the ground state binding energy. However, it quickly drops to become negligible for $\varepsilon \geq 0.5$ MeV, showing that it will not be dominant for the integrated breakup cross sections. The $s$-wave breakup cross section on the other hand, peaks at 0.5 MeV, and is even more important than the total breakup cross section for excitation energies in the 0.5 MeV $\leq \varepsilon \leq 1.5$ MeV interval. The $d$-and $f$-wave breakup cross sections happen to be important at high excitation energies. For the $^8\text{B} + ^{208}\text{Pb}$ reaction
Figure 6.3: Differential partial wave breakup cross sections for the $^8$B + $^{58}$Ni reaction.

Figure 6.4: Differential partial wave breakup cross sections for the $^8$B + $^{208}$Pb reaction.

however, which involves a heavy target and an incident energy much above the Coulomb barrier, the differential partial wave breakup cross sections are presented in Fig. 6.4. The results indicate, unlike for the $^8$B + $^{58}$Ni reaction, that the total breakup cross section is more dominant at the whole range of excitation energies displayed, followed by the $s$-wave and $p$-wave breakup cross sections. Again the $d$-wave breakup cross section slightly dominates at high excitation energies. It is observed that the $f$- and $g$-wave breakup cross sections are small, although not negligible. These results show that the breakup cross sections for the two breakup reactions are both qualitatively and quantitatively different. It is important to investigate whether this difference is due to the different target masses or incident energies, by either considering the same incident energy or the same target.
Figure 6.5: Energy distributions differential total, Coulomb and nuclear breakup cross sections for the $^8\text{B} + ^{58}\text{Ni}$ reaction, (a) in the presence of all the different couplings, (b) when the continuum-continuum couplings are excluded.

### 6.3.2 Effects of the continuum-continuum couplings

We now consider the effects of the ccc on the differential total, Coulomb and nuclear breakup cross sections. For the $^8\text{B} + ^{58}\text{Ni}$ reaction, the differential total, Coulomb and nuclear breakup cross sections are presented in Fig. 6.5 (a), for the case where the ccc are included and in Fig. 6.5 (b), when they are excluded. When the ccc are included, it is seen that the Coulomb breakup cross section dominates both the total and nuclear breakup cross sections, at the whole range of excitation energies, similar to the results shown in Fig. 5.6. Considering the case where the ccc are excluded, one observes a spectacular rise of the nuclear breakup cross section, which becomes more important than
its Coulomb breakup counterpart, and slightly important than the total breakup cross section at $\varepsilon \geq 0.5$ MeV. This shows again that the total and nuclear breakups are the most affected by the ccc than the Coulomb breakup, in line with the conclusions drawn in chapter 5.

For the $^8$B + $^{208}$Pb reaction, the breakup cross sections obtained are displayed in Figs. 6.6 (a) and (b), in the presence and absence of the ccc, respectively. It is also seen that the total and nuclear breakup cross sections are substantially reduced by the ccc. Again, when the ccc are excluded, we observe that the total and nuclear breakup cross sections are more important than the Coulomb breakup cross section and are much extended to
high excitation energies. These results, together with the results in chapter\textsuperscript{5} serve to conclude that the qualitative effects of the ccc on the energy distributions breakup cross sections do not depend on the nucleon charge, target mass and the incident energy, at least for the three reactions. It is noticed that the dependence of the breakup cross sections on the three parameters is rather quantitative.

6.3.3 Energy integrated breakup cross sections

Let us now consider the quantitative effects of the ccc on the integrated total, Coulomb and nuclear nuclear breakup cross sections as well as on the Coulomb-nuclear interference. The energy integrated breakup cross sections are presented in Table\textsuperscript{6.4} for the $^8\text{B} + ^{58}\text{Ni}$ reaction and for both cases. Looking first at the case where the ccc are included, it is shown that for the total and Coulomb breakups, the $s$-wave integrated breakup cross sections are more important, followed by the $d$-wave for the total breakup and by the $f$-wave for the Coulomb breakup. For the nuclear breakup on the other hand, the $d$-wave integrated breakup cross section is dominant. As already seen, the integrated Coulomb breakup cross section prevails over both the integrated total and nuclear breakup cross sections. As a result, the Coulomb-nuclear interference is exclusively destructive. It is noticed that the exclusion of the ccc, leads to a dominance of the nuclear breakup over both the total and Coulomb breakups. Quantitatively, we deduce that the exclusion of the ccc, increases the integrated total and nuclear breakup cross section, respectively by 81.00\% and 93.68\%, while it increases the by 46.78\% the integrated Coulomb breakup cross section. Again the Coulomb-nuclear interference remains destructive in all the partial waves other than the $p$-waves. These qualitative conclusions are the same as those drawn from Tables\textsuperscript{5.2} and\textsuperscript{5.3} for the $^{19}\text{C} + ^{208}\text{Pb}$ reaction. However, here we find that the ccc have a much pronounced effect on the integrated Coulomb breakup cross section.

In Table\textsuperscript{6.5} we show the integrated breakup cross sections for the $^8\text{B} + ^{208}\text{Pb}$ reaction in the presence and absence of the ccc. It is seen that for the Coulomb breakup, the $p$-wave breakup cross section becomes more dominant followed by the $s$-wave breakup cross section. A much reduced nuclear breakup cross section compared to the $^8\text{B} + ^{58}\text{Ni}$
Table 6.4: Different partial waves contributions to the integrated breakup cross sections (in barns) in the presence and absence of the ccc for the $^8$B+$^{58}$Ni reaction. The numerical integration is performed up to $\varepsilon_{\text{max}}=8$ MeV, $\sigma_{\text{CN}}^f = \sigma_C^f + \sigma_N^f$.

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<th>Part. waves</th>
<th>All couplings</th>
<th>No ccc</th>
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<tbody>
<tr>
<td>s</td>
<td>$\sigma_T^f$</td>
<td>$\sigma_C^f$</td>
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<tr>
<td></td>
<td>0.042</td>
<td>0.041</td>
</tr>
<tr>
<td>p</td>
<td>0.017</td>
<td>0.026</td>
</tr>
<tr>
<td>d</td>
<td>0.024</td>
<td>0.027</td>
</tr>
<tr>
<td>f</td>
<td>0.016</td>
<td>0.030</td>
</tr>
<tr>
<td>$\hat{\sigma}_s$</td>
<td>0.099</td>
<td>0.124</td>
</tr>
</tbody>
</table>

Table 6.5: Different partial waves contributions to the integrated breakup cross sections (in barns) in the presence and absence of the ccc for the $^8$B+$^{208}$Pb reaction.

<table>
<thead>
<tr>
<th>Part. waves</th>
<th>All couplings</th>
<th>No ccc</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>$\sigma_T^f$</td>
<td>$\sigma_C^f$</td>
</tr>
<tr>
<td></td>
<td>0.255</td>
<td>0.311</td>
</tr>
<tr>
<td>p</td>
<td>0.174</td>
<td>0.416</td>
</tr>
<tr>
<td>d</td>
<td>0.159</td>
<td>0.250</td>
</tr>
<tr>
<td>f</td>
<td>0.042</td>
<td>0.094</td>
</tr>
<tr>
<td>g</td>
<td>0.026</td>
<td>0.054</td>
</tr>
<tr>
<td>$\hat{\sigma}_s$</td>
<td>0.656</td>
<td>1.125</td>
</tr>
</tbody>
</table>

is observed, and the total breakup cross section is reduced such that $\hat{\sigma}_C = 1.715\hat{\sigma}_T$, instead of $\hat{\sigma}_C = 1.253\hat{\sigma}_T$ for the $^8$B+$^{58}$Ni reaction. The exclusion of the ccc increases the both total and nuclear breakup cross sections by 92.95% and 99.67%, respectively, and by 39.06% the Coulomb breakup cross section. It can then be concluded that for this reaction, the ccc are much stronger for the total and nuclear breakups than in the $^8$B+$^{58}$Ni reaction case, where these couplings are more stronger for the Coulomb breakup. The consequence of the ccc strength is a more destructive Coulomb-nuclear interference.
6.4 Angular distributions cross sections

We now turn to the angular distributions cross sections, where we consider first the effects of the ccc on the elastic scattering cross sections.

6.4.1 Elastic scattering cross sections

In Fig 6.7, we present the results obtained for the elastic scattering cross sections, for the three reactions. The case where all the couplings are included (full curves), and the case where only the dccc (dotted curves) are included are presented. A closer look at this figure shows that, the effect of the ccc decreases as the charge of the target increases. Similar conclusions were drawn in Refs [71, 80].

![Figure 6.7: Elastic scattering cross sections for the $^8$B $+$ $^{58}$Ni reaction (left panel) and for the $^8$B $+$ $^{208}$Pb reaction (right panel) in the presence of all the couplings and in the absence of the off-diagonal couplings. The experimental data are from [83, 84].](image)

6.4.2 Differential breakup cross sections

The differential angular distributions elastic breakup cross sections, are presented in Figs 6.8 and 6.9 for the $^8$B$+$ $^{58}$Ni, $^8$B$+$ $^{208}$Pb reactions, respectively. Let us first consider the $^8$B$+$ $^{58}$Ni reaction, with the case where all the Coulomb and nuclear interactions are included coherently [Fig 6.8(a)]. This figure shows that, the ncct breakup cross section is much extended to larger angles, starting in the vicinity of the grazing angle ($\theta_{gr} \sim 50^\circ$),
Figure 6.8: Differential total (upper panel), Coulomb (middle panel) and nuclear (lower panel) breakup cross sections for the $^8\text{B} + ^{58}\text{Ni}$ reaction, for the different continuum-continuum couplings.

and exhibits a minimum around 30°, similar to the results of [57,76]. When the dccc are included, the results show that, this extension is completely removed, and the resulting
breakup cross section drops rapidly starting as well in the vicinity of the grazing angle, to become negligible beyond 100°. However, this breakup cross section is increased at forward angles (10° ≤ θ ≤ θ_gr). The same figure shows that, including the odccc (that is to have all the ccc included), the corresponding breakup cross section (which is compared to the one obtained in the dccc case), increases at θ ≥ 70° and decreases at 10° ≤ θ ≤ 70°. On the light of these results, it can be concluded that, the dramatic decrease of the Coulomb+nuclear breakup cross section beyond the grazing angle, is largely an effect of the dccc. On the other hand, its decrease below the grazing angle, is an exclusive effect of the odccc. The Coulomb breakup results, as presented in Fig. 6.8(b) (where the nuclear interactions are switched off) show that, the inclusion of the dccc increases the breakup cross section at the whole range of angles starting from 10°. It can be observed that, the inclusion of the odccc decreases the breakup cross section, also starting from 10°, while their effect beyond 100° is negligible. Looking at [Fig. 6.8(c)] for the nuclear breakup, we can draw similar conclusions as in the Coulomb+nuclear case. However, at θ ≤ 20°, the effect of the dccc is negligible. Moreover, the inclusion of the odccc shows an increase of the breakup cross section at θ ≤ 10°. The results in Figs. 6.8(b) and (c) serve to conclude that, the effect of the ccc on the Coulomb+nuclear breakup cross section is much dominated by the nuclear breakup.

We now come to the 8B+208Pb reaction, where the results are presented in Fig. 6.9. It is interesting to see that, the effect of the ccc show some similarities with the 8B+58Ni reaction. However, for the Coulomb breakup+nuclear case [Fig. 6.9(a)], the odccc reduce the breakup cross section at the whole range of angles, starting from 5°. Concerning the Coulomb breakup [Fig. 6.9(b)], it can be seen that, the ccc have small effect, in comparison with 8B+58Ni reaction and the oscillatory pattern of the accc breakup cross section is due to the dccc. For the nuclear breakup [Fig. 6.9(c)], the observation is that, the dccc increase the nuclear breakup cross section at θ ≤ 5°, and results in its huge reduction at θ ≥ 15°. The inclusion of the odccc, leads to a large decrease of the breakup cross section at 5° ≤ θ ≤ 40°.
Figure 6.9: Differential total (upper panel), Coulomb (middle panel) and nuclear (lower panel) breakup cross sections for the $^8B + ^{208}Pb$ reaction, for the different continuum-continuum couplings.
Table 6.6: Integrated angular distributions cross sections in barns. The numerical integrations are performed up to $\theta_{\text{max}} = 180^\circ$ for $^8\text{B}^+\text{Ni}$ reaction and $\theta_{\text{max}} = 40^\circ$ for $^8\text{B}^+\text{Pb}$ reaction.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Total $\sigma^nccc$</th>
<th>Total $\sigma^dccc$</th>
<th>Total $\sigma^{acc}$</th>
<th>Coulomb $\sigma^nccc$</th>
<th>Coulomb $\sigma^dccc$</th>
<th>Coulomb $\sigma^{acc}$</th>
<th>Nuclear $\sigma^nccc$</th>
<th>Nuclear $\sigma^dccc$</th>
<th>Nuclear $\sigma^{acc}$</th>
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</thead>
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<tr>
<td>$^8\text{B}^+\text{Ni}$</td>
<td>54.106</td>
<td>17.143</td>
<td>9.900</td>
<td>21.040</td>
<td>28.378</td>
<td>14.042</td>
<td>52.814</td>
<td>17.087</td>
<td>3.870</td>
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<tr>
<td>$^8\text{B}^+\text{Pb}$</td>
<td>236.791</td>
<td>72.198</td>
<td>42.094</td>
<td>90.684</td>
<td>114.403</td>
<td>82.830</td>
<td>207.336</td>
<td>39.563</td>
<td>4.638</td>
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</tbody>
</table>

6.5 Integrated breakup cross sections

Qualitatively, we have seen in the above discussions that the odccc play an important role at small scattering angles. For a quantitative understanding of the effects of dccc and odccc, we obtain again the different integrated breakup cross sections by computing numerically the integral (5.2), for these reactions under investigation. The computed integrated breakup cross sections are given in Table 6.6. As already observed for the differential breakup cross sections, the integrated total and nuclear breakup cross sections are substantially reduced as the dccc and odccc are included. For the Coulomb breakup, one notices that the Coulomb breakup cross section is increase as a result of the inclusion of the dccc, which is afterwards reduced when the odccc are included.

Concerning the Coulomb and nuclear contributions, it is clearly that in the nccc case, the nuclear breakup the more dominant process and one deduces that the corresponding integrated breakup cross sections amount to 71.51% and 69.75% of the integrated incoherent Coulomb+nuclear sum, for the $^8\text{B}^+\text{Ni}$ and $^8\text{B}^+\text{Pb}$ reactions, respectively. In the presence of the ccc, however, it is seen that the Coulomb breakup prevails, and the resulting integrated breakup cross sections contribute respectively up to 78.39% and 94.70% of the incoherent Coulomb+nuclear sum. This shows again that the nuclear breakup is the most affected by the ccc.

We use again equation (5.3), for a further analysis of the effects of the dccc and odccc on the total and nuclear integrated breakup cross sections. It is obtained for the $^8\text{B}^+\text{Ni}$
reaction that, the ccc reduce by 81.70% the total integrated breakup cross section, distributed as: 68.34% due to the dcce and 13.39% due to the odccc. For the Coulomb breakup on the other hand, the dcce increase the integrated breakup cross section by 34.88%, which is turn is reduced by 50.52%, owing to the odccc. Finally for the nuclear breakup, it is deduced that the ccc dramatically reduce the breakup cross section by 92.67%, distributed as follows: 67.65%, due to the dcce and 25.02%, due to the odccc. Turning to the $^8$B+$^{208}$Pb reaction, one obtains that the ccc reduce by 82.22%, the total integrated breakup cross section, distributed as follows: 69.51%, due to the dcce and 12.71%, to the odccc. It is obtained that for the Coulomb breakup, the dcce again increase the integrated breakup cross section by 26.16%, which is reduced by 25.85%, owing to the odccc. Looking finally at the nuclear breakup, a dramatic reduction by 97.76% of the breakup cross section, due to the ccc, where 80.92% is due to the dcce and 16.84% to the odccc is obtained.

Considering the effects of the dcce and odccc on the Coulomb-nuclear interference, and observing once again the results in Table 5.6, one has that, $\sigma_{I}^{cccc} = 54.106 \text{ b} - (21.040 \text{ b} + 52.814 \text{ b}) = -19.748 \text{ b}$, $\sigma_{I}^{dccc} = 17.143 \text{ b} - (28.378 \text{ b} + 17.087 \text{ b}) = -28.322 \text{ b}$, and $\sigma_{I}^{accc} = 9.900 \text{ b} - (14.042 \text{ b} + 3.870 \text{ b}) = -8.012 \text{ b}$, for the $^8$B+$^{58}$Ni Reaction. For the $^8$B+$^{208}$Pb Reaction, we obtain the following results: $\sigma_{I}^{cccc} = 236.791 \text{ b} - (90.684 \text{ b} + 207.336 \text{ b}) = -61.229 \text{ b}$, $\sigma_{I}^{dccc} = 72.198 \text{ b} - (114.403 \text{ b} + 39.563 \text{ b}) = -81.768 \text{ b}$ and $\sigma_{I}^{accc} = 42.094 \text{ b} - (82.830 \text{ b} + 4.638 \text{ b}) = -45.374 \text{ b}$. These results show for both reactions an increase of the Coulomb-nuclear interference due to the dcce, while this interference is reduced, owing to the odccc. For the $^{19}$C + $^{208}$Pb reaction, we have rather seen that the dcce slightly decrease the Coulomb breakup cross section, and the Coulomb-nuclear interference is increased due to the odccc, contrary to what is observed for the two reactions. We can say that this is the major difference so far observed between the $^{19}$C and $^8$B breakups. One can argue that this difference, among other reasons is due to the valence nucleon charge in the $^8$B, since the different incident energies and target masses considered in for the $^8$B+$^{58}$Ni and $^8$B+$^{208}$Pb reactions do not affect the results. However, a study where the same energy is considered for both the $^{19}$C + $^{208}$Pb and $^8$B+$^{208}$Pb reactions, for example is needed to enforce this conclusion.
Figure 6.10: Effects of the different continuum-continuum couplings on the Coulomb-nuclear interferences, for the $^8\text{B} + ^{58}\text{Ni}$ reaction (upper panel) and for the $^8\text{B} + ^{208}\text{Pb}$ reaction (lower panel).

We consider the qualitative effect of the dccc and odccc on the Coulomb-nuclear interference, where the angular distributions of the Coulomb-nuclear interference are displayed in Fig. 6.10. The results show that, for both the $^8\text{B} + ^{58}\text{Ni}$ (upper panel) and $^8\text{B} + ^{208}\text{Pb}$ (lower panel) reactions, and in the nccc case, this interference is strongly destructive where the $\sigma_{\text{dccc}}$ crosses the $\sigma_{\text{nccc}}$ (i.e in the vicinity of the grazing angles). This shows that, in the vicinity of the grazing angle, the nuclear breakup is more important than the total breakup. Also for the $^8\text{B} + ^{58}\text{Ni}$ reaction, one sees that the Coulomb-nuclear interference is exclusively constructive at $\theta \geq 80^\circ$. This, among other reasons, is due to the fact that the Coulomb breakup cross section significantly decreases in this region. As for the $^8\text{B} + ^{208}\text{Pb}$ reaction, it is seen that at $\theta \geq 20^\circ$, this interference is comparatively much less
Table 6.7: Estimated total, Coulomb and nuclear integrated breakup cross sections inside and outside the Coulomb barrier (in barns).

<table>
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</thead>
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<td>nccc  dccc accc</td>
<td>nccc  dccc accc</td>
<td>nccc  dccc accc</td>
</tr>
<tr>
<td>$\sigma_{IB}$</td>
<td>39.421</td>
<td>1.969</td>
<td>1.725</td>
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<td>$\sigma_{OB}$</td>
<td>14.758</td>
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<td>8.178</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Coulomb</th>
<th>Nuclear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>nccc  dccc accc</td>
<td>nccc  dccc accc</td>
<td>nccc  dccc accc</td>
</tr>
<tr>
<td>$\sigma_{IB}$</td>
<td>164.434</td>
<td>2.933</td>
<td>0.778</td>
</tr>
<tr>
<td>$\sigma_{OB}$</td>
<td>72.618</td>
<td>69.307</td>
<td>41.328</td>
</tr>
</tbody>
</table>

constructive, which is not surprising given the importance of the Coulomb breakup cross section at these angles.

6.6 Effects of the continuum-continuum couplings on the Coulomb barrier penetration

Finally in this chapter, we analyze the effects of the dccc and odccc on the Coulomb barrier penetration. The inside and outside the Coulomb barrier breakup cross sections [obtained using equations (5.4)] are summarized in Table 6.7. Looking at this table, we firstly notice that for both reactions, $\sigma_{IB}^{nccc} \gg \sigma_{OB}^{nccc}$, except for the Coulomb breakup and for the $^8B+^{58}Ni$ reaction, where $\sigma_{IB}^{nccc} \approx \frac{1}{2} \sigma_{OB}^{nccc}$. However, the situation turns around when the ccc are included, as we observe that $\sigma_{IB}^{acc} \ll \sigma_{OB}^{acc}$. It is seen that this reduction of $\sigma_{IB}^{acc}$ is largely due to the dccc. These results highlight again the fact that the ccc, once included in the potential matrix element, they reduce the penetrability of the Coulomb barrier, which therefore results in the reduction of the fusion cross sections.
Chapter 7

Concluding Remarks

In this thesis, we have analyzed in more details the dynamics of the breakups the $^{11}$Be, $^{15}$C and $^{19}$C neutron-halo nuclei on the $^{208}$Pb target and the $^8$B proton halo on the $^{58}$Ni and $^{208}$Pb targets, at different incident energies. For both the $^{11}$Be+$^{208}$Pb and $^{15}$C+$^{208}$Pb breakup reactions, we considered the same incident energy, such that the only major different was the ground state binding energies. We were mostly interested in analyzing the effects of the first-and higher-order interferences on the total, Coulomb and nuclear breakup cross sections as well as on the magnitude and nature of the Coulomb-nuclear interference. A partial wave analysis was first performed in order to check the importance of each single partial wave included in the CDCC model space. It is shown that the $p$-wave breakup cross sections are dominant for both reactions, and dictate the shape of the differential total breakup cross section. However, considered alone, they overestimated the experimental data, hence the importance of the other partial waves, and consequently, all the outgoing neutrons are not in the $p$-waves.

The Total, Coulomb and nuclear breakups were analyzed separately, in order to understand especially the important of the nuclear breakup in a Coulomb breakup dominated reaction. To obtain a pure Coulomb/nuclear breakup, we switched off/on the nuclear interactions. Comparing with the experimental data, for the $^{11}$Be+$^{208}$Pb reaction, we observed that the differential total breakup cross section fits well the data at low excitation energies ($\varepsilon \leq 0.5$ MeV). However, the differential Coulomb breakup cross section provided a fair fit of the data at higher excitation energies. We then concluded that the disagreement between the data and the theory at high excitation energies in [13], could not be attributed to the nuclear and/or higher-order effects as pointed out in that reference. For the $^{15}$C+$^{208}$Pb reaction, however, we found that the differential total breakup
cross section fits well the experimental data, while they are significantly overestimated by the differential Coulomb breakup cross section at the whole energy spectrum considered. This fitness was found to be an effect of the Coulomb-nuclear interference, which indicated that in this reaction, the nuclear breakup contribution cannot simply be ignored.

To analyze the effects of the first-order interference for these two reactions, we performed the first-order CDCC calculations by selecting single ($\lambda = 0, 1$) multipoles and their coherent sum $\lambda_{\text{max}} = 1$. To obtain the effects of the higher-order interference, we compared the $\lambda_{\text{max}} = 1$, and the all-order ($\lambda_{\text{max}} = 4$) breakup cross sections. We showed that when the $\lambda = 0, 1$ multipoles are summed incoherently, the nuclear breakup cross section and hence the total breakup cross section are much larger than the Coulomb breakup cross section. However, we noticed that the $\lambda_{\text{max}} = 1$ nuclear breakup cross section is negligible compared to the Coulomb breakup cross section for both reactions, which becomes even more important than its total breakup counterpart, due largely to the first-order interference for the $^{11}\text{Be} + ^{208}\text{Pb}$ reaction. For the $^{15}\text{C} + ^{208}\text{Pb}$ reaction on the other hand, we obtained that the higher-order interference effect is needed for the Coulomb breakup cross section to be more important than the total breakup cross section. The conclusion was that the first-and higher-order interference effects could be the reason why the total breakup cross section was found to be lesser than the Coulomb breakup cross section, not only in this study but also in [57], for example for the $^{15}\text{C}+^{208}\text{Pb}$ reaction. We noticed that these conclusions are valid even for angular distributions breakup cross sections, where we found that for these two reactions, the effect of the first-order interference is to suppress the large extension of the first-order breakup cross sections to large angles.

For further insight into the first-and higher-order interference effects, we analyzed the amounts reduced from the total, Coulomb and nuclear breakup cross sections, owing to these interferences. For the $^{11}\text{Be}+^{208}\text{Pb}$ reaction, 78.50% of the total breakup cross section is reduced due to the all-order interference, in the following distribution: 71.91% due to the first-order interference and 6.59% to the higher-order interference. The Coulomb breakup cross section is reduced by 3.07%, where 2.73% is due to the first-order interference and 0.34% to the higher-order interference. Finally, the all-order interference
reduces by 94.76% the nuclear breakup cross section, distributed as follows: 91.47% due
to the first-order interference and 3.29% to the higher-order interference. Concerning the
\(^{15}\text{C} + ^{208}\text{Pb}\) reaction, we obtained the following results. The all-order interference reduces
by 72.50% the total breakup cross section, distributed as follows: 63.75% due to the
first-order interference and 8.74% to the high-order interference. It reduces by 2.86% the
Coulomb breakup cross section, where 2.69% is due to the first-order interference and
0.17% to the higher-order interference. For the nuclear breakup cross section on the other
hand, it is reduced by 92.36%, with 85.97% due to the first-order interference and 6.39%
to the high-order interference. The results indicated for both reactions that, although the
higher-order effects fall below 10%, the first-order results alone overestimate the data at
low excitation energies and hence the importance of the higher-order interference effects,
especially at low excitation energies.

The effects of multipole transitions and the first-and higher-order interferences on the
Coulomb nuclear interferences were also investigated. The results showed for the \(^{11}\text{Be} + ^{208}\text{Pb}\)
reaction, that at zero-order, the Coulomb-nuclear interference is destructive and equal to
the integrated Coulomb breakup cross section in magnitude. At first-order on the other
hand, this interference was found to be strongly constructive, owing to its s-wave contri-
bution. We found that the first-order interference affects both the nature and magnitude
of the Coulomb-nuclear interference, which becomes destructive and slightly reduced by
20.34%. The higher-order interference was seen to increase the Coulomb-nuclear inter-
ference by 36.00%, without modifying its nature. We concluded that, quantitatively, the
higher-order interference has a pronounced effect on the Coulomb-nuclear interference
than its first-order counterpart. Turning to the \(^{15}\text{C} + ^{208}\text{Pb}\) reaction, we obtained that at
zero-order, the Coulomb-nuclear interference is destructive and again equal to the inte-
grated Coulomb breakup cross section. At first-order, this interference becomes strongly
destructive, owing to its p-wave contribution. We noticed that the first-order interfer-
ence reduces the Coulomb-nuclear interference by 27.56%, without affecting its nature.
The higher-order interference also increases the Coulomb-nuclear interference by 25.26%,
keeping its nature as well.

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For the $^{19}$C + $^{208}$Pb breakup reaction, our study focused on the effects of the continuum-continuum couplings (ccc) on the energy and angular distributions breakup cross sections and as well on the Coulomb-nuclear interference. A partial wave analysis also showed a dominant $p$-wave breakup cross section, and this dominance was seen to be independent of the ccc. Considering the total, Coulomb and nuclear breakups separately, and in the presence of all the different couplings, we also found that the differential nuclear breakup cross section is negligible compared to its differential Coulomb breakup counterpart, which is also dominant over the differential total breakup cross section. However, the incoherent difference of the differential total and nuclear breakup cross sections resulted in a fair description of the data for low excitation energies, whereas the differential total breakup cross section provided a good fit of the data at higher excitation energies, showing the again the importance of the nuclear breakup contribution. Removing the ccc from the potential matrix element, it is noticed that both the differential total and nuclear breakup cross sections become more important than the differential Coulomb breakup cross section. We concluded that the ccc have a pronounced effect on the differential total and nuclear breakup cross sections than on the differential Coulomb breakup cross section.

To assess the quantitative effects of these ccc on the Coulomb-nuclear interference, we also integrated numerically the differential total, Coulomb and nuclear breakup cross sections in the presence and absence of these couplings. In the presence of the ccc, we obtained that this interference is exclusively destructive in each partial wave. In the absence of the ccc, we found that the Coulomb-nuclear interference remains destructive, but increased in magnitude in all the partial waves, other than the $p$-waves, where this interference becomes constructive. We observed that the overall nature of this interference is destructive regardless whether the ccc are included or not.

A further analysis of this reaction led us to the angular distributions breakup cross sections. In this case, analyzed separately the effects of the diagonal continuum-continuum couplings (dccc) and off-diagonal continuum-continuum couplings (odccc). To analyze the role of the odccc, we compared the results obtained when the all the ccc are included, and when the results when only the dccc are included. Qualitatively, we found that, the dccc
substantially reduce the total and nuclear breakup cross section at large angles, starting in the vicinity of the grazing angle, while the odccc reduce these breakup cross sections mostly at small angles. We found that the ccc have no effect on the Coulomb breakup cross section other than removing its oscillatory behavior. Quantitatively, considering the total breakup, we deduced that the ccc reduce by 70.39% the total breakup cross section, distributed as follows: 59.28% due to the dccc and 11.11% due to the odccc. For the Coulomb breakup, it is deduced that the breakup cross section is slightly reduced by 4.71%, where 1.32% is due to the dccc and 3.39% to the odccc. Looking at the nuclear breakup, it is obtained that the breakup cross section is substantially reduced by 88.16%, owing to the ccc, where 55.53% is due to the dccc and 32.63% to the odccc. The results also showed that the dccc decrease the destructiveness of the Coulomb-nuclear interference, which is increased by the odccc. This revealed that the odccc have a significant effect on this interference than the dccc.

Finally in this reaction, we investigated the effects of these different ccc on the penetrability of the Coulomb barrier, for a understanding of the dependence of the fusion cross sections on the breakup process. To this end, we first determined the integrated breakup cross sections inside and outside the Coulomb barrier. It was seen that the ccc reduce substantially the integrated breakup cross section inside the barrier, by increasing the Coulomb barrier, therefore lowering of the tunneling for the total breakup. On the other hand, for the nuclear breakup, where there is absence of the natural Coulomb barrier, the results indicated that the ccc create a kind of barrier that hinders the penetration of the projectile flux.

For the breakup of the proton-halo $^8$B projectile on $^{58}$Ni and $^{208}$Pb targets, the prime motivation was to check whether the conclusions drawn for the $^{19}$C + $^{208}$Pb reaction can be extended to these particular reactions, thus providing an opportunity of investigate the dependence of the ccc effects on the valence nucleon charge, on the target mass and on the incident energy regime. We then followed the same steps as for the $^{19}$C + $^{208}$Pb reaction. Regarding the partial wave contributions, for the $^{8}$B + $^{58}$Ni reaction, the results indicated that the p-wave differential total breakup cross section is leading at low
excitation energies. However, it amounted to the smallest contribution at high excitation energies, where the s-wave breakup cross section is leading. For the $^8\text{B} + ^{208}\text{Ni}$ reaction on the other hand, it is obtained that the s-wave differential total breakup cross section is more dominant at low excitation energies, and the d-wave differential total breakup cross section happens to be slightly dominant. Considering the effects of the ccc (dccc and odccc together), we found also that the total and nuclear breakup cross sections are largely affected by these couplings than the Coulomb breakup cross section. More precisely, we found that these couplings reduce substantially the integrated total and nuclear breakup cross section by 81.00% and 93.68%, respectively and by 46.78% the integrated Coulomb breakup cross section, for the $^8\text{B} + ^{58}\text{Ni}$ reaction. For the $^8\text{B} + ^{208}\text{Pb}$ reaction on the other hand, the integrated total and nuclear breakup cross sections were dramatically reduced respectively by 92.95% and 99.67%, while the integrated Coulomb breakup cross section was reduced by 39.06%. On the light of these results, we concluded that the difference between the effects of the ccc on the energy distributions breakup cross sections for these reactions and for the $^{19}\text{C} + ^{208}\text{Pb}$ reaction is rather quantitative. It is important to further investigate which parameter precisely between the valence nucleon charge and the energy regime is responsible for this quantitative difference. One can for instance, analyze the $^8\text{B} + ^{208}\text{Pb}$ and $^{19}\text{C} + ^{208}\text{Pb}$ reactions at the same incident energy. As for the Coulomb-nuclear interference, it was shown that the ccc don’t also affect the nature of this interference.

Concerning the effects of the dccc and odccc on the angular distributions breakup cross sections, qualitatively, we found again that the dccc are largely responsible for the substantial reduction of the differential total and nuclear breakup cross section at large angles. At small angles, the reduction of these breakup cross sections, is due as well to the odccc. Regarding the Coulomb breakup, it is concluded that the inclusion of the dccc gives rise to an increase of the breakup cross sections. Quantitatively, we deduced for the $^8\text{B}+^{58}\text{Ni}$ reaction that, the ccc reduce by 81.70% the integrated total breakup cross section, distributed as: 68.34% due to the dccc and 13.39% due to the odccc. For the Coulomb breakup on the other hand, the dccc increase the integrated breakup cross section by 34.88%, which is turn is reduced by 50.52%, owing to the odccc. Finally for
the nuclear breakup, it is deduced that the ccc dramatically reduce the breakup cross section by 92.67%, distributed as follows: 67.65%, due to the dccc and 25.02%, due to the odccc. Turning to the $^8\text{B}+^{208}\text{Pb}$ reaction, we found that the ccc reduce by 82.22% the total integrated breakup cross section, distributed as follows: 69.51%, due to the dccc and 12.71%, to the odccc. It is obtained that for the Coulomb breakup, the dccc increase the integrated breakup cross section by 26.16%, which is reduced by 25.85%, owing to the odccc. Looking finally at the nuclear breakup, a dramatic reduction by 97.76% of the breakup cross section, due to the ccc, where 80.92% is due to the dccc and 16.84% to the odccc is obtained. The major difference between these reactions and the $^{19}\text{C}+^{208}\text{Pb}$ reaction in this case, was that here the Coulomb breakup cross sections are rather significant increased, owing to the dccc, while for the $^{19}\text{C}+^{208}\text{Pb}$ reaction, the Coulomb breakup cross section was slightly decreased due to the dccc. A complete understanding of this difference require further analysis. We also obtained that the ccc significantly reduce the penetrability of the Coulomb barrier for these two reactions.

As pointed out already, it is seen that the first-order interference effect is to suppress the extension of the angular distributions differential total and nuclear breakup cross sections at large angles. Similarly, we also saw that the effect of the ccc is to suppress the same extension behavior of the angular distributions differential total and nuclear breakup cross sections at large angles. A more detailed analysis is therefore needed to understand the interplay of the first-order interference and the ccc.
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