

**THE INFLUENCE OF IRRATIONAL BELIEFS ON THE MATHEMATICS
ACHIEVEMENT OF SECONDARY SCHOOL LEARNERS IN ZIMBABWE**

by

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- More than 2 000 years ago, Epictetus said, *“People are disturbed not by events that happen to them, but by their view of these events”* (Ellis, 2004a: 74).
- *“The nature of people is such that when they think, they also feel and behave; when they feel, they also think and behave; and when they behave, they also think and feel”* (Najafi & Baranovich, 2014:2).
- *“In our march towards scientific and technological advancement, we need nothing short of good performance in Mathematics at all levels of schooling. Unfortunately performance of students in Mathematics at the end of secondary education has not improved in the past decade”* (Tella, 2007:149).
- *“That Mathematics is an important subject is indubitable but it is very sad to note that the performance by secondary school students ... in the subject in recent times is not encouraging”* (Asikhia, 2010:205).

DECLARATION

STUDENT NUMBER: 4739-029-8

I, **Moses Kufakunesu**, declare that the thesis entitled **THE INFLUENCE OF IRRATIONAL BELIEFS ON THE MATHEMATICS ACHIEVEMENT OF SECONDARY SCHOOL LEARNERS IN ZIMBABWE** is my own work and all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

I further declare that I have not previously submitted this work, or part of it, for examination at UNISA for another qualification or at any other higher education institution.



12 NOVEMBER 2015

M. Kufakunesu

DATE

DEDICATION

This thesis is dedicated to my two daughters, Monica, who was an adolescent secondary school learner during the time of the study, and Isabel, who was born when the empirical investigation was in progress. I also dedicate the thesis to all adolescent secondary school learners who aspire to pass Mathematics.

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- The Provincial Education Director and the District Education Officer in Masvingo, Zimbabwe, who facilitated the data collection process
- School heads (Principals) for facilitating the empirical investigation and availing documents with learners' Mathematics examination scores
- Mathematics teachers who acted as my research assistants at the schools where the empirical investigation was undertaken
- Parents and guardians of Mathematics secondary school learners who permitted their underage children and dependants to take part in the study
- Form 3 and form 4 learners who made the empirical investigation a success through their participation in the study.

I would also like take this opportunity to thank my wife Gladys and my two daughters, Monica and Isabel, for inspiring me to continue working hard throughout the study period.

I am also indebted to Lyn E. Voigt, the professional language editor for meticulously editing the entire thesis.

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SUMMARY

This study explored the influence of irrational beliefs on adolescent secondary school learners' Mathematics achievement in Zimbabwe. Learner, home and school factors which influence secondary school learners' Mathematics achievement were discussed and relevant studies were scrutinised. The theoretical views of Albert Ellis regarding the characteristics, effects, acquisition and maintenance of irrational beliefs were discussed together with the major irrational beliefs and their possible relationship with learners' Mathematics achievement. A sample of 306 randomly selected adolescent Mathematics learners comprising 182 girls and 124 boys in the 14 to 18 year age range participated in the study. A composite questionnaire with subscales on learners' irrational beliefs, socio-affective variables and perceptions was used during the empirical investigation. Six major hypotheses were tested. The study established that learners' irrational thoughts about Mathematics correlate negatively with their Mathematics achievement. Learners' irrational thoughts about Mathematics correlated negatively with motivation, self-concept, parental involvement, and teacher-learner relationships and positively with stress, anxiety and faulty perceptions. Regression analysis proved that learners' irrational beliefs, socio-affective variables and perceptions jointly explain a greater proportion of the variance in Mathematics achievement than any one of these factors on its own. Therefore, learners' Mathematics achievement is affected by irrational beliefs together with their socio-affective variables and perceptions. Practical recommendations were given to Mathematics education stakeholders such as teachers, school counsellors, parents and learners to minimise poor Mathematics achievement attributable to irrational beliefs and the allied variables explored in this study.

KEY WORDS:

- Irrational beliefs
- Rational emotive behaviour therapy
- Mathematics achievement
- Socio-affective Variables
- Motivation
- Stress
- Anxiety
- Self-concept
- Parental involvement
- Teacher-learner relationships
- Faulty perceptions

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CHAPTER ONE

STATEMENT OF THE PROBLEM AND PROGRAMME OF RESEARCH

1.1 AWARENESS OF THE PROBLEM

Beliefs normally have a bearing on the way people conduct themselves as well as the manner in which they interpret situations and express their emotions (Mpofu, 2006:135; Egbochuku, Obodo & Obadan, 2008:154). According to Mukangi (2010:54-55), the beliefs held by people are mostly congruent with their behaviour and can also be manifested in people's attitudes. Petty (2009:510) emphasises that beliefs are often self-fulfilling. Mpofu (2006:136) asserts that human beings are consciously or unconsciously influenced by beliefs in one way or the other. It seems credible that beliefs are a critical aspect of human functioning which pervades almost all the domains of human existence including teaching and learning situations.

Among the various psychological theorists who have explored the subject of beliefs is Albert Ellis whose theory is known as the rational emotive behaviour therapy (Prout & Brown, 2007:279; Melgosa, 2008:135). Albert Ellis was an American psychologist who originally trained as a Freudian psychoanalyst before starting his own rational therapy in 1955 (Mukangi, 2010:54). Ellis renamed the theory the rational emotive therapy before eventually calling it the rational emotive behaviour therapy in 1993 (Egbochuku et al, 2008:153). Ellis advanced his rational emotive behaviour therapy after being strongly inspired by ancient Greek philosophers. In particular, the statement by Epictetus, a Greek philosopher, that human beings are not disturbed by events but by the view they take of the events, strongly inspired Ellis to develop the rational emotive behaviour therapy (Ellis, 2004a: 74; Prout & Brown, 2007:281). Palmer (2000:278) viewed the rational emotive behaviour therapy as a system of psychotherapy which teaches individuals how their belief systems largely determine the way they feel about and towards events in their lives. This therapy propounded by Ellis is usually one of the major theoretical underpinnings in studies involving irrational beliefs.

In this therapy, Ellis identified two major categories of beliefs; namely, rational and irrational beliefs (Masvaure & Zenda, 2010:55; Mpofu, 2006:136). Rational beliefs encompass thoughts, feelings and behaviour that lead to the achievement of personally chosen goals (Mukangi, 2010:55; Masvaure & Zenda, 2010:55). Rational beliefs are essentially based on objective facts and are definitely necessary for survival and direct achievement of the individual's set goals (Mpofu, 2006:135; Egbochuku et al, 2008:153). Ellis claimed that irrational beliefs are the root cause of human emotional, behavioural and cognitive dysfunction through demanding ideal experiences. Irrational beliefs tend to turn wishes and preferences into necessities (Mpofu, 2006:138; Sporrle, Strobel & Tumasjan, 2010:543). Sporrle et al (2010:543) maintained that irrational beliefs cause a person to place emphasis on thoroughness, competence and being adequate, an exercise which Ellis terms masturbation. Ellis viewed masturbation as a situation in which people turn their wishes or preferences into necessities and absolutes which are expressed through awfulising, self-damnation and low frustration tolerance leading to self-inflicted suffering (Mpofu, 2006:138). According to Ellis, the four fundamental irrational beliefs around which all the other irrational beliefs revolve are demandingness, awfulising beliefs, low frustration tolerance beliefs, and depreciation beliefs (David, Lynn & Ellis, 2010:13; Bridges & Harnish, 2010:863) as shown in Figure 1.1.

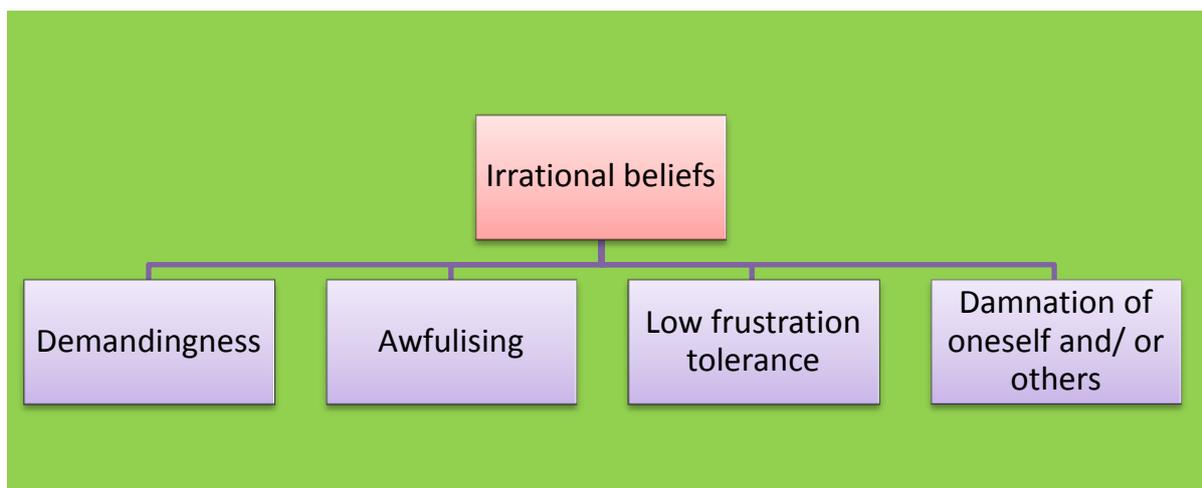


Figure 1.1: Ellis's core irrational beliefs

Ellis amplified the importance of beliefs in general and irrational beliefs in particular through the ABC model (Khaledian, Saghafi, Pour & Moradian, 2013:759). The ABC model can be linked to the already stated stoic philosophical dictum advanced by

Epictetus who postulates that emotional disturbance is normally triggered by beliefs and not by real life events. In the ABC model, 'A' denotes the activating environmental stimulus or event which an individual experiences (Mukangi, 2010:57; Froggatt, 2005:1; Khaledian et al, 2013:759; Thompson & Henderson, 2007:215; Austad, 2009:275; Feldman, 2009:559). The activating event can be a real or imaginary life event (Zionts & Zionts, 1997:105). For example, scoring a low mark in a Mathematics test can be a form of adversity which qualifies as an activating event. Another example of an activating event is an adolescent learner being reprimanded by the teacher for not doing homework.

'B' represents an individual's array of beliefs which is used for interpreting the activating event (Rosner, 2011:83; Froggatt, 2005:1). Ellis posited that the beliefs held by people determine the manner in which they interpret the activating environmental stimuli (Austad, 2009:275; Thompson & Henderson, 2007:207). This implies that the same activating event can be interpreted rationally or irrationally by different people depending on their orientations towards rationality or irrationality (Mukangi, 2010: 57; Khaledian et al, 2013:760, 762). Such a scenario indicates that beliefs mediate between an activating event and its consequences. The consequences of any form of adversity encountered in the environment are more determined by a person's evaluative beliefs than by the original adversity itself (Kosslyn & Rosenberg, 2006:699).

'C' stands for the emotional and behavioural consequences which have been mediated by an individual's beliefs (Froggatt, 2005:1; Kosslyn & Rosenberg, 2006:699; Thompson & Henderson, 2007:207; Mukangi, 2010:57). For people with irrational beliefs, 'C' is dominated by negative, maladaptive, unrealistic and inappropriate emotions which are self-defeating in nature (Rosner, 2011:83). Examples of emotional consequences which can be triggered by irrational beliefs include embarrassment, depression, shame, anxiety and hostility (Austad, 2009:276; Feldman, 2009:559). On the other hand, an individual who is guided by rational beliefs will acknowledge the inconveniences brought about by the activating event without being gripped by despair and anxiety. As illustrated in Figure 1.2, a learner who gets a low score in a Mathematics test will engage in self-damnation as a result of irrational thinking. Alternatively, the learner will make a rational resolution to try to improve in the next Mathematics test.

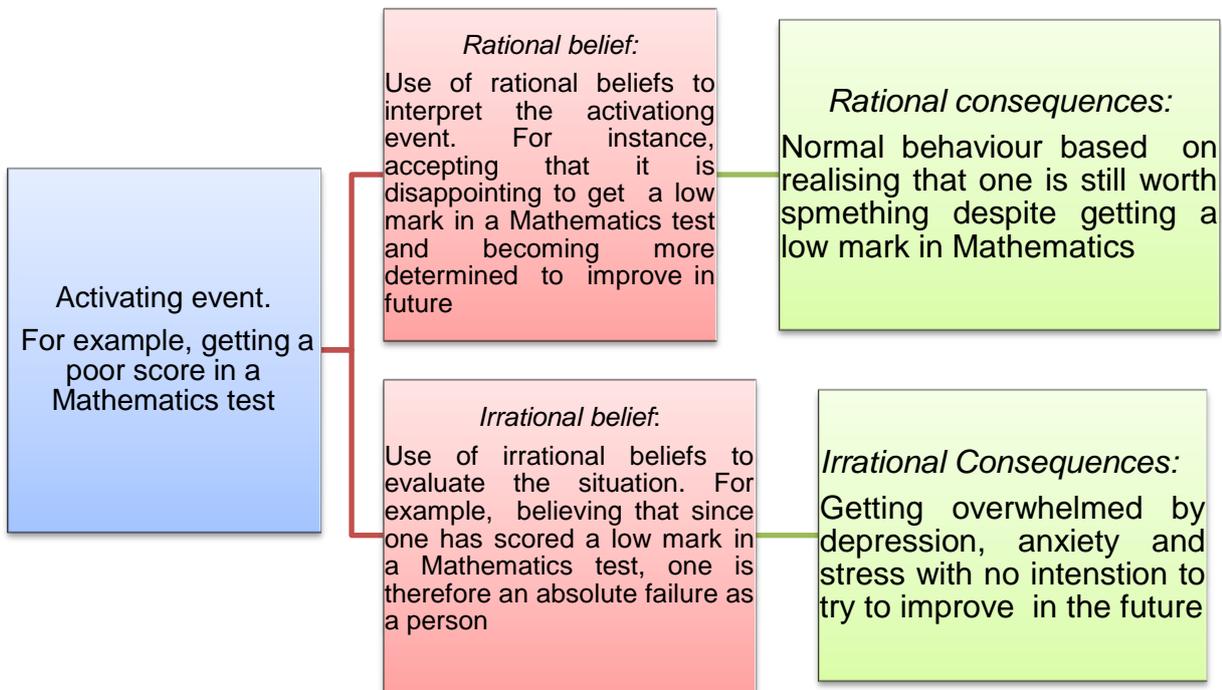


Figure 1.2: The ABC model in a Mathematics learning context

If left unchecked, irrational beliefs can lead to emotionally charged consequences which in turn become activating events themselves (Feldman, 2009:559; Austad, 2009:276). This becomes a self-defeating cycle in which the individual can become very anxious, depressed, guilty or hopeless (Thompson & Henderson, 2007:209; Austad, 2009:276; Feldman, 2009:559).

Ellis maintained that there are certain clinically observable core irrational beliefs which account for most neurotic disturbances in human beings (Melgosa, 2008:137). Ellis described more than ten such irrational beliefs most of which revolve around demandingness (Al-Salameh, 2011:139). One such irrational belief is the belief that it is a dire necessity for people to be loved by significant others for virtually everything they do (Egan, 2007:141; Austad, 2009:277). Another irrational belief is the notion that human happiness can be achieved through inertia and inaction when in actual If left unchecked, irrational beliefs can lead to emotionally charged consequences

which in turn become activating events themselves (Feldman, 2009:559; Austad, 2009:276). This becomes a self-defeating cycle in which the individual can become very anxious, depressed, guilt-ridden or hopeless (Thompson & Henderson, 2007:209; Austad, 2009:276; Feldman, 2009:559). In actual fact happiness can be derived from being actively involved in constructive and purposeful activities.

Other irrational beliefs according to Ellis include the belief that misery is externally caused (Mpofu, 2006:137,138) and is imposed upon us by other people or events and the belief that individuals have virtually no control over their emotions and that they remain susceptible to being disturbed by events (Melgosa, 2008:137). The notion that people should be upset and perennially obsess about anything which is potentially dangerous or fearsome is one more irrational belief according to Ellis (Prout & Brown, 2007:289). Moreover, the belief that it is horrible when situations are not the way we want them to be (Sporrle et al, 2010:543) and the notion that we absolutely need to rely fully on others for us to achieve our goals are other irrational beliefs which Ellis described as limiting and self-defeating thinking errors which need to be altered. On the basis of the characteristics of irrational beliefs outlined above, it can be contended that there is a possibility that irrational beliefs on their own can relate to Mathematics achievement. Spangler (1992:19) posits that there is a two way relationship between beliefs and Mathematics education. While learners' educational experiences can influence their beliefs about learning Mathematics, their beliefs about Mathematics as an academic discipline are likely to affect the way they learn Mathematics (Spangler, 1992:19). Given this pervasive and dynamic nature of beliefs, one can contend that more studies which explore the relationship between beliefs and Mathematics achievement can be beneficial to the various stakeholders in the education fraternity.

According to Ellis, human beings are more biologically predisposed to develop irrational beliefs than rational beliefs (Bernard, 2009:66; Ellis, 1998:22; Han, 2011:47). Ellis claimed that people normally acquire irrational beliefs through early childhood social experiences (Masvaure & Zenda, 2010:59). Some people acquire irrational beliefs through intentionally choosing to adopt them regardless of the quality of the social environment in which they live. Furthermore, inability to develop the capacity to behave rationally can also expose one to a dysfunctional belief system. Once acquired, irrational beliefs can be sustained through self-talk as

people rehearse the irrational beliefs and indoctrinate themselves with their painful past experiences (Prout & Brown, 2007:283). Prolonged focus on unfortunate past experiences or the belief that change is virtually impossible can also reinforce irrational beliefs. Irrational beliefs can be sustained through lack of scientific thinking. This occurs when the concerned individuals do not make any effort to verify their beliefs against the known scientific principles (Melgosa, 2008: 136). Mpofo (2006: 135) claimed that individuals of low intelligence or who lack training in thinking intelligently and are neurotic are bound to be tormented by irrational beliefs. Sporrle et al (2010:547) asserted that neuroticism is positively associated with irrationality. Given these various ways through which irrational beliefs can be acquired, it could be necessary to examine the relationship between irrational beliefs and Mathematics achievement at secondary school level.

In 1998, a commission called the Nziramasanga Commission was tasked to investigate the state of education and training in Zimbabwe (Nziramasanga, 1999:1). The commission generated a comprehensive report and a set of recommendations on many educational aspects one of which was Mathematics education (Nziramasanga, 1999:323). It established that many Mathematics learners in Zimbabwe hated Mathematics and many adults gave negative accounts of their school experiences of Mathematics (Nziramasanga, 1999:323). The study by Nziramasanga (1999:332) discovered that learners' negative attitudes towards Mathematics could be attributed to poor Mathematics instruction, a shaky foundation of lower order concepts and poor pass rates of previous learners. The inquiry into the state of education in Zimbabwe also revealed that the Zimbabwean education system was examination-oriented (Nziramasanga, 1999:339). The emphasis on the need to pass examinations can expose learners to stress, anxiety and irrational beliefs pertaining to low frustration tolerance and demandingness associated with desperately wanting to pass Mathematics. Moreover, diagnostic tests were not being used in Zimbabwe both at primary and secondary school levels (Nziramasanga, 1999:340). A comprehensive study exploring the relationship between irrational beliefs and Mathematics achievement can to some extent play a diagnostic role. To add to the aforementioned, the Nziramasanga Commission recommended that learners be motivated to learn Mathematics and to have positive attitudes towards the subject and its many uses in solving existential problems (Nziramasanga,

1999:324). Motivation is one of the affective variables which can be explored in connection with the relationship between irrational beliefs and Mathematics achievement as a partial endeavour to follow up on some of the recommendations made by the Nziramasanga Commission.

It has been established by many researchers and authorities that numerous learners the world over have negative perceptions about Mathematics. According to Asikhia (2010:205), some learners associate Mathematics with pain and frustration. Mathematics is perceived with a strange air of mystery by a significant number of learners and it has been labelled a 'killer subject' (Saraswathi, 2003:326). Petty (2009:510) reiterated that many learners confess that they cannot do Mathematics. A significant proportion of learners dislike Mathematics (Asikhia, 2010:206). Faulty and negative perceptions about Mathematics are not confined to learners. Haylock and Cockburn (2003:x) maintained that many adults view Mathematics as a subject that triggers feelings of tension and insecurity. Learners also have various perceptions pertaining to the way they must solve mathematical problems. According to Spangler (1992:19), some learners believe that failure to solve mathematical problems quickly is an indication that something is wrong with them. Some learners also believe that the chief goal of solving a mathematical problem is to obtain a correct answer (Spangler, 1992:19). Such beliefs might be oriented towards irrationality.

Some studies have been conducted to explore the relationship between learners' faulty perceptions and their Mathematics achievement. Tachie and Chireshe (2013:68, 71) established that some learners justify their poor Mathematics achievement by using the claim that Mathematics is a difficult academic discipline. A study to explore the relationship between learners' perceptions of Mathematics and their Mathematics achievement was undertaken by Anderson, Rogers, Klinger, Ungerleider, Glickman and Anderson (2006:718). The study established a positive correlation between learners' interest in Mathematics and their Mathematics achievement. In another study, Githua (2013:175) revealed that learners' negative attitudes towards Mathematics have been implicated in causing poor academic achievement in the subject. Research projects scrutinising the relationship between learners' faulty perceptions about Mathematics, irrational beliefs and Mathematics achievement appear to be quite scarce. The exploration of learners' faulty

perceptions of Mathematics should, therefore, be undertaken to verify a possible relationship between irrational beliefs and Mathematics achievement.

Mathematics is universally rated as a vital academic discipline because of its utility in the industrial, technological and economic advancement of any country (Awolola, 2011:91; Nziramasanga, 1999:323; Mahanta & Islam, 2012:713). According to Githua (2013:174) and Mbugua, Kibet, Muthaa and Nkonke (2012:87), Mathematics acts as the foundation for scientific and technological concepts which are needed by virtually all nations for their socioeconomic progress. Mathematics is a crucial subject useful in solving real life problems (Mahanta & Islam, 2012:713; Nziramasanga, 1999:323). Apart from being a fundamental foundation for lifelong learning, Mathematics makes young people better prepared to meet the numeracy demands of many modern workplaces (Lamb & Fullarton, 2001:3). Tachie and Chireshe (2013:67) reiterated that Mathematics serves as the gateway to future occupations in a variety of domains. Nyaumwe (2006:40) posited that the utilitarian values of Mathematics such as its role in opening career opportunities for learners make Mathematics a very important subject in the curriculum. According to Tachie and Chireshe (2013:67), at elementary level Mathematics is a prerequisite for science, computer science and engineering courses. Given the importance of Mathematics as outlined above, it is arguably necessary for a study pertaining to some variables which relate to Mathematics achievement to be undertaken in the hope of generating research findings which can enhance the quality of Mathematics teaching and learning in Zimbabwe and beyond.

Adolescence as a developmental stage is unique in the sense that it is characterised by physiological changes which normally produce psychological turmoil and mood swings (Melgosa, 2008:91; Swartz, De la Rey, Duncan, Townsend & O'Neill 2011:87; Feldman, 2009:423). It can be claimed that adolescence is a developmental stage which is characterised by turbulence and trouble in virtually all cultures (Swartz et al, 2011:87; Kufakunesu, Ganga, Chinyoka, Hlupo & Denhere, 2013:827). Blackwell, Trzesniewski and Dweck (2007:248) indicated that adolescence is a demanding and frustrating developmental stage for a significant number of individuals. Stanley G. Hall, a 19th century psychologist who is popularly called the father of adolescent psychology, described adolescence as a period of storm and stress (Kufakunesu et al, 2013:827; Swartz et al, 2011:87). According to

Erikson's psychosocial theory, adolescents are in the stage called identity versus role confusion (Meggitt, 2006:163). There is a notably genuine increase in friction between teenagers and those in authority particularly during early adolescence (Steinberg & Morris, 2001:88). Apart from wanting to establish their personal identities, adolescents tend to critically question some of the beliefs which they have been holding since childhood (Santrock, 2004:71). Upon reaching adolescence, some learners become less focused on their academic work, become isolated and detached from the proceedings in lessons while others tend to engage in truancy and disruptive behaviour (Sullivan, McDonough & Harrison, 2004:289).

It is during adolescence that learners are mostly influenced by their peers (Kufakunesu et al, 2013:829; Tuckman & Monetti, 2011:117; Melgosa, 2008:94; Keenan, 2002:205). It has been established that adolescents typically tend to spend significantly more time with their peers than with their parents (Steinberg & Morris, 2001:93). According to Tuckman and Monetti (2011:117), adolescents tend to reveal their genuine feelings to their peers. Moreover, adolescents strive to gain the approval of their peers. This is to some extent related to the irrational belief that it is a dire necessity for one to be approved by significant others in virtually all situations (Thompson & Henderson, 2007:213). The demand for approval and respect within peer group contexts as part of identity formation is a common feature among adolescents which is related to irrational thinking (Civitci, 2007:3; Cardenoso & Calvete, 2004:289). Consequently, it can be argued that exploring the relationship between peer relations and irrational beliefs in a Mathematics learning context is necessary.

Beliefs in general and irrational beliefs in particular do not exist in a vacuum. They can influence and be influenced by a number of variables which can either be internal or external relative to the learner. The factors in the affective domain can be very pertinent to the influence of irrational beliefs on scholastic achievement. According to Holtman, Julie, Mbekwa, Mtetwa & Ngcoba (2011:121), the affective domain is a broad field that entails aspects such as attitudes, interest, beliefs, emotions, values and motives. The affective domain focuses on the individual's internal factors with regard to emotional factors such as beliefs, attitudes, fears and ambitions (Reece & Walker, 2003:17). Emotions, which are part of the affective domain, are defined by Feldman (2009:335) as feelings which constitute both

cognitive and psychological elements which can impinge on human behaviour. Emotions have also been viewed as a link between environmental events and people's responses to such stimuli as already outlined in the ABC model (Feldman, 2009:336; Lahey, 2009:387). Holtman et al (2011:121) claimed that it is well established that manipulating the affective domain can be more crucial than subject matter when one wants to improve learners' Mathematics achievement. Learners' affective factors such as motivation, study habits, self-concept, anxiety and stress can be explored in relation to irrational beliefs and Mathematics achievement.

External factors such as the influence of parental involvement and teacher-learner interaction can be scrutinised when one attempts to establish the relationship between irrational beliefs and secondary school learners' Mathematics achievement. A study by Gonzalez and Wolters (2006:203) established a positive correlation between authoritative parenting and learners' Mathematics achievement because of high levels of motivation. On the contrary, Levpuscek and Zupancic (2009:541) demonstrated that high parental expectations tend to weaken learners' academic performance in Mathematics. A study by Anderson et al (2006:718) found a negative correlation between parental involvement and Mathematics achievement. The presence of such contradictory research findings can act as an impetus for further research pertaining to the relationship between parental involvement and Mathematics achievement. As very few studies have examined the relationship between variables such as irrational beliefs, parental involvement and Mathematics achievement, research projects that explore the relationship between such external learner factors and irrational beliefs as they separately and jointly relate to Mathematics achievement are needed.

Some studies have been undertaken to explore the relationship between teacher-learner interaction and Mathematics achievement. For instance, Jaiyeoba and Atanda (2011:91) established that the instructional materials employed by teachers have a strong bearing on learners' Mathematics achievement. Hinnant, O'Brien and Ghazarian (2009:662) explored the extent to which teacher expectations can predict learners' future Mathematics achievement at primary school level. Some research projects scrutinised the relationship between teachers' personality attributes and their learners' Mathematics achievement (Gilles & Bailleux, 2001:3; Garcia, Kupczynski & Holland, 2011:3). Studies which attempt to link teacher-learner

interaction to irrational beliefs and Mathematics achievement seem to be quite scarce. This implies that it is necessary for the relationship between teacher-learner interaction and irrational beliefs to be explored relative to Mathematics achievement at secondary school level.

A number of studies have been undertaken to explore the extent to which gender mediates Mathematics achievement at secondary school level (Grootenboer & Hemmings, 2007:6). Some of these studies have demonstrated that male secondary school learners are more motivated to study Mathematics than their female counterparts (Frenzel, Goetz, Pekrun & Watt, 2010:511; Watt, 2004:1556; Hannula, 2002:125). However, other studies have found no variability in Mathematics achievement attributable to gender (Abubaka & Adegboyega, 2012:122). A study by Penner (2008:162) established that although males were better in Mathematics than females in some countries, in other countries females did better than males. Frenzel et al (2010:507) explored how variables such as gender, home factors and school-related factors influenced learners' interest in Mathematics in Germany. Some studies such as that by Mubeen, Saeed and Arif (2013:41) and Aunio (2006:10) revealed that female secondary school learners performed better in Mathematics than boys (section 3.4.1.1.1). There seems to be virtually no research which explores the influence of or relationship between irrational beliefs and Mathematics achievement with gender as one of the mediating variables. Consequently, there is a need to examine the relationship between irrational beliefs and Mathematics achievement of secondary school male and female learners.

Many studies involving irrational beliefs as one of the independent variables have been undertaken in different parts of the world. For instance, Dilmac, Aydogan, Koruklu and Deniz (2009:720-728) carried out a research study on the extent to which irrational beliefs can predict perfectionism as a personality attribute. Al-Salameh (2011:137-144) examined the relationship between irrational beliefs among Jordanian college students and their self-efficacy. Sporrle et al (2010:543-548) undertook a study concerning the incremental validity of irrational beliefs in predicting subjective well-being while controlling for personality variables. A study to establish the extent to which early adolescent learners' irrational beliefs can predict their test anxiety was undertaken by Boyacioglu and Kucuk (2011:454). However, the studies referred to above have never explored the relationship between irrational

beliefs and Mathematics achievement. Such a gap warrants more research projects which directly explore the relationship between irrational beliefs as postulated by Ellis and learners' scholastic achievement in Mathematics.

Studies which explore the relationship between irrational beliefs as postulated by Ellis and Mathematics achievement are significantly few. One example of such a research project was undertaken by Boehnke (2008:149) to explore the effects of demand for peer approval by adolescent learners on their Mathematics achievement in Canada, Israel and Germany. The irrational belief that it is a dire necessity for people to be loved by significant others in virtually everything they do is a reflection of the demand for approval and respect (Civitci, 2007:3). The current study established that the demand for approval and respect from peers can impinge upon adolescent learners' Mathematics achievement. Although the study by Boehnke (2008:148) focused on the irrational belief of demandingness, it did not focus on the other three core irrational beliefs as portrayed in Figure 1.1. The study did not consider global rating of oneself and/or others, awfulising and low frustration tolerance. Moreover, the research project did not attempt to scrutinise the relationship between Mathematics achievement and Ellis's major irrational beliefs. This underscores the need to study all the main irrational beliefs postulated by Ellis relative to Mathematics achievement.

Another study which attempted to investigate the relationship between one irrational belief and Mathematics achievement was undertaken by Tsui and Mazzocco (2007:132). This study explored the influence of the demand for perfectionism and Mathematics anxiety on the Mathematics achievement of primary school learners. Perfectionism can be viewed as an irrational resolution to do everything in a clinically flawless manner (Macsinga & Dobrita, 2010:80). Apart from establishing a negative correlation between Mathematics achievement and both perfectionism and Mathematics anxiety, the study also confirmed that Mathematics anxiety and perfectionism independently account for variability in Mathematics achievement (Tsui & Mazzocco, 2007:139). Just as in the study by Boehnke (2008:148), the core irrational beliefs such as awfulising, low frustration tolerance and global rating of oneself and/or others were not considered. Furthermore, the research project did not endeavour to explore the other irrational beliefs propounded by Ellis in relation to Mathematics achievement. Consequently, it can be argued that research examining

the relationship between all Ellis's main irrational beliefs and secondary school learners' Mathematics achievement should be undertaken.

As already pointed out, it is possible for irrational beliefs on their own to relate to Mathematics achievement. However, it can be argued that this is not the only perspective through which the relationship between irrational beliefs and Mathematics achievement can be examined. Variables such as learners' affective factors, teacher-learner interaction, learners' faulty perceptions about Mathematics, parental involvement and irrational beliefs can separately and singly relate to Mathematics achievement as shown in Figure 1.3.

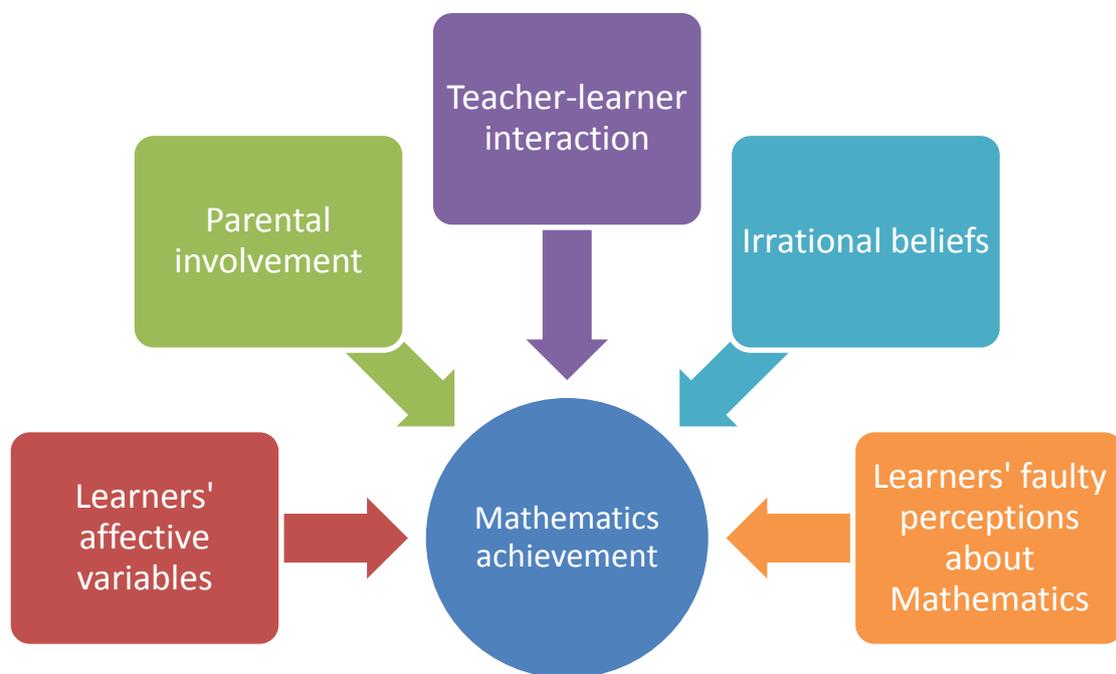


Figure 1.3: Variables which can separately relate to Mathematics achievement

The perspective suggested in Figure 1.3 portrays Mathematics achievement as a dependent variable which is related to several mutually exclusive independent variables. For instance, a research project carried out by Lamb and Fullarton (2001:2) revealed that the interaction between teachers and learners in the classroom strongly influences Mathematics achievement more than any other school factors.

The pervasive nature of irrational beliefs makes it possible for them to be related to learners' internal and external variables. It is possible that parental involvement, teacher-learner interaction, learners' faulty perceptions about Mathematics and

learners' affective variables such as self-concept, motivation, anxiety and stress each relate to irrational beliefs. Figure 1.4 depicts a situation in which irrational beliefs are viewed as a central variable around which factors such as the affective domain of learners, parental involvement, learners' faulty perceptions about Mathematics and teacher-learner interaction revolve. Such a perspective seems to propose that learners' affective variables, teacher-learner interaction, learners' faulty perceptions about Mathematics and parental involvement are mutually exclusive.

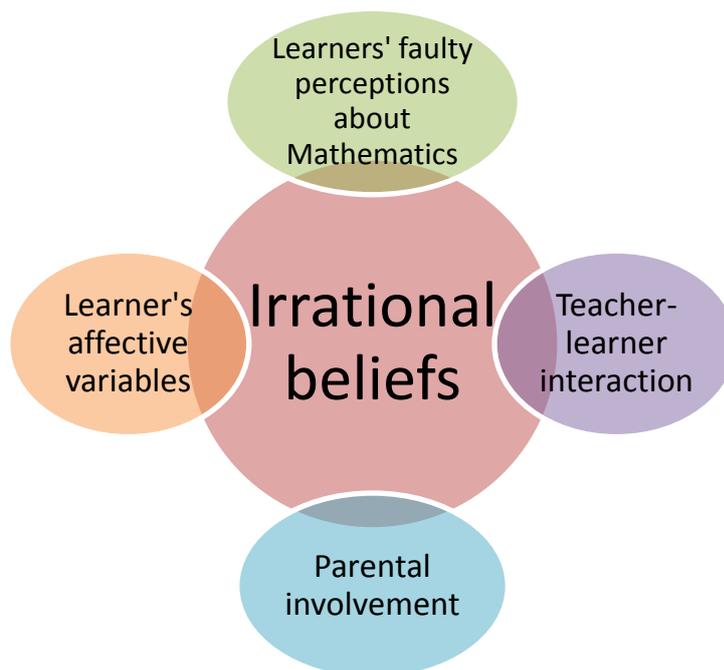


Figure 1.4: Possible relationships between irrational beliefs and learner variables

There seems to be virtually no previous research projects in which factors such as learners' faulty perceptions about how Mathematics, teacher-learner interaction, parental involvement and learners' affective variables relate as shown in Figure 1.4. Therefore, studies exploring the possibility of such relationships should be considered.

Mathematics achievement can be mediated by a multiplicity of variables. Therefore, for one to explore the relationship between irrational beliefs and Mathematics achievement comprehensively, it is imperative that the relationship between irrational beliefs and other learner variables be established first. Thereafter, the joint influence of irrational beliefs and learners' internal and external variables on Mathematics

achievement can be examined. Figure 1.5 shows a situation in which irrational beliefs, learners' affective factors, teacher-learner interaction, learners' faulty perceptions about Mathematics and parental involvement are considered as mutually related.

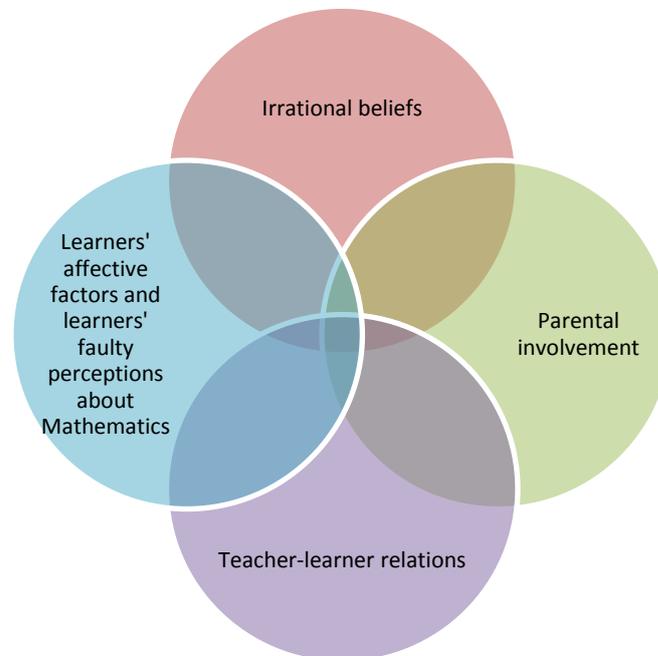


Figure 1.5: Possible interwoven nature of variables which can relate to Mathematics achievement

Examples of studies which attempted to explore several variables include a study by Wang, Osterlind and Bergin (2012:1215) in which variables such as learners' Mathematics self-concepts, teachers' perceptions and other school-related variables were linked to Mathematics achievement. Ferla, Valckle and Cai (2009:499-505) also confirmed that learners' affective variables such as academic self-concept, self-efficacy, Mathematics interest and Mathematics anxiety are mutually related and jointly mediate learners' Mathematics achievement. Moreover, Md.Yunus and Ali (2009:93) explored learners' socio-affective variables such as motivation, gender, self-efficacy and worry in relation to Mathematics achievement. Another endeavour to explore a number of factors related to Mathematics achievement was made by Mbugua et al (2010:88). Variables such as learners' socio-affective factors which include motivation, attitudes and parents' socioeconomic status as well as teacher attributes and the availability of teaching materials were scrutinised by Mbugua et al (2010:88) in relation to secondary school learners' Mathematics achievement in

Kenya. However, all these studies did not consider irrational beliefs thereby leaving a conceptual gap. It can still be validly argued that these studies did not adequately explore the relationship between Mathematics achievement and the variables depicted in Figure 1.5. Virtually no studies have endeavoured to explore the relationship between Mathematics achievement and the other variables as portrayed in Figure 1.5. Consequently, it seems necessary for a study which explores variables such as learners' affective variables, teacher-learner interaction, parental involvement, irrational beliefs and Mathematics achievement at secondary school level to be undertaken.

The fact that there are many permutations through which Mathematics achievement, teacher-learner interaction, learners' faulty perceptions of Mathematics, irrational beliefs and parental involvement can relate necessitates a study in which the way these variables relate can be empirically verified. On the basis of the research studies outlined above, it can be argued that there still exist some conceptual, time and geographical gaps which at least necessitate research in which the relationship between irrational beliefs and Mathematics achievement is explored factoring other allied variables. A credible possibility still remains that a new research project will at least generate novel insights which can be added to the already existing pool of knowledge in the domains of Psychology of Education and Mathematics education.

1.2 FORMAL STATEMENT OF THE PROBLEM

As already outlined, beliefs, whether rational or irrational, are an important variable in determining the success or failure of most human endeavours in various aspects of life. As postulated by Ellis, irrational beliefs can lead to human emotional and behavioural distress as well as psychological dysfunction. This also applies to learners in a school context, more specifically in a Mathematics environment. Consequently the influence of irrational beliefs on Mathematics achievement can be better understood by exploring the internal and external factors affecting Mathematics learners. Learners' socio-affective variables need to be explored to establish whether they relate to Mathematics achievement separately and also jointly with irrational beliefs. Gender is a variable which can be considered in a study which explores the relationship between irrational beliefs and Mathematics achievement.

Mathematics has been rated as one of the most vital academic disciplines in many countries because of its utilitarian values particularly for problem solving in technological and industrial sectors as well as in lifelong learning and training. Paradoxically, numerous studies have established that Mathematics is viewed by a significant number of learners and adults as a dreadful, stressful and anxiety-provoking subject. Learners' faulty perceptions of Mathematics need to be explored to establish whether they relate to irrational beliefs and Mathematics achievement. It is through investigating the relationship between Mathematics achievement and irrational beliefs that one may establish why an important academic subject such as Mathematics is so unpopular. The current investigation is also a partial attempt to explore the variables which relate to Mathematics achievement as recommended by the Nziramasanga Commission. Furthermore, some of the already highlighted studies generate conflicting research findings, thus acting as an impetus for further research regarding the factors which relate to Mathematics achievement. Therefore, it can be contended that there still exist some conceptual, contextual, time, geographical and methodological gaps which to some extent necessitate further research. Hence the current study will endeavour to provide responses to the research questions stated below:

- What is the relationship between irrational beliefs and achievement in Mathematics?
- How do irrational beliefs relate to socio-affective variables such as self-concept, motivation, anxiety, teacher-learner relationships and parental involvement which on their own part also relate to achievement in Mathematics?
- How do irrational beliefs relate to faulty perceptions of Mathematics?
- How do irrational beliefs, faulty perceptions and socio-affective variables jointly explain variation in Mathematics achievement?

1.3 AIMS OF THE INVESTIGATION

The aims of the investigation are outlined relative to the literature review and the empirical investigation.

1.3.1 Literature review related aims

A literature review will be conducted with the aim to:

- Examine the factors which influence Mathematics achievement at secondary school level
- Identify the major irrational beliefs as postulated by Albert Ellis
- Establish how human beings acquire and maintain irrational beliefs
- Explore the relationship between affective factors, teacher-learner interaction, parental involvement and irrational beliefs relative to achievement in Mathematics at secondary school level
- Establish the extent to which irrational beliefs relate to Mathematics achievement.

1.3.2 Empirical investigation related aims

An empirical investigation will be conducted with the aim to:

- Establish the relationship between irrational beliefs and Mathematics achievement
- Determine how irrational beliefs relate to learners' socio-affective variables which also relate to Mathematics achievement
- Explore the relationship between irrational beliefs and faulty perceptions of Mathematics as an academic discipline
- Determine the manner in which irrational beliefs, faulty perceptions and socio-affective variables jointly explain variability in Mathematics achievement.

1.4 PROGRAMME OF THE RESEARCH

The study is reported in six chapters.

Chapter 2 explores the factors which influence Mathematics achievement at secondary school level. Learners' internal and external factors are discussed relative to how they impinge on Mathematics achievement, and relevant psychological theories are referred to. Among learners' internal factors are affective variables such as motivation, self-concept, stress and anxiety. Home factors and school factors are the external variables discussed in conjunction with their relationship with irrational beliefs and Mathematics achievement.

In the third chapter, the nature, acquisition, maintenance and effects of irrational beliefs as postulated by Ellis are discussed. Ellis's major irrational beliefs and some of Beck's cognitive distortions are outlined relative to their influence on Mathematics achievement. Attention is also directed at irrational beliefs in the school context, particularly focusing on their influence on Mathematics achievement and how irrational beliefs relate to variables such as gender, intelligence, self-concept, anxiety, perfectionism, procrastination, motivation and peer influence.

A detailed explanation of the method of empirical investigation employed in the study is given in chapter 4. Several research hypotheses pertaining to the relationship between Mathematics achievement, irrational beliefs and other allied variables are formulated. Thereafter the research design is discussed. The research design includes the composition, characteristics and size of the population and the sample, the data gathering instrument and the data collection procedure. A large sample of more than 300 respondents is used. The main data gathering instrument is a questionnaire with Likert scale items.

The outcomes of the empirical investigation are scrutinised in the fifth chapter. Since the questionnaire to be used is an adaptation of several questionnaires, effort is made to establish the reliability of the resultant questionnaire. Thereafter, the procedure for testing the hypotheses outlined in the fourth chapter is discussed. The final decisions and conclusions emanating from the tested hypotheses are explained.

In the final chapter, conclusions highlighting the extent to which the aims of the study have been achieved are given. Recommendations based on the research findings are made in relation to the influence of irrational beliefs on the Mathematics achievement of secondary school learners to the various stakeholders. An outline of the limitations of the study and possible areas of further empirical enquiry are also made in the sixth chapter.

CHAPTER TWO

FACTORS WHICH INFLUENCE MATHEMATICS ACHIEVEMENT AT SECONDARY SCHOOL LEVEL

2.1 INTRODUCTION

This chapter explores the various factors which can influence the Mathematics achievement of secondary school learners. Psychological theories and studies related to the different factors are discussed. Since the current study focuses on the influence of irrational beliefs on Mathematics achievement, the relationship between some of the factors which influence Mathematics achievement and irrational beliefs is deliberated on. The variables which influence secondary school learners' academic performance in Mathematics can be broadly categorised into internal factors and external factors relative to the learners as shown in Figure 2.1. Under internal factors, the current study focuses on cognitive and affective factors as well as the learners' study habits and learning styles. It can be argued that it is through exploring internal factors that one can gain a better understanding of how Mathematics achievement is influenced by the variables which are inherently embedded in the learners (McLean, 2003:40). The cognitive factors of the learners entail intelligence, aptitude, information processing, language and previous academic performance (Gerrig & Zimbardo, 2005:314; Mwamwenda, 2004:252-253). The various theoretical perspectives on intelligence including Gardner's theory of multiple intelligences are discussed as well as the relationship between aptitude and intelligence (Sternberg, 2009:414). Moreover, the views of Piaget and Vygotsky on the development of thought and its possible influence on Mathematics achievement are elaborated (Bruce, 2006:98; Gravett & Geysler, 2004:71). With regard to information processing, the Atkinson-Shiffrin model, as outlined by Feldman (2009:217) and Kosslyn and Rosenberg (2006: 279) is examined together with the various memory enhancing techniques which learners can employ to boost their Mathematics achievement. Regarding language, the influence of language on cognition is discussed in conjunction with Vygotsky's sociocultural theory and the Sapir-Whorf hypothesis (Bhatt, 2007:37; Matsumoto & Juang, 2008:241; Nisbett & Norenzayan, 2002:6). One major justification for focusing on previous academic

performance is that it normally acts as a credible benchmark upon which one determines the probable standard of performance in future tasks (Snowman, McCown & Biehler, 2009:274).

The affective variables discussed in this chapter include motivation, self-concept, stress and anxiety. Intrinsic motivation, extrinsic motivation, the self-determination theory and McClelland's need theory are examined under motivation as affective variables which can influence Mathematics achievement (Rao & Rao, 2003:25; Gagne & Deci, 2005:335; Visser, 2009:11; Yates, 2002:4). The mutual relationship between affect, beliefs, behaviour and cognition necessitated the exploration of the stated internal factors (Forgas, 2001:17; Al-Salameh, 2011:137). Identity formation as theorised by Erikson, Allport's views on self-concept, Rogers' theory of self-concept development and Harter's self-perception profile for adolescents act as the theoretical bases for the self-concept (Birjandi, Mahmoudi & Abdolahi, 2012:8719; Tuckman & Monetti, 2011:383; Manning, 2007:11). Selye's and Lazarus's theories of stress together with the Yerkes-Dodson law are used to elaborate the concept of stress (Stickle, 2010:40; Lahey, 2009:367). Anxiety in general and Mathematics anxiety in particular, as pointed out by Feldman (2009:521) and Zakaria and Nordin (2008:28) respectively, are discussed in relation to their possible impact on Mathematics achievement. Study habits and learning styles such as the VARK model (Saadi, 2012:35) and Kolb's experiential learning are also discussed relative to their possible influence on achievement in Mathematics at secondary school level. It was deemed necessary to scrutinise study habits and learning styles because individual differences among learners relative to the manner in which they respond to intellectual stimuli can to some extent account for the variance in their academic performance.

The external factors, the variables which the learner cannot directly control, are broadly divided into home factors and school factors. Bronfenbrenner's ecological systems theory, as portrayed by Donald, Lazarus and Lolwana (2010:40) is discussed as a broad theoretical overview of the external factors which can impinge upon secondary school learners' academic performance in Mathematics. The exploration of external factors is justified by Tuckman and Monetti (2011:171) who argue that internal factors such as a learner's aptitude and intelligence cannot be realised unless home and school factors are adequately stimulating and supportive.

Under the home-related variables, parenting styles mainly as outlined by Baumrind and parental involvement and socioeconomic background are examined (Santrock, 2004:74; Woolfolk, 2007:165). The home, being the setting in which children are nurtured even before they enrol for formal schooling, arguably remains a critical determinant of children's ultimate academic success (Rivkin, Hanushek & Kain, 2005:417). It is in the home that children gain their early childhood experiences which can lead to the development of irrational beliefs (Birjandi et al, 2012:8719; Al-Salameh, 2011:137). Parents, just like teachers, are important adults who can influence learners' academic performance in ways which correlate positively with the level of their involvement (Ozmete & Bayo-Lu, 2009:314).

The school-related factors include classroom management and teacher-related factors. Lewin's leadership styles together with Blake and Mouton's managerial grid form the theoretical underpinnings of classroom management as a variable which can influence Mathematics achievement (Dhameja & Dhameja, 2009:82). By virtue of being the setting in which teaching and learning occurs, the school context and its allied variables were deemed to be important in the current study (Rivkin et al, 2005:417). Attention is also directed at teacher-related variables such as the expectations of teachers, the quality of teaching instructions and the personality of teachers (Awolola, 2011:91; Snowman et al, 2009:105). Ausubel's and Gagne's theories shed more light on the quality of teaching instructions (Ifamuyiwa, 2011:129; Mwamwenda, 2004:200). Under the personality of teachers, Bandura's social learning theory and Eysenck's three factor personality model are highlighted (Feldman, 2009:458; Bee & Boyd, 2004:20). Teacher-related factors are discussed because the teacher is a qualified professional who manipulates the variables in the school setting to facilitate learning (Berns, 2010:235; Mohd, Mahmood & Ismail, 2011:49). The chapter ends with a conclusion in which the variables ultimately to be considered during the empirical investigation are specified.

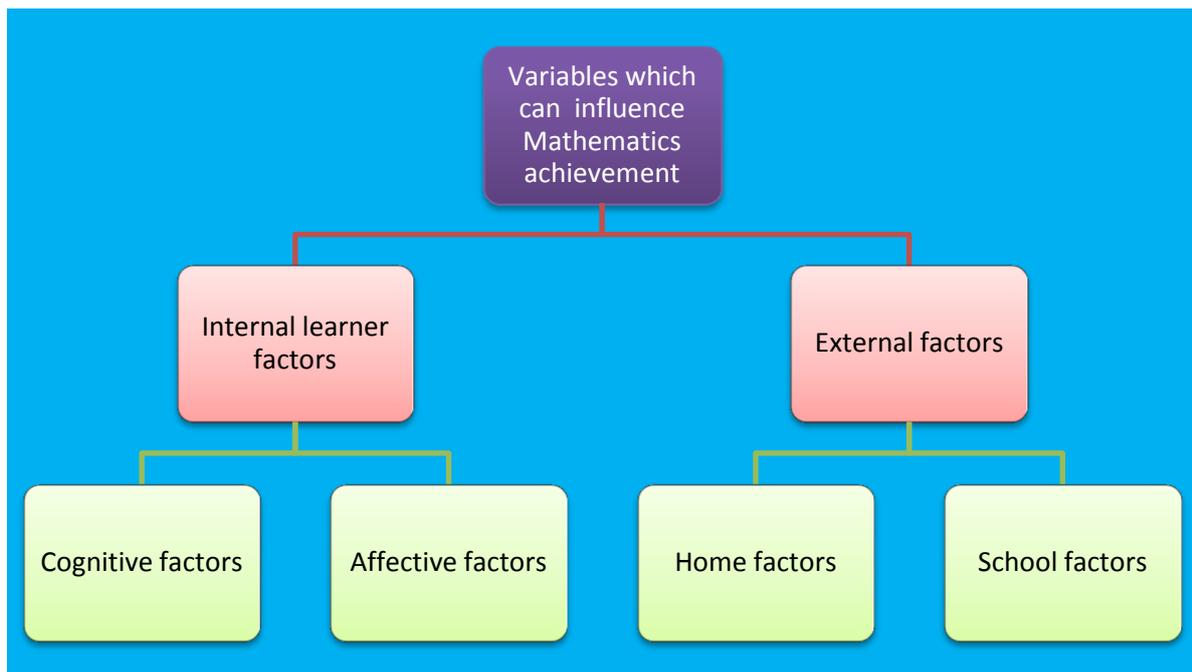


Figure 2.1: Factors which can influence learners' Mathematics achievement.

2.2 INTERNAL FACTORS

In this study, internal factors refer to the variables which can be directly attributed to each individual learner. The factors are categorised into cognitive factors, affective factors, study habits and learning styles.

2.2.1 Cognitive Factors

In a bid to establish the influence of cognitive factors on achievement in Mathematics at secondary school levels, cognitive factors such as intelligence, aptitude, the development of thought, information processing models, language issues in Mathematics education and the learners' previous academic performance are considered as shown in Figure 2.2.

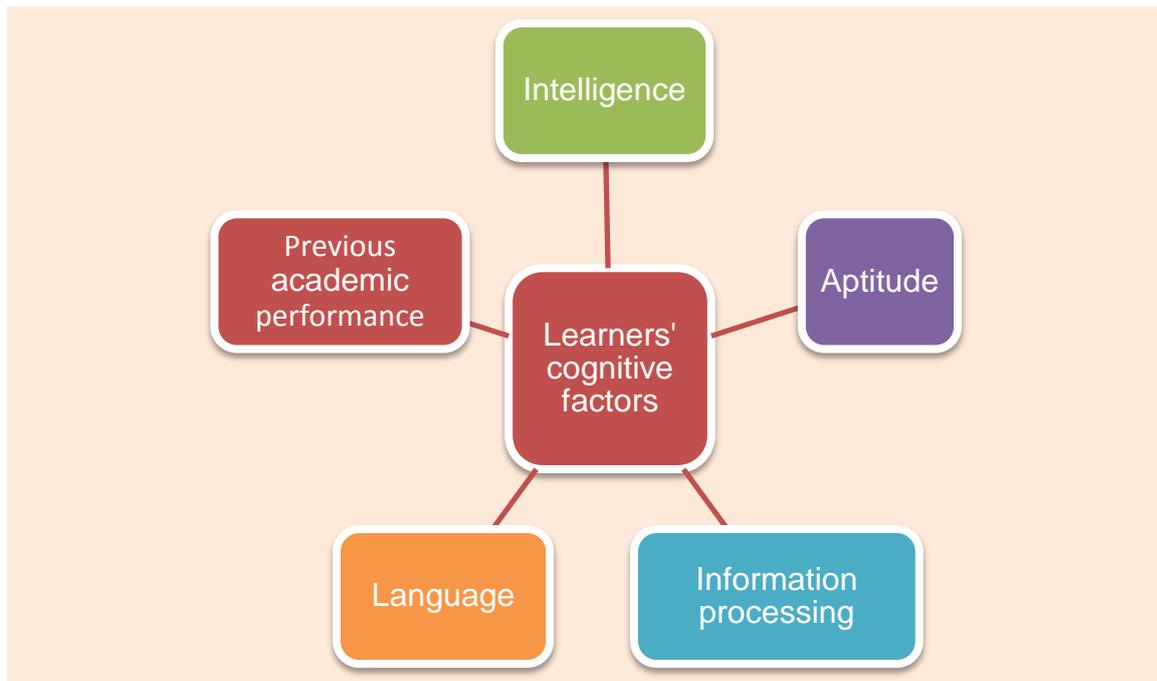


Figure 2.2: Cognitive factors which can influence learners' Mathematics achievement

2.2.1.1 Intelligence and aptitude

The terms *intelligence* and *aptitude* have been used both separately and interchangeably by different authorities to portray human ability. Intelligence and aptitude have a lot in common. According to Kinra (2008:25), intelligence is concerned with general mental ability while aptitude connotes specific aspects of intelligence such as mechanical, artistic, professional, perceptual and motor abilities. Mwamwenda (2004:252-253) posits that aptitude and intelligence are influenced by both hereditary and environmental factors. The terms *intelligence* and *aptitude*, which are related but not necessarily synonymous, are separately outlined.

2.2.1.1.1 Description of the concept of aptitude

According to Sethi (2011:1), aptitude is a set of characteristics or a condition regarded as symptomatic of a person's ability to acquire skills such as the ability to speak a language or play a musical instrument as a result of training. This means aptitude is an individual's specific ability or capacity to profit from future experience (Mwamwenda, 2004:480). Sujata (2005:09) observed that aptitude is a person's acquired or innate ability to learn or develop knowledge or a skill in some specific area. An aptitude focuses on an individual's potentialities to acquire knowledge or

skills in a specific domain at some point in the future (Kinra, 2008:24). More comprehensively, an aptitude is an acquired, innate or developed competence for a specific subset of mental skills which offers vital information on an individual's potential, particularly with regard to education and employment. This implies that aptitude and achievement, not only in Mathematics but in other disciplines, should correlate positively.

2.2.1.1.2 Description of the concept of intelligence

Intelligence has proved to be an intricate concept which is difficult to define to everyone's satisfaction (Feldman, 2009:285). According to Kosslyn and Rosenberg (2006:380), psychologists have offered many definitions of intelligence which are not in total agreement. While some psychologists view intelligence as a single quantifiable score, others contend that intelligence has many components which must be separately explored (Gerrig & Zimbardo, 2005:314). As a result of these ambiguities, intelligence has in some cases been defined as a single entity and in other cases as a collection of elemental attributes. According to Sternberg (2009:395), intelligence can be broadly viewed as the capacity to learn from experience and the ability to adapt to the demands of the environment. Intelligence can also be viewed as a general mental capability that, among other things, involves the ability to plan, solve problems, reason, think abstractly, comprehend intricate ideas, learn quickly and learn from experience (Gerrig & Zimbardo, 2005:314). This is supported by Kosslyn and Rosenberg (2006:381) who define intelligence as the individual's ability to solve problems adequately and to learn and understand intricate material. The above definitions concur that an intelligent person has minimum difficulties in learning new content, solving problems and adapting to novel environmental phenomena if all other variables are held constant. If this is related to achievement in Mathematics, it can be claimed that there should be a strong positive correlation between intelligence and achievement in Mathematics.

Intelligence has also been expressed as a single numerical index called the intelligence quotient (IQ). The intelligence quotient is an index obtained by dividing the mental age by the chronological age so as to enable the comparison of intellectual capabilities of children of different ages (Tuckman & Monetti, 2011:169; Lahey, 2009:294). However, Kosslyn and Rosenberg (2006:381) observed that the IQ has evolved to the extent of incorporating norms which measure general

intelligence. People believe that aptitude and the IQ are closely related but represent opposing views of human mental ability, that is, an IQ denotes intelligence as a single quantifiable trait whereas aptitude disintegrates that intelligence into several different characteristics that are relatively independent of each other. It is also important to note that although aptitude and intelligence are conceptually different, a significantly strong positive correlation between IQ scores and aptitude scores exists. One aspect of intelligence is the 'general factor' which Spearman developed and denoted by 'g'. According to Kosslyn and Rosenberg (2006:386), 'g', the general factor is a single intellectual capacity that underlies the positive correlations among a variety of tests of intelligence. It has to be admitted that the use of a single IQ to measure intelligence has proved to be inadequate (Sternberg, 2009:388).

Intelligence can also be broadly divided into two categories; namely, fluid intelligence and crystallised intelligence. According to Lahey (2009:293) and Feldman (2009:291), crystallised intelligence is one's ability to utilise previously learned information and strategies as well as the already mastered skills to handle familiar problems. On the other hand, fluid intelligence is the ability to rapidly process information and devise strategies for dealing with novel problems (Feldman, 2009:291; Lahey, 2009:293). Given the cumulative nature of some mathematical concepts, both crystallised and fluid intelligence are needed for one to score high marks in Mathematics.

Among the theories of intelligence which attempt to scrutinise intelligence in its elemental form is Howard Gardner's theory of multiple intelligences. As indicated by Gerrig and Zimbardo (2005:321), Gardner identifies eight intelligences that address the diverse human experiences. Gardner's eight multiple intelligences are musical intelligence, naturalistic intelligence, spatial intelligence, linguistic intelligence, interpersonal intelligence, intrapersonal intelligence, bodily kinaesthetic intelligence and logical-mathematical intelligence (Santrock, 2004:106; Tuckman & Monetti, 2011:167; Feldman, 2009:288). Of particular importance in the current study is logical-mathematical intelligence which Santrock (2004:108) defined as the ability to undertake mathematical and related scientific operations. Logical-mathematical intelligence is used for undertaking logical reasoning and solving mathematical problems (Sternberg, 2009:414). Logical-mathematical intelligence can be defined as an individual's sensitivity to and ability to identify logical and numerical patterns,

that is, the ability to handle long chains of reasoning (Gerrig & Zimbardo, 2005:322; Tuckman & Monetti, 2011:167). One can safely contend that amid other things, the higher the level of one's mathematical intelligence, the greater the probability of scoring high marks in Mathematics. However, it is important to note that other types of intelligence such as spatial intelligence and linguistic intelligence can also influence achievement in Mathematics.

2.2.1.1.3 Research on intelligence, aptitude and achievement in Mathematics

A study on mathematical aptitude and intelligence was undertaken by Gehlawat (2011:75) using a sample of 120 respondents comprising 60 girls and 60 boys from urban, rural and private schools. One of the key findings was that there is a significant relationship between mathematical intelligence and mathematical aptitude. The importance of identifying learners' mathematical aptitude at an early stage so as to capitalise on it to enhance their academic achievement in Mathematics at a later stage is emphasised by Gehlawat (2011:75). An allied research cited by Sethi (2011:2) established a significant positive correlation between mathematics aptitude and achievement in mathematics.

In an attempt to establish the relationship between Mathematics aptitude and achievement in Mathematics, Sethi (2011:2) undertook a study with a sample of 700 learners in the 9th grade. Interpreting the findings, Sethi (2011:2) indicated that mathematical aptitude is a strong determinant of learners' achievement in Mathematics.

The direct and indirect effects of general intelligence and broad cognitive abilities on Mathematics achievement were explored by Taub, Floyd, Keith and McGrew (2008:187). Structural equation modelling was employed to explore the simultaneous effects of both general and broad cognitive abilities on students' Mathematics achievement. Fluid reasoning, crystallised intelligence, and processing speed were found to be directly related to Mathematics achievement. On the other hand, the general intelligence factor was found to indirectly influence the Mathematics achievement of learners across all age levels (Taub et al, 2008:187). The study indicated that intelligence influences Mathematics achievement whether it is considered as a single entity or in its elemental form.

2.2.1.2 Development of thought

Many theories have been developed in a bid to outline explicitly how cognitive development occurs in human beings. While some theorists suggest that human cognitive development takes place in stages, others advance a non-stage theory. For the purpose of this study, Piaget's theory of cognitive development and Vygotsky's sociocultural theory are discussed.

2.2.1.2.1 Piaget's theory of cognitive development

Jean Piaget's theory of cognitive development has strongly impacted on the way cognitivists, psychologists and educationists understand mental development (Bruce, 2006:98). Piaget's theory of cognitive development postulated that cognitive development follows a sequence of four stages which are the sensory-motor stage, the pre-operational stage, the concrete operational stage and the formal operational stage (Mwamwenda, 2004:89; Bruce, 2006:98). More emphasis is placed on the concrete operational and formal operational stages since most secondary school learners fall within these last two stages of Piaget's theory.

According to Gazzaniga and Heatherton (2006:447), Piaget postulated that cognitive development occurs when the child cumulatively adds new information to his or her already existing schemata. A schema is a unit of information that a person possesses (Ashford & Le Croy, 2010:93). Schemata, as described by Gazzaniga and Heatherton (2006:447), are conceptual models or mental structures formed by children as they interact with their environment. Children normally discover and adapt to their surroundings through manipulating and exploring what is new and unfamiliar (Bhatt, 2007:19). Ashford and Le Croy (2010:93) commented that the actual cognitive development occurs through adaptation. Adaptation is a genetically inherent process of adjusting to environmental demands by coping with the new situations through problem solving and thinking (Bhatt, 2007:19; Swartz et al, 2011:71). Adaptation entails two complementary processes of assimilation and accommodation as outlined below.

Upon perceiving something slightly or totally novel, disequilibrium occurs. The state of cognitive conflict that occurs when an individual's already existing way of thinking is not confirmed by the experience is what is referred to as disequilibrium (Bhatt,

2007:22). Either assimilation or accommodation is used to know what the perceived object or situation is. While assimilation is the adaptation process in which a novel experience is interpreted using already existing schemata, accommodation is the process of modifying a schema or developing a new schema in order to incorporate a new experience that does not smartly fit into an existing schema (Gazzaniga & Heatherton, 2006:447; Bhatt, 2007:21; Swartz et al, 2011:68). Thereafter, equilibration is regained and the process continues.

The concrete operational stage normally covers children in the 7 to 11 year age range. However, some children and even adults remain at the concrete operational stage although they might be chronologically older than eleven years. According to William-Boyd (2003:67), children at the concrete operational stage are capable of solving concrete tasks in the present. Moreover, learners at this stage are less egocentric, can organise objects serially and can fluently use language. The development of the concrete operational stage can be fostered through exposing learners to multiple ways of representing mathematical solutions and giving them the opportunity to handle a variety of teaching aids (Burns & Silbey, 2000:55).

The formal operation stage, which coincides with adolescence, is characterised by the ability to undertake deductive reasoning and manipulating abstract concepts (William-Boyd, 2003:67). According to Gazzaniga and Heatherton (2006:448), formal operations involve abstract thinking which is characterised by the ability to formulate a hypothesis about a situation and subsequently test it through deductive logic. As claimed by Stickle (2010:18), adolescents have large volumes of mathematical knowledge and can undertake verbal reasoning since they possess a large vocabulary.

The provision of teaching aids, teaching from the known to the unknown and properly sequencing mathematical ideas are some of the ways in which teachers can deliberately endeavour to enhance the learners' Mathematics achievement at secondary school level. Schwartz (2000:62-65) argued that Mathematics teachers should teach from simple to complex. On the other hand, learners who learn in settings in which teaching aids are not used for whatever reason or in which Mathematics content is not properly sequenced may underachieve in Mathematics.

2.2.1.2.2 Vygotsky's sociocultural theory

Levy Vygotsky advanced a non-stage theory of cognitive development which has had a significant impact on teaching and learning activities at different educational levels. Vygotsky's sociocultural theory essentially postulated that cognitive development occurs in a social context in which knowledge is transmitted from those who are more competent to the less competent ones through social interaction using language as a medium of instruction (Baumann, Bloomfield & Roughton, 2006:63; Gravett & Geysler, 2004:71; Swartz et al, 2011: 80). This view was supported by Daly, Byers and Taylor (2004:75) who indicated that children are active learners who need to be immersed in a favourable environment in which they gain new ideas through interacting with more skilled people such as teachers, parents, peers and siblings. Many studies have confirmed that learning rather occurs during social interaction than through solitary efforts (Walters, 2000:3; Jones & Jones, 2008:63).

Vygotsky referred to the zone of proximal development. It can be defined as the difference between what the learner can handle independently and what the learner can achieve after being assisted by more competent adults or peers (Riley, 2007:19; Daly et al, 2004:75). Vygotsky postulated that the child's level of cognitive development should not be measured on the basis of what the child can do independently, but on the basis of what the child can do with the assistance of others (Vygotsky, 1978:85). The learners can be helped to navigate their zones of proximal development through scaffolding, which occurs when adults or more competent peers alter the level of assistance during social interaction till the learner masters the concept or skill being taught (Swartz et al, 2011:81; Daly et al, 2004:75).

At secondary school level, learners' scholastic achievement in Mathematics can be compromised or enhanced depending on the extent to which the principles of Vygotsky's sociocultural theory are employed. Swartz et al (2011:80) contended that Vygotsky's sociocultural theory has significantly impacted on the teaching and learning of adolescents and primary school learners. According to Vygotsky, allowing learners to solve some mathematical problems in groups can enhance the academic performance of some learners as they benefit from the knowledge of their more competent peers. This implies that overemphasising individual activities can deny learners the opportunity to learn from one another. Teachers can also enhance the scholastic achievement of secondary school learners by giving them useful hints

which help them to navigate their zones of proximal development. Teachers' ability to use examples and teaching aids from the learners' culture together with employing cooperative learning activities can boost the learners' Mathematics achievement.

2.2.1.2.3 Research on level of thought and achievement in mathematics

A research conducted in the Philippines to ascertain whether a group of newly recruited college students had actually reached the formal operational stage as was expected of people of their chronological ages was outlined by Reedal (2010:19). The study focused on the Mathematics achievement of 59 Philippine students who were tasked to answer some questions on sequences and angles of triangles. Scores were awarded on the basis of the completeness of the responses as well as the level of understanding demonstrated. The study revealed that 61.02% of the students were actually in the concrete operational stage while 38.98% were in the formal operational stage. Although it was expected that the respondents would be in the formal operational stage, the study revealed that they were actually in the concrete operational stage, thereby emphasising the importance of not making assumptions about what children are capable of understanding without first assessing their cognitive ability (Reedal, 2010:19). The research suggested that some secondary school learners could be stuck at the concrete operational stage for longer than was expected. The extent to which teachers attempt to meet learners' cognitive developmental needs can influence their achievement in Mathematics.

Reedal (2010:19) discussed the four developmental stages of Piaget and the educational implications of each stage with regard to the learning of Mathematics. She placed emphasis on the concepts of one-to-one correspondence and comparisons. The paper revealed that learners at the formal operational stage could answer many abstract Mathematical concepts such as comparing infinite sets. On the basis of her research findings, Reedal (2010:19) encouraged teachers to ascertain learners' cognitive developmental stages so as to know how best to teach them. Wrong assumptions regarding Mathematics learners' developmental stage could negatively affect their achievement in Mathematics especially if the assumption was an overstatement of their developmental stage. A similar paper was developed by Blake and Pope (2008:66) who focused on the applicability of both Piaget and Vygotsky's theories in teaching learners at primary, intermediate and advanced

levels. Blake and Pope (2008:66) concluded that the simultaneous application of Piaget and Vygotsky's theories in the teaching of various subjects including Mathematics could enhance the learners' academic performance and further prepare them for future learning.

2.2.1.3 Memory and information processing

The quest for knowledge regarding the functions of the brain and human information processing has put the study of memory processes under the spotlight. Several information processing models have been advanced in an attempt to shed more light on this subject. For the purpose of this study, emphasis was placed on the Atkinson-Shiffrin model, also called the three-system approach to memory (Kosslyn & Rosenberg, 2006:279). According to the Atkinson-Shiffrin model of information processing whose key principles are shown in Figure 2.3, information processing entails three stages and their separate memory stores. The three memory processes are encoding, storage and retrieval which are implicitly implied in the definitions of memory and information processing (Feldman, 2009:213). According to Feldman (2009:213), memory is the process by which human beings encode, store and retrieve information. Ashford and Le Croy (2010:96) claimed that information processing focuses on the capture, selection, coding storage and retrieval of information. As indicated by Mwamwenda (2004:212) and Kosslyn and Rosenberg (2006:279), the three memory stores are the sensory register, the short-term memory and the long-term memory.

Encoding was defined by Kosslyn and Rosenberg (2006:278) as the process of transforming and organising incoming information so that it can be entered into memory, either to be stored or to be compared with previously stored information. While storage is concerned with the retention of information over time, retrieval refers to the recovering of information out of storage for use (Santrock, 2004:248)

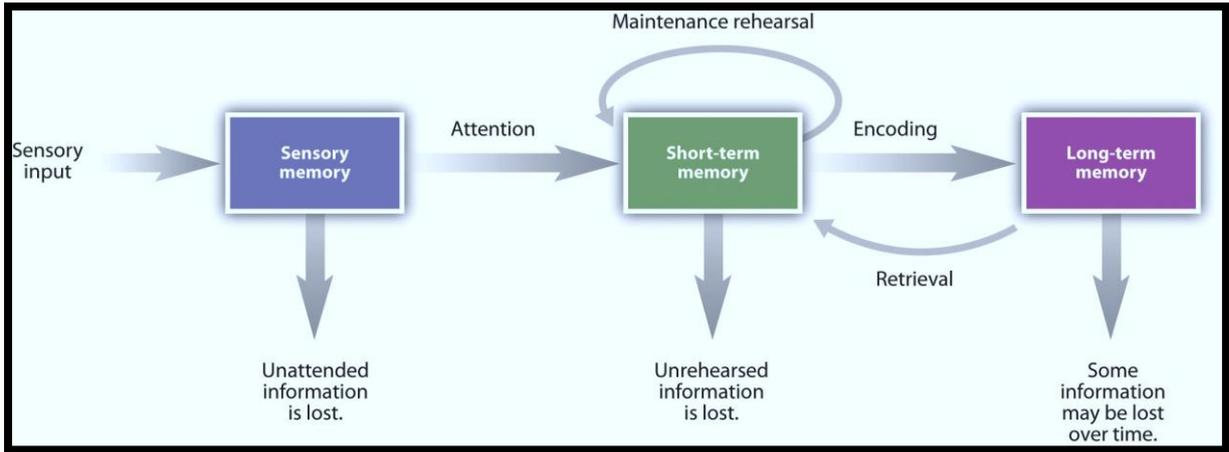


Figure 2.3: The Atkinson-Shiffrin information processing model (Freberg, 2010:418)

The information from the environment is stored by the sensory register only for a split second after it has been detected. The bulk of the information that comes into the sensory register is lost or discarded (Mwamwenda, 2004:212). The information can either be relayed to the short-term memory or is lost through a rapid process called decay.

In the short-term memory, which is also called the working memory or the immediate memory, the information can be lost, renewed or stored. The working memory deals with the things which the individual is consciously thinking about at a given moment in time and it comprises information from the long-term memory, external stimuli or both (Swartz et al, 2011:281). Displacement is the process through which information is lost by being pushed out of storage by other incoming information. According to Tuckman and Monetti (2011:479), the short-term memory has a limited capacity relative to both the amount of time information can be held and the number of items that it can store simultaneously. The working memory has a limited capacity which can only keep approximately five to nine items at a time (Mwamwenda, 2004:212; Kosslyn & Rosenberg, 2006:280). Keeping information in the short-term memory can be intentionally prolonged through rehearsal. Rehearsal is the deliberate and conscious repetition of information over time in a bid to lengthen the time it remains in the working memory (Santrock, 2004:251). According to Sternberg (2009:151), the Atkinson-Shiffrin model posits that the short-term memory does not only hold a limited number of items from the sensory register but also regulates the flow of information to or from the long-term memory.

The information in the short-term memory can be encoded into the long-term memory where it is subsequently stored. According to Swartz et al (2011:273), the long-term memory keeps information which people are not currently conscious of but is stored and is retrievable when there is need. The information that is stored in the long-term memory can stay there for one's lifetime. This is backed by Mwamwenda (2004:212) who indicates that there is credible evidence based on the findings of clinical, experimental and neurological research that the long-term memory is not only unlimited in capacity, but is also capable of storing the entire experiences of an individual from birth to death. Santrock (2004:257) reiterated that the long-term memory has a capacity to hold enormous amounts of information for a long period of time in a relatively permanent fashion. When needed, the information in the long-term memory may be retrieved. However, not all the information in the long-term memory can be retrieved for several reasons. Some of the reasons are discussed below in relation to achievement in Mathematics.

The sequential nature of the information processing model outlined above suggests that the manner in which the environmental stimuli are initially detected by the senses determines how the information will be subsequently processed in the mind. This means the circumstances surrounding the perception, encoding and storage of the information are some of the variables which determine an individual's ability to remember particular information. This has some implications to achievement in Mathematics. The academic performance of learners in Mathematics can be influenced by the extent to which teachers attempt to create circumstances which are conducive to remembering the content. For instance, the frequency with which learners are given Mathematics exercises and the type of teaching aids teachers use can determine the extent to which learners can accurately remember the subject matter. If teachers attempt to harness the attention of the learners during instruction to ensure that the concepts are initially encoded and stored accurately, learners will stand a better chance of remembering the content. This is important because inaccurately stored information is likely to be retrieved in that distorted state. More implications are discussed under the views of Gagne and Ausubel (sections 2.3.3.2.2 and 2.3.3.2.3 respectively).

A variety of memory enhancing techniques can be used to facilitate the storage and subsequent retrieval of information from the long-term memory. According to

Feldman (2009:217), examples of such memory enhancing techniques include chunking and mnemonics such as peg word, acronyms, method of loci and the use of rhyming words. According to Mwamwenda (2004:212) and Santrock (2004:254), chunking is the act of combining the items to be remembered into organised single entities which can easily be remembered. A chunk is a meaningful entity of stimuli that can be stored as a single unit in the short-term memory (Feldman, 2009:215). As far as Conderman, Bresnahan and Pedersen (2009:53) are concerned, a mnemonic is an information processing device for aiding memory. Mnemonic devices are techniques in which learners associate less meaningful content with more meaningful and memorable statements, words and images (Tuckman & Monetti, 2011:281). Acronyms are abbreviations or words which are formed from the first letters of words. Examples include *CHASHOTAO* and *SOHCAHTOA*, which some Mathematics learners use to remember the three basic trigonometrical ratios. While the loci method involves the use of objects which are situated in a particular familiar position to remember certain ideas, peg word entails the use of specific words to remember a certain concept or item (Tuckman & Monetti, 2011:281). The suitability of the memory enhancing techniques employed by teachers can influence the academic achievement of secondary school learners in Mathematics. Failure by teachers to familiarise the learners with appropriate memory enhancing techniques can negatively affect the concerned learners' achievement in Mathematics. The use of memory enhancing techniques can be quite useful in secondary school Mathematics given the diversity of topics which entail mensuration, geometry, trigonometry, statistics and probability.

Other variables related to memory and information processing can ultimately affect learners' achievement in Mathematics. For instance, Feldman (2009:218) posited that research has established that the effectiveness of an individual's working memory can be reduced by stress. The influence of stress on learners' achievement is deliberated on in a later section (section 2.2.2.3).

2.2.1.3.1 Research on information processing and achievement in Mathematics

A longitudinal research study was undertaken by De Smedt, Janssen, Bouwens et al (2009:186) to determine the link between the working memory and the learners' individual differences in Mathematics achievement. A sample of 63 boys and 43 girls from five primary schools in Flanders, Belgium, was used. The study revealed

the existence of a significant relationship between working memory and Mathematics achievement as the working memory was found to be a predictor of Mathematics achievement. Amplifying on the research findings, De Smedt et al (2009:186) reported that the central executive, that is, the working memory, was a unique predictor of the Mathematics achievement of both first grade and second grade learners. Although the study was conducted with primary school respondents, its findings can be informative to the current study which used a sample of secondary school learners.

Using a sample of 66 respondents who were undergraduate university students, Ashcraft and Kirk (2001:227) carried out a study to unravel the relationships in the working memory, Mathematics anxiety and academic performance. A strong negative correlation was obtained between Mathematics anxiety and working memory capacity. Moreover, the study confirmed that Mathematics anxiety, which varies from individual to individual, affects on-line performance of Mathematics-related activities (Ashcraft & Kirk, 2001:224). Although the current study also focuses on the factors which influence academic performance in Mathematics such as Mathematics anxiety and working memory capacity, the research participants are secondary school learners, not undergraduate students.

A study undertaken by Iguchi (2008:5-6) to examine the relationship between scholastic achievements in Mathematics as learners progress through the various educational levels and the working memory adds to the already cited findings. A sample of 136 respondents was used. The study also explored the specific effects of the working memory on the knowledge of basic Mathematics concepts, computational skills and the application of mathematical concepts. The research revealed that academic achievement in Mathematics is predicted more explicitly by the auditory working memory than by verbal and nonverbal memory as well as processing speed. Iguchi (2008:6) reported that there is sufficient evidence to support the theory that the auditory working memory affects Mathematics achievement both at elementary and secondary school levels.

2.2.1.4 Language

Language has captured the attention of many classroom practitioners for several reasons including the fact that language is used as a medium of instruction during teaching. The idea of language as a transparent medium of meaning is embedded in the belief that language can accurately portray reality (Gravett & Geysler, 2004:69). For instance, Abedi, Courtney, Leon, Kao and Azzam (2006:1) implicitly indicated that language is a critical variable in the learning of Mathematics. Moreover, some psychologists have linked language issues with mental functions such as information processing. Baumann et al (2006:221) intimated that teachers should consider the implications of the language they use in the classroom. In Zimbabwean secondary schools, English is mostly used as the medium of instruction in Mathematics although English is not the mother tongue of the majority of the learners. Moreover, all the Mathematics textbooks used in Zimbabwean secondary schools have been written in English. According to Gutierrez (2002:1047) and Mosqueda and Tellez (2008:416), English proficiency is a vital factor in determining Mathematics achievement since several studies have established that Mathematics is itself a language that is more complex than everyday English. After a consideration of the views such as the ones raised above, it was deemed appropriate to explore the influence of language on Mathematics achievement in the current study. The concept of language is explored first before a few theories of language and cognition are scrutinised.

2.2.1.4.1 *The concept of language*

Language can be viewed as a form of communication, whether spoken, written or in the form of signs, that is, arranged according to systematic rules (Santrock, 2004:56; Feldman, 2009:271). Language is defined by Goldstein (2008:389) as a system of communication that uses symbols or sounds for expressing feelings, concepts, thoughts and experiences. The above definitions basically portray language as a medium which is used to transmit ideas through speech, signs or written symbols guided by certain rules.

2.2.1.4.2 *Theories of language and cognition*

A multiplicity of theories has been developed in a bid to unravel the relationship between language and thought. According to Baumann et al (2006:221), language,

thought and action develop in intricate interrelationships. For the purpose of this study, Vygotsky's views on language and thought are discussed together with the Sapir-Whorf hypothesis.

2.2.1.4.2.1 Vygotsky's views on language and cognition

The work of Lev Vygotsky tried to clarify the relationship between cognitive processes and language. Santrock (2004:52) posited that Vygotsky claimed that although language and thought initially developed as separate entities, they later merged. According to Bhatt (2007:37), language is a tool for thinking, that is, inner speech is verbal thinking. This means language is viewed as an important instrument which facilitates thinking. In support of this, Baumann et al (2006:66) pointed out that concepts first develop in the child at the inter-mental level and subsequently become internalised at an intra-mental terrain. After mastering the ability to engage in self-talk, the individual uses language as internal thought for regulating behaviour and cognitive processes. An individual's private speech, it is believed, is that which the individual uses to communicate with herself or himself for purposes of self-guidance, self-direction and problem solving (Bhatt, 2006:37). Since Vygotsky claimed that linguistic fluency is related to mental sharpness, one could argue that learners who are proficient linguistically have a higher chance of achieving well in Mathematics. On the basis of such a perspective, it could be hypothesised that academic achievement in Mathematics can be influenced by language issues.

2.2.1.4.2.2 The Sapir-Whorf Hypothesis

One of the most popular and yet controversial theories pertaining to the association between language and cognition is the Sapir-Whorf hypothesis, which is sometimes referred to as Whorfianism (Matsumoto & Juang, 2008:241; Nisbett & Norenzayan, 2002:6; Ji, Zhang & Nisbett, 2004:58). According to Park (2000:19), the idea that language determines the nature of thought has a long history within psychology, expressed in its most renowned form via the so-called Sapir-Whorf hypothesis which postulates that the manner in which people see and speak of the world is significantly determined by their mother languages. Basically, the Sapir-Whorf hypothesis posits that the structures of a language strongly influence or even determine the manner in which its native speakers perceive the world in a highly habituated manner (Wang et al, 2008:27). According to Goldstein (2008:387), the Sapir-Whorf hypothesis was proposed by anthropologist Edward Sapir and linguist

Benjamin Lee Whorf and it postulates that the way people think is affected by the nature of their culture's language.

More specifically, the Sapir-Whorf hypothesis has two versions; namely, linguistic determinism and linguistic relativity. Linguistic determinism, which was developed by Sapir and is referred to as the extreme or strong version of the hypothesis, states that language determines thought. The weaker version, which was developed by Whorf, is linguistic relativity which suggests that language affects thought, that is, the structure of language affects cognition. Linguistic relativity has been backed by more research evidence than linguistic determinism (Nisbett & Norenzayan, 2002:8; Feldman, 2009:275). Wang et al (2008:27) and Park (2000:19) concurred that while linguistic determinism has not been adequately backed by research evidence, many studies on linguistic relativity have been undertaken.

The Sapir-Whorf hypothesis has some connotations regarding the relationship between language and Mathematics achievement especially if the learners are not taught Mathematics in their mother language as is the case in most Zimbabwean schools. Different languages differ in the way numbers are named as evidenced by the procedural irregularities in counting and numerical notation systems (Nisbett & Norenzayan, 2002:9-10). Examples of such differences can be noted between Roman numerals and Arabic numerals. Some educationists and researchers have developed an interest in determining the extent to which cultural differences in numerical notation systems and number naming systems influence mathematical cognition. The focus on the variance which can be attributed to cultural differences is related to the Sapir-Whorf hypothesis (Ji et al, 2004:58; Nisbett & Norenzayan, 2002:9). A significant number of psychologists and Mathematics educators argue that the debate concerning language and number concepts has persuaded them to believe that linguistic differences play an important part in explaining the higher scores of, for instance, Korean, Chinese, and Japanese learners on number tasks (Park, 2000:19; Matsumoto & Juang, 2008:240). The above sentiments suggest that language has a bearing on Mathematics achievement.

2.2.1.4.3 Research on language and achievement in Mathematics

A study to explore the influence of language proficiency on Mathematics achievement was conducted by Mosqueda and Tellez (2008:416) with a sample of 2 234 Latino 10th graders. The study revealed that the English language proficiency of non-English speakers is very important in predicting the learners' Mathematics achievement. One could, therefore, conclude that language proficiency can be a serious determinant of Mathematics achievement in Zimbabwe. As already indicated, secondary school Mathematics is mostly taught in English, yet English is not the mother language of the generality of the learners in Zimbabwe. The conclusion made above was supported by Barton and Neville-Barton (2003:19) who argued that language problems are a critical contributing factor towards poor scholastic achievement in Mathematics particularly in bilingual and multilingual settings.

A longitudinal study undertaken by Chang, Sing and Filler (2009:27) in the United States of America revealed that the Mathematics achievement of English language learners can be aggravated by ability grouping. The research also established that the Mathematics achievement of English language learners was worse than that of English-speaking learners. This tallies with the findings of Mosqueda and Tellez (2008:416) outlined above. Although the study by Chang et al (2009:27-45) involved ability grouping as an additional variable, it confirms that language can influence learners' Mathematics achievement.

Prins and Ulijn (1998:139-159) undertook research in an attempt to determine the linguistic and cultural factors in the ordinary language of Mathematics literature which influenced readability (Prins & Ulijn, 1998:139-159). The study theoretically revolved around the Sapir-Whorf hypothesis and it was necessitated by the multilingual nature of South Africa as a country. Protocol analysis was employed on the data obtained from a sample of 108 learners in the 17 to 18 year age range to identify readability problems. The results of the study confirmed the assertion that Mathematics achievement can be improved by improving the readability of Mathematics assignments texts. Linguistic and cultural factors in ordinary language are implicated in influencing learners' achievement in Mathematics.

2.2.1.5 Previous academic performance

The Zimbabwean education system is structured in such a way that learners enter secondary education after spending at least seven years at primary school level. By the time they embark on secondary education, many learners would have developed a subjective idea of their self-efficacy on the basis of their performance at primary school level. According to Snowman et al (2009:274), one apparent way in which people develop a sense of what they can and cannot do in various areas is thinking about how well they have performed in the past on a given task or a set of allied tasks. This assertion is suggestive of the view that learners' Mathematics achievement can be influenced by their previous performance in the same subject or other allied subjects. In the same vein, Jagero and Masasi (2012:2) alluded to the assertion that learners' previous performance can influence their later achievement. A consistent record of past academic success can lead learners to attribute their success to their own intellectual abilities (Tuckman & Monetti, 2011:406).

The sequential and hierarchical nature of mathematical concepts makes a learner's previous knowledge and previous academic achievement very critical determinants of the learner's current and future academic performance (Sternberg, 2009:341). David Ausubel's subsumption theory placed emphasis on the importance of previous knowledge, as outlined in section 2.3.3.2.3. Moreover, Robert Mills Gagne advanced the hierarchical learning theory in which the importance of previous knowledge is portrayed as a vital determinant of the mastery of more sophisticated concepts and skills, as discussed in the next section.

2.2.1.5.1 Gagne's hierarchical learning theory

Gagne adopted an eclectic approach in formulating the hierarchical learning theory by making reference to Pavlov's classical conditioning, and operant conditioning advanced by Skinner after modifying Thorndike's work. Gagne also referred to cognitive processes. According to Gagne, learning is basically sequential in nature and it builds on previously acquired knowledge and skills. As far as Gagne is concerned, higher order learning depends on previously learnt skills and knowledge since simple concepts and skills act as a foundation for more intricate ones.

Gagne advanced the hierarchical theory of learning which he also referred to as the eight conditions of learning or the hierarchy of learning. The hierarchy of learning in ascending order of complexity of the skills, as suggested by Westwood (2004:04), includes signal learning, stimulus-response association, chaining, verbal association, discrimination learning, concept learning, learning of rules and finally problem solving, as shown in Figure 2.4. The majority of the mathematical concepts learned at secondary school level require discrimination skills, concept learning, learning of rules and problem solving. It can be argued that the hallmark of studying Mathematics is problem solving (Feldman, 2009:257). Consequently, in the current study, more emphasis is placed on discrimination learning, concept learning, learning of rules and problem solving.

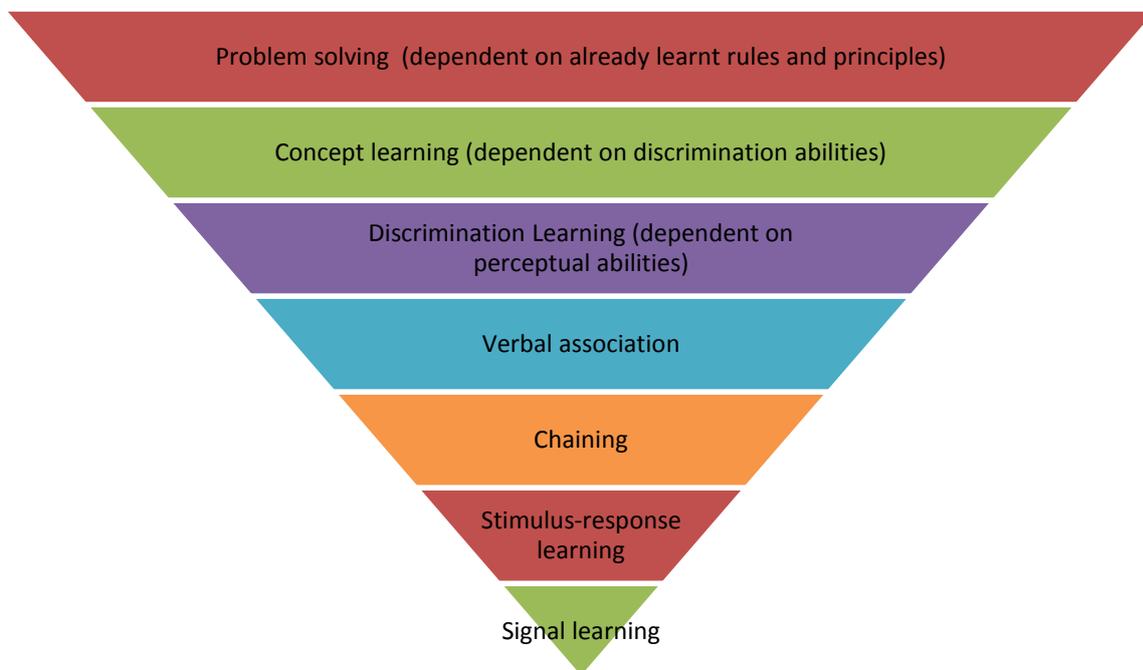


Figure 2.4: Gagne's hierarchical learning theory arranged in ascending levels of complexity

Signal learning is the most elementary version of learning and is similar to classical conditioning as theorised by Pavlov and Watson. Signal learning is a type of learning in which a stimulus gains the capacity to elicit a reflexive response (Swartz et al, 2011:194; Tuckman & Monetti, 2011:227-228). It's a version of learning in which a neutral stimulus gains the ability to evoke a response after being paired with that which naturally elicits the response (Feldman, 2009:178; Driscoll, 2000:341). Stimulus-response association is basically similar to Thorndike's theory of

instrumental learning and Skinner's operant conditioning. This is a type of learning in which the outcomes of a behaviour determine the probability of the repetition of the behaviour in similar situations in the future (Snowman et al, 2009:191; Tuckman & Monetti, 2011:233; Bergh & Theron, 2009:5). Chaining is the ability to link at least two previously learned distinct stimulus-response connections into a coherent sequence. According to Tuckman and Monetti (2011:253), chaining refers to the connecting of simple responses in a sequence to form a more intricate response that would be difficult to learn simultaneously. Verbal association is a version of chaining in which a learner uses verbal connections to make associations.

Discrimination is the ability to respond in different ways to similar stimuli (Snowman et al, 2009:194). Discrimination learning occurs when the learner gains the ability to detect the unique aspects of seemingly similar situations and subsequently responds differently to each phenomenon (Driscoll, 2005:47; Swartz et al, 2011:26). Gagne viewed the ability to discriminate between stimuli as a crucial aspect of the learning process. Discrimination is made more intricate by the possibility of interference which occurs when there is a mix-up of different but allied concepts and this can cause learners to forget the concepts. For instance, some learners may encounter difficulties in evaluating $\log\left(\frac{16}{8}\right)$ and $\frac{\log 16}{\log 8}$ as a result of failing to discriminate between the two mathematical problems.

A concept is a fundamental unit of symbolic knowledge which in essence is an idea about something that provides meaning in order to understand situations (Sternberg, 2009:237). Concept learning takes place when the learners master the ability to generalise responses on the basis of a set of stimuli which belongs to a particular category or class such as odd and even numbers or regular and irregular polygons. On the other hand, rule learning is a sophisticated cognitive process in which the learner is able to identify the relationships between at least two concepts and subsequently apply them in both familiar and novel situations. The learning of rules involves the formation of a chain of several concepts which enables the learner to demonstrate a particular behaviour.

Problem solving can be defined as an attempt to overcome hindrances obstructing the path to a solution (Sternberg, 2009:326). It entails devising a suitable way to achieve a goal (Santrock, 2004:298). According to Kosslyn and Rosenberg

(2006:357), a problem is a snag that must be overcome to attain a goal. Problem solving is attained by a learner who becomes capable of utilising the previously learned rules to deal with challenges in new situations. According to Gagne, problem solving is the highest level of cognitive functioning in the learning hierarchy and involves developing the ability to formulate procedures, rules and algorithms to solve similar but slightly different problems (Kosslyn & Rosenberg, 2006:358; Sternberg, 2009:331). For example, at secondary school level, problem solving occurs when learners become capable of solving quadratic equations by using the quadratic formula which has been derived from the method of completing the square.

2.2.1.5.2 Research on the relationship between current achievement and previous achievement in Mathematics

A research undertaken by Yates (2000:4-15) in South Australia with respondents in the 8 to 12 year age range confirmed that learners' previous Mathematics achievement was a significant predictor of their Mathematics achievement at a later stage. Rafi and Samsudin (2007:53-67) established that previous Mathematics achievement correlates moderately with learners' experience in engineering.

An allied study was conducted by Sasanguie, Van den Bussche and Reynvoet (2012:119-128) in an attempt to determine the predictive influence of learners' earlier mastery of the basic number processing tasks which include number comparison and number line estimation on academic achievement in Mathematics at a later stage. The relationship between the academic performance of children on a variety of basic number processing tasks and their individual Mathematics achievement scores on a curriculum-based test measured one year later was explored. A large sample of primary school learners in the 5 to 7 year age range participated in the study. It was found that most of the differences in academic performance in Mathematics are attributable to the level of mastery of non-symbolic number line estimations at an earlier stage. Therefore, one can conclude that earlier mastery of mathematical concepts can influence the academic performance of learners at a later stage.

On the basis of a longitudinal research study conducted in Britain, it was concluded by Aubrey, Dahl and Godfrey (2006:27-46) that Mathematics competence during the early primary school years is critical for sound Mathematics achievement at a later

stage. Thomas (2002:423-442) and Hemmings and Kay (2009:1-10) backed the view that previous academic performance in Mathematics is a determinant of current or future Mathematics achievement.

It has been established that a learner's previous academic achievement in a given subject can influence their future scholastic achievement in the same subject (Yates, 2000:4-15; Jagero & Masasi, 2012:2; Snowman et al, 2011:406). This is a basis for the claim that there is a relationship between irrational beliefs and learners' previous academic achievement in Mathematics. This is suggested by the irrational belief that *because something once strongly affected our life, it should indefinitely affect it*. The existence of such a relationship is examined in the current study.

2.2.2 Affective Factors

Affective factors play a critical role in learning in general and in the learning of Mathematics in particular (Wei, 2010:7). Stakeholders in Mathematics education have always been concerned about affective variables in the learning of Mathematics (Wei, 2010:7). In the current study, affective factors of secondary learners which are explored entail motivation, self-concept, stress and anxiety which are shown in Figure 2.5. For each of these affective variables, some selected theories are highlighted before relevant research studies pertaining to achievement in Mathematics are discussed.

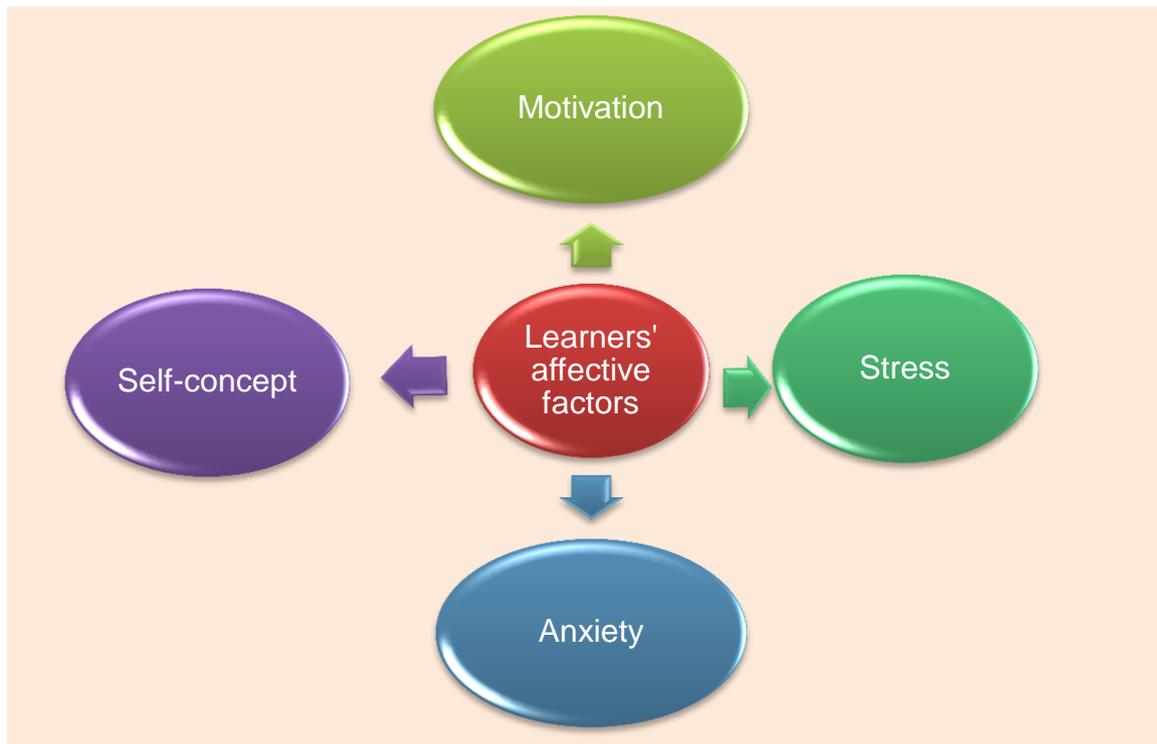


Figure 2. 5: Affective variables which can influence Mathematics achievement

2.2.2.1 Motivation

Motivating learners to learn in education is a topic of great concern to many classroom practitioners today due to the positive relationship between motivation and learning (Awan, Noureen & Naz, 2011:72). Among the various psychological and personal factors which have been explored by researchers in connection with academic achievement, motivation seems to be gaining popularity (Shihusa & Keraro, 2009:414). It is against this background that the concept of motivation is explored in this study.

2.2.2.1.1 Description of the concept of motivation

According to Gazzaniga and Heatherton (2006:343) and Crawford, Kydd and Riches (2011:89), the word *motivation* is a derivative of the Latin word *movere* which means to move. Motivation can be defined as all the phenomena that are involved in the stimulation of action towards particular objectives where there was previously little or no movement towards these goals (Gerrig & Zimbardo, 2006:386; Kumar, 2004:135). In the education context, motivation was defined by Md.Yunus and Ali (2009:93) as the learner's willingness, desire, need and compulsion to take part in

and be successful in the learning process. Rao and Rao (2003:23) and Abuameerh and Saudi (2012:313) viewed motivation as the force which energises and gives direction to behaviour and can determine the individual's degree of persistence. In teaching and learning situations, motivation is concerned with the cognitive, emotional, and behavioural indicators of student investment in and attachment to education (Tucker, Zayco & Herman, 2002:477). Therefore, academic motivation is the psychological process that influences the direction, intensity and persistence of behaviour related to learning (Shihusa & Keraro, 2009:414).

From the above definitions of motivation, it can be deduced that learners' scholastic achievement in Mathematics can be influenced by their degree of motivation. This was backed by Rao and Rao (2003:25) who asserted that low motivation on the part of learners could lead to poor achievement in Mathematics. This means there is a positive correlation between a learner's level of motivation and his or her academic performance if the other variables are held constant. The arousal, direction, choice and maintenance of behaviour on the part of the learners are to a large extent determined by their levels of motivation (Shihusa & Keraro, 2009:414). Motivated school learners normally choose tasks at the border of their competencies, initiate action when given the chance and exert intense effort and concentration in the implementation of learning tasks (Md.Yunus & Ali, 2009:93). Such learners generally show positive emotions such as enthusiasm, optimism, curiosity, and interest when performing academic activities. It can be contended that learners who exhibit the above-mentioned attributes in the learning of Mathematics are likely to enhance their chances of scoring high marks. The success and achievement of a learner in life in general and in learning in particular sometimes depend on the learner's level of motivation (Rao & Rao, 2003:23). It is through motivation that learners remain riveted to their scholastic activities despite many other enticing and tempting activities. The role of motivation in determining the learners' academic achievement in Mathematics is further scrutinised when allied research studies are explored.

In broad terms, motivation has been categorised into two types which are intrinsic motivation and extrinsic motivation (Mwamwenda, 2004:236). According to McLean (2003:9), people are intrinsically motivated when they want to do something for its own sake, interest and enjoyment, that is, when they get a feeling of satisfaction

during rather than after an activity. Intrinsic motivation is when people are motivated by the inherent nature of the task to be performed, their pleasure in mastering something novel or the natural outcomes of the activity (Lahey, 2009:372). Similarly, McLean (2003:9) reiterated that intrinsically motivated learners embark on academic activities for the sake of satisfying their desire to gain knowledge and understanding, the quest for gaining a sense of competence, accomplishment and self-determination. Intrinsic motivation refers to the act of spontaneously engaging in an activity for its own sake without any response to external pressure. Variables such as enthusiasm to learn, the level of difficulty of the task to be accomplished and the need for control can influence intrinsic motivation (McLean, 2003:9).

On the other hand, extrinsic motivation is the drive to embark on an activity in response to the desire to earn external rewards like recognition, financial benefits, praise and medals and not on the basis of the individual's internal interests (Lahey, 2009:372). According to Santrock (2004:418) and McLean (2003:9), extrinsic motivation is the desire to embark on a particular activity to gain an external reward or shun an undesirable outcome. Educators are encouraged by McLean (2003:9) to utilise both intrinsic and extrinsic motivation. As a matter of principle, Mwamwenda (2004:236) recommended that classroom practitioners begin by extrinsically motivating learners through rewards such as marks and praises and then leading the learners to be ultimately intrinsically motivated to experience pleasure and satisfaction from engaging in academic activities.

Donald et al (2010:95) emphasised that motivation has captured the attention of the various stakeholders in the education fraternity probably because it provides valuable insights into and explanations for the learning and behaviour of learners in all subjects including mathematics. In support of this view, Lahey (2009:372) observed that learners who hate to do their Mathematics homework will often do it diligently if they are rewarded with extra allowance money. Information such as when and how to use intrinsic and/or extrinsic motivation to boost the learners' scholastic achievement can be obtained when one studies the concept of motivation (McLean, 2003:9; Mwamwenda, 2004:236). Extrinsic motivation can be used to boost the frequency of a particular behaviour for which an individual is intrinsically motivated (Lahey, 2009:372). However, Snowman et al (2009:342) argued that studies such as

the one conducted by Akin-Little, Eckert, Lovett and Little (2004:344) reveal that the use of rewards in education can reduce the level of intrinsic motivation under certain conditions. It is with these considerations in mind that one can argue that motivation is one of the key determinants of scholastic achievement in Mathematics.

2.2.2.1.2 Self-determination theory

The self-determination theory, which was developed by Deci and Ryan, is a macro theory of motivation which focuses on a person's self-motivated and self-determined behaviour (Deci & Ryan, 2000:68-78; Standage & Treasure, 2002:88). In a school setting, the self-determination theory helps to distinguish between the behaviours that learners perform by choice driven by the need for autonomy and those behaviours which learners are forced to do (Tuckman & Monetti, 2011:433). The self-determination theory portrays motivation as a fluid entity which lies in a continuum ranging from amotivation to intrinsic motivation with extrinsic motivation centrally positioned between the two extreme ends (Visser, 2009:8; Deci & Ryan, 2000:68-78). According to Tuckman and Monetti (2011:433), a person's volitionally selected behaviours reflect intrinsic motivation while those behaviours which are instrumental to some outcome represent extrinsic motivation. Between amotivation and intrinsic motivation are four types of extrinsic motivation; namely, external regulation, introjected regulation, identified regulation and integrated motivation (Standage & Treasure, 2002:89; Gagne & Deci, 2005:335). Figure 2.6 summarises the self-determination continuum.

Amotivation is characterised by virtually little or practically no motivation to undertake a given task or activity. The sheer lack of intention, self-determination and motivation have been described by Gagne and Deci (2005:334-335) as amotivation. Visser (2009:9) agreed with the already outlined ideas that extrinsic motivation is the type of motivation in which someone engages in an activity to gain a reward which is separable from the activity performed (Visser, 2009:9).

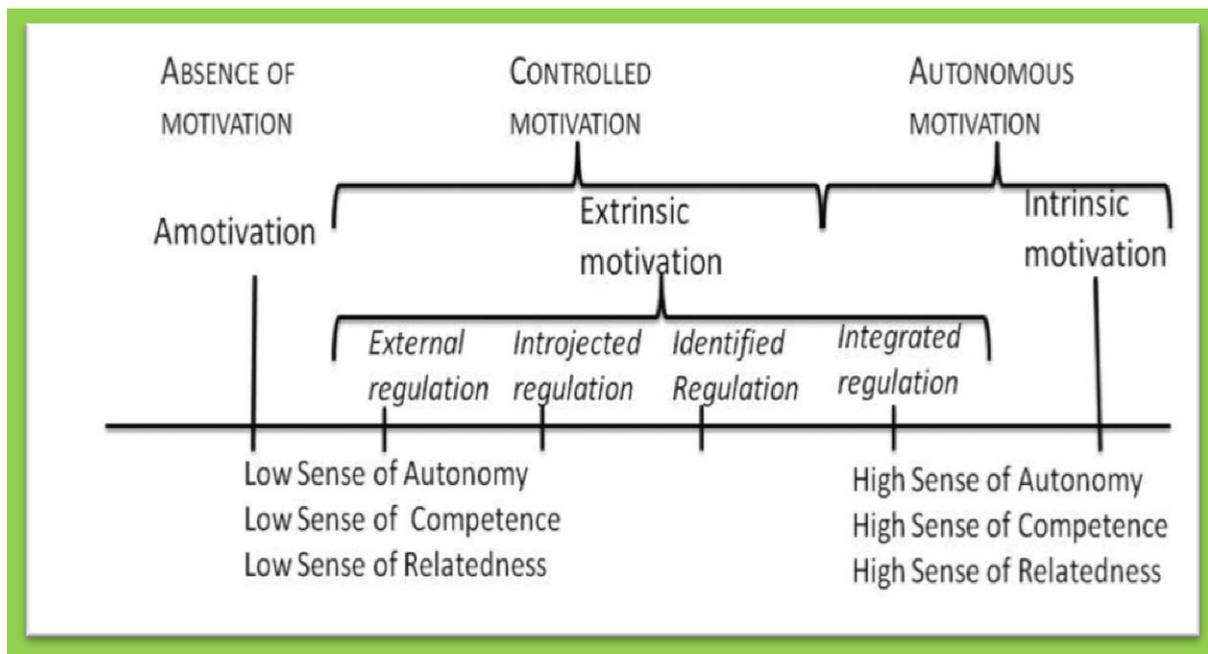


Figure 2.6: The self-determination continuum (Visser, 2009:10)

According to Visser (2009:8), external regulation is a variation of extrinsic motivation in which an individual engages in an activity as a result of external coercion exerted by external variables such as rewards or punishments. Externally regulated behaviour is maintained by contingencies external to the person and the person's participation is fuelled by the desire to get a favourable outcome or to avoid an aversive one (Gagne & Deci, 2005:334). Introjected motivation is a type of motivation in which the individual has partially internalised the external justification for embarking on an activity (Standage & Treasure, 2002:90).

Identified regulation is a type of motivation in which the task to be performed by an individual to some extent resembles the individual's personal identity, values and goals thereby allowing the individual to make some choices (Visser, 2009:9). Integrated regulation is attained by a person whose personal system of values and convictions encompasses the task to be performed so much that the task defines who the person is (Visser, 2009:9).

There are three innate, universal and psychological intrinsic needs which are involved in self-determination and these are competence, autonomy and psychological relatedness (Chen & Jang, 2010:742; Standage & Treasure, 2002:89; Lynch, 2010:1; Quested & Duda, 2010:40). Studies conducted in various countries

confirm that the satisfaction of the need for competence, autonomy and relatedness is a fundamental prerequisite for optimal functioning (Deci & Ryan, 2008:183). It is through gratifying these three basic psychological needs that an individual can develop and retain intrinsic motivation even when the individual is engaged in non-intrinsically motivating activities (Chen & Jang, 2010:750). However, it has to be acknowledged that the three psychological needs are strongly influenced by the social context in which the concerned individual is operating (Niemi & Ryan, 2009:135).

According to Visser (2009:9) together with Deci and Vansteenkiste (2004:17), the need for competence is the perception of being effective in tackling challenges in one's environment. Competence can be viewed as knowing how to achieve certain results and feeling efficacious in doing so (Wang & Holcombe, 2010:635). While objective competence refers to what one is actually good at, perceived competence is concerned with what the individual thinks is within his or her level of mastery. Research has established that perceived competence is more vital for instilling motivation than objective competence.

Competence can be enhanced through giving positive feedback, giving challenging but manageable tasks, giving a clear structure of the task to be performed and expressing positive expectations (Niemi & Ryan, 2009:139; Visser, 2009:11). According to Deci, Guardia, Moller, Scheiner and Ryan (2006:315-316), the provision of informative positive feedback which specifically elaborates on one's domains of mastery boosts feelings of competence thereby facilitating the development of intrinsic motivation. Burt, Young-Jones, Yardon and Carr (2013:51) alluded to the idea that fragmenting an intricate and challenging task into manageable sub-tasks could facilitate a sense of competence. According to Visser (2009:11), factors such as negative feedback, attributing success to chance and voicing negative expectations have been found to militate against the development of healthy perceptions of competence.

Autonomy as a motivational psychological need refers to the ability to be the causal agent of one's own life (Chen & Jang, 2010:742; Deci & Vansteenkiste, 2004:18). Visser (2009:9) observed that the need for autonomy is the sense of being able to exercise choice and experience psychological freedom in the commencement and

sustained engagement in a particular task. This means autonomy is the need to have control over what one is doing and be free to regulate oneself when it comes to making choices. Therefore, autonomy can be viewed as having an internal locus of control and the freedom to determine one's own behaviour.

Autonomy can be boosted through providing and emphasising choice rather than control; permitting people to follow their own individual approaches; giving a meaningful justification for any suggestions made and promoting experimentation and self-initiative (Gagne & Deci, 2005:338; Visser, 2009:11). Autonomy can also be increased by exercising originality in undertaking the task at hand. This can be done through introducing variations in aims and objectives, the procedures of undertaking the task at hand or even crafting new justifications for the activities. Deadlines and stringent pressured evaluations have been found to be detrimental to the individual's sense of autonomy. This is backed by Visser (2009:11) who posited that perceptions of autonomy are normally stifled by variables such as punishments, grades, imposition of goals and stringent deadlines. This implies that autonomy can be boosted by avoiding deadlines and rigorous evaluation procedures where possible. Some of these ideas are discussed in relation to their influence on Mathematics achievement under classroom management and teaching instructions (sections 2.3.3.1 and 2.3.3.2 respectively).

The need for relatedness can be defined as the perception of being cared for and well linked to other people (Chen & Jang, 2010:742; Quested & Duda, 2010:40; Visser, 2009:9). This is backed by Wang and Holcombe (2010:636) who viewed relatedness as the strength of one's affiliations and connections to other people in a particular social context. According to the self-determination theory, human beings usually integrate and internalise the values of the social group to which they are affiliated, thus incorporating the group values into their own value systems. Combined with a sense of competence, relating to a social group can motivate people. Relatedness can be enhanced by finding social groups whose values and behaviours mirror the ambitions of the individual. Improving the quality of social interaction within social groups can enhance relatedness. According to Visser (2009:11), a sense of relatedness can be promoted through appreciating different people's perspectives, placing emphasis on the uniqueness of each individual, seeking different people's opinions and encouraging cooperative learning activities.

The psychological need for relatedness can be thwarted by non-responsiveness and insensitivity to each individual's concerns, excessive competition and lack of respect (Visser, 2009:11). It can, therefore, be argued that the achievement of learners in Mathematics can be negatively affected by excessive competition in class and being belittled by significant others such as peers and teachers. Some of these ideas are examined in detail under home factors and school factors which can influence the learners' achievement in Mathematics at secondary school level (sections 2.3.2 and 2.3.3 respectively).

For a person to accomplish a given task, he or she has to try to integrate autonomy, competence and relatedness into the task to be performed. This implies that competence can be considered as the need to succeed in whatever a person is engaged in, with relatedness being the extent to which the person is connected to others while autonomy focuses on the individual's ability to be in control of his or her own life. The self-determination theory sheds more light on the concept of motivation since it goes beyond categorising motivation simply as intrinsic or extrinsic.

2.2.2.1.3 McClelland's Need Theory

David McClelland is an American psychologist who extensively researched the subject of human motivation. Awan et al (2011:72) alluded to the fact that McClelland developed the Achievement Theory in which people are viewed as having three needs: the need for achievement, the need for affiliation and the need for power. According to Baumann et al (2006:80), achievement motivation is the need to attain goals that require competence to be accomplished. Achievement motivation can be defined as the extent to which people differ in their need to strive to attain rewards such as physical satisfaction, praise from others and feelings of personal mastery (Awan et al, 2011:73). Achievement motivation is the psychological need to succeed in various areas of life which include school and work (Lahey, 2009:369). McClelland viewed the need for achievement as the most important need among the three identified needs. Consequently, for the purpose of the current study, attention is directed at achievement motivation.

Zenzen (2002:10) and Awan et al (2011:73) argued that learners with high levels of achievement motivation normally work hard to attain the desired level of success. Linking this assertion to achievement in Mathematics, it can be assumed that

learners who have a high need for achievement will probably obtain high marks in Mathematics. Moreover, people with a high need for achievement are risk-takers who embark on reasonably challenging but potentially achievable goals (Abuameerh & Saudi, 2012:314). People with high achievement motivation normally behave in ways which help them to out-perform others, meet or surpass some standard of excellence, or do something unique (Awan et al, 2011:73). Moreover, people driven by the need for achievement are comfortable working either as individuals or within a team with other people who are as ambitious as they are. All the attributes outlined above seem to back the notion that there is a strong positive correlation between one's level of achievement motivation and the level of effort one exerts which is likely to yield the desired success. This is backed by Rao and Rao (2003:25) who declared that the level of achievement motivation of a learner in a given academic discipline accounted for that learner's achievement in that subject. Hence achievement motivation can be considered as one of the important determinants of Mathematics achievement.

2.2.2.1.4 Research on motivation and achievement in Mathematics

Tella (2007:154) undertook a study to determine whether the academic performance of highly motivated and lowly motivated learners in Mathematics would differ significantly. A sample of 450 randomly selected male and female learners drawn from 10 schools in Ibadan was used. Secondary school learners differed significantly in their academic achievement as a result of their different levels of motivation (Tella, 2007:154). The results revealed that highly motivated learners perform better academically than the lowly motivated counterparts.

Md.Yunus and Ali (2009:93) conducted a related study to explore the relationship between motivation and Mathematics achievement as well as the association between variables such as effort, self-efficacy, gender and cumulative grade point average. The sample comprised 195 final year students. The study established that significant positive correlations exist between effort, self-efficacy, overall motivation and students' overall academic achievement. Similarly, significant positive correlations were found between effort, self-efficacy, worry, and overall motivation with students' average Mathematics achievement. However, Md.Yunus and Ali (2009:99) acknowledged that their study could not establish whether a high level of

motivation is the result of high achievement or whether high achievement is the result of a high level of motivation. Despite the study's inability to indicate the direction of causality, it managed to establish that variables such as motivation, effort and self-efficacy positively correlate with achievement in Mathematics.

Mousoulides and Philippou (2005:321) examined the associations between motivational beliefs, the use of self-regulation strategies and academic achievement in Mathematics. The respondents were 194 pre-service teachers in Cyprus. Motivational beliefs related to self-efficacy were found to be strong predictors of achievement in Mathematics. On the other hand, the use of self-regulation strategies was found to have a negative effect on Mathematics achievement. This study served to indicate the importance of motivation as a determinant of academic achievement in Mathematics.

Studies by Tella (2007:154), Md.Yunus and Ali (2009:93) and Mousoulides and Philippou (2005:321) confirmed that motivation is a major determinant of Mathematics achievement. There is a possibility that a relationship exists between motivation and some irrational beliefs. For example, the irrational beliefs that *it is easier to avoid than to face life difficulties and self-responsibilities* and that *people should be thoroughly competent, intelligent, and achieving in all possible respects* are likely to have a relationship with learners' motivation to study Mathematics. The existence of such a relationship is explored in subsequent chapters.

2.2.2.2 Self-concept

According to Ayodele (2011:176), self-concept as a phenomenon has received much attention from many researchers and has accordingly been defined in different ways. In the current study, the developmental stage of the respondents necessitated a thorough exploration of the development of self-concept. After a definition of self-concept is given, the views of several theorists regarding self-concept are discussed followed by a review of some research concerning the influence of self-concept on mathematics achievement.

2.2.2.2.1 Description of the term self-concept

According to Manning (2007:11), self-concept refers to an individual's perceptions of competence or adequacy in scholastic and non-academic domains such as social and behavioural areas and is best represented by a profile of self-perceptions across the various domains. A person's self-concept is his or her subjective perception of who he or she is and what he or she would like to be (Lahey, 2009:405). As far as Ayodele (2011:176) is concerned, self-concept in a Mathematics context is the way people think, feel, act, value and evaluate themselves relative to Mathematics achievement.

2.2.2.2.2 Erikson's theory on identity formation

Erik Erikson is a Neo-Freudian ego psychologist whose psychosocial theory is composed of eight lifelong developmental stages (Kail & Cavanaugh, 2004:16). In their correct order, the eight bipolar psychosocial stages are basic trust versus mistrust; autonomy versus shame and doubt; initiative versus guilt; industry versus inferiority; identity versus role confusion; intimacy versus isolation; generativity versus stagnation and finally ego integrity versus despair (Berns, 2010:40; Austad, 2009:101). The psychosocial theory is characterised by the quest for an identity as the individual resolves a psychosocial crisis at each of the eight stages. Erikson claimed that failure to resolve a crisis at a particular stage has an effect on the individual's subsequent development relative to several developmental aspects although a crisis can be resolved at a later stage (Meggitt, 2006:162).

Erikson maintained that once self-concept development commences during childhood, especially during the second stage, it continues to develop and mutate as the individual resolves all the stages of his or her life (Austad, 2009:101). According to Ashford and Le Croy (2010:90), the progression through the stages is determined by both biological factors and age-related sociocultural expectations. This was supported by Keenan (2002:41) who described the theory as epigenetic, that is, a theory which acknowledges the interaction of genetic and environmental factors during development. A child who gains a sense of autonomy develops a good level of self-esteem while feelings of shame and doubt weaken the child's self-esteem. Erikson maintains that through socialisation, the child's self-concept is further moulded in the third and fourth stages in proportion to the extent to which a child experiences and exercises mastery over some tasks and gains positive recognition

from significant others such as parents and siblings. As pointed out by Meggitt (2006:163) and Berns (2010:41), during the industry versus inferiority stage, a child's self-concept can be boosted by gaining a sense of competence while a child who is engulfed by feelings of inferiority tends to develop a fractured concept of the self.

The fifth stage of Erikson's psychosocial theory, the identity versus role confusion stage, was particularly singled out by Erikson as a critical developmental stage (Swartz et al, 2011:90). As already indicated, this is the stage which is critical to the current study since most secondary school learners are adolescents. Erikson proposed that the fifth psychosocial stage is a landmark developmental stage because it enables the developing individual to condense the key aspects of the first four stages while simultaneously anticipating how to find his or her way through the final three psychosocial stages (Swartz et al, 2011:90; Berns, 2010:41-42). This was backed by Feldman (2009:427) and Tuckman and Monetti (2011:129) who posited that Erikson placed a great deal of emphasis on identity formation during adolescence. According to Stickle (2010:18), identity formation entails the examination of personal likes and dislikes relative to factors such as political, religious and moral values, occupational interests, gender roles and sexual behaviours. This means the self-concept of an adolescent is affected by his or her potential adult roles as he or she commences to visualise the future relative to aspects such as career, relationships and family responsibilities.

Prior to the development of a sense of identity, the adolescent in most cases experiences role confusion. Austad (2009:101) claimed that failure by the adolescent to successfully resolve the problem of role confusion results in identity confusion and uncertainty about his or her place in the world. According to Berns (2010:41), the danger associated with this stage is that young people going through the normal process of trying out many roles may be unable to choose an identity or make a commitment and consequently will not know who they are. Furthermore, the identity formation of adolescents is sometimes influenced by their conflicts with the authority enforced by parents at home and by teachers at school. People who eventually resolve the fifth stage develop a sense of identity as well as an integrated self-concept (Meggitt, 2006:163).

The extent to which an adolescent resolves identity confusion and gains a clear picture of who he or she is can influence his or her achievement in Mathematics. For instance, an adolescent with a clear future career perspective in which a pass in Mathematics is a prerequisite is likely to exert effort in learning Mathematics. Conversely, an adolescent who is engulfed in identity diffusion may underachieve in Mathematics owing to lack of commitment and lack of vision. Hence Erikson's theory on identity formation can help to explain the relationship between the self-concept of learners and their achievement in Mathematics.

2.2.2.2.3 Allport's theory on self-concept development

Apart from being known as a trait personality theorist, Gordon Allport is also well known for advancing a theory on proprium development (Buskist, 2011:154; Lahey, 2009:406). *Proprium* is the term which Allport used to refer to the self. According to Dumont (2010:207), the proprium is the unifying principle in the personhood of each person. Carducci (2009:265) described the proprium as the centrepiece of an individual's personality which serves to give the individual a sense of self. Allport viewed the proprium as the totality of each individual as an entity containing the basic psychological, social and physical aspects which are critical in defining each individual's sense of self (Dandapani, 2001:321).

According to Allport, proprium development normally occurs in a sequential manner in which the individual passes through eight stages. In their correct order, the stages are development of a bodily sense, self-identity, self-esteem, self-extension, self-image, self as a rational copier, proprium striving and self as knower. The development of the bodily self occurs in the first year of life when the individual becomes aware of his or her body, its sensations and its separateness from other bodies. In the second year of life, the sense of identity starts to develop and language acquisition helps in this regard.

The sense of self-esteem in which the child focuses on self-evaluation and tries to exercise control and autonomy in performing certain tasks occurs in the third year of life. According to Dandapani (2001:321), the sense of self-extension develops when the child realises that some things are a critical part of one's life despite their being only possessions which are not part of the body. The self-image develops when the child is in the 4 to 6 year age range. The child realises that there are expectations of

significant others which need to be satisfied and this becomes the basis of self-evaluation either as good or bad.

The individual directs energy at becoming a rational copier with problem-solving skills between 6 and 12 years of age. Rational coping is a function that enables children to solve problems rather than escape from them (Dumont, 2010:208). This stage coincides with Erikson's industry versus inferiority stage (Dandapani, 2001:321).

According to Allport, the seventh stage of proprium development is called proprium striving and it occurs when the individual is 12 to 20 years old. This implies that proprium striving stretches from early adolescence to early young adulthood. This is the developmental stage of interest in the current study because the generality of secondary school learners are adolescents. It is during this stage that the adolescent becomes engrossed in undertaking strategic planning in matters such as education, relationships and career options. According to Dumont (2010:208), proprium striving is concerned with the development of a sense of ownership or proprietorship of feelings, aspirations, values and both short-term and long-term decisions. The individual becomes capable of distinguishing between peripheral motives and appropriate motives. While peripheral motives are a person's impulses, drives and strivings towards immediate gratification of needs, appropriate motives are strivings for important goals in life. Functional autonomy, which is the ability to convert peripheral motives to appropriate motives, develops and the adolescent undertakes tasks in a more self-directed manner even in the absence of external rewards so as to perfect his or her value system and self-image (Dandapani, 2001:321).

If Allport's theory is considered in a Mathematics learning context, one can claim that the extent to which the adolescent manages to strive for a healthy proprium can influence his or her achievement in Mathematics. For instance, an adolescent who aspires to be a medical doctor is likely to put much effort into science subjects which include Mathematics so as to increase his or her chances of ultimately becoming a medical doctor. Failure to develop sound career aspirations may lead to complacency which can ultimately lead to underachievement.

The last stage identified by Allport portrays the self as a knower and it commences during young adulthood (Dandapani, 2001:321). The individual at this stage is a motivated independent thinker who has a strong tendency to engage in problem

solving. Since the current study focuses on adolescent learners, less emphasis is placed on Allport's last stage.

2.2.2.2.4 Rogers' theory on self-concept development

As argued by Lahey (2009:425), the development of self-concept is a fundamental principle of Carl Rogers' theory. According to Rogers, all human beings have two basic needs which are the tendency to actualise and the need to receive positive regard. Positive regard is the recognition and loving warmth which an individual receives from significant others during social interaction. In the development of the self-concept, an individual incorporates meanings and values which are based on his or her personal experiences as well as meanings and values which are based on other people's experiences. The significant others, who are the important people in the individual's social network from whom the individual can receive positive regard, are very important in the development of the individual's self-concept. The child's parents and siblings initially act as the significant others before the list extends to members of the extended family, friends, neighbours, teachers and even pastors.

Rogers claimed that the sound development of self-concept hinges upon the nature of the positive regard received from significant others. The imposition of conditions of worth can stifle sound self-concept development. According to Austad (2009:157), conditions of worth are a set of specific standards imposed on a person and are used to judge the person's value by significant others such as parents, family, loved ones, teachers and persons in authority. According to Rogers, the conditions of worth are likely to cause the individual to ignore his or her original behavioural intentions for the sake of being accepted by the significant others. Attempting to fulfil the imposed conditions of worth unfortunately causes people to incorporate values and meanings of other people into their self-concepts at the expense of their own personal characteristics, experiences and preferences. Such a phenomenon can lead to incongruence between the individual's self-image and the ideal self which ultimately translates to a distorted self-concept and subsequent difficulties to actualise. People who live their lives according to other people's conditions of worth are not likely to achieve their full human potential (Kosslyn & Rosenberg, 2006:488).

For the optimum development of a healthy self-concept, Rogers proposes that an individual should receive unconditional positive regard. Giving someone

unconditional positive regard is an indication of accepting and tolerating the person regardless of his or her weaknesses, failures, shortcomings and inabilities (Mpofu, 2006:22; Kirshenbaum & Jourdan, 2005:37). The individual receives recognition and acceptance from significant others without the imposition of conditions of worth (Feldman, 2009:563). The individual is accepted and recognised as a peculiar person with unique needs and aspirations which do not need to be evaluated relative to those of significant others. An individual who receives unconditional positive regard has a high probability of attaining congruence between the ideal self and self-image because he or she is free to incorporate as many of his or her experiences into the self-concept as possible (Austad, 2009:156). This implies that people who receive unconditional positive regard are expected to actualise and become fully-functioning organisms since all their innate potentialities are not thwarted by the imposition of conditions of worth by significant others.

If Mathematics teachers give learners unconditional positive regard, it is likely that the learners will develop positive self-concepts thereby standing a better chance of doing well academically. If teachers impose conditions of worth, the learners may have distorted self-concepts which can interfere with their scholastic achievement. However, it is not always easy for teachers to accept all learners as they are because the teachers sometimes have to enforce discipline as part of classroom management.

2.2.2.2.5 Harter's self-perception profile for adolescents

The need to shed more light on the development of the self has led some researchers to shift from the one-dimensional representation of the self to a multi-dimensional approach. Susan Harter developed a self-perception profile for adolescents among other profiles to highlight the multi-faceted nature of the self (Berns, 2010:40). According to Harter, adolescents have eight domains of competence; namely, job competence, athletic competence, scholastic achievement, physical appearance, romantic appearance, social appearance, behavioural conduct and close friendships (Eapen, Naqvi & Al-Dhaheeri, 2000:8). Studies have revealed that an individual's self-perception is a function of competence in the already stated domains as well as the individual's interpretation of the attitude of significant others towards the individual. As a result of the explicit categorisation of the attributes of the self, it was established that adolescents exhibit different fronts depending on the

contextual situation. Moreover, adolescents have to devise ways of resolving some of the conflicting roles which they are expected to play. Eapen et al (2000:8) report that physical appearance was found to have a higher correlation coefficient with self-esteem during adolescence than during pre-adolescence.

The manner in which adolescents try to obtain success in each of the domains of competence can have implications for their achievement in Mathematics. Learners who do not register success in most of the domains may have low self-esteem which can interfere with their academic efforts. For instance, the expectations of the adolescent's significant others who include parents, peers and teachers can have an effect on the adolescent's academic performance in Mathematics.

2.2.2.2.6 Research on self-concept and achievement in Mathematics

Wang et al (2012:1215) used the Trends in International Mathematics and Science Study 2003 data to generate Mathematics achievement models for grade 8 learners in the United States of America, Singapore, Russia and South Africa. Using hierarchical linear modelling, a Mathematics achievement model was developed in each of the four countries using learners' concepts of ability in Mathematics, mathematics values, perceptions of teachers and principals of schools and other allied factors pertaining to school and classroom activities. The developed models indicated that self-concept of ability in Mathematics was highly related to the respondents' Mathematics achievement in all four countries. It was also established that the relationship between Mathematics achievement and other learner attributes as well as teacher and school factors differed in the four countries. In conclusion, Wang et al (2012:1242) intimated that self-concept of ability is an important determinant of scholastic achievement.

A wide range of research studies have been conducted in an attempt to unravel the relationship between learners' self-concept and their academic achievement in Mathematics. Skaalvik and Skaalvik (2006:51-74) conducted two longitudinal studies to explore the relationship between Mathematics self-concept and the learners' subsequent achievement in Mathematics. The two samples comprised 246 middle school learners and 484 high school learners. The studies revealed that self-concept influences academic performance in Mathematics.

In a bid to determine the nature of the relationship between academic self-concept and self-efficacy as well as their mediating and predictive roles relative to Mathematics interest, Mathematics anxiety and Mathematics performance, Ferla et al (2009:499-505) undertook a study using scores of 15-year old learners in Belgium. The study revealed that academic self-concept influences an individual's academic self-efficacy beliefs. A conceptual difference between Mathematics self-efficacy and Mathematics self-concept was found. While academic self-concept refers to past-oriented perceptions of the self, self-efficacy shows relatively future-oriented perceptions of the learner (Ferla et al, 2009:500). The study also revealed that the Mathematics self-concept can strongly influence Mathematics anxiety and Mathematics interest.

It has been established that self-concept can influence learners' Mathematics achievement. On the basis of the outlined studies pertaining to self-concept, it seems possible for self-concept to be related to irrational beliefs. For example, irrational beliefs pertaining to self-damnation and conditional self-acceptance can affect the way Mathematics learners rate themselves. The existence of a relationship between self-concept and irrational beliefs is explored in the current study.

2.2.2.3 Stress

The chaotic events of life are bound to exert some physiological and psychological demands on the human body to levels which exceed the body's elastic limit to bear the demands. Such a situation triggers what is generally known as stress. At secondary school level, stress is experienced by learners with more intensity as a result of the emotional and biological transformations associated with adolescence. After the description of the concept of stress, some theories of stress are outlined before attention is directed at some research studies on stress and achievement in Mathematics.

2.2.2.3.1 Description of the concept stress

Stress can be defined as real or imagined environmental events that threaten the physiological or psychological well-being leading to physiological and/or behavioural responses (Stickle, 2010:40; McEwen, 2000:508). According to Swartz et al (2011:408), stress is a normal physical response to events that make one feel

threatened or a reaction to events which upset an individual's balance in one way or the other. Stress is defined by Melgosa (2008:19) as the act of being under outside pressures or forces which can yield positive or negative effects on an individual.

Stress involves a stressor and a stress response (McEwen, 2000:508). Stressors are the events which trigger stress and they entail a diversity of situations such as physical jeopardy, attending an interview, making a class presentation or preparing for an important examination or test in a seemingly or actually difficult subject. A stress response, which is also called a 'fight or flight' response, is the body's natural automatic reaction to a stressful phenomenon in a bid to fully arm the body to confront the potentially threatening situation (Swartz et al, 2011:408).

The human body reacts to stressors by activating the nervous system which subsequently releases hormones such as adrenaline and cortisol into the bloodstream. These hormones increase the pulse rate, breathing rate, blood pressure, and the body's metabolism (McEwen, 2000:508). Moreover blood vessels dilate to allow more blood to flow to large muscle groups, putting the muscles on alert (Swartz et al, 2011:408).

The effects of stress on human thoughts and feelings include depression, anxiety, feelings of insecurity, fragmented concentration, fatigue, sadness and restlessness. The effects of stress on the cognitive domain include alterations in the information processing as well as attentional and memory functions which can subsequently have a bearing on the individual's performance and decision-making capabilities (Steptoe, 2000:510). The behavioural manifestations of stress entail excessive eating; sudden anger outbursts; drug abuse; social withdrawal; relationship problems; loss of appetite and habitual crying (Steptoe, 2000:510).

Stress is not always harmful (Swartz et al, 2011:408). It can be argued that without stress, we cannot achieve certain tasks. A certain amount of stress is necessary for optimum performance (Swartz et al, 2011:409). The stress response helps an individual to rise to meet challenges. Stress is what keeps an individual alert, disciplined and focused on vital impending issues such as examinations and tests rather than relaxing and engaging in irrelevant behaviour. Some authorities argue

that without any stress, life would be virtually boring and probably hollow (Swartz et al, 2011:409).

2.2.2.3.2 Selye's theory on stress

According to Kosslyn and Rosenberg (2006:589) and Dhameja and Dhameja (2009:64), Hans Selye is viewed by many authorities as one of the founding fathers of the study of stress since he is credited with coining the term *stress* in 1936. Borrowing from his rich knowledge of psychobiology and physiology in general and the endocrine system in particular, Selye developed the General Adaptation Syndrome model which elaborates the phases which a person passes through in battling with stress (Swartz et al, 2011:408; Feldman, 2009:485). According to Selye, human beings are genetically pre-programmed to combat stress to a certain level.

Selye indicated that upon being confronted by a stressor, human beings go through three sequential stages which are the alarm stage, resistance stage and the exhaustion stage as shown in Figure 2.7. The alarm stage, which is the initial reaction to the stressor, is characterised by the mobilisation of the body's internal mechanism for combating stress (Swartz et al, 2011:413). Kosslyn and Rosenberg (2006:589) described the alarm phase as one in which the individual's fight or flight mode is activated after a stressor is perceived. As already outlined, this is when the body responds by secreting hormones.

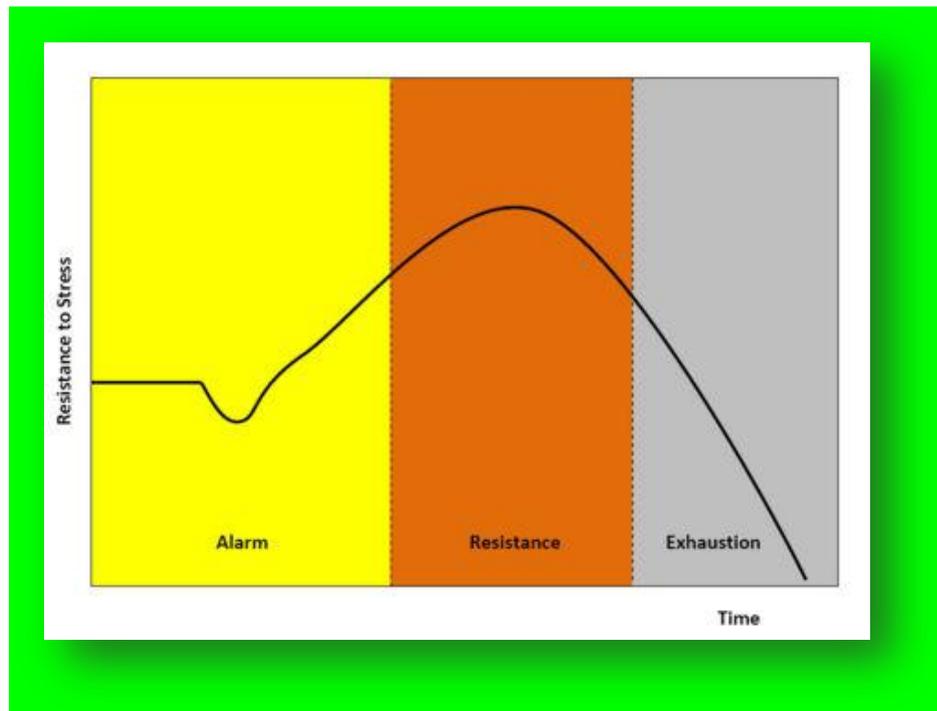


Figure 2.7: General Adaptation Syndrome model of stress (Melgosa, 2008:22)

During the resistance stage, the level of arousal remains high with persistent hormonal circulation in the bloodstream. During the resistance phase, which is also known as the adaptation phase, the body mobilises its resources to attain equilibrium, despite the continued presence of the stressor (Kosslyn & Rosenberg, 2006:590). This means the body harnesses virtually all the physiological and psychological resources to combat the stressful phenomenon. Although the individual appears to be outwardly unruffled, internally the nervous system is engaged in a battle against the stressor.

According to Kosslyn and Rosenberg (2006:591) and McEwen (2000:508), the exhaustion stage is the third and final stage which is reached when the individual remains perennially under stress to the extent of exhausting all the bodily mechanisms to cope with the stressful situation. During the exhaustion phase, the body finally succumbs to the pressure of the stressor and the individual may fall sick or even succumb to death.

Apart from outlining the Generalised Adaptation Syndrome, Selye also identified some phases of stress which include eustress, distress and under-stress (Swartz et al, 2011:408). Under-stress, which can also be called boredom, occurs when the

task to be performed is not adequately challenging. Distress, which can also be called negative stress, is experienced when the demands of the situation to be handled are overwhelming and prolonged to the extent of causing sickness (Swartz et al, 2011:408). Eustress, which can also be called positive or good stress, is experienced when the task to perform is sufficiently challenging to trigger interest on the part of the individual concerned thereby encouraging the individual to perform the task. The relationship between stress and performance is further clarified using the Yerkes-Dodson curve shown in Figure 2.8 (section 2.2.2.3.4).

2.2.2.3.3 Lazarus's theory on stress

Richard Lazarus and his colleagues advanced a transactional theory of stress which is called the psychological appraisal theory or the cognitive-relational approach (Mark & Smith, 2008:10; Swartz, 2011:408). The term *transaction* is indicative of the subjective interaction between the individual and the environmental event. Connors, Putwain and Nicholson (2009:2) argued that the transactional model of stress and anxiety is a product of the interaction between the situation and the individual's personal attributes such as optimism, pessimism and self-esteem (Connors et al, 2009:2). The psychological appraisal theory broadly dwells on two aspects which are cognitive appraisal and coping.

Appraisal refers to an individual's subjective assessment of a situation resulting in establishing the meaning and perceived impact of the situation on the individual (Kosslyn & Rosenberg, 2006:592). How one sees the stressful event is normally the largest single factor that impacts on one's physical and mental health. In support of the views of Lazarus and Folkman (1984:141), Kosslyn and Rosenberg (2006:592) remarked that many psychological, social and physical stimuli are stressors only if people perceive them as stressful. Lazarus argued that an individual normally appraises an event as a stressor not only when the event is personally relevant to the individual, but also when the individual perceives a mismatch between the demands of the event and the individual's ability to cope with the event (Kosslyn & Rosenberg, 2006:594).

According to Kosslyn and Rosenberg (2006:593), there are two types of appraisal; namely, primary appraisal and secondary appraisal. Primary appraisal is concerned with using one's personal experiences to determine whether a given event can be

categorised as stressful, beneficial or irrelevant (Swartz et al, 2011:408). After the individual has assessed the nature of the event relative to its ability to trigger stress, the individual then engages in secondary appraisal. According to Lazarus, secondary appraisal involves assessing the extent to which a person can cope with the event. Secondary appraisal is described by Kosslyn and Rosenberg (2006:593) as the determination of the resources which are available to deal with the stressor.

Lazarus and Folkman (1984:141) and Stickle (2010:43) defined coping as constantly changing behavioural and cognitive efforts exerted to manage specific external and/or internal demands that are appraised as taxing or exceeding a person's resources (Stickle, 2010:43). According to Mark and Smith (2008:10), coping basically refers to the behavioural and cognitive attempts to minimise, tolerate or master the internal and external demands of an environmental phenomenon which the individual would have appraised as taxing on the part of his or her resources. In more elaborate terms, Stickle (2010:43) described coping as conscious and volitional efforts exerted by people to regulate their behaviour, thoughts, emotions, physiology and the environment in response to a stressor. Essentially, Lazarus and his colleagues postulated that each human being has a unique way of responding to stress as well as coping with a stressful situation. This was backed by Kosslyn and Rosenberg (2006:616) who argued that different people employed different coping strategies when experiencing stress since the coping strategy used could be determined by the individual's personality (Swartz et al, 2011:408). For instance, a person's position on a pessimistic-optimistic continuum can determine the extent to which the individual appraises an event as stressful. Kosslyn and Rosenberg (2006:616-617) pointed out that while optimists have a positive future perspective, pessimists tend to expect negative results.

2.2.2.3.4 The Yerkes-Dodson law

This is a model which endeavours to explain the curvilinear association between an individual's level of arousal and his or her ability to perform a given task. Lahey (2009:367) viewed arousal as an individual's overall state of activation and alertness. *Arousal* is a generic term used to describe physiological activation such as increased brain activity or increased autonomic responses such as quickened heart rate, increased sweating or muscle tension (Gazzaniga & Heatherton, 2006:345). The law was named after two American psychologists, Robert Yerkes and John Dodson in

recognition of their article on an allied principle which was published in 1908 and since then it has to some extent stood the test of time (Yerkes & Dodson, 1908:459; Kosslyn & Rosenberg, 2006:451).

The Yerkes-Dodson law postulates that performance of a given task increases with arousal until an optimum point after which arousal interferes with performance (Gazzaniga & Heatherton, 2006:346; Hayes, 2000:401). This means the Yerkes-Dodson law maintains that optimal task performance occurs at an intermediate level of arousal, with relatively poorer performance at both lower and higher arousal levels, resulting in an inverted U curvilinear relationship between arousal and performance (Contrada & Baum, 2011:288; McMorris, 2004:243; Mateo, 2007: 582-590). The optimal level of performance occurs at a lower level of arousal for simple tasks than for difficult tasks. The resultant curve is called the Yerkes-Dodson curve and it depicts the level of arousal as the independent variable on the horizontal axis and the level of performance on the vertical axis as the dependent variable, as shown in Figure 2.8. Research has established that the level of difficulty of a task determines the optimum level of arousal, thereby leading to variations in the resultant shapes of the Yerkes-Dodson curves. A low arousal level is associated with a low level of performance because the organism will be under the influence of fatigue, poor motivation, low body temperature, and poor concentration. This is supported by Kosslyn and Rosenberg (2006:451) who posit that under-arousal leads to sluggishness. It is the lethargy associated with low arousal that leads to poor performance. This implies that secondary school learners need to be stimulated in order to obtain good grades in Mathematics. Another implication of the Yerkes-Dodson law for Mathematics achievement relates to low marks not because of learners' lack of intellectual potential, but because their levels of arousal are too low.

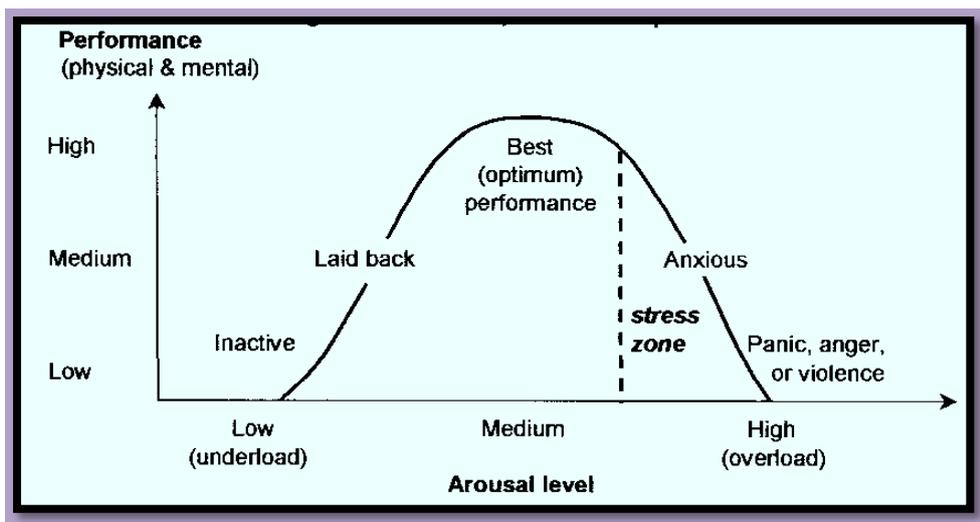


Figure 2.8: The Yerkes-Dodson curve (Contrada & Baum, 2011:288)

Excessive arousal can be counter-productive because it triggers panic, exhaustion, indecision, fear, anxiety, physical hypertension and fragmented concentration (Lahey, 2009:368). The Yerkes-Dodson law can also be used to explain the relationship between stress and human performance as shown in Figure 2.8. Over-arousal normally causes people to focus on one or fewer variables at the expense of other equally important factors, leading to a compromise in the level of performance. People who are over-aroused usually have difficulty in focusing and sustaining attention (Kosslyn & Rosenberg, 2006:451). Hence the academic achievement of learners in Mathematics can be negatively affected by over-arousal. Excessive competition in class and over-emphasising grades can trigger over arousal on the part of the Mathematics learners which can ultimately translate to poor academic achievement.

2.2.2.3.5 Stress during adolescence

While stress affects virtually everyone, adolescents are affected by stress in a unique way since adolescence is a peculiar developmental stage. This view is backed by Sulaiman, Hassan, Sopian and Abdullah (2009:179) when they indicate that learners in their teens experience stress particularly because of the biological, physical, mental and emotional changes as well as the changes in their responsibility on their way to adulthood. The developmental changes which characterise an individual's transition from childhood to adulthood are a scientifically proven source of stress (Centre for Adolescent Health, 2006:3).

There are other sources of teenage stress apart from the factors which directly relate to bodily changes. Kempf (2011:11) and Magwa (2013:373) alluded to the fact that the sources of adolescent stress can be broadly categorised into three groups; namely, school factors, home factors and peer-related factors. Sulaiman et al (2008:180) argued that since adolescence is normally experienced during schooling years, adolescents are always under pressure from themselves, their teachers and their parents to succeed academically. Lohman and Jarvis (2000:15) confirmed that adolescents ascribe stress to school factors such as tests, grades, homework, academic achievement and expectations of teachers and parents. Typical stressors in a school context are unstructured classrooms, high teacher expectations as well as fear of academic failure at school; at home adolescents are stressed by strife in families, parents' expectations, poverty and sibling rivalry (Kempf, 2011:11; Magwa, 2013:373; Fergusson & Woodward, 2002:225-231).

Centre for Adolescent Health (2006:4) referred to a research study in which teenagers in the 14 to 15 year age range supplied information through questionnaires and interview sessions. Reporting the findings, Centre for Adolescent Health (2006:4) reported that the five most frequently experienced sources of stress in the lives of the youths who participated in the study were school work (78%), parents (68%), friends' problems (64%), romantic relationships (64%) and younger siblings (64%). Failure to make friends, being victimised by bullies and peer pressure are credible stressors to adolescents (Verma, Sharma, & Larson, 2002:500-508). In an attempt to hint at how adolescent stress can negatively influence academic performance, Waterhouse and Dickinson (2001:82) commented that teachers deal with young people who may not be capable of coping with all the stressors in their lives and who often react with laziness, defiance, mischief, insubordination, aggression or destructiveness. In reaction to stressors, some adolescents may end up harbouring irrational beliefs such as the idea that life is horrible when things are not the way they would like them to be and the idea that human misery is invariably externally caused and is forced on them by the external world (Austad, 2009:277; Thompson & Henderson, 2007:211).

2.2.2.3.6 Research on stress and achievement in Mathematics

To gain information regarding stress, Kauts and Sharma (2009:39) investigated the effects of yoga on the academic performance of learners in Mathematics, science and social studies. The experimental research design was employed with one experimental group and one control group who were tested in the stated subjects before and after yoga sessions. A sample of 800 adolescent respondents comprised of 159 high-stress learners and 142 low-stress learners participated in the study. Reporting the results of the study, Kauts and Sharma (2009:39) pointed out that the learners who practised yoga performed better in academics. The study further revealed that low-stress learners out-performed high-stress learners, meaning that stress affects the learners' performance (Kauts & Sharma, 2009:39). The findings of the study confirmed that stress can influence secondary school learners' academic performance in Mathematics and other subjects especially during adolescence.

A study to determine the relationship between the level of stress and scholastic achievement among rural and urban Malaysian secondary school learners was conducted by Sulaiman et al (2009:179-185). A total of 155 form four learners of whom 63 were from rural schools and 92 from urban schools took part in the study. Significant differences were obtained in levels of stress relative to gender as well as between rural and urban secondary school learners. It was established that 71% of the learners experienced low stress, 29% experienced medium stress while no learner was diagnosed with high stress. It was generally concluded that the stress levels of adolescent secondary school learners is influenced by a large number of variables which entail parenting styles, school climate and the education level of parents. Although this study did not directly focus on secondary school learners' Mathematics achievement, the results help to highlight how stress and other factors can influence academic performance.

Byrne, Davenport and Mazanov (2007:393) undertook a study in which they used factor analysis to generate ten categories of stressors from a list of stressors supplied by adolescents in the 13 to 18 year age range in Australia. The study revealed that 40% of the categories were related to academic work while the remaining categories included identity diffusion, relationships with parents and siblings, financial challenges and conflicting personality attributes. The study

confirmed that adolescents are surrounded by many stressors which may also interfere with their achievement in Mathematics.

Researches exploring the individual differences among learners who have a high probability of failing Mathematics were reviewed by Beilock (2008: 339-343). The review revealed that high-stress situations trigger worries which are self-sustaining. The worries occupy the working memory space which is normally supposed to be available for performance. The study generated the conclusion that the higher the level of stress, the lower the working memory space which subsequently leads to poor academic performance.

Kaplan, Liu and Kaplan (2005:3) conducted an allied study to test the hypothesis that excessive academic expectations of junior high school learners combined with school-related stress during adolescence would negatively affect the quality of the learners' academic scores. A sample of 1034 respondents was used. The respondents were initially interviewed at home during junior high school and were interviewed for the second time after three years during high school. Concerning the findings of the study, Kaplan et al (2005:3) found sufficient evidence to support the view that for learners in high stress school environments, an increase in academic expectations may increase their school-related stress and impede their academic achievement. This study confirmed that stress can influence learners' academic achievement in various academic disciplines including Mathematics.

There is a possibility that stress emanating from studying Mathematics can be related to irrational beliefs. Ellis, the cognitive-behavioural psychologist who propounded irrational beliefs, claimed that irrational beliefs trigger negative emotions such as stress (Bermejo-Toro & Prieto-Ursua, 2006:89). The existence of a relationship between stress and irrational beliefs in a Mathematics learning context is examined in subsequent sections of the current study.

2.2.2.4 Anxiety

Numerous studies have been undertaken to establish the impact of anxiety as an affective factor on academic achievement not only in mathematics, but in practically all the domains of learning. Anxiety, which is mostly taken to be synonymous with

stress, is also reported as a symptom of stress. The concept of general anxiety is explained before specific attention is directed at Mathematics anxiety, the factors which cause Mathematics anxiety and research on anxiety and achievement in Mathematics.

2.2.2.4.1 Description of the concept anxiety

Anxiety can be defined as an unrealistic fear resulting in physiological arousal and is accompanied by the behavioural signs of avoidance or escape (Hey, Bailey & Stouffer, 2001:81). The difference between fear and anxiety is that fear is triggered by a realistic objective source of danger while anxiety is a generalised subjective and unrealistic sense of fear about something out there that seems menacing. This implies that general anxiety is an apprehensive feeling of tension or worry which can manifest itself as a symptom of stress. This was backed by Feldman (2009:521) who defined anxiety as a feeling of tension or apprehension in reaction to stressful situations. Some of the symptoms of anxiety according to Hey et al (2001:82) include trembling, feeling shaky, muscular tension, aches, soreness, restlessness, fatigue, shortness of breath, palpitations, fast heartbeat, sweating, dryness of the mouth, dizziness, light-headedness, nausea, flushes, frequenting the toilet, fragmented concentration, irritability, and trouble falling or staying asleep. Experiencing several of the above symptoms simultaneously is likely to interfere with one's achievement in Mathematics. However, Feldman (2009:521) argued that anxiety, just like stress, is not totally bad because without anxiety many learners will not be adequately motivated to study in preparation for examinations.

2.2.2.4.2 Anxiety in Mathematics

Mathematics anxiety can be defined as an inconceivable tension, apprehension or dread of Mathematics that can interfere with a person's ability to manipulate numbers and solve mathematical problems within a variety of everyday life and academic situations (Smith, 2004:4; Sherman & Wither, 2003:138; Sheffield & Hunt, 2007:19). Unpleasant feelings of fear or tension that sabotage a learner's Mathematics problem-solving ability or other activities related to Mathematics can be described as Mathematics anxiety (Wei, 2010:6). Mathematics anxiety has been defined by Zakaria and Nordin (2008:28) as the feeling of tension, helplessness, mental disorganisation and fear which grips someone when he or she is tasked to solve mathematical problems through manipulating numbers and shapes. Sufficient

research evidence indicates that learners with high levels of Mathematics anxiety achieve at a lower level in Mathematics (Scarpello, 2007:34).

2.2.2.4.3 Factors causing anxiety in Mathematics

Anxiety may be caused by a variety of stressful life events, numerous hereditary factors, personal background, personality and physical illness (Hey et al, 2001:82). Mathematics anxiety can be caused by parental influences, past classroom experiences and remembering poor past achievement in Mathematics (Scarpello, 2007:34). Hey et al (2001:82) claimed that anxiety is contagious since children and adolescents living with anxious caregivers are more likely to have the same anxiety disorders. Mathematics anxiety can be attributed to a host of variables which entail truancy, poor self-image, inadequate coping skills and teaching methods which emphasise rote learning at the expense of meaningful learning (Zakaria & Nordin, 2008:28). According to Stuart (2000: 330-335), Mathematics anxiety can stem from lack of confidence when manipulating mathematical problems. Smith (2004:7) maintained that Mathematics anxiety is caused by poor test scores, reluctance or inability to complete challenging assignments, negative parental predispositions and the Mathematics teacher's personality and attitudes. According to the above beliefs, the conclusion can be made that Mathematics anxiety can be caused by many factors which can be categorised into learner-related factors, school-related factors and home-related factors. The influence of some of these factors on Mathematics achievement is scrutinised in the current study.

2.2.2.4.4 Research on anxiety and achievement in Mathematics

Zakaria and Nordin (2008:27-30) undertook a research in Malaysia to establish the effects of Mathematics anxiety on motivation and achievement with a sample of 88 learners comprising 73 females and 15 males. The study established a strong negative correlation of -0.72 between Mathematics anxiety and achievement. The results of this study provided evidence that Mathematics anxiety has an important effect on academic achievement and it cannot be ignored (Zakaria & Nordin, 2008:30).

A longitudinal study to explore the relationship between anxiety and Mathematics achievement was undertaken by Sherman and Wither (2003:149). Starting from the

well-documented finding that a negative correlation exists between Mathematics achievement and anxiety, the study tested three additional hypotheses. The three hypotheses revolved around establishing whether Mathematics anxiety is a cause or an effect of Mathematics achievement as well as the possibility of a third underlying variable which influences the relationship between Mathematics anxiety and Mathematics achievement. A sample of 66 respondents was used. The study confirmed that there is a negative correlation between Mathematics anxiety and Mathematics achievement. However, challenges were encountered in trying to establish the direction of causality. The study statistically failed to establish whether Mathematics anxiety really causes poor Mathematics achievement. Insufficient evidence was also found to confirm that lack of achievement causes Mathematics anxiety or that the presence of a third factor influences the relationship between Mathematics achievement and Mathematics anxiety. As a result of the unclear findings, Sherman and Wither (2003:149) recommended that more specific studies involving these variables be undertaken. The researchers advised that the experiment be replicated or a more direct line of inquiry such as measuring the changes in the Mathematics achievement of people undergoing an anxiety reduction programme be conducted so as to generate more insights regarding the relationship between Mathematics achievement and Mathematics anxiety.

Wei (2010:07) studied the impact of treatment messages on learners' Mathematics anxiety and Mathematics learning in a computer-based environment. Anxiety treatment messages are statements based on psychological theories which are delivered to people for the sake of helping them to cope with anxiety. For instance, the statements can be based on cognitive-behavioural principles of Beck's cognitive therapy and disputation techniques derived from the rational emotive behaviour therapy advanced by Ellis. A pre-test, post-test experimental research design was used. A sample of 128 grade nine respondents participated in the study. Technologically animated human-like characters called pedagogical agents who acted as tutors interacted with the research participants in a computer-based environment. The respondents were randomly assigned to the experimental and control groups. While the pedagogical agents who interacted with the experimental group presented anxiety treatment messages, the pedagogical agent of the control group did not present any anxiety treatment messages (Wei, 2010: iii). The study

revealed that there were significant interaction effects between treatment messages and the learners' prior levels of Mathematics anxiety on current Mathematics anxiety. It was discovered that treatment messages reduced the anxiety levels of highly anxious learners in the treatment group. It was also established that the anxiety of the low-anxious learners in the control group increased probably because they had interacted with non-human pedagogical agents who did not equip them with any mechanisms to cope with anxiety. On the other hand, the anxiety of the low-anxious group in the experimental group remained constant tentatively because they benefited from the treatment messages presented by the pedagogical agents, despite being subjected to an artificial environment.

Working on the premise that Mathematics anxiety affects learners' achievement in Mathematics, Smith (2004:15-17) undertook a study to establish the role played by teachers in determining their learners' Mathematics anxiety. Using two different classes, each with a different teacher, Smith (2004:15-17) used the Minnesota Mathematics Attitude Inventory (MAI) to measure learners' attitudes to Mathematics. Seventh graders participated in the study. The attitudes of the two Mathematics teachers of the participating classes were also measured. The study established that the attitudes of the two teachers towards Mathematics were different. The teacher with a negative attitude towards Mathematics was found to have a higher level of Mathematics anxiety than the teacher with a favourable attitude towards Mathematics (Smith, 2004:28). After comparing the results of the seventh graders and their respective teachers, it was established that the attitudes of teachers towards Mathematics have an influence on their learners' attitudes towards Mathematics. According to Smith (2004:3), the difference between the two teachers' attitudes implies that teachers play a role regarding the extent to which learners can be affected by Mathematics anxiety. Such a conclusion was based on the argument that teachers who understand Mathematics anxiety better are better positioned to prevent it and assist learners to overcome it (Smith, 2004:3).

It has been established that Mathematics achievement can be influenced by anxiety in general and Mathematics anxiety in particular (Zakaria & Nordin, 2008:30; Wei, 2010:7; Scarpello, 2007:34). It is possible for some irrational beliefs to be related to anxiety which emanates from studying Mathematics. For example, the irrational

belief that *if something is or may be dangerous or fearsome, one should be terribly upset and endlessly obsess about it* can foster anxiety among learners. The irrational belief that *human emotional misery is mostly externally caused and that people have little or virtually no ability to regulate their emotional sorrows and disturbances* also seems to be related to anxiety as a learner affective variable. Subsequent chapters in the current verify the existence of relationships between irrational beliefs and anxiety in a Mathematics learning context.

2.2.3 Study Habits and Learning Styles of Learners

This section focuses on study habits and the various learning styles which can be adopted by learners. These two aspects will be considered with a bias towards highlighting how they can influence Mathematics achievement at secondary school level.

2.2.3.1 Study habits

According to Yahaya (2003:221), study habits can be defined as a series of activities undertaken by learners in a bid to ensure effective learning. A study habit is a set of routines which entail the frequency of study sessions, review of subject matter, self-testing, rehearsal of learnt content and studying in a favourable setting (Crede & Kuneel, 2008:425; Sheikh & Jahan, 2012:120; Ozsoy, Memis & Temur, 2009:156). This implies that a study habit is the number and kind of study routines which are regularly employed by a learner in an environment conducive to learning in order to enhance content mastery (Ozsoy et al, 2009:156). Effective study habits involve monitoring the time and place of study, using a definite timetable and writing down well-organised short notes of the content being studied (Sheikh & Jahan, 2012:120; Bajwa, Gujjar, Shaheen & Ramzan, 2011:175). Study skills, which are closely related to study habits, are a learner's ability to manage time and other resources to accomplish an academic task (Ozsoy et al, 2009:156).

Numerous studies have revealed that poor academic performance can be attributed to lack of effective study habits which is normally evidenced by poor time management, lack of understanding and failure to find an environment conducive to study (Osa-Edoh & Alutu, 2012:228; Yahaya, 2003:221). On the other hand, it has

been confirmed that learners who employ good study habits tend to be academically successful (Ozsoy et al, 2009:157). Learning becomes successful when learners employ good study habits and utilise effective skills which include sustained concentration (Bajwa et al, 2011:175).

According to Ozsoy et al (2009:157), study habits are influenced by a wide range of variables which include good time management, personality, metacognition, note-taking skills, study habits training, teacher-related factors, family, library use and the availability of an environment conducive to study. Regarding time management, Ozsoy et al (2009:157) intimated that making a plan and conforming to it is normally the first step in effective time management. Metacognition can be defined as people's knowledge of how they think as individuals (Snowman et al, 2009:225). It is people's knowledge of and beliefs in their mental resources and their awareness about what to do in dealing with life situations (Ozsoy et al, 2009:155). Some of the factors which influence study habits such as the home setting and teacher-related factors and their influence on Mathematics achievement are discussed in sections 2.3.2 and 2.3.3 respectively. Moreover, the variables are also discussed in conjunction with irrational beliefs and Mathematics achievement in chapter 3.

In an attempt to come up with effective study habits, learners can employ a variety of study techniques which include overlearning, distributed practice instead of massed practice, good note-taking skills, the *MURDER* approach and the *SQ4R* method. Overlearning is the act of revising content several times so as to improve the storage of the information in the long-term memory (Feldman, 2009:242). It can be argued that overlearning is necessary in the study of Mathematics for learners to remember the various formulae or rules in order to solve mathematical problems.

The frequency and duration of study sessions can determine the learners' ultimate level of content mastery. While distributed practice is regularly revising subject matter in bits and pieces over a long period of time, massed practice occurs when the content is revised in bulk over a short period of time (Snowman et al, 2009:229; Tuckman & Monetti, 2011:279). Research has established that distributed practice is more effective than massed practice especially in Mathematics (Tuckman & Monetti, 2011:279; Snowman et al, 2009:228).

MURDER is an acronym representing *Mood, Understand, Recall, Digest, Expand* and *Review* (Bajwa et al, 2011:176). *Mood* reminds learners to assess their psychological and emotional preparedness to study while the other terms are verbs which prescribe what learners have to do to enhance their chances of mastering the content. *SQ3R* is another acronym denoting the verbs *Survey, Question, Read, Recite* and *Review*. It has been modified by many authors some of whom have upgraded it to *SQ4R* with the fourth R representing *Relate* (Bajwa et al, 2011:176; Osa-Edoh & Alutu, 2012:231). The *SQ4R* method is a study technique which ensures massive learner participation because it is actually a hybrid of learning techniques. While surveying focuses on skimming the content to be studied, questioning deals with setting questions for oneself or utilising past examination papers. Reading is gaining the actual meaning of the subject matter and reciting is rehearsing the learnt content. According to Osa-Edoh and Alutu (2012:231), reciting was recommended by psychologist Lovell who established that the greater the time one spends reciting study material, the quicker the rate of learning. Reviewing comprises recalling and rehearsing the content encountered in a study session (Osa-Edoh & Alutu, 2012:231). Many elementary psychology courses normally expose learners to the *SQ4R* study technique because it is useful when preparing for tests and examinations in virtually all subjects (Bajwa et al, 2011:176).

Learners can enhance their Mathematics achievement by deliberately adopting the study habits and the study techniques outlined above. Siklos (2012:1) emphasises that the way learners learn Mathematics is different from the way other subjects are learned. One relatively unique attribute about studying Mathematics is that it mostly involves solving mathematical problems rather than reading non-numerical content. This means there is a need for learners to solve mathematical problems regularly, that is, to apply distributed practice, so as to be well-versed in formulae and other procedures. Furthermore, the productive use of memory enhancing techniques as discussed in section 2.2.1.3 can help learners to improve their marks in Mathematics.

Learners who study related Mathematics topics in succession may have a higher chance of attaining a higher level of mastery as postulated by Ausubel and Gagne in sections 2.3.3.2.3 and 2.2.1.5.1 respectively. However, this may depend on each learner's learning style. Where possible, learners can form study groups so that

there is cross-pollination of ideas. As suggested by Vygotsky, less competent learners can benefit from their more competent peers during social interaction in study groups (Bee & Boyd, 2004:17). However, learners still need to study individually since the final examinations require individual effort.

The study guidelines suggested above can positively influence the academic performance of secondary learners in Mathematics. Learners who manage to implement some of these study guidelines may improve their achievement in Mathematics. Conversely, learners who fail to adopt clearly defined study habits may under-achieve, thereby scoring lower marks. Therefore, it can be argued that the extent to which secondary school learners utilise systematic study habits can influence their Mathematics achievement.

2.2.3.2 Description of the concept of learning styles

The need to cater for the individual needs of each learner has necessitated the study of learning styles in virtually all disciplines of learning including Mathematics. According to Kalnishkan (2005:2) and Sriphai, Damrongpanit and Sakulku (2011:835), a multiplicity of learning styles has been developed, some focusing on a single attribute of a learner while others are bipolar in nature. A learning style is a consistent preference over time and subject matter for perceiving, organising and thinking about information in a particular way (Snowman et al 2009:139). Basically learning styles are preferences for solving intellectual tasks in a specific manner, that is, an individual's unique way of perceiving intellectual stimuli and subsequently processing them (Le Fever, 1998:26; Sternberg, 2012:5). This was backed by Kalnishkan (2005:2) who defined a learning style as a collection of preferences for the ways in which an individual receives and processes information. In an attempt to be more elaborate, Sriphai et al (2011:836) defined a learning style as a combination of cognitive characteristics, affective and psychological factors that influence the way individuals interact and respond to learning environments. Learning styles indicate people's preferred environments for learning or habits for processing information to be learned (Kopsovich, 2001:15). This implies that different learners perceive and process mathematical content differently and this can lead to differences in academic performance. For the purpose of this study, the VARK model, Kolb's experiential learning and some bipolar learning styles are discussed.

2.2.3.2.1 The VARK model of learning styles

The VARK model was developed by a group of scholars led by Neil Flemming of Lincoln University in New Zealand and is sometimes called the Flemming VARK model (Kalnishkan, 2005:3). According to Kalnishkan (2005:4), the VARK model derives its name from the four different media in which learners may prefer to receive information. These four distinct media subsequently generate four corresponding learning styles which are visual, aural, reading and kinaesthetic styles which can be used to describe learners. As far as Saadi (2012:35) is concerned, learners are actually not necessarily confined to a single learning style since some learners can have multiple learning styles. A model in which each learner adheres to a single learning style is described as a uni-model while the version of the VARK model in which learners prefer exactly any two learning styles is called a bi-model (Saadi, 2012:35). A tri-model occurs when learners make use of any three learning styles while quad-model is a version of the VARK model in which the learners employ all four learning styles. Figure 2.9 gives the various combinations of learning styles which can emanate from the VARK model.

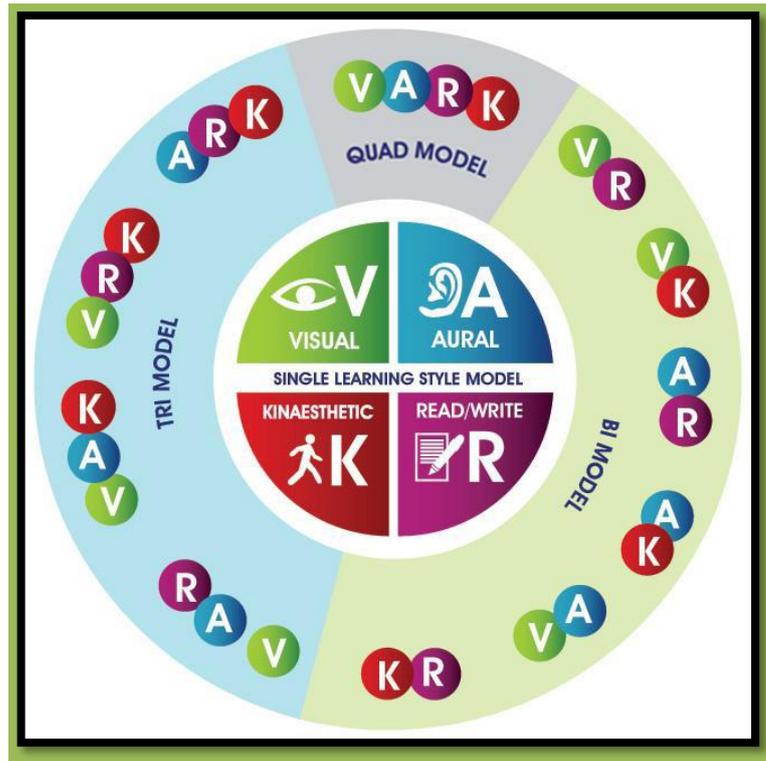


Figure 2.9: The VARK model (Adapted from Saadi, 2012:35)

According to Petty (2009:149) and Saadi (2012:35), learners who prefer the visual medium tend to learn best when they are presented with visual stimuli such as graphs, diagrams, maps, charts, handouts and flowcharts. In Mathematics, visual learners are likely to score high marks in topics such as mensuration of shapes, probability tree diagrams and geometrical construction. As claimed by Saadi (2012:35), aural learners perceive intellectual stimuli through listening to sounds either from others or through self-talk. The strength of auditory learners lies in their ability to attentively listen, subsequently repeat the fundamental concepts to themselves in their own minds and recall the information they have heard (Kopsovich, 2001:26). They mostly thrive when teaching methods such as group discussions, debates and lectures are employed.

According to Tennent, Becker and Kehoe (2005:649-659) and Saadi (2012:35), reading and writing learners prefer to write down information in note form and later read it in its written form. Such learners are likely to write down some mathematical formulae which they revise as a way of boosting their probability of remembering them whenever need arises. Kinesthetic learners prefer to manipulate objects or to

do hands-on exercises to enhance their understanding (Petty, 2009:149). They prefer feeling, holding and doing some activities as suggested in Gardner's theory of multiple intelligences. In Mathematics, kinaesthetic learners are likely to enjoy learning topics such as geometrical constructions and locus, scale drawing and manipulating three-dimensional shapes. They are likely to encounter challenges in mastering purely abstract concepts such as logarithms and factorisation.

2.2.3.2.2 Kolb's theory of experiential learning and learning styles

Kolb's learning styles are derived from his experiential learning theory. On one continuum of the experiential model, concrete experience and abstract conceptualisation address how an experience leads to the acquisition of knowledge (Kopsovich, 2001:26). As illustrated in Figure 2.10, the second continuum has reflective observation and active experimentation on its ends which mainly indicate how the learning experience can lead to transformation (Petty, 2009:150; Martin, Cashel, Wagstaff & Breunig, 2006:172; Moon, 2004:25). Kolb postulated that learning most effectively occurs when the four modes are sequentially followed starting at any point on the cycle. The four quadrants formed by the two continuums represent four learning styles (Fleming & Baume, 2006:4-7; Saadi, 2012:20). The four learning styles are the converger, the diverger, the assimilator and the accommodator.

Characteristically convergent learners have an acclivity towards abstract conceptualisation and active experimentation (Kolb, Boyatzis & Mainemelis, 2000:193-210; Saadi, 2012:20). They are likely to do well in Mathematics since they normally employ deductive reasoning when solving problems. The divergent learning style is characterised by a preference for concrete experience and reflective observation. Divergent learners, who are also called reflectors, view phenomena from different perspectives and normally explore a variety of alternative ways of solving challenges (Martin et al, 2006:172; Pont, 2003:61; Danish & Awan, 2009:162-168; Saadi, 2012:19). Such learners are likely to thrive in humanities where open-ended answers are highly cherished.

According to Kolb et al (2000:193-210) and Saadi (2012:20), accommodating learners, who are also called activists, prefer experimentation and concrete experience. Assimilators are defined by Russian (2005:1-6) and Saadi (2012:20) as

learners who prefer reflective observation and abstract conceptualisation. This implies that assimilators are capable of hypothesis generation while accommodating learners are action-oriented and pragmatic in their style.

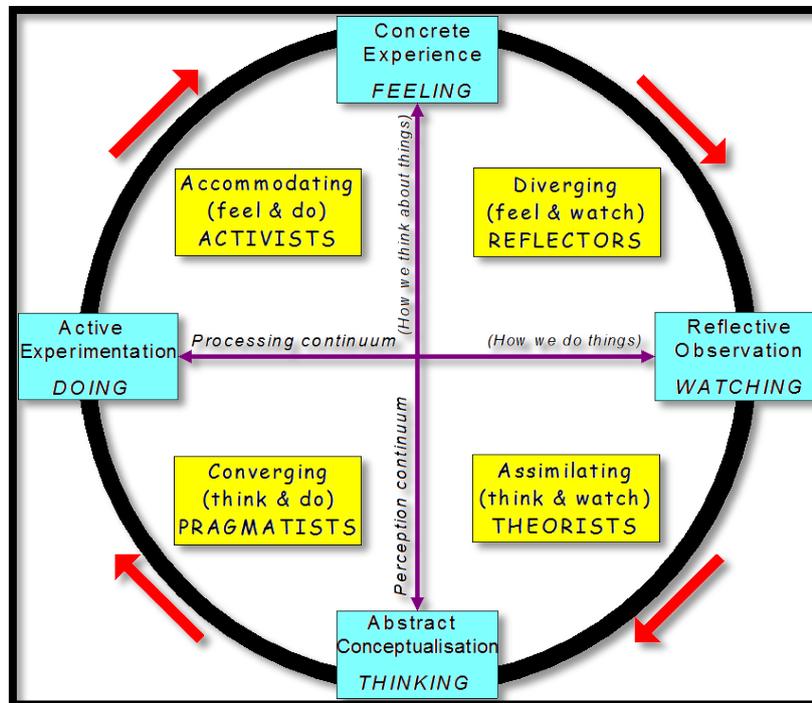


Figure 2.10: Kolb's learning styles (Adapted from Martin et al, 2006:172)

2.2.3.2.3 Other bipolar learning styles

Apart from the learning styles already outlined in the two models above, many other learning styles clarify how learners' achievement in Mathematics can be influenced by learning styles. Examples include impulsive versus reflective learners; serial learners versus holists; and field dependent versus field independent learners. According to Snowman et al (2009:139), impulsive learners rush to blurt out the first response which comes to their minds and risk giving incorrect responses since they do not take the time to meditate on their answer before providing it. Reflective learners evaluate their responses before they provide their answers. While serial learners cognitively process information in small connected chunks, holists process stimuli as a whole. Field dependent learners are implicitly defined by Snowman et al (2009:140) as learners who are sensitive to background elements which can distract them while field independent learners are capable of ignoring distracters and focusing on the critical elements of a situation. Dembo (1994:406) claimed that field

independent learners prefer subjects like Mathematics and Physics as they place more emphasis on impersonal and abstract concepts.

2.2.3.2.4 Research on study habits, learning styles and achievement in Mathematics

Osa-Edoh and Alutu (2012:228) conducted a study in Nigeria to establish the utility of study habits in enhancing the academic performance of learners. The research was primed by the falling educational standards as testified by stakeholders such as parents, learners and teachers. A sample of 50 learners from five private schools participated in the study (Osa-Edoh & Alutu, 2012:232). A strong positive correlation between the quality of study habits and learner academic performance was found. The study also established that ignorance and lack of proper study habits accounted for high failure rates on the part of the learners (Osa-Edoh & Alutu, 2012:234). The study by Osa-Edoh and Alutu (2012:228) generated evidence to back the argument that Mathematics achievement can be influenced by the study habits adopted by the learners.

A research project to determine the difference between the study habits of learners from formal and non-formal education systems in Pakistan was undertaken by Bajwa et al (2011:175). A sample of 1000 learners comprising 500 learners from the formal system and 500 from the non-formal system participated in the study. Variables such as time management, note taking, goal setting, motivation and general study strategies were considered. The study revealed that learners from the formal education system were better at time management and note taking. On the other hand, learners from the non-formal system were found to be better at general study strategies, goal setting and motivation. By virtue of wanting to be in control of their study habits, learners from the non-formal system were found to be significantly better than their formal system counterparts.

An allied research to explore the relationship between metacognition, study habits and attitudes was undertaken by Ozsoy et al (2009:154) in Turkey. A sample of 221 fifth grade learners participated in the study. Moderate positive correlations were found between study habits, knowledge, metacognition and skills. The study also revealed a significant relationship between attitudes, study habits and metacognition

for high achievers and no significant relationship between the variables for low and medium achievers (Ozsoy et al, 2009:154).

Kopsovich (2001:2-3) undertook a study to examine the influence of learning styles on the Mathematics achievement of fifth grade learners. A sample of 500 male and female fifth graders was used. The respondents' standardised Mathematics test scores were generated using the Texas Assessment of Academic Skills test. It was established that the learning style preferences of all learners in the area of persistence significantly affected their academic achievement in Mathematics.

According to Sriphai et al (2011:835), the influence of learning styles treated as exogenous variables has a greater coefficient of determination than learning styles treated as endogenous variables. This implies that learning styles are found to be a moderator variable affecting learners' Mathematics achievement. It may also be possible that irrational beliefs can affect the learning style and study habits of learners and can thus directly or indirectly have an influence on their Mathematics achievement at secondary school level in Zimbabwe.

In a bid to investigate how Mathematics achievement, studying and attitudes are affected by two study habits; namely, the traditional study strategies and the learning-style-responsive study strategies, Geiser, Dunn, Deckinger, Denig, Sklar, Beasley and Nelson (2000:38) conducted a research using a sample of 130 grade eight learners. The study strategies were employed by respondents in separate groups. The study revealed that the learners who applied learning-style-responsive strategies had significantly higher Mathematics achievement and attitude scores than the learners who applied traditional study strategies (Geiser et al, 2000:38). The findings of the study suggest that Mathematics achievement can be enhanced through employing systematic study habits or strategies.

2.3 EXTERNAL FACTORS

In this section, attention is directed at exploring how the variables which are not internally related to secondary school learners but are found within their home and school settings can influence their Mathematics achievement. Bronfenbrenner's ecological systems theory is first discussed since it gives an overview of the most

important external factors. Home factors are scrutinised before school-related factors.

2.3.1 Bronfenbrenner's Ecological Systems Theory

Urie Bronfenbrenner is an influential Russian-American psychologist whose ecological systems theory highlights how the development of a child is socially influenced by a diversity of systems. Bronfenbrenner's ecological model of human development undertakes a qualitative and contextual analysis of the developing child's environment. The ecological systems theory places much emphasis on environmental factors and how they impinge upon human development despite varying from one culture to another (Saraswathi, 2003:142). According to Donald et al (2010:40), Bronfenbrenner has established that the intimate face-to-face interactions and long-term relationships which he termed proximal interactions are virtually the most critical variables in shaping enduring aspects of development. This can be attributed to the power of reciprocal influences in settings such as families, schools, peer groups, classrooms and local communities (Donald et al, 2010:40). The developing child is at the core of the overlapping and continuously mutating systems whose interactions substantially impinge upon the child's ultimate development. In the current research, Bronfenbrenner's ecological systems theory is discussed so as to shed more light on important home-related and school-related factors which can influence achievement in Mathematics.

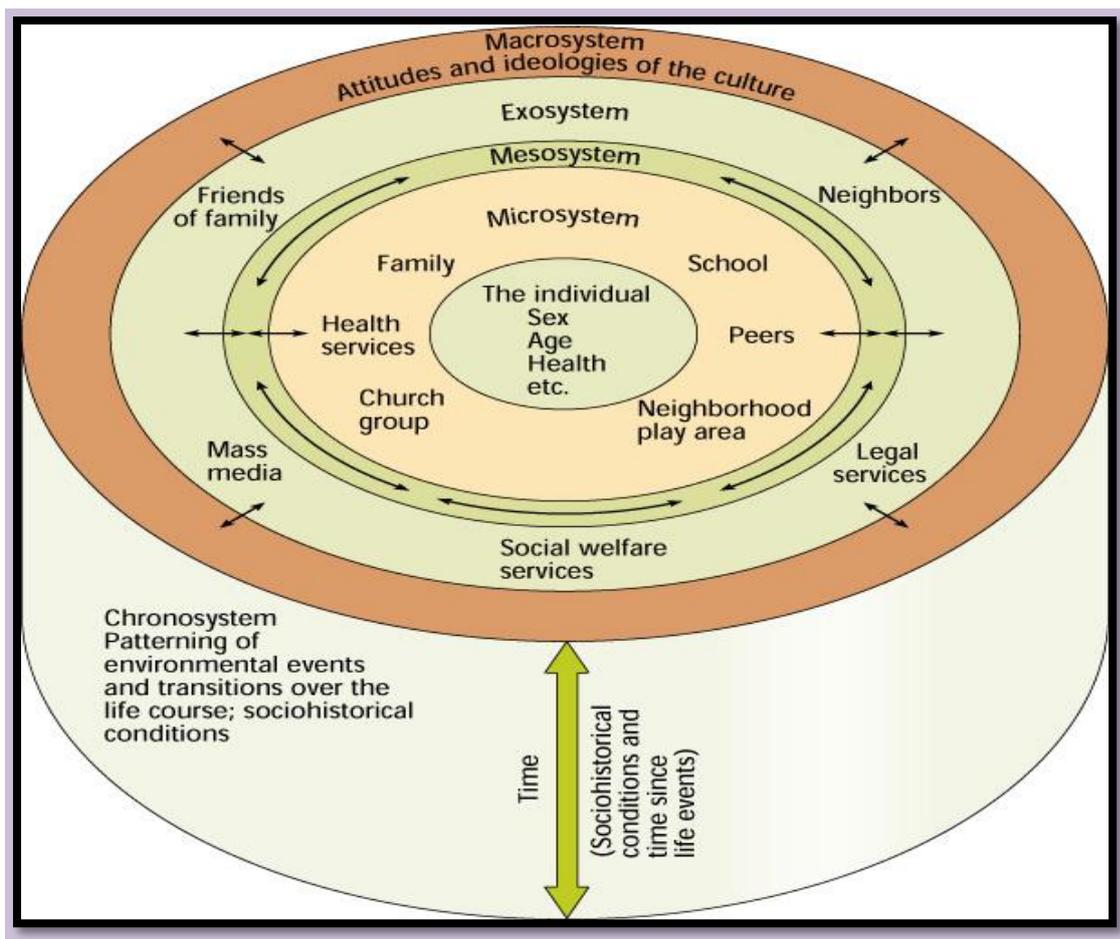


Figure 2.11: Bronfenbrenner's ecological systems theory (Santrock, 2004:69)

According to Donald et al (2010:40), Bronfenbrenner's ecological model of child development has fundamental intersecting dimensions: the person, process, context and time factors. Bronfenbrenner contended that the proper exploration of human development should be done through observing children and adults in their natural settings (Keenan, 2002:29). Bronfenbrenner indicated that the development of the child occurs within four systems which are all influenced by an overarching fifth system of time. As shown in Figure 2.11, the four systems are the microsystem, the mesosystem, the exosystem and the macrosystem which all interact with the chronosystem (Keenan, 2002:30). The systems are nested, that is, they can be represented by concentric circles in which the smallest one is at the centre with each successive level as a subset of the levels above it (Saraswathi, 2003:142).

The microsystem is the developing individual's immediate social environment which comprises his or her family, school, peer groups and the entire neighbourhood (Johnson, 2008:2; Berk, 2000:23-38). The child directly interacts with social agents

such as parents, siblings, teachers and classmates thereby seriously shaping the child's development (Neal & Neal, 2013: 725; Penn, 2005:45). Bronfenbrenner contended that the child is substantially influential in determining the social settings in the microsystem.

The second system as outlined in Bronfenbrenner's ecological systems model is the mesosystem in which the various microsystems interact socially as well as contextually (Neal & Neal, 2013: 725). This means the mesosystem focuses on the relationships between microsystems such as school life, family experiences, church experiences and peer group experiences (Bronfenbrenner, 1989:227).

The third layer of Bronfenbrenner's ecological model of child development is the exosystem. This layer is concerned with the relationship between a social setting in which the developing individual does not have an active role and his or her immediate context (Puroila & Karila, 2001:223-224). The exosystem represents settings in which the child is not directly involved (Berns, 2010:22; Neal & Neal, 2013: 725). The exosystem is external to the child's experiential world but still affects the child despite having no active role in determining the settings (Bronfenbrenner, 1989:227).

The large system which comprises the society and the culture in which a child is developing is called the macrosystem. This system is influenced by broad variables such as religion, the law and the prevailing ideology which determines how resources are distributed in society (Bronfenbrenner, 1989:228). The macrosystem entails the dominant and economic variables such as beliefs, values and activities which impinge upon other systems (Neal & Neal, 2013: 725; Donald et al, 2010:41). Although the macrosystem has a direct influence on the individual's development, the individual can influence it to a limited extent.

The progressive development of the child under the influence of the various interacting systems is ultimately influenced by time (Donald et al, 2010:41). The chronosystem is patterning of environmental events and changes throughout the life of the developing individual together with sociohistorical situations (Bronfenbrenner, 1989:201-202). This means the various developmental milestones and social contexts all change over time. By referring to Bronfenbrenner's ecological systems

theory, it is possible to obtain a better understanding of the variables which can influence learners' achievement in Mathematics.

2.3.2 Home Factors

Before each learner is even enrolled into the education system, the home will have started imparting a number of skills, concepts and attitudes in the child. The home, where siblings, parents and other relatives are found, remains an important influence in whatever the learner does. The current study explores home-related factors such as parenting styles, socioeconomic background and parental involvement which are summarised in Figure 2.12.

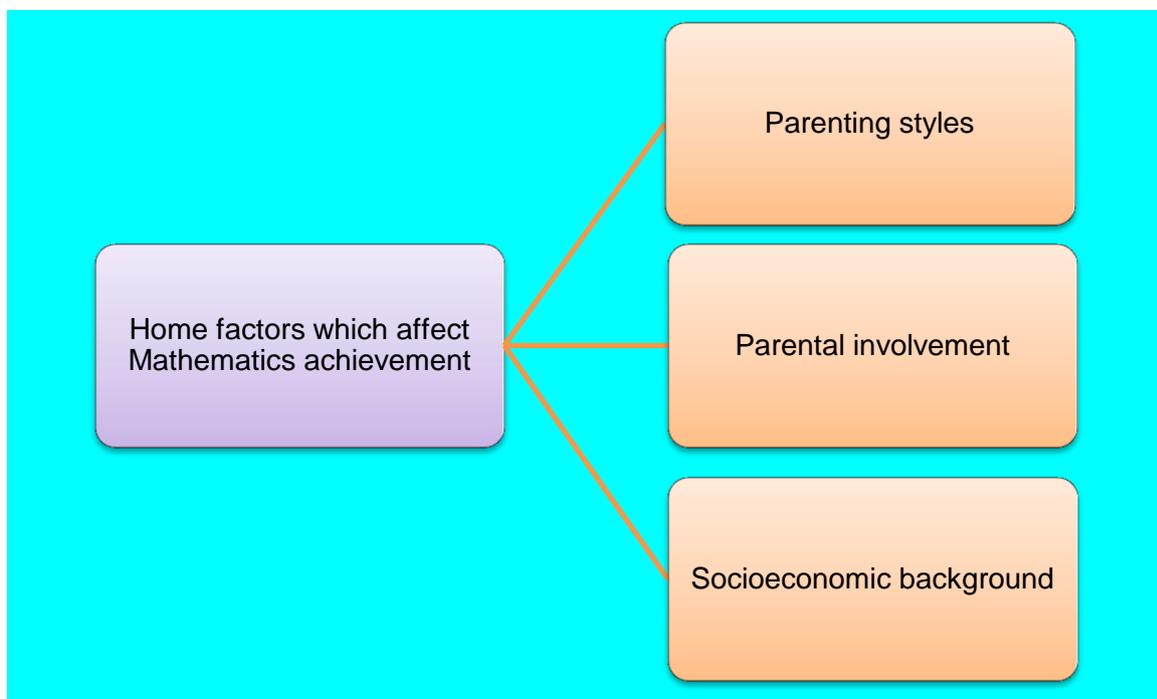


Figure 2.12: Home factors which influence Mathematics achievement

2.3.2.1 Parenting styles

According to McLean (2003:41), several aspects of parenting have been found to be influential on the academic achievement of learners. The concept of parenting styles and the theories of parenting styles are elaborated below together with some research studies pertaining to parenting styles and academic achievement in Mathematics.

2.3.2.1.1 Description of the concept of parenting styles

Tassoni, Beith, Bulman and Eldridge (2007:327) intimated that parenting styles are the various ways in which different parents discipline and rear their children. According to Berns (2010:119), parenting can be defined as the act of implementing a series of decisions pertaining to the socialisation of the child. Normally the parenting style adopted is defined by considering the extent to which the parents exercise control over the child as well as the responsiveness of such parents to the needs of the children (Tuckman & Monetti, 2011:119). While parental responsiveness is concerned with the extent to which parents respond to the needs of their children, parental demandingness is the degree to which parents expect their children to exhibit responsible and mature behaviour.

2.3.2.1.2 The four common parenting styles

Diana Baumrind is a developmental psychologist who has extensively studied parenting styles (Santrock, 2004:74; Tuckman & Monetti, 2011:119; Swartz et al, 2011:65). Originally she advanced three parenting styles which are authoritarian parenting, authoritative parenting and permissive parenting (Swartz et al, 2011:65). Her work was later revised by Maccoby and Martin who ended up with four parenting styles which are authoritarian parenting, authoritative parenting, indulgent parenting and uninvolved parenting.

Authoritarian parents are demanding and unfriendly to their children (Snowman et al, 2009:26). According to Tassoni et al (2007:327), authoritarian parents normally have high expectations of their children and are bound to control and limit the behaviour of their children by means of strict rules. Authoritarian parents are rigid, punitive and demand unquestionable obedience from their children (Feldman, 2009:411; Swartz et al, 2011:65). Authoritarian parents rarely devote time to convincingly explain the rules which they are imposing on their children. Owing to lack of warmth and being denied the opportunity to air their views, children subjected to authoritarian parenting develop resentment and a sense of insecurity as well as lack of initiative, social incompetence and anxiety when faced with competition (Santrock, 2004:74; Snowman et al, 2009:411). The given of this argument connotes

that authoritarian parenting can lead to poor academic performance in Mathematics owing to inferiority complex, lack of confidence and lack of initiative.

Feldman (2009:412) indicated that authoritative parents are firm and set clear limits but reason with their children and take time to explain situations to them (Tuckman & Monetti, 2011:119). There is moderate demandingness and moderate responsiveness. Tassoni et al (2007:327) asserted that the authoritative parenting style is generally believed to be beneficial to children since they feel secure as a result of knowing the boundaries set for them. Baumrind indicated that she preferred the authoritative parenting style because of its flexibility among other things (Santrock, 2004:74). The authoritative parenting style normally produces happy, successful and capable children who have high levels of self-esteem, intrinsic motivation and curiosity. This is supported by Santrock (2004:74) and Snowman et al (2009:26) who posited that children of authoritative parents tend to be socially competent, self-reliant and exhibit high levels of self-esteem. On the basis of the positive outcomes of authoritative parenting outlined above, it can be argued that children of authoritative parents are likely to do well in Mathematics provided all the other variables are in place as well.

The indulgent parenting style can also be called the permissive parenting style. According to Tassoni et al (2007:327), permissive parents may not take control of their children's behaviour but allow their children many choices and responsibilities. Permissive parents tend to shun confrontation with their children and they are more responsive to the demands of their children. This parenting style can be viewed as 'too soft' since the parent is excessively responsive to the child's demands with little emphasis on enforcing the rules (Tuckman & Monetti, 2011:119). Indulgent parents give too much autonomy to children at a premature stage and children operate with almost no rules at all (Lahey, 2009:351). Children of indulgent parents are less assertive and less intellectually skilled than those from authoritative homes (Snowman et al, 2009:26).

The fourth parenting style is the uninvolved or neglectful parenting style in which the parents are emotionally detached and only concentrate on providing for the children's material needs such as food and clothes (Feldman, 2009:412). Uninvolved

parents neither make demands on their children nor respond to their children's emotional needs (Tuckman & Monetti, 2011:119).

Parents who adopt the uninvolved parenting style place low demands on their children and low responsiveness to their children's needs. Communication is at its lowest ebb and the parents are physically, psychologically and emotionally detached from their children's lives. On average, an uninvolved parent meets some of the basic needs of the children although such parents can neglect or even reject their children's needs (Snowman et al, 2009:26). Santrock (2004:75) posited that children who have uninvolved parents usually lack achievement motivation. This implies that uninvolved parenting may lead to underachievement in Mathematics because of lack of direction, motivation and intellectual stimulation.

2.3.2.2 Parental involvement in learner's academic work

In this section, attention is directed at examining how secondary school learners' achievement in Mathematics can be influenced by the contribution of their parents to their school work in ways which are not related to parenting styles in general. Abdullah, Seede, Alzaidiyeen, Al-Shabatat, Alzeydeen, and Al-Awabdeh (2011:1403) defined parental involvement as the entire set of activities occurring between parents and their children at home or between parents and teachers at school that may contribute to the children's educational attainment and development. According to Sheldon (2003:149-165) and Van Voorhis (2003:323-339), sufficient evidence exists to prove that there is a positive correlation between parental involvement and children's achievement in school. Cai (2003:87-106) observed that parental support is a critical predictor of children's scholastic achievement in Mathematics.

The subject of parental involvement has been closely linked to parents' level of education and socioeconomic status. Learners' scholastic achievement in Mathematics is positively influenced by the educational level of the parents (Pezdek, Berry & Renno, 2002:776; Quintos, Bratton & Civil, 2005: 1184, 1189). Ma and Klinger (2000:41) pointed out that factors which correlate positively with Mathematics achievement are parental education and occupation as well as the socioeconomic status of the family. If parents are not well educated, they may not be knowledgeable

enough to directly help their children to tackle some mathematical problems especially when their children are in secondary school. In some cases, parents realise that the way Mathematics is taught is different from the way it was taught during their time as school learners (Qualifications and Curriculum Development Agency, 2010:3). This can minimise the help the parents can extend to their children in solving Mathematics problems, thereby reducing the learners' chances of scoring high marks in Mathematics.

2.3.2.3 Research on parental involvement and achievement in Mathematics

Gonzalez and Wolters (2006:203) undertook a study to establish the extent to which perceived parenting styles and parental involvement predict algebra learners' levels of motivation based on the self-determination theory and achievement motivation. A sample of 140 high school learners completed self-report surveys and the responses were subjected to multivariate analyses. The study found a positive correlation between authoritative parenting and mastery goal orientation as well as between authoritative parenting and relative autonomy on the part of the children. Gonzalez and Wolters (2006:203) confirmed that permissive parenting correlates negatively with mastery orientation and correlates positively with the performance approach orientation. It was further revealed that authoritarian parenting positively correlates with emphasis on performance. This implies that parenting styles can influence learners' academic achievement in Mathematics.

A study to examine the influence of parental involvement and Mathematics teachers' classroom behaviour on learners' motivation and academic performance in Mathematics was undertaken by Levpuscek and Zupancic (2009:541). A sample of 365 Slovene learners in grade eight participated in the study in which interviews and questionnaires were used together with the respondents' final Mathematics scores from the school records. The study revealed that both parental involvement and Mathematics teachers' classroom behaviour influence learners' academic performance. Moreover, Levpuscek and Zupancic (2009:541) reported that learners' perceptions of their Mathematics teachers' behaviour are predictive of both motivational beliefs and achievement in Mathematics, over and above the account of learners' evaluations of the involvement of their parents. A negative correlation was found between high parental expectations and the learners' academic performance.

This evidence backs the view that the Mathematics achievement of secondary school learners can be influenced by both teacher behaviour and parental involvement.

Parental involvement has been identified as one of the variables which can influence learners' Mathematics achievement. It seems possible for a relationship between parental involvement and irrational beliefs to exist. For example, the way parents interact with their children can foster irrational beliefs regarding the dependence mentality and the demand for love and approval. The extent to which such a relationship exists is verified in the current study.

2.3.2.4 Socioeconomic background

Marks, McMillan, Jones, and Ainley (2000:7) remarked that researchers, educators and policy makers are concerned about the extent to which learners from lower socioeconomic backgrounds are consistently disadvantaged with regard to performance at school, school completion, and participation in post-secondary education and training. This hypothetical statement is suggestive of the notion that learners' academic performance is to some extent influenced by the socioeconomic position of the families to which they belong. Such a notion was backed by Newbold, Peace, Swain and McNeil (2008:145) who claimed that attention has been directed at exploring how the family's economic position can account for learners' academic achievement. Socioeconomic background and allied concepts were scrutinised relative to their influence on learners' academic achievement in Mathematics. Research shows that both parents' educational level and family income correlate positively with parents' expectations for their children's education (Neuenschwander, Vida, Garrett & Eccles, 2007:595).

2.3.2.4.1 Elaboration of the concept of socioeconomic status

According to Woolfolk (2007:165), an individual's socioeconomic status is a comparative position in society on the basis of income, power, background and prestige. Santrock (2004:136) added an educational perspective by defining socioeconomic status as the categorisation of people on the basis of their educational, economic and occupational characteristics. Closely related to the

concept of socioeconomic status is one's socioeconomic position. Marks et al (2000:9) portrayed socioeconomic position as the relative position of a family or a person in a social structure on the basis of their access to scarce and precious resources such as education, wealth and prestige. The socioeconomic background of an individual's family of origin is normally referred to as the socioeconomic background (Marks et al, 2000:9).

According to Ceballo and McLoyd (2002:1310), children who hail from poverty-stricken families regularly face challenges both at home and at school which can ultimately influence their academic efforts in a negative way. Santrock (2004:136) argues that learners with low socioeconomic positions may have parents who do not set high educational standards for them, who are unable to help their children do homework or who do not have sufficient financial resources to pay for educational materials or to manage the timely payment of tuition fees. In support of this argument, Marks et al (2000:7) contended that when a relationship between social background and educational achievement is present, it can be concluded that learners from economically disadvantaged backgrounds face disadvantages at school.

To further explain how learners' socioeconomic position can affect their scholastic achievement, Santrock (2004:137) postulated that children from poor families mostly attend schools with fewer material resources and facilities such as crumbling classrooms and depleted laboratories than those attended by their richer counterparts. Schools in high income areas normally encourage meaningful learning while schools in low-income surroundings mostly encourage rote learning. It remains arguably true that in Mathematics, meaningful learning is more likely to enhance academic performance than rote learning (Orton, 2004:187). However, Krauss and Keltner (2008:99) refuted the above sentiments. They argued that sometimes learners from high socioeconomic backgrounds sabotage their chances of doing well academically through refusing to interact with their peers whom they deem to be of no significant assistance in the future. This to some extent implies that such learners may not benefit from cooperative learning activities which can enhance achievement in Mathematics.

2.3.2.4.2 Research on socioeconomic background and achievement in Mathematics

Eamon (2002:49) conducted a study with a large sample of more than 1200 adolescent respondents in the 12 to 14 year age range to explore the impact of poverty on achievement in Reading and Mathematics (Santrock, 2004:137). The study established a relationship between poverty and low Mathematics scores resulting from low cognitive stimulation and unsupportive home surroundings. The study also revealed that poverty is strongly linked to misbehaviour at school.

A study to compare the relationships among the family socioeconomic status and the parental educational expectations during early adolescence and the adolescents' self-concept, Mathematics achievement and language was conducted by Neuenschwander et al (2007:594-602) in Switzerland and the United States of America. A large sample of 2 535 respondents comprising parents and learners completed questionnaires. The study showed that the adolescent learners' self-concepts and Mathematics achievement are influenced by both the family socioeconomic status and parents' educational expectations.

A longitudinal study was conducted by Goosby and Cheadle (2009:1291-1320) to investigate the relationship between the birth weight, family context and Mathematics achievement of respondents aged between 5 and 14 years within and between families. The study examined the association between birth weight and subsequent academic achievement in Mathematics. It was established that the home is not only related to achievement during early childhood, but is also responsible for shaping achievement in Mathematics across childhood and into adolescence. The findings confirmed that the socioeconomic position of an individual's family has vital developmental consequences starting from the womb through early childhood and into adolescence. The obtained results confirmed that the academic performance of adolescent learners can be influenced by home factors which include socioeconomic position.

Ma and Kingler (2000:41) undertook a research on grade six learners to determine the variables which impinge upon the learners' achievement in Mathematics. The study confirmed that socioeconomic status can be considered a significant predictor of achievement in Mathematics. Moreover, they found that parental involvement

significantly affected the relationship between Mathematics achievement and socioeconomic status.

2.3.3 School Factors

It can be argued that the school is virtually learners' academic home where they come into direct contact with intellectual stimuli from their teachers. The events which occur within the school premises as learners interact with their teachers can determine the extent to which their intellectual potential can be fully utilised. Studies investigating differences in Mathematics achievement highlight the importance of classroom, teacher and school factors (Lamb & Fullarton, 2001:2). School factors such as the quality of teaching, teacher expectations, personality of teachers and classroom management which are shown in Figure 2.13, are analysed in the current study in an attempt to establish the factors which influence Mathematics achievement at secondary school level.



Figure 2.13: School factors which influence Mathematics achievement

2.3.3.1 Classroom management

Patrick, Ryan and Kaplan (2007:83) found a strong positive correlation between learners' levels of motivation and their perceptions of the classroom environment as being socially supportive. The way teachers interact with learners can determine the quality of teaching and learning. In an attempt to shed more light on classroom management, leadership and management are described before attention is directed at some leadership styles which may be employed in the classroom. The implications of each leadership style with regard to learners' academic achievement in Mathematics are also highlighted.

2.3.3.1.1 Description of the concept of classroom management

According to Robbin and Coutler (2009:22), management entails coordinating and overseeing the tasks being performed by others so that they complete the tasks efficiently and effectively. Coleman (2003:8) indicated that school management includes activities such as planning, organising, guiding and controlling of school activities. Leadership is a management function which involves working with and through people to accomplish predetermined organisational goals (Robbins & Coutler, 2009:25). Northouse (2010:12) reiterated that leadership is a process by which one person influences a group of individuals for the sake of achieving a common goal. While a leader is someone who guides and takes charge of the performance of an activity, followers are the people who carry out the activity under the guidance and instruction of a leader (Pierce & Newstrom, 2008:5). In the current study, teachers are taken as classroom managers who employ various leadership styles with the adolescent learners as the followers influenced by the leadership styles adopted. The leadership styles are discussed particularly in relation to how they could influence Mathematics achievement at secondary school level.

2.3.3.1.2 Lewin's leadership styles

Psychologist Kurt Lewin was credited with establishing three key leadership styles after he partnered with his colleagues to undertake a research study on leadership styles in 1939. According to Berns (2010:235) and McLean (2003:17), the three leadership styles, which are akin to Baumrind's parenting styles, are democratic

leadership, autocratic leadership and laissez-faire leadership (Tuckman & Monetti, 2011:384).

Under the autocratic leadership style, which is also called the authoritarian leadership style, virtually all the decision-making power is in the hands of the leader (Robbin & Coutler, 2009:388). The followers are expected to stick to the stipulations of the leader regarding the task to be performed. In the context of Mathematics education, an autocratic teacher will give learners some exercises which are to be completed within the stipulated time. Although the autocratic leadership style thrives in situations in which the leader knows what needs to be done as well as when there is insufficient time for group consensus, it can lead to apathy and boredom on the part of the followers. This is quite likely to happen when one is dealing with adolescent learners who are bound to revolt, thereby jeopardising their chances of doing well in Mathematics. Smith (2004:14) remarked that Mathematics teachers need to exercise authority in control and management of the class, but should be careful not to restrict thinking (Smith, 2004:14). As suggested by the self-determination theory, learners may not be intrinsically motivated to learn under autocratic leadership. This was supported by Hamre and Pianta (2001:625) who observed that research has confirmed that poor teacher-learner relationships can undermine the learners' academic performance.

Lewin's democratic leadership style, which is also called the authoritative leadership style, has generally been viewed as the most effective of the three since followers are allowed to make contributions (Kumar, 2004:114). This coincides with Baumrind's view that authoritative parenting is better than other parenting styles. Secondary school Mathematics teachers can establish sound teacher-learners relationships which can translate to sound academic achievement if they adopt a democratic leadership style. In support of this view, Wayne and Youngs (2003:89) argued that teachers who demonstrate warmth and friendship during their interaction with learners are more likely to foster positive attitudes towards learning than their aggressive counterparts. However, excessive democracy may undermine academic performance through delays in decision-making and failure to complete the syllabus.

The laissez-faire leadership style is characterised by group decision-making with minimal or no guidance from the leader (Berns, 2010:237; Kumar, 2004:115). This

approach is likely to yield positive results if mature and dedicated students are involved. Since the current study focuses on adolescent learners, the laissez-faire leadership style is not likely to enhance the learners' scholastic achievement in Mathematics. Adolescent learners may not be knowledgeable and competent enough to put all the logistical modalities in place for optimal academic achievement in Mathematics. Tentatively, it can be argued that the selective and contingent use of Lewin's leadership styles can contribute to a better understanding of classroom management of the teacher teaching Mathematics. Classroom management in turn has consequences for achievement of learners in Mathematics (Tuckman & Monetti, 2011:384).

2.3.3.1.3 Blake and Mouton's managerial grid

Blake and Mouton described five leadership styles on a managerial grid which was named after them (Robbin & Coutler, 2009:390). The grid is composed of two dimensions; namely, the concern for results on the horizontal axis and the concern for people on the vertical axis as shown in Figure 2.14.

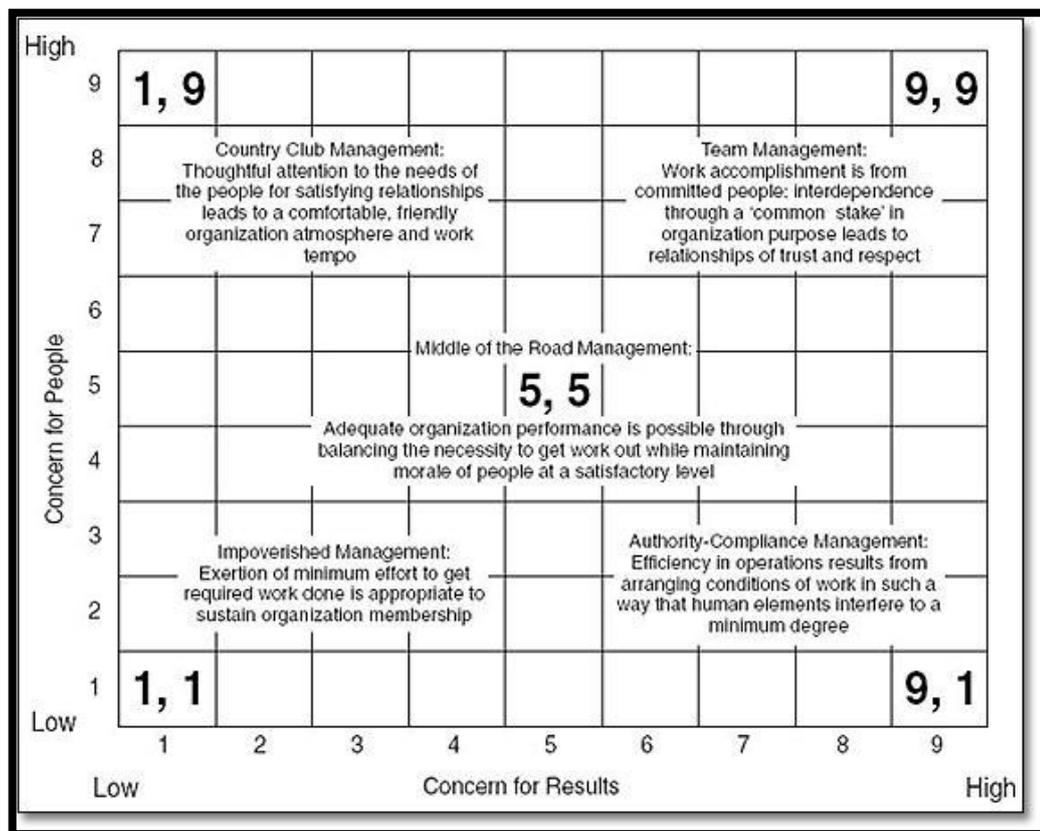


Figure 2.14: Blake and Mouton's managerial grid (Northouse, 2010:74)

In the context of classroom teaching and learning, the concern for production or results relates to emphasis on high academic achievement and having school work done. On the other hand, the concern for people refers to the teacher's efforts to address teacher-learner relationships and meet the learners' psychological, physiological and emotional needs.

Applying the managerial grid to classroom practice, the impoverished teacher (1, 1) neither worries about having scholastic work done nor about teacher-learner relationships (Dhameja & Dhameja, 2009:82). This style is undeniably not conducive to good academic achievement. This is backed by Waterhouse and Dickinson (2001:79) who warn that poor leadership can easily change the classroom into a haven of laziness, disruption and untoward attitudes. The authority-compliance management style, which is also called the task management style (9, 1) is dictatorial in nature and is employed by a teacher who is insensitive to the human needs of the learners. Such a teacher wants to have homework done and deadlines met. To some extent, some learners may underachieve because of fear, apathy and lack of motivation. In support of this, Osler (2010:77) commented that a study conducted in England revealed that learners prefer teachers who at least listen to them instead of being quick to find fault.

A teacher who adopts the country club style (1, 9) in the classroom places emphasis on teacher-learner relations at the expense of the achievement of educational objectives. Such a teacher focuses on introducing and maintaining healthy teacher-learner relations while the learners' academic efforts are not adequately monitored. This leadership style is not likely to enhance achievement in Mathematics especially for adolescents whose behaviour is erratic because of developmental turbulences which are characteristic of their age (William-Boyd, 2003:97). The middle of the road approach (5.5) refers to a leadership style in which the concern to establish good relations and the concern for productivity are not fully emphasised (Dhameja & Dhameja, 2009:83). It is adopted by a cautious teacher who places moderate emphasis on both teacher-learner relationships and the attainment of educational goals. This means neither scholastic achievement goals are fully met nor the teacher-learner relationships made as healthy as is possible. Moderate academic performance in Mathematics can be the outcome of such a leadership style since the two concerns are not fully met.

The last leadership style, the democratic leadership style (9, 9) places high emphasis on both achievement of educational objectives and good teacher-learner relationships. Numerous research studies confirm Blake and Mouton's claim that the democratic leadership style is the best since commitment, trust, respect and satisfaction prevail (Dhameja & Dhameja, 2009:82). According to Kumar (2004:115), the democratic leadership style has come to be generally regarded as the chief leadership style that a professional teacher should use. This was confirmed by Osler (2010:80) who intimates that learners prefer a situation in which the respect they show their teachers is shown back to them. Good relationships between learners and teachers often boost the learners' academic performance (Davis, 2006:193-224; Garcia, Kupczynski & Holland, 2011:2). Therefore, the extent to which a teacher endeavours to meet both educational goals and healthy teacher-learner relationships has a bearing on secondary school learners' academic achievement in Mathematics.

2.3.3.1.4 Research on classroom management and achievement in Mathematics

A study by Lamb and Fullarton (2001:2) examined the learner, classroom and school factors affecting Mathematics achievement in the United States and Australia. Data from the Third International Mathematics and Science Study (TIMSS) was used. Large samples of 7 087 United States respondents and 6 916 Australian respondents were used. The study established that approximately one-third of the variance in Mathematics achievement in the United States and more than one-quarter of the variance in Australia were a result of differences in classroom conditions. The study also revealed that classroom factors which include classroom management strongly influence learners' Mathematics achievement more than other school factors in general.

Similarly, Freiberg, Huzinec and Templeton (2009:63) conducted a study to investigate how the Consistency Management and Cooperative Discipline (CMCD), which is a prosocial instructional and classroom management programme, impacted on schools. The quasi-experimental research design was used with a sample of 350 respondents in the upper elementary school obtained through stratified random sampling. The study was undertaken over a period of two years. Test data on Mathematics was collected at the beginning and at the end of the experiment. The study revealed that learners in the CMCD schools did better than those in the control

group. Freiberg et al (2009:63) remarked that in Mathematics, the CMCD learners ranked at the 67th percentile while the learners in the control group ranked at the 50th percentile. On the basis of studies such as these, it can be argued that classroom management is a factor which can influence learners' academic achievement in Mathematics amid other variables.

2.3.3.2 Quality of teaching instructions

The manner in which classroom practitioners attempt to impart concepts to learners has long been implicated in determining the academic achievement of learners in virtually all the disciplines of learning. In support of this, Awolola (2011:91) asserted that one critical factor in determining how well learners learn Mathematics is the quality of teaching. As a warning, Conderman et al (2009:106) remarked that today's learners are used to a fast-paced technology-driven lifestyle in which information is quickly shared and altered. This implies that teachers need to adopt teaching methods in which the learners actively take part. In the same vein, Orton (2004:175) alluded to the idea that the degree to which Mathematics teachers employ theoretical principles of teaching and learning can determine the ultimate achievement of learners in the subject.

2.3.3.2.1 Description of the concept of teaching instruction

According to Mwamwenda (2004:201), the hallmark of instruction should be to impart concepts in such a way that learners will be capable of transferring them in future learning experiences or in approaching everyday problems. Teaching can be viewed as one person's effort to assist others to acquire, develop skill and realise their potential (Bhatt, 2007:1). Teaching instruction basically refers to the way teachers attempt to impart skills, knowledge and attitude to learners. It relates to the activities which are arranged by teachers so as to ensure the impartation of subject matter to the learners (Kumar, 2004:107).

2.3.3.2.2 Gagne's theory of instruction

Gagne has been credited with developing nine events of learning which have to a large extent made a positive impact on teaching and learning efforts (Tuckman & Monetti, 2011:480). Combining the principles of behaviourism and cognitivism, Gagne developed the nine events of instruction on the basis of how information

processing occurs when learners are exposed to intellectual stimuli (Pinfan & St. Amant, 2010:337). In their correct sequence, the nine events of instruction as postulated by Gagne are gaining attention, informing the learners of the instructional objectives, stimulating recall prior to learning, presenting the content as the stimulus, providing learning guidance, eliciting performance, providing feedback, assessing performance and finally enhancing retention and transfer (Mwamwenda, 2004:200). Figure 2.15 shows the nine events of instruction.

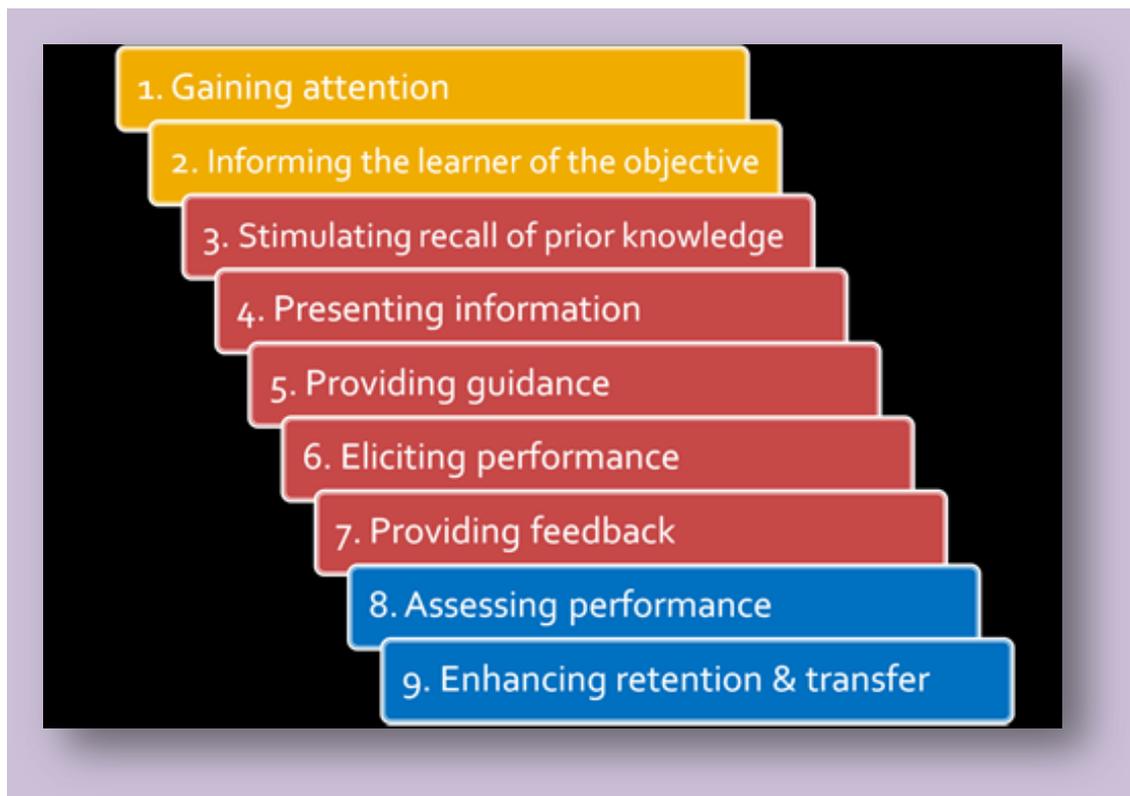


Figure 2.15: Gagne's nine events of learning (Adapted from Weyers, 2006:53; Orange, 2002:99)

Capturing the attention of the learners is crucial because it plays a receptive role in the learners' cognitive capacities (Pinfan & St. Amant, 2010:337). Providing the learners with a list of learning objectives triggers the cognitive process of expectancy which boosts the learners' motivation to learn. The learning objectives highlight the new skills or concepts learners will gain as a result of being exposed to the new subject matter (Tuckman & Monetti, 2011:481). Stimulating recall of prior learning is geared to help learners retrieve information from the long-term memory to the

working memory so that the retrieved information becomes the anchoring concepts of the new concepts to be learned. This is similar to Ausubel's subsumption theory.

Presenting the subject matter as the stimulus is the fourth step of Gagne's instructional events (Yazdani, 2008:60). Gagne prescribed that the new content should be meaningfully chunked and organised and its delivery should be spiced by the use of a diversity of teaching aids which appeal to the learners' various senses. Learning guidance in the form of examples, case studies and graphical representations together with the use of memory enhancing techniques can facilitate the storage of the concepts in the long-term memory. Eliciting performance takes place when the learners are challenged to practise using the newly learnt concepts or skills through responding to questions (Tuckman & Monetti, 2011:482).

The learners are given immediate and specific feedback on their performance in the application of the newly acquired skills and knowledge (Alutu, 2006:45). The feedback reinforces comprehension and uproots any misconceptions held by the learners. The performance of the learners is determined through the administration of a post-test. This helps in determining the learner's level of proficiency (Tuckman & Monetti, 2011:482). Finally, the teacher tries to boost the learners' ability to generalise the learnt concepts through providing a variety of practice activities.

The methodical procedure suggested by Gagne is relevant to the teaching and learning of Mathematics. This is backed by Yazdani (2008:59) who posits that proficiency in Mathematics is unattainable without mastery of the prerequisite concepts and skills. Teachers who take their time to follow the suggested nine instructional events are likely to foster meaningful learning among their learners. It is important to harness the attention of learners whenever new concepts are being introduced. Giving practice questions during and after the lesson can enhance learners' performance in Mathematics. Therefore, the learners' level of performance in Mathematics can be viewed as directly proportional to the teacher's ability to implement Gagne's nine events of instruction.

2.3.3.2.3 Ausubel's theory

After being influenced by Piaget's theory of cognitive development, David Ausubel crafted his own theory which entails concepts such as meaningful learning, advance

organisers, and the subsumption theory. Although the principles of Ausubel's theory were not specifically crafted for Mathematics education, they remain useful especially if one considers the hierarchical and sequential nature of mathematical concepts (Ifamuyiwa, 2011:129).

According to Orton (2004:185) and Mwamwenda (2004:196), meaningful learning as suggested by Ausubel is the process through which new knowledge is absorbed by linking it to some existing relevant aspect of the learner's pre-existing knowledge structure. Ausubel suggests that meaningful learning can only occur if three conditions are satisfied. According to Orton (2004:186), the learner must first exhibit readiness to learn, that is, the learner must adopt a supportive learning mentality. Second, the subject matter must be at least logically connected to the learner. Last, the learner's mental structures must have specific relevant concepts which are associated with the new subject matter to be learned.

Ausubel postulated that meaningful learning occurs through the subsumption theory. Subsumption is the incorporation of new material into one's cognitive structures which are called subsuming concepts. A subsumer is any idea, principle or generalisation that the learner has already mastered that provides a foundation upon which new concepts are laid (Ifamuyiwa, 2011:129). Orton (2004:187) indicated that subsumption is characterised by the review and reorganisation of knowledge in the light of new concepts encountered. Ausubel attempted to summarise the essence of educational psychology by the principle that the most important single factor influencing learning is what the learner already knows (Mwamwenda, 2004:194). The existing structure acts as a framework into which new concepts are hierarchically reorganised and related (Ausubel, 1978:251). This is related to Piaget's concepts of assimilation and accommodation.

Ausubel's significant original contribution to actual classroom practice came in the form of advance organisers. According to Shihusa and Keraro (2009:414), an advance organiser is a type of cognitive bridge which teachers employ to assist learners to make a link between what is already known and what is to be learnt. An advance organiser is a framework that helps learners to learn new concepts and meaningfully associate them with the already existing mental structures (Shihusa & Keraro, 2009:414). This means that an advance organiser is information that a

teacher presents in an attempt to help the learner organise new incoming information. Therefore, an advance organiser is any material that is introduced before novel content to act as an anchor for the reception of the new subject matter. Mwamwenda (2004:196) commented that the key purpose of advance organisers is to act as a link between what the learner already knows and the prerequisite insights which are required before new concepts are fully learnt (Mwamwenda, 2004:196). Woolfolk, Winnie, Pierry and Shapka (2010:289) suggested that the instructor must focus on highlighting the key elements of the new content to be introduced as well as the relationships which the new content to be introduced has with prior relevant knowledge. An advance organiser can be used for such a purpose.

Theoretically, it can be argued that the extent to which teachers heed the principles of Ausubel's principles determine the academic achievement of learners. For instance, the teacher's ability to formulate relevant advance organisers can determine the extent to which Mathematics concepts are mastered. According to Baxendell (2003:46), an advance organiser should be relevant and consistent with the content to be taught. Moreover, the teacher's sensitivity to the conditions necessary for meaningful learning to take place can influence secondary school learners' Mathematics scholastic achievement (Sipe, 2001:336).

2.3.3.2.4 Research on teaching instructions and achievement in Mathematics

A study to explore the importance of teacher effectiveness in boosting learners' academic achievement in reading and Mathematics was undertaken by Heck (2009:227). A large sample of 9 196 from 156 elementary schools was used. The study established that the effectiveness of successive teachers positively correlated with learners' academic achievement in reading and Mathematics. It was also found that collective teacher effectiveness as an organisational attribute of schools was positively correlated with achievement levels. The study revealed that stability of the school's teaching staff and the quality of its academic organisation and teaching processes positively correlate with achievement levels (Heck, 2009:227). The findings of the study reinforce the principle that teachers play a critical role in determining the academic performance of learners in Mathematics and other subjects.

Another allied study was conducted by Ifamuyiwa (2011:129) to ascertain the utility of behavioural objectives as advance organisers in the teaching of secondary school Mathematics. The pre-test, post-test control group quasi-experimental research design was employed using a purposively chosen sample of nine respondents. The experimental class which received advance organisers obtained a mean post-test score which was significantly higher than the mean post-test score of the control class (Ifamuyiwa, 2011:129). The study revealed that using behavioural objectives as advance organisers is a significantly effective method of teaching and learning Mathematics. Therefore, it can be argued that the extent to which advance organisers are used by teachers can influence the degree to which meaningful learning can occur.

Jaiyeoba and Atanda (2011:91) conducted a study in Nigeria, with a sample of 1014 Mathematics teachers and principals to explore the school factors that are likely to influence learners' academic performance in Mathematics. Instructional materials were found to be a critical determinant of the learners' academic achievement in Mathematics (Jaiyeoba & Atanda, 2011:91).

According to Lamb and Fullarton (2001:2), a study was undertaken in the United Kingdom to ascertain the extent to which teachers accounted for variance in academic performance of learners. In the study, 80 schools and 170 teachers were involved and the academic achievement growth of learners over a period of one academic year was measured. The study established that more than 30% of the variance in learner progress was attributable to teachers. The study concluded that teacher quality and teacher effectiveness contributed more significantly to learners' academic progress than other variables related to the classroom, the school or even the learners themselves.

In an attempt to investigate the impact of brain-based teaching on Mathematics achievement, Awolola (2011:91-106) carried out a study using a pre-test post-test non-equivalent quasi-experimental research design. In the study, Mathematics achievement was the dependent variable, instructional strategy was the independent variable while cognitive style acted as the moderator variable. The sample was composed of 522 secondary school learners in Oyo State, Nigeria. The outcomes of

the study revealed that brain-based teaching boosted Mathematics achievement more than the traditional lecture method. This is evidence in support of the notion that teaching methods which promote participation and mental activity on the part of the learners can positively influence Mathematics achievement.

Hill, Rowan and Ball (2005:371) focused on examining the extent and manner in which teachers' mathematical knowledge for teaching is contributory towards gains in learners' Mathematics achievement. A linear mixed-model methodology involving the gains in Mathematics achievement of first and third graders over a period of one year was used. Emphasis was placed on the teacher's knowledge of specific mathematical skills which are used during instruction (Hill et al, 2005:371). The study established that teachers' mathematical knowledge is significantly related to both first and third graders' Mathematics achievement. It can be argued that the extent to which Mathematics teachers know mathematical concepts can determine their ability to utilise constructive teaching principles suggested by psychologists such as Ausubel and Gagne.

Ares and Gorrell (2002:275) undertook a study to investigate the views of middle school teachers and learners towards meaningful learning and traditional learning methods. They employed the descriptive survey research design in which interviews were the data collection instruments. The learners who participated in the study emphasised that active participation in learning activities was beneficial to them as it enabled them to gain deep insights regarding the content they were learning as well as mastering the relationships between the various concepts presented to them. On the other hand, the respondents acknowledged that passive reading and listening as well as mechanical note taking without participation were not conducive to meaningful learning. Ares and Gorrell (2002:275) encouraged classroom practitioners to increase interactive activities in which learners have many opportunities to participate so as to promote meaningful learning (Ares & Gorrell, 2002:275). The findings of the study imply that the academic performance of learners in Mathematics can be influenced by the frequency with which learners are given the opportunity to take part in learning exercises actively.

In an attempt to examine the impact of using advance organisers on biology learners, Shihusa and Keraro (2009:413) conducted a study in Kenya. A sample of 166 form 3 learners participated in the study. It was established that learners who were exposed to advance organisers had a higher level of motivation than those subjected to the conventional teaching methods (Shihusa & Keraro, 2009:413). This implies that the use of advance organisers as an instructional tool can boost learners' motivational levels which can ultimately translate to better academic performance in Mathematics and other subjects.

2.3.3.3 Teachers' expectations

According to Snowman et al (2009:105), Jussim and Harber (2005: 131-135) and Hinnant et al (2009:662), the importance of teachers' expectations regarding learners' abilities above and beyond what can be attributed to their actual achievement is a topic that has received considerable attention from researchers. Because of the importance of teacher expectations in determining learners' academic performance in general, the current study examines the influence of the expectations of teachers on the academic performance of secondary school learners in Mathematics.

2.3.3.3.1 Description of the concept of teacher expectations

The concept of teacher expectations has been closely linked to or even equated with a self-fulfilling prophecy. Feldman (2009:603) viewed self-fulfilling prophecies as expectations about the occurrence of a future event or behaviour which act to boost the probability of the occurrence of that very event or behaviour (Feldman, 2009:603). The self-fulfilling prophecy prevails when beliefs lead to their fulfilment (Hinnant et al, 2009:662). A self-fulfilling prophecy prevails only if the original expectation was wrong and a change was brought about in the learners' behaviour on the basis of that erroneous expectation. Myers (2008:110) described a self-fulfilling prophecy as a belief that triggers its own fulfilment. To shed more light on the subject in question, Snowman et al (2009:105-106) claimed that teachers begin by forming expectancies pertaining to the learners' future academic performance on the basis of attributes such as socioeconomic position, dress, speech pattern and academic scores in tests and other exercises. Thereafter, the teachers proceed, subtly and sometimes unconsciously, to communicate those expectations to the

learners in a number of ways. The learners in turn decode the signals from the teachers and eventually behave in a manner which is consistent with the original expectations of the teachers.

2.3.3.3.2 Impact of teacher expectations on academic achievement

According to Good and Nicholls (2001:113-126) and Snowman et al (2009:106), many studies exploring the impact of teacher expectations on the academic performance and classroom participation of learners have established that the expectations of teachers can generate positive or negative consequences. Hinnant et al (2009:662) indicated that positive self-fulfilling prophecies occur when a teacher overestimates a learner's abilities while negative self-fulfilling prophecies are a result of the underestimation of a learner's abilities by the teacher (Hinnant et al, 2009:662). Teacher expectations are more likely to reinforce and maintain already existing tendencies than to radically change a learner's inveterate behaviours (Snowman et al, 2009:106). Hence it can be concluded that the academic performance of secondary school learners can be influenced by the expectations of their teachers. However, Myer (2008:111) warns that low teacher expectations do not necessarily doom a capable child, nor do high expectations magically transform a slow learner into a valedictorian.

2.3.3.3.3 Research on expectations of teachers and Mathematics achievement

In an attempt to establish the extent to which teacher expectations act as a predictor of future academic achievement in Reading and Mathematics, Hinnant et al (2009:662) embarked on a study with a sample of nearly 1000 primary school learners. The study revealed that teacher expectations were related to learners' academic achievement in Reading and Mathematics. Although the study found no evidence to suggest that teacher expectations are cumulative in nature, it was established that teacher expectations are relatively enduring for achievement in Mathematics (Hinnant et al, 2009:662).

2.3.3.4 Personality of teachers

The way teachers behave and handle themselves in the presence of learners can either enhance or sabotage their learners' academic performance. The reason for this is that teachers are arguably one of the key role models in the lives of learners.

The role of personality traits of teachers is even more critical at secondary school level where the generality of the learners are adolescents who are seriously searching for their own identities as postulated by psychological theorists such as Erikson, Rogers and Allport. For the purpose of the current study, the personality theories advanced by Hans Eysenck and Albert Bandura are discussed in an effort to understand how the personality of teachers can influence the academic achievement of secondary school learners in Mathematics. The three factor personality model postulated by Eysenck was opted for because numerous studies have been undertaken to explore the relationship between academic performance and the five variables outlined in the big five personality model (Blau & Fuller, 2006:87; Gilles & Bailleux, 2001:3; Sava, 2009:135). Focusing on Eysenck's three factor model is a variation to such studies. The social learning theory was deemed suitable for the current study because the focus of the current study is adolescent secondary school learners who are likely to imitate the behaviour of teachers as role models, as postulated by Bandura (Santrock, 2004:227).

2.3.3.4.1 Description of the concept of personality

According to Parkinson (2000:6) and Mwamwenda (2004:282), personality can be defined as the characteristic way in which someone responds to situations or their preferred way of behaving towards other people and particular circumstances. Personality is the pattern of relatively stable characteristics that generate individuality and consistency in a given person (Feldman, 2009:446). Personality can be viewed as the totality of all the distinctive thoughts, feelings and behaviours that characterise a person thereby making each person different from others (Lahey, 2009:405; Santrock, 2004:126). Hence the essence of personality is individuality in a relatively consistent way.

2.3.3.4.2 Eysenck's three factor theory of personality

According to Kosslyn and Rosenberg (2006:493), Hans Eysenck used factor analysis to develop the three factor personality model. The three personality dimensions of the model, which can also be called super traits, are extraversion, neuroticism and psychoticism. Only extraversion and neuroticism are considered in the current study since they are part of the Big Five personality model (Kosslyn & Rosenberg, 2006:493). While the extraversion dimension relates to the extent of sociability, the neurotic dimension encompasses emotional stability (Feldman,

2009:458). Emotional stability is expressed in a continuum stretching from a neurotic personality at one extreme end to a stable personality at the other extreme end. The sociability continuum stretches from introversion to extraversion. The two continua can be combined to produce four personality types; namely, unstable introvert (introverted-neurotic); stable introvert (introverted-stable); unstable extrovert (extroverted-neurotic) and stable extrovert (extroverted-stable). Figure 2.16 outlines some of the unique attributes of people with different personalities relative to neuroticism and extraversion.

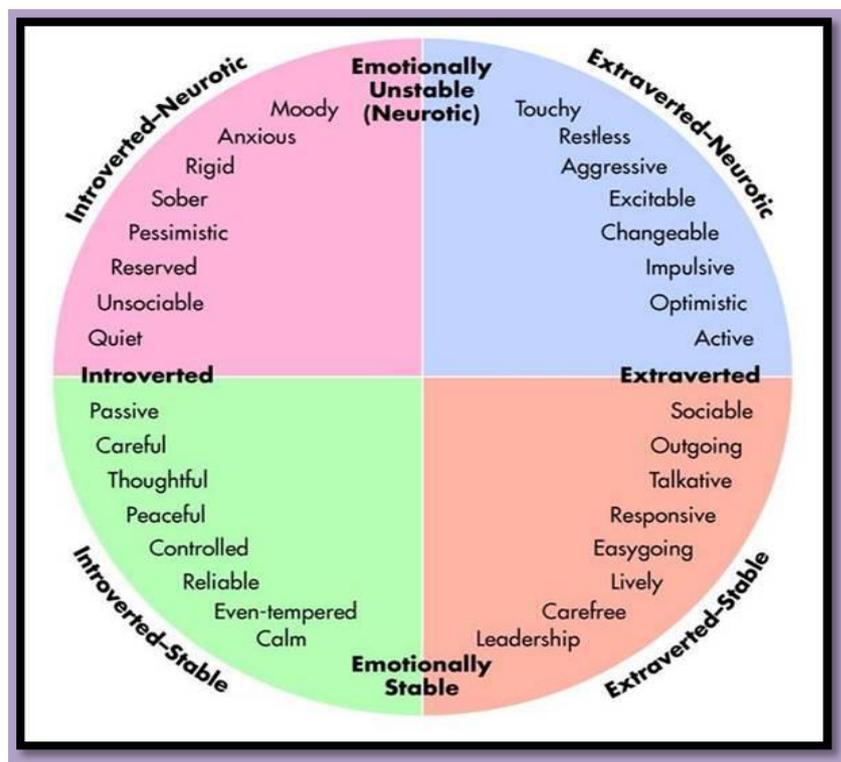


Figure 2.16: Eysenck's three factor theory of personality (Adapted from Houston, 2005:301)

The unique attributes of each teacher determine the nature of the teacher-learner interaction in the classroom which can have a bearing on the learners' ultimate scholastic achievement. For instance, moody, touchy and hostile teachers are not likely to be effective especially when they teach adolescents whose behaviour is also erratic and volatile. Extremely reserved teachers may have difficulty in sufficiently motivating learners and harnessing their curiosity and attention. On the other hand, cheerful, sociable and friendly teachers may easily gain the attention and appreciation of adolescent learners. Moreover, such teachers are likely to use

cooperative learning in a tension-free atmosphere. Such a phenomenon can arguably translate to significantly high scholastic achievement in Mathematics and even in other academic disciplines.

2.3.3.4.3 Bandura's social learning theory

Albert Bandura is a neo-behavioural psychologist who added a cognitive dimension to the principles of mainstream behaviourism. Basing his results on his experiments with Bobo dolls, Bandura established that learning can take place through observational learning with or without reinforcement (Bee & Boyd, 2004:20; Swartz et al, 2011:133). Modelling or observational learning is defined by Santrock (2004:227) as learning that happens when someone observes and subsequently imitates the other person's behaviour. Some studies have obtained evidence to suggest that human beings are genetically pre-programmed to imitate other people (Feldman, 2009:201). Mwamwenda (2004:187) and Berns (2010:235) concurred that prestigious and competent people with the capacity to reward or punish are potential role models whose behaviour is likely to be imitated. Teachers, as already suggested above, are likely to be significant role models at secondary school level (Berns, 2010:235). Observational learning is a long process in which every action of the teacher manifested in the presence of the learners has a probability of being imitated (Mwamwenda, 2004:185).

On the basis of the above principles, Mathematics teachers whose personalities encompass attributes such as diligence, perseverance, determination, confidence, patience and thoroughness are likely to be emulated by learners who will ultimately exhibit the same characteristics. This is likely to result in good academic achievement in Mathematics. On the other hand, teachers who are lazy, unfocused, impatient and unfriendly are likely to be poor role models for their learners. Research has shown that learners tend to internalise their teacher's interest in and enthusiasm for teaching Mathematics (Smith, 2004:11). Since attitudes can also be imitated, it implies that to some extent, teachers who exhibit positive attitudes towards Mathematics are likely to transmit such attitudes to their learners while negative attitudes can also be equally transmitted.

Apart from observational learning, Bandura also referred to the role of self-efficacy in learning (Bandura, 2002). Self-efficacy is the belief by the individual that he or she can undertake a given task and produce the desirable results (McLean, 2003:31; Swartz et al, 2011:133). People with a high level of self-efficacy normally exert much effort to ensure that their initial belief in their own abilities is fulfilled (Santrock, 2004:226). Self-efficacy is not only important to learners. Teacher self-efficacy is implicated by some scholars as a determinant of learners' own concepts of self-efficacy. According to Tschannen-Moran and Hoy (2001:800), the self-efficacy beliefs of teachers relate to the teachers' judgements of their capabilities to produce learner engagement and meaningful learning among unmotivated learners. A teacher with a high level of self-efficacy is open-minded, creative and has good organisational skills (Tschannen-Moran & Hoy, 2001:800). The extent to which teachers exude and encourage self-efficacy can determine the learners' levels of self-efficacy through modelling. It was maintained by Snowman et al (2009:273) that learners with the same mathematical skills can perform differently in Mathematics owing to differences in their self-efficacy levels. On the basis of such explanations, it can be claimed that there is a strong positive correlation between one's self-efficacy and one's academic achievement in Mathematics.

2.3.3.4.4 Research on personality of teachers and achievement in Mathematics

In an endeavour to determine whether a relationship exists between the personality of teachers and academic achievement, Garcia et al (2011:3) carried out a study using a sample of secondary school learners in the tenth and eleventh grades. The study focused on the impact of teacher personality styles on secondary learners in the areas of Mathematics, science, English language, arts and social studies for two academic school years (Garcia et al, 2011:3). Large samples of teachers and learners in these subjects were used. The study was based on the Big Five personality model which focuses on five personality dimensions; namely, openness to experience, conscientiousness, extraversion, agreeableness and neuroticism. The study revealed a significant difference among the academic excellence of secondary school learners in the already stated subjects relative to the personality types of the teachers.

Due to overwhelming evidence of reduced female participation and performance in Mathematics and computer science, Wiest and Johnson (2005:12) embarked on a study to explore ways of boosting the participation and performance of females in the two subjects. As a suggestion, Wiest and Johnson (2005:12) intimated that teachers can help to dispel such myths by introducing learners to historic and contemporary female role models in Mathematics and computer science. It was also recommended that exposing boys to female role models in the fields of Mathematics and computer science could help to foster a classroom climate which is conducive to encouraging both male and female learners. Encouraging both male and female learners to believe that virtually all career opportunities are open to anyone independent of gender was mentioned as a way of boosting female participation and performance in Mathematics through modelling.

Tschannen-Moran and Hoy (2001:783) conducted a study aimed at reviewing many of the ways in which teacher self-efficacy has been measured by various authorities. In developing a new instrument for measuring teacher self-efficacy, a sample of 10 respondents who were attending a seminar on self-efficacy and learning was used. The sample was composed of eight graduate students and two researchers. The method generated for measuring teacher self-efficacy addressed teacher efficacy for student engagement and classroom management (Tschannen-Moran & Hoy, 2001:800). Although the above study did not directly focus on Mathematics achievement, the conclusion can be made that the way teachers present themselves in the presence of students can ultimately influence the students' self-efficacy beliefs in virtually all subjects including Mathematics.

In the same vein, Gilles and Bailleux (2001:3) carried out a study to investigate the effects of personality traits and cognitive abilities on academic achievement, numerical ability, g factor and spatial ability. The research participants were adolescent learners with a mean age of 12 years. Three cognitive tests on the areas cited above were administered as well as a personality questionnaire based on the Big Five personality model. The study established that neuroticism had a more significant effect on Mathematics than on other disciplines. Personality was also found to be a critical variable in predicting learners' academic performance.

Teacher expectations, classroom management and teachers' personality attributes can determine the quality of the relationship between teachers and learners. It is possible for a relationship to exist between teacher-learners relationships and irrational beliefs. The way learners interpret their teachers' expectations can make them vulnerable to irrational beliefs pertaining to the demand for approval, perfectionism and low frustration tolerance. The existence of such a relationship is verified in the subsequent chapters of the current study

2.4 CONCLUSION

The first part of the literature study confirmed that learners' Mathematics achievement at secondary school level can be influenced by a myriad of variables. Psychological theories and studies outlining how Mathematics achievement can be influenced by some of the variables were also explored. Taub et al (2008:187) established that intelligence, whether considered as a single entity or in its elemental form, influences learners' Mathematics achievement. It was found that stage of learners' intellectual development can affect their academic performance in Mathematics (Reedal, 2010:19). A positive correlation was found between aptitude and Mathematics achievement in a secondary school context (Gehlawat, 2011:75; Sethi, 2012:2). Learners' Mathematics achievement can be influenced by the operations of the working memory (De Smith et al, 2009:186; Ashcraft & Kirk, 2001:227). Language proficiency is another variable which was confirmed to be capable of influencing learners' Mathematics achievement (Mosqueda & Tellez, 2008:416; Barton & Neville-Barton, 2003:19). The literature study also revealed that previous Mathematics achievement can predict learners' Mathematics achievement at a later stage (Yates, 2000:4-15; Rafi & Samsudin, 2007:53-67; Sasanguie et al, 2012:119-128).

High levels of motivation can lead to self-efficacy which can subsequently promote good Mathematics achievement (Mousoulides & Philippou, 2005:321; Tella, 2007:154; Md.Yunus & Ali, 2009:99). It was revealed that self-concept can influence learners' Mathematics achievement by indirectly affecting other variables such as Mathematics anxiety and perceptions of Mathematics (Ferla et al, 2009:499-505; Wang et al, 2012:1242). It was confirmed that stress and anxiety can influence

learners' academic performance in Mathematics both positively and negatively (Kauts & Sharma, 2009:39; Feldman, 2009:521).

Parenting styles can affect the academic attainment of secondary school learners in Mathematics (Gonzalez & Wolters, 2006:203). The literature study established that learners' self-concept and Mathematics achievement can be influenced by their socioeconomic background (Neuenschwander et al, 2007:594-602; Eamon, 2002:49). Authoritative parenting was found to be conducive to high academic achievement in Mathematics (Santrock, 2004:74; Snowman et al, 2009:26).

Studies by Lamb and Fullarton (2001:2) and Freiberg et al (2009:63) confirmed that learners' Mathematics achievement can be influenced by the way teachers manage activities in the classroom. The quality of teaching instructions can affect secondary school learners' Mathematics achievement (Lamb & Fullarton, 2001:2; Jaiyeoba & Atanda, 2011:91). The literature study also revealed that the use of participatory teaching methods can improve learners' Mathematics achievement (Awolola, 2011:91-106; Ares & Gorrell, 2002:275). Teacher expectations were found to be a predictor of learners' Mathematics achievement (Hinnant et al, 2009:662). A study by Giles and Bailleux (2001:3) confirmed that the personality attributes of teachers can influence the academic achievement of secondary school learners in Mathematics.

Since Mathematics achievement can be influenced by several variables, studies regarding Mathematics achievement can be made more comprehensive by considering learners' internal factors as well as their home and school factors. In line with the current study, it is necessary to explore the relationship between irrational beliefs and some of the identified variables in a Mathematics learning context. Feasibility of measurement of the variables is considered in determining the variables to be discussed in relation to irrational beliefs in a Mathematics learning context. The possible relationship between irrational beliefs and some of the variables discussed above is explored in the third chapter of the current study. The next chapter, which constitutes the second part of the literature study, is an in-depth exploration of irrational beliefs and how they can influence Mathematics achievement at secondary school level.

CHAPTER THREE

IRRATIONAL BELIEFS AND MATHEMATICS ACHIEVEMENT

3.1 INTRODUCTION

Before irrational beliefs are explored in detail, this chapter commences by giving an exploration of the nature of beliefs in general. Types of beliefs are outlined relative to rationality and irrationality. The characteristics of rational beliefs and the four categories of rational beliefs which entail preference, anti-awfulising, high frustration tolerance and unconditional acceptance of oneself, others and the world are examined (Ellis & Dryden, 2003:2; Feltham & Horton, 2012:298-299). The characteristics of irrational beliefs are highlighted before an outline of the four categories of irrational beliefs as theorised by Ellis is given. The categories include demandingness, awfulising, low frustration tolerance and global evaluation of oneself, others and the world (Seligman & Reichenberg, 2010:255; David et al, 2010:13; Bridges & Harnish, 2010:863).

Two theories pertaining to the development of irrational beliefs are discussed. These are Ellis's rational emotive behaviour therapy and Beck's Cognitive theory (Gazzaniga & Heatherton, 2006:570; Kaygusuz, 2013:143; Froggatt, 2006:2). An outline of the acquisition, maintenance and effects of irrational beliefs as postulated by Ellis is given. Thirteen irrational beliefs advanced by Ellis are discussed together with their influence on Mathematics achievement. Beck's schemas, cognitive triad and cognitive distortions are also examined in conjunction with their influence on Mathematics achievement. The theories of Ellis and Beck are then briefly compared. In an attempt to explore the influence of irrational beliefs on learners' Mathematics achievement, irrational beliefs related to the learner are discussed. Learner variables such as gender, intelligence, self-concept, anxiety, study habits, perfectionism, procrastination and motivation are elaborated relative to irrational beliefs and Mathematics achievement. Some accompanying studies are also highlighted. The association between peer relations and irrational beliefs is discussed in line with their influence on Mathematics achievement.

The irrational beliefs which may be held by learners and parents regarding school and schooling are explored. Parents' irrational beliefs related to their school-going children are also examined. The ultimate influence of irrational beliefs on learners' Mathematics achievement at secondary school level is discussed as are the irrational beliefs of both learners and parents related to the teacher, with the intention of exploring how irrational beliefs can impinge upon Mathematics achievement. The role of teachers in moulding beliefs is deliberated on before the issues discussed in the entire chapter are rounded off by a summary.

3.2 EXPLORATION OF THE NATURE OF BELIEFS

Definitions of beliefs are provided together with the characteristics of beliefs as a way of exploring the concept of a belief. Thereafter, types of beliefs are examined relative to rationality and irrationality.

3.2.1 Description of the Concept of a Belief

According to Liu (2003:32), beliefs are generalisations which are learned through observation and inference. Beliefs are an individual's hypothetical propositions which the person holds as the truth with or without verification (Grootenboer & Hemmings, 2007:4; Ashford & LeCroy, 2010:109). In support of this, Philipp (2007:259) defined beliefs as the understandings, hypotheses, or propositions held by a person, which act as the lenses governing the person's view of the world. Beliefs are units of cognition which constitute a person's knowledge in the form of faith, facts, opinions and premises (Leder, Pehkonen & Torner, 2002b:12). A belief system is the manner in which an individual's beliefs are organised to form an entity (Philipp, 2007:259). Characteristically beliefs are normally very structured, highly enduring and cognitive in nature (Hannula, Evans, Philippou & Zan, 2004:112). Najafi, Jamaluddin and Lea-Baranovich (2012:312) acknowledged that beliefs play a vital role in a human being's life since they may lead to happiness or sadness. Beliefs can affect the way people understand themselves and how they behave in various situations (Palsdottir, 2007:118). Beliefs may determine people's attitudes and may also act as cognitive elements of attitudes (Bergh & Theron, 2009:147). Once established, beliefs normally influence people's perceptions of life situations (Meyer, 2008:78). Learners' beliefs remain a critical determinant of their academic performance (Blackwell et al,

2007:259). The current study is an endeavour to explore the influence of irrational beliefs on the Mathematics achievement of secondary school learners.

3.2.2 Types of Beliefs

While beliefs can be categorised in different ways (Leder et al, 2002b:15), the current study focuses on beliefs relative to rationality and irrationality as postulated by cognitive-behavioural theorists. In an attempt to distinguish between rational and irrational beliefs, Sarvestani (2011:432) suggested that one should consider the extent of a given belief's flexibility, consistence with reality, logical nature and inclination of the belief towards productivity. It is important to note that rational and irrational are actually not at the opposite ends of the same continuum (Bernard, 2009:66). This means rational and irrational beliefs are not jointly bipolar since rationality has its own continuum which ranges from little to intense rationality and irrationality has a similar continuum (David et al, 2010:13). The dominance of irrational beliefs renders rational beliefs dormant and unable to reduce the negative effects of the irrational beliefs (David et al, 2010: 59).

3.2.2.1 Rational beliefs

Rational beliefs are defined and some of their characteristics are highlighted, followed by an outline of the four categories of rational beliefs.

3.2.2.1.1 Description of the concept of rational beliefs

According to Davies (2008:2), rational beliefs are flexible, logical and have a high degree of consistence with reality. Mousavi and Basavarajappa (2012:853) asserted that apart from being based on empirical evidence, rational beliefs are practical and/or logical. People who are guided by rational beliefs normally have fruitful and healthy lifestyles (Rosner, 2011:82; Ellis & Dryden, 2003:2). Davies (2008:102) in Najafi et al (2012:313) argued that rational beliefs, which are an outcome of a healthy way of thinking, yield healthier emotions, more functional behaviours and greater acceptance of oneself and others. Moreover, rational beliefs are at the core of many solutions which people generate when faced by some problems (Dryden, 2006:14; Najafi et al, 2012:313). It has been claimed that thinking rationally yields appropriate emotional and behavioural responses (Ellis, 2007:13). On the basis of

the outlined characteristics of rational beliefs, it can be argued that learners with strong rational beliefs are likely to do well in Mathematics.

3.2.2.1.2 The four categories of rational beliefs

Just like irrational beliefs, rational beliefs can be divided into four categories. The four classifications of rational beliefs are preferences, anti-awfulising, high frustration tolerance and unconditional acceptance of oneself, others and the world (Feltham & Horton, 2012: 298-299; Ellis & Dryden, 2003:2).

3.2.2.1.2.1 Preferences

People who are rational prefer to achieve their goals without taking achievement as a dire necessity (Ellis & Dryden, 2003:2). The wish is expressed in a non-dogmatic manner which does not lead to neurosis and frustration if it does not materialise. According to Palmer (2000:281), preferences lead to healthy negative emotions in which people admit that they have failed without viewing themselves as useless. For instance, learners may indicate that it is good for them to score high marks in a Mathematics test although they realise that at certain occasions it may not be possible to do so.

3.2.2.1.2.2 Anti-awfulising

This expresses the realisation that although situations may not be as ideal as initially imagined, all hope is not lost (Ellis & Dryden, 2003:2). Unpleasant experiences do not signal the end of one's life. According to Palmer (2000:280), anti-awfulising is rating events as reasonably and understandably bad but not as hopelessly unimaginable (Feltham & Horton, 2012:298). Moreover, unfortunate experiences do not mean that the other people involved are worthless. People interpret unfriendly events on a realistic scale and treat such events as part of life. For example, a Mathematics learner may acknowledge that a mark of 55% is acceptable since it is a pass mark although not a distinction.

3.2.2.1.2.3 High frustration tolerance

People with a high frustration tolerance try to be happy even if the circumstances are not as good as they may have preferred (Ellis & Dryden, 2003:2). High frustration tolerance enables people to tolerate or endure real life difficulties or discomforts (Palmer, 2000:280; Bernard, 2008:3; Feltham & Horton, 2012:298). According to Bernard (2008:8), learners with a high frustration tolerance believe that school work

and homework do not necessarily have to be fascinating all the time and it is normal to encounter challenging academic work.

3.2.2.1.2.4 Unconditional acceptance of oneself, others and the world

According to Ellis (1977:101), unconditional self-acceptance occurs when people accept themselves regardless of variables such as their intellectual capabilities, competence or other people's approval. Such individuals accept themselves with all their shortcomings and imperfections (Palmer, 2000:280; Austad, 2009:278). Unconditional self-acceptance is one of the key rational principles emphasised by the rational emotive behaviour therapy as a way of curbing unhealthy self-rating (Ellis, 1977:101; Kosslyn & Rosenberg, 2006:698). Unconditional other acceptance is demonstrated when a person accepts other people despite their weaknesses (Palmer, 2000:280; Austad, 2009:278). This implies that people view others as fallible (Bernard, 2008:3, 8). Accepting other people unconditionally involves disapproving of their bad behaviour and personality attributes without condemning them totally (Bernard, 2008:8; Feltham & Horton, 2012:299). Both Rogers and Ellis posited that unconditional self-acceptance can lead to emotional well-being (Flett, Besser, Davis & Hewitt, 2003:120). Unconditional life acceptance is concerned with viewing life as something punctuated by both good and bad experiences (Feltham & Horton, 2012:299). According to Ellis, people with rational beliefs are emotionally healthy and they demonstrate normal behaviour like being self-accepting, other-accepting and life-accepting (Austad, 2009:275). Mathematics learners who exercise unconditional self-acceptance continue to rate themselves highly despite scoring low marks in Mathematics in some situations. Such learners also accept their Mathematics teachers, parents and peers despite their having some shortcomings. Such a level of self and other acceptance can have a bearing on one's Mathematics achievement.

3.2.2.2 Irrational beliefs

A description of the concept of irrational beliefs precedes an outline of the four categories of irrational beliefs as theorised by Ellis.

3.2.2.2.1 Description of the concept of irrational beliefs

According to Froggatt (2006:12), a belief is said to be irrational if it mars individuals from achieving their goals through distorting reality, creating enduring emotions that distress and paralyse the individuals and subsequently driving them to engage in behaviours which are harmful to themselves, to others and to life in general. In support of this, Sarvestani (2011:432) and Davies (2008:2) posited that characteristically irrational beliefs are rigid and extreme in nature, not consistent with reality, illogical and tend to thwart goal attainment. This means irrational beliefs lack empirical backing and are illogical and/or non-practical (Mousavi & Basavarajappa, 2012:853; Ellis & Dryden, 2003:4; Kaygusuz, 2013:143; Bernard, 2008:2). According to Steel (2007:68), irrational beliefs are a cluster of anxiety-provoking perspectives which are dysfunctional in nature. Irrational beliefs are not utterly psychological disturbances in their own right, but are illogical and/or dogmatic beliefs that have turned into deeply held values as a result of social conditioning and emotionally-oriented reasoning, which subsequently trigger emotional disturbances when an individual's life experiences clash with those beliefs (Heimisson, 2011:2-3). Moreover, irrational belief contains illogical mechanisms of evaluating oneself, other individuals and the entire life in general (Froggatt, 2006:12; Bermejo-Toro & Prieto-Ursua, 2006:89). One attribute of irrational beliefs which is worth emphasising is their pervasive nature. According to David et al (2010:54), irrational beliefs influence people regardless of their intelligence, level of education or culture. Ellis made the assertion that irrational thinking occurs in virtually all people regardless of culture because they are essentially human (Lega & Ellis, 2001:204). The above characteristics of irrational beliefs seem to suggest that irrational beliefs can be detrimental to good Mathematics achievement.

3.2.2.2.2 The four main categories of irrational beliefs

In broad terms, Ellis, in his rational emotive behaviour therapy, considered four main categories of irrational beliefs that are at the centre of psychological disturbances (Sarvestani, 2011:432). The four core irrational beliefs which subsume all the other irrational beliefs are demandingness, awfulising beliefs, low frustration tolerance beliefs, and depreciation beliefs (David et al, 2010:13; Bridges & Harnish, 2010:863). These four categories of irrational beliefs have been used to explore various aspects

of human functioning which include academic achievement, affiliation and comfort (Szentagotai & Jones, 2010:83; David et al, 2010:13).

3.2.2.2.1 Demandingness

According to Ciarrochi (2004:175), demandingness is a category of irrational beliefs within the rational emotive behaviour therapy which is characterised by unrealistic and absolute expectations of individuals or events. It is believed that at the core of all human disturbances lies the tendency of making devout, absolutistic evaluations of perceived events that come in the form of dogmatic statements (Bernard, 2008:2; Corey, 2009:145; Macavei, 2005:74). Demandingness has been identified as the core irrational belief which causes people to express their wishes in the form of peremptory and unqualified commands and demands (David et al, 2010:13; Bridges & Harnish, 2010:863; Austad, 2009:276; Szentagotai & Jones, 2010:83). Bistamam and Nasir (2009:335) also maintained that demandingness is the most important and most pervasive of all the themes of irrational beliefs. Demandingness is characterised by the use of terms such as *must*, *should*, *ought* and *have to* (Ciarrochi, 2004:175; Kosslyn & Rosenberg, 2006:697; Corey, 2009:145). Examples of some forms of demandingness include the demand for approval, success, fairness and justice (Hayes, 1998:209). The absolutistic nature of demandingness has been linked with Karen Horney's 1950 idea of the *tyranny of the shoulds* (Corey, 2009:276). For example, learners may be guided by the belief that they must obtain a high mark in a Mathematics examination (Sava, 2009:137).

3.2.2.2.2 Awfulising

According to Rosner (2011:84) and Bishop (2001:96), awfulising, which can also be called catastrophising, is a way of thinking in which an individual evaluates situations in an exaggerated and negative manner. Awfulising involves viewing events as so horrible that they cannot be imagined and conceived by human beings (Palmer, 2000:279; Cosman, Macavei, Sucala & David, 2013:179; Bridges & Harnish, 2010:863). It is characterised by rating an event as more than catastrophic and more than 100% bad (Macavei, 2005:74; Kosslyn & Rosenberg, 2006:697; Ellis & Dryden, 2003:5; David et al, 2010:13). Catastrophising is also regarded as one of Beck's cognitive distortions (Kottler & Shepard, 2008:107). Some learners believe that they can never comprehend Mathematics and they rate their Mathematics experiences as

awful (Hannula, 2002:32). Believing that failing a Mathematics examination would be the most tragic experience that could ever happen is an example of an awfulising mentality (Sava, 2009:137).

3.2.2.2.3 Low frustration tolerance

Low frustration tolerance, which can also be referred to as frustration intolerance, means the individual believes that happiness is impossible if an unwanted event occurs (Macavei, 2005:74; Palmer, 2000:280). It involves the rating of an event as unbearable and intolerable (Bridges & Harnish, 2010:863; David et al, 2010:14). This means people with low frustration tolerance tend to give up on their goals whenever they meet challenges (Ellis & Dryden, 2003:5; Ciarrochi, 2004:185). According to Bermejo-Toro and Prieto-Ursua (2006:89), low frustration tolerance has been linked to high levels of stress which can negatively influence learners' Mathematics achievement as discussed in section 2.2.2.3.6. Although giving up prematurely may reduce immediate frustration, it is likely to lead to frustration in the future when one misses opportunities to prosper in life (Ciarrochi, 2004:185). Claiming that one cannot in any way stand failing Mathematics is an example of frustration intolerance (Sava, 2009:137).

3.2.2.2.4 Global evaluation of oneself, others and the world

This category of irrational beliefs is characterised by the tendency to label oneself, other people or life events as bad or hopeless whenever failure is experienced (Macavei, 2005:74; Ellis & Dryden, 2003:5; Palmer, 2000:280; Bridges & Harnish, 2010:863). According to Bermejo-Toro and Prieto-Ursua (2006:90), a self-downing mentality is demonstrated by setting unrealistically high standards for oneself, exhibiting a high need for gaining other people's approval and believing that one's value as a person is reduced when one makes a mistake. The wish to gain other people's approval in virtually all situations can lead to self-damnation when one's actions are disapproved of (Ciarrochi, 2004:175). In support of this, Flett et al (2003:120) indicated that both Ellis and Rogers maintained that conditions of self-worth normally lead to cognitive and emotional distress. Viewing oneself as an absolute failure as a result of not passing a Mathematics test is an example of global evaluation of oneself (Sava, 2009:137).

3.3 THEORIES ON THE DEVELOPMENT OF IRRATIONAL BELIEFS

The two main theories discussed in connection with the development of irrational beliefs are Albert Ellis's rational emotive theory and Aaron Beck's cognitive theory. The differences and similarities between these two theoretical orientations are also briefly explored.

3.3.1 Albert Ellis's rational emotive behaviour therapy

Albert Ellis is a cognitive behavioural theorist who abandoned psychoanalysis to develop the rational emotive behaviour theory after being influenced by philosophical ideas (Farley, 2009:215; Bridges & Harnish, 2010:86; Feltham & Horton, 2012:298; Corey, 2009:275-276). According to Feltham and Horton (2012:298), the rational emotive behaviour therapy has been applied in the education fraternity on several occasions. Ellis's theory is anchored on the premise that people's belief systems are normally responsible for triggering negative feelings, not the actual events which people encounter (Ullusoy & Duy, 2013:1441). More details regarding the rational emotive behaviour therapy were discussed in section 1.2. Ellis maintained that human beings have a biological propensity to develop both rational and irrational beliefs with a greater inclination towards irrationality (Bernard, 2008:2; Gunduz, 2013:2081). Ellis claimed that people have an inherent desire to seek happiness while simultaneously trying to avoid pain (Obodo & Obadan, 2008:153). He developed a list of irrational beliefs which are responsible for causing human cognitive, behavioural and emotional dysfunction. Some of the main irrational beliefs suggested by Ellis are discussed in section 3.3.1.3. He later condensed the list of irrational beliefs into four major categories as discussed in section 3.2.2.2.2.

3.3.1.1 Acquisition and maintenance of irrational beliefs

There is sufficient evidence to back the hypothesis that human beings are biologically prewired to develop irrational beliefs (Corey, 2009:276; Ellis, 1998:22; Han, 2011:47). Ciarrochi (2004:175) maintained that virtually all people have a natural tendency to develop dysfunctional irrational beliefs. Bernard (2009:66) posited that Ellis theorised that irrational beliefs have a greater biological basis than rational beliefs. Ellis postulated that 80% of the acclivity to engage in irrational thinking is biological while 20% is environmental (Bernard, 2008:2). Irrational thinking patterns are normally transferred from parents, teachers and members of the society

to which the developing individual belongs (Feltham & Horton, 2012:299; Birjandi, Mahmoudi & Abdolahi, 2012:8719). People's early childhood experiences, especially their interaction with their parents and/or caregivers have been implicated for leading to the acquisition of irrational thinking patterns (Corey, 2009:145; Rosner, 2011:83; Kaygusuz, 2013:143; Obodo & Obadan, 2008:153). Parents who severely condemn their children for their shortcomings in behaviour and recommend that they should be penalised for any mischief expose their children to irrational thinking patterns (Ellis, Wolfe & Moseley, 1966:107; Bernard, 2008:4). Beliefs can emanate from people's personal and direct experiences at home or at school (Kaygusuz, 2013:143; Pierce & Newstrom, 2008: xxix). Irrational tendencies can also be strengthened by peer, school and other social relations (Gunduz, 2013:2083). Moreover, stressful life events play an important role in inducing irrational beliefs (Najafi et al, 2012:314-315). It has also been established that adolescents are vulnerable to developing irrational beliefs because of the volatile and negative emotions associated with their developmental stage (Najafi et al, 2012:314). Furthermore, Najafi et al (2012:313) pointed out that among the many causes of irrational thinking are stressful life events whose effects on scholastic achievement in Mathematics were discussed in section 2.2.2.3

The symbiotic relationship between cognition, emotions and behaviour maintains irrational thinking patterns once they are acquired (Birjandi et al, 2012:8719; Neenan & Dryden, 2010:8). People normally reinforce their irrationalities through internal self-talk and this in turn influences their emotions and behaviour (Obodo & Obadan, 2008:154; Rosner, 2011:83). Regularly confessing one's incompetence in Mathematics can negatively influence one's Mathematics achievement. Furthermore, the way people are socialised in a particular culture can maintain irrational beliefs (Han 2011:46; Kaygusuz, 2013:143; Ellis, 1998:22). Social conditioning and emotionally-oriented reasoning, which subsequently trigger emotional disturbances when an individual's life experiences clash with the person's beliefs have been held responsible for sustaining irrational thinking patterns in human beings (Heimisson, 2011:2-3).

3.3.1.2 Effects of irrational beliefs

Irrational beliefs have been implicated for inducing depression, anger and feelings of inferiority (Ellis, 2007:13; Kaygusuz, 2013:143; Palmer, 2000:281). According to Al-Salameh (2011:137), Ellis (1962) suggested that irrational beliefs are the major factor causing emotional and behaviour disorders. Many cognitive behavioural theorists subscribe to the idea that anxiety is a consequence of dysfunctional beliefs (Palmer, 2000:281; Austad, 2009:275). Ciarrochi (2004:173) claimed that there is now substantial evidence to support the view that dysfunctional attitudes are associated with increased levels of depression and anxiety. Individuals with irrational beliefs manifest negative emotions such as depression, shame, irritability, embarrassment, guilt and hostility (Austad, 2009:275). Irrational beliefs can also hamper learners' ability to cope with stress (Bermejo-Toro & Prieto-Ursua, 2006:89). Failure to cope with stress can negatively influence learners' Mathematics achievement as discussed in section 2.2.2.3.6.

According to Ellis (2003a:247), individuals who regularly hold more intense irrational beliefs normally become more disturbed than those who harbour fewer or less intense irrational beliefs. This implies a positive correlation between the intensity of irrational beliefs and the level of emotional disturbance experienced (Najafi et al, 2012:312). Ample evidence now supports the assertion that there is a strong association between dysfunctional beliefs and low levels of well-being (Ciarrochi, 2004:172; Dryden, 2006:14; Thompson & Henderson, 2007:211; Amutio & Smith, 2007:321). Davies (2008:2) maintains that harbouring irrational beliefs can lead to dysfunctional behaviours and a lack of self-acceptance. According to Najafi et al (2012:312), irrational beliefs mostly lead to self-disturbing behaviours and are frequently associated with poor individual functioning and individual adjustment. People with irrational beliefs mostly have difficulty in solving real life problems (Kaygusuz, 2013:143). Irrational beliefs sometimes trap people in a cycle of self-destructive and self-sustaining arguments which prevent them from dealing with real life challenges realistically and with resilience (Hayes, 1998:210). According to Flett and Hewitt (2008:127-133) and Najafi et al (2012:313), one of the adverse effects of irrational beliefs in adolescents is perfectionism which uniquely contributes to adolescent depressive symptoms (section 3.4.1.6). A strong positive correlation

between the intensity of people's irrational beliefs and their levels of hopelessness has also been established (Ullusoy & Duy, 2013:1441). The above views confirm that irrational beliefs can be detrimental to normal human functioning and can cause people to fail to achieve their goals (Bermejo-Toro & Prieto-Ursua, 2006:89). For secondary school Mathematics learners, such goals will most probably be on attaining an acceptable level of Mathematics achievement.

3.3.1.3 Ellis's main irrational beliefs and their possible influence on Mathematics achievement

The major irrational beliefs advanced by Ellis relative to adolescent secondary school learners are discussed below before the potential influence of each irrational belief on Mathematics achievement is outlined.

3.3.1.3.1 The belief that it is a dire necessity for people to be loved by significant others for almost everything they do

This is the irrational belief which normally comes first on Ellis's list of irrational beliefs. It expresses people's desire to gain the appreciation, love and approval of significant others in everything they do (Austad, 2009:277; Egan, 2007:141). People who are guided by this belief always try to earn the approval and love of other people and avoid disapproval as much as possible (Rosner, 2011:83). This is expressed through the demand for other people's approval and respect for boosting one's feelings of worth (Boyacioglu & Kucuk, 2011:447; Civitci, 2007:3; Cardenoso & Calvete, 2004:289; Woodward, Carless & Findlay, 2001:258). This demand for approval can cause people to measure their personal worth on the basis of their level of achievement (Palmer, 2008:281; Sue, Sue & Sue, 2006: xvii; Froggatt, 2006:4; Alizadeh, 2012:2824). However, viewing the need for other people's approval in all one's actions as an absolute necessity becomes an unhealthy form of demandingness (Froggatt, 2005:3).

Given the developmental turbulence of adolescence which coincides with identity formation as outlined in section 2.2.2.2, adolescent learners seriously yearn for love, recognition and approval from significant others. It is better for Mathematics learners

to concentrate on loving and accepting their parents, peers, and teachers rather than demanding approval. As discussed in section 2.3.3.1, the parenting styles adopted can determine the extent to which adolescent learners gain the approval of their parents. Learners need to develop the ability to seek love and recognition only in cases where such approval is earned through practical and realistic means. The possible influence of this irrational belief is further discussed in sections 3.4.1.3, 3.4.2.2 and 3.4.4.1.

3.3.1.3.2 The belief that certain acts are awful or wicked and that people who perform such acts should be severely damned

According to Austad (2009:277) and Froggatt (2005:3), the above irrational belief is based on the notion that people's behaviours should always be perfect. Any weaknesses in character such as selfishness or impatience are taken to warrant condemnation and punishment (Egan, 2007:141; Rosner, 2011:84). The demand for fairness and justice normally reflects the existence of this irrational belief (Seligman & Reichenberg, 2010:255). This irrational belief can also be linked to conditional other-acceptance as discussed in section 3.2.2.2.4. Instead, it would be better to realise that other people's antisocial conduct can be an indication that they need assistance or counselling to change and behave in socially acceptable ways. Moreover, behaving in a socially deplorable manner does not necessarily render an individual a complete social misfit.

When learners interpret the bad behaviour of everyone around them as utter rottenness of character, it implies that they may have difficulty in accepting the individuality and weaknesses of their peers, teachers and parents. This is likely to strain their interpersonal relationships with significant others (Thompson & Henderson, 2007:211). In a way, having strained relationships with significant others can sometimes have a negative effect on one's academic performance in Mathematics. The influence of this irrational belief on Mathematics achievement is explored again in sections 3.4.4.1 and 3.4.4.2.

3.3.1.3.3 The belief that it is horrible and catastrophic when things are not the way we like them to be

The above belief reflects an idealistic mentality based on demandingness and awfulising (Rosner, 2011:84). It is based on the wish to have situations as good as one imagines and the declaration that one cannot survive any unexpected challenges in life (Froggatt, 2005:3; Austad, 2009:277). People who hold this belief expect to see their plans always materialising (Egan, 2007:141; Froggatt, 2006:4). This belief is sometimes reflected in the demand for comfort (Civitci, 2007:3; Boyacioglu & Kucuk, 2011:447). The above irrational belief is to a large extent so idealistic that one is bound to be habitually frustrated when one's wishful thinking does not materialise. Instead of awfulising, it would be better to try to grapple with the situation so as to change it for the better. In situations where the variables are beyond one's control, one has to tolerate the situation as it is.

If secondary school Mathematics learners fail to achieve scores which they were expecting to attain, they may be frustrated even though the obtained marks may not be poor. If a Mathematics learner who was expecting to get recognition from the teacher regularly fails to achieve such recognition, the learner may resent the teacher and Mathematics as a subject thereby jeopardising his or her chances of succeeding in the subject.

3.3.1.3.4 The belief that human misery is invariably externally caused and is forced on us by outside people and events

Having an external locus of control regarding the source of our misery is another irrational belief propounded by Ellis (Austad, 2009:277). People who are influenced by the above irrational belief attribute all their unhappiness to other people and variables beyond their control (Rosner, 2011:84; Egan, 2007:141). Such people believe that they cannot do anything to allay their unhappiness (Froggatt, 2005:3). Instead of adopting such an external locus of control, it would be more empowering to realise that the way we interpret the events of life may determine our levels of anxiety and misery. According to Beck (2008:970), human anxiety and depression are mostly triggered by people's erroneous information processing. Adopting a positive and tolerating view of other people and events can yield positive outcomes.

Having an external locus of control can cause some Mathematics learners not to exhaust their intellectual potential since they believe that there is no hope to improve no matter how much effort they exert (section 3.4.1.2.1). Mathematics by its very nature requires regular practice on the part of the learners especially at secondary school level. Learners who have an external locus of control normally have low self-efficacy levels since they view the task to be undertaken to be beyond their reach (Steel, 2007:69). Such learners are likely to shift the blame for their failure to other stakeholders such as parents, siblings, teachers or even their peers (sections 3.4.1.1.1, 3.4.1.4 and 3.4.4.2).

3.3.1.3.5 The belief that if something is or may be dangerous or fearsome we should be terribly upset and endlessly obsess about it

Harbouring the notion that one should be terribly upset and perpetually obsessed about a phenomenon that may be fearsome or dangerous is another irrational belief put forward by Ellis (Froggatt, 2005:4; Froggatt, 2006:4). Subscribing to this belief causes one to worry needlessly about situations or events one perceives as challenging or dangerous (Austad, 2009:277; Egan, 2007:141). Moreover, the belief can also trigger feelings of helplessness (Cardenoso & Calvete, 2004:289). It would be better to confront the problem in order to reduce the danger and the fear it poses. In cases where one cannot bring about a desirable change, it is better to accept it as it is (Austad, 2009:277).

Mathematics, by its very notorious nature, can be viewed by some learners as a threatening or a frightening subject which is too demanding (Saraswathi, 2003:326; Asikhia, 2010:206). Continually obsessing about the somewhat intricate nature of Mathematics is not likely to make it less frightening or less challenging. This excessive obsession is likely to lead to hopelessness which can in turn erode one's self-efficacy (Steel, 2007:69; Ullusoy & Duy, 2013:1441). Mathematics learners should try to meet the intellectual demands of the subject so as to pass it or to make it less frightening. In cases where some concepts remain elusive, they should accept the situation and concentrate on the topics which they can manage. The influence of this irrational belief on Mathematics achievement is further examined in sections 3.4.1.4 and 3.4.3.1.

3.3.1.3.6 The belief that it is easier to avoid than to face life difficulties and self-responsibilities

Individuals who are led by this belief deliberately try to evade challenges as well as personal responsibilities (Froggatt, 2005:4; Froggatt, 2006:4; Austad, 2009:277). Problem avoidance can be attributed to this irrational belief (Cardenoso & Calvete, 2004:289; Egan, 2007:141). It can be linked to Freudian hedonism in which people are assumed to have an inherent desire to seek pleasure and avoid pain (Rosner, 2011:84). This irrational belief reflects the demand for comfort as outlined in the Irrational Beliefs Scale for Adolescents (Civitci, 2007:3). Task avoidance normally leads to procrastination. Although procrastination and inaction may seem pleasurable, they are likely to yield undesirable outcomes. The influence of procrastination on Mathematics achievement is elaborated in detail in section 3.4.1.7.

The belief is likely to persuade Mathematics learners who view Mathematics as a difficult subject to shun the subject to the extent of almost dropping it (Asikhia, 2010:206). Avoiding taking responsibility such as homework; organising study groups; seeking additional assistance from Mathematics teachers and more competent peers; researching in libraries; and seeking past examination papers to use during revision sessions may compromise one's chances of doing well in Mathematics.

3.3.1.3.7 The belief that we absolutely need something other or stronger or greater than ourselves on which to rely

This belief suggests that people can only achieve their goals when they depend on more competent people (Austad, 2009:277; Froggatt, 2006:4). The belief emphasises that it is an absolute necessity for one to rely on other people for protection and guidance (Froggatt, 2005:3; Rosner, 2011:84). Instead of adopting such a dependent mentality, it can be more beneficial to believe that it pays to be an assertive risk-taker who acts with a reasonable degree of autonomy and independence.

Absolute dependence on other people may not be conducive to good individual academic achievement. While it is arguably true that a learner can be helped to tackle mathematical problems by more competent peers, teachers, siblings and even parents as suggested by Vygotsky's sociocultural theory, this should only be done to help the learner to master the concepts to the extent of eventually solving the problems independently (sections 2.2.1.2.2 and 2.3.2.2). The individual learner must learn to solve some mathematical problems independently and should only need the assistance of others to explore concepts in the zone of proximal development. Mathematics learners who always want to rely on more competent individuals jeopardise their chances of developing reasonable levels of confidence, self-efficacy, self-esteem and problem-solving skills. This irrational belief associated with dependence is elaborated on again in sections 3.4.1.1.1, 3.4.1.5, 3.4.4.1 and 3.4.4.2.

3.3.1.3.8 The belief that we should be thoroughly competent, intelligent, and achieving in all possible respects

The above belief places excessive emphasis on thorough competence, intelligence and being always able to undertake any tasks perfectly (Thompson & Henderson, 2007:210; Austad, 2009:277; Egan, 2007:141). The belief expresses high self-expectations and the demand for success (Cardenoso & Calvete, 2004:289; Civitci, 2007:3; Boyacioglu & Kucuk, 2011:447) and is one of the root causes of perfectionism (Sue et al, 2006: xxiv). An example of such a belief is the resolution that one must always outdo one's classmates in Mathematics (Sue et al, 2006: xxiv). It would be better to believe that we are differently endowed and we are bound to be imperfect and fallible in some aspects of our lives. The belief denies the fallible nature of human beings (Domino, 2005:7; Ellis, 1994:130).

As a result of the diverse nature of the topics in the Mathematics syllabuses at secondary school level, it is common even for intelligent learners to have difficulties in mastering some topics. Encountering a few hitches in solving some mathematical problems may not be sufficient for learners to rate themselves as incompetent or dull. Holding the above irrational belief can lead to poor Mathematics achievement when intelligent learners become frustrated, stressed and/or depressed as a result of failing to solve a few problems. The impact of perfectionism on secondary school

learners' academic achievement in Mathematics is discussed at length in section 3.4.1.6.

3.3.1.3.9 The belief that because something once strongly affected our life, it should indefinitely affect it

This belief suggests that past tragedies and challenges will perennially influence individuals who once encountered them (Thompson & Henderson, 2007:21; Austad, 2009:277; Froggatt, 2006:4). People who are led by this belief do not have any hope of improving in future even if the variables which led to their initial failure change (Rosner, 2011:84; Froggatt, 2005:5). Such a phenomenon has been viewed by Egan (2007:141) as the tyranny of the past. Chiesi and Primi (2010:19) argue that previous Mathematics achievement has a strong influence on future academic performance. As elaborated in section 2.2.1.5, learners can use their previous achievement to rate their current and future Mathematics achievement (Sternberg, 2009:341; Snowman et al, 2009:274). However, people are likely to achieve more in life if they believe that their past experiences can serve as an impetus to do better in the present and in future. Unfortunate past experiences should not be used as evidence that one can never succeed in the present or in future.

While acknowledging that past achievement is important in determining achievement at a later stage, Carpraro, Young, Lewis, Yetkiner and Woods (2009:49) claimed that it is possible for learners to overcome past failures and master mathematical concepts just like their initially well-positioned counterparts. There are many reasons why people can improve in a domain in which they previously encountered difficulties. For instance, progression from one stage of cognitive development to the next can increase one's intellectual capacity to handle mathematical problems. Learners who have reached the formal operational stage as postulated by Piaget virtually become capable of solving problems better than the way they used to attempt them when they were at the concrete operational stage (section 2.2.1.2.1). Moreover, positive changes in home, school, and personal factors such as better study habits and motivation as well as ability to handle stress and anxiety, can enable a learner to overcome the negative effects of past failures in Mathematics.

This irrational belief is further discussed in sections 3.4.2.2.1, 3.4.1.3, 3.4.1.4, 3.4.3.2 and 3.4.4.2.

3.3.1.3.10 *The belief that we must have certain and perfect control over things*

According to Ellis, it is irrational to have perfect control over various life situations and variables (Austad, 2009:277; Egan, 2007:141). The use of the word *must* indicates the peremptory and unrealistic expectations of the demandingness of such individuals. It would be more realistic to acknowledge that there are many variables which affect people in life some of which are beyond their control. Real life events are probabilistic in nature and uncertainty is part and parcel of how people live (Austad, 2009:277). Learners who want to be utterly in control of all the factors which influence their academic efforts to the extent of being certain of their perfect performance may be frustrated at some point. It may not be practically possible to be in control of some variables which influence achievement in Mathematics. For instance, learners may not choose teachers for themselves and they may not influence the sequence in which Mathematics concepts are presented to them. Looking for absolute control and certainty is almost impossible given the challenging nature of Mathematics.

3.3.1.3.11 *The belief that human happiness can be achieved by inertia and inaction*

The notion that human happiness can be achieved by inertia and inaction can lead to procrastination (Austad, 2009:277). Holding such a belief can lead to passivity and lack of commitment (Egan, 2007:141). It can be attributed to people's inherent quest to avoid pain and discomfort (Froggatt, 2005:3; Thompson & Henderson, 2007:211). According to Ulusoy and Duy (2013:1441), inertia can lead to anxiety which can negatively impinge upon Mathematics achievement. It is more rational and more fulfilling for learners to engage in activities which can produce a positive difference in their Mathematics achievement.

Inertia, which is the reluctance or inability to act, when adopted by Mathematics learners is likely to lead to poor academic achievement. Mathematics can be viewed as a practical subject in the sense that it requires regular exercises so as to sharpen

the concerned individual's problem-solving skills (Riley, 2007:125). Upon encountering some mathematical problems, learners should devise ways of improving their skills. Inactivity or evasive efforts are to a large extent likely to militate against scoring high marks in Mathematics especially at secondary school level where a variety of difficult concepts are taught. Task avoidance can lead to procrastination whose influence on Mathematics achievement is explored in depth in section 3.4.1.7.1. To increase one's probability of performing well in Mathematics, one has to be assertive, active and willing to expend intellectual energy in an attempt to master concepts.

3.3.1.3.12 The belief that we have virtually no control over our emotions and that we cannot help feeling disturbed about things

This irrational belief which is related to the affective domain is the notion that we have virtually no control over our emotions and that we cannot help feeling disturbed about things (Austad, 2009:277). Failure to exercise control over one's emotions is likely to lead to loss of control over one's cognition and behaviour since emotions, behaviour and cognition are intertwined (Neenan & Dryden, 2010:8). Moreover, this form of emotional irresponsibility can foster learned helplessness (Woodward et al, 2001:258). Emotional irresponsibility can cause people to be neurotic as described in section 2.3.3.4.2. People who declare that they have no control over their emotions are likely to remain vulnerable to stress and anxiety. It is better to be guided by the belief that it is practically possible to control our affective domain by deliberately changing the way we think. This can be achieved through curbing our self-inflicted emotional disturbance which Ellis has named masturbation. The influence of anxiety on Mathematics achievement was discussed in section 2.2.2.4 and is further explored in section 3.4.1.4.

3.3.1.3.13 The belief that there is invariably a right, precise and perfect solution to human problems and it is catastrophic if this perfect solution is not found

This irrational belief emphasises the need to obtain a perfect and ideal single solution to every challenge one encounters in life (Ellis, 1994:129; Froggatt, 2005:3; Rosner, 2011:84). In cases in which such a solution is not found, the belief suggests that one should view the situation as catastrophic (Froggatt, 2006:4). This irrational

belief is one of the root causes of perfectionism on the part of learners in particular and all human beings in general. The absolutistic and perfectionistic nature of this belief is likely to lead to frustration which can subsequently trigger negative emotions in the learners. Negative emotions such as depression, distress and anxiety have been found to be detrimental to Mathematics achievement (Scarpello, 2007:34).

Real life challenges encountered by learners at home or at school may have a variety of possible solutions depending on the manner in which the problems are interpreted (Alberta Education, 2008:2). Moreover, solutions to some problems may not be immediately available. In situations in which one cannot find an immediate solution, one has to continue trying without necessarily viewing the situation as hopeless. There are some situations in which one has to improvise because the right and perfect solution to a problem cannot be found. For example, learners who are taught by an incompetent Mathematics teacher may improvise through organising their own study groups.

3.3.2 Beck's Cognitive Theory

Beck's cognitive theory has three fundamental principles; namely, cognitive schemas, the cognitive triad and cognitive distortions (Najafi & Lea-Baranovich, 2011:1447).

3.3.2.1 Description of cognitive schemas

Beck claimed that painful early childhood experiences trigger the development of negative attitudes, beliefs and biases towards life (Corey, 2009:274; Beck, 2008:970). The most crucial early childhood experiences are the ones involving interaction with parents or other caregivers. The negative biases, attitudes and beliefs combine to form cognitive schemas. According to Friedman, Thase and Wright (2008:1922), cognitive schemas are the underlying assumptions which an individual uses to process information from the environment. An individual's cognitive schemas can comprise the individual's perceptions of self, others and the world, the person's future goals, memories, assumptions and beliefs. This means cognitive schemas are relatively stable cognitive patterns that stem from an individual's beliefs, attitudes and behavioural responses (Friedman et al, 2008:1921; Froggatt, 2006:4).

Any later allied negative event activates these schemas as a result of automatic thinking. Automatic thoughts are cognitions that rapidly permeate a person's mental awareness spontaneously or after being triggered by an external event (Neenan & Dryden, 2010:5; Friedman et al, 2008:1921; Beck, 2008:972). Automatic thoughts are part of an individual's spontaneous internal monologue and can be in the form of words, pictorial images or both which precede emotions and behaviour (Neenan & Dryden, 2010:4). Such a manner of information processing makes the individual vulnerable to depression and anxiety (McGinn, 2000:257; Najafi & Lea-Baranovich, 2011:1446). Depression has the potential to impinge on the adolescents' overall well-being, interpersonal relationships and scholastic achievement (McCarthy, Downes & Sherman, 2008:49). Poor achievement in Mathematics at one point can lead to depression and anxiety which can lead to future poor Mathematics achievement because of errors in the way the learners evaluate their circumstances.

3.3.2.2 Description of a cognitive triad

According to Friedman et al (2008:1922), Beck used the term *cognitive triad* to describe the content of automatic negative thoughts pertaining to the self, the future and the world. A negative cognitive triad can cause learners to have negative views about their mathematical ability, the way they are rated by others relative to Mathematics achievement and negative views about the future attributable to perceived poor mathematical competence as shown in Figure 3.1. Such a negative cognitive triad can trigger poor Mathematics achievement as a result of anxiety and lack of motivation (sections 2.2.2.4.4 and 2.2.2.1 respectively).

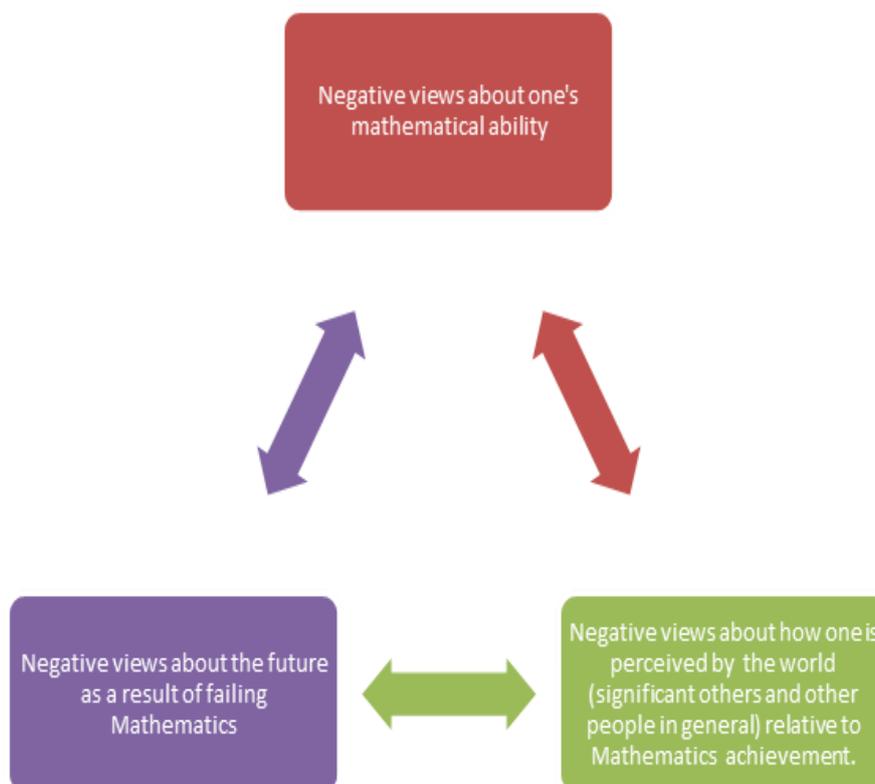


Figure 3.1: Cognitive triad in a Mathematics learning context

3.3.2.3 Description of cognitive distortions

Cognitive distortions are cognitive errors in information processing which reflect an individual's automatic thoughts (Corey, 2009:274). They are a consequence of faulty schemas. Friedman et al (2008:1922) claim that Beck and his colleagues theorise that cognitive distortions, which can also be called cognitive errors, are stereotypic errors in logic which shape the content of automatic thoughts. Beck postulates that cognitive distortions cause and maintain dysfunctional thinking.

3.3.2.4 Beck's cognitive distortions and their possible influence on Mathematics achievement

Beck developed a number of cognitive distortions which can interfere with the individual's psychological operations as a result of depression. Some cognitive distortions are discussed below relative to their possible influence on secondary school learners' Mathematics achievement.

3.3.2.4.1 Dichotomous thinking

This is a cognitive distortion which is characterised by thinking in extremes in such a way that people rate themselves as either utterly perfect or a total failure (Kosslyn & Rosenberg, 2006:698; Wendland, 2004:7). Dichotomous thinking involves viewing a situation in only two mutually exclusively categories rather than a continuum (Knapp & Beck, 2008:57). Such individuals adopt an 'all or nothing' thinking pattern (Bridges & Harnish, 2010:863; Neenan & Dryden, 2010:4; Corey, 2009:289). Dichotomous thinking is also called dualistic thinking, polarised thinking or 'black and white' thinking (Bernard, 2008:2; Froggatt, 2006:3; Kottler & Shepard, 2008:107).

In a Mathematics context, learners with dualistic thinking patterns may rate themselves either as perfectly competent in Mathematics or utterly incompetent. They tend to view moderate performance as substandard thereby denying themselves the opportunity to improve. Moreover, such learners may not realise that passing Mathematics does not necessarily imply getting everything correct since there are varying grades of passing. At secondary school level, getting all Mathematics assignments continually correct is practically impossible even for bright learners. Therefore, learners who engage in dualistic thinking may give up after having minor Mathematics challenges.

3.3.2.4.2 Magnification and Minimisation

According to Corey (2009:289), magnification and minimisation occur when a person evaluates situations either as far more important or less important than what they really are. While magnification means amplifying a negative situation more than is really necessary, minimisation occurs when one diminishes the positive aspects of a situation (Bridges & Harnish, 2010:863; Knapp & Beck, 2008:57).

Secondary school learners with a magnification and minimisation thinking pattern tend to get depressed because of failing to solve some mathematical problems.

Rather than being satisfied with being successful in some areas, such learners tend to be discouraged by the Mathematics tasks which they had failed to solve. Such a way of thinking is likely to affect their Mathematics achievement negatively through having a low level of self-efficacy (Steel, 2007:69).

3.3.2.4.3 Personalisation

Personalisation is a cognitive error which occurs when people hold themselves responsible for causing unfortunate events (Wendland, 2004:7). Personalisation also occurs when people believe that whatever other people do should be seen as a deliberate attack on them (Froggatt, 2006:3).

In the classroom context, Mathematics learners who are prone to personalisation may interpret the teacher's general remarks as a personal attack on them specifically. Such learners are very sensitive to the actions or statements of their teachers and peers. This can negatively influence their Mathematics achievement.

3.3.2.4.4 Overgeneralisation

Overgeneralisation occurs when an individual draws a broad and sweeping conclusion about all situations on the basis of a single and probably trivial incident (Wendland, 2004:7; Bernard, 2008:2; Froggatt, 2006:3). Such people tend to erroneously believe that if something unfortunate happens, then it is heraldic of future tragedies since the event is likely to recur.

At secondary school level, there are many topics in the Mathematics syllabus which include geometry, trigonometry, statistics, algebra and mensuration. Performing poorly in a Mathematics test focusing on a single topic does not automatically imply that one will definitely perform dismally in the other topics. Moreover, given the high number of Mathematics exercises given to learners at secondary school level, it is to a large extent unfortunate to evaluate one's academic ability on the basis of one's performance in a single exercise. Hence overgeneralisation can negatively impinge on learners' Mathematics achievement when they do not realise that one can

ultimately pass the examination even after encountering some hitches along the way.

3.3.2.4.5 *Should statements*

This is a cognitive distortion which is also called wishful thinking (Knapp & Beck, 2008:57). An individual tries to live under the guidance of definite rules which are expected to govern the behaviour of the individual and other people. Instead of facing the reality of the situation, the individual obsesses over what ought to prevail (Knapp & Beck, 2008:57). According to Kottler and Shepard (2008:107), *the should statements* theorised by Beck are equivalent to the masturbatory statements theorised by Ellis. Mathematics learners who rely on these imperative statements may wish to score high marks in Mathematics always. In cases in which such marks cannot be attained, learners may become anxious and depressed, thereby further jeopardising their chances to do well in Mathematics.

3.3.2.5 Comparison of Ellis's irrational beliefs and Beck's cognitive distortions

Ellis's rational emotive behaviour therapy and Beck's cognitive therapy are two mature and popular cognitive therapeutic models which have some similarities (Austad, 2009:275; Seligman & Reichenberg, 2010:252). Ellis and Beck are both former psychoanalysts who proceeded separately to develop allied theories which fall under the cognitive behavioural paradigm (Heimisson, 2011:14; Knapp & Beck, 2008:56; Froggatt, 2006:1). Moreover, both Ellis and Beck are progenitors of cognitive behavioural therapy (Najafi & Lea-Baranovich, 2011:1446; Knapp & Beck, 2008:56). Both Ellis and Beck agree that human maladjustment is a consequence of defective belief systems rather than objective causes (Gazzaniga & Heatherton, 2006:570; Bridges & Harnish, 2010:863). Another area of theoretical harmony between Ellis and Beck is that they both concur that people's biological inclinations influence their behaviour and emotions (Froggatt, 2006:2). According to Kaygusuz (2013:143), there is a relationship between irrational beliefs and cognitive distortions. The cognitive distortions theorised by Beck are actually special irrational beliefs

which emanate from an individual's consistent bias in information processing (Kosslyn & Rosenberg, 2006:698). Asikhia (2010:207) maintained that both irrational beliefs and cognitive distortions can cause perfectionism and procrastination which in turn lead to low self-efficacy and neurotic task avoidance.

There are similarities between some of the irrational beliefs propounded by Ellis and some cognitive distortions suggested by Beck. Overgeneralisation postulated by Beck is to some extent similar to Ellis's irrational belief that people should be indefinitely affected by something which once affected them in the past. Demandingness which Ellis described as the core irrational belief is similar to *the should* statements theorised by Beck (Kottler & Shepard, 2008:107). Dichotomous thinking as a cognitive distortion overlaps with the irrational beliefs pertaining to demandingness and perfectionism. According to Hayes (1998:209), damnation of oneself, other people and the world as theorised by Ellis is similar to Beck's cognitive triad which focuses on the individual, the world and the future. As indicated in section 3.2.2.2.2, awfulising advanced by Ellis has been postulated by Beck as a cognitive distortion called catastrophising (Kottler & Shepard, 2008:107).

However, Beck and Ellis differ in the way they understand irrational beliefs and their impact on psychopathological maladjustment (Bridges & Harnish, 2010:863). While Ellis's theory is more inclined to particular thoughts that generate emotional dysfunction, Beck's theory emphasises the thought processes that actually trigger emotional distress (Bridges & Harnish, 2010:863). Beck's cognitive distortions are inferential while most irrational beliefs theorised by Ellis are evaluative (Froggatt, 2006:2). Moreover, Beck criticised Ellis for using the term *irrational* instead of describing beliefs as absolutistic and extreme (Corey, 2009:290).

3.4 IRRATIONAL BELIEFS IN A SCHOOL CONTEXT

Since most teaching and learning activities take place in the school setting, irrational beliefs which can influence Mathematics achievement in the school context are now explored. Reference is also made to some of the school factors which influence Mathematics achievement as outlined in section 2.3.3.

3.4.1 Irrational Beliefs Related to the Learner

A number of learner factors are outlined below in conjunction with irrational beliefs and Mathematics achievement.

3.4.1.1 Gender

The concept of gender is explored first before reference is made to the influence of gender and irrational beliefs on Mathematics achievement.

3.4.1.1.1 Gender issues and Mathematics achievement

According to Wedege (2011:6) and Crawford (2006:26), gender refers to the masculine and feminine characteristics which are ascribed to males and females by members of a particular society guided by their culture. Gender has been found to be one of the key factors which influence learners' beliefs, motivation and determination to learn Mathematics (Palsdottir, 2007:118). The association between Mathematics achievement and gender has been analysed by various researchers in different parts of the world (Grootenboer & Hemmings, 2007:6). Many studies undertaken over the past decades have established variability in Mathematics achievement as a result of gender (Penner, 2008:S138; LaLonde, Leedy & Runk, 2003:286). A sizeable number of empirical researches have documented that boys have more interest in Mathematics than girls (Watt, 2004:1556; Frenzel, Goetz, Pekrun & Watt, 2010:508; Hannula, 2002:25). This is backed by Palsdottir, 2007:117) who reported that girls' interest in Mathematics tends to decline during adolescence. According to Grootenboer and Hemmings (2007:6), several studies undertaken in Australia and New Zealand reveal that the number of girls who viewed themselves as good at Mathematics was significantly lower than that of boys. Moreover, Frenzel et al (2010:511) argued that a substantial number of studies confirmed that by adolescence, boys are more intrinsically motivated to perform well in Mathematics than girls. However, Dreber, Von Essen and Ranehill (2012:1) contended that recent studies noted that the gender gap in choosing to compete in mathematical activities is dwindling and is no longer significant where there is a control for relative performance beliefs. This is supported by Abubakar and Adegboyega (2012:122) who posited that several studies found no significant gender differences in Mathematics achievement. Some recent researches have actually established that girls are now outclassing boys in Mathematics (Aunio, 2006:10;

Mubeen, Saeed & Arif, 2013:41). In the current study, gender issues are explored relative to irrational beliefs and their ultimate influence on Mathematics achievement.

Secondary school female learners are likely to be influenced by the irrational belief that *we absolutely need something other or stronger or greater than ourselves on which to rely*. This is so because of the patriarchal nature of the society in which they are being nurtured to believe that their male counterparts are more mathematically gifted and capable than they are (Haylock & Thangata, 2007:77; Haylock, 2006:3). Consequently, some girls may irrationally believe that they need to depend on their male counterparts to do well in Mathematics. Such a belief is likely to weaken the female learners' self-efficacy thereby negatively affecting their Mathematics achievement. On the other hand, learners who have high levels of self-efficacy may do well in Mathematics independent of gender (Wigfield & Eccles, 2002:71).

Female learners who are influenced by the irrational belief that *human misery is almost always caused and imposed on them by outside people and events* may believe that they are being compelled by their parents and teachers to study Mathematics, a subject which is traditionally believed to be a male domain (Haylock & Thangata, 2007:76; Haylock, 2006:3). Such an approach may compromise their Mathematics achievement. However, female learners who view Mathematics as a useful subject may study it without complaining thereby enhancing their chances of doing well.

3.4.1.1.2 Research on gender, irrational beliefs and Mathematics achievement

Bojman, Duggan and Mezo (2011:1) undertook a study to examine gender differences in irrational beliefs among undergraduate students. A sample of 410 undergraduate students took part in the study. The research revealed that the male respondents endorsed irrational beliefs significantly less than their female counterparts. It was deduced from the research that female undergraduate students are more vulnerable to having irrational beliefs than their male counterparts.

A longitudinal research exploring the variables which relate to female adolescent learners' participation in Mathematics at secondary school level was undertaken by Watt, Eccles and Durik (2006:642). Two samples comprising of adolescents from coeducational schools were used in the research. The first sample from the United States of America had 266 respondents while the other sample with 459

respondents was from Sydney in Australia. The study revealed that in the Australian context, more boys chose Mathematics at higher levels than their female counterparts. Moreover, it was established that in Australia, the decision to pursue Mathematics at higher levels was influenced by self-efficacy beliefs and interest in the subject while the adolescents' previous Mathematics achievement was a peripheral determinant. On the other hand, the decision to choose Mathematics at higher levels in the USA was found to be independent of gender. However, self-efficacy beliefs influenced female respondents' decisions to pursue Mathematics in the USA more strongly than their male counterparts.

LaLonde et al (2003:285) conducted a study to examine the attitudes of male and female learners, their teachers and parents towards taking part in a regional Mathematics competition. A sample of 74 mathematically talented learners, 133 parents, 35 Mathematics coaches and 14 Mathematics teachers participated in the study. It was revealed that even the mathematically endowed learners continued to hold the traditional gender stereotyped beliefs pertaining to Mathematics. The contributions of the different groups of respondents confirmed the existence of the negative influence of the belief that Mathematics is a male domain despite the high levels of motivation and mathematical intellectual endowment of the female learners.

3.4.2.2 Intelligence, aptitude and academic performance

The implicit theory of intelligence advanced by Dweck is relevant to the current study because it portrays the learners' beliefs regarding the nature of intelligence. As part of her self-theories, Dweck postulated the implicit theory of intelligence with two frameworks; namely, the entity theory of intelligence and the incremental theory of intelligence (Blackwell et al, 2007:247). According to Blackwell et al (2007:259), learners' beliefs about their intelligence can influence their levels of motivation.

3.4.2.2.1 Entity theory of intelligence

Individuals who are influenced by the entity theory of intelligence believe that their talents, abilities and intelligence are permanently fixed and will remain constant regardless of how much effort they exert (Stipek, Givvin, Salmon & MacGyver, 2001:215; Petty, 2009:510). This fixed mind-set is to some extent pessimistic since such people do not optimally use learning opportunities (McLean, 2003:31). Failure

at any point is interpreted as an indelible indicator of inability. This corresponds with the irrational belief that past setbacks indefinitely affect people for the rest of their lives. Owing to lack of confidence and assertiveness, learners who believe in the entity theory of intelligence tend to over-depend on their teachers for feedback in the form of praise and grades (McLean, 2003:31).

Learners who subscribe to the entity theory of intelligence believe that tasks should be accomplished easily without applying any sustained effort to prove that one is naturally intellectually gifted. They believe that learning should be perfectly easy and hassle-free (Stipek et al, 2001:215). This mentality seems to be based on the irrational belief that *people should be extremely competent, intelligent and achieving in virtually all aspects*. This is backed by Stipek et al (2001:215) who posited that demandingness is sometimes a consequence of believing in the entity theory of ability. Such learners feel doomed by their mistakes rather than viewing their mistakes as an opportunity to learn and subsequently improve (Stipek et al, 2001:215).

During moments of adversity, people who subscribe to the entity theory of intelligence easily concede defeat and accept failure amid feelings of helplessness, depression and anxiety. Failure is blamed on everyone and everything, including themselves. As already indicated, the hallmark of irrationality revolves around absolute demands which trigger extortionate self-criticism, damnation of other people and global evaluation of life circumstances (Kosslyn & Rosenberg, 2006:697; Seligman & Reichenberg, 2010:255). As a result of their passive and pessimistic view of life, the Mathematics achievement of learners who believe in the entity theory of intelligence is likely to be poor.

3.4.2.2 Incremental theory of intelligence

People who follow the incremental theory of intelligence believe that their intelligence can be nurtured and developed during their lifespan through exerting persistent effort and learning (Stipek et al, 2001:215). While acknowledging that people may learn at different rates, the incremental theory of intelligence claims that the greater the effort people exert the more talented and competent they are bound to become. According to Sternberg (2009:418), research has confirmed that human intelligence is malleable regardless of one's intellectual endowment. Uttal (1997:170) contended

that Mathematics as a subject requires persistent effort even when learners face intermittent setbacks.

People whose beliefs are oriented towards the incremental theory of ability are mastery-oriented, that is, they strive to acquire new skills and master new tasks as a way of improving their abilities (McLean, 2003:31). Such people believe in exerting effort for the sake of perfecting their skills and abilities. They also study hard using various study techniques such as distributed practice, the SQ4R method or the MURDER approach as outlined in section 2.2.3.1. Learners who are guided by the incremental theory of intelligence courageously attempt to deal with challenges and difficulties. Such learners are likely to seek the assistance of teachers when they are stuck and thereafter continue to work independently. They believe that it is possible for them to achieve success at some point in the future as a result of the effort they are applying. Their optimistic mentality when confronted by challenges tends to lead to an improvement in their scholastic achievement as a result of regular practice as confirmed by Sternberg (2009:419). Consequently, it can be argued that learners who believe in the incremental theory of intelligence have a high chance of doing well in Mathematics.

3.4.1.2.3 Research on intelligence, aptitude, irrational beliefs and Mathematics achievement

Da Fonseca, Cury, Santos, Payen, Bounoua, Brisswalter, Rufo, Poinso and Deruelle (2009:213-222) undertook a study to determine the extent to which Mathematics achievement correlates with the belief that intelligence is a fixed trait. A sample of 353 French volunteer adolescent learners in the 11 to 16 year age range participated in the study (Da Fonseca et al, 2009:213-222). The learners' scores in Mathematics were used to reflect their academic performance. It was established that the entity theory of intelligence negatively correlates with Mathematics achievement and positively with depression. The study revealed that learners who view their intelligence as an unchangeable entity are more vulnerable to experience depression which subsequently hampers their academic performance especially during adolescence and childhood (Da Fonseca et al, 2009:213-222).

In an attempt to establish the impact of learners' beliefs on the malleability of intelligence on their Mathematics achievement, Blackwell et al (2007:248) explored

junior high school learners and their beliefs about the nature of intelligence (Sternberg, 2009:419). A sample of 373 respondents comprising 175 males and 198 females took part in the research (Blackwell et al, 2007:248; Sternberg, 2009:419). The research revealed that the belief in the malleability of intelligence predicted a significant improvement in the respondents' Mathematics achievement (Sternberg, 2009:419; Blackwell et al, 2007:253). The respondents who believed in the incremental theory of intelligence did not lean towards helplessness and they did not attribute their failure to lack of ability (Blackwell et al, 2007:250). On the other hand, learners who subscribed to the entity theory of intelligence did not register any significant improvement in their Mathematics achievement over the same period and they expressed feelings of helplessness (Blackwell et al, 2007:250; Sternberg, 2009: 419).

To further ascertain the importance of believing in the malleability of intelligence, Blackwell et al (2007:253) experimented with another sample of 99 secondary school learners comprising 50 males and 49 females whose Mathematics achievement was declining (Sternberg, 2009:419). The experimental group received information on the changeability of intelligence while such information was not given to the control group (Sternberg, 2009:419; Blackwell et al, 2007:253). It was observed that the Mathematics achievement of learners in the experimental group stopped declining and their self-determination improved (Blackwell et al, 2007:258). The Mathematics achievement of the learners in the control group continued to deteriorate. According to Sternberg (2009:419), these studies support the possibility that apart from being malleable, intelligence can also be significantly influenced by learners' beliefs and expectations.

3.4.1.3 Self-concept

Self-concept has been identified as one of the factors which can influence secondary school learners' Mathematics achievement (section 2.2.2.2). The first irrational belief outlined by Ellis can be linked to self-concept. Learners who always seek approval, recognition and acceptance from others may have a distorted self-concept when some of their actions are disapproved of or ignored by significant others. As suggested by Rogers, imposition of conditions of worth by significant others may lead to the development of a distorted self-concept (section 2.2.2.2.4). Such

learners' self-concept formation will be under threat resulting in low Mathematics achievement.

Self-concept can also be influenced by the irrational belief that one will always be affected by past events and situations which once affected one. Viewing oneself as a victim can lead to complacency and despair which can lead to underachievement in Mathematics. A distorted self-concept can interfere with Mathematics achievement.

On the other hand, if learners view themselves as survivors of past disasters, they can develop positive self-concepts which can boost their self-efficacy levels. As postulated by Santrock (2004:226), high levels of self-efficacy normally cause a person to exert more effort in a bid to accomplish a given task. Hence such learners are likely to do well in Mathematics. Therefore, the nature of the irrational beliefs held by some learners may not have any influence on their self-concept which ultimately influences their Mathematics achievement as discussed in section 2.2.2.2.6

3.4.1.3.1 Research on self-concept, irrational beliefs and Mathematics achievement

A study exploring the fluctuations in secondary school learners' competence beliefs, utility value and achievement goals in Mathematics was undertaken by Chouinard and Roy (2008:31). The research also considered the respondents' gender and academic year. A sample of 1130 learners from 18 secondary schools participated in the study (Chouinard & Roy, 2008:31). The study revealed that there was a significant decline in learners' competence beliefs, utility value and achievement goals in Mathematics. It was also established that the decline in the stated variables was more pronounced in boys than in girls.

Arai (2001:315) conducted a study to investigate the effects of irrational beliefs and the difference between learners' ideal self-concepts and their actual self-concepts on their self-acceptance. A sample of 464 tertiary students took part in the study. Some students had ideal self-concepts which were greater than their real self-concepts and others had real self-concepts which exceeded their ideal self-concepts. It was established that students whose ideal self-concepts were greater than their actual self-concepts had low self-acceptance scores. The study also revealed that students

whose real self-concepts exceeded their ideal self-concepts had high self-acceptance scores. It was found that the intensity of irrational beliefs influenced students whose ideal self-concepts exceeded their actual self-concepts more than the students whose actual self-concepts were greater than their real self-concepts. The research project confirmed that there is a relationship between irrational beliefs and self-concept and these two variables can influence Mathematics achievement.

The mutual relationship among learners' Mathematics identity, technological identity, racial identity and Mathematics achievement was explored by Nzuki (2010:77). A sample of 30 respondents took part in the study (Nzuki, 2010:83). The study established that the ultimate quality of learners' Mathematics achievement is determined by the identity formation level of the learners (Nzuki, 2010:109). The identity formation of the learners was found to be dependent on their beliefs, which emanated from the way they interpreted their teachers' expectations, their socioeconomic statuses and their ability to use technological devices. Nzuki (2010:109) remarked that with suitable encouragement from Mathematics educators, Mathematics learners could overcome the various obstacles they encountered in everyday life. According to Nzuki (2010:105), it is through identity formation based on the learner's ability to belong to and to function within a particular community that learning takes place.

3.4.1.4 Anxiety

Anxiety in general and Mathematics anxiety in particular were discussed in depth in connection with their influence on Mathematics achievement in sections 2.2.2.4.1 and 2.2.2.4.2 respectively. The focus of this section is to explore the irrational beliefs which are related to anxiety and their ultimate influence on Mathematics achievement. Mathematics anxiety has been linked to learners' beliefs (Zakaria, Zain, Ahmad & Erlina, 2012:1828). According to Thompson and Henderson (2007:212), Ellis emphasised that anxiety itself is not irrational but it is an unnecessary emotion that emanates from irrational thinking.

Mathematics learners who process information guided by the belief that *it is horrible when things are not the way we like them to be* may remain needlessly anxious. Obviously real life situations are rarely as perfect and ideal as we prefer them to be.

In a Mathematics context, learners may not like the personality of their Mathematics teacher, the scarcity of textbooks, the time of the day when Mathematics is slotted on the timetable and the frequency with which they write tests. As a result of such imperfect situations, learners may feel helpless and agitated thereby exposing themselves to anxiety. As discussed in section 2.2.2.4.2, excessive anxiety is not conducive to high academic performance in Mathematics and other subjects (Zakaria & Nordin, 2008:29; Scarpello, 2007:34).

Anxiety, which is a symptom of stress, can be triggered by viewing situations as overwhelming and over demanding. Learners who believe that one should be terribly upset and endlessly obsess about the challenges associated with learning Mathematics can end up being victims of anxiety. This implies that anxiety can be fostered by irrational beliefs as theorised by Ellis and Beck. Such perpetual obsessions may degenerate into a self-defeating vicious cycle which can interfere with one's short-term memory as discussed in section 2.2.1.3.1.

Another irrational belief which can make Mathematics learners more vulnerable to anxiety is the belief that people have virtually no control over their emotions and, therefore, they cannot help feeling disturbed about things. Anxiety is a condition which can be allayed by employing a variety of techniques such as disputation as theorised by Ellis (Thompson & Henderson, 2007:215). As postulated by Feldman (2009:218), excessive anxiety militates against the proper functioning of an individual's short-term memory which can ultimately translate to underachievement in Mathematics. On the other hand, learners who believe that human beings can to some extent control their emotions can deliberately employ some anxiety-reducing techniques. Such techniques are likely to regulate their anxiety to levels which do not interfere with the operations of the working memory, thereby enhancing their Mathematics achievement.

Believing that if something once affected one strongly at some point in the past implies that it will indefinitely affect one can be both a cause and effect of anxiety. Learners who during their formative years believe that Mathematics is an intricate subject are likely to develop Mathematics anxiety (Zakaria et al, 2012:1829). Perennially thinking about past traumatic experiences can be a source of anxiety on the part of learners. Poor Mathematics achievement will ensue and will act as

another unfortunate experience which will continue to haunt the learners. Being anxious about past experiences will destroy a learner's chances of doing well, thereby reinforcing the belief that past experiences will always affect one's academic performance. Hence anxiety can reinforce and be reinforced by some irrational beliefs, thus preventing good Mathematics achievement. Learners who deliberately attempt to ignore past anxiety-provoking experiences may stand a better chance to concentrate on the new concepts and subsequently score high marks in Mathematics.

One of the irrational beliefs outlined by Ellis is that *human suffering is predominantly externally caused and imposed on us by outside people and events*. This is in agreement with the views of Zakaria et al (2012:1829) who posit that external figures such as peers, teachers and parents are usually responsible for causing anxiety among Mathematics learners through emphasising high levels of performance. Learners who make decisions based on this belief are likely to be victims of high levels of anxiety. Such anxiety emanates from their negative expectations of encountering difficulties from external sources. They are likely to view homework, tests and other Mathematics tasks as unbearable suffering imposed on them by teachers and the entire education system, resulting in poor Mathematics achievement.

3.4.1.4.1 Research on anxiety, irrational beliefs and Mathematics achievement

Obodo and Obadan (2008:152) undertook a study to establish the efficacy of Ellis's rational emotive behaviour therapy to curb test anxiety among secondary school adolescent learners. A quasi-experimental research design in which the rational emotive behaviour therapy was the primary independent variable was employed. The other variables which were considered included gender, locus of control and entry test anxiety level (Obodo & Obadan, 2008:155). From a population of all the secondary schools in Lagos, Nigeria, a sample of 125 respondents of whom 80 were test anxious, took part in the study. The experimental group was exposed to rational emotive behaviour therapy treatment procedures. The study established that the rational emotive behaviour therapy can significantly curb test anxiety. This implies that the rational emotive behaviour therapy, which mainly aims to dispute irrational beliefs and replace them with empowering rational beliefs, is effective for the treatment of test anxiety (Obodo & Obadan, 2008:153, 161). Test anxiety was found

to be independent of gender and locus of control (Obodo & Obadan, 2008:162). The findings of the study point out that the higher the learner's ability to dispute irrational beliefs, the lower the learner's level of test anxiety. This implies that irrational beliefs can foster test anxiety thereby interfering with learners' Mathematics achievement.

In an effort to establish whether irrational beliefs are related to emotional adjustment and life satisfaction among college students, Matta, Bizarro and Reppold (2009:71) undertook a study with a sample of 157 college students. Irrational beliefs correlated negatively with both age and life satisfaction (Matta et al, 2009:71). The research outcome is relevant to the current study owing to the positive correlation obtained between irrational beliefs and anxiety. As already discussed in section 2.2.2.4.2, excessive anxiety is detrimental to Mathematics achievement. Therefore, one can conclude that irrational beliefs can trigger anxiety which can subsequently influence Mathematics achievement.

A research project to explore the relationship between test anxiety in learners and two allied variables, which are irrational beliefs and cognitive distortions, was undertaken by Erfani, Soheili and Ahmadkhami (2013:110). A sample of 393 respondents selected from a population of 15 400 learners took part in the study. The components of cognitive distortions which were explored include dichotomous thinking, overgeneralisation, 'should statements' and personalisation (Erfani et al, 2013:113). The irrational beliefs components which were examined entail the need for approval, emotional irresponsibility, dependence, perfectionism and the tendency to blame (Erfani et al, 2013:113). The study established that a direct relationship exists between learners' irrational beliefs and cognitive distortions. The research also revealed that learners' test anxiety is strongly associated with both their irrational beliefs and cognitive distortions (Erfani et al, 2013:110).

A descriptive and correlational study was undertaken by Boyacioglu and Kucuk (2011:454) to establish the extent to which early adolescent learners' irrational beliefs could predict test anxiety. A sample of 557 Turkish primary school learners took part in the study. Irrationality was explored by considering the demand for success, demand for comfort and demand for respect. The study established a strong positive correlation between test anxiety, the overall irrationality score and each of the three forms of demandingness. Demand for comfort and success

predicted test anxiety more strongly than the demand for respect (Boyacioglu & Kucuk, 2011:447). It can be concluded that irrational beliefs can trigger test anxiety which can eventually impinge upon Mathematics achievement.

3.4.1.5 Study habits

As outlined in section 2.2.3.1, the study habits adopted by learners can ultimately influence their Mathematics achievement positively or negatively. In this section, the association between study habits and some irrational beliefs are discussed together with their ultimate impact on Mathematics achievement.

Good study habits normally require learners to plan their study schedules properly and to find an environment conducive to study (Ozsoy et al, 2009:156) (section 2.2.3.1). The ideal study environment may be difficult to introduce especially if there are disturbances at home and/or at school. Learners who are influenced by the irrational belief that *it is horrible when things are not the way one likes them to be* are likely to catastrophise when the home or school conditions do not allow them a favourable study environment. This might negatively impinge upon their Mathematics achievement. On the other hand, learners who do not view challenges as horrible and unbearable may improvise and study Mathematics even if the environment is not ideally conducive to studying. This is likely to enhance their Mathematics achievement.

The temptation to avoid undertaking difficult tasks may result in poor academic performance on the part of learners (Haylock, 2006:5). Mathematics learners who believe that *it is easier to avoid than to confront life's challenges and self-responsibilities* may have difficulty in adopting fruitful study habits. Another irrational belief which can affect study habits in a similar manner is the opinion that *human happiness can be achieved by inertia and inaction*. Mathematics is viewed as a difficult subject by a significant number of people (Haylock, 2006:3; Saraswathi, 2003:326). Learners who believe that Mathematics is a challenging subject may avoid studying Mathematics on a regular basis as a result of the irrational belief associated with problem avoidance. This might have a negative effect on learners' Mathematics achievement since distributed practice yields better results than massed practice (Tuckman & Monetti, 2011:279) (section 2.2.3.1).

Depending on the parenting styles and classroom management styles adopted by parents and teachers respectively, some parents and teachers may not monitor learners' study techniques. Learners who accept the responsibility of crafting and employing study techniques are likely to improve their Mathematics achievement (section 2.2.3.2.4). Conversely, learners who evade the responsibility of generating and implementing study habits as a result of irrational beliefs are likely to underachieve in Mathematics.

As already indicated by Bajwa et al (2011:175), it is normally the responsibility of learners to devise and implement a particular set of study techniques. Learners who believe that they always need other people to rely on may encounter difficulties in drafting personal study timetables and regularly solving mathematical problems on their own. This can interfere with their Mathematics achievement. The quality of the assistance learners receive from their parents can determine the quality of their Mathematics achievement (section 2.3.2.2). However, learners who believe in their own abilities to employ various study habits independently stand a better chance of scoring high marks in Mathematics.

3.4.1.6 Perfectionism

A description of the concept of perfectionism is provided before the relationship between irrational beliefs and perfectionism is explored. Thereafter, the influence of perfectionism on Mathematics achievement is examined.

3.4.1.6.1 Perfectionism and irrational beliefs

According to Macinga and Dobrita (2010:80), perfectionism can be viewed as an irrational resolution to undertake everything in an accurate and flawless manner. It is the propensity to set extortionately high standards for oneself and for other people (Macinga & Dobrita, 2010:80). In previous research, a relationship between perfectionism and irrational beliefs has been established (Flett & Hewitt, 2004:234). Perfectionism has also been attributed to family aspects such as parenting styles (Besser, Flett & Hewitt, 2004:302). Perfectionism can be directed towards oneself, towards others or assumed to be imposed by others. People normally become perfectionists so as to protect themselves from self-criticism and disapproval from other people (Otto & Stoeber, 2006:295; Flett et al, 2003:120). Self-oriented

perfectionism has been attributed to the irrational belief that one must be perfect to be worthwhile (Macsinga & Dobrita, 2010:80; Flett & Hewitt, 2004:233). Global evaluation of oneself, others and the world can lead to perfectionism which is oriented towards oneself and/or other people.

According to Macsinga and Dobrita (2010:80), low frustration tolerance has been implicated in causing other-oriented perfectionism. Ellis argued that when perfectionism is viewed as an absolute and dire necessity, it forces the individual to pursue an unattainable set of goals and the whole pursuit is frustrating and self-defeating in many ways (Flett & Hewitt, 2004:234). Perfectionism which emanates from irrational beliefs weakens people's mental operations and makes them more vulnerable to stress (Gunduz, 2013:2081). This was backed by Domino (2005:8) who maintained that Ellis asserted that perfectionism can trigger paralysis in one's operations and can cause psychological harm. However, Tsui and Mazzocco (2007:133) maintained that research has established that mathematically gifted learners are perfectionistic. This implies that excessive perfectionism can influence learners' Mathematics achievement.

3.4.1.6.2 Research on perfectionism, irrational beliefs and Mathematics achievement

A research project to explore the influence of perfectionism and Mathematics anxiety on Mathematics achievement was undertaken by Tsui and Mazzocco (2007:132). A sample of mathematically gifted sixth graders took part in the study (Tsui & Mazzocco, 2007:134). The research confirmed that there is a negative correlation between Mathematics achievement and both Mathematics anxiety and perfectionism. Apart from identifying a strong positive correlation between Mathematics anxiety and perfectionism, the study also revealed that perfectionism and Mathematics anxiety independently account for variability in Mathematics achievement (Tsui & Mazzocco, 2007:139). No significant gender differences were found in Mathematics achievement, perfectionism and Mathematics anxiety (Tsui & Mazzocco, 2007:132).

3.4.1.7 Procrastination

The association between irrational beliefs and procrastination is examined before the ultimate influence of procrastination on Mathematics achievement is explored.

3.4.1.7.1 Procrastination and irrational beliefs

According to Sirin (2011:447), procrastination is a behavioural propensity to postpone imminent activities. Procrastination occurs when an individual evades doing a task which needs to be performed and uses excuses to justify the evasive behaviour (Asikhia, 2010:205; Akinsola, Tella & Tella, 2007:364). It is the act of postponing the undertaking of tasks till it is too late (Sunitha & Muhammeduni, 2013:101). Ellis and other cognitive behavioural theorists attributed procrastination to irrational beliefs and cognitive distortions (Akinsola et al, 2007:364). Procrastination has been linked to irrational beliefs related to task avoidance, rating oneself as inadequate and viewing the world as over-demanding and cruel (Steel, 2007:68). If there is a high realistic or imagined probability that failure to accomplish a particular task will lead to rejection by significant others, people tend to be anxious and ultimately avoid or delay doing the task (Steel, 2007:68). Sunitha and Muhammedunni (2013:101) backed the idea that fear of failure triggers procrastination as learners fear performing awkwardly thereby failing to meet the expected standards of significant others.

Academic procrastination is of importance in the current study. Academic procrastination can be defined as postponing preparing for impending examinations or tests and delaying doing homework till very little time is left (Sunitha & Muhammedunni, 2013:101). According to Sirin (2011:448), academic procrastination occurs when a learner delays undertaking academic tasks such as preparing for paper presentations, examinations and revision exercises. Numerous researches have confirmed that there is a strong negative correlation between the tendency to engage in academic procrastination and variables such as self-efficacy, motivation and self-esteem (Seo, 2008:753; Sirin, 2011:448; Steel, 2007:69). For instance, Steel (2007:70) argued that high achievement motivation levels can lower an individual's tendency to engage in procrastination. A strong positive correlation between irrational thinking and academic procrastination has been found (Sirin, 2011:448). According to Seo (2008:753) and Sirin (2011:448), there is a strong positive correlation between perfectionism and academic procrastination. Learners with a strong propensity to procrastinate tend to perform more poorly than their

assertive counterparts (Asikhia, 2010:205). Academic procrastination has been found to be a prevalent challenge at secondary school level (Sunitha & Muhammedunni, 2013:101; Asikhia, 2010:205-210). Asikhia (2010:206) contended that the majority of secondary school Mathematics learners have been found to be procrastinators principally owing to lack of academic counselling. Since Mathematics is viewed as a challenging subject by a significant proportion of learners (Haylock, 2006:3), there is a high probability that some learners may engage in academic procrastination when it comes to performing Mathematics tasks (Asikhia, 2010:205; Sunitha & Muhammedunni, 2013:101). The more the learners procrastinate, the more their Mathematics achievement is likely to deteriorate (Asikhia, 2010:205).

3.4.1.7.1 Research on procrastination, irrational beliefs and mathematics achievement

In a bid to examine gender differences in Mathematics anxiety and academic procrastination, Sunitha and Muhammedunni (2013:102) undertook a study in Kerala in India. A sample of 352 secondary school learners took part in the study (Sunitha & Muhammedunni, 2013:102). Two measurement scales; namely, the Scale of Academic Procrastination for Secondary School Students and the Scale of Mathematics Anxiety were used to collect data (Sunitha & Muhammedunni, 2013:102). Significant gender differences in academic procrastination and Mathematics anxiety were found. Girls were found to be less prone to academic procrastination than their male counterparts. The study also established that girls experience more Mathematics anxiety than boys (Sunitha & Muhammedunni, 2013:104).

3.4.1.8 Motivation

The impact of motivation on Mathematics achievement was discussed in section 2.2.2.1. The reviewed studies confirmed that Mathematics achievement can be significantly influenced by learners' motivation (Mbugua et al, 2012:88). Some of the irrational beliefs proposed by Ellis can influence learners' motivation which can subsequently affect their Mathematics achievement as discussed below.

3.4.1.8.1 Motivation and irrational beliefs

The irrational belief that *human suffering and discomfort are usually externally caused* can impinge upon Mathematics learners' motivation. Learners who subscribe to this belief are likely to have low achievement motivation and may lack intrinsic motivation. Having an external locus of control can lead to amotivation as postulated by the self-determination theory. As outlined in section 2.2.2.1.2, this can lead to poor Mathematics achievement. Moreover, learners who are guided by the irrational belief that *it is easier to avoid self-responsibilities and life challenges than to face them* are likely to have low self-determination and low motivation owing to learned helplessness (Johnson, 2003:18). Learners who have high achievement motivation do not normally avoid challenging tasks (Eccles & Wigfield, 2002:117). Intrinsically motivated learners usually take responsibility for their academic work (Waege, 2010:85).

Learners' motivation can be reduced by the irrational belief that *human happiness can be attained by task avoidance*. Inertia and inaction normally act as indications that one is not intrinsically motivated to undertake a given task. Task avoidance can be a cause or effect of low achievement motivation and amotivation. This can negatively affect learners' Mathematics achievement. On the other hand, learners who are determined to take action are likely to be highly motivated to actively solve the problems they experience in Mathematics and increase their achievement. The belief that past failures indefinitely influence them can affect learners' motivation. Eccles and Wigfield (2002:116) contend that sometimes learners evade certain tasks for fear of being confirmed as incompetent. According to Waege (2010:89), events which interfere with the satisfaction of competence and autonomy usually reduce an individual's intrinsic motivation. Mathematics learners who do not believe in their ability to surpass their past failures are likely to have little or no intrinsic motivation resulting in poor Mathematics achievement.

The irrational belief *that we need to be highly competent, intelligent and achieving in virtually all domains* can to some extent motivate learners to work hard in an attempt to accomplish such a feat. Learners who believe in the need to be competent and to achieve may be highly motivated and may have high levels of self-efficacy which will spur them to continue working hard. The need for competence as outlined under the self-determination theory (section 2.2.2.1.2), spurs learners to engage in challenging

tasks (Waege, 2010:89; Eccles & Wigfield, 2002:112). However, as discussed in sections 3.3.1.3.8 and 3.3.1.3.13, the absolutistic nature of the above irrational belief may lead to frustration, stress and anxiety as a result of failing to meet such a high standard. Failure to accomplish some tasks can also reduce the learners' achievement motivation. All this is likely to have a negative effect on learners' Mathematics achievement.

3.4.2 Irrational Beliefs and Peers

The possible association between irrational beliefs and peer relations and the way in which they influence Mathematics achievement at secondary school level is explored below.

3.4.2.1 Peer group influence during adolescence

According to Tuckman and Monetti (2011:117), peers are individuals who share the same status. During adolescence teenagers try to gain autonomy from their parents and become affiliated to a peer group (Swartz et al, 2011:93; Kufakunesu et al, 2013:829). Peer groups serve as avenues through which adolescents learn to share information, solve problems and gain emotional support (Tuckman & Monetti, 2011:117; Santrock, 2004:81). Moreover, adolescents normally feel comfortable about expressing their most sincere feelings freely when they are with their peers (Tuckman & Monetti, 2011:117).

Depending on their motivation orientation, peer groups have been found to have a positive or negative effect on the members' academic achievement (Santrock, 2004:433; Gazzaniga & Heatherton, 2006:465). Snowman et al (2009:44) posited that academic performance can be influenced by the values of the peer group to which learners belong. Conformity to the dictates of peers is not necessarily bad because some peer groups can actually enforce desirable behaviour such as doing homework, working hard and employing good study habits (Swartz et al, 2011:94). Peer groups that place high emphasis on academic achievement tend to enhance the academic achievement of their members. According to Santrock (2004:433), low achieving peer groups tend to weaken the scholastic achievement of their members.

3.4.2.2 Peer relations and irrational beliefs

According to Thompson and Henderson (2007:213), adolescents can irrationally believe that it is a dire necessity for them to conform to their peers. This belief is a derivative of the view *that it is an utter necessity to be approved by significant others such as peers in all the things which they do*. Learners who are guided by this belief may have difficulty in developing individual identities as outlined in section 2.2.2.2.2. They will only live to fulfil what their peers want even if the peer influence is not consistent with their individual aspirations and personalities. As theorised by Rogers, attempting to fulfil the conditions of worth imposed by other people can lead to incongruence in the self of an individual (section 2.2.2.2.4). Depending on the nature of the peer values, attempting to adhere to the demands of one's peer group can be conducive or detrimental to good Mathematics achievement (Ornstein & Hunkins, 2009:142; Swartz et al, 2011:94). Learners whose peer group encourages hardworking may do well in Mathematics while learners whose peer group does not place high emphasis on academic achievement may perform poorly in Mathematics. However, viewing the desire to conform to one's peers as an absolute necessity is bound to exert pressure on the learners leading to stress, depression and anxiety. As discussed in sections 2.2.2.3.6 and 2.2.2.4.4 respectively, stress and anxiety can influence Mathematics achievement.

One irrational belief which can influence adolescents as they relate to their peers is the belief that one should not make mistakes, especially social ones (Thompson & Henderson, 2007:212). The wish to be perfect in one's social behaviour is likely to induce undue pressure on the adolescents as they attempt to please their peers (Haylock, 2006:6). Human beings, by virtue of being fallible (Domino, 2005:7), are bound to make mistakes as they interact with their peers. Learners who are guided by this perfectionistic perspective of social behaviour are likely to expose themselves to anxiety, stress, inferiority complex and identity crisis. These variables have been found to be detrimental to Mathematics achievement. Hence irrationality has a probability of impinging upon academic performance in Mathematics.

3.4.2.3 Research on peers, irrational beliefs and Mathematics achievement

A research project was undertaken by Boehnke (2008:149) to explore the effects of high peer pressure on the Mathematics achievement of adolescents in Canada, Germany and Israel. The study was anchored on the premise that the desire to avoid

social exclusion by high ability Mathematics learners led to underachievement in the subject (Boehnke, 2008:149). A sample of 1700 adolescent learners participated in the study. The research established that peer pressure can influence Mathematics achievement although the negative effects of peer pressure can be neutralised by the other learner variables. To some extent, the study confirmed that the demand for approval and respect from peers can influence Mathematics achievement. The study also revealed that peer pressure mostly victimises girls and that more boys than girls perpetrate the peer pressure (Boehnke, 2008:149).

The interrelations among social acceptance, peer-related mental health and irrational beliefs were explored by Lee, Sohn and Park (2004:1144). Lee et al (2004) hypothesised that adolescents who were rated by their peers as comparatively more mentally stable were more likely to receive higher social acceptance from their peers and demonstrate fewer irrational beliefs than their counterparts who were deemed to be less mentally stable. A sample of 476 respondents all taken from one upper middle-class area in Seoul in Korea participated in the study. Apart from completing a questionnaire measuring irrational beliefs, the research participants also rated one another's mental health in domains such as creativity and industry, sound interpersonal relationships, ability to cope with stressful situations, social acceptance, contentment with life, and concern with other people. The study revealed a positive correlation between peer rating of mental stability and social acceptance by peers during adolescence (Lee et al, 2004:1144). The study also established that the manner in which adolescents are rated by their peers can determine their vulnerability to irrational beliefs. The findings confirmed the idea that the higher the perceived rate of mental health by the peers, the higher the social acceptance received and the fewer the irrational beliefs manifested by the individual concerned.

The effects of peer group influence on learners' Mathematics achievement was explored by McEwan (2003:131) who conducted a study focusing on the academic performance of learners in Mathematics and Spanish in Chile. A sample of 443 eighth graders took part in the study. The Mathematics scores were taken from a census conducted by Chile's Ministry of Education in 1997 (McEwan, 2003:133). Variables such as gender, parental school experiences, ethnicity, family income and the number of books in the family home were also considered (McEwan, 2003:133).

The study established that peer group characteristics such as parental education of the peers and the peer group members' general cognitive ability had an effect on the learners' academic achievement in Mathematics and Spanish (McEwan, 2003:131,136). The study revealed that there is a positive correlation between learners' Mathematics and Spanish achievement and the attributes of their school or classroom peers (McEwan, 2003:139).

3.4.3 Irrational Beliefs Related to School and Schooling

The school is the setting in which the school factors outlined in section 2.3.3 can influence the Mathematics achievement of secondary school learners. Some irrational beliefs of learners and parents pertaining to school and schooling are explored below.

3.4.3.1 Learners' irrational beliefs related to school and schooling

Some learners may irrationally believe that it is unfair and horrible when things are not the way they ideally want them to be (Alizadeh, 2012:2824; Thompson & Henderson, 2007:212). This implies that learners may catastrophise over school variables such as school management, the quality of teaching instructions, teachers' expectations and teachers' personalities as outlined in sections 2.3.3.1, 2.3.3.2, 2.3.3.3 and 2.3.3.4 respectively. While there may be some imperfections associated with the stated school variables, awfulising normally interferes with learners' Mathematics achievement. On the other hand, learners who are capable of tolerating challenges and frustrations associated with school variables have a high probability of doing well in Mathematics.

Learners who derive happiness from actively taking part during Mathematics lessons and diligently studying Mathematics independently are likely to score high marks in the subject. Conversely, learners who believe that they can achieve human happiness through inaction and inertia are likely to sabotage their chances of scoring high marks in Mathematics. Mastering already learnt concepts to some extent enhances learners' chances of understanding newly introduced concepts as theorised by Gagne and Ausubel (sections 2.3.3.2.2 and 2.3.3.2.3 respectively). Moreover, it is through active participation that one experiences meaningful learning.

Actively cooperating with more competent peers can lead to better Mathematics achievement as conceived by Vygotsky (section 2.2.1.4.2.1).

Adolescent learners may cling to the irrational belief that one should not allow oneself to be criticised (Thompson & Henderson, 2007:212). This belief is linked to demand for approval, low frustration tolerance and awfulising. According to Mubeen, Saeed and Arif (2013:39), Mathematics learners normally develop positive attitudes towards Mathematics when they earn the approval of their teachers. Learners who are too sensitive to the remarks made about them by their teachers or peers at school may end up emotionally agitated. Szentagotai and Jones (2010:83) contended that demandingness can lead to poor social relationships. Ashcraft and Kirk (2001:224) claimed that anger, frustration, hostility, anxiety and stress can congest the working memory thereby interfering with the individual's information processing (section 2.2.1.3.1). This may in turn negatively affect the learners' Mathematics achievement. Conversely, tolerant learners can remain focused on their studies in spite of criticism. Such learners stand a higher chance of doing well in Mathematics.

Some adolescent learners may follow the irrational belief that if something is perilous or fear-provoking, one should be terribly agitated and endlessly obsessive about it. Viewing challenges associated with schooling as fearsome or dangerous and subsequently remaining perpetually worried about them, is likely to foster stress and anxiety (Haylock, 2006:3). As postulated by the General Adaptation Syndrome model of stress in section 2.2.2.3.2, prolonged stress can lead to exhaustion which can subsequently lead to physical illnesses such as hypertension, high blood pressure and many others (Feldman, 2009:485). This is likely to lead to poor Mathematics achievement as discussed in section 2.2.2.3.

3.4.3.2 Parents' irrational beliefs related to school and schooling

As explained in section 2.3.2.2, parental involvement can influence learners' Mathematics achievement. The quality of parental involvement can be determined by the nature of the experiences which the parents have had during their own schooling days. The irrational belief that *because something once seriously affected a person's life, it will continue affecting it* can be held by the parents of some learners on the

basis of their own school experiences. According to Ramirez (1999:22), some parents feel too insecure to participate in their children's academic activities. Parents who encountered difficulties in Mathematics during their own school days may not be involved in their children's Mathematics activities especially if they believe that their own experiences still affect them negatively. Mbugua et al (2012:90) posited that poorly educated parents are not likely to be good role models for their children in Mathematics. Some parents may not be happy with their own levels of mathematical understanding and are afraid of assisting their children in this regard (Mooney, Briggs, Fletcher, Hansen & McCulloch, 2007:99; Cushner, McClelland & Safford, 2009:111). Moreover, parents who encountered academic difficulties during their school days may transfer their negative beliefs and attitudes to their children (Compton, Feilding & Scott, 2007:7). Haylock (2006:6) reported a case in which a mother persuaded her child to believe that it was socially acceptable to perform poorly in Mathematics. Such parental beliefs can negatively influence learners' Mathematics achievement. Conversely, parents who are resilient enough to shrug off their own unpleasant school experiences have a high probability of fostering rational beliefs and attitudes about school and schooling in their children. Such parents can encourage their children to study Mathematics seriously despite having encountered difficulties in Mathematics during their own school days (Pekrun, Vom Hofe, Blum, Frenzel, Goetz, & Wartha, 2007:32).

3.4.3.2.1 Research on effect of parents' beliefs on school and schooling

A longitudinal research was conducted by Sonnenschein, Baker, Moyer and LeFevre (2005: slide 2) to establish the association between parental beliefs about children's Mathematics development and reading and how they relate to their children's academic achievement. A sample of 80 families comprising 26 low income European American families and 29 low income African American families as well as 14 middle income European American families and 11 middle income African American families participated in the study (Sonnenschein et al, 2005:slide 6). Among other things, the study revealed that low income parents strongly encouraged their children to master mathematical skills more than their middle income counterparts. On the other hand, middle income parents encouraged their children to apply mathematical concepts in real life situations more than their low income counterparts. Therefore, it can be argued that the socioeconomic status of parents can influence the beliefs

they emphasise and relay to their children, thus influencing their children's Mathematics achievement differently.

3.4.3.3 Parents' irrational beliefs related to school-going children

Two allied irrational beliefs which can be held by parents regarding their children are the belief that children should always do what their parents want them to do and that children should not at all disagree with their parents (Thompson & Henderson, 2007:212). These beliefs are likely to be held by authoritarian parents who demand passive obedience from their children (section 2.3.2.1.2). Parents who believe that their children should be utterly obedient may be frustrated when their children question some of their expectations. Habitual conflicts are witnessed between adolescent children and their parents principally because the adolescents search for autonomy as part of identity formation (Kufakunesu et al, 2013:828-829). As pointed out in section 2.3.2.1.2, the authoritarian parenting style is bound to produce children who are insecure, passive, anxious and socially incompetent (Santrock, 2004:74). Such attributes can negatively impinge upon the adolescents' Mathematics achievement. On the other hand, authoritative parents are likely to view conflicts with their adolescent children as a normal developmental process which when handled properly can lead to identity formation on the part of the children. Authoritative parenting is likely to enhance Mathematics achievement as discussed in section 2.3.2.1.2.

3.4.3.3.1 Research on parents' beliefs related to school-going children

Aunola, Nurmi, Lerkka and Rasku-Puttonen (2003:403) undertook a study exploring the changes between learners' Mathematics achievement relative to the learners' purposefulness and task-avoidant behaviours. The beliefs of the parents regarding their children's academic competence in Mathematics were also considered. A sample of 111 primary school learners in the 6 to 7 year old age range participated in the study. The teachers of the participating learners rated their task-focused versus task-avoidant behaviours on four occasions during the learners' first school year. The study showed that parents' beliefs in their children's general school ability boosted their children's task-focused behaviours at school, which subsequently related to the children's high level of Mathematics achievement (Aunola et al, 2003:403). It was further established that parents' beliefs in their children's

competence in undertaking mathematical tasks significantly contributed to children's high Mathematics achievement.

Chen and Uttal (1998:351-352) conducted an investigation which compared parental beliefs and cultural values and their impact on Mathematics achievement in the United States of America and China. A sample of 390 Chinese and 580 American mothers participated in the study with interviews and questionnaires as data collection instruments (Chen & Uttal, 1998:354). The study revealed that the Chinese parents set higher standards and worked more frequently with their children at home than their American counterparts. It was also established that Chinese cultural values and beliefs helped to foster diligence on the part of the children. The research confirmed that the nature of the beliefs held by parents regarding schooling could impinge upon their children's Mathematics achievement. Parents who harbour irrational beliefs are likely to relay them to their children who will in turn behave and think irrationally, thereby negatively affecting their Mathematics achievement.

3.4.4 Irrational Beliefs and the Teacher

According to Orton (2004:175), teachers are a critical determinant of the entire teaching and learning process as discussed in section 2.3.3.2. Learners' and parents' irrational beliefs related to the teacher and the role of teachers in moulding beliefs are examined below.

3.4.4.1 Learners' irrational beliefs related to teachers

Adolescents' demand for the approval of teachers can cause them to believe irrationally that it is catastrophic for them to be criticised by teachers (Thompson & Henderson, 2007:212). However, it is unrealistic for learners to expect teachers to approve of all their actions. Since teachers are in most cases more knowledgeable than learners relative to Mathematics content, it is likely that teachers may criticise learners not for the sake of condemning them but as a way of guiding them (Osler, 2010:77). Teachers who employ the authoritarian and authoritative classroom management styles are likely to disapprove of some of their learners' actions (section 2.3.3.1.2). When such learners feel that they have failed to earn the approval of their teachers, they may develop negative attitudes towards Mathematics

leading to poor Mathematics achievement. Only teachers who employ the laissez-faire or country club management styles are likely to ignore learners' unacceptable behaviour. As already established in sections 2.3.3.1.2 and 2.3.3.1.3 respectively, the laissez-faire and country club classroom management styles are not very conducive to good Mathematics achievement at secondary school level (William-Boyd, 2003:97).

It is possible that Mathematics teachers may execute their professional duties in ways which fall short of their learners' expectations. Examples of teacher activities which may disappoint learners include subjective marking, giving learners too much homework, delays in giving learners feedback and adopting an authoritarian management style (Osler, 2010:77). Learners who hold the irrational belief that *certain actions are awful or wicked, and that people who engage in such activities should be thoroughly penalised* are likely to develop hostility towards teachers. According to Osler (2010:78, 80), some learners do not approve of the authoritarian way in which some teachers conduct lessons. Learners who view people's weaknesses as an indication that such people should be utterly damned may disrespect their teachers. This is likely to strain teacher-learner relations leading to poor Mathematics achievement. On the other hand, learners who are not influenced by this irrational belief may unconditionally accept and respect their teachers as role models. This may enhance the learners' Mathematics achievement as a result of healthy teacher-learner relationships.

Mathematics learners who irrationally believe *that human misery is always externally caused* may believe that Mathematics, which is viewed by many people as a difficult subject (Haylock, 2006:3), is a form of suffering imposed upon them by curriculum implementers who happen to be teachers (Osler, 2010:79). The effort exerted by Mathematics teachers to ensure that learners participate in class and complete class exercises, homework and adequately prepare for tests can be viewed by learners as external misery imposed upon them by the teachers (Osler, 2010:78). Learners with such a perspective are not likely to respect and identify with their Mathematics teachers. Moreover, such learners may perform poorly in Mathematics because of low achievement motivation, reduced self-determination and limited intrinsic motivation (sections 2.2.2.1 and 3.4.1.8.1). However, learners who do not view the

teachers' operations as a source of external misery have a high probability of doing well in Mathematics.

For secondary school learners to score high marks in Mathematics, they need to have a certain degree of confidence and autonomy to work independently. Mathematics learners who believe that they can only do well in Mathematics when they rely on their teachers may underachieve owing to low self-efficacy, and limited achievement motivation as well as low intrinsic motivation (section 3.4.1.8.1). Nevertheless, learners who do not view teachers as their perennial source of support may explore their zones of proximal development more frequently as postulated by Vygotsky's sociocultural theory (section 2.2.1.4.2.1). Such learners are likely to adopt beneficial study habits, thus enhancing their chances of doing well in Mathematics (section 3.4.1.5).

3.4.4.2 Parents' irrational beliefs related to the teacher

The irrational belief that *people absolutely need something other or more powerful than they are to succeed* may affect the way some parents view teachers. Some parents may feel that they are intellectually inadequate and incompetent, therefore nursing the belief that they really need to rely on teachers who are competent professionals (Cushner et al, 2009:111; Arends, 2009:308). This was backed by Mooney, Knox and Schacht (2007:264) who contended that some parents, especially low-income ones, are in most cases low academic achievers who impart limited academic skills to their children. It can be argued that parents who have such a mentality may not adequately attempt to assist their children in academic matters. This is likely to affect the learners' Mathematics achievement negatively.

Some parents may be influenced by the irrational belief that *human suffering is almost always externally caused and is imposed on individuals by outside people and events*. When children seek their parents' help in solving any mathematical problems, some parents may view Mathematics teachers as the eternal source of misery not only on the part of the children but on the entire family (Haylock, 2006:2; Mooney et al, 2007:264). Arends (2009:308) contended that homework can sometimes cause strife and stress in the family. It is likely that parents who hold such views do not complement the teachers' efforts to ensure that learners do well in

Mathematics. Such poor parental involvement can negatively affect secondary school learners' Mathematics achievement as discussed in section 2.3.2.2.

One irrational belief which can be held by some parents is the opinion that *certain acts are awful or wicked, and that people who perform such acts should be thoroughly damned*. Such parents may view teachers as sophisticated adults who are too strict with their children (Arends, 2009:308). In cases in which teachers employ authoritarian classroom management or when teachers make mistakes, parents may label them as wicked and learners may actually adopt such beliefs from their parents (Compton et al, 2007:7). This is likely to translate to poor Mathematics achievement on the part of the learners. On the other hand, parents who accept the fallible nature of teachers may complement the teachers' efforts to promote good Mathematics achievement on the part of the learners.

The irrational belief that *something which once seriously affected our life will indefinitely affect it* may influence the way some parents view teachers. Parents who, during their schooling days did not relate well with teachers may hold on to the belief that they will never be able to relate well with their children's teachers (Osborne & Dyer, 2000:51). According to Haylock (2006:3), the unsympathetic attitudes of some teachers propagate prolonged Mathematics learners' anxiety which could affect them even during adulthood. This reduces such parents' chances of meaningfully assisting teachers to monitor the academic performance of their children in Mathematics. Such a phenomenon can negatively affect their children's Mathematics achievement. On the other hand, parents who put aside their negative interactions with teachers are more likely to assist their children thereby enhancing their Mathematics achievement (Arends, 2009:308).

3.4.4.3 Role of teachers in moulding learners' beliefs

According to Shein and Chiou (2011:1097), human conduct is largely transmitted when individuals are exposed to role models as postulated by Bandura in his social learning theory (section 2.3.3.4.3). In the education fraternity, teachers are mostly role models who are bound to be imitated by learners (Shein & Chiou, 2011:1097; Philipp, 2007:257). Teachers transmit their Mathematics beliefs to their learners mainly through observation during the long period of teacher-learner interaction in

the classroom (Stipek et al, 2001:224; Handal, 2003:48; Bol & Berry III, 2005:36). Learners acquire their teachers' beliefs in the same manner an apprentice learns the skills of a trade from the mentor (Handal, 2003:49; Pierce & Newstrom, 2008: xxix).

According to Compton et al (2007:7), the attitudes of teachers can strongly influence the attitudes of learners. The nature of the attitudes portrayed by teachers regarding Mathematics learning can influence learners' beliefs about Mathematics (Compton et al, 2007:7). The way learners think is strongly influenced by teachers (Beedle & Burkill, 2008:47). Teachers are capable of influencing learners' attitudes towards academic performance, self-efficacy beliefs and their beliefs about their own intelligence (McLean, 2003:51; Miller & Satchwell, 2006:136). The ways teachers give feedback to learners regarding their scholastic efforts impinge upon the way the learners interpret their own academic progress. Teachers' expectations can be self-fulfilling, thus triggering irrational beliefs in learners (sections 2.3.3.3 and 2.3.3.3.2). On the basis of the above arguments, it can be concluded that teachers are crucial in influencing learners' beliefs in general and irrational beliefs in particular especially at secondary school level. This can ultimately influence the learners' Mathematics achievement depending on the learners' positions on the rationality and irrationality continua.

3.5 SUMMARY

The characteristics, acquisition, maintenance and effects of irrational beliefs have been discussed in line with Ellis's theoretical underpinnings. The biological basis of irrationality which claims that human beings are born with an inherent tendency to think irrationally goes a long way towards supporting the claim that Mathematics achievement can be influenced by irrational beliefs. The studies cited in this chapter serve to indicate that irrational beliefs can permeate through learners' internal, home and school factors thereby influencing the learners' Mathematics achievement. Learners' internal factors such as anxiety, self-concept, perfectionism, procrastination, motivation, gender and study habits were found to relate to some irrational beliefs in ways which can ultimately influence Mathematics achievement. Some irrational beliefs were also found to be capable of influencing the way parents viewed school activities and teachers.

It can be argued on the basis of the studies outlined above that irrational beliefs can negatively impinge upon learners' academic performance in Mathematics. Irrational beliefs which can broadly be categorised into demandingness, awfulising, low frustration tolerance and global rating of oneself, others and the world, can influence Mathematics achievement more negatively than positively since irrational beliefs generally trigger cognitive, emotional and behavioural dysfunction (Ellis, 2007:13; Kaygusuz, 2013:143; Ciarrochi, 2004:173; Palmer, 2000:281). By virtue of their pervasive nature, irrational beliefs have also been found to be influential in determining the manner in which learners relate with their peers. It has also been established that teachers can be influential in moulding learners' beliefs (McLean, 2003:51; Stipek et al, 2001:224; Handal, 2003:49; Miller & Satchwell, 2006:136). Depending on the way they conduct themselves, teachers can promote learners to think and behave rationally or irrationally.

CHAPTER FOUR

METHOD OF EMPIRICAL INVESTIGATION

4.1 INTRODUCTION

The main thrust of this chapter is to outline the method of empirical investigation employed in the current study. The aim of this study is to explore the relationship between irrational beliefs and secondary school learners' Mathematics achievement. Since irrational thinking and Mathematics achievement do not exist in isolation, a variety of allied variables have been explored so as to identify any possible relationships as suggested in section 1.1.

Six hypotheses were formulated for consideration in the exploration of the relationship between irrational thinking among adolescents and their Mathematics achievement. Some of the hypotheses focus on the relationship between irrational thinking and allied variables such as learners' gender, socio-affective variables and learners' perceptions in a Mathematics teaching and learning context. The sixth hypothesis asserts that a number of variables jointly explain a greater proportion of variability in Mathematics achievement than any one of the factors on its own.

The data gathering instrument used for the empirical investigation is discussed in detail. A single questionnaire with subsections was administered to each research participant. The subsections were irrational belief tests; learners' socio-affective variables; parental involvement; perceptions and Mathematics achievement. A composite questionnaire was developed by making a few adaptations in already existing research instruments. Each subsection of the questionnaire is discussed with some examples of items being given as they appeared after adaptation.

A large sample of adolescent secondary school learners was chosen to participate in the study. The sampling procedure, the sample composition and sample characteristics are highlighted in this chapter. Effort was made to outline the extent to which the chosen sample reflects the characteristics of the entire population. The ethical principles which were observed during the empirical investigation are elaborated. An outline of the data collection procedure is given before the whole chapter is summarised.

4.2 HYPOTHESES

As already pointed out, Mathematics achievement does not take place in a vacuum. It is influenced by and/or related to a variety of factors. In an attempt to establish the influence of irrational beliefs on Mathematics achievement, several hypotheses were generated. The hypotheses focus on the possible relationships between irrational thinking and learner variables such as gender, grade level (form), perceptions of Mathematics and socio-affective variables all considered in a Mathematics teaching and learning context at secondary school level. On the basis of the several findings of previous studies, evidence apparently supports the hypothesis that differences in learners' Mathematics achievement is better understood by considering the joint effect of a number of variables instead of considering each variable in isolation. Such a hypothesis was formulated at the end. Below is an outline of each of the six hypotheses:

4.2.1 Hypothesis 1

There is a significant difference between the irrational thoughts of boys and girls with regard to Mathematics.

Rationale

Numerous studies have been undertaken the world over to explore the relationship between learners' gender and their scholastic achievement in Mathematics (Grootenboer & Hemmings, 2007:6). The fact that gender can cause variability in Mathematics achievement has been established by many researchers (Penner, 2008:S138; LaLonde et al, 2003:286). However, such studies generate conflicting findings in some cases. Many researches confirm that relative to Mathematics, male learners normally perform better than their female counterparts (Watt, 2004:1556; Frenzel et al, 2010:508; Hannula, 2002:25). The Mathematics achievement of pre-adolescents was found to be independent of gender while that of adolescents was seen to be gender-determined as adolescent boys tend to outperform adolescent girls in Mathematics (Okoiye, Ukah & Nwoga, 2013:87). A study by Bojman et al (2011:1) involving university students showed that female students harbour more irrational beliefs than their male counterparts. However, there seems to be a scarcity of studies exploring the difference between the irrational thoughts of adolescent boys

and girls with regard to Mathematics. Therefore, the need to verify the above hypothesis remains.

It seems that male learners tend to be more enthusiastic about learning Mathematics than female learners (Palsdottir, 2007:117; Watt, 2004:1556; Frenzel et al, 2010:508; Hannula, 2002:25). In certain cases teachers themselves were found to believe that boys are more intelligent and logical when solving mathematical problems than girls (Gunderson et al, 2012:153). The patriarchal values enforced in many societies tend to socialise learners to believe that Mathematics is a male domain (Haylock & Thangata, 2007:77; Haylock, 2006:3; Wedege, 2011:107). Substandard Mathematics achievements on the part of female learners has also been ascribed to the notion that female learners are less assertive, very irrational and too emotional to excel in Mathematics (Wedege, 2011:96; Walkerdine, 1998:15; Henningsen, 2008:35-36; Dreber et al, 2012:2). Gunderson et al (2012:153) contended that female learners are normally associated with more negative emotions such as anxiety, negative attitudes and negative Mathematics self-concepts. According to Abubakar and Adegboyega (2012:122), female learners normally have significantly lower levels of self-efficacy than their male counterparts regarding Mathematics and allied domains such as computer studies. While it is arguably true that female learners are intellectually capable of matching their male counterparts, it seems female learners' vulnerability to high levels of anxiety lowers their Mathematics achievement (Okoiye et al, 2013:87; Kenny-Benson, Pomerantz, Ryan & Patrick, 2006:11-26). The presence of such gender differences suggests that there is a possibility that the irrational thoughts of male and female learners pertaining to Mathematics may differ.

4.2.2 Hypothesis 2

There is a significant difference between the irrational thoughts of Form 3 and Form 4 learners with regard to Mathematics.

Rationale

Cognitive developmental theorists such as Piaget maintain that learners at different stages of mental development learn and process information differently (section 2.2.1.2.1). While learners at the concrete operational stage are incapable of deductive reasoning, learners at the formal operational stage can undertake abstract

thinking, engage in hypothetical deductions and solve problems requiring the use of logical principles (William-Boyd, 2003:67; Gazzaniga & Heatherton, 2006:448). Form 3 learners, by virtue of being generally chronologically younger than their form 4 counterparts, are more likely to be at the concrete operational stage while form 4 learners are more likely to be at the formal operational stage. This can translate to differences in Mathematics achievement between form 3 and form 4 learners.

Conflicting findings regarding the relationship between age and academic performance in Mathematics have been obtained. Some researches reveal that as learners grow older, the differences in Mathematics achievement tend to diminish owing to the equalising effect of common school experiences (Josiah & Adejoke, 2014:474; White, 1982:461-481). Other studies, however, have acknowledged that older learners outperform younger learners in academic tasks (Josiah & Adejoke, 2014:475; La Paro & Pianta, 2000:443-484; Walker, Greenwood, Hart & Carta, 1994:606-621). In his trait theory, Allport contended that learners' sense of self-concept varies with age. While pre-adolescents are mostly rational copers who endeavour to solve problems without evading them, adolescents and young adults tend to perform tasks in a more self-directed manner for the sake of attaining crucial long-term life goals (Dandapani, 2001:321; Dumont, 2010:208). This seems to suggest that learners of different ages may have different levels of self-determination when it comes to studying Mathematics.

A study by Okoiye et al (2013:93) tested the null hypothesis that there is no significant effect of age on the Mathematics anxiety of senior secondary school learners. However, there is a significant effect of age on Mathematics anxiety scores between younger and older adolescents (Okoiye et al, 2013:93). Older learners appear to be less equipped to overcome their Mathematics anxiety than their younger counterparts (Okoiye et al, 2013:93). This is attributed to the fact that older learners are more disappointed by their past Mathematics failures than their younger counterparts (Arem, 2009:215; Okoiye et al, 2013:94). Although the aforementioned study seems to suggest that a significant relationship exists between age and Mathematics anxiety, a variable which has been linked with irrational thinking (Thompson & Henderson, 2007:212; Palmer, 2000:281; Austad, 2009:275), the need remains to ascertain whether a significant difference exists between Form 3 and Form 4 learners' irrational thoughts pertaining to Mathematics.

4.2.3 Hypothesis 3

There is a significant relationship between the irrational thoughts of learners in Mathematics and achievement in Mathematics.

Rationale

It has been established that irrational thinking can trigger a host of negative emotions such as embarrassment, irritability, depression, guilt and hostility (Austad, 2009:275; Ellis, 2003a:247; Najafi et al, 2012:312). Research also shows that people who engage in irrational thinking normally have difficulty in solving existential problems (Kaygusuz, 2013:143). Furthermore, irrational thinking usually leads to poor human functioning, inability to cope with stress and failure to adjust when faced with adversity (Najafi et al, 2012:312; Bermejo-Toro & Prieto-Ursua, 2006:89; Hayes, 1998:210). The negative emotions outlined above are likely to impinge negatively upon learners' Mathematics achievement. Moreover, Mathematics as a subject basically involves a lot of problem solving (Feldman, 2009:257; Kosslyn & Rosenberg, 2006:358; Sternberg, 2009:231) (section 2.2.1.5.1). Failure to solve problems can translate into inability to deal with a variety of mathematical tasks, thereby resulting in poor Mathematics achievement.

A negative correlation has been recognised between stress and Mathematics achievement (Kaplan et al, 2005:3; Beilock, 2008:339-343; Byrne et al, 2007:393; Kauts & Sharma, 2009:39). Therefore, if irrational thinking interferes with an individual's ability to cope with stress, it can be argued that irrational thinking can negatively correlate with Mathematics achievement. Furthermore, irrational thinking has been implicated in causing perfectionism among adolescents (Najafi et al, 2012:313; Flett & Hewitt, 2004:234; Macinga & Dobrita, 2010:80). Perfectionism triggers stress which can interfere with Mathematics achievement as already outlined in section 3.4.1.6 (Gunduz, 2013:2081; Domino, 2005:8). It has actually been confirmed that there is a negative correlation between perfectionism and Mathematics achievement (Tsui & Mazzocco, 2007:134).

Closely linked to perfectionism is the concept of academic procrastination. Irrational thinking, perfectionism and academic procrastination have been found to be mutually positively correlated (Sirin, 2011:448; Seo, 2008:753). Academic procrastination is detrimental to good Mathematics achievement (Asikhia, 2010:205). Consequently, it

can be contended that irrationality can lead to academic procrastination which in turn negatively correlates with Mathematics achievement. This implies that irrational thinking can correlate negatively with Mathematics achievement. However, there remains the need to verify such a possibility.

4.2.4 Hypothesis 4

There is a significant relationship between the irrational thoughts of learners in Mathematics and socio-affective variables such as motivation, stress, anxiety, self-concept, teacher-learner relationships and parental involvement.

Rationale

As outlined in chapter 2, the Mathematics achievement of learners can be influenced by a variety of socio-affective variables which include self-concept, motivation, anxiety, teacher-learner relationships and parental involvement. Efforts have already been made to explore the relationship between motivation and Mathematics achievement (section 2.2.2.1). It has been established that there is a strong positive correlation between learners' levels of motivation and their Mathematics achievement (Tella, 2007:154; Mousoulides & Philippou, 2005:321; Mbugua et al, 2012:88; Md.Yunus & Ali, 2009:93). The tentative relationship between irrational beliefs and motivation has also been explored relative to Mathematics achievement (section 3.4.1.8.1). For instance, it can be argued that irrational thinking based on task avoidance can reduce learners' self-determination and motivation as a result of learned helplessness (Johnson, 2003:18). It has also been shown that learners sometimes avoid engaging in Mathematics tasks for fear of being labelled as incompetent (Eccles & Wigfield, 2002:116; Waege, 2010:89). On the other hand, it has been claimed that intrinsically motivated learners or learners with high levels of achievement motivation do not normally procrastinate when given tasks (Waege, 2010:85; Eccles & Wigfield, 2002:117). Studies exploring irrational thinking related to Mathematics and motivation in relation to Mathematics seem to be scarce. Therefore, the existence of a relationship between motivation as a socio-affective variable and learners' irrational thoughts in Mathematics needs to be determined.

The concept of stress was extensively explored in chapter 2 together with the allied concept of the Yerkes-Dodson law (sections 2.2.2.3.1, 2.2.2.3.2, 2.2.2.3.3 and 2.2.2.3.4). Adolescents are particularly vulnerable to stress owing to the maturational

turbulence associated with adolescence (Lohman & Jarvis, 2000:15; Sulaiman et al, 2008:180). Some studies have also confirmed that learners' Mathematics achievement can be influenced either positively or negatively by stress (Kauts & Sharma, 2009:39; Byrne et al, 2007:393; Beilock, 2008:359-343; Kaplan et al, 2005:3). Ellis maintained that harbouring irrational beliefs can trigger and sustain stress as irrationality interferes with a person's ability to cope with stress (Bermejo-Toro & Prieto-Ursua, 2006:89). Therefore, a relationship between learners' irrational thoughts in Mathematics and stress can be expected.

Ellis indicated that anxiety is a negative emotion emanating from irrational thinking (Thompson & Henderson, 2007:212; Austad, 2009:275; Palmer, 2000:281; Ciarrochi, 2004:173). A negative correlation was seen between learners' ability to dispute irrational beliefs and the test anxiety they experience (Obodo & Obadan, 2008:162). Studies by Boyacioglu and Kucuk (2011:454) and Matta et al (2009:71) established a strong positive correlation between learners' irrational thinking and their levels of anxiety. A related study by Erfani et al (2013: 10) did not only discover a direct relationship between irrational beliefs and cognitive distortions but acknowledged that both irrational beliefs and cognitive distortions strongly correlate with learners' test anxiety. Moreover, irrational thinking can foster test anxiety which can negatively affect learners' Mathematics achievement. On the basis of such findings, it seems logical to anticipate obtaining a relationship between learners' irrational thoughts in Mathematics and anxiety as one of the socio-affective variables.

Wang et al (2012:1215) acknowledged that self-concept is related to learners' Mathematics achievement. Another study by Ferla et al (2009: 499-505) found that academic self-concept in Mathematics can influence learners' Mathematics self-efficacy. This means self-concept can ultimately be related to Mathematics achievement (Ferla et al, 2009: 499-505). Arai (2001:315) confirmed the existence of a relationship between irrational beliefs and learners' self-concept (section 3.4.1.3.1). However, the study by Arai (2001:315) did not particularly focus on the relationship between irrational beliefs and Mathematics learners' self-concept. Therefore, there is need to verify if there exists a relationship between irrational thoughts about Mathematics and learners' Mathematics achievement.

The way teachers attempt to regulate classroom activities can determine the quality and nature of the teacher-learner relationships which prevail in the classroom (Tuckman & Monetti, 2011:234). Poor teacher-learner relationships can weaken learners' scholastic achievement (Smith, 2004:14; Hamre & Pianta, 2001:625). On the other hand, research has found a positive correlation between healthy teacher-learner relationships and learners' academic achievement (Garcia et al, 2011:2). Relative to classroom management, research has revealed that teacher-learner relationships based on the authoritative leadership style are conducive to good scholastic achievement (Kumar, 2004:114; Wayne & Young, 2003:89). Good teacher-learner relationships are conducive to good Mathematics achievement (Freiberg et al, 2009:63). Several authorities have also claimed that teachers are capable of influencing learners' beliefs about Mathematics (Shein & Chiou, 2011:1097; Bol & Berry III, 2005:36; Handal, 2003:48; Campton et al, 2007:7; Stipek et al, 2001:224; Pierce & Newstrom, 2008:xxix). The way learners rate themselves relative to academic achievement can be influenced by their teachers (Beedle & Burkill, 2008:47; McLean, 2003:51). In as much as teachers are capable of influencing learners' beliefs, attitudes and motivation, few studies directly focus on the relationship between learners' irrational thoughts pertaining to Mathematics achievement and teacher-learner relationships as a socio-affective variable. As a result, such a relationship needs to be verified through hypothesis testing.

A positive correlation has been discovered between learners' Mathematics achievement and their parents' occupations (Ma & Kingler, 2000:41; Levpuscek & Zupancic, 2009:541). Parental attributes such as level of education, educational expectations and socioeconomic status have been found to be related to learners' Mathematics achievement (Neuenschwander et al, 2007:594-602; Eamon, 2002:49). Closely related to the above parental considerations is the issue of parenting styles. The parenting style adopted by parents is related to learners' Mathematics achievement (Gonzalez & Wolters, 2006:203). The authoritative parenting style is suitable for nurturing learners' academic needs and development (Snowman et al, 2009:26; Tassoni et al, 2007:327). The quality of assistance which parents can give to their children when studying Mathematics is determined by a number of variables. Parents who encountered difficulties with Mathematics during their schooling days were reluctant to assist their children in doing Mathematics assignments (Mooney et

al, 2007:99; Ramirez, 1999:22; Mbugua et al, 2012:90). Parents with the irrational belief that something which once strongly affected an individual at some point in life would indefinitely affect that person were likely to be unwilling to help their children in doing Mathematics. In addition, it is clear that negative academic beliefs and attitudes can be transferred from parents to their school-going children (Haylock, 2006:6; Campton et al, 2007:7; Chen & Uttal, 1998:354). There might be a relationship between parental involvement as a socio-affective factor and the irrational thoughts of learners regarding Mathematics. However, the existence of such a relationship has not been adequately explored; hence the need to examine it further.

4.2.5 Hypothesis 5

There is a significant relationship between learners' irrational thoughts in Mathematics and their perceptions of Mathematics.

Rationale

It can be argued that the way learners perceive Mathematics as an academic discipline can determine their Mathematics achievement. According to Mutodi and Ngirande (2014:432), learners' perceptions of Mathematics broadly entail all the visual and verbal schemata, metaphorical images and associations, beliefs, attitudes and sentiments related to Mathematics as a subject and Mathematics learning experiences. Sprangler (1992:19) conceded that learners' perceptions of Mathematics determine learner variables such as enthusiasm to learn Mathematics and attitudes towards Mathematics. It was highlighted in section 1.1 that Mathematics is to some extent a notorious subject since it is rated as difficult and demanding (Saraswathi, 2003:326; Asikhia, 2010:206; Meremikwu & Erukoha, 2011:87-88; Mutodi & Ngirande, 2014:432). A significant number of learners perceive Mathematics as an intricate subject with a variety of elusive concepts (Tachie & Chireshe, 2013:68, 71; Okoiye et al, 2013:86). For a substantial number of learners, learning Mathematics is characterised by anguish and stress (Asikhia, 2010:205). Some learners' perceptions of Mathematics are so negative that they openly confess that they cannot do Mathematics (Petty, 2009:510). Tachie and Chireshe (2013:68, 71) alluded to the fact that learners' faulty perceptions of Mathematics can trigger complacency and underachievement as learners justify their

poor Mathematics achievement using the notion that Mathematics is unbearably difficult.

A few studies exploring the relationship between learners' perceptions of Mathematics and their Mathematics achievement have been undertaken. For example, Anderson et al (2006:718) established that there is a positive correlation between learners' Mathematics perceptions and their Mathematics achievement. Githua (2013:175) contended that harbouring negative perceptions of Mathematics has been found to be one of the main causes of poor Mathematics achievement. Okoiye et al (2013:88) reiterated the same sentiments by claiming that as long as learners harbour negative perceptions of Mathematics, they remain vulnerable to Mathematics failure.

It can be argued that holding faulty perceptions of Mathematics can make learners vulnerable to irrational beliefs pertaining to task avoidance, overdependence and emotional irresponsibility (section 3.3.1.3). The belief that human misery is invariably externally caused and forced on us by outside people and events and the irrational belief that because something once strongly affected our life, it should affect it indefinitely are two irrational beliefs which can be held by learners with faulty perceptions of Mathematics. However, the existence of a relationship between learners' irrational thoughts in Mathematics and their perceptions of Mathematics needs to be empirically determined.

4.2.6 Hypothesis 6

Irrational thoughts, perceptions and socio-affective variables jointly explain a larger proportion of the variance in Mathematics achievement than any one of these factors on its own.

Rationale

A number of studies have simultaneously explored several variables which can account for variability in Mathematics achievement. For instance, Wang et al (2012:1215) examined how Mathematics self-concept, teacher perceptions and other school factors related to learners' Mathematics achievement. Learners' affective factors such as self-efficacy, scholastic self-concept, Mathematics interest and Mathematics anxiety were scrutinised by Ferla et al (2009:499-505) relative to their

mutual relationship and how they jointly mediate learners' Mathematics achievement. Mbugua et al (2010:88) examined the joint influence of variables such as teachers' attitudes, availability of teaching materials, motivation, learners' attitudes and parents' socioeconomic status on learners' Mathematics achievement (section 1.1).

An allied study by Nzuki (2010:109) established that the ultimate quality of learners' Mathematics achievement can be influenced by the identity formation which is mediated by a diversity of variables such as their teachers' expectations and socioeconomic status. In a study exploring university students' achievement goals and approaches to learning Mathematics, Cano and Berben (2009:149) acknowledged that the Mathematics achievement of learners is jointly influenced by a variety of variables which entail learners' conceptions of Mathematics as a subject, their personal goals of learning and their perceptions of the quality of instructions. A holistic approach for exploring the variables which influence Mathematics learners' experiences in the classroom was recommended (Cano & Berben, 2009:149). Abubakar and Adegboyega (2012:121) reiterated the same view by recommending that a number of variables need to be analysed simultaneously to find out the determinants of Mathematics achievement. The multivariate nature of the studies to some extent provides a rationale for the assertion that irrational thoughts, perceptions and socio-affective variables jointly explain a greater proportion of the variability in Mathematics achievement than any one of these factors considered singly.

4.3 RESEARCH DESIGN

In this section, an outline of the population, the sampling method and the sample composition and characteristics are given.

4.3.1 Population, Sample and Sampling Procedure

4.3.1.1 Population

A population can be defined as the total number of people, items or events from which particular research information could be gathered (Chiromo, 2006:16; Haralambos & Holborn, 2008:817). This means that a population is a set of all the individuals, objects or units that acts as a group of interest in a particular study

(Feldman, 2009:A15; Shastri, 2008:116; Kosslyn & Rosenberg, 2006:59). In the current study, the population comprised all the form 3 and form 4 learners studying Mathematics in Masvingo Province in Zimbabwe. In Zimbabwe, secondary education has three levels; namely, junior level, Ordinary level and Advanced level. Form 3 and form 4, jointly called Ordinary level, follow the junior secondary school level which is almost equivalent to grade 10 in some countries. These grades are done in the third and fourth years of learners' secondary education respectively. The target population had approximately 2 700 Ordinary level learners.

4.3.1.2 Sampling procedure

Sampling is a process of choosing individuals to participate in a study after establishing that they can supply information which is relevant to the research (Oppong, 2013:203; Onwuegbuzie & Collins, 2007:281). The simple random sampling method was employed in the selection of secondary schools and the research participants. This method was used as a deliberate endeavour to make the sample sufficiently representative of the population and reduce bias (Chiromo, 2006:17; Cohen et al, 2011:153). A representative sample is one in which the sample members are typical of the entire population, that is, the sample members accurately and fairly reflect the characteristics of the population (Giddens, 2009:53; Chiromo, 2006:16).

Using the letter from the Secretary of the Ministry of Primary and Secondary Education in Zimbabwe, the researcher visited the Provincial Education Director of Masvingo Province to be granted permission to visit secondary schools in the province. Thereafter, the researcher approached the District Education Officer so as to be granted written permission to visit specific secondary schools in Masvingo Province. A list of co-educational secondary schools, that is, schools in which male and female learners learned together in Masvingo Province, was drawn up. This was important because gender is one of the variables to be scrutinised in the exploration of the relationship between irrational beliefs and Mathematics achievement. After generating a list of such secondary schools, the researcher assigned a number to each secondary school. Thereafter, the table of random numbers was used to select five secondary schools from the list.

The principal of each of the five randomly chosen secondary schools was given a letter seeking permission to undertake the study at the school. The letter highlighted issues such as the thesis title, the nature of the study, the fact that questionnaires were the main data gathering instruments with secondary school learners as the main respondents, and the need to have access to the learners' Mathematics examination scores which were kept at the schools. The letter also outlined the fact that participation in the study was voluntary and that the respondents retained the right to withdraw from the study at any time without any form of reprisal. The letter also specified that confidentiality and anonymity would be upheld. Principals of three of the five randomly selected secondary schools visited by the researcher agreed to have the empirical investigation undertaken at their schools.

The first school, whose pseudonym is School A, was a relatively small day school with an approximate Ordinary level enrolment of 450 learners. By virtue of being a day school, School A learners lived with their parents or legal guardians. The second school, School B, had both day scholars and boarders. This school had a large enrolment of approximately 750 Ordinary level learners. By virtue of being a boarding school, School B had learners from virtually all over Zimbabwe. To some extent, this ensured that the learners at this school had a semblance of national representation. It was for this reason that the researcher sampled more respondents from School B than from the other two schools. The third school, identified as School C, was a day school which had a total enrolment of about 300 Ordinary level learners. Some of the learners lived in Masvingo peri-urban where they had to travel at most 10 kilometres to get to the school. Just like the learners at School A, the learners at School C lived at their respective homes with their parents or legal guardians.

With the help of secondary school teachers and school heads at each of the three abovementioned schools, the researcher distributed letters to the form 3 and form 4 day scholars addressed to their parents or legal guardians seeking the permission of the parents or legal guardians to allow their children or dependents to participate in the study. The letter supplied the parents or legal guardians with the necessary information for them to make informed decisions. The voluntary nature of the study was spelt out together with the fact that the learners retained the right to withdraw from the study prematurely without any penalty. The letter also stressed the fact that

the data collected would be used strictly for research purposes. After the distribution of the letter to the learners' parents or legal guardians, the researcher waited for the learners to return the parents' or legal guardians' signed letters confirming that they had allowed their children or dependants to participate in the study. After the parents or legal guardians' signed assent forms were returned to the schools by the day scholars, the researcher ascertained their number.

Thereafter, child assent forms were given to the learners whose parents or legal guardians had granted them permission to participate in the study. The child assent forms provided the learners with sufficient information to make informed decisions regarding whether to participate in the study or not. School teachers in the Mathematics Department at each school assisted with the distribution of the child assent forms. The researcher explained every aspect on the child assent form at each of the three secondary schools. The assisting Mathematics teachers also helped to clarify the contents of the forms. At each of the three secondary schools, the researcher did not ask the learners to sign the child assent forms promptly. The learners were allowed to take the child assent forms home or to their halls of residence and read them several times before deciding whether to participate in the empirical investigation or not. The signed child assent forms were collected from the learners a day after their distribution. A total of 306 secondary school learners expressed their willingness to take part in the study by signing the child assent forms.

Secondary school learners who signed the child assent forms after their parents or legal guardians had granted them permission to take part in the study eventually constituted the sample: 72 such learners from School A, 177 from School B and 57 from School C, giving a total sample size of 306 respondents.

4.3.1.3 Sample composition and characteristics

A sample of 306 adolescent secondary school Mathematics learners was chosen using the simple random sampling method to take part in the study as outlined in section 4.3.1.2. Please find below an outline of the sample composition and the characteristics of the respondents.

4.3.1.3.1 Distribution of respondents by gender and school

Table 4.1 shows the distribution of research participants with respect to gender and school:

Table 4.1: Gender-School Cross-tabulation

		School			Total
		A	B	C	
Gender	Male	33	64	27	124
	Female	39	113	30	182
Total		72	177	57	306

Table 4.1 reveals that 177 of the respondents attended School B while 72 and 57 respondents attended School A and School C respectively. Relative to gender, 124 respondents were male and 182 female. At School A and School C, the number of female sample members was marginally above that of their male counterparts. The number of female respondents at School B was almost twice that of boys at that school. This gender distribution is consistent with the overall demographic distribution of the entire Zimbabwean population in which females constitute 51.9% of the population while males comprise 48.1% (Zimbabwe National Statistics Agency, 2013:17). However, the proportion of boys, that is, 124 of 306 in the sample is quite significant. Therefore, it can be argued that the sample was reasonably balanced relative to gender.

4.3.1.3.2 Distribution of respondents by gender and form

The simultaneous consideration of the research participants' gender and form produced the following distribution:

Table 4.2: Gender-Form Cross-tabulation

		Form		Total
		Form 3	Form 4	
Gender	Male	75	49	124
	Female	116	66	182
Total		191	115	306

The research participants were doing their Ordinary level course, that is, forms 3 and 4 according to the Zimbabwean education system. Table 4.2 shows that 191 research participants were in form 3 while 115 were in form 4. It can be argued that the number of form 4 learners, that is, 115 out of 306, is so significantly high that their contribution could not be ignored. By virtue of having been in school for more years than their form 3 counterparts, the contributions of the 115 form 4 learners who took part in the study could be informative. In both forms 3 and 4, the number of male respondents was fewer than the number of girls. However, the difference between the number of girls and boys in form 4 was smaller than the difference between the frequencies of form 3 girls and boys. The drop in the difference between the number of female and male respondents in form 4 can be accounted for by the fact that female learners tend to drop Mathematics at form 4 level as a result of internal and external variables.

4.3.1.3.3 Distribution of respondents by age

Relative to age, the composition of the respondents was as shown below:

Table 4.3: Distribution by Age

Age in years	Frequency
14	9
15	103
16	145
17	40
18	9
Total	306

Table 4.3 shows that 103 research participants were 15, while 145 were 16 years old. Forty learners out of 306 were 17 years old while equal numbers were 14 and 18. The modal age of the research participants was 16 years. Such a distribution confirms that the majority of the respondents were in middle adolescence. The ages of the respondents seem to be relatively normally distributed with a mean of 15.79 years and a median of 16 years. The variance and the standard deviation of the respondents' ages were 0.662 and 0.814 respectively as shown in Table 4.4. These two measures of variability indicate that the research participants' ages were closely packed around the mean.

Table 4.4: Descriptive Statistics of Respondents' Ages

Mean	Median	Mode	Standard deviation	Variance	Total frequency
15.79	16	16	0.814	0.662	306

4.3.1.3.4 Distribution of respondents by form and age

The following is the composition of the research participants relative to form and age:

Table 4.5: Form-Age Cross-tabulation

		Age in years					Total
		14	15	16	17	18	
Form	Form 3	9	100	68	9	5	191
	Form 4	0	3	77	31	4	115
Total		9	103	145	40	9	306

Table 4.5 shows that no form 4 learner who participated in the study was 14 years old. Of the 103 respondents who were 15 years old, only three were in form 4 while 100 were in form 3. In the 16 year age group, 68 research participants were in form 3 while 77 were in Form 4. This means most of the form 3 learners were 15 years old while the modal age for the form 4 learners was 16 years old. More form 4 than form 3 learners were at least 17 years of age. This can be explained by the fact that form 4 learners had been in school for more years than their form 3 counterparts.

4.3.1.3.5 Distribution of respondents by gender and age

The cross-tabulation showing the respondents' distribution by gender and age is given below:

Table 4.6: Gender-Age Cross-tabulation

		Age in years					Total
		14	15	16	17	18	
Gender	Male	3	34	59	22	6	124
	Female	6	69	86	18	3	182
Total		9	103	145	40	9	306

Table 4.6 shows that 34 boys were 15 years old while 69 girls were the same age. The group of 59 16-year-old boys who took part in the study was the same age as the 86 16-year-old girls. Of the 40 adolescents who were 17 years old, 22 were male

and 18 female. In the 14 year old category, the number of girls was twice the number of boys while the number of boys was twice the number of girls in the 18 year old category. The table shows a larger number of older boys than older girls but a smaller number of younger girls than younger boys.

4.3.1.3.6 Distribution of respondents by form and school

The distribution by the research participants relative to form and school was as follows:

Table 4.7: Form-School Cross-tabulation

		School			Total
		A	B	C	
Form	Form 3	72	99	20	191
	Form 4	0	78	37	115
Total		72	177	57	306

All 72 research participants at School A were in form 3 as shown in Table 4.7. This was so because the form 4 learners at School A were said to be busy with their studies and the school authorities suggested that inviting the form 4 learners to participate in the study would negatively affect their academic work. At School B, 99 form 3 learners and 78 form 4 learners participated in the study. At School C the number of form 4 learners who took part in the study was almost twice the number of form 3 participants. No restrictions were imposed by the authorities at School B and School C regarding the participation of form 4 learners.

4.3.1.3.7 Distribution of respondents by form and home language

The composition of the research participants relative to form and home language is given below:

Table 4.8: Form-Home Language Cross-tabulation

		Home Language			Total
		Shona	Ndebele	English	
Form	Form 3	161	6	24	191
	Form 4	87	4	24	115
Total		248	10	48	306

Table 4.8 confirms that the majority of the respondents, that is, 248 of 306, used Shona as their home language. Of the 248 Shona-speaking respondents, 161 were in form 3 and 87 in form 4. Only 10 research participants used Ndebele as their home language while 48 respondents, that is, 24 in form 3 and 24 in form 4, used English as their home language. Such a distribution occurred because Shona – a language that has many dialects – is essentially the most prevalent native language not only in Masvingo Province in which the study was undertaken, but in Zimbabwe as a whole. Ndebele is another relatively prevalent language in Zimbabwe but it is mostly used in the provinces in the western side of Zimbabwe. Shona and Ndebele are Zimbabwean native languages while English is an international language. Zimbabwean learners are exposed to English upon commencing elementary education at the tender age of about five or six. Although most of the learners who took part in the study did not use English as their home language, almost all of them were fluent in it. This is so because English is prevalently used as a medium of instruction in Zimbabwe in the majority of subjects from kindergarten to university. Although the distribution of the respondents' home languages is not uniform, it can be contended that the three most dominant languages in Zimbabwe are at least represented in the sample.

4.3.2 Measuring Instruments: Questionnaires

The only data gathering instrument employed in the current study was a questionnaire. A questionnaire was opted for because of its suitability for generating quantitative data which could be subjected to statistical analysis. The various sections of the questionnaire are outlined below.

General instructions which guided the respondents as they completed the questionnaire were on the first page of the questionnaire. The instructions indicated that the questionnaire was not a test but a mere information exercise in which the research participants were expected to express their opinions in approximately one hour. The need to respond spontaneously and honestly to each statement was emphasised. The respondents were also informed that the entire research process would remain confidential and that the supplied information would be statistically processed by a computer. They were also told that in section A, one number was to be entered in each block as they completed the questionnaire. All the questionnaire items were in English. In Zimbabwe, most learners are quite fluent in English by the time they reach secondary school level, that is, after having studied it for at least seven years during primary education.

4.3.2.1 Section A: Secondary school learners' demographic data

The first section of the questionnaire contained items on the demographic details of secondary school learners. Such personal information entailed school, gender, form and home language. Regarding schools, three options in the form of pseudonyms featured on the questionnaire. There were only two options for grade level, that is, form, since the respondents were either in form 3 or form 4. Shona, Ndebele and English were given as the three options for learners' home language.

4.3.2.2 Section B

Section B of the questionnaire contained 282 items to be answered using a six point Likert scale as shown below:

This is definitely the case 6 5 4 3 2 1. This is not the case at all

The Likert scale key featured as a footnote on all the pages of Section B of the questionnaire to assist the respondents as they completed the questionnaire. Although section B of the questionnaire had no obvious subsections, the items requiring direct input from the respondents were divided into four categories as shown in Table 4.9.

Table 4.9: Variables Explored in Section B of the Questionnaire

Category	Variables covered	Item numbers	Number of items
1	Socio-Affective Factors <ul style="list-style-type: none"> • Self-concept • Motivation • Stress • Anxiety • Teacher-Learner relationships • Peer relations 	1-122	122
2	Irrational Belief Test	123- 222	100
3	Parental Involvement	223-232	10
4	Perceptions <ul style="list-style-type: none"> • The Inevitable • The Role of Rewards • School and Schooling • Teachers and Parents 	233- 282	50
Total			282

4.3.2.2.1 Irrational Belief Test

The items for the irrational belief test were developed based on Albert Ellis's ten major irrational beliefs discussed in section 3.3.1.3. This part of the questionnaire

arguably remains the fulcrum of the questionnaire because it contains items which explore irrational beliefs, a key variable in the current study. The original Irrational Belief Test contained 100 items which were distributed in such a way that ten items were generated from each of the ten irrational beliefs advanced by Ellis. The original Irrational Belief Test had ten subscales focusing on the ten main irrational beliefs as theorised by Ellis. The items were arranged in a periodic and cyclic manner, so that items 1, 11, 21, up to 91 focused on the first irrational belief while items 2, 22, 32, up to 92 dealt with the second irrational belief and so on.

All 100 items in the original Irrational Belief Test were adapted so as to deal with irrational thoughts in Mathematics. The cyclic and periodic nature of the items was not altered during the adaptation. Consequently, the reliability of the resultant subscales for each of the 10 irrational beliefs is established in the fifth chapter of the current study using the data collected during the empirical investigation. Each of the 10 subscales is briefly discussed below relative to Ellis's major irrational beliefs and in each instance three examples of items are given.

4.3.2.2.1.1 The belief that it is a dire necessity for people to be loved by significant others in virtually everything they do

This belief revolves around the demand for approval, respect, recognition and/or love (Austad, 2009:277; Egan, 2007:141; Boyacioglu & Kucuk, 2011:447). Emphasis is placed on trying to evade disapproval and criticism in virtually all situations (Rosner, 2011:83) (section 3.3.1.3.1). The ten items based on the above irrational belief were modified to deal with irrational beliefs applied to the Mathematics experiences of adolescent secondary school learners. Below are some examples of adapted items belonging to the first subscale:

- It is important that Mathematics teachers always approve the work I do.
- I want to receive approval from my Mathematics teachers.
- I am very concerned about what my Mathematics peers say about me.

4.3.2.2.1.2 The belief that we should be thoroughly competent, intelligent, and achieving in all possible respects.

The belief demands that an individual demonstrate high levels of competence and intelligence as well as being able to perform given tasks perfectly (Civitci, 2007:3; Thompson & Henderson, 2007:10) (section 3.3.1.3.8). The irrational belief expresses a learner's unrealistic expectation to be utterly infallible when solving Mathematics problems (Ellis, 1994:130; Domino, 2005:7). Examples of items which dealt with the second irrational belief in a Mathematics learning context at secondary school level are given below:

- I hate to fail Mathematics tasks at school.
- It is highly important for me to be successful in all the Mathematics activities I do.
- I often become annoyed over the slightest failures I experience in Mathematics.

4.3.2.2.1.3 The belief that certain acts are awful or wicked and that people who perform such acts should be severely damned

As outlined in section 3.3.1.3.2, the above irrational belief revolves around the demand for fairness and justice (Seligman & Reichenberg, 2010:255). People who are guided by this irrational belief exercise conditional other-acceptance and are prone to blaming other people with the intention of punishing or condemning them (Egan, 2007:141; Rosner, 2011:84). Below are some of the adapted items as they appeared on the questionnaire:

- Those Mathematics learners doing wrong at school deserve to be penalised.
- I am quick to blame other Mathematics learners when they do something wrong.
- 'It is unfair that Mathematics learners who do wrong get away with it.

4.3.2.2.1.4 The belief that it is horrible and catastrophic when things are not the way we like them to be

This is an irrational belief in which demandingness and awfulising are manifested through being idealistic (Froggatt, 2005:3; Rosner, 2011:83) (section 3.3.1.3.3). The

irrational belief portrays the demand for comfort and being reactive to frustration. Examples of items in this subscale include the following:

- My experience in Mathematics is a disaster because my ideals will never materialise.
- I find it impossible to accept things which happen to me during Mathematics lessons.
- My ideals in Mathematics and reality will never meet.

4.3.2.2.1.5 Irrational beliefs pertaining to emotional irresponsibility

The irrational belief *that human misery is invariably externally caused and is forced on us by outside people and events* (section 3.3.1.3.4) is sometimes combined with the irrational belief that *we have virtually no control over our emotions and that we cannot help feeling disturbed about things* (section 3.3.1.3.12). The resultant compound irrational belief is the belief that *human emotional misery is mostly externally caused and that people have little or virtually no ability to regulate their emotional sorrows and disturbances*. This combined irrational belief expresses gross emotional irresponsibility on the part of the bearer of the belief. Attributing one's misery to other people and variables beyond one's control is a form of irrationality which can trigger perennial sadness, anxiety and dependence (Froggatt, 2005:3; Egan, 2007:141; Rosner, 2011:84). Moreover, confessing one's inability to regulate one's affective domain fosters anxiety and stress (Woodward et al, 2001:258; Austad, 2009:277). In Mathematics, learners who suffer from emotional irresponsibility are likely to be upset and anxious when their Mathematics achievement fails to meet their expectations.

Adaptations were made to the statements on the original Irrational Belief Test to suit a secondary school Mathematics teaching and learning context. For example:

- Other people in my Mathematics class cause me to be depressed.
- I find it impossible to be happy at school because of the challenges concerning Mathematic.
- Studying Mathematics causes my entire life to unhappy.

4.3.2.2.1.6 The belief that if something is or may be dangerous or fearsome we should be terribly upset and endlessly obsess about it

As discussed in section 3.3.1.3.5, the above irrational belief can cause an individual to obsess about a seemingly threatening situation thereby becoming vulnerable to anxiety and feelings of helplessness (Cardenoso & Calvete, 2004:289; Austad, 2009:277; Egan, 2007:141). Asikhia (2010:206) and Saraswathi (2003:326) asserted that Mathematics is viewed by some learners as a demanding or threatening subject thereby increasing the probability of anxiety on the part of the learners who anxiously obsess over the threats of life (section 3.3.1.3.5). Below are three examples of the modified items on the sixth irrational belief which featured on the questionnaire used in the current study:

- Mathematics tasks can cause me to be miserable for days.
- Mathematics problems at school continually bother me.
- I experience anxiety wherever I think about my Mathematics achievement.

4.3.2.2.1.7 The belief that it is easier to avoid than to face life difficulties and self-responsibilities

As discussed in section 3.3.1.3.6, the above irrational belief revolves around problem avoidance which can lead to inaction and/or procrastination (Froggatt, 2006:4; Austad, 2009:277; Egan, 2007:141; Rosner, 2011:84; Cardenoso & Calvete, 2004:289). The above irrational belief is also closely related to the belief that *human happiness can be achieved by inertia and inaction* (section 3.3.1.3.11). Inertia can trigger anxiety which can interfere with the functioning of the working memory (Ullusoy & Duy, 2013:1441). Below are some examples of items in this subscale:

- I avoid solving difficult Mathematics tasks.
- I normally ignore Mathematics tasks that are difficult and unfamiliar.
- I do not like Mathematics tasks which require hard work.

4.3.2.2.1.8 The belief that we absolutely need something other or stronger or greater than ourselves on which to rely

The above irrational belief expresses an individual's dependence mentality whenever there is a task to be undertaken (Froggatt, 2006:4; Austad, 2009:277). People with such a belief are likely to have low levels of self-efficacy as discussed in section 3.3.1.3.7. Examples of such items as they featured on the modified questionnaire are given below:

- I cannot do Mathematics assignments without the support of my peers.
- There are certain people at school that I depend on greatly when doing Mathematics.
- Without someone to support you in doing Mathematics assignments, you are doomed.

4.3.2.2.1.9 The belief that because something once strongly affected our life, it should indefinitely affect it

People whose mental philosophies entail this belief do not have the hope of recovering from past tragedies and disasters (Froggatt, 2006:4; Thompson & Henderson, 2007:10; Rosner, 2011:84) (see section 3.3.1.3.9). Learners' previous academic performance in Mathematics can be related to their later scholastic achievement especially if they are guided by the above irrational belief (Snowman, et al, 2009:274; Sternberg, 2009:341) (section 2.2.1.5). Some of the resultant adaptations of the original Irrational Belief Test to suit a secondary school Mathematics teaching and learning context are given below:

- It is almost impossible to overcome your past failures in Mathematics.
- Things from my past have a major influence on my Mathematics achievement.
- I cannot succeed in freeing myself from my previous bad experiences in Mathematics.

4.3.2.2.1.10 The belief that there is invariably a right, precise and perfect solution to human problems and it is catastrophic if this perfect solution is not found

The above irrational belief is centred on perfectionism and an absolutistic thinking philosophy in which one is guided by 'oughts', 'musts' and 'shoulds' as discussed in

sections 3.3.1.3.10 and 3.3.1.3.13. The irrational belief is an expression of demandingness and if applied in a Mathematics context, one's Mathematics achievement can be compromised owing to negative emotions such as depression, anxiety and distress (Scarpello, 2007:34). In an attempt to explore the irrational beliefs on perfectionism in a Mathematics context at secondary school level, some adaptations were made to the original Irrational Belief Test. Below are examples of some adapted items which were used in the current study:

- To be successful in Mathematics, you have to do everything in the correct way.
- You are only successful in Mathematics if you do everything perfectly.
- Practical mathematical solutions are of no help if they are not ideal solutions.

4.3.2.2.2 Learners' perceptions

It was highlighted in section 1.1 that learners' perceptions can be related to their scholastic achievement in Mathematics. Consequently, the questionnaire contained items concerning learners' perceptions of the teaching and learning of Mathematics. Allied socio-affective factors were also incorporated into the items which examined learners' perceptions. The items constituting the fourth part of section B of the questionnaire were adapted from section B of a perception questionnaire which explored learners' perceptions of schooling in general (Chetty, 1996:138). The original perception questionnaire used a four point Likert scale with the following key:

Agree entirely 4 3 2 1 Disagree entirely

As already pointed out, the questionnaire used for empirical investigation in the current study had a six point Likert scale as shown below:

This is definitely the case 6 5 4 3 2 1 This is not the case at all

Chetty (1996:138) developed the original perception questionnaire using factor analysis and four main factors were identified. The four factors were The Inevitable, The Role of Rewards, School and Schooling and Teachers and Parents. To some extent, the perceptions under each of the four factors were faulty. Since the items on the original questionnaire were altered for use in the current study, the reliability of the new items are established statistically in the next chapter.

Table 4.10: The Four Perception Factors

Factor	Number of items
The Inevitable	15
The Role of Rewards	8
School and Schooling	15
Teachers and Parents	12
Total	50

Table 4.10 shows the number of items in each of the four perception factors. Each of the above perceptions is discussed below and three examples of modified items under each factor are given.

4.3.2.2.2.1 The Inevitable

According to Chetty (1996:138), The Inevitable refers to learners' perceptions of scholastic achievement which significant others in society have imposed on the learners. The norms and values acquired by learners during socialisation are part of The Inevitable which influences the way they interpret events (Chetty, 1996:138). The Inevitable focuses on the perceptions which seem to be beyond the control of the learners (Chetty, 1996:138). Issues pertaining to gender are one such example. Fifteen items belong to the first factor as shown in Table 4.10. Below are examples of items which belong to The Inevitable:

- Teachers expect better Mathematics results from boys than from girls.
- The atmosphere in the classroom plays an important role in determining how one performs in Mathematics.
- It is important for boys to do well in Mathematics as they are the future breadwinners.

4.3.2.2.2.2 The Role of Rewards

In essence, this factor deals with learners' perceptions of the anticipated rewards from significant others when the learners attain good academic results (Chetty, 1996:139). Learners' perceptions of the justification for striving to attain good academic results are part of The Role of Rewards (Chetty, 1996:139). Table 4.10

shows that eight items pertain to The Role of Rewards. The following are some of the items dealing with The Role of Rewards:

- One should study hard and do well in Mathematics to get social recognition and status.
- Doing well in Mathematics is important because society places such a great emphasis on good Mathematics achievement.
- Parents should always reward children whenever they do well in Mathematics.

4.3.2.2.2.3 School and Schooling

According to Chetty (1996:140), learners' perceptions of School and Schooling focus on the extent to which learners perceive school and schooling as a vital aspect of their lives. This factor deals with the atmosphere cultivated by school administrators at the school in general and in the classroom in particular (Chetty, 1996:140). School and Schooling also covers the usefulness of school activities in solving real life existential problems as perceived by learners (Chetty, 1996:140). In the current study, the aspects of school and classroom management are discussed in section 2.3.3.1. Fifteen items pertaining to School and Schooling feature on the questionnaire as shown on Table 4.10. Below are examples of items which are classified under School and Schooling in a Mathematics learning context:

- Much of what is taught during Mathematics lessons nowadays is irrelevant and unimportant.
- The Mathematics curriculum does not allow for one's independent thoughts and ideas.
- One gets better Mathematics results in a small school than in a large school.

4.3.2.2.2.4 Teachers and Parents

The fourth and final factor focused on the attitudes of teachers and parents towards learners and their scholastic achievement. Other aspects covered under Teachers and Parents include the instructional approaches employed by teachers, parenting styles and parental involvement in learners' academic work (Chetty, 1996:140). Teachers and Parents are the adults in learners' lives who play a significant role in

shaping learners' beliefs and perceptions of Mathematics in particular and education in general. In the current study, parenting styles are discussed in section 2.3.2.1 while parental involvement is elaborated in section 2.3.2.2. The quality of teaching instructions, teacher expectations, the personality of teachers and the role of teachers in moulding beliefs are highlighted in sections 2.3.3.2, 2.3.3.3, 2.3.3.4 and 3.4.4.3 respectively. Twelve items on the questionnaire measure this factor as shown in Table 4.10. Examples of items pertaining to Teachers and Parents are given below:

- Mathematics teachers largely determine one's opinion of oneself.
- Good Mathematics teachers present their lessons in an exciting and challenging way.
- Parents place greater emphasis on Mathematics achievement than anything else.

4.3.2.2.3 Learners' socio-affective variables

Learners' socio-affective factors were explored in relation to Mathematics teaching and learning at secondary school level. The adapted questionnaire was initially designed by Bester (2003: 187-190) to explore drama learners' affective experiences as they undertook their drama studies. The drama questionnaire has 122 items which were responded to using the following six point Likert scale:

This is exactly how I experience it 6 5 4 3 2 1 This is not at all how I experience it

All 122 items on the drama questionnaire were adapted to deal with the teaching and learning of Mathematics at secondary school level. The items attempt to measure Mathematics learners' socio-affective variables such as motivation, anxiety, stress, self-concept, teacher-learner relations and peer relationships. The frequency distribution of these socio-affective variables is shown in Table 4.11.

Table 4.11: Learners' Socio-Affective Variables

Socio-Affective Variable	Number of items
Motivation	20
Stress	20
Anxiety	20
Self-concept	20
Teacher-learner relations	21
Peer relations	21
Total	122

4.3.2.2.3.1 Motivation

As already discussed under hypothesis 4, motivation is a crucial affective variable in determining learners' Mathematics achievement (sections 2.2.2.1 and 3.4.1.8.1). Several studies have established that there is a positive correlation between learners' motivation and their academic performance in Mathematics (Mbugua et al, 2012:88; Tella, 2007:154; Mousoulides & Philippou, 2005:321). Table 4.11 shows that 20 of the first 122 items in section B of the questionnaire measure the motivation of Mathematics learners. Below are examples of such items after they have been adapted for use in the current study:

- I am always motivated to attend Mathematics lessons.
- I always come up with reasons not to study Mathematics.
- I am usually enthusiastic when I begin to learn a new Mathematics topic but after a while my enthusiasm declines.

4.3.2.2.3.2 Stress

Stress as a socio-affective variable was explored under hypothesis 4 as well as in chapter 2. The maturational turbulence associated with adolescence makes adolescent learners particularly prone to stress (Sulaiman et al, 2008:180). Common stressors during adolescence include academic work such as tests and homework, strained relationships with parents and siblings, high teacher expectations and

identity diffusion (Kauts & Sharma, 2009:39; Lohman & Jarvis, 2000:15; Kaplan et al, 2005:3; Beilock, 2008:359-343; Byrne et al, 2007:393). It was on the basis of such a background that some items concerning stress were included in the questionnaire. Examples of items in this category are given below as they appeared after adaptation:

- My Mathematics studies frustrate me because of too much work and too little time.
- I often become tense because of the amount of work in Mathematics.
- Because of Mathematics I cannot relax any more.

4.3.2.2.3.3 Anxiety

Anxiety in general and Mathematics anxiety in particular were explored in chapter 2 relative to their influence on Mathematics achievement (sections 2.2.2.4.1 and 2.2.2.4.2). Anxiety has been found to be related to Mathematics achievement (Zakaria & Nordin, 2008:30; Scarpello, 2007:34; Smith, 2004:3). In section 3.3.1.2, it was revealed that anxiety is one of the outcomes of irrational thinking (Ciarrochi, 2004:173; Palmer, 2000:81; Austad, 2009:275). Anxiety was also discussed in conjunction with a number of irrational beliefs and Mathematics achievement (section 3.4.1.4). In the current study, the anxiety of secondary school Mathematics learners was measured by means of 20 items as shown in Table 4.11. Below are examples of such items:

- I experience more anxiety during Mathematics tests than other Mathematics learners because of my own insecurity.
- I feel anxious when I write Mathematics tests because I do not want to disappoint my Mathematics teacher.
- Before a Mathematics test I am almost in tears because of anxiety.

4.3.2.2.3.4 Self-concept

Among the learner affective variables which can relate to Mathematics achievement is self-concept (section 2.2.2.2). Self-concept has also been associated with irrational beliefs pertaining to the demand for approval and acceptance as well as the

irrational belief that one will always be influenced by past disasters (section 3.4.1.3). Moreover, some studies have revealed that Mathematics achievement can be mediated by learners' self-concept (Wang et al, 2012:1242; Ferla et al, 2009:499-505; Nzuki, 2010:109). Consequently, the questionnaire used for empirical investigation in the current study has some items that measure the self-concept of Mathematics learners at secondary school level. Twenty of the 122 items measure self-concept as shown in Table 4.11. Examples of items which measure self-concept in a Mathematics context are given below:

- I have hope for myself as a Mathematics learner.
- As a Mathematics learner I often doubt myself and what I can achieve.
- I do not have enough self-confidence to participate during Mathematics lessons.

4.3.2.2.3.5 Teacher-learner relationships

Teaching and learning activities involve the interaction between teachers and learners. The teaching methods employed by teachers and the classroom management styles they adopt can determine the quality of teacher-learner relationships which prevail in the classroom (sections 2.3.3.1 and 2.3.3.2). Learners prefer authoritative to authoritarian classroom management (Osler, 2010:80; Kumar, 2004:115). As hinted in chapter 1, adolescents normally want to rebel against figures of authority who include teachers (Steinberg & Morris, 2001:88). The nature and quality of teacher-learner relationships can also be influenced by teachers' expectations regarding learners' Mathematics achievement (Hinnant et al, 2009:662; Snowman et al, 2009:106) (section 2.3.3.3.2). Many authorities acknowledge that the instructional principles based on the theoretical perspectives of Ausubel and Gagne are quite conducive to good Mathematics achievement (Ifamuyiwa, 2011:129; Sipe, 2001:336; Hill et al, 2005:371; Awolola, 2011:91-106) (sections 2.3.3.2.2 and 2.3.3.2.3). As discussed in section 2.3.3.4, teachers' personality attributes can also influence the quality of the prevailing teacher-learner relationships (Garcia et al, 2011:3; Miller & Satchwell, 2006:136). In a bid to explore teacher-learner relationships in a Mathematics teaching and learning context, 21 of the 122 items

feature in the questionnaire. Below are examples of items examining teacher-learner relationships:

- I often clash with my Mathematics teacher about instructions given to me.
- My Mathematics teacher is unnecessarily critical of my performance.
- I do not concern myself with my Mathematics teacher's point of view; I mostly do what I want.

4.3.2.2.3.6 Peer relations

It has been confirmed that adolescents have a tendency to spend more time with their peers than with their parents or siblings (Tuckman & Monetti, 2011:117; Kufakunesu et al, 2013:829; Steinberg & Morris, 2001:93). Peer relations act as a bureau in which adolescents learn to solve problems, express their emotions and gain vital values about life away from their parents or guardians (Tuckman & Monetti, 2011:117). In section 3.4.2.1, peer relations during adolescence and their possible influence on the learners' Mathematics achievement were explored. In sections 3.4.2.2 and 3.4.2.3 the possible inter-relationship between adolescent peer relations, irrational beliefs and Mathematics achievement were examined. Therefore, 21 items pertaining to peer relations in the secondary school Mathematics teaching and learning context feature in the questionnaire. Examples of statements in this group are stated below:

- I am a cheerful Mathematics learner who easily gets along with others.
- I definitely have fewer friends than most of my Mathematics peers.
- Friends make me feel inferior regarding my Mathematics achievement.

4.3.2.2.4 Parental involvement

The role of parental involvement in determining learners' scholastic achievement in Mathematics was discussed in sections 2.3.2.1, 2.3.2.2 and 2.3.2.3. The third part of section B of the questionnaire attempts to expose the respondents' views regarding the contribution of parental involvement to learners' Mathematics experiences. Just like the other sections of the questionnaire, these items were adapted from a questionnaire which examined parental involvement and the school in general.

Adaptations were made to measure the level of involvement of Mathematics learners' parents in their children's academic work.

The ten items which constituted the third part of section B of the questionnaire addressed issues such as the support and encouragement which parents extend to their children regarding studying Mathematics; the efforts made by parents to contact their children's schools; and the extent to which the parents request updates on their children's academic progress in Mathematics. Five items in this category requested learners to rate their parents' encouragement and support meant to enhance the learners' Mathematics achievement. Three statements explored the efforts made by parents to get in contact with the children's teachers and school authorities. The remaining two items from this part of the questionnaire dealt with establishing the extent to which parents request learners to furnish them with updates concerning the learners' progress in Mathematics. Examples of such items are:

- My parents often show appreciation when I have done well in Mathematics.
- My parents often encourage me to do well in Mathematics.
- My parents frequently contact the school if they are uncertain about my Mathematics achievement.

4.3.2.2.5 Mathematics achievement

Secondary school learners' Mathematics achievement remains a fundamental dependent variable in the current study. In an attempt to gain a better understanding of each respondent's Mathematics achievement, a Mathematics examination score for each research participant was obtained from the respondents' respective school authorities after permission was sought and granted. The Mathematics scores were outcomes of the examinations written by the respondents at their schools under formal examination conditions. The examinations covered a wide range of Ordinary level Mathematics concepts. The respondents did not record their Mathematics examination scores on the questionnaires. The researcher recorded each learner's respective Mathematics examination score on the corresponding questionnaire after the learners had completed all the other sections of the questionnaire. This was done to ensure that the authenticity of the information was not compromised as some learners might have entered inaccurate Mathematics scores.

4.3.3 Ethical Considerations

In research, ethics are principles of right and wrong that guide people undertaking research in a bid to protect the respondents, the researchers and the researchers' professions (Chiromo, 2006:10; Kufakunesu et al, 2013:833). In the current study, the main ethical principles observed were informed consent, confidentiality, anonymity and non-maleficence. Informed consent is an ethical principle which stipulates that the potential research participant should agree to take part in the study after being furnished with all the relevant research details such as the procedure, risks and benefits of participating in the study (Cohen et al, 2011:77; Keenan, 2002:66; Kufakunesu et al, 2013:833; Lahey, 2009:43; Feldman, 2009:49; Kosslyn & Rosenberg, 2006:29). In an attempt to gain the respondents' informed consent, the researcher supplied all the relevant information pertaining to the study as evidenced by the letters to secondary school principals, parents or legal guardians as well as the child assent forms.

Anonymity is an ethical principle which requires that the names of the research participants should not be disclosed especially during the analysis and discussion of the research findings (Kufakunesu et al, 2013:833-834; Kufakunesu, Dzingo & Dekeza, 2013:1561). The actual names of the research participants and the secondary schools were not used as already discussed in section 4.3.1.3. The names of the respondents were only used before and during the recording of the learners' Mathematics examination scores on their respective questionnaires. Thereafter, the record of their actual names was destroyed. Anonymity normally supports the principle of confidentiality. In the research fraternity, confidentiality stipulates that the information supplied by the research informants should not be revealed to third parties unless prior permission is sought and granted (Lahey, 2009:43; Kufakunesu, 2011:39). Moreover, the collected data should be safely kept to prevent future abuse or misuse of the information (Lahey, 2009:43). After collecting the completed questionnaires from the respondents, the researcher safely kept the questionnaires before personally delivering them to the University for data capturing and statistical analysis. After the data capturing and statistical analysis, the questionnaires were destroyed. Non-maleficence stipulates that respondents must not be subjected to physical, emotional or psychological harm during the study. The

researcher tried to ensure that the respondents completed the questionnaires without incurring any form of harm.

4.3.4 Data Collection Procedure

The researcher sought the assistance of at least two teachers in the Mathematics department at each of the three secondary schools to distribute the questionnaires. After distributing the questionnaires to the respondents, the researcher took the learners through the first three instructions on the first page of the questionnaire. The need to express their spontaneous feelings and opinion in response to each item was emphasised. The researcher also informed the respondents that a single number must be entered in each box as they completed the questionnaire. The respondents were then informed about how to respond to the items in section A of the questionnaire which focused on their demographic data. Thereafter, the respondents were told how to complete the items in section B of the questionnaire by indicating their degree of agreement or disagreement with each statement using code numbers sliding down from 6 to 1. The Likert scale used as a code was fully interpreted to the respondents and it was brought to their attention that the scale featured at the bottom of each questionnaire from page 3 to page 23 as a footnote. The respondents were asked to respond to all the items up to item 282 on page 23 of the questionnaire. The researcher took time to remind the respondents to seek clarification when necessary as they completed the questionnaires. After clarification of the instructions, the researcher then tasked the learners with going through the questionnaires. As the learners completed the questionnaires, the assisting Mathematics teachers and the researcher stayed in the classrooms to help the learners whenever there was need.

A few respondents in form 3 sought clarification on the meanings of words such as, *anxious*, *nervous* and *postpone* which featured in some questionnaire items. The researcher paraphrased the statements in which the words were used to ensure that the research participants fully understood the items before responding to them. In some cases, clarification was given to the respondents in the vernacular to ensure that the questionnaire items were explicitly clear to them.

After the completion of the questionnaires, the questionnaires were collected by the researcher and the research assistants. Thereafter, the researcher thanked the

learners and the teachers who had acted as research assistants for participating in the study. At all of the three participating secondary schools, the questionnaires were completed during the morning and the respondents took about 55 to 65 minutes to complete them. The questionnaires registered a 100% return rate because they were administered and collected soon after completion at each of the three secondary schools.

4.4 SUMMARY

The chapter outlined six hypotheses pertaining to irrational thinking in a Mathematics learning context in relation to several allied variables. The population, sample composition and characteristics as well as the sampling procedure were also delineated. The various components of the questionnaire used for the empirical investigation were discussed. Finally, the ethical principles which were adhered to and the procedure which was followed during the collection of data were discussed. The results of the empirical investigation are analysed and discussed in the next chapter.

CHAPTER FIVE

RESULTS OF THE EMPIRICAL INVESTIGATION

5.1 INTRODUCTION

The method of empirical investigation was outlined in the previous chapter. Six hypotheses pertaining to the differences and/or relationships between learners' irrational thoughts regarding Mathematics and a number of variables were stated and justified. The various sections of the Questionnaire for Secondary School Learners were also discussed. This chapter is composed of two sections, one focusing on item analysis and the reliability of the research instrument and the other dealing with the testing of the hypotheses stated in chapter 4.

It is through item analysis that any weak or poorly formulated items can be identified and omitted from the questionnaire (Marais, 2012:119). The quality of the items of the various sections of the questionnaire was established. Each of the six hypotheses stated in chapter 4 (section 4.2) was tested using the data collected during the empirical investigation. During the hypotheses testing, the existence of differences and/or relationships between learners' irrational thoughts about Mathematics and the learners' socio-affective and academic variables as well as their perceptions were ascertained. The differences between learners' irrational thoughts in Mathematics relative to gender and age were verified using the t-test. The presence of a significant relationship between learners' irrational thoughts regarding Mathematics and each of the socio-affective, academic and perception factors were verified by calculating Pearson's product-moment correlation coefficients. Regression analysis was used to determine whether learners' irrational thoughts, perceptions and socio-affective variables jointly explained a larger proportion of the variance in Mathematics achievement than any of these factors on its own.

5.2 ITEM ANALYSIS AND RELIABILITY

For each section of the questionnaire, an item analysis was undertaken. As part of the item analysis, the correlation coefficient between each item and the total (that is, the item-total correlation) was calculated. Items with significantly large positive

correlations with the total were retained while items with weak positive correlations or negative correlations with the total were omitted (Chetty, 1996:141-142; Marais, 2012:200). The Cronbach Alpha reliability coefficient was also calculated for each section. Items whose omission would result in a significant increase in the Cronbach Alpha reliability coefficient were omitted while items whose exclusion did not lead to a substantial increase in the Cronbach Alpha reliability coefficient were retained. As outlined by Marais (2012:200) and Chetty (1996:141-142), items which correlated positively with the total and simultaneously provided high reliability were retained in the final research instrument. The item analyses and calculation of reliability coefficients were necessary because all the items of the questionnaire were adapted from various original instruments (sections 4.3.2.2.1, 4.3.2.2.2, 4.3.2.2.3 and 4.3.2.2.4).

5.2.1 Irrational Belief Test

As indicated in section 4.3.2.2.1, the Irrational Belief Test arguably remains a crucial part of the questionnaire because it contains items which explore irrational beliefs, a key variable in the current study. Table 5.1 shows each of the 10 irrational beliefs stated in full and the corresponding brief statements which were used as the names of the irrational belief subscales in the current study.

Table 5.1: Irrational Belief Test Subscales

Irrational Belief Subscale	Irrational Belief in Full	Brief Statement of Irrational Belief
1	The belief that it is a dire necessity for people to be loved and approved by significant others in virtually everything they do	Demand for Love and Approval
2	The belief that we should be thoroughly competent, intelligent, and achieving in all possible respects	Demand for Competence
3	The belief that certain acts are awful or wicked and that people who perform such acts should be severely damned	Demand for Justice
4	The belief that it is horrible and catastrophic when things are not the way we like them to be	Wishful Thinking
5	The belief that human emotional misery is mostly externally caused and that people have little or virtually no ability to regulate their emotional sorrows and disturbances	Emotional Irresponsibility and Luck
6	The belief that if something is or may be dangerous or fearsome we should be terribly upset and endlessly obsess about it	Anxious and Endless Obsession over a Problem
7	The belief that it is easier to avoid than to face life difficulties and self-responsibilities	Task or Problem Avoidance
8	The belief that we absolutely need something other or stronger or greater than ourselves on which to rely	Dependence Mentality
9	The belief that because something once strongly affected our life, it should indefinitely affect it	Perennial Influence of a Past Problem
10	The belief that there is invariably a right, precise and perfect solution to human problems and it is catastrophic if this perfect solution is not found	Perfectionistic and Absolutistic Mentality

Below is an outline of the reliability and item analysis of each of the 10 irrational belief subscales:

5.2.1.1 Demand for Love and Approval subscale

This subscale is based on the irrational belief *that it is a dire necessity for people to be loved and approved by significant others in virtually every aspect of their lives*. The irrational belief was discussed at length in sections 3.3.1.3.1 and 4.3.2.2.1.1.

Table 5.2: Reliability of the Demand for Love and Approval Subscale

N	306
Number of items	10
Cronbach Alpha Reliability Coefficient	0.51

The Demand for Love and Approval subscale had a Cronbach Alpha reliability coefficient of 0.51. Such a reliability coefficient is poor and below what is normally acceptable. In an attempt to establish if no individual items had a weak or negative correlation with the total, an item analysis was carried out.

Table 5.3: Item Analysis of the Demand for Love and Approval Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
123	0.051	0.529
133	0.267	0.467
143	0.215	0.484
153	0.228	0.480
163	0.319	0.449
173	0.209	0.496
183	0.257	0.473
193	0.252	0.473
203	0.310	0.453
213	0.041	0.535

Two items, that is, 123 and 213 were omitted because they correlated very lowly with the total and their omission caused the Cronbach Alpha reliability coefficients to increase. A new reliability coefficient was calculated after the omission of the two items. The new Cronbach Alpha reliability coefficient increased from 0.51 to 0.56.

5.2.1.2 Demand for Competence Subscale

As shown in Table 5.1, the Demand for Competence subscale focuses on the irrational belief that people should be thoroughly competent, intelligent, and achieving in all possible respects. The irrational belief was elaborated in detail in the two preceding chapters (sections 3.3.1.3.8 and 4.3.2.2.1.2).

Table 5.4: Reliability of the Demand for Competence Subscale

N	306
Number of items	10
Cronbach Alpha Reliability Coefficient	0.45

As shown in Table 5.4, the Cronbach Alpha reliability coefficient of the Demand for Competence subscale was 0.45. This means the internal consistency of this subscale was unacceptably low. Items which correlate negatively or weakly with the total tend to reduce the overall Cronbach Alpha reliability coefficient. These items were removed.

Table 5.5: Item Analysis of the Demand for Competence Subscale

Item	Item Correlation with Total	Cronbach Alpha if item is omitted
124	0.144	0.433
134	0.252	0.392
144	0.023	0.479
154	0.102	0.450
164	0.156	0.429

174	0.133	0.438
184	0.275	0.385
194	0.299	0.378
204	0.242	0.399
214	0.180	0.421

Items 144 and 154 correlated lowly with the total and when both were omitted, the reliability increased. Consequently, a new item analysis excluding the two items was undertaken and the Cronbach Alpha reliability coefficient increased from 0.45 to 0.52.

5.2.1.3 Demand for Justice Subscale

The irrational belief that certain acts are awful or wicked and that people who perform such acts should be severely damned is the basis of the Demand for Justice irrational belief subscale. More details of the irrational belief were outlined in sections 3.3.1.3.2 and 4.3.2.2.1.3.

Table 5.6: Reliability of the Demand for Justice Subscale

N	306
Number of items	10
Cronbach Alpha Reliability Coefficient	0.43

A low Cronbach Alpha reliability coefficient of 0.43 was found for the Demand for Justice irrational belief subscale. Such a reliability coefficient signifies an unacceptably low level of internal consistency. The items which weakly or negatively correlated with the total were identified.

Table 5.7: Item Analysis of the Demand for Justice Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
125	0.255	0.377

135	0.258	0.378
145	0.215	0.393
155	0.080	0.439
165	0.226	0.391
175	0.248	0.379
185	0.158	0.414
195	0.266	0.371
205	0.029	0.460
215	-0.006	0.470

Of the ten items which constituted this subscale, three items, that is, 155, 205 and 215 were ultimately omitted because they correlated very lowly with the total. The Cronbach Alpha reliability coefficient increased if they were omitted. After omitting the three items, the new item analysis generated a higher Cronbach Alpha reliability coefficient of 0.50.

5.2.1.4 Wishful Thinking Subscale

The irrational belief that it is horrible and catastrophic when things are not the way we like them to be is the basis of the Wishful Thinking subscale. More details pertaining to this irrational belief were given in chapters 3 and 4 (sections 3.3.1.3.3 and 4.3.2.2.1.4).

Table 5.8: Reliability of the Wishful Thinking Subscale

N	306
Number of items	10
Cronbach Alpha Reliability Coefficient	0.71

The Wishful Thinking subscale had a Cronbach Alpha reliability coefficient of 0.71. Such a reliability coefficient indicates that the subscale had a good level of internal consistency.

Table 5. 9: Item Analysis of the Wishful Thinking Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
126	0.382	0.680
136	0.413	0.674
146	0.241	0.703
156	0.343	0.686
166	0.318	0.690
176	0.341	0.687
186	0.374	0.681
196	0.469	0.665
206	0.351	0.685
216	0.412	0.675

As shown in Table 5.9, none of the items had a very small or negative correlation with the total and the Cronbach Alpha reliability coefficient did not increase significantly if any of the items was omitted. Consequently, no items were deleted in this subscale.

5.2.1.5 Emotional Irresponsibility and Luck Subscale

The Emotional Irresponsibility and Luck subscale is a derivative of the irrational belief that *human emotional misery is mostly externally caused and that people have little or virtually no ability to regulate their emotional sorrows and disturbances*. The irrational belief was discussed in a Mathematics learning context in sections 3.3.1.3.4, 3.3.1.3.12 and 4.3.2.2.1.5.

Table 5.10: Reliability of the Emotional Irresponsibility and Luck Subscale

N	306
Number of items	10
Cronbach Alpha Reliability Coefficient	0.56

As shown in Table 5.10, the Emotional Irresponsibility and Luck subscale had a Cronbach Alpha reliability coefficient of 0.56 which signifies poor internal consistency. Effort was made to identify items with a weak or negative correlation with the total.

Table 5.11: Item Analysis of the Emotional Irresponsibility and Luck Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
127	0.276	0.521
137	0.233	0.533
147	0.377	0.490
157	0.239	0.532
167	0.212	0.539
177	0.191	0.544
187	0.355	0.504
197	0.172	0.550
207	0.210	0.540
217	0.184	0.546

None of the 10 items in this subscale was omitted because each of the 10 items had a positive correlation with the total. Moreover, the Cronbach Alpha reliability coefficient decreased whenever an item was omitted.

5.2.1.6 Anxious and Endless Obsession over a Problem Subscale

The basis of the Anxious and Endless Obsession over a Problem Subscale is the irrational belief that if something is or may be dangerous or fearsome we should be terribly upset and endlessly obsess about it. Details pertaining to this irrational belief were outlined in sections 3.3.1.3.5 and 4.3.2.2.1.6.

Table 5.12: Reliability of the Anxious and Endless Obsession over a Problem Scale

N	306
Number of items	10
Cronbach Alpha Reliability Coefficient	0.71

The Anxious and Endless Obsession Subscale had a Cronbach Alpha reliability coefficient of 0.71. A Cronbach Alpha reliability coefficient of 0.71 shows a good and acceptable level of internal consistency among the items constituting the subscale.

Table 5.13: Item Analysis of the Anxious and Endless Obsession over a Problem Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
128	0.333	0.699
138	0.437	0.681
148	0.386	0.690
158	0.417	0.684
168	0.457	0.678
178	0.375	0.691
188	0.360	0.694
198	0.148	0.728
208	0.471	0.674
218	0.355	0.695

Only one item, that is, 198, correlated lowly with the total and when it was omitted, the Cronbach Alpha reliability coefficient increased. Consequently, a new item analysis was done with the remaining nine items and the Cronbach Alpha reliability coefficient increased from 0.71 to 0.73.

5.2.1.7 Task or Problem Avoidance Subscale

The Task or Problem Avoidance irrational belief subscale is derived from the irrational belief that it is easier to avoid than to face life's difficulties and self-responsibilities. The irrational belief was elaborated in sections 3.3.1.3.6, 3.3.1.3.11 and 4.3.2.2.1.7.

Table 5.14: Reliability of the Task or Problem Avoidance Subscale

N	306
Number of Items	10
Cronbach Alpha Reliability Coefficient	0.75

A Cronbach Alpha reliability coefficient of 0.75 was found for the Task or Problem Avoidance subscale. Such a reliability coefficient shows a good and acceptable level of internal consistency among the items in this subscale.

Table 5.15: Item Analysis of the Task or Problem Avoidance Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
129	0.430	0.727
139	0.526	0.712
149	0.504	0.716
159	0.427	0.728
169	0.425	0.728
179	0.346	0.739
189	0.280	0.749
199	0.365	0.737
209	0.444	0.725
219	0.365	0.736

All 10 items in the Task or Problem Avoidance subscale were retained since they all showed significant positive correlations with the total. If any one of the items was to be omitted, the Cronbach Alpha reliability coefficient would be lower than the overall Cronbach alpha reliability coefficient of 0.75. Consequently, all the items were retained.

5.2.1.8 Dependence Mentality Subscale

The irrational belief that we absolutely need something other or stronger or greater than ourselves on which to rely is the basis of the Dependence Mentality subscale. Detailed outlines of the irrational belief were given in sections 3.3.1.3.7 and 4.3.2.2.1.8 in the two preceding chapters.

Table 5.16: Reliability of the Dependence Mentality Subscale

N	306
Number of items	10
Cronbach Alpha Reliability Coefficient	0.62

The Cronbach Alpha reliability coefficient of the Dependence Mentality subscale was 0.62. Such a reliability coefficient shows a minimally reliable but acceptable level of internal consistency. Items with a low positive correlation or a negative correlation with the total tend to reduce the internal consistency of a research instrument. Consequently, such items were identified.

Table 5.17: Item Analysis of the Dependence Mentality Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
130	0.218	0.614
140	0.388	0.574
150	0.350	0.583

160	0.297	0.595
170	0.318	0.591
180	0.349	0.583
190	0.294	0.596
200	0.258	0.604
210	0.273	0.601
220	0.163	0.624

Only one item in the Dependence Mentality subscale, that is 220, was eventually omitted because it showed a comparatively low correlation with the total, coupled with the fact that when the item was omitted, the Cronbach Alpha reliability coefficient increased to 0.624, which is greater than the overall Cronbach Alpha reliability coefficient of 0.622.

5.2.1.9 Perennial Influence of a Past Subscale

The Perennial Influence of a past problem subscale is based on the irrational belief that because something once strongly affected our life, it should indefinitely affect it. The irrational belief was elaborated in sections 3.3.1.3.9 and 4.3.2.2.1.9.

Table 5.18: Reliability of the Perennial Influence of a Past Problem Subscale

N	306
Number of items	10
Cronbach Alpha Reliability Coefficient	0.70

A good and acceptable Cronbach Alpha reliability coefficient of 0.70 was found for the Perennial Influence of a Past Problem Subscale. A reliability coefficient of 0.70 shows an acceptable level of internal consistency among the items. An attempt was made to identify any items which correlated weakly or negatively with the total.

Table 5.19: Item Analysis of the Perennial Influence of a Past Problem Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
131	0.423	0.664
141	0.413	0.664
151	0.328	0.680
161	0.436	0.660
171	0.294	0.685
181	0.463	0.655
191	0.358	0.674
201	0.224	0.696
211	0.301	0.684
221	0.320	0.681

All 10 items in the Perennial Influence of a Past Problem subscale were retained because they all had significant positive correlations with the total. When each item was omitted, the Cronbach Alpha reliability coefficient did not increase significantly.

5.2.1.10 Perfectionistic and Absolutistic Mentality Subscale

The Perfectionistic and Absolutistic Mentality Subscale is a derivative of the irrational belief that there is invariably a right, precise and perfect solution to human problems and it is catastrophic if this perfect solution is not found. The irrational belief was discussed in detail in sections 3.3.1.3.10, 3.3.1.3.13 and 4.3.2.2.1.10.

Table 5.20: Reliability of the Perfectionistic and Absolutistic Mentality Subscale

N	306
Number of items	10
Cronbach Alpha Reliability Coefficient	0.45

Table 5.20 shows that the Cronbach Alpha reliability coefficient of the Perfectionistic and Absolutistic Mentality subscale was 0.45. Such a reliability coefficient indicates an unacceptably low level of internal consistency among the items constituting the subscale. An attempt was made to identify any items with a weak or negative correlation with the total.

Table 5.21: Item Analysis of the Perfectionistic and Absolutistic Mentality Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
132	0.259	0.400
142	0.285	0.388
152	0.007	0.484
162	0.144	0.440
172	0.299	0.384
182	0.203	0.419
192	0.157	0.435
202	0.057	0.468
212	0.169	0.431
222	0.236	0.407

Items 152 and 202 correlated very lowly with the total and the Cronbach Alpha reliability coefficient increased if each of the two items was omitted. Consequently, the two items were deleted. The value of the Cronbach Alpha reliability coefficient increased from 0.43 to 0.52 for the remaining eight items.

5.2.2 Perceptions

As discussed in section 4.3.2.2.2, the four factors regarding the perceptions of learners were identified. They are The Inevitable, The Role of Rewards, School and Schooling and Teachers and Parents. The reliability and the item analysis of each of the four factors of perceptions are outlined below:

5.2.2.1 The Inevitable

In section 4.3.2.2.1, it was outlined that The Inevitable refers to learners' perceptions of scholastic achievement which significant others in society have imposed on them and which seem to be beyond the control of the learners (Chetty, 1996:138).

Table 5.22: Reliability of The Inevitable Subscale

N	306
Number of Items	15
Cronbach Alpha Reliability Coefficient	0.63

Table 5.22 shows that the Cronbach Alpha reliability coefficient of The Inevitable subscale was 0.63. Such a reliability coefficient shows a marginally reliable but acceptable level of internal consistency. Effort was made to identify items with a negative or weak correlation with the total.

Table 5.23: Item Analysis of The Inevitable Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
260	0.319	0.606
237	0.176	0.631
234	0.332	0.604
258	0.247	0.618
267	0.291	0.612
281	0.282	0.612
268	0.168	0.630
263	0.404	0.593
235	0.389	0.595
239	0.277	0.613
240	0.244	0.618
251	-0.002	0.653

236	0.289	0.611
269	0.276	0.614
247	0.130	0.635

Only one item, that is, 251, was eventually omitted since it had a negative correlation with the total and also because the Cronbach Alpha reliability coefficient increased if the item was omitted. After omitting the item, the Cronbach Alpha reliability coefficient increased from 0.63 to 0.65.

5.2.2.2 The Role of Rewards

The Role of Rewards, discussed in section 4.3.2.2.2, is a factor which is concerned with learners' perceptions of the anticipated rewards from significant others when the learners attain good academic results (Chetty, 1996:139).

Table 5.24: Reliability of the Role of Rewards Subscale

N	306
Number of Items	8
Cronbach Alpha Reliability Coefficient	0.59

A Cronbach Alpha reliability coefficient of 0.59 was obtained for the Role of Rewards subscale. A Cronbach Alpha reliability coefficient of 0.59 indicates that the items constituting the subscale had a poor inter-item correlation. An item analysis was done to establish if any items had a negative or weak correlation with the total.

Table 5.25: Item Analysis of the Role of Rewards Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
265	0.399	0.529
266	0.361	0.538
261	0.290	0.560

264	0.416	0.524
245	0.328	0.547
249	0.321	0.550
259	0.289	0.560
241	-0.002	0.647

Of the eight items which constituted the Rewards subscale, only one, that is, 241, was deleted because not only did the item have a negative correlation with the total but its omission caused the Cronbach Alpha reliability coefficient to increase. A second item analysis which was undertaken with the remaining seven items caused the Cronbach Alpha reliability coefficient to increase from 0.59 to 0.65.

5.2.2.3 School and Schooling

This factor of perceptions deals with the utility of school activities in solving real life existential problems as perceived by learners (Chetty, 1996:140). As indicated in section 4.3.2.2.2.3, it also deals with the way learners interpret the prevailing school and classroom atmosphere cultivated by school administrators (Chetty, 1996:140).

Table 5.26: Reliability of the School and Schooling Subscale

N	306
Number of Items	15
Cronbach Alpha Reliability Coefficient	0.68

The School and Schooling subscale had a Cronbach Alpha reliability coefficient of 0.68. This implies that the items had a marginally reliable but acceptable inter-item correlation. An attempt was made to identify any items with a weak or negative correlation with the total.

Table 5.27: Item Analysis of the School and Schooling Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
280	0.202	0.679
242	0.373	0.657
246	0.355	0.659
254	0.372	0.657
253	0.182	0.681
248	0.237	0.674
244	0.364	0.657
252	0.318	0.664
255	0.149	0.685
257	0.224	0.675
250	0.272	0.670
262	0.426	0.649
243	0.307	0.665
233	0.328	0.663
238	0.255	0.672

Table 5.27 shows that only one of the 15 items, that is, 255, had a low correlation with the total and its omission caused the Cronbach Alpha reliability coefficient to increase. Therefore, item 255 was omitted. A second item analysis performed with the remaining 14 items in the School and Schooling Subscale caused the Cronbach Alpha reliability coefficient to increase from 0.68 to 0.69.

5.2.2.4 Teachers and Parents

According to Chetty (1996:140), the Teachers and Parents subscale deals with the instructional approaches employed by the teachers, parenting styles, and parental involvement in learners' scholastic work (section 4.3.2.2.2.4).

Table 5.28: Reliability of the Teachers and Parents Subscale

N	306
Number of Items	12
Cronbach Alpha Reliability Coefficient	0.58

As shown in Table 5.28, the Cronbach Alpha reliability coefficient for the Teachers and Parents subscales was 0.58. Such a reliability coefficient shows a poor level of internal consistency. Items with a weak or negative correlation with the total were identified.

Table 5.29: Item Analysis of the Teachers and Parents Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
279	0.297	0.576
273	0.242	0.566
272	0.395	0.531
274	0.354	0.540
282	0.231	0.568
277	0.301	0.553
270	0.304	0.552
278	0.112	0.592
276	0.074	0.599
275	0.268	0.560
271	0.280	0.558
256	0.174	0.580

Table 5.29 shows that the Teachers and Parents subscale had 15 items. Two items, that is, 278 and 276, had low correlation coefficients with the total and both caused the Cronbach Alpha reliability coefficient to increase if they were omitted. A second

item analysis which was performed after omitting the two items led to an increase in the value of the Cronbach Alpha reliability coefficient from 0.58 to 0.59.

5.2.3 Socio-Affective Variables

Below is an outline of the reliability and item analysis of the subscales which measured Mathematics learners' socio-affective variables:

5.2.3.1 Motivation

The importance of motivation as a socio-affective variable in determining learners' beliefs and their academic performance in Mathematics was discussed in detail in sections 2.2.2.1, 3.4.1.8.1 and 4.3.2.2.3.1.

Table 5.30: Reliability of the Motivation Subscale

N	306
Number of Items	20
Cronbach Alpha Reliability Coefficient	0.76

The Motivation subscale had a Cronbach Alpha reliability coefficient of 0.76. Such a reliability coefficient shows an acceptable and good level of inter-item correlation. To improve the internal consistency of the Motivation subscale further, items with a weak or negative correlation with the total were identified.

Table 5.31: Item Analysis of the Motivation Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
1	0.337	0.746
5	0.449	0.736
7	0.444	0.738
12	0.176	0.757
17	0.371	0.743
24	0.475	0.735
31	0.201	0.757

36	0.330	0.746
47	0.454	0.736
48	0.270	0.750
52	0.411	0.739
55	0.506	0.733
59	0.388	0.743
83	0.407	0.740
86	0.329	0.746
92	-0.024	0.775
104	0.345	0.745
108	0.456	0.736
118	0.220	0.755
121	0.003	0.769

Item 92 was omitted from the Motivation subscale because it had a negative correlation with the total coupled with the fact that its omission led to an increase in the value of the Cronbach Alpha reliability coefficient. Item 121 was also omitted because it had a low positive correlation with the total and its omission caused the Cronbach Alpha reliability coefficient to increase. The second item analysis which was undertaken with the remaining 18 items led to a significant increase in the Cronbach Alpha reliability coefficient from 0.76 to 0.79.

5.2.3.2 Stress

It was highlighted in chapter 2 that adolescent learners are particularly susceptible to stress as a result of the maturational turmoil associated with the transition from childhood to adulthood. Stress is a crucial variable which can influence learners' academic attainment, hence the development of the Stress subscale. Detailed discussions of stress as an affective variable may be found in sections 2.2.2.3 and 4.3.2.2.3.2.

Table 5.32: Reliability of the Stress Subscale

N	306
Number of Items	20
Cronbach Alpha Reliability Coefficient	0.76

As shown in Table 5.32, the Cronbach Alpha reliability coefficient for the Stress subscale was 0.76. A reliability coefficient of 0.76 signifies a good and acceptable level of internal consistency among the items in the subscale. Items with a weak or negative correlation with the total were identified.

Table 5.33: Item Analysis of the Stress Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
6	0.271	0.750
10	0.332	0.746
13	-0.033	0.769
16	0.361	0.744
19	0.498	0.734
23	0.388	0.742
25	0.175	0.757
65	0.384	0.742
73	0.452	0.737
80	0.360	0.744
88	0.353	0.745
90	0.354	0.745
95	0.105	0.763
98	-0.163	0.774
100	0.351	0.745
107	0.294	0.749
111	0.497	0.734
114	0.332	0.746

117	0.473	0.735
120	0.491	0.734

Table 5.33 shows that two items, that is, 13 and 98, had negative correlations with the total. When each of the two items was omitted, the Cronbach Alpha reliability coefficient increased. Therefore, the two items were omitted. A second item analysis was undertaken with the remaining 18 items and the Cronbach Alpha reliability coefficient increased from 0.76 to 0.79.

5.2.3.3 Anxiety

Relationships have been found between anxiety and other factors such as stress, irrational thinking and Mathematics achievement (sections 2.2.2.4.1, 2.2.2.4.2 and 4.3.2.2.3.3). Therefore, an Anxiety subscale was developed for use in the current study as part of the questionnaire.

Table 5.34: Reliability of the Anxiety Subscale

N	306
Number of Items	20
Cronbach Alpha Reliability Coefficient	0.85

The Anxiety subscale had a high Cronbach Alpha reliability coefficient of 0.85. This implies that the items constituting the Anxiety subscale had a high internal consistency. Therefore, the subscale was quite good for use as it could generate the same results over several administrations. An item analysis was undertaken to establish if no items correlated weakly or negatively with the total.

Table 5.35: Item Analysis of the Anxiety Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
4	0.240	0.848
9	0.422	0.841

18	0.432	0.840
30	0.387	0.842
37	0.585	0.833
40	0.532	0.836
42	0.375	0.843
45	0.459	0.839
54	0.581	0.833
57	0.463	0.839
61	0.497	0.837
64	0.360	0.843
72	0.250	0.847
76	0.399	0.842
82	0.428	0.840
84	0.334	0.844
94	0.414	0.841
106	0.499	0.837
112	0.380	0.842
115	0.484	0.838

All the items in the Anxiety subscale had high positive correlations with the total. If any item was omitted, the Cronbach Alpha reliability coefficient decreased. Consequently, all the 20 items in the Anxiety subscale were retained and the Cronbach Alpha reliability coefficient remained 0.85.

5.2.3.4 Self-concept

The importance of secondary school Mathematics learners having a sound self-concept was highlighted in sections 2.2.2.2, 3.4.1.3 and 4.3.2.2.3.4. That necessitated the development of a Self-concept subscale in the current study.

Table 5.36: Reliability of the Self-concept Subscale

N	306
Number of Items	20
Cronbach Alpha Reliability Coefficient	0.82

A very high Cronbach Alpha reliability coefficient of 0.82 was found for the Self-concept subscale. Such a reliability coefficient confirms that the items which constituted the Self-concept subscale had a very high level of internal consistency. Items with a weak or negative correlation with the total were identified.

Table 5.37: Item Analysis of the Self-concept Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
2	0.334	0.812
8	0.487	0.805
14	0.501	0.803
20	0.437	0.806
26	0.459	0.806
32	0.418	0.807
38	0.200	0.819
43	0.284	0.814
49	0.569	0.799
62	0.537	0.801
67	0.359	0.811
69	0.303	0.814
70	0.553	0.800
74	0.509	0.802
77	0.508	0.802
91	0.477	0.804
96	0.038	0.823
101	0.349	0.811

105	0.101	0.823
116	0.229	0.817

Table 5.37 shows that items 96 and 105 had very low correlation coefficients with the total. The Cronbach Alpha reliability coefficient increased if one of the two items was omitted. Consequently, the two items were omitted. A second item analysis was done with the remaining 18 items resulting in an increase in the value of the Cronbach Alpha reliability coefficient from 0.82 to 0.83.

5.2.3.5 Teacher-Learner Relationships

The way Mathematics learners interact with their teachers can to some extent influence their academic performance in Mathematics as outlined in sections 2.3.3.1, 2.3.3.2 and 4.3.2.2.3.5. The questionnaire used in the current study had a Teacher-Learner Relationships subscale.

Table 5.38: Reliability of the Teacher-Learner Relationships Subscale

N	306
Number of items	21
Cronbach Alpha Reliability Coefficient	0.83

Table 5.38 shows that the Teacher-Learner Relationships subscale had a Cronbach Alpha reliability coefficient of 0.83. This means the internal consistency of this subscale was very high and the subscale was a good research instrument. Items with a weak or negative correlation with the total were identified.

Table 5.39: Item Analysis of the Teacher-Learner Relationships Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
3	0.558	0.819
11	0.609	0.815
22	0.366	0.826

28	0.285	0.830
29	0.327	0.828
34	0.364	0.827
35	0.518	0.819
41	0.432	0.823
46	0.357	0.827
50	0.581	0.817
53	0.441	0.823
58	0.172	0.837
60	0.427	0.824
79	0.353	0.827
81	0.206	0.834
85	0.561	0.818
89	0.407	0.825
93	0.364	0.827
97	0.491	0.821
103	0.430	0.824
110	0.228	0.834

One item, that is, 58, had a low correlation with the total and its omission caused the Cronbach Alpha reliability coefficient to increase. Therefore, the item was deleted. A second item analysis was undertaken with the remaining 20 items and the Cronbach Alpha reliability coefficient increased from 0.83 to 0.84.

5.2.4 Parental Involvement

Parental Involvement qualifies as a socio-affective variable which explores the contribution of parents in the academic work of their children. Aspects of parental involvement were discussed in detail in sections 2.3.2.1, 2.3.2.2, 2.3.2.3, 3.4.3.3 and 4.3.2.2.4.

Table 5.40: Reliability of the Parental Involvement Subscale

N	306
Number of Items	10
Cronbach Alpha Reliability Coefficient	0.76

An acceptable Cronbach Alpha reliability coefficient of 0.76 was found for the Parental Involvement Subscale. Such a reliability coefficient shows that the items which constituted the Parental Involvement subscale had a good internal consistency. An attempt was made to identify items with a weak or negative correlation with the total.

Table 5.41: Item Analysis of the Parental Involvement Subscale

Item	Item Correlation with Total	Cronbach Alpha if Item is Omitted
223	0.475	0.731
224	0.592	0.715
225	0.283	0.760
226	0.352	0.749
227	0.470	0.733
228	0.441	0.736
229	0.470	0.736
230	0.457	0.733
231	0.413	0.740
232	0.323	0.753

Table 5.41 shows that each of the 10 items in the Parental Involvement subscale had a positive correlation with the total. The omission of any of the 10 items did not significantly increase the numerical value of the Cronbach Alpha reliability coefficient. Therefore, all 10 items in this subscale were retained.

5.3 SUMMARY OF RELIABILITY OF THE QUESTIONNAIRE SUBSECTIONS

5.3.1 Irrational Belief Test

Table 5.42: Reliability of Irrational Belief Subscales

Irrational Belief Test Subscale	Number of Respondents, N	Number of Items in the Final Subscale	Cronbach Alpha Reliability Coefficient
Demand for Love and Approval	306	8	0.56
Demand for Competence	306	8	0.52
Demand for Justice	306	7	0.50
Wishful Thinking	306	10	0.71
Emotional Irresponsibility and Luck	306	10	0.56
Anxious and Endless Obsession over a Problem	306	9	0.73
Task or Problem Avoidance	306	10	0.75
Dependence Mentality	306	9	0.62
Perennial Influence of a Past Problem	306	10	0.70
Perfectionistic and Absolutistic Mentality	306	8	0.52

Table 5.42 shows that five of the ten Irrational Beliefs Test subscales whose Cronbach Alpha reliability coefficients lay between 0.50 and 0.59 were inclusive. This means the Demand for Love and Approval subscale, the Demand for

Competence subscale, the Demand for Justice subscale, the Emotional Irresponsibility and Luck subscale and the Perfectionistic and Absolutistic Mentality subscale each had a poor internal consistency. Their low level of internal consistency is a limitation which will be acknowledged in the next chapter.

The internal consistency for the Dependence Mentality subscale was minimal and marginally acceptable since it had a Cronbach Alpha reliability coefficient of 0.62. The remaining four subscales, that is, the Wishful Thinking subscale, the Anxious and Endless Obsession over a Problem subscale, the Task or Problem Avoidance subscale and the Perennial Influence of a Past Problem subscale had good and acceptable levels of internal consistency.

5.3.2 Perceptions

Table 5.43: Reliability of Perception Factors

Perceptions Subscale	Number of Respondents, N	Number of Items in the Final Subscale	Cronbach Alpha Reliability Coefficient
The Inevitable	306	14	0.65
The Role of Rewards	306	7	0.65
School and Schooling	306	10	0.69
Teachers and Parents	306	14	0.59

Three Perceptions subscales, that is, The Inevitable subscale, The Role of Rewards subscale and the School and Schooling subscale had moderate but acceptable levels of internal consistency. Only one subscale, that is, the Teachers and Parents subscale, had poor internal consistency since its Cronbach Alpha reliability coefficient was 0.59.

5.3.3. Socio-Affective Variables

Table 5.44: Reliability of Socio-Affective Variables

Socio-Affective Subscale	Number of	Number of	Cronbach

	Respondents, N	items in the Final Subscale	Alpha Reliability Coefficient
Motivation	306	18	0.79
Stress	306	18	0.79
Anxiety	306	20	0.85
Self-concept	306	18	0.83
Teacher-Learner Relationships	306	20	0.84
Parental Involvement	306	10	0.76

Three of the six socio-affective subscales, that is, the Motivation subscale, the Stress subscale and the Parental Involvement subscale had acceptable levels of internal consistency. The other three subscales, that is, the Anxiety subscale, the Self-concept subscale and the Teacher-Learner Relationships subscale had high levels of internal consistency. The Anxiety subscale had the highest Cronbach Alpha reliability coefficient of 0.85. All the six subscales in Table 5.45 were quite suitable for use in the empirical investigation.

5.4 TESTING OF HYPOTHESES

5.4.1 Hypothesis 1

The following null hypothesis which corresponds to hypothesis 1 stated in chapter 4 was tested:

There is no significant difference between the irrational thoughts of boys and girls with regard to Mathematics.

The null hypothesis was tested for each subscale of the irrational belief section of the questionnaire.

Table 5.45: Differences between the Irrational Thoughts of Boys and Girls with Regard to Mathematics

Irrational Belief Test Subscale	Gender	N	Mean	S	df	t	p-value
Demand for Love and Approval	Boys	124	27.15	6.49	304	0.56	p>0.05
	Girls	182	26.68	7.64			
Demand for Competence	Boys	124	28.87	5.89	304	3.51	p<0.01
	Girls	182	31.38	6.32			
Demand for Justice	Boys	124	21.18	6.56	304	0.66	p>0.05
	Girls	182	20.67	6.55			
Wishful Thinking	Boys	124	28.31	9.78	304	0.06	p>0.05
	Girls	182	28.25	9.20			
Emotional Irresponsibility and Luck	Boys	124	31.80	7.98	304	0.89	p>0.05
	Girls	182	32.64	8.11			

Anxious and Endless Obsession over a Problem	Boys	124	28.13	8.39	304	2.09	p<0.05
	Girls	182	30.36	9.66			
Task or Problem Avoidance	Boys	124	28.83	9.37	304	0.51	p>0.05
	Girls	182	29.43	10.75			
Dependence Mentality	Boys	124	30.60	7.66	304	1.25	p>0.05
	Girls	182	31.80	8.59			
Perennial Influence of a Past Problem	Boys	124	30.69	9.28	304	0.79	p>0.05
	Girls	182	29.81	9.77			
Perfectionistic and Absolutistic Mentality	Boys	124	31.60	7.43	304	1.31	p>0.05
	Girls	182	32.64	6.35			

Table 5.45 shows that for eight of the ten irrational belief test subscales, no significant differences were found between the irrational thoughts of boys and girls with regard to Mathematics. Consequently, for the eight irrational beliefs, the null hypothesis cannot be rejected. This implies that for the eight irrational belief subscales, there is no sufficient evidence to support the alternative hypothesis that

there is a significant difference between the irrational thoughts of boys and girls regarding Mathematics. Male and female learners were found to have the same level of irrational thinking about Mathematics relative to the eight irrational belief subscales.

The above results are consistent with the studies cited in sections 3.4.1.1.1 and 4.2 which established that there are no longer significant gender differences in Mathematics (Abubakar & Adegboyega, 2012:122; Dreber et al, 2012:2; Gunderson et al, 2012:153; Josiah & Adejoke, 2014:474; Abubakar, 2010:210-215; Habibollah, Abdullahi, Arizan, Sharir & Kurma, 2009:101-111). The results contradict the claim that boys are more vulnerable to perfectionism and procrastination than girls (Sunitha & Muhammedunni, 2013:104).

According to Table 5.45, there were two irrational belief test subscales for which a significant difference between the irrational thoughts of boys and girls in a Mathematics context was found. The first subscale for which gender differences in irrational thinking were found is Demand for Competence. The subscale is based on the irrational belief that *people should be thoroughly competent, intelligent, and achieving in all possible respects*. A t-value of 3.51 with $p < 0.01$ was obtained. The null hypothesis is rejected at the 1% significance level. The results suggest that girls, whose mean was 31.38, are more vulnerable to irrational thoughts about Mathematics based on the Demand for Competence subscale than boys who had a mean of 28.87. This implies that female Mathematics learners are more vulnerable to the irrational belief that *people should be thoroughly competent, intelligent, and achieving in all possible respects* than their male counterparts.

A significant difference was also found between the irrational thoughts of male and female Mathematics learners relative to the Anxious and Endless Obsession over a Problem subscale. A t-value of 2.09 with $p < 0.05$ was found. Consequently, the null hypothesis is rejected at the 5% significance level. Girls had a mean of 30.36 while boys had a mean of 28.13. This implies that with regard to Mathematics, girls are more vulnerable to the irrational belief that *if something is or may be dangerous or fearsome people should be terribly upset and endlessly obsess about it* than boys.

These results support the view that girls are more irrational and more emotional than boys (Wedegge, 2011:96; Walkerdine, 1998:15; Henningsen, 2008:35-36; Dreber et

al, 2012:2). This is also in agreement with the findings of a study undertaken by Bojman et al (2011:1) with a sample of university students which established that female students are more vulnerable to irrational thinking than male students. High levels of irrationality on the part of female learners can be explained by the claim that they are more susceptible to negative emotions such as depression, anxiety, negative self-concepts, low self-efficacy and stress than boys (Gunderson et al, 2012:153; Okoiye et al, 2013:87; Kenny-Benson et al, 2006:11-26; Abubakar & Adegboyega, 2012:122). Low levels of irrational thinking for boys can be ascribed to the fact that male learners tend to be more enthusiastic about learning Mathematics than female learners (Palsdottir, 2007:117; Watt, 2004:1556; Frenzel et al, 2010:508; Hannula, 2002:25). The rejection of the null hypothesis also supports the widely held belief that male Mathematics learners are academically better than their female counterparts (Watt, 2004:1556; Frenzel et al, 2010:508; Hannula, 2002:25; Okoiye et al, 2013:87). However, the results contradict the findings of some studies which reveal that female learners now outclass male learners in Mathematics (Aunio, 2006:10; Mubeen et al, 2013:41).

5.4.2 Hypothesis 2

The following null hypothesis regarding hypothesis 2 stated in chapter 4 was tested:

There is no significant difference between the irrational thoughts of form 3 and form 4 learners with regard to Mathematics.

The null hypothesis was tested for each subscale of the irrational belief section of the questionnaire.

Table 5.46: Difference between the Irrational Thoughts of Form 3 and Form 4 Learners with Regard to Mathematics

Irrational Belief Test Subscale	Form	N	Mean	S	df	t	p-value
Demand for Love and Approval	Form 3	191	26.92	7.65	304	0.16	p>0.05

	Form 4	115	26.78	6.39			
Demand for Competence	Form 3	191	31.196	6.34	304	3.00	p<0.01
	Form 4	115	29.00	5.90			
Demand for Justice	Form 3	191	20.42	6.52	304	1.58	p>0.05
	Form 4	115	21.63	6.54			
Wishful Thinking	Form 3	191	26.22	9.16	304	5.11	p<0.01
	Form 4	115	31.69	8.88			
Emotional Irresponsibility and Luck	Form 3	191	31.85	8.08	304	1.24	p>0.05
	Form 4	115	33.03	7.98			
Anxious and Endless Obsession over a Problem	Form 3	191	28.31	9.21	304	2.84	p<0.01
	Form 4	115	31.37	8.96			
Task or Problem Avoidance	Form 3	191	26.87	9.92	304	5.36	p<0.01
	Form 4	115	33.04	9.51			
Dependence Mentality	Form 3	191	30.83	8.53	304	1.31	p>0.05
	Form 4	115	32.10	7.71			
Perennial Influence of a Past Problem	Form 3	191	28.38	9.49	304	4.35	p<0.01

	Form 4	115	33.15	8.96			
Perfectionistic and Absolutistic Mentality	Form 3	191	33.21	6.54	304	3.34	p<0.01
	Form 4	115	30.57	6.96			

Table 5.46 shows that for four of the ten irrational belief subscales, there was no evidence to suggest a significant difference between the irrational thoughts of form 3 and form 4 learners with regard to Mathematics. The four subscales are Demand for Love and Approval, Demand for Justice, Emotional Irresponsibility and Luck and Dependence Mentality. For the four irrational belief subscales, the null hypothesis cannot be rejected.

The current study to some extent confirmed the results of earlier researches which revealed that as learners grow older, their attitudes towards Mathematics and their Mathematics experiences tend to be the same owing to the equalising effect of common school experiences (Josiah & Adejoke, 2014:474; White, 1982:461-481) (section 4.2). Nevertheless, the above results to some extent contradict the notion suggested by Okoiye et al (2013:93), Josiah and Adejoke (2014:475), La Paro and Pianta (2000:443-484) and Walker et al (1994:606-621) that there is a significant difference between learners' socioemotional experiences in Mathematics attributable to age differences.

Of the six irrational belief subscales in which a significant difference was found between the irrational thoughts of form 3 and form 4 learners with regard to Mathematics, there were two subscales in which form 3 learners were found to be more irrational than form 4 learners. The two subscales are Demand for Competence and Perfectionistic and Absolutistic Mentality. For the Demand for Competence subscale, a t-value of 3.00 with $p < 0.01$ was found. The null hypothesis is, therefore, rejected at the 1% level of significance. The mean of the form 3 learners was 31.20 while that of the form 4 learners was 29.00. This implies that form 3 Mathematics learners are more likely to be affected by the irrational belief that *people should be thoroughly competent, intelligent, and achieving in all possible respects* than their form 4 counterparts.

For the Perfectionistic and Absolutistic Mentality subscale, a t-value of 3.34 with $p < 0.01$ was found. Relative to this irrational belief subscale, the null hypothesis is rejected at the 1% significance level. As shown in Table 5.46, the mean of the form 3 learners was 33.31 while the mean of the form 4 learners was 30.57. This suggests that with regard to Mathematics, form 4 learners are less vulnerable to the irrational belief that *there is invariably a right, precise and perfect solution to human problems and it is catastrophic if this perfect solution is not found* than their form 3 counterparts. This means that form 3 learners tend to be more perfectionistic and absolutistic when doing Mathematics than form 4 learners. Moreover, the results imply that form 3 Mathematics learners tend to awfulise more than form 4 learners when they fail to obtain perfect solutions to mathematical problems.

The fact that form 3 learners have been found to be more irrational than form 4 learners relative to the Demand for Competence and the Perfectionistic and Absolutistic Mentality subscales can be explained by a number of factors. First, form 3 learners, by virtue of being chronologically younger than form 4 learners, are likely to be in the concrete operational stage where they cannot undertake deductive reasoning as discussed in section 2.2.1.2.1. On the other hand, form 4 learners mostly operate at the formal operational stage where they can deal with deductive reasoning and hypothetical concepts such as beliefs (William-Boyd, 2003:67; Gazzaniga & Heatherton, 2006:448). The claim by Allport that pre-adolescents are less self-directed in attaining crucial long-term life goals than adolescents and young adults (Dandapani, 2001:321; Dumont, 2010:208) can be another reason for the differences in the irrational thoughts of form 3 and form 4 learners regarding Mathematics.

There were four irrational belief subscales in which form 4 Mathematics learners were found to be more irrational than their form 3 counterparts as shown in Table 5.46. The first one is the Wishful Thinking subscale in which a t-value of 5.11 with $p < 0.01$ was obtained. This implies that relative to the Wishful Thinking subscale, the null hypothesis is rejected at the 1% level of significance. Form 3 learners had a mean of 26.22 while the mean of form 4 learners was 31.69. Consequently, it can be deduced that form 4 Mathematics learners are more susceptible than form 3 learners to the irrational belief that *it is horrible and catastrophic when things are not the way we like them to be*.

Relative to the Anxious and Endless Obsession over a Problem irrational belief subscale, a t-value of 2.34 with $p < 0.01$ was obtained. This suggests that for this subscale, there is statistical evidence to support the rejection of the null hypothesis at the 1% significance level. Consequently, there is a significant difference between the irrational thoughts of form 3 and form 4 learners with regard to Mathematics. The mean of form 3 learners was 28.31 while that of form 4 learners was 31.37. The arithmetic means show that form 4 learners are more vulnerable than form 3 learners to the irrational belief that *if something is or may be dangerous or fearsome we should be terribly upset and endlessly obsess about it*.

A t-value of 5.36 with $p < 0.01$ was found for the Task or Problem Avoidance subscale. The null hypothesis is rejected at the 1% level of significance. Therefore, a significant difference between the irrational thoughts of form 3 and form 4 Mathematics learners with regard to the Task or Problem Avoidance subscale exists. According to Table 5.46, the mean of form 3 learners was 26.87 and that of form 4 learners 33.04. The results indicate that form 4 learners are more inclined to believe that *it is easier to avoid than to face life difficulties and self-responsibilities* in a Mathematics learning context than form 3 learners.

The Perennial Influence of a Past Problem subscale is the fourth irrational belief subscale for which the mean of form 4 learners was greater than that of form 3 learners. For this subscale, a t-value of 4.35 with $p < 0.01$ was obtained. The results mean that the test is significant and the null hypothesis is rejected at the 1% level of significance. Form 3 learners had a mean of 28.38 and form 4 learners a mean of 33.15. This suggests that form 4 learners are more susceptible than form 3 learners to the irrational belief that *because something once strongly affected our life, it should indefinitely affect it* with regard to Mathematics.

The fact that form 4 learners are more irrational than form 3 learners concurs with results from some previous studies. It was established by Okoiye et al (2013:93) that older learners tend to be less equipped to overcome their Mathematics anxiety than their younger counterparts. The higher level of irrational thinking on the part of older learners has been ascribed to the fact they are more disappointed by their past Mathematics failures than younger learners are (Arem, 2009:215; Okoiye et al, 2013:94). Form 4 learners experienced more Mathematics by virtue of having been

in school for more years than form 3 and had, therefore, developed irrational thoughts with regard to Mathematics as an academic discipline.

5.4.3 Hypothesis 3

The following null hypothesis which corresponds to hypothesis 3 stated in chapter 4 was tested:

There is no significant relationship between the irrational thoughts of learners regarding Mathematics and their achievement in Mathematics.

The null hypothesis was tested for each of the 10 irrational belief subscales.

Table 5.47: Correlation between Irrational Belief Subscales and Mathematics Achievement

N=306	Mathematics Achievement	
Irrational Belief Test Subscales	<i>Pearson, r</i>	p-value
Demand for Love and Approval	-0.15	p<0.05
Demand for Competence	-0.01	p>0.05
Demand for Justice	-0.09	p<0.01
Wishful Thinking	-0.29	p<0.01
Emotional Irresponsibility and Luck	-0.29	p<0.01
Anxious and Endless Obsession over a Problem	-0.29	p<0.01
Task or Problem Avoidance	-0.27	p<0.01
Dependence Mentality	-0.22	p<0.01
Perennial Influence of a Past Problem	-0.26	p<0.01
Perfectionistic and Absolutistic Mentality	-0.04	p>0.05

Table 5.47 shows that all ten irrational beliefs subscales correlate negatively with achievement in Mathematics. However, for two of the ten subscales, that is, Demand for Competence ($r = -0.01$) and Perfectionistic and Absolutistic Mentality ($r = -0.04$), the results are not statistically significant. This implies that for these two subscales,

the null hypothesis cannot be rejected. Therefore, relative to the irrational beliefs that *people should be thoroughly competent, intelligent, and achieving in all possible respects and that there is invariably a right, precise and perfect solution to human problems and it is catastrophic if this perfect solution is not found*, there is no significant relationship between the irrational thoughts of learners and their achievement in Mathematics.

The results contradict the claim by Tsui and Mazzocco (2007:133) that earlier studies established that mathematically gifted learners are perfectionistic. The suggestion made in section 3.3.1.3.13 that the Perfectionistic and Absolutistic Mentality subscale can trigger negative emotions which are detrimental to Mathematics achievement is not supported by the results of the current study. The results are inconsistent with the assertion that the demand for success causes learners to have too high expectations in Mathematics resulting in perfectionism (Cardenoso & Calvete, 2004:289; Boyacioglu & Kucuk, 2011:447; Sue et al, 2006: xvii) thereby negatively affecting their Mathematics achievement because perfectionism denies the infallible nature of human beings (Domino, 2005:8; Ellis, 1994:130).

Eight of the ten irrational belief subscales had a significant negative relationship with learners' achievement in Mathematics. The Demand for Love and Approval subscale had a significant negative correlation of $r = -0.15$ with Mathematics achievement with $p < 0.05$. This means that the null hypothesis is rejected at the 5% significance level. The other seven irrational belief subscales all had significant correlations with achievement in Mathematics with $p < 0.01$. The subscales are Demand for Justice ($r = -0.09$), Wishful Thinking ($r = -0.29$), Emotional Irresponsibility and Luck ($r = -0.29$), Anxious and Endless Obsession over a Problem ($r = -0.29$), Task or Problem Avoidance ($r = -0.29$), Dependence Mentality ($r = -0.22$) and Perennial Influence of a Past Problem ($r = -0.26$). For all seven irrational belief subscales, the null hypothesis is rejected at the 1% significance level.

The rejection of the null hypothesis implies that there is a significant relationship between the irrational thoughts of learners in Mathematics and their achievement in Mathematics. The results precisely show that there is a negative correlation between learners' irrational thoughts in a Mathematics context and their academic

achievement in Mathematics. This implies that the higher the level of irrational thinking regarding Mathematics, the lower the learners' academic achievement in Mathematics. The other meaning is that Mathematics learners with low levels of irrational thinking tend to have high levels of Mathematics achievement. As shown in Table 5.47, three irrational belief subscales had the strongest negative correlation with achievement. The three subscales whose common correlation coefficient with achievement in Mathematics was $r = -0.29$, are Wishful Thinking, Emotional Irresponsibility and Luck and Anxious and Endless Obsession over a Problem. This implies that learners who are influenced by any one of these three irrational beliefs are more likely to have poor Mathematics achievement than learners who are influenced by the other irrational beliefs.

To some extent, the results support the claim made in chapter 3 (section 3.3.1.2) that irrational beliefs trigger negative emotions such as depression, shame, irritability, embarrassment, guilt and hostility (Austad, 2009:275) which can hamper learners' ability to cope with stress (Bermejo-Toro & Prieto-Ursua, 2006:89) and thus lead to poor Mathematics achievement. The negative correlation between irrational thinking and achievement in Mathematics is consistent with the assertion made in section 3.3.1.2 that irrational beliefs relate to poor human functioning and well-being (Ciarrochi, 2004:172; Dryden, 2006:14; Thompson & Henderson, 2007:211; Amutio & Smith, 2007:321). The obtained statistically significant negative correlation between learners' irrational thoughts regarding Mathematics and their scholastic achievement in Mathematics supports the notion that irrational thinking fosters hopelessness (Ullusoy & Duy, 2013:1441) and prevents people from achieving their life goals (Bermejo-Toro & Prieto-Ursua, 2006:89). The assertion that people with irrational beliefs normally have difficulties in solving real life problems (Kaygusuz, 2013:143; Hayes, 1998:210) is supported by the results of the current study. Since studying Mathematics involves problem solving (Feldman, 2009:257; Kosslyn & Rosenberg, 2006:358; Sternberg, 2009:231) as outlined in section 2.2.1.5.1, irrational thinking is bound to interfere with studying Mathematics, hence the negative correlation.

The confirmed negative correlation between the Wishful Thinking subscale and achievement in Mathematics corresponds with the claim that Mathematics learners who fail to obtain the Mathematics grades they are expecting may become frustrated

even though the obtained scores are not very low (section 3.3.1.3.3). The results also support the assertion made in section 3.3.1.3.3 that Mathematics learners wishing to gain regular recognition from their teachers may resent the teachers if they fail to obtain such recognition and this can reduce their chances of doing well in Mathematics. The inference made in section 3.3.1.3.6 that avoiding taking responsibilities like doing homework, organising study groups, seeking extra help from Mathematics teachers and more competent peers, adopting good study habits and researching in libraries may compromise learners' Mathematics achievement is supported by the obtained statistically significant negative correlation between the Task or Problem Avoidance subscale and achievement in Mathematics. The results are also consistent with the view that task avoidance can lead to procrastination which is detrimental to good Mathematics achievement (Sunitha & Muhammedunni, 2013:104; Asikhia, 2010:205-210).

The significant negative correlation between the Dependence Mentality subscale and achievement in Mathematics backs the assertion made in sections 3.3.1.4.7, 3.4.1.1, 3.4.1.4, 3.4.4.1 and 3.4.4.2 that Mathematics learners who regularly want to rely on others reduce their chances of developing reasonable levels of confidence, self-efficacy, self-esteem and problem-solving skills. As far as the Demand for Love and Approval subscale is concerned, the deduction that viewing other people's love and approval in all one's actions as an absolute necessity becomes an unhealthy form of demandingness (Froggatt, 2005:3; Boyacioglu & Kucuk, 2011:447; Civitci, 2007:3; Cardenoso & Calvete, 2004:289; Woodward et al, 2001:258) in a Mathematics context has been supported by results of the current study.

The results obtained in connection with the Demand for Justice subscale in the current study corresponds with the claim made in section 3.3.1.3.2 that the demand for justice and fairness can strain a Mathematics learners' interpersonal relations with significant others such as teachers, parents and peers, which can then result in poor Mathematics achievement. The established significant negative correlation between the Anxious and Endless Obsession over a Problem subscale and achievement in Mathematics supports the deduction made in section 3.3.1.3.5 that continuously obsessing about the intricate nature of Mathematics (Saraswathi, 2003:326; Asikhia, 2010:206) can trigger helplessness (Cardenoso & Calvete,

2004:289) and hopelessness which can reduce one's self-efficacy (Steel, 2007:69; Ullusoy & Duy, 2013:1441) thereby leading to poor Mathematics achievement.

With reference to the Perennial Influence of a Past Problem subscale, the results are consistent with the inference that irrational thinking based on the Perennial Influence of a Past Problem subscale causes learners not to anticipate improving regardless of exerting more effort or developing better study habits (Rosner, 2011:84; Froggatt, 2005:5; Chiesi & Primi, 2010:19) thereby performing poorly in Mathematics. The significant negative correlation between the Perennial Influence of a Past Problem subscale and achievement in Mathematics confirms that the entity theory of intelligence (section 3.4.2.2.1) has weaknesses while simultaneously confirming that the incremental theory of intelligence (section 3.4.2.2.2) can be supported.

The claim made in section 3.3.1.3.12 that emotional irresponsibility can trigger learned helplessness (Woodward et al, 2001:258), stress and anxiety which in turn negatively affect learners' Mathematics achievement (Scarpello, 2007:34) is to some extent supported by the obtained negative correlation between the Emotional Irresponsibility and Luck subscale and achievement in Mathematics. The results are consistent with the claim that learners who attribute all the challenges they encounter to external people and events tend to view Mathematics, their parents, peers or teachers as the external sources of their misery (Egan, 2007:141; Rosner, 2011:84) (section 3.3.1.3.4) thus becoming vulnerable to depression and anxiety which can ultimately lead to poor Mathematics achievement (Beck, 2008:970).

5.4.4 Hypothesis 4

The following null hypothesis regarding hypothesis 4 stated in chapter 4 was tested:

There is no significant relationship between the irrational thoughts of learners in Mathematics and their socio-affective variables such as motivation, stress, anxiety, self-concept, teacher-learner relationships and parental involvement.

A sub-hypothesis was tested for each of the socio-affective variables in the order in which the socio-affective variables appeared in the above null hypothesis.

5.4.4.1 Sub-hypothesis on Irrational Thoughts and Motivation

The following null hypothesis regarding learners' irrational thoughts in Mathematics and motivation was tested:

There is no significant relationship between the irrational thoughts of learners in Mathematics and their motivation to study Mathematics.

The null hypothesis was tested for each of the 10 irrational belief subscales.

Table 5.48: Correlation between Irrational Belief Subscales and Motivation

N=306 Irrational Belief Test subscales	Motivation	
	<i>Pearson, r</i>	p-value
Demand for Love and Approval	-0.19	p<0.01
Demand for Competence	-0.04	p>0.05
Demand for Justice	-0.15	p<0.01
Wishful Thinking	-0.49	p<0.01
Emotional Irresponsibility and Luck	-0.32	p<0.01
Anxious and Endless Obsession over a Problem	-0.36	p<0.01
Task or Problem Avoidance	-0.56	p<0.01
Dependence Mentality	-0.29	p<0.01
Perennial Influence of a Past Problem	-0.37	p<0.01
Perfectionistic and Absolutistic Mentality	0.12	p<0.05

Of the ten irrational belief subscales, only one had no significant relationship with learners' motivation in a Mathematics context. The subscale is Demand for Competence ($r = -0.04$) with $p > 0.05$. For this subscale, the null hypothesis cannot be rejected. This implies that learners' irrational thinking based on the irrational belief that *people should be thoroughly competent, intelligent, and achieving in all possible respects* is not related to their motivation to study Mathematics.

The other nine irrational belief subscales all have significant relationships with motivation. Only one of these nine irrational belief subscales has a positive

correlation with learners' motivation to study Mathematics. The subscale is Perfectionistic and Absolutistic Mentality ($r = 0.12$) with $p < 0.05$. Therefore, the null hypothesis is rejected at the 5% significance level. The results mean that Mathematics learners who believe that *there is invariably a right, precise and perfect solution to human problems and it is catastrophic if this perfect solution is not found* tend to be motivated to study Mathematics. This implies that the greater the level of learners' perfectionism in Mathematics, the more they are motivated to study the subject. This suggests that perfectionism in a Mathematics learning context can be a source of motivation; that is, perfectionists are motivated to study Mathematics. The other meaning of the results is that motivated Mathematics learners tend to be perfectionistic.

The fact that motivated learners tend to work very hard in an attempt to be successful (Awan et al, 2011:73) is consistent with the results of the current study. The positive correlation between the Perfectionistic and Absolutistic Mentality subscale and motivation agrees with the findings of a study by Md.Yunus and Ali (2009:93) which established that there is a positive correlation between effort, self-efficacy and motivation. The results support the remark made in section 2.2.2.1.1 that motivated learners normally show positive emotions such as enthusiasm, optimism, curiosity and interest and tend to exert intense effort and concentration when performing academic activities (Skinner & Belmont, 1991:3; Md.Yunus & Ali, 2009:93).

The remaining eight irrational belief subscales all have a significant negative correlation with motivation. The eight subscales are Demand for Love and Approval ($r = -0.19$), Demand for Justice ($r = -0.15$), Wishful Thinking ($r = -0.49$), Emotional Irresponsibility and Luck ($r = -0.32$), Anxious and Endless Obsession over a Problem ($r = -0.36$), Task or Problem Avoidance ($r = -0.56$), Dependence Mentality ($r = -0.29$) and Perennial Influence of a Past Problem ($r = -0.37$). As shown in Table 5.48, the null hypothesis is rejected at the 5% significance level for two irrational belief subscales and at the 1% level of significance for six subscales. The negative correlation between each of the eight irrational belief subscales and motivation has two broad implications. First, the results indicate that learners who think irrationally about Mathematics tend to have low levels of motivation to study Mathematics. Second, Mathematics learners with low levels of motivation tend to think irrationally

about Mathematics. This implies that unmotivated Mathematics learners are prone to irrational thinking.

The Wishful Thinking subscale has a relatively strong moderate negative correlation coefficient of $r = -0.49$ with motivation. This suggests that Mathematics learners who engage in wishful thinking tend to have low levels of motivation to study Mathematics. The other meaning is that Mathematics learners with low levels of motivation are susceptible to the irrational belief that *it is horrible and catastrophic when things are not the way we like them to be*. The irrational belief subscale which had the strongest significant negative correlation with motivation is Task or Problem Avoidance ($r = -0.56$). The resultant coefficient of determination, r^2 , between the Task or Problem Avoidance subscale and motivation is 0.3136. This implies that about 31% of the variance in learners' motivation to study Mathematics can be explained by the Task or Problem Avoidance subscale. This means that unmotivated Mathematics learners are more prone to the irrational belief that *it is easier to avoid than to face life difficulties and self-responsibilities* than any other irrational belief. The other implication is that learners who follow the irrational belief that *it is easier to avoid than to face life difficulties and self-responsibilities* are least motivated to study Mathematics relative to other irrational beliefs.

The obtained significant negative correlation between irrational thinking in Mathematics and learners' motivation is consistent with the assertion that intrinsically motivated learners or learners with high levels of achievement motivation do not normally engage in task avoidance (Waege, 2010:85; Eccles & Wigfield, 2002:117). The results also support the claim that irrational thinking based on task avoidance can reduce learners' self-determination and motivation as a result of learned helplessness (Johnson, 2003:18). It has been confirmed that irrational beliefs correlate negatively with Mathematics achievement. Therefore, the results support the view that there is a positive correlation between Mathematics learners' levels of motivation and their Mathematics achievement (Tella, 2007:154; Mousoulides & Philippou, 2005:321; Mbugua et al, 2012:88; Md.Yunus & Ali, 2009:93). Consequently, the tentative relationship between irrational beliefs and motivation explored in chapter 3 (section 3.4.1.8.1) relative to Mathematics achievement has been confirmed by the results of the current study.

5.4.4.2 Sub-hypothesis on Irrational Thoughts and Stress

The following null hypothesis pertaining to learners' irrational thoughts about Mathematics and stress was tested:

There is no significant relationship between the irrational thoughts of learners in Mathematics and stress emanating from studying Mathematics.

The null hypothesis was tested for each of the 10 irrational beliefs.

Table 5.49: Correlation between Irrational Belief Subscales and Stress

N=306	Stress	
	<i>Pearson, r</i>	p-value
Irrational Belief Test subscales		
Demand for Love and Approval	0.37	p<0.01
Demand for Competence	0.31	p<0.01
Demand for Justice	0.26	p<0.01
Wishful Thinking	0.56	p<0.01
Emotional Irresponsibility and Luck	0.52	p<0.01
Anxious and Endless Obsession over a Problem	0.63	p<0.01
Task or Problem Avoidance	0.61	p<0.01
Dependence Mentality	0.45	p<0.01
Perennial Influence of a Past Problem	0.55	p<0.01
Perfectionistic and Absolutistic Mentality	0.23	p<0.01

Table 5.49 shows that all 10 irrational belief subscales correlated positively with stress in a Mathematics learning context. For all 10 irrational belief subscales, the null hypothesis is rejected at the 1% level of significance. The results imply that there is a significant positive correlation between learners' irrational thoughts in Mathematics and stress which emanates from studying Mathematics. Learners who think irrationally about Mathematics tend to have high levels of stress. In fact, the

higher the level of a Mathematics learner's irrational thinking the higher the level of that learner's stress and vice-versa.

The Perfectionistic and Absolutistic Mentality subscale ($r = 0.23$) had the weakest positive correlation with stress while several subscales had high positive correlations with stress. Examples of such subscales include Emotional Irresponsibility and Luck ($r = 0.52$); Perennial Influence of a Past Problem ($r = 0.55$); Wishful Thinking ($r = 0.56$); Task or Problem Avoidance ($r = 0.61$) and Anxious and Endless Obsession over a Problem ($r = 0.63$). The coefficient of determination between the Task or Problem Avoidance subscale and stress is 0.3721 and 0.3969 between the Anxious and Endless Obsession over a Problem subscale and stress. This implies that approximately 37% of the variance in Mathematics learners' stress can be explained by the Task or Problem Avoidance subscale. Similarly, about 40% of the variance in Mathematics learners' stress can be explained by the Anxious and Endless Obsession over a Problem subscale. The fact that the Anxious and Endless Obsession over a Problem subscale has the greatest positive correlation with stress has two implications. First, it means that Mathematics learners who believe that *if something is or may be dangerous or fearsome people should be terribly upset and endlessly obsess about it* are most vulnerable to stress relative to the other irrational beliefs. Second, learners who experience stress emanating from studying Mathematics are more vulnerable to the irrational belief that *if something is or may be dangerous or fearsome people should be terribly upset and endlessly obsess about it* than any other irrational belief.

The results support the claim made by Ellis that harbouring irrational beliefs can trigger and sustain stress as irrationality interferes with a person's ability to cope with stress (Bermejo-Toro & Prieto-Ursua, 2006:89). The inference made in section 4.2 that one can expect to find a relationship between learners' irrational thoughts in Mathematics and stress has also been confirmed. The assertion that there is a positive correlation between the intensity of people's irrational thinking and the level of emotional disturbance in the form of the stress which they experience (Najafi et al, 2012:312) is supported by the results of the current study (section 3.3.1.2).

A study by Kauts and Sharma (2009:39) established that learners with low levels of stress perform academically better than high-stress learners. Another study by

Beilock (2008:339-343) revealed that the higher the level of stress, the lower the working memory space, which subsequently leads to poor Mathematics achievement. The findings of these two studies are consistent with the results of the current study since it has already been established (section 5.4.3) that poor Mathematics achievement can expose learners to irrational thinking.

5.4.4.3 Sub-hypothesis on Irrational Thoughts and Anxiety

The following null hypothesis regarding learners' irrational thoughts about Mathematics and anxiety was tested:

There is no significant relationship between the irrational thoughts of learners in Mathematics and their anxiety regarding Mathematics.

The null hypothesis was tested for each of the 10 irrational beliefs.

Table 5.50: Correlation between Irrational Belief Subscales and Anxiety

N=306		Anxiety
Irrational Belief Test subscales	<i>Pearson, r</i>	p-value
Demand for Love and Approval	0.37	p<0.01
Demand for Competence	0.36	p<0.01
Demand for Justice	0.20	p<0.01
Wishful Thinking	0.51	p<0.01
Emotional Irresponsibility and Luck	0.46	p<0.01
Anxious and Endless Obsession over a Problem	0.64	p<0.01
Task or Problem Avoidance	0.58	p<0.01
Dependence Mentality	0.44	p<0.01
Perennial Influence of a Past Problem	0.54	p<0.01
Perfectionistic and Absolutistic Mentality	0.19	p<0.01

A significant positive correlation was found between each of the 10 irrational belief subscales and anxiety as an affective variable as shown in Table 5.50. The null

hypothesis is rejected at the 1% significance level for all the 10 subscales. There is a significant relationship between the irrational thoughts of learners about Mathematics and their anxiety about Mathematics. The positive correlation between the learners' irrational thoughts regarding Mathematics and anxiety implies that the higher the level of irrational thinking regarding Mathematics, the higher the level of anxiety which they experience as a result of studying Mathematics and vice-versa.

Table 5.50 shows that the Perfectionistic and Absolutistic Mentality subscale ($r=0.19$) had the weakest positive correlation with anxiety while several subscales had comparatively strong positive correlations with anxiety. Examples of such subscales include Wishful Thinking ($r=0.51$); Perennial Influence of a Past Problem ($r=0.54$) and Task or Problem Avoidance ($r=0.58$). The Anxious and Endless Obsession over a Problem subscale ($r=0.64$) had the strongest positive correlation with anxiety. The coefficient of determination between the Anxious and Endless Obsession over a Problem subscale and learners' Mathematics anxiety is 0.4096. Almost 41% of the variance in learners' Mathematics anxiety can be attributed to the Anxious and Endless Obsession over a Problem Irrational Belief subscale. This implies that Mathematics learners who believe that *if something is or may be dangerous or fearsome people should be terribly upset and endlessly obsess about it* are most likely to experience anxiety emanating from studying Mathematics relative to other irrational beliefs. The results also mean that Mathematics learners who have high levels of anxiety are more prone to the irrational belief that *if something is or may be dangerous or fearsome people should be terribly upset and endlessly obsess about it* than any other irrational belief.

The results agree with the studies by Boyacioglu and Kucuk (2011:454) and Matta et al (2009:71) which established that there is a strong positive correlation between learners' irrational thinking and their levels of anxiety. The results of the current study are also consistent with the findings of an allied study by Erfani et al (2013:10) which found that both irrational beliefs and cognitive distortions strongly correlate with learners' test anxiety. Moreover, Ellis's claim that anxiety is a negative emotion emanating from irrational thinking (Thompson & Henderson, 2007:212; Austad, 2009:275; Palmer, 2000:281; Ciarrochi, 2004:173) is supported by the results of the current study. Therefore, the speculation made in chapter 4 that there seems to be a relationship between learners' irrational thoughts in Mathematics and anxiety as one

of the socio-affective variables has been supported by the results of the empirical investigation.

5.4.4.4 Sub-hypothesis on Irrational Thoughts and Self-concept

The following null hypothesis pertaining to learners' irrational thoughts about Mathematics and their self-concept was tested:

There is no significant relationship between the irrational thoughts of learners in Mathematics and their self-concept.

The null hypothesis was tested for each of the 10 irrational beliefs.

Table 5.51: Correlation between Irrational Belief Subscales and Self-concept

Irrational Belief Test subscales	Self-concept	
	<i>Pearson, r</i>	p-value
Demand for Love and Approval	-0.20	p<0.01
Demand for Competence	-0.25	p<0.01
Demand for Justice	-0.10	p>0.05
Wishful Thinking	-0.48	p<0.01
Emotional Irresponsibility and Luck	-0.40	p<0.01
Anxious and Endless Obsession over a Problem	-0.54	p<0.01
Task or Problem Avoidance	-0.55	p<0.01
Dependence Mentality	-0.38	p<0.01
Perennial Influence of a Past Problem	-0.44	p<0.01
Perfectionistic and Absolutistic Mentality	-0.05	p>0.05

All 10 irrational belief subscales correlated negatively with self-concept as shown in Table 5.51. However, for two irrational belief subscales, the correlation coefficients were insignificant. The two subscales are Demand for Justice ($r = -0.10$ with $p > 0.05$) and Perfectionistic and Absolutistic Mentality ($r = -0.05$ with $p > 0.05$). For these two irrational belief subscales, the null hypothesis cannot be rejected.

As shown in Table 5.51, there is a significant relationship between each of the remaining eight irrational belief subscales and learners' self-concepts. This implies that there is sufficient statistical evidence to reject the null hypothesis for the eight irrational belief subscales. For the eight irrational belief subscales, the null hypothesis is rejected at the 1% level of significance. The rejection of the null hypothesis means that the higher the level of learners' irrational thinking about Mathematics, the more distorted their self-concepts become. This suggests that learners who engage in irrational thinking regarding Mathematics tend to have poor self-concepts. The results also mean that Mathematics learners with good self-concepts are not likely to engage in irrational thinking about Mathematics and vice-versa.

The two irrational belief subscales which had the strongest negative correlation with self-concept are Anxious and Endless Obsession over a Problem ($r = -0.54$) and Task or Problem Avoidance ($r = -0.55$). The coefficient of determination for the Anxious and Endless Obsession over a Problem subscale is 0.2916 and 0.3025 for the Task or Problem Avoidance subscale. This implies that about 29% of the variance in Mathematics learners' self-concept can be explained by the Anxious and Endless Obsession over a Problem subscale while approximately 30% of the variance in Mathematics learners' self-concept can be explained by the Task or Problem Avoidance subscale. Therefore, learners who engage in irrational thinking about Mathematics based on Anxious and Endless Obsession over a Problem and Task or Problem Avoidance subscales tend to have the most distorted self-concepts relative to the other irrational belief subscales. The results also mean that Mathematics learners with distorted self-concepts are more vulnerable to the irrational beliefs that *if something is or may be dangerous or fearsome people should be terribly upset and endlessly obsess about it* and that *it is easier to avoid than to face life difficulties and self-responsibilities*.

The confirmation of the assertion that there is a significant relationship between irrational thoughts of learners regarding Mathematics and self-concept as an affective variable in the current study is consistent with a study by Arai (2001:315) which established that there is a relationship between irrational beliefs in general and learners' self-concepts (section 3.4.1.3.1). Unconditional self-acceptance, which is part of an individual's self-concept, occurs when people accept themselves

regardless of factors such as their intelligence, competence or other people's approval (Ellis, 1977:101; Palmer, 2000:280; Austad, 2009:278) (section 3.2.2.1.2.4). The obtained negative correlation between irrational thinking and self-concept confirms that unconditional self-acceptance is a rational belief (Ellis, 1977:101; Kosslyn & Rosenberg, 2006:698). On the other hand, conditional self-acceptance in which people condemn themselves for being imperfect and yearn for other people's approval, reflects a poor sense of self-worth and a distorted self-concept (Macavei, 2005:74; Ellis & Dryden, 2003:5; Palmer, 2000:280; Bridges & Harnish, 2010:863) (section 3.2.2.2.2.4). Both Ellis and Rogers asserted that conditions of self-worth normally lead to cognitive and emotional distress (Flett et al, 2003:120). The results of the current empirical investigation confirm that conditional self-acceptance is a form of irrationality which is normally accompanied by a distorted self-concept.

5.4.4.5 Sub-hypothesis on Irrational Thoughts and Teacher-Learner Relationships

The following null hypothesis regarding learners' irrational thoughts pertaining to Mathematics and Teacher-Learner Relationships was tested:

There is no significant relationship between the irrational thoughts of learners about Mathematics and Teacher-Learner Relationships.

The null hypothesis was tested for each of the 10 irrational beliefs.

Table 5.52: Correlation between Irrational Belief Subscales and Teacher-Learner Relationships

N=306	Teacher-Learner Relationships	
Irrational Belief Test subscales	<i>Pearson, r</i>	p-value
Demand for Love and Approval	-0.04	p>0.05
Demand for Competence	-0.02	p>0.05
Demand for Justice	-0.18	p<0.01
Wishful Thinking	-0.40	p<0.01

Emotional Irresponsibility and Luck	-0.29	$p < 0.01$
Anxious and Endless Obsession over a Problem	-0.31	$p < 0.01$
Task or Problem Avoidance	-0.39	$p < 0.01$
Dependence Mentality	-0.12	$p < 0.05$
Perennial Influence of a Past Problem	-0.27	$p < 0.01$
Perfectionistic and Absolutistic Mentality	0.06	$p > 0.05$

As shown in Table 5.52, nine of the ten irrational belief subscales each had a negative correlation with Teacher-Learner Relationships. The only subscale which correlated positively with Teacher-Learner Relationships was Perfectionistic and Absolutistic Mentality ($r = 0.06$) and the correlation was insignificant. Two other subscales for which there was no statistical evidence to reject the null hypothesis were Demand for Love and Approval ($r = -0.04$) and Demand for Competence ($r = -0.02$).

Seven of the ten irrational belief subscales had significant negative correlations with Teacher-Learner Relationships in a Mathematics learning context. The null hypothesis is rejected at the 5% level of significance for the Dependence Mentality subscale ($r = -0.12$). For the remaining six subscales, that is, Demand for Justice ($r = -0.18$); Wishful Thinking ($r = -0.40$); Emotional Irresponsibility and Luck ($r = -0.29$); Anxious and Endless Obsession over a Problem ($r = -0.31$); Task or Problem Avoidance ($r = -0.39$) and Perennial Influence of a Past Problem ($r = -0.27$), the null hypothesis is rejected at the 1% level of significance. The results indicate that learners with a high level of irrational thinking normally experience poor relationships with their teachers. Therefore, learners who do not engage in irrational thinking normally have good relationships with their Mathematics teachers. Conversely, Mathematics learners who have poor relationships with their teachers tend to think irrationally about Mathematics. These interpretations apply mostly to Task or Problem Avoidance ($r = -0.39$) and Wishful Thinking ($r = -0.40$) subscales since they have the strongest negative correlation coefficients with Teacher-Learner Relationships. This implies that 16% of the variance in Teacher-Learner Relationships can be explained by the Wishful Thinking irrational belief subscale

since the coefficient of determination between the Wishful Thinking subscale and Teacher-Learner relationships is 0.16.

The negative correlation between irrational thinking and Teacher-Learner Relationships is consistent with the studies which revealed that teacher-learner relationships based on the authoritative leadership style are conducive to good scholastic achievement (Kumar, 2004:114; Wayne & Young, 2003:89; Dhameja & Dhameja, 2009:83). Irrational thinking regarding Mathematics has been found to correlate negatively with both Mathematics achievement (section 5.4.3) and Teacher-Learner relationships thereby suggesting that there is a positive correlation between good teacher-learner relationships and achievement in Mathematics. This agrees with the results of a study by Freiberg et al (2009:63) which established that good teacher-learner relationships are conducive to good Mathematics achievement. A study by Garcia et al (2011:2) also found a positive correlation between healthy teacher-learner relationships and learners' academic achievement.

On the other hand, it has been established that poor teacher-learner relationships can weaken learners' scholastic achievement (Smith, 2004:14; Hamre & Pianta, 2001:625), thus exposing the learners to irrational thinking. Poor teacher-learner relationships are prevalent in cases in which the authoritarian classroom management style is used (Osler, 2010:77; Dhameja & Dhameja, 2009:82). This is in agreement with the obtained results that irrational thinking in Mathematics correlates negatively with the quality of teacher-learner relationships.

It has already been established that teachers are capable of influencing Mathematics learners' beliefs, attitudes and motivation (Shein & Chiou, 2011:1097; Bol & Berry III, 2005:36; Handal, 2003:48; Campton et al, 2007:7; Stipek et al, 2001:224; Pierce & Newstrom, 2008:xxix). As outlined in section 3.4.4.3, teachers are capable of influencing learners' attitudes towards academic performance, self-efficacy beliefs and their beliefs about their own intelligence (McLean, 2003:51; Miller & Satchwell, 2006:136; Compton et al, 2007:7; Beedle & Burkill, 2008:47). The results of the current study also confirm that the way teachers interact with learners is related to the learners' level of irrational thinking regarding Mathematics.

5.4.4.6 Sub-hypothesis on Irrational Thoughts and Parental Involvement

The following null hypothesis pertaining to learners' irrational thoughts regarding Mathematics and parental involvement was tested:

There is no significant relationship between the irrational thoughts of learners about Mathematics and parental involvement.

The null hypothesis was tested for each of the 10 irrational beliefs subscales.

Table 5.53: Correlation between Irrational Belief Subscales and Parental Involvement

N=306	Parental Involvement	
	<i>Pearson, r</i>	p-value
Irrational Belief Test subscales		
Demand for Love and Approval	0.08	p>0.05
Demand for Competence	0.28	p<0.01
Demand for Justice	-0.07	p>0.05
Wishful Thinking	-0.19	p<0.01
Emotional Irresponsibility and Luck	-0.08	p>0.05
Anxious and Endless Obsession over a Problem	-0.12	p<0.05
Task or Problem Avoidance	-0.23	p<0.01
Dependence Mentality	0.06	p>0.05
Perennial Influence of a Past Problem	-0.12	p<0.05
Perfectionistic and Absolutistic Mentality	0.25	p<0.01

According to Table 5.53, there are four irrational belief subscales for which there is no significant relationship between Mathematics learners' irrational thoughts and parental involvement. The subscales are Demand for Love and Approval ($r = 0.08$); Demand for Justice ($r = -0.07$); Emotional Irresponsibility and Luck ($r = -0.08$) and Dependence Mentality ($r = 0.06$). In the instances, the null hypothesis cannot be rejected.

The relationship between the irrational thoughts of learners in Mathematics and parental involvement was found to be significant for six subscales. Two of the six subscales had significant positive correlations with Parental Involvement while the other four subscales correlated negatively with Parental Involvement. The two subscales which had a significant positive correlation with Parental Involvement were Demand for Competence ($r = 0.28$) and Perfectionistic and Absolutistic Mentality ($r = 0.25$). For these two subscales, the null hypothesis is rejected at the 1% level of significance. The results indicate that a high level of irrational thinking in a Mathematics context is accompanied by high levels of parental involvement. The higher the level of parental involvement the more Mathematics learners become prone to the irrational beliefs that *people should be thoroughly competent, intelligent, and achieving in all possible respects* and that *there is invariably a right, precise and perfect solution to human problems and it is catastrophic if this perfect solution is not found*.

The above results are consistent with the authoritarian parenting style. It was highlighted in section 2.3.2.1.2 that authoritarian parents have high expectations (Tassoni et al, 2007:327) and are rigid, punitive, hostile and demand unquestionable obedience from their children (Feldman, 2009:411; Swartz et al, 2011:65; Snowman et al, 2009:26). Learners who are subjected to authoritarian parenting tend to develop resentment and a sense of insecurity as well as lack of initiative, social incompetence and anxiety when faced with competition (Santrock, 2004:74; Snowman et al, 2009:411) owing to lack of warmth and being denied the opportunity to air their views. This is likely to promote irrational thinking among such learners thereby giving a positive correlation between irrational thinking and parental involvement. The results of the current study also concur with the findings of a study by Levpuscek and Zupancic (2009:541) which established that there is a negative correlation between high parental expectations and learners' academic performance.

The four irrational belief subscales which correlated negatively with Parental Involvement are Wishful Thinking ($r = -0.19$); Anxious and Endless Obsession over a Problem ($r = -0.12$); Task or Problem Avoidance ($r = -0.23$) and Perennial Influence of a Past Problem ($r = -0.12$). For these four subscales, the null hypothesis is rejected at the 1% level of significance for the Wishful Thinking subscale and at the 5% level of significance for the other three subscales. This implies that irrational thinking in

Mathematics occurs if learners experience poor parental involvement. The results also indicate that learners who are exposed to poor parental involvement are likely to think irrationally about Mathematics. These results are more applicable to the relationship between the Task or Problem subscale and Parental Involvement since the Task or Problem Avoidance subscale had the strongest negative correlation of $r = -0.23$ with Parental Involvement.

A negative correlation between learners' irrational thoughts regarding Mathematics and parental involvement is consistent with the authoritative parenting style. As discussed in section 2.3.2.1.2, authoritative parents are firm and set clear limits but reason with their children and take time to explain situations to them (Feldman, 2009:412; Tuckman & Monetti, 2011:119; Gonzalez & Wolters, 2006:203). There is moderate demandingness and moderate responsiveness. Baumrind specifically indicated that the authoritative parenting style is the best parenting style because it normally produces happy, successful, intrinsically motivated, curious, socially competent, self-reliant and capable children who have high levels of self-esteem (Santrock, 2004:74; Snowman et al, 2009:26; Tassoni et al, 2007:327). This implies that authoritative parenting tends to foster rational thinking in children, thus generating a negative correlation between irrational thinking and Parental Involvement.

5.4.5 Hypothesis 5

The following null hypothesis regarding hypothesis 5 stated in chapter 4 was tested:

There is no significant relationship between learners' irrational thoughts about Mathematics and their perceptions of Mathematics.

A null hypothesis was tested for each of the ten irrational beliefs and the four perception factors as well as the overall perceptions of learners regarding Mathematics.

5.4.5.1 Sub-hypothesis on Irrational Thoughts about Mathematics and The Inevitable

The following null hypothesis pertaining to learners' irrational thoughts regarding Mathematics and The Inevitable was tested:

There is no significant relationship between learners' irrational thoughts regarding Mathematics and The Inevitable

Table 5.54: Correlation between Irrational Belief Subscales and The Inevitable

Irrational Belief Test subscales	The Inevitable	
	<i>Pearson, r</i>	p-value
N=306		
Demand for Love and Approval	0.29	p<0.01
Demand for Competence	-0.01	p>0.05
Demand for Justice	0.31	p<0.01
Wishful Thinking	0.39	p<0.01
Emotional Irresponsibility and Luck	0.38	p<0.01
Anxious and Endless Obsession over a Problem	0.31	p<0.01
Task or Problem Avoidance	0.36	p<0.01
Dependence Mentality	0.25	p<0.01
Perennial Influence of a Past Problem	0.43	p<0.01
Perfectionistic and Absolutistic Mentality	0.12	p<0.05

There is only one irrational belief subscale for which no significant relationship was found between learners' irrational thoughts regarding Mathematics and The Inevitable. The subscale is Demand for Competence ($r = -0.01$ with $p > 0.05$). For this subscale the null hypothesis cannot be rejected. This implies that the way Mathematics learners are influenced by the irrational belief that *people should be thoroughly competent, intelligent, and achieving in all possible respects* has no significant relationship with The Inevitable.

Nine of the ten irrational belief subscales had significant relationships with The Inevitable. A positive correlation was found between each of the nine irrational belief subscales and The Inevitable. For the nine subscales, there is sufficient statistical evidence to reject the null hypothesis. As shown in Table 5.54, the null hypothesis is

rejected at the 5% level of significance for only one subscale, that is, Perfectionistic and Absolutistic Mentality, and at the 1% level of significance for the other eight subscales. The results indicate that the higher the irrational thinking of learners regarding Mathematics based on the nine irrational beliefs, the more they bear faulty perceptions of issues such as gender, the belief that Mathematics is naturally difficult and the school atmosphere, all of which seem to be beyond their control. The faulty perceptions are strongest in learners' irrational thoughts about Mathematics based on the Perennial Influence of a Past Problem subscale ($r= 0.43$). The coefficient of determination for the Perennial Influence of a Past Problem subscale is 0.1849. This indicates that approximately 18% of the variance in learners' faulty perceptions of The Inevitable is explained by the Perennial Influence of a Past Problem subscale.

The positive correlation between learners' irrational thinking about Mathematics and The Inevitable supports the assertion that many learners have faulty perceptions of Mathematics since they view it as an academic subject which has been imposed upon them. It has already been pointed out that Mathematics is often rated as a challenging subject (Saraswathi, 2003:326; Asikhia, 2010:206; Meremikwu & Eukoha, 2011:87-88; Mutodi & Ngirande, 2014:432). The results are also consistent with the widely held perception that male learners often outclass their female counterparts in Mathematics (Haylock & Thangata, 2007:77; Haylock, 2006:3; Watt, 2004:1556; Frenzel et al, 2010:508; Hannula, 2002:25; Palsdottir, 2007:117). Moreover, the results are in agreement with the view that teacher expectations as part of The Inevitable can lead to negative self-fulfilling prophecies on the part of the learners (Hinnant et al, 2009:662) which can promote irrational thinking.

5.4.5.2 Sub-hypothesis on irrational thoughts about Mathematics and the Role of Rewards

The following null hypothesis regarding learners' irrational thoughts pertaining to Mathematics and The Role of Rewards was tested:

There is no significant relationship between learners' irrational thoughts about Mathematics and The Role of Rewards

Table 5.55: Correlation between Irrational Belief Subscales and the Role of Rewards

N=306	The Role of Rewards	
Irrational Belief Test subscales	<i>Pearson, r</i>	p-value
Demand for Love and Approval	0.18	p<0.01
Demand for Competence	0.27	p<0.01
Demand for Justice	0.05	p>0.05
Wishful Thinking	-0.06	p>0.05
Emotional Irresponsibility and Luck	0.16	p<0.01
Anxious and Endless Obsession over a Problem	0.08	p>0.05
Task or Problem Avoidance	-0.06	p>0.05
Dependence Mentality	0.13	p<0.05
Perennial Influence of a Past Problem	0.07	p>0.05
Perfectionistic and Absolutistic Mentality	0.40	p<0.01

Table 5.55 shows that there are five irrational belief subscales for which no significant relationship between the irrational thoughts of learners regarding Mathematics and The Role of Rewards was found. The subscales are Demand for Justice; Wishful Thinking; Anxious and Endless Obsession over a Problem; Task or Problem Avoidance and Perennial Influence of a Past Problem. For these five subscales, the null hypothesis cannot be rejected. The results indicate that learners' faulty perceptions of The Role of Rewards are not related to learners' irrational thoughts pertaining to Mathematics.

There are also five irrational belief subscales for which significant relationships were found between learners' thoughts regarding Mathematics and The Role of Rewards. The subscales are Demand for Love and Approval ($r= 0.18$); Demand for Competence ($r= 0.27$); Emotional Irresponsibility and Luck ($r= 0.16$); Dependence Mentality ($r= 0.13$) and Perfectionistic and Absolutistic Mentality ($r= 0.40$). The null hypothesis is rejected at the 5% level of significance for the Dependence Mentality

subscale and at the 1% significance level for the other four subscales. The results imply that learners who have low levels of irrational thinking regarding Mathematics tend to have low levels of faulty perceptions of The Role of Rewards. The higher the level of learners' irrational thinking in Mathematics, the more they have faulty perceptions of The Role of Rewards. The results also mean that Mathematics learners who have faulty perceptions of The Role of Rewards tend to engage in irrational thinking in Mathematics based on the five subscales. The Perfectionistic and Absolutistic Mentality ($r= 0.40$) had the strongest positive correlation with The Role of Rewards. Consequently, 16% of the variance in The Role of Rewards can be explained by the Perfectionistic and Absolutistic Mentality irrational belief subscale. This implies that Mathematics learners who irrationally believe that *there is invariably a right, precise and perfect solution to human problems and it is catastrophic if this perfect solution is not found* are more likely to have faulty perceptions of The Role of Rewards relative to other irrational beliefs.

Perceptions of The Role of Rewards are characterised by an excessive desire to do well in Mathematics so as to gain rewards such as recognition and status from significant others (Chetty, 1996:139) (section 4.3.2.2.2.2). This excessive desire to attain high grades in Mathematics in a bid to be rewarded exerts pressure on the learners and it becomes a form of perfectionism and demandingness. Ellis maintains that when perfectionism is viewed as an absolute and a dire necessity, it forces the individual to pursue an unattainable set of goals and the whole pursuit is frustrating and self-defeating in many ways (Flett & Hewitt, 2004:234) thereby fostering irrationality (Gunduz, 2013:2081; Domino, 2005:8; Flett & Hewitt, 2004:234). This is supported by the results of the current study because the Perfectionistic and Absolutistic Mentality subscale ($r= 0.40$) had the greatest positive correlation with The Role of Rewards. Demandingness is a core irrational belief which causes people to express their wishes in the form of peremptory and unqualified commands and demands (David et al, 2010:13; Bridges & Harnish, 2010:863; Austad, 2009:276; Szentagotai & Jones, 2010:83; Ciarrochi, 2004:175; Kosslyn & Rosenberg, 2006:697; Corey, 2009:145) (section 3.2.2.2.2.1). Perceptions of the Role of Rewards are related to the demand for success. The positive correlation between irrational thinking in Mathematics and The Role of Rewards confirms that

demandingness is a fundamental irrational belief which is related to learners' faulty perceptions of Mathematics.

5.4.5.3 Sub-hypothesis on irrational thoughts in Mathematics and School and Schooling

The following null hypothesis pertaining to learners' irrational thoughts regarding Mathematics and School and Schooling as a factor of perceptions was tested:

There is no significant relationship between learners' irrational thoughts in Mathematics and their perceptions of School and Schooling.

Table 5.56: Correlation between Irrational Belief Subscales and School and Schooling

N=306	School and Schooling	
Irrational Belief Test subscales	<i>Pearson, r</i>	p-value
Demand for Love and Approval	0.33	p<0.01
Demand for Competence	-0.08	p>0.05
Demand for Justice	0.37	p<0.01
Wishful Thinking	0.48	p<0.01
Emotional Irresponsibility and Luck	0.42	p<0.01
Anxious and Endless Obsession over a Problem	0.33	p<0.01
Task or Problem Avoidance	0.45	p<0.01
Dependence Mentality	0.28	p<0.01
Perennial Influence of a Past Problem	0.51	p<0.01
Perfectionistic and Absolutistic Mentality	0.01	p>0.05

There were two irrational belief subscales for which there was no significant relationship between learners' irrational thoughts in Mathematics and faulty perceptions of School and Schooling. The subscales are Demand for Competence and Perfectionistic and Absolutistic Mentality. In both cases, the null hypothesis

cannot be rejected. Therefore, the perceptions of Mathematics learners pertaining to School and Schooling are not related to learners' beliefs that *people should be thoroughly competent, intelligent, and achieving in all possible respects* and that *there is invariably a right, precise and perfect solution to human problems and it is catastrophic if this perfect solution is not found*.

According to Table 5.56, eight irrational belief subscales have significant relationships with learners' faulty perceptions of School and Schooling. For the eight subscales, the null hypothesis is rejected at the 1% level of significance. There is a significant positive correlation between learners' irrational thoughts regarding Mathematics and their faulty perceptions of School and Schooling. As discussed in section 4.3.2.2.3, the School and Schooling subscale deals with perceptions of the school and classroom atmosphere and the utility of schooling in solving real life problems (Chetty, 1996:140). The results indicate that high levels of irrational thinking on the part of Mathematics learners are accompanied by high levels of faulty perceptions of the school and classroom atmosphere and the utility of schooling in solving real life problems. Learners who have faulty perceptions of School and Schooling are prone to irrational thinking about Mathematics based on the eight irrational belief subscales.

Examples of irrational belief subscales with comparatively high positive correlations with School and Schooling include Emotional Irresponsibility and Luck ($r = 0.42$); Task or Problem Avoidance ($r = 0.45$) and Wishful Thinking ($r = 0.48$). The Perennial Influence of a Past Problem subscale ($r = 0.51$) had the strongest positive correlation with School and Schooling. The resultant coefficient of determination (r^2) between the Perennial Influence of a Past Problem subscale and School and Schooling is 0.2601. Therefore, about 26% of the variance in School and Schooling can be attributed to the Perennial Influence of a Past Problem irrational belief subscale. This implies that Mathematics learners who believe that *because something once strongly affected our life, it should indefinitely affect it* tend to have the faultiest perceptions of School and Schooling.

The obtained significant positive correlation between learners' irrational thoughts about Mathematics and faulty perceptions of School and Schooling reinforces the view that irrational beliefs promote human dysfunction and negative feelings about

life (Ciarrochi, 2004:172; Dryden, 2006:14; Thompson & Henderson, 2007:211; Amutio & Smith, 2007:321; Davies, 2008:2). To some extent, the results are also consistent with the studies by Githua (2013:175) and Tachie and Chireshe (2013:68) which established that negative perceptions of Mathematics are detrimental to Mathematics achievement since irrational thinking has been found to correlate negatively with Mathematics achievement (section 5.4.3).

5.4.5.4 Sub-hypothesis on irrational thoughts about Mathematics and Teachers and Parents

The following null hypothesis pertaining to learners' irrational thoughts regarding Mathematics and Teachers and Parents was tested:

There is no significant relationship between learners' irrational thoughts about Mathematics and their perceptions of Teachers and Parents

Table 5.57: Correlation between Irrational Belief Subscales and Teachers and Parents

N=306	Teachers and Parents	
Irrational Belief Test subscales	<i>Pearson, r</i>	p-value
Demand for Love and Approval	0.30	p<0.01
Demand for Competence	0.12	p<0.05
Demand for Justice	0.26	p<0.01
Wishful Thinking	0.39	p<0.01
Emotional Irresponsibility and Luck	0.44	p<0.01
Anxious and Endless Obsession over a Problem	0.37	p<0.01
Task or Problem Avoidance	0.39	p<0.01
Dependence Mentality	0.36	p<0.01
Perennial Influence of a Past Problem	0.46	p<0.01
Perfectionistic and Absolutistic Mentality	0.23	p<0.01

It was highlighted in section 4.3.2.2.4 that Teachers and Parents as a factor of perceptions focuses on issues such as the instructional approaches employed by teachers, parenting styles, and the attitudes of teachers and parents towards learners' scholastic work (Chetty, 1996:140). Table 5.57 shows that all the 10 irrational belief subscales had positive correlations with faulty perceptions of Teachers and Parents. Moreover, relative to each subscale, there is a significant relationship between learners' irrational thoughts regarding Mathematics and their faulty perceptions of Teachers and Parents. There is sufficient evidence to reject the null hypothesis for all 10 subscales. The null hypothesis is rejected at the 5% level of significance for the Demand for Competence subscale and at the 1% level of significance for the other nine subscales. The results indicate that the more learners engage in irrational thinking about Mathematics, the more they have faulty perceptions of Teachers and Parents. The results also suggest that the more Mathematics learners have faulty perceptions of Teachers and Parents, the more they become susceptible to irrational thinking about Mathematics. The outlined relationships are weakest for the Demand for Competence subscale ($r = 0.12$). The Perennial Influence of a Past Problem subscale ($r = 0.46$) had the highest Pearson product moment correlation coefficient with Teachers and Parents and a coefficient of determination of 0.2116. Such a coefficient of determination indicates that approximately 21% of the variance in learners' perceptions of Teachers and Parents can be explained by the Perennial Influence of a Past Problem subscale. This implies that Mathematics learners who are influenced by the irrational belief that *because something once strongly affected our life, it should indefinitely affect it* have the faultiest perceptions of Teachers and Parents.

Several studies have established that teachers are a critical determinant of learners' academic achievement (Lamb & Fullarton, 2001:2; Jaiyeoba & Atanda, 2011:91; Heck, 2009:227). The results support the view that learners who have faulty perceptions of the way teachers conduct their professional duties normally perform poorly in scholastic tasks thereby becoming prone to irrational thinking. The findings of the current study are consistent with the assertion that faulty perceptions of teachers' expectations can result in negative self-fulfilling prophecies and poor academic achievement (Levpuscek & Zupancic, 2009:541; Hinnant et al, 2009:662). The findings also concur with the view that learners who have negative perceptions

of the role of their parents in their education tend to become frustrated owing to lack of achievement motivation as hinted by Snowman et al (2009:26), Gonzalez and Wolters (2006:203) and Santrock (2004:75), thereby becoming susceptible to irrational thinking.

5.4.5.5 Sub-hypothesis on irrational thoughts in Mathematics and learners' overall perceptions of Mathematics.

The following null hypothesis pertaining to learners' irrational thoughts regarding Mathematics and their overall Perceptions of Mathematics was tested:

There is no significant relationship between learners' irrational thoughts regarding Mathematics and their overall Perceptions of Mathematics.

Table 5.58: Correlation between Irrational Belief Subscales and Overall Perceptions of Mathematics

N=306	Overall Perceptions of Mathematics	
Irrational Belief Test subscales	<i>Pearson, r</i>	p-value
Demand for Love and Approval	0.23	p< 0.01
Demand for Competence	-0.16	p< 0.01
Demand for Justice	0.25	p< 0.01
Wishful Thinking	0.25	p< 0.01
Emotional Irresponsibility and Luck	0.24	p< 0.01
Anxious and Endless Obsession over a Problem	0.21	p< 0.01
Task or Problem Avoidance	0.23	p< 0.01
Dependence Mentality	0.14	p< 0.05
Perennial Influence of a Past Problem	0.32	p< 0.01
Perfectionistic and Absolutistic Mentality	0.07	p>0.05

No significant relationship was found between learners' irrational thoughts regarding Mathematics relative to the Perfectionistic and Absolutistic Mentality subscale (r=

0.07) and their overall perceptions of Mathematics. With regard to the Perfectionistic and Absolutistic Mentality subscale, the null hypothesis cannot be rejected. This implies that Mathematics learners' belief that *there is invariably a right, precise and perfect solution to human problems and it is catastrophic if this perfect solution is not found* is not related to their faulty perceptions of Mathematics.

Table 5.58 shows that a significant relationship was found between nine irrational belief subscales and learners' overall perceptions of Mathematics. One irrational belief subscale, the Demand for Competence ($r = -0.16$), had a negative correlation with learners' overall perceptions of Mathematics. For this subscale, the null hypothesis is rejected at the 1% significance level. The results imply that the higher the degree of Mathematics learners' irrational thinking relative to the irrational belief that *people should be thoroughly competent, intelligent, and achieving in all possible respects*, the fewer the faulty perceptions the learners have about Mathematics.

Each of the eight remaining irrational belief subscales had a significant positive correlation with learners' overall perceptions of Mathematics. There is ample evidence to reject the null hypothesis. For the Dependence Mentality subscale ($r = 0.14$), the null hypothesis is rejected at the 5% significance level and at the 1% significance level for the other seven irrational belief subscales. The results indicate that the higher the degree of learners' irrational thinking regarding Mathematics, the higher the level of learners' faulty perceptions of Mathematics. The other implication is that Mathematics learners with faulty perceptions of Mathematics are prone to irrational thinking regarding Mathematics based on the eight irrational beliefs.

For the eight irrational belief subscales, the Dependence Mentality ($r = 0.14$) had the lowest positive correlation with learners' overall perceptions of Mathematics. The Perennial Influence of a Past Problem subscale ($r = 0.32$) had the strongest positive correlation with learners' faulty perceptions of Mathematics and a coefficient of determination of 0.1024. Therefore, approximately 10% of the variance in learners' faulty perceptions of Mathematics can be explained by the Perennial Influence of a Past Problem subscale. This implies that learners who are influenced by the irrational belief that *because something once strongly affected our life, it should indefinitely affect it* are most likely to harbour strong faulty perceptions of Mathematics. It is worth noting that the Perennial Influence of a Past Problem

subscale had the highest correlation coefficients with three perception factors, which are, The Inevitable, School and Schooling, and Teachers and Parents

The significant positive correlation between learners' irrational thinking about Mathematics and their perceptions of Mathematics supports the already established view that many learners have negative perceptions of Mathematics as an academic discipline (Asikhia, 2010:205-6; Saraswathi, 2003:326; Petty, 2009:510; Meremikwu & Eukoha, 2011:87-88; Mutodi & Ngirande, 2014:432). The results also back the findings of studies by Githua (2013:175) and Tachie and Chireshe (2013: 71) which show that faulty perceptions of Mathematics correlate negatively with Mathematics achievement since Mathematics achievement has been found to correlate negatively with irrational thinking (section 5.4.3). The results of the current study support the claim made in chapter 1 (section 1.2) that the faulty perceptions which some learners have regarding studying Mathematics (Spangler, 1992:19) are to some extent related to their irrational beliefs. Furthermore, the assertion by Okoiye et al (2013:88) that learners who harbour negative perceptions of Mathematics are vulnerable to Mathematics failure is also supported by the results of the current study since failure in Mathematics can expose the learners to irrational thinking.

5.4.6 Hypothesis 6

The following null hypothesis regarding hypothesis 6 stated in chapter 4 was tested:

Irrational thoughts, perceptions and socio-affective variables do not jointly explain a larger proportion of the variance in Mathematics achievement than any one of these factors on its own.

In order to ascertain whether irrational thoughts, perceptions and socio-affective variables jointly explain a larger proportion of the variance in learners' Mathematics achievement than any one of these factors on its own, a stepwise regression analysis was performed. Learners' Mathematics achievement was used as the dependent variable while irrational beliefs, perceptions and socio-affective factors were used as the independent variables.

Below are the 10 irrational beliefs, four factors regarding perceptions and six socio-affective variables which were used as independent variables in the initial regression analysis:

➤ Irrational beliefs:

- Demand for Love and Approval
- Demand for Competence
- Demand for Justice
- Wishful Thinking
- Emotional Irresponsibility and Luck
- Anxious and Endless over a Problem
- Task Avoidance
- Dependence Mentality
- Perennial Influence of a Past Problem
- Perfectionistic and Absolutistic Mentality

➤ Perceptions regarding

- The Inevitable
- The Role of Rewards
- School and Schooling
- Teachers and Parents

➤ Socio-affective variables

- Motivation
- Stress
- Anxiety
- Self-concept
- Teacher-learners Relationships

- Parental Involvement

The first variable to enter the regression model was Self-concept. For Self-concept, the coefficient of determination, $R^2 = 0.160$ was significant with $F(1; 304) = 58.04$; $p < 0.01$. The second variable to enter the regression analysis was perceptions of School and Schooling. The joint proportion of the variance in Mathematics achievement explained by Self-concept and perceptions of School and Schooling was 18.4%. This implies that perceptions of School and Schooling explained an additional 2.4% of the variance in Mathematics achievement not explained by Self-concept. As shown in Table 5.59, for Self-concept and perceptions of School and Schooling, R^2 was significant with $F(2; 303) = 8.92$; $p < 0.01$.

The third variable to enter the regression analysis was Emotional Irresponsibility and Luck. Self-concept, perceptions of School and Schooling and Emotional Irresponsibility and Luck jointly explained a total variance of 19.3% in secondary school learners' Mathematics achievement. This indicates that Emotional Irresponsibility and Luck explained only an additional 1% of the proportion of the variance in Mathematics achievement not jointly explained by Self-concept and perceptions of School and Schooling. For the first three independent variables, a model R^2 was significant with $F(3; 302) = 3.04$; $p < 0.05$.

The fourth independent variable to enter the regression analysis was Demand for Competence. Demand for Competence increased the coefficient of determination, R^2 , from 0.193 to 0.203. This indicates that Demand for Competence explained approximately a further 1% of the variance in Mathematics achievement which was not already jointly explained by Self-concept, perceptions of School and Schooling and Emotional Irresponsibility and Luck. The coefficient of determination, R^2 of 0.203 was significant with $F(4; 301) = 3.93$; $p < 0.05$.

None of the remaining independent variables could explain a significantly larger proportion of the variance in learners' Mathematics achievement than that already explained by the first four variables. The variables are:

- Self-concept
- Perceptions of School and Schooling
- Emotional Irresponsibility and Luck

- Demand for Competence

The four independent variables comprised one socio-affective variable, that is, Self-concept and one factor concerning learners' perceptions of School and Schooling. The other two variables, that is, Demand for Competence and Emotional Irresponsibility and Luck, are both irrational beliefs.

The results of the regression analysis are summarised in Table 5.59.

Table 5.59: Proportion of the Variance in Mathematics Achievement as the Dependent Variable Jointly Explained by Irrational Thoughts, Perceptions and Socio-Affective Variables

Step	Independent Variable	Model R-Square, R^2	F	df	p-value
1	Self-concept	0.160	58.04	(1;304)	p< 0.01
2	Perceptions of School and Schooling	0.184	8.92	(2;303)	p<0.01
3	Emotional Irresponsibility and Luck	0.193	3.04	(3;302)	p<0.05
4	Demand for competence	0.203	3.93	(4;301)	p<0.05

The results of the regression analysis shown in Table 5.59 indicate that Self-concept as a socio-affective variable explained the largest proportion, that is, 16% of the variance in learners' Mathematics achievement. Table 5.59 also shows that Self-concept, perceptions of School and Schooling, Emotional Irresponsibility and Luck and Demand for Competence jointly explained 20.3% of the variance in learners' Mathematics achievement. Not one of the independent variables explained such a large proportion of the variance in learners' Mathematics achievement on its own.

The regression analysis confirmed that the proportion of the variance in Mathematics achievement jointly explained by some of the irrational thoughts, perceptions and socio-affective variables is greater than that explained by any one of these factors on its own. Consequently, the null hypothesis is rejected.

The results are consistent with the findings of studies which established that variability in Mathematics achievement is better understood when several factors are considered simultaneously (Wang et al, 2012:1215; Ferla et al, 2009:499-505; Mbugua et al, 2010:88). The recommendation by Cano and Berben (2009:149) and Abubakar and Adegboyega (2012:121) that many variables need to be analysed at the same time to establish the determinants of variability in Mathematic achievement was justified.

5.4.7 Relationship between Irrational Beliefs and Self-concept

Although no formal hypothesis was originally stated in chapter 4, another regression analysis was performed with self-concept as the dependent variable and the 10 irrational belief subscales as the independent variables. This was done since the self-concept of Mathematics learners explained the greatest proportion of variance in their Mathematics achievement relative to the other socio-affective variables, irrational beliefs and perceptions as shown in Table 5.59.

Below are the 10 irrational beliefs which were used as the independent variables in the initial regression analysis:

- Irrational beliefs:
 - Demand for Love and Approval
 - Demand for Competence
 - Demand for Justice
 - Wishful Thinking
 - Emotional Irresponsibility and Luck
 - Anxious and Endless over a Problem
 - Task Avoidance

- Dependence Mentality
- Perennial Influence of a Past Problem
- Perfectionistic and Absolutistic Mentality

The first variable to enter the regression model was Task or Problem Avoidance. For this irrational belief, the coefficient of determination, $R^2 = 0.307$ was significant with $F(1; 304) = 134.69; p < 0.01$. The Anxious and Endless Obsession over a Problem subscale was the second irrational belief to enter the regression analysis. For Task or Problem Avoidance and Anxious and Endless Obsession over a Problem, the coefficient of determination, $R^2 = 0.379$ was significant with $F(2; 303) = 35.18; p < 0.01$.

The third irrational belief to enter the regression analysis was Demand for Justice. As shown in Table 5.60, the joint proportion of the variance in self-concept jointly explained by the Task or Problem Avoidance, the Anxious and Endless Obsession over a Problem and the Demand for Justice subscales was 39.2%. The coefficient of determination, R^2 was significant with $F(3; 302) = 6.42; p < 0.05$.

The fourth irrational belief to enter the regression model was Wishful Thinking. The proportion of the variance in learners' self-concept jointly explained by Task or Problem Avoidance, Anxious and Endless Obsession over a Problem, Demand for Justice and Wishful Thinking is 40.2%. R^2 was significant with $F(4; 301) = 4.79; p < 0.05$.

The fifth irrational belief to enter the regression analysis was Demand for Competence. For the five irrational beliefs, that is, Task or Problem Avoidance, Anxious and Endless Obsession over a Problem, Demand for Justice, Wishful Thinking and Demand for Competence, the coefficient of determination, $R^2 = 0.414$ was significant with $F(5; 300) = 4.01; p < 0.05$.

None of the remaining five irrational beliefs could explain a significantly larger proportion of the variance in learners' self-concept than that already explained by the five irrational beliefs already mentioned. The irrational beliefs are:

- Task or Problem Avoidance

- Anxious and Endless Obsession over a Problem
- Demand for Justice
- Wishful Thinking
- Demand for Competence

The results of the regression analysis are summarised in Table 5.60.

Table 5.60: Proportion of the Variance in Self-concept as the Dependent Variable Jointly Explained by Irrational Beliefs

Step	Independent Variable	Model R-square, R^2	F	df	p-value
1	Task or Problem Avoidance	0.307	134.69	(1;304)	p< 0.01
2	Anxious and Endless Obsession over a Problem	0.379	35.18	(2;303)	p< 0.01
3	Demand for Justice	0.392	6.42	(3;302)	p< 0.05
4	Wishful Thinking	0.402	4.79	(4;301)	p< 0.05
5	Demand for Competence	0.414	4.01	(5;300)	p< 0.05

As shown in Table 5.60, the results of the regression analysis established that 30.7% of the variance in secondary school learners' self-concept was explained by the irrational belief pertaining to Task or Problem Avoidance. This implies that the Task or Problem Avoidance irrational belief subscale explained the largest proportion of the variance in learners' self-concept relative to other irrational beliefs. The Anxious and Endless Obsession over a Problem subscale explained 7.2% of the variance in learners' self-concept that was not already explained by the Task or Problem

Avoidance subscale. Demand for Justice explained a further 1.3% of the variance in learners' self-concept, Wishful Thinking explained an additional 1% and Demand for Competence accounted for a further 1.2% of the variance in secondary school Mathematics learners' self-concept. All five irrational beliefs, that is, Task or Problem Avoidance, Anxious and Endless Obsession over a Problem, Demand for Justice, Wishful Thinking and Demand for Competence jointly explained 41.4% of the variance in secondary school learners' self-concept in a Mathematics context.

The regression analysis confirmed that the proportion of the variance in learners' self-concept jointly explained by several irrational beliefs is greater than that which is explained by any one of the irrational beliefs on its own. This implies that a combination of a number of irrational beliefs can influence learners' self-concept which in turn explains a significant proportion of the learners' Mathematics achievement as shown in Table 5.59.

With regard to Task or Problem Avoidance, which is based on the irrational belief that *it is easier to avoid than to face life difficulties and self-responsibilities*, learners with a low self-concept are typically likely to express themselves using statements such as:

- I usually postpone doing Mathematics activities.
- I find it difficult to finish Mathematics assignments.

The irrational belief that *if something is or may be dangerous or fearsome we should be terribly upset and endlessly obsess about it*, is the basis of the Anxious and Endless Obsession over a Problem irrational belief subscale. Guided by this irrational belief, Mathematics learners with low self-concepts are likely to express themselves using statements such as the ones below:

- I worry about mathematical issues which are beyond my control.
- Things that upset me regarding my Mathematics achievement continuously haunt me.

The Demand for Justice irrational belief subscale is based on the irrational belief that *certain acts are awful or wicked and that people who perform such acts should be*

severely damned. Learners with low Mathematics academic achievement and low self-concepts are likely to express themselves in statements like those below:

- Mathematics learners who do wrong must be punished.
- Bad behaviour often reflects rottenness of character.

The irrational belief that *it is horrible and catastrophic when things are not the way we like them to be*, is the basis of the Wishful Thinking irrational belief subscale. Below are typical examples of how learners with low self-concepts are likely to express themselves guided by Wishful Thinking:

- My Mathematics wishes and dreams will never materialise.
- I have given up hope to reach any of my aims in Mathematics.

Demand for Competence is based on the irrational belief that *we should be thoroughly competent, intelligent, and achieving in all possible respects*. Typically, low Mathematics achievement and low self-concept based on Demand for Competence can lead learners to express themselves as follows:

- I ignore Mathematics tasks if I am not good at them.
- It upsets me when I make mistakes when solving Mathematics problems.

5.5 SUMMARY OF FINDINGS

The following is a summary of the results which emerged from the hypotheses testing:

- A significant difference was found between the irrational thoughts of boys and girls regarding Mathematics for two irrational belief subscales, that is, the Demand for Competence and the Anxious and Endless Obsession over a Problem subscales. For both the Demand for Competence and the Anxious and Endless Obsession over a Problem irrational belief subscales, girls were found to be more irrational than boys. This indicates that female Mathematics learners were found to be more prone to the irrational beliefs that *people should be thoroughly competent, intelligent, and achieving in all possible*

respects and that if something is or may be dangerous or fearsome people should be terribly upset and endlessly obsess about it than their male counterparts.

- There were six irrational belief subscales for which significant differences were found between the irrational thoughts of form 3 and form 4 learners regarding Mathematics. Form 3 learners were more irrational than form 4 learners with regard to Demand for Competence and Perfectionistic and Absolutistic Mentality. This implies that form 3 learners were more vulnerable than form 4 learners to the irrational beliefs that *people should be thoroughly competent, intelligent, and achieving in all possible respects and that there is invariably a right, precise and perfect solution to human problems and it is catastrophic if this perfect solution is not found.* It was also established that form 4 learners were more irrational than form 3 learners with regard to the following four irrational belief subscales: Wishful Thinking, Anxious and Endless Obsession over a Problem, Task or Problem Avoidance and Perennial Influence of a Past Problem. Form 4 learners were found to be more prone to the irrational beliefs that *it is horrible and catastrophic when things are not the way we like them to be and that if something is or may be dangerous or fearsome we should be terribly upset and endlessly obsess about it than form 3 learners.* The other two irrational beliefs for which form 4 learners were found to be more irrational than their form 3 counterparts were the beliefs that *it is easier to avoid than to face life difficulties and self-responsibilities and that because something once strongly affected our life, it should indefinitely affect it.*
- For eight of the ten irrational belief subscales, significant negative correlations were found between the irrational thoughts of learners regarding Mathematics and their academic achievement in Mathematics. Generally, this indicates that the higher the level of learners' irrational thinking regarding Mathematics, the lower their academic achievement in Mathematics. The negative correlation ($r=-0.29$) between learners' irrational thoughts about Mathematics and their academic achievement in Mathematics was strongest for three irrational belief subscales; namely, Wishful Thinking, Emotional Irresponsibility and Luck and Anxious and Endless Obsession over a Problem. This reveals that Mathematics learners who believe that *it is horrible and catastrophic when things are not the way we like them to be, that human emotional misery is*

mostly externally caused and that people have little or virtually no ability to regulate their emotional sorrows and disturbances or that if something is or may be dangerous or fearsome we should be terribly upset and endlessly obsess about it are most likely to have poor academic achievement in Mathematics.

- Significant relationships were found between learners' irrational thoughts in Mathematics and motivation relative to nine irrational belief subscales. In general, the irrational beliefs of learners regarding Mathematics correlated negatively with their motivation indicating that the higher the possibility of irrational beliefs, the lower the learners' motivation to study Mathematics. The strongest negative correlation ($r = -0.56$) was obtained between motivation and the irrational belief that *it is easier to avoid than to face life difficulties and self-responsibilities*.
- All of the 10 irrational belief subscales correlated positively with stress, thus confirming that there is a significant relationship between learners' irrational thoughts regarding Mathematics and stress. Generally, this indicates that the higher the level of learners' irrational thoughts about Mathematics, the more they become vulnerable to stress. Comparatively high positive correlations were found between stress and Wishful Thinking ($r = 0.56$) and Task or Problem Avoidance ($r = 0.61$). The strongest positive correlation ($r = 0.63$) was obtained between stress and the Anxious and Endless Obsession over a Problem subscale. This indicates that learners who believe that *if something is or may be dangerous or fearsome we should be terribly upset and endlessly obsess about it* are most vulnerable to stress emanating from studying Mathematics.
- A significant positive correlation was found between each of the 10 irrational belief subscales and anxiety. In general, the results of the current study established that the higher the level of learners' irrational thinking, the more they experience Mathematics anxiety. Relatively strong positive correlations were obtained between anxiety and Perennial Influence of a Past Problem ($r = 0.54$) and Task or Problem Avoidance ($r = 0.58$). The strongest positive correlation ($r = 0.64$) was obtained between anxiety and the irrational belief that *if something is or may be dangerous or fearsome we should be terribly upset and endlessly obsess about it*.

- Significant negative correlations were found between learners' self-concept and eight irrational belief subscales. This indicates that the higher the possibility of irrational beliefs, the lower the levels of Mathematics learners' self-concepts. Comparatively high negative correlations were obtained between Self-concept and Wishful Thinking ($r = -0.48$) and Anxious and Endless Obsession over a Problem ($r = -0.54$). The strongest negative correlation ($r = -0.55$) was obtained between Self-concept and Task or Problem Avoidance. This indicates that Mathematics learners with low self-concepts are most prone to the irrational belief that *it is easier to avoid than to face life difficulties and self-responsibilities*.
- Seven of the ten irrational belief subscales had significant negative correlations with Teacher-Learner Relationships in a Mathematics learning context. The results generally signify that Mathematics learners who have poor relationships with their teachers tend to have high levels of irrational thinking regarding Mathematics. The strongest negative correlation ($r = -0.40$) was obtained between Teacher-Learner Relationships and the irrational belief that *it is horrible and catastrophic when things are not the way we like them to be*.
- The relationships between the irrational thoughts of learners in Mathematics and Parental Involvement were found to be significant for six irrational belief subscales. Two of the six subscales had significant positive correlations with Parental Involvement while the other four subscales correlated negatively with Parental Involvement. The strongest positive correlation ($r = 0.28$) was obtained between Parental involvement and the irrational belief that *people should be thoroughly competent, intelligent, and achieving in all possible respects*. The strongest negative correlation ($r = -0.23$) was obtained between Parental Involvement and the irrational belief that *it is easier to avoid than to face life difficulties and self-responsibilities*.
- Nine of the ten irrational belief subscales correlated positively with perceptions of The Inevitable. In general, the results signify that the higher the level of learners' irrational thinking regarding Mathematics, the more they tend to have faulty perceptions of The Inevitable. Relatively strong positive correlations were found between perceptions of The Inevitable and Emotional Irresponsibility and Luck ($r = 0.38$) and Wishful Thinking ($r = 0.39$). The highest positive correlation ($r = 0.43$) was obtained between perceptions of The

Inevitable and the irrational belief that *because something once strongly affected our life, it should indefinitely affect it*.

- Significant positive correlations were found between five irrational belief subscales and perceptions of The Role of Rewards. The results generally indicate that the higher the level of learners' irrational thinking pertaining to Mathematics, the more they have faulty perceptions of The Role of Rewards. The highest positive correlation ($r = 0.40$) was obtained between perceptions of The Role of Rewards and the irrational belief that *there is invariably a right, precise and perfect solution to human problems and it is catastrophic if this perfect solution is not found*.
- Eight irrational belief subscales revealed significant positive correlations between learners' irrational thoughts about Mathematics and their faulty perceptions of School and Schooling. The results generally indicated that the higher the level of learners' irrational thinking pertaining to Mathematics, the more they were prone to faulty perceptions of School and Schooling. Comparatively strong positive correlations were obtained between perceptions of School and Schooling and Task or Problem Avoidance ($r = 0.45$) and Wishful Thinking ($r = 0.48$). The highest positive correlation ($r = 0.51$) was found between Mathematics learners' perceptions of School and Schooling and the irrational belief that *because something once strongly affected our life, it should indefinitely affect it*.
- All 10 irrational belief subscales had significant positive correlations with learners' faulty perceptions of Teachers and Parents. In general, the results revealed that the more learners engage in irrational thinking about Mathematics, the more they have faulty perceptions of Teachers and Parents. Relatively high positive correlations were found between perceptions of Teachers and Parents and Wishful Thinking ($r = 0.39$), Task or Problem Avoidance ($r = 0.39$) and Emotional Irresponsibility and Luck ($r = 0.44$). The highest positive correlation ($r = 0.46$) was obtained between perceptions of Teachers and Parents and the Perennial Influence of a Past Problem irrational belief subscale. This indicates that Mathematics learners with faulty perceptions of Teachers and Parents are most prone to the irrational belief that *because something once strongly affected our life, it should indefinitely affect it*.

- Only one irrational belief subscale, Demand for Competence ($r = -0.16$), had a negative correlation with learners' overall perceptions of Mathematics. Eight irrational belief subscales had significant positive correlations with learners' overall perceptions of Mathematics. For the eight irrational beliefs, the results generally indicate that the higher the level of learners' irrational thinking regarding Mathematics, the stronger their faulty perceptions of Mathematics. The highest positive correlation ($r = 0.32$) was found between learners' faulty perceptions of Mathematics and the irrational belief that *because something once strongly affected our life, it should indefinitely affect it*.
- The regression analysis undertaken established that the proportion of the variance in Mathematics achievement jointly explained by some irrational thoughts, perceptions and socio-affective variables is greater than that explained by any one of these factors on its own. Four independent variables, which are, Self-concept, perceptions of School and Schooling, Emotional Irresponsibility and Luck and Demand for Competence jointly explained 20.3% of the variance in learners' academic achievement in Mathematics. Perceptions of School and Schooling explained 2.4% of the variance in learners' Mathematics achievement. Self-concept explained the largest proportion, that is, 16% of the variance in learners' Mathematics achievement.
- Regression analysis established that the proportion of the variance in learners' self-concept jointly explained by several irrational beliefs is greater than that which is explained by any one of these irrational beliefs on its own. Five irrational belief subscales, which are, Task or Problem Avoidance; Anxious and Endless Obsession over a Problem; Demand for Justice; Wishful Thinking and Demand for Competence jointly explained 41.4% of the variance in secondary school learners' self-concepts in a Mathematics context. The Anxious and Endless Obsession over a Problem irrational belief subscale explained 7.2% of the variance in secondary school Mathematics learners' self-concepts. The largest proportion, that is, 30.7% of the variance in learners' self-concept was explained by the Task or Problem Avoidance irrational belief subscale.

CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

6.1 INTRODUCTION

The main purpose of this chapter is to draw conclusions based on the extent to which the aims of the current study as outlined in chapter 1 (section 1.3) were achieved and to give recommendations derived from the results of the empirical investigation. In the first chapter, two sets of aims were stated in both the literature review and the empirical investigation. The literature review related aims focused on examining factors which influence Mathematics achievement in a secondary school context. It also aimed to explore the major irrational beliefs as theorised by Ellis and to examine the extent to which irrational beliefs relate to and influence Mathematics achievement. The aim of the empirical investigation was to establish the relationship between irrational beliefs and learners' Mathematics achievement. The study also aimed to determine how irrational beliefs relate to Mathematics learners' socio-affective variables and their faulty perceptions of Mathematics. Another aim of the empirical investigation was to determine whether the proportion of the variance in learners' Mathematics achievement jointly explained by irrational beliefs, socio-affective variables and faulty perceptions is greater than that explained by any one of these variables on its own. These are the aims which form the basis of the main conclusions in this chapter. Recommendations are made based on the results of the six hypotheses which were tested in the current study. The first two hypotheses focused on the differences in learners' irrational thoughts about Mathematics relative to gender and grade level. The other hypotheses tested the relationships between learners' irrational thoughts about Mathematics and their socio-affective variables and their perceptions of Mathematics. The recommendations are meant to encourage and educate Mathematics education stakeholders such as teachers, school counsellors, parents and learners to manipulate the variables which the current study found to be related to learners' irrational thoughts about Mathematics. This is done with the intention of helping and encouraging the stakeholders to work towards minimising the negative influence of irrational beliefs and allied variables on learners' Mathematics achievement. This implies that the recommendations are

meant to reduce poor Mathematics achievement at secondary school level which can be attributed to learners' irrational thoughts about Mathematics. Some limitations of the study and possibilities for further research are also highlighted.

6.2 CONCLUSIONS

The main conclusions regarding the current study are made relative to the literature study and the empirical investigation.

6.2.1 Conclusions Reached From the Literature Study

The first part of the literature study discussed learners' internal variables, that is, secondary school learners' cognitive and affective factors. The learners' cognitive factors discussed in the literature review were intelligence, aptitude, information processing and previous academic achievement (Gerrig & Zimbardo, 2005:314; Mwamwenda, 2004:252-253). Learners' affective variables explored include stress, anxiety, motivation and self-concept. The influence of learners' home factors such as parenting styles, parental involvement and socioeconomic status on learners' Mathematics achievement was also examined. The contribution of school factors to learners' Mathematics achievement was examined by considering instructional approaches, classroom management and the personality of teachers. Most of these factors were considered during the development of the questionnaire used for the empirical investigation.

The second part of the literature review highlighted the characteristics, effects, acquisition and maintenance of irrational beliefs. The theoretical views of Ellis were used to outline the major irrational beliefs and the manner in which each irrational belief is likely to influence or relate to learners' academic achievement in Mathematics. The relationships between irrational beliefs and variables such as gender, intelligence, motivation, perfectionism and procrastination were highlighted and studies pertaining to these factors were provided. However, very few studies directly explored the influence of and/or the relationship between irrational beliefs and Mathematics achievement. Tentative deductions regarding the relationship between irrational beliefs and learners' academic achievement in Mathematics were made. Thus the majority of the literature review's aims stated in the first chapter were achieved.

6.2.2 Conclusions Reached From the Empirical Investigation

The results of the first five hypotheses tested in the empirical investigation revealed relationships shown in Figure 6.1.

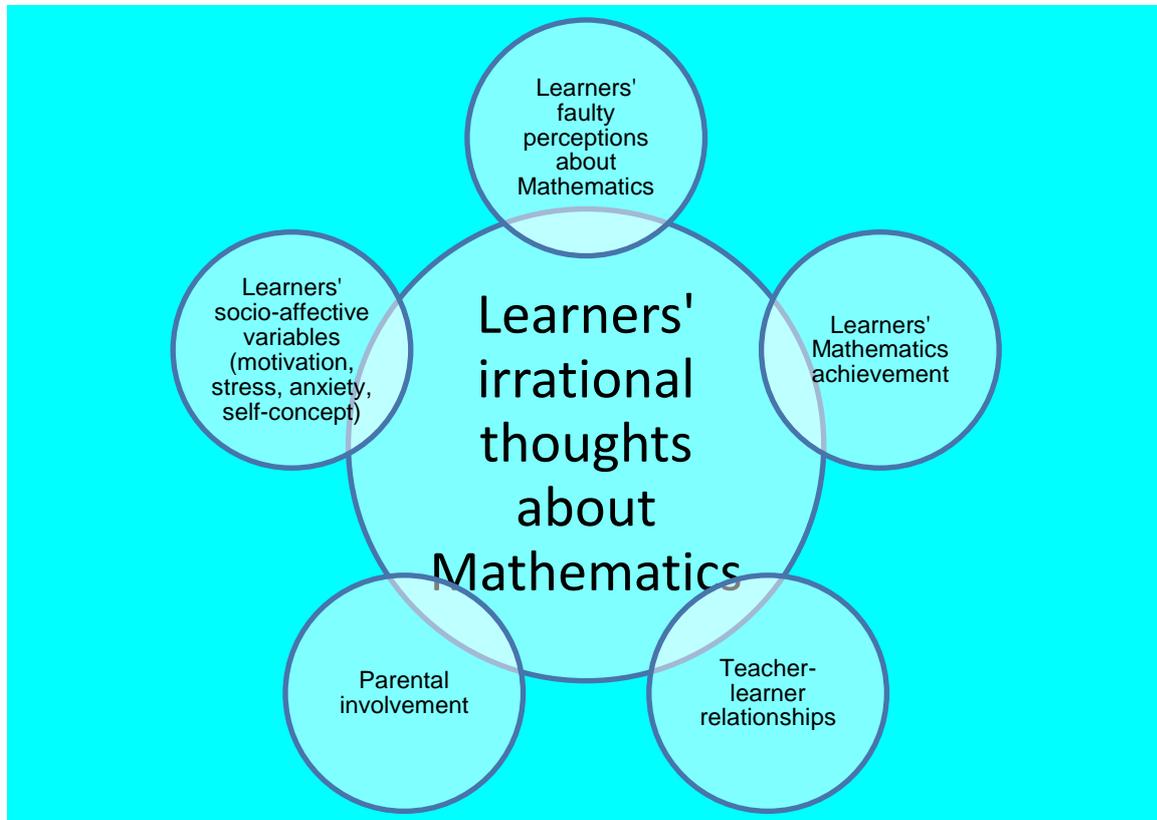


Figure 6.1: Relationships between the variables as found in the current study

The research results established a significant relationship among eight of the ten major irrational beliefs advanced by Ellis and learners' academic achievement in Mathematics. All eight irrational beliefs correlated negatively with learners' academic performance in Mathematics. Therefore, the current study managed to empirically confirm that irrational beliefs relate to learners' Mathematics achievement. The research findings backed the assertion made in chapter 3 (section 3.3.1.2) that irrational beliefs relate to human dysfunction (Ciarrochi, 2004:172; Dryden, 2006:14; Thompson & Henderson, 2007:211; Amutio & Smith, 2007:321). This is important since very few studies pertaining to the relationship between irrational beliefs and Mathematics achievement were found during the literature study. The results of the empirical investigation also confirmed that the irrational thoughts of learners regarding Mathematics differ on the basis of both age and gender. Female learners were found to have more irrational thoughts about Mathematics than their male

counterparts. The results are consistent with the literature explored in chapter 3 (section 3.4.1.1.1).

Three irrational belief subscales, that is, Wishful Thinking, Emotional Irresponsibility and Luck, and Anxious and Endless Obsession over a Problem, each had a coefficient of determination of 0.0841 with Mathematics achievement. This implies that each of these three irrational beliefs explained approximately 8% of the variance in learners' academic achievement in Mathematics. The regression analysis undertaken in the current study also confirmed that two irrational belief subscales; namely, Emotional Irresponsibility and Luck and Demand for Competence, each explained about 1% of the variance in learners' Mathematics achievement. Consequently, the current study established that irrational beliefs influence secondary school learners' Mathematics achievement.

The research results confirmed that irrational beliefs relate to Mathematics learners' socio-affective variables such as motivation, stress, anxiety, self-concept, teacher-learner relationships and parental involvement. Learners' motivation to study Mathematics generally correlated negatively with their irrational thoughts about Mathematics. The assertion made in section 3.4.1.8.1 that learners who think irrationally are likely to have low self-determination and low motivation owing to learned helplessness was confirmed by the results of the current study. All the 10 irrational belief subscales correlated positively with both Mathematics learners' stress and anxiety. This is a confirmation of the claim made in section 3.3.1.2 that irrational beliefs normally foster negative emotions thereby making learners vulnerable to stress and anxiety.

Eight irrational beliefs subscales correlated negatively with Mathematics learners' self-concepts. The research findings pertaining to the relationship between self-concept and irrational thoughts about Mathematics are a confirmation of the view that conditional self-acceptance is a form of irrational thinking as discussed in section 3.2.2.2.4. Moreover, seven irrational belief subscales correlated negatively with teacher-learner relationships in a Mathematics context. A relationship was found between the irrational thoughts of learners regarding Mathematics and parental involvement as a socioemotional variable.

The empirical findings ascertained that irrational beliefs correlated positively with all four factors of Mathematics learners' faulty perceptions of Mathematics; namely, The Inevitable, The Role of Rewards, School and Schooling and Teachers and Parents. Learners' overall faulty perceptions of Mathematics also correlated positively with irrational thinking in a Mathematics learning context. The established relationship between learners' irrational thoughts about Mathematics and their faulty perceptions of Mathematics is consistent with the findings of earlier studies cited in section 1.1 in which faulty perceptions of Mathematics were found to be a cause for poor Mathematics achievement (Githua, 2013:175; Asikhia, 2010:205). Moreover, the research findings confirmed the assertion made in chapter 1 that there is a possibility that learners' faulty perceptions of Mathematics are related to irrational beliefs.

Figure 6.2 shows the results of the sixth hypothesis in which regression analysis was undertaken.



Figure 6.2: Variance in Mathematics achievement jointly explained by self-concept, perceptions of School and Schooling and irrational beliefs

Regression analysis established that the proportion of the variance in learners' Mathematics achievement jointly explained by irrational thoughts, perceptions and socio-affective variables is greater than that explained by any one of these factors on its own as shown in Figure 6.2. Self-concept explained the largest proportion of the variance in Mathematics achievement. The research results confirmed the assertion made in chapter 1 (section 1.1) that Mathematics achievement is determined by a multiplicity of variables. It was also confirmed in the current study that the proportion of the variance in Mathematics learners' self-concept jointly explained by several irrational beliefs is greater than that which is explained by any one of the irrational beliefs on its own. The fact that irrational beliefs correlated with learners' academic performance in Mathematics and other allied socio-affective variables confirmed that irrational beliefs are pervasive in nature (Corey, 2009:276; Ellis, 1998:22; Han, 2011:47; Bernard, 2009:66; Ciarrochi, 2004:175).

Conclusively, the current study managed to ascertain the relationships between irrational beliefs and secondary school learners' Mathematics achievement, socio-affective variables and faulty perceptions of Mathematics. Through regression analysis, the empirical findings ascertained that the proportion of the variance in learners' Mathematics achievement jointly explained by irrational thoughts, perceptions and socio-affective variables is greater than that explained by any one of these factors on its own. Therefore, it can be argued that all the empirical investigation's related aims stated in the first chapter in section 1.3.2 were achieved.

6.3 RECOMMENDATIONS

Several practical implications and recommendations for the various Mathematics education stakeholders can be derived from the results of the literature review and the empirical investigation. The recommendations and educational implications apply to Mathematics teachers, school counsellors, parents of Mathematics learners and Mathematics learners.

6.3.1 Recommendations to Mathematics Teachers

It has already been mentioned that teachers play a significant role in determining learners' academic performance in virtually all subjects (Compton et al, 2007:7; McLean, 2003:51; Shein & Chiou, 2011:1097; Philipp, 2007:257; Miller & Satchwell,

2006:136). Consequently, several recommendations to Mathematics teachers based on the results of the current study follow.

The results of the empirical study revealed that there is a significant difference between the irrational thoughts of boys and girls regarding Mathematics relative to two irrational belief subscales: Demand for Competence and Anxious and Endless Obsession over a Problem. For the two irrational belief subscales, girls were found to be more irrational than boys. To minimise the irrational thoughts of female learners relative to Demand for Competence at secondary school level, Mathematics teachers can:

- Encourage female learners to express their wishes as preferences rather than as dire necessities
- Urge girls to set realistic Mathematics standards for themselves
- Give group activities in which both male and female learners work together and receive the same feedback. This can help female learners view themselves as equal in status to their male counterparts
- Invite prominent female Mathematics academics to schools to encourage female Mathematics learners to work hard and become successful academically and socially. Female Mathematics learners may end up imitating such role models.

The second hypothesis tested in the current study revealed that there is a significant difference between the irrational thoughts of form 3 and form 4 learners regarding Mathematics for six irrational belief subscales. For the Demand for Competence and the Perfectionistic and Absolutistic Mentality irrational belief subscales, form 3 learners were found to have more irrational thoughts about Mathematics than form 4 learners. In this regard, Mathematics teachers are recommended to:

- Train form 3 learners to acknowledge that it is part of human nature to make mistakes and to be incompetent in some areas in the Mathematics syllabus
- Urge form 3 Mathematics learners to set realistic academic standards which are consistent with their personal efforts

- Encourage form 3 learners to accept the fact that sometimes their classmates will outclass them in Mathematics
- Teach Mathematics learners in early adolescence that one can be successful in Mathematics without necessarily doing everything correctly and perfectly.

Form 4 learners were found to have more irrational thoughts regarding Mathematics than form 3 learners relative to four irrational belief subscales: Wishful Thinking, Anxious and Endless Obsession over a Problem, Task or Problem Avoidance and Perennial Influence of a Past Problem. Based on some items pertaining to the above irrational belief subscales, Mathematics teachers can try to improve the situation by:

- Explaining to Mathematics learners in late adolescence that real life requires that people tolerate situations which they cannot alter because people's ideals do not always materialise
- Emphasising the need to persevere in one's Mathematics studies despite various forms of adversity
- Indicating to form 4 learners that new concepts will always be introduced in Mathematics and learners need to be assertive and confident about learning and mastering such concepts.
- Emphasising to form 4 Mathematics learners that it is possible for one to recover from unfortunate past experiences in Mathematics if one exerts effort in one's studies as postulated by the incremental theory of intelligence (see section 3.4.2.2.2)
- Encouraging form 4 learners to seek help from school counsellors on how to deal with anxiety emanating from studying Mathematics.

The empirical findings revealed that eight irrational belief subscales correlated negatively with learners' Mathematics achievement. The five irrational beliefs subscales which had the highest negative correlation with learners' Mathematics achievement were Wishful Thinking ($r = -0.29$); Emotional Irresponsibility and Luck ($r = -0.29$); Anxious and Endless Obsession over a Problem ($r = -0.29$); Task or Problem Avoidance ($r = -0.27$) and Perennial Influence of a Past Problem ($r = -0.26$). The regression analysis performed in this study established that Demand for

Competence and Emotional Irresponsibility and Luck each explain a statistically significant proportion of the variance in learners' Mathematics achievement.

Several recommendations can be made to Mathematics teachers in connection with the irrational beliefs which had the most prominent correlation with learners' Mathematics achievement.

With regard to Wishful Thinking, Mathematics classroom practitioners are urged to:

- Emphasise that it is possible for people's wishes, aspirations, dreams and ideals in Mathematics to materialise if they apply effort, employ good study habits (Liu & Lin, 2010:232) and use memory enhancing techniques
- Urge Mathematics learners to have high frustration tolerance since real life situations are not always the way we wish them to be.

Regarding Emotional Irresponsibility and Luck, Mathematics teachers are recommended to:

- Encourage learners to be accountable for their own studies and refrain from blaming their teachers, family members or peers for their failure in Mathematics
- Indicate to Mathematics learners that they can deliberately decide to be happy regardless of their Mathematics achievement or how they are treated by their teachers, parents, siblings or peers
- Refer Mathematics learners to school counsellors to be helped to deal with their emotional sorrows and disturbances emanating from studying Mathematics.

To reduce irrational thinking regarding the Anxious and Endless Obsession over a Problem subscale, Mathematics teachers are recommended to:

- Notify learners of test and examination dates while there is enough time for them to prepare
- Give learners challenging but manageable Mathematics tasks which do not make them unnecessarily anxious

- Refer learners to school counsellors to be taught various anxiety reduction techniques.

Concerning Task or Problem Avoidance, Mathematics teachers are recommended to:

- Expose learners to problem solving skills and encourage them to apply such skills in Mathematics learning contexts
- Encourage learners to be assertive whenever they are given Mathematics tasks
- Give learners a reasonable amount of work to do since too much work can foster task avoidance.

Resilience is the antidote to the irrational belief that *because something once strongly affected our life, it should indefinitely affect it*. By definition, resilience is an individual's ability to recover from adversity and subsequently thrive despite serious developmental threats (Prince-Embury, 2013:9; Donald et al, 2010:159; Swartz et al, 2011:345; Masten, 2001:228; Connor & Davidson, 2003:76; Tugade & Fredrickson, 2004:320). In an attempt to reduce learners' irrational thoughts regarding the Perennial Influence of a Past Problem irrational belief subscale, resilience can be fostered. In this regard, Mathematics teachers can:

- Be humorous and outgoing during lessons since humour and an outgoing personality tend to promote positive emotions and psychological resilience (Tugade & Fredrickson, 2004:320; Swartz et al, 2011:70; Prince-Embury, 2013:9). Learners may eventually imitate such teacher behaviours and equally become resilient in the face of the challenges emanating from studying Mathematics
- Demonstrate good communication skills for learners to imitate. According to Prince-Embury (2013:11) and Donald et al (2010:160), learners with good communication skills tend to be resilient
- Use the authoritative leadership style and offer social support and warmth to learners. Such support from teachers can increase learners' chances of

recovering after experiencing failures in their Mathematics studies (Prince-Embury, 2013:14).

The results of the empirical study established that learners' motivation to study Mathematics correlated negatively with their irrational thoughts about Mathematics. The negative correlation was most prominent between motivation and two irrational belief subscales: Wishful Thinking ($r = -0.49$) and Task or Problem Avoidance ($r = -0.56$). Some questionnaire items concerning motivation and Task or Problem Avoidance are given below:

- If a Mathematics topic is difficult, I give up easily.
- If a Mathematics topic is too difficult, I do not learn it.

In an attempt to minimise learners' tendency to engage in task or problem avoidance, Mathematics teachers can impart problem solving skills to learners. Problem solving can be an antidote to task or problem avoidance. A problem is a setback that one has to overcome in order to attain a predetermined goal (Kosslyn & Rosenberg, 2006:357; Snowman et al, 2009:248). Problem solving entails the identification and application of skills and knowledge which result in the attainment of a goal that is blocked by an obstacle (Lahey, 2009:277; Snowman et al, 2009:248). Essentially, problem solving involves steps such as acknowledging the existence of a problem, understanding what the problem entails, gathering relevant information, identifying and implementing a solution before eventually evaluating the implemented solution (Tuckman & Monetti, 2011:325; Donald et al, 2010:143-144; Feldman, 2009:270; Snowman et al, 2009:250; Lahey, 2009:277). Mathematics classroom practitioners can encourage learners to view any challenges they encounter as they study Mathematics as obstacles which can be overcome by implementing the outlined problem solving steps. It is hoped that if learners gain the ability to implement problem solving skills, their tendency to engage in task avoidance decreases thereby elevating their levels of motivation to study Mathematics.

According to Steel (2007:68), there is a relationship between task avoidance and procrastination (section 3.4.1.7.1). The following are some questionnaire items pertaining to motivation, Task or Problem Avoidance and procrastination:

- I constantly postpone Mathematics revision times
- I always come up with reasons not to study Mathematics

It can, therefore, be argued that the level of learners' motivation to study Mathematics can be elevated by training them to overcome procrastination and irrational thoughts pertaining to Task or Problem Avoidance. Mathematics teachers can try to reduce procrastination by:

- Training learners to manage their time wisely and to set their priorities in the correct order (Asikhia, 2010:207, 209)
- Giving learners challenging but manageable Mathematics tasks which will enable them to experience success and subsequently develop high levels of self-efficacy (Asikhia, 2010:207)
- Employing a variety of instructional approaches which cater for different learners' diverse learning styles (Sunitha & Muhamedunni, 2013:104; Asikhia, 2010:209)
- Teaching mathematical concepts from simple to complex as a way of helping learners to develop confidence and to lean less towards procrastination (Asikhia, 2010:210)
- Portraying Mathematics as a vital subject as a means of encouraging learners to develop positive attitudes towards the subject (Sunitha & Muhamedunni, 2013:104). This can ultimately reduce procrastination and enhance learners' motivation to study Mathematics.

Apart from attempting to motivate learners through problem solving and minimising procrastination, Mathematics teachers can use other ways to motivate learners in the hope that this will eventually reduce their irrational thoughts about Mathematics. Classroom practitioners can:

- Encourage Mathematics learners to be academic risk takers who view making mistakes as an integral part of the learning process (Snowman et al, 2009:357).
- Attempt to make Mathematics lessons captivating by placing emphasis on learner participation, indicating the utility of Mathematics and by being humorous (Snowman et al, 2009:358, 366)
- Train Mathematics learners to employ memory enhancing techniques (Snowman et al, 2009:360)
- Develop new mathematical concepts on what learners have already mastered as theorised by Ausubel (Snowman et al, 2009:358)
- Urge secondary school learners to join Mathematics clubs where they can enhance their problem solving skills, learn more about the utility of Mathematics and benefit from the contributions of their peers (Hossain & Robinson, 2012:448).
- Encourage Mathematics learners to be guided by the incremental theory of intelligence which stresses that ability is a set of cognitive skills which can be increased through applying effort (Sternberg, 2009:419; Blackwell et al, 2007:253; Snowman et al, 2009:356). Learners should be discouraged from perceiving ability as a static entity which is independent of the effort one applies as claimed by the entity theory of intelligence (section 3.4.2.2.1)
- Notify learners of the content they are to be taught, how they will be taught and the expected outcomes (Snowman et al, 2009:355). This is consistent with Gagne's theory of instruction (section 2.3.3.2.2)
- Give learners feedback based on criterion referenced assessment in which their academic achievement is evaluated relative to a predetermined level of proficiency. This can be better than norm referenced assessment in which learners' scholastic achievement is rated relative to that of their classmates (Tuckman & Monetti, 2011:400; Snowman et al, 2009:356)
- Urge Mathematics learners to set mastery goals which focus on using already learnt concepts to gain new skills and knowledge rather than performance goals in which learners mainly aim to outclass their peers in Mathematics tests and examinations (Tuckman & Monetti, 2011:414; Snowman et al, 2009: 356)

- Place emphasis on Mathematics learners' strengths rather than amplifying their weaknesses (Snowman et al, 2009:353; Tuckman & Monetti, 2011:400).

The empirical investigation established that stress correlated positively with all 10 irrational beliefs. The most prominent irrational belief subscales were Task or Problem Avoidance ($r= 0.61$) and Anxious and Endless Obsession over a Problem ($r= 0.63$). Some items pertaining to stress and the Anxious and Endless Obsession over a Problem subscale as they appeared in the questionnaire used in the current study are given below:

- Preparations prior to Mathematics tests put a lot of pressure on me.
- My Mathematics studies frustrate me because of too much work and too little time.
- Continuous competition with friends in Mathematics makes me worried.
- Mathematics makes my life difficult and out of control

On the basis of the above items, it is recommended that Mathematics teachers should:

- Encourage secondary school learners to effectively manage their time through preparing revision timetables and spending less time playing and seeking entertainment
- Minimise competition in classes since excessive competition among learners can foster stress. Competition can be minimised through the use of cooperative learning methods (Snowman & Biehler, 2006:379-381).
- Regularly give revision exercises to Mathematics learners instead of relying on tests administered after a long time. This is so because it was established during the literature study in section 2.2.3.1 that distributed practice is more effective than massed practice (Tuckman & Monetti, 2011:279; Snowman et al, 2009:228).

Anxiety generally correlated positively with learners' irrational thoughts about Mathematics. Task or Problem Avoidance ($r= 0.58$) and Anxious and Endless

Obsession over a Problem ($r= 0.64$) had the highest positive correlations with anxiety. Some items concerning anxiety and the Anxious and Endless Obsession over a Problem irrational belief subscale are stated below:

- I often become very anxious before I write a Mathematics test.
- I become anxious when I realise that other people are going to judge my Mathematics achievement.
- To solve Mathematics problems in front of others is one of my greatest fears.
- After a Mathematics test, I experience anxiety because I do not know what criticism to expect.
- I am always afraid that others will mock me because of the mistakes I make in a Mathematics test.

Therefore, Mathematics teachers can try to minimise anxiety emanating from studying Mathematics by:

- Notifying learners of test and examination dates in advance so that they can mentally adjust and be ready for the tests or examinations
- Encouraging learners to set their own individual realistic standards pertaining to their academic performance in Mathematics
- Regularly giving Mathematics learners the opportunity to solve mathematical problems in front of the entire class so as to reduce anxiety which emanates from stage fright as they become accustomed to the practice
- Discouraging learners from mocking one another for making mistakes during Mathematics lessons
- Refraining from condemning learners for poor Mathematics achievement since this can trigger anxiety before and after writing tests.

The empirical findings confirmed that the Task or Problem Avoidance irrational belief subscale had the strongest negative correlation of $r= -0.55$ with self-concept. Through regression analysis, five irrational belief subscales; namely, Task or Problem avoidance; Anxious and Endless Obsession over a Problem; Demand for Justice; Wishful Thinking and Demand for Competence were found to jointly explain

41.4% of the variance in Mathematics learners' self-concepts at secondary school level. The Task or Problem Avoidance irrational belief subscale explained the largest proportion, that is, 30.7% of the variance in Mathematics learners' self-concepts. This makes it necessary to derive recommendations from the relationship between self-concept and the Task or Problem Avoidance irrational belief subscale. Examples of self-concept items pertaining to Task or Problem Avoidance are given below:

- I am unwilling to learn new demanding Mathematics concepts.
- As a Mathematics learner I do not accept new ways of doing things.
- I do not have enough self-confidence to participate during Mathematics lessons.
- I feel ashamed of my shortcomings in Mathematics.

In an endeavour to reduce negative self-concepts and the tendency to engage in Task or Problem Avoidance on the part of learners, Mathematics teachers can:

- Emphasise the need for flexibility in studying Mathematics since different mathematical concepts can be solved by using different methods
- Encourage learners to be assertive so as to be successful in their studies
- Urge learners to accept and love themselves despite their shortcomings and previous failures. Mathematics teachers can stress that it is possible to be successful in life despite one's weaknesses. When Mathematics teachers accept learners who have shortcomings, the learners are likely to exercise unconditional self-acceptance eventually
- Discourage learners from engaging in self-damnation and self-downing since this can negatively affect their Mathematics achievement owing to cognitive and emotional distress (Flett et al, 2003:120).

The current study established through regression analysis that self-concept explained the greatest proportion, that is, 16% of the variance in learners' Mathematics achievement. Therefore, it remains imperative that Mathematics teachers should endeavour to boost learners' self-concepts. Mathematics teachers

can adopt the following Rogerian principles to help learners develop positive self-concepts:

- Giving learners unconditional positive regard for them to develop unconditional self-acceptance and unconditional others-acceptance (Najafi & Lea-Baranovich, 2014:3). Teachers can give learners unconditional positive regard by accepting their uniqueness and individuality (Choi, 2015:1; Brophy, 2004:30)
- Showing empathy and respecting Mathematics learners as well as encouraging them to praise themselves and others (Manning, 2007:12; Githua & Mwangi, 2003:495)
- Demonstrating positive attitudes towards life in general and Mathematics learning in particular. Such positive attitudes can be imitated by learners leading to healthy self-concepts on the part of the learners
- Creating opportunities for learners to experience success through giving them challenging but manageable Mathematics tasks (Manning, 2007:12)
- Giving positive comments and allowing learners to exercise freedom of expression (Swartz et al, 2011:78; Mwamwenda, 2004:313, 315).

In general, negative correlations were found between teacher-learner relationships and learners' irrational thoughts about Mathematics in the current study. The two irrational belief subscales which had the highest negative correlations with teacher-learner relationships were Wishful Thinking ($r = -0.40$) and Task or Problem Avoidance ($r = -0.39$). Some items relating to teacher-learner relationships and Wishful Thinking are stated below:

- I do not think that my Mathematics teacher really understands me.
- My Mathematics teacher is unnecessarily critical of my performance.
- My Mathematics teacher is often unfair.
- My Mathematics teacher is often dissatisfied without even listening to my explanation.

On the basis of the items stated above, Mathematics teachers can attempt to improve the quality of teacher-learner relationships by:

- Setting realistic and attainable performance standards for learners
- Demonstrating empathy to learners and being friendly to them with the intention of understanding their circumstances (Githua & Mwangi, 2003: 495)
- Giving constructive comments to learners without condemning them
- Identifying learners' strengths and weaknesses with the intention of giving them unconditional positive regard since positive regard is a basic need (Laryea, Saani, & Dawson-Brew, 2014:2)
- Objectively assessing learners' academic work and refraining from showing favouritism
- Demonstrating genuineness and transparency when interacting with learners
- Allowing learners to explain their situations before judging them
- Creating a tension-free and non-threatening classroom environment which caters for learners' security needs
- Assisting learners to solve mathematical problems as much as possible.

A positive correlation was found between learners' faulty perceptions of Mathematics and their irrational thoughts about Mathematics. The Perennial Influence of a Past Problem subscale had the highest positive correlation with three factors of perceptions, that is, The Inevitable, School and Schooling and Teachers and Parents. The regression analysis undertaken in the current study revealed that faulty perceptions of School and Schooling were the most prominent among the other factors of perceptions since they explained variability in learners' Mathematics achievement. Below are some recommendations to Mathematics teachers mainly derived from items pertaining to faulty perceptions of School and Schooling and the Perennial Influence of a Past Problem irrational belief subscales:

- Teachers need to emphasise the importance of applying effort in one's Mathematics studies since effort is likely to lead to better Mathematics

achievement as suggested by the incremental theory of intelligence (Blackwell et al, 2007:250; Sternberg, 2009:419).

- Mathematics teachers should demonstrate creativity and try to be cheerful and humorous during Mathematics lessons so as to make Mathematics lessons interesting. This can reduce learners' faulty perceptions of Mathematics and persuade them to believe that their past mistakes in Mathematics can be rectified.
- Mathematics teachers need to try to create a good classroom climate. This can be done by being authoritative, humanistic, approachable and friendly to learners.
- Teachers should respect the uniqueness and individuality of each learner by giving each learner as much as possible unconditional positive regard.
- Mathematics teachers should regularly inform learners about the importance of Mathematics as an academic discipline for economic, industrial, academic and technological development (Awolola, 2011:91; Tachie & Chireshe, 2013:67; Nyaumwe, 2006:40; Nziramasanga, 1999:323; Mahanta & Islam, 2012:713; Githua, 2013:174; Mbugua et al, 2012:87). Such advice can help Mathematics learners realise that they learn Mathematics not only to satisfy their parents or teachers but also to increase their chances of being successful in their future career pursuits.
- Teachers can use participatory instructional approaches in the hope that learners will realise that they can still do well in Mathematics despite having failed the subject in the past.
- Using teaching aids and examples from learners' immediate settings can help to reduce learners' faulty perceptions of Mathematics and foster positive attitudes towards the subject.
- Teachers are encouraged to sequence Mathematics content properly so that they teach simple concepts before complex ones and use advance organisers as recommended by Ausubel (Shihusa & Keraro, 2009:414). Such a practice can assist learners to recover from their past failures.
- Mathematics teachers can employ mastery learning in which the teacher ensures that a given concept is understood by virtually all learners before

embarking on the next concept. To some extent, this can help learners to believe that they are not slaves of their past failures in Mathematics.

The proportion of the variance in learners' Mathematics achievement jointly explained by irrational thoughts, perceptions and socio-affective variables was found to be greater than that explained by any one of these factors on its own. Consequently, Mathematics teachers should try to manipulate many variables to reduce learners' irrational thoughts regarding Mathematics and improve the way learners perceive Mathematics as well as boosting learners' self-concepts. This can be done by deliberately attempting to apply a number of the already mentioned recommendations simultaneously as they teach. For instance, Mathematics teachers can teach from simple to complex, vary instructional approaches, use a variety of teaching aids, emphasise the utility of Mathematics for economic, industrial, technological and professional development and work in liaison with school counsellors and parents of Mathematics learners.

6.3.2 Recommendations to School Counsellors

The third hypothesis tested in the current study found a negative correlation between learners' irrational thoughts about Mathematics and their Mathematics achievement. It is hoped that reducing learners' irrational thoughts about Mathematics will promote good academic performance in the subject. Ellis theorised that irrational beliefs can be dealt with by employing the rational emotive behaviour therapy which he developed as discussed in section 1.1 Therefore, school counsellors can use the rational emotive behaviour therapy to reduce secondary school learners' irrational thoughts about Mathematics. According to Rosner (2011:82, 85), the rational emotive behaviour therapy has been found to be particularly effective for counselling children and adolescents. Corey (2009:123) viewed the rational emotive behaviour therapy as an active, directive, time-limited and organised technique for treating various disorders like phobias, anxiety and stress. The rational emotive behaviour therapy is a psychoeducational approach in which people are trained to challenge their faulty beliefs leading to healthy emotions, cognitions and behaviours (Mukangi, 2010:154).

The rational emotive behaviour therapy can also be referred to as the ABCDEF model of cognitive behavioural counselling. The ABC part of the model was outlined in chapter 1 (section 1.1) as the diagnostic part of the entire therapy (Thompson & Henderson, 2007:215). As pointed out in section 1.1, 'A' denotes the activating event, 'B' stands for an individual's set of beliefs and 'C' represents the consequences of the activating event which are mediated by the individual's beliefs. As claimed by Epictetus, one of the ancient Greek philosophers whose ideas appealed to Ellis, the way one reacts to an event is not determined by the event itself but by one's beliefs regarding the event (Najafi & Lea-Baranovich, 2014:2; Prout & Brown, 2007:281). The therapeutic part of the ABCDEF model is represented by the letters 'D', 'E' and 'F' (Rosner, 2011:83; Thompson & Henderson, 2007:215). 'D' represents disputation; 'E' stands for an effective new array of beliefs and life philosophies while 'F' denotes further action meant to consolidate an individual's new belief system (Kosslyn & Rosenberg, 2006:699; Thompson & Henderson, 2007:215).

Disputation is the act of deliberately challenging one's irrational beliefs which trigger cognitive, behavioural and emotional ill-health with the intention of replacing the irrational beliefs with realistic and rational beliefs which promote healthy functioning (Austad, 2009:278; Kosslyn & Rosenberg, 2006:699; Tomotake, Okura, Taniguchi & Ishimoto, 2002: 51-52; Melgosa, 2008:136). According to Thompson and Henderson (2007:215), disputation entails the use of logical arguments to attack and challenge irrational self-messages which get into one's mind after an activating event. Feldman (2009:559) indicates that in an attempt to curb irrational thinking, counsellors can train counselees to dispute irrational beliefs and adopt an active and directive approach in which they openly challenge the dysfunctional beliefs of the clients. Disputation can be cognitive, behavioural or emotive. According to David, Kangas, Schnur and Montgomery (2004:10), cognitive disputation focuses on adjusting people's negative unhelpful thoughts regarding a particular situation. Behavioural disputation refers to the practical activities undertaken by an individual in an attempt to deal with the negative effects of irrational beliefs; emotive disputation uses feelings to change people's negative thoughts and replace them with positive ones (David et al, 2004:10).

School counsellors can train secondary school learners to dispute various irrational beliefs which they may have regarding Mathematics. Below are some examples of

statements which Mathematics learners can be trained to use to dispute irrational thoughts about Mathematics based on the irrational beliefs subscales which had the most prominent negative correlations with Mathematics achievement in the current study:

- Is it really catastrophic when my academic achievement in Mathematics is not as good as I originally wished?
- How valid is the belief that my emotional misery is mostly externally caused and that I have little or virtually no ability to regulate my emotional sorrows and disturbances emanating from studying Mathematics?
- Should I be terribly upset and endlessly obsess about the challenges which I encounter in my Mathematics studies?
- Is it better for me to avoid than to face the difficulties which arise as I study Mathematics?
- How reasonable is the view that my past failures in Mathematics will affect me indefinitely?

As already pointed out, 'E' represents an effective new philosophy of life which is based on rational beliefs. According to Austad (2009:278), successful disputation leads to the adoption of an effective array of beliefs and life principles which are consistent with reality, empirically verifiable and logical. Adopting rational beliefs can allow Mathematics learners to be healthy in their cognitive, behavioural and emotional domains. Austad (2009:278) claimed that the new set of beliefs can eventually lead to unconditional self-acceptance, unconditional other-acceptance and unconditional life acceptance as discussed in chapter 3 (section 3.2.2.1.2.4). Therefore, through the intervention of school counsellors, secondary school learners can dispute their irrational thoughts about Mathematics and increase their chances of doing well in Mathematics. The 'F' part of the ABCDEF model requires school counsellors to reinforce Mathematics learners' newly acquired belief systems and life philosophies regarding Mathematics in particular and life in general.

School counsellors should undertake career guidance sessions with secondary school Mathematics learners. It is during such career guidance sessions that the utility of Mathematics for economic, industrial, technological and scientific

advancement can be highlighted. School counsellors can inform adolescent learners that a pass in secondary school Mathematics, particularly at Ordinary level, is normally a prerequisite for admission into many career fields (Lamb & Fullarton, 2001:3; Nyaumwe, 2006:40; Tachie & Chireshe, 2013:67). Where possible, school counsellors can invite prominent people in the society and some alumni to talk to secondary school learners about the importance of Mathematics as an academic discipline.

In an attempt to reduce learners' irrational thoughts about Mathematics, school counsellors can encourage Mathematics learners to use self-talk to determine how they feel, behave and think. School counsellors should clarify to Mathematics learners that positive self-talk can foster positive emotions and well-being which can reduce their irrational thoughts regarding Mathematics (Rosner, 2011:83; Obodo & Obadan, 2008:154). On the other hand, negative self-talk can trigger a wide range of negative emotions such as anxiety, stress, helplessness, irritability and hopelessness (Obodo & Obadan, 2008:154; Rosner, 2011:83). Such negative emotions can subsequently lead to ill-health and irrationality which can ultimately affect learners' academic performance in Mathematics.

The research results revealed that the Anxious and Endless Obsession over a Problem irrational belief subscale which is based on the irrational belief that *if something is or may be dangerous or fearsome we should be terribly upset and endlessly obsess about it* was prominent in a number of instances. Below are the cases in which the Anxious and Endless Obsession over a Problem irrational belief subscale had a significant contribution in the current study:

- Girls were found to be more prone to the irrational belief that *if something is or may be dangerous or fearsome we should be terribly upset and endlessly obsess about it* than boys
- Form 4 Mathematics learners were found to be more vulnerable to the irrational thoughts concerning the Anxious and Endless Obsession over a Problem irrational belief subscale than their form 3 counterparts.
- A comparatively prominent negative correlation was found between learners' Mathematics achievement and the Anxious and Endless Obsession over a Problem irrational belief subscale.

- The Anxious and Endless Obsession over a Problem irrational belief subscale had the highest positive correlation with both stress and anxiety experienced by learners as a result of studying Mathematics.
- The Anxious and Endless Obsession over a Problem irrational belief subscale had one of the highest negative correlations with learners' self-concept.
- The regression analysis undertaken confirmed that the Anxious and Endless Obsession over a Problem irrational belief subscale explained 7.2% of the variance in Mathematics learners' self-concept.

The stated research findings make it necessary for school counsellors to pay special attention to some techniques for reducing Mathematics learners' stress and anxiety which are related to the Anxious and Endless Obsession over a Problem irrational belief subscale. Some questionnaire items on stress and anxiety claimed that learners were nervous and tense because of studying Mathematics. Stress and anxiety can be reduced by employing a variety of relaxation techniques (Melgosa, 2008:132). School counsellors can train Mathematics learners to use the following relaxation techniques:

- Progressive muscle relaxation which is meant to train the individual to nurture a habit of responding with relaxation when he or she faces anxiety-provoking situations (Austad, 2009:384; Lowe, Breining, Wilke, Wellmann, Zipfel & Eich, 2002: 179-191)
- Visualisation in which individuals vividly imagine scenes in which they feel relaxed and peaceful as a way of diffusing anxiety and bodily tension (David et al, 2004:10; Austad, 2009:385). For example, school counsellors can encourage Mathematics learners to visualise themselves writing a Mathematics test in a tension-free and non-threatening atmosphere
- Taking a deep breath before a seemingly stressful event such as writing a Mathematics test or examination
- Recommending to Mathematics learners that anxiety and stress can be diffused by sleeping for at least seven or eight hours a day (Lowe et al, 2002:179-191).

Form 3 learners were found to be more irrational than their form 4 counterparts relative to the Demand for Competence and the Perfectionistic and Absolutistic irrational belief subscales. School counsellors can attempt to reduce irrational thinking related to the two irrational belief subscales by:

- Training form 3 learners to exercise unconditional self-acceptance. School counsellors can emphasise to adolescent Mathematics learners that the personal worth of an individual does not necessarily deteriorate as a result of making minor mistakes in Mathematics
- Discouraging adolescent secondary school learners from setting unrealistically high standards for themselves since this is likely to promote a self-downing mentality when the standards are not met (Bermejo-Toro & Prieto-Ursua, 2006:90)
- Encouraging form 3 learners to accept their individual differences in terms of academic performance so that they will stop comparing their academic achievement with that of their peers so as to determine their self-worth.

6.3.3 Recommendations to Parents of Mathematics Learners

The empirical findings revealed that female learners are more vulnerable to irrational thinking about Mathematics than boys relative to the Demand for Competence and the Anxious and Endless Obsession over a Problem irrational belief subscales. Therefore, parents need to give special material and socio-emotional support to their daughters who study Mathematics at secondary school level. Parents can assist their daughters by guiding them to set realistic academic standards which do not make them vulnerable to anxiety and perfectionism. Just like Mathematics teachers, parents need to encourage their children, both female and male, to work hard so as to do well in Mathematics regardless of gender. Parents can emphasise to their daughters that with the right attitude and commitment, they can equal or even outclass their male counterparts in Mathematics.

In general, a negative correlation was found between the irrational thoughts of learners regarding Mathematics and their Mathematics achievement. It was also revealed during the literature study that an individual's early childhood experiences, particularly one's interaction with one's parents, can determine the individual's

vulnerability to the acquisition of irrational thinking patterns (Obodo & Obadan, 2008:153; Rosner, 2011:83; Corey, 2009:145; Kaygusuz, 2013:143). Therefore, it is recommended that parents try to enrich their children's early childhood experiences so as to minimise the acquisition of irrational beliefs by their children. Parents should avoid ruthlessly condemning and punishing their children for their shortcomings (Ellis et al, 1966:107; Bernard, 2008:4) since human beings are naturally fallible.

The results of the empirical investigation revealed a significant negative correlation between learners' irrational thoughts about Mathematics and their motivation to study the subject. Therefore, it is recommended that parents should:

- Try to motivate their children extrinsically to study Mathematics through the use of verbal comments and material rewards in appreciation of their children's academic efforts
- Teach their secondary school children some problem solving skills for application in life in general and in their Mathematics studies in particular
- Help their adolescent children to move progressively along the self-determination continuum, that is, from amotivation to intrinsic motivation. This can be done by making their children aware of their own individual strengths and the importance of Mathematics in various facets of human existence. This can reduce the children's levels of irrational thinking regarding Mathematics as suggested by the results of the current study.

Parental Involvement correlated positively with the Demand for Competence and the Perfectionistic and Absolutistic Mentality irrational belief subscales in the current study. On the basis of these results, it is recommended that parents of Mathematics learners should try:

- To set moderately high and attainable academic standards for their children. Parents of Mathematics learners should not force their children to be thoroughly competent, intelligent and achieving in all possible respects since this is unrealistic and irrational
- To avoid excessively meddling in their children's academic work since this can promote irrational thinking regarding the Demand for Competence.

The Task or Problem Avoidance irrational belief subscale had the most prominent negative correlation of $r = -0.23$ with Parental Involvement in the current study. This implies that good parental involvement is normally accompanied by low level of task avoidance in the study of Mathematics. Some questionnaire items concerning Parental Involvement are given below:

- My parents give me adequate support in my Mathematics work.
- My parents often show appreciation when I have done well in Mathematics.
- My parents regularly attend meetings and social functions at my school.

Therefore, it is recommended that parents of Mathematics learners should:

- Give sufficient material and academic support to their children regarding Mathematics
- Show appreciation when their children do well in Mathematics
- Attend meetings and social functions at their children's schools.

Learners' faulty perceptions of Mathematics correlated positively with their irrational thoughts regarding the subject. The regression analysis undertaken confirmed that learners' faulty perceptions of School and Schooling contributed to the explanation of the variance in learners' Mathematics achievement. In an attempt to reduce faulty perceptions of Mathematics, parents of Mathematics learners can emphasise the importance of Mathematics in life in general and for their children's future career opportunities in particular. As already mentioned, Mathematics as an academic discipline is vital for economic, technological, industrial, scientific and career development (Tachie & Chireshe, 2013:67; Awolola, 2011:91; Nziramasanga, 1999:323; Mahanta & Islam, 2012:713; Githua, 2013:174; Mbugua et al, 2012:87).

Parents should try to convince their children that with focus and determination, they can do well in Mathematics just as in any other subject. Parents should try to explain the incremental theory of intelligence to their children (section 3.4.1.2.2). They should try to reassure their children that it is possible to recover from past failures if

one exerts effort, seeks assistance from teachers and more competent peers or believes that passing Mathematics is a possibility. It was established during the literature study that poor academic performance in Mathematics can be attributed to failure to adopt effective study habits (Osa-Edoh & Alutu, 2012:228; Yahaya, 2003:221). Some studies confirm that the adoption of good study habits can enhance learners' academic achievement in Mathematics (Ozsoy et al, 2009:157; Bajwa et al, 2011:175). This can also reduce faulty perceptions of Mathematics on the part of Mathematics learners. Therefore, parents of Mathematics learners should encourage their children to adopt good study habits so as to boost their Mathematics achievement.

6.3.4 Recommendations to Mathematics Learners

The empirical findings confirmed that learners' irrational thoughts about Mathematics correlated negatively with their Mathematics achievement. Ellis and other cognitive behavioural theorists maintained that disputation is the key antidote to irrational thinking (Melgosa, 2008:136; Austad, 2009:278; Tomotake et al, 2002:51-52; Kosslyn & Rosenberg, 2006:699). As already pointed out, an individual's irrational beliefs can be strengthened by regular negative self-talk while positive self-talk, as a form of cognitive disputation, can weaken an individual's irrational thoughts (Obodo & Obadan, 2008:154; Rosner, 2011:83). Therefore, Mathematics learners should train themselves to engage in positive self-talk regarding their Mathematics studies. For example, Mathematics learners can recite the following statements in their minds as positive self-talk:

- I will eventually do well in Mathematics if I continue exerting effort.
- The fact that I scored a low mark in a Mathematics test today does not mean that I will always score low marks in future Mathematics tests.

Mathematics learners can craft slogans in an attempt to reinforce their commitment and intrinsic motivation to study Mathematics. Below are some examples of slogans Mathematics learners can use to enhance their academic achievement in Mathematics:

- No matter what happens, I will continue studying Mathematics.

- Mathematics helps my mind to be smart.
- Yes, female learners can do well in Mathematics.
- Now it is known that girls can do well in Mathematics.
- I enjoy the beauty of Mathematics.

Secondary school learners, by virtue of being adolescents operating at Piaget's formal operational stage, should try to evaluate their beliefs so as to ascertain whether they are logical, empirically verifiable and consistent with reality. The learners should deliberately implement the disputation techniques which they are taught by their school counsellors so as to replace irrational beliefs with rational beliefs which are logical, consistent with reality and empirically verifiable. This is likely to promote better academic achievement in Mathematics since low levels of irrational thinking are normally accompanied by high levels of Mathematics achievement as revealed by the empirical findings.

The literature review established that study habits are one of the factors which can influence the academic performance of learners in Mathematics. It was outlined in the literature study that failure to apply study habits can weaken learners' Mathematics achievement (Osa-Edoh & Alutu, 2012:228; Yahaya, 2003:221) while the adoption of effective study habits can make a positive difference. This implies that Mathematics learners should monitor the time and place of study using a definite timetable and write down well organised short notes of the content being studied (Sheikh & Jahan, 2012:120; Ozsoy et al, 2009:156; Bajwa et al, 2011:175). Study techniques such as overlearning, distributed practice instead of massed practice, good note-taking skills, the *MURDER* approach and the *SQ4R* method were found to be quite effective (Bajwa et al, 2011:176; Snowman et al, 2009:229; Tuckman & Monetti, 2011:279; Feldman, 2009:242) (section 2.2.3.1). Mathematics learners can use these study techniques to enhance their academic achievement in Mathematics in a bid to minimise their irrational thoughts regarding Mathematics.

As already pointed out, both stress and anxiety correlated positively with learners' irrational thoughts regarding Mathematics in the current study. Consequently, Mathematics learners should consult their school counsellors to be taught the various techniques for reducing both stress and anxiety which when implemented

can lead to a reduction in their levels of irrational thinking regarding Mathematics. As already outlined in section 6.3.2, stress and anxiety can be reduced through the use of techniques such as relaxation techniques, visualisation and being optimistic (Melgosa, 2008:132; Kosslyn & Rosenberg, 2006:617).

The Perennial Influence of a Past Problem irrational belief subscale had prominent positive correlations with learners' faulty perceptions of The Inevitable, School and Schooling, Teachers and Parents and overall perceptions of Mathematics. This implies that Mathematics learners who believe that *because something once strongly affected one's life, it should indefinitely affect it* tend to have the faultiest perceptions of Mathematics. To reduce their vulnerability to irrational thoughts regarding the Perennial Influence of a Past Problem, Mathematics learners need to be resilient. Mathematics learners can enhance their abilities to exercise resilience by regularly seeking help from their teachers, parents, school counsellors, peers and siblings since resilience can be fostered by having a strong support base comprising significant others (Donald et al, 2010:162-163; Swartz et al, 2011:70; Manning, 2007:15).

6.4 LIMITATIONS OF THE STUDY

Although the current study has managed to generate useful research findings, it has some limitations. Firstly, some subscales of the irrational belief test had weak internal consistency. For instance, the irrational belief Demand for Justice had a Cronbach Alpha reliability coefficient of 0.50 and the Demand for Competence had a Cronbach Alpha reliability coefficient of 0.52. The irrational belief Demand for Love and Approval had Cronbach Alpha reliability coefficients of 0.56. More items for each of these subscales need to be developed in an attempt to improve the internal consistency.

Two irrational belief subscales did not have a significant relationship with learners' academic achievement in Mathematics. They are the Demand for Competence and the Perfectionistic and Absolutistic Mentality subscales. This implies that the finding that irrational beliefs correlate negatively with Mathematics achievement cannot be generalised to all the irrational beliefs advanced by Ellis.

Thirdly, there were few form 3 participants who indicated that the questionnaire was a little bit too long

The current study only targeted form 3 and form 4 learners for several reasons. Firstly, the records of the Mathematics achievement of form 3 and form 4 learners were found to be more readily available in schools than those of learners in the other levels. Secondly, form 3 and form 4 learners were considered to have been in secondary school for a reasonably long period of time and their beliefs arguably incorporated their secondary school experiences. However, the fact that the research participants in the current study were only form 3 and form 4 learners can be viewed as a limitation. A sample involving learners from all the tiers of secondary school education could have been particularly useful in the identification of differences in learners' irrational thoughts about Mathematics attributable to age.

6.5 POSSIBILITIES FOR FURTHER RESEARCH

The current study explored the relationship between learners' irrational beliefs and their academic achievement in Mathematics by considering the learners' socio-affective variables. More research on the relationship between irrational beliefs and learners' academic performance in Mathematics can be done by considering cognitive variables such as intelligence, information processing and aptitude.

An attempt can be made to validate the results of the current study by replicating the study using qualitative research designs in which data gathering instruments such as interviews and focus group discussions can be used. Through interviews and focus group discussions, the research participants can explain why they have certain irrational beliefs regarding Mathematics.

In future, researchers may repeat the current study with research participants at all the secondary school grade levels, that is, from Form 1 up to Form 6. As already pointed out, the current study only sampled Form 3 and Form 4 Mathematics learners. Involving learners at all the secondary school grade levels will to some extent enable researchers to validate the obtained differences between learners' irrational thoughts regarding Mathematics and age.

The current study can also be replicated with primary school respondents. This can help to ascertain the age at which irrational beliefs start to relate with learners' Mathematics achievement. It is also possible for other researchers to undertake longitudinal studies in which learners' irrational beliefs are explored throughout their secondary education, that is, from the time they commence Form 1 till they complete Form 6. This can help to establish trends in irrational thinking as learners are exposed to more intricate mathematical concepts.

It is also possible to embark on an allied study exploring the influence of and/ or the relationship between irrational beliefs and learners' academic achievement in other subjects apart from Mathematics. Such research will establish the extent to which irrational beliefs can be viewed as pervasive in the academic arena.

There remains a possibility that researchers can undertake a related study in which the questionnaire is not theoretically restricted to Ellis's irrational beliefs but also includes Beck's cognitive distortions. This is important because the theories of Ellis and Beck are closely related and they both belong to the cognitive-behavioural psychological paradigm. Researchers who intend to further explore irrational beliefs in a Mathematics learning context can develop a questionnaire with more items on the irrational belief test subscales which had low reliability in the current study. This can help to improve the internal consistency of the subscales stated in section 6.4.

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APPENDICES

APPENDIX I:

LETTER TO THE PROVINCIAL EDUCATION DIRECTOR, MASVINGO, ZIMBABWE

Great Zimbabwe University

Department of Educational Foundations

P.O. Box 1235

Masvingo

27 November 2013

The Provincial Education Director

Ministry of Primary and Secondary Education

P.O. Box 89

Masvingo

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN SECONDARY SCHOOLS IN MASVINGO DISTRICT

I hereby apply for permission to carry out research in secondary schools in Masvingo District. I am a lecturer at Great Zimbabwe University who is studying a D ED in Psychology of Education with the University of South Africa (UNISA). The topic of my doctoral thesis is:

The influence of irrational beliefs on the achievement of Mathematics of secondary school learners in Zimbabwe.

The targeted research participants are secondary school learners. Each participant will complete a questionnaire containing items concerning irrational beliefs and Mathematics achievement in about one hour. The questionnaire will be administered during school hours and the respondents will complete the Likert scale questionnaire using numbers. There is voluntary participation and participants can withdraw at any time without reprisal. Confidentiality and anonymity will be upheld by the researcher.

The end of year Mathematics scores for each of the learners who will participate in the study will also be required for correlation purposes.

I hope the research will generate vital insights which will ultimately enhance the quality of Mathematics teaching and learning in Zimbabwe and beyond.

I will be looking forward to getting a favourable response from you.

Yours faithfully

A handwritten signature in blue ink, appearing to read 'Kufakunesu Moses'.

Kufakunesu Moses
moseskufakunesu@gmail.com

Ref: +263 772 831 054;

LETTER FROM THE PROVINCIAL EDUCATION DIRECTOR (MASVINGO) TO THE SECRETARY OF THE MINISTRY OF PRIMARY AND SECONDARY EDUCATION, ZIMBABWE

ALL communications should be addressed to
"The Provincial Education Director for Primary and
Secondary Education"
Telephone: 263585/264331
Fax: 039-263261



Reference:

Ministry of Primary and Education
P.O Box 89
MASVINGO

18 December 2013.

The Secretary
Ministry of Primary and Secondary Education

Att: Mr Muzawazi

**RE: SEEKING PERMISSION TO CARRY OUT RESEARCH: KUFAKUNESU
MOSES: UNIVERSITY OF SOUTH AFRICA (UNISA) STUDENT**

The above matter refers.

Kufakunesu Moses, a University of South Africa (UNISA) student, is seeking permission to carryout research on;

Title: 'The influence of irrational beliefs on the achievement of Mathematics of Secondary School learners in Zimbabwe'.

Attached please find the applicant's letter. The application is supported therefore permission is sought.

A handwritten signature in blue ink, appearing to read 'F. R. Jirivengwa'.

F. R. JIRIVENGWA
ACTING PROVINCIAL EDUCATION DIRECTOR: MASVINGO



APPENDIX III:

LETTER TO THE SECRETARY FOR THE MINISTRY OF PRIMARY AND SECONDARY EDUCATION, ZIMBABWE

Great Zimbabwe University

Department of Educational Foundations

P.O Box 1235

Masvingo

06 January 2014

The Secretary

Ministry of Primary and Secondary Education

P.O. Box CY 121

Causeway

Harare

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN SECONDARY SCHOOLS IN MASVINGO DISTRICT

I hereby apply for permission to carry out research in secondary schools in Masvingo District. I am a lecturer at Great Zimbabwe University who is studying a D ED in Psychology of Education with the University of South Africa (UNISA). The topic of my doctoral thesis is:

The influence of irrational beliefs on the achievement of Mathematics of secondary school learners in Zimbabwe.

The targeted research participants are secondary school learners. Each participant will complete a questionnaire containing items pertaining to irrational beliefs and Mathematics achievement in about one hour. There is voluntary participation and participants can withdraw at any time without reprisal. Confidentiality and anonymity will be upheld by the researcher. The end of year Mathematics scores for each of the respondents will also be required for use during data analysis.

I hope the research will generate important insights which will ultimately enhance the quality of Mathematics teaching and learning in Zimbabwe.

I will be looking forward to getting a favourable response from you.

Yours faithfully



Kufakunesu Moses

Ref: +263 772 831 054;

moseskufakunesu@gmail.com

LETTER OF PERMISSION TO UNDERTAKE RESEARCH FROM THE SECRETARY FOR THE MINISTRY OF PRIMARY AND SECONDARY EDUCATION, ZIMBABWE

all communications should be addressed to
"The Secretary for Education Sport and Culture"
Telephone: 732006
Telegraphic address : "EDUCATION"
Fax: 794505



REFERENCE: C/426/3

Ministry of Primary and
Secondary Education
P.O Box CY 121
Causeway
HARARE

07 January 2014

Mr. Kufakunesu Moses
Great Zimbabwe University
Department of Educational Foundations
P.O. Box 1235
Masvingo

Re: **PERMISSION TO CARRY OUT RESEARCH IN MASVINGO
DISTRICT, MASVINGO**

Reference is made to your application to carry out research in Masvingo District, Masvingo on the title:

**THE INFLUENCE OF IRRATIONAL BELIEFS ON THE ACHIEVEMENT OF
MATHEMATICS OF SECONDARY SCHOOL LEARNERS IN ZIMBABWE**

Permission is hereby granted. However, you are required to liaise with the Provincial Education Director Masvingo who is responsible for the schools which you want to involve in your research.

You are also required to provide a copy of your final report to the Secretary for Primary and Secondary Education.

MINISTRY OF EDUCATION, SPORT,
ARTS AND CULTURE
POLICY PLANNING

Z.M. Chitiga
00 DEC 2013
P.O. BOX CY 121, CAUSEWAY
HARARE

Z.M Chitiga
Acting Director: Policy, Planning, Research and Development
For: **SECRETARY FOR PRIMARY AND SECONDARY EDUCATION**

UNISA RESEARCH ETHICS CLEARANCE CERTIFICATE



Research Ethics Clearance Certificate

This is to certify that the application for ethical clearance submitted by

M Kufakunesu [47390298]

for a D Ed study entitled

**The influence of irrational beliefs on the achievement of Mathematics of
Secondary School learners in Zimbabwe**

has met the ethical requirements as specified by the University of South Africa
College of Education Research Ethics Committee. This certificate is valid for two
years from the date of issue.

Prof KP Dzvimbob
Executive Dean : CEDU

Dr M Claassens
CEDU REC (Chairperson)
mcdtc@netactive.co.za

Reference number: 2014 APRIL/47390298/MC

14 April 2014

TESTIMONIAL LETTER FROM GREAT ZIMBABWE UNIVERSITY



Department of Educational Foundations
P O Box 1235
MASVINGO
Tel: 039-253507
Fax: 039-253504

Off Old Great Zimbabwe Road
MASVINGO
E mail: hr@gzu.ac.zw

GREAT ZIMBABWE UNIVERSITY

25 November, 2013

To whom it may concern

This serves to confirm that Mr M. Kufakunesu, I.D. Number 22-151427-M-22, Employee number 0876, is a lecturer in Educational Psychology at Great Zimbabwe University.

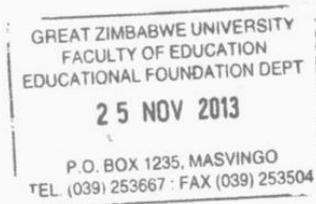
He is a registered student of UNISA pursuing a D.ED (Psychology of Education).

Please give him any assistance he may require.

Thank you



A. Chingombe
Chairperson – Educational Foundations



APPENDIX VII:

CONSENT FORM FOR SECONDARY SCHOOL HEADS

Great Zimbabwe University

Department of Educational Foundations

P.O. Box 1235

Masvingo

06 February 2014

The School Head

.....

Dear Sir/Madam

RE: REQUEST FOR PERMISSION TO UNDERTAKE RESEARCH AT YOUR SCHOOL

I am a Psychology of Education lecturer at Great Zimbabwe University who is pursuing a Doctor of Education (Psychology of Education) with the University of South Africa (UNISA). I hereby write to kindly request for the permission to conduct a research study at your school. My study focuses on the influence of irrational beliefs on the Mathematics achievement of secondary school learners. The title of my thesis is: *The influence of irrational beliefs on the achievement of Mathematics of secondary school learners in Zimbabwe.*

The study employs questionnaires as data gathering instruments. The learners are expected to complete a questionnaire containing items on irrational beliefs and Mathematics achievement in about one hour. There is voluntary participation and participants can withdraw at any time without reprisal. Confidentiality and anonymity will be upheld by the researcher. The end of year Mathematics scores for each of the learners who will take part in the study will also be required for correlation purposes. My research supervisor is Professor G. Bester, telephone number +27 12 429 4337; E-mail: besteg@unisa.ac.za .

If you would like the study to be conducted at your school as outlined in this letter, please sign below as a way of registering your consent.

Signature of School Head Date

Signature of ResearcherDate

Ref: +263 772 831 054; moseskufakunesu@gmail.com

CONSENT FORM FOR PARENTS/ LEGAL GUARDIANS

Principal Researcher: Moses Kufakunesu

Topic:

The influence of irrational beliefs on the achievement of Mathematics of secondary school learners in Zimbabwe

Dear Parent/Legal Guardian

I am a Doctor of Education (Psychology of Education) student with the University of South Africa (UNISA) exploring the influence of irrational beliefs on the Mathematics achievement of secondary school learners in Zimbabwe. Since your child/dependent is legally a minor, I hereby kindly ask for permission for your child/dependent to take part in the study. Your child/dependent and other learners will complete a questionnaire containing items pertaining to variables related to irrational beliefs and their influence on Mathematics achievement. The study will be undertaken in line with the UNISA policy on research ethics.

The research will be conducted in a risk-free manner and all the rights of the participating secondary school learners will be respected. Participation in this study is purely voluntary and the participants are free to withdraw from the research study at any time without any penalty. The data which will be collected will be used for research purposes only.

For more information, please contact Moses Kufakunesu, telephone number +263 772 831 054; E-mail; moseskufakunesu@gmail.com or my research supervisor Professor G. Bester, telephone number +27 12 429 4337; E-mail: besteg@unisa.ac.za

Parent/Legal Guardian's Confirmation and Signature

If you would like your child/dependent to participate in this study, please sign below as a way of giving your consent.

Signature of Parent/Legal Guardian

Date

Signature of Investigator

Date

CHILD ASSENT FORM

Principal Investigator: Moses Kufakunesu

Topic:

The influence of irrational beliefs on the achievement of Mathematics of secondary school learners in Zimbabwe

Introduction

I am a Doctor of Education (Psychology of Education) student with the University of South Africa (UNISA). I am carrying out a study on the influence of irrational beliefs on the Mathematics achievement of secondary school learners in Zimbabwe. The purpose of this assent form is to clarify what the participants in the study are expected to do. It is also an invitation to learners to take part in this study if they are willing to do so. You are kindly requested to carefully read the form before you decide whether to participate in the study or not.

Purpose of the study

The purpose of the study is to examine the influence of irrational beliefs on the Mathematics achievement of secondary school learners in Zimbabwe. Factors such as learners' home factors, school factors and learners' internal factors will be explored in conjunction with irrational beliefs and Mathematics achievement.

Procedure

By virtue of being a secondary school learner who is studying Mathematics in Zimbabwe, you have been invited to take part in this study. You and other secondary school learners at your school will be kindly requested to respond to a questionnaire with items concerning the influence of irrational beliefs on Mathematics achievement. You will complete the questionnaire in the morning during school hours. It will take you about one hour to complete the questionnaire. Your end of year Mathematics scores will also be required for use in the final data analysis.

Confidentiality

Your responses will be used for research purposes only. Data analysis will be done using a computer. Your views and your identity will not be disclosed to other people. The data which will be finally presented in the thesis report will not give specific names of the research participants.

Possible Risks and Discomforts

This is a risk-free research study. The researcher shall ensure that the concerns raised by you and other participants are respected. The questionnaire is not difficult to complete since responses are given as numbers.

Benefits of the Study

Participation in the study does not entitle the participants to any monetary or material benefits. Nevertheless, it is the researcher's expectation that the participants may benefit later when the outcomes of the study are published.

Participants' Rights

Participants have the right to ask questions regarding the study and their participation in the study. For more information, please contact Moses Kufakunesu, telephone number +263 772 831 054; E-mail; moseskufakunesu@gmail.com or my research supervisor Professor G. Bester, telephone number +27 12 429 4337; E-mail: besteg@unisa.ac.za .

Role of the Parent/Legal Guardian

You are free to discuss with your parent/legal guardian any issue outlined in this assent form before you volunteer to participate. Permission for you to participate will be requested from your parent/legal guardian. Your parent/ legal guardian will receive a copy of the signed consent form.

Duration

The participants will complete a questionnaire for about one hour. As already pointed out, the questionnaire contains Likert scale type items in which participants respond using numbers.

Invitation to Participate

I kindly ask you to participate in the study now that I have outlined what the study focuses on and the procedures involved.

Voluntary Participation

You are not forced to participate in the study. This means your participation is voluntary. You are free to withdraw from the study at any time without being punished in any way.

Assent to Participate in the Study

May you kindly sign in the space provided below as a way of indicating that you have read and understood the assent form and you are willing to take part in the study.

Signature of Participant Date

Signature of InvestigatorDate

LANGUAGE EDITOR'S DECLARATION

LYN VOIGT LITERARY SERVICES

Lyn Voigt: B. Mus. (Eng Hons) [Wits] H. Dip. Ed. [JCE]

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35 El Prado

Randpark Ridge

Edit of the thesis: *The Influence of Irrational Beliefs on the Mathematics Achievement of Secondary School Learners in Zimbabwe* by Moses Kufakunesu, submitted in accordance with the requirements for the degree of Doctor of Education in the subject Psychology of Education at the University of South Africa. Supervisor: Professor G. Bester

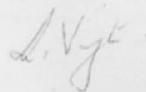
The edit that I carried out **included** the following:

- Spelling - Tense
- Vocabulary - Number
- Punctuation - Pronoun matches
- Word usage - Correct acronyms (matching your furnished list)
- Sentence structure - Figure numbers
- References (checking against reference list)

The edit that I carried out **excluded** the following:

- Content
- Correctness or truth of information (unless obvious)
- Correctness/spelling of specific technical terms and words (unless obvious)
- Correctness/spelling of unfamiliar names and proper nouns (unless obvious)
- Correctness of specific formulae or symbols, or illustrations.
- Formatting

LE Voigt



Language Practitioner
September 2015

QUESTIONNAIRE FOR SECONDARY SCHOOL LEARNERS

QUESTIONNAIRE FOR SECONDARY SCHOOL LEARNERS

Please read the following instructions:

1. The questionnaire that you are going to answer concerns you as a secondary school Mathematics learner. Please note that the questionnaire is not a test with right or wrong answers. It is simply an information exercise in which your answer is the right one. It will take you about one hour to complete the questionnaire.
2. The purpose of this questionnaire is to gather information pertaining to the influence of irrational beliefs on the Mathematics achievement of secondary school learners. Please answer each item independently and according to how you normally behave, think or feel. You are kindly asked to evaluate each statement according to the first feeling you experience upon reading it.
3. Please answer the statements honestly and not according to how others would expect you to do. Remember that this is a confidential questionnaire in which no one has access to your answers. The information will be processed by a computer.

SECTION A

Kindly provide your personal information by writing the applicable number in the block provided next to each statement. Write only one number in each block.

SCHOOL (MXC = 1; VHS=2; NHS = 3)

k1

1. Gender (boy = 1 ; girl = 2)

k2

2. State your age (to the nearest whole number in years) in the space provided

k3	k4

1. In which form are you at present? (03 or 04)

k5	k6

2. Home Language (Shona=1; Ndebele=2; English=3)

k7

3. State the number of children in your family

k8

4. What is your position in the family?

Youngest child =1

Middle child =2

Eldest child =3

Only child =4

k9

5. Father's Occupation

Professional (Teacher, doctor, lawyer, etc.) =1

Artisan (Carpenter, painter, welder, bricklayer, etc.) =2

Business Management =3

Secretarial =4

Technical (technician, engineer, mechanic, etc.) =5

Self-employed =6;

Unemployed =7

Father absent from home =8

Other =9

k10

6. Mother's Occupation

Professional= 1

Artisan =2

Business management =3

Secretarial =4

Technical =5

--

Self-employed =6
Unemployed =7
Mother absent from home =8
Other =9

k11

For office use

7.

k12 k13

8.

k14 k15

SECTION B

Answer each statement by honestly indicating your degree of agreement or disagreement with each statement by means of a code number between 1 and 6 as outlined in the scale below:

This is definitely the case 6 5 4 3 2 1 This is not the case at all

Place a code number in the block next to each statement.

To assist you to remember the code as you complete the questionnaire, a key has been given to you at the bottom of each page.

1. I am always motivated to attend Mathematics lessons

k16

2. I often feel insecure in a Mathematics lesson

k17

3. My Mathematics teacher makes me feel safe and at ease

k18

4. A leading role in Mathematics discussions is much more terrifying to me than group discussions because all the attention will be fixed on me
k19
5. If a Mathematics topic is difficult to learn, I give up easily
k20
6. I lose control over my Mathematics studies
k21
7. I hate studying Mathematics
k22
8. I feel that I achieve something with my Mathematics performance
k23
9. I become very anxious before I write a Mathematics test
k24
10. My Mathematics studies frustrate me because of too much work and too little time
k25
11. My Mathematics teacher supports me sufficiently
k26
12. It bothers me when I neglect my Mathematics studies
k27
13. As far as Mathematics is concerned, I expect too much of myself
k28
14. As a Mathematics learner I am mostly disappointed in myself
k29

15. I love to lend a helping hand to Mathematics learners in need

k30

16. Because of Mathematics obligations, I often feel that I underrate myself

k31

17. Where Mathematics is concerned, I put duty before pleasure

k32

18. I become anxious when I realise that other people are going to judge my Mathematics achievement

k33

19. I often become tense because of the amount of work in Mathematics

k34

20. I am proud of what I have already achieved in Mathematics

k35

21. I find it difficult to socialise with other Mathematics learners

k36

22. My Mathematics teacher allows me to voice my opinion

k37

23. Continuous competition with friends in Mathematics makes me worried

k38

24. I always come up with reasons not to study Mathematics

k39

25. Other people expect too much of me as a Mathematics learner

k40

26. I believe in myself whenever I have to write a Mathematics test

k41

27. I do not need friends in my Mathematics career

k42

28. I often clash with my Mathematics teacher about instructions given to me

k43

29. My Mathematics teacher sets goals which I cannot reach

k44

30. The moment I think about studying Mathematics, I get very nervous.

k45

31. I revise Mathematics when I feel like it

k46

32. My Mathematics studies are under control - I know where I am going

k47

33. I like to reach out to other Mathematics learners

k48

34. The rules laid down by my Mathematics teacher are acceptable to me

k49

35. I do not think that my Mathematics teacher really understands me

k50

36. I set goals in Mathematics and try to achieve them k51
37. To solve mathematical problems in front of others is one of my greatest fears k52
38. I am unwilling to learn new, demanding Mathematics concepts k53
39. I am a cheerful Mathematics learner who easily gets along with others k54
40. I have difficulty to calm down before a Mathematics test k55
41. My Mathematics teacher accepts the way I am k56
42. During a Mathematics test I make unnecessary mistakes because of nervousness k57
43. I have hope for myself as a Mathematics learner k58
44. I find it difficult to make friends k59
45. To be the first to be assessed in a Mathematics task is terrifying k60
46. My Mathematics teacher is unnecessarily critical of my performance k61

47. If a Mathematics topic is too difficult, I do not even try to learn it k62
48. I am usually enthusiastic when I begin to learn a new Mathematics topic, but after a while my enthusiasm declines k63
49. I often feel that I will never be able to do well in Mathematics k64
50. My Mathematics teacher is honest and sincere towards me k65
51. I often feel lonely as a Mathematics learner k66
52. It bothers me if I did not adequately revise Mathematics for the day k67
53. My Mathematics teacher is familiar with my abilities and limitations k68
54. I am more afraid of Mathematics examinations than any other examinations k69
55. As far as Mathematics is concerned, I spend my time productively k70
56. I am very critical of other Mathematics learners k71
57. I experience more anxiety during Mathematics tests than other Mathematics

learners due to my own insecurity

k72

58. My Mathematics teacher gives me credit whenever I perform well

k73

59. I love to learn new Mathematics concepts and to sharpen my Mathematics skills

k74

60. My Mathematics teacher is often unfair

k75

61. I am always afraid that others will mock me because of the mistakes I make during a Mathematics test

k76

62. I sometimes feel that I will not achieve anything with my Mathematics performance

k77

63. I definitely have fewer friends than most of my Mathematics peers

k78

64. My heart misses a beat whenever there is a Mathematics specialist in the Mathematics lesson

k79

65. I feel overburdened by my Mathematics studies

k80

66. I sometimes distrust other Mathematics learners

k81

67. I fail most of the Mathematics topics that I want to learn k82
68. I do not really have friends to whom I can turn k83
69. I feel ashamed about my shortcomings in Mathematics k84
70. My Mathematics achievement is acceptable to me k85
71. My social relationships with other Mathematics learners are superficial k86
72. I feel anxious when I write Mathematics tests because I do not want to disappoint my Mathematics teacher k87
73. Preparations prior to Mathematics tests put a lot of pressure on me k88
74. As a Mathematics learner I often doubt myself and what I can achieve k89
75. Friends make me feel inferior regarding my Mathematics achievement k90
76. If I were not so afraid to solve mathematical problems, learning Mathematics would be more enjoyable k91
77. I do not have enough self-confidence to participate during Mathematics lessons k92

78. I often get angry with my friends because I am sensitive to what they say about me

k93

79. I do not concern myself with my Mathematics teacher's point of view; I mostly do what I want

k94

80. Because of Mathematics, I cannot relax any more

k95

81. I hide many things from my Mathematics teacher

k96

82. I am anxious during Mathematics tests because I know that there are people who would secretly be happy if I make mistakes

k97

83. As far as Mathematics is concerned, I regard myself as a hard worker

k98

84. Before a Mathematics test I am almost in tears due to anxiety

k99

85. My Mathematics teacher trusts me

k100

86. I am determined to revise Mathematics on a high standard

k101

87. I feel at ease amongst my Mathematics peers

k102

88. Because of my workload in Mathematics, I often become impatient with people

k103

89. I can rely on my Mathematics teacher when I need help

k104

90. I am under pressure to neglect other school work in order to make time for Mathematics

k105

91. I feel ashamed about the standard of my Mathematics achievement

k106

92. I do not need to be told to revise Mathematics

k107

93. My Mathematics teacher is often dissatisfied without even listening to my explanation

k108

94. I experience anxiety because I never know how good my Mathematics achievement will be

k109

95. It seems to me that my Mathematics responsibilities will never end

k110

96. To pursue Mathematics studies gives meaning to my life

k111

97. I can trust my Mathematics teacher

k112

98. I constantly motivate myself to solve mathematical problems perfectly
k113
99. I do my share to keep friendships with other Mathematics learners alive
k114
100. Mathematics makes my life difficult and out of my control
k115
101. I can overcome challenges in Mathematics because I believe in myself
k116
102. I accept my Mathematics peers the way they are
k117
103. I respect my Mathematics teacher's opinions and rules
k118
104. I catch up on Mathematics revision times that I have missed
k119
105. As a Mathematics learner I do not accept new ways of doing things
k120
106. I become nervous while participating during Mathematics lessons because I do not know how good my contributions will seem to my classmates
k121
107. My Mathematics studies cause me to feel guilty as soon as I relax
k122
108. I am motivated to learn difficult and challenging mathematical concepts
k123

109. I can give to other Mathematics learners without expecting anything in return
k124
110. My Mathematics teacher takes a personal interest in me
k125
111. I experience my Mathematics workload as too heavy
k126
112. Before writing a Mathematics test, I think of all sorts of ridiculous things that can go wrong
k127
113. Before I can stop myself, I say hurtful things to my Mathematics peers
k128
114. I cannot master the mathematical concepts expected from me
k129
115. After a Mathematics test, I experience anxiety because I do not know what criticism to expect
k130
116. If I could, I would change a number of things about myself as a Mathematics learner
k131
117. I experience conflict between the demands of my Mathematics studies and my own private interests
k132
118. I constantly postpone Mathematics revision times
k133

119. It gives me more pleasure to do Mathematics on my own than in a group
k134
120. My Mathematics studies cause me tension because of a lack of time
k135
121. As far as my Mathematics achievement is concerned, I do what I am supposed to do, but nothing more
k136
122. My parents take an interest in my Mathematics studies and are aware of my progress
k137
123. It is important that Mathematics teachers always approve the work I do
k138
124. I hate to fail Mathematics tasks at school
k139
125. Mathematics learners who do wrong must be punished
k140
126. I find it impossible to accept things which happen to me during Mathematics lessons
k141
127. I find it impossible to be happy at school because of the challenges concerning Mathematics
k142
128. Things that regularly happen during Mathematics lessons cause fear in my life
k143

129. I usually postpone doing Mathematics activities
k144
130. Everyone needs someone at school on whom to depend when solving mathematical problems
k145
131. I have no control over the experiences which I encounter during Mathematics lessons
k146
132. To be successful in Mathematics, you have to do everything the correct way
k147
133. I worry if my fellow Mathematics learners or teachers do not respect me
k148
134. I avoid mathematical problems which I cannot solve
k149
135. Too many Mathematics learners escape the punishments they deserve
k150
136. Frustration associated with Mathematics learning makes me feel miserable
k151
137. I get upset because my Mathematics aspirations at school never materialise
k152
138. Mathematics tasks can cause me to be miserable for days
k153
139. I find it difficult to finish Mathematics assignments

k154

140. I strongly depend on my friends when solving mathematical problems

k155

141. It is almost impossible to overcome your past failures in Mathematics

k156

142. You are only successful in Mathematics if you do everything perfectly

k157

143. I want everyone to like me at school

k158

144. I ignore competition if I know others who can do Mathematics better than me

k159

145. Those Mathematics learners doing wrong at school deserve to be penalised

k160

146. The Mathematics concepts which I learn at school will never be able to prepare me properly for the future

k161

147. Other people create my unhappiness at school

k162

148. Mathematical problems at school continuously bother me

k163

149. I avoid solving difficult Mathematics tasks

k164

150. I cannot do Mathematics assignments without the support of my peers
k165
151. Things from my past have a major influence on my Mathematics achievement
k166
152. All Mathematics activities at school are stressful
k167
153. I doubt myself if I suspect that my Mathematics peers do not like me
k168
154. I feel pressurised to be successful in Mathematics
k169
155. Bad behaviour at school should be punished
k170
156. I often get disturbed over Mathematics-related situations that I do not like
k171
157. You can only be happy in Mathematics lessons when you have the right peers
k172
158. I worry about mathematical issues which are beyond my control
k173
159. I usually postpone decisions which I have to take in connection with Mathematics learning
k174
160. There are certain people at school whom I depend on greatly when doing Mathematics
k175

161. My past counts against me in Mathematics
k176
162. It is one of those things- all problems concerning Mathematics learning must be overcome
k177
163. If other Mathematics learners do not like me, it bothers me tremendously
k178
164. It is highly important for me to be successful in all the Mathematics activities I do
k179
165. I am quick to blame other Mathematics learners when they do something wrong
k180
166. It is difficult to accept certain mathematical issues at school even if others accept them
k181
167. Studying Mathematics causes my entire life to be unhappy
k182
168. I experience anxiety whenever I think about my Mathematics achievement
k183
169. I normally ignore Mathematics tasks that are difficult and unfamiliar
k184
170. At school I do not succeed in solving Mathematics tasks on my own
k185

171. If I had different Mathematics circumstances, I would be better off

k186

172. You can only be happy in Mathematics lessons if you adhere to all prescriptions

k187

173. I find it difficult to disagree with my peers when doing Mathematics tasks

k188

174. I ignore Mathematics tasks if I am not good at them

k189

175. The fear of punishment helps learners to do well in Mathematics

k190

176. I have given up hope to reach any of my aims in Mathematics

k191

177. A person can only be happy during Mathematics lessons if Mathematics teachers create the ideal circumstances

k192

178. Things that upset me regarding my Mathematics achievement continuously haunt me

k193

179. I often defer my Mathematics assignments

k194

180. I need the help of other people to deal with Mathematics problems

k195

181. I cannot succeed to free myself from my previous bad experiences in Mathematics

k196

182. If you experience a Mathematics-related problem, it is best not to do anything before you have identified the ideal solution

k197

183. I want to receive approval from my Mathematics teachers

k198

184. It bothers me if my friends' Mathematics achievement is better than mine

k199

185. Everybody in my Mathematics class has something bad in them

k200

186. My experience in Mathematics is a disaster because my ideals will never materialise

k201

187. Other people in my Mathematics class cause me to be depressed

k202

188. I worry a lot about the future and my Mathematics career

k203

189. It is difficult for me to do unpleasant Mathematics tasks

k204

190. I do not like to make Mathematics-related decisions on my own

k205

191. Everybody is a slave of his/her own Mathematics history

k206

192. There are many Mathematics-related problems but a few ways to solve them
k207
193. I often worry if people in my Mathematics class will accept me
k208
194. It upsets me when I make mistakes when solving Mathematics problems
k209
195. It is unfair that Mathematics learners who do wrong get away with it
k210
196. I will never meet the requirements of Mathematics as a subject
k211
197. To be happy in Mathematics lessons depends a lot on the Mathematics teachers
k212
198. I cannot get rid of Mathematics in my thoughts
k213
199. I will rather do nothing in a Mathematics lesson before I do something wrong
k214
200. Without someone to support you in doing Mathematics assignments, you are doomed
k215
201. Once you do something wrong in Mathematics, it will always count against you
k216
202. Practical mathematical solutions are of no help if they are not ideal solutions
k217

203. I am very concerned about what my Mathematics peers say about me
k218
204. I often become annoyed over the slightest failures I experience in Mathematics
k219
205. A Mathematics learner who has done me wrong must not enjoy a second chance
k220
206. My Mathematics wishes and dreams will never materialise
k221
207. Your Mathematics progress is determined by the support of your parents
k222
208. For long periods of time, I am worried about my Mathematics achievement
k223
209. I do not like Mathematics tasks which require hard work
k224
210. I cannot do well in Mathematics without the support of my Mathematics teachers
k225
211. The mistakes one makes in Mathematics are difficult to rectify
k226
212. I demand from myself to do all Mathematics tasks in the correct way
k227
213. To be criticised during Mathematics lessons is always annoying
k228

214. I feel disappointed if I do not achieve in various Mathematics exercises

k229

215. Bad behaviour often reflects rottenness of character

k230

216. My ideals in Mathematics and reality will never meet

k231

217. If luck is not on your side, you will never be successful in Mathematics

k232

218. I often worry about Mathematics-related situations I might find myself in

k233

219. I do not do Mathematics tasks which are not really necessary

k234

220. Mathematics teachers should be more worried about the welfare of Mathematics learners

k235

221. I look back at my past Mathematics achievement with regrets

k236

222. One can only achieve in Mathematics if the circumstances are absolutely right

k237

223. My parents often show their appreciation when I have done well in Mathematics

k238

224. My parents often ask about my Mathematics achievement

k239

225. My parents know who most of my Mathematics teachers are

k240

226. My parents regularly attend meetings and social functions at my school

k241

227. My parents give me adequate support in my Mathematics work

k242

228. My parents would be quick to notice if I were to neglect my Mathematics studies

k243

229. My parents often encourage me to do well in Mathematics

k244

230. My parents make regular suggestions as to how I could organise and do my Mathematics assignments more efficiently

k245

231. My parents insist on being informed about my Mathematics progress at school

k246

232. My parents frequently contact the school if they are uncertain about my Mathematics achievement

k247

233. One's success or failure in Mathematics can be attributed to luck

k248

234. Boys do better in Mathematics than girls

k249

235. Teachers expect better Mathematics results from boys than from girls

k250

236. Parents expect better Mathematics results from girls than they do from boys

k251

237. It is important for boys to do well in Mathematics as they are the future breadwinners

k252

238. Certain subjects offered in schools are meant to be taken by boys and others by girls. For example, boys are expected to do Mathematics while girls do Home Economics

k253

239. Learners whose parents check on their progress in Mathematics on daily basis become frustrated and perform badly in Mathematics

k254

240. Girls are not expected to enter professions which are related to Mathematics

k255

241. For girls, entry into college or university is important because they need to be make friends and find marriage partners

k256

242. Much of what is taught during Mathematics lessons nowadays is irrelevant and unimportant

k257

243. The Mathematics curriculum does not allow for one's independent thoughts and ideas

k258

244. If a parent dislikes Mathematics, the child will do so as well
k259
245. A learner who chooses to study Mathematics is an intelligent learner
k260
246. Even if one tried hard, one will never understand the Mathematics concepts which are taught at school
k261
247. The atmosphere of the classroom plays an important role in determining how one performs in Mathematics
k262
248. Mathematics learners who do not abide by the school rules are rejected by staff and by other learners
k263
249. It is important to be accepted at school to be successful in Mathematics
k264
250. One gets better Mathematics results in a small school than in a large school
k265
251. Learners in schools that are more academically-oriented produce better Mathematics results
k266
252. If the Mathematics concepts are not interesting, the teacher is to be blamed
k267
253. The attitude of the teachers and the school head determines how well one performs in Mathematics
k268

254. Certain Mathematics learners are openly rejected by the teachers because the way they speak, dress and behave is different from the teachers'

k269

255. The atmosphere of the classroom primarily depends on the attitude of the Mathematics teacher

k270

256. One communicates better with one's Mathematics peers than with one's parents

k271

257. One studies Mathematics in order satisfy one's parents

k272

258. Learners who are very highly intelligent in Mathematics find it difficult to get into a relationship

k273

259. One should compete with others academically in order to improve one's Mathematics results

k274

260. Girls who are very intelligent in Mathematics are often rejected by their friends

k275

261. One should study hard and do well in Mathematics to get social recognition and status

k276

262. For a learner who later wants to buy and sell goods for a living, Mathematics has no value

k277

263. The mathematically gifted learner usually finds it difficult to communicate and has very few friends
k278

264. Doing well in Mathematics is important because society places such a great emphasis on good Mathematics achievement
k279

265. One should do well in Mathematics in order to gain the approval of one's parents and teachers
k280

266. Parents should always reward children whenever they do well in Mathematics
k281

267. If you perform poorly in Mathematics, it is always because of bad luck
k282

268. If one is afraid of failing Mathematics, one should not take any chances and avoid any Mathematics activity that is unfamiliar
k283

269. One should not question the teachers' authority on the Mathematics subject matter
k284

270. Parents are never satisfied with the Mathematics results of their children
k285

271. Mathematics homework will only be done if parents check whether it is done
k286

272. Parents wrongly believe that learning Mathematics is enjoyable
k287

273. Mathematics teachers largely determine one's opinion of oneself
k288
274. Teachers are only interested in how one achieves in Mathematics
k289
275. Almost all Mathematics teachers have favourites in class
k290
276. Good Mathematics teachers present their lessons in an exciting and challenging way
k291
277. Mathematics teachers fail to realise that learners can make decisions on their own
k292
278. Mathematics teachers cannot be trusted
k293
279. Mathematics teachers like to draw attention to learners' failures
k294
280. Earning money is far more important than getting a diploma or degree in Mathematics
k295
281. Studying Mathematics is full of negative experiences
k296
282. Parents place greater emphasis on Mathematics achievement than anything else
k297

For office use

283. Current Mathematics achievement score (as a percentage)

k298	k299

284. Previous Mathematics achievement score (as a percentage)

k300	k301

285.

k302	k303

286.

k304	k305